High precision motion control for hard disk drive systems

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High Precision Motion Control for Hard Disk Drive Systems

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A thesis submitted to the Nanyang Technological University

in fulfilment of the requirements for the degree of

Doctor of Philosophy

2008

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Date

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Teoh Jul Nee

*To my family,
for their encouragement and love.*

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Summary

The demand for better servo control technology for hard disk drives (HDDs) never ceases. One important element for HDDs servo control is better precision for future drives with ultrahigh densities. As the track density increases, the amount of time required for the process of writing servo information onto the disks will also increase accordingly. However, the cost of production must be kept low.

Since HDDs are manufactured in mass production, the servo control design for the voice coil motor (VCM) actuator has to be robust towards uncertainty. At the same time, the track misregistration (TMR) during track following has to be minimized. These two requirements can be formulated into a mixed H_2/H_∞ control problem. Therefore, a new mixed H_2/H_∞ control design via a linear matrix inequality (LMI) based approach is proposed. By an equivalent transformation of Lyapunov matrix inequality, additional slack variables are introduced to characterize both the H_2 and H_∞ performances. This offers additional degree of freedom in optimizing the mixed H_2/H_∞ performance, and thus, resulting in equal or better performance than existing LMI based methods.

It is generally difficult to attenuate high frequency disturbances of a system due to waterbed effect of the sensitivity function. However, attenuation of dominant narrow-band disturbances is possible. Therefore, a technique that combines a disturbance observer with a phase stabilized controller is proposed for rejection of narrow-band disturbances at two different frequencies, which are higher than the servo bandwidth. The phase stabilized controller is designed first to attenuate a high frequency disturbance. Then, the disturbance observer is added into the loop in order to reject another high frequency disturbance, without affecting the stability of the feedback control loop.

Previous research has shown that the sensitivity function can be shaped such

that the error rejection at certain frequencies can be improved. A nonlinear least-squares optimization (NLSO) method is applied in sensitivity function shaping to design a feedback controller for a microactuator such that its sensitivity function fulfills desired specifications in order to attenuate disturbances at specific frequencies. Due to the limited improvement of the NLSO method, an approach based on the generalized Kalman-Yakubovic-Popov (KYP) Lemma together with the Youla parameterization is proposed which allows us to suppress disturbances of particular frequencies within or beyond the servo bandwidth via convex optimization. Rejections of high-frequency and low-frequency disturbances are both achievable using this approach. This method is simple and effective compared to the traditional frequency weighting method. As the objective of rejecting disturbances is to obtain higher positioning accuracy (smaller TMR), the generalized KYP lemma is combined with the H_2 control to design a controller which provides narrow-band disturbance rejection and simultaneously ensures that TMR is minimized.

Since multiple microactuators are attached onto the same arm for the hybrid type servo track writer (STW), interactions amongst the microactuators are expected. The interactions due to the close proximity of the microactuators may cause degradation or error of the servo information written on the disks' surfaces. Moreover, due to the presence of uncertainty in the microactuator plant, a conventional feedforward decoupling control may not be sufficient to cancel the interactions when there are variations in the resonance modes of the plant and interaction. Therefore, an H_∞ feedforward decoupler is presented.

In addition, we also investigate the self-servo track writing (SSTW) technology, which was invented by Chainer, et al. to save the production cost of the HDDs as it does not require a clean room environment. This process actually consists of axial (or time) and radial (or track) dimensions. Therefore, it is formulated into a 2-D system framework, where the radial error propagation containment is formulated as

a 2-D H_2 control problem. The adopted 2-D controller is designed with the easily implemented LMI approach. With the stored error information of the preceding track, the 2-D control scheme is realizable. Servo performance is evaluated with the 1-D controller in time dimension, obtained from the designed 2-D controller.

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Nomenclature

The following abbreviations are frequently used throughout this thesis:

ARE	Algebraic Riccati equation
BPI	Bits Per Inch
CNF	Composite Nonlinear Feedback
DSA	Digital Signal Analyzer
DSP	Digital Signal Processor
FIR	Finite Impulse Response
FPC	Flexible Printed Circuit
HDD	Hard Disk Drive
IVC	Initial Value Compensation
KYP	Kalman-Yakubovich-Popov
LDV	Laser Doppler Vibrometer
LMI	Linear Matrix Inequality
LQG	Linear Quadratic Gaussian
MIMO	Multiple Input and Multiple Output
MSC	Mode-switching Control
NLSO	Nonlinear Least Squares Optimization
NPID	Nonlinear Proportional Integral Derivative
NRRO	Non Repeatable Runout
PES	Position Error Signal
PD	Proportional Derivative
PID	Proportional Integral Derivative
PTC	Perfect Tracking Control

PTOS	Proximate Time Optimal Servomechanism
PZT	Lead-zirconate-titanate
RRO	Repeatable Runout
R/W	Read/Write
SSTW	Self-servo Track Writing
STW	Servo Track Writer
TMR	Track Misregistration
TPI	Track Per Inch
VCM	Voice Coil Motor

Chapter 1

Introduction

1.1 Hard Disk Drive Technology and Trends

The magnetic recording industry has been going through a series of dramatic transformations ever since IBM invented the first disk drive as part of a business machine, widely known as RAMAC (Random Access Method of Accounting and Control) in 1956 [1, 2]. It consists of a motor running at a speed of 1200 rpm which powered a stack of 50 aluminum platters of two feet in diameter and an actuator with a single pair of heads that moved both vertically and radially to access data. The areal density of the disks was 2 Kbits/in² and a total capacity of about 5 MBytes. From then onwards, disk drives have become such an important part of our daily lives. Today, disk drives have been incorporated into video game console, television program recorder, MP3-player and many other applications. These contribute to the great demand for development of better hard disk drive technology.

In year 2007, a high-end enterprise hard disk drive (HDD) is able to pack as much as 1 TBytes of data, with media transfer rate of over 1070 Mbits/s [3, 4]. The trend in the industry is to increase the storage capacity by increasing the areal

density, which is the amount of data that can be stored in a square inch of the disk media. The areal density is calculated by the multiplication of the track density and the bit density (or linear density). The track density is the reciprocal of the track width in the direction of a radius, defined as tracks per inch (TPI), whereas, the bit density is the number of bits which is stored along a unit distance of a track, and is defined as bits per inch (BPI). From the viewpoint of servo control technology, the increase of areal density, in particular the TPI, contributes to greater challenges in high precision head positioning.

Besides capacity, the performance of disk drives such as the average seeking time and data transfer rate has also improved tremendously due to many technological innovations in recording media, magnetic head, data channel, servomechanism and others. Moreover, the cost has gone down significantly due to the mass production nature of the hard disk drive industry. Therefore, disk drives will continue to play an important role in the data storage technology.

In the following sections, we will provide some background information about HDD servo systems and control algorithms, and servo track writing technology.

1.2 Servo Control in HDD Systems

A typical 3.5 inch disk drive with its cover removed is shown in Fig. 1.1 with the main components labeled. This HDD consists of two disks, sputtered with a thin magnetic layer as the recording medium. The disks are rotated by an electric motor, called the spindle motor, at 7200 rpm. There are four read/write (R/W) heads attached to the sliders in this HDD that read and record information onto the disk tracks (concentric circles on disks). The sliders are aerodynamically shaped block, designed to allow the R/W heads to maintain a consistent fly height of several nanometers on the disk surface. The sliders are connected to the arm (E-block),

which will be driven by the voice coil motor (VCM). The VCM actuator is a rotary actuator that rotates about the pivot bearing axis to move the R/W head over the disk surface. The control signal for the VCM actuator is transmitted by the flexible printed circuit (FPC) from the electronics board fixed on the baseplate of the disk drive.

Figure 1.1: Components of a typical 3.5 inch HDD with cover removed [5].

The HDD servo control loop refers to the motion control of the VCM actuator such that the R/W head can be positioned accurately on the disk tracks. The schematic block diagram of a HDD servo control loop is illustrated in Fig. 1.2. The system includes the actuator and driver, position error signal (PES) demodulator, timing generation and position control subsystems. As the disk is rotating in an embedded servo system, the recording head, which is capable of either writing data or reading data while following the track, will cross servo information at regular intervals. These intervals are referred to as system sampling periods. Each time a servo wedge is crossed; specially coded servo data is read and amplified. Then, the PES demodulator converts the encoded servo data to a digital position error signal. The PES is then fed into the controller, which is loaded with several control

algorithms. Subsequently, the output of the servo controller is converted and held to an equivalent analog value that is smoothed and converted to a current by the VCM driver in order to move the actuator in the direction that reduces the PES.

Figure 1.2: Hard disk drive servo control loop [6].

Essentially, there are three primary control objectives for the servo system. The first one is to move the R/W head from the initial track to the target track in the shortest possible time (seeking control). When the target track is near, the R/W head has to settle into the target track as quickly and smoothly as possible (settling control). And finally, when the target track is reached, the objective is to maintain the head on the destination track with minimum error. Similarly, this analogy can be applied to the servo track writing technology. From the viewpoint of servo track writing technology, the seeking control is to move the write head from the initial track to next track that is to be written, and the track-following control is to maintain the positioning of the write head such that servo signals will be written in concentric circles on the disk media surface.

1.2.1 Track-Seeking Control

The objective of track-seeking control is to position the R/W head to the designated track quickly and smoothly using bounded control effort. Fast seeking would reduce the time in preparation for data reading/writing; and smooth seeking such that the ideal angular velocity profile with minimum jerk will be guaranteed.

The proximate time optimal servomechanism (PTOS) [7,8] has been a popular and successful method for long-distance track-seeking. In the acceleration mode, the head is driven by maximum force until the velocity error becomes so large that saturation occurs. Once the error has become small, the inner velocity feedback loop can follow the reference velocity trajectory that is based on the time optimal curve of the inertia model. When the distance to the target track is under a certain threshold, the actuator is decelerated and prepares for switching to track-following. Adaptive PTOS [9] is proposed to improve the robustness against actuator uncertainties.

In short-distance track-seeking, where the power amplifier is not saturated, the two-degree-of-freedom (2 DOF) [10,11] is a popular servo method. The method is to design a stable feedforward compensator that produces an inverse dynamics of the plant to improve the step response of a stable feedback servo loop. The key problem in the 2 DOF method is how to produce a precise inverse model of the plant. Hence, the zero-phase error tracking control (ZPETC) has been used to design a single-rate feedforward compensator. Since the ZPETC is basically a design method for a single-rate system, the perfect tracking control (PTC) was proposed to improve the transient response using multi-rate control method [12].

1.2.2 Track-Settling Control

Track settling control is to provide fast and smooth settling on the target track when the servo mode is switched from the track-seeking mode to the track following mode.

The mode-switching control (MSC) in disk drives is done in only one direction; from track seeking to track following. Therefore, it can be treated as a design with nonzero initial values of track-following mode. The initial value compensation (IVC) [13] for MSC is now widely used in disk drives to take into account the nonzero system state variables. The IVC method requires precise estimation of the initial values while keeping the disturbance rejection capability. Therefore, the optimal mode switching control based on IVC is proposed in [14].

Recent research papers for track settling control eliminate the need for mode-switching between the separate track seeking, settling and following controllers, such as the unified control scheme, called composite nonlinear feedback (CNF) control [15], and the nonlinear proportional integral derivative (NPID) [16] control. The CNF [15] control scheme combines a linear feedback law to yield a fast response and a nonlinear feedback law to increase the damping ratio of the closed-loop system as the system approaches command input. The NPID [16] control scheme consists of a linear proportional derivative (PD) controller and a nonlinear integrator shaped by a nonlinear Gaussian function of PES. When the PES is large, the integrator is almost deactivated to achieve fast response with an approximate PD control. As PES decreases, the integrator component will increase exponentially for steady state performance.

1.2.3 Track-Following Control

The aim of track-following control is to maintain the R/W head stationary over the center of the data track under variations of environment around the HDD. During track-following, various disturbances and noises can cause the head to deviate off the target track. Thus, the controller must move the actuator swiftly to reduce the effects of disturbances and noises and keep the head on the track within allowable error tolerance. The track-following performance is evaluated by the track misreg-

istration (TMR), which is the offset between the actual head position and the track center and is defined as follows:

$$\text{TMR} = 3\sigma_{pes} \quad (1.1)$$

where σ_{pes} is the standard deviation of the PES.

The TMR must be less than 10% of the track pitch for acceptable tracking performance [7, 17]. Correspondingly, the maximum achievable track density can be approximated by,

$$\text{Track density (kTPI)} \approx 2.54 \times 10^4 \cdot \frac{10\%}{3\sigma_{pes}} = \frac{846.67}{\sigma_{pes}}, \quad (1.2)$$

where the unit of σ_{pes} is nanometer. Therefore, to increase the TPI for future disk drives, σ_{pes} has to be reduced as much as possible.

There are generally two types of disturbances that contribute to TMR; the internal and external disturbances. The internal disturbances are largely due to the rotation of the disks on the spindle and the reaction forces on the drive baseplate and housing. These disturbances cause the synchronous position error (periodic) and non-synchronous position error (random). The synchronous position error due to repeatable runout (RRO) is the result of eccentric data track with reference to the axis of rotation of the disk. On the other hand, the non-synchronous position error is largely due to the non-repeatable runout (NRRO) from PES generation noise, disk vibrations, windage, and reaction forces from the actuator [18, 19]. The external disturbances are from environmental shock and vibration such as the factory floor environment, and sudden motion or shock.

Various feedback control strategies have been developed to reduce the effects of the disturbances and noises, so that TMR may be lowered. In a feedback system, the sensitivity function from the disturbance to the output indicates the capability

of disturbance rejection. If the largest singular value of the sensitivity function is less than unity over a frequency range, the effects of disturbance at the plant output are effectively desensitized over that frequency range.

- *Classical control*

Classical design approaches have been used for many years. The common ones are proportional-integral-derivative (PID) control, lead-lag compensator to increase the servo bandwidth. There are two major obstacles to increase the servo bandwidth, which are the sampling frequency and mechanical resonance. Notch filtering is a popular gain stabilization method that has been widely used to compensate for mechanical resonances. Notch filters ensure a large stability margin according to small gain theorem but the drawback is the phase shift at around the crossover frequency which limits the servo bandwidth [20].

- *Multirate control*

Although the sampling period of the position signal is restricted, the sampling period of the control input is only limited by the execution speed of the microprocessor. As a result, the sampling period of the control input is much shorter than that of the position signal. A multirate feedback using an observer that outputs an estimated position signal two to four times during a sampling period can reduce phase shift and increase servo bandwidth [21, 22].

- *Advanced optimal control*

Advanced control methodologies such as linear quadratic Gaussian (LQG) control, loop transfer recovery (LTR) [23], and H_2/H_∞ control [19, 24, 25], have been used to optimize the servo loop. These methods are usually used to optimize the servo system such that specified control objectives are achieved, such as, robustness towards uncertainty, error minimization, disturbance rejection and etc. LQG design optimizes the linear quadratic criterion while the LTR method generates an estimator to make the output feedback loop to be the same with the full state feedback

system. H_2/H_∞ controllers are derived based on the minimization of the 2-norm and ∞ -norm of the certain system transfer function. H_∞ robust control deals with systems having uncertainty in the model definition and when taken with the structured singular value analysis method, this technique provides a design procedure with certifiable results.

- *Adaptive feedback control*

In the HDD servo, it is not easy to realize robust performance under external vibration uncertainties using fixed controllers, especially where high performance is required. Adaptive feedback control [6, 26] was designed to compensate selective harmonics and NRRO, as well as to reject narrow band vibrations due to windage and disk fluttering.

1.2.4 Dual-stage Actuator

The crossover frequency of current 3.5 inch disk drives with single-stage actuation is around 1.0 – 1.5 kHz, which makes it difficult to improve the positioning accuracy to keep up with increasing track density [7]. Hence, a microactuator is used to increase the positioning accuracy and servo bandwidth. The VCM actuator is used as the first stage actuation to generate large but relatively slow movement, while the microactuator is used as the secondary actuation to provide fine and fast positioning.

A few types of microactuators that have been studied over the years include the actuated suspension (see Fig. 1.3) [27], actuated slider and actuated head [28]. Several control schemes, such as the parallel [29], master-slave [30], decoupled master-slave, and modified decoupled master-slave [28] were proposed for dual-stage actuation.

Figure 1.3: Dual-stage actuator with PZT actuated suspension as secondary actuator.

1.3 Servo Track Writing Technology

The performance of a HDD servomechanism is greatly affected by the precision at which the position feedback signal can be generated from the servo patterns [31]. There are three different ways that the hard disk servo patterns has been implemented; wedged, dedicated and embedded servo. However, some of these techniques had become obsolete. Today, virtually all drives use the embedded servo method to read the relative position of the head to the track directly from the disk media. User data and position information are multiplexed in space around the disk, which puts a practical limit on the sample rate. However, its main advantage is that it is largely immune to the mechanical offsets in the actuator assembly [32]. The servo signals that are written by the servo track writer (STW) on the disks include the servo sync mark, track address, and offset information. Position detector circuit of HDDs detects them and the HDD controller uses them to move the actuator with magnetic heads to a target track address.

The amount of time required to servo write one disk is given by [31]

$$\text{Time (second)} = LT (60/\omega + t_s) d, \quad (1.3)$$

where

Figure 1.4: Disk stacks for dedicated and sectored servo systems [32].

ω is the RPM of the disks during the STW process;

T is the desired track density (TPI) of the drive to be servo written;

L is the radial length of disk which servo tracks are to be created in unit inch;

d is the number of steps involved in writing one servo track with each step involving one rotation of the disk;

t_s is the seek-settle time taken by the position control loop of the STW process.

1.3.1 Servo Track Writer

Conventionally, servo tracks are written by pushpin STWs, as shown in Fig. 1.5. The apparatus holds a disk drive and pushes the drive's actuator to move it to a desired position to write servo signals on the disks. Servo track writing must be done inside a clean room because the pushpin STW needs constant contact with the disk drive actuator through an opening in the covers of a HDD. Since it has a large moving mass, it cannot fully resist disturbances with a servo bandwidth of 200 to 300 Hz, and the component vibration and friction between the drive's head arm and

the STW pushpin can cause degradation [33] and repeatable runout (RRO) of the servo signals [34].

Figure 1.5: Pushpin servo track writer [33].

Hence, a push-pin free STW (Fig. 1.6) was introduced in [33]. It does not require a large mass pushpin actuator to control the position of the drive actuator and thus eliminate the coupled vibration and friction. The pushpin free STW uses an interferometer detector to detect the position of the write head from a sheet scale mounted on the head arm so that this can be used as a feedback to control the disk drive voice coil motor (VCM) actuator. However, the system servo bandwidth of this head-positioning mechanism is limited by the vibration modes of the suspension and resonance modes of the VCM actuator, which could usually be found in the kHz range. The track density is thus limited. For development of future HDDs, the system servo bandwidth will have to be increased.

1.3.2 Media-level Servo Track Writer

Due to higher track densities, the production duration of a HDD will be inevitably increased because more tracks will have to be written into each disk. In order to shorten the throughput time, media-level servo track writing technology [35] has

Figure 1.6: Pushpin-free servo track writer [33].

been proposed because the number of disks handled by the media-level STW is larger than that handled by conventional STW. Fig. 1.7 illustrates the media-level servo track writing method. The apparatus is composed of an air-bearing spindle motor, which holds the magnetic disks, and an actuator with magnetic heads. The servo controller of the apparatus positions the actuator at the proper place to write the servo signals on each disk surface before the disks are mounted in the drive separately. This method could also avoid the disk-drive vibration during the servo track writing process. Using this method, the eccentricity of the servo signals should be suppressed during assembly.

Fig. 1.8 shows a new magnetic contact duplication technology using a lithographically patterned master disk proposed in [36]. A magnetic film patterned according to the information signal to be printed on slave disks is processed by a lithography technique on the master disk. Then, the master disk is placed facing a dc erased slave disk, an external field is applied to the master disk. Due to the shielding effect of the individual patterned magnetic film on the master, flux

Figure 1.7: Media-level servo track writing [35].

leaks and correspondingly reverses the initial magnetization on the slave disk at the portion between the neighboring patterned films, and magnetic signal information according to the magnetic film pattern on the master is printed on the slave disk.

Figure 1.8: Magnetic printing method by using a lithographically patterned master disk [36].

Although both the media-level STW and magnetic printing method improves the productivity and the efficiency of clean room usage, another problem arises because the servo track trajectory does not always coincide with the rotation trajectory of an externally written disk when it is mounted in an HDD after media STW and magnetic printing STW [35].

On the other hand, hybrid-type servo track writer has also been introduced [37]. There are two main stages involved in the hybrid STW process. In the first stage, master disks are made using an independent STW called the master STW. Several raw disks are stacked in the master STW and master patterns are written on all the disks at the same time. The first stage takes place in a clean room to maximize precision. In the second stage (called the master following stage), one master disk is assembled with several raw disks in a head disk assembly (HDA). Servo patterns are written simultaneously on all disks by following the master disk.

Given that the dual-stage actuation can improve the servo bandwidth (as mentioned in Section 1.2.4), it can also be applied to the servo track writing technology. Since previously proposed control schemes for dual-stage actuators are all applied to HDDs and not for STWs, where at a particular instant, only one microactuator is activated to read or write signals on the disk surface, the interaction dynamics among other microactuators on the arm tips of the VCM actuator are neglected. If dual-stage actuators are used in STW, where all microactuators have to read and write servo signals simultaneously on the multiple disks surfaces, the interactions among the microactuators have to be considered when designing a servo controller.

1.3.3 Self-servo Track Writing

Self-servo track writing has also been proposed [19,38]. Such system does not require a STW to write the servo tracks but uses its own VCM actuator and magnetic heads to write the servo signals. The self-servo track writing system no longer uses any external radial positioning devices; but instead radial information has to be generated from previous written tracks, which leads to the problem of radial error propagation. A few studies have been done by [19,39–41] to investigate this problem. Vibration analysis has been carried out in [19] to identify sources of disturbances that could affect the accuracy of the self-servo track writing process. The radial error

propagation problem is formulated into a two-dimensional control problem [40] and a compensator with an observer based structure is designed in [41]. However, [41] has not considered the disturbance distribution of the system. Further research where disturbances and noise models of the systems are also considered, should be carried out.

1.4 Motivation and Objectives of the Thesis

Currently, the recording density is more than 130 Gb/in² for commercial product and over 230 Gb/in² for those of the laboratory level [42] using perpendicular magnetic recording (PMR) technology. Experts estimate that this technology is likely to improve the areal density up to 500 Gb/in² or slightly beyond before reaching the thermal stability limitations [42]. In order to meet the current consumer demands and extend the annual areal density growth rate at about 30–40%, new technologies will be required.

One of the costs of producing a hard disk drive comes from the duration of the throughput. The number of STWs in the production line must be increased at an annual rate of at least 50%, because it takes more time for the STWs to write the increased number of tracks. To lower the production costs, the servo track writing process should write servo signals quickly and efficiently. To achieve densities of 1 Tb/in² for future disk drives, track density of more than 540 kTPI is necessary. Along with higher TPI, comes the need for better servo control for the R/W head to read (or write) data onto (or from) the hard disk accurately. Hence, it is absolutely essential to improve the current servo control technology so that TMR can be minimized.

This thesis is to study the various HDD servo technologies and propose new control strategies for the servo control of HDD systems so that the positioning

accuracy of the R/W head can be increased in order to achieve higher TPI. As such, the research work presented in this thesis is mainly related to high precision servo control in the track following mode for application in HDD, STW and SSTW.

Since HDDs are produced in bulk, the controller has to be robust towards uncertainties in the actuator plant. At the same time, it also has to ensure minimal TMR. Further, as the rotation speed of HDDs is increased for higher data transfer rate, the amplitude of air-flow induced vibrations will increase as well. These high frequency vibrations will affect the accuracy of the R/W head. Therefore, there is a need to investigate disturbance rejection methods to reject high frequency disturbances.

It is known that the disturbance rejection property of a closed-loop system is related to the sensitivity function of the system. In view of this, loop shaping techniques must be studied so that the sensitivity function of the system can be exploited to attenuate disturbances at specific frequencies.

For development of future high performance HDDs or STWs, system with multiple dual-stage microactuators will also be investigated. Since the interactions among the multiple microactuators can affect the accuracy or even the stability of the other microactuators, great care has to be taken when designing suitable controllers.

As discussed in the previous sections, a high percentage of the disk drive production costs lies in the servo track writing process that is required to be done in a clean room environment. On the other hand, this requirement is not needed for the SSTW process. Therefore, investigation of the SSTW process will be carried out and new control strategies will be proposed.

Thus, our research objectives can be briefly summarized as in the following:

1. Develop controllers for TMR minimization under unmodelled plant dynamics;

2. Study existing disturbance rejection methods and develop new methods for high frequency disturbance rejection;
3. Improve disturbance rejection by better shaping the sensitivity function of the systems;
4. Develop a decoupling control schemes for interaction compensation;
5. Investigate the SSTW process, formulate the servo control problem and propose suitable control algorithms.

1.5 Contributions of the Thesis

The contributions of this thesis are as follows:

- An improved mixed H_2/H_∞ control algorithm is proposed to reduce the TMR in the presence of unmodelled dynamics of the VCM actuator. The TMR is formulated as an H_2 control problem, which is optimized under the H_∞ constraint to meet robustness requirements. In the new algorithm, additional scaling parameters are introduced to offer additional freedom during optimization and this leads to a less conservative design compared to previous research work.
- A disturbance observer with a phase stabilized controller is presented to deal with two narrow band disturbances at frequencies beyond the bandwidth of the HDD servo control loop. A high frequency bandpass filter is designed for the disturbance observer, unlike the lowpass filter used in conventional disturbance observer, therefore, enabling high frequency disturbance attenuation.
- Based on an initial sensitivity function using a basic PID controller with notch filters, a nonlinear least-squares optimization (NLSO) method is applied to

shape the sensitivity function of the system so that higher attenuation of the disk induced vibrations is achieved.

- The generalized Kalman-Yakubovic-Popov (KYP) lemma is applied to shape the sensitivity function to attenuate disturbances at desired frequency ranges. Since the generalized KYP lemma involves a matrix variable that is not necessarily positive definite, the Schur complement cannot be applied to convexify the controller design. Thus, the Youla parameterization approach is used to parameterize the closed loop function. A finite impulse response (FIR) filter is used to design the controller so that the sensitivity shaping problem can be solved using linear matrix inequality (LMI). The generalized KYP lemma is also combined with H_2 control to obtain PES minimization.
- An H_∞ feedforward decoupler is proposed as a means to cancel the interaction between microactuators in a hybrid-type STW. The feedforward decoupler is designed by an H_∞ design method.
- In order to contain the radial error propagation, the SSTW process is formulated into a 2-D H_2 control problem to minimize the error with due consideration given to the various disturbances and noise of the system (i.e., torque disturbances, quantization noise, air turbulence, and etc.). The 2-D controller is then designed with an easily implemented LMI approach.

1.6 Organization of the Thesis

This chapter has given the overview of the HDD trends and some background of the HDD servo systems and servo track writing technology. The research motivation, objectives and contributions are also described here.

Chapter 2 explains the plant measurement, modeling of the VCM actuator

and PZT microactuator. Noises and disturbances modeling are also included here.

Chapter 3 presents an improved mixed H_2/H_∞ algorithm to reduce the TMR while meeting the robustness constraints against unmodelled high frequency dynamics of the VCM actuator. This chapter also includes the simulation and implementation results of this algorithm.

Chapter 4 proposes a combined disturbance observer and phase stabilized controller to attenuate two narrow-band disturbances, both with frequencies higher than servo bandwidth. The selection of the filter for the disturbance observer is also detailed here.

Chapter 5 demonstrates the application of a nonlinear least-squares optimization (NLSO) method in sensitivity function shaping to design a feedback controller for a microactuator such that its sensitivity function fulfills desired specifications in order to attenuate disturbances at specific frequencies.

Chapter 6 gives detailed information on how disturbance attenuation at desired frequency can be achieved using the generalized KYP lemma. The Youla parameterization method is used to parameterize the controller such that the controller design problem becomes a convex optimization problem. Simulation and experimental results for low frequency and high frequency disturbance rejection are also found here. Then, we combine the generalized KYP lemma with H_2 control in order to attenuate a low frequency disturbance and minimize the TMR. Simulation results have shown some improvement of the TMR.

Chapter 7 proposes an H_∞ feedforward decoupler to cancel the interactions between two PZT microactuators. The feedforward decoupler is designed such that the effects of noises and disturbances from the second microactuator loop in the PES of the first microactuator is attenuated. Simulation results show the effectiveness of this method.

Chapter 8 formulates the SSTW process into a 2-D system framework. Then, a 2-D H_2 control scheme is designed for the system. Simulation and experimental results are also shown here.

Chapter 9 concludes the entire thesis and suggests some future work for HDDs servo control.

Chapter 2

Measurement and Modeling of Plant and Disturbance

This chapter studies the modeling of the plants under investigation and disturbances acting on the plants.

2.1 Setup for Measurement and Testing

Fig. 2.1 and Fig. 2.2 show the configuration of a HDD system during experiment. The equipment used for experiments are:

- Scanning laser Doppler vibrometer (LDV) - POLYTEC Scanning vibrometer PSV-200 [43],
- Digital signal processor (DSP) - dSPACE 1103 on TMS320C240 DSP board,
- Digital signal analyzer (DSA) - HP 35670A [44],
- Oscilloscope - LeCroy 9354 500MHz Oscilloscope [45].

Figure 2.1: Experimental setup for plant identification.

2.1 Setup for Measurement and Testing

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Figure 2.2: Experimental setup for designed controller verification.

Figure 2.3: VCM actuator.

Figure 2.4: PZT microactuator.

The side of the baseplate of the HDD is partially removed so that the laser beam from the LDV can be directed at a right angle (90 degrees) to the tip of the VCM arm. This is to ensure accuracy in the position measurement. The cover of the disk drive has also been partially removed to facilitate the mounting of a PZT microactuator suspension on the E-block of the VCM. The close up views of the VCM actuator and PZT microactuator [46] are shown in Fig. 2.3 and Fig. 2.4, respectively.

2.2 Plant Modeling

Fig. 2.5 shows the equivalent block diagram of the setup shown in Fig. 2.1. The frequency response of a HDD actuator, i.e., VCM actuator and PZT microactuator, is measured by giving a reference input signal into the VCM or PZT through their respective amplifiers. The actuator dynamics, $P(s)$ is measured in the frequency domain by giving an input sine wave, u , which sweeps across the required frequency range into the plant and the output, y , is measured by the Laser Doppler Vibrometer (LDV) and analyzed using a Digital Signal Analyzer (DSA). The LDV is used

Figure 2.5: Block diagram of system dynamics measurements.

because it allows non-contacting displacement and velocity measurements.

The measured frequency response of the VCM actuator (see Fig. 2.6) contains the VCM dynamics, the VCM driver and the gains of the LDV equipment. Similarly, the frequency response of the PZT microactuator, as shown in Fig. 2.7 contains the dynamics of microactuator, the piezo power amplifier and the gains of the LDV equipment.

Many resonance modes of the plant are usually seen from the measured frequency response. However, only those that are of significant magnitude are modeled because those that have insignificant magnitude usually do not affect the closed-loop system stability.

2.2.1 Frequency Domain Modeling for VCM Actuator

After obtaining the frequency response of the plant, the plant model of a system is easily identified using the curve-fitting method. The resonance modes of a plant are determined by the poles of the systems, while the antiresonance modes are determined by the zeros of the plant. The resonance modes of a system will usually contribute a decreasing phase. However, the antiresonance modes can be categorized into two categories. A minimum phase antiresonance will give rise to an increasing phase while a non-minimum phase antiresonance will contribute a decreasing phase.

Thus, the curve-fitting method is most widely used for HDD plant identification.

The plant model of a VCM actuator can be expressed as the following:

$$P_{vcm}(s) = K \times P_o(s) \times P_r(s) \times P_d(s), \quad (2.1)$$

$$P_o(s) = \frac{\omega_o^2}{s^2 + 2\zeta_o\omega_o s + \omega_o^2}, \quad (2.2)$$

$$P_r(s) = \prod_{i=1, j=1}^{m, n} \frac{\omega_{pi}^2 (s^2 + 2\zeta_{zj}\omega_{zj}s + \omega_{zj}^2)}{\omega_{zj}^2 (s^2 + 2\zeta_{pi}\omega_{pi}s + \omega_{pi}^2)}, \quad (2.3)$$

$$P_d(s) = e^{-T_d s}, \quad (2.4)$$

where K is the dc gain of the plant model; $P_o(s)$ describes the rigid body mode in the low frequency with damping ratio ζ_o and frequency ω_o in rad/s; $P_r(s)$ describes the resonances and antiresonances, where ζ_{zj} and ω_{zj} ($j = 1, 2, \dots, n$) are the damping ratio and frequency of the antiresonance modes, respectively; and ζ_{pi} and ω_{pi} ($i = 1, 2, \dots, m$) are the damping ratio and frequency of the resonance modes, respectively. ω_{pi} (or ω_{zj}) are determined by the frequency at which the peaks (or notches) of the magnitude in the frequency response are located. ζ_{pi} and ζ_{zj} are tuned to match the magnitude and phase response curves. $P_d(s)$ is the time delay term to approximate the phase lag where T_d is the time in second.

Fig. 2.6 shows the measured frequency response of the VCM actuator. The identified plant model is also plotted on the same graph for comparison. $R1, R2, R3, R4, R5$ indicate the resonance modes and $N1, N2, N3$ indicate the antiresonance modes.

Figure 2.6: Frequency response of the VCM actuator.

2.2.2 Frequency Domain Modeling for PZT Microactuator

Similarly, the plant model of a PZT microactuator can be described as:

$$P_{pzt}(s) = K \times P_r(s) \times P_d(s), \quad (2.5)$$

$$P_r(s) = \prod_{i=1, j=1}^{m, n} \frac{\omega_{pi}^2 (s^2 + 2\zeta_{zj}\omega_{zj}s + \omega_{zj}^2)}{\omega_{zj}^2 (s^2 + 2\zeta_{pi}\omega_{pi}s + \omega_{pi}^2)}, \quad (2.6)$$

$$P_d(s) = e^{-T_d s}. \quad (2.7)$$

Fig. 2.7 shows the measured frequency responses of the PZT microactuator and the identified plant model. $R1, R2, R3, R4$ indicate the resonance modes and $N1, N2, N3, N4$ indicate the antiresonance modes.

Figure 2.7: Frequency response of the PZT microactuator.

2.3 Disturbance Modeling

Fig. 2.8 shows the servo control loop for HDD systems where

r is the reference signal;

$C(z)$ is the controller;

$P(z)$ is the plant model;

d_i is all the torque disturbances and windage, power-amplifier noise and D/A quantization error;

d_o is the disk flutter and motor vibrations;

n is the PES demodulation noise and A/D quantization error.

The servo control loop in Fig. 2.8 can be represented with the disturbance and noise models as in Fig. 2.9 where $D_i(z)$, $D_o(z)$ and $N(z)$ describe the transfer

Figure 2.8: HDD servo control loop.

Figure 2.9: Servo control loop with disturbance/noise models.

function for the input and output disturbances, and noise, respectively, and w_i ($i = 1, 2, 3$) is a white noise with variance 1.

Several studies have been carried out to obtain the disturbance/noise models [19, 47, 48]. In this thesis, to reflect the different HDD systems in discussions, such as the self-servo track writer, VCM actuator, PZT microactuator and servo track writer, different disturbances and noise models are used.

Figure 2.10: PES NRRO spectrum.

2.3.1 Modeling of Noises and Disturbances in Closed-Loop

Fig. 2.10 shows the closed-loop NRRO spectrum of a 3.5-inch HDD with a VCM actuator. The NRRO spectrum is given by

$$S_{pes} = |P(z)S(z)|^2 |D_i(z)|^2 + |S(z)|^2 |N(z)|^2 + |S(z)|^2 |D_o(z)|^2 \quad (2.8)$$

where $S(z)$ is the sensitivity function.

The procedure to obtain $D_i(z)$, $D_o(z)$ and $N(z)$, as shown in [47], is listed in the following:

- i. Find the baseline curve S_b of the spectrum

$$S_b(j) = \min_{i=1+(j-1)m}^{jm} S_{pes}(i), \quad j = 1, 2, \dots, L/m \quad (2.9)$$

where L is the length of S_{pes} and m is a sufficiently small integer;

- ii. Compute $D_i(z)$ and $N(z)$ using

$$|D_i(z)|^2 = \frac{W_L S_b}{|P(z) S(z)|^2}, \quad |N(z)|^2 = \frac{W_H S_b}{|S(z)|^2} \quad (2.10)$$

where W_L (W_H) is a low (high)-pass weighting function used to select S_b of low (high) frequencies corresponding to the two dominant peaks. Till now, S_b can be fit well by the identified $D_i(z)$ and $N(z)$.

- iii. The spikes on the spectrum, especially between 0 to 2.5 kHz, are regarded as d_o effect. Thus,

$$|D_o(z)|^2 = \frac{\{S_{pes} - [|P(z) S(z)|^2 |D_i(z)|^2 + |S(z)|^2 |N(z)|^2]\}}{|S(z)|^2} \quad (2.11)$$

where S_{pes} is the whole NRRO spectrum as in Fig. 2.10. For simplicity, a low order model is used to describe d_o .

The identified disturbances and noise models are as follows [49]:

$$\begin{aligned} D_i(s) &= \frac{1.3916 \times 10^{-5} (s+575.8)(s+575.6)(s^2+0.04389s+161.6)}{(s^2+315.5s+8.178 \times 10^4)(s^2+315.4s+8.178 \times 10^4)}, \\ D_o(s) &= \frac{0.701588 (s+1.271 \times 10^4)^2 (s^2+6.125 \times 10^{-5}s+4.373 \times 10^8)}{(s+708.4)^2 (s^2+0.0001317s+4.376 \times 10^8)}, \\ N(s) &= \frac{1.1695 (s+1.431 \times 10^4)(s+766.2)(s^2+8609s+4.672 \times 10^7)}{(s+4630)(s+1538)(s^2+4451s+1.507 \times 10^7)}. \end{aligned} \quad (2.12)$$

2.3.2 Modeling of Noises and Disturbances in Open-Loop

Fig. 2.11 shows the PES NRRO power spectrum measured from our STW platform [50]. The PES NRRO reflects the disturbance and noise distribution of the STW platform.

Using methods in [47, 48], the sources of noise and disturbances that are reflected in the PES NRRO are determined. The baseline of the NRRO reflects the noise of the system, n , while the peaks between frequency 0 – 100 Hz are due to the

Figure 2.11: Nonrepeatable runout (NRRO) power spectrum calculated from measured PES signal without servo control, reflecting the disturbance distribution of the STW platform.

input disturbance, d_i , and the peaks above 100 Hz are due to the output disturbance, d_o . The noise and disturbance models of the system are given by

$$\begin{aligned}
 D_i(s) &= \frac{12.79}{(s^2 + 113.1s + 3.198 \times 10^5)}, \\
 D_o(s) &= \frac{0.0019(s^2 + 381s + 7.012 \times 10^5)(s^2 + 2276s + 1.602 \times 10^7)(s^2 + 2640s + 5.647 \times 10^8)}{(s^2 + 8.168s + 6.672 \times 10^5)(s^2 + 245s + 1.668 \times 10^7)(s^2 + 477.5s + 5.701 \times 10^8)}, \\
 N(s) &= 0.0023.
 \end{aligned} \tag{2.13}$$

Chapter 3

Improved Mixed H_2/H_∞ Control Design

As previously mentioned in Chapter 1, HDDs are manufactured in mass production. Thus, the designed servo control has to be robust towards unmodelled uncertainty. For future disk drives with ultra high track densities, the TMR has to be very small. A mixed H_2/H_∞ controller will duly achieve both of these requirements.

3.1 Introduction

The mixed H_2/H_∞ control problem is concerned with the design of a controller that minimizes the H_2 norm of certain closed-loop transfer function while satisfying a H_∞ norm constraint on the same or another closed-loop transfer function; see, e.g. [51–53]. One of the important applications of this problem is to address the optimal nominal performance subject to a robust stability constraint.

The first formulation of the mixed H_2/H_∞ control was reported in [51]. A Riccati equation based approach to solve the mixed H_2/H_∞ control problem was

adopted in [52, 54, 55]. The results of these works either provide an over design or involve solving coupled Riccati equations, which is not an easy task in general. An exact solution was given in [56] via convex optimization. A Banach duality theory was developed in [57] which gives a ‘nearly’ optimal solution by reducing the dual and predual representations to finite variable convex optimization. An alternative based on the LMI approach has gained popularity [57, 58] since the development of the interior point algorithm for convex optimization. In the early development, to solve the problem in terms of LMIs, a single Lyapunov matrix is adopted for both the H_2 and H_∞ performances, which is very conservative in general. A significant improvement was made in [59] where a slack variable technique is introduced which separates the Lyapunov matrices from the controller parameters and hence allows them to be different for the H_2 and H_∞ performances. This result, though, alleviates the design conservatism but remains to be a sufficient condition [59]. On the other hand, it is noted that the result of [59] is based on a special case of the robust stability result given in [60], indicating that a better mixed H_2/H_∞ control may be designed by removing or relaxing the specialization. Hence, this is the motivation of our study here.

In this chapter, we first present a new characterization of the H_2 and H_∞ performances of linear discrete time-invariant systems by extending the result of [60]. The characterization has the advantages: (a) it unifies the treatment of H_2 and H_∞ control; (b) it contains a number of additional slack variables than existing characterizations of H_2 and H_∞ performances. Those additional degrees of freedom are then explored to give a less or equally conservative design of mixed H_2/H_∞ control. In fact, when those parameters are set to be zero or one, our result reduces to the well known result of [59], implying that the proposed design will achieve an equal or better performance. Our result involves searching for those variables and carrying out convex optimization subject to two LMIs.

We then apply our proposed mixed H_2/H_∞ design to disk drive control. One of the important performance measures for hard disk drives (HDDs) is the track misregistration or TMR, the total amount of random fluctuation of the read/write head off the desired track location. In order to achieve a high storage capacity in hard disk drives, TMR is to be minimized via a proper design of servo system. Since TMR determines the disk capacity, various ways such as vibration analysis and control [18, 61–64], high bandwidth servo control [65, 66], and dual-stage actuation [67, 68] have been studied for hard disk drives over the years. [69] treated the control design problem to minimize TMR as an optimal H_2 control problem and a Riccati equation approach was used to design the optimal H_2 controller. On the other hand, it is difficult to involve all high-frequency dynamics of VCM actuator in a model-based servo control design. To meet the robustness requirement against unmodelled high frequency dynamics of the VCM actuator, an H_∞ constraint is to be satisfied when minimizing the H_2 performance of the nominal system. Hence, a mixed H_2/H_∞ control is formulated for disk drive control in this chapter. Both the simulation and experiment results demonstrate that the proposed design gives a significant performance improvement of TMR over the existing mixed H_2/H_∞ control method of [59] even though an iterative procedure is applied to refine the controller designed by the latter. A comparison with the conventional PID plus notch filter control also reveals the advantages of our method.

This chapter is organized as follows. Section 3.2 presents a novel design for the mixed H_2/H_∞ control. The application of the new design method in hard disk drives will be presented in Section 3.3. Finally, we shall draw some conclusions in Section 3.4.

3.2 An Improved Mixed H_2/H_∞ Control Method

This section aims to derive a new design method for the mixed H_2/H_∞ control which will give rise to an equal or better performance than existing LMI results.

We consider the state-space representation for linear time-invariant systems:

$$x(k+1) = Ax(k) + B_2u(k) + B_1w(k) \quad (3.1)$$

$$y(k) = Cx(k) + D_1w(k) \quad (3.2)$$

$$z_1(k) = L_1x(k) + H_{12}u(k) + H_{11}w(k) \quad (3.3)$$

$$z_2(k) = L_2x(k) + H_{22}u(k) + H_{21}w(k) \quad (3.4)$$

where $x(k) \in \mathcal{R}^n$ is the state, $y(k) \in \mathcal{R}^l$ is the measurement output, $z_i(k) \in \mathcal{R}^r$ ($i = 1, 2$) are the controlled outputs, $w(k) \in \mathcal{R}^p$ is the disturbance input, $u(k) \in \mathcal{R}^m$ is the control input, and $A, B_1, B_2, C, D_1, L_1, L_2, H_{11}, H_{12}, H_{21}, H_{22}$ are of appropriate dimensions.

Let a controller of the same dimension as that of the system (3.1)-(3.4) be of the form:

$$\hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}y(k), \quad (3.5)$$

$$u(k) = \hat{C}\hat{x}(k) + \hat{D}y(k) \quad (3.6)$$

where the matrices $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$ are to be determined.

Denote T_{z_1w} the transfer function matrix from w to z_1 , and T_{z_2w} the transfer function matrix from w to z_2 . The mixed H_2/H_∞ control problem is then stated as: *Given a positive scalar γ , design a controller of the form (3.5)-(3.6) such that the H_2 norm, $\|T_{z_1w}\|_2$, is minimized subject to the constraint $\|T_{z_2w}\|_\infty < \gamma$.*

Remark 3.2.1. *In recent years, LMI based approaches to control design have at-*

tracted significant interests due to their numerical tractability. However, existing LMI based solutions for the mixed H_2/H_∞ control present sufficient solvability conditions which are generally conservative with various degrees of conservatism. [59] presents a slack variable technique which is considered to be the most effective LMI based approach so far. In this section, we aim to give a design with better or equal performance than that of [59].

First, denote $\xi = [x^T \ \hat{x}^T]^T$. It follows from (3.1)-(3.4) and (3.5)-(3.6) that

$$\xi(k+1) = \bar{A}\xi(k) + \bar{B}w(k) \quad (3.7)$$

$$z_1(k) = \bar{C}_1\xi(k) + \bar{D}_1w(k) \quad (3.8)$$

$$z_2(k) = \bar{C}_2\xi(k) + \bar{D}_2w(k) \quad (3.9)$$

where

$$\bar{A} = \begin{bmatrix} A + B_2\hat{D}C & B_2\hat{C} \\ \hat{B}C & \hat{A} \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_2\hat{D}D_1 + B_1 \\ \hat{B}D_1 \end{bmatrix}, \quad (3.10)$$

$$\bar{C}_1 = \begin{bmatrix} L_1 + H_{12}\hat{D}C & H_{12}\hat{C} \end{bmatrix}, \quad \bar{D}_1 = H_{12}\hat{D}D_1 + H_{11}, \quad (3.11)$$

$$\bar{C}_2 = \begin{bmatrix} L_2 + H_{22}\hat{D}C & H_{22}\hat{C} \end{bmatrix}, \quad \bar{D}_2 = H_{22}\hat{D}D_1 + H_{21}. \quad (3.12)$$

Recall that for a given controller that stabilizes the system (3.7)-(3.9), the H_2 norm square, $\|T_{z_1w}\|_2^2$, can be computed by the following minimization [54]:

$$\min_{(Q \geq 0, \Pi = \Pi^T)} \text{trace}(\Pi) \quad (3.13)$$

subject to

$$\bar{A}^T Q \bar{A} - Q + \bar{C}_1^T \bar{C}_1 < 0 \quad (3.14)$$

$$\bar{B}^T Q \bar{B} + \bar{D}_1^T \bar{D}_1 < \Pi \quad (3.15)$$

The following lemma leads to an alternative for computing the H_2 norm of the system (3.7)-(3.8).

Lemma 3.2.1. *The minimization of the H_2 norm square in (3.13) is equivalent to the following minimization:*

$$\min_{(Q \geq 0, \Pi = \Pi^T, \Sigma)} \text{trace}(\Pi) \quad (3.16)$$

subject to

$$\tilde{A}^T \text{diag}\{Q, I\} \tilde{A} - \begin{bmatrix} Q & \Sigma \\ \Sigma^T & \Pi \end{bmatrix} < 0 \quad (3.17)$$

where

$$\tilde{A} = \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C}_1 & \bar{D}_1 \end{bmatrix}.$$

Proof First of all, (3.17) can be rewritten as

$$\begin{bmatrix} \bar{A}^T Q \bar{A} - Q + \bar{C}_1^T \bar{C}_1 & \bar{A}^T Q \bar{B} + \bar{C}_1^T \bar{D}_1 - \Sigma \\ \bar{B}^T Q \bar{A} + \bar{D}_1^T \bar{C}_1 - \Sigma^T & \bar{B}^T Q \bar{B} + \bar{D}_1^T \bar{D}_1 - \Pi \end{bmatrix} < 0 \quad (3.18)$$

It is then clear that if there exists a solution (Q, Π, Σ) to (3.17), the same Q and Π also satisfy (3.14) and (3.15), respectively. On the other hand, if there exist Q and Π satisfying (3.14) and (3.15), then (3.17) is also met by the same Q and Π and $\Sigma = \bar{A}^T Q \bar{B} + \bar{C}_1^T \bar{D}_1$.

Remark 3.2.2. *The characterization of the H_2 norm in the above lemma has the advantage that it provides a unified treatment of H_2 and H_∞ designs via an LMI approach as seen later. Furthermore, the additional parameter Σ in (3.17) will offer an additional freedom in optimization of performance when a mixed H_2 and H_∞ control design is concerned.*

Without loss of generality, we shall assume $l = p$, i.e. the disturbance input

and the measured output have the same dimension. Note that, if this is not the case, some simple modification can render the requirement satisfied. For example, if $l < p$, the matrices B_1 , D_1 , H_{11} and H_{21} can be extended as $B'_1 = [B_1 \ 0_{n \times (p-l)}]$, $D'_1 = [D_1 \ 0_{l \times (p-l)}]$, $H'_{11} = [H_{11} \ 0_{p \times (p-l)}]$ and $H'_{21} = [H_{21} \ 0_{p \times (p-l)}]$.

Lemma 3.2.2. *There exists a solution (Σ, Q, Π) with $Q = Q^T \geq 0$ to (3.17) if and only if there exist matrices (Σ, Π, Q, F, G) with $Q = Q^T \geq 0$ and $\Pi = \Pi^T \geq 0$ such that*

$$\begin{bmatrix} -\begin{bmatrix} Q & \Sigma \\ \Sigma^T & \Pi \end{bmatrix} + \tilde{A}^T F + F^T \tilde{A} & -F^T + \tilde{A}^T G \\ -F + G^T \tilde{A} & \text{diag}\{Q, I\} - (G + G^T) \end{bmatrix} < 0. \quad (3.19)$$

Proof The proof is similar to the proof of a result in [60]. First, if (3.17) holds for some $Q > 0$, by applying the Schur complement, it is easy to know that (3.19) is satisfied with $F = 0$ and $G^T = G = \text{diag}\{Q, I\}$. On the other hand, if (3.19) holds for some (Σ, Q, F, G) , multiplying (3.19) from the left and from the right by Γ^T and Γ , respectively, where

$$\Gamma = \begin{bmatrix} I \\ \tilde{A} \end{bmatrix},$$

(3.17) follows.

Remark 3.2.3. *It should be pointed out that (3.13)-(3.15) and Lemma 3.2.2 give equivalent computations of the H_2 norm of the system. For systems without uncertainty, it is well known that (3.13)-(3.15) can be applied to derive the optimal H_2 controller [54]. Hence, there is no advantage of using Lemma 3.2.2. However, as will be seen later, when additional performances such as H_∞ performance are to be met in addition to the H_2 performance, the latter will result in a less or equally conservative design due to the additional variables F and G . We observe that when $F = 0$ and $\Sigma = 0$, Lemma 3.2.2 reduces to the result of [59].*

While Lemma 3.2.2 can be applied to compute the H_2 norm for the system (3.7)-(3.8) when a controller (3.5)-(3.6) is given, it may not be directly applicable to the H_2 control design problem due to the presence of the products of F with $\tilde{A}$ and G with $\tilde{A}$. To overcome this difficulty, we specialize the matrices F and G as follows:

$$F = \begin{bmatrix} \lambda_1 \Phi & 0 \\ 0 & \lambda_2 I \end{bmatrix} \quad G = \begin{bmatrix} \Phi & 0 \\ 0 & \lambda_3 I \end{bmatrix} \quad (3.20)$$

where $\Phi \in \mathcal{R}^{n \times n}$ and λ_i , $i = 1, 2, 3$ are real scaling parameters. While this specialization of F and G generally introduces some conservatism, it contains three additional variables λ_i , $i = 1, 2, 3$ as compared to the result of [59] which help reduce the design conservatism in [59].

Substituting (3.20) into (3.19) leads to

$$\begin{bmatrix} -Q + \lambda_1 \bar{A}^T \Phi + \lambda_1 \Phi^T \bar{A} & \lambda_2 \bar{C}_1^T + \lambda_1 \Phi^T \bar{B} - \Sigma \\ \lambda_2 \bar{C}_1 + \lambda_1 \bar{B}^T \Phi - \Sigma^T & -\Pi + \lambda_2 (\bar{D}_1^T + \bar{D}_1) \\ -\lambda_1 \Phi + \Phi^T \bar{A} & \Phi^T \bar{B} \\ \lambda_3 \bar{C}_1 & -\lambda_2 I + \lambda_3 \bar{D}_1 \\ -\lambda_1 \Phi^T + \bar{A}^T \Phi & \lambda_3 \bar{C}_1^T \\ \bar{B}^T \Phi & -\lambda_2 I + \lambda_3 \bar{D}_1^T \\ Q - (\Phi + \Phi^T) & 0 \\ 0 & (1 - 2\lambda_3)I \end{bmatrix} < 0. \quad (3.21)$$

A similar characterization for H_∞ performance can be derived. Indeed, recall that when the system (3.7) and (3.9) is known, it is stable with its H_∞ norm less than γ if and only if there exists a matrix $P = P^T > 0$ such that [59]

$$\tilde{A}^T \text{diag}\{P, I\} \tilde{A} - \text{diag}\{P, \gamma^2 I\} < 0. \quad (3.22)$$

where

$$\tilde{A} = \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C}_2 & \bar{D}_2 \end{bmatrix}.$$

Observe that (3.22) turns out to be a special case of (3.17) with $\Pi = \gamma^2 I$ and $\Sigma = 0$. Thus, following a similar procedure for deriving (3.21), it can be shown that the system (3.7) and (3.9) has an H_∞ performance γ if and only if for some real scalars ε_i , $i = 1, 2, 3$, there exist matrices $P = P^T > 0$ and Φ such that

$$\begin{bmatrix} -P + \varepsilon_1(\bar{A}^T \Phi + \Phi^T \bar{A}) & \varepsilon_2 \bar{C}_2^T + \varepsilon_1 \Phi^T \bar{B} \\ \varepsilon_2 \bar{C}_2 + \varepsilon_1 \bar{B}^T \Phi & -\gamma^2 I + \varepsilon_2(\bar{D}_2^T + \bar{D}_2) \\ -\varepsilon_1 \Phi + \Phi^T \bar{A} & \Phi^T \bar{B} \\ \varepsilon_3 \bar{C}_2 & -\varepsilon_2 I + \varepsilon_3 \bar{D}_2 \\ -\varepsilon_1 \Phi^T + \bar{A}^T \Phi & \varepsilon_3 \bar{C}_2^T \\ \bar{B}^T \Phi & -\varepsilon_2 I + \varepsilon_3 \bar{D}_2^T \\ P - (\Phi + \Phi^T) & 0 \\ 0 & (1 - 2\varepsilon_3)I \end{bmatrix} < 0. \quad (3.23)$$

Remark 3.2.4. When setting $\varepsilon_1 = \varepsilon_2 = 0$ (i.e. setting $F = 0$) and $\varepsilon_3 = 1$ and by some row-column exchanges, the above inequality reduces to

$$\begin{bmatrix} P - \Phi - \Phi^T & \Phi^T \bar{A} & \Phi^T \bar{B} & 0 \\ \bar{A}^T \Phi & -P & 0 & \bar{C}_2^T \\ \bar{B}^T \Phi & 0 & -\gamma^2 I & \bar{D}_2^T \\ 0 & \bar{C}_2 & \bar{D}_2 & -I \end{bmatrix} < 0$$

which is the result of [59]. Therefore, ε_i , $i = 1, 2, 3$ are additional variables which can be exploited to alleviate the conservatism in the mixed H_2/H_∞ design of [59].

Denote

$$\Omega (\Pi, X, Y, E, U, W, Z, \hat{D}, \bar{Q}_{11} = \bar{Q}_{11}^T, \bar{Q}_{12}, \bar{Q}_{22} = \bar{Q}_{22}^T, \bar{\Sigma}_1,$$

$$\bar{\Sigma}_2, \lambda_i, i = 1, 2, 3) =$$

$$\left[\begin{array}{l} -\bar{Q}_{11} + \lambda_1(AY + Y^T A^T + E^T B_2^T + B_2 E) \\ -\bar{Q}_{12}^T + \lambda_1(A^T + C^T \hat{D}^T B_2^T) + \lambda_1 U^T \\ \lambda_2(L_1 Y + H_{21} E) + \lambda_1(B_1^T + D_1^T \hat{D}^T B_2^T) - \bar{\Sigma}_1^T \\ -\lambda_1 Y^T + AY + B_2 E \\ -\lambda_1 I + U \\ \lambda_3(L_1 Y + H_{21} E) \end{array} \right]$$

*

$$\begin{aligned} & -\bar{Q}_{22} + \lambda_1(A^T X + X^T A) + \lambda_1(C^T W^T + WC) \\ & \lambda_2(L_1 + H_{21} \hat{D} C) + \lambda_1 B^T X + \lambda_1 D_1^T W^T - \bar{\Sigma}_2^T \\ & -\lambda_1 Z + A + B_2 \hat{D} C \\ & -\lambda_1 X + X^T A + WC \\ & \lambda_3(L_1 + H_{21} \hat{D} C) \end{aligned}$$

*

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$$\begin{array}{ll} -\Pi + \lambda_2(D_1^T \hat{D}^T H_{21}^T + H_{21} \hat{D} D_1 + H_{11}^T + H_{11}) & * \\ B_1 + B_2 \hat{D} D_1 & \bar{Q}_{11} - (Y + Y^T) \\ X^T B_1 + W D_1 & \bar{Q}_{12}^T - (I + Z^T) \\ -\lambda_2 I + \lambda_3(H_{11} + H_{21} \hat{D} D_1) & 0 \end{array}$$

$$\begin{bmatrix} * & * \\ * & * \\ * & * \\ * & * \\ \bar{Q}_{22} - (X + X^T) & * \\ 0 & (1 - 2\lambda_3)I \end{bmatrix}, \quad (3.24)$$

where ‘*’ denotes entries that can be deduced from the symmetry of the matrix. We have the following solution to the mixed H_2/H_∞ control.

Theorem 3.2.1. *Consider the system (3.1)-(3.4). A controller of the form (3.5)-(3.6) that minimizes an upper-bound of the H_2 norm of the system subject to a given H_∞ norm constraint exists if for some $\lambda_i, \varepsilon_i, i = 1, 2, 3$, there exists a solution $(\bar{P}_{11}, \bar{P}_{12}, \bar{P}_{22}, \bar{Q}_{11}, \bar{Q}_{12}, \bar{Q}_{22}, \bar{\Sigma}_1, \bar{\Sigma}_2, X, Y, \Pi > 0, U, E, Z, W, \Psi, \hat{D})$ with $\bar{Q} = [\bar{Q}_{lj}] = \bar{Q}^T > 0$ and $\bar{P} = [\bar{P}_{lj}] = \bar{P}^T > 0$ to the following optimization:*

$$\min \text{trace}(\Pi)$$

subject to

$$\Omega(\Pi, X, Y, E, U, W, Z, \hat{D}, \bar{Q}_{11}, \bar{Q}_{12}, \bar{Q}_{22}, \bar{\Sigma}_1, \bar{\Sigma}_2, \lambda_i, i = 1, 2, 3) < 0 \quad (3.25)$$

$$\Omega(\gamma^2 I, X, Y, E, U, W, Z, \hat{D}, \bar{P}_{11}, \bar{P}_{12}, \bar{P}_{22}, \bar{\Sigma}_1, \bar{\Sigma}_2, \varepsilon_i, i = 1, 2, 3)|_{\bar{\Sigma}_1 = \bar{\Sigma}_2 = 0} < 0 \quad (3.26)$$

In this situation, a mixed H_2/H_∞ controller is given by

$$\begin{aligned} \hat{A} &= M_1^{-T}(U - X^T(A + B_2 \hat{D}C)Y - X^T B_2 \hat{C} N_1 \\ &\quad - M_1^T \hat{B} C Y) N_1^{-1}, \end{aligned} \quad (3.27)$$

$$\hat{B} = M_1^{-T}(W - X^T B_2 \hat{D}), \quad (3.28)$$

$$\hat{C} = (E - \hat{D} C Y) N_1^{-1}, \quad \hat{D} = \hat{D} \quad (3.29)$$

where M_1, N_1 satisfy $N_1^T M_1 = Z - Y^T X$.

Proof First, observe from (3.21) that Φ is invertible since $\Phi + \Phi^T > Q > 0$. Denote

$$\Phi = \begin{bmatrix} X & M \\ M_1 & U \end{bmatrix}, \quad \Phi^{-1} = \begin{bmatrix} Y & H \\ N_1 & E \end{bmatrix}$$

and

$$J = \begin{bmatrix} Y & I_n \\ N_1 & 0 \end{bmatrix}, \quad J_1 = \text{diag}\{J, I, J, I\}$$

Denote

$$E = \hat{D}CY + \hat{C}N_1 \quad (3.30)$$

$$U = X^T(A + B_2\hat{D}C)Y + X^TB_2\hat{C}N_1 + M_1^T\hat{B}CY + M_1^T\hat{A}N_1 \quad (3.31)$$

$$W = X^TB_2\hat{D} + M_1^T\hat{B} \quad (3.32)$$

$$Z = Y^TX + N_1^TM_1. \quad (3.33)$$

Multiplying from the left and the right of (3.21) by J_1^T and J_1 respectively and applying (3.10), we obtain (3.25) where $\bar{Q} = [\bar{Q}_{ij}] = J^T Q J$ and $[\bar{\Sigma}_1^T \quad \bar{\Sigma}_2^T] = \Sigma^T J$.

By applying a similar procedure to (3.23), (3.26) can be obtained.

Further, if there exists a solution for the LMIs (3.25) and (3.26), it is easy to see that

$$\begin{bmatrix} Y + Y^T & I + Z \\ I + Z^T & X + X^T \end{bmatrix} > \bar{Q} > 0.$$

Multiply the above from the left by $[Y^{-T} \quad -I]$ and from the right by $[Y^{-T} \quad -I]^T$, we obtain that

$$(X - Y^{-T}Z) + (X - Y^{-T}Z)^T > 0.$$

It is then clear that $Z - Y^TX$ is invertible. Hence, there exist invertible matrices

M_1 and N_1 such that $Z - Y^T X = N_1^T M_1$. Thus, it follows from (3.30)-(3.33) that the controller parameters of (3.5)-(3.6) can be obtained as in (3.27)-(3.29).

Remark 3.2.5. *It is worth noting that when setting $\lambda_1 = \lambda_2 = \varepsilon_1 = \varepsilon_2 = 0$, $\lambda_3 = \varepsilon_3 = 1$ and $\bar{\Sigma}_1 = \bar{\Sigma}_2 = 0$, Theorem 3 will reduce to that of [59]. Hence, by exploring the freedoms offered by these parameters, a less or equally conservative mixed H_2/H_∞ design can be achieved. Certainly, the controller of [59] is derived by a convex optimization and can be further refined by an iterative procedure. In our disk drive application to be presented later, we demonstrate that our approach can give a much better performance than [59] together with an iterative refinement in the latter design.*

Remark 3.2.6. *Observe that for given $\lambda_i, \varepsilon_i, i = 1, 2, 3$, (3.25) and (3.26) are linear in $(\bar{P}_{11}, \bar{P}_{12}, \bar{P}_{22}, \bar{Q}_{11}, \bar{Q}_{12}, \bar{Q}_{22}, \bar{\Sigma}_1, \bar{\Sigma}_2, X, Y, \Pi, U, E, Z, W, \hat{D})$, and hence can be solved by employing the LMI Tool [70]. The problem of searching for the optimal scaling parameters $\lambda_i, \varepsilon_i, i = 1, 2, 3$ in general may be numerically costly although the function **fminsearch** in Matlab Optimization Toolbox may be applied.*

3.3 Application in Servo Loop Design for HDDs

3.3.1 Problem Formulation

A block-diagram representation of a typical HDD servo loop is shown in Fig. 3.1 with disturbances injected. $P(z)$ and $C(z)$ represent transfer functions of the plant and controller, respectively. d_i represents all torque disturbances. d_o represents disturbances that are due to non-repeatable disk and slider motions. n denotes the PES demodulation and measurement noise. z_1 is the true position error, and y or pes is the measured position error. D_i, D_o , and N are the disturbance and noise models, and w_1, w_2 and w_3 are white noises of zero mean and unit variance.

Figure 3.1: Mixed H_2/H_∞ control scheme for the HDD servo loop with disturbance models.

Through experiments, the frequency responses of the actual VCM is obtained and is shown in Fig. 3.2. A 5th order model is used to approximate the actual frequency responses of the VCM actuator and is given by

$$P(s) = \frac{5.172 \times 10^{12}s^2 + 1.82 \times 10^{17}s + 3.267 \times 10^{21}}{s^5 + 2.117 \times 10^4 s^4 + 1.032 \times 10^9 s^3 + 1.906 \times 10^{13} s^2 + 8.587 \times 10^{15} s + 7.345 \times 10^{18}} \quad (3.34)$$

Fig. 3.2 shows the comparison between the frequency responses of the actual data (rough lines) and those of $P(s)$ (smooth lines). It is clear that their difference is more significant for the frequency range of over 4 kHz. To capture the unmodelled dynamics in high frequencies, dozens of frequency response measurements are carried out and Fig. 3.3 shows the multiplicative uncertainty of the VCM actuator defined by

$$\Delta(\omega) = \max_{i=1,2,\dots,N_{mea}} \left| \frac{P_i(j\omega) - P(j\omega)}{P(j\omega)} \right| \quad (3.35)$$

where N_{mea} is the number of measurements and $P_i(j\omega)$ is the actual frequency response of the plant in the i -th measurement, and $P(j\omega)$ is the frequency response of the model in (3.34). An approximate bounding function $W_u(s)$, i.e. the smooth line in Fig. 3.3, is obtained as

$$W_u(s) = \frac{3s^2 + 2.903 \times 10^4 s + 1.433 \times 10^8}{s^2 + 3.016 \times 10^4 s + 1.421 \times 10^9}. \quad (3.36)$$

From Fig. 3.3, it is clear that the uncertainty at frequency over 5 kHz is the major concern. We observe that the actual uncertainties at some frequencies below 5 kHz exceed the bounding function, which, however, will not cause any major problem. In fact, we verify by simulation that the robust stability of our designed system is guaranteed even with the worst case of uncertainty.

Figure 3.2: Frequency responses of the VCM actuator.

By discretization using the zero-order hold, the corresponding z-domain models of the VCM and the bounding function, i.e., $P(z)$ and $W_u(z)$ can be obtained.

Based on the power spectrum of the measured PES in Fig. 3.4 and the disturbance modeling method in [47], the identified $D_i(z)$, $D_o(z)$ and $N(z)$ based on a 20 kHz sampling frequency are given by

$$D_i(z) = \frac{8.199 \times 10^{-5}z + 8.008 \times 10^{-5}}{z^2 - 1.931z + 0.9319}, \quad (3.37)$$

$$D_o(z) = \frac{0.07333z - 0.007806}{z - 0.8058}, \quad (3.38)$$

$$N(z) = \frac{8.58 \times 10^{-5}z^2 - 1.802 \times 10^{-5}z - 3.937 \times 10^{-5}}{z^3 - 1.958z^2 + 1.265z - 0.2702}. \quad (3.39)$$

Figure 3.3: Multiplicative uncertainty of the VCM actuator.

Figure 3.4: PES power spectrum (rough line) and its baseline curve (smooth line).

The models of $D_i(z)$, and $N(z)$ are obtained by modeling the baseline of the PES power spectrum. The spikes in the spectrum are then modeled by $D_o(z)$. However, for simplicity of the servo control design, a low order model of $D_o(z)$ is used.

As mentioned, one of the most important performance measures for HDDs is the track mis-registration or TMR, the total amount of random fluctuation about the desired track location. TMR is used to judge the required accuracy of positioning and thus to scale the disk capacity. To achieve a high capacity disk drive, one way in servo control is to minimize TMR, which is expressed as the standard deviation of the true PES, i.e.

$$3\sigma_{z_1} = 3\sqrt{\frac{1}{q-1} \sum_{i=1}^q z_1(i)^2} \quad (3.40)$$

where q is the number of true PES samples and z_1 is the position error signal of the HDD servo loop.

Let $\tilde{w} = [w_1 \ w_2 \ w_3]^T$ and $T_{z_1\tilde{w}}$ denote the transfer function matrix from $\tilde{w}$ to z_1 . When q is large enough, the H_2 norm of $T_{z_1\tilde{w}}$ is given by [69]

$$\|T_{z_1\tilde{w}}\|_2 = \sqrt{\frac{1}{q-1} \sum_{i=1}^q z_1(i)^2} \quad (3.41)$$

Thus, the control design problem to minimize TMR can be treated as an H_2 optimal control problem.

On the other hand, we need to ensure the system stability against the unmodelled high frequency dynamics of the VCM actuator, i.e. the constraint $\|TW_u\|_\infty < 1$ is to be met, where T is the closed-loop transfer function and W_u is the bounding function of the unmodelled dynamics which was derived earlier. Therefore, we have the mixed H_2/H_∞ control scheme as shown in Fig. 3.1, where $w_4 \in \ell_2[0, \infty)$, a disturbance input or as a reference is used to generate the closed-loop T for the H_∞

constraint. Clearly, the transfer function from w_4 to z_2 is TW_u .

We now derive the state-space representation (3.1)-(3.4) for the system in Fig. 3.1 with

$$x = \begin{bmatrix} x_p^T & x_{d_i}^T & x_{d_o}^T & x_n^T & x_u^T \end{bmatrix}^T, \quad (3.42)$$

$$w = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 \end{bmatrix}^T, \quad (3.43)$$

$$A = \begin{bmatrix} A_p & B_p C_{d_i} & 0 & 0 & 0 \\ 0 & A_{d_i} & 0 & 0 & 0 \\ 0 & 0 & A_{d_o} & 0 & 0 \\ 0 & 0 & 0 & A_n & 0 \\ B_u C_p & 0 & B_u C_{d_o} & 0 & A_u \end{bmatrix}, \quad (3.44)$$

$$B_1 = \begin{bmatrix} B_p D_{d_i} & 0 & 0 & 0 \\ B_{d_i} & 0 & 0 & 0 \\ 0 & B_{d_o} & 0 & 0 \\ 0 & 0 & B_n & 0 \\ 0 & B_u D_{d_o} & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} B_p \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.45)$$

$$C = \begin{bmatrix} -C_p & 0 & -C_{d_o} & C_n & 0 \end{bmatrix}, \quad (3.46)$$

$$D_1 = \begin{bmatrix} 0 & -D_{d_o} & D_n & 1 \end{bmatrix}, L_1 = \begin{bmatrix} -C_p & 0 & -C_{d_o} & 0 & 0 \end{bmatrix}, \quad (3.47)$$

$$H_{11} = \begin{bmatrix} 0 & -D_{d_o} & 0 & 0 \end{bmatrix}, H_{12} = 0 \quad (3.48)$$

$$L_2 = \begin{bmatrix} D_u C_p & 0 & D_u C_{d_o} & 0 & C_u \end{bmatrix}, \quad (3.49)$$

$$H_{21} = \begin{bmatrix} 0 & D_u D_{d_o} & 0 & 0 \end{bmatrix}, H_{22} = 0 \quad (3.50)$$

and x_p , x_{d_i} , x_{d_o} , x_n , and x_u are respectively the state variables of the VCM actuator $P(z)$, the input disturbance model $D_i(z)$, the output disturbance model $D_o(z)$, the measurement noise model $N(z)$, and the uncertainty $W_u(z)$. (A_p, B_p, C_p, D_p),

$(A_{d_i}, B_{d_i}, C_{d_i}, D_{d_i})$, $(A_{d_o}, B_{d_o}, C_{d_o}, D_{d_o})$, (A_n, B_n, C_n, D_n) and (A_u, B_u, C_u, D_u) are respectively the state-space models of $P(z)$, $D_i(z)$, $D_o(z)$, $N(z)$, and $W_u(z)$. Note that for the servo control shown in Fig. 3.1, $p = 4$, $r = 1$ and $D_p = 0$ for our VCM model.

3.3.2 Design Results

In this section, we will apply Theorem 3.2.1 to solve the mixed H_2/H_∞ control problem of hard disk drives as formulated previously. The sampling frequency being used is 20 kHz. By applying Theorem 3.2.1 and searching for the optimal scaling parameters, we obtain $\lambda_1 = 0.3$, $\lambda_2 = 0.31$, $\lambda_3 = 0.9$, $\varepsilon_1 = 0.3$, $\varepsilon_2 = 0.28$, $\varepsilon_3 = 1.1$ and the minimum H_2 norm, i.e., σ value of true PES z_1 , of $0.00748\mu m$. The parameters for the mixed H_2/H_∞ controller are given as follows

$$\begin{aligned} \text{Zeros} = & [-0.7749 \pm 0.5068i, -0.7749 \pm 0.5068i, 0.0006 \pm 0.9540i, \\ & 0.2590 \pm 0.7334i, 0.6699 \pm 0.4889i, 0.5348 \pm 0.4207i, \\ & 0.7754 \pm 0.2484i, 0.9694 \pm 0.0650i, 0.9752, 0.9394, \\ & 0.9114, 0.7834, 0.5996, 0.5752, 0.4728, 0.3897, 0.0314], \end{aligned} \quad (3.51)$$

$$\begin{aligned} \text{Poles} = & [-0.6252 \pm 0.4219i, -0.6252 \pm 0.4219i, 0.6137 \pm 0.4813i, \\ & 0.2587 \pm 0.3228i, 0.5469 \pm 0.1536i, 0.6172 \pm 0.0301i, \\ & 0.0640 \pm 0.0822i, -0.7248, -0.3718, 0.9937, 0.9654, 0.9653, \\ & 0.8058, 0.7835, 0.5577, 0.4092, 0.0389, 0], \end{aligned} \quad (3.52)$$

$$\text{Gain} = 3.1322. \quad (3.53)$$

For the purpose of comparison, we also design a mixed H_2/H_∞ controller for the disk drive using the approach in [59]. The minimum H_2 norm of $0.01013\mu m$ is obtained. Starting from this controller, a further iterative procedure between controller variables and Lyapunov parameters gives a controller that produces a slight

improvement of the H_2 norm, i.e., $0.01002\mu m$. Hence, our proposed approach represents about 25.3% improvement on TMR than the design method in [59] together with an iterative refinement.

Fig. 3.5 shows the comparison of sensitivity functions, where we can see the sensitivity function designed based on the proposed design method is better than that from [59], except that its hump is slightly higher. The comparison of control performances obtained by the proposed method and that of [59] is given as in Table 3.2, where the bandwidth from the proposed method is much higher. Although the H_∞ norm of TW_u of the proposed design is slightly higher than that of [59], it is below one as required, implying the designed controller makes the closed-loop system robustly stable in the presence of uncertainty bounded by W_u . Further, from Fig. 3.4, the disturbance mainly concentrates on frequencies below 1 kHz, hence, the slight higher peak does not degrade the disturbance rejection performance much. Fig. 3.6 shows the testing result of sensitivity functions, which is consistent with the simulation results in Fig. 3.5. Table 3.1 shows the designed and achieved H_2 and H_∞ norm of the system solved using Theorem 3.2.1. The designed and achieved H_2 norms have shown that Theorem 3.2.1 is a sufficient condition that solves the mixed H_2/H_∞ control problem. Note that the values in Table 3.1 are unscaled while the ones in Table 3.2 have been scaled according to our experimental setup.

Table 3.1: Designed and Achieved H_2 and H_∞ Performance

Criteria	Designed	Achieved
H_2 norm	0.3662	0.1193
H_∞ norm	1	0.89

Remark 3.3.1. From Table 3.2, one may argue that a reduced H_2 norm for the existing method of [59] may be obtained by a γ value of greater than one as the actual H_∞ norm is 0.78, lower than that of the proposed method. However, based on our simulations, no obvious improvement on the H_2 performance has been observed. For

Table 3.2: Comparison of Control Performance

Method	Proposed	Existing method	PID
Open loop crossover frequency (Hz)	1.3k	969	976
H_2 norm ($\sigma(\text{nm})$)	7.48	10.13	12.11
$\ TW_u\ _\infty$	0.89	0.78	0.27

example, with $\gamma = 1.5$, a slightly reduced H_2 norm of 10.11nm is obtained whereas the actual H_∞ norm is 0.79. With $\gamma = 5$, the H_2 norm is reduced to 10.08nm with an unchanged H_∞ norm of 0.78. This means that with a larger γ , the improvement on the H_2 norm for the existing method of [59] is negligible.

The proposed method is also compared with conventional PID design. As shown in Fig. 3.5 and Fig. 3.6, the sensitivity function by PID control has a lower bandwidth and an almost same peak value, which gives a higher TMR as listed in Table 3.2.

The previous simulation is carried out with $\bar{\Sigma}_1$ and $\bar{\Sigma}_2$ as solutions to the LMI (3.24). When (3,1) and (3,2) blocks in (3.24) are set as zeros, it is found that the designed controller gives a 9% lower bandwidth for the closed-loop system, leading to a worse TMR. This demonstrates that the parameter Σ in Lemma 3.2.1 is useful in achieving a better performance.

Next, we shall calculate the H_2 norm using the measured plant frequency response as in Fig. 3.2 and sensitivity function as in Fig. 3.6. The spectrum of z_1 is given by

$$\begin{aligned}
 |z_1(f_k)|^2 &= |P(f_k)S(f_k)|^2|D_i(f_k)|^2 + |S(f_k)|^2|D_o(f_k)|^2 \\
 &\quad + |1 - S(f_k)|^2|N(f_k)|^2
 \end{aligned} \tag{3.54}$$

where $f_k (k = 1, 2, 3, \dots, N')$ are frequency points, $P(f_k)$ represents the measured

Figure 3.5: Frequency response of sensitivity functions.

Figure 3.6: Frequency response of sensitivity functions.

frequency response of the plant, and $S(f_k)$ is the frequency response of the sensitivity function. The resultant σ value of z_1 is 0.01155, better than 0.02714 with the controller designed by the method in [59].

3.4 Conclusions

This chapter has presented a less or equally conservative design of mixed H_2 and H_∞ controllers in terms of LMIs that contain more free variables than existing LMI approaches. Those variables offer additional freedoms in optimization, resulting in a less or equally conservative control design. The proposed H_2/H_∞ control design has been applied to hard disk drives to minimize the track mis-registration while guaranteeing the system robustness in the presence of actuator uncertainties. Compared with an existing method, our design result for hard disk drives has indicated a marked improvement of 25.3% in the H_2 norm or the TMR performance while guaranteeing the system robustness by satisfying the required H_∞ constraint. The experimental result validates the advantage of the proposed design.

Chapter 4

Rejecting High Frequency Disturbances with Disturbance Observer and Phase Stabilized Control

For future HDD with high density, a higher data transfer rate is required. Therefore, the disk rotation speed will have to be increased. However, an increase in the disk rotation speed increases the amplitude of air-flow-induced high frequency vibrations such as windage and disk flutter and, hence, causes head position error. These vibrations are the main cause of TMR that prevents a high track density. Hence, a servo control design where high frequency disturbances are attenuated is required.

4.1 Introduction

As track density increases, higher control bandwidth is required to attain sufficient positioning accuracy while suppressing the effects of these high frequency disturbances of the system [66]. To obtain higher bandwidth and better vibration attenuation than notch filter servo control, phase stabilized compensation [65] has been applied to design a controller that takes into account both the gain and phase characteristics of the main mechanical resonances. The results are higher bandwidth, and less vibratory signals compared to lag compensator with notch filter servo control. On the other hand, a disturbance observer without using additional sensors to attenuate the effects of disturbances in the low frequency range from 10 Hz up to 500 Hz is designed in [71]. Soft sensors, such as observers, are attractive alternatives as they are highly cost effective [72].

Existing literature has shown that both methods, i.e., designing disturbance observer first and then followed by designing external controller [73] and vice versa [71, 74], are feasible. Here, the phase stabilized controller design is applied to the feedback control of a microactuator control system in order to fulfill the requirements of the sensitivity function, so that a high frequency disturbance can be attenuated. A disturbance observer is then added to the system to reject another high frequency disturbance and not to affect the stability of the closed loop. A high frequency bandpass filter is used in the disturbance observer, which is different from the existing disturbance observer method with low pass filter to attenuate low frequency disturbance [71]. It is shown that the proposed design is able to combine the benefits of phase stable feedback control and observer in suppressing high frequency disturbances. The contributions of this chapter can be clarified as follows:

- i. The disturbance observer structure with a high-frequency bandpass filter Q is used to reject narrow-band disturbance at the frequency higher than servo

bandwidth.

- ii. Narrow-band disturbances at two frequencies beyond servo bandwidth are rejected through the phase stabilized feedback controller combined with the disturbance observer.
- iii. Compared with the phase stabilized compensator, the combined control can lead to better stability margins.

This chapter is organized as follows: The high frequency disturbances rejection problem is formulated in Section 4.2. Section 4.3.1 describes the application of phase stabilized controller design in order to fulfill the bandwidth and sensitivity function requirements. Section 4.3.2 shows the design of Q filter for the disturbance observer method. Section 4.4 follows with the simulation and experiment results of the system with (1) phase stabilized controller, (2) phase stabilized controller with disturbance observer, and (3) PID controller with notch filters. The conclusions are presented in Section 4.5.

4.2 Problem Formulation

The control system for the microactuator loop is shown in Fig. 4.1 where r is the reference signal; C is the controller; P is the microactuator plant model; d_i is all torque disturbances and windage; d_o is disk and motor vibrations; and n is the demodulation and measurement noise of the position error signal (PES). PES is the relative position between the R/W head and the disk track.

Fig. 4.2 shows the power spectrum of the NRRO of a servo track writing system. Existing literature [48, 64] has identified the contributors to the PES. High frequency dominant disturbances at about 6 kHz and 12 kHz, induced by air turbulence to suspensions in hard disk drives (HDDs), can be clearly seen. It is important

Figure 4.1: Block diagram of microactuator plant with nominal feedback control system.

that disturbances at these frequencies, represented by d_o in Fig. 4.1, be attenuated to enhance the accuracy of the system during track following mode. For easier manipulation, d_o is lumped together with d_i to form equivalent input disturbance d_i .

Figure 4.2: Power spectrum of PES NRRO.

From Figure Fig. 4.1, it is easy to know that

$$PES = S(z)n - P(z)S(z)d_i - S(z)d_o \quad (4.1)$$

where $S(z) = 1/(1 + P(z)C(z))$ is the sensitivity function. This implies that the sensitivity function $S(z)$ is important in determining the disturbance attenuation of the servo control loop. For disturbance attenuation, it is desired that $|S(z)| < 1$. Based on NRRO power spectrum in Fig. 4.2, the servo control is to be designed such that $|S(z)| < 1$ for frequency ranges around 6 kHz and 12 kHz so that the disturbances can be attenuated.

4.3 High Frequency Disturbance Rejection

As seen from the measured frequency response data in Fig. 4.3 there are many resonance modes in the plant. However, only those that are of significant magnitude are modeled because those that have insignificant magnitude usually do not affect the closed-loop system stability. The identified plant model of the microactuator is a 12th-order model with a time delay term. The parameters for the identified transfer function of the microactuator in continuous-time are:

$$\begin{aligned} Zeros = 10^5 \cdot & [-0.4521 \pm 1.0359i, -0.1427 \pm 0.9407i, \\ & -0.0125 \pm 0.6923i, -0.0179 \pm 0.4712i, \\ & -0.0200 \pm 0.2844i, -0.0100 \pm 0.1991i]; \end{aligned} \quad (4.2)$$

$$\begin{aligned} Poles = 10^4 \cdot & [-0.9901 \pm 9.8510i, -0.1519 \pm 7.5950i, \\ & -0.0809 \pm 6.7423i, -0.1183 \pm 3.8130i, \\ & -0.1892 \pm 2.6968i, -0.1165 \pm 1.9378i]; \end{aligned} \quad (4.3)$$

$$Gain = 0.0658; \quad (4.4)$$

$$Delay = 1.5 \times 10^{-5} s. \quad (4.5)$$

The identified model is plotted against the measured frequency response as a comparison in Fig. 4.3.

Figure 4.3: Frequency response of microactuator (solid line: measured; dashed line: modeled).

4.3.1 Phase Stabilized Control Design

In general, the HDD servo loop is a unity feedback control system because only the PES can be measured. To reduce the effect of the disturbance at certain frequency, the sensitivity function of the servo loop must have sufficiently low magnitude at that frequency. Correspondingly, a high gain will appear in the open-loop transfer function of the servo system around the frequency [63].

To attenuate the high frequency disturbance at 12 kHz, we have to design a controller such that the magnitude of the sensitivity function is less than 0 dB at 12 kHz. To satisfy this specification, a phase-stabilized controller [65] is applied, which consists of a PID controller, notch filters, a peak filter and a low pass filter given by

$$C_{ps} = C_{pid} \times C_{notch\ filters} \times C_{peak\ filter} \times C_{lowpass\ filter} \quad (4.6)$$

where,

$$C_{pid} = K_p + K_i \frac{1}{s} + K_d s \quad (4.7)$$

$$C_{notch\ filters} = \prod_{m=1}^6 \frac{s^2 + 2\zeta_{z_m}\omega_{z_m}s + \omega_{z_m}^2}{s^2 + 2\zeta_{p_m}\omega_{p_m}s + \omega_{p_m}^2} \quad (4.8)$$

$$C_{peak\ filter} = \frac{s^2 + 2\zeta_{pf1}\omega_{pf1}s + \omega_{pf1}^2}{s^2 + 2\zeta_{pf2}\omega_{pf2}s + \omega_{pf2}^2} \quad (4.9)$$

$$C_{lowpass\ filter} = \frac{1}{Ts + 1} \quad (4.10)$$

The resonance peaks of the plant model are first suppressed with notch filters and then, a PID controller is designed to obtain an open loop bandwidth around 2 kHz. After that, a 12 kHz peak filter and a low pass filter are added. The former is to obtain a peak in the open loop frequency response, resulting in a notch at the gain of the sensitivity function around 12 kHz, while the latter is to increase the phase delay so that the second phase margin of the open loop is secured within -360 ± 90 degrees. The second phase margin is defined as the angle from the negative real axis to a vector from the origin O to point M , where the Nyquist plot passes through a unit circle from the inside to the outside [65]. Firstly, the resonance modes of the plant are compensated by $C_{notch\ filters}$, where the parameters of the notch filters are determined by inverting the pole-zero pairs of the model of the microactuator in (4.2)-(4.3), since all zeros of the microactuator are in the left half-plane. As a result, the rest of the parameters are designed as $K_p = 2.25$, $K_i = 5.86 \times 10^4$, $K_d = 1.5 \times 10^{-5}$, $\omega_{pf1} = 2\pi 15000$, $\omega_{pf2} = 2\pi 12000$, $\zeta_{pf1} = 0.1$, $\zeta_{pf2} = 0.02$, $T = 1/(2\pi 2000)$. Then, C_{ps} is discretized to obtain a discrete-time controller with a sampling frequency of 40 kHz.

The frequency responses of the phase stabilized controller, open loop, and sensitivity transfer function are shown in Fig. 4.4, 4.7 and 4.8, respectively. For comparison purposes, the sensitivity function of the system with phase stabilized

Figure 4.4: Frequency response of phase stabilized controller.

controller is plotted together with the sensitivity function of the system with a PID controller and notch filters that cancel the resonance peaks of the microactuator plant in Fig. 4.8. With phase stabilized controller, the magnitude of the sensitivity function is decreased from 1.2 dB to -4.8 dB around 12 kHz. However, the magnitude reduction around 6 kHz is insufficient because the magnitude of the transfer function from d_i to the PES is still greater than 0 dB, as seen in Fig. 4.9. Thus, next we will design a disturbance observer to increase the attenuation of the disturbance with frequencies around 6 kHz.

4.3.2 Disturbance Observer Design

In [71], a disturbance observer is designed to attenuate disturbances in the low frequency range from 10 Hz to 500 Hz. However, as stated in Section 4.2 on the basis of the PES power spectrum in Fig. 4.2, it is important to consider disturbances in higher frequency range. In Section 4.3.1, the phase stabilized controller can only reject disturbances with frequencies between 100 Hz to 1.9 kHz and 11 kHz to 14

kHz. The objective of the disturbance observer design is to make the magnitude of the transfer function from the input disturbance to the PES around frequency 6 kHz to be less than 0 dB so that the disturbances around this frequency can be attenuated.

Fig. 4.5 shows the block diagram of the disturbance observer structure, where C_{ps} is the previously designed phase stabilized controller; P_n is the nominal model of P ; $\hat{d}_i$ is an estimate of d_i . Ignoring the nominal feedback loop with C_{ps} , the transfer functions from the disturbance d_i and noise n to the true PES are given respectively by

$$G_{d_i\varepsilon} = \frac{PP_n(1 - Qz^{-d})}{P_n + Q(P - P_nz^{-d})} \quad (4.11)$$

$$G_{n\varepsilon} = \frac{-PQ}{P_n + Q(P - P_nz^{-d})} \quad (4.12)$$

Note that the nominal feedback loop with C_{ps} can be ignored because the total disturbance attenuation of the two loops is essentially the addition of the magnitudes of both in dB when the nominal model of the plant P_n is very similar to the plant P . Therefore the overall transfer function of the disturbance d_i to the true PES ε is equals to $G_{d_i\varepsilon} \times S_{nom}$ where S_{nom} is the sensitivity function of the nominal loop with C_{ps} .

To reject the disturbance d_i , Q can be set as unity because $G_{d_i\varepsilon} \approx 0$ when the delay is negligible. However, when $Q = 1$, $G_{n\varepsilon} \approx -1$ which means that the measurement noise n is not attenuated. Thus to eliminate the noise effect, it is known from (4.12) that an ideal solution of Q is zero, but this will mean $G_{d_i\varepsilon} \approx P$ and hence the disturbance d_i will be amplified.

Considering the overall system in Fig. 4.5, the sensitivity function is given by

$$S(z) = \frac{1 - Qz^{-d}}{1 - Qz^{-d} + PC_{ps} + PQP_n^{-1}}. \quad (4.13)$$

Figure 4.5: Block diagram with disturbance observer.

Theoretically, the microactuator plant model is $P = P_n \cdot z^{-d}$, thus (4.13) becomes $S(z) = (1 - Qz^{-d}) / (1 + PC_{ps})$ which is stable as long as Q is stable and the loop is stable before the disturbance observer is added on. Moreover, it is deduced from (4.13) that when $Qz^{-d} = 1$ with zero phase around the disturbance frequency, the disturbance can be rejected because $S(z) \rightarrow 0$.

Since the targeted disturbance frequency is around 6 kHz and it is also known that the magnitude of $z^{-d} = 1$ for all frequencies, Q filter must be designed such that it has a magnitude close to 1 around 6 kHz in order to obtain $|Qz^{-d}| \approx 1$ at 6 kHz. On the other hand, the phase of Q has to be the opposite of the phase of z^{-d} so that the phase of $Qz^{-d} \approx 0^\circ$. Because z^{-d} has decreasing phase, Q must have increasing phase. Therefore, a low pass filter that is used in conventional disturbance observer cannot be used. Since it is desired that the magnitude of the sensitivity function around other frequency region is not changed too much, Q is designed as a bandpass filter. A second order bandpass filter is chosen so that the overall controller has as low order as possible. Hence, Q is initially designed as a second order bandpass filter with a pass band centered at 6 kHz. Then, Q is adjusted until Qz^{-d} has a magnitude close to 1 and phase close to 0° . Q is finally designed having a pass band

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from 6 kHz to 6.7 kHz and is given by

$$Q = K \frac{0.05216z^2 - 0.05216}{z^2 - 1.03z + 0.8957} \quad (4.14)$$

where K is used to adjust the gain of Q . The frequency responses of Q , z^{-d} and Qz^{-d} are plotted in Fig. 4.6. With the designed Q , $|S(z)| < 1$ around 6 kHz, as shown later in Fig. 4.8.

Figure 4.6: Bode plot of Q , z^{-d} and Qz^{-d} .

Moreover, from Fig. 4.5 the transfer function from the disturbance to the output is derived as

$$H_{d_i\varepsilon} = \frac{\varepsilon}{d_i} = \frac{P(1 - Qz^{-d})}{1 - Qz^{-d} + PC_{ps} + PQP_n^{-1}} = S(z) \cdot P(z) \quad (4.15)$$

Note that there is a dominant peak at 6 kHz in the plant model in Fig. 4.3. It can be seen from (4.1) that the disturbance can only be attenuated around 6 kHz if $|S(z) \cdot P(z)| < 1$. Thus Q must be designed so that $|S(z)|$ is low enough in order to meet this requirement.

This method differs from the peak filter method [75], because the frequency response of the sensitivity function is shaped indirectly when the filter Q is designed. When the disturbance observer is designed at the frequency three times higher than the crossover frequency, it may cause instability to the overall system. Hence, the pass band of Q has to be chosen carefully, in order to maintain the stability of the overall system when satisfying the requirements of the sensitivity function.

4.4 Simulation and Experiment Results

4.4.1 Simulation Results

With the previously designed phase stabilized control, the open loop frequency responses of the overall system with and without the disturbance observer are shown in Fig. 4.7. It shows that the stability of the closed loop is maintained when the disturbance observer is added. The control performances are summarized and compared in Table 4.1. The control scheme with disturbance observer gives better gain and phase margins, while the servo bandwidths are similar.

Table 4.1: Comparison of Control Performances

Control scheme	Open loop crossover frequency (kHz)	Gain margin (dB)	Phase margin (deg)
Phase stabilized control (for 12 kHz disturbance attenuation)	1.96	5.7	64.9
Phase stabilized control + Disturbance observer	1.93	5.8	66.2
Phase stabilized control (for 6 kHz and 12 kHz disturbance attenuation)	1.98	5.0	44.8

Figure 4.7: Open loop frequency response of the system (solid line with two peaks at 6 and 12 kHz: phase stabilized controller and disturbance observer; dotted line with single peak at 12 kHz: phase stabilized controller).

Figure 4.8: Simulated frequency response of sensitivity function (solid line: phase stabilized control without disturbance observer; dashed line: phase stabilized control with disturbance observer; dash-dotted line: PID control and notch filters).

Fig. 4.8 shows the simulated frequency responses of the sensitivity function (4.13) for the control system with the phase stabilized controller with and without the disturbance observer, and the PID controller with notch filters. It is observed that with the disturbance observer, the magnitude of the sensitivity function is lower than 0 dB around 6 kHz. However, it is increased slightly between 6.5 kHz and 10 kHz. The magnitude of the transfer function (4.15) from the disturbance to the PES is shown in Fig. 4.9, where we can clearly see that the magnitude is reduced considerably from 1.3 dB to -3.8 dB around 6 kHz when the disturbance observer is added-on. Although at 12 kHz, the magnitude is slightly higher than that without disturbance observer, it is still much lower than using the PID control with notch filters.

Figure 4.9: Simulated frequency response from the disturbance d_i to the PES.

Remark 4.4.1. *The phase stabilized compensator alone is able to attenuate both disturbances at 6 kHz and 12 kHz. However, as shown in the Table 4.1 and Fig. 4.10, it leads to much lower stability margins with similar bandwidth.*

Figure 4.10: Open loop frequency response of the system with phase stabilized compensator to attenuate both 6 kHz and 12 kHz disturbances.

4.4.2 Experimental Results

Unlike the simulation, during real implementation, the actual micro actuator plant $P \neq P_n z^{-d}$. K in Q has to be tuned increasingly from zero. It turns out that when $K = 0.4$, the system remains stable and is able to attenuate disturbance around 6 kHz. When K reaches one, the system becomes unstable. Note that the saturation of the microactuator plant in the experiment is ± 30 V.

Fig. 4.11 and Fig. 4.12 show that the measured sensitivity functions obtained from experiments are consistent with the simulations. Fig. 4.13 and 4.14 are the experimental results of the system output when an input disturbance is injected into the system. It can be clearly seen in Fig. 4.13 that the disturbance observer can reduce the error significantly for a sinusoidal disturbance with 6 kHz. Compared to the system with PID control and notch filters, the amplitude of the error is improved by 25.0% for the system with phase stabilized control and disturbance observer. For a 12 kHz sinusoidal input disturbance, as shown in Fig. 4.14 the amplitude of the error signal with the phase stabilized controller and disturbance observer is similar to

that with only the phase stabilized controller. However, it is much better compared to the PID control with notch filters and a 32.6% improvement is achieved.

In contrast to the spectrum in Fig. 4.2 without servo control, Fig. 4.15 shows the power spectrum of the error with PID controller with notch filters, phase stabilized controller with and without disturbance observer, respectively. Table 4.2 shows the improvement percentage for each control scheme for comparison. The proposed phase stabilized controller with disturbance observer shows the highest improvement compared to the other schemes.

Figure 4.11: Sensitivity function of the system with phase stabilized control.

Table 4.2: Comparison of PES 3σ and Improvement Percentage Compared to the Case Without Servo Control

Control scheme	3σ (nm)	Improvement (%)
PID with notch filters	10.23	29
Phase stabilized control	9.39	35
Phase stabilized control + Disturbance observer	9.09	37

Figure 4.12: Sensitivity function of the system with phase stabilized control and disturbance observer.

Figure 4.13: Position error when a 6 kHz sinusoidal input disturbance is injected.

Figure 4.14: Position error when a 12 kHz sinusoidal input disturbance is injected.

Figure 4.15: Comparison of NRRO power spectrum of the system with (a) PID + notch filters, (b) phase stabilized controller, and (c) phase stabilized controller + disturbance observer.

Fig. 4.16 shows the transient responses of the system with a 250 mV step reference. It has 27% overshoot and the settling time is 0.8 ms. It should be mentioned that in order to achieve more disturbance rejection, higher servo bandwidth is needed. However, in the time domain high servo bandwidth will cause high overshoot in step response. Here in this chapter, the PID control is designed so that a 2 kHz bandwidth is achieved. Thus such a high overshoot appears inevitably.

The robustness of the phase stabilized controller with disturbance observer against changes of parameters in the actual microactuator plant is evaluated by simulation and illustrated in Fig. 4.17. This is obtained by varying the damping ratios and resonance frequencies of the plant model at 6 kHz and 12 kHz while using the phase stabilized controller with disturbance observer that was designed earlier to obtain the standard deviation of the error signal. It is almost maintained for 5% variation of the resonance frequencies at 6 and 12 kHz, and insensitive towards variations of the damping ratios.

Figure 4.16: Step response of the system with phase stable control and disturbance observer.

Figure 4.17: Simulated results for performance robustness against variation of the 6 kHz and 12 kHz resonance modes of the microactuator.

4.5 Conclusion

The phase-stabilized control combined with the disturbance observer has been used to achieve the sensitivity function requirements such that the narrow band disturbances at two frequencies higher than the servo bandwidth can be attenuated. In the disturbance observer design, the high frequency bandpass filter Q has been used in order to reject the high frequency disturbance, and simultaneously the stability of the whole system has been maintained when the disturbance observer is added on to the pre-designed phase stabilized control. Therefore, without the need of redesigning the external phase stabilized feedback controller, a better disturbance rejection has been accomplished and ultimately the positioning accuracy has been improved by 37%. Additionally, it has been illustrated that the proposed control scheme has given better stability margins than the phase stabilized compensator.

Chapter 5

Nonlinear Least-Squares Optimization of Sensitivity Function for Disturbance Attenuation

In the previous chapters, it is known that the disturbance rejection/attenuation properties of a closed loop system is determined by the sensitivity function. Thus, in this chapter, we shall demonstrate a sensitivity shaping method by optimizing an initial sensitivity function to obtain the desired sensitivity function.

5.1 Introduction

Nagamune and Blomqvist [76] has developed a very effective method to shape the sensitivity function by optimizing the weighted sum of squares of the distances between the ‘desired’ sensitivity function and the ‘best approximate’ sensitivity func-

tion of the overall system. Their methodology often generates controllers of lower orders compared to the conventional H_∞ method.

In this chapter, the nonlinear least-squares optimization (NLSO) method [76] is applied in sensitivity function shaping to design a feedback controller for a lead-zirconate-titanate (PZT) microactuator, which is to be facilitated as a secondary actuator in a servo track writer to enhance its positioning accuracy. Based on the disturbance distribution information known from NRRO spectrum, a few specifications of the targeted sensitivity function are determined at first in order to reduce the contribution from the disturbances on the position error. The NLSO method is then applied to design a closed-loop control system of the microactuator to accomplish the specifications. Simulation and experimental results demonstrate the effectiveness of this sensitivity function shaping method. Additionally, the designed controller is investigated in terms of robustness of disturbance attenuation capability against frequency variation of the disturbances, which will happen due to different spindle rotational speed such as in variable speed hard disk drives [77].

5.2 Problem Description and Sensitivity Function Shaping by NLSO Method

5.2.1 Problem Description

Equation (4.1) implies that the sensitivity function $S(z)$ is important in determining the disturbance attenuation of a servo control loop. Therefore, the design of a servo control $C(z)$ that gives rise to a sensitivity function which can attenuate disturbances with known frequencies becomes rather significant. More specifically, a servo control

5.2 Problem Description and Sensitivity Function Shaping by NLSO Method 81

$C(z)$ is to be designed such that for the frequency ranges $f_{i1} < f_i < f_{i2}, i = 1, \dots, q$,

$$|S(f_i)| < r_i, \quad (5.1)$$

are satisfied, where $S(f) = 1/(1 + C(f)P(f))$ and r_i is required to be smaller so that more rejection or less amplification to the disturbances within $[f_{i1}, f_{i2}]$ can be achieved. Next, the nonlinear least-squares optimization (NLSO) method proposed in [76] will be applied in the control design to meet a few specifications with the description in (5.1) for the resultant sensitivity function.

5.2.2 Sensitivity Function Shaping by NLSO Method

Define the weighted sum of squares of the distances between the ‘desired’ and ‘best-approximate’ of the sensitivity function as

$$d_w((\theta, \mathbf{s}), S) := \frac{1}{2} \sum_{k=1}^N \frac{w_k}{|s_k|^2} |S(\theta_k) - s_k|^2 \quad (5.2)$$

where $\theta := \{\theta_k\}_{k=1}^N$ is a vector of selected frequency points, $\mathbf{s} := \{s_k\}_{k=1}^N \subset \mathcal{C}$ is the frequency response of the ‘desired’ sensitivity function with the set $\mathcal{C}$ of complex numbers, $S(\theta_k)$ is the resultant ‘best-approximate’ sensitivity function, and $\mathbf{w} := \{w_k\}_{k=1}^N$ are positive scalar weights to be chosen by the designer. If better approximation is required for frequency θ_k , w_k has to be larger, relatively to weights at other frequencies.

The ‘best approximate’ sensitivity function S has to fulfill the following conditions [76]:

- i. internal stability condition;
- ii. n_e conditions $S(\lambda_j) = \eta_j, j = 1, \dots, n_e$, which are specified at points $\lambda_j \in \mathcal{C}$

outside the unit disc;

- iii. H_∞ norm bound condition $\|S\|_\infty < \gamma$ for some given $\gamma > 0$;
- iv. rationality and a degree condition, i.e., S must be real rational and $\deg S \leq \tau := n_p + n_z + n_e - 1$, where n_p and n_z are the number of unstable poles and zeros of plant $P(z)$, respectively.

Hence, the sensitivity function shaping problem is converted to a nonlinear least-squares optimization one [76]:

$$\inf_{S \in \ell} d_w((\theta, \mathbf{s}), S) \quad (5.3)$$

where $\ell := \{S : S \text{ satisfies conditions (i)-(iv)}\}$ is the class of allowable sensitivity functions.

Since S satisfies condition (iv), it can be factored as $S(z) = b(z)/a(z)$, where $a(z) := \mathbf{z}^T \alpha$, $b(z) := \mathbf{z}^T K \alpha$, $\alpha \in \mathcal{R}^{n+1}$, $z = [z^n, \dots, z, 1]^T$ and K is a uniquely determined real matrix. The construction of matrix K is shown in Appendix A of the technical report [78] and reproduced in the Appendix A of this thesis. Stability condition states that the denominator vector α needs to be in the Schur stability region:

$$\Psi := \left\{ \begin{array}{l} \alpha := [\alpha_0, \dots, \alpha_n]^T \in \mathcal{R}^{n+1} : \alpha_0 > 0 \\ \mathbf{z}^T \alpha \neq 0, \forall z \in \mathcal{D}^c \end{array} \right\} \quad (5.4)$$

where, $\mathcal{D}^c := \{z \in \mathcal{C} : |z| \geq 1\}$. Since S also satisfies (iii), there exists a spectral factorization for

$$\gamma^2 a(z)a(z^{-1}) - b(z)b(z^{-1}) = \rho(z)\rho(z^{-1}) \quad (5.5)$$

for a unique spectral factor $\rho(z) := \mathbf{z}^T \rho$ with $\rho \in \Psi$. Therefore, the optimization

5.2 Problem Description and Sensitivity Function Shaping by NLSO Method 83

of (5.3) can be converted into the following finite-dimensional constrained nonlinear least-squares problem,

$$\inf_{\rho \in \Psi} \frac{1}{2} \sum_{k=1}^N \frac{w_k}{|s_k|^2} \left| \frac{\mathbf{e}_k^T K \mathbf{h}(\rho)}{\mathbf{e}_k^T \mathbf{h}(\rho)} - s_k \right|^2 \quad (5.6)$$

where, $\mathbf{e}_k := \mathbf{e}(\theta_k) = [e^{in\theta_k}, e^{i(n-1)\theta_k}, \dots, 1]^T$, and $\mathbf{h}$ is a diffeomorphism which maps ρ to α .

The finite-dimensional constrained nonlinear least-squares optimization problem (5.6) can be rewritten as

$$\inf_{\rho \in \Psi} \frac{1}{2} \mathbf{F}(\rho)^T \mathbf{F}(\rho) \quad (5.7)$$

where, $\mathbf{F} : \Psi \mapsto \mathcal{R}^{2N}$ is the vector-valued residual map

$$\mathbf{F}(\rho) := [\operatorname{Re}\{f_1(\rho)\}, \dots, \operatorname{Re}\{f_N(\rho)\}, \\ \operatorname{Im}\{f_1(\rho)\}, \dots, \operatorname{Im}\{f_N(\rho)\}]^T,$$

$$f_k(\rho) := \frac{\sqrt{w_k}}{|s_k|} \left(\frac{\mathbf{e}_k^T K \mathbf{h}(\rho)}{\mathbf{e}_k^T \mathbf{h}(\rho)} - s_k \right), k = 1, \dots, N. \quad (5.8)$$

Then, (5.7) can be solved by derivative-based algorithms [78]. However, non-convexity of derivative-based algorithms means that the solution will not converge to a global minimizer unless it is initialized properly. Hence, the *approximate peak solution* is proposed by [76] as an initial point.

Remark 5.2.1. *The construction of matrix K in (5.8) is detailed in Appendix A in [78]. During the optimization, given an initial value of ρ , K is updated iteratively until the performance (5.7) is minimized. With the final matrix K and thus $S(z) = \mathbf{z}^T K \alpha / \mathbf{z}^T \alpha$, the controller is calculated from $C(z) = (1 - S(z)) / (P(z)S(z))$. The Matlab based software named as “Sshaper” [79] is available to perform the*

optimization and yield the controller $C(z)$ ultimately.

To summarize, the NLSO controller $C(z)$ can be designed by the following procedure for plant $P(z)$ such that the specifications in (5.1) imposed on the sensitivity function $S(z)$ are fulfilled.

- i. Obtain the plant model $P(z)$.
- ii. Determine the specifications of $S(z)$ to be fulfilled.
- iii. Select an initial approximate sensitivity function, $S(z)$, that we can optimize on. For example, in this chapter, the sensitivity function from a proportional-integral-derivative (PID) controller in cascade with notch filters, is used as the initial sensitivity function.
- iv. Select desired frequency range and generate N frequency points θ_k .
- v. Generate the corresponding complex values of the sensitivity function $S(z)$ using the *freqresp* function in *Matlab* based on θ_k and let this be $\mathbf{s}_k$.
- vi. Use “Sshaper” Matlab based computational tool [79] to optimize and design a controller.
- vii. If the sensitivity function obtained does not meet all specifications (5.1), add additional constraints and/or adjust the complex values of $S(z)$ to meet the specifications, and let the new generated complex values be the ‘desired’ sensitivity function, $\mathbf{s}_k$.
- viii. Repeat step (vi) and (vii) until all sensitivity function specifications (5.1) are met.

5.3 Application Results for a Microactuator

In this section, the NLSO $S(z)$ shaping method will be used in the control design for the microactuator in a servo track writing platform for hard disk drives. The microactuator, as a secondary actuator, is attached to the arm of a commercial high-precision MicroE [80] which is controlled by its internal PID controller and considered rigid enough, and thus the positioning error is suppressed mainly through the microactuator servo loop. Fig. 5.1 shows the block diagram of the STW servo loop, where $P_{MicroE}(s)$ and $C_{MicroE}(z)$ are the MicroE plant and controller, and $P(s)$ and $C(z)$ are the microactuator plant and controller. The transfer function of the primary actuator, $P_{MicroE}(s)$ is:

$$P_{MicroE}(s) = 3.4 \times 10^6 \frac{(s^2 + 3.519 \times 10^4 s + 6.317 \times 10^8)(s^2 + 1.96 \times 10^4 s + 2.402 \times 10^9)}{(s^2 + 131.9s + 4.836 \times 10^4)(s^2 + 1508s + 3.553 \times 10^8)(s^2 + 816.8s + 1.668 \times 10^9)} \quad (5.9)$$

$C_{MicroE}(z)$ with sampling time of $49.6 \mu\text{sec}$ consists of notch filters that pre-compensate the resonance modes at 3 kHz and 6.5 kHz and a PID controller, where $K_p = 0.0044$, $K_i = 3.44 \times 10^{-4}$ and $K_d = 0.0025$.

Figure 5.1: Block diagram of servo track writing platform for hard disk drives.

The frequency response of the PZT microactuator is shown in Fig. 5.2, where the main resonance modes of the plant are at frequencies 7.0 kHz, 9.2 kHz, 11.1 kHz,

and 20 kHz. The identified plant model of the microactuator, $P(s)$ has the following parameters:

$$\begin{aligned} Zeros = 10^5 \cdot [-0.0213 \pm 1.0648i, -0.6505 \pm 0.6636i, \\ -0.0060 \pm 0.6032i, -0.0173 \pm 0.4932i]; \end{aligned} \quad (5.10)$$

$$\begin{aligned} Poles = 10^5 \cdot [-0.0451 \pm 1.2891i, -0.0105 \pm 0.7005i, \\ -0.0030 \pm 0.5812i, -0.0176 \pm 0.4395i]; \end{aligned} \quad (5.11)$$

$$Gain = 0.6147; \quad (5.12)$$

$$Delay = 20\mu s \quad (5.13)$$

The frequency response of the plant model is plotted against the measured data in Fig. 5.2 for comparison. $P(z)$ is obtained by discretizing $P(s)$ with ‘zoh’ method and 40 kHz sampling frequency.

Figure 5.2: Frequency response of PZT microactuator.

The NRRO power spectrum of PES signal of our current STW platform at 5400 RPM with no servo control is shown in Fig. 5.3. It reflects the disturbance and noise information in the platform, and is similar to that in real hard disk drives.

In Fig. 5.3, disturbances in low-frequency range up to 200 Hz are mostly due to air-flow induced and external vibrations, 650 Hz mode is from disk vibration, and 3.8 kHz mode is excited by air turbulence to suspension.

Figure 5.3: NRRO power spectrum without servo control (calculated from measured PES signals ($3\sigma = 21$ nm)).

5.3.1 Design and Simulation Results

For the design of $C(z)$, the specifications for the resultant sensitivity function $S(z)$ are described at first:

- i. $|S(f)| < 0$ dB, $f \leq 1$ kHz,
- ii. $|S(f)| < -12$ dB, 610 Hz $\leq f \leq 670$ Hz,
- iii. $|S(f)| < 6$ dB, $f \geq 19$ kHz.

The first specification means that the crossover frequency of the sensitivity function is 1 kHz at least, and the open loop crossover frequency is known to be

nearby. Accordingly to the disk vibration mode at 650 Hz in Fig. 5.3, the second specification is set to particularly reject disturbances around 650 Hz further. The third specification limits the maximum gain of the sensitivity function to restrict the amplification to disturbances of frequencies above system bandwidth.

We design a PID controller for the microactuator (5.10)-(5.13) before using the NLSO method to optimize. The transfer function of the designed PID controller is given by,

$$C_{pid}(z) = \frac{0.3235z^2 - 0.121z + 0.06}{z^2 - z} \quad (5.14)$$

In addition to the PID controller, notch filters need to be used to compensate the resonance modes of the microactuator at 7.0 kHz, 9.2 kHz, 11.1 kHz, and 20 kHz. The transfer function of the overall notch filter is as follows.

$$C_{notch}(z) = \frac{0.1711z^8 + 0.1756z^7 + 0.3631z^6 + 0.473z^5 + 0.3747z^4 + 0.4604z^3 + 0.2776z^2 + 0.1651z + 0.1169}{z^8 - 0.005765z^7 + 0.8289z^6 + 0.2373z^5 + 0.2455z^4 + 0.1781z^3 + 0.04751z^2 + 0.03563z + 0.01044} \quad (5.15)$$

The frequency response of $C(z) = C_{pid}(z) \cdot C_{notch}(z)$ is shown in Fig. 5.5. With the PID controller and notch filters, the open loop frequency response is plotted in Fig. 5.6 with 1.6 kHz crossover frequency. The sensitivity function

$$S(z) = 1/(1 + C_{pid}(z)C_{notch}(z)P(z)) \quad (5.16)$$

is shown in Fig. 5.7 with the peak below 6 dB. However, it is clearly observed in Fig. 5.7 that the sensitivity function does not meet the second specification.

Hence, in order to further attenuate the disturbances around 650 Hz, we shall use the NLSO method to optimize the controller by following the steps given in Section 5.2.2. 100 logarithmically equally spaced points between frequency 10 Hz to 20 kHz are generated for $\theta_{\mathbf{k}}$. Then, we generate equivalent ‘desired sensitivity’ points using the sensitivity function (5.16).

Remark 5.3.1. *Here in order to save computation memory and time, 100 points are selected in the optimization, and the ultimately designed controller is able to meet the specifications and will be verified implementable. Certainly more points can be added if there is no restriction in computation.*

Since the second specification for the desired $|S(z)|$ from 610 Hz to 670 Hz is not satisfied by $C(z) = C_{pid}(z) \cdot C_{notch}(z)$, we adjust the complex value of $S(670\text{Hz})$ as a constraint in the NLSO optimization. As seen in Fig. 5.4, the magnitude needs to be lowered to meet specification (ii) and correspondingly the phase has to be lifted. Thus the magnitude is fixed at -12 dB, and the phase is determined to be 87deg to ensure enough phase margin. Another two constraints are also listed in Table 5.1 and similar to the value from the previous design, as seen in Fig. 5.4. The constraint at 10 Hz is to keep the magnitude of the sensitivity function low at low frequency portion. The one at 9400 Hz is to ensure that the magnitude does not grow too much as it is lowered around 650 Hz. As a result, a 14th-order controller is obtained as

$$C_{nlso}(z) = \frac{0.1344z^{14} - 0.1146z^{13} + 0.2754z^{12} - 0.1217z^{11} + 0.01645z^{10} + 0.1339z^9 - 0.2966z^8 + 0.1762z^7 - 0.1478z^6 - 0.03538z^5 + 0.07733z^4 - 0.07912z^3 + 0.05159z^2 - 0.01664z}{z^{14} - 1.804z^{13} + 2.047z^{12} - 1.735z^{11} - 0.01969z^{10} + 0.584z^9 - 0.6682z^8 + 0.4844z^7 + 0.1232z^6 + 0.2522z^5 - 0.2974z^4 + 0.207z^3 - 0.1877z^2 + 0.01448z + 0.0005602} \quad (5.17)$$

The frequency response of the NLSO controller is then plotted with comparison to the PID controller in Fig. 5.5.

Figure 5.4: Sensitivity function from PID and notch filters design and additional constraints.

Table 5.1: Additional Constraints for Optimization (Values in the brackets are original values from $C_{pid}(z)C_{notch}(z)$ design)

Point	1	2	3
Frequency (Hz)	10	670	9400
Magnitude (dB)	-44 (-43)	-12 (-8)	0.7 (0)
Phase (deg)	60 (57.5)	87 (79)	0 (0)

The frequency responses of the open loop ($P(z)C_{nlso}(z)$) and ($P(z)C_{pid}(z)C_{notch}(z)$) are shown in Fig. 5.6. The control performance comparison is tabulated in Table 5.2. With the NLSO controller, the bandwidth becomes higher, and the gain margin is also higher, while the phase margin is lowered to 46 degrees, which is due to the tradeoff between the phase and gain margins of the system.

From Fig. 5.7, the sensitivity function shows better disturbance attenuation at frequencies between 10 Hz and 1.4 kHz, and between 2.5 kHz to 5.5 kHz after

5.3 Application Results for a Microactuator

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Figure 5.5: Frequency response of controller (solid line: NLSO method; dashed line: PID control with notch filters).

Figure 5.6: Frequency response of open loop from simulation (solid line: NLSO method; dashed line: PID control with notch filters).

Table 5.2: Control Performance of PID Controller with Notch Filters and NLSO Controller

Controller	$C_{pid}(z).C_{notch}(z)$	$C_{nlso}(z)$
Open loop bandwidth (Hz)	1660	1820
Gain margin (dB)	9.4	15.4
Phase margin (deg)	60	46

fine tuning the sensitivity function from the PID controller with notch filters using the NLSO method. The magnitude of the sensitivity function around 650 Hz is decreased from -8 dB to -12 dB when the NLSO method is used. This means that the disturbance due to the disk vibration, which is around 650 Hz can be better attenuated. The magnitude above 6 kHz is increased, which is attributed to the higher magnitude 0.7 of the constraint at 9400 Hz in Table I. This also correspondingly causes the higher magnitude from 6 kHz and above in Fig.5.6, the open loop frequency response. In Fig. 5.7, the magnitude between 1.5 kHz to 2.5 kHz slightly increases, which however will not affect the overall improvement, as seen in the next section.

5.3.2 Implementation Results

Experiment based on dSPACE 1103 on TMS320C240 DSP board is carried out to test the designed controllers. The sensitivity function obtained from the experiment, as shown in Fig. 5.8, agrees with the simulation results in Fig. 5.7 and demonstrates the effectiveness of the sensitivity shaping by NLSO method.

Fig. 5.9 shows the power spectrum of the error after the servo controls, in contrast to the spectrum before servo control in Fig. 5.3. The error is measured with zero reference when the input disturbance d_i , the output disturbance d_o , and the noise n , which are modeled according to the NRRO spectrum in Fig. 5.3, are injected into the closed loop system. It is clearly seen that the NLSO controller has a

Figure 5.7: Simulated sensitivity function (solid line: NLSO method; dashed line: PID control with notch filters).

Figure 5.8: Sensitivity function from experiment (solid line: NLSO method; dashed line: PID control with notch filters).

better disturbance attenuation around frequency 650 Hz, with a 64% improvement. This means that the disturbances around this frequency, which is due to the disk vibration, can be better suppressed using the NLSO design method. Although both PID controller with notch filters and NLSO controller amplify the disturbance at 3.8 kHz, the amplification is much smaller with the NLSO controller. Overall, the 3σ value of error with NLSO controller is 17.29 nm, which is slightly better than 17.52 nm obtained from the PID controller with notch filters. As observed in Fig. 5.9, the spectrum is improved greatly before 1 kHz. However, because it is a small portion in the whole frequency range up to 20 kHz (Nyquist frequency), only a slight improvement is achieved for the error signal. Thus our future work will target more overall improvement by developing other control algorithms.

Figure 5.9: Power spectrum of measured error signal ((a) PID control and notch filters, (b) NLSO method).

In our design specification (ii), we set the attenuation of at least -12 dB for frequency ranging from 610 Hz to 670 Hz which will take care of possible variations of disk vibration disturbance frequency around 650 Hz due to different spindle speeds. Additionally the error signal against $\pm 20\%$ frequency variation of the disk vibration

is also evaluated and shown in Fig. 5.10. It is indicated that after optimization with NLSO method, the 3σ of PES is maintained better than before optimization. Therefore the designed NLSO controller is more robust against frequency variation of the disturbance.

Figure 5.10: Error signal 3σ against frequency variation of the disk vibration.

5.4 Conclusion

The NLSO method for sensitivity function shaping has been applied to design a feedback controller for the PZT microactuator. With the known disturbance characteristic, the specifications for the targeted sensitivity function have been determined. Then the PID and notch filters have been designed so that the sensitivity function is close to the targeted one. Later, the NLSO method has been applied to fine tune the controller so that the specifications are fulfilled completely. By using the NLSO method, we were able to improve the performance given by the PID controller with notch filter. It has also been demonstrated that the robustness of the system against frequency variation of the disturbances is improved using the NLSO method.

Chapter 6

Generalized KYP Lemma Based Approach for Disturbance Rejection and PES Minimization

Due to the small percentage of improvement of the overall PES 3σ value using the NLSO method in the previous chapter, further research is carried out. In this chapter, another sensitivity shaping method using the generalized KYP lemma is presented to attenuate disturbances at desired frequency ranges.

6.1 Introduction

To shape frequency responses of closed-loop transfer functions such as sensitivity/complementary sensitivity functions, an H_∞ optimization together with frequency weighting is a commonly used method. The additional weighting functions, however, increase the system and controller complexity since the weighting functions usually have to be of high orders in order to capture the desired specifications

accurately. This is especially so when a servo controller in a HDD is to be designed, aiming at achieving a wider bandwidth while simultaneously rejecting disturbances of frequencies beyond the bandwidth. Further, the process of choosing appropriate weights is tedious and time-consuming. The KYP lemma [81], being one of the most fundamental results in control and signal processing, establishes the equivalence between a frequency domain inequality (FDI) for a transfer function and a linear matrix inequality (LMI) associated with its state space realization. It allows us to characterize various properties of dynamic systems in the frequency domain in terms of LMIs. However, the standard KYP lemma is only applicable for the infinite frequency range. Recently, a very significant development was made by Iwasaki et al [82] which establishes the equivalence between a frequency domain property and an LMI over a finite frequency range, allowing designers to impose performance requirements over chosen finite or infinite frequency ranges. Hence, it is very suitable for analysis and synthesis problems in practical applications where different specifications over different frequency ranges are usually required.

In this chapter, the generalized KYP lemma is applied to design a feedback control such that the specifications of the sensitivity function, required to suppress some specific frequency disturbances, are satisfied. Unlike the standard KYP lemma, the matrix inequality in the generalized KYP lemma involves a matrix variable which is not necessarily positive definite and thus the Schur complement cannot be applied to convexify the controller design. To overcome this difficulty, the Youla parameterization approach [83, 84] is used to parameterize the closed-loop transfer function. The search for the coefficients of the parameter $Q(z)$ is then converted to the LMI problem within the generalized KYP lemma framework. Two applications of the method in the rejection of low-frequency and high-frequency disturbances are presented in Section 6.4 to demonstrate the improvement of the positioning accuracy by the resultant controller and the simplicity of the design.

Although the generalized KYP lemma is effective in shaping the sensitivity function such that narrow-band frequency disturbances can be attenuated, it does not ensure that the PES is minimized. To ensure that the PES is minimized, optimal H_2 control method can be applied. On the other hand, the H_2 control design method does not allow a designer to have the flexibility to specifically reject disturbance at certain frequency ranges as desired. Hence, there is a clear motivation for a control design which incorporates both the generalized KYP lemma and H_2 control. In Section 6.5, the generalized KYP lemma and H_2 control is applied to design a feedback control such that the specifications of the sensitivity function, required to suppress some specific frequency disturbances are satisfied while minimizing the standard deviation of the true PES simultaneously.

In addition to that, we also demonstrate the application of the generalized KYP lemma to limit the bandwidth of the system while attenuating a low frequency disturbance (before the open loop bandwidth) and another disturbance (situated around the usual sensitivity function hump) simultaneously, in Section 6.6. This method is useful because it will keep the magnitude of the sensitivity function at the high frequency region low so that measurement noise will not be amplified too much.

6.2 Problem Formulation

With reference to the servo control loop in Fig. 4.1, the objective here is to design a dynamic feedback controller $C(z)$ for plant $P(z)$ such that the closed-loop system is stable and for some prescribed positive scalars r_i and frequency ranges $(f_{i1}, f_{i2}), i = 1, \dots, N$,

$$|S(f)| < r_i, \quad f_{i1} \leq f \leq f_{i2} \quad (6.1)$$

where $S(f) = 1/(1 + C(f)P(f))$. Smaller r_i means the less contribution of the disturbance of frequency within range (f_{i1}, f_{i2}) to the position error.

In view of the constraint stated in the Bode integral theorem, it is impossible to achieve disturbance rejection at all frequencies higher than the bandwidth of the control loop for the VCM actuators or the microactuators used in data storage systems such as hard disk drives [85]. The specification (6.1) is considered for a specific frequency range, which is however possible to be achieved through shaping the sensitivity function. The above design problem may be approached by selecting a proper frequency weighting function and carrying out an H_∞ optimization. However, the problem of how to select a proper frequency weighting function that can give an accurate shaping of the sensitivity function is generally difficult and time consuming. Further, the resultant controller order will depend on the order of the weighting function and the plant. Note that a more accurate frequency shaping usually requires a higher order weighting function. The generalized KYP lemma [82] gives a necessary and sufficient condition for a given transfer function to satisfy a required frequency domain property over a finite frequency range in terms of a matrix inequality condition. Thus, it may be applied to address the above design problem. In what follows, we employ the generalized KYP lemma to design a feedback control such that the sensitivity function satisfies the required specifications so as to reject

disturbances of specific frequencies. In order to convexify the matrix inequality, the Youla parameterization approach as shown in Fig. 6.1 is used to design a controller with the generalized KYP lemma [86].

Figure 6.1: Q parameterization for servo control design.

6.3 Generalized KYP Lemma Based Disturbance Rejection

The analysis in the last section indicates that the sensitivity function plays a key role in disturbance rejection. To reject disturbance of frequency within certain frequency range, a proper shaping of the sensitivity function can be carried out. In this section, we shall present a frequency shaping method based on the generalized KYP lemma.

First, it is easy to see that the sensitivity function $S(z)$ equals to the transfer function from r to z in Fig. 6.1. The state-space model of the microactuator under consideration is denoted as (A_p, B_p, C_p, D_p) . Then, a state-space representation of the system in Fig. 6.1 is given below.

$$x(k+1) = A_p x(k) + B_p u(k), \quad (6.2)$$

$$z(k) = -C_p x(k) + r(k) - D_p u(k), \quad (6.3)$$

where $x \in \mathcal{R}^n$ is the state.

Let a state-space representation of the controller $C(z)$ be given by (A_c, B_c, C_c, D_c) . Assuming that $D = 1 + D_c D_p$ is invertible, then a state-space representation of the sensitivity function can be given by $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$, where

$$\tilde{A} = \begin{bmatrix} A_p - B_p D^{-1} D_c C_p & B_p D^{-1} C_c \\ -B_c C_p + B_c D_p D^{-1} D_c D_p & A_c - B_c D_p D^{-1} C_c \end{bmatrix} \quad (6.4)$$

$$\tilde{B} = \begin{bmatrix} B_p D^{-1} D_c \\ B_c - B_c D_p D^{-1} D_c \end{bmatrix} \quad (6.5)$$

$$\tilde{C} = \begin{bmatrix} -C_p + D_p D^{-1} D_c C_p & -D_p D^{-1} C_c \end{bmatrix} \quad (6.6)$$

$$\tilde{D} = 1 - D_p D^{-1} D_c \quad (6.7)$$

A special case of the generalized KYP lemma [82] that relates the bounded realness of the sensitivity function over finite frequency ranges to its state space representation is given below.

Lemma 6.3.1. [82] Consider the sensitivity function $S(z) = \tilde{C}(zI - \tilde{A})^{-1}\tilde{B} + \tilde{D}$ with $\tilde{A}$ being stable. Then, for a given scalar $r > 0$, $|S(e^{j\theta})| \leq r$ over a finite frequency range if and only if there exist Hermitian matrices P and $V \geq 0$ such that

$$\begin{bmatrix} \begin{bmatrix} \tilde{A} & \tilde{B} \\ I & 0 \end{bmatrix}^* \Sigma \begin{bmatrix} \tilde{A} & \tilde{B} \\ I & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -r^2 \end{bmatrix} & \begin{bmatrix} \tilde{C} & \tilde{D} \end{bmatrix}^* \\ \begin{bmatrix} \tilde{C} & \tilde{D} \end{bmatrix} & -I \end{bmatrix} \leq 0 \quad (6.8)$$

where (i) for low frequency range $|\theta| \leq \theta_l$,

$$\Sigma = \begin{bmatrix} -P & V \\ V & P - (2 \cos \theta_l) V \end{bmatrix}; \quad (6.9)$$

(ii) for middle frequency range $|\theta_1| \leq \theta \leq |\theta_2|$,

$$\Sigma = \begin{bmatrix} -P & e^{j\theta_c} V \\ e^{-j\theta_c} V & P - (2 \cos \theta_d) V \end{bmatrix}, \quad (6.10)$$

$$\theta_c = (\theta_1 + \theta_2)/2, \theta_d = (\theta_2 - \theta_1)/2;$$

(iii) for high frequency range $|\theta| \geq \theta_h$,

$$\Sigma = \begin{bmatrix} -P & -V \\ -V & P + (2 \cos \theta_h) V \end{bmatrix}. \quad (6.11)$$

For a given controller $C(z)$, the above gives a necessary and sufficient condition for evaluating if $|S(z)| \leq r$ over some given frequency range in terms of LMI. However, (6.8) is no longer an LMI when the controller $C(z) = (A_c, B_c, C_c, D_c)$ is to be designed since $\tilde{A}$ and $\tilde{B}$ involve the unknown parameters A_c, B_c, C_c and D_c . Further, it is not possible to be converted to an LMI by Schur complement since the matrix Σ is not definite. To overcome this difficulty, in the following we apply a Youla parameterization approach where the controller $C(z)$ is now of the structure as shown in Fig. 6.1.

Let $K(z)$ be a given observer based controller that can be designed using approaches such as the LQG control:

$$\hat{x}(k+1) = A_p \hat{x}(k) + B_p u(k) + L(z(k) + C_p \hat{x}(k)) \quad (6.12)$$

$$u(k) = -M \hat{x}(k) \quad (6.13)$$

Note that it is assumed that $D_p = 0$ for the plant.

Then, the set of sensitivity functions can be parameterized as [83, 84]

$$S(z) = T_{11}(z) + T_{12}(z)Q(z)T_{21}(z) \quad (6.14)$$

where $Q(z)$ is a stable transfer function to be designed and

$$\begin{aligned} \begin{bmatrix} T_{11}(z) & T_{12}(z) \\ T_{21}(z) & 0 \end{bmatrix} &= C_T(zI - A_T)^{-1} B_T + D_T, \\ \begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} &= \left[\begin{array}{cc|cc} A_p & -B_p M & 0 & B_p \\ -LC_p & A_p - B_p M + LC_p & L & B_p \\ \hline -C_p & D_p M & 1 & -D_p \\ -C_p & C_p & 1 & 0 \end{array} \right] \end{aligned} \quad (6.15)$$

If $Q(z)$ has the state-space realization (A_q, B_q, C_q, D_q) , then the designed controller $C(z)$ is given by

$$\begin{aligned} A_c &= \begin{bmatrix} A_p - B_p M + LC_p + B_p D_q C_p & B_p C_q \\ B_q C_p & A_q \end{bmatrix}, \\ B_c &= \begin{bmatrix} L + B_p D_q \\ B_q \end{bmatrix}, \\ C_c &= \begin{bmatrix} -M + D_q C_p & C_q \end{bmatrix}, \\ D_c &= D_q. \end{aligned} \quad (6.16)$$

Denote the state space representations of $T_{11}(z)$ and $T_{12}(z)T_{21}(z)$ by $(A_{t11}, B_{t11}, C_{t11}, D_{t11})$ and (A_t, B_t, C_t, D_t) , respectively, then from (6.14) a state space model of $S(z)$ can be written as

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} A_{t11} & 0 & 0 \\ 0 & A_t & 0 \\ 0 & B_q C_t & A_q \end{bmatrix}, \tilde{B} = \begin{bmatrix} B_{t11} \\ B_t \\ B_q D_t \end{bmatrix}, \\ \tilde{C} &= \begin{bmatrix} C_{t11} & D_q C_t & C_q \end{bmatrix}, \tilde{D} = D_{t11} + D_q D_t. \end{aligned} \quad (6.17)$$

Let $Q(z)$ be a FIR filter:

$$Q(z) = q_0 + q_1 z^{-1} + q_2 z^{-2} + \dots + q_\tau z^{-\tau}, \quad (6.18)$$

$$q = [q_0 \ q_1 \ q_2, \ \dots, \ q_\tau] \quad (6.19)$$

which is to be designed so that the required bounded realness of the sensitivity function is satisfied. It is well known that a state space realization for $Q(z)$ can be given by:

$$\begin{aligned} A_q &= \begin{bmatrix} 0 & I_{\tau-1} \\ 0 & 0 \end{bmatrix}, B_q = \begin{bmatrix} 0_{(\tau-1) \times 1} \\ 1 \end{bmatrix}, \\ C_q &= [q_\tau \ q_{\tau-1} \ \dots \ q_1], D_q = q_0 \end{aligned} \quad (6.20)$$

where $I_{\tau-1}$ is the identity matrix of dimension $(\tau - 1) \times (\tau - 1)$ and $0_{(\tau-1) \times 1}$ is the zero matrix of dimension $(\tau - 1) \times 1$. Note that the filter parameter q to be designed only appears in C_q and D_q . Therefore, from (6.20), we know that q exists in $\tilde{C}$ and $\tilde{D}$ only. In this case, (6.8) defines an LMI in terms of the variables P, V , and q . Hence, P, V , and the design parameter q can be computed via a convex optimization [70].

The KYP lemma based control design can be carried out following the steps:

- i. Compute M and L using Matlab commands $M = dlqr(A_p, B_p, C_p' C_p, R)$ and $L = A_p \cdot dlqe(A_p, B_p, -C_p, W_d, W_v)$, where R is the weighting for control input in the cost function $J = \Sigma(\hat{x}' C_p' C_p \hat{x} + u' R u)$ for the linear-quadratic regulator design, and W_d and W_v are the variance matrices of the process noise and measurement noise for the Kalman estimator design, respectively. Here in the KYP lemma based control design, R, W_d , and W_v can be chosen as identity matrices.

- ii. Compute $T_{11}(z), T_{12}(z)$ and $T_{21}(z)$ from (6.15).

- iii. Obtain the state space model $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ in (6.17).
- iv. Based on disturbance spectrum, specify the positive scalars r_i and frequency points $f_i (i = 1, \dots, N)$ for the sensitivity function

$$|S(f_i)| < r_i, f_{i1} \leq f_i \leq f_{i2} \quad (6.21)$$

where f_{i1} and f_{i2} define the frequency range. For each specification on the resultant sensitivity function in the frequency range, construct the LMI (6.8) in terms of the variables P, V , and q with $r = r_i$.

- v. Obtain q, P and V by solving these LMIs using Matlab LMI toolbox. If the LMIs are not solvable, the specifications given in step (iv) are to be adjusted.
- vi. Obtain the controller parameters from (6.16).

6.4 Disturbance Rejection in Data Storage Systems

In this section, we shall provide two examples to demonstrate the design and effectiveness of our proposed KYP based control to reject disturbances at high and low frequency, respectively.

6.4.1 Rejecting High-frequency Disturbances

6.4.1.1 System Models

Fig. 6.2 shows one kind of microactuators for HDDs, which is a PZT microactuator attached to a VCM actuator arm. Moreover, Fig. 6.3 shows the main disturbances that exist in hard disk drive servo systems. Besides the disturbances of

low frequencies, the system suffers from the high frequency disturbances around 8 kHz and 10 kHz, which are induced by air turbulence to suspensions or sliders in HDDs. For higher rotational speed HDDs, these disturbances appear more prominent and impose significant impact on the positioning accuracy of the R/W head. Here we employ the generalized KYP lemma to design a feedback control $C(z)$ for the microactuator such that the sensitivity function $S(z)$ satisfies the required specifications so as to reject disturbances around 8 kHz and 10 kHz. Thus the desired specifications for the sensitivity function $S(z)$ are set as:

$$\text{Spec. (a)} \quad |S(f)| < 0 \text{ dB}, \quad f \leq 2 \text{ kHz}$$

$$\text{Spec. (b)} \quad |S(f)| < -0.35 \text{ dB}, \quad 9950 \leq f \leq 10050 \text{ Hz}$$

$$\text{Spec. (c)} \quad |S(f)| < -1.41 \text{ dB}, \quad 7950 \leq f \leq 8050 \text{ Hz}$$

Spec. (a) means to guarantee a 2 kHz bandwidth at least.

Figure 6.2: PZT microactuator attached to VCM actuator arm in hard disk drives.

The PZT microactuator is modeled by the following transfer function

$$Pzt(s) = \frac{1.81 \times 10^{14} (s^2 + 3079s + 1.934 \times 10^9)}{(s + 1.257 \times 10^5) (s^2 + 791.7s + 1.567 \times 10^9) (s^2 + 5089s + 7.195 \times 10^9)} \quad (6.22)$$

A comparison between the measured and the modeled frequency responses is shown in Fig. 6.4. $Pzt(s)$ is discretized using the zero-order-hold method to obtain $Pzt(z)$. The sampling frequency being used is 40 kHz.

Figure 6.3: Power spectrum of the position error before servo control.

Figure 6.4: PZT microactuator frequency response.

Although the measured frequency response is higher than the model from 7 to 11 kHz, the model is sufficient to represent the actual plant. This is because two notch filters will be included to compensate the two resonance modes around 6.1 kHz and 12 kHz in the next section, thereby lowering the magnitude of the open loop frequency response from 7 to 11 kHz, as well. Experimental results have also verified that closed loop system remains stable.

6.4.1.2 Design and LDV Based Experiment Results

The plant $Pzt(s)$ is pre-compensated by an integrator and two notch filters as follows:

$$\begin{aligned} Int(z) &= \frac{6.3z}{z-1}, \\ Notch1(z) &= \frac{1.177z^2 - 1.279z + 1.154}{z^2 - 0.8751z + 0.9259}, \\ Notch2(z) &= \frac{0.4011z^2 + 0.3972z + 0.3532}{z^2 + 0.1391z + 0.007922} \end{aligned}$$

before the proposed approach is applied to design a controller so that the specifications (a), (b) and (c) can be satisfied.

(A_p, B_p, C_p, D_p) in (6.2)-(6.3) is the state space description of the combined system $P(z) = Int(z) \cdot Notch1(z) \cdot Notch2(z) \cdot Pzt(z)$. $Q(z)$ is chosen as the 1st-order FIR filter (6.18), and q_0 and q_1 are to be solved via the KYP lemma.

As mentioned previously in step (i) of the control design procedure, M and L are obtained using Matlab commands [87], i.e. $M = dlqr(A, B_2, C_1' C_1, R)$ and $L = A \cdot dlqe(A, B_2, C_1, W_d, W_v)$.

Next we use the KYP lemma to search for the coefficients of $Q(z)$. Spec. (a), (b) and (c) require to solve three LMIs of the form (6.8) with Σ in (6.9), (6.10) and (6.11), respectively. The obtained q_0 and q_1 are: $q_0 = -0.4117, q_1 = 0.6371$.

Therefore, the overall controller is given by the following parameters:

$$\begin{aligned} Zeros = & [-0.0740 \pm 0.9488i, 0.6913 \pm 1.0317i, 0.7016 \pm 0.7015i, \\ & 0.4394 \pm 0.8573i, -0.0696 \pm 0.0555i, 0.0432]; \end{aligned} \quad (6.23)$$

$$\begin{aligned} Poles = & [-0.8203, 0.9990, -0.1222, 0.4383 \pm 0.8587i, \\ & 0.4375 \pm 0.8570i, 0.2007 \pm 0.8145i, -0.0696 \pm 0.0555i]; \end{aligned} \quad (6.24)$$

$$Gain = 2.6192. \quad (6.25)$$

As such, $C(z)$ can be obtained via (6.16). The resulting controller for the PZT microactuator is $C(z) \cdot Int(z) \cdot Notch1(z) \cdot Notch2(z)$, and the sensitivity function is shown in Fig. 6.5. It can be observed in Fig. 6.5 that with $Q(z)$, the sensitivity function is less than -4 dB and -2 dB at 8 kHz and 10 kHz respectively, which means that specifications (a)-(c) have been satisfied by the searched $Q(z)$. The sensitivity function with a lead-lag control and notch filters is also shown in Fig. 6.5 where it can be seen that the required specifications are not met. Moreover, as seen as in Fig. 6.6, the gain margin and the phase margin are 12 dB and 67 deg, higher than 7.7 dB and 63 deg with the lead-lag control.

In the experiment, the dSpace 1103 on *TMS320C240* DSP board was used to implement the controller, and LDV was used to measure the actuator displacement. Fig. 6.7 shows the experimental sensitivity functions, which agree with the simulation results in Fig. 6.5. From (4.1), σ (i.e. standard deviation) values of the position errors with control laws designed by the lead-lag control with notch filters and the KYP Lemma based control versus frequencies are obtained and shown in Fig. 6.8. It can be seen that the performance with the KYP lemma based control is much better from 5 kHz onwards, and slightly worse before 5 kHz, which is consistent with the sensitivity functions in Fig. 6.5.

Figure 6.5: Sensitivity functions with KYP design, lead-lag control and notch filters: simulation result.

Figure 6.6: Open loop frequency response with KYP design, and lead-lag control with notch filters.

Figure 6.7: Sensitivity functions with the lead-lag control with notch filters and the KYP based control: experimental results.

Figure 6.8: PES σ value versus frequency.

6.4.2 Rejecting Low-frequency Disturbances for a Servo Track Writer

6.4.2.1 System Models

Fig. 6.9 shows the zoom-in head-disk-spindle assembly of our newly developed servo track writer with microactuators for hard disk drives. The frequency response of the microactuator is shown in Fig. 6.10. The transfer function of the PZT microactuator is obtained by curve fitting to the measured frequency responses and its zeros, poles and gain are given by

$$\begin{aligned} \text{Zeros} = 10^4 \cdot [& 6.0256 \pm 6.1473i, -0.2168 \pm 7.2224i, \\ & -0.3016 \pm 5.0175i, -0.1100 \pm 3.1397i, \\ & -0.0282 \pm 2.3497i]; \end{aligned} \quad (6.26)$$

$$\begin{aligned} \text{Poles} = 10^4 \cdot [& -0.3316 \pm 9.4692i, -0.1596 \pm 7.9780i, \\ & -0.1131 \pm 5.6537i, -0.4335 \pm 4.3137i, \\ & -0.1539 \pm 3.0749i, -0.0372 \pm 2.3245i]; \end{aligned} \quad (6.27)$$

$$\text{Gain} = 1.8580 \times 10^{10}. \quad (6.28)$$

It is seen that six resonance modes at 3.7, 4.9, 6.9, 9, 12.7 and 15 kHz are included in the model. The disturbance distribution of the STW platform is reflected in the non-repeatable run out (NRRO) power spectrum in Fig. 5.3, where 650 Hz mode is due to disk vibration and is to be suppressed. Thus the objective here is to design a dynamic feedback controller for the microactuator such that its closed-loop system is stable and the disturbances centering at 650 Hz is suppressed.

Figure 6.9: Zoom-in head-disk-spindle assembly of the platform.

Figure 6.10: Frequency response of the microactuator.

6.4.2.2 Design and Implementation Results

The servo track writer writes servo pattern into 501 sectors using 5400 rpm spindle rotational speed. Thus 45 (*i.e.*, $501 \times 5400 \div 60$) kHz sampling rate is used in the servo control design. Currently due to the limitation by the DSP speed, with this sampling rate the platform can support up to 10th order controller.

Since our objective is to attenuate the low frequency disturbance centered at 650 Hz, only the static part of the microactuator is taken into account for the controller designed using the generalized KYP design method. The static part of the microactuator can be represented by a Pade delay model. Meanwhile, notch filters will be used to compensate the high frequency resonance modes at 3.7, 9 and 15 kHz. The notch filters will not affect the performance obtained by KYP design method greatly since it is at least 5 times away from 650 Hz. The Pade delay model is given by

$$P_{Pade-delay} = -5.6234 \frac{s - 2 \cdot \pi \cdot 17000}{s + 2 \cdot \pi \cdot 17000}, \quad (6.29)$$

which is pre-compensated by the proportional-integral (PI) controller:

$$Int(z) = 0.027 \left(\frac{z}{z - 0.999} - 0.5 \right). \quad (6.30)$$

Since only the first order Pade-delay model is involved, the order of the sensitivity function $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ is much lower. The desired specifications for the sensitivity function $S(z)$ are set as:

$$Spec. (a) \quad |S(f)| < 0 \text{ dB}, \quad f \leq 500 \text{ Hz}$$

$$Spec. (b) \quad |S(f)| < -10 \text{ dB}, \quad 610 \text{ Hz} \leq f \leq 670 \text{ Hz}$$

$$Spec. (c) \quad |S(f)| < -9.54 \text{ dB}, \quad f \geq 19 \text{ kHz}$$

Spec. (b) means to attenuate the disturbances centering at 650 Hz by 10 dB at least. The parameters of $Q(z)$ in (6.18) with $\tau = 1$ are attained by solving three LMIs of the form (6.8) corresponding to Spec. (a), (b) and (c). The resultant controller $C(z)$ is a 10th order controller which is described by the following parameters:

$$\begin{aligned} \text{Zeros} = & [-0.9990, 0.2754 \pm 0.9560i, -0.5092 \pm 0.8484i, \\ & 0.0936, 0.8659 \pm 0.4971i, 0.8925 \pm 0.0487i], \end{aligned} \quad (6.31)$$

$$\begin{aligned} \text{Poles} = & [-0.9999, 0.8554 \pm 0.4974i, 0.2827 \pm 0.8590i, \\ & 0.9990, 0.9569 \pm 0.0776i, -0.0869 \pm 0.3382i], \end{aligned} \quad (6.32)$$

$$\text{Gain} = 0.0144. \quad (6.33)$$

For the purpose of comparison, the phase-lead peak filter (PLPF) method [75] is also applied to design a controller to suppress the low frequency disturbances around 650 Hz, and the sensitivity function comparison is shown in Fig. 6.11, where the magnitude between 100 Hz and 1 kHz, especially at 650 Hz, is much higher than that given by the KYP lemma method. It is seen in the open loop comparison in Fig. 6.12 that the phase margin (PM) with the PLPF method is higher, while the bandwidth is lower and the gain margin (GM) is comparable with our KYP lemma method. The NRRO power spectrum comparison is given in Fig. 6.13 which shows consistent results with the sensitivity functions in Fig. 6.11. The 3σ value of true PES NRRO is improved from 6.4 nm (395 kTPI) with the PLPF method to 6 nm (425 kTPI) with the KYP lemma method.

The above examples show that even with a first order FIR filter $Q(z)$, the sensitivity function can be shaped very efficiently for rejecting disturbances at high and low frequency.

Figure 6.11: Comparison of sensitivity functions (magnitude at 650 Hz is much lower).

Figure 6.12: Open loop frequency responses (PLPF (GM: 9 dB, PM: 68 deg, Bandwidth 1.1 kHz); KYP (GM: 6 dB, PM: 34 deg, Bandwidth: 1.7 kHz)).

Figure 6.13: NRRO power spectrum with PLPF and KYP (52% reduction before 1 kHz, TPI improved from 395 to 425 kTPI (10% track width tolerance)).

6.5 Combined Generalized KYP Lemma and H_2 Control for PES Minimization

6.5.1 System Models and Problem Formulation

Figure 6.14: H_2 control scheme with Q parameterization for servo control design.

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Consider the HDD servo loop with disturbances injected as in Fig. 6.14, where $P(z)$ is an actuator model, $C(z)$ is the controller to be designed, $D_i(z)$, $D_o(z)$, $N(z)$ are the models representing all torque disturbances, disturbances due to non-repeatable disk motions, and PES demodulation and measurement noise, respectively. z_1 is the true PES, y is the measured PES, $w_i (i = 1, 2, 3)$ are independent Gaussian white noises with zero-mean and unit covariance.

As shown in equation (3.40)-(3.41) of Chapter 3, the PES 3σ minimization can be formulated into a H_2 control problem. Therefore, the servo control design to reject disturbances at particular frequency ranges and minimize TMR can be treated as combined generalized KYP lemma and H_2 control problem.

6.5.2 H_2 Control

Based on Fig. 6.14 and the generalized KYP lemma controller design with Q parameterization explained in Section 6.3, a H_2 control problem can be formulated as follows:

$$-z_1 = Nw_3 + S(PD_iw_1 + D_ow_2 - Nw_3) \quad (6.34)$$

Denote the state space representations of $P(z)D_i(z)$, $D_o(z)$ and $N(z)$ by (A_1, B_1, C_1, D_1) , (A_2, B_2, C_2, D_2) , and (A_3, B_3, C_3, D_3) . It follows from (6.2)-(6.3) and (6.12)-(6.13) that

$$x(k+1) = \bar{A}x(k) + \bar{B}\tilde{w}(k) \quad (6.35)$$

$$-z_1(k) = \bar{C}x(k) + \bar{D}\tilde{w}(k) \quad (6.36)$$

where,

$$\bar{A} = \begin{bmatrix} A_1 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 \\ 0 & 0 & A_3 & 0 \\ \tilde{B}C_1 & \tilde{B}C_2 & -\tilde{B}C_3 & \tilde{A} \end{bmatrix} \quad (6.37)$$

$$\bar{B} = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \\ \tilde{B}D_1 & \tilde{B}D_2 & -\tilde{B}D_3 \end{bmatrix} \quad (6.38)$$

$$\bar{C} = \begin{bmatrix} \tilde{D}C_1 & \tilde{D}C_2 & -\tilde{D}C_3 + C_3 & \tilde{C} \end{bmatrix} \quad (6.39)$$

$$\bar{D} = \begin{bmatrix} \tilde{D}D_1 & \tilde{D}D_2 & -\tilde{D}D_3 + D_3 \end{bmatrix} \quad (6.40)$$

From (6.17), the state-space representation for the H_2 control problem in terms of (A_q, B_q, C_q, D_q) can be described as the following:

$$\bar{A} = \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_3 & 0 & 0 & 0 \\ B_{t11}C_1 & B_{t11}C_2 & -B_{t11}C_3 & A_{t11} & 0 & 0 \\ B_tC_1 & B_tC_2 & -B_tC_3 & 0 & A_t & 0 \\ B_qD_tC_1 & B_qD_tC & -B_qD_tC_3 & 0 & B_qC_t & A_q \end{bmatrix} \quad (6.41)$$

$$\bar{B} = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \\ B_{t11}D_1 & B_{t11}D_2 & -B_{t11}D_3 \\ B_tD_1 & B_tD_2 & -B_tD_3 \\ B_qD_tD_1 & B_qD_tD_2 & -B_qD_tD_3 \end{bmatrix} \quad (6.42)$$

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$$\bar{C} = \begin{bmatrix} (D_{t11} + D_q D_t) C_1 & (D_{t11} + D_q D_t) C_2 & -(D_{t11} + D_q D_t) C_3 \\ & C_{t11} & D_q C_t & C_q \end{bmatrix} \quad (6.43)$$

$$\bar{D} = \begin{bmatrix} (D_{t11} + D_q D_t) D_1 & (D_{t11} + D_q D_t) D_2 \\ & -(D_{t11} + D_q D_t) D_3 + D_3 \end{bmatrix} \quad (6.44)$$

For a given controller that stabilizes the system (6.35)-(6.36), the H_2 norm square, $\|T_{z_1 w}\|_2^2$ can be computed by the following minimization:

$$\min_{(\Xi=\Xi^T, \Omega=\Omega^T)} \text{trace}(\Omega) \quad (6.45)$$

subject to

$$\bar{A}^T \Xi \bar{A} - \Xi + \bar{C}^T \bar{C} < 0 \quad (6.46)$$

$$\bar{B}^T \Xi \bar{B} + \bar{D}^T \bar{D} < \Omega \quad (6.47)$$

From (6.46) and (6.47),

$$\begin{bmatrix} \bar{A}^T \Xi \bar{A} - \Xi & \bar{C}^T \\ \bar{C} & -I \end{bmatrix} < 0 \quad (6.48)$$

$$\begin{bmatrix} -\Omega + \bar{B}^T \Xi \bar{B} & \bar{D}^T \\ \bar{D} & -I \end{bmatrix} < 0 \quad (6.49)$$

Note that the parameters of $Q(z)$, C_q and D_q , which are to be designed only appear in $\bar{C}$ and $\bar{D}$. Hence, (6.48)-(6.49) are LMIs that can be solved by convex optimization [70].

The KYP lemma based control with H_2 control design steps are listed in the

following:

- i. Compute M and L using Matlab commands $M = dlqr(A_p, B_p, C_p' C_p, R)$ and $L = A_p \cdot dlqe(A_p, B_p, -C_p, W_d, W_v)$, where R is the weighting for control input in the cost function $J = \sum (\hat{x}' C_p' C_p \hat{x} + u' R u)$ for the linear-quadratic regulator design, and W_d and W_v are the variance matrices of process noise and measurement noise for the Kalman estimator design, respectively. Here in the KYP lemma based control design, R , W_d , and W_v can be chosen as identity matrices.
- ii. Compute $T_{11}(z)$, $T_{12}(z)$ and $T_{21}(z)$ from (6.15).
- iii. Obtain the state space model $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ in (6.17).
- iv. Based on disturbance spectrum, specify the positive scalars r_i and frequency points $f_i (i = 1, \dots, N)$ for sensitivity function

$$|S(f_i)| < r_i, f_{i1} \leq f_i \leq f_{i2} \quad (6.50)$$

where f_{i1} and f_{i2} define the frequency range. For each specification on the resultant sensitivity function in the frequency range, construct the LMIs (6.8) and (6.46)-(6.47) in terms of the variables Φ , Λ , and q with $r = r_i$.

- v. Obtain q , Φ and Λ by solving these LMIs using Matlab LMI toolbox. If the LMIs are not solvable, the specifications given in Step (iv) are to be adjusted.
- vi. Obtain the controller parameters from (6.16).

6.5.3 Design and Implementation Results

This section demonstrates the effectiveness of the generalized KYP lemma and H_2 control method to reject specific disturbances and minimize the H_2 norm of the PES.

As shown in Fig. 5.3, the disturbance around 650 Hz which is due to the disk vibration is to be suppressed, while ensuring that the H_2 norm of the position error signal is minimized. Hence, the desired specifications of the sensitivity function $S(z)$ is $|S(f)| < -10$ dB for $610 \text{ Hz} \leq f \leq 670 \text{ Hz}$ and at the same time minimize the error signal.

The frequency response of the PZT microactuator, shown in Fig. 6.15, is measured using a laser Doppler vibrometer (LDV) and a digital signal analyzer (DSA). The main resonance modes of the plant are at frequencies 6.5 kHz, 9.5 kHz, 11.3 kHz, and 20 kHz. The identified plant model of the microactuator, $P(s)$ has the following parameters:

$$\begin{aligned} \text{Zeros} = 10^5 \cdot & [-0.0296 \pm 0.4927i, -0.0110 \pm 1.0964i, \\ & -0.7116 \pm 0.6275i, -0.0093 \pm 0.6198i, \\ & 0.9245], \end{aligned} \quad (6.51)$$

$$\begin{aligned} \text{Poles} = 10^5 \cdot & [-0.0255 \pm 1.2733i, -0.0050 \pm 0.7100i, \\ & -0.0048 \pm 0.5981i, -0.0327 \pm 0.4071i, \\ & -0.9245], \end{aligned} \quad (6.52)$$

$$\text{Gain} = 0.4819. \quad (6.53)$$

The frequency response of the plant model is plotted against the measured data in Fig. 6.15 for comparison and it is subsequently discretized in Matlab, using the ‘zoh’ method for a 40 kHz sampling frequency.

Figure 6.15: Frequency response of PZT microactuator.

The parameters for $Q(z)$ in (6.20) with $\tau = 1$ are obtained by solving the three LMIs (6.8), (6.46) and (6.47). Therefore, the final controller designed using the combined generalized KYP lemma and H_2 control method has the following parameters:

$$\begin{aligned} \text{Zeros} = & [-0.9990, 0.0948, -0.9375 \pm 0.0392i, -0.2002 \pm 0.9671i, \\ & 0.0745 \pm 0.9853i, 0.4929 \pm 0.8011i, 0.8923 \pm 0.0508i], \end{aligned} \quad (6.54)$$

$$\begin{aligned} \text{Poles} = & [-0.9999, 0.9990, 0.9517 \pm 0.0875i, 0.3943 \pm 0.5351i, \\ & 0.0713 \pm 0.8581i, -0.1403 \pm 0.7533i, -0.5289 \pm 0.0116i], \end{aligned} \quad (6.55)$$

$$\text{Gain} = -0.0847. \quad (6.56)$$

For comparison, the generalized KYP lemma based control [50] is also applied to design a controller to suppress the disk vibration around 650 Hz. The frequency response of the open loop functions, controllers and sensitivity functions for both the generalized KYP lemma based control and our proposed method are depicted in Fig. 6.16, Fig. 6.17, and Fig. 6.18, respectively. The performance specifications

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on Table 6.1 show that the proposed method does not compromise the stability of the overall system. The 3σ value of true PES NRRO has a 4% improvement, from 7.5 nm with the KYP method to 7.2 nm with the KYP+ H_2 method.

Table 6.1: Comparison of Performance Specifications

Method	KYP	KYP+ H_2
Crossover frequency (kHz)	1.85	1.78
Gain margin (dB)	7.9	12.6
Phase margin (deg)	35.4	45.9

Figure 6.16: Open loop frequency responses.

Experiments are carried out for the KYP controller and the KYP+ H_2 controller to verify the simulation results. The controllers are implemented using *dSPACE* 1103 on *TMS320C240* DSP board. The sensitivity function obtained from the experiment, as shown in Fig. 6.19, demonstrates the effectiveness of the proposed method. Fig. 6.20 shows the closed-loop NRRO power spectrum. It can be seen from this figure that the spectrum coincides with the results of the sensitivity function where the KYP+ H_2 offers better improvement for frequency between 2 and

Figure 6.17: Frequency response of the controller.

Figure 6.18: Designed sensitivity functions.

6.6 Generalized KYP Lemma for Disturbance Rejection Near Open Loop Bandwidth

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4 kHz. This is expected since the sensitivity function designed using the KYP+ H_2 method has a lower hump.

Figure 6.19: Comparison of sensitivity functions obtained from experiment.

6.6 Generalized KYP Lemma for Disturbance Rejection Near Open Loop Bandwidth

If attenuation of disturbance at both low frequency and frequency above bandwidth where the sensitivity hump is situated are required, we may add another LMI for the frequency above bandwidth using the middle frequency range criterion (6.10) in (6.8). However, the resultant controller from the optimization of these LMIs will make the open loop bandwidth undesirably high. A very high open loop is undesirable because the resultant sensitivity hump will be very high, causing high amplification of measurement noise. Hence, in order to limit the open loop bandwidth, we use another KYP specification by letting $S(z) \geq 1$ for $f_{i1} \leq f_i \leq f_{i2}$.

Figure 6.20: NRRO power spectrum with KYP and KYP+ H_2 control.

6.6.1 Generalized KYP Lemma for $|S(z)| \geq 1$

The generalized KYP lemma that gives the relationship between the specification $|S(z)| \geq 1$ for $f_{i1} \leq f_i \leq f_{i2}$ is explained in the following lemma.

Lemma 6.6.1. [82] $|S(e^{j\theta})| \geq 1$ over a finite frequency range if and only if there exist Hermitian matrices Φ and $\Lambda \geq 0$ such that

$$\begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^* \Sigma \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} + \begin{bmatrix} C & D \\ 0 & I \end{bmatrix}^* \Pi \begin{bmatrix} C & D \\ 0 & I \end{bmatrix} \leq 0 \quad (6.57)$$

where for the frequency range $\theta_1 \leq \theta \leq \theta_2$

$$\Sigma = \begin{bmatrix} -\Phi & e^{j\theta_c} \Lambda \\ e^{-j\theta_c} \Lambda & \Phi - (2 \cos \theta_d) \Lambda \end{bmatrix}, \quad (6.58)$$

$$\theta_c = (\theta_1 + \theta_2)/2, \theta_d = (\theta_2 - \theta_1)/2. \quad (6.59)$$

and

$$\sigma(G, \Pi) = a\mathcal{R}(G) + b\mathcal{I}(G) + c, \quad \Pi := \begin{bmatrix} 0 & a + jb \\ a - jb & 2c \end{bmatrix} \quad (6.60)$$

for a half plane with normal vector $a + jb$.

Thus, for $|S(e^{j\theta})| \geq 1$,

$$\begin{aligned} \sigma(G, \Pi) = \text{Im}(G) - 1 \geq 0 &\Leftrightarrow 1 - \text{Im}(G) \leq 0 \\ \Pi &:= \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^* & \Pi_{22} \end{bmatrix} = \begin{bmatrix} 0 & -j \\ j & 2 \end{bmatrix}. \end{aligned} \quad (6.61)$$

6.6.2 Design Specifications

Here, we demonstrate that we can attenuate a disturbance at low frequency and another disturbance at frequency above bandwidth without increasing the bandwidth undesirably using the same PZT microactuator plant as in Section 6.5.3. The desired sensitivity function should be able to attenuate disturbances at both 650 Hz and 2 kHz, while the bandwidth of the open loop should be around 1 kHz. These requirements are equivalent to the following specifications:

$$\text{Spec. (a)} \quad |S(f)| < -10 \text{ dB}, \quad 610\text{Hz} \leq f \leq 670\text{Hz}$$

$$\text{Spec. (b)} \quad |S(f)| < -3 \text{ dB}, \quad 1950\text{Hz} \leq f \leq 2050\text{Hz}$$

$$\text{Spec. (c)} \quad |S(f)| > 0 \text{ dB}, \quad 990\text{Hz} \leq f \leq 1010\text{Hz}$$

In order to achieve Spec. (a) and (b), we use the LMI (6.8) twice with the middle frequency range parameters (6.10). To set the open loop bandwidth (i.e., Spec. (c)), the LMI from Lemma 6.6.1 is used. A controller which satisfies the requirements of the sensitivity function can be obtained by solving these LMIs.

6.6.3 Design and Simulation Results

A feasible solution is obtained with Q as a fourth-order FIR filter. The controller obtained is given by the following parameters

$$\begin{aligned} \text{Zeros} = & [-0.1892 \pm 0.9553i, 0.1296 \pm 0.9630i, 0.5147 \pm 0.8396i, \\ & 0.9456 \pm 0.2743i, 0.9303, 0.8889], \end{aligned} \quad (6.62)$$

$$\begin{aligned} \text{Poles} = & [-0.1557 \pm 0.8235i, 0.1214 \pm 0.7403i, 0.4402 \pm 0.6862i \\ & 0.9247 \pm 0.3158i, 0.9618 \pm 0.0894i, 0.9992], \end{aligned} \quad (6.63)$$

$$\text{Gain} = 0.3942. \quad (6.64)$$

The frequency response of the controller is plotted in Fig. 6.21, and the corresponding open loop and sensitivity function is shown in Fig. 6.22 and 6.23, respectively.

We also plot the sensitivity function obtained by solving the LMIs without Spec. (c) in Fig. 6.23 for comparison. It can be seen from this figure that the bandwidth of the system is increased, thus, causing the sensitivity hump to be much higher than desired at the high frequency range. Although the controller solved with Spec. (a), (b) and (c) has less attenuation from 100 Hz to 4 kHz, its magnitude is much lower from 4 kHz to 20 kHz. This implies that the system noise amplification will be lesser than the controller designed without using Lemma 6.6.1.

6.7 Conclusion

This chapter has applied the generalized KYP lemma in the microactuator closed-loop design to suppress the narrow-band disturbances for hard disk drives. The system design problem with multiple specifications on the gain properties of the sensitivity function over several frequency ranges has been solved by the LMI optimization based on the KYP lemma. The Youla parameterization approach has

Figure 6.21: Frequency response of controller.

Figure 6.22: Open loop frequency response. The bandwidth is around 1.2 kHz.

Figure 6.23: Designed sensitivity function. (Solid line: With Spec. (a), (b) and (c); Dashed line: With Spec. (a) and (b) only)

been used in the feedback controller design. Two practical applications have been demonstrated for low-frequency and high-frequency disturbance rejection. The resultant controller verifies that the desired specifications to reject the disturbances have been satisfied via the search for the coefficients of $Q(z)$.

Since H_2 control alone does not guarantee high servo bandwidth, and the generalized KYP lemma approach does not guarantee PES minimization, we combined both control methods to design a controller that can suppress the gain of the sensitivity function at specific frequency range in order to attenuate narrow-band disturbance, and minimize the H_2 norm of the PES concurrently. This method is applied to design a controller for a PZT microactuator to attenuate disturbance due to disk vibrations around 650 Hz while minimizing the 3σ value of the true PES. Based on simulation and experimental results, this method has demonstrated its effectiveness and shown acceptable improvement for the 3σ value of the PES. In addition, we also demonstrated that the KYP lemma can be used to attenuate disturbance at two different frequencies simultaneously, i.e., 650 Hz and 2 kHz,

even if one of the frequency band for attenuation is very near the usual open loop bandwidth.

Both the NLSO method (Chapter 5) and the KYP lemma method presented in this chapter deal with disturbance attenuation. The NLSO method requires the user to design a sensitivity function and obtain the frequency response data. This is then followed by adding in additional constraints so that the final sensitivity function obtained from the NLSO controller fits the desired sensitivity function of the user. The difficulty for using this method stems from determining the correct phase required for the corresponding additional constraints, since the user is only concerned about shaping the magnitude of the sensitivity function. However, the NLSO method's computational time is much faster than the KYP lemma method. On the other hand, the KYP lemma method allows the user to set several desired specifications for the magnitude of the sensitivity function on several frequency ranges without the need to specify the exact corresponding phase angle. Since these two methods have both the advantages and disadvantages, and it is hard to distinguish a clear winner.

Chapter 7

H_∞ Feedforward Decoupler for Hybrid-type Servo Track Writer

As shown in Chapters 5 and 6, the hybrid-type STW, which uses a PZT microactuator as its secondary actuator, is highly beneficial in improving the TPI for the development of future HDDs. Based on the NRRO power spectrum of the system, sensitivity function shaping was done to attenuate disturbances at specific frequency ranges. However, the experiments were done for writing one disk surface at a time. In order to reduce the manufacturing duration of HDDs, the hybrid-type servo track writer must be able to write servo information accurately onto multiple disks simultaneously, therefore requiring multiple PZT microactuators. Due to the close proximity of the microactuators, interactions amongst the microactuators are expected. The interaction, if not suppressed, may cause instability or affect the accuracy of the system. Thus, in this chapter, we will design an H_∞ feedforward decoupler to cancel the interactions among the multiple microactuators.

7.1 Introduction

A multi-disk media-level servo track writing system is shown in Fig. 7.1. It consists of a primary actuator with multiple PZT microactuator suspensions on the arm tips. The system will be able to write servo signals simultaneously on multiple disks with its multiple heads on the PZT suspensions. Dual-stage actuation is utilized because it has a higher bandwidth than single-stage actuation and this could improve the positioning accuracy of the R/W heads, reduce TMR and increase the track density.

Figure 7.1: Hybrid-type servo track writer with multiple microactuators.

Fig. 7.2 shows the block diagram of the servo control loop for the microactuator. C_j , P_j ($j = 1, 2, \dots, m$) represent the controller and plant model for j th microactuator, respectively. n_j ($j = 1, 2, \dots, m$) represents the PES demodulation noise. d_{ij} ($j = 1, 2, \dots, m$) represents all torque disturbance and windage, power-amplifier noise and D/A quantization error. d_{oj} ($j = 1, 2, \dots, m$) represents disk and motor vibrations. $P_{int\ j}$ ($j = 1, 2, \dots, m$) represents the interaction between the microactuators.

In order to cancel the interaction between the two actuators in a dual-actuator HDD, a feedforward decoupling control is proposed in [88]. However, due to the presence of non-minimum phase zeros and possible model uncertainty of the plant, it is not possible to cancel the coupling effect directly. Hence, the approach in [88]

Figure 7.2: Servo loop for multiple microactuator.

may not be suitable for the hybrid-type STW with multiple microactuators. In this chapter, we shall formulate the decoupling control as a H_∞ control problem and aim to minimize the effect of coupling on PES.

7.2 H_∞ Feedforward Decoupler

Fig. 7.3 shows the feedforward decoupling structure for two microactuators. Since both microactuators used are from the same production batch, they have similar resonance modes and their interactions affecting the other microactuator are similar too. Therefore, for simplicity, we shall consider the interaction originating from the second microactuator loop which is affecting the positioning accuracy of the first microactuator only. Similarly, our proposed decoupling can be applied to the other microactuator too. $F_1(s)$ is to be designed to cancel the interactions originating from the second microactuator to the first microactuator loop. We shall demonstrate the H_∞ feedforward decoupler design for a servo track writer with two microactuators,

where the controllers $C_1(s)$ and $C_2(s)$ for the feedback loop of the two microactuators have been pre-designed, which are PID controllers with notch filters.

Figure 7.3: Feedforward decouplers for interaction between two microactuators' servo loops.

Associated with the system in Fig. 7.3, we consider the state-space representation:

$$\dot{x}(t) = Ax(t) + B_1w(t) + B_2f_1(t) \quad (7.1)$$

$$u_2(t) = Cx(t) + Dw(t) \quad (7.2)$$

$$e_1(t) = Lx(t) + H_1w(t) + H_2f_1(t) \quad (7.3)$$

where $x(t) \in \mathcal{R}^n$ is the state; $e_1(t) \in \mathcal{R}^r$ is the controlled output; $u_2(t) \in \mathcal{R}^m$ is the measurement output; $w = \begin{bmatrix} d_{i2} & d_{o2} & n_2 \end{bmatrix}^T$; A , B_1 , B_2 , C , D , L , H_1 and H_2 are constant matrices of appropriate dimensions that describe the system. Note that r_2 is not included because it has similar effect on PES, e_1 , as n_2 . Also, we are considering track following, where r_1 is set to be zero.

Denote the state space representation of $C_1(s)$, $C_2(s)$, $P_1(s)$, $P_2(s)$ and $P_{int1}(s)$

by $(A_{c_1}, B_{c_1}, C_{c_1}, D_{c_1})$, $(A_{c_2}, B_{c_2}, C_{c_2}, D_{c_2})$, $(A_{p_1}, B_{p_1}, C_{p_1}, D_{p_1})$, $(A_{p_2}, B_{p_2}, C_{p_2}, D_{p_2})$ and $(A_{i_1}, B_{i_1}, C_{i_1}, D_{i_1})$, respectively. Define $x = \begin{bmatrix} x_{p_1}^T & x_{p_2}^T & x_{c_1}^T & x_{c_2}^T & x_{i_1}^T \end{bmatrix}^T$. Assuming that $\Psi = 1 + D_{c_1}D_{p_1}$ and $\Phi = 1 + D_{c_2}D_{p_2}$ are invertible, then, the system (7.1)-(7.3) can be derived as

$$A = \begin{bmatrix} A_{p_1} - B_{p_1}D_{c_1}\Psi^{-1}C_{p_1} & B_{p_1}D_{c_1}\Psi^{-1}D_{i_1}\Phi^{-1}D_{c_2}C_{p_2} & & & & & & & & \\ 0 & A_{p_2} - B_{p_2}\Phi^{-1}D_{c_2}C_{p_2} & & & & & & & & \\ -B_{c_1}\Psi^{-1}C_{p_1} & B_{c_1}\Psi^{-1}D_{i_1}\Phi^{-1}D_{c_2}C_{p_2} & & & & & & & & \\ 0 & -B_{c_2}C_{p_2} + B_{c_2}D_{p_2}\Phi^{-1}D_{c_2}C_{p_2} & & & & & & & & \\ 0 & -B_{i_1}\Phi^{-1}D_{c_2}C_{p_2} & & & & & & & & \\ B_{p_1}C_{c_1} - B_{p_1}D_{c_1}\Psi^{-1}D_{p_1}C_{c_1} & -B_{p_1}D_{c_1}\Psi^{-1}D_{i_1}\Phi^{-1}C_{c_2} & & & & & & & & \\ 0 & B_{p_2}\Phi^{-1}C_{c_2} & & & & & & & & \\ A_{c_1} - B_{c_1}\Psi^{-1}D_{p_1}C_{c_1} & -B_{c_1}\Psi^{-1}D_{i_1}\Phi^{-1}C_{c_2} & & & & & & & & \\ 0 & A_{c_2} - B_{c_2}D_{p_2}\Phi^{-1}C_{c_2} & & & & & & & & \\ 0 & B_{i_1}\Phi^{-1}C_{c_2} & & & & & & & & \\ & -B_{p_1}D_{c_1}\Psi^{-1}C_{i_1} & & & & & & & & \\ & 0 & & & & & & & & \\ & -B_{c_1}\Psi^{-1}C_{i_1} & & & & & & & & \\ & 0 & & & & & & & & \\ & A_{i_1} & & & & & & & & \end{bmatrix}, \quad (7.4)$$

$$\begin{aligned}
B_1 &= \begin{bmatrix} B_{p1}D_{c1}\Psi^{-1}D_{i1}\Phi^{-1}D_{c2}D_{p2} & B_{p1}D_{c1}\Psi^{-1}D_{i1}\Phi^{-1}D_{c2} \\ B_{p2} - B_{p2}\Phi^{-1}D_{c2}D_{p2} & -B_{p2}\Phi^{-1}D_{c2} \\ B_{c1}\Psi^{-1}D_{i1}\Phi^{-1}D_{c2}D_{p2} & B_{c1}\Psi^{-1}D_{i1}\Phi^{-1}D_{c2} \\ -B_{c2}D_{p2} + B_{c2}D_{p2}\Phi^{-1}D_{c2}D_{p2} & -B_{c2} + B_{c2}D_{p2}\Phi^{-1}D_{c2} \\ -B_{i1}\Phi^{-1}D_{c2}D_{p2} & -B_{i1}\Phi^{-1}D_{c2} \\ -B_{p1}D_{c1}\Psi^{-1}D_{i1}\Phi^{-1}D_{c2} & \\ B_{p2}\Phi^{-1}D_{c2} & \\ -B_{c1}\Psi^{-1}D_{i1}\Phi^{-1}D_{c2} & \\ B_{c2} - B_{c2}D_{p2}\Phi^{-1}D_{c2} & \\ B_{i1}\Phi^{-1}D_{c2} & \end{bmatrix}, \\
B_2 &= \begin{bmatrix} B_{p1} - B_{p1}D_{c1}\Psi^{-1}D_{p1} \\ 0 \\ -B_{c1}\Psi^{-1}D_{p1} \\ 0 \\ 0 \end{bmatrix}, \\
C &= \begin{bmatrix} 0 & -\Phi^{-1}D_{c2}C_{p2} & 0 & \Phi^{-1}C_{c2} & 0 \end{bmatrix}, \\
D &= \begin{bmatrix} -\Phi^{-1}D_{c2}D_{p2} & -\Phi^{-1}D_{c2} & \Phi^{-1}D_{c2} \end{bmatrix}, \\
L &= \begin{bmatrix} -\Psi^{-1}C_{p1} & \Psi^{-1}D_{i1}\Phi^{-1}D_{c2}C_{p2} & -\Psi^{-1}D_{p1} & C_{c1} \\ -\Psi^{-1}D_{i1}\Phi^{-1}C_{c2} & -\Psi^{-1}C_{i1} \end{bmatrix}, \\
H_1 &= \begin{bmatrix} \Psi^{-1}D_{i1}\Phi^{-1}D_{c2}D_{p2} & \Psi^{-1}D_{i1}\Phi^{-1}D_{c2} & -\Psi^{-1}D_{i1}\Phi^{-1}D_{c2} \end{bmatrix}, \\
H_2 &= [-\Psi^{-1}D_{p1}].
\end{aligned} \tag{7.5}$$

Let a feedforward decoupler of the same dimension as the system (7.1)-(7.3)

be of the form:

$$\dot{x}_{f1}(t) = A_{f1}x_{f1}(t) + B_{f1}u_2(t) \quad (7.6)$$

$$f_1(t) = C_{f1}x_{f1}(t) + D_{f1}u_2(t) \quad (7.7)$$

where the matrices $(A_{f1}, B_{f1}, C_{f1}, D_{f1})$ are to be determined.

Denote T_{e_1w} as the transfer function matrix from w to e_1 . The H_∞ control design for the feedforward decoupler is then stated as: *Given a positive scalar, γ , design a feedforward controller of the form (7.6)-(7.7) that stabilizes the system such that $\|T_{e_1w}\|_\infty < \gamma$. Further, γ is to be minimized.*

Denote $\xi = \begin{bmatrix} x^T & x_{f1}^T \end{bmatrix}^T$. It follows from (7.1)-(7.3) and (7.6)-(7.7) that

$$\dot{\xi}(t) = \bar{A}\xi(t) + \bar{B}w(t) \quad (7.8)$$

$$e_1(t) = \bar{C}\xi(t) + \bar{D}w(t) \quad (7.9)$$

where

$$\bar{A} = \begin{bmatrix} A + B_2D_{f1}C & B_2C_{f1} \\ B_{f1}C & A_{f1} \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_1 + B_2D_{f1}D_1 \\ B_{f1}D_1 \end{bmatrix}, \quad (7.10)$$

$$\bar{C} = \begin{bmatrix} L + H_2D_{f1}C & H_2C_{f1} \end{bmatrix}, \quad \bar{D} = H_1 + H_2D_{f1}D_1. \quad (7.11)$$

Given a scalar $\gamma > 0$, the system is stable with H_∞ disturbance attenuation γ if and only if there exists a real constant matrix $X = X^T > 0$, such that [89]

$$\begin{bmatrix} \bar{A}^T X + X \bar{A} & X \bar{B} & \bar{C}^T \\ \bar{B}^T X & -\gamma I & \bar{D}^T \\ \bar{C} & \bar{D} & -\gamma I \end{bmatrix} < 0. \quad (7.12)$$

Define

$$U_k = SA^T R + SC^T D_{f_1}^T B_2^T R + \Lambda C_{f_1}^T B_2^T R + SC^T B_{f_1}^T \Xi^T + \Lambda A_{f_1}^T \Xi^T, \quad (7.13)$$

$$V_k = D_{f_1}^T B_2^T R + B_{f_1}^T \Xi^T, \quad (7.14)$$

$$Z_k = D_{f_1} CS + C_{f_1} \Lambda^T. \quad (7.15)$$

Theorem 7.2.1. *Consider the system (7.1)-(7.2). There exists a full order feedforward controller in the form of (7.6)-(7.7) that asymptotically stabilizes the system (7.1)-(7.2) and $\|T_{e1w}\|_\infty < \gamma$ if there exist matrices $R > 0$, $S > 0$, D_{f_1} , U_k , V_k , and Z_k such that the following LMI holds:*

$$\begin{bmatrix} A^T R + RA + C^T V_k + V_k^T C & * & * & * & * & * \\ U_k + A + B_2 D_{f_1} C & SA^T + AS + Z_k^T B_2^T + B_2 Z_k & * & * & * & * \\ B_1^T R + D^T V_k & B_1^T + D^T D_{f_1}^T B_2^T & * & * & * & * \\ L + H_2 D_{f_1} C & LS + H_2 Z_k & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ -\gamma I & * & * & * & * & * \\ H_1 + H_2 D_{f_1} D & -\gamma I & * & * & * & * \end{bmatrix} < 0. \quad (7.16)$$

where ‘*’ denotes entries that can be deduced from the symmetry of the matrix.

In this situation, the controller parameters of (7.6)-(7.7) can be given as

$$\begin{aligned} C_{f_1} &= (Z_k - D_{f_1} CS) \Lambda^{-T} \\ B_{f_1} &= \Xi^{-1} (V_k^T - RB_2 D_{f_1}) \\ A_{f_1} &= \Xi^{-1} (U_k^T - RAS - RB_2 D_{f_1} CS - RB_2 C_{f_1} \Lambda^T - \Xi B_{f_1} CS) \Lambda^{-T} \end{aligned} \quad (7.17)$$

Proof First, partition X and X^{-1} as [90]

$$X^{-1} = \begin{bmatrix} S & \Lambda \\ \Lambda^T & U \end{bmatrix}, X = \begin{bmatrix} R & \Xi \\ \Xi^T & V \end{bmatrix} \quad (7.18)$$

where $R, S, \Lambda, \Xi \in \mathcal{R}^{2n \times 2n}$. It is easy to show from (7.18) that $\Xi \Lambda^T = I - RS$.

Let

$$\Omega = \begin{bmatrix} I & S \\ 0 & \Lambda^T \end{bmatrix}. \quad (7.19)$$

Pre and post-multiplying (7.12) by $\text{diag}\{\Omega^T, I, I\}$ and $\text{diag}\{\Omega, I, I\}$, respectively, we have

$$\begin{bmatrix} \Omega^T \bar{A}^T X \Omega + \Omega^T X \bar{A} \Omega & \Omega^T X \bar{B} & \Omega^T \bar{C}^T \\ \bar{B}^T X \Omega & -\gamma I & \bar{D}^T \\ \bar{C} \Omega & \bar{D} & -\gamma I \end{bmatrix} < 0 \quad (7.20)$$

The above matrix inequality is not convex in the controller parameters which are contained in $\bar{A}$. To resolve this non-convexity problem, we shall introduce the variables (7.13)-(7.15) [90]. In view of Ω, X and $\bar{A}$ in (7.19), (7.18) and (7.10), respectively, it can be verified that

$$\begin{aligned} \Omega^T \bar{A}^T X \Omega + \Omega^T X \bar{A} \Omega &= \\ & \begin{bmatrix} A^T R + RA + C^T V_k + V_k^T C & A^T + C^T D_{f_1}^T B_2^T + U_k^T \\ U_k + A + B_2 D_{f_1} C & SA^T + AS + Z_k^T B_2^T + B_2 Z_k \end{bmatrix}, \\ \bar{B}^T X \Omega &= \begin{bmatrix} B_1^T R + D^T V_k & B_1^T + D^T D_{f_1}^T B_2^T \end{bmatrix}, \\ \bar{C} \Omega &= \begin{bmatrix} L + H_2 D_{f_1} C & LS + H_2 Z_k \end{bmatrix}, \end{aligned} \quad (7.21)$$

where U_k, V_k and Z_k are defined as in (7.13)-(7.15). Thus, (7.16) follows directly

from (7.20). This completes the proof.

7.3 System Models and Design Results

7.3.1 System Models

$P_1(s)$ and $P_2(s)$ are from the same batch of products, therefore they have similar dynamics as shown in Fig. 7.4. The transfer functions for $P_1(s)$ and $P_2(s)$ are described by the following parameters:

$$\begin{aligned} \text{Zeros} = & 10^5 \cdot [2.5133, -0.0267 \pm 1.1621i, -0.0138 \pm 0.6910i \\ & -0.0440 + 0.4376i], \end{aligned} \quad (7.22)$$

$$\begin{aligned} \text{Poles} = & 10^5 \cdot [-2.5133, -0.0251 \pm 1.2564i, -0.0151 \pm 0.7538i \\ & -0.0118 \pm 0.6565i, -0.0260 \pm 0.3761i], \end{aligned} \quad (7.23)$$

$$\text{Gain} = 1.2335 \times 10^9. \quad (7.24)$$

Fig. 7.5 shows the interaction between the two microactuators. It is the frequency response of the measured displacement of the first microactuator, $P_1(s)$, when the second microactuator, $P_2(s)$, is injected with a swept sinusoidal signal. The interaction model $P_{int\ 1}(s)$ is described by the following:

$$\text{Zeros} = [6.7293 \times 10^{18}, -1590.3 \pm 86988i, -492.52 \pm 46954i], \quad (7.25)$$

$$\text{Poles} = [-1885 \pm 94229i, -1102.7 \pm 73505i, -439.82 \pm 43980i], \quad (7.26)$$

$$\text{Gain} = -9.0949 \times 10^{-13}. \quad (7.27)$$

Based on the two similar microactuator plants, we designed a simple lag control with a notch filter to stabilize the individual microactuator loops. $C_1(s)$ and $C_2(s)$

Figure 7.4: Frequency response of PZT microactuator.

Figure 7.5: Frequency response of interaction P_{int1} .

are described by the following parameters

$$Zeros = 10^4 \cdot [-0.1267 \pm 7.0360i], \quad (7.28)$$

$$Poles = 10^4 \cdot [-5.2779 \pm 5.3845i, -0.0063], \quad (7.29)$$

$$Gain = 5.049 \times 10^4. \quad (7.30)$$

7.3.2 Design Results

The full order feedforward decoupler $F_1(s)$ designed by solving (7.16) is 34th-order and the minimal achievable γ is given by $\gamma = 0.00739$.

$$\begin{aligned}
 F_1(s) = & -5.637s^{34} - 6.905 \times 10^{13}s^{33} - 4.366 \times 10^{19}s^{32} - 1.663 \times 10^{25}s^{31} \\
 & -5.575 \times 10^{30}s^{30} - 1.454 \times 10^{36}s^{29} - 3.3 \times 10^{41}s^{28} - 6.629 \times 10^{46}s^{27} \\
 & -1.141 \times 10^{52}s^{26} - 1.827 \times 10^{57}s^{25} - 2.523 \times 10^{62}s^{24} - 3.296 \times 10^{67}s^{23} \\
 & -3.757 \times 10^{72}s^{22} - 4.075 \times 10^{77}s^{21} - 3.899 \times 10^{82}s^{20} - 3.556 \times 10^{87}s^{19} \\
 & -2.88 \times 10^{92}s^{18} - 2.227 \times 10^{97}s^{17} - 1.532 \times 10^{102}s^{16} - 1.008 \times 10^{107}s^{15} \\
 & -5.879 \times 10^{111}s^{14} - 3.291 \times 10^{116}s^{13} - 1.618 \times 10^{121}s^{12} - 7.664 \times 10^{125}s^{11} \\
 & -3.138 \times 10^{130}s^{10} - 1.244 \times 10^{135}s^9 - 4.15 \times 10^{139}s^8 - 1.354 \times 10^{144}s^7 \\
 & -3.534 \times 10^{148}s^6 - 9.209 \times 10^{152}s^5 - 1.731 \times 10^{157}s^4 - 3.426 \times 10^{161}s^3 \\
 & - 3.669 \times 10^{165}s^2 - 5 \times 10^{169}s + 5.839 \times 10^{171} \quad (7.31)
 \end{aligned}$$

$$\begin{aligned}
 & s^{34} + 1.827 \times 10^{10}s^{33} + 3.181 \times 10^{17}s^{32} + 2.229 \times 10^{26}s^{31} + 2.545 \times 10^{32}s^{30} \\
 & +1.007 \times 10^{38}s^{29} + 3.003 \times 10^{43}s^{28} + 7.411 \times 10^{48}s^{27} + 1.427 \times 10^{54}s^{26} \\
 & +2.568 \times 10^{59}s^{25} + 3.755 \times 10^{64}s^{24} + 5.275 \times 10^{69}s^{23} + 6.226 \times 10^{74}s^{22} \\
 & +7.097 \times 10^{79}s^{21} + 6.953 \times 10^{84}s^{20} + 6.585 \times 10s^{19} + 5.43 \times 10^{94}s^{18} \\
 & +4.332 \times 10^{99}s^{17} + 3.024 \times 10^{104}s^{16} + 2.047 \times 10^{109}s^{15} + 1.21 \times 10^{114}s^{14} \\
 & +6.96 \times 10^{118}s^{13} + 3.465 \times 10^{123}s^{12} + 1.688 \times 10^{128}s^{11} + 6.993 \times 10^{132}s^{10} \\
 & +2.858 \times 10^{137}s^9 + 9.647 \times 10^{141}s^8 + 3.253 \times 10^{146}s^7 + 8.595 \times 10^{150}s^6 \\
 & +2.328 \times 10^{155}s^5 + 4.428 \times 10^{159}s^4 + 9.185 \times 10^{163}s^3 + 9.954 \times 10^{167}s^2 \\
 & + 1.444 \times 10^{172}s + 9.038 \times 10^{173}
 \end{aligned}$$

$F_1(s)$ is discretized in Matlab using a 40 kHz sampling rate to obtain $F_1(z)$. For comparison, we have also designed a feedforward decoupler using the direct calculation

method, where

$$F_{dec}(s) = -\frac{P_{\text{int } 1}(s)}{P_1(s)}. \quad (7.32)$$

Due to an unstable zero in $P_1(s)$, we have to design $F_d(s)$ using the curve fitting method against the frequency response of $F_{dec}(s)$. Then, $F_d(z)$ is obtained by discretizing $F_d(s)$ in Matlab using a 40 kHz sampling rate. The parameters for $F_d(z)$ are given by

$$\begin{aligned} \text{Zeros} = & [-0.1120 \pm 0.9873i, 0.5079 \pm 0.7893i, \\ & -0.4077 \pm 0.8472i, -0.7126 \pm 0.2857i, \\ & 0.0736 \pm 0.5584i, -0.8167, -0.1921], \end{aligned} \quad (7.33)$$

$$\begin{aligned} \text{Poles} = & [-0.8490 \pm 0.2248i, 0.4496 \pm 0.8822i, \\ & -0.6206 \pm 0.6215i, -0.2359 \pm 0.8990i, \\ & -0.1448 \pm 0.8577i, -0.0604 \pm 0.0212i], \end{aligned} \quad (7.34)$$

$$\text{Gain} = -0.0055. \quad (7.35)$$

Since $F_d(z)$ is a 12th-order plant, we reduce the order of $F_1(z)$ using the **balmr** (balanced model reduction) method in Matlab into a 12th-order plant to obtain $F_{1(\text{red})}(z)$ for a fair comparison with the direct calculation method. The parameters for $F_{1(\text{red})}(z)$ are given by

$$\begin{aligned} \text{Zeros} = & [-0.9669 \pm 0.1481i, -0.4898 \pm 0.6399i, \\ & -0.1811 \pm 0.8806i, 0.5887 \pm 0.7840i, \\ & 0.3720 \pm 0.7483i, 0.0480, -1.603 \times 10^{-6}], \end{aligned} \quad (7.36)$$

$$\begin{aligned} \text{Poles} = & [0.5516 \pm 0.7540i, 0.4470 \pm 0.8794i, \\ & -0.2198 \pm 0.9543i, -0.6696 \pm 0.6769i, \end{aligned}$$

$$-0.9143 \pm 0.2239i, 0, 3.146 \times 10^{-8}], \quad (7.37)$$

$$\text{Gain} = -0.0051. \quad (7.38)$$

The bode plots of $F_d(z)$, $F_1(z)$ and $F_{1(red)}(z)$ are illustrated in Fig. 7.6.

Figure 7.6: Frequency response of feedforward decoupler.

Fig. 7.7, Fig. 7.8 and Fig. 7.9 show the magnitude responses of the transfer functions from d_{i_2} to e_1 without decoupling, and with decouplers $F_d(z)$, $F_1(z)$ and $F_{1(red)}(z)$, when there are no parameter variations in $P_1(z)$ and $P_2(z)$, and ± 200 Hz variation to the 12 kHz mode of $P_1(z)$ and $P_2(z)$, respectively. It can be seen from Fig. 7.7, Fig. 7.8 and Fig. 7.9 that $F_1(z)$ has better interaction decoupling property compared to $F_d(z)$ from 560 Hz onwards. However, after model order reduction, the performance of $F_{1(red)}(z)$ has been slightly degraded although it is still able to keep the interaction low from 100 Hz to 20 kHz.

Figure 7.7: Magnitude response from d_{i_2} to e_1 with nominal $P_1(z)$ and $P_2(z)$.

Figure 7.8: Magnitude response from d_{i_2} to e_1 when the 12 kHz resonance modes of $P_1(z)$ and $P_2(z)$ vary by +200 Hz.

Figure 7.9: Magnitude response from d_{i2} to e_1 when the 12 kHz resonance modes of $P_1(z)$ and $P_2(z)$ vary by -200 Hz.

Fig. 7.10, Fig. 7.11 and Fig. 7.12 show the magnitude responses of the transfer functions from d_{o2} (or n_2 , or r_2) to e_1 without decoupling, as well as with $F_d(z)$, $F_1(z)$ and $F_{1(red)}(z)$, when there are no parameter variations in $P_1(z)$ and $P_2(z)$, and ± 200 Hz variation to the 12 kHz mode of $P_1(z)$ and $P_2(z)$, respectively. Only $F_{1(red)}(z)$ achieves an attenuation of at least -65 dB from 100 Hz to 20 kHz even when the resonance frequency of the 12 kHz mode varies by ± 200 Hz.

Fig. 7.13 shows the transient response of the PES of $P_1(z)$, e_1 , when a step input is injected into $P_2(z)$. The step input disturbance is almost decoupled completely when $F_{1(red)}(z)$ is used and the performance is much better than $F_d(z)$. The time response of e_1 due to the noise and disturbance (see Section 2.3.2) from the second microactuator loop is shown in Fig. 7.14. From Fig. 7.14, it is clearly seen that $F_{1(red)}(z)$ is much better in rejecting the interaction caused by the noise and disturbances from the second microactuator loop compared to $F_d(z)$. Table 7.1 shows the 3σ of the PES of the first microactuator loop. The interaction rejection capability is improved by 60% when the H_∞ feedforward decoupler, $F_{1(red)}(z)$ is used.

Figure 7.10: Magnitude response from d_{o2} to e_1 with nominal $P_1(z)$ and $P_2(z)$.

Figure 7.11: Magnitude response from d_{o2} to e_1 when the 12 kHz resonance modes of $P_1(z)$ and $P_2(z)$ vary by +200 Hz.

Figure 7.12: Magnitude response from d_{i_2} to e_1 when the 12 kHz resonance modes of $P_1(z)$ and $P_2(z)$ vary by -200 Hz.

Figure 7.13: Time response of e_1 when a step input is injected in the reference of $P_2(z)$.

Figure 7.14: Time response of e_1 when disturbance and noise are injected into the second microactuator loop.

Table 7.1: 3σ of e_1 Due to Disturbances and Noise in P_2 Servo Loop When Resonance Frequency of the 12 kHz Mode in P_1 and P_2 Vary

Frequency change (Hz)	0	+200	-200
Without decoupling (nm)	4.17×10^{-2}	4.17×10^{-2}	4.21×10^{-2}
With $F_d(z)$ (nm)	5.90×10^{-3}	5.90×10^{-3}	5.46×10^{-3}
With $F_{1(red)}(z)$ (nm)	2.33×10^{-3}	2.33×10^{-3}	2.11×10^{-3}

7.4 Conclusion

In this chapter, we have formulated the design of a feedforward decoupling problem into a H_∞ control problem. Using the formulation for a H_∞ feedforward decoupler, we designed a feedforward decoupler to attenuate the interaction/disturbance caused by the second microactuator servo control loop on the first microactuator. Similarly, a H_∞ feedforward decoupler can also be designed for the second microactuator loop to attenuate the interaction from the first microactuator. Based on the simulation done using the reduced order H_∞ feedforward decoupler, it is shown that this design method is effective in reducing the effect of the disturbances and noise from the second microactuator loop acting on the first microactuator feedback loop.

Chapter 8

H_2 Control for Head Positioning in Axial and Radial Dimensions for Self-servo Track Writing

As one of the cost of HDD's production is the requirement of clean room environment, we shall study the process of self-servo track writing (SSTW), which eliminates this requirement. Ideally, the servo tracks written are circular and concentric to the spindle axis. Nevertheless, servo tracks may become non-circular due to various disturbances and noise. The position error in one track will affect the accuracy of the next track and this error when compounded results in the radial error propagation. Therefore, a control design will be given here to contain the error propagation.

8.1 Introduction

SSTW regenerates timing and radial information from previously written track using the existing HDA GMR head. The external equipment is no longer needed in

servopattern writing and thus no open access is required for HDA and servo track writing does not have to be carried out in clean room environment. However, along with the SSTW, many new challenges arise such as radial error propagation which causes the SSTW process to fail.

In SSTW [38, 91], due to vibrations and noise the servo controller causes the actuator to follow the resulting non-circular trajectory in a next burst writing step, so that the new bursts are written at locations reflecting the errors present in the preceding track via the closed-loop response of the servo loop, as well as in the present track. Consequently, each step in the process carries a “memory” of all preceding track shape errors. This “memory” depends on the particular closed-loop response of the servo loop. Because of the interdependency of propagation tracks, track shape errors may be amplified from one track to the next through the closed-loop response when writing the propagation tracks. Thus self-servo writing systems must provide a means of accurately writing servo-patterns while controlling the propagation of track shape errors. Therefore, error propagation containment is significantly proposed and studied [39, 91]. The target of preventing error propagation is to reject the track shape error due to track non-circularity recorded in propagated tracks so that the circular concentric tracks are achieved in every propagation trace.

One performance for self-propagation is referred to as track mis-registration or TMR, the total amount of random fluctuation about the desired track location. One of the largest sources of TMR is the turbulent wind generated by the spinning disks during servo track writing operation. TMR is used to judge the required accuracy of servo-pattern placement and thus to scale the disk capacity. Note that to achieve a high TPI, lots of work is dedicated to reduce TMR through any possible ways such as vibration analysis and control [92], high bandwidth servo control and dual-stage actuators [63]. [69] treated the control design problem to minimize TMR as an optimal H_2 control problem and a Riccati equation approach was used to design

the H_2 controller. In [93], to meet the robustness requirement against unmodelled high frequency dynamics of the VCM actuator, an H_∞ constraint is satisfied when minimizing the H_2 performance of the nominal system.

In this chapter we describe the SSTW process with a two-dimensional (2-D) model [94]. Then the error propagation containment problem of the SSTW process is formulated as a 2-D fundamental stabilization problem. The 2-D stabilizing control [90] is able to prevent the error propagation and a feedforward control as in [47, 91] to deal with the error propagation is not required. Furthermore, the TMR minimization problem of the SSTW process is formulated as a 2-D H_2 control problem. Linear matrix inequality (LMI) approaches are applied to design the 2-D controllers [90]. The designed 2-D H_2 controller is able to prevent the error propagation and minimize the TMR as well.

We note that in [47], based on a pre-designed feedback control, a feedforward control is designed to contain the error propagation and in [41], an observer-based 2-D control is designed by using 1-D pole placement method. Different from these ad-hoc methods, this chapter proposes a systematic 2-D control design method. With the stored error information of the preceding track, the 2-D control scheme is realizable and no additional channel to the plant is required. Obtained from the designed 2-D controller, the 1-D controller in the axial dimension is implemented on a laser doppler vibrometer(LDV)-dSpace experiment platform and its performance is evaluated through both simulations and experiments. The main contributions in this chapter is listed as follows:

- i. SSTW process is modelled as a 2-D model associated with disturbance and noise models;
- ii. Based on the 2-D model, the error propagation problem in SSTW process is formulated as a 2-D stabilization problem;

- iii. The minimization problem of position error is formulated as a 2-D H_2 control problem;
- iv. 2-D H_2 controller is designed for the SSTW process, and simulation and experiment results demonstrate that the designed controller is applicable.

8.2 Modeling and Control Formulation for Self-servo Track Writing in Axial and Radial Dimensions

8.2.1 Error Propagation in SSTW

Fig. 8.1 shows the SSTW process with the axial and radial direction indicated. During the process, SSTW generates radial information progressively to deploy servo-pattern. The radial error in Track N is inevitably compounded to the tracks that follow. This leads to the error propagation in SSTW. Fig. 8.2 shows the SSTW servo loop with disturbances and noises. In Fig. 8.2, $C(z)$ and $P(s)$ represent the controller and plant respectively. d_i represents all torque disturbances, including D/A quantization noise, poweramp noise, and any torque due to air-turbulence impinging upon the actuator, the R/W head suspension, or the slider. d_o represents disturbances that are due to non-repeatable disk motions. n represents the PES demodulation noise, which includes actual electrical noise and A/D quantization noise. $D_i(s)$, $D_o(s)$, and $N(s)$ are the models of d_i , d_o and n , and $w_i (i = 1, 2, 3)$ is a white noise with variance 1.

8.2 Modeling and Control Formulation for Self-servo Track Writing in Axial and Radial Dimensions

Figure 8.1: SSTW process.

Figure 8.2: SSTW servo loop with disturbances and noise models.

In this chapter, the frequency response of the VCM plant under investigation is shown in Fig. 8.3, and its transfer function $P(s)$ is described by the following zeros, poles and gain, identified by using curve-fitting method to the measured frequency response:

$$\begin{aligned} \text{Zeros} = 10^4 \cdot [3.4558, -0.6158 \pm 8.7749i, 0.7540 \pm 4.9697i, \\ -0.1960 \pm 3.2614i, -0.9425 \pm 1.6324i] \end{aligned} \quad (8.1)$$

$$\begin{aligned} \text{Poles} = 10^5 \cdot [-0.2513, -0.3456, -0.0726 \pm 1.0342i, -0.0090 \pm 0.5968i, \\ -0.0377 \pm 0.3751i, -0.0283 \pm 0.2813i, -0.0011 \pm 0.0036i] \end{aligned} \quad (8.2)$$

$$\text{Gain} = 5.8987 \times 10^{12}. \quad (8.3)$$

Figure 8.3: Frequency response of VCM actuator.

The noise and disturbance models of the system are given in Section 2.3.2 of Chapter 2. Fig. 8.4 shows PES NRRO in the time domain and its sigma value versus track number when a 1-D feedback controller $C(z)$ is used in the closed loop. The error propagation mentioned above is clearly observed. Thus, the error propagation problem of the SSTW must be addressed in servo control design.

Figure 8.4: PES NRRO and its sigma values (standard deviation) versus track number during propagation.

8.2.2 Modeling and Control Formulation in Axial and Radial Dimensions

For the 2-D modeling, the following notations are used:

T_p : one revolution period of spindle.

T_s : sampling time.

$i = 0, 1, 2, \dots, L$: the i th track.

$K = T_p/T_s$: sector number.

$j \cdot T_s$: time with $j = 0, 1, 2, \dots, K - 1$.

Then as shown in Fig. 8.5, $y(i, j)$ is the position of the write head at track i in the radial dimension and time $j \cdot T_s$ in the axial dimension and $pes(i, j)$ the position error. $y(i - 1, j)$ (i.e., $y(j - K)$) represents the track profile of the $(i - 1)$ -th track. Similarly, $pes(i - 1, j)$ represents the position error of the $(i - 1)$ -th track. The read head follows the track $y(i - 1, j)$ which is the reference input for the SSTW servo system, i.e., one revolution of $y(i, j)$ becomes the reference of the next written track due to the action of self-servo track writing.

Figure 8.5: SSTW servo loop modeling in two dimensions.

$P(z)$ is the discrete-time model of $P(s)$ by discretizing it with the zero-order-hold method. Denote (A_p, B_p, C_p, D_p) as the state-space realization of $P(z)$. From Fig. 8.5, we have

$$x_p(i, j + 1) = A_p x_p(i, j) + B_p(u(i, j) + d_1(i, j)) \quad (8.4)$$

$$y(i, j) = C_p x_p(i, j) + D_p(u(i, j) + d_1(i, j)) + d_2(i, j) \quad (8.5)$$

$$pes(i, j) = y(i - 1, j) - y(i, j) + n(i, j). \quad (8.6)$$

Denote $(A_{D1}, B_{D1}, C_{D1}, D_{D1})$, $(A_{D2}, B_{D2}, C_{D2}, D_{D2})$, and (A_N, B_N, C_N, D_N) respectively the state-space realizations of $D_i(z)$, $D_o(z)$, and $N(z)$ and x_{Di} , x_{Do} and x_N their corresponding state vectors. Let

$$\begin{aligned} x^h(i + 1, j) &= y(i, j), \quad x^h \in \mathcal{R}^{n_h}, \\ x^v(i, j + 1)^T &= [x_p(i, j + 1)^T \ x_{D1}(i, j + 1)^T \ x_{D2}(i, j + 1)^T \ x_N(i, j + 1)^T], \quad x^v \in \mathcal{R}^{n_v}, \\ e(i, j) &= pes(i, j), \quad w(i, j)^T = [w_1(i, j)^T \ w_2(i, j)^T \ w_3(i, j)^T], \end{aligned} \quad (8.7)$$

it follows from (8.4)-(8.6) that

8.2 Modeling and Control Formulation for Self-servo Track Writing in Axial and Radial Dimensions

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$$\begin{bmatrix} x^h(i+1, j) \\ x^v(i, j+1) \end{bmatrix} = A \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix} + B_1 w(i, j) + B_2 u(i, j) \quad (8.8)$$

$$y(i, j) = C_1 \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix} + D_{11} w(i, j) + D_{12} u(i, j) \quad (8.9)$$

$$e(i, j) = C_2 \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix} + D_{21} w(i, j) + D_{22} u(i, j) \quad (8.10)$$

where

$$A = \begin{bmatrix} 0 & C_p & D_p C_{D1} & C_{D2} & 0 \\ 0 & A_p & B_p C_{D1} & 0 & 0 \\ 0 & 0 & A_{D1} & 0 & 0 \\ 0 & 0 & 0 & A_{D2} & 0 \\ 0 & 0 & 0 & 0 & A_N \end{bmatrix}, \quad (8.11)$$

$$B_1 = \begin{bmatrix} D_p D_{D1} & D_{D2} & 0 \\ B_p D_{D1} & 0 & 0 \\ B_{D1} & 0 & 0 \\ 0 & B_{D2} & 0 \\ 0 & 0 & B_N \end{bmatrix}, \quad B_2 = \begin{bmatrix} D_p \\ B_p \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (8.12)$$

$$C_1 = \begin{bmatrix} 0 & C_p & D_p C_{D1} & C_{D2} & 0 \end{bmatrix}, \quad (8.13)$$

$$D_{11} = \begin{bmatrix} D_p D_{D1} & D_{D2} & 0 \end{bmatrix}, \quad D_{12} = D_p, \quad (8.14)$$

$$C_2 = \begin{bmatrix} 1 & -C_p & -D_p C_{D1} & -C_{D2} & C_N \end{bmatrix}, \quad (8.15)$$

$$D_{21} = \begin{bmatrix} -D_p D_{D1} & -D_{D2} & D_N \end{bmatrix}, \quad D_{22} = -D_p. \quad (8.16)$$

As such, the SSTW servo loop is modeled as the 2-D Roesser model (8.8)-(8.16) [94] where disturbance and noise models are taken into consideration.

Introduce the following 2-D output feedback controller $C(z_1, z_2)$ [95]:

$$\begin{aligned} x_c(i+1, j+1) &= A_{c1}x_c(i, j+1) + A_{c2}x_c(i+1, j) \\ &\quad + B_{c1}e(i, j+1) + B_{c2}e(i+1, j), \end{aligned} \quad (8.17)$$

$$u(i, j) = C_c x_c(i, j) + D_c e(i, j). \quad (8.18)$$

Remark 8.2.1. *It is not difficult to see from (8.17)-(8.18) that $(A_{c2}, B_{c2}, C_c, D_c)$ is acting along the time direction only, and thus actually it works like 1-D feedback controller we are concerned with conventionally.*

With the model (8.8)-(8.16), the SSTW error propagation problem is formulated as the stabilization problem of a 2-D system, that is to design a dynamic output feedback controller of the form in (8.17)-(8.18) for the model (8.8)-(8.10) with $w(i, j) = 0$ such that the resulting closed-loop SSTW servo system is asymptotically stable. Unlike the design of 1-D feedback plus feedforward control in [47], our approach does not need an additional feedforward controller to prevent the error propagation in the SSTW.

As mentioned, one of the most important performance measures for SSTW is the track mis-registration or TMR, namely the total amount of random fluctuation about the desired track location. TMR is used to judge the required accuracy of positioning. To achieve a high positioning accuracy, one way in servo control is to minimize TMR, which is expressed as the standard deviation of the true PES, i.e.

$$\sigma_{y(i,j)} = \sqrt{\frac{1}{L} \frac{1}{K-1} \sum_{i,j=0}^{L,K-1} y(i,j)^2}. \quad (8.19)$$

Let $T_{yw} : w \rightarrow y$ denote the closed loop system subject to the white noise w . When L and K are large enough, the H_2 norm of T_{yw} is given by [96, 97]

$$\|T_{yw}\|_2 = \sqrt{\frac{1}{L} \frac{1}{K-1} \sum_{i,j=1}^{L,K-1} y(i,j)^2} \quad (8.20)$$

Thus, the control design problem to minimize TMR can be treated as a 2-D H_2 optimal control problem, which can be stated as follows: *For a given constant $\lambda > 0$, find a 2-D output feedback controller of the form in (8.17)-(8.18) for the SSTW plant $P(s)$ such that the closed-loop system is stable and has the specified H_2 performance $\|T_{yw}\|_2 < \lambda$.*

The problem of minimizing the TMR of the SSTW process is thus formulated as a 2-D H_2 control problem. In what follows, a 2-D stabilizing controller will be designed to minimize the track mis-registration. Note that since the 2-D controller stabilizes the system, it also contains the error propagation. Thus, our approach simultaneously addresses the error propagation and TMR minimization problems, which is different from that in [47] where these problems are addressed separately by feedforward and feedback controllers.

8.3 2-D H_2 Controller Design

In this section, we will use an LMI approach to design the 2-D H_2 controller for the SSTW plant such that the closed loop system is stable with error propagation containment and the TMR is minimized as well.

Theorem 8.3.1. [97] *The 2-D H_2 control problem for the plant (8.8)-(8.16) is solvable if there exist matrices $S > 0$, Θ , Λ , Γ , D_c and block-diagonal matrices $X = \text{diag}\{X^h, X^v\}$, $Y = \text{diag}\{Y^h, Y^v\}$, $M = \text{diag}\{M^h, M^v\}$, $H_{11} = \text{diag}\{H_{11}^h, H_{11}^v\} > 0$,*

$H_{22} = \text{diag}\{H_{22}^h, H_{22}^v\} > 0$, and $H_{12} = \text{diag}\{H_{12}^h, H_{12}^v\}$, of appropriate dimensions, such that

$$\begin{bmatrix} H_{11} & * & * & * & * \\ H_{12}^T & H_{22} & * & * & * \\ XA + \Gamma C_2 & \Theta & X + X^T - H_{11} & * & * \\ A + B_2 D_c C_2 & AY^T + B_2 \Lambda & I + M^T - H_{12}^T & Y + Y^T - H_{22} & * \\ C_1 + D_{12} D_c C_2 & C_1 Y^T + D_{12} \Lambda & 0 & 0 & I \end{bmatrix} > 0 \quad (8.21)$$

$$\begin{bmatrix} S & * & * & * \\ XB_1 + \Gamma D_{21} & X + X^T - H_{11} & * & * \\ B_1 + B_2 D_c D_{21} & I + M^T - H_{12}^T & Y + Y^T - H_{22} & * \\ D_{11} + D_{12} D_c D_{21} & 0 & 0 & I \end{bmatrix} > 0 \quad (8.22)$$

$$\text{trace}(S) < \lambda^2 \quad (8.23)$$

are satisfied, where $*$ denotes an entry that can be deduced from the symmetry of the matrix. If the above stated conditions are satisfied, a feasible H_2 controller is given by

$$\begin{bmatrix} x_c^h(i+1, j) \\ x_c^v(i, j+1) \end{bmatrix} = A_c \begin{bmatrix} x_c^h(i, j) \\ x_c^v(i, j) \end{bmatrix} + B_c e(i, j), x_c^h \in R^{n_h}, x_c^v \in R^{n_v} \quad (8.24)$$

$$u(i, j) = C_c \begin{bmatrix} x_c^h(i, j) \\ x_c^v(i, j) \end{bmatrix} + D_c e(i, j) \quad (8.25)$$

with

$$A_c = U^{-1}[\Theta - X(A + B_2 D_c C_2)Y^T - XB_2 C_c V^T - UB_c C_2 Y^T]V^{-T}, \quad (8.26)$$

$$C_c = (\Lambda - D_c C_2 Y^T)V^{-T}, B_c = U^{-1}(\Gamma - XB_2 D_c), \quad (8.27)$$

where $U = \text{diag}\{U^h, U^v\}$ and $V = \text{diag}\{V^h, V^v\}$ satisfy $XY^T + UV^T = M$.

Remark 8.3.1. *The solution of the 2-D controller in the above theorem is in terms of LMIs which can be efficiently solved by convex optimization just like other 1-D and 2-D control problems involving LMIs [59, 90]. In [97], it is proved that if there exist solutions $X, Y, M, D_c, S > 0, \Theta, \Lambda, \Gamma, H_{11}, H_{22}$, and H_{12} to the LMIs (8.21)-(8.23), then $M - XY^T$ is invertible. From $UV^T = M - XY^T$, nonsingular matrices U and V can be computed. Then, the computation of the controller parameters C_c, A_c, B_c can be carried out by solving (8.26) and (8.27).*

With the consideration of Remark 8.2.1, we rewrite the 2-D controller (8.24)-(8.25) equivalently to the form (8.17)-(8.18) so as to easily understand the usual control performance in time dimension of the 2-D controller. Let

$$A_{c1} = \begin{bmatrix} I_{n_h} & 0 \\ 0 & 0 \end{bmatrix} A_c, \quad A_{c2} = \begin{bmatrix} 0 & 0 \\ 0 & I_{n_v} \end{bmatrix} A_c, \quad (8.28)$$

$$B_{c1} = \begin{bmatrix} I_{n_h} & 0 \\ 0 & 0 \end{bmatrix} B_c, \quad B_{c2} = \begin{bmatrix} 0 & 0 \\ 0 & I_{n_v} \end{bmatrix} B_c. \quad (8.29)$$

Then, according to [90], (8.24)-(8.25) is equivalent to (8.17)-(8.18). In the next section, the performance of $(A_{c2}, B_{c2}, C_c, D_c)$, as a 1-D control in the time dimension, will be evaluated.

8.4 Simulation and Experiment Results

8.4.1 Simulation of 2-D Control in SSTW

The simulation block diagram for the 2-D control is shown in Fig. 8.6. The simulation is carried out in Matlab/Simulink. In the simulation, the sector number

$K = 270$, the spindle rotational speed is 7200 rpm, and thus the sampling frequency is $270 \cdot (7200/60) = 32400$ Hz.

Figure 8.6: 2-D controller for SSTW servo loop.

To show the capability of the designed 2-D controller to prevent the error propagation, 100 tracks are propagated. The σ value of PES NRRO is plotted versus the track number in Fig. 8.7, where the error amplitude is oscillating steadily, which implies that error propagation is contained. Additionally, the position error has been minimized in the H_2 norm sense.

For comparison, the 2-D stabilization control without considering the H_2 performance is designed. The σ values of PES NRRO with the stabilization control and the H_2 control are compared in Fig. 8.7 and it is seen that the error is reduced by an average of 60% via the 2-D H_2 control.

In [47], conventional 1-D H_2 feedback control is designed to minimize the error signal, and based on the feedback control a feedforward control is further designed to contain the error propagation. This method is also used here in the control design for the SSTW. The performance is compared with the above two cases in Fig. 8.7. It is seen that the proposed 2-D control method is comparable with the previous control method in [47].

Figure 8.7: Sigma of PES NRRO versus track number.

8.4.2 1-D Evaluation of the Designed 2-D Controller

It is noted that in the 2-D controller (8.17)-(8.18), $(A_{c2}, B_{c2}, C_c, D_c)$ acts in the time dimension, and runs on the same track as a 1-D controller $C(z)$ we have usually considered. Thus in this section we particularly take into account the 1-D controller $C(z) : (A_{c2}, B_{c2}, C_c, D_c)$ for the plant $P(s)$ (8.1)-(8.3) in the time dimension.

Straightforwardly from the designed 2-D controller $C(z_1, z_2) : (A_{c1}, A_{c2}, B_{c1}, B_{c2}, C_c, D_c)$, the 1-D controller $C(z) : (A_{c2}, B_{c2}, C_c, D_c)$ can be obtained. The parameters for the 1-D controller is as follows

$$\begin{aligned} \text{Zeros} = & [0.9958 \pm 0.0096i, -0.4033 \pm 0.8082i, -0.6187 \pm 0.0136i, \\ & 0.7317 \pm 0.6748i, 0.6050 \pm 0.6262i, 0.9981 \pm 0.0173i, \\ & 0.9990, 0.9646, 0.9144, 0.6158, 0.4039, -1.3143], \end{aligned} \quad (8.30)$$

$$\begin{aligned} \text{Poles} = & [-0.7301 \pm 0.5191i, -0.0693 \pm 0.6440i, 0.1961 \pm 0.7394i, \\ & 0.7318 \pm 0.6676i, 0.6305 \pm 0.3163i, 0.9981 \pm 0.0173i, \end{aligned}$$

$$0.9990 \pm 0.0002i, 0.9638, 0.9410, 0.3976, -0.6221], \quad (8.31)$$

$$\text{Gain} = 3.6099. \quad (8.32)$$

and its frequency response is shown in Fig. 8.8, where the notches around 4, 6 and 9 kHz are to suppress the resonances as observed in Fig. 8.3. With the 1-D controller $C(z)$, the simulated frequency responses of the open loop $C(z)P(z)$ and the sensitivity function $S(z) = 1/(1 + C(z)P(z))$ are drawn in Fig. 8.9 and 8.10.

To practically verify the 1-D controller $C(z)$, experiment is done with dSpace 1103 on TMS320C240 DSP board and the LDV is used to measure the displacement of the actuator. The measured open loop frequency response compared with the simulated one is shown in Fig. 8.9, and the crossover frequency is 1.2 kHz, the gain margin 9 dB, and the phase margin 54 deg. Fig. 8.10 also shows the measured sensitivity function with comparison to the simulated one. Additionally, the step response and the control signal of the closed loop are captured from the oscilloscope as a picture shown in Fig. 8.11, which means that the 1-D controller drawn from the designed 2-D controller can work properly.

The results in this subsection show that the 1-D controller $C(z)$ can stabilize the plant $P(s)$ in the time dimension. However as shown in Fig. 8.4, one 1-D feedback controller $C(z)$ is not capable enough to contain error propagation and stabilize the SSTW process modeled in (8.8)-(8.16) in two dimensions. On the basis of the 2-D model, the 2-D controller designed previously can stabilize the SSTW process with plant $P(s)$ as the actuator and minimize the position error simultaneously.

Figure 8.8: Frequency response of the 1-D controller (A_{c2}, B_{c2}, C_c, D_c) obtained from 2-D H_2 controller.

Figure 8.9: Open loop frequency response with the 1-D controller (A_{c2}, B_{c2}, C_c, D_c).

Figure 8.10: Frequency response of the sensitivity function with the 1-D controller (A_{c2} , B_{c2} , C_c , D_c).

Figure 8.11: Step response and control signal (Channel 1: Step reference signal; Channel 2: Output; Channel 3: Control signal).

8.5 Conclusion

This chapter has considered the control of the self-servo track writing process within a 2-D framework. By applying a two-dimensional model, the error propagation containment problem and the TMR minimization problem of the self-servo track writing process has been formulated as a 2-D H_2 control problem. With the stored error information of the preceding track, the adopted 2-D controller is practically applicable. Our approach provides a systematic way of achieving both the error propagation containment and the TMR minimization simultaneously. Simulation results have demonstrated that the error propagation is prevented by the 2-D control scheme, and good positioning accuracy is achieved with the 2-D H_2 control scheme. Also, the time dimensional part, as a 1-D controller, of the designed 2-D controller has been implemented via LDV and dSpace and the implementation results have verified that the 1-D controller can work as usual.

Chapter 9

Conclusions & Future Work

9.1 Conclusions

We had investigated various control strategies for the track following control of several HDD systems (i.e., the HDD, SSTW and STW). We also proposed some solutions to minimize the duration required for writing servo information onto disk tracks and enhance the track following accuracy of the systems.

Due to the dual objectives of the servo control for HDDs where TMR had to be minimized while ensuring robustness against unmodelled uncertainties, an improved mixed H_2/H_∞ control was proposed in Chapter 3. In the new algorithm, additional scaling parameters were introduced to offer additional freedom during optimization and this led to a less conservative design compared to previous research work. Our results had shown a large improvement of the TMR performance.

Higher data transfer rate requires higher spindle rotation speed. This causes high frequency vibrations in the HDDs. As a result of this, previous research using the disturbance observer to reject low frequency disturbances was not applicable. Therefore, we designed a disturbance observer with a phase stabilized controller in

Chapter 4 as a means to attenuate disturbances at dominant frequencies beyond the bandwidth of the HDD servo control loop. The implementation results had illustrated the effectiveness and efficiency of this method.

Several loop shaping servo control algorithms were investigated in order to attenuate the disk induced vibration present in our hybrid STW system. Based on an initial sensitivity function designed for the system using a basic PID controller with notch filters, the controller was fine-tuned using the NLSO method to shape the sensitivity function of the system such that higher attenuation of the disk induced vibrations were achieved in Chapter 5. Simulation and experimental results showed some improvement of the PES 3σ and the resultant controller is robust against frequency variation of the disk induced vibrations.

Besides that, the application of the generalized KYP lemma to shape the sensitivity function to attenuate disturbances at desired frequency ranges was also investigated in Chapter 6. Since the generalized KYP lemma involves a matrix variable that is not necessarily positive definite, the Schur complement could not be applied to convexify the controller design. Hence, the Youla parameterization approach was used to parameterize the closed loop function. The FIR filter was used to design the controller so that the sensitivity shaping problem could be written in LMIs. We had demonstrated that this method could successfully design controllers that attenuate narrow-band disturbances at low or high frequencies as desired. In addition to that, we had combined the generalized KYP lemma with H_2 control so that we could achieve both, disturbance attenuation at specified frequency ranges, as well as, TMR minimization. The experimental and simulation results had illustrated the effectiveness of the proposed methods.

The interaction between the microactuators had to be canceled to improve the accuracy of the multiple disks STW. We had formulated an H_∞ feedforward decoupler in Chapter 7 so that the interaction between the microactuators can be de-

coupled or attenuated. Simulation results had shown that the designed feedforward decoupler managed to attenuate the interaction caused by the other microactuator.

To contain the radial error propagation of the SSTW, we had proposed a 2-D H_2 control in Chapter 8, where the PES was minimized based on the distribution of disturbances and noise of the system. The simulation and experimental results based on our proposed method had indicated a marked improvement in the standard deviation of the PES NRRO, and therefore implied higher track density for the HDD.

Finally, we shall end this thesis with some suggestions for possible future work.

9.2 Future Work

9.2.1 Robust Feedforward Decoupler

In Chapter 7, we had proposed a H_∞ feedforward decoupler to cancel the interactions between two microactuators in the hybrid-type STW. Although this method is effective, large parameter variations in the multi-actuator servo track writing system may have severe impact on performance and stability. Therefore, the possibility of designing a robust feedforward decoupler that takes into account the modeling uncertainties in the interaction and plant models should be explored in the future.

9.2.2 Rejection of NRRO Disturbances with Phase Variation

In this thesis, NRRO with known dominant frequencies are dealt with narrow-band attenuation/rejection method and the sources of the NRRO (i.e., noises and disturbances) are modeled according to the magnitude of the power spectrum. However, for better tracking accuracy, phase identification of the NRRO signals should not

be neglected. Since the phases of NRRO disturbances are usually time-variant, relevant research should be carried out. Adaptive control [98–101] can be adopted for phase recognition and automatically redesigns the controller in order to reject the disturbances.

9.2.3 Nonlinear Control & Multisensing Servo for Small Form Factor HDDs

The trend of miniaturization of hard disks' form factor requires miniaturization of sliders as well. Currently, femto sliders with nano-meter level fly height [102–104] have been developed and used successfully to manufacture various products [104] for mobile applications. As fly height is decreased, nonlinear dynamics is expected and this phenomena has been studied in [105]. Hence, nonlinear control methodologies, such as, the sliding mode control [106] and neural network based control [107, 108], should be studied. Moreover, soft intelligent sensors/observers, which detect the disturbances/vibrations, should be investigated. Soft sensors are attractive because they do not require additional sensor circuitry, and therefore do not increase the price of HDDs.

Appendix A

Construction of the matrix K

The derivation of the matrix K is based on interpolation conditions of a sensitivity function, which are caused by the internal stability conditions, as well as by additional interpolation constraints.

Suppose that a real rational plant P has relative degree r , unstable poles p_j of multiplicity $l_j, j = 1, \dots, N_p$, and unstable zeros z_j of multiplicity $m_j, j = 1, \dots, N_z$. Then, for the internal stability condition (condition (i) of Section 5.2.2), the sensitivity function must satisfy the following interpolation and derivative conditions (see e.g., [109, 110]):

$$\begin{aligned} S(p_j) &= S'(p_j) = \dots = S^{(l_j-1)}(p_j) = 0, \quad j = 1, \dots, N_p, \\ S(z_j) &= 1, S'(z_j) = \dots = S^{(m_j-1)}(z_j) = 0, \quad j = 1, \dots, N_z, \\ S(\infty) &= 1, \frac{d}{dz} S(z^{-1})|_{z=0} = \dots = \frac{d^{r-1}}{dz^{r-1}} S(z^{-1})|_{z=0} = 0. \end{aligned} \tag{A-1}$$

In addition, we assume that the interpolation conditions in condition (ii) of Section 5.2.2 are given as

$$S(\lambda_j) = \eta_j, \quad j = 1, \dots, n_e. \tag{A-2}$$

We require that $\{\lambda_j\}_{j=1}^{n_e}$ is a distinct set, and

$$\{\lambda_j\}_{j=1}^{n_e} \cap \left[\{p_j\}_{j=1}^{N_p} \cup \{z_j\}_{j=1}^{N_z} \right] = \emptyset. \tag{A-3}$$

Denoting the total number of the interpolation and derivative conditions in (A-1) and (A-2) by $n+1$, our goal here is, by fixing the structure of the sensitivity function

as

$$S(z) = \frac{z^T \beta}{z^T \alpha}, \quad \alpha \in \mathcal{R}^{n+1}, \quad \beta \in \mathcal{R}^{n+1}, \quad z := [z^n, z^{n-1}, \dots, 1]^T, \quad (\text{A-4})$$

to derive a relation between the vectors α and β so that S in (A-4) satisfies (A-1) and (A-2).

To express the conditions (A-1) and (A-2) in a simpler form, we introduce a function

$$f(z) := S(z^{-1}) = \frac{\hat{z}^T \beta}{\hat{z}^T \alpha}, \quad \hat{z} := [1, z, \dots, z^n]^T. \quad (\text{A-5})$$

Then, using chain rules, the conditions (A-1) and (A-2) are translated into the conditions on f as

$$\begin{aligned} f(1/p_j) &= f'(1/p_j) = \dots = f^{(l_j-1)}(1/p_j) = 0, \quad j = 1, \dots, N_p, \\ f(1/z_j) &= 1, f'(1/z_j) = \dots = f^{(m_j-1)}(1/z_j) = 0, \quad j = 1, \dots, N_z, \\ f(0) &= 1, \quad f'(0) = \dots = f^{(r-1)}(0) = 0, \\ f(1/\lambda_j) &= \eta_j, \quad j = 1, \dots, n_e. \end{aligned} \quad (\text{A-6})$$

We will present, for more general conditions on f , the relation between α and β in A-5 (or equivalently, in (A-4)).

Proposition A.1 *Suppose that a function f has a structure in (A-5) and satisfies the interpolation and derivative conditions*

$$\frac{f^{(k)}(\zeta_j)}{k!} = \omega_{jk}, \quad k = 0, 1, \dots, M_j, \quad j = 0, 1, \dots, l, \quad (\text{A-7})$$

where $\zeta_j \neq \zeta_k$ whenever, $j \neq k$. Here, the degree n in (A-4) is taken as the total

number of conditions in (A-7) minus one:

$$n := \sum_{j=0}^l M_j - 1. \quad (\text{A-8})$$

Then, the vectors α and β satisfies the relation

$$\beta = K\alpha, \quad K := \Gamma^{-1}W\Gamma, \quad (\text{A-9})$$

where Γ and W are defined as follows: Γ is the controllability matrix:

$$\Gamma := [B, AB, \dots, A^n B], \quad (\text{A-10})$$

for the pair (A, B) defined by

$$A := \begin{bmatrix} A_0 & & & \\ & A_1 & & \\ & & \ddots & \\ & & & A_l \end{bmatrix}, \text{ where } A_j := \begin{bmatrix} \zeta_j & & & \\ 1 & \zeta_j & & \\ & \ddots & \ddots & \\ & & 1 & \zeta_j \end{bmatrix} \in C^{M_j \times M_j},$$

$$B := \begin{bmatrix} B_0 \\ B_1 \\ \vdots \\ B_l \end{bmatrix}, \quad \text{where } B_j := \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \notin R^{M_j}, \quad (\text{A-11})$$

and W is a block diagonal matrix:

$$W := \begin{bmatrix} W_0 & & & \\ & W_1 & & \\ & & \ddots & \\ & & & W_l \end{bmatrix}, \text{ where } W_j := \begin{bmatrix} \omega_{j0} & & & \\ \omega_{j1} & \omega_{j0} & & \\ \vdots & \ddots & \ddots & \\ \omega_{jM_j} & \cdots & \omega_{j1} & \omega_{j0} \end{bmatrix}. \quad (\text{A-12})$$

Remark A.2 Note that this proposition assumes neither any analyticity of the interpolant f nor the region that the interpolation points and values lie.

Proof. Since f is supposed to have a structure in (A-5), we have $\hat{z}^T \beta = f(z) \hat{z}^T \alpha$. By taking the first $(M_j - 1)$ -derivatives of this equation and by evaluating them at $z = \zeta_j$, it is straightforward to obtain

$$\Gamma_j \beta = W_j \Gamma_j \alpha, \quad \Gamma_j := [B_j, A_j B_j, \dots, A_j^n B_j], \quad (\text{A-13})$$

where W_j is defined in (A-12). From the structures of A_j and B_j , it is easy to verify that the matrix Γ_j is full row rank. By stacking the equation (A-13) for $j = 0, 1, \dots, l$, we can have

$$\Gamma \beta = W \Gamma \alpha, \quad (\text{A-14})$$

where Γ and W are defined in (A-10) and (A-12), respectively. Since, the matrix Γ_j is full row rank and $\{\zeta_j\}_{j=0}^l$ is assumed to be distinct, the matrix Γ is nonsingular, and thus we have completed the proof.

From this proposition, we have an immediate corollary.

Corollary A.3 Suppose that a sensitivity function S satisfies the conditions (i)-(iv) in Section 5.2.2. Let K be a matrix defined in (A-9) corresponding to the conditions (i) and (ii) (or equivalently, (A-1) and (A-2)). Then, the matrices $\gamma I + K$ and $\gamma I - K$ are invertible.

Proof. Due to the norm condition (iii) in Section 5.2.2, the absolute values of the diagonal elements of the matrix W in (A-12) are less than γ . Noting that W is triangular and that the eigenvalues of K coincide with those of W , we conclude that the matrices $\gamma I + K$ and $\gamma I - K$ are invertible.

Author's Publications

The research work related to this thesis has been reported in the following publications:

1. C. Du, L. Xie, J. N. Teoh and G. Guo, "An improved mixed H_2/H_∞ control design for hard disk drives," *IEEE Transactions on Control Systems Technology*, vol. 13, no. 5, pp. 832-839, 2005.
2. J. N. Teoh, C. Du, G. Guo and L. Xie, "Narrow-band high frequency disturbance attenuation with disturbance estimator and phase stable feedback control," in *Proceedings of the 3rd International Conference on Computational Intelligence, Robotics and Autonomous Systems*, (Singapore), December 2005.
3. C. Du, L. Xie, G. Guo, J. Zhang, Q. Li, B. Hredzak and J. N. Teoh, "A generalized KYP lemma based control design and application for 425 kTPI servo track writing," in *Proceedings of the American Control Conference*, (Minneapolis, Minnesota, USA), pp. 1303-1308, June 2006.
4. J. N. Teoh, C. Du, L. Xie and Y. Wang, "Sensitivity function shaping by nonlinear least-squares optimization for disturbance rejection on hard disk drives," in *Proceedings of The First International Conference on Computer Science & Education*, (Xiamen, China), pp. 126-130, July 2006.
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6. J. N. Teoh, C. Du, L. Xie and Y. Wang, "Nonlinear least-squares optimization of sensitivity function for disturbance attenuation on hard disk drives," *IET Control Theory & Applications*, vol. 1, no. 5, pp. 1364-1369, 2007.
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Other publication that is not included in the thesis:

1. J. N. Teoh, C. Du, G. Guo and L. Xie, "R/W gap delay estimation for self-servo track writing," in *Proceedings of The 9th IEEE International Workshop on Advanced Motion Control*, (Istanbul, Turkey), pp. 149-152, March 2006.

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