Determination of Mn-Zn ferrites' dimension-independent complex permeability and permittivity

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**NANYANG
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UNIVERSITY**

**DETERMINATION OF MN-ZN FERRITES' DIMENSION-
INDEPENDENT COMPLEX PERMEABILITY AND
PERMITTIVITY**

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Summary

In this thesis, three currently available field-circuit coupled methods to determine the Mn-Zn ferrites' dimension-independent, or intrinsic, complex permeability and permittivity are reviewed. The application criterion, the accuracy and the computational efficiency of the methods are compared. A few mathematical measures are taken to improve the currently available methods. Furthermore, to avoid the stray capacitance between the coils in the wire wound ferrite core inductor in the measurements, a set of short-ended coaxial test fixture is designed and manufactured. Experimental results of different field-circuit coupled methods are compared up to 10 MHz.

Two new field-circuit coupled methods derivative from the currently available field-circuit coupled methods are proposed. The two methods determine the two intrinsic values from the impedances of two ferrite core capacitors and two ferrite core inductors respectively. Since the two methods need only one kind of test fixtures, the measurement process is simplified in terms of the number of test fixtures used. Experimental results of the two methods are given. Advantages and limitations of the two methods are disclosed.

After the common features of the field-circuit coupled methods are briefly described, a general field-circuit coupled method is presented, which use a single toroidal Mn-Zn ferrite core as the measurement sample. In the general method, both the ferrite core capacitor and the ferrite core inductor can be constructed in many different ways. Thus the measurement process is simplified in terms of the sample preparation and the choice of the test fixtures. Experiments are performed to verify the method up to 10 MHz.

A general mode matching method is presented to tackle common axisymmetric coaxial discontinuity structures. To efficiently solve the propagation constants of the multi-layered axisymmetric coaxial structures, a 1-D finite difference method is proposed. To overcome the limitations of the general mode matching method which is not applicable for the structures with open-area sections, and to improve the computational efficiency of the two-dimensional (2-D) finite difference time domain (FDTD) method in solving the frequency response of the axisymmetric coaxial structures, a 2-D finite difference frequency domain (FDFD) method is proposed.

By virtue of the general mode matching method and the 2-D FDFD method, the great influence of the air gaps between the material under test (MUT) and the coaxial transmission line test fixture is studied when the coaxial transmission line method is used to measure the Mn-Zn ferrites' intrinsic complex permeability and permittivity. Measures are presented to effectively minimize that influence. The influence of the finite conductivity of the conductors of the coaxial transmission line test fixture is also discussed. A coaxial transmission line test fixture is manufactured. Its calibration issues are discussed in detail. With the proposed measures taken to effectively minimize the influence of the air gaps, experimental results of the intrinsic complex permeability and permittivity of a Mn-Zn ferrite core are obtained by using the manufactured coaxial transmission line test fixture in the frequency range from 10 MHz to 200 MHz.

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List of Abbreviations

1-D	One-Dimensional
2-D	Two-Dimensional
3-D	Three-Dimensional
AC	Alternating Current
ADSL	Asymmetric Digital Subscriber Line
CPU	Central Processing Unit
DC	Direct Current
DD	Domain Decomposition
DUT	Device under Test
FD	Finite Difference
FDFD	Finite Difference Frequency Domain
FDM	Finite Difference Method
FDTD	Finite Difference Time Domain
FEA	Finite Element Analysis
FEM	Finite Element Method
HFCT	High Frequency Current Transducer
ISDN	Integrated Services Digital Network
MMM	Mode Matching Method
Mn-Zn	Manganese-Zinc
MUT	Material under Test
Ni-Zn	Nickel-Zinc
NRW	Nicolson, Ross and Weir
OSL	Open-Short-Load
PEC	Perfectly Electric Conductor
PML	Perfectly Matched Layer
RAM	Random Access Memory
SEM	Scanning Electron Microscopic
SFR	Scattered Field Region
TE	Transverse Electric
TEM	Transverse Electric and Magnetic

List of Abbreviations

TFR	Total Field Region
TM	Tansverse Magnetic
VNA	Vector Network Analyzer

List of Principle Symbols

B	Magnetic flux density
D	Electric flux density
E	Electric field strength
E_r	Radial direction component of the electric field strength
E_z	z direction component of the electric field strength
f	Frequency
H	Magnetic field strength
H_ϕ	Azimuthal direction component of the magnetic field strength
I	Current
J_0	First-kind zero-order Bessel function
J_1	First-kind first-order Bessel function
J_n	First-kind n th-order Bessel function
k	Wavenumber
k_0	Wavenumber of free space
N	Number of coil turns
V	Voltage
Y_0	Second-kind zero-order Bessel function
Y_1	Second-kind first-order Bessel function
Y_n	Second-kind n th-order Bessel function
Z	Impedance
Z_C	Impedance of the capacitor
Z_{DUT}	Impedance of the device under test
Z_L	Impedance of the inductor
ε	Complex permittivity ($\varepsilon = \varepsilon' - j\varepsilon''$)
ε_0	Permittivity of free space
ε'	Real part of complex permittivity
ε''	Negative imaginary part of complex permittivity

List of Principle Symbols

γ	Propagation constant
λ	Wavelength
μ	Complex permeability ($\mu = \mu' - j\mu''$)
μ_0	Permeability of free space
μ'	Real part of complex permeability
μ''	Negative imaginary part of complex permeability
ω	Angular frequency

CHAPTER 1

INTRODUCTION

1.1 Motivation

Ferrites may be defined as magnetic materials composed of oxides containing ferric ions as the main constituent [1]. Ferrites can be classified into the hard ferrites and the soft ferrites according to the coercive field required to demagnetize them from remanence to zero. Hard ferrites are generally permanent magnets. Up to 2000 oersted coercive field can be required to demagnetize hard ferrites from remanence to zero. For soft ferrites, only a small coercive field, typically from 0.05 to 4 oersted, is needed [2]. There are two major groups in soft ferrites, i.e. Ni-Zn and Mn-Zn ferrites. They are developed for a wide range of applications where low loss and high permeability are the main requirements. Compared with the Ni-Zn ferrites, the Mn-Zn ferrites have higher permeability up to the MHz frequency range, but relatively lower resistivity [1]. The bulk DC resistivity is usually 0.1 to 10 $\Omega\cdot\text{m}$ for Mn-Zn ferrites and 10^4 to 10^6 $\Omega\cdot\text{m}$ for Ni-Zn ferrites [3]. The scope of this thesis is restricted to the Mn-Zn ferrites.

The Mn-Zn ferrites have wide applications in the industry. The main applications of the Mn-Zn ferrites can be summarized as follows:

1. Inductors for resonant circuits (approximate frequency range: <0.2 MHz) [1];
2. Inductors for resonant circuits, ferrite antennas for medium and long wave broadcast bands (approximate frequency range: 0.1~2 MHz) [1];
3. Low-power wide-band transformers and low-power pulse transformers (approximate frequency range: up to a few hundred MHz) [1, 4];
4. Application requiring high saturation flux density and low loss at high flux densities in the approximate frequency range from 10 kHz to 1 MHz, in particular

- transformers and chokes for switched mode power supplies and line scanning transformers in television receivers [1];
5. TV picture tube deflection yokes [2];
 6. EMI suppression applications (approximate frequency range: up to a few hundred MHz) [3];
 7. Sensors for detection of weak signals [2] such as the high frequency current transducers (HFCT) used for the detection of partial discharge signals generated in the power apparatuses (approximate frequency range: up to a few tens of MHz);
 8. Coupling and impedance matching components in digital applications, such as integrated services digital network (ISDN) interface transformers [2] and asymmetric digital subscriber line (ADSL) wideband transformers (approximate frequency range: from 2.6 kHz to 1.1 MHz) [5].

All ferrimagnetic and ferromagnetic materials possess to a lesser or greater degree a crystal direction or set of directions in which the magnetization prefers to be oriented [2]. In other words, they are anisotropic and can be described in terms of certain anisotropy constants. But for most soft ferrites, the anisotropy is quite small in absolute magnitude [2]. Thus in this thesis, the Mn-Zn ferrites are considered isotropic for simplicity.

The permeability and the permittivity are the two important parameters used to describe the electromagnetic properties of materials. In the low power level application of the Mn-Zn ferrite cores, the level of the alternating current (AC) magnetic fields inside the core is very low. The Mn-Zn ferrite core operates in a special case of a minor loop-a loop in which the material is not magnetized to saturation. Under that condition, the ferrite core is said to be working in a linear region called Rayleigh region which can be characterized by the initial complex permeability. 200 Gauss is a conservative limit for determination whether the flux density is in the Rayleigh range or not [2]. The initial permeability and the permittivity under that condition are named the intrinsic complex permeability and the intrinsic complex permittivity respectively in this thesis, since they belong to the intrinsic properties of the Mn-Zn ferrites, and they are both complex numbers. Obviously, these two intrinsic values are not dependent on the physical dimensions of the Mn-Zn

ferrite cores. In other words, the two intrinsic values are dimension-independent. In this thesis, the imaginary part of the complex permeability includes the effects of both the dielectric loss and the conductivity.

Over the last a few decades, manufacturers have been devoting great efforts to provide standardized material data sheets, which usually include the initial complex permeability as a function of frequency, the initial permeability as a function of temperature, and the typical B-H loops, to assist the users in comparing the performance of different materials in common applications. For the EMI suppression application Mn-Zn ferrites, the impedance magnitude as a function of frequency is also given, which is measured on a specified bead [3]. But there is still important material information which is not covered by the datasheet, i.e. the intrinsic complex permeability.

Since the early history of the Mn-Zn ferrites, it has been found that combination of the high intrinsic complex permeability and the high intrinsic complex permittivity of the Mn-Zn ferrites may cause dimensional effects [6] such as dimensional resonance and skin effects in the Mn-Zn ferrite-core-based magnetic devices to deteriorate their performance above a few hundred kHz [1]. Also, it is well known that both the imaginary parts of the intrinsic complex permeability and the intrinsic complex permittivity contribute to the losses in the ferrite cores in those devices. Thus both the intrinsic complex permeability and the intrinsic complex permittivity are the critical parameters for the performance examination [7-11] and the optimized design of the Mn-Zn ferrite-core-based magnetic devices. At the low-power level applications, since the Mn-Zn ferrites can be fully characterized by using the intrinsic complex permeability and permittivity, determination of the intrinsic complex permeability and permittivity can not only provide scientists and engineers with important information to incorporate the material into its intended application for more solid design, but also help to monitor the manufacturing process for improved quality control from the point of view of the manufacturers [12, 13].

Two simple examples are used below to further illustrate that, besides the intrinsic complex permeability, the intrinsic complex permittivity is also critical information for the solid design of the Mn-Zn ferrite-core-based magnetic devices. In the first example, we assume that a Mn-Zn ferrite core shown in Figure 1.1 (a) is used for a common mode

choke. The dimensions of the core are $R_1=5$ mm, $R_2=10$ mm, $d=9$ mm. The material is the same as the sample used in Section 3.5. The measurable impedance of the one-turn common mode choke is shown in dashed line in Figure 1.2, which is predicted by using (3.19) and the determined intrinsic complex permeability and permittivity of the material shown in Section 3.5. If we uniformly cut the core into three layers and separate the layers with non-magnetic isolators, e.g. air, as illustrated in Figure 1.1 (b), the magnitude of the measurable impedance of the one-turn common mode choke made from the laminated core is shown in a solid line in Figure 1.2. For the laminated core based common mode choke, the impedance is assumed to be the sum of the impedance of three smaller common mode chokes, each of which is made from the sub-layered cores separately. It can be seen from Figure 1.2 that due to the influence of the intrinsic complex permittivity, the impedances of the two common mode chokes are obviously different above a few hundred kHz. The discrepancy is larger at higher frequencies. The laminated core in Figure 1.1 (b) apparently has better performance than the core in Figure 1.1 (a) in suppression of common mode EMI noise above about 3 MHz. This simple example illustrates that with the intrinsic complex permeability and permittivity known, we can always alter the structure of the core to optimize its performance.

In the second example, the design of a high-frequency wide-band transformer [14] is considered. The simplified equivalent circuit of the transformer and its ideal transmission characteristic are shown in Figure 1.3 and Figure 1.4 respectively [14]. The fundamental function of the cores used in the transformers is to provide the maximum magnetic coupling between the windings of the transformers. A high permeability core results in a large additional flux, most of which is confined to the core and is thus constrained to couple completely with the windings. The higher the permeability, the more complete is the coupling of the total flux [14].

Figure 1.1 Toroidal Mn-Zn ferrite cores under consideration

Figure 1.2 Impedance magnitude versus frequency for the one-turn inductor constructed from the core in Figure 1.1 (solid line: Figure 1.1 (b), dashed line: Figure 1.1 (a))

- E_a : source voltage
- R_a : source resistance
- R_s : total winding resistance referred to the primary
- L_s : total leakage inductance referred to the primary
- R_p : the shunt loss resistance representing the core loss
- L_p : open circuit inductance
- S_1, S_2 : the primary and referred self or stray capacitance respectively
- R_b : load resistance referred to the primary
- r : turn ratio

Figure 1.3 Simplified equivalent circuit of a transformer

Figure 1.4 Ideal transmission characteristic of a wide band transformer

The magnetic core is described by R_p and L_p in the simplified equivalent circuit. From

(3.17), (3.20), (4.2) and (5.9) in [14], R_p and L_p can be expressed as follows:

$$L_p = L_0 \mu_p', \quad (1.1)$$

$$R_p = \omega L_0 \mu_p'', \quad (1.2)$$

$$L_0 = N^2 / C_1, \quad (1.3)$$

$$C_1 = l_e / A_e, \quad (1.4)$$

$$\mu_p' = \mu_e' (1 + \tan^2 \delta_m), \quad (1.5)$$

$$\mu_p'' = \mu_e'' (1 + 1 / \tan^2 \delta_m), \quad (1.6)$$

$$\tan \delta_m = \mu_e'' / \mu_e', \quad (1.7)$$

where C_1 is called the core factor, l_e and A_e are the effective length of the magnetic path in the core and its effective cross sectional area respectively, δ_m is the loss angle, μ_e is the effective, or apparent, complex permeability of the core. From (1.1), (1.2), (1.5) and (1.6), it can be seen that L_p and R_p are related to the effective complex permeability.

Detailed description of the relationship between the performance of the transformer and the elements in the equivalent circuit can be found in [14]. Generally speaking, an ideal low power wide band transformer should have low f_1 , high f_2 , and low mid-band attenuation. The lower frequency of the transmission band, f_1 , depends on the ratio $(R_a/R_b)/L_p$, where $R_a/R_b = (R_a + R_b)/(R_a R_b)$. The smaller the ratio, the lower f_1 . Increasing the turn number of the winding can definitely increase L_p , but also increase R_s which causes the mid-band attenuation. For a given core, R_s is proportional to L_p . Furthermore, R_s/L_p is independent of the number of turns, and dependent on the effective complex permeability and the geometry of the core. The bandwidth of the wide band transformer ($f_2 - f_1$) depends on the ratio L_p/L_s which is independent of the turn number of the windings. Thus increase of the turn number of the windings will not only decrease f_1 and increase the mid-band attenuation, as stated before, but also decrease f_2 . Thus as far as the core is concerned, the obvious options for optimization of the transformer is to increase the effective complex permeability and to optimize the geometry to decrease

$(R_a/R_b)/L_p$ and R_s/L_p , and to increase L_p/L_s . It is noted that the effective complex permeability is related to the intrinsic complex permeability due to the dimensional effects caused by the combination of the high intrinsic complex permeability and the high intrinsic complex permittivity, which will be explained in detail in Chapter 2.

The measured transmission characteristics of a Mn-Zn ferrite core based wide-band transformer [1] are shown in Figure 1.5. The traditional design method described in [1] does not take into account the intrinsic complex permittivity. At the lower end of the frequency band, since the permeability may be regarded as constant and the influence of the permittivity are negligible, the performance can be easily predicted by using the analytical formulae [1]. But the performance of the wide-band transformer at the upper end of the frequency band cannot be predicted in that design method [1].

Figure 1.5 Measured transmission characteristics of a Mn-Zn ferrite core wide-band transformer [1]

From the two simple examples above, it can be seen that accurate prediction of the performance of the Mn-Zn ferrite-core-based magnetic devices in the design process will require the information of both the intrinsic complex permeability and the intrinsic complex permittivity. As long as the intrinsic complex permeability and permittivity of the Mn-Zn ferrite are determined, we can take into account those electrical properties of the Mn-Zn ferrite in the designing process of the magnetic devices. Actually, the performance of the Mn-Zn ferrite-core-based magnetic devices can be conveniently predicted by using the numerical software, such as the finite element analysis (FEA) tools, if the intrinsic complex permeability and permittivity of the Mn-Zn ferrite core are determined [8, 15]. Thus more accurate prediction and optimization of the performance of the magnetic devices can be done.

Due to the dimensional effects, the complex permeability and the complex permittivity measured by using the traditional methods [16] which assume that the ferrite cores are non-dielectric and non-magnetic respectively can cause great errors. Those measured complex permeability and permittivity are actually the effective, or the apparent,

complex permeability and permittivity, which will be explained in Section 2.3 and Section 2.4. Thus methods for measuring both the intrinsic complex permeability and the intrinsic complex permittivity are needed.

1.2 Objectives

The objectives of this research project are listed as follows:

1. To investigate the application criterion, the accuracy and the computational efficiency of the currently available field-circuit coupled methods for determination of the intrinsic complex permeability and permittivity of the Mn-Zn ferrites;
2. To improve the currently available field-circuit coupled methods in terms of the application criterion, the accuracy and the computational efficiency;
3. To propose new field-circuit coupled methods not only with high accuracy and high computational efficiency but also with flexible application;
4. To examine the factors which can greatly influence the accuracy of the results when the coaxial transmission line method is used to measure the intrinsic complex permeability and permittivity of the Mn-Zn ferrites at high frequencies;
5. To propose new methods to improve the accuracy of the coaxial transmission line method when used to measure the intrinsic complex permeability and permittivity of the Mn-Zn ferrites at high frequencies.

1.3 Major Contributions of the Research

The major contributions of this research can be summarized as follows:

1. Careful examination of the currently available field-circuit coupled methods reveals their limitations in terms of application criterion, result accuracy and computational efficiency. A few measures are taken to improve the field-circuit coupled methods. A set of short-ended coaxial test fixtures is manufactured to measure the effective complex permeability of the toroidal Mn-Zn ferrite core.

Thus the stray capacitance between the coils when the wire wound inductor is used to measure the effective complex permeability can be avoided.

2. Two derivative field-circuit coupled methods are proposed to determine the intrinsic complex permeability and permittivity of the Mn-Zn ferrites, which use two ferrite core capacitors and two ferrite core inductors respectively. Since the first derivative method only needs to measure the impedances of the capacitors, and the second derivative method only needs to measure the impedances of the inductors, only one type of test fixtures is needed for each derivative method. Thus the measurement process is simplified in terms of the number of test fixtures used.
3. The features of the field-circuit coupled methods are generalized. A general field-circuit coupled method is presented, with a single toroidal Mn-Zn ferrite core as the measurement sample. Similar to the currently available field-circuit coupled methods, the general method needs to measure the impedances of a capacitor and an inductor made from the sample. But in the general method, both the capacitor and the inductor can be constructed in many different ways. Thus, with the general method, the measurement process is simplified in terms of the sample preparation and the choice of the test fixtures.
4. A general mode matching method is presented to tackle common axisymmetric coaxial discontinuity structures. A set of linear equations are derived, which describes the matching conditions of the transverse fields on the discontinuity interfaces, and can be easily applied to different specific discontinuity structures of that type to solve the field distributions in the structures. A one-dimensional (1-D) finite difference method (FDM) is proposed to efficiently solve the propagation constants of the multi-layer filled axisymmetric coaxial structures.
5. A two-dimensional (2-D) finite difference frequency domain (FDFD) method is presented to versatily solve the axisymmetric discontinuity structures, including the structures containing open-area sections, e.g. open-ended coaxial probes. The accuracy and the versatility of the method are the same as the 2-D finite difference time domain (FDTD) method, while the computational efficiency of

the 2-D FDFD method is much higher in solving the frequency response of the structures.

6. By virtue of the general mode matching method and the 2-D FDFD method, the important factor influencing the accuracy of the results of the coaxial transmission line method when used to measure the intrinsic complex permeability and permittivity of the Mn-Zn ferrites is discussed, which is the air gaps between the sample and the coaxial transmission line test fixture. Measures are proposed to minimize the errors caused by the air gaps. The influence of the finite conductivity of the coaxial transmission line test fixture is also evaluated.
7. A coaxial transmission line test fixture is designed and manufactured to determine the Mn-Zn ferrites' intrinsic complex permeability and permittivity. Its calibration issues are discussed in detail. With the proposed measures taken to effectively minimize the influence of the air gaps, experiments are performed to determine the intrinsic complex permeability and permittivity of a Mn-Zn ferrite core in the frequency range from 10 MHz to 200 MHz.

1.4 Organization of the Thesis

Given below is a brief introduction of what is covered in each chapter in this thesis.

In chapter 1, after motivation of our work is described, the objectives and the major contributions of this research are presented. The outline of this thesis then follows.

In Chapter 2, the phenomenon of the dimensional effects in the Mn-Zn ferrites is explained in detail. Explanations on the difference between the effective, or apparent, complex permeability and permittivity and the intrinsic, or dimension-independent, complex permeability and permittivity are also given. Three currently available field-circuit coupled methods to determine the Mn-Zn ferrites' intrinsic complex permeability and permittivity are reviewed. The application criterion, the accuracy of the results and the computational efficiency of the methods are compared.

In Chapter 3, a few mathematical measures are taken to improve the currently available methods in terms of application criterion, accuracy and computational efficiency. Furthermore, to avoid the influence of the stray capacitance between the coils in the wire

wound ferrite core inductor on the impedance measurement, a set of short-ended coaxial test fixtures is manufactured. The structure of the test fixtures and the calibration issues are presented. Experimental results of different field-circuit coupled methods are compared.

In Chapter 4, two new field-circuit coupled methods derived from the currently available field-circuit coupled methods are presented. Experimental results of the two methods are given. Advantages and limitations of the two methods are also disclosed.

In Chapter 5, after the common features of the field-circuit coupled methods are briefly described, a general field-circuit coupled method is presented, which uses a single toroidal Mn-Zn ferrite core as the measurement sample. Experiments are performed to verify the method.

In Chapter 6, a general mode matching method is presented to tackle common axisymmetric coaxial discontinuity structures. To efficiently solve the propagation constants of the multi-layer filled axisymmetric coaxial structures, a 1-D finite difference method is proposed. Numerical examples are given to verify the methods.

In Chapter 7, to overcome the limitations of the general mode matching method which is not applicable for the structures with open-area sections, and to improve the computational efficiency of the 2-D FDTD method in solving the frequency response of the axisymmetric coaxial structures, a 2-D FDFD method is proposed. The high accuracy and the high computational efficiency of the method are carefully examined through several numerical examples.

In Chapter 8, by virtue of the general mode matching method and the 2-D FDFD method, the influence of the air gaps between the sample and the coaxial transmission line test fixture is studied when the coaxial transmission line method is used to measure the Mn-Zn ferrites' intrinsic complex permeability and permittivity. The measures taken to minimize that influence are described. The influence of the finite conductivity of the conductors of the transmission line is also discussed. A coaxial transmission line test fixture is designed and manufactured. The structures and the calibration issues of the test fixture are presented in detail. Experimental results of the intrinsic complex permeability and permittivity of a Mn-Zn ferrite core measured with the test fixture are presented.

In Chapter 9, the concluding remarks of this thesis and the recommendations for the future research are provided.

CHAPTER 2

THEORETICAL ANALYSIS OF AVAILABLE FIELD-CIRCUIT COUPLED METHODS

2.1 Introduction

In this chapter, the basic concepts of the dimensional effects, the apparent complex permeability, and the apparent complex permittivity are explained in detail in Section 2.2, Section 2.3 and Section 2.4 respectively. It is theoretically revealed that due to the dimensional effects which are caused by the combination of both the high intrinsic complex permeability and the high intrinsic permittivity of the Mn-Zn ferrites, both the apparent complex permeability and the apparent complex permittivity are dimension-dependent [6]. To trace the dimension-independent intrinsic complex permeability and permittivity, a few field-circuit coupled methods are presented in [6, 17, 18]. In Section 2.5, those methods are analyzed thoroughly to disclose their limitations. Computational efficiency and accuracy of those methods are also compared theoretically in Section 2.5.

2.2 Dimensional Effects

Figure 2.1 Scanning electron microscopic (SEM) photograph of a Mn-Zn ferrite sample [19]

In the view of their microstructures as shown in Figure 2.1 [19], the Mn-Zn ferrites are composed of conductive grains, i.e. crystallites, separated by very thin organic binders which are highly resistive [3]. The DC resistivity of the crystallites is in the order of 10^{-3}

$\Omega \cdot m$ [3]. The highly resistive binders can effectively prevent the direct currents to flow inside the ferrites. Thus the DC resistivity of the Mn-Zn ferrites is high. But with the frequency increased, the structure becomes ineffective since the displacement currents can easily go through the equivalent capacitors formed by the conductive grains and the thin insulation layers between them [17]. Thus the Mn-Zn ferrites have very high complex permittivity [3].

Usually the relative complex permittivity of the Mn-Zn ferrites up to a few MHz is in the order of 1×10^5 , and the relative complex permeability in the order of 1000 [15]. Due to the high complex permeability as well as the high complex permittivity of the Mn-Zn ferrites within the kilohertz-to-megahertz frequency range [20, 21], the dimensional effects [6, 22], such as the dimensional resonance due to the electromagnetic wave propagation and the skin effect due to the electromagnetic wave attenuation, could occur, which will be explained in detail in the following paragraphs.

The propagation constant of the electromagnetic wave in any given medium under time harmonic excitations can be expressed as [23]:

$$\gamma = j\omega\sqrt{\mu\varepsilon} = j\omega\sqrt{(\mu' - j\mu'')(\varepsilon' - j\varepsilon'')} = \alpha + j\beta, \quad (2.1)$$

where μ and ε are the complex permeability and the complex permittivity of the medium respectively, α is the attenuation constant, and β is the phase constant. The wavelength λ and the penetration depth δ can be written as below [1]:

$$\lambda = \frac{2\pi}{\beta} = \frac{2\sqrt{2}\pi}{\omega\sqrt{|\mu||\varepsilon| + \mu'\varepsilon'(1 - \tan\delta_m \tan\delta_d)}}, \quad (2.2)$$

$$\delta = \frac{1}{\alpha} = \frac{\sqrt{2}}{\omega\sqrt{|\mu||\varepsilon| - \mu'\varepsilon'(1 - \tan\delta_m \tan\delta_d)}}, \quad (2.3)$$

where

$$\tan\delta_m = \mu''/\mu', \quad (2.4)$$

$$\tan\delta_d = \varepsilon''/\varepsilon'. \quad (2.5)$$

If we assume in a Mn-Zn ferrite $\mu = 2000\mu_0$ and $\varepsilon = 5 \times 10^4 \varepsilon_0$ at 1 MHz, according to (2.2), the wavelength will be approximately 3 cm. A standing wave will be set up, or in other words the dimensional resonance will occur, if the smallest dimension of the cross section of the core (perpendicular to the incident electric or magnetic field) equals half wavelength, i.e. 1.5 cm [1].

In the example mentioned above, the ferrite is assumed to be lossless; the electromagnetic waves will propagate from the surface into the center of the core without attenuation. In reality, the Mn-Zn ferrites will always be lossy. When the electromagnetic waves travel inside the core, it will be attenuated, which results in the phenomena of skin effect. The penetration depth in (2.3) describes how far the electromagnetic waves can travel before they are attenuated to $1/e$ of their original magnitudes. Especially, under the condition $\tan \delta_m \tan \delta_d \gg 1$, we can see from (2.3) that the penetration depth is small, which means that the electromagnetic waves will be concentrated in the region near the surface and the standing waves will not be supported [1]. In other words, there will be no dimensional resonance under that condition.

To further illustrate the concept of the dimensional effects, a mathematical model for an infinite slab [6] is used as below. The infinite slab shown in Figure 2.2 is parallel with the x-y plane. The exciting magnetic field on the two surfaces of the slab, i.e. H_0 , is in the x direction. The solution of the magnetic fields inside the slab can be written as [6]:

$$H(z) = H_0 \frac{\cos j\gamma z}{\cos j\gamma t / 2}, \quad (2.6)$$

where γ is the propagation constant of the electromagnetic wave in the slab, and t is the thickness of the slab.

Figure 2.2 An infinite Mn-Zn ferrite slab parallel with the x-y plane

The intrinsic complex permeability and permittivity of the ferrite slab are taken as the values in Figure 2.3, which are measured from a Mn-Zn ferrite core (RS Components Pte Ltd, Part No. 5978001901) by using Method 3 in Section 3.5. We assume that the thickness of the slab, i.e. t , is 20 mm, and H_0 is 0.01 A/m. It can be seen from Figure 2.3 (a) that the frequency dispersion of the intrinsic complex permeability is a resonant type. Due to the intrinsic ferrimagnetic resonance of the Mn-Zn ferrite [2], the real and imaginary parts of the relative complex permeability increase to the maximum and then decrease to a certain value, as shown in Figure 2.3.

Figure 2.3 Measured intrinsic complex permeability and permittivity of a Mn-Zn ferrite core at 23°C

If we assume that the ferrite slab is lossless by setting μ'' and ϵ'' in Figure 2.3 to be zeros, the half wavelength according to (2.2) is shown in Figure 2.4. In this case, according to

(2.3) the penetration depth is infinity, which means the electromagnetic waves will not be attenuated during the propagation process. The distributions of the magnetic field along the z direction and the magnitude of the magnetic field at $z=0$ in this lossless case are shown in Figure 2.5 and Figure 2.6 respectively. From Figure 2.5 and Figure 2.6, we can see that the dimensional resonance occurs at around 600 kHz and 5 MHz. From Figure 2.4, it can be found that at those dimensional resonance frequencies, the thickness of the slab exactly equals $\lambda/2$ and $3\lambda/2$ respectively.

Figure 2.4 Half wavelength of the electromagnetic waves in the lossless ferrite slab

Figure 2.5 Magnetic field distribution in the lossless ferrite slab

When the actual μ'' and ϵ'' are taken into account, the half wavelength and the penetration depth in the dissipative Mn-Zn ferrite slab are shown in Figure 2.7. The distributions of the magnetic field along the z direction and the magnitude of the magnetic field at $z=0$ are shown in Figure 2.8 and Figure 2.9 respectively. From Figure 2.8 and Figure 2.9, it

can be seen that the distributions of the magnetic fields are rather uniform below 100 kHz. But at higher frequencies, the distributions of the magnetic fields are distorted by the dimensional resonance phenomena and the skin effect. At the frequencies above a few MHz, the magnetic field is concentrated in a skin region near the surface of the slab due to the prominent skin effect.

Figure 2.6 Magnitude of the magnetic field at $z=0$ in the lossless ferrite slab

Figure 2.7 Half wavelength and penetration depth in the dissipative ferrite slab

From Figure 2.7 we can see that at around 600 kHz and 3 MHz, the thickness of slab equals $\lambda/2$ and $3\lambda/2$ respectively. Thus it may be expected that the dimensional resonance could occur at around 600 kHz and 3 MHz. But from Figure 2.8 and Figure 2.9, we can see that the second resonance disappears. The reason is that the penetration depth at the second resonance frequency is too small to support standing waves, as is stated before.

To illustrate the variation of the dimensional effects with the sizes of the ferrite core, the

magnetic distributions at $z=0$ in the dissipative slab with different thickness are shown in Figure 2.10. From Figure 2.10, it can be concluded that dimensional effects are less prominent, and the dimensional resonance frequency is higher in the ferrite core with smaller dimensions [6].

Figure 2.8 Magnetic field distributions along the z direction in the dissipative ferrite slab

Figure 2.9 Magnitudes of the magnetic field at $z=0$ in the dissipative ferrite slab

2.3 Apparent Complex Permeability

Ferrite core inductors as illustrated in Figure 2.11 are commonly used by the researchers and the manufacturers to measure the complex permeability of the Mn-Zn ferrite cores [6, 13, 17, 18]. To avoid the stray capacitance between the coils, short-ended coaxial test fixtures can be used [13], which is equivalent to a one-turn inductor. It is assumed that the inner radius, the outer radius, and the area of the cross section (the shaded area in Figure 2.11(b)) are R_1 , R_2 , and A respectively.

Figure 2.10 Magnitudes of the magnetic field at $z=0$ in the dissipative ferrite slabs with different thickness t

Figure 2.11 Illustration of a ferrite core inductor and the ferrite core

Based on the Maxwell's Equations as follows [16]:

$$\nabla \times \vec{H} = j\omega\epsilon\vec{E}, \quad (2.7)$$

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}, \quad (2.8)$$

The magnetic fields in the ferrite core satisfy the following equation:

$$\nabla \times \nabla \times \vec{H} - \omega^2 \mu\epsilon \vec{H} = 0. \quad (2.9)$$

By utilization of the following equations [16]:

$$\nabla \times \nabla \times \vec{H} = \nabla(\nabla \cdot \vec{H}) - \nabla^2 \vec{H}, \quad (2.10)$$

$$\nabla \cdot \vec{H} = 0, \quad (2.11)$$

Eq. (2.9) can be rewritten as:

$$\nabla^2 \vec{H} - \gamma^2 \vec{H} = 0. \quad (2.12)$$

On the surface of the ferrite core, the magnetic field can be expressed as [7]:

$$H_0(r) = \frac{NI}{2\pi r}, \quad (2.13)$$

where N is the number of the coil turns, I is the current in the coil, and r is the distance between the axis and the concerned point on the surface of the core.

As is stated in Section 2.2, the dimensional effects are due to the electromagnetic wave propagation and the electromagnetic wave attenuation during the propagation process. Neglecting the dimensional effects means setting the propagation constant γ to be zero. The solution to (2.12) and (2.13) with γ being zero can be simply written as:

$$H(r) = \frac{NI}{2\pi r}. \quad (2.14)$$

For the ferrite core with rectangular cross sections, the induced electromagnetic force in the ferrite core inductor can be derived as follows:

$$V = N(-j\omega \int_{R_1}^{R_2} \vec{B} \cdot d\vec{S}) = Nj\omega d \int_{R_1}^{R_2} \mu \frac{NI}{2\pi r} dr = j\omega\mu \frac{N^2 A \ln(R_2 / R_1)}{2\pi(R_2 - R_1)} I. \quad (2.15)$$

Thus the impedance of the ferrite core inductor can be expressed as [16]:

$$Z = j\omega\mu \frac{N^2 A}{L_{ave}} = \omega\mu'' \frac{N^2 A}{L_{ave}} + j\omega\mu' \frac{N^2 A}{L_{ave}} = R + j\omega L, \quad (2.16)$$

where

$$L_{ave} = \frac{2\pi(R_2 - R_1)}{\ln(R_2 / R_1)}. \quad (2.17)$$

From (2.16), the complex permeability can be expressed from the impedance of the ferrite core inductors as follows [16]:

$$\mu = -j \frac{Z L_{ave}}{\omega N^2 A}. \quad (2.18)$$

Figure 2.12 Measured apparent complex permeability of two Mn-Zn ferrite cores at 23°C (Solid lines: Core No.1; Dotted lines: Core No.2)

Table 2.1 Dimensions of the two toroidal Mn-Zn ferrite cores

Core No.	$2 \times R_2$ (mm)	$2 \times R_1$ (mm)	d(mm)
1	12.90	8.05	7.80
2	12.90	8.05	2.05

For non-dielectric and non-conductive magnetic core, the dimensional effects can be ignored. Thus the measured permeability according to (2.18) will reflect the true complex permeability of the magnetic core, as is pointed out in [18]. But for the Mn-Zn ferrite core, the complex permeability according to (2.18) may be different from the true value due to the dimensional effects. In this thesis, for differentiation of the complex permeability according to (2.18) and the true complex permeability of the material, the former is termed the effective, or apparent, complex permeability and the latter the dimension-independent, or intrinsic, complex permeability. Obviously, since the dimensional effects in the ferrite cores with different dimensions are different, the apparent complex permeability is dimension-dependent, as is illustrated in Figure 2.12. The apparent complex permeabilities in Figure 2.12 are measured from two Mn-Zn ferrite cores which are cut from a larger Mn-Zn ferrite core (RS Components Pte Ltd; Part No. 5978001901) by using a high precision diamond blade cutter (ISOMET 1000). The physical dimensions of the two cores are shown in Table 2.1. Details on the measurements of the apparent complex permeability are presented in Section 4.3.

2.4 Apparent Complex Permittivity

To measure the complex permittivity of material, parallel plate capacitors as illustrated in Figure 2.13 are commonly used. By substituting (2.8) into (2.7), we can obtain the equation governing the electric fields in the capacitor as follows:

$$\nabla \times \nabla \times \vec{E} - \omega^2 \mu \epsilon \vec{E} = 0. \quad (2.19)$$

Figure 2.13 A parallel plate capacitor for characterization of complex permittivity of dielectrics

By utilization of the following equations [16]:

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}, \quad (2.20)$$

$$\nabla \cdot \vec{E} = 0, \quad (2.21)$$

Eq. (2.19) can be rewritten as:

$$\nabla^2 \vec{E} - \gamma^2 \vec{E} = 0. \quad (2.22)$$

If the fringing effect can be neglected, the electric field on the lateral surface of the capacitor can be written as:

$$E_0 = \frac{V}{d}. \quad (2.23)$$

If the dimensional effects are neglected by setting γ in (2.22) to be zero, the solution to (2.22) can be easily obtained as follows:

$$E(r) = \frac{V}{d}. \quad (2.24)$$

The current flowing through the capacitor can be derived from (2.24) as below:

$$I = j\omega \int \vec{D} \cdot d\vec{S} = j\omega\epsilon \frac{V}{d} A, \quad (2.25)$$

where A is the area of the capacitor electrode.

Thus the impedance of the capacitor can be written as:

$$Z = \frac{V}{I} = \frac{d}{j\omega\epsilon A}. \quad (2.26)$$

From (2.26), the complex permittivity can be expressed by the measured impedance of the capacitor as follows [16]:

$$\epsilon = \frac{d}{j\omega ZA} = -j \frac{dZ^*}{\omega |Z|^2 A}. \quad (2.27)$$

From the above analysis, we can see that there are two assumptions for the expression of the complex permittivity in (2.27). The first is that the fringing effect can be neglected. The second is that the dimensional effects can be neglected. For the satisfaction of the first assumption, the guard electrode shown in Figure 2.14 can be used when the common dielectrics are measured. Since the common dielectrics are non-magnetic, the second assumption can be easily satisfied. Thus for the common non-magnetic dielectrics, (2.27) can be simply used to obtain the material's complex permittivity.

Figure 2.14 A parallel plate capacitor with a guard electrode [24]

For the Mn-Zn ferrites, on account of the high permittivity, no guard rings are necessary to minimize the fringing effect [6] if only the ferrite sample does not extrude outside of the parallel plate electrodes. But due to the dimensional effects, the distribution of the electric fields in the capacitor may not be uniform. Thus the complex permittivity according to (2.27), which is termed effective, or apparent, complex permittivity, may be deviated from the true value which is termed the dimension-independent, or intrinsic,

complex permittivity. The dimension-dependent characteristic of the apparent complex permittivity is shown clearly in Figure 2.15. The apparent complex permittivities in Figure 2.15 are measured from two rectangular Mn-Zn ferrite blocks as shown in Figure 2.16. The two blocks are cut from a larger Mn-Zn ferrite bar (TDK H7C4) by using a high precision diamond blade cutter (ISOMET 1000). The physical dimensions of the two blocks are shown in Table 2.2. Details on the measurements of the apparent complex permittivity are presented in Section 4.2.

Figure 2.15 Measured apparent complex permittivity of two Mn-Zn ferrite blocks at 23°C (Dotted lines: Block No.1; Solid lines: Block No.2)

Figure 2.16 Illustration of a Mn-Zn ferrite block capacitor

Table 2.2 Dimensions of the two Mn-Zn ferrite blocks

Block No.	a(mm)	b(mm)	d(mm)
1	10.90	4.55	5.55
2	10.90	9.50	5.55

2.5 Available Field-Circuit Coupled Methods to Measure the Mn-Zn Ferrites' Intrinsic Complex Permeability and Permittivity

As is stated in the above sections, the apparent complex permeability and permittivity measured by using the traditional methods could deviate significantly from the intrinsic values due to the dimensional effects. Since the dimensional effects are less significant in the ferrite cores with smaller sizes, the obvious method to measure the intrinsic complex permeability and permittivity is to use a small ferrite core inductor and a small ferrite core capacitor and take the measured apparent complex permeability and permittivity as the intrinsic values respectively. The complex permeability in the manufacturers' datasheet is usually obtained by using that method [13, 17]. Since the dimensional effects depend on the intrinsic complex permeability and permittivity as well as the dimensions of the cores, it is unpractical to decide how small the ferrite cores should be to eliminate the dimensional effects for a given measurement frequency range before the intrinsic complex permeability and permittivity are known. Furthermore, for the ferrite cores with small sizes, the measured apparent complex permeability and permittivity are very sensitive to the dimensional measurement errors. Thus special attention should be paid to the preparation of the samples to minimize the measurement errors of the physical dimensions. Also, since at low frequencies the impedance of the small ferrite core inductor is very small, and the impedance of the small ferrite core capacitor is very large, significant impedance measurement errors could occur. Thus precise measurements will be difficult [13].

Different from the method using small samples as stated above, the general principle of the field-circuit coupled methods presented in [6, 17, 18] is to derive the two unknowns, i.e. the intrinsic complex permeability and the intrinsic complex permittivity, from two measured values, i.e. the apparent complex permeability and permittivity, or the impedances of a ferrite core inductor and a ferrite core capacitor. Determination of the intrinsic values from the measured impedances is an inverse problem. The direct problem is to set up appropriate mathematical models to express the impedances of the inductor and the capacitor in terms of the intrinsic values. The main features of these methods are summarized in Table 2.3. One of the main differences between these methods is that the ferrite core capacitor and the ferrite core inductor in Method B and Method C are made

from the same sample and they are made from different samples in Method A. The other main difference between these methods is the mathematical models of the ferrite core inductor and the ferrite core capacitor.

Figure 2.17 A sample used in [6]

Figure 2.18 Simplified sample models

Table 2.3 Summarization of the main features of the available field-circuit coupled methods

Methods	Method A	Method B	Method C
References	[6]	[17]	[18]
Inductor Samples	Figure 2.11 (b)	Figure 2.11 (b)	Figure 2.11 (b)
Capacitor Samples	Figure 2.17	Figure 2.11 (b)	Figure 2.11 (b)
Dimensional Assumptions	$R_2 - R_1 \ll 2R_2$; $d \ll R_2 - R_1$ $d \ll w$	$R_2 - R_1 \ll 2R_2$ $d \ll 2R_2$	$R_2 - R_1 \ll 2R_2$
Inductor Models	Figure 2.18 (b) 1-D Cartesian	Figure 2.18 (c) 1-D Cartesian	Figure 2.18 (a) 2-D Cartesian
Capacitor Models	Figure 2.18 (b) 1-D Cartesian	Figure 2.18 (c) 1-D Cartesian	Figure 2.11 (b) 1-D cylindrical
Computational Efficiency	high	high	low
Accuracy	low	low	high

2.5.1 Comparison of the mathematical inductor models

For easy formulation of the mathematical model for the ferrite core inductor, toroidal ferrite cores with rectangular cross sections shown in Figure 2.11 (b) are used in those methods without exception.

In Method A, three assumptions are taken as follows, which imposes restrictions on the dimensions of the toroidal core:

1. The width of the cross section, i.e. $R_2 - R_1$, must be small compared to the diameter of the toroid.
2. The thickness, d , must be small compared to the width of the cross section.
3. All the dimensions must be small compared to the wavelength in vacuum of the electromagnetic wave set up by the applied fields.

Based on the first assumption, the toroidal core can be unwound without much change in the internal fields. The result of the unwound toroidal core is a bar as shown in Figure 2.18 (a). Under the second assumption, the ferrite bar can be further seen as an infinite ferrite slab parallel with the x - z plane as in Figure 2.18 (b). The mathematical model of the inductor can be simplified into a 1-D one. The fields in the infinite slab are assumed to vary only in the y direction. The third assumption is necessary to guarantee the validation of the field-circuit coupled method. The solution to the magnetic fields in the infinite slab is given by [6]:

$$H(y) = H_s \frac{\cos j\gamma y}{\cos j\gamma d/2}, \quad (2.28)$$

where

$$H_s = \frac{NI}{L_{ave}}, \quad (2.29)$$

and the expression of L_{ave} is given in (2.17).

The impedance of the inductor can be derived as follows [6]:

$$Z_L = \frac{V}{I} = \frac{N}{I} (-j\omega \int \vec{B} \cdot d\vec{S}) = \frac{Nj\omega}{I} \int_{-d/2}^{d/2} \int_{R_1}^{R_2} \mu H_s \frac{\cos j\gamma y}{\cos j\gamma d/2} dx dy$$

$$= \frac{\omega\mu N^2 (R_2 - R_1)}{L_{ave}} \frac{2}{\gamma} \tan j\gamma d/2. \quad (2.30)$$

As shown in Table 2.3, the dimensional assumptions in Method B are very similar to those in Method A. However, in Method B, the upper and the lower flat surfaces of the core are deposited with aluminum. There are no exciting magnetic fields on the two flat surfaces. Thus it is more reasonable to assume that the fields in the unwound bar in Figure 2.18 (a) vary only in the x direction. The ultimate mathematical model of the toroidal core is an infinite ferrite slab parallel with the y-z plane in Figure 2.18 (c). Based on (2.28), the magnetic fields in the infinite slab in Figure 2.18 (c) can be written as:

$$H(x) = H_s \frac{\cos j\gamma x}{\cos[j\gamma(R_2 - R_1)/2]}. \quad (2.31)$$

The impedance of the inductor can be derived as follows [17]:

$$Z_L = \frac{V}{I} = \frac{N}{I} (-j\omega \int \vec{B} \cdot d\vec{S}) = \frac{Nj\omega}{I} \int_{-d/2}^{d/2} \int_{-(R_2-R_1)/2}^{(R_2-R_1)/2} \mu H_s \frac{\cos j\gamma x}{\cos[j\gamma(R_2 - R_1)/2]} dx dy$$

$$= \frac{\omega\mu N^2 d}{L_{ave}} \frac{2}{\gamma} \tan[j\gamma(R_2 - R_1)/2]. \quad (2.32)$$

In Method C, only the first dimensional assumption in Method A is needed. The ultimate mathematical model of the toroidal core is the ferrite bar shown in Figure 2.18 (a). The model allows that the fields vary in the x direction, as well as in the y direction, thus conforming more closely to the physical reality. The solution to the magnetic fields in the bar is given by [18]:

$$H(x, y) = H_{ind}(x, y) + \frac{NI}{2\pi(x + R_1)}, \quad (2.33)$$

where

$$H_{ind}(x, y) = \int_0^d \int_0^{R_2-R_1} f(w, v) \frac{4}{(R_2 - R_1)d}$$

$$\times \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin \frac{m\pi x}{R_2 - R_1} \sin \frac{n\pi y}{d} \sin \frac{m\pi w}{R_2 - R_1} \sin \frac{n\pi v}{d}}{\left(\frac{m\pi}{R_2 - R_1}\right)^2 + \left(\frac{n\pi}{d}\right)^2 - k^2} dw dv, \quad (2.34)$$

$$f(w, v) = \frac{k^2 NI}{2\pi(w + R_1)}, \quad (2.35)$$

$$k^2 = \omega^2 \mu \varepsilon. \quad (2.36)$$

The electric fields in the core can be solved by using the following equations [18]:

$$E_x = \frac{1}{j\omega\varepsilon} \frac{\partial H(x, y)}{\partial y}, \quad (2.37)$$

$$E_y = -\frac{1}{j\omega\varepsilon} \frac{\partial H(x, y)}{\partial x}. \quad (2.38)$$

The impedance of the ferrite core inductor can be written as [18]:

$$Z_L = \frac{S}{I^2} = \frac{S_E + S_H}{I^2}, \quad (2.39)$$

where S_E and S_H are the complex power related with the electric field and the complex power related with the magnetic field respectively, and are written as follows:

$$S_H = \iiint j\omega\mu |H(x, y)|^2 dv = 2 \int_0^{d/2} \int_0^{R_2 - R_1} j\omega\mu |H(x, y)|^2 2\pi(x + R_1) dx dy, \quad (2.40)$$

$$\begin{aligned} S_E &= \iiint (j\omega\varepsilon)^* |E(x, y)|^2 dv \\ &= 2 \int_0^{d/2} \int_0^{R_2 - R_1} j\omega\mu (|E_x(x, y)|^2 + |E_y(x, y)|^2) 2\pi(x + R_1) dx dy. \end{aligned} \quad (2.41)$$

2.5.2 Comparison of the mathematical capacitor models

The ferrite core capacitor in Method A is constructed from a ferrite block as shown in Figure 2.17. The electrodes of the capacitor are formed directly on two parallel surfaces of the block, e.g. the left and the right shaded surfaces in Figure 2.17. Based on the assumption that $d \ll w$, the fields in the capacitor can be assumed to be varied only in the

y direction in Figure 2.17. The ferrite block can now be seen as an infinite slab parallel with the x-z plane with the exciting electric field $E_s = V/h$ in the z direction on the surfaces of the slab in Figure 2.18 (b). Thus the mathematical model for the ferrite block capacitor can be simplified into a 1-D one; and easy solution to the electric fields inside the ferrite block can be obtained as follows [6]:

$$E(y) = E_s \frac{\cos j\gamma y}{\cos j\gamma d/2}. \quad (2.42)$$

The impedance of the capacitor can be derived as follows [6]:

$$Z_c = \frac{V}{I} = \frac{V}{j\omega\epsilon \int_{-d/2}^{d/2} \int_{-w/2}^{w/2} E(y) dx dy} = \frac{\gamma h}{2\omega\epsilon w \tan(j\gamma d/2)}. \quad (2.43)$$

To eliminate the dimensional assumption $d \ll w$, a new mathematical model of the capacitor made from a ferrite block is presented in Section 4.2. Besides the dimensional assumptions, another factor that limits the accuracy of Method A is that the unavoidable inconsistency of the two different samples caused by the manufacturing dispersion. Thus the idea that both the inductor and the capacitor are made from the same core is preferred. As mentioned before, in Method B, the ferrite core capacitor is constructed from the same core in the ferrite core inductor. The metallic electrodes are deposited on the upper and the lower flat surfaces of the toroidal core. Based on the same dimensional assumptions as taken in the mathematical inductor model, the ultimate model of the ferrite core capacitor is a ferrite slab parallel with the y-z plane shown in Figure 2.18 (c) with the exciting electric field $E_s = V/d$ in the y direction on the surfaces of the slab. The fields in the ferrite vary only in the x direction. Thus the solution to the electric fields in the ferrite is similar to (2.42) as follows:

$$E(x) = E_s \frac{\cos j\gamma x}{\cos j\gamma (R_2 - R_1)/2}. \quad (2.44)$$

From (2.44), the impedance of the ferrite core capacitor can be written as [17]:

$$Z_c = \frac{V}{I} = \frac{V}{j\omega\epsilon L_{ave} \int_{-(R_2-R_1)/2}^{(R_2-R_1)/2} E(x) dx} = \frac{\gamma d}{2\omega\epsilon L_{ave} \tan(j\gamma (R_2 - R_1)/2)}. \quad (2.45)$$

Figure 2.19 Top view of the core and the coordinates adopted

In Method C, the ferrite core capacitor is made from the toroidal core in the same way as in Method B. However, in Method C, no dimensional assumptions are needed in the mathematical capacitor model. Thus it can be expected that a more accurate, and more complicated, solution to the fields is obtained. The top view of the toroidal core and the coordinates adopted are shown in Figure 2.19. The electric fields in the core are given by [18]:

$$E(r) = \frac{1}{j\omega\epsilon} \frac{1}{r} \frac{\partial}{\partial r} [rH(r)] = \frac{1}{j\omega\epsilon} \frac{H(r)}{r} + \frac{1}{j\omega\epsilon} \frac{\partial H(r)}{\partial r}, \quad (2.46)$$

where

$$H(r) = CJ_1(kr) + DY_1(kr), \quad (2.47)$$

$$C = \frac{\begin{vmatrix} H(R_1) & Y_1(kR_1) \\ H(R_2) & Y_1(kR_2) \end{vmatrix}}{\Delta}, \quad (2.48)$$

$$D = \frac{\begin{vmatrix} J_1(kR_1) & H(R_1) \\ J_1(kR_2) & H(R_2) \end{vmatrix}}{\Delta}, \quad (2.49)$$

$$\Delta = \begin{vmatrix} J_1(kR_1) & Y_1(kR_1) \\ J_1(kR_2) & Y_1(kR_2) \end{vmatrix}, \quad (2.50)$$

$$H(R_2) = \frac{I}{2\pi R_2}, \quad (2.51)$$

$$H(R_1) = \frac{kJ_1(kR_1)H(R_2)Y_1'(kR_1) - kY_1(kR_1)H(R_2)J_1'(kR_1)}{\frac{\varepsilon}{\varepsilon_0} \frac{\Delta}{J_1(kR_1)} \left[\frac{J_1(k_0 R_1)}{R_1} + k_0 J_1'(k_0 R_1) \right] - \frac{\Delta}{R_1} - kY_1(kR_2)J_1'(kR_1) + kJ_1(kR_2)Y_1'(kR_1)}, \quad (2.52)$$

$$k = \gamma / j = \omega \sqrt{\mu \varepsilon}, \quad (2.53)$$

$$k_0 = \gamma_0 / j = \omega \sqrt{\mu_0 \varepsilon_0}. \quad (2.54)$$

The impedance of the ferrite core capacitor can be written as [18]:

$$Z_c = \frac{S}{I^2} = \frac{S_E + S_H}{I^2}, \quad (2.55)$$

where

$$S_E = \iiint (j\omega\varepsilon)^* |E(r)|^2 dv = 2\pi d(j\omega\varepsilon)^* \int_{R_1}^{R_2} r |E(r)|^2 dr, \quad (2.56)$$

$$S_H = \iiint j\omega\mu |H(r)|^2 dv = 2\pi dj\omega\mu \int_{R_1}^{R_2} r |H(r)|^2 dr. \quad (2.57)$$

Figure 2.20 Electric field distributions along the r direction

Suppose the inner radius, the outer radius and the height of a toroidal ferrite core are 5 mm, 7 mm and 1 mm respectively. The permeability and the permittivity of the core are assumed to be the values shown in Figure 2.3. According to (2.46), the electric field distributions along the r direction in the ferrite core are calculated for a given current amplitude, e.g. 1 mA, in the frequency range from 10 kHz to 10 MHz. The electric field

distributions normalized to the electric fields on the outer lateral surface of the core are shown in Figure 2.20. From Figure 2.20, it can be seen that the electric fields on the inner and the outer lateral surfaces, i.e. $E(R_1)$ and $E(R_2)$, are different above about 1 MHz due to the dimensional resonance and the skin effects. But in [17], it is assumed that both $E(R_1)$ and $E(R_2)$ are equal to V/d . Thus the mathematical capacitor model in Method B cannot be expected to be very accurate at high frequencies even if its dimensional assumptions are satisfied.

2.5.3 Comparison of the inverse processes

Due to the simple 1-D mathematical models of the ferrite core inductors and the ferrite core capacitors in Method A and Method B, the inverse processes are very computationally efficient, as shown in (9)-(10) in [6] and (7)-(9) in [17] respectively. It can be seen from (2.39) and (2.55) that the mathematical models of the ferrite core inductors and the ferrite core capacitors in Method C are very complex. The inverse process to determine the intrinsic complex permeability and permittivity is solved iteratively by using the Newton-Raphson method as shown below [18]:

$$\begin{bmatrix} \mu'(i+1) \\ \mu''(i+1) \\ \varepsilon'(i+1) \\ \varepsilon''(i+1) \end{bmatrix} = \begin{bmatrix} \mu'(i) \\ \mu''(i) \\ \varepsilon'(i) \\ \varepsilon''(i) \end{bmatrix} - \mathbf{J}^{-1} \begin{bmatrix} f_1(i) - y_1 \\ f_2(i) - y_2 \\ f_3(i) - y_3 \\ f_4(i) - y_4 \end{bmatrix}, \quad (2.58)$$

$$\mathbf{J} = \begin{bmatrix} \partial f_1 / \partial \mu' & \partial f_1 / \partial \mu'' & \partial f_1 / \partial \varepsilon' & \partial f_1 / \partial \varepsilon'' \\ \partial f_2 / \partial \mu' & \partial f_2 / \partial \mu'' & \partial f_2 / \partial \varepsilon' & \partial f_2 / \partial \varepsilon'' \\ \partial f_3 / \partial \mu' & \partial f_3 / \partial \mu'' & \partial f_3 / \partial \varepsilon' & \partial f_3 / \partial \varepsilon'' \\ \partial f_4 / \partial \mu' & \partial f_4 / \partial \mu'' & \partial f_4 / \partial \varepsilon' & \partial f_4 / \partial \varepsilon'' \end{bmatrix}, \quad (2.59)$$

where y_1 and y_2 are the real part and the imaginary part of the measured impedance of the ferrite core inductor respectively, y_3 and y_4 are the real part and the imaginary part of the measured impedance of the ferrite core capacitor respectively, f_1 and f_2 are the real part and the imaginary part of the calculated impedance of the ferrite core inductor respectively, and f_3 and f_4 are the real part and the imaginary part of the calculated impedance of the ferrite core capacitor respectively. The elements in the matrix $\mathbf{J}$ are expressed in (60)-(63) in [18]. It can be seen from (60)-(63) in [18] that calculation of the

elements in the matrix $\mathbf{J}$ is time-consuming, since numerical integrations of the fields over the cross section of the core are needed for the elements in the first two rows, and numerical integrations of the fields along the radius direction of the core are needed for the elements in the last two rows.

From the above analysis, we can see that the computational efficiency of Method C is much lower than that of Method A and Method B, as shown in Table 2.3.

In terms of accuracy, it can be reasonably concluded that the method which has less dimensional assumptions and less simplifications to the mathematical models of the ferrite core inductor and the ferrite core capacitor is more accurate. Thus Method C is much more accurate than Method A and Method B. In terms of the accuracy comparison of Method A and Method B, since the latter, like Method C, needs only one sample, and thus avoids the potential material difference between different samples, it is more accurate than the former. The methods needing only one sample also have the advantage of simple sample preparation over the method which needs two samples.

2.6 Conclusions

The dimensional assumptions, the computational efficiency and the accuracy of three available field-circuit coupled methods are compared in this chapter. In Method A, based on the strict dimensional assumptions on the samples, the mathematical models of the ferrite core inductor and the ferrite core capacitor are simple. Thus the inverse process to extract the intrinsic values from the measured impedances is very computationally efficient. However, Method A needs two samples. Besides the strict dimensional assumptions, the potential material difference between the two samples with different shapes may cause great errors. Furthermore, great efforts have to be taken to prepare two samples with different shapes. Method B needs only one sample, and avoids the difficulty in preparing two different samples as in Method A. Based on almost the same strict dimensional assumptions as in Method A, high computational efficiency to extract the intrinsic values can also be obtained. In the mathematical model of the ferrite core capacitor, Method B assumes that in the ferrite core capacitor the electric fields on the inner and the outer lateral surfaces of the toroidal core are the same. However, theoretical analysis reveals that the assumption is violated at the frequencies above about 1 MHz.

Method C also needs only one toroidal sample. The dimensional assumptions in Method C are much less strict than those in Method A and Method B. As can be expected, the mathematical models of the ferrite core inductor and the ferrite core capacitor are much more accurate, and much more complex, than those in Method A and Method B. Thus the computational efficiency of Method C is low. Researches on new methods with less dimensional assumptions and higher computational efficiency are justified.

CHAPTER 3

IMPROVEMENT OF THE AVAILABLE FIELD-CIRCUIT COUPLED METHODS

3.1 Introduction

From the analysis in Chapter 2, it can be seen that the mathematical models of the ferrite core inductor and the ferrite core capacitor in [18] are more accurate than those in [6] and [17]. Numerical results in [18] show that the Newton-Raphson method can be used to solve the inverse problem of determining the intrinsic complex permeability and permittivity of the Mn-Zn ferrite cores after only a few iteration steps. But the analytical expressions for the electric and the magnetic fields in the ferrite core inductor given in [18] are very complicated and time consuming for evaluation. Usually it takes hours for a Pentium 4 PC (Central Processing Unit (CPU): 3.4 GHz, Random Access Memory (RAM): 1024 MB) to determine the intrinsic values at a single frequency by using the method in [18]. For higher computational efficiency of the method, the expressions of the impedances of the ferrite core inductor and the ferrite core capacitor are improved in Section 3.2. Furthermore, the finite difference method is introduced in Section 3.2 to evaluate numerically the partial derivatives in the Newton-Raphson method. As will be seen in the numerical example in Section 3.2, this improved technique can determine the intrinsic values much more efficiently. It takes less than one second to obtain the intrinsic values at each frequency point.

As is stated in Section 2.5, the inductor model in [18] is based on the assumption that the width of the cross section ($R_2 - R_1$) is small when compared to the diameter of the toroidal core. Thus the accuracy of the model depends on how closely the actual dimensions of the ferrite core under test conform to that assumption. For more accurate results, a cylindrical coordinate inductor model is presented in Section 3.3 to replace the Cartesian coordinate inductor models presented in Section 3.2. Thus theoretically it can be

expected that the method incorporating with the cylindrical coordinate inductor model will be more accurate than other field-circuit coupled methods. Experiments are done to compare different field-circuit coupled methods in terms of accuracy and computational efficiency. Experimental results are shown in Section 3.5.

In the field-circuit coupled methods mentioned above, the impedance of a Mn-Zn ferrite core inductor needs to be measured. For the wire wound ferrite core inductor, the stray capacitance among the coils may have significant influence on the measured impedance especially at high frequencies [17]. How to choose the turn number of the coils and what is the favorable disposition of the turns are discussed in [17] to minimize the stray capacitance. To totally eliminate the stray capacitance and to simplify the measurement procedure, a set of short-ended coaxial test fixtures is manufactured. The structure of the test fixture and the calibration issues are presented in Section 3.4.

3.2 Improvement of the Field-Circuit Coupled Method in [18]

From (2.34), it can be seen that evaluation of the magnetic fields is very time consuming, since the formula include double integrations whose integrands are double summations of infinite terms. Actually, (2.34) can be further simplified to the following equation:

$$\begin{aligned}
 H_{ind}(x, y) &= \int_0^{R_2-R_1} f(w) \frac{4}{(R_2-R_1)d} \times \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{d}{n\pi} [1 - (-1)^n] \frac{\sin \frac{m\pi x}{R_2-R_1} \sin \frac{n\pi y}{d} \sin \frac{m\pi w}{R_2-R_1}}{\left(\frac{m\pi}{R_2-R_1}\right)^2 + \left(\frac{n\pi}{d}\right)^2 - k^2} dw \\
 &= \frac{4}{(R_2-R_1)d} \times \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{d}{n\pi} [1 - (-1)^n] \frac{\sin \frac{m\pi x}{R_2-R_1} \sin \frac{n\pi y}{d}}{\left(\frac{m\pi}{R_2-R_1}\right)^2 + \left(\frac{n\pi}{d}\right)^2 - k^2} \int_0^{R_2-R_1} f(w) \sin \frac{m\pi w}{R_2-R_1} dw \\
 &= \frac{4k^2 NI}{\pi(R_2-R_1)} \times \sum_{m=1}^{\infty} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n\pi} \frac{\sin \frac{m\pi x}{R_2-R_1} \sin \frac{n\pi y}{d}}{\left(\frac{m\pi}{R_2-R_1}\right)^2 + \left(\frac{n\pi}{d}\right)^2 - k^2} \int_0^{R_2-R_1} \frac{1}{w+R_1} \sin \frac{m\pi w}{R_2-R_1} dw. \quad (3.1)
 \end{aligned}$$

The integration in (3.1) can be solved numerically or by using the following integration equation:

$$\begin{aligned}
 \int_0^b \frac{\sin cw}{w+a} dw &= \int_a^{a+b} \frac{\sin c(x-a)}{x} dx = \int_a^{a+b} \frac{\sin cx \cos ca - \cos cx \sin ca}{x} dx \\
 &= \cos ca \int_a^{a+b} \frac{\sin cx}{x} dx - \sin ca \int_a^{a+b} \frac{\cos cx}{x} dx = \left\{ \cos ca \left[cx - \frac{(cx)^3}{3 \cdot 3!} + \frac{(cx)^5}{5 \cdot 5!} - \frac{(cx)^7}{7 \cdot 7!} + \dots \right] \right. \\
 &\quad \left. - \sin ca \left[\ln cx - \frac{(cx)^2}{2 \cdot 2!} + \frac{(cx)^4}{4 \cdot 4!} - \frac{(cx)^6}{6 \cdot 6!} + \dots \right] \right\} \Big|_a^{(a+b)}. \quad (3.2)
 \end{aligned}$$

The expressions of the impedances of the ferrite core inductor and the ferrite core capacitor shown in (2.39) and (2.55) respectively are obtained by making use of the Law of Energy Conservation. From (2.39) and (2.55), it can be seen that great efforts are needed for evaluation of the impedances. Actually, the impedance of the inductor can be derived by using the Faraday's Law of induction as follows:

$$Z_L = \frac{V}{I} = \frac{N}{I} (-j\omega \int \vec{B} \cdot d\vec{S}) = \frac{2Nj\omega\mu}{I} \int_0^{d/2} \int_0^{R_2-R_1} H_{ind}(x, y) dx dy + \frac{j\omega\mu N^2 d}{2\pi} \ln\left(\frac{R_2}{R_1}\right), \quad (3.3)$$

where

$$\begin{aligned}
 &\int_0^{d/2} \int_0^{R_2-R_1} H_{ind}(x, y) dx dy \\
 &= \frac{8k^2 NI}{\pi(R_2 - R_1)} \times \sum_{m=1,3,5,\dots}^{\infty} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n\pi} \frac{\frac{R_2 - R_1}{m\pi} \cdot \frac{d}{n\pi}}{\left(\frac{m\pi}{R_2 - R_1}\right)^2 + \left(\frac{n\pi}{d}\right)^2 - k^2} \int_0^{R_2-R_1} \frac{1}{w + R_1} \sin \frac{m\pi w}{R_2 - R_1} dw. \quad (3.4)
 \end{aligned}$$

The impedance of the ferrite core capacitor can be written as follows:

$$Z_C = \frac{V}{I} = \frac{E(R_2)d}{I}, \quad (3.5)$$

where $E(R_2)$ is the electric field strength on the outer lateral surface of the toroidal ferrite core, and can be evaluated by substituting $r=R_2$ into (2.46).

To further improve the computational efficiency of the field-circuit coupled method in [18], the matrix $\mathbf{J}$ in formulation of the Newton-Raphson method shown in (2.59) is calculated numerically by using the finite difference method. The elements in the 4×4 matrix are computed as follows (for $k=1, 2, 3, 4$):

$$\frac{\partial f_k(i)}{\partial \mu'} \approx \frac{\Delta f_k(i)}{\Delta \mu'} = \frac{f_k(\mu'(i) + \mu_0, \mu''(i), \varepsilon'(i), \varepsilon''(i)) - f_k(i)}{\mu_0}, \quad (3.6)$$

$$\frac{\partial f_k(i)}{\partial \mu''} \approx \frac{\Delta f_k(i)}{\Delta \mu''} = \frac{f_k(\mu'(i), \mu''(i) + \mu_0, \varepsilon'(i), \varepsilon''(i)) - f_k(i)}{\mu_0}, \quad (3.7)$$

$$\frac{\partial f_k(i)}{\partial \varepsilon'} \approx \frac{\Delta f_k(i)}{\Delta \varepsilon'} = \frac{f_k(\mu'(i), \mu''(i), \varepsilon'(i) + \varepsilon_0, \varepsilon''(i)) - f_k(i)}{\varepsilon_0}, \quad (3.8)$$

$$\frac{\partial f_k(i)}{\partial \varepsilon''} \approx \frac{\Delta f_k(i)}{\Delta \varepsilon''} = \frac{f_k(\mu'(i), \mu''(i), \varepsilon'(i), \varepsilon''(i) + \varepsilon_0) - f_k(i)}{\varepsilon_0}. \quad (3.9)$$

(a) μ'/μ_0 (b) μ''/μ_0 **Figure 3.1 Convergence of relative complex permeability versus iteration numbers**

(a) ϵ'/ϵ_0 (b) ϵ''/ϵ_0 **Figure 3.2 Convergence of relative complex permittivity versus iteration numbers**

To test the efficiency of the improved method, the dimensions of the ferrite core are assumed as $R_1=15$ mm, $R_2=20$ mm, and $b=5$ mm. The intrinsic complex permeability and permittivity are assumed as $\mu'=2000\mu_0$, $\mu''=500\mu_0$, $\epsilon'=1\times 10^5\epsilon_0$, and $\epsilon''=2\times 10^4\epsilon_0$. The frequency is 1 MHz. The calculated impedances of the ferrite core inductor and the ferrite core capacitor according to (3.3) and (3.5) are assumed to be the measured values. The initial values for the iteration are taken as $\mu=1000\mu_0$, $\epsilon=1.5\times 10^5\epsilon_0$. From Figure 3.1 and Figure 3.2, it can be seen that the improved method can determine the intrinsic complex permeability and permittivity after only a few iteration steps, even though the initial values for the iteration deviate a lot from the convergence values. The time needed is only about 0.4 seconds for a Pentium 4 PC (CPU: 3.4 GHz, RAM: 1024 MB). The

efficiency of the iteration is considerably improved.

In the practical application, for the fast convergence of the Newton-Raphson method, the determined intrinsic complex permeability and permittivity at a frequency point become the initial values for the next frequency point. For the first frequency point, the measured apparent complex permeability and permittivity are treated as the initial values.

3.3 Cylindrical Coordinate Model of the Ferrite Core Inductor

The inductor model in [18] is based on the assumption that the width of the cross section ($R_2 - R_1$) is small when compared to the diameter of the toroidal core. Thus the accuracy of the model depends on how closely the actual dimensions of the ferrite core under test conform to that assumption. To eliminate that dimensional assumption, a cylindrical coordinate inductor model is presented in this section.

For derivation of the cylindrical coordinate model of the ferrite core inductor, the cylindrical coordinates shown in Figure 3.3 are adopted. The total magnetic fields $\mathbf{H}$ can be decomposed into the induced magnetic fields $\mathbf{H}_\phi$ and the incident magnetic fields $\mathbf{H}_{\phi 0}$ (i.e. $\mathbf{H} = \mathbf{H}_\phi + \mathbf{H}_{\phi 0}$). Eq. (2.12) can be rewritten as follows [7]:

$$\frac{\partial^2 H_\phi}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial H_\phi}{\partial \rho} - \frac{H_\phi}{\rho^2} + \frac{\partial^2 H_\phi}{\partial z^2} - \gamma^2 H_\phi = \gamma^2 H_{\phi 0}, \quad (3.10)$$

where

$$H_{\phi 0} = \frac{NI}{2\pi\rho}. \quad (3.11)$$

Figure 3.3 Cylindrical coordinates adopted in the ferrite core

According to (3.10) and the original boundary condition (2.13), on the surface of the ferrite core, the induced magnetic fields are zeros [7].

The solution to (3.10) is given by [7]:

$$H_{\phi} = \sum_{s=1}^{\infty} \sum_{n=0}^{\infty} A_{sn} \cos \frac{(2n+1)\pi z}{h} \times \left\{ \frac{J_1(\alpha_s \rho)}{J_1(\alpha_s R_1)} - \frac{Y_1(\alpha_s \rho)}{Y_1(\alpha_s R_1)} \right\}, \quad (3.12)$$

where

$$A_{sn} = -\frac{2k^2 NI}{\pi} \frac{(-1)^n}{2n+1} \frac{1}{\left[\frac{(2n+1)\pi}{h} \right]^2 + \alpha_s^2 + \gamma^2} \times \frac{\kappa(R_1 - \kappa R_2)}{(1 - \kappa^2) R_1 R_2} J_1(\alpha_s R_1) Y_1(\alpha_s R_1), \quad (3.13)$$

$$\kappa = \frac{J_1(\alpha_s R_2)}{J_1(\alpha_s R_1)}. \quad (3.14)$$

Here, J and Y are the Bessel functions [25] of the first kind and the second kind respectively. $\alpha_s R_1$ is the sth zeros of the cross product as follows:

$$J_1(\alpha_s R_1) Y_1(\alpha_s R_2) - J_1(\alpha_s R_2) Y_1(\alpha_s R_1) = 0. \quad (3.15)$$

The voltage across the ferrite core inductor for a given supplied current I can be derived as follows:

$$\begin{aligned} V &= -j\omega N \int \vec{B} \cdot d\vec{S} = j\omega N \mu \int (H_{\phi} + H_{\phi 0}) dS \\ &= 2j\omega N \mu \left(\int_{R_1}^{R_2} \int_0^{h/2} H_{\phi} dz d\rho + \int_{R_1}^{R_2} \int_0^{h/2} H_{\phi 0} dz d\rho \right), \end{aligned} \quad (3.16)$$

where

$$\int_{R_1}^{R_2} \int_0^{h/2} H_{\phi 0} dz d\rho = \frac{NI}{2\pi} \frac{h}{2} \ln \frac{R_2}{R_1}, \quad (3.17)$$

$$\begin{aligned} \int_{R_1}^{R_2} \int_0^{h/2} H_{\phi} dz d\rho &= \sum_{s=1}^{\infty} \sum_{n=0}^{\infty} A_{sn} \frac{h}{(2n+1)\pi} \frac{(-1)^n}{\alpha_s} \\ &\times \left\{ \frac{Y_0(\alpha_s R_2) - Y_0(\alpha_s R_1)}{Y_0(\alpha_s R_1)} - \frac{J_0(\alpha_s R_2) - J_0(\alpha_s R_1)}{J_0(\alpha_s R_1)} \right\}. \end{aligned} \quad (3.18)$$

The impedance of the inductor can be written as follows:

$$Z_L = \frac{V}{I}. \quad (3.19)$$

3.4 Short-Ended Coaxial Test Fixture

Figure 3.4 Structure of the short-ended coaxial test fixture

Table 3.1 Dimensions of Part 3 and Part 4

Piece No.	Part 3		Part 4	
	d (mm)	l (mm)	D (mm)	L (mm)
1	4	42	10.5	44.9
2	7	42	13.5	44.55
3	12.5	42.2	23	45
4	22	42.2	37	45

To avoid the influence of the stray capacitance among the coils of the wire wound inductor, a set of short-ended coaxial test fixtures shown in Figure 3.4 was manufactured to perform the measurement when the Mn-Zn ferrite core is under the magnetic field excitation. Four pieces of Part 3 and Part 4 with different dimensions shown in Table 3.1 were manufactured for the convenience of measuring cores with different sizes.

The test fixture is connected to the impedance analyzer HP4194A [26] through a 300 mm coaxial cable. The open-short-load (OSL) correction technique [27] is used to obtain the

impedance of the device under test (DUT) which is below the OSL correction plane shown in Figure 3.4. The impedance of the DUT after the correction is expressed as [27]:

$$Z_{DUT} = \frac{Z_L(Z_{Om} - Z_{Lm})(Z_{DUTm} - Z_{Sm})}{(Z_{Lm} - Z_{Sm})(Z_{Om} - Z_{DUTm})}, \quad (3.20)$$

where Z_{Om} and Z_{Sm} are the measured impedances in the open correction and the short correction respectively, Z_L is the true impedance of the load standard, Z_{Lm} is the measured impedance in the load correction, and Z_{DUTm} is the measured impedance when the DUT is connected. Details on the layout of the measurement system and the OSL correction process are shown in the following paragraphs.

Figure 3.5 Layout of the measurement system (A: HP4194A impedance analyzer; B: 16085B terminal adapter [28], C: 16092A spring clip fixture; D: 300 mm coaxial cable; E: coaxial test fixture)

Figure 3.6 Connection between the 300 mm coaxial cable and the 16092A spring clip fixture

The whole layout of the measurement system is shown in Figure 3.5. The transition part connecting the 300 mm coaxial cable to the 16092A spring clip fixture [29] is shown in Figure 3.6 in detail.

The OSL correction technique [27] allows using (3.20) to eliminate from the measured impedance the influence of the sections between the calibration plane of the 16092A spring clip fixture and the OSL correction plane shown in Figure 3.4, including the BNC JK/JK adapter, i.e. Part 1, of the coaxial test fixture, the 300 mm coaxial cable, and the transition part connecting the 300 mm coaxial cable to the 16092A spring clip fixture. The 50 Ω load standard and the short standard used in the OSL correction are shown in Figure 3.7 (a) and Figure 3.8 respectively. The 50 Ω load standard in Figure 3.7 (a) is made by removing the outer part of a BNC termination plug shown in Figure 3.7 (b) (RS Components Pte Ltd, Stock No. 112-3148, 50 Ω , 1% tolerance). The setups of the open correction and the load correction are shown in Figure 3.9 (a) and (b) respectively. For the short correction, the short standard in Figure 3.8 is plugged in the JK/JK adapter of the coaxial test fixture first, as shown in Figure 3.9 (c). Then Piece No. 2 of Part 4 in Table 3.1 is screwed in to press the short standard tight, as shown in Figure 3.9 (d). Z_{Om} , Z_{Sm} and Z_{Lm} in (3.20) are the measured impedances in the setups shown in Figure 3.9 (a), Figure 3.9 (d) and Figure 3.9 (b) respectively. Z_{DUTm} in (3.20) is the measured impedance in the setup shown in Figure 3.5, where the DUT is connected. The Z_{DUT} obtained by using (3.20) is the input impedance of the DUT.

(a) The load standard

(b) BNC termination plug

Figure 3.7 The load standard used in the OSL correction **Figure 3.8 The short standard used in the OSL correction**

For validation of the correction technique shown in (3.20), the impedance of the empty coaxial test fixture consisting of piece No.4 of Part 4 and piece No.2 of Part 3 is measured by using the impedance analyzer HP 4194A, and calculated as well by using the mode matching method [30] with the detailed physical dimensions of the test fixture

taken into account. The corrected measured impedances of the empty test fixture and the calculated ones are shown in Figure 3.10. Very good agreement between the corrected measured values and the calculated values can be observed up to 20 MHz.

Figure 3.9 OSL correction setups

Figure 3.10 Comparison of the corrected measured impedances of the empty coaxial test fixture and the calculated values

When (3.3) or (3.19) is used to calculate the impedance of the inductor, only the magnetic fields in the ferrite core are taken into account. But in the actual measurement, both the magnetic fields in the ferrite core and in the empty space between the ferrite core and the test fixture contribute to the measured impedance. Thus further calibration is needed to remove from the measured impedance the influence of the magnetic fields in the empty space inside the test fixture. For that purpose, the impedance of the empty test fixture, Z_{air} , is measured. The impedance due to exclusively the magnetic fields in the ferrite core can then be expressed as:

$$Z_{\text{core}} = Z_{\text{core+air}} - [Z_{\text{air}} - j \frac{\mu_0 \omega d}{2\pi} \ln(\frac{R_2}{R_1})], \quad (3.21)$$

where d , R_1 and R_2 are the height, the inner radius and the outer radius of the core respectively, and $Z_{\text{core+air}}$ is the corrected measured impedance of the test fixture loaded with the Mn-Zn ferrite core. The terms inside the square brackets in (3.21) represent the impedance due to the fields in the air gap between the ferrite core and the test fixture.

Figure 3.11 Calculated impedances of the empty test fixture and the test fixture loaded with the Mn-Zn ferrite core

Figure 3.12 Comparison of the calibrated impedance and the impedance calculated by using the cylindrical inductor model in Section 3.3 (circle: calibrated impedance; solid line: cylindrical inductor model in Section 3.3)

For validation of the calibration method shown in (3.21), we assume that the test fixture composed of piece No. 2 of Part 3 and piece No. 3 of Part 4 in Table 3.1 is loaded with a Mn-Zn ferrite core with the dimensions: $R_1=5$ mm, $R_2=10$ mm, and $d=8$ mm. It is assumed that the intrinsic complex permeability and permittivity of the Mn-Zn ferrite

core in the structure are respectively the same as shown in Fig. 2 and Fig. 3 in [31]. With the detailed physical dimensions of the test fixture taken into account, $Z_{\text{core+air}}$ and Z_{air} shown in Figure 3.11 are calculated by using the 2-D FDFD method presented in Chapter 7 and the general mode matching method [30] presented in Chapter 6 respectively. The calibrated impedance Z_{core} obtained by using (3.21) is shown in Figure 3.12 as circles. The impedance calculated by using (3.19) is shown in Figure 3.12 as solid lines. It can be seen from Figure 3.12 that in the frequency range of interest Z_{core} is in very good agreement with the impedance due to exclusively the magnetic fields in the core calculated by using (3.19).

Since the impedance of the empty test fixture is subtracted in (3.21), it can be seen that the influence of the losses of the test fixture is also eliminated from Z_{core} .

3.5 Experimental Comparison of Different Field-Circuit Coupled Methods

In this section, three different methods are compared experimentally, which are the method in [17], the improved method presented in Section 3.2, and the method which eliminates the dimensional assumption of the method in Section 3.2 by replacing the Cartesian coordinate inductor model of the improved method in [18] with the cylindrical coordinate model presented in Section 3.3. The three methods are named Method 1, Method 2, and Method 3 for conciseness. The temperature at which the experiments are conducted is 23°C.

Before the experiment, the upper and the lower surfaces of a toroidal Mn-Zn ferrite core (RS Components Pte Ltd, Part No. 5978001901) are flattened and polished by using a variable speed grinder/polisher (Ecomet 6). The outer diameter, the inner diameter and the height of the processed core are 12.9 mm, 8.05 mm, and 12.25 mm respectively. After the apparent μ of the core is measured, it is cut into three smaller toroidal cores by using a high precision diamond blade cutter (ISOMET 1000). The upper and the lower surfaces of the three smaller cores are all polished by using the grinder/polisher. The heights of the three processed smaller cores are 0.70 mm, 2.05 mm and 7.80 mm respectively. For conciseness, the four cores mentioned above, including the uncut core with the height equal to 12.25mm, from the smallest to the largest, are named Sample 1, Sample 2,

Sample 3 and Sample 4 respectively. As mentioned in Chapter 2, for a small sample the dimensional effects can be neglected. In that case the apparent complex permeability will be very close to the intrinsic complex permeability. The height of Sample 1 is chosen to be as small as possible so that its measured apparent complex permeability can be treated as the intrinsic complex permeability. That fact will be further justified in the following paragraphs. After Sample 1 is cut from Sample 4, the remaining part of Sample 4 is split into two samples, i.e. Sample 2 and Sample 3. To make the two samples different, their heights should be different. In the case here, the ratio of their heights is chosen to be around 1:4. Method 1, Method 2, and Method 3 are used to determine the intrinsic μ and ϵ with Sample 3. The other three samples are used to verify the determined intrinsic values. In the experiment, the apparent μ is measured by using the short-ended coaxial test fixture presented in Section 3.4. The apparent ϵ is measured by using the HP 16451B dielectric test fixture [24, 32]. When the apparent ϵ is measured, the silver conductive paint is used to effectively eliminate the air gaps between the sample and the capacitor electrodes. The layout of the measurement system is shown in Figure 3.13 when the apparent ϵ is measured.

Figure 3.13 Layout of the measurement system when the apparent ε is measured

Figure 3.14 Measured apparent μ and determined intrinsic μ by using the three different methods (Dashed lines: apparent μ ; Dotted lines: method 1; Solid lines: method 2; Dots: Method 3)

The measured apparent μ and ϵ of Sample 3, together with the determined intrinsic values by using the three different methods, are shown in Figure 3.14 and Figure 3.15. Due to the dimensional effects, the resonant phenomenon of the frequency dispersion of the apparent complex permeability is more obvious than the intrinsic complex permeability as shown in Figure 3.14. The real part of the relative complex permeability can become negative at high frequencies. The frequency dispersion of the intrinsic complex permittivity is not a resonant type in the considered frequency range. But due to the dimensional effects, the frequency dispersion of the apparent complex permittivity is a resonant type as shown in Figure 3.15.

In this case, the dimensional assumption of Method 2 is satisfied. Thus it can be seen that the results of Method 2 and Method 3 are in very good agreement. However, the dimensional assumptions of Method 1 are obviously violated. Thus above about 1 MHz, obvious discrepancy can be found between the result of Method 1 and the other two results. The time needed for the three different methods to determine the intrinsic values at 215 different frequency points in this case is shown in Table 3.2.

Figure 3.15 Measured apparent ϵ and determined intrinsic ϵ by using the three different methods (Dashed lines: apparent ϵ ; Dotted lines: method 1; Solid lines: method 2; Dots: Method 3)

Table 3.2 Time needed to determine the intrinsic values at 215 frequency points

Methods	Method 1	Method 2	Method 3
Time (s)	<1	15	68

To validate the determined intrinsic values of permeability, the apparent μ of sample 1 was measured. The apparent μ of sample 1 was calculated as well from the intrinsic values yielded by Method 3. Very good agreement between the determined intrinsic μ and the calculated apparent μ of sample 1 can be observed (see Figure 3.16), which indicates that for such a small core as Sample 1 dimensional effects can be neglected in the frequency range of interest. Thus the measured apparent μ of Sample 1 actually is the intrinsic μ .

For further validation of the result yielded by Method 3, the measured apparent μ of Sample 2 and Sample 4 is compared respectively with the calculated apparent μ from the

intrinsic values yielded by Method 3. From Figure 3.17 and Figure 3.18, very good agreement between the measured and the calculated values is noted.

Figure 3.16 Comparison of measured apparent μ of Sample 1 and determined intrinsic μ by using Method 3 (Solid lines: measured apparent μ ; Circles: calculated apparent μ ; Dots: Determined intrinsic μ by using method 3)

Figure 3.17 Comparison of measured and calculated apparent μ of Sample 2 (Solid lines: measured; Circles: calculated)

To show that Method 2 will have great errors if its dimensional assumption is not satisfied, R_2 , R_1 and d of a toroidal ferrite core are assumed to be 15 mm, 5 mm, and 8 mm respectively. The intrinsic μ and ϵ of the core are assumed to be the same as the core used in the experiment. The measurable inductor and capacitor impedances are calculated by using the models in Method 3. Method 2 is then used to determine the intrinsic μ and ϵ . By comparison of the assumed intrinsic values and the determined intrinsic values by using Method 2 in Figure 3.19 and Figure 3.20, great errors of Method 2 are easily observed at certain frequencies above a few MHz.

Figure 3.18 Comparison of measured and calculated apparent μ of Sample 4 (Solid lines: measured; Circles: calculated)

Figure 3.19 Comparison of the assumed intrinsic μ (solid lines) and the determined intrinsic μ of a virtual ferrite core by using Method 2 (dashed lines)

Figure 3.20 Comparison of the assumed intrinsic ϵ (solid lines) and the determined intrinsic ϵ of a virtual ferrite core by using Method 2 (dashed lines)

3.6 Conclusions

In this chapter, a few measures are taken to effectively improve the computational efficiency of the method in [18], which is the most accurate among the three available methods as mentioned in Section 2.5. To further improve the accuracy of the method in [18], a cylindrical coordinate inductor model is presented in this chapter, which can be used to replace the Cartesian coordinate inductor model in [18] to eliminate the dimensional assumption on the measurement sample. Finally, to avoid the influence of the stray capacitance of the wire wound ferrite core inductor and to simplify the measurement procedure for the field-circuit couple methods, a set of short-ended coaxial test fixtures is manufactured. Details on the structure of the test fixture and the calibration issues are also presented in this chapter.

CHAPTER 4

DERIVATIVE FIELD-CIRCUIT COUPLED METHODS USING TWO SAMPLES

4.1 Introduction

From the analysis in Chapter 2, it can be seen that the dimensional effects, which are caused by the combination of the high intrinsic complex permeability and the high intrinsic complex permittivity, cause the apparent complex permeability and permittivity to deviate from their intrinsic values. Therefore, either the apparent complex permeability or the apparent complex permittivity includes the information of both the intrinsic values. Thus theoretically it is possible to determine the intrinsic values from either two differently deviated apparent complex permeabilities or two differently deviated apparent complex permittivities.

Based on the principle stated above, in Section 4.2 and Section 4.3, the Newton-Raphson method is used to solve the inverse problem of tracing the intrinsic values from the impedances of two different-sized ferrite core capacitors and from the impedances of two different-sized toroidal ferrite core inductors respectively. The methods in Section 4.2 and Section 4.3 are experimentally validated. The experiments are conducted at 23°C. Advantages and limitations of the two methods are also revealed through theoretical and experimental analysis.

4.2 Derivative Field-Circuit Coupled Method Using Two Rectangular-Shaped Ferrite Core Capacitors

Following setting up a mathematical model for the rectangular-shaped Mn-Zn ferrite core capacitors, this section investigates the dimensional effects on the difference between the intrinsic complex permittivity and the measured apparent complex permittivity of the Mn-Zn ferrite blocks. The simple block structure of the samples guarantees their easy

and precise preparation.

In this section, the Newton-Raphson method is used to trace the intrinsic complex permittivity and permeability from the measured impedances of two rectangular-shaped Mn-Zn ferrite core capacitors. It takes only about 10 seconds to trace the intrinsic values for a hundred measured points in a Pentium 4 PC (CPU: 3.4 GHz, RAM: 1024MB). Based on the traced intrinsic complex permittivity and permeability of the two mentioned above samples, the apparent complex permittivity of a third sample can be calculated. Experiments are also carried out to measure the apparent complex permittivity of the third sample. Very good agreement between the measured and the calculated values is found.

4.2.1 Mathematical model of rectangular parallel plate Mn-Zn ferrite core capacitor

Figure 4.1 Ferrite core capacitor under study

The rectangular parallel plate Mn-Zn ferrite core capacitor under study is shown in Figure 4.1 (a). The top view of the ferrite core capacitor and the corresponding coordinate system is shown in Figure 4.1 (b).

Due to the high permittivity of Mn-Zn ferrites, fringing effect of the capacitor can be neglected [6, 33]. Based on (2.22), the equation governing the electric field in the capacitor can be written as:

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \omega^2 \mu \epsilon E_z = 0. \quad (4.1)$$

According to [6], the boundary condition can be written as:

$$E_z = \frac{V}{d}, (x = \pm a, y = \pm b). \quad (4.2)$$

Figure 4.2 Illustration for the boundary condition of the ferrite core capacitor

For further justification of the boundary condition in (4.2), the capacitor shown in Figure 4.2 is considered. Due to the high permittivity of the Mn-Zn ferrites, the electric fields on the interface between the air and the Mn-Zn ferrites are in parallel with the interface and perpendicular to the electrodes. In the air region just next to the interface, the electric fields are uniform and can be simply written as:

$$E_{t_air} = \frac{V}{d}, \quad (4.3)$$

where V is the supplied voltage of the capacitor.

Since the traverse electric fields on the interface must be continuous, the following equation holds:

$$E_{t_air} = E_{t_ferrite}, \quad (4.4)$$

where $E_{t_ferrite}$ is the electric fields in the Mn-Zn ferrites and just next to the interface. From (4.3) and (4.4), the boundary condition in (4.2) can be obtained.

Suppose that the total electric field is decomposed into the incident electric field E_0 and the induced electric field E_i , and let E_0 be V/d . The above equation and the boundary condition can be rewritten as:

$$\frac{\partial^2 E_i}{\partial x^2} + \frac{\partial^2 E_i}{\partial y^2} + k^2 E_i = -k^2 E_0, \quad (4.5)$$

$$E_i = 0, (x = \pm a, y = \pm b), \quad (4.6)$$

where

$$k^2 = \omega^2 \mu \epsilon . \quad (4.7)$$

For the solution of (4.5), the green's function satisfying the following Helmholtz's equation is considered first:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2 \right) G = -\delta(x-w)\delta(x-v) , \quad (4.8)$$

$$G = 0, (x = \pm a, y = \pm b) . \quad (4.9)$$

The solution to (4.8) and (4.9) can be assumed to be

$$G(x, y | w, v) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} u_{mn}(x, y) , \quad (4.10)$$

where

$$u_{mn}(x, y) = \cos \frac{(2m-1)\pi x}{2a} \cos \frac{(2n-1)\pi y}{2b} . \quad (4.11)$$

By substituting (4.10) into (4.8), the coefficient C_{mn} can be obtained as follows by utilization of the orthogonality of the cosine functions:

$$C_{mn} = \frac{1}{ab(k_{mn}^2 - k^2)} \cos \frac{(2m-1)\pi w}{2a} \cos \frac{(2n-1)\pi v}{2b} , \quad (4.12)$$

where

$$k_{mn}^2 = \left[\frac{(2m-1)\pi}{2a} \right]^2 + \left[\frac{(2n-1)\pi}{2b} \right]^2 . \quad (4.13)$$

Thus, (4.10) can be rewritten as:

$$\begin{aligned} G(x, y | w, v) = \frac{1}{ab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\cos \frac{(2m-1)\pi x}{2a} \cos \frac{(2n-1)\pi y}{2b}}{\left[\frac{(2m-1)\pi}{2a} \right]^2 + \left[\frac{(2n-1)\pi}{2b} \right]^2 - k^2} \\ \times \cos \frac{(2m-1)\pi w}{2a} \cos \frac{(2n-1)\pi v}{2b} . \end{aligned} \quad (4.14)$$

The solution to (4.5) and (4.6) can be obtained by performing the following integration:

$$E_i(x, y) = \int_{-b}^b \int_{-a}^a G(x, y | w, v) k^2 E_0 dx dy = k^2 E_0 \frac{16}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^{m+n}}{(2m-1)(2n-1)} \times \frac{\cos \frac{(2m-1)\pi x}{2a} \cos \frac{(2n-1)\pi y}{2b}}{\left[\frac{(2m-1)\pi}{2a}\right]^2 + \left[\frac{(2n-1)\pi}{2b}\right]^2 - k^2}. \quad (4.15)$$

The electric field inside the capacitor can be written as:

$$E_z = E_i + E_0. \quad (4.16)$$

Once the electric field is obtained, the magnetic field inside the capacitor can be derived without difficulty as follows:

$$H_x = \frac{j}{\omega\mu} \frac{\partial E_z}{\partial y} = -\frac{j}{\omega\mu} k^2 E_0 \frac{8}{b\pi} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^{m+n}}{2m-1} \times \frac{\cos \frac{(2m-1)\pi x}{2a} \sin \frac{(2n-1)\pi y}{2b}}{\left[\frac{(2m-1)\pi}{2a}\right]^2 + \left[\frac{(2n-1)\pi}{2b}\right]^2 - k^2}, \quad (4.17)$$

$$H_y = -\frac{j}{\omega\mu} \frac{\partial E_z}{\partial x} = \frac{j}{\omega\mu} k^2 E_0 \frac{8}{a\pi} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^{m+n}}{2n-1} \times \frac{\sin \frac{(2m-1)\pi x}{2a} \cos \frac{(2n-1)\pi y}{2b}}{\left[\frac{(2m-1)\pi}{2a}\right]^2 + \left[\frac{(2n-1)\pi}{2b}\right]^2 - k^2}. \quad (4.18)$$

The current flowing through the capacitor for a given incident electric field E_0 can be derived as follows:

$$\begin{aligned} I &= j\omega\epsilon \int_{-b}^b \int_{-a}^a E_z dx dy = j\omega\epsilon \int_{-b}^b \int_{-a}^a (E_i + E_0) dx dy \\ &= j\omega\epsilon k^2 E_0 \left(\frac{16}{\pi^2}\right)^2 ab \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{(2m-1)^2 (2n-1)^2} \frac{1}{\left[\frac{(2m-1)\pi}{2a}\right]^2 + \left[\frac{(2n-1)\pi}{2b}\right]^2 - k^2} \\ &\quad + j\omega\epsilon 4ab E_0. \end{aligned} \quad (4.19)$$

The impedance of the capacitor can be written as follows:

$$Z_c = \frac{V}{I}. \quad (4.20)$$

4.2.2 Influence of the sample's dimensions on the apparent permittivity

According to (2.27), the apparent complex permittivity can be calculated as follows:

$$\varepsilon_{app} = -j \frac{Z_c^* d}{|Z_c|^2 \omega ab}. \quad (4.21)$$

The apparent complex permittivity could deviate from the intrinsic value greatly. The deviation is due to the distortion of the distribution of the electric and magnetic fields in the ferrite core capacitor [33], which are caused by the dimensional effects. For further illustration of the influence of the sample's dimension on the deviation, the intrinsic complex permeability and permittivity are assumed to be the intrinsic values determined by using Method 3 in Figure 3.14 and Figure 3.15 respectively.

Table 4.1 Dimensions of the four assumed samples

Sample No.	2a (mm)	2b (mm)	d (mm)
1	5	5	5
2	10	10	5
3	10	40	5
4	20	20	5

To study the influence of the area and the length ratio of the upper and the lower surfaces of the samples on the apparent complex permittivity, four samples are assumed to have the dimensions shown in Table 4.1. Sample 1 and Sample 2 have difference surface areas, but the same length ratio. Sample 3 and Sample 4 have the same surface area, but different length ratio. The intrinsic complex permittivity and the calculated apparent complex permittivities of the four samples are shown in Figure 4.3. The dimensional effects on the apparent complex permittivity can be easily identified in the frequency range above around 200 kHz. As mentioned before, due to the dimensional effects, the frequency dispersion of the apparent complex permittivity is a resonant type as shown in Figure 4.3. For a larger sample, the resonant frequency is lower, and the resonant phenomenon is more obvious. The real part of the relative complex permittivity can become negative at the frequencies near the resonant frequency.

Figure 4.3 Calculated apparent complex permittivities of the four samples

As we can see from the analysis in subsection 4.2.1, for a given incident electric field, the electric field and the magnetic field distributions in the ferrite core capacitor are not affected by the thickness of the ferrite, i.e. d . In other words, for two ferrite core capacitors with the same electrode area and shape but different electrode distances, the measured apparent permittivities are the same. Thus it can be concluded that the differences among the apparent permittivities in Figure 4.3 are caused by the different upper and lower surfaces of the four samples. Comparing the apparent complex permittivities of the four samples can lead to the conclusion that both the surface area ($2a \times 2b$) and the length ratio ($a : b$) have influence on the apparent complex permittivity. The electric and the magnetic field distributions of sample 2 at 1.28 MHz are shown in Figure 4.4 and Figure 4.5 respectively. It can be seen that the electric and the magnetic

fields are highly distorted. As shown in (4.16), the electric fields inside the capacitor are the summation of the induced electric fields and the incident electric fields. Since the incident electric fields are constant, the highly distorted electric fields are due to the large induced electric fields. As shown in (2.24) in Section 2.4, the apparent complex permittivity will equal to the intrinsic complex permittivity only when the electric fields inside the capacitor equal to the incident electric fields. Thus when there are large induced electric fields inside the capacitor, large difference between the apparent complex permittivity and the intrinsic complex permittivity can be observed, as shown in Figure 4.3.

Figure 4.4 Electric field distribution in sample 2 at 1.28 MHz

Figure 4.5 Magnetic field distribution in sample 2 at 1.28 MHz

4.2.3 Determination of intrinsic complex permeability and permittivity

To determine the intrinsic complex permeability and permittivity from the measured impedances of two ferrite capacitors, the Newton-Raphson method [18, 34] is used. The iteration equation of the Newton-Raphson method can be written as:

$$\begin{bmatrix} \mu'(i+1) \\ \mu''(i+1) \\ \varepsilon'(i+1) \\ \varepsilon''(i+1) \end{bmatrix} = \begin{bmatrix} \mu'(i) \\ \mu''(i) \\ \varepsilon'(i) \\ \varepsilon''(i) \end{bmatrix} - \mathbf{J}^{-1} \begin{bmatrix} f_1(i) - \text{Re}(Z_1) \\ f_2(i) - \text{Im}(Z_1) \\ f_3(i) - \text{Re}(Z_2) \\ f_4(i) - \text{Im}(Z_2) \end{bmatrix}, \quad (4.22)$$

where Z_1 and Z_2 are the measured impedances of the two capacitors respectively, and

$$f_1 = \text{Re}[Z_C(\mu, \varepsilon, a_1, b_1, d_1)], \quad (4.23)$$

$$f_2 = \text{Im}[Z_C(\mu, \varepsilon, a_1, b_1, d_1)], \quad (4.24)$$

$$f_3 = \text{Re}[Z_C(\mu, \varepsilon, a_2, b_2, d_2)], \quad (4.25)$$

$$f_4 = \text{Im}[Z_C(\mu, \varepsilon, a_2, b_2, d_2)]. \quad (4.26)$$

The expression of $\mathbf{J}$ in (4.22) is the same as in (2.59). The elements in the matrix $\mathbf{J}$ are numerically calculated as shown in (3.6)-(3.9).

Figure 4.6 Convergence of relative complex permeability versus iteration number

Figure 4.7 Convergence of relative complex permittivity versus iteration number

As one example to illustrate the convergence of the method, two ferrite core capacitors are assumed to have the dimensions of sample 2 and sample 4 in Table 4.1 respectively. The intrinsic complex permeability and permittivity of the ferrite at 1 MHz are assumed as $\mu = (2500 - j500)\mu_0$, $\epsilon = (8 \times 10^4 - j4 \times 10^4)\epsilon_0$. The initial values for the iteration are taken as $\mu = 1000\mu_0$, $\epsilon = 1 \times 10^5\epsilon_0$. From Figure 4.6 and Figure 4.7, it can be seen that the method can trace the intrinsic complex permeability and permittivity accurately after only a few iteration steps, even though the initial values for the iteration deviate a lot from the convergence values.

4.2.4 Experimental results

In this section, results from three samples in the shape of rectangular bar are described. Two of them are used to trace the intrinsic complex permittivity and permeability. The apparent complex permittivity of the third bar is calculated from the traced intrinsic complex permittivity and permeability. The calculated and measured apparent complex permittivities of the third bar are compared to validate the intrinsic values.

The three bars were cut from a TDK H7C4 ferrite bar by using a high precision diamond blade cutter (ISOMET 1000). The lateral surfaces of the three bars were polished by using a variable speed grinder/polisher (Ecomet 6). The dimensions of the three bars are shown in Table 4.2. Since the three bars are cut from a large bar, their thickness (i.e. d) and one of their two surface lengths (i.e. $2a$) are all the same. To make the upper and the lower surface areas of the three samples as different as possible, the ratio of the other

surface lengths (i.e. $2b$) of the three bars is chosen to be around 1:2:3, as shown in the third column in Table 4.2.

Table 4.2 Dimensions of the three bars cut from a large ferrite bar

Bar No.	2a (mm)	2b (mm)	d (mm)
1	10.9	4.55	5.55
2	10.9	9.5	5.55
3	10.9	14.65	5.55

Figure 4.8 Measured impedances of the three ferrite core capacitors

The air gaps between the Mn-Zn ferrites and the capacitor electrodes are eliminated by painting the upper and the lower surfaces ($2a \times 2b$) of the ferrite bars with silver conductive paint. The impedances of the capacitors are measured with HP 4194A

impedance analyzer equipped with the HP 16451B dielectric test fixture. The measured impedances of the three ferrite core capacitors are shown in Figure 4.8. The measured apparent complex permittivities obtained by using (4.21) are shown in Figure 4.9. The difference in the measured apparent complex permittivities of the two samples in the frequency range above around a few hundred kHz is due to the dimensional effects which have been analyzed in subsection 4.2.2.

Figure 4.9 Measured apparent complex permittivities of the three bars

The measured impedances of Bar 1 and Bar 2 in the frequency range above 800 kHz are used to trace the intrinsic complex permittivity and permeability by using the method described in subsection 4.2.3. The results are shown in Figure 4.10 and Figure 4.11.

To validate the traced intrinsic values, the apparent complex permittivity of Bar 3 is

calculated from the traced intrinsic complex permittivity and permeability. The calculated and the measured apparent complex permittivities of Bar 3 in the frequency range from 800 kHz to 10 MHz are shown in Figure 4.12. It can be seen that the calculated apparent complex permittivity coincides with the measured one very well.

In the frequency range below 800 kHz, when the method is used to trace the intrinsic values, it is found that the traced intrinsic permeability is apparently wrong. The reason is that in the low frequency range, the influence of the permeability on the apparent complex permittivity is negligible. In other words, a minor variation of the apparent complex permittivity in the low frequency will need a large variation of the intrinsic complex permeability. Thus the traced intrinsic complex permeability is too sensitive to the measurement error to make the method practical in the low frequency range.

Figure 4.10 Traced intrinsic complex permeability of the ferrite bar

Figure 4.11 Traced intrinsic complex permittivity of the ferrite bar

Figure 4.12 Comparison of measured and calculated apparent complex permittivity

4.3 Derivative Field-Circuit Coupled Method Using Two Toroidal Ferrite Core Inductors

This section presents a technique to trace the Mn-Zn ferrites' intrinsic complex permeability and permittivity by using two toroidal ferrite core inductors with rectangular cross sections. Experimental results show good agreement between the traced intrinsic complex permeability and the apparent complex permeability of a very small toroidal Mn-Zn ferrite core. Also observed is the good agreement between the calculated and the measured apparent complex permeabilities of the Mn-Zn ferrite cores of different sizes. The technique needs no difficult preparation of the samples, and has no special requirement on the sizes of the samples if only the sizes of the two samples are different. For more accurate result of the traced intrinsic complex permittivity, large samples are preferred. One advantage of using large samples is that the measurement errors of the dimensional parameters and the impedance can be minimized.

4.3.1 Determination of intrinsic complex permeability and permittivity

To determine the intrinsic complex permeability and permittivity from the apparent complex permeabilities, or equivalently from the measured impedances, of two ferrite core inductors of different sizes, the Newton-Raphson method [18, 34] is used. The iteration equation of the Newton-Raphson method can be written as:

$$\begin{bmatrix} \mu'(i+1) \\ \mu''(i+1) \\ \varepsilon'(i+1) \\ \varepsilon''(i+1) \end{bmatrix} = \begin{bmatrix} \mu'(i) \\ \mu''(i) \\ \varepsilon'(i) \\ \varepsilon''(i) \end{bmatrix} - \mathbf{J}^{-1} \begin{bmatrix} f_1(i) - \text{Re}(Z_1) \\ f_2(i) - \text{Im}(Z_1) \\ f_3(i) - \text{Re}(Z_2) \\ f_4(i) - \text{Im}(Z_2) \end{bmatrix}, \quad (4.27)$$

where Z_1 and Z_2 are the measured impedances of the two ferrite core inductors respectively, and

$$f_1 = \text{Re}[Z_L(\mu, \varepsilon, R_{11}, R_{21}, h_1, N_1)], \quad (4.28)$$

$$f_2 = \text{Im}[Z_L(\mu, \varepsilon, R_{11}, R_{21}, h_1, N_1)], \quad (4.29)$$

$$f_3 = \text{Re}[Z_L(\mu, \varepsilon, R_{12}, R_{22}, h_2, N_2)], \quad (4.30)$$

$$f_4 = \text{Im}[Z_L(\mu, \varepsilon, R_{12}, R_{22}, h_2, N_2)]. \quad (4.31)$$

In (4.28)-(4.31), R_{11} , R_{21} , h_1 , and N_1 are the inner radius, the outer radius, the height, and the coil numbers of the first core respectively. R_{12} , R_{22} , h_2 , and N_2 are the inner radius, the outer radius, the height, and the coil numbers of the second core respectively. Z_L in (4.28)-(4.31) is the same as in (3.19). The expression of $\mathbf{J}$ in (4.27) is the same as in (2.59). The elements in the matrix $\mathbf{J}$ are numerically calculated as shown in (3.6)-(3.9).

As one example to illustrate the convergence of the method, two ferrite core inductors are assumed to have the dimensions as $R_{11} = 5$ mm, $R_{21} = 7.5$ mm, $h_1 = 5$ mm, $R_{12} = 7.5$ mm, $R_{22} = 12.5$ mm, and $h_2 = 10$ mm. The intrinsic complex permeability and permittivity of the Mn-Zn ferrite at 1 MHz are assumed to be $\mu = (2500 - j500)\mu_0$, $\varepsilon = (1 \times 10^5 - j5 \times 10^4)\varepsilon_0$. The initial values for the iteration are taken as $\mu = 1000\mu_0$, $\varepsilon = 1.5 \times 10^5\varepsilon_0$. From Figure 4.13 and Figure 4.14, it can be seen that the method can trace the intrinsic complex permeability and permittivity accurately after only a few iteration steps, even though the initial values for the iteration deviate a lot from the convergence values.

4.3.2 Experimental results

The four toroidal cores in Section 3.5, i.e. Sample 1, Sample 2, Sample 3 and Sample 4, are used as the experimental samples. The dimensions of the four cores are summarized

in Table 4.3 for clarity, where $2R_2$, $2R_1$ and h denote the outer diameter, the inner diameter and the height of a toroidal core respectively. Sample 2 and Sample 3 are used to trace the intrinsic complex permeability and permittivity. The apparent complex permeabilities of the other two cores are calculated from the traced intrinsic complex permittivity and permeability. The calculated apparent complex permeabilities are compared with the measured apparent complex permeabilities. One of the validation cores, i.e. Sample 1, is very small. Its measured apparent complex permeability can thus be compared with the traced intrinsic complex permeability for the validation of the traced intrinsic values, as mentioned in Section 3.5.

Figure 4.13 Convergence of relative complex permeability versus iteration number

Figure 4.14 Convergence of relative complex permittivity versus iteration number

The impedances of the ferrite core inductors are measured by using the short-ended coaxial test fixture presented in Section 3.4. The magnitudes and the phase angles of the

measured impedances of the four ferrite core inductors are shown in Figure 4.15 and Figure 4.16 respectively.

Table 4.3 Dimensions of the four cores

Sample No.	$2R_2$ (mm)	$2R_1$ (mm)	h (mm)
1	12.9	8.05	0.70
2	12.9	8.05	2.05
3	12.9	8.05	7.80
4	12.9	8.05	12.25

Figure 4.15 Modules of the measured impedances of the four ferrite core inductors

Figure 4.16 Phase angles of the measured impedances of the four ferrite core inductors

From Figure 4.15 and Figure 4.16, we can see that the ferrite core inductors can become capacitive at high frequencies. At a certain resonance frequency the impedance of the

ferrite core inductor reaches its maximum value, and the phase angle is zero. In other words, the ferrite core inductor becomes resistive at that resonance frequency. For the core of smaller size, the resonance frequency is higher.

Figure 4.17 Traced intrinsic complex permeability and measured apparent complex permeabilities of Sample 2 and Sample 3 (solid lines: Sample 3; dashed lines: Sample 2; dotted lines: intrinsic)

Figure 4.18 Traced intrinsic complex permittivity.

The measured apparent complex permeabilities of Sample 2 and Sample 3 and the resulting traced intrinsic complex permeability are shown in Figure 4.17. The traced intrinsic complex permittivity is shown in Figure 4.18. The difference between the measured apparent permeabilities of the two cores and the intrinsic values in the frequency range above 1 MHz is due to the dimensional effects, as mentioned in Section 2.3. For larger cores, the influence of the high permittivity is more prominent, as we can see from Figure 4.17.

Figure 4.19 Comparison of measured and calculated apparent complex permeability of Sample 1 and traced intrinsic complex permeability (solid lines: measured; circles: calculated; crosses: intrinsic)

Figure 4.20 Comparison of measured and calculated apparent complex permeability of Sample 4 (solid lines: measured; circles: calculated)

As mentioned before, since Sample 1 is very small, the influence of the intrinsic complex permittivity on the apparent complex permeability is negligible. Thus the traced intrinsic complex permeability shown as crosses in Figure 4.19 coincides with the measured apparent complex permeability shown as solid lines in Figure 4.19. To validate the technique to predict the apparent complex permeability, the apparent complex permeabilities of Sample 1 and Sample 4 are calculated from the traced intrinsic complex permeability and permittivity. The calculated and the measured apparent complex permeabilities of Sample 1 and Sample 4 are shown as circles and solid lines in Figure 4.19 and Figure 4.20 respectively. It can be seen that the calculated apparent complex permeabilities coincides with the measured values very well.

The result of the traced intrinsic complex permittivity in Figure 4.18 has great errors in the low frequency range. The reason is similar as stated in subsection 4.2.4 where great errors of the determined intrinsic complex permeability are found in the low frequency range when the technique in Section 4.2 is used. The accuracy of the intrinsic complex permittivity can be improved when large cores are used to trace the intrinsic values, since for larger cores dimensional effects are more prominent in the low frequency range.

The reason why the apparent complex permeability of Sample 4 can be successfully predicted even though the determined intrinsic complex permittivity has great errors in the low frequency range is that since dimensional effects in Sample 4, which is not very large, in the low frequency range are negligible, the apparent complex permeability in the frequency range is mostly only dependent on the intrinsic complex permeability.

4.4 Conclusions

This Chapter presents two novel field-circuit coupled methods to determine the intrinsic complex permeability and permittivity of the Mn-Zn ferrite cores. In the first method, two rectangular parallel plate ferrite core capacitors are used. The simple bar structure of the samples guarantees their easy and precise preparation. The apparent complex permeability of the ferrites does not need to be measured in that method. Thus the method is especially useful when the test fixtures for measuring the apparent complex permeability are not available, or the shapes of the samples do not permit the direct measurement of the apparent complex permeability. In contrast to the first method, the second method needs two different-sized toroidal ferrite cores. Since the method does not need to measure the apparent complex permittivity of the samples, it avoids the difficulty in making solid electrical contacts on the surfaces of the ferrite material. From the experimental results we can see that the limitations of the two methods are as obvious as their advantages. They cannot be applied in the frequency range usually below a few hundred kHz where the dimensional effects are negligible. Great errors of the intrinsic complex permeability and the intrinsic complex permittivity in that frequency range are found respectively in the first method and the second method.

CHAPTER 5

GENERAL FIELD-CIRCUIT COUPLED METHODS USING ONE SAMPLE

5.1 Introduction

In Chapter 2 and Chapter 3, different field-circuit coupled methods using a Mn-Zn ferrite core inductor and a Mn-Zn ferrite core capacitor are compared in terms of accuracy and computational efficiency. In Chapter 4, two field-circuit coupled methods using two Mn-Zn ferrite core capacitors and two Mn-Zn ferrite core inductors respectively are also presented.

Since there are two parameters, i.e. μ and ϵ , to be determined for the Mn-Zn ferrites, at least two independent measurements should be performed. For the two measurements to be independent, distributions of the fields in the core(s) in the two measurements should be different. To make the distribution of the fields different, one way is to change the excitation mode of the exciting fields on the ferrite core. The other way is to change the physical dimensions of the core. The methods in Chapter 2 and Chapter 3 belong to the former. The methods presented in Chapter 4 fall into the latter. Compared to the methods in Chapter 4 needing two measurement samples, the methods using only one measurement sample obviously have the advantage of easy sample preparation and eliminating the potential errors caused by the nonhomogeneity between the samples.

The objective of this Chapter is to look for as many as possible field-circuit coupled methods to suit different measurement situations, i.e. the availability of different test fixtures, to determine the intrinsic complex permeability and permittivity of the Mn-Zn ferrite core by using one measurement sample [35]. From the above analysis, it can be seen that as long as in the two measurements the distributions of the fields in the core are different and can be predicted mathematically when both μ and ϵ of the core are given, μ and ϵ can then be determined from the two measurements by using an appropriate inverse

approach like the Newton-Raphson method. A toroidal sample is assumed in this chapter due to both its versatility to construct either an inductor or a capacitor, in other words the simplicity to change the excitation mode of the core, and the simple symmetry of the configuration which may result in an easy mathematical model to calculate the fields in the core. Different electric and magnetic field excitation modes of the toroidal core are introduced in Section 5.2. A simple and general finite difference method for solving the fields in the core under different magnetic field excitation modes is presented in Section 5.3. The fields in the core under the electric field excitation are solved by using the currently available methods, as presented in Section 5.4. Theoretically any combination of two different exciting modes can be used to determine the intrinsic complex permeability and permittivity of the Mn-Zn ferrite core. A brief discussion about the practical combinations is presented in Section 5.5. Experimental results of two practical combinations and an impractical combination are also compared in that section.

5.2 Different Excitation Modes of the Toroidal Mn-Zn Ferrite Core

Figure 5.1 Different magnetic field excitation modes (▨ Mn-Zn ferrite core; ▤ PEC; $\dot{I}$ excitation current)

Figure 5.2 Different electric field excitation modes (▨ Mn-Zn ferrite core; ▤ PEC; $\dot{U}$ excitation voltage)

The electromagnetic excitation modes of the toroidal Mn-Zn ferrite core can be classified into the electric field excitation and the magnetic field excitation. The exciting fields have to be axially symmetric to maintain the symmetry of the fields in the core. Different excitation modes are shown in Figure 5.1 and Figure 5.2. It is noted that on the interface

between the Mn-Zn ferrite core and the PEC the tangential electric fields are zero. In the magnetic field excitation modes shown in Figure 5.1, the exciting fields are generated by the exciting current located on the axis of the toroidal core. In Figure 5.1 (b), the upper and the lower flat surfaces of the core have solid electrical contacts with the PEC. In Figure 5.1 (c), the inner and the outer lateral surfaces of the core have solid electrical contacts with the PEC. In the electric field excitation modes shown in Figure 5.2, two different capacitors are constructed. In Figure 5.2 (a), the two capacitor electrodes are on the upper and the lower flat surfaces of the core. In Figure 5.2 (b), the two capacitor electrodes are on the inner and the outer lateral surfaces of the core.

5.3 General Finite Difference Method for Toroidal Mn-Zn Ferrite Cores under Magnetic Field Excitation

Let us take the magnetic field excitation mode in Figure 5.1 (b) as an example to illustrate the basic principle of the general finite difference method to solve the fields in the core under magnetic field excitations. The toroidal Mn-Zn ferrite core under study is shown in Figure 5.3. Due to the cylindrical symmetry of the structure and the exciting fields, only the fields in the cross section need to be solved. Furthermore, with the symmetry along the plane $z=0$ taken into account, only half of the cross section, which is enclosed by the dashed rectangle shown in Figure 5.3, needs to be analyzed.

Figure 5.3 Toroidal Mn-Zn ferrite core under study

The electric and the magnetic fields in the core are governed by the following equations [36]:

$$-\frac{\partial H_{\phi}}{\partial z} = j\omega\epsilon_0\epsilon_r E_r, \quad (5.1)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r H_{\phi}) = j\omega\epsilon_0\epsilon_r E_z, \quad (5.2)$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu_0\mu_r H_\phi, \quad (5.3)$$

where μ_r and ϵ_r are the relative complex permeability and permittivity of the Mn-Zn ferrite core respectively.

The discretization form of (5.1)-(5.3) can be written as:

$$j\omega\epsilon_0\epsilon_r E_r^{i,j+1/2} = \frac{H_\phi^{i,j} - H_\phi^{i,j+1}}{\Delta z}, \quad (5.4)$$

$$j\omega\epsilon_0\epsilon_r E_z^{i+1/2,j} = \frac{1}{r_{i+1/2}} \frac{r_{i+1} H_\phi^{i+1,j} - r_i H_\phi^{i,j}}{\Delta r}, \quad (5.5)$$

$$-j\omega\mu_0\mu_r H_\phi^{i,j} = \frac{E_r^{i,j+1/2} - E_r^{i,j-1/2}}{\Delta z} - \frac{E_z^{i+1/2,j} - E_z^{i-1/2,j}}{\Delta r}, \quad (5.6)$$

where Δr and Δz are the lengths of the differential meshes in the r and the z directions respectively. The locations of the discrete field components are shown in Figure 5.4.

Figure 5.4 Locations of the discrete electric and magnetic field components

Substituting (5.4) and (5.5) into (5.6) to eliminate the electric field components results in the following equation:

$$\frac{H_\phi^{i,j} - H_\phi^{i,j+1}}{\Delta z^2} + \frac{H_\phi^{i,j} - H_\phi^{i,j-1}}{\Delta z^2} + \frac{r_i H_\phi^{i,j} - r_{i-1} H_\phi^{i-1,j}}{r_{i-1/2} \Delta r^2} + \frac{r_i H_\phi^{i,j} - r_{i+1} H_\phi^{i+1,j}}{r_{i+1/2} \Delta r^2} - \omega^2 \mu_0 \epsilon_0 \mu_r \epsilon_r H_\phi^{i,j} = 0. \quad (5.7)$$

Of the five terms in (5.7), the first four terms correspond to the contribution of the

electric field components around the magnetic field component. The fifth term is the contribution of the magnetic field component.

On the inner and the outer lateral surfaces of the core, the tangential magnetic fields, i.e. H_ϕ , must be continuous. Thus according to the Ampere's law, the magnetic field components located on those surfaces can be written as:

$$H_\phi^{i,j} = \frac{NI}{2\pi R_1}, \quad i = 1, \quad (5.8)$$

$$H_\phi^{i,j} = \frac{NI}{2\pi R_2}, \quad i = m, \quad (5.9)$$

where N is the turn number of the exciting coils, and I is the current in the coils.

On the upper and the lower surfaces of the core, which have solid electric contact with the PEC, the tangential electric fields are zero. Thus we have:

$$E_r^{i,j+1/2} = 0, \quad j = n. \quad (5.10)$$

Since the structure of the core and the fields are symmetrical along the plane $z=0$, on the basis of (5.7) the magnetic field components located on the plane $z=0$ satisfy the following equation:

$$2 \frac{H_\phi^{i,j} - H_\phi^{i,j+1}}{\Delta z^2} + \frac{r_i H_\phi^{i,j} - r_{i-1} H_\phi^{i-1,j}}{r_{i-1/2} \Delta r^2} + \frac{r_i H_\phi^{i,j} - r_{i+1} H_\phi^{i+1,j}}{r_{i+1/2} \Delta r^2} - \omega^2 \mu_0 \epsilon_0 \mu_r \epsilon_r H_\phi^{i,j} = 0, \quad j = 1. \quad (5.11)$$

From (5.7)-(5.11), a set of simultaneous linear equations containing only the magnetic field components in the solution region can be obtained as follows:

$$[A][X] = [B], \quad (5.12)$$

where $[A]$ is a sparse coefficient matrix, $[X]$ is a one-column matrix consisting of the unknown magnetic field components, and the column matrix $[B]$ is the source vector, whose non-zero terms are due to (5.8) and (5.9). There are many efficient methods to solve (5.12) such as the BICG method and the Gaussian elimination method. The programs in this chapter are all written in Matlab 7. The backslash '\ ' Matlab command

can be used to efficiently solve (5.12). In Matlab, $[X] = [A] \backslash [B]$ is the solution to (5.12) computed by using the Gaussian elimination method.

Based on the Faraday's law of induction, the induced electromagnetic force in the inductor can be expressed as:

$$V = -j\omega N \int \vec{B} \cdot d\vec{S} = j\omega N \mu_0 \mu_r \int H_\phi dS = j\omega N \mu_0 \mu_r 2\Delta r \Delta z \left(\frac{1}{4} \sum_{i=1,m} \sum_{j=1} H_\phi^{i,j} + \frac{1}{2} \sum_{i=1,m} \sum_{j=2}^n H_\phi^{i,j} + \frac{1}{2} \sum_{i=2}^{m-1} \sum_{j=1} H_\phi^{i,j} + \sum_{i=2}^{m-1} \sum_{j=2}^n H_\phi^{i,j} \right). \quad (5.13)$$

The impedance of the inductor can then be obtained by dividing the induced voltage by the current in the coils.

Taking a look back at the solution procedures above, we will see that the fields are solved by combining a general equation in (5.7) which govern the relations between the neighboring magnetic field components in the finite difference meshes, and its corresponding boundary equations including the given magnetic field boundary equation in (5.8) and (5.9), the given electric field boundary equation in (5.10), and the symmetry boundary equation in (5.11). The general equation (5.7) is the same for all the magnetic field excitation modes in Figure 5.1. Thus the crux of the problem is on how to create the finite difference meshes so that the boundary conditions can be easily implemented. The general rule of creating the finite difference meshes is to set the known field components on the boundary, and the magnetic field components on the symmetry plane. For example the finite difference meshes for Figure 5.1 (c) are shown in Figure 5.5.

The general finite difference method presented above can be conveniently verified by comparison of the impedance calculated by using the proposed method and the impedance obtained by using other methods.

The short-ended coaxial structure shown in Figure 5.6 is actually equivalent to a one-turn inductor. The impedance of the inductor can be solved by using the finite difference method presented above. The input impedance of the coaxial structure from a certain reference plane can also be solved by using the 2-D FDFD method presented in Chapter 7.

Figure 5.5 Finite difference meshes of the case in Figure 5.1 (c)

Figure 5.6 Short-ended coaxial line loaded with the Mn-Zn ferrite core sample in Figure 5.1 (b)

Figure 5.7 Comparison of impedances calculated by using different methods for the magnetic field excitation mode in Figure 5.1 (b) (solid lines: method in this section; circles: 2-D FDFD method)

It is assumed that the intrinsic complex permeability and permittivity of the Mn-Zn ferrite core in the structure are respectively the same as shown in Fig. 2 and Fig. 3 in [31]. The inner and the outer radii of the core are 5 mm and 10 mm respectively. The height of the core is 8 mm. The thickness of the PEC on the upper and the lower surfaces of the core is 0.2 mm. The air gaps between the lateral surfaces of the core and the coaxial line are 0.5 mm wide. The distance between short end of the coaxial line and the reference plane of the input impedance is 9 mm. The lengths of the differential grids of the general finite difference method and the 2-D FDFD method are set to be 0.05 mm.

Figure 5.8 Comparison of impedances calculated by using different methods for the magnetic field excitation modes in Figure 5.1 (a) (solid lines: method in this section; circles: 2-D FDFD method; crosses: method in Section 3.3)

Figure 5.9 Comparison of impedances calculated by using different methods for the magnetic field excitation modes in Figure 5.1 (c) (solid lines: method in this section; circles: 2-D FDFD method)

The results of the impedance calculated by using the two methods are shown in Figure 5.7. In the next two numerical examples, the coaxial line is loaded with the samples in the manner shown in Figure 5.1 (a) and Figure 5.1 (c) respectively. In those cases, the air gap between the lower surface of the sample and the short end of the coaxial line is 0.2 mm. The numerical results of the two cases are shown in Figure 5.8 and Figure 5.9 respectively. For the magnetic field excitation mode in Figure 5.1 (a), the analytical method presented in Section 3.3 can also be used to efficiently solve the problem. The result of the method in Section 3.3 is also shown in Figure 5.8. The very good agreement between the results of different methods as shown in Figure 5.7-Figure 5.9 validates the

general finite difference method proposed here. It is also interesting to note from Figure 5.7-Figure 5.9 that at the frequencies above around 1-3 MHz, the imaginary parts of the impedances are negative, which implies that due to the dimensional effects the inductors shown in Figure 5.1 behave as capacitors at high frequencies.

5.4 Computation Methods for Toroidal Mn-Zn Ferrite Cores under Electric Field Excitation

Figure 5.10 Different models derived from the model in Figure 5.2 (b)

Figure 5.11 Calculated impedances of the models in Figure 5.10 (solid lines: model in Figure 5.10 (a); crosses: model in Figure 5.10 (b); circles: model in Figure 5.10 (c))

For the electric field excitation mode shown in Figure 5.2 (a), the fringing effects of the ferrite core capacitor can be totally neglected due to the high permittivity of the Mn-Zn ferrite core [31]. The analytical method presented in [18] or Equation (3.5) can efficiently solve the fields in the core and the impedance of the Mn-Zn ferrite core capacitor. For computation of the impedance of the Mn-Zn ferrite core capacitor shown in Figure 5.2 (b), the three different models shown in Figure 5.10 are derived. The first model is an infinite-flanged open-ended coaxial probe. Its input impedance can be calculated by using the full-wave analysis method presented in [37]. The last two models

are coaxial discontinuity structures. Their input impedance can be solved by using the mode matching method [30, 38]. It is assumed that the Mn-Zn ferrite cores in the three models are the same as the one in Figure 5.6. The air gap at the short end of the coaxial line in Figure 5.10 (b) is 1 mm long. The calculated input impedances of the three models are shown in Figure 5.11. The very good agreement among those results shows that the fields at the non-source end have negligible influence on the input impedance in the frequency range of interest due to the high permittivity of the Mn-Zn ferrite core. Thus we can use any of the three computational models to replace the original one in Figure 5.2 (b). But obviously the input impedances of the models in Figure 5.10 (b) and Figure 5.10 (c) can be more efficiently and simply solved by using the mode matching method than that in Figure 5.10 (a) solved by using the method in [37]. It is also noted from Figure 5.11 that at the frequencies around 1 MHz the imaginary parts of the impedances become positive, which implies that the capacitor models shown in Figure 5.10 behave as inductors at those frequencies due to the dimensional effects.

5.5 Experimental Results

Table 5.1 Practical combination of different excitation modes

Combination	Magnetic field excitation	Electric field excitation
A	Figure 5.1 (a)	Figure 5.2 (a)
B	Figure 5.1 (a)	Figure 5.2 (b)
C	Figure 5.1 (b)	Figure 5.2 (a)
D	Figure 5.1 (c)	Figure 5.2 (b)

As we mentioned in Section 5.1, as long as the distributions of the fields in the core are different in the two measurements, μ and ϵ of the Mn-Zn ferrite core can then be determined. Thus theoretically any two of the excitation modes shown in Figure 5.1 and Figure 5.2 can be used for our purpose. But in practice, in order to minimize the influence of the imperfection of the sample and the measurement errors, significant difference between the field distributions in the two measurements are preferred. The combination of one magnetic field excitation and one electric field excitation will ensure the significant difference in the two measurements. Thus the practical combinations of

the two different excitation modes can be summarized as in Table 5.1. The reason that the combinations such as Figure 5.1 (b) & Figure 5.2 (b) and Figure 5.1 (c) & Figure 5.2 (a) are not included in the table is due to their inconvenient sample preparation.

Experimental results of Combination A, Combination B, and the combination of Figure 5.1 (a) and Figure 5.1 (b) are compared. For conciseness, the combination of Figure 5.1 (a) and Figure 5.1 (b) is named Combination E. The temperature at which the experiments are conducted is 23°C. Sample 1, Sample 2 and Sample 3 in Table 4.3 are used in the experiment. Sample 3 is used to determine the intrinsic complex permeability and permittivity. The other two samples are used to validate the results.

Figure 5.12 Calibrated measured impedances with the Mn-Zn ferrite core under the magnetic field excitations in Figure 5.1 (a) (solid lines) and Figure 5.1 (b) (dotted lines)

Figure 5.13 Measured impedance with the Mn-Zn ferrite core under the electric field excitation in Figure 5.2 (a)

Figure 5.14 Determined intrinsic complex permeabilities (solid lines: Combination A; dotted lines: Combination B; dashed lines: Combination E; circles: measured apparent complex permeability of Sample 1)

In the measurements, the HP4194A impedance analyzer is used. In the first measurement, Sample 3 is under the magnetic field excitation in Figure 5.1 (a). The calibrated impedance Z_{core} (see (3.21)) measured with the short-ended coaxial test fixture presented in Chapter 3 is shown in Figure 5.12 as solid lines. In the second measurement, the upper and the lower surfaces of Sample 3 are painted with silver conductive paint. With Sample 3 under the magnetic field excitation in Figure 5.1 (b), the calibrated measured impedance is shown in Figure 5.12 as dotted lines. In the third measurement, Sample 3 is under the electric field excitation in Figure 5.2 (a). The conductive paint on the upper and the lower surfaces effectively eliminates the air gaps between those surfaces and the capacitor electrodes. The measured impedance of the Mn-Zn ferrite core capacitor by using the HP 16451B dielectric test fixture is shown in Figure 5.13. For conciseness, the

impedances obtained in the three successive measurements are named Z_a , Z_b and Z_c respectively.

Figure 5.15 Determined intrinsic complex permittivities (solid lines: Combination A; dotted lines: Combination B; dashed lines: Combination E)

The determined intrinsic complex permeabilities and permittivities by using the Newton-Raphson method from the measurement results of Combination A (Z_a & Z_c), Combination B (Z_b & Z_c), and Combination E (Z_a & Z_b) are shown in Figure 5.14 and Figure 5.15. It is worth mentioning that Combination A actually is equivalent to method 3 in Section 3.5. For validation of the determined intrinsic values, the apparent complex permeability of Sample 1 is measured and shown in Figure 5.14 as circles. As we mentioned in subsection 4.3.2 or Section 3.5, the measured apparent complex permeability of Sample 1 actually can be taken as the intrinsic complex permeability. From Figure 5.14, very

good agreement can be observed between the measured apparent complex permeability of Sample 1 and the result of either Combination A or Combination B up to 10 MHz.

From Figure 5.14 and Figure 5.15 obvious discrepancy can be seen between the result of Combination E and those of the other two combinations. It can also be seen from Figure 5.14 and Figure 5.15 that the result of Combination E at the high frequencies is more accurate than that at the low frequencies. The reason is that the difference between the field distributions in the core in the two measurements at the high frequencies is more obvious, as we mentioned before. In other words, the measured impedances in the two measurements are more different at the high frequencies, as we can see from Figure 5.12.

For further validation of the results of Combination A and Combination B, the impedance with Sample 2 under the magnetic field excitation in Figure 5.1 (a) is calculated from the determined intrinsic values, and measured by using the short-ended coaxial test fixture as well. Very good agreement between the predicted values and the measured values can be observed (see Figure 5.16).

Figure 5.16 Measured and predicted impedances with Sample 2 under the magnetic field excitations in Figure 5.1 (a) (circles: measured; solid lines: Combination A; dotted lines: Combination B)

5.6 Conclusions

From the practical combination of the different excitation modes of a toroidal Mn-Zn ferrite core, a set of general field-circuit coupled methods is presented in this chapter. The simple structure of the sample needed guarantees the convenient preparation of the sample and the easy availability of the test fixtures. The two measurements of the

methods use one magnetic field excitation mode and one electric field excitation mode respectively. The different types of the excitation modes in the two measurements ensure the high accuracy of the methods in principle. The accuracy of the methods is shown experimentally by the good agreement between the determined and the measured intrinsic complex permeability, and the accurate prediction of the impedance when another ferrite core is under magnetic field excitation. The measurement result of another method which combines two magnetic field excitation modes is also shown in this chapter. By comparison, one can see that the accuracy of the combination of two different excitation modes in the general methods is higher than that of the combination of the same excitation modes.

CHAPTER 6

GENERAL MODE MATCHING METHOD FOR AXISYMMETRIC COAXIAL DISCONTINUITY STRUCTURES

6.1 Introduction

10 MHz is typically an upper frequency limit when the field-circuit coupled method is used to analyze the experimental results. This fact will be explained in details in Section 8.1. At high frequencies, axisymmetric coaxial discontinuity structures are commonly used in the permeability and/or permittivity measurement for materials [39-44]. This kind of structures are usually analyzed in the frequency domain by using the mode matching method [38, 41, 42, 45], which are commonly used to solve the waveguide discontinuity problems [38]. In the mode matching method, the fields in each discontinuity section are decomposed into discrete independent modes. According to the boundary condition that the transverse fields on the discontinuity interfaces must be continuous, a set of equations can be derived, whose unknowns are the amplitudes of the discrete modes. After the equations are solved, the fields in the structure can be obtained by assembling the discrete modes.

For certain axisymmetric coaxial discontinuity structures, the mode matching method has been successfully applied [41, 42, 45] to derive the specific governing equations for a specific structure. It is the purpose of this chapter to present a general approach based on the mode matching method to deal with this kind of structures. As we will see in Section 6.4, the electromagnetic waves in this kind of discontinuous structures can be described by a set of rather simple linear simultaneous equations which can be applied to different structures and it can be easily solved.

Since the mode matching method is based on the modal expansion of the total

electromagnetic field, the propagation constants of the modal fields in the structure have to be solved first. Usually, the propagation constants of waveguides or transmission lines are solved by using the analytical/semianalytical method for simple structures or by using the numerical methods such as the finite difference method [46-50] and the finite element method (FEM) [51, 52] for complicated structures. It is worthwhile to mention that in [46] a compact 2-D finite difference method is presented to solve the propagation constants of a general waveguide. In that method the longitudinal field components are eliminated and only four transverse field components are involved in the final resulting eigenvalue equation. In [47, 48], the compact 2-D finite difference method is further improved with the resultant eigenvalue problem involving only two field components.

In this chapter, for the uniformly filled coaxial and circular waveguides, the propagation constants are solved by using the existent semianalytical methods. For the multi-layer filled axisymmetric coaxial waveguides and the multi-layer filled axisymmetric circular waveguides, a 1-D finite difference method is presented to efficiently solve the propagation constants. In the 1-D finite difference method, with the axisymmetric geometry of the considered structures taken into account, the resultant eigenvalue problem involves only one field component. Thus it considerably reduces the required CPU time. For the multi-layer filled axisymmetric coaxial waveguides, a semianalytical approach, the field matching technique, is presented in [53] to solve the propagation constants. Since that method has to search the propagation constants in the complex plane by numerical routines with small step widths, as is described in detail in Section 6.5, by comparison the 1-D finite difference method presented in this chapter has much higher computational efficiency.

The organization of the chapter is as follows. The propagation constants of the axisymmetric structures under consideration are solved in Section 6.2. Based on the solutions of the propagation constants, in Section 6.3, the modal field patterns for different modes are determined. In Section 6.4, a set of general formulae which describe the matching relations of the fields on the discontinuity interfaces in the structures are derived. Several numerical examples are given in Section 6.5 to verify the methods in this chapter. Finally, a brief conclusion is drawn in Section 6.6.

6.2 Solving Propagation Constants in Axisymmetric Waveguides

In this chapter, among the high order modes, we are only interested in the dispersion characteristics of the symmetrical transverse magnetic (TM) modes, i.e. those with axial symmetry, which are the only high order modes excited in the axisymmetric discontinuity structures under the transverse electric and magnetic (TEM) wave excitation.

For the uniformly filled coaxial waveguides, the propagation constant γ of the TEM mode can be expressed as [54]:

$$\gamma = j\omega\sqrt{\mu\varepsilon}, \quad (6.1)$$

where μ and ε are the complex permeability and the complex permittivity of the filling material respectively. The propagation constants γ of the symmetrical TM modes satisfy the following equation [54]:

$$J_0(a)Y_0(ak) - Y_0(a)J_0(ak) = 0, \quad (6.2)$$

where J and Y are the Bessels functions of the first kind and the second kind respectively, and

$$a = k_c R_1, \quad (6.3)$$

$$k = R_2 / R_1, \quad (6.4)$$

$$k_c = \sqrt{\omega^2 \mu \varepsilon + \gamma^2}. \quad (6.5)$$

Since the roots of (6.2), i.e. a , are all real, they can be easily searched by numerical methods. Thus γ can be solved without much difficulty.

For the uniformly filled circular waveguide, the propagation constants γ of the symmetrical TM modes are solved by finding the roots of the following equation [54]:

$$J_0(b) = 0, \quad (6.6)$$

where

$$b = k_c R_2, \quad (6.7)$$

and k_c is defined in (6.5). Again, since the roots of (6.6), i.e. b , are all real, γ can be easily solved.

For the multi-layer filled coaxial or circular waveguides, solving the propagation constants is not as easy as for the uniformly filled ones. A 1-D finite difference method is presented below to efficiently solve the propagation constants of this kind of structures. Without loss of generality, an axisymmetric structure shown in Figure 6.1 is considered. The inner and the outer conductors are assumed to be PEC. Without the inner conductor, the structure becomes a circular waveguide.

Figure 6.1 Cross section of a multi-layer filled axisymmetric waveguide

Based on the Maxwell's curl equations in the cylindrical coordinates, the following equations governing the field components of the TM modes can be obtained [36]:

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu H_\phi, \quad (6.8)$$

$$-\frac{\partial H_\phi}{\partial z} = j\omega\epsilon E_r, \quad (6.9)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r H_\phi) = j\omega\epsilon E_z. \quad (6.10)$$

Since in the problem of dispersion characteristics analysis, the structure is considered to be uniform along the z axis, (6.8) and (6.9) can be rewritten as follows:

$$-\gamma E_r - \frac{\partial E_z}{\partial r} = -j\omega\mu H_\phi, \quad (6.11)$$

$$E_r = \frac{\gamma}{j\omega\epsilon} H_\phi, \quad (6.12)$$

where γ is the propagation constant to be solved.

Eliminating the E_r components in the two equations above by substituting (6.12) into (6.11) results in the following equation:

$$(\omega^2 \mu \varepsilon + \gamma^2) H_\phi + j\omega \varepsilon \frac{\partial E_z}{\partial r} = 0. \quad (6.13)$$

(a) 3-D FD cell

(b) 2-D TM mode FD cell

(c) 1-D TM mode FD cell

Figure 6.2 Finite difference cells in different dimensions in cylindrical coordinates

In the finite difference method, finite difference cells are used to discretize the differential equations. The conventional three-dimensional (3-D) finite difference cell in the cylindrical coordinates is shown in Figure 6.2 (a). Since the field components are independent of the azimuthal angle Φ , the 2-D finite difference cell for TM modes in Figure 6.2 (b) can be obtained by projecting the 3-D finite difference cell onto r - z plane and neglecting the field components of the transverse electric (TE) modes. For the differential equation in (6.13), the E_r components are eliminated. The 1-D finite difference cell in Figure 6.2 (c), which originates from neglecting the E_r components in the 2-D finite difference cell, can be simply used to discretize (6.13) as below:

$$(\omega^2 \mu_i \varepsilon_i + \gamma^2) H_\phi^i + j\omega \varepsilon_i \frac{E_z^{i+1/2} - E_z^{i-1/2}}{\Delta r} = 0, \quad (6.14)$$

where μ_i and ε_i are the complex permeability and the complex permittivity of the i th cell respectively. For simplicity, we assume that all the cells have the same width of Δr .

Discretizing (6.10) by using the 1-D finite difference cell, we then get

$$j\omega \varepsilon_i \frac{E_z^{i+1/2}}{\Delta r} = \frac{\varepsilon_i (r_{i+1} H_\phi^{i+1} - r_i H_\phi^i)}{\varepsilon_{i+1/2} r_{i+1/2} \Delta r^2}, \quad (6.15)$$

$$-j\omega\epsilon_i \frac{E_z^{i-1/2}}{\Delta r} = \frac{\epsilon_i (r_{i-1} H_\phi^{i-1} - r_i H_\phi^i)}{\epsilon_{i-1/2} r_{i-1/2} \Delta r^2}, \quad (6.16)$$

where $r_{i+1/2}$ is the distance between the center of the i th cell and the axis of the structure, and $\epsilon_{i+1/2}$ the complex permittivity at the interface between the i th cell and the $(i+1)$ th cell, which can be calculated as below by employing spatial average [46, 47, 55-57]:

$$\epsilon_{i+1/2} = \frac{\epsilon_i + \epsilon_{i+1}}{2}. \quad (6.17)$$

Equation (6.16) is derived from (6.15) by setting $i=i-1$. The E_z components at the surface of the PEC are set to zero in (6.14).

For the circular waveguides without the inner conductor, (6.16) cannot be used to evaluate the E_z component at $r = 0$, since there is no cell to the left of the axis in the computation area. To evaluate this component, the integral form of the Maxwell's equation [36] shown below is used:

$$\oint_l \mathbf{H} d\mathbf{l} = j\omega \int_s \epsilon \mathbf{E} d\mathbf{S}. \quad (6.18)$$

From (6.18), the E_z component at $r = 0$ can be expressed as:

$$-j\omega\epsilon_i \frac{E_z^{i-1/2}}{\Delta r} = \frac{-4H_\phi^i}{\Delta r^2}, \quad i = 1. \quad (6.19)$$

Eliminating the E_z components by substituting (6.15), (6.16), and (6.19) into (6.14), and implementing the PEC boundary conditions results in the following standard eigenvalue problem:

$$[A][H_\phi] = \gamma^2 [H_\phi], \quad (6.20)$$

where $[A]$ is a sparse and banded matrix.

Solving the eigenvalue problem above, we can obtain the propagation constants and the corresponding magnetic field distribution in the r direction, i.e. the eigenvectors, which can be used to determine the modal field patterns as presented in the following section. Since the eigenvalue problem (6.20) is only related to H_ϕ , it can be seen that the

propagation constant of the quasi-TEM mode is also included in the solution.

6.3 Determination of the Modal Field Patterns

For the TEM mode in the uniformly filled coaxial waveguides, the field components E_r and H_ϕ can be expressed as below [41, 42]:

$$E_r = \frac{C^1}{r} e^{-\gamma z}, \quad (6.21)$$

$$H_\phi = \sqrt{\frac{\epsilon}{\mu}} \frac{C^1}{r} e^{-\gamma z}, \quad (6.22)$$

where C^1 denotes a constant.

For the TM or the quasi-TEM modes, from Maxwell's equations in the cylindrical coordinates, the equation governing E_z can be written as [36]:

$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + k_c^2 \right] E_z = 0, \quad (6.23)$$

where

$$k_c^2 = \omega^2 \mu \epsilon + \gamma^2. \quad (6.24)$$

Based on the assumption that the field components are independent of the azimuthal angle Φ , the solution of (6.23) can be expressed as follows [58]:

$$E_z = \frac{k_c^x}{\gamma} [C^x J_0(k_c^x r) + D^x Y_0(k_c^x r)] e^{-\gamma z}, \quad (6.25)$$

where the superscript x is the layer index ($x=1, 2, \dots$).

The transverse field components E_r and H_ϕ can be expressed in terms of the field components E_z and H_z as follows [36]:

$$E_r = -\frac{1}{k_c^2} \left[\gamma \frac{\partial E_z}{\partial r} + \frac{j\omega\mu}{r} \frac{\partial H_z}{\partial \phi} \right], \quad (6.26)$$

$$H_\phi = -\frac{1}{k_c^2} \left[\frac{\gamma}{r} \frac{\partial H_z}{\partial \phi} + j\omega\epsilon \frac{\partial E_z}{\partial r} \right]. \quad (6.27)$$

Substituting (6.25) into (6.26) and (6.27), and ignoring the magnetic field components H_z of the TE modes, we can obtain:

$$E_r = [C^x J_1(k_c^x r) + D^x Y_1(k_c^x r)] e^{-\gamma z}, \quad (6.28)$$

$$H_\phi = \frac{j\omega\epsilon^x}{\gamma} [C^x J_1(k_c^x r) + D^x Y_1(k_c^x r)] e^{-\gamma z}. \quad (6.29)$$

For the uniformly filled coaxial waveguides, since $E_z|_{r=R_1}=0$ and $E_z|_{r=R_2}=0$, from (6.25), the coefficient D^1 in (6.25), (6.28) and (6.29) can be expressed as:

$$D^1 = -\frac{J_0(k_c R_1)}{Y_0(k_c R_1)} C^1 = -\frac{J_0(k_c R_2)}{Y_0(k_c R_2)} C^1. \quad (6.30)$$

For the uniformly filled circular waveguides, since $E_z|_{r=0} \neq \infty$, from (6.25), the coefficient D^1 in (6.25), (6.28) and (6.29) is zero.

For the multi-layer filled coaxial and circular waveguides, as mentioned in the last section, the magnetic field distribution along the r direction corresponding to the propagation constants can be solved as the eigenvectors of (6.20). Thus from (6.29), C^x and D^x can be solved by using the least square fitting method, which is shown in the numerical examples in Section 6.5.

6.4 Mode Matching Method for Axisymmetric Coaxial Discontinuity Structures

Figure 6.3 Axisymmetric discontinuity structure under consideration

For the general axisymmetric discontinuity structure shown in Figure 6.3, both sections may have multiple layers of filling materials. Also both sections may not necessarily have the inner conductors. A coaxial discontinuity structure may have many such discontinuities. Naturally, the first section of the structure, to which the source is connected, must have the inner conductor to support the TEM wave excitation.

From (6.21), (6.22), (6.28), and (6.29), the transverse electromagnetic field components of the two sections can be written as follows:

$$E_{Ar} = 0, \quad 0 < r < R_{1a}, \quad (6.31)$$

$$E_{Ar} = \frac{A_0}{r} [e^{\gamma_{A0}(z-z_0)} + \Gamma_{A0} e^{-\gamma_{A0}(z-z_0)}] + \sum_{i=1}^{\infty} A_i Z_{A1}(k_{ci}^x r) [e^{\gamma_i(z-z_0)} + \Gamma_{Ai} e^{-\gamma_i(z-z_0)}],$$

$$R_{1a} < r < R_{2a}, \quad (6.32)$$

$$H_{A\phi} = 0, \quad 0 < r < R_{1a}, \quad (6.33)$$

$$H_{A\phi} = -\sqrt{\frac{\epsilon_A}{\mu_A}} \frac{A_0}{r} [e^{\gamma_{A0}(z-z_0)} - \Gamma_{A0} e^{-\gamma_{A0}(z-z_0)}] + \sum_{i=1}^{\infty} \frac{-j\omega\epsilon^x}{\gamma_i} A_i Z_{A1}(k_{ci}^x r) [e^{\gamma_i(z-z_0)} - \Gamma_{Ai} e^{-\gamma_i(z-z_0)}],$$

$$R_{1a} < r < R_{2a}, \quad (6.34)$$

$$E_{Br} = 0, \quad 0 < r < R_{1b} \quad (6.35)$$

$$E_{Br} = \frac{B_0}{r} [e^{\gamma_{B0}(z-z_0+d)} + \Gamma_{B0} e^{-\gamma_{B0}(z-z_0+d)}] + \sum_{m=1}^{\infty} B_m Z_{B1}(k_{cm}^y r) [e^{\gamma_m(z-z_0+d)} + \Gamma_{Bm} e^{-\gamma_m(z-z_0+d)}],$$

$$R_{1b} < r < R_{2b}, \quad (6.36)$$

$$H_{B\phi} = 0, \quad 0 < r < R_{1b}, \quad (6.37)$$

$$H_{B\phi} = -\sqrt{\frac{\epsilon_B}{\mu_B}} \frac{B_0}{r} [e^{\gamma_{B0}(z-z_0+d)} - \Gamma_{B0} e^{-\gamma_{B0}(z-z_0+d)}] +$$

$$\sum_{m=1}^{\infty} \frac{-j\omega\epsilon^y}{\gamma_m} B_m Z_{B1}(k_{cm}^y r) [e^{\gamma_m(z-z_0+d)} - \Gamma_{Bm} e^{-\gamma_m(z-z_0+d)}], \quad R_{1b} < r < R_{2b}, \quad (6.38)$$

where d is the length of Section B in Figure 6.3, γ_{A0} is the propagation constant of the TEM mode in Section A when Section A is a uniformly filled coaxial waveguide with the

permeability and the permittivity of the filling material being μ_A and ϵ_A respectively, A_0 is zero except in the case where Section A is a uniformly filled coaxial waveguide, γ_{B0} , B_0 , μ_B and ϵ_B in Section B are the counterparts of γ_{A0} , A_0 , μ_A and ϵ_A respectively, and

$$Z_{An}(k_{ci}^x r) = [C_i^x J_n(k_{ci}^x r) + D_i^x Y_n(k_{ci}^x r)], \quad (6.39)$$

$$Z_{Bn}(k_{cm}^y r) = [C_m^y J_n(k_{cm}^y r) + D_m^y Y_n(k_{cm}^y r)]. \quad (6.40)$$

The continuity of the transverse electromagnetic field components on the discontinuity interface provides the following equations:

$$E_{Ar} \big|_{z=z_0} = E_{Br} \big|_{z=z_0}, \quad (\min(R_{1a}, R_{1b}) < r < \max(R_{2a}, R_{2b})), \quad (6.41)$$

$$H_{A\phi} \big|_{z=z_0} = H_{B\phi} \big|_{z=z_0}, \quad (\max(R_{1a}, R_{1b}) < r < \min(R_{2a}, R_{2b})). \quad (6.42)$$

The following orthogonality relation between different modes in Section A or Section B holds when the PEC boundary conditions are assumed [38]:

$$\int_{R_1}^{R_2} (\mathbf{E}_p \times \mathbf{H}_q) \cdot \mathbf{u}_z 2\pi r dr = 0, \quad (6.43)$$

where R_2 is the inner radius of the outer conductor, R_1 is the outer radius of the inner conductor, and $p \neq q$. In the case where there is no inner conductor, R_1 is zero.

The first step of the mode matching procedure is to perform the following integral at both sides of (6.41):

$$\int_{R_{1a}}^{R_{2a}} E_{Ar} H_{A\phi i} 2\pi r dr. \quad (6.44)$$

By making use of the orthogonality relation in (6.43), together with the Lommel integrals, the following equations can be obtained:

$$\begin{aligned} A_0(1 + \Gamma_{A0}) \int_{R_{1a}}^{R_{2a}} \frac{1}{r} dr &= A_0(1 + \Gamma_{A0}) \ln \frac{R_{2a}}{R_{1a}} \\ &= B_0(e^{\gamma_{B0}d} + \Gamma_{B0}e^{-\gamma_{B0}d}) \int_{\max(R_{1a}, R_{1b})}^{\min(R_{2a}, R_{2b})} \frac{1}{r} dr + \sum_{m=1}^{\infty} B_m(e^{\gamma_m d} + \Gamma_{Bm}e^{-\gamma_m d}) \int_{\max(R_{1a}, R_{1b})}^{\min(R_{2a}, R_{2b})} Z_{B1}(k_{cm}^y r) dr \end{aligned}$$

$$\begin{aligned}
 &= B_0(e^{\gamma_{B0}d} + \Gamma_{B0}e^{-\gamma_{B0}d}) \ln \frac{\min(R_{2a}, R_{2b})}{\max(R_{1a}, R_{1b})} + \sum_{m=1}^{\infty} B_m(e^{\gamma_m d} + \Gamma_{Bm}e^{-\gamma_m d}) \sum_{y=y_1}^{y_t} \int_{R_{1y}}^{R_{2y}} Z_{B1}(k_{cm}^y r) dr \\
 &= B_0(e^{\gamma_{B0}d} + \Gamma_{B0}e^{-\gamma_{B0}d}) \ln \frac{\min(R_{2a}, R_{2b})}{\max(R_{1a}, R_{1b})} \\
 &\quad + \sum_{m=1}^{\infty} B_m(e^{\gamma_m d} + \Gamma_{Bm}e^{-\gamma_m d}) \sum_{y=y_1}^{y_t} [-Z_{B0}(k_{cm}^y R_{2y}) + Z_{B0}(k_{cm}^y R_{1y})] / k_{cm}^y, \quad (6.45)
 \end{aligned}$$

$$\begin{aligned}
 &A_i(1 + \Gamma_{Ai}) \int_{R_{1a}}^{R_{2a}} \mathcal{E}^x Z_{A1}^2(k_{ci}^x r) r dr = A_i(1 + \Gamma_{Ai}) \sum_{x=1}^{t_A} \mathcal{E}^x [f_{Ai}^x(R_{2x}) - f_{Ai}^x(R_{1x})] \\
 &= B_0[e^{\gamma_{B0}d} + \Gamma_{B0}e^{-\gamma_{B0}d}] \int_{\max(R_{1a}, R_{1b})}^{\min(R_{2a}, R_{2b})} \mathcal{E}^x Z_{A1}(k_{ci}^x r) dr \\
 &\quad + \sum_{m=1}^{\infty} B_m[e^{\gamma_m d} + \Gamma_{Bm}e^{-\gamma_m d}] \int_{\max(R_{1a}, R_{1b})}^{\min(R_{2a}, R_{2b})} \mathcal{E}^x Z_{A1}(k_{ci}^x r) Z_{B1}(k_{cm}^y r) r dr \\
 &= B_0[e^{\gamma_{B0}d} + \Gamma_{B0}e^{-\gamma_{B0}d}] \sum_{x=x_1}^{x_t} \mathcal{E}^x \int_{R_{1x}}^{R_{2x}} Z_{A1}(k_{ci}^x r) dr \\
 &\quad + \sum_{m=1}^{\infty} B_m[e^{\gamma_m d} + \Gamma_{Bm}e^{-\gamma_m d}] \sum_{x=x_1}^{x_t} \mathcal{E}^x \int_{R_{1x}}^{R_{2x}} Z_{A1}(k_{ci}^x r) Z_{B1}(k_{cm}^y r) r dr \\
 &= B_0[e^{\gamma_{B0}d} + \Gamma_{B0}e^{-\gamma_{B0}d}] \sum_{x=x_1}^{x_t} \mathcal{E}^x [-Z_{A0}(k_{ci}^x R_{2x}) + Z_{A0}(k_{ci}^x R_{1x})] / k_{ci}^x \\
 &\quad + \sum_{m=1}^{\infty} B_m[e^{\gamma_m d} + \Gamma_{Bm}e^{-\gamma_m d}] \sum_{xy=x_1y_1}^{xy_t} \mathcal{E}^{xy} [f_{AiBm}^{xy}(R_{2xy}) - f_{AiBm}^{xy}(R_{1xy})], \quad (6.46)
 \end{aligned}$$

where,

t_A is the total layer number of the filling material in Section A;

x_1 and x_t are respectively the first layer index and the last layer index in the range from $\max(R_{1a}, R_{1b})$ to $\min(R_{2a}, R_{2b})$ in Section A;

y_1 and y_t are respectively the first layer index and the last layer index in the range from $\max(R_{1a}, R_{1b})$ to $\min(R_{2a}, R_{2b})$ in Section B;

xy_1 and xy_t are respectively the first and the last layer index in the range from $\max(R_{1a}, R_{1b})$ to $\min(R_{2a}, R_{2b})$ on the interface between Section A and Section B;

R_{2x} and R_{1x} are respectively the outer and the inner radius of the x th layer in Section A;

R_{2y} and R_{1y} are respectively the outer and the inner radius of the y th layer in Section B;

R_{2xy} and R_{1xy} are respectively the outer and the inner radius of the xy th layer on the interface between Section A and Section B;

ε_A^{xy} is Section A's permittivity of the xy th discontinuity layer on the interface between Section A and Section B, and

$$f_{Ai}^x(r) = \frac{r^2}{2} [Z_{A1}^2(k_{ci}^x r) - Z_{A0}(k_{ci}^x r) Z_{A2}(k_{ci}^x r)], \quad (6.47)$$

$$f_{AiBm}^{xy}(r) = \int Z_{A1}(k_{ci}^x r) Z_{B1}(k_{cm}^y r) r dr = \frac{r}{(k_{ci}^x)^2 - (k_{cm}^y)^2} [k_{cm}^y Z_{B0}(k_{cm}^y r) Z_{A1}(k_{ci}^x r) - k_{ci}^x Z_{A0}(k_{ci}^x r) Z_{B1}(k_{cm}^y r)]. \quad (6.48)$$

The second step of the mode matching procedure is to perform the following integral at both sides of (6.42):

$$\int_{R_{1b}}^{R_{2b}} E_{Brm} H_{B\phi} 2\pi r dr. \quad (6.49)$$

From the integral above, the following equations can be obtained:

$$\begin{aligned} & -\sqrt{\frac{\varepsilon_B}{\mu_B}} B_0 [e^{\gamma_{B0} d} - \Gamma_{B0} e^{-\gamma_{B0} d}] \int_{R_{1b}}^{R_{2b}} \frac{1}{r} dr = -\sqrt{\frac{\varepsilon_B}{\mu_B}} B_0 [e^{\gamma_{B0} d} - \Gamma_{B0} e^{-\gamma_{B0} d}] \ln \frac{R_{2b}}{R_{1b}} \\ & = -\sqrt{\frac{\varepsilon_A}{\mu_A}} A_0 (1 - \Gamma_{A0}) \int_{\max(R_{1a}, R_{1b})}^{\min(R_{2a}, R_{2b})} \frac{1}{r} dr \\ & \quad + \sum_{i=1}^{\infty} A_i (1 - \Gamma_{Ai}) \frac{-j\omega}{\gamma_i} \int_{\max(R_{1a}, R_{1b})}^{\min(R_{2a}, R_{2b})} \varepsilon^x Z_{A1}(k_{ci}^x r) dr \\ & = -\sqrt{\frac{\varepsilon_A}{\mu_A}} A_0 (1 - \Gamma_{A0}) \ln \frac{\min(R_{2a}, R_{2b})}{\max(R_{1a}, R_{1b})} \\ & \quad + \sum_{i=1}^{\infty} A_i (1 - \Gamma_{Ai}) \frac{-j\omega}{\gamma_i} \sum_{x=x_1}^{x_i} \varepsilon^x [-Z_{A0}(k_{ci}^x R_{2x}) + Z_{A0}(k_{ci}^x R_{1x})] / k_{ci}^x, \end{aligned} \quad (6.50)$$

$$\begin{aligned}
 & B_m [e^{\gamma_m d} - \Gamma_{Bm} e^{-\gamma_m d}] \int_{R_{1b}}^{R_{2b}} \frac{-j\omega \varepsilon^y}{\gamma_m} Z_{B1}^2(k_{cm}^y r) r dr \\
 &= B_m [e^{\gamma_m d} - \Gamma_{Bm} e^{-\gamma_m d}] \frac{-j\omega}{\gamma_m} \sum_{y=1}^{t_B} \varepsilon^y [f_{Bm}^y(R_{2y}) - f_{Bm}^y(R_{1y})] \\
 &= -\sqrt{\frac{\varepsilon_A}{\mu_A}} A_0 (1 - \Gamma_{A0}) \int_{\max(R_{1a}, R_{1b})}^{\min(R_{2a}, R_{2b})} Z_{B1}(k_{cm}^y r) dr \\
 &\quad + \sum_{i=1}^{\infty} A_i (1 - \Gamma_{Ai}) \frac{-j\omega}{\gamma_i} \sum_{x=x_1}^{x_i} \varepsilon^x \int_{R_{1x}}^{R_{2x}} Z_{A1}(k_{ci}^x r) Z_{B1}(k_{cm}^y r) r dr \\
 &= -\sqrt{\frac{\varepsilon_A}{\mu_A}} A_0 (1 - \Gamma_{A0}) \sum_{y=y_1}^{y_i} [-Z_{B0}(k_{cm}^y R_{2y}) + Z_{B0}(k_{cm}^y R_{1y})] / k_{cm}^y \\
 &\quad + \sum_{i=1}^{\infty} A_i (1 - \Gamma_{Ai}) \frac{-j\omega}{\gamma_i} \sum_{xy=xy_1}^{xy_i} \varepsilon_A^{xy} [f_{AiBm}^{xy}(R_{2xy}) - f_{AiBm}^{xy}(R_{1xy})], \tag{6.51}
 \end{aligned}$$

where t_B is the total layer number of the filling material in Section B, and

$$f_{Bm}^y(r) = \frac{r^2}{2} [Z_{B1}^2(k_{cm}^y r) - Z_{B0}(k_{cm}^y r) Z_{B2}(k_{cm}^y r)]. \tag{6.52}$$

Figure 6.4 Axisymmetric discontinuity structure where Section B has a larger cross section

The two-step mode matching procedure shown in (6.44) and (6.49) is valid as long as the cross section of Section A is not larger than that of Section B. In the case where the cross section of Section B is larger than that of Section A, as shown in Figure 6.4, the mode matching procedure as follows should be adopted instead of (6.44) and (6.49) to ensure correctly enforcing the transverse electric field continuity on the interface between Section A and Section B [59]:

$$\int_{R_{1b}}^{R_{2b}} E_{Br} H_{B\phi m} 2\pi r dr, \tag{6.53}$$

$$\int_{R_{1a}}^{R_{2a}} E_{Ari} H_{A\phi} 2\pi r dr. \quad (6.54)$$

Based on (6.53) and (6.54), the following equations can be obtained:

$$B_0 [e^{\gamma_{B0}d} + \Gamma_{B0} e^{-\gamma_{B0}d}] \ln \frac{R_{2b}}{R_{1b}} = A_0 (1 + \Gamma_{A0}) \ln \frac{\min(R_{2a}, R_{2b})}{\max(R_{1a}, R_{1b})} + \sum_{i=1}^{\infty} A_i (1 + \Gamma_{Ai}) \sum_{x=x_1}^{x_i} [-Z_{A0}(k_{ci}^x R_{2x}) + Z_{A0}(k_{ci}^x R_{1x})] / k_{ci}^x, \quad (6.55)$$

$$B_m [e^{\gamma_{m0}d} + \Gamma_{Bm} e^{-\gamma_{m0}d}] \sum_{y=1}^{t_B} \varepsilon^y [f_{Bm}^y(R_{2y}) - f_{Bm}^y(R_{1y})] = A_0 (1 + \Gamma_{A0}) \sum_{y=y_1}^{y_i} \varepsilon^y [-Z_{B0}(k_{cm}^y R_{2y}) + Z_{B0}(k_{cm}^y R_{1y})] / k_{cm}^y + \sum_{i=1}^{\infty} A_i (1 + \Gamma_{Ai}) \sum_{xy=xy_1}^{xy_i} \varepsilon_B^{xy} [f_{AiBm}^{xy}(R_{2xy}) - f_{AiBm}^{xy}(R_{1xy})], \quad (6.56)$$

$$-\sqrt{\frac{\varepsilon_A}{\mu_A}} A_0 (1 - \Gamma_{A0}) \ln \frac{R_{2a}}{R_{1a}} = -\sqrt{\frac{\varepsilon_B}{\mu_B}} B_0 (e^{\gamma_{B0}d} - \Gamma_{B0} e^{-\gamma_{B0}d}) \ln \frac{\min(R_{2a}, R_{2b})}{\max(R_{1a}, R_{1b})} + \sum_{m=1}^{\infty} \frac{-j\omega}{\gamma_m} B_m (e^{\gamma_{m0}d} - \Gamma_{Bm} e^{-\gamma_{m0}d}) \sum_{y=y_1}^{y_i} \varepsilon^y [-Z_{B0}(k_{cm}^y R_{2y}) + Z_{B0}(k_{cm}^y R_{1y})] / k_{cm}^y, \quad (6.57)$$

$$\frac{-j\omega}{\gamma_i} A_i (1 - \Gamma_{Ai}) \sum_{x=1}^{t_A} \varepsilon^x [f_{Ai}^x(R_{2x}) - f_{Ai}^x(R_{1x})] = -\sqrt{\frac{\varepsilon_B}{\mu_B}} B_0 [e^{\gamma_{B0}d} - \Gamma_{B0} e^{-\gamma_{B0}d}] \sum_{x=x_1}^{x_i} [-Z_{A0}(k_{ci}^x R_{2x}) + Z_{A0}(k_{ci}^x R_{1x})] / k_{ci}^x + \sum_{m=1}^{\infty} \frac{-j\omega}{\gamma_m} B_m [e^{\gamma_{m0}d} - \Gamma_{Bm} e^{-\gamma_{m0}d}] \sum_{xy=xy_1}^{xy_i} \varepsilon_B^{xy} [f_{AiBm}^{xy}(R_{2xy}) - f_{AiBm}^{xy}(R_{1xy})], \quad (6.58)$$

where ε_B^{xy} is Section B's permittivity of the xyth discontinuity layer on the interface.

Suppose that the total number of the sections of the structure is s_n , and the number of modes retained for the i th section is m_i . Then the total number of discontinuities is $s_n - 1$. It is worthwhile to mention that the number of modes retained in a section with a larger cross section should be greater than that in a section with a smaller cross section [59].

After the two steps of the mode matching procedure presented above are performed at each discontinuity interface, a set of simultaneous equations which contain the unknown variables, ..., A_i , $A_i \Gamma_{A_i}$, ..., B_m , $B_m \Gamma_{B_m}$, ..., can be obtained. Apparently, the number of the unknown variables is $2 \times (m_1 + m_2 + \dots + m_{s_n})$, and the number of the simultaneous equations is $m_1 + 2 \times (m_2 + \dots + m_{s_n-1}) + m_{s_n}$. The equations cannot be solved yet since the number of equations is less than the number of the unknown variables.

Figure 6.5 Termination conditions of the axisymmetric structure

For the termination condition of the first section shown in Figure 6.5 (a), it is reasonable to assume that all the modes will only travel in the z direction in the first section, except for the incident TEM mode. Actually, it is usually the case that a coaxial waveguide used as sample holders is designed only for the propagation of the TEM wave, and all the high order modes will be attenuated to zero very fast far before they reach the source impedance. Furthermore, in the general cases, the characteristic impedance of the first section of the coaxial structure is designed to be equal to the source impedance. The reflected TEM wave from the first discontinuity will not be reflected again when they reach the source impedance. Thus, from (6.32) and (6.34), the transverse electromagnetic field components in the first section can be written as follows:

$$E_{Ar} = \frac{A_0}{r} [e^{\gamma_{A0}(z-z_0)} + \Gamma_{A0} e^{-\gamma_{A0}(z-z_0)}] + \sum_{i=1}^{\infty} A_i Z_{A1}(k_{ci}^x r) \Gamma_{Ai} e^{-\gamma_i(z-z_0)}, \quad R_{1a} < r < R_{2a}, \quad (6.59)$$

$$H_{A\phi} = -\sqrt{\frac{\epsilon_A}{\mu_A}} \frac{A_0}{r} [e^{\gamma_{A0}(z-z_0)} - \Gamma_{A0} e^{-\gamma_{A0}(z-z_0)}] + \sum_{i=1}^{\infty} \frac{j\omega\epsilon^x}{\gamma_i} A_i Z_{A1}(k_{ci}^x r) \Gamma_{Ai} e^{-\gamma_i(z-z_0)},$$

$$R_{1a} < r < R_{2a}. \quad (6.60)$$

It can be seen from the two equations above that the number of the unknown variables is reduced by $m_1 - 1$. The number of unknown variables can be further reduced by 1 since the amplitude of the incident wave can be assumed known.

An electric wall indicated in Figure 6.5 (b) means a perfectly electric conducting wall. In the case of normal incidence, since the total electric field must be zero on the surface of an electric wall, the phase change of the reflected electric field must be 180° . Thus when the last section is terminated by an electric wall as shown in Figure 6.5 (b), Γ_{Bm} is -1 . The magnetic wall indicated Figure 6.5 (b) means a perfectly magnetic conducting wall. In the case of normal incidence, the total magnetic field is zeros on the surface of the magnetic wall. The phase change of the reflected magnetic field is 180° . The phase change of the reflected electric field is 0° . Thus when the last section is terminated by a magnetic wall as shown in Figure 6.5 (b), Γ_{Bm} is 1. For the termination condition in Figure 6.5 (c), the last section is terminated by the lumped impedance. Again, in the usual case, the termination impedance is equal to the characteristic impedance of the last section of the coaxial waveguide. The high order modes will be attenuated to zero quickly. Under these assumptions, all the waves will travel only toward the $-z$ direction. Thus the reflection coefficients Γ_{Bm} are all zeros. In the case of Figure 6.5 (c), it can be seen that changing the length of the last section will not affect the traveling of the waves inside. Thus the length of that section can be set to be zero. When the last section is stretching to infinity along the $-z$ direction, the termination condition is equivalent to Figure 6.5 (c). Apparently, the termination conditions of the last section reduce the number of the unknown variables by m_{sn} . Up to now, with the termination conditions taken into account, the number of unknown variables is equal to the number of the simultaneous equations. The simultaneous equations can be solved. The distribution of the electromagnetic field in the structure can be readily obtained.

The scattering parameters (s parameters) of the discontinuity structure can be easily obtained, since the amplitudes of the TEM modes are solved. Let us take the structure in Figure 6.6 as an example for illustration of the solution of the s parameters.

Figure 6.6 An axisymmetric coaxial discontinuity structure with three sections

If the reference planes for the s parameters are chosen to be the discontinuity interfaces of the structure shown in Figure 6.6, the s parameter can be expressed as follows:

$$s_{11} = \Gamma_{A0}, \quad (6.61)$$

$$s_{21} = \frac{C_0 \ln(R_{2C} / R_{1C})}{A_0 \ln(R_{2A} / R_{1A})}, \quad (6.62)$$

where R_{2C} and R_{1C} are the outer and the inner radii of the last section respectively, and R_{2A} and R_{1A} are the outer and the inner radii of the first section respectively.

If the reference planes for the s parameters are shifted to the dotted lines shown in Figure 6.6, the s parameters can be written as:

$$s'_{11} = s_{11} e^{-2\gamma_{A0}l_1}, \quad (6.63)$$

$$s'_{21} = s_{21} e^{-\gamma_{A0}l_1 - \gamma_{C0}l_2}, \quad (6.64)$$

where γ_{A0} and γ_{C0} are the propagation constants of the TEM modes in Section A and Section C respectively.

6.5 Numerical Results

In this section, we apply the method above to calculate the s parameters of two kinds of coaxial discontinuity structures shown in Figure 6.6 and Figure 6.7 which are used in the complex permittivity and permeability measurement [39, 40, 42]. The outer diameter of the inner conductor of the coaxial structures, i.e. d , is 3.04 mm. The inner diameter of the

outer conductor, i.e. D, is 7 mm. In the two structures, the material under test (MUT) is assumed to be Mn-Zn ferrite. The height of the MUT in Figure 6.6 and Figure 6.7 are 2 mm and 1 mm respectively.

The permeability and the permittivity of the Mn-Zn ferrite are assumed as below [20], both of which are frequency dependent:

$$\varepsilon = \varepsilon' - j\varepsilon'', \quad (6.65)$$

$$\varepsilon' = \frac{\varepsilon_0 \varepsilon_r g^2}{\omega^2 (\varepsilon_0 \varepsilon_r)^2 + g^2}, \quad (6.66)$$

$$\varepsilon'' = \frac{\omega (\varepsilon_0 \varepsilon_r)^2 g}{\omega^2 (\varepsilon_0 \varepsilon_r)^2 + g^2}, \quad (6.67)$$

$$\mu = \mu' - j\mu'', \quad (6.68)$$

$$\mu' = \frac{\mu_0 \mu_r \lambda^2}{\omega^2 (\mu_0 \mu_r)^2 + \lambda^2}, \quad (6.69)$$

$$\mu'' = \frac{\omega (\mu_0 \mu_r)^2 \lambda}{\omega^2 (\mu_0 \mu_r)^2 + \lambda^2}, \quad (6.70)$$

$$\lambda = \frac{\lambda_h \lambda_f}{\lambda_h + \lambda_f}, \quad (6.71)$$

$$\lambda_h = \lambda_{h0} \frac{\omega}{\omega_0}, \quad (6.72)$$

where $g=10$ S/m, $\varepsilon_r=10^5$, $\mu_r=3080$, $\lambda_f=57 \times 10^3$ Ω /m, $\lambda_{h0}=615$ Ω /m, and $\omega_0=2\pi \times 1000$ rad/s.

Figure 6.7 A coaxial discontinuity structure used for material characterization

Before the mode matching method is used to solve the s parameters, the propagation constants of all the sections have to be solved first.

Table 6.1 Comparison of propagation constants of the symmetrical TM modes in the air filled coaxial waveguide ($f = 1$ GHz, $\Delta r = 1 \times 10^{-5}$ m).

Mode	By (6.2)	1-D finite difference method
TM ₀₁	1573.1	1573.1
TM ₀₂	3166.1	3165.9
TM ₀₃	4755.1	4754.6
TM ₀₄	6342.9	6341.9
TM ₀₅	7930.3	7928.3
TM ₀₆	9517.5	9513.9

Table 6.2 Comparison of propagation constants of the symmetrical TM modes in the circular waveguide uniformly filled with the MUT ($f = 1$ GHz, $\Delta r = 1 \times 10^{-5}$ m).

Mode	By (6.6)	1-D finite difference method
TM ₀₁	1021.2 + j0.90642	1021.2 + j0.90642
TM ₀₂	1748.8+j0.52931	1748.8+j0.52931
TM ₀₃	2585.3+j0.35803	2585.3+j0.35804
TM ₀₄	3452.7+j0.26809	3452.5+j0.2681
TM ₀₅	4332.4+j0.21366	4332+j0.21367
TM ₀₆	5218.1+j0.17739	5217.6+j0.17741

A numerical routine is used to solve the propagation constants of the symmetrical TM modes in the air-filled sections in Figure 6.6 and Figure 6.7 by finding the roots of (6.2). At the same time, those propagation constants are solved by using the 1-D finite difference method in Section 6.2. The results of these two methods at 1 GHz are shown in Table 6.1. It can be seen that the two sets of results are in excellent agreement. From Table 6.1, it can be seen that the propagation constants of all the TM modes are real, which means that the air-filled sections do not allow the propagation of the TM modes at the frequency of interest, i.e. 1 GHz.

The propagation constants of the symmetrical TM modes at 1 GHz in the MUT-filled section in Figure 6.7 solved by using (6.6) and the 1-D finite difference method are shown in Table 6.2. Again, good agreement between the two sets of results can be observed.

For the section filled by the MUT in Figure 6.6, we assume that the coaxial waveguide is not fully filled by the MUT. The widths of both the inner air gap and the outer air gap are 0.1 mm. The propagation constants of the quasi-TEM mode and the symmetrical TM modes can be solved by using the field matching technique given in [53]. Based on the continuity conditions of the transverse field components at the discontinuity interface, a matrix $[A(\gamma)]$ is formed in [53]. The elements of the matrix contain the propagation constants γ . Let us take the coaxial structure shown in Figure 6.1 as an example to illustrate the principle of the field matching technique. There are three layers inside that coaxial structure. We assume that the inner and the outer radius of the coaxial structure are R_1 and R_2 respectively, and the inner and the outer radius of the second layer is r_1 and r_2 respectively. The fields in the structure can be expressed by (6.25), (6.28) and (6.29). According to the continuity conditions of the transverse field components at the discontinuity interface and the PEC assumption on the inner and the outer conductors, we have:

$$C^x J_0(k_c^x r) + D^x Y_0(k_c^x r) = 0, \quad (x=1, r=R_1; x=3, r=R_2), \quad (6.73)$$

$$\frac{k_c^x}{\gamma} [C^x J_0(k_c^x r) + D^x Y_0(k_c^x r)] = \frac{k_c^y}{\gamma} [C^y J_0(k_c^y r) + D^y Y_0(k_c^y r)],$$

$$(x=1, y=2, r=r_1; x=2, y=3, r=r_2), \quad (6.74)$$

$$\frac{j\omega\epsilon^x}{\gamma} [C^x J_1(k_c^x r) + D^x Y_1(k_c^x r)] = \frac{j\omega\epsilon^y}{\gamma} [C^y J_1(k_c^y r) + D^y Y_1(k_c^y r)],$$

$$(x=1, y=2, r=r_1; x=2, y=3, r=r_2). \quad (6.75)$$

From the above 6 equations, we can obtain:

$$[A(\gamma)]_{6 \times 6} \begin{bmatrix} C^x \\ D^x \end{bmatrix}_{6 \times 1} = 0. \quad (6.76)$$

The method for solving the propagation constants is to try different values of γ and test for the determinant of the matrix $[A(\gamma)]$ to be zero [53]. But the actual implementation of searching is a difficult task because of the fact that $\det([A(\gamma)])$ is a rapidly changing function with γ , containing not only poles and zeros in close neighborhood but also extremely steep gradients. A method to circumvent this problem is presented in [60] where the singular-value decomposition of the matrix $[A(\gamma)]$ is used instead of $\det([A(\gamma)])$.

The propagation constants at 0.1 GHz solved by using the 1-D finite difference method and the field matching technique with the singular value decomposition searching technique are shown in Table 6.3. The two sets of results are in excellent agreement. Since the propagation constants are not real or purely imaginary, it takes time to search for the propagation constants in the complex plane using the field matching technique. It is found that even with the singular-value decomposition searching technique, it is hard to find some high order modes by using the field matching technique due to the fact that the sharpness of the minimum singular value increases as its location approaches a pole of the determinant [61]. Therefore, the numerical search algorithm has to operate at very small step widths. The minimum singular values around some propagation constants are shown in Figure 6.8. It can be seen from Figure 6.8 that the minimum singular values around the propagation constant of the TM_{05} mode are much sharper than those around the propagation constant of the TM_{01} mode. Thus to search the propagation constant of the TM_{05} mode, smaller searching step widths will be needed. Given that the searching step widths in the real and the imaginary axes are 1.0 when the field matching technique is used to roughly search the propagation constants, the required number of searching for all the propagation constants in Table 6.3 will be at least $7110 \times 60 = 426600$. The average time needed for each searching operation, including the construction and the singular-value decomposition of the matrix $[A(\gamma)]$, is approximately 0.0025 seconds when a P4 3.4 GHz PC is used. Thus the time needed to roughly search the propagation constants can be estimated to be about 18 minutes. After the approximate values of the propagation constants are obtained, the simplex method can be used to refine the results. For the 1-D finite difference method, the time needed to solve all the propagation constants is less than 1 second when the same PC is used. Apparently, the computational efficiency of the 1-D finite difference method is much higher.

Table 6.3 Comparison of propagation constants of the coaxial waveguide partly filled with the MUT
($f = 0.1$ GHz, $\Delta r = 1 \times 10^{-5}$ m)

Mode	Field matching technique	1-D finite difference method
Quasi-TEM	32.927+j33.523	32.927+j33.523
TM ₀₁	60.272+j59.924	60.271+j59.924
TM ₀₂	1952+j7.2872	1952+j7.2871
TM ₀₃	3628.1+j4.0874	3627.9+j4.0872
TM ₀₄	5360.9+j2.738	5360.3+j2.7378
TM ₀₅	7109.5+j2.0066	7108.1+j2.0064

(a) Around γ of TM₀₁

(b) Around γ of TM₀₅

Figure 6.8 Minimum singular values of the matrix used to find γ in Table 6.3

Although the 1-D finite difference method seems quite accurate and computationally efficient from the above numerical examples, there are a few points needed to be mentioned.

First, the accuracy of the 1-D finite difference method is related to the length of the finite difference cell, i.e. Δr . The smaller the length, the higher is the accuracy of the results. For example, if we change the value of Δr in Table 6.1 and take the results obtained by using (6.2) as the true values, the errors of the results of the 1-D finite difference method are shown in Table 6.4. From Table 6.4, it also can be seen that the error of the higher order TM mode is greater, and the accuracy of the higher order TM mode is more sensitive to the changing of Δr .

Table 6.4 Errors (%) of the results of the 1-D finite difference method for different Δr

Mode	$\Delta r = 1 \times 10^{-5} \text{m}$	$\Delta r = 5 \times 10^{-5} \text{m}$	$\Delta r = 1 \times 10^{-4} \text{m}$
TM ₀₁	0.0009503	1.0343	1.1033
TM ₀₂	0.0040979	1.1025	1.4002
TM ₀₃	0.0093426	1.2279	1.9054
TM ₀₄	0.016682	1.4051	2.6119
TM ₀₅	0.026124	1.633	3.5162
TM ₀₆	0.03766	1.9115	4.6148

Second, according to eigenvalue problem in (6.20), the number of eigenvalues solved will be equal to the dimensions of the matrix $[A]$ in (6.20). Generally speaking, the eigenvalues solved by using a numerical program will not be sorted according to the order of the TM modes. Since only a few of the relative low order TM modes will be retained in the mode matching method, additional numerical programs are needed to roughly sort the solved eigenvalues according to the order of the TM modes. A quick and easy method is to sort those eigenvalues according to their real parts, since the higher order TM modes usually are attenuated faster.

Third, there may be some unphysical modes among the eigenvalues solved. The unphysical modes should not be used in the mode matching method. Since the signs of the real part and the imaginary part of the propagation constants of the unphysical modes

are opposite, and usually the E_z components of the unphysical modes are not zero at $r=R_1$ and $r=R_2$, identification of the unphysical modes is not a difficult problem.

(a) H_ϕ (b) E_r (c) E_z

Figure 6.9 Modal field patterns determined by least square fitting method for TM_{03} in Table 6.3

For the multi-layered structures, after the propagation constants are solved, the modal field patterns can be readily obtained by using the least square fitting method as mentioned in Section 6.3. As one example, for the mode TM_{03} in Table 6.3, the eigenvectors of (6.20) and the least square fitting results of (6.29) are shown in Figure 6.9 (a). The E_r field distribution by (6.28) is shown in Figure 6.9 (b). The E_z field distribution by (6.25) is shown in Figure 6.9 (c).

Figure 6.10 Result comparison of the general mode matching method in this chapter (solid lines), the method in [42] (crosses) and the 2-D FDFD method (circles) for the structure in Figure 6.7

For the structure in Figure 6.7, the s parameters are respectively calculated by using the general mode matching method in this chapter, the method in [42], and the 2-D FDFD method presented in Chapter 7. The results are shown in Figure 6.10.

For the structure in Figure 6.6, in the first case, we assume that the MUT perfectly fills up the cross section of the coaxial sample holder. In this case, it can be predicted that only TEM mode exists in the whole structure. The s parameters can be easily calculated by the analytical method [39, 40]. Results of the analytical method are shown in Figure 6.11. In the second case, we assume that there is a 0.1 mm air gap between the MUT and the inner conductor of the coaxial holder. Results of the general mode matching method in this chapter and the 2-D FDFD method in Chapter 7 are shown in Figure 6.12. By comparison of Figure 6.11 and Figure 6.12, it is interesting to note that a small air gap between the MUT and the conductor of the coaxial holder makes the s parameters totally different.

Figure 6.11 Results of the analytical method for the first case of the structure in Figure 6.6

Figure 6.12 Result comparison of the mode matching method (solid lines) and the 2-D FDFD method (circles) for the second case of the structure in Figure 6.6

6.6 Conclusions

In this chapter, the mode matching method has been applied to the analysis of axisymmetric coaxial discontinuity structures commonly used in the permeability and/or permittivity measurement. The general solution provided by using the approach in this chapter makes the analysis of this kind of structures very convenient.

In combination with the approach in this chapter, the permeability and/or permittivity of materials in the measurement can be solved by solving an inverse problem. As we will see in Section 8.5, the inverse problem can be efficiently solved by combining the

Newton-Raphson method and the approach in this chapter.

However, there are still a few constraints on the application of the approach in this chapter, which is worth being mentioned. First, the conductors of the structures have to be assumed to be PECs for the orthogonality relation between different modes shown in (6.43) to be correct. Second, the method cannot be applied to open-area structures such as the open-ended coaxial probes [37, 62-68], since the orthogonality relation shown in (6.43) only holds for waveguides with PEC as their outer coats. Third, if the number of modes retained in each section is not properly chosen, the mode matching method will fail to converge [59]. To overcome these constraints, a 2-D FDFD method is presented in Chapter 7.

CHAPTER 7

2-D FINITE DIFFERENCE FREQUENCY DOMAIN METHOD WITH NONUNIFORM GRIDS AND PML

7.1 Introduction

As mentioned in Chapter 6, the axisymmetric coaxial discontinuity structures can be conveniently analyzed by using the mode matching method. But compared to the analytical/semi-analytical methods, the numerical methods such as the FEM [43, 69, 70] and the FDTD method [55-57, 71-73] have more flexibility in tackling complicated structures. For some structures such as the finite-flanged open-ended coaxial probes [74, 75], we have to resort to only the numerical methods since the analytical/semi-analytical solutions are impossible if no assumption is introduced to simplify the original models [76]. It is worthwhile to mention that a 2-D FDTD method [55, 71] for the analysis of the axisymmetric structures reduces the original problem into an equivalent 2-D one by taking advantage of the azimuthal symmetry of the structures, thereby greatly improving computational efficiency. While in many cases, the frequency response of the structure is of interest. To obtain the response at a certain frequency, the 2-D FDTD method has to run for a number of time steps to obtain enough time domain data for FFT [55, 75]. Meanwhile, it is well known that the finite difference frequency domain (FDFD) method [77-80] has been widely used in many areas to efficiently solve the frequency response of many structures. In this chapter, the main features of the 2-D FDTD method and the FDFD method are combined to originate a 2-D FDFD method [81] with nonuniform grids and perfectly matched layers (PML) [82-84] for analyzing coaxial discontinuity structures possessing azimuthal symmetry. In the large scale problems, a domain decomposition technique is proposed to improve the computational efficiency and to lessen the burden on the computer memories by decomposing the computational region into many small sub-domains which can be handled independently. Several numerical examples are presented in Section 7.4 to compare the accuracy and the computational

efficiency of the 2-D FDFD method and the 2-D FDTD method. The effect of different domain decomposition manner on the computational efficiency is also demonstrated through numerical simulation.

7.2 2-D FDFD Method with Nonuniform Grids and PML

Figure 7.1 A coaxial discontinuity structure used for illustration of the principle of the 2-D FDFD method (TFR: total field region; SFR: scattered field region; MUT: material under test; PEC: perfectly electric conductor)

Without loss of generality, a simple coaxial step discontinuity structure [42] shown in Figure 7.1 (a) is used to illustrate the principle of the method. Due to the cylindrical symmetry, only half of the longitudinal cross section needs to be analyzed, as shown in Figure 7.1 (b). For easy simulation of the incident wave and to minimize the load on the absorbing boundary conditions, the whole computational region is divided into two regions, i.e. the total field region and the scattered field region [73], which are indicated in Figure 7.1 (b). The incident waves are assumed to be generated at the total-scattered field interface. At the two ends of the coaxial structure, anisotropic PML conditions [82] are used to effectively absorb the fields. The general form of the constitutive tensors of the cylindrical anisotropic PML formulation is given by [82]:

$$\bar{\mu} = \mu \bar{\Lambda}, \quad (7.1)$$

$$\bar{\varepsilon} = \varepsilon \bar{\Lambda}, \quad (7.2)$$

where

$$\bar{\Lambda} = \hat{r}\hat{r}\frac{S_\phi S_z}{S_r} + \hat{\phi}\hat{\phi}\frac{S_r S_z}{S_\phi} + \hat{z}\hat{z}\frac{S_r S_\phi}{S_z} = \hat{r}\hat{r}PML_r + \hat{\phi}\hat{\phi}PML_\phi + \hat{z}\hat{z}PML_z, \quad (7.3)$$

$$s_\phi = \frac{\tilde{r}}{r} = \frac{1}{r} \int_0^r s_r(r') dr'. \quad (7.4)$$

In (7.3) and (7.4), s_r and s_z are the cylindrical coordinate stretching variables [83].

Usually, the coaxial structures are excited by TEM waves. It can be predicted that with the TEM wave excitation only the TM modes independent of the azimuthal angle Φ will be excited in the structure due to the cylindrical symmetry [42]. Based on the Maxwell's equations, the electromagnetic fields inside the structure can be expressed as:

$$-\frac{\partial H_\phi}{\partial z} = j\omega\epsilon_0\epsilon_r PML_r E_r, \quad (7.5)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r H_\phi) = j\omega\epsilon_0\epsilon_r PML_z E_z, \quad (7.6)$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu_0\mu_r PML_\phi H_\phi. \quad (7.7)$$

Figure 7.2 The 2-D FD grid for TM modes in the cylindrical coordinates

The 2-D finite difference (FD) grid for the TM modes shown in Figure 7.2 is obtained by projecting the conventional 3-D FD grid in the cylindrical coordinates shown in Figure 6.2 (a) onto the r - z plane and neglecting the field components of the TE modes. Discretizing (7.5)-(7.7) by employing the 2-D FD grid leads to the following equations:

$$j\omega\epsilon_0 \frac{\epsilon_r^{i,j} dz_j + \epsilon_r^{i,j+1} dz_{j+1}}{dz_j + dz_{j+1}} PML_r^{i,j+1/2} E_r^{i,j+1/2} = -\frac{1}{0.5(dz_j + dz_{j+1})} (H_\phi^{i,j+1} - H_\phi^{i,j}), \quad (7.8)$$

$$j\omega\epsilon_0 \frac{\epsilon_r^{i,j} dr_i + \epsilon_r^{i+1,j} dr_{i+1}}{dr_i + dr_{i+1}} PML_z^{i+1/2,j} E_z^{i+1/2,j} = \frac{1}{r_{i+1/2}} \frac{1}{0.5(dr_i + dr_{i+1})} (r_{i+1} H_\phi^{i+1,j} - r_i H_\phi^{i,j}), \quad (7.9)$$

$$\frac{1}{dz_j}(E_r^{i,j+1/2} - E_r^{i,j-1/2}) - \frac{1}{dr_i}(E_z^{i+1/2,j} - E_z^{i-1/2,j}) = -j\omega\mu_0\mu_r^{i,j}PML_\phi^{i,j}H_\phi^{i,j}, \quad (7.10)$$

where r_i is the distance between the axis of the structure and the center of the cell. The relative complex permittivity at the edge of the cell where the electric field components E_r and E_z are located are calculated by using spatial average, as indicated in (7.8) and (7.9).

To reduce the number of variables, the electric field components are eliminated by substituting (7.8) and (7.9) into (7.10). The resultant equation is:

$$T_1 - T_2 - T_3 + T_4 - T_5 = 0, \quad (7.11)$$

where T_1 - T_4 are the contributions of the electric field components at the edge of the finite difference cell, and

$$T_1 = \frac{1}{dz_j} \frac{dz_j + dz_{j+1}}{\varepsilon_r^{i,j} dz_j + \varepsilon_r^{i,j+1} dz_{j+1}} \frac{1}{PML_r^{i,j+1/2}} \frac{H_\phi^{i,j} - H_\phi^{i,j+1}}{0.5(dz_j + dz_{j+1})}, \quad (7.12)$$

$$T_2 = \frac{1}{dz_j} \frac{dz_{j-1} + dz_j}{\varepsilon_r^{i,j-1} dz_{j-1} + \varepsilon_r^{i,j} dz_j} \frac{1}{PML_r^{i,j-1/2}} \frac{H_\phi^{i,j-1} - H_\phi^{i,j}}{0.5(dz_{j-1} + dz_j)}, \quad (7.13)$$

$$T_3 = \frac{1}{dr_i} \frac{dr_i + dr_{i+1}}{\varepsilon_r^{i,j} dr_i + \varepsilon_r^{i+1,j} dr_{i+1}} \frac{1}{PML_z^{i+1/2,j}} \times \frac{1}{r_{i+1/2}} \frac{r_{i+1} H_\phi^{i+1,j} - r_i H_\phi^{i,j}}{0.5(dr_i + dr_{i+1})}, \quad (7.14)$$

$$T_4 = \frac{1}{dr_i} \frac{dr_{i-1} + dr_i}{\varepsilon_r^{i-1,j} dr_{i-1} + \varepsilon_r^{i,j} dr_i} \frac{1}{PML_z^{i-1/2,j}} \times \frac{1}{r_{i-1/2}} \frac{r_i H_\phi^{i,j} - r_{i-1} H_\phi^{i-1,j}}{0.5(dr_{i-1} + dr_i)}, \quad (7.15)$$

$$T_5 = \omega^2 \varepsilon_0 \mu_0 \mu_r^{i,j} PML_\phi^{i,j} H_\phi^{i,j}. \quad (7.16)$$

Eq. (7.11) describes the relation between the magnetic field component of a cell (i, j) and the magnetic field components of the cells around. Since the electric field is zero on the surface of the PEC, in the cases that the cell (i, j) is next to the PEC, the term in (7.11) which is corresponding to the contribution of the electric field on the surface of the PEC should be set to zero. In the cases that one or more electric field components of a cell is given, the given electric field components should be treated as source terms and moved

to the right hand side of (7.11). For example, if $E_r^{i,j+1/2}$ and $E_z^{i-1/2,j}$ in Figure 7.2 are given, (7.11) should be rewritten as:

$$-T_2 - T_3 - T_5 = -\frac{1}{dz_j} j\omega\epsilon_0 E_r^{i,j+1/2} - \frac{1}{dr_i} j\omega\epsilon_0 E_z^{i-1/2,j}. \quad (7.17)$$

It is noted that (7.15) cannot be used to evaluate E_z at $r = 0$, e.g. the $E_z^{1/2,j}$ component of cell A shown in Figure 7.3. To eliminate that singularity along the z axis, the following integral form of the Maxwell's equation is used:

$$\oint_l \mathbf{H} d\mathbf{l} = j\omega \int_S \epsilon \mathbf{E} d\mathbf{S}. \quad (7.18)$$

Figure 7.3 Illustration of the cell next to the z axis

If the integrals of (7.18) are performed along the circle indicated in Figure 7.3, and the electric fields in the z direction are assumed uniform inside the circle, the following equation can be obtained:

$$H_\phi^{1,j} 2\pi(0.5dr_1) = j\omega\epsilon_0 \epsilon_r^{1,j} PML_z^{1/2,j} E_z^{1/2,j} \pi(0.5dr_1)^2. \quad (7.19)$$

From (7.19), E_z at $r = 0$ can be expressed as:

$$E_z^{i-1/2,j} = \frac{1}{dr_i} \frac{4H_\phi^{i,j}}{j\omega\epsilon_0 \epsilon_r^{i,j} PML_z^{i-1/2,j}}, \quad i = 1. \quad (7.20)$$

Based on (7.20) and (7.10), for the cell just next to the z axis, (7.15) should be revised to:

$$T_4 = \left(\frac{1}{dr_i}\right)^2 \frac{4H_\phi^{i,j}}{\epsilon_r^{i,j} PML_z^{i-1/2,j}}, \quad i = 1. \quad (7.21)$$

As stated above, the computational domain is divided into the total field region and the scattered field region [85, 86]. The TEM wave source is located at the interface between the total field region and the scattered field region, as illustrated in Figure 7.4. Suppose that the magnitude of the applied incident voltage is U_{inc} . The electric and magnetic field components of the incident TEM wave can be written as:

$$E_{rinc}^{i,j+1/2} = \frac{U_{inc}}{\ln(R_2/R_1)} \frac{1}{r_i + R_1} \exp[-j\omega\sqrt{\mu_c\epsilon_c}(z_j + 0.5dz_j)], \quad (7.22)$$

$$H_{rinc}^{i,j} = \sqrt{\epsilon_c/\mu_c} \frac{U_{inc}}{\ln(R_2/R_1)} \frac{1}{r_i + R_1} \exp(-j\omega\sqrt{\mu_c\epsilon_c}z_j), \quad (7.23)$$

where z_j is the distance between the bottom end of the coaxial structure and the center of the cell. Notice that the initial phase of the incident wave is not important. Since the s parameters which we are interested in are defined to be the reflected or the transmitted TEM wave over the incident TEM wave, the influence of the initial phase of the incident wave will be factored out.

Figure 7.4 TEM wave source located at the total-scattered field region interface

For the cell which the total-scattered field region interface crosses, e.g. cell B in Figure 7.4, supposing that the index of that cell is (i, j') , all the field components of the cell are in the total field region except $E_r^{i,j'-1/2}$. Based on (7.8), the following equation for the $E_r^{i,j'-1/2}$ component can be obtained:

$$j\omega\epsilon_0 \frac{\epsilon_r^{i,j-1} dz_{j-1} + \epsilon_r^{i,j} dz_j}{dz_{j-1} + dz_j} PML_r^{i,j-1/2} E_r^{i,j-1/2} = \frac{1}{0.5(dz_j + dz_{j-1})} [H_\phi^{i,j-1} - (H_\phi^{i,j} - H_{\phi inc}^{i,j})], \quad (j = j'). \quad (7.24)$$

For such cells, (7.10) should be modified to:

$$\frac{1}{dz_j} [E_r^{i,j+1/2} - (E_r^{i,j-1/2} + E_{rinc}^{i,j-1/2})] - \frac{1}{dr_i} (E_z^{i+1/2,j} - E_z^{i-1/2,j}) = -j\omega\mu_0\mu_r^{i,j} PML_\phi^{i,j} H_\phi^{i,j}. \quad (7.25)$$

Substituting (7.8) for the $E_r^{i,j'+1/2}$ component, (7.24) for the $E_r^{i,j'-1/2}$ component, and (7.9) for the $E_z^{i+1/2,j'}$ and the $E_z^{i-1/2,j'}$ components into (7.25) leads to the following equation:

$$T_1 - T_2 - T_3 + T_4 - T_5 = \frac{1}{dz_j} \frac{dz_{j-1} + dz_j}{\varepsilon_r^{i,j-1} dz_{j-1} + \varepsilon_r^{i,j} dz_j} \times \frac{1}{PML_r^{i,j-1/2}} \frac{H_{\phi inc}^{i,j}}{0.5(dz_{j-1} + dz_j)} + \frac{1}{dz_j} j\omega\varepsilon_0 E_{rinc}^{i,j-1/2}. \quad (7.26)$$

Thus for the cell which the total-scattered field region interface crosses, (7.26) should be used instead of (7.11) to describe the relation between its magnetic field component and the magnetic field components of the surrounding cells.

For the cell just below the total-scattered field region interface, e.g. cell C in Figure 7.4, all the field components are in the scattered field region. Suppose that the index of that cell is (i, j''). According to the fact that the H_ϕ component of the cell above, i.e. $H_\phi^{i,j'}$, is in the total field region, based on (7.8), the following equation can be obtained for the $E_r^{i,j''+1/2}$ component:

$$j\omega\varepsilon_0 \frac{\varepsilon_r^{i,j} dz_j + \varepsilon_r^{i,j+1} dz_{j+1}}{dz_j + dz_{j+1}} PML_r^{i,j+1/2} E_r^{i,j+1/2} = \frac{1}{0.5(dz_j + dz_{j+1})} [H_\phi^{i,j} - (H_\phi^{i,j+1} - H_{\phi inc}^{i,j+1})], \quad (j = j''). \quad (7.27)$$

Substituting (7.8) for the $E_r^{i,j''-1/2}$ component, (7.27) for the $E_r^{i,j''+1/2}$ component, and (7.9) for the $E_z^{i+1/2,j'}$ and the $E_z^{i-1/2,j'}$ components into (7.10) leads to the following equation:

$$T_1 - T_2 - T_3 + T_4 - T_5 = \frac{1}{dz_j} \frac{dz_j + dz_{j+1}}{\varepsilon_r^{i,j} dz_j + \varepsilon_r^{i,j+1} dz_{j+1}} \frac{1}{PML_r^{i,j+1/2}} \frac{-H_{\phi inc}^{i,j+1}}{0.5(dz_j + dz_{j+1})}. \quad (7.28)$$

Thus for the cell just below the total-scattered field region interface, (7.11) should be replaced by (7.28).

Notice that the expressions of the left hand sides of (7.11), (7.26), and (7.28) are the same. The only difference lies on their right hand sides, which indicate the contribution of the TEM wave source.

To summarize, the cells in the computational domain can be classified into four types, i.e. the cells next to the z axis like cell A in Figure 7.3, the cells which the total-scattered field region interface crosses like cell B in Figure 7.4, the cells just below the total-scattered field region interface like cell C in Figure 7.4, and the other cells. The four types of cells are described by (7.11) with (7.21) replacing (7.15), (7.26), (7.28) and (7.11) respectively. We name those four equations *the cell equations* for the sake of conciseness.

A set of linear simultaneous equations can be obtained by assembling all the cell equations. This set of equations is denoted as follows:

$$[A][X]=[B], \quad (7.29)$$

where $[A]$ is a sparse matrix, $[X]$ is a column matrix consisting of all the unknown variables H_ϕ , and $[B]$ denotes the contribution of the TEM wave source. It is noted that $[A]$ is also banded since the H_ϕ component of a cell are only related to the H_ϕ components of its neighbor cells. Eq. (7.29) can be efficiently solved by using the Gaussian elimination method.

Figure 7.5 A simple example for illustration of the assembling process

For clear illustration of the assembling process of (7.29), let us consider the example given in Figure 7.5 which contains only 3 cells in the computational domain. The cell equations of Cell 1, Cell 2, and Cell 3 can be respectively written as follows:

$$A_1^{1,1} H_\phi^{1,1} + A_1^{1,2} H_\phi^{1,2} = B_1, \quad (7.30)$$

$$A_2^{1,1} H_\phi^{1,1} + A_2^{1,2} H_\phi^{1,2} + A_2^{2,2} H_\phi^{2,2} = B_2, \quad (7.31)$$

$$A_3^{1,2} H_\phi^{1,2} + A_3^{2,2} H_\phi^{2,2} = B_3. \quad (7.32)$$

Assembling the three cell equations, we can obtain:

$$\begin{bmatrix} A_1^{1,1} & A_1^{1,2} & 0 \\ A_2^{1,1} & A_2^{1,2} & A_2^{2,2} \\ 0 & A_3^{1,2} & A_3^{2,2} \end{bmatrix} \begin{bmatrix} H_\phi^{1,1} \\ H_\phi^{1,2} \\ H_\phi^{2,2} \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}, \quad (7.33)$$

which is the specific form of (7.29) in this example.

The electric field components E_r and E_z can be solved by substituting the solutions of H_ϕ into (7.8), (7.9), (7.20), (7.24) and (7.27).

It can be seen from the above formulation process that the 2-D FDFD method does not add any more complexity into the formulation of the problems than the 2-D FDTD method. In the 2-D FDTD method, performing the Fourier transforms to obtain the frequency-domain information, updating the time-domain source during each iteration step, and setting an appropriate criterion to terminate the iteration are needed. In the 2-D FDFD method, an assembling process to obtain (7.29) is required. Thus the effort and time needed to formulate the problems with the 2-D FDFD method is just comparable to that needed in the 2-D FDTD method.

The s parameters can be written as:

$$S_{11} = \frac{V_{ref}^{j+1/2}}{V_{inc}^{j+1/2}}, \quad (7.34)$$

$$S_{21} = \frac{V_{tran}^{k+1/2}}{V_{inc}^{j+1/2}}, \quad (7.35)$$

where V_{ref} , V_{tran} , and V_{inc} are the reflected voltage wave, the transmitted voltage wave and the incident voltage wave respectively, which can be calculated as follows:

$$V_{ref}^{j+1/2} = \sum_i E_{r_scattered}^{i,j+1/2} dr_i, \quad (7.36)$$

$$V_{tran}^{k+1/2} = \sum_i E_{r_total}^{i,k+1/2} dr_i, \quad (7.37)$$

$$V_{inc}^{j+1/2} = \sum_i E_{r_inc}^{i,j+1/2} dr_i. \quad (7.38)$$

Figure 7.6 Illustration of reference plane translation

In some cases, the reference planes of the s parameters may need to be translated after the s parameters are calculated. For example, we may want to obtain the s parameters with the reference planes on the flat surfaces of the MUT in Figure 7.1, i.e. s'_{11} and s'_{21} shown in Figure 7.6, whereas in the calculation the s parameter s_{11} and s_{21} in Figure 7.6 are obtained by using (7.34) and (7.35). The reason why the reference planes cannot be set to be the flat surfaces of the MUT in the calculation will be stated in the following paragraphs. To obtain the s parameters with the reference planes translated, the following equations can be used:

$$s'_{11} = \frac{V'_{ref}}{V'_{inc}} = \frac{V_{ref} e^{\gamma_0 l_1}}{V_{inc} e^{-\gamma_0 l_1}} = s_{11} e^{2\gamma_0 l_1}, \quad (7.39)$$

$$s'_{21} = \frac{V'_{tran}}{V'_{inc}} = \frac{V_{tran} e^{\gamma_0 l_2}}{V_{inc} e^{-\gamma_0 l_1}} = s_{21} e^{2\gamma_0 (l_1 + l_2)}, \quad (7.40)$$

where

$$\gamma_0 = j\omega\sqrt{\mu_c \epsilon_c}. \quad (7.41)$$

It is noted that in this chapter the s parameters are calculated in terms of the voltages rather than the electric and the magnetic field components at certain sampling points [75]. The reason is stated as below.

Let us take the calculation of s_{11} as an example. The radial electric fields in Section 3 shown in Figure 7.6 can be written as [37]:

$$E_r = A_0 \frac{1}{r} (e^{-\gamma_0 z} + \Gamma e^{\gamma_0 z}) + \sum_{m=1}^{\infty} A_m R(k_{cm} r) e^{\gamma_m z}, \quad (7.42)$$

where k_{cm} is the m th root of the following equation:

$$J_0(k_{cm} R_1) N_0(k_{cm} R_2) - N_0(k_{cm} R_1) J_0(k_{cm} R_2) = 0, \quad (7.43)$$

and

$$\gamma_m = \sqrt{k_{cm}^2 - \omega^2 \mu_c \epsilon_c}, \quad (7.44)$$

$$R(k_{cm} r) = J_1(k_{cm} r) + G_m N_1(k_{cm} r), \quad (7.45)$$

$$G_m = -\frac{J_0(k_{cm} R_1)}{N_0(k_{cm} R_1)} = -\frac{J_0(k_{cm} R_2)}{N_0(k_{cm} R_2)}. \quad (7.46)$$

From (7.42), by making use of the orthogonality relationship between different modes [87], the voltage of the coaxial structure can be written as:

$$V(z) = \int_{R_1}^{R_2} E_r(r, z) dr = A_0 e^{-\gamma_0 z} \ln(R_2 / R_1) + A_0 \Gamma e^{\gamma_0 z} \ln(R_2 / R_1) = V_{inc}(z) + V_{ref}(z). \quad (7.47)$$

From (7.47), s_{11} can be rigorously written as:

$$s_{11}(z) = \frac{V_{ref}(z)}{V_{inc}(z)}. \quad (7.48)$$

From the above analysis, it can be seen that the high order TM modes actually do not affect s_{11} defined in (7.48). Thus the reference plane for the calculation of s_{11} can be chosen with more freedom, and do not need to be located far away enough from the MUT where the high order TM modes are attenuated to negligible values. Thus s_{11} calculated from (7.34) has higher accuracy than that calculated in terms of the electric and the magnetic field components at a sampling point, and has higher computational efficiency since the original reference plane, i.e. the total-scattered field region interface, can be chosen to be nearer to the MUT, thus the computational domain can be smaller.

Certainly, even with s_{11} calculated by using (7.34), the original reference plane cannot be set to be the lower flat surface of the MUT. This is because the reflected and incident voltage practically are approximately calculated in a numerical way as shown in (7.36) and (7.38). The high order TM modes will have some kind of influence due to the finite spatial discretization. The reason why the reference plane of s_{21} cannot be set to be the upper flat surface of the MUT is similar to what is stated above.

7.3 Domain Decomposition Technique

To lessen the heavy burden on the computer memory and the CPU time in solving large scale problems, for different numerical methods, different domain decomposition techniques [88-97] are employed. For the 2-D FDFD method presented in the last section, in some cases where the computational domain is large or very fine grids are needed, the number of the unknown variables H_ϕ can be huge. Assembling all the cell equations to construct (7.29) may not be a good idea because the dimensions of the matrix $[A]$ can be so large that a large number of computer memories are needed, and solving (7.29) will be very time-consuming. In this section, a domain decomposition technique is presented to divide the whole computational domain into some small sub-domains. Each sub-domain can be handled independently. Thus the computer memory required is greatly reduced. Furthermore, parallel computation can be applied to obtain high computational efficiency.

As shown in Figure 7.7 (a), a computational domain can be divided into many sub-domains. Let us take sub-domain A as an example. Suppose that the number of grids in sub-domain A is t , and that the numbers of grids in the r and the z directions in sub-domain A are m and n respectively. Besides the potential TEM wave source in sub-domain A, the electric fields on the boundary of sub-domain A, i.e. E_{rA-B} , E_{zA-C} , E_{rA-D} , E_{zA-E} shown in Figure 7.7 (b), are treated as known sources. Assembling all the cell equations in sub-domain A results in the following equation:

$$[S_A]_{t \times t} [H_{\phi A}]_{t \times 1} = [S_B]_{t \times m} [E_{rA-B}]_{m \times 1} + [S_C]_{t \times n} [E_{zA-C}]_{n \times 1} + [S_D]_{t \times m} [E_{rA-D}]_{m \times 1} + [S_E]_{t \times n} [E_{zA-E}]_{n \times 1} + [F_A]_{t \times 1}, \quad (7.49)$$

where $[S_x]$ ($x=A, B, C, D, E$) is a coefficient matrix, and $[F_A]$ is a constant matrix due to the TEM wave source inside sub-domain A.

Figure 7.7 Schematic presentation of the domain decomposition technique

From (7.49), the following equation can be obtained:

$$\begin{aligned}
 [H_{\phi A}]_{l \times 1} &= [S_A]_{l \times l}^{-1} [S_B]_{l \times m} [E_{rA-B}]_{m \times 1} + [S_A]_{l \times l}^{-1} [S_C]_{l \times n} [E_{zA-C}]_{n \times 1} \\
 &\quad + [S_A]_{l \times l}^{-1} [S_D]_{l \times m} [E_{rA-D}]_{m \times 1} + [S_A]_{l \times l}^{-1} [S_E]_{l \times n} [E_{zA-E}]_{n \times 1} + [S_A]_{l \times l}^{-1} [F_A]_{l \times 1} \\
 &= [S_{A-B}]_{l \times m} [E_{rA-B}]_{m \times 1} + [S_{A-C}]_{l \times n} [E_{zA-C}]_{n \times 1} \\
 &\quad + [S_{A-D}]_{l \times m} [E_{rA-D}]_{m \times 1} + [S_{A-E}]_{l \times n} [E_{zA-E}]_{n \times 1} + [F_A']_{l \times 1}. \quad (7.50)
 \end{aligned}$$

Eq. (7.50) shows that the each magnetic field component in sub-domain A is a function of the electric field components on the boundary of sub-domain A and the TEM wave source inside sub-domain A. Thus from (7.50) the magnetic field components in sub-domain A and next to the sub-domain boundaries can be written as:

$$[H_{\phi A-B, C, D, E}] = [\tilde{S}_{A-B}] [E_{rA-B}] + [\tilde{S}_{A-C}] [E_{zA-C}] + [\tilde{S}_{A-D}] [E_{rA-D}] + [\tilde{S}_{A-E}] [E_{zA-E}] + [\tilde{F}_A], \quad (7.51)$$

where $[\tilde{S}_{x-y}]$ is a subset of $[S_{x-y}]$, $[\tilde{F}_x]$ is a subset of $[F_x]$, and $[H_{\phi_{x-y}}]$ is the magnetic field components of the cells in sub-domain x and next to sub-domain y .

For clearness, we name (7.51) *the sub-domain boundary equation*. It can be seen that the sub-domain boundary equation for each sub-domain is calculated independently. Thus the sub-domain boundary equation can be obtained by application of parallel computation.

The electric field components on the sub-domain boundary can be expressed in terms of

the magnetic field components next to them by making use of (7.8) and (7.9). For example, the electric field components on the boundary between sub-domains A and B in Figure 7.7 can be written as:

$$[E_{rA_B}] = [\epsilon_{A_B}]^{-1} ([H_{\phi A_B}] - [H_{\phi B_A}]), \quad (7.52)$$

where $[\epsilon_{x_y}]$ is a diagonal coefficient matrix containing the space-averaged complex permittivity on the boundary between sub-domain x and sub-domain y . We name (7.52) *the sub-domain connection equation*, since it relates the magnetic field components of the two neighbor sub-domains.

From the sub-domain boundary equation and the sub-domain connection equation, all the magnetic field components next to the sub-domain boundaries, and the electric field components on the sub-domain boundaries can be solved. Then all the magnetic field components inside the sub-domains can be obtained by using (7.50).

The domain decomposition procedure stated above is suitable for parallel computation and serial computation as well. Just like other domain decomposition techniques, when the domain decomposition technique presented above is implemented in a serial computer, it is helpful in reducing the required computer memories, whereas the computational efficiency is usually even lower. For more efficient application of the domain decomposition technique in the serial computation, the procedure is a little different from what is stated above, and is stated as below.

As shown in Figure 7.8, the computational domain is decomposed into N sub-domains, or layers. The TEM wave source is located at the N th sub-domain. Computation is performed from the first sub-domain to the N th sub-domain. Based on (7.49)-(7.51), the sub-domain boundary equation for the first sub-domain can be written as:

Figure 7.8 Domain decomposition for serial computation

$$[H_{\phi_{1_2}}] = [\tilde{S}_{1_2}][E_{r_{1_2}}]. \quad (7.53)$$

The sub-domain boundary equation for the second sub-domain can be written as:

$$[H_{\phi_{2_3,1}}] = [\tilde{S}_{2_1}][E_{r_{1_2}}] + [\tilde{S}_{2_3}][E_{r_{2_3}}]. \quad (7.54)$$

The sub-domain connection equation for the first and the second sub-domains can be written as:

$$[E_{r_{1_2}}] = [\varepsilon_{1_2}]^{-1}([H_{\phi_{2_1}}] - [H_{\phi_{1_2}}]). \quad (7.55)$$

From (7.53)-(7.55), the following equations can be obtained:

$$[H_{\phi_{2_1}}] = ([\varepsilon_{1_2}] + [\tilde{S}_{1_2}])[E_{r_{1_2}}], \quad (7.56)$$

$$[H_{\phi_{2_3,1}}] = [\tilde{S}_{2_1}]([\varepsilon_{1_2}] + [\tilde{S}_{1_2}])^{-1} [H_{\phi_{2_1}}] + [\tilde{S}_{2_3}][E_{r_{2_3}}]. \quad (7.57)$$

Eq. (7.57) can also be written as:

$$\begin{bmatrix} \frac{H_{\phi_{2_3}}}{H_{\phi_{2_1}}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \kappa_{1_2} \end{bmatrix} \begin{bmatrix} \frac{H_{\phi_{2_3}}}{H_{\phi_{2_1}}} \end{bmatrix} + [\tilde{S}_{2_3}][E_{r_{2_3}}], \quad (7.58)$$

where

$$\kappa_{1_2} = [\tilde{S}_{2_1}]([\varepsilon_{1_2}] + [\tilde{S}_{1_2}])^{-1}. \quad (7.59)$$

From (7.58), the following equation can be obtained:

$$\begin{bmatrix} \frac{H_{\phi_{2_3}}}{H_{\phi_{2_1}}} \end{bmatrix} = \left(\mathbf{I} - \begin{bmatrix} \mathbf{0} & \kappa_{1_2} \end{bmatrix} \right)^{-1} [\tilde{S}_{2_3}][E_{r_{2_3}}] = \begin{bmatrix} \hat{S}_{2_3} \\ \hat{S}_{2_1} \end{bmatrix} [E_{r_{2_3}}], \quad (7.60)$$

where $\mathbf{I}$ is an identity matrix. Thus we can obtain:

$$[H_{\phi_{2_3}}] = [\hat{S}_{2_3}][E_{r_{2_3}}]. \quad (7.61)$$

Continue the above computational procedure until the i th ($i < N$) sub-domain and we can obtain:

$$[H_{\phi_{i-(i+1)}}] = [\hat{S}_{i-(i+1)}][E_{r_{i-(i+1)}}]. \quad (7.62)$$

According to (7.50), the magnetic field components in the N th sub-domain can be expressed as:

$$[H_{\phi_N}] = [S_{N-(N-1)}][E_{r(N-1)_N}] + [F'_N]. \quad (7.63)$$

The sub-domain boundary equation for the N th sub-domain can be written as:

$$[H_{\phi_{N-(N-1)}}] = [\tilde{S}_{N-(N-1)}][E_{r(N-1)_N}] + [\tilde{F}_N]. \quad (7.64)$$

Substitute (7.62) for the $(N-1)$ th sub-domain and the sub-domain connection equation for the $(N-1)$ th and the N th sub-domain into (7.64) and we can obtain:

$$[E_{r(N-1)_N}] = ([\varepsilon_{(N-1)_N}] + [\hat{S}_{(N-1)_N}] - [\tilde{S}_{N-(N-1)}])^{-1}[\tilde{F}_N]. \quad (7.65)$$

From (7.63) and (7.65), the magnetic field components in the N th sub-domain can be solved. Thus the reflection coefficient of the TEM mode can be solved.

It is worth pointing out that the domain decomposition technique shown above is particular useful when repeated analysis at a certain frequency is performed. Since for the sub-domain whose permeability and permittivity do not change during the repeated analysis, the sub-domain boundary equation is needed to be calculated only once. For example, the whole region in Section 3 in Figure 7.6 can be treated as the N th sub-domain. Eq. (7.63) and (7.64) are needed to be calculated only once during the repeated analysis. It is also worth to notice that for the same sub-domains, their sub-domain boundary equations are the same and needed to be calculated only once. Even in a serial computer, with these features taken into account, in some cases the 2-D FDFD method with the domain decomposition technique can be much more efficient than that without the domain decomposition technique, as will be shown in the numerical examples in the next section.

7.4 Numerical Results

In this section, the accuracy and the computational efficiency of the 2-D FDFD method are thoroughly examined through several numerical examples by comparison of the

results of other methods including the 2-D FDTD method. In the 2-D FDTD method, the TEM wave source is a Gaussian pulse. The frequency response of the structure is obtained by using the discrete Fourier transform at the frequency of interest [73]. The s parameters in the time domain is also calculated in terms of voltages according to (7.34) and (7.35). In the 2-D FDTD method, the unsplit field formulation of PML [98] is employed. The PML configurations in both the 2-D FDTD method and the 2-D FDFD method are the same. The mesh structures are the same in the two methods. Both the 2-D FDTD method and the 2-D FDFD method are programmed in MATLAB 7. Then the Matlab command `mcc` is used to generate C codes, and build stand-alone binary files for both the programs. The two stand-alone binary files run in the same personal computer (P4 1GB RAM). Thus the fairness for comparison of the efficiency of the two numerical methods is guaranteed. For simplicity, in the numerical examples in this section uniform finite difference cells are adopted in the 2-D FDTD method and the 2-D FDFD method. There are 10 layers of PML in either the radial or the axial direction.

7.4.1 The first numerical example

The structure in the first numerical example is shown in Figure 7.1 (a), which is used for the complex permeability and permittivity measurement in [42]. The inner and the outer radii of the air-filled coaxial line, i.e. R_1 and R_2 , are 1.52 mm and 3.5 mm respectively. The thickness of the MUT, i.e. h , is 1 mm. The complex permeability and permittivity of the MUT are μ_0 and $2\epsilon_0$ respectively. The s parameters calculated by using the 2-D FDFD method and the mode matching method are shown in Figure 7.9. The reference planes of the s parameters have been transferred to the two flat surfaces of the MUT. From Figure 7.9, very good agreement between the results of the 2-D FDFD method and the mode matching method can be observed.

The 2-D FDTD method [55] is then used to calculate s_{11} at 10 GHz. The time step of the 2-D FDTD method is $1 \times 10^{-5} / 3 \times 10^8$ second. The results of the 2-D FDTD method at different iteration steps are shown in Figure 7.10. Good agreement among the results of the 2-D FDTD method after 2500 iteration steps, the 2-D FDFD method and the mode matching method can be observed. The time needed for the 2-D FDFD method to calculate s_{11} at 10 GHz is about 3.5 seconds, whereas the time needed for the 2-D FDTD method to perform the 2500 iteration steps is about 157 seconds. The reason why the 2-D

FDTD method is not well converged even after 2500 iterations is due to the structure of the simulation model. In that structure, there is a slab of MUT. The electromagnetic waves will travel back and forth inside the MUT many times before the waves inside the MUT are attenuated to zero. The discontinuous inner conductors further enhance the reflection of the electromagnetic waves inside the MUT, since when the electromagnetic waves reach the interface between the MUT and the inner conductors, the waves are totally reflected.

(a)

(b)

Figure 7.9 Results of the s parameters calculated by using the 2-D FDFD method and the mode matching method (MMM)

Figure 7.10 Results of the 2-D FDTD method at different iteration steps, the 2-D FDFD method and the mode matching method at 10 GHz (O: 2-D FDFD method; *: mode matching method)

Figure 7.11 Computational domains of the 2-D FDFD method

The following numerical examples involve the infinite-flanged probe and the finite-flanged probe shown in Figure 7.11. The distance between the total-scattered field interface

and the coaxial probe aperture is 2 mm. The dimensional parameters of the coaxial probe indicated in Figure 7.11 (a) and Figure 7.11 (b) are: $a=1.5$ mm, and $b=3.5$ mm. The coaxial probe is air-filled. The frequency of interest is 10 GHz. The MUT is assumed to be homogenous for simplicity.

For the infinite-flanged probe, the reflection coefficient of the TEM mode can be easily solved by using the semi-analytical full-wave analysis method in [37]. Thus the accuracy of the 2-D FDTD method and the 2-D FDFD method can be conveniently checked. In the semi-analytical full-wave analysis method, the number of modes retained, i.e. the number of basis functions for the method of moments to solve the resultant integral equation in [37], is 15 for high accuracy. For the finite-flanged probe, whose analytical/semi-analytical full-wave analysis solution is unfeasible, the results of the 2-D FDTD method and the 2-D FDFD method are compared.

7.4.2 The second numerical example

In the second numerical example, the distance between the flange and the surface of the PML outside the probe in the axial direction, i.e. d indicated in Figure 7.11 (a), is 2 mm. The radius of the PEC backing the PML in the radial direction, i.e. c indicated in Figure 7.11 (a), is $1.2 \times b$. The relative complex permittivity of the MUT is $\epsilon_r=10-10j$.

In this example, the wavelength in the MUT is approximately 1 cm. Usually, in the finite difference method, the length of the finite difference cells has to be at least $1/15$ to $1/20$ the wave length, i.e. about 0.50 mm to 0.67 mm in this example, to minimize the spatial discretization error. However, it is noted that the fields are highly nonuniform in the computational domain shown in Figure 7.11, especially in the area near the probe aperture. Thus smaller finite difference cells have to be adopted. For different cell lengths (Δ), the reflection coefficients of the TEM mode (Γ) calculated by using the 2-D FDTD method are shown in Figure 7.12. The reference plane of the reflection coefficients has been translated to the probe aperture. From Figure 7.12, we can see that the results of the 2-D FDTD method will always converge to certain values. The converged values (up to four digits after the decimal point) of the 2-D FDTD method and the result of the 2-D FDFD method are shown in Table 7.1. The reflection coefficient calculated by using the semi-analytical full-wave analysis method in [37] is $-0.4857-j0.1012$, which is taken as the reference for calculation of the errors in Table 7.1. It can

be seen from Table 7.1 that no matter how the computational domain is discretized, as long as the mesh structures are the same, the accuracy of the 2-D FDFD method and the 2-D FDTD method are approximately the same. When smaller finite difference cells are adopted, the accuracy of the two methods is higher.

Figure 7.12 Reflection coefficients of the TEM mode (Γ) calculated by using the 2-D FDTD method at different iteration steps for different cell lengths (Δ)

Since the 2-D FDTD method is performed in the time domain, we have to use certain criterion to terminate the iteration. For example, we can halt the iteration after the first reflected pulse has reached negligible values [75]. Certainly, the time needed for the 2-D FDTD method to obtain the result is different when different termination criteria are used. For fairness when the time needed for the 2-D FDTD method is determined, we run the 2-D FDTD method twice. The first run is to obtain the converged result up to four digits after the decimal point. From the results at different iteration steps and the

converged result, we can determine the minimum iteration step for the 2-D FDTD method to obtain the result within a given error with reference to the converged result, which we assume to be 1.0%. In the second run, the time needed for the 2-D FDTD method to perform that determined iteration steps is recorded, which will be the minimum time for the 2-D FDTD method to obtain the result within the given error, no matter what termination criteria are used. In all the numerical examples below, the time needed for the 2-D FDTD method is determined by this means.

Table 7.1 Results of the 2-D FDTD method and the 2-D FDFD method

Δ (mm)	2-D FDFD Method			2-D FDTD Method		
	Γ	Error (%)	Time	Γ	Error (%)	Time
0.5	-0.5291-j0.0974	8.78	0.05s	-0.5293-j0.0985	8.81	1.31s
0.25	-0.5075-j0.0986	4.43	0.10s	-0.5069-j0.0995	4.29	2.73s
0.1	-0.4932-j0.1001	1.53	0.33s	-0.4925-j0.1006	1.38	10.0s
0.05	-0.4883-j0.1008	0.53	1.14s	-0.4876-j0.1012	0.38	33.3s
0.025	-0.4858-j0.1012	0.03	6.18s	-0.4852-j0.1012	0.1	135s

In this numerical example, the time needed for the 2-D FDFD method and the time recorded by using the above procedure for the 2-D FDTD method are shown in Table 7.1. It is worthwhile to point out again that the reflection coefficients obtained by using the 2-D FDTD method with the time shown in Table 7.1 have 1.0% discrepancy from those converged values (Γ) in Table 7.1. To obtain the converged values, a much longer time has to be taken for the 2-D FDTD method. By comparison of the time needed for the two methods shown in Table 7.1, we can see that the 2-D FDFD is apparently much more computationally efficient than the 2-D FDTD method. It is obvious that when the cell length (Δ) decreases, the errors of both the 2-D FDFD method and the 2-D FDTD method decrease. As shown in Table 7.1, the decrease of the errors is not linear with the decrease of Δ . Actually the decrease rate of the errors is slightly faster than the decrease rate of Δ . However, with Δ decreased, the computational time needed for both the two methods increases. Thus the optimum Δ for a specific problem is the compromise between the computational errors and the computational time. It is worthwhile to point

out that we can always adopt nonuniform cells to improve the computational efficiency of both the 2-D FDFD method and the 2-D FDTD method.

7.4.3 The third numerical example

In the third numerical example, the MUT is backed by an infinite PEC. The distance between the flange and the PEC is 3 mm. The relative complex permittivity of the MUT is $\epsilon_r = 10 - 10j$. The length of the finite difference cells is 0.1 mm. From the results shown in Figure 7.13, we can see that the converged result of the 2-D FDTD method has very good agreement with the results of the 2-D FDFD method and the semi-analytical full-wave analysis method [37]. In this example, the 2-D FDTD method takes 11.36 seconds to perform 580 iteration steps to obtain a result within 1.0% discrepancy from the converged value. Whereas the time needed for the 2-D FDFD method is only 0.34 seconds. It can be seen that the computational domain and the mesh structure in this example are the same as in the second example when the 0.1 mm cell length is adopted. Referring to the data corresponding to the 0.1 mm cell length in Table 7.1, we can see that the time needed for the 2-D FDFD method in this example is approximately the same as in the second example. However, in this example, since the EM wave will travel back and forth between the probe and the backing PEC, the 2-D FDTD method has to take more time to run for a larger number of time steps than in the second example to obtain the result within the given error from the converged value, as we can also see from the comparison of Figure 7.13 and the curves in Figure 7.12 which correspond to the 0.1 mm cell length.

Figure 7.13 Results of the 2-D FDTD method at different iteration steps, the 2-D FDFD method and the semi-analytical full-wave analysis method in the third example (O: 2-D FDFD; *: semi-analytical)

7.4.4 The fourth numerical example

It can be expected that if the MUT is of low loss and backed by the PEC, the 2-D FDTD method has to run for a much larger number of iteration steps than in the highly lossy MUT case to obtain an accurate result, since the EM wave traveling back and forth between the flange and the backing PEC will be attenuated very slowly if the MUT is of low loss. For clear illustration of this conclusion, in the fourth example, the relative complex permittivity of the MUT in the second example is changed to $\epsilon_r=10-0.1j$. The results of the three methods are shown in Figure 7.14. In this case, the FDTD method has to take 19.4 seconds to run 984 iteration steps to obtain the result within 1.0% discrepancy from the converged value. The time needed for the FDFD method is only about 0.30 second, approximately the same as in the first and the second examples.

Figure 7.14 Results of the 2-D FDTD method at different iteration steps, the 2-D FDFD method and the semi-analytical full-wave analysis method [37] in the fourth example (O: 2-D FDFD; *: semi-analytical)

7.4.5 The fifth numerical example

In the fifth example, a finite-flanged probe shown in Figure 7.11 (b) is considered. The relative complex permittivity of the MUT is $\epsilon_r=10-0.1j$. The dimensional parameters indicated in Figure 7.11 (b) are: $B=4.5$ mm, $R=6$ mm, $w=1$ mm, $c=7$ mm, $d=2$ mm and $e=2$ mm. The length of the uniform finite difference cells is 0.1 mm. The reflection coefficients calculated by using the 2-D FDTD method and the 2-D FDFD method are shown in Figure 7.15. It can be seen that the converged result of the 2-D FDTD method is in very good agreement with the result of the 2-D FDFD method. In this example, the

time needed for the 2-D FDTD method to perform 812 iteration steps to obtain the result within 1.0% discrepancy from the converged value is 25.7 seconds, whereas the 2-D FDFD method needs only 0.63 seconds to obtain the result.

Figure 7.15 Results of the 2-D FDTD method at different iteration steps and the 2-D FDFD method for the finite-flanged probe (O: 2-D FDFD method)

7.4.6 The sixth numerical example

Figure 7.16 Efficient computational model of the finite-flanged open-ended coaxial probe

In the sixth example, “e” indicated in Figure 7.11 (b) is set to be zero. The resultant computational model is shown in Figure 7.16. In this case, the 2-D FDTD method takes 21.7 seconds to run 836 iteration steps to obtain the result within 1.0% discrepancy from the converged value. The 2-D FDFD method takes only 0.53 seconds to obtain the result. The reflection coefficients calculated by using the 2-D FDTD method and the 2-D FDFD

method are shown in Table 7.2, together with the results in the fifth example. From comparison of the reflection coefficients of the two computational models in Figure 7.11 (b) and Figure 7.16, it can be seen that changes in the flange thickness, i.e. w in Figure 7.11 (b), will have little effect on the reflection coefficient, as is pointed out in [75]. Thus for the finite-flanged probes, the computational model in Figure 7.16 is preferred, since the computational region is smaller, thus the computational efficiency is higher.

Table 7.2 Comparison of the results of different methods

Models	2-D FDFD	2-D FDTD (Converged Value)
Figure 7.11 (a)	-0.6202-j0.3769	-0.6197-j0.3775
Figure 7.16	-0.6224-j0.3797	-0.6216-j0.3803

7.4.7 The seventh numerical example

As we mention in the last section, when the computational domain is large or very fine grids are needed so that the number of finite difference cells is huge, the domain decomposition technique can be employed to improve the computational efficiency and to reduce the computer memories needed. For demonstration of this conclusion, in the seventh example it is assumed that the distance between the flange and the backing PEC is 8 mm, and $c=1.5b$ in the model in the fourth example. The length of the finite difference cell is 0.02 mm. The other parameters are the same as in the fourth example. This numerical example may not be very practical since the length of the finite difference cell is much smaller than necessary and c is much large than necessary. But it can clearly demonstrate the advantage of the domain decomposition technique in solving large-scale problems.

The domain decomposition technique for serial computation is employed. The region inside the probe is treated as one sub-domain. The region outside the probe is evenly decomposed into 40 sub-domains. Since all the 40 sub-domains are the same, their sub-domain boundary equations are the same, and needed to be solved only once. The results of the 2-D FDTD method at different time steps are shown in Figure 7.17. The results and the time needed for the 2-D FDFD method with and without the domain decomposition (DD) technique are shown in Table 7.3, together with the results of the 2-D FDTD method after 40000 time steps, and the full-wave analysis method [37].

Figure 7.17 Results of the FDTD method at different time steps in the seventh numerical example

Table 7.3 Comparison of accuracy and computational efficiency of different methods in the seventh numerical example

Methods	Reflection coefficient	Time needed (s)
FDTD (40000 steps)	- 0.6005 – j0.4299	16891
FDFD without DD	- 0.6052 – j0.4223	78
FDFD with DD	- 0.6052 – j0.4223	42
Full-wave analysis [37]	- 0.6044 - j0.4263	-

Figure 7.18 Time needed for the FDFD method to obtain the reflection coefficient for different numbers of sub-domains outside the probe

From Table 7.3, it can be seen that the computational efficiency is greatly improved when the domain decomposition technique is employed, while high accuracy of the result

is retained. Actually, the results of the 2-D FDFD method with and without the domain decomposition technique are the same up to the 10 digits after the decimal point, no matter how the computational domain is decomposed.

It is found that the computational efficiencies are different for different domain decomposition configurations, if we change the numbers of the uniform sub-domains outside the coaxial probe, and investigate the time needed to obtain the result. The time needed to obtain the result for different numbers of sub-domains outside the probe is shown in Figure 7.18. It can be seen from Figure 7.18 that there is obviously an optimal number of the uniform sub-domains outside the probe. When the number of sub-domains is too large, the computational efficiency for the 2-D FDFD method with the domain decomposition technique will be even lower than that needed for the 2-D FDFD method without the domain decomposition technique.

If the length of the uniform finite difference cells is changed to 0.01 mm, the time needed for the FDFD method with and without the domain decomposition technique is shown in Table 7.4. From comparison of the ratio of the time needed for the FDFD method with and without the domain decomposition technique in Table 7.3 and Table 7.4, it can be seen that for larger scale systems, the effect of the domain decomposition technique on improving the computational efficiency is more obvious.

Table 7.4 Efficiency comparison of the 2-D FDFD method with and without the domain decomposition (DD) technique

Methods	Reflection coefficient	Time needed (s)
FDFD without DD	- 0.6044 – j0.4229	1354
FDFD with DD	- 0.6044 - j0.4229	309

7.5 Conclusions

In this chapter a domain decomposition 2-D FDFD method with nonuniform grids and PML is presented to analyze axisymmetric coaxial discontinuity structures. Numerical examples show that the accuracy of the method is high. Numerical examples also show that the method has much higher computational efficiency than the 2-D FDTD method in solving the frequency response of the structures, while retaining the versatility and

ability of the 2-D FDTD method to handle complex geometries. In the large scale systems, the presented domain decomposition technique can effectively improve the computational efficiency and lessen the burden on the computer memories. With the domain decomposition technique introduced, parallel computation of the method can be employed.

CHAPTER 8

ISSUES RELATED TO THE COAXIAL TRANSMISSION LINE METHOD

8.1 Introduction

The bandwidth of the field-circuit coupled methods presented in Chapter2-Chapter5 is limited by many factors, such as the test fixtures used and the manners to construct the ferrite core inductor and the ferrite core capacitor. For example, if the HP 16451B dielectric test fixture [24] is used to measure the impedance of the ferrite core capacitor, the bandwidth is limited to 15 MHz. If a wire wound ferrite core inductor is used in the measurement, the stray capacitance between the coils and the unsymmetrical disposition of the coils will definitely limit the bandwidth of the measurement [17]. Solid electrical contacts on the ferrite core are required when the impedance of the ferrite core capacitor is measured. But in practice, the electrical contacts made by using either the silver conductive paint or aluminium deposition [17] are not perfectly conducting at high frequencies. Usually the upper frequency of the measurement results by using the field-circuit coupled methods is limited to 10 MHz [6], [17], [99].

At high frequencies, the coaxial transmission line method is commonly used to measure the intrinsic complex permeability and permittivity of materials, including some soft ferrites [100], [101]. For the coaxial transmission line method, one important factor that can greatly influence the accuracy of the results is the air gaps between the MUT and the coaxial transmission line test fixture [102], [103]. The influence of the air gaps and the measures taken to minimize that influence are discussed in this chapter when the Mn-Zn ferrites are measured. The influence of the finite conductivity of the coaxial transmission line test fixture is also examined in this chapter. A coaxial transmission line test fixture is manufactured to measure the intrinsic complex permeability and permittivity of a Mn-Zn ferrite core up to 200 MHz. The structure of the test fixture, its calibration issues, and the

measurement results are presented in this chapter.

8.2 Basic Principle of the Transmission Line Method

Figure 8.1 A coaxial transmission line test fixture holding the MUT

A coaxial transmission line test fixture holding the MUT is shown in Figure 8.1. The s parameters s'_{11} and s'_{21} , which are referenced to the calibration reference planes, can be measured with a vector network analyzer (VNA). The s parameters referenced to the two flat surfaces of the MUT can be written as [104]:

$$s_{11} = s'_{11} e^{2jk_0 l_1}, \quad (8.1)$$

$$s_{21} = s'_{21} e^{jk_0 (l_1 + l_2)}, \quad (8.2)$$

where

$$k_0 = \omega \sqrt{\mu_0 \epsilon_0}. \quad (8.3)$$

The s parameters s_{11} and s_{21} are related to the material properties μ and ϵ and the thickness of the sample, and can be written as follows [104]:

$$s_{11} = \frac{(1 - T^2)\Gamma}{1 - \Gamma^2 T^2}, \quad (8.4)$$

$$s_{21} = \frac{(1 - \Gamma^2)T}{1 - \Gamma^2 T^2}, \quad (8.5)$$

where

$$T = e^{-jkd}, \quad (8.6)$$

$$\Gamma = \frac{Z_s - Z_0}{Z_s + Z_0}, \quad (8.7)$$

$$Z_s = \sqrt{\mu / \varepsilon}, \quad (8.8)$$

$$Z_0 = \sqrt{\mu_0 / \varepsilon_0}, \quad (8.9)$$

$$k = \omega \sqrt{\mu \varepsilon}. \quad (8.10)$$

Given the measured s parameters s_{11} and s_{21} in (8.4) and (8.5), the Nicolson, Ross and Weir (NRW) method is commonly used to determine the material properties μ and ε . The procedure to extract μ and ε is given below [39], [40], [104].

Based on (8.4) and (8.5), the following relations hold:

$$\Gamma = X \pm \sqrt{X^2 - 1}, \quad (8.11)$$

$$T = \frac{s_{11} + s_{21} - \Gamma}{1 - (s_{11} + s_{21})\Gamma}, \quad (8.12)$$

where

$$X = \frac{1 - (s_{21}^2 - s_{11}^2)}{2s_{11}}. \quad (8.13)$$

The proper root choice in (8.11) is based on the requirement that the magnitude of the reflection coefficient must be less than unity, i.e. $|\Gamma| \leq 1$.

From (8.6) and (8.10), we can obtain:

$$\sqrt{\mu \varepsilon} = \frac{j \ln T}{\omega d}. \quad (8.14)$$

From (8.7)-(8.9), the following equation can be obtained:

$$\sqrt{\mu / \varepsilon} = \sqrt{\mu_0 / \varepsilon_0} \frac{1 + \Gamma}{1 - \Gamma}. \quad (8.15)$$

By multiplying (8.14) and (8.15), we can obtain:

$$\mu = \frac{j \ln T}{\omega d} \sqrt{\mu_0 / \varepsilon_0} \frac{1 + \Gamma}{1 - \Gamma}, \quad (8.16)$$

where Γ and T are readily obtained from the measured s parameters s_{11} and s_{21} by using (8.11) and (8.12) respectively.

Similarly, ε can be obtained from (8.14) and (8.15) as follows:

$$\varepsilon = \frac{j \ln T}{\omega d} \sqrt{\varepsilon_0 / \mu_0} \frac{1 - \Gamma}{1 + \Gamma}. \quad (8.17)$$

From the above extraction process, we can see that determination of the material properties μ and ε by using the coaxial transmission line method is quite simple in principle. However, a few points have to be noted. First, in the coaxial transmission line method, it is required that there should be no air gaps between the MUT and the coaxial transmission line test fixture. It has been found that a small air gap between the sample and the coaxial transmission line test fixture may cause great errors when the material with high permittivity ($\varepsilon_r > 10$) is measured [102]. A few correction techniques with the air gaps taken into account can be found in [102] and [103] when common dielectric materials are measured. However, there are few researches on the influence of the air gaps when a sample with very high permittivity as the Mn-Zn ferrites is measured. Second, the conductors of the coaxial transmission line test fixture are assumed to be perfectly conducting. Study is needed on the influence of the finite conductivity of the conductors when the Mn-Zn ferrites are measured. These issues will be covered in the following sections. In the numerical examples in the following sections, the intrinsic complex permeability and permittivity of the Mn-Zn ferrite MUT are assumed to be expressed by (6.68) and (6.65) respectively. Those two formulae are only very rough estimation of the intrinsic values of a Mn-Zn ferrite. But the rough estimation is good enough for our purpose, which is to analyze the influence of the air gaps when the MUT is with very high permeability as well as very high permittivity. The inner and the outer diameters of the coaxial transmission line test fixture are assumed to be 3.04 mm and 7 mm respectively. The s parameters in all the numerical examples are referenced to the two flat surfaces of the MUT.

8.3 Influence of the Air Gaps

In the last paragraph in Section 6.5, the influence of the air gaps between the MUT and the coaxial transmission line test fixture is briefly mentioned. In this section, more details are given.

If the thickness of the MUT in Figure 8.1 is assumed to be 2 mm, the s parameters calculated by using (8.4) and (8.5) are shown in Figure 8.2 when there is no air gap.

Figure 8.2 Calculated s parameters when there is no air gap (solid lines: real parts; dotted lines: imaginary parts)

Figure 8.3 Illustration of air gaps between the MUT and the coaxial transmission line test fixture

We assume the air gaps between the MUT and the coaxial transmission line test fixture shown in Figure 8.3 are 0.05 mm wide. The thickness of the MUT is 2 mm. The s parameters calculated by using the 2-D FDFD method presented in Chapter 7 are shown in Figure 8.4 and Figure 8.5 respectively. If we take the results in Figure 8.2 as the references, the calculated errors of the s parameters shown in Figure 8.4 and Figure 8.5 are shown in Figure 8.6. From Figure 8.6, we can see that a tiny air gap between the MUT and the coaxial transmission line test fixture can lead to significant errors. The existence of the inner air gap shown in Figure 8.3 (a) causes greater errors than the existence of the outer air gap shown in Figure 8.3 (b) especially at higher frequencies.

Figure 8.4 Calculated s parameters for the structure shown in Figure 8.3 (a) (solid lines: real parts; dotted lines: imaginary parts)

Figure 8.5 Calculated s parameters for the structure shown in Figure 8.3 (b) (solid lines: real parts; dotted lines: imaginary parts)

Figure 8.6 Errors of the s parameters for the structures shown in Figure 8.3 (solid lines: Figure 8.3 (a); dotted lines: Figure 8.3 (b))

Figure 8.7 The assumed and the determined intrinsic complex permeability and permittivity (solid lines and dotted lines: assumed values; circles and crosses: determined values)

Figure 8.8 Errors of the determined intrinsic complex permeability and permittivity for the structures shown in Figure 8.3 (b)

Since there are great errors of the s parameters when there are air gaps, the determined intrinsic complex permeability and permittivity by using the NRW method shown in (8.16) and (8.17) can also be expected to have great errors. For example, if we use the s parameters in Figure 8.5 to extract the intrinsic complex permeability and permittivity by using the NRW method, the determined intrinsic complex permeability and permittivity are shown in Figure 8.7. The errors of the determined intrinsic values are shown in Figure 8.8. From Figure 8.7 and Figure 8.8, an interesting phenomenon which can be observed is that the error of the determined intrinsic complex permeability is much less than that of the determined intrinsic complex permittivity. From Figure 8.7 (b), it can also be seen that the real part of the determined intrinsic complex permittivity does not vary much with the frequency.

8.4 Dimensional Correction Techniques

To improve the accuracy of the determined intrinsic complex permeability and permittivity when there are air gaps, dimensional correction techniques presented in [102] and [103] can be used for common materials when the air gap widths are measured accurately. In [102], only the inner air gap is taken into consideration, as shown in Figure 8.3 (a). The corrected intrinsic complex permeability and permittivity are expressed as follows [102]:

$$\mu_c = m(\mu_r - 1) + 1, \quad (8.18)$$

$$\varepsilon_c = \frac{\varepsilon_r}{m(1 - \varepsilon_r \Delta / \alpha)}, \quad (8.19)$$

where μ_r and ε_r are the determined intrinsic relative complex permeability and permittivity respectively, μ_c and ε_c are the corrected intrinsic relative complex permeability and permittivity respectively, and

$$m = \log(R_2 / R_1) / \log(R_2 / r_1) \approx 1 + \Delta / \alpha, \quad (8.20)$$

$$\alpha = \log(R_2 / R_1), \quad (8.21)$$

$$\Delta = 2(r_1 - R_1) / (r_1 + R_1). \quad (8.22)$$

In (8.20)-(8.22), R_1 , R_2 , and r_1 are the inner radius of the coaxial transmission line test fixture, the outer radius of the coaxial transmission line test fixture, and the inner radius of the MUT respectively.

As an example to illustrate the effectiveness of the correction technique shown above, we assume that the relative complex permeability and permittivity of the MUT are 15 and 25 respectively, the thickness of the MUT is 5 mm, and the inner air gap is 0.05 mm wide. The determined intrinsic complex permeability and permittivity by using the NRW method and the corrected intrinsic values by using (8.18) and (8.19) are shown in Figure 8.9. The errors of the determined intrinsic complex permeability and permittivity before and after the correction are shown in Figure 8.10. From Figure 8.9 and Figure 8.10, the effectiveness of the dimensional correction technique can be seen obviously.

Figure 8.9 Determined intrinsic complex permeability and permittivity before and after correction (solid lines: assumed values; dashed lines: determined values before correction; circles: determined values after correction)

Figure 8.10 Errors of the determined intrinsic complex permeability and permittivity before and after correction (dashed lines: before correction; circles: after correction)

We apply the above correction technique to the example in Figure 8.3 (a). The determined intrinsic complex permeability and permittivity after correction are shown in Figure 8.11. The errors of the determined intrinsic complex permeability and permittivity before and after the correction are shown in Figure 8.12. From Figure 8.11 and Figure 8.12, we can see that the dimensional correction technique can improve a little the accuracy of the intrinsic complex permeability, but help little the result of the intrinsic complex permittivity. From Figure 8.11 (b), it can also be seen that the real parts of the intrinsic complex permittivity after correction become negative, and are apparently wrong.

Figure 8.11 Determined intrinsic complex permeability and permittivity after correction

Figure 8.12 Errors of the determined intrinsic complex permeability and permittivity before and after dimensional correction (dashed lines: before correction; circles: after correction)

In [103], both the inner and the outer air gaps are taken into consideration. Only the corrected intrinsic complex permittivity are given in [103], which can be expressed as follows:

$$\varepsilon_c = \frac{\varepsilon_r \log(r_2 / r_1)}{\log(R_2 / R_1) - \varepsilon_r \log\left(\frac{R_2 / R_1}{r_2 / r_1}\right)}, \quad (8.23)$$

where R_1 , R_2 , r_1 , and r_2 are the inner radius of the coaxial transmission line test fixture, the outer radius of the coaxial transmission line test fixture, the inner radius of the MUT and the outer radius of the MUT respectively, ε_r is the determined intrinsic relative complex permittivity, and ε_c is the corrected intrinsic relative complex permittivity.

As an example, we assume that both the inner and the outer air gaps are 0.05 mm wide.

The thickness of the Mn-Zn ferrite MUT is 2 mm. The corrected intrinsic complex permittivity according to (8.23) is shown in Figure 8.13. The errors of the determined intrinsic complex permittivity before and after correction are shown in Figure 8.14. From Figure 8.11 (b), Figure 8.12 (b), Figure 8.13 and Figure 8.14, we can see that the effectiveness of the correction techniques shown in (8.19) and (8.23) are similar. Actually, if we set $r_2=R_2$ in (8.23), and make use of the approximation equation in (8.20), (8.23) can be transformed into (8.19) exactly.

Figure 8.13 Determined intrinsic complex permittivity after correction

Figure 8.14 Errors of the determined intrinsic complex permittivity before and after correction
(dashed lines: before correction; circles: after correction)

From the above analysis, we can see that only the accuracy of the intrinsic complex permeability can be improved a little by using the dimensional correction techniques when the Mn-Zn ferrite MUT is measured. Furthermore, the requirement of the dimensional correction techniques to accurately measure the air gap width imposes a strict constrain on the application of the techniques.

8.5 Eliminate the Air Gaps with Conductive Materials

A practical method to effectively improve the accuracy of the determined intrinsic complex permeability and permittivity is to eliminate the air gaps by using conductive materials. For this purpose, the silver conductive paint or the metal deposition can be used to coat the lateral cylindrical surfaces of the MUT. To evaluate the effectiveness of that method, the influence of the finite conductivity of the conductive materials has to be studied. In the following numerical examples in this section, we assume that the inner and the outer air gaps are of the same width. The thickness of the Mn-Zn ferrite MUT is 2 mm. The conductive materials fully fill the air gaps. According to the data sheet of the silver conductive paint (RS 186-3600), the maximum conductivity of the paint is 10^5 S/m when the paint is fully cured. When the metal deposition technique is used to coat the MUT, the conductivity of the coating material is the conductivity of the metal used, which is apparently greater than 10^5 S/m. For conservative estimation, we assume the conductivity of the conductive materials is 1×10^4 S/m. The s parameters of the structure are calculated by using the 2-D FDFD method.

Figure 8.15 Errors of the determined intrinsic complex permeability and permittivity for different conductor-filled gap width (solid lines: 0.05 mm; dashed lines: 0.08 mm; circles: 0.10 mm; dotted lines: 0.15 mm)

The errors of the determined intrinsic complex permeability and permittivity by using the NRW method for different gap width are shown in Figure 8.15. By comparison of Figure 8.15 and Figure 8.8, we can see that the accuracy of both the determined intrinsic complex permeability and permittivity is considerably improved. It can also be seen from Figure 8.15 that the errors of the results increase almost linearly with the increase of the

gap width. The errors do not vary much with the frequency. The determined intrinsic complex permeability and permittivity of the Mn-Zn ferrite MUT when the gap width is 0.15 mm are shown in Figure 8.16.

Figure 8.16 The determined intrinsic complex permeability and permittivity (solid lines and dotted lines: assumed values; circles and crosses: determined values)

Figure 8.17 Improved model of the coaxial transmission line test fixture holding the MUT

The NRW method assumes that the MUT perfectly fills the coaxial transmission line test fixture, and does not take into account the actual inner and outer radii of the MUT. To further improve the accuracy of the results, the improved model of the coaxial transmission line test fixture holding the MUT shown in Figure 8.17 is adopted. That model is a coaxial discontinuity structure, which contains three sections. The MUT perfectly fills the middle section. The inner convex and the outer concave parts of the other two sections have the same thickness, which is called equivalent conductor-filled gap width in the following paragraphs. The intrinsic complex permeability and permittivity are determined by using the Newton-Raphson method as follows:

$$\begin{bmatrix} \mu'(i+1) \\ \mu''(i+1) \\ \varepsilon'(i+1) \\ \varepsilon''(i+1) \end{bmatrix} = \begin{bmatrix} \mu'(i) \\ \mu''(i) \\ \varepsilon'(i) \\ \varepsilon''(i) \end{bmatrix} - \mathbf{J}^{-1} \begin{bmatrix} f_1(i) - \text{Re}(s_{11m}) \\ f_2(i) - \text{Im}(s_{11m}) \\ f_3(i) - \text{Re}(s_{21m}) \\ f_4(i) - \text{Im}(s_{21m}) \end{bmatrix}, \quad (8.24)$$

$$\mathbf{J} = \begin{bmatrix} \partial f_1 / \partial \mu' & \partial f_1 / \partial \mu'' & \partial f_1 / \partial \varepsilon' & \partial f_1 / \partial \varepsilon'' \\ \partial f_2 / \partial \mu' & \partial f_2 / \partial \mu'' & \partial f_2 / \partial \varepsilon' & \partial f_2 / \partial \varepsilon'' \\ \partial f_3 / \partial \mu' & \partial f_3 / \partial \mu'' & \partial f_3 / \partial \varepsilon' & \partial f_3 / \partial \varepsilon'' \\ \partial f_4 / \partial \mu' & \partial f_4 / \partial \mu'' & \partial f_4 / \partial \varepsilon' & \partial f_4 / \partial \varepsilon'' \end{bmatrix}, \quad (8.25)$$

$$f_1 = \text{Re}(s_{11c}), \quad (8.26)$$

$$f_2 = \text{Im}(s_{11c}), \quad (8.27)$$

$$f_3 = \text{Re}(s_{21c}), \quad (8.28)$$

$$f_4 = \text{Im}(s_{21c}), \quad (8.29)$$

where s_{11m} and s_{21m} are the measured s parameters, s_{11c} and s_{21c} are the calculated s parameters of the model in Figure 8.17 by using the mode matching method presented in Chapter 6. The elements in the matrix $\mathbf{J}$ are numerically calculated as shown in (3.6)-(3.9). For the fast convergence of the Newton-Raphson method, the determined intrinsic complex permeability and permittivity at a frequency point become the initial values for the next frequency point for the iteration in (8.24). For the first frequency point, the intrinsic values determined by using the NRW method can be taken as the initial values.

Figure 8.18 Errors of the determined intrinsic complex permeability and permittivity for different equivalent conductor-filled gap width (solid lines: 0.05 mm; dashed lines: 0.08 mm; circles: 0.10 mm; dotted lines: 0.15 mm; crosses: 0.50 mm)

In the following numerical examples, s_{11m} and s_{21m} are the predicted s parameters by using the 2-D FDFD method with the conductivity of the filling conductive materials

taken into account. The errors of the determined intrinsic complex permeability and permittivity by using the Newton-Raphson method presented above for different equivalent conductor-filled gap width are shown in Figure 8.18. It can be seen from Figure 8.18 that even for rather wide equivalent conductor-filled gaps, the errors of the results are within acceptable limits. By comparison of Figure 8.15 and Figure 8.18, we can see that the accuracy of the results obtained by using the Newton-Raphson method is greatly improved.

8.6 Influence of the Finite Conductivity of the Coaxial Transmission Line Test Fixture

Figure 8.19 Coaxial discontinuity model for the analysis of the influence of the finite conductivity of the coaxial transmission line test fixture

In the above analysis, the conductors of the coaxial transmission line test fixture are assumed to be PEC. The influence of the finite conductivity of the coaxial transmission line test fixture is analyzed in this section. The model adopted for the analysis is shown in Figure 8.19, which is a coaxial discontinuity structure composed of five discontinuity sections. The length of the lossy coaxial transmission line test fixture under study is L as indicated in Figure 8.19. The conductivity of the conductors of the lossy coaxial transmission line test fixture is assumed to be σ . The model takes the conductors of the lossy coaxial transmission line test fixture as a kind of medium whose complex permittivity is $\epsilon_0 - j\sigma/\omega$. Perfectly matched layers (PML) in the radial direction are used to wrap the lossy coaxial transmission line test fixture so that only a thin layer of the outer conductor of the lossy coaxial transmission line test fixture is needed in the model for less computational effort.

Suppose the coaxial transmission line test fixture is made of aluminum. For conservative estimation, the conductivity of the aluminum is assumed to be 3.5×10^7 S/m. In the model

in Figure 8.19, the thickness of the outer conductor of the lossy coaxial transmission line test fixture is set to be 0.2 mm. The thickness of the Mn-Zn ferrite MUT, i.e. d , is 2 mm. The length of the lossy coaxial transmission line test fixture is 40 mm. The s parameters calculated by using the 2-D FDFD method presented in Chapter 7 are shown in Figure 8.20, together with the s parameters predicted by using (8.4) and (8.5). It can be seen from Figure 8.20 that the finite conductivity of the coaxial transmission line test fixture has little influence on the resultant s parameters in the frequency range of interest. If the calculated s parameters by using the 2-D FDFD method are used to determine the intrinsic complex permeability and permittivity by using the NRW method shown in (8.16) and (8.17), it is found that the maximum error of the determined intrinsic values is far below 1.0%.

Figure 8.20 The s parameters of the coaxial transmission line test fixture loaded with the MUT (solid lines: 2-D FDFD, real parts; dotted lines: 2-D FDFD, imaginary parts; circles: (8.4)–(8.5), real parts; crosses: (8.4)–(8.5), imaginary parts)

8.7 Manufactured Coaxial Transmission Line Test Fixture

The structure of the manufactured coaxial transmission line test fixture is shown in Figure 8.21. Part 1 is a BNC Jack-Jack adaptor. Part 2 and part 3 of the test fixture are made of aluminum. Part 4 is made of copper. The dimensions of the test fixture are: $D=13.5$ mm, $L=44.55$ mm, $d=7$ mm, $l=32.2$ mm. The measurement planes of the test fixture are shown in Figure 8.21. The test fixture is connected to the calibration planes of the VNA via interseries adaptors and cables. The influence of the sections between the calibration planes of the VNA and the measurement planes of the test fixture has to be removed from the measurement results. The error box measurement model is shown in

Figure 8.22 (a). The DUT in Figure 8.22 (a) is the section between the measurement planes. Details of the measurement model in terms of the s parameters are shown in Figure 8.22 (b).

Figure 8.21 Structure of the manufactured coaxial transmission line test fixture

To avoid using a load standard which is usually unavailable over a wide frequency range, the open-short-through line de-embedding technique [105, 106] is used to solve the error boxes L and R shown in Figure 8.22. Details of the de-embedding process are shown in the following paragraphs.

The following equations in terms of the s parameters can be found for the measurement network shown in Figure 8.22 [106, 107]:

$$(1 - L_{22}s_{11})M_{11} - R_{22} \frac{L_{21}}{R_{21}} s_{21} M_{21} = L_{11} - \Delta L s_{11}, \quad (8.30)$$

$$-L_{22}s_{12}M_{11} + (1 - R_{22}s_{22}) \frac{L_{21}}{R_{21}} M_{12} = -\Delta L s_{12}, \quad (8.31)$$

$$(1 - L_{22}s_{11})M_{21} - R_{22} \frac{L_{21}}{R_{21}} s_{21} M_{22} = -\Delta R \frac{L_{21}}{R_{21}} s_{21}, \quad (8.32)$$

Figure 8.22 Error box measurement model

$$-L_{22}s_{12}M_{21} + (1 - R_{22}s_{22})\frac{L_{21}}{R_{21}}M_{22} = (R_{11} - \Delta R s_{22})\frac{L_{21}}{R_{21}}, \quad (8.33)$$

where $\Delta L = L_{11}L_{22} - L_{12}L_{21}$, $\Delta R = R_{11}R_{22} - R_{12}R_{21}$, and M_{11} , M_{21} , M_{12} and M_{22} are the s parameters read from the VNA.

Suppose the three standards used to replace the DUT in Figure 8.22 are A, B and C respectively. The error networks L and R can be identified in the general case by using (8.30A), (8.33A), (8.30B), (8.33B), (8.30C), (8.31C) or (8.32C), and (8.33C), where (8.30A) is (8.30) with standard A. They form a set of seven linear equations for L_{11} , L_{22} , ΔL , kR_{11} , kR_{22} , $k\Delta R$, and $k(=L_{21}/R_{21})$, from which the unknown two-ports L and R can be solved [106]. In our case here, the standards A, B and C are the open, the short, and the through line standards respectively. The seven linear equations can be further expressed as follows:

$$(1 - L_{22}s_{A11})M_{A11} = L_{11} - \Delta L s_{A11}, \quad (8.34)$$

$$(1 - R_{22}s_{A22})\frac{L_{21}}{R_{21}}M_{A22} = (R_{11} - \Delta R s_{A22})\frac{L_{21}}{R_{21}}, \quad (8.35)$$

$$(1 - L_{22}s_{B11})M_{B11} = L_{11} - \Delta L s_{B11}, \quad (8.36)$$

$$(1 - R_{22}s_{B22})\frac{L_{21}}{R_{21}}M_{B22} = (R_{11} - \Delta R s_{B22})\frac{L_{21}}{R_{21}}, \quad (8.37)$$

$$(1 - L_{22}s_{C11})M_{C11} - R_{22}\frac{L_{21}}{R_{21}}s_{C21}M_{C21} = L_{11} - \Delta L s_{C11}, \quad (8.38)$$

$$(1 - L_{22}s_{C11})M_{C21} - R_{22}\frac{L_{21}}{R_{21}}s_{C21}M_{C22} = -\Delta R \frac{L_{21}}{R_{21}}s_{C21}, \quad (8.39)$$

$$-L_{22}s_{C12}M_{C21} + (1 - R_{22}s_{C22})\frac{L_{21}}{R_{21}}M_{C22} = (R_{11} - \Delta R s_{C22})\frac{L_{21}}{R_{21}}, \quad (8.40)$$

where s_{A11} and M_{A11} are respectively s_{11} and M_{11} in Figure 8.22 (b) when the DUT in Figure 8.22 (a) is replaced by the standard A. Obviously, we have $s_{A11} = s_{A22} = 1$,

$s_{B11} = s_{B22} = -1$, $s_{C11} = s_{C22} = 0$, $s_{C21} = s_{C12} = e^{-j\omega\sqrt{\mu_0\epsilon_0}l_C}$, where l_C is the length of the through line. As mentioned above, the seven unknowns: L_{11} , L_{22} , ΔL , kR_{11} , kR_{22} , $k\Delta R$, and $k(=L_{21}/R_{21})$ can be solved from the seven linear equations (8.34)-(8.40).

The transmission matrix of the measurement model shown in Figure 8.22 can be expressed as [108]:

$$[T_M] = \frac{1}{L_{21}R_{12}}[Y_L][T_{DUT}][Y_R], \quad (8.41)$$

where

$$[Y_L] = \begin{bmatrix} -\Delta L & L_{11} \\ -L_{22} & 1 \end{bmatrix}, \quad (8.42)$$

$$[Y_R] = \begin{bmatrix} -\Delta R & R_{22} \\ -R_{11} & 1 \end{bmatrix}, \quad (8.43)$$

$$[T_{DUT}] = \frac{1}{s_{21}} \begin{bmatrix} s_{21}s_{12} - s_{11}s_{22} & s_{11} \\ -s_{22} & 1 \end{bmatrix} \triangleq \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}, \quad (8.44)$$

$$[T_M] = \frac{1}{M_{21}} \begin{bmatrix} M_{21}M_{12} - M_{11}M_{22} & M_{11} \\ -M_{22} & 1 \end{bmatrix}. \quad (8.45)$$

Equation (8.41) can also be expressed as follows:

$$[T_{DUT}] = L_{21}R_{12}[Y_L]^{-1}[T_M][Y_R]^{-1}. \quad (8.46)$$

Let us define:

$$[Y_L]^{-1}[T_M][Y_R]^{-1} \triangleq \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}, \quad (8.47)$$

where $[T_{MC}]$ is the transmission matrix $[T_M]$ when the DUT is the standard C, i.e. the through line.

From (8.44), (8.46) and (8.47), we have:

$$L_{21}R_{12} = \frac{1}{s_{C21}t_{22}} = \frac{1}{e^{-j\omega\sqrt{\mu_0\epsilon_0}l_C}t_{22}}. \quad (8.48)$$

Up to now, the transmission matrix of the DUT, i.e. $[T_{DUT}]$, can be obtained by using (8.46). From (8.44), the s parameters of the DUT can be expressed from the components of the transmission matrix $[T_{DUT}]$ as follows:

$$s_{11} = \frac{T_{12}}{T_{22}}, \quad (8.49)$$

$$s_{21} = \frac{1}{T_{22}}, \quad (8.50)$$

$$s_{12} = \frac{T_{11}T_{22} - T_{21}T_{12}}{T_{22}}, \quad (8.51)$$

$$s_{22} = -\frac{T_{21}}{T_{22}}. \quad (8.52)$$

Figure 8.23 Configuration of the open, the short, and the through line standards

The open, the short and the through line standards used in the de-embedding process are shown in Figure 8.23. Since the inner conductor of the BNC Jack-Jack connector is shorter than its outer bound, for the open standard to be ideal, a short copper bar, i.e. the shaded part in Figure 8.23 (a), is plugged into the BNC Jack-Jack connector. The end of the short bar is made to align with the end of the outer bound of the BNC Jack-Jack connector, as shown in Figure 8.23 (a). The short standard, i.e. the shaded part in Figure 8.23 (b), is the same as shown in Figure 3.8. Part 3 of the test fixture is screwed in to press the short standard tight, as shown in Figure 3.9 (c) and (d). The through line standard is the same as the test fixture shown in Figure 8.21 except that part 4 of the test fixture is replaced with a straight copper bar, i.e. the shaded part in Figure 8.23 (c).

8.8 Experimental Results

Figure 8.24 Illustration of preparation of the MUT

Before the measurement of the intrinsic complex permeability and permittivity of a toroidal Mn-Zn ferrite core, the inner and the outer lateral surfaces of the core (RS Components Pte Ltd, Part No. 5978001901) are respectively deposited with a thin layer of copper. A high precision diamond blade cutter (ISOMET 1000) is then used to separate the core into two cores, as illustrated in Figure 8.24. We choose the smaller one of the two cores as our measurement sample, i.e. the MUT. The two flat surfaces of the MUT are further polished by using a variable speed grinder/polisher (Ecomet 6). The physical dimensions of the MUT are: $2R_1=8.05$ mm, $2R_2=12.90$ mm, and $d=1.9$ mm. The temperature at which the experiments are conducted is 23°C .

Figure 8.25 Configuration of the MUT in the test fixture

To eliminate the air gaps between the MUT and the test fixture, as illustrated in Figure 8.25, conductive adhesive copper foils are used to wrap the outer surface of the MUT and

the middle section of part 4 of the test fixture. The position of the MUT in the test fixture can be measured by using a depth caliper.

Figure 8.26 Layout of the measurement system

The VNA used in the experiments is Agilent E5062A. The calibration kit is HP 85033D. The layout of the measurement system is shown in Figure 8.26. By using the open-short-through line de-embedding technique presented above, the resultant s parameters s_{11} and s_{21} referenced to the measurement planes are shown in Figure 8.27. From Figure 8.27 we can see that s_{11} is close to -1 and s_{21} is close to zero, which means that most of the incident waves from the VNA are reflected back; only a small portion of the waves can be transmitted through the MUT. Thus theoretically for more accurate results, a thinner MUT should be used, so that s_{11} and s_{21} can be measured with higher accuracy. But the Mn-Zn ferrites are hard and brittle, machining a very thin Mn-Zn ferrite core becomes difficult. Wrapping a very thin core with the copper foils is also quite difficult.

From the analysis in Section 8.5 and Section 8.6, it can be seen that the conductive adhesive copper foil, the deposited copper layers, and the test fixture can be seen as PEC. The resultant mathematical model seen from the measurement planes is shown in Figure 8.28. The mathematical model is an axisymmetric coaxial discontinuity structure with eight discontinuity sections. By using the Newton-Raphson method shown in Section 8.5, the intrinsic complex permeability and permittivity of the MUT can be determined from the mathematical model shown in Figure 8.28 and the measured s parameters shown in Figure 8.27. The determined results are shown in Figure 8.29.

Figure 8.27 Measured s parameters referenced to the measurement planes (solid lines: real parts; dotted line: imaginary parts)

Figure 8.28 Mathematical model of the test fixture loaded with MUT seen from measurement planes

Figure 8.29 Determined intrinsic complex permeability and permittivity of a Mn-Zn ferrite core by using the manufactured coaxial transmission line test fixture

Figure 8.30 A test fixture used to validate the determined intrinsic values

Figure 8.31 Measured and calculated s_{11} of the structure in Figure 8.30 (solid line: measured real parts; dotted line: measured imaginary parts; circles: calculated real parts; crosses: calculated imaginary parts)

For validation of the determined intrinsic values shown in Figure 8.29, a test fixture shown in Figure 8.30 is used. Its physical dimensions are: $d_1=1.4$ mm, $d_2=5$ mm, $d_3=7$ mm, $L_1=38$ mm and $L_2=32.1$ mm. Part 1, part 2 and part 3 indicated in Figure 8.30 are the same as those in Figure 8.21. A toroidal Mn-Zn ferrite core (RS Components Pte Ltd, Part No. 5978001901), which is made of the same material as the core shown in Figure 8.24, was measured using the test fixture. The inner and the outer radii of the Mn-Zn ferrite core are the same as the core in Figure 8.24. The height of the Mn-Zn ferrite core is 10.30 mm. The s parameter s_{11} referenced to the measurement plane shown in Figure 8.30 is measured by using the open-short-through line de-embedding technique mentioned above, and calculated as well by using the 2-D FDFD method presented in Chapter 7. In the calculation, the air gap between the lower flat surface of the ferrite core and the test fixture is assumed to be 0.05 mm wide. When that air gap width is changed to be 0.1 mm or 0.2 mm in the calculation, the calculated results have little difference. The measured and the calculated s_{11} are shown in Figure 8.31. Good agreement between the measured and the calculated results can be observed.

8.9 Conclusions

In this chapter, issues related to the determination of the Mn-Zn ferrites' intrinsic complex permeability and permittivity by using the coaxial transmission line method is

discussed. Following the introduction of the basic principle of the coaxial transmission line method, the influence of the air gaps between the MUT and the coaxial transmission line test fixture is studied. It is found that a small air gap will cause great errors to the determined Mn-Zn ferrites' intrinsic values by using the NRW method. The effectiveness of two dimensional correction techniques is examined, which can be used to improve the accuracy of the determined intrinsic values when there are air gaps in the measurement of common materials. It is found that for the Mn-Zn ferrite MUT the dimensional correction techniques help to improve only a little the accuracy of the determined intrinsic complex permeability, and make the errors of the determined intrinsic complex permittivity even worse. It is found from the numerical examples that the accuracy of both the determined intrinsic values is greatly improved when the air gaps are filled with conductive materials, even if the conductivity of the conductive materials is only 1×10^4 S/m. To further improve the accuracy of the results, a new coaxial discontinuity model is proposed to take into account the actual inner and outer radii of the MUT when the air gaps are filled with conductive materials. Instead of the NRW method, the Newton-Raphson method and the mode matching method are used to determine the intrinsic values. It is found that the accuracy of the determined intrinsic values with the new model is greatly improved, even if the width of the gaps is rather large. In this chapter, the influence of the finite conductivity of the conductors of the coaxial transmission line test fixture is also examined and found to be negligible in the frequency range of interest. For carrying out the measurement by using the coaxial transmission line method, a coaxial transmission line test fixture is manufactured. The intrinsic complex permeability and permittivity of a Mn-Zn ferrite core are experimentally determined by using the coaxial transmission line test fixture in the frequency range from 10 MHz to 200 MHz. The determined intrinsic values are also experimentally verified.

CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS

9.1 Conclusions

The aim of this work is to develop the measurement and analysis procedures which allow determining the intrinsic complex permeability and permittivity of Mn-Zn ferrites. The research includes a review of the available field-circuit coupled methods for determination of the Mn-Zn ferrites' intrinsic complex permeability and permittivity; the measures taken to improve the computational efficiency and the accuracy of the available field-circuit coupled methods; the presentation of two derivative field-circuit coupled methods which can simplify the measurement process in the sense that only one test fixture is needed, instead of two in the available field-circuit coupled methods; the generalization of the field-circuit coupled methods using only one toroidal sample; the presentation of a general mode matching method which can efficiently tackle common axially symmetric coaxial discontinuity structures; the proposal of a 2-D FDFD method to versatily solve all kinds of axially symmetric coaxial discontinuity structures; the examination, by virtue of the general mode matching method and the 2-D FDFD method, of the common issues related to the coaxial transmission line method for the determination of the Mn-Zn ferrites' intrinsic complex permeability and permittivity above 10 MHz, and determination of the intrinsic complex permeability and permittivity of a Mn-Zn ferrite core by using a manufactured coaxial transmission line test fixture in the frequency range from 10 MHz to 200 MHz where the field-circuit coupled methods cannot be applied.

In the review of the available field-circuit coupled methods for determination of the Mn-Zn ferrites' intrinsic complex permeability and permittivity, the physical phenomenon of the dimensional effects is firstly explained in detail to show clearly why the traditional methods for measuring the permeability of the magnetic materials with low permittivity and for measuring the permittivity of the dielectrics with low permeability are unsuitable

for the Mn-Zn ferrites which have high permeability as well as high permittivity. The advantages and the disadvantages of the available field-circuit coupled methods for determination of the Mn-Zn ferrites' intrinsic values are then closely examined theoretically and experimentally. It is found that among the three available field-circuit coupled methods in [6, 17, 18], the method in [18] is the most accurate and the most computationally time-consuming.

To improve the computational efficiency of the method in [18], a few measures are taken. Firstly, the expressions of the magnetic fields in the toroidal Mn-Zn ferrite cores are further simplified. Secondly, the impedance of the ferrite core inductor is expressed directly from the magnetic fields by using the Faraday's Law of induction, instead of the complex power by using the Law of Energy Conservation. Thirdly, the Jacobian Matrix in the iteration equation of the Newton-Raphson method is numerically evaluated by using the finite difference method. With these measures taken, the computational efficiency of the method in [18] is greatly improved. The method in [18] is based on the dimensional assumption that the toroidal Mn-Zn ferrite core inductor can be unwound into a bar without much change of the internal fields. To improve the accuracy of the method in [18], a cylindrical coordinate model of the ferrite core inductor is presented, which can be used to replace the Cartesian coordinate model of the ferrite core inductor to eliminate that dimensional assumption. In the available field-circuit coupled methods, the impedance of the ferrite core inductor is usually measured from a wire wound inductor. Above a few hundred kHz, the stray capacitance between the coils can have great influence on the measured impedance of the inductor if the coils are not appropriately disposed [17]. To eliminate that influence and to simplify the measurement process, a set of short-ended coaxial test fixture is manufactured.

Two novel derivative field-circuit coupled methods are presented. Both of them need two samples with different sizes. The first method determines the intrinsic values from the impedances of two ferrite core capacitors. The second determines the intrinsic values from the impedances of two ferrite core inductors. Thus they need only one kind of test fixtures. The accuracy of the two methods depends on the difference between the measured apparent values of the two samples, i.e. the apparent complex permittivity for the first method and the apparent complex permeability for the second method. Since the apparent values of the two samples are more different at higher frequencies, the accuracy

of the two methods is low at the low frequencies. Experiments are carried out, showing that the methods are only applicable above a few hundred kHz.

The field-circuit coupled method using one toroidal sample is generalized. The basic principle of the generalization is that the two intrinsic values can be determined accurately as long as the sample is under different kinds of exciting modes in the two measurements. A general finite difference method is presented to solve the impedance of the ferrite core inductor in different magnetic field exciting modes. For the calculation of the impedance of the ferrite core capacitor, different available methods are carefully chosen for different electric field exciting modes. Experiments are carried out to verify the generalization principle. The general method allows different combinations of the two kinds of exciting modes. Thus the preparation of the samples, the measurement process and the choice of the test fixtures can be performed with more freedom.

A general mode matching method is presented to efficiently tackle common axisymmetric coaxial discontinuity structures. Since the mode matching method is based on the modal expansion of the total electromagnetic field, the propagation constants of the modal fields in the structure have to be solved first. The propagation constants of the uniformly filled coaxial or circular waveguides can be easily solved by using the available semi-analytical methods. A 1-D finite difference method is proposed to efficiently solve the propagations of the multi-layer filled coaxial or circular waveguides. For the orthogonal relation between different modes to be true, the general mode matching method cannot be applied to the waveguides either with open areas or with lossy conductor walls.

To overcome the limitations of the mode matching method, a 2-D FDFD method is proposed. With the axially symmetry of the structure taken into account, the method reduces the original three dimensional problem into an equivalent two dimensional one. Numerical examples show that the 2-D FDFD method is much more efficient than the 2-D FDTD method in solving the frequency response of the structures, while maintaining the high accuracy and the versatility of the 2-D FDTD method.

By virtue of the general mode matching method and the 2-D FDFD method, a few important issues are discussed theoretically, which are related to the coaxial transmission line method when it is used to measure the intrinsic values of the Mn-Zn ferrites. Firstly,

the great influence of the air gaps on the accuracy of the determined intrinsic values is demonstrated. It is found from numerical results that even if the air gap width is strictly controlled, the intrinsic complex permittivity of the Mn-Zn ferrites cannot be obtained by using the NRW method. For the Mn-Zn ferrites, the available dimensional correction techniques, which correct the determined intrinsic values according to the actual air gap width, can only improve a little the accuracy of the determined intrinsic complex permeability, but are helpless in improving the accuracy of the determined intrinsic complex permittivity. A practical method to improve the accuracy of the determined intrinsic values is to fill the air gaps with conductive materials. Numerical examples show that even if the conductivity of the material used to fill the air gaps is only 1×10^4 S/m, the accuracy of both the determined intrinsic values can be improved greatly. To further improve the accuracy of the results, a novel model of the coaxial transmission line test fixture holding the MUT with the air gaps filled with conductive materials is proposed. Based on the novel model, the Newton-Raphson method and the general mode matching method are used to accurately determine the intrinsic values from the measured s parameters of the coaxial transmission line test fixture holding the MUT. The influence of the finite conductivity of the coaxial transmission line test fixture is also theoretically examined and found to be negligible. A coaxial transmission line test fixture is manufactured. Its calibration issues are presented in detail. Experimental results of the intrinsic complex permeability and permittivity of a Mn-Zn ferrite core in the frequency range from 10 MHz to 200 MHz are obtained by using the coaxial transmission line test fixture. The determined intrinsic complex permeability and permittivity are experimentally verified.

9.2 Recommendations

Further researches are recommended in the following areas:

1. Investigation into the temperature dependence and the stress dependence of the intrinsic complex permeability and permittivity

The Mn-Zn ferrite cores used in the magnetic devices may be subjected to different temperatures and different stress conditions. The temperature and the stress dependence of the intrinsic complex permeability and permittivity need to be studied for accurate

modelling of the magnetic devices.

The temperature dependence and the stress dependence of the initial permeability of the Mn-Zn ferrites at the low frequencies, where the initial permeability varies little with the frequency, has been investigated by many researchers [1, 2, 14]. The temperature dependence and the stress dependence of the intrinsic complex permeability above a few hundred kHz need to be studied.

A modified coaxial transmission line test fixture designed to measure the intrinsic complex permeability of soft ferrites under stress at high frequencies is presented in [101], as shown in Figure 9.1. But only the experimental results of the Ni-Zn ferrites are presented in that paper. Investigations into the feasibility to apply the test fixture to the Mn-Zn ferrites are necessary.

Figure 9.1 Modified coaxial transmission line test fixture [101]

An experimental investigation of the temperature dependence of the Mn-Zn ferrites is presented in Section 4.3 in [109]. The experimental test fixture is shown in Figure 9.2. The measurement does not take into account the dimensional effects of the Mn-Zn ferrites. The experimental results up to only a few hundred kHz are obtained. Methods and the test fixtures for determination of the temperature dependence of the intrinsic complex permittivity of the Mn-Zn ferrites at high frequencies need to be investigated.

2. Accurate modelling and systematic optimization of the Mn-Zn ferrite core based devices in the low power level applications with the intrinsic complex permeability and permittivity taken into account

As it is mentioned in Chapter 1, the Mn-Zn ferrite cores can be fully characterized with the intrinsic complex permeability and permittivity in the low power level applications. It can be seen from the numerical examples in [8, 15] that, with the intrinsic complex permeability and permittivity known, the commercial 3-D Finite Element Analysis (FEA) tools can be successfully used in examining the flux distributions in the ferrite magnetic structures with both the core geometry and the windings taken into account. However, different parts of the core may be subjected to different temperature and different stress conditions. How to take into account these factors for more accurate modeling is required to be studied.

Figure 9.2 Test fixture used for dielectric measurements [109]

Based on the accurate modeling of the magnetic devices, optimization of the structure of the devices may be done. However, the performance of the magnetic devices is determined by many different factors. Those factors may also be related. For example, changing the geometry of the core will change not only the effective permeability of the core, but also the stray capacitance and the leakage inductance of the windings. Researches on a systematic guide for the optimization of the magnetic devices are recommended.

3. Characterization of the Mn-Zn ferrites with the complex permeability and permittivity for the high power level applications up to a few MHz

Nowadays, the application of the Mn-Zn power ferrites is advancing into the MHz frequency range. For a given core, there are a few methods that can be used to characterize the B versus H loop and the core loss up to the MHz frequency range in high power level [110]. Apparently, the characteristics measured by using those methods are not the intrinsic properties of the materials, thus cannot be applied to other cores with different dimensions from the core under test, even if their materials are the same.

Since the dimensional effects play an important role in the performance of the Mn-Zn ferrite core devices above a few hundred kHz, determination of the complex permeability and permittivity of the Mn-Zn power ferrite up to a few MHz is necessary for accurate modelling and optimization of the high power level Mn-Zn ferrite core devices. However, in the high power level application, the complex permeability of the Mn-Zn ferrites is no longer in the linear Rayleigh region. In [11], since the frequencies of interest are limited to a few MHz, a thin-walled toroidal core, which can ensure a uniform flux density and eliminate the influence of the complex permittivity, can be used to determine the nonlinear complex permeability under different excitation levels. In [11], the complex permittivity is measured by using a small disk capacitor made of the Mn-Zn ferrites. Similarly, the small size of the disk capacitor can ensure the uniform distribution of the electric fields and eliminate the influence of the complex permeability in the frequency range of interest. As we mentioned before, Mn-Zn ferrites are hard and brittle. Preparing such two small samples is not easy. How to measure the nonlinear complex permeability and the complex permittivity under different excitation levels with a single relative large sample is recommended for the future researches. The study on the stress dependence and the temperature dependence of the nonlinear complex permeability and the complex permittivity is also necessary for the high power level application.

4. Modification of the chemical composition, the crystal structure and the grain structure to improve the performance of the Mn-Zn ferrite cores

As illustrated in Figure 9.3, the performance of the Mn-Zn ferrite core are determined by many different factors including the electrical component structure, the core structure and the intrinsic properties of the core. The intrinsic properties of the Mn-Zn ferrites are

mainly determined by the chemical composition, the crystal structure and the grain structure [2]. Thus to improve the performance of the core in a specific application situation, one effective way is to modify the chemical composition, the crystal structure and the grain structure to optimize the intrinsic properties for that application. It is an interesting interdisciplinary research area to study the influence of the chemical composition, the crystal structure and the grain structure of the Mn-Zn ferrites on the intrinsic properties. In [19], very good researches on modification of the chemical composition of the Mn-Zn ferrites to reduce the dielectric constant and the dielectric loss have been reported. Further researches in this interdisciplinary area may be promising to greatly improve the performance of the Mn-Zn ferrite cores. Since the intrinsic μ and ϵ provide important information on the properties of the material, the methods presented in the thesis will definitely benefit the researchers in this interdisciplinary research area.

Figure 9.3 Illustration of the factors determining the core performance

Author's Publications

1. **R. F. Huang**, D. M. Zhang and K. J. Tseng, "An efficient finite-difference-based Newton-Raphson method to determine intrinsic complex permeabilities and permittivities for Mn-Zn ferrites," *IEEE Trans. Magn.*, vol. 42, no. 6, pp. 1655-1660, Jun. 2006.
2. **R. F. Huang** and D. M. Zhang, "Experimentally verified Mn-Zn ferrites' intrinsic complex permittivity and permeability tracing technique using two ferrite core capacitors," *IEEE Trans. Magn.*, vol. 43, no. 3, pp. 974-981, Mar. 2007.
3. **R. F. Huang** and D. M. Zhang, "Using a single toroidal sample to determine Mn-Zn ferrites' intrinsic complex permeability and permittivity," *IEEE Trans. Magn.*, vol. 43, no. 10, pp. 3807-3815, Oct. 2007.
4. **R. F. Huang** and D. M. Zhang, "Application of mode matching method to analysis of axisymmetric coaxial discontinuity structures used in permeability and/or permittivity measurement," *Progress in Electromagnetics Research*, No. PIER 67, pp. 205-230, 2007.
5. **R. F. Huang** and D. M. Zhang, "Theoretical and experimental comparison of different lumped circuit methods for determination of Mn-Zn ferrites' intrinsic complex permeability and permittivity," *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 846-849, 2008.
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8. **R. F. Huang** and D. M. Zhang, "Characterization of Mn-Zn ferrites using the coaxial transmission line method," *IEEE Trans. Magn.*, vol. 44, no. 7, pp. 1703-1710, 2008.

9. **R. F. Huang**, D. M. Zhang and K. J. Tseng, "Determination of dimension-independent magnetic and dielectric properties for Mn-Zn ferrite cores and its EMI applications," *IEEE Trans. Electromagn. Compat.* (Revised).
10. J. J. Yang, **R. F. Huang** and D. M. Zhang, "A field-circuit coupled method to accurately determine intrinsic complex permittivity for XLPE insulation material," *IEEE Trans. Dielectr. Electr. Insul.*, vol. 15, no. 2, pp. 334-341, Apr. 2008.
11. D. M. Zhang and **R. F. Huang**, "Calculation of effective impedance of common mode choke made of Mn-Zn ferrite", in *Proceedings of the 17th International Zurich Symposium on Electromagnetic Compatibility*, Feb. 27- Mar. 3, 2006, pp. 391-394.

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