Low complexity multiuser detection algorithm for CDMA systems

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2008

Li, Z. (2008). Low complexity multiuser detection algorithm for CDMA systems. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/13351>

<https://doi.org/10.32657/10356/13351>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

LOW COMPLEXITY MULTIUSER DETECTION
ALGORITHM FOR CDMA SYSTEMS

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ALGORITHM FOR CDMA SYSTEMS**

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SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING

2008

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Low Complexity Multiuser Detection Algorithm For CDMA Systems

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctor of Philosophy

2008

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Date

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Li Zhongjie

Acknowledgments

I would like to express my sincere thanks to my advisor, Associate Professor Ser Wee, for his valuable supervision throughout the course of the project. His encouragement and patience inspire my confidence to finish this project.

My stay at Center for Signal Processing has been most memorable with many staff and postgraduate students. I have learned much from the discussions with them.

Finally, I wish to acknowledge the encouragement and support from my family.

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Summary

In the study, the research emphasis has been focused on the methods for reducing the complexity of multiuser detection algorithms for code division multiple access (CDMA) communications systems. Several low complexity multiuser detection algorithms and interference cancellation algorithms have been proposed for direct sequence code division multiple access (DS-CDMA) systems and their performances have been tested through computer simulations.

The first proposed algorithm is weighted polynomial expansion (WPE) detector. The optimal weight vector for the L stage WPE detector has been derived based on MMSE criteria. An adaptive WPE detector based on least squares (LS) criteria and the recursive least squares (RLS) algorithm have been proposed to adjust the weight vector iteratively until it becomes the exact optimal weight vector. Complexity analysis shows that the reduction in the implementation complexity for the multi-stage WPE detector increases significantly as the number of users increases. Simulation results show that the optimal WPE detector and adaptive WPE detector can exactly implement the decorrelating detector and MMSE detector in low system load situation. Although there are some performance loss in high system load situation, the optimal WPE detector and adaptive WPE detector perform better than the linear Chebyshev parallel interference cancellation (PIC) detector and asymptotic WPE detector.

A blind cascaded modular structure detector has been proposed to implement the blind minimum output energy (MOE) detector in Chapter 4. The blind cas-

caded modular structure detector is a reduced rank detector which is relatively simple and has a low complexity compared with blind MOE detector. A vector filter has been introduced for adaptive interference cancellation in each module of the proposed detector. The weight vector has been derived based on maximum magnitude cross correlation criteria which maximizes the magnitude of the cross correlation between the output of the nonadaptive filter and that of the weight vector filter. Simulation results show that the proposed cascaded modular detector is able to capture and reduce most interference by using only the first few stage weight vector filters and achieve performance close to that of blind MOE detector even in large system, high load and near-far situation. The proposed detector is able to achieve the same performance as that of the full rank MOE detector with few stages compared with principal component (PC), principal component partitioned linear interference cancellation (PC-PLIC) and cross spectral (CS) detectors. Simulation results show that both the proposed detector and multistage wiener filter (MFWF) detector have nearly the same performance when the number of stages is large, while the proposed detector has better performance than that of the MFWF detector when the number of stages is small.

In Chapter 5, a RAKE structure cascaded modular detector has been proposed to handle the signature mismatch problem in DS-CDMA systems over multipath fading channel. The detector does not require the knowledge of the fading channel and can achieve good performance with simple structure compared with other blind receivers that need to estimate the channel information. The proposed cascaded detector is used in each arm of the RAKE receiver to extract part of the desired signal and the output of all arms is coherently combined to detect the desired signal. Simulation results on bit error rate (BER) performance show that the RAKE cascaded modular receiver has nearly the same performance as that of the RAKE MOE receiver and has better convergence performance than that of the adaptive max-min generalized sidelobe canceller (GSC) detector.

List of Abbreviations

3G	third generation
AWGN	additive white Gaussian noise
BER	bit error rate
BPSK	binary phase shift keying
CDMA	code division multiple access
CS	cross spectral
DS-CDMA	direct sequence code division multiple access
DS-SS	direct-sequence spread spectrum
FDMA	frequency division multiple access
flops	floating point operations
FIR	finite impulse response
GSC	generalized sidelobe canceller
i.i.d.	independent and identically distributed
LHS	left hand side
LS	least squares
MAI	multiple Access interference
MIMO	multiple input multiple output
MLS	maximum likelihood sequence
MMSE	minimum mean squared error
MOE	minimum output energy
MSE	mean squared error

MSWF	multistage wiener filter
MUD	multiuser detection
MVDR	minimum variance distortionless response
PC	principal component
PE	polynomial expansion
PIC	parallel interference cancellation
PLIC	partitioned linear interference cancellation
RHS	right hand side
RLS	recursive least squares
SIC	successive interference cancellation
SINR	signal to interference plus noise ratio
SNR	signal to noise ratio
TDMA	time division multiple access
WPE	weighted polynomial expansion
ZF-DF	zero forcing decision feedback

List of Symbols

a	scalar
$\mathbf{a}$	vector
$\mathbf{A}$	matrix
A_k	amplitude of the k th user
$(\cdot)^H$	complex conjugate transpose of a vector or matrix
$(\cdot)^T$	transpose of a vector or matrix
$(\cdot)^{-1}$	inverse operation
$\ \cdot\ _2$	Euclidean norm
β	system load
$\mathbf{C}$	constrain matrix
$\text{diag}\{\mathbf{x}\}$	diagonal matrix whose i th diagonal element is the i th element of vector $\mathbf{x}$
λ	eigenvalue
$E\{\cdot\}$	expectation operator
$g_k(t)$	impulse response of channel
$h_k(t)$	effective signature waveform of the k th user
Λ	eigenvalue matrix
$\mathbf{I}$	identity matrix
N	spreading gain
$\mathbf{R}$	correlation matrix
$r(t)$	received signal
σ^2	the noise power

$s_k(t)$	signature waveform
$\star$	linear convolution
τ_k	time delay
T	symbol interval
T_c	chip interval
$tr(\cdot)$	trace operator
$v(t)$	noise
$\mathbf{y}$	output of matched filter bank

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Chapter 1

Introduction

1.1 Motivation

The goal for the third generation (3G) mobile wireless communications systems is to seamlessly integrate a wide variety of communication services such as high speed data, video and multimedia traffic as well as voice signals. The demand for increased capacity and improved performance is a motivating factor in the study of receiver structures for mobile wireless communications applications. Because of its attractive properties such as multipath rejection and inherently robust to jamming and interference [1]-[5], direct sequence spread spectrum (DS-SS), an efficient multi-access technique is experiencing widespread adoption in wireless communications. In direct sequence code division multiple access (DS-CDMA) channels, the messages of several users are multiplexed by distinct spreading codes and transmitted concurrently in the same radio frequency band.

In conventional CDMA receivers, multiple access interference (MAI) combined with all other interference and noise is often modelled as additive white Gaussian noise (AWGN), especially for large number of users. Therefore, the optimal receiver corresponds to a matched filter receiver from the fundamental result in communication theory [7]. Therefore, as long as the interference is Gaussian the

optimal receiver takes the form of a correlation receiver or a filter matched to the signature sequence of the transmitter. However, MAI is not quite Gaussian. The conventional approach is only optimal for the case of single user transmissions and orthogonal synchronous multiuser communications in AWGN channels. The existence of MAI limits the capacity and performance of the conventional DS-CDMA receiver communications systems. Potentially significant capacity increase and near-far resistance can be achieved if the negative effect of MAI can be reduced.

Multiuser detection (MUD) is a technique where the information of multiple users is used jointly to reduce the MAI and detect the desired user, and it has gained significant recognition in recent years as an advanced enabling technology for CDMA wireless communications systems. An optimal multiuser receiver for Gaussian channels derived in [6] significantly outperforms the conventional receiver and solves the near-far problem. The computational complexity of the optimal multiuser receiver drastically limits its potential application. Consequently, the cost of multiuser detection must be as low as possible so that there is a performance cost tradeoff advantage [41]. Suboptimal algorithms aimed at reducing complexity and maintaining good performance have been proposed in [18], [19], [41]. Decorrelating detector and MMSE detector are two important suboptimal linear multiuser detectors for CDMA communications that have been well studied in the literature [19]-[23]. The problem with such detectors is the need to compute the inverse of the correlation matrix, which can be very complex ($O(K^3)$) when the number of users is large and the signature sequences are changing from symbol interval to symbol interval. Multistage weighted polynomial expansion detectors have been proposed in [41], [42] to approximate the decorrelating detector and MMSE detector. The problem of WPE detector reduces to choose the weight vector. Reference [39] derives the optimal weight vector for the WPE detector to maximize the total large system SINR over all users for fixed stage L and the asymptotic weight vector is obtained based on the results of the use of random

matrices for equal received power case. However, the use of an asymptotic weight vector, instead of the exact optimal weight vector, still causes some noticeable performance loss. Thus, how to find a simplified method to obtain the weight vector is one of research work in the thesis.

Much of the research work in MUD has been focused on blind adaptive multiuser detection schemes [53], [55], [56] in the past two decades. These blind adaptive schemes are especially attractive for downlink CDMA systems because it is very difficult for mobile stations to obtain accurate knowledge of all active users in the channel. The blind minimum output energy (MOE) detector can be studied as a linear constraint optimization problem [53]. A partitioned linear interference cancellation (PLIC) structure can now be applied to solve the constrained optimization problem [53]. Reduced rank methods for the realization of the adaptive module are commonly used to reduce computational complexity. The study of reduced rank methods will be one of research work in the thesis.

1.2 Objectives

The purpose of this thesis is to study low complexity multiuser detection algorithms that can be implemented in advanced DS-CDMA receivers. Specifically, three research objectives have been identified and they are, (1) to find methods to obtain weight vector for WPE detector to approximate decorrelating detector and MMSE detector, (2) to find reduce rank methods to reduce computation complexity for the realization of blind MOE detector, (3) to find receivers to overcome the signature mismatch problem in multi-path fading channel environment.

1.3 Major Contributions of Thesis

The main contributions of this thesis are summarized below.

In Chapter 3, a linear weighted polynomial expansion (WPE) multistage detector that approximates the decorrelating detector and MMSE detector has been discussed. The optimal weight vector has been derived based on the minimum mean squared error (MSE) criteria. An adaptive WPE detector based on least squares (LS) criteria and the recursive least squares (RLS) algorithm have been proposed in this chapter. Simulation results show that the proposed WPE multiuser detector can achieve the performance of the decorrelating detector and MMSE detector with considerable complexity reduction.

In Chapter 4, we propose a cascaded modular structure detector for the implementation of the blind MOE detector. In each module of the detector, a vector filter is introduced for interference cancellation and the weight vector is determined based on maximum magnitude cross correlation criteria. The cascaded modular structure detector is relatively simple and has a low complexity compared with that of the blind MOE detector because the total number of adaptive degrees of freedom is distributed throughout the individual modules.

In Chapter 5, a RAKE structure combined with the blind cascaded modular structure detector proposed in Chapter 4 has been proposed for interference cancellation in multipath fading channels. The proposed receiver only needs the knowledge of the desired user's spreading code. It does not need to explicitly estimate the channel characteristics. The cascaded modular structure detector is used in each arm of the RAKE structure to extract part of the desired signal. The output of all arms is then coherently combined to enhance the desired signal for the final detection.

1.4 Organization of Thesis

The remaining parts of this thesis are organized as follows. Chapter 2 provides the background materials relevant to the thesis. Literature survey on multiuser detec-

tion algorithms is also included. Chapter 3 studies the WPE detector. Chapter 4 proposes the blind cascaded modular structure interference cancellation detector. Chapter 5 proposes a blind receiver that combines the cascaded modular structure detector with the RAKE structure for interference cancellation in multipath fading channels. Finally, Chapter 6 provides some concluding remarks and future work.

Chapter 2

Literature Study

2.1 CDMA Systems

Code division multiple access which uses spread spectrum techniques to achieve efficient multiple access communications is commonly used for wireless communications systems. There are many kinds of CDMA systems [9] and this thesis concentrates on DS-CDMA systems. In DS-CDMA systems, each user is assigned a fixed and distinct signature sequence. The signature sequences are used to generate information-bearing spread spectrum signal. The resulting signal has wide bandwidth, low power spectral density and noise-like properties [3].

Fig. 2.1 shows a block diagram of a typical K user DS-CDMA system. In the transmitter, the information data $b_k(i)$ of user k is modulated by its signature waveform, $s_k(t)$, to generate the transmitted signal. $s_k(t)$ has finite interval $[0, T]$. and T denotes the information symbol interval. τ_k denotes the delay of the k th user where $0 \leq \tau_k < T$. The signature waveform $s_k(t)$ in DS-CDMA systems is given by

$$s_k(t) = \sum_{l=0}^{N-1} c_k(l) \varphi(t - lT_c), \quad 0 \leq t \leq T \quad (2.1)$$

where T_c denotes the chip interval and $N = T/T_c$ denotes the processing gain.

Figure 2.1: The basic model for DS-CDMA system

$c_k(l) \in \{-1/\sqrt{N}, +1/\sqrt{N}\}$ is the normalized signature sequence for the k th user, $\varphi(t)$ is the chip waveform defined on $[0, T_c]$. Then the signal transmitted by the k th user is given by

$$x_k(t) = A_k \sum_{i=-\infty}^{\infty} b_k(i) s_k(t - iT - \tau_k) \quad (2.2)$$

where A_k is the amplitude of the k th user.

$g_k(t)$ is the impulse response of dispersive channel of the k th user and $v(t)$ is the additive noise. Then the received signal (without noise) due to the k th user is given by

$$y_k(t) = x_k(t) * g_k(t) = A_k \sum_{i=-\infty}^{\infty} b_k(i) \sum_{l=0}^{N-1} c_k(l) \underline{g}_k(t - iT - lT_c) \quad (2.3)$$

where $*$ denotes the convolution operation and $\underline{g}_k(t)$ is the convolution of chip waveform $\varphi(t)$ and the channel impulse response $g_k(t)$.

$$\underline{g}_k(t) = \varphi(t - \tau_k) * g_k(t) \quad (2.4)$$

In the receiver, the received signal, $r(t)$, can be expressed as

$$r(t) = \sum_{k=1}^K y_k(t) + v(t) \quad (2.5)$$

DS-CDMA receivers employ the signature sequence of the desired user as the key to recover the transmitted information. Fig. 2.1 shows that the received signal $r(t)$ is first filtered by a matched filter bank. The matched filter corresponds to a correlation demodulator driven by a synchronized replica of the signature sequence. It has been proved that the matched filter bank sacrifices no information relevant to demodulation [19]. The output of the matched filter bank, $y_k(i)$, given by (2.6), is then sent to the multiuser detector to obtain the demodulated sequence. Finally, the transmitted information signal $b_k(i)$ are detected.

$$y_k(i) = A_k b_k(i) + \sum_{j=1, j \neq k}^K A_j b_j(i) \rho_{jk} + \hat{v}(i) \quad (2.6)$$

where

$$\rho_{jk} = \langle \mathbf{s}_j, \mathbf{s}_k \rangle \quad (2.7)$$

The output of the matched filter bank for all users in matrix notation is given by

$$\mathbf{y} = \mathbf{R}\mathbf{A}\mathbf{b} + \mathbf{v} \quad (2.8)$$

where $\mathbf{R}$ is the cross correlation matrix of the signature waveforms, $\mathbf{A}$ is the diagonal matrix of signal amplitudes, $\mathbf{b}$ is the vector of user bits and $\mathbf{v}$ is the vector of noise samples. $\mathbf{s}_j$ is the spreading code of user j .

CDMA systems are interference limited systems and have a soft capacity limit. As the number of users in the same channel grows, the amount of interference experienced at the receiver increases, eventually degrading the received signal's quality

to unacceptable levels. In addition to the performance degradation experienced when numerous users share the channel, the conventional CDMA receivers suffer from near-far effect problem. If interfering signals are received with much larger power than the signal desired, they have the potential of overwhelming the desired user.

Because the conventional CDMA receivers treat MAI as additive noise, the overall capacity of conventional CDMA systems is limited. In the past two decades, multiuser detection techniques have gained significant recognition because of their ability to combat MAI at the receiver. Therefore, they have the potential to increase the capacity of CDMA systems.

In single-cell case, ideal multiuser detection can remove all interference other than background noise and create an equivalent of a single user channel. In multi-cell case, if multiuser detection only deals with interference from users in the same cell while neglecting the out-of-cell interference, an upper limit to the capacity improvement achievable with multiuser detection can be computed assuming that the multiuser detection cancels all the interference within the cell. In this case, the amount of effective interference reduces from $I_{ic} + I_{oc}$ to I_{oc} , which results in a capacity increase of $(1 + \eta)/\eta$. where, I_{ic} is the interference within the cell, I_{oc} is the interference out-of-cell, and $\eta = I_{ic}/I_{oc}$ [10].

2.2 Multiuser Detection

Multiuser detection is the jointly demodulation and interference cancellation technique to improve detection of desired signal in the presence of multiple access interference, intersymbol interference and noise. In DS-CDMA systems, multiuser detection is proposed to mitigate the influence of MAI and increase the capacity of CDMA systems by exploiting the inherent structure of the multiple access interference.

CDMA interference cancellation was firstly proposed in [11] and [12]. The two papers describe a number of ideas presented in much of research work in the past two decades. Estimation based on mean squared error and maximum likelihood was discussed in [11]. How cancellation can be implemented by solving simultaneous equations was shown in [12].

The fundamental theory of multiuser detection was presented in Verdu's work [6]. The paper proposed and analyzed the structure and complexity of optimal multiuser detector, maximum-likelihood sequence detector, for asynchronous CDMA channels in AWGN. Although the optimal detector has huge performance and capacity gain over the conventional detection, it is too complex for practical DS-SS-CDMA systems. Therefore, over the past two decades or so, most of the research has been focused on finding suboptimal multiuser detectors and finding an appropriate tradeoff between complexity and performance.

Most of the suboptimal multiuser detectors fall into two categories: linear multiuser detectors and subtractive interference cancellation detectors. In linear multiuser detection, a linear mapping is applied to the soft output of the conventional detector to reduce the MAI. Decorrelating detector [20], [21] and MMSE detector [22] are two most important linear multiuser detectors. The decorrelating detector applies the inverse of the correlation matrix to the conventional detector output in order to decouple the data. The MMSE detector implements the linear mapping which minimizes the mean squared error between the actual data and the soft output of the conventional detector. Both the decorrelating detector and MMSE detector need to compute the inverse of matrix for implementation. To avoid the matrix inverse computation, the polynomial expansion (PE) detector [42] applies a polynomial expansion in the correlation matrix $\mathbf{R}$ to the matched filter bank output.

The subtractive interference cancellation detectors attempt to estimate and subtract out some or all of the MAI in the received signal. These detectors include

the successive interference cancellation (SIC) [25], parallel interference cancellation (PIC) [26], [27], and zero-forcing decision-feedback (ZF-DF) detectors [17]. The bit decisions used to estimate the MAI in the subtractive interference cancellation detectors may be either hard decisions or soft decisions. Soft decision approach provides a joint estimate of data and amplitude and is easier to implement. The hard decision approach feeds back a bit decision and is nonlinear. If reliable channel estimates are available, however, hard decision schemes perform better than their soft decision counterparts.

The adaptive multiuser detectors have the potential of providing the anticipated multiuser detection performance gains with a complexity that would be manageable for wireless communications systems. The adaptive multiuser detectors include data-aided MMSE detector and blind MOE detector. Since the channel is time variant in practice, training sequence has to be sent periodically to update the detector coefficients to track the variation of the channel. This will reduce the bandwidth efficiency of the system. Therefore blind multiuser detection techniques become very attractive. It has been shown in [55] that minimum output energy and minimum mean squared error are directly related, and minimizing one is equivalent to minimizing the other. The linear blind multiuser detectors have been studied within a constrained optimization framework in [53] and the partitioned linear interference canceller (PLIC) structure can be implemented for the constrained optimization detector.

In the following sections, we will simply describe the operation of several of the most noted multiuser detectors for DS-CDMA systems.

2.3 Optimal Multiuser Detector

The optimal multiuser detector requires knowledge of the spreading sequence, the timing of users, and the received amplitudes of users. The optimal multiuser de-

tection includes two optimum criterions, individually optimum decision and jointly optimum decision. The former criterion results in the minimum bit-error-rate. In this case the solution is based on a forward-backward dynamic programming algorithm [19]. Jointly optimum decisions are obtained by a maximum likelihood sequence (MLS) detector which selects the most likely sequence of transmitted bits given the observations to maximize the joint *a posteriori probability*, $P(\mathbf{b}|r(t))$ [7]. Because the complexity of the maximum-likelihood solution is lower than individual optimum decision, and, unless the signal-to-noise ratio is very low, the probability of error achieved by the maximum-likelihood solution is extremely close to the minimum, we often consider the maximum-likelihood sequence detector as the optimal detector that achieves minimum bit error rate (BER).

For the synchronous channel, consider the optimum joint demodulation of the transmitted symbol vector $\mathbf{b} = [b_1, b_2, \dots, b_K]^T$ as a 2^K hypotheses decision process for different value of $\mathbf{b}$,

$$H : r(t) = \sum_{k=1}^{2^K} A_k b_k s_k + \sigma v(t) \quad (2.9)$$

where σ^2 is the power spectral density of noise $v(t)$. The optimum joint demodulation of $\mathbf{b}$ is given by

$$[b_1, \dots, b_K] = \max_{b_1, \dots, b_K} \exp\left(-\frac{1}{2\sigma^2} \int_0^T [y(t) - \sum_{k=1}^K A_k b_k s_k(t)]^2 dt\right) \quad (2.10)$$

The maximization of (2.10) is a combinatorial optimization problem, which can always be solved by exhaustive search. This means that the selection of the optimum $\mathbf{b}$ requires $O(2^K)$ operations. This computation complexity is clearly impractical for large number users systems. For certain special cases of the correlation matrix $\mathbf{R}$, it has been shown that MLS detection can be obtained by polynomial time algorithms [13], [14]. An approximation to the MLS problem has been sug-

gested in [15] based on iterative transformations of the quadratic minimization problem.

For asynchronous CDMA channel, we should consider the whole message for the optimum detection. Hence, there are $O(2^{LK})$ possible vectors for the maximum-likelihood sequence detector (L is the length of transmitted bits per user). By using Viterbi algorithm to solve the MLS detection [6], the required complexity is still exponential in the number of users, that is, on the order of $O(2^K)$. Sequential decoding method is an alternative method for the MLS detection for asynchronous channel [16]. Sequential decoding algorithm has lower complexity than Viterbi algorithm.

2.4 Suboptimal Multiuser Detector

2.4.1 Decorrelating Detector

The decorrelating detector applies the inverse of the channel matrix $\mathbf{R}^{-1}$ to the matched filter bank output $\mathbf{y}$ in an attempt to cancel the MAI resulting from cross-correlations among the signature sequences. The bit decisions of this detector are given by

$$\hat{\mathbf{b}} = \text{sgn}[\mathbf{A}\mathbf{b} + \mathbf{R}^{-1}\mathbf{v}] \quad (2.11)$$

We can see that the decorrelating detector completely eliminates the MAI. The detector does not require knowledge of the users' received energies and each user can be demodulated completely independently. From (2.11), we also can see that the noise terms are no longer independent and the correlation between noise creates noise enhancement effect that can degrade receiver's performance.

2.4.2 MMSE Detector

MMSE detector takes into account the background noise and utilizes knowledge of the received signal energies. It implements the linear mapping which minimizes the mean squared error between the actual data and the soft output of the conventional detector $E\{|\mathbf{b} - L_{mmse}\mathbf{y}|^2\}$. The solution to this minimization problem is given by

$$L_{mmse} = (\mathbf{R} + (N_0/2)\mathbf{A}^{-2})^{-1} \quad (2.12)$$

The bit decisions of the MMSE detector are given by

$$\hat{\mathbf{b}} = \text{sgn}[L_{mmse}\mathbf{y}] \quad (2.13)$$

The operation of the MMSE detector can be viewed as offering a balance between interference cancellation and noise enhancement. Thus, MMSE detector generally provides better probability of error performance than decorrelating detector. Because MMSE detector depends on the energies of the interfering users, there is some loss of resistance to the near-far problem.

2.4.3 SIC Detector

SIC detector takes a serial approach to cancel interference. In this technique, estimation and interference cancellation are performed on user at a time in order of decreasing received energies. Interference cancellation is achieved by subtracting from the received signal an estimate of the multiple access interference generated by users with higher received energies than that of currently being detected user k .

For synchronous systems, the received signal of k th user after users 1 through $k - 1$ have been estimated and cancelled can be given by

$$r_k(t) = r(t) - \sum_{i=1}^{k-1} A_i \hat{b}(i) s_i(t) \quad (2.14)$$

The bit decision of user k is given by

$$\hat{b}_k = \text{sgn}[y_k - \sum_{i=1}^{k-1} A_i \rho_{ik} \hat{b}_i] \quad (2.15)$$

2.4.4 PIC Detector

PIC detector is an iterative interference cancellation method in which it estimates and subtracts all of the MAI for each user in parallel at each stage. Improved estimates are obtained for next stage from each user's enhanced signal which exhibits reduced interference.

For synchronous systems, the received signal of k th user after s stages can be given by

$$r_k^s(t) = r(t) - \sum_{i=1, i \neq k}^K A_i \hat{b}^{s-1}(i) s_i(t) \quad (2.16)$$

The bit decision of user k after s stage is given by

$$\hat{b}_k^s = \text{sgn}[y_k - \sum_{i=1, i \neq k}^K A_i \rho_{ik} \hat{b}_i^{s-1}] \quad (2.17)$$

The improved PIC detector proposed in [28] does a partial MAI cancellation at each stage, with the amount of cancellation increasing for each successive stage. Thus, the MAI estimate is first scaled by a fraction before cancellation. The value of the fraction increases for successive stages. This takes into account the fact that the tentative decisions of the earlier stages are less reliable than those of the later stages.

2.4.5 ZF-DF Detector

The detector performs two operations. The linear operation partially decorrelates the users and SIC operation decides and subtracts out the interference from one additional user at a time in descending order of signal strength.

The linear operation of ZF-DF detector is also called whitening forward filter. The forward filter is obtained by first factoring the cross-correlation matrix $\mathbf{R}$ using Cholesky decomposition $\mathbf{R} = \mathbf{F}^T \mathbf{F}$ ($\mathbf{F}$ is lower triangular matrix). The optimal forward filter is the noise whitening filter $(\mathbf{F}^T)^{-1}$, which eliminates multi-user interference due to future inputs. The output of the forward filter is given by

$$\mathbf{y}_w = \mathbf{F} \mathbf{A} \mathbf{b} + \mathbf{v}_w \quad (2.18)$$

The feedback filter, $\mathbf{B} = \mathbf{F} - \text{diag}(\mathbf{F})$, uses previously bit decisions to eliminate MAI provided that feedback data is correct.

The bit decision of k th user after feedback filter is

$$\hat{b}_k = \text{sgn}[y_{w,k} - \sum_{i=1}^{k-1} \mathbf{F}_{k,i} A_i \hat{b}_i] \quad (2.19)$$

2.5 Blind Multiuser Detector

Blind adaptive multiuser detection was first introduced in [55]. The blind approach is based on decomposing the linear multiuser detectors into two orthogonal components. One of the components is chosen as the signature sequence of the desired user. Consider the linear detector for desired user 1 which is characterized by the filter response $\mathbf{w}_1$

$$\hat{b}_1 = \text{sgn}[\langle \mathbf{y}, \mathbf{w}_1 \rangle] \quad (2.20)$$

the canonical representation of $\mathbf{w}_1$ is

$$\mathbf{w}_1 = \mathbf{s}_1 + \mathbf{x}_1 \quad \text{subject to} \quad \langle \mathbf{s}_1, \mathbf{x}_1 \rangle = 0 \quad (2.21)$$

where, $\mathbf{s}_1$ is the signature waveform of user 1 and is assumed to be known.

The variance or output energy at the output of the linear detector is given by

$$E\{(\langle \mathbf{y}, \mathbf{s}_1 + \mathbf{x}_1 \rangle)^2\} = A_1^2 + \sum_{k=2}^K A_k^2 (\rho_{1k} + \langle \mathbf{s}_k, \mathbf{x}_1 \rangle)^2 + \sigma^2(1 + \|\mathbf{x}_1\|^2) \quad (2.22)$$

We have used the fact that the bits of user 1 are uncorrelated with the bits of the other users and the noise.

We can see from (2.22), the output energy has three additive independent components: the first term is the signal energy of the desired user 1, the second term represents the energy of the multiple access interference, and the third term represents the energy of the background noise. The first component is transparent to the choice of $\mathbf{x}_1$. Thus, by varying $\mathbf{x}_1$, we can only change the energy of the second and third components. We can intuitively see that choosing $\mathbf{x}_1$ to minimize the output energy would be a sensible idea since this would just minimize the interference energy. Accordingly, a very simple and sensible detection strategy is to choose $\mathbf{x}_1$ to minimize the output energy.

$$MOE(\mathbf{x}_1) = E\{(\langle \mathbf{y}, \mathbf{s}_1 + \mathbf{x}_1 \rangle)^2\} \quad (2.23)$$

The steepest descent algorithm is used to derive the update rule of the adaptive algorithm. The update rule for the blind adaptive MUD is given by

$$\mathbf{x}_1(i) = \mathbf{x}_1(i-1) - \mu \mathbf{Z}(i)(\mathbf{y}(i) - \mathbf{Z}_{MF}(i)\mathbf{s}_1) \quad (2.24)$$

where,

$$\mathbf{Z}(i) = \langle \mathbf{y}(i), \mathbf{s}_1 + \mathbf{x}_1(i-1) \rangle \quad (2.25)$$

$$\mathbf{Z}_{MF}(i) = \langle \mathbf{y}(i), \mathbf{s}_1 \rangle \quad (2.26)$$

Finite precision implementation of the above update rule can have effects that are not immediately visible but that have a cumulative effect. It may happen that finite precision round off errors drive the updates of $\mathbf{x}_1$ to be outside the orthogonal space. This problem can be solved by replacing the update $\mathbf{x}_1$ by its orthogonal projection:

$$\mathbf{x}_1(i) - \langle \mathbf{x}_1(i), \mathbf{s}_1 \rangle \mathbf{s}_1 \quad (2.27)$$

In Chapter 4 and 5, we will give a detailed introduction of several proposed blind MUD algorithms [65]-[78] and compare the performance of these algorithms with the algorithms proposed in the thesis.

Chapter 3

WPE Multiuser Detector

3.1 Background

Decorrelating detector and MMSE detector are two important suboptimal linear multiuser detectors for CDMA communications that have been well studied in the literature [19]-[23] due to their vastly reduced complexity compared with the optimal maximum-likelihood receiver. One problem with such detectors is the need to compute the inverse of the correlation matrix, which can be very complex ($O(K^3)$) when the number of users is large and if the signature sequences are changing from symbol interval to symbol interval. As such, linear multistage multiuser receivers are often proposed to approximate the decorrelating detector and MMSE detector due to their simple structure and ability to achieve close to linear MMSE performance with less computational complexity.

Linear multistage interference cancellation detectors such as SIC detector [24], [25], PIC detector [26] and the improved weighted PIC detector [28]-[34] are presented to approximate the decorrelating detector and MMSE detector. The performance of the multistage linear PIC detector is analyzed in [35]. Iterative methods to solve linear equation system are introduced to discuss the convergence of the multistage interference cancellation detector in [36], [37].

In this thesis, we discuss a linear multistage detector that approximates the decorrelating detector and MMSE detector with the use of a polynomial expansion series. The first order Taylor approximation for decorrelating detector has been presented and analyzed in [19], [40]. Although the approximate detector reported in these papers can produce better detection performance than that of the matched filter, it has considerable performance loss compared to the decorrelating detector. As shown in [36], the multistage Taylor series expansion detector is equivalent to the Jacobi iteration methods for the linear equation system. The multistage detector only converges for system loads of less than about 17% in large systems. In order to improve the detection performance, multistage weighted polynomial expansion detectors have been proposed in [41], [42] to approximate the decorrelating detector and MMSE detector. It is shown in [42] that the WPE detector can exactly implement the decorrelating detector and MMSE detector. The problem of WPE detector reduces to choose the weight vector such that it effectively approximates the decorrelating detector and MMSE detector. Reference [39] derives the optimal weight vector for the WPE detector to maximize the total large system SINR over all users for fixed stage L and the asymptotic weight vector is obtained based on the results of the use of random matrices for equal received power case. However, the use of the asymptotic weight vector, instead of the exact weight vector, still causes some noticeable performance loss. In this thesis, we derive the optimal weight vector based on the minimum MSE criteria. An adaptive WPE detector based on the least squares criteria and the RLS algorithm have been proposed in this chapter to adjust the weight vector iteratively until it becomes the exact optimal weight vector.

In the remaining parts of this chapter, Section 3.2 describes the system model, Section 3.3 gives an introduction of the WPE detector, Section 3.4 discusses the convergence of the WPE detector to the decorrelating detector, Section 3.5 derives the optimal weight vector based on MSE criteria, Section 3.6 gives a simple in-

introduction of asymptotic WPE detector for comparison, Section 3.7 proposes the adaptive WPE detector and gives the RLS algorithm, Section 3.8 discusses the complexity of WPE detector, and Section 3.9 provides some simulation results on the performance of the proposed WPE detector. A summary of the findings is given in the conclusions in Section 3.10.

3.2 Signal Model

In this chapter, for brevity and simplicity, we consider a symbol synchronous DS-SS-CDMA system with K users and a processing gain of N . We assume that users employ BPSK modulation and consider a real baseband model. The results can be extended to asynchronous system readily. The spreading sequence used for each user is chosen randomly according to the rule that the elements of the $N \times K$ spreading sequence matrix $\mathbf{S}$ are independent and identically distributed (i.i.d.) random variables taking values of $\pm 1/\sqrt{N}$ with equal probability. Such a system can be described in matrix-vector form by

$$\mathbf{r}(n) = \mathbf{S}\mathbf{A}\mathbf{b}(n) + \mathbf{v}(n) \quad (3.1)$$

The sample index n is omitted in the following discussion for simplified derivation and clarification. Equation (3.1) will be described by

$$\mathbf{r} = \mathbf{S}\mathbf{A}\mathbf{b} + \mathbf{v} \quad (3.2)$$

where $\mathbf{r}$ is the N -dimensional chip matched filter vector for each symbol interval, $\mathbf{A} = \text{diag}(A_1, \dots, A_K)$ is the diagonal matrix of the users' amplitudes, $\mathbf{b}$ is the K -dimensional transmitted data symbol vector of the K users with each element taking values of ± 1 and satisfies $E\{\mathbf{b}\mathbf{b}^T\} = \mathbf{I}$, $\mathbf{v}$ is AWGN with zero mean and covariance $E\{\mathbf{v}\mathbf{v}^T\} = \sigma^2\mathbf{I}$. We assume that the spreading sequences are known at

the receiver and that we have perfect estimation of the received user powers and the noise variance. Then the output vector $\mathbf{y}$ of a bank of matched filter is given by

$$\mathbf{y} = \mathbf{S}^T \mathbf{r} = \mathbf{R} \mathbf{A} \mathbf{b} + \mathbf{S}^T \mathbf{v} \quad (3.3)$$

where $\mathbf{R} = \mathbf{S}^T \mathbf{S}$ is a $K \times K$ correlation matrix, whose entries contain the values of the correlations between the various pairs of the spreading codes.

For the CDMA system defined above, the output vector of decorrelating detector and MMSE detector is given by

$$\mathbf{b}_{dec} = \mathbf{A}^{-1} \mathbf{R}^{-1} \mathbf{y} \quad (3.4)$$

$$\begin{aligned} \mathbf{b}_{mmse} &= (\mathbf{A} \mathbf{R} \mathbf{A} + \sigma^2 \mathbf{I})^{-1} \mathbf{A} \mathbf{y} \\ &= \mathbf{A}^{-1} (\mathbf{R} + \sigma^2 \mathbf{A}^{-2})^{-1} \mathbf{y} \end{aligned} \quad (3.5)$$

respectively.

In asynchronous system, we assume that the delays of all users can be obtained. Instead of one transmitted symbol of each user in synchronous system, a block M transmitted sequences of each user would be considered to process together. The dimension of the correlation matrix $\mathbf{R}_a$ for the block transmitted sequences will be $MK \times MK$ [19].

$$\mathbf{R}_a = \begin{pmatrix} \mathbf{R}(0) & \mathbf{R}(-1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{R}(1) & \mathbf{R}(0) & \mathbf{R}(-1) & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}(1) & \mathbf{R}(0) & \ddots & \mathbf{0} \\ \vdots & \cdots & \ddots & \ddots & \mathbf{R}(-1) \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{R}(1) & \mathbf{R}(0) \end{pmatrix} \quad (3.6)$$

where $\mathbf{R}(1)$ is an upper triangular matrix with zero diagonal, $\mathbf{R}(-1) = \mathbf{R}^T(1)$.

The entries of the matrices $\mathbf{R}(i)$, $i = -1, 0, 1$ are obtained by cross-correlating appropriately delayed replicas of the normalized signature waveforms according to equation (3.7)

$$\mathbf{R}_{kj}(\ell) = \int_{-\infty}^{+\infty} s_k(t - \tau_k) s_j(t + \ell T - \tau_j) dt \quad (3.7)$$

We can obtain the output vector of decorrelating detector and MMSE detector in asynchronous system by

$$\mathbf{b}_{dec}^a = \mathbf{A}_a^{-1} \mathbf{R}_a^{-1} \mathbf{y}_a \quad (3.8)$$

$$\begin{aligned} \mathbf{b}_{mmse}^a &= (\mathbf{A}_a \mathbf{R}_a \mathbf{A}_a + \sigma^2 \mathbf{I})^{-1} \mathbf{A}_a \mathbf{y}_a \\ &= \mathbf{A}_a^{-1} (\mathbf{R}_a + \sigma^2 \mathbf{A}_a^{-2})^{-1} \mathbf{y}_a \end{aligned} \quad (3.9)$$

respectively.

3.3 WPE Detector

In this section, we will give an introduction of the weighted polynomial expansion method to approximate the decorrelating detector and MMSE detector based on Taylor series approximation.

Based on Taylor series approximation $(1+x)^{-1} = \sum_{\ell=0}^{\infty} (-x)^\ell, |x| < 1$, the decorrelating detector (3.4) can be approximated by L th order polynomial expansion detector

$$\mathbf{b}_{dec,L} = \mathbf{A}^{-1} \sum_{\ell=0}^L (-(\mathbf{R} - \mathbf{I}))^\ell \mathbf{y} \quad (3.10)$$

The approximation converges to the exact solution of decorrelating detector

$$\lim_{L \rightarrow \infty} \mathbf{b}_{dec,L} = \mathbf{b}_{dec} \quad (3.11)$$

if

$$|\rho(\mathbf{R} - \mathbf{I})| < 1 \quad (3.12)$$

where $\rho(\mathbf{R})$ is the maximal eigenvalue λ_{max} of the correlation matrix $\mathbf{R}$.

The MMSE detector (3.5) can be approximated by L th order polynomial expansion detector

$$\mathbf{b}_{mmse,L} = \mathbf{A}^{-1} \sum_{\ell=0}^L (-(\mathbf{R} + \sigma^2 \mathbf{A}^{-2} - \mathbf{I}))^\ell \mathbf{y} \quad (3.13)$$

The approximation converges to the exact solution of MMSE detector

$$\lim_{L \rightarrow \infty} \mathbf{b}_{mmse,L} = \mathbf{b}_{mmse} \quad (3.14)$$

if

$$|\rho(\mathbf{R} + \sigma^2 \mathbf{A}^{-2} - \mathbf{I})| < 1 \quad (3.15)$$

For the finite L th order polynomial expansion approximation, the weighted polynomial expansion detector should has better approximation to the decorrelating detector and MMSE detector than the L th Taylor series approximation.

The L th order WPE detector for decorrelating detector is given by

$$L_{dec,PE} = \sum_{\ell=0}^L w_\ell (\mathbf{R})^\ell \quad (3.16)$$

where w_ℓ is the weight coefficient of $\mathbf{R}^\ell$.

The L th order WPE detector for MMSE detector should consider two situations: equal received power case and unequal received power case. In the equal received power case, all received users with the same power $A_1^2 = A_2^2 = \dots = A_K^2 = A_p^2$, the L th order WPE detector for MMSE detector is the same as that for decorrelating detector. The term $(\sigma^2 A_p^{-2} - 1)\mathbf{I}$ in (3.13) is absorbed in the weight coefficients.

In unequal received power case, $(\sigma^2 \mathbf{A}^{-2} - \mathbf{I})$ is a diagonal matrix. Thus, the weight coefficient of the L th order WPE detector for each stage will be a polynomial expression of $(\sigma^2 \mathbf{A}^{-2} - \mathbf{I})$ which is a diagonal matrix and can be substituted by a diagonal weight matrix $\mathbf{W}_\ell$.

$$L_{mmse,PE} = \sum_{\ell=0}^L \mathbf{W}_\ell(\mathbf{R})^\ell \quad (3.17)$$

In high SNR situation, $\sigma^2 \mathbf{A}^{-2} \rightarrow \mathbf{0}$, the WPE detector for MMSE detector in the unequal received power case comes to the same as (3.16). Then, in the following, we will focus on the discussion of the WPE detector (3.16) and for ease of notation, we define the WPE detector as L_{PE} .

The output vector of the L stage WPE detector can be written as

$$\begin{aligned} \mathbf{b}_{PE} &= \mathbf{A}^{-1} L_{PE} \mathbf{y} \\ &= \mathbf{A}^{-1} \sum_{\ell=0}^L w_\ell(\mathbf{R})^\ell \mathbf{y} \end{aligned} \quad (3.18)$$

where $\mathbf{b}_{PE}$ is the output vector of the WPE detector.

In asynchronous system, the WPE detector can be obtained based on equation (3.6) to equation (3.9) by the same method discussed above.

The structure of two stage WPE detector is shown in Fig. 3.1.

3.4 Convergence Analysis

In this section, theoretical analysis will be provided to prove the convergence of the WPE detector to the decorrelating detector based on the Cayley-Hamilton theorem.

Consider a square matrix $\mathbf{B}$ with dimension n and with a characteristic poly-

Figure 3.1: The structure of two stage WPE detector

nomial

$$\Delta(s) = |s\mathbf{I} - \mathbf{B}| = s^n + c_{n-1}s^{n-1} + \dots + c_0$$

and define a corresponding matrix polynomial, formed by substituting $\mathbf{B}$ for s above, we can get

$$\Delta(\mathbf{B}) = \mathbf{B}^n + c_{n-1}\mathbf{B}^{n-1} + \dots + c_0\mathbf{I}$$

The Cayley-Hamilton theorem states that every matrix satisfies its own characteristic equation, that is

$$\Delta(\mathbf{B}) = \mathbf{0} \quad (3.19)$$

Note that the normal characteristic equation $\Delta(s) = 0$ is satisfied only at the eigenvalues $(\lambda_1, \lambda_2, \dots, \lambda_n)$.

The Taylor series expansion for the decorrelating detector is given by

$$L_{dec} = \mathbf{R}^{-1} = \sum_{\ell=0}^{\infty} \beta_{\ell}(\mathbf{R})^{\ell} \quad (3.20)$$

Implying the Cayley-Hamilton theorem (3.19) to the Taylor series expansion of the linear decorrelating detector, we can get

$$L_{dec} = Q(\mathbf{R})\Delta(\mathbf{R}) + P(\mathbf{R}) \quad (3.21)$$

$$\Delta(\mathbf{R}) = \mathbf{R}^K + c_{n-1}\mathbf{R}^{K-1} + \cdots + c_0\mathbf{I} \quad (3.22)$$

$Q(\mathbf{R})$ is found by long division of $\Delta(\mathbf{R})$, and the remainder polynomial $P(\mathbf{R})$ is of degree $K - 1$ or less. Because $\Delta(\mathbf{R}) = \mathbf{0}$, we can obtain

$$L_{dec} = \mathbf{R}^{-1} = P(\mathbf{R}) = \sum_{\ell=0}^{K-1} w_{\ell}(\mathbf{R})^{\ell} \quad (3.23)$$

This means that the WPE detector can exactly implement the decorrelating detector for $L \geq K - 1$. The weights depend on the eigenvalues, $(\lambda_1, \lambda_2, \dots, \lambda_K)$, of the matrix $\mathbf{R}$ and can be obtained by substituting the eigenvalues in equation (3.23) and solving the following linear equation group.

$$\begin{aligned} (\lambda_1)^{-1} &= \sum_{\ell=0}^{K-1} w_{\ell} \lambda_1^{\ell} \\ (\lambda_2)^{-1} &= \sum_{\ell=0}^{K-1} w_{\ell} \lambda_2^{\ell} \\ &\vdots \\ (\lambda_K)^{-1} &= \sum_{\ell=0}^{K-1} w_{\ell} \lambda_K^{\ell} \end{aligned}$$

The weight coefficients can be obtained for $L = K - 1$ order WPE detector by

$$\mathbf{w}_{L=K-1} = \left(\begin{pmatrix} 1 & \lambda_1 & \cdots & \lambda_1^{K-1} \\ 1 & \lambda_2 & \cdots & \lambda_2^{K-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \lambda_{K-1} & \cdots & \lambda_{K-1}^{K-1} \end{pmatrix} \right)^{-1} \begin{pmatrix} \lambda_1^{-1} \\ \lambda_2^{-1} \\ \vdots \\ \lambda_{K-1}^{-1} \end{pmatrix} \quad (3.24)$$

3.5 Optimal Weight Vector Based on MSE Criteria

In the above section, we discuss that the WPE detector can approximate the decorrelating detector and MMSE detector when the order of WPE $L \geq K - 1$. While the number of users K is large, the $K - 1$ order WPE detector may be also too complex for practical implementation. Reference [41], [42] have derived a vector of weight that minimizes the MSE between the output signals of the exact MMSE detector and its polynomial approximation for users with equal amplitudes. In this section, we will discuss how to obtain the optimal weight vector for the L th order WPE detector when $L \ll K$ based on MSE criteria without the assumption that all users have equal amplitudes.

From (3.18), for convenience, we consider that the mean squared error between the output of L th order WPE detector $\mathbf{A}\mathbf{b}_{PE}$ and the transmitted vector $\mathbf{A}\mathbf{b}$ is given by

$$\varepsilon = E\{(\mathbf{A}\mathbf{b} - \mathbf{A}\mathbf{b}_{PE})^T(\mathbf{A}\mathbf{b} - \mathbf{A}\mathbf{b}_{PE})\} \quad (3.25)$$

Substituting (3.18) into (3.25) and removing the terms that do not depend on the weight vector, we arrive at

$$\varepsilon = E\{\mathbf{y}^T L_{PE}^T L_{PE} \mathbf{y}\} - E\{(\mathbf{y})^T L_{PE}^T \mathbf{A}\mathbf{b}\} - E\{\mathbf{b}^T \mathbf{A} L_{PE} \mathbf{y}\} \quad (3.26)$$

Let $\mathbf{R} = \mathbf{T}\mathbf{\Lambda}\mathbf{T}^T$ be the eigenvalue decomposition of $\mathbf{R}$ ($\mathbf{R}$ is symmetric matrix)

and substitute (3.3) into (3.26). Since ε is a scalar, we can apply trace operator to the right hand side of (3.26). Utilizing the property $tr(\mathbf{AB}) = tr(\mathbf{BA})$, we obtain (The derivation is given in Appendix A)

$$\begin{aligned}\varepsilon &= tr\{(\mathbf{A}^2\Lambda^2 + \sigma^2\Lambda)(\sum_{\ell=0}^L w_\ell\Lambda^\ell)^2\} - 2tr\{\mathbf{A}^2\Lambda \sum_{\ell=0}^L w_\ell\Lambda^\ell\} \\ &= \mathbf{w}_L^T \mathbf{C}^T (\mathbf{A}^2\Lambda^2 + \sigma^2\Lambda) \mathbf{C} \mathbf{w}_L - 2\mathbf{w}_L^T \mathbf{C}^T \mathbf{A}^2 \lambda\end{aligned}\quad (3.27)$$

where,

$$\mathbf{w}_L = [w_0, w_1, \dots, w_L]^T$$

$$\lambda = [\lambda_1, \dots, \lambda_K]^T$$

$$\mathbf{C} = \begin{bmatrix} 1 & \lambda_1 & \lambda_1^2 & \cdots & \lambda_1^L \\ 1 & \lambda_2 & \lambda_2^2 & \cdots & \lambda_2^L \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \lambda_K & \lambda_K^2 & \cdots & \lambda_K^L \end{bmatrix}$$

The minimum MSE solution now involves taking the derivatives of ε with respect to the weight vector and setting it to zero (3.28).

$$\frac{\partial \varepsilon}{\partial \mathbf{w}_L} = \mathbf{C}^T (\mathbf{A}^2\Lambda^2 + \sigma^2\Lambda) \mathbf{C} \mathbf{w}_L - \mathbf{C}^T \mathbf{A}^2 \lambda = 0 \quad (3.28)$$

The resultant optimal weight vector is

$$\mathbf{w}_L = (\mathbf{C}^T (\mathbf{A}^2\Lambda^2 + \sigma^2\Lambda) \mathbf{C})^{-1} \mathbf{C}^T \mathbf{A}^2 \lambda \quad (3.29)$$

3.6 Asymptotic Weight Vector Based on Random Matrix Theory

In the above section, we derive the optimal weight vector $\mathbf{w}_L$ for the L th order WPE detector. We can see from (3.29), the optimal weight coefficients are determined by the eigenvalues of the correlation matrix $\mathbf{R}$, the received power of users and the covariance of AWGN. In order to get the optimal weight vector, we have to compute the eigenvalues of the $\mathbf{R}$. As we know, the computation for the eigenvalues is also too complex. In this section, we can get asymptotic weight vector for equal power case from (3.29) by making use of the result of random matrix theory (3.30), (3.31) which is cited from the reference [39].

Theorem: Let $\mathbf{S}$ be an $N \times K$ matrix whose entries are i.i.d random variables with zeros mean and variance $1/N$, and let λ_k be the eigenvalues of $\mathbf{S}^T \mathbf{S}$. Moreover, let $K \rightarrow \infty$ and $N \rightarrow \infty$, but $0 < \beta = K/N < \infty$. Then, the moments of the eigenvalues

$$\frac{1}{K} \sum_{\ell=1}^K \lambda_{\ell}^m = \frac{1}{K} \text{tr}(\mathbf{S}^T \mathbf{S})_m \quad (3.30)$$

converge almost surely to the nonrandom limits [39]

$$\sum_{i=0}^m \binom{m}{i} \binom{m}{i+1} \frac{\beta^i}{m} \quad (3.31)$$

This nice convergence property of the eigenvalues of large dimensional random covariance matrices allows for an explicit analytic expression of the asymptotically optimum weight vector which depends only on the noise power density and the load.

Then, for the equal power case and assume that $\mathbf{A} = \mathbf{I}$, the optimal weight

vector given in (3.29) can be expressed as

$$\mathbf{w}_L = (\Phi_L(\beta, \sigma^2))^{-1} \mathbf{m}_L(\beta) \quad (3.32)$$

where

$$\begin{aligned} \mathbf{m}_L(\beta) &= \lim_{K \rightarrow \infty} \frac{1}{K} \mathbf{C}^T \lambda \\ &= \frac{1}{K} \sum \{[\lambda, \lambda^2, \dots, \lambda^{L+1}]^T\} \end{aligned}$$

where $\lambda^\ell = [\lambda_1^\ell, \dots, \lambda_K^\ell]^T$.

$$\begin{aligned} \Phi_L(\beta, \sigma^2) &= \lim_{K \rightarrow \infty} \frac{1}{K} \mathbf{C}^T (\Lambda^2 + \sigma^2 \Lambda) \mathbf{C} \\ &= \sum \left\{ \begin{pmatrix} \lambda^2 + \sigma^2 \lambda & \lambda^3 + \sigma^2 \lambda^2 & \dots & \lambda^{L+2} + \sigma^2 \lambda^{L+1} \\ \lambda^3 + \sigma^2 \lambda^2 & \lambda^4 + \sigma^2 \lambda^3 & \dots & \lambda^{L+3} + \sigma^2 \lambda^{L+2} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda^{L+2} + \sigma^2 \lambda^{L+1} & \lambda^{L+3} + \sigma^2 \lambda^{L+2} & \dots & \lambda^{2L+2} + \sigma^2 \lambda^{2L+1} \end{pmatrix} \right\} \end{aligned}$$

The optimal vector can be rescaled by a scalar and does not lose any performance of the WPE detector. When we choose the scalar by the determinant of $\Phi_L(\beta, \sigma^2)$ and a scalar function of the load $\beta^{-L(L+1)/2}$, we can get the new expression of the optimal vector in (3.33).

$$\mathbf{w}_L = \frac{|\Phi_L(\beta, \sigma^2)|}{\beta^{L(L+1)/2}} \Phi_L(\beta, \sigma^2) \mathbf{m}_L(\beta) \quad (3.33)$$

This scaled weight vector's components only include L th order polynomials in β and σ^2 . Table 3.1 gives the asymptotic weight vectors for 1 to 8 stage WPE detector.

Table 3.1: Asymptotic weight vectors for L=1 to 8 stage WPE detector

$\mathbf{W}_{asy,1}$	$w_0 = -\sigma^2 w_1 + 2 + 2\beta$ $w_1 = -1$
$\mathbf{W}_{asy,2}$	$w_0 = -\sigma^2 w_1 + 3 + 4\beta + 3\beta^2$ $w_1 = -\sigma^2 w_2 - 3 - 3\beta$ $w_2 = 1$
$\mathbf{W}_{asy,3}$	$w_0 = -\sigma^2 w_1 + 4 + 6\beta + 6\beta^2 + 4\beta^3$ $w_1 = -\sigma^2 w_2 - 6 - 9\beta - 6\beta^2$ $w_2 = -\sigma^2 w_3 + 4 + 4\beta$ $w_3 = -1$
$\mathbf{W}_{asy,4}$	$w_0 = -\sigma^2 w_1 + 5 + 8\beta + 9\beta^2 + 8\beta^3 + 5\beta^4$ $w_1 = -\sigma^2 w_2 - 10 - 18\beta - 18\beta^2 - 10\beta^3$ $w_2 = -\sigma^2 w_3 + 10 + 16\beta + 10\beta^2$ $w_3 = -\sigma^2 w_4 - 5 - 5\beta$ $w_4 = 1$
$\mathbf{W}_{asy,5}$	$w_0 = -\sigma^2 w_1 + 6 + 10\beta + 12\beta^2 + 12\beta^3 + 10\beta^4 + 6\beta^5$ $w_1 = -\sigma^2 w_2 - 15 - 30\beta - 36\beta^2 - 30\beta^3 - 15\beta^4$ $w_2 = -\sigma^2 w_3 + 20 + 40\beta + 40\beta^2 + 20\beta^3$ $w_3 = -\sigma^2 w_4 - 15 - 25\beta - 15\beta^2$ $w_4 = -\sigma^2 w_5 + 6 + 6\beta$ $w_5 = -1$
$\mathbf{W}_{asy,6}$	$w_0 = -\sigma^2 w_1 + 7 + 12\beta + 15\beta^2 + 16\beta^3 + 15\beta^4 + 12\beta^5 + 7\beta^6$ $w_1 = -\sigma^2 w_2 - 21 - 45\beta - 60\beta^2 - 60\beta^3 - 45\beta^4 - 21\beta^5$ $w_2 = -\sigma^2 w_3 + 35 + 80\beta + 100\beta^2 + 80\beta^3 + 35\beta^4$ $w_3 = -\sigma^2 w_4 - 35 - 75\beta - 75\beta^2 - 35\beta^3$ $w_4 = -\sigma^2 w_5 + 21 + 36\beta + 21\beta^2$ $w_5 = -\sigma^2 w_6 - 7 - 7\beta$ $w_6 = 1$
$\mathbf{W}_{asy,7}$	$w_0 = -\sigma^2 w_1 + 8 + 14\beta + 18\beta^2 + 20\beta^3 + 20\beta^4 + 18\beta^5 + 14\beta^6 + 8\beta^7$ $w_1 = -\sigma^2 w_2 - 28 - 63\beta - 90\beta^2 - 100\beta^3 - 90\beta^4 - 63\beta^5 - 28\beta^6$ $w_2 = -\sigma^2 w_3 + 56 + 140\beta + 200\beta^2 + 200\beta^3 + 140\beta^4 + 56\beta^5$ $w_3 = -\sigma^2 w_4 - 70 - 175\beta - 225\beta^2 - 175\beta^3 - 70\beta^4$ $w_4 = -\sigma^2 w_5 + 56 + 126\beta + 126\beta^2 + 56\beta^3$ $w_5 = -\sigma^2 w_6 - 28 - 49\beta - 28\beta^2$ $w_6 = -\sigma^2 w_7 + 8 + 8\beta$ $w_7 = -1$
$\mathbf{W}_{asy,8}$	$w_0 = -\sigma^2 w_1 + 9 + 16\beta + 21\beta^2 + 24\beta^3 + 25\beta^4 + 24\beta^5 + 21\beta^6 + 16\beta^7 + 9\beta^8$ $w_1 = -\sigma^2 w_2 - 36 - 84\beta - 126\beta^2 - 150\beta^3 - 150\beta^4 - 126\beta^5 - 84\beta^6 - 36\beta^7$ $w_2 = -\sigma^2 w_3 + 84 + 224\beta + 350\beta^2 + 400\beta^3 + 350\beta^4 + 224\beta^5 + 84\beta^6$ $w_3 = -\sigma^2 w_4 - 126 - 350\beta - 525\beta^2 - 525\beta^3 - 350\beta^4 - 126\beta^5$ $w_4 = -\sigma^2 w_5 + 126 + 336\beta + 441\beta^2 + 336\beta^3 + 126\beta^4$ $w_5 = -\sigma^2 w_6 - 84 - 196\beta - 196\beta^2 - 84\beta^3$ $w_6 = -\sigma^2 w_7 + 36 + 64\beta + 36\beta^2$ $w_7 = -\sigma^2 w_8 - 9 - 9\beta$ $w_8 = 1$

3.7 Adaptive Weight Vector Based on LS Criterion

Figure 3.2: The structure of two stage adaptive WPE detector

The adaptive WPE detector proposed in this section is shown in Fig. 3.2.

Let $\mathbf{L}_y = [\mathbf{y}, \mathbf{R}\mathbf{y}, \dots, (\mathbf{R})^L\mathbf{y}]$, equation (3.18) can be modified to

$$\mathbf{b}_{PE} = L_{PE}\mathbf{y} = \mathbf{L}_y\mathbf{w} \quad (3.34)$$

Two cases are considered: equal received power and unequal received power. For the equal received power case, the weight vector for all the users is assumed to be the same. For the unequal received power case, the weight vector for each user is assumed to be different in general. Corresponding to the two cases, two adaptive weight vector WPE detectors are proposed. The first adaptive WPE detector assumes that the weight vector is the same for all users, then the transmitted bits of all users can be used in the adaptive procedure.

The objective function for the weight vector $\mathbf{w}$ is given by

$$J = \min_{\mathbf{w}} (\mathbf{b} - \mathbf{L}_y \mathbf{w})^T (\mathbf{b} - \mathbf{L}_y \mathbf{w}) \quad (3.35)$$

The second adaptive WPE detector assumes that the weight vector is different for each user, then the weight coefficients for each user is adapted by their own transmitted bits for the adaptive procedure.

If $\mathbf{L}_y(k)$ is the k th row vector of matrix $\mathbf{L}_y$, then the objective function for each user's weight vector $\mathbf{w}^k$ is given by

$$J_k = \min_{\mathbf{w}^k} (b_k - \mathbf{L}_y(k) \mathbf{w}^k)^2 \quad (3.36)$$

The second adaptive WPE detector should have better performance compared with the first adaptive WPE detector in the unequal power case, but with additional complexity and more transmitted training sequence.

The equations used for the RLS adaptive process for the unequal received power case are given from (3.37) to (3.41). Replacing $\mathbf{w}^k$ by $\mathbf{w}$ will lead to the solution for the special case of the equal received power case.

$$\mathbf{w}^k(0) = [0, \dots, 0]^T, \mathbf{P}(0) = \delta^{-1} \mathbf{I} \quad (3.37)$$

$$e_k(n) = b_k - \mathbf{L}_y(k) \mathbf{w}^k(n-1) \quad (3.38)$$

$$\mathbf{k}(n) = \frac{\mu^{-1} \mathbf{P}(n-1) \mathbf{L}_y^T(k)}{1 + \mu^{-1} \mathbf{L}_y(k) \mathbf{P}(n-1) \mathbf{L}_y^T(k)} \quad (3.39)$$

$$\mathbf{w}^k(n) = \mathbf{w}^k(n-1) + \mathbf{k}(n) e_k(n) \quad (3.40)$$

$$\mathbf{P}(n) = \mu^{-1}\mathbf{P}(n-1) - \mu^{-1}\mathbf{k}(n)\mathbf{L}_y(k)\mathbf{P}(n-1) \quad (3.41)$$

3.8 Complexity Analysis

In the section, the DSP based implementation complexity of the decorrelating detector and the multistage WPE detector will now be compared. A processing engine capable of performing single-cycle multiply and add instruction is assumed. The computational complexity per bit is defined as the number of real floating point operations (flops). The number of flops required per bit for the decorrelating detector Z_{decor} and that for the WPE detector Z_{WPE} can be computed to be,

$$Z_{decor} = KN(N-1) + K^2(K-1) + N(N-1)K^2 + O(K^3) \quad (3.42)$$

$$Z_{WPE} = KN(N-1) + K + (NK + N^2K - N + 2K)L \quad (3.43)$$

K denotes the number of active users in the systems. For a spreading factor of 31, the number of flops per bit for the decorrelating detector can be computed approximately from (3.42) as 104,200($K = 10$), 406,200($K = 20$) and 918,000($K = 30$). Similarly, the number of flops per bit for a 6-stage WPE detector can be computed from (3.43) as 68,764($K = 10$), 137,714($K = 20$) and 206,664($K = 30$). It can be seen from these Figures that, the reduction in the implementation complexity for the multi-stage WPE detector increases significantly as the number of users increases.

Now, we give some analysis of the complexity of the proposed adaptive WPE detector. The computational complexity of the RLS adaptive process is of the order of $O(ML^2)$ [8], which is dependent on the number of iterations M and the

dimension of the weight vector ($\mathbf{w}$) L . Usually, the number of stages of the WPE detector can be chosen to be small and the adaptive algorithm converges fast. For example, for the number of iterations $M = 50$ and the number of weight vectors $L = 6$, the additional complexity of the adaptive WPE detector is $O(1800)$ which can be neglected compared to the number of flops per bit for the 6-stage WPE detector. As such, the complexity of the proposed adaptive WPE detector is only marginally higher than that of the approximate WPE detector.

3.9 Numerical Results

A number of numerical experiments have been conducted to validate the performance of the WPE detector. The bit error rate (BER) and Mean Squared Error (MSE) are selected as the performance indicator for the performance measure. The system load and the channel environment will be considered in the experiments to show the performance of the WPE detectors discussed in this chapter. In the simulation test, a synchronous CDMA AWGN channel model with perfect power control is assumed. The spreading code is a random spreading sequence with a spreading factor $N = 31$. The results are averaged over 200 independent simulations.

Experiment 1: MSE of optimal WPE detector

This experiment examines the mean squared error between the output of optimal WPE detector and the transmitted bit sequence. Fig. 3.3 - Fig. 3.6 show the mean squared error of optimal WPE detector when the number of users $K = 10$ and $E_b/N_0 = 10\text{dB}$, 15dB , 20dB and 30dB respectively. From these Figures, it can be seen that the optimal WPE detector can exactly implement the MMSE detector in low system load situation. And the stage for the convergence is less than 7. Fig. 3.7 - Fig. 3.10 show the mean squared error when the number of users $K = 20$ and $E_b/N_0 = 10\text{dB}$, 15dB , 20dB and 30dB respectively. From these

Figures, it can be seen that in high system load situation, the optimal WPE detector can't converge to the MMSE detector as the SNR increases, instead it will diverge when the stage is greater than 7.

The reason for the divergence is that the Taylor series expansion of decorrelating detector and MMSE detector can't satisfy the convergence condition (3.12) and (3.15) in high system load. In the following, we will give the theoretical analysis to the divergence of WPE detector. According to the lemma in [44], [45], the extremal eigenvalues of $\mathbf{R}$ converge to specific values when $\beta = K/N < 1$ constant, and $K \rightarrow \infty$. The largest eigenvalue of $\mathbf{R}$ converges to

$$\lambda_{max} \rightarrow (\sqrt{\beta} + 1)^2 \quad (3.44)$$

Then, to satisfy (3.12) and (3.15), the system load β should be limited by

$$\beta < (\sqrt{2} - 1)^2$$

and

$$\beta < \left(\sqrt{2 - \frac{\sigma^2}{A^2}} - 1 \right)^2$$

We observe that the performance of the optimal WPE detector degrades when the number of stages exceeds 7 if the optimal WPE detector can't converge to the MMSE detector. Fig. 3.7 -Fig. 3.10 show the performance of the optimal WPE detector when we change the parameter SNR. Fig. 3.11 - Fig. 3.13 show the performance (MSE) of the optimal WPE detector when we change the parameters such as the number of users and spreading gain. We find that the stage value 7 is universal for different parameters. From these simulations, we can make a conclusion that a 7 stage optimal WPE detector is a better choice to approximate decorrelating detector or MMSE detector.

The simulation show that although the WPE detector can't converge to decor-

relating detector and MMSE in high system load, the performance of WPE detector has significant improvement compared with matched filter which corresponds to zero stage WPE detector in the Figures (Fig. 3.7 - Fig. 3.10). And the WPE detector is a performance and complexity tradeoff compared with decorrelating detector and MMSE detector.

Experiment 2: Convergence of adaptive WPE detector

This experiment shows the convergence of the adaptive WPE detector. In the simulation test, the number of users K is 20 and $E_b/N_0 = 15\text{dB}$. Fig. 3.14 - Fig. 3.21 show the learning curve of RLS adaptive algorithm from 1 to 8 stage adaptive WPE detector. It can be seen from these Figures that the number of stages has no effect on the number of iterations for the RLS adaptive algorithm to converge to the optimal WPE detector. The number of iterations M for the convergence is about 40. This means that the additional complexity of adaption of weight vector can be neglected in the total detection algorithm.

Experiment 3: The effect of the load

This experiment examines the effect of the system load to the WPE detector. In the simulation test, BER performance is shown for six stage optimal WPE detector, adaptive WPE detector and asymptotic WPE detector proposed in reference [39]. The simulation results of linear PIC detector, linear Chebyshev PIC detector, decorrelating detector and MMSE detector are also shown for the comparison. The length of the spreading sequence is fixed, $N = 31$. Fig. 3.22 - Fig. 3.24 show the BER performance versus system load in $E_b/N_0 = 10\text{dB}$, $E_b/N_0 = 15\text{dB}$ and $E_b/N_0 = 20\text{dB}$ respectively. From these Figures, we can see that the performance of linear PIC detector, which is exactly six stage no weighted polynomial expansion approximation of decorrelating detector, diverges from decorrelating detector and MMSE detector for system load $\beta > 0.15$. Fig. 3.22 shows that the optimal WPE detector, adaptive WPE detector, asymptotic WPE detector and linear Chebyshev PIC detector can nearly implement the decorrelating detector and MMSE detector

without any performance loss for system load in low E_b/N_0 situations. When the signal to noise ratio of E_b/N_0 increases as shown in Fig. 3.23 and Fig. 3.24, there will be some performance loss on the three WPE detectors and Chebyshev PIC detector compared with the decorrelating detector and MMSE detector. These Figures show that the optimal WPE detector and adaptive WPE detector can approximate decorrelating detector and MMSE detector better than the asymptotic WPE detector and linear Chebyshev PIC detector.

Experiment 4: BER performance versus the number of stages

Fig. 3.25 - Fig. 3.30 show the simulated BER versus stage for optimal WPE detector, adaptive WPE detector, asymptotic WPE detector. Simulation results of decorrelating detector, MMSE detector and linear Chebyshev PIC detector are also shown for comparison. The simulation parameters for Fig.3.25 - Fig.3.27 are $K = 15$ and $E_b/N_0 = 10\text{dB}, 15\text{dB}, 20\text{dB}$ respectively. The simulation parameters for Fig. 3.28 - Fig. 3.30 are $K = 20$ and $E_b/N_0 = 10\text{dB}, 15\text{dB}, 20\text{dB}$ respectively. From these Figures, it can be seen that the BER performance of optimal WPE detector is consistent with the observation in experiment 1. The BER performance of optimal WPE detector can't converge to decorrelating detector and MMSE detector in high system load situation. The adaptive WPE detector can achieve the similar BER performance as that of optimal WPE detector with considerable computation complexity reduced. The asymptotic WPE detector has similar performance as that of optimal WPE detector and adaptive WPE detector. but the performance degrades significantly at high SNR. Compared with the linear Chebyshev PIC detector, the optimal WPE detector and adaptive WPE detector converge fast to the optimal performance and the number of stages for the convergence is no more than 7.

Experiment 5: Near-far effect

In this experiment, the power control is not assumed to be perfect. The performance of WPE detector in near-far environment will be examined. Fig.

3.31 - Fig. 3.33 show the convergence properties of optimal WPE detector, two kinds of adaptive WPE detectors and asymptotic WPE detector. Fig. 3.34 - Fig. 3.36 show the BER performance of optimal WPE detector, two kinds of adaptive WPE detectors and asymptotic WPE detector in different near-far environment. simulation results of Chebyshev PIC detector, decorrelating detector and MMSE detector are also shown for comparison. In the simulation test, user 1 is assumed as the desired user and two kinds of adaptive WPE detectors given in Section 3.7 are considered. The adaptive WPE detector 1 assumes that the weight vector for all users is the same and the transmitted bits of all users are used as training sequence in the adaptive procedure. While the adaptive WPE detector 2 assumes that the weight vector is different for each user, then the weight coefficients for each user are adapted by their own transmitted training sequence. The simulation parameters for Fig. 3.31 - Fig. 3.33 are that the signal to noise ratio (SNR) of user 1 is 10dB while the E_b/N_0 of the remaining users is 20dB, and the number of users is $K = 10, 15, 20$ respectively. The simulation parameters for Fig. 3.34 - Fig. 3.36 are that E_b/N_0 of user 1 is 10dB while the E_b/N_0 of the remaining users is $(E_b/N_0)_{user1} + ISR$ dB, the number of stages is 6 and the number of users is $K = 10, 15, 20$ respectively. From these Figures, we can see that the adaptive WPE detector 2 can have significant performance improvement and near-far resistance in low system load situation. While the system load increase, the performance of adaptive WPE detector 2 will degrade and is similar to the adaptive WPE detector 1. The optimal WPE detector and adaptive WPE detector perform better than the linear Chebyshev PIC detector and asymptotic WPE detector but worse than the decorrelating detector and MMSE detector under near-far effect.

Figure 3.3: Mean squared error of optimal WPE detector ($K = 10$, $E_b/N_0 = 10\text{dB}$)

Figure 3.4: Mean squared error of optimal WPE detector ($K = 10$, $E_b/N_0 = 15\text{dB}$)

Figure 3.5: Mean squared error of optimal WPE detector ($K = 10$, $E_b/N_0 = 20\text{dB}$)

Figure 3.6: Mean squared error of optimal WPE detector ($K = 10, E_b/N_0 = 30\text{dB}$)

Figure 3.7: Mean squared error of optimal WPE detector ($K = 20, E_b/N_0 = 10\text{dB}$)

Figure 3.8: Mean squared error of optimal WPE detector ($K = 20$, $E_b/N_0 = 15\text{dB}$)

Figure 3.9: Mean squared error of optimal WPE detector ($K = 20$, $E_b/N_0 = 20\text{dB}$)

Figure 3.10: Mean squared error of optimal WPE detector ($K = 20$, $E_b/N_0 = 30\text{dB}$)

Figure 3.11: Mean squared error of optimal WPE detector ($N = 31$, $K = 15$, $E_b/N_0 = 30\text{dB}$)

Figure 3.12: Mean squared error of optimal WPE detector ($N = 63$, $K = 50$, $E_b/N_0 = 20\text{dB}$)

Figure 3.13: Mean squared error of optimal WPE detector ($N = 127$, $K = 100$, $E_b/N_0 = 20\text{dB}$)

Figure 3.14: Learning curve of one stage adaptive WPE detector

Figure 3.15: Learning curve of two stage adaptive WPE detector

Figure 3.16: Learning curve of three stage adaptive WPE detector

Figure 3.17: Learning curve of four stage adaptive WPE detector

Figure 3.18: Learning curve of five stage adaptive WPE detector

Figure 3.19: Learning curve of six stage adaptive WPE detector

Figure 3.20: Learning curve of seven stage adaptive WPE detector

Figure 3.21: Learning curve of eight stage adaptive WPE detector

Figure 3.22: BER performance of 6 stage WPE detector versus the system load ($E_b/N_0 = 10\text{dB}$)

Figure 3.23: BER performance of 6 stage WPE detector versus the system load ($E_b/N_0 = 15\text{dB}$)

Figure 3.24: BER performance of 6 stage WPE detector versus the system load ($E_b/N_0 = 20\text{dB}$)

Figure 3.25: BER convergence properties ($K = 15$, $E_b/N_0 = 10\text{dB}$)

Figure 3.26: BER convergence properties ($K = 15$, $E_b/N_0 = 15\text{dB}$)

Figure 3.27: BER convergence properties ($K = 15$, $E_b/N_0 = 20\text{dB}$)

Figure 3.28: BER convergence properties ($K = 20$, $E_b/N_0 = 10\text{dB}$)

Figure 3.29: BER convergence properties ($K = 20$, $E_b/N_0 = 15\text{dB}$)

Figure 3.30: BER convergence properties ($K = 20$, $E_b/N_0 = 20\text{dB}$)

Figure 3.31: BER convergence properties in near-far environment ($K = 10$, $(E_b/N_0)_{\text{user}1} = 10\text{dB}$, $(E_b/N_0)_{\text{user}2-K} = 20\text{dB}$)

Figure 3.32: BER convergence properties in near-far environment ($K = 15$, $(Eb/N0)_{user1} = 10\text{dB}$, $(Eb/N0)_{user2-K} = 20\text{dB}$)

Figure 3.33: BER convergence properties in near-far environment ($K = 20$, $(Eb/N0)_{user1} = 10\text{dB}$, $(Eb/N0)_{user2-K} = 20\text{dB}$)

Figure 3.34: BER performance in near-far environment ($K = 10$, the number of stages $L = 6$, $(E_b/N_0)_{user1} = 10\text{dB}$)

Figure 3.35: BER performance in near-far environment ($K = 15$, the number of stages $L = 6$, $(E_b/N_0)_{user1} = 10\text{dB}$)

Figure 3.36: BER performance in near-far environment ($K = 20$, the number of stages $L = 6$, $(Eb/N0)_{user1} = 10\text{dB}$)

3.10 Conclusions

In summary, this chapter presents a derivation of the optimal weight vector for the WPE detector based on the minimum MSE criteria. An adaptive WPE detector based on the least squares criteria and RLS adaptive algorithm is then proposed to determine the optimum weight vector. Simulation results show that the WPE detector has similar performance as that of the decorrelating detector and MMSE detector, but it has a significantly lower complexity. In near-far situation, the optimal WPE detector and adaptive WPE detector perform better than the linear Chebyshev PIC detector and asymptotic WPE detector but worse than the decorrelating detector and MMSE detector for high ISR.

Chapter 4

Blind Cascaded Modular Structure Interference Cancellation Detector

4.1 Background

Much of the research work in MUD has been focused on blind adaptive multiuser detection schemes [53], [55], [56] in the past two decades. These blind adaptive schemes are especially attractive for downlink CDMA systems because it is very difficult for mobile stations to obtain accurate knowledge of all active users in the channel.

The blind MOE detector [55], which is also referred to as the minimum variance distortionless response (MVDR) detector in array signal processing [63], is designed based on the criterion that the minimum output energy and the minimum mean squared error are directly related, and hence minimizing one is equivalent to minimizing the other. The MOE detector has an advantage over the MMSE detector in that the MMSE solution can be implemented without the transmission of training sequences. The blind MOE detector can be studied as a linear con-

straint optimization problem [53]. By using the knowledge of the desired user's spreading code, the detector minimizes the receiver output energy while constraining the response of the desired user to a constant. Partitioned linear interference cancellation (PLIC) structure can be applied to solve the constrained optimization problem [53]. The structure decomposes the detector into two orthogonal components. One component corresponding to the constraint condition is a nonadaptive module, while the other component is an adaptive module designed to reduce the interference that exists at the output of the nonadaptive module. It should be noted that the PLIC structure is equivalent to Frost's algorithm [64] where it is capable of automatically correcting small deviations due to finite precision arithmetic.

Reduced rank methods for the realization of the adaptive module are commonly used to reduce computational complexity. As the computational burden and convergence speed of adaptive algorithms are determined by the number of adaptive weights, reduced rank methods are designed to reduce the dimension of the adaptive weight vector. This design allows the adaptive module to converge to the steady-state performance quickly. It can also allow the filter coefficients to be estimated with a relatively small amount of observation data. Principal component (PC) methods [49], [50] and cross spectral (CS) methods [51], [52] are two commonly used reduced rank methods that are designed based on eigen-decomposition technique. PC methods involve a lower dimensional signal subspace and are very suitable for small loaded system. CS methods take into account the energy in the subspace contributed by the desired user, and thus it does not require the dimension of the estimated subspace to be greater than that of the signal subspace. A nested chain structure reduced rank multistage Wiener filter (MSWF) has been discussed and analyzed in [57]. The MSWF generates a set of orthogonal basis vectors from a successive refinement procedure and uses the vectors to form an estimated signal subspace without the need of eigen-decomposition. The MSWF

can attain near full rank MMSE detector performance by using subspace that has much less dimension than that of the signal subspace [59]. Several adaptive algorithms on MSWF have been described in [58]. An iterative algorithm that uses an "Auxiliary Vector" filter has been reported to be capable of generating the same subspace as that of the reduced rank MSWF [60], [61], [62].

In this thesis, we propose a cascaded modular structure detector to implement the blind MOE detector. The proposed cascaded modular structure interference cancellation detector is a reduced rank method. Like the MSWF, the proposed detector does not need to explicitly estimate the subspace. In each module of the structure, a vector filter is introduced for adaptive interference cancellation. The weight vector is determined based on maximum magnitude cross correlation criterion which maximizes the magnitude of the cross correlation between the output of the nonadaptive filter and that of the weight vector filter. Each module of the structure is able to reduce most of the interference presented in the output of the preceding module such that very few modules or stages are required to achieve the performance of the full rank MVDR detector. The cascaded modular structure detector is relatively simple and has a low complexity. As each module represents one adaptive degree of freedom, the adaptive procedure in each module converges rapidly.

In the remaining parts of this chapter, Section 4.2 describes the general framework of the constrained optimization problem, Section 4.3 introduces the eigen-decomposition and MSWF reduced rank methods, Section 4.4 discusses the design of the proposed cascaded modular structure interference detector, the performance analysis of the detector and its adaptive implementation, and Section 4.5 provides some simulation results on the performance of the proposed interference cancellation detector. A summary of the findings is given in Section 4.6.

4.2 Framework of Constrained Optimization

4.2.1 Channel Model

A K -user synchronous DS-CDMA system model is assumed in this chapter. The received continuous time baseband signal can be represented as

$$y(t) = \sum_{n=0}^{\infty} \sum_{k=1}^K a_k b_k(n) s_k(t - nT_s) + v(t) \quad (4.1)$$

where

- $b_k(n)$ n th data symbol of the k th user,
- a_k real-valued amplitude of the k th user,
- $s_k(t)$ spreading waveform of the k th user,
- $v(t)$ Additive White Gaussian Noise (AWGN) with power spectral density σ_v^2 ,
- T_s symbol period.

It is assumed that the data symbols have a statistics of zero mean, and an independently and identically distributed (i.i.d) BPSK modulation.

At the receiver, the received signal is sampled at the chip rate. The chip rate discrete time signal of (4.1) is given by

$$\mathbf{y}(n) = \mathbf{S}\mathbf{A}\mathbf{b}(n) + \mathbf{v}(n) \quad (4.2)$$

where

- $\mathbf{y}(n)$ $N \times 1$ vector of signal samples at time n ,
- $\mathbf{S}$ $N \times K$ matrix whose columns contain the users's spreading codes,
- $\mathbf{A}$ $K \times K$ diagonal matrix of signal amplitudes,
- $\mathbf{b}(n)$ $K \times 1$ vector of user bits at time n ,
- $\mathbf{v}(n)$ $N \times 1$ vector of noise samples at time n .

4.2.2 Code Constrained Optimization

The blind linear adaptive multiuser detector is designed to suppress the interference by minimizing the output power of the receiver, while at the same time maintaining a distortionless response toward the specific input of interest [53]. This is a constrained optimization problem. Consider a linear detector at the receiver. Let the weight vector of the filter be $\mathbf{w}$. The output signal of the detector can be expressed as

$$\mathbf{z} = \mathbf{w}^H \mathbf{y} \quad (4.3)$$

The weight vector of the constrained optimization detector can be derived by minimizing the output power subject to a set of constraints.

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{yy} \mathbf{w} \quad \text{subject to} \quad \mathbf{C}^H \mathbf{w} = \mathbf{f} \quad (4.4)$$

where $\mathbf{R}_{yy}$ is the $N \times N$ data correlation matrix, $\mathbf{C}$ is a $P \times N$ matrix whose rows specify the constraints, $\mathbf{f}$ is a $P \times 1$ vector of constraint values, and P is the number of specified constraints. The optimal solution to the problem can be obtained via the Lagrange multipliers procedure,

$$\mathbf{w}_{opt} = \mathbf{R}_{yy}^{-1} \mathbf{C}^H (\mathbf{C} \mathbf{R}_{yy}^{-1} \mathbf{C}^H)^{-1} \mathbf{f} \quad (4.5)$$

We can see from (4.5) that the $N \times N$ data correlation matrix $\mathbf{R}_{yy}$, needs to be estimated and then inverted before arriving at the solution. Thus, it is computationally complex to directly compute the optimal weight vector of the constrained optimization detector. The PLIC structure can be implemented to compute the solution.

Figure 4.1: The structure of partitioned linear interference canceller

4.2.3 PLIC Structure

The PLIC structure [53] is depicted in Fig. 4.1. The upper branch implements the constraint equation with the nonadaptive weight vector $\mathbf{w}_c$. The lower branch first removes the portion of the received signal that satisfies the constraints and then adaptively removes the remaining interference present in the output of the upper branch nonadaptive filter with another weight vector $\mathbf{w}_a$.

The weight vector of the constrained optimization detector can be decomposed into two orthogonal components via a projection matrix associated with the constraint equation. The nonadaptive weight vector $\mathbf{w}_c$ in the upper branch is given by

$$\mathbf{w}_c = \mathbf{C}^H (\mathbf{C}\mathbf{C}^H)^{-1} \mathbf{f} \quad (4.6)$$

The blocking matrix, $\mathbf{M}$, inserted in the lower branch, is designed such that $\mathbf{M}\mathbf{w}_c = 0$. This ensures that no component of the received signal that satisfies

the constraints pass through the lower branch. The dimension of matrix $\mathbf{M}$ is $(N - P) \times N$. Hence, the weight vector $\mathbf{w}_a$ in the lower branch has dimension $(N - P) \times 1$ and can be adapted according to any conventional unconstrained weight update scheme. The optimal adaptive weight vector $\mathbf{w}_a$ can be obtained by solving the unconstrained optimization problem:

$$\min_{\mathbf{w}_a} (\mathbf{w}_c - \mathbf{M}^H \mathbf{w}_a)^H \mathbf{R}_{yy} (\mathbf{w}_c - \mathbf{M}^H \mathbf{w}_a) \quad (4.7)$$

and is given by

$$\mathbf{w}_{opt_a} = (\mathbf{M} \mathbf{R}_{yy} \mathbf{M}^H)^{-1} \mathbf{M} \mathbf{R}_{yy} \mathbf{w}_c \quad (4.8)$$

4.3 Reduced-Rank Interference Suppression Methods

In Section 4.2, we see that the lower branch adaptive weight vector $\mathbf{w}_a$ has a dimension of $(N - P) \times 1$, which is proportional to that of the spreading code. Since the spreading code length is often quite large, the response time of the fully adaptive algorithm can be quite slow. Consequently, rank reduction methods are often used where a lower dimensional filter is designed to yield a steady-state performance that is as close as possible to that obtained by the full-rank solution. In this section, we will describe some reduced-rank techniques that have been proposed in the literature.

4.3.1 Reduced-Rank Methods Based on Eigen-Decomposition

PC reduced-rank [49], [50] and CS reduced-rank [51], [52] are two reduced-rank techniques based on eigen-decomposition. PC reduced-rank method functions by performing an eigen-decomposition on the covariance matrix

$$\mathbf{R} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H \quad (4.9)$$

where $\mathbf{V}$ is the orthonormal matrix whose columns are eigen-vectors of $\mathbf{R}$, and $\mathbf{\Lambda}$ is the diagonal matrix of eigenvalues. Let us assume that the eigenvalues are arranged in order, i.e., $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N$. For a given subspace dimension D , the first D columns of $\mathbf{V}$ are chosen as the projection matrix for the PC reduced-rank method. For $K < N$, the eigenvalues $\lambda_1, \dots, \lambda_K$ are associated with the signal subspace, and the remaining eigenvalues are associated with the noise subspace. By selecting $D \geq K$, the PC reduced-rank method is able to achieve the same performance as that of the full rank filter. However, the performance can degrade quite rapidly when $D \leq K$, as the associated signal subspace may not contain sufficient signal energy. This is especially problematic in a near-far environment, since for small D , the subspace that contains most of the energy is likely to correspond to the interference and not the desired signal.

In order to overcome the above shortfall of the PC reduced-rank method, the CS reduced-rank method select the set of D eigenvectors for the projection matrix such that the mean squared error is the minimum. The subspace that minimizes the MSE has basis vectors which are the eigenvectors of R associated with the D largest values of $|\mathbf{v}_k^H \mathbf{c}_1|^2 / \lambda_k$, where $\mathbf{v}_k$ is the k th column of $\mathbf{V}$, $\mathbf{c}_1$ is the signature sequence of the desired user 1, and λ_k is the corresponding eigenvalue. This technique can perform well for $D < K$ since it takes into account the energy in the subspace contributed by the desired user.

Realizing the PC or CS reduced-rank method and using the PLIC structure produces the partially adaptive PLIC detector shown in Fig. 4.2 [53]. In this design, the lower branch applies the rank-reduction projection matrix $\mathbf{Q}$ so that fewer weights are required for interference cancellation. The columns of $\mathbf{Q}$ are the eigenvectors of $\mathbf{R}_{MM} = E\{\mathbf{M}\mathbf{Y}\mathbf{Y}^H\mathbf{M}^H\}$ corresponding to the D largest eigenvalues in the PC rank-reduction method. For the CS method, the columns of $\mathbf{Q}$ are the D

Figure 4.2: The structure of partially adaptive partitioned linear interference canceller

eigenvectors that maximize the metric $|\mathbf{r}_{Mc}^H \mathbf{v}_k|^2 / \lambda_k$, where $\mathbf{v}_k$ is the k th eigenvector of $\mathbf{R}_{MM}$, λ_k is the associated eigenvalue and $\mathbf{r}_{Mc} = E\{\mathbf{M}\mathbf{Y}(\mathbf{w}_c^H \mathbf{Y})^H\}$ is the cross correlation between the lower branch data and the upper branch output.

4.3.2 Multistage Wiener Filter (MSWF) Method

One disadvantage of the eigen-decomposition techniques is the complexity associated with the estimation of the signal subspace, as it involves eigen-decomposition. A sequence of nested filters called multistage Wiener filter (MSWF) has been proposed to overcome this disadvantage [57], [59]. The block diagram of a three-stage MSWF is shown in Fig. 4.3. The stages are associated with the sequence of nested filters $\mathbf{c}_1, \dots, \mathbf{c}_D$, where D is the order of the filter. The matrix $\mathbf{B}_1, \dots, \mathbf{B}_D$ shown in the Figure are blocking matrix satisfying

$$\mathbf{B}_n^H \mathbf{c}_n = 0 \quad (4.10)$$

Figure 4.3: Three stage multistage Wiener filter

In Fig. 4.3, the filter generates a desired sequence $d_n(i)$ at stage n at the output of the filter $\mathbf{c}_n$, and an observation sequence $\mathbf{y}_n(i)$ at the output of the blocking matrix $\mathbf{B}_n$. The MSWF is constructed by repeating the same structure at each stage. The $n + 1$ stage filter is determined by correlating the outputs of the preceding stage

$$\mathbf{c}_{n+1} = E\{d_n \mathbf{y}_n\} \quad (4.11)$$

The MSWF output is obtained by linearly combining the outputs of the filters $\mathbf{c}_1, \dots, \mathbf{c}_D$ via the weights $w_1, \dots, w_{D-1}$. Continuing the procedure for N iterations gives the full-rank MMSE filter. Terminating after D iterations gives a rank D filter.

4.4 Proposed Cascaded Structure Interference Cancellation Detector

4.4.1 Design of Cascaded Structure Interference Cancellation Detector

Figure 4.4: D stage cascaded modular structure interference cancellation detector

The structure of D stage cascaded modular detector is shown in Fig. 4.4. In Fig. 4.4, $\mathbf{y}$ is the input vector, $\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_D$ are output vector of each stage and x is the desired output symbol. The blind MOE detector is organized in D-stage cascaded modules. Each module is designed by one vector weight filter $\mathbf{g}_l$ and only one nonadaptive filter $\mathbf{w}_c$ is inserted after the D-stage module. In the remaining parts of this section, we will show how the weight vector $\mathbf{g}_l$ in each module can be determined so that the performance of the cascaded modular structure detector can converge to that of the blind MOE detector with a few number of stages.

Consider one dimensional weight vector PLIC structure detector shown in Fig.

Figure 4.5: One dimensional weight vector PLIC structure detector

4.5. The weight vector $\mathbf{g}_1$ is selected by maximizing the magnitude of the cross correlation of the output of the nonadaptive constraint filter $\mathbf{w}_c$ and the output of the weight vector filter $\mathbf{g}_1$,

$$\mathbf{g}_1 = \arg \max_{\mathbf{g}_1} |E\{(\mathbf{g}_1^H \mathbf{y})(\mathbf{w}_c^H \mathbf{y})^H\}| \quad (4.12)$$

The weight vector $\mathbf{g}_1$ needs to satisfy the following constraints.

$$\mathbf{C}^H \mathbf{g}_1 = \mathbf{0} \quad \text{and} \quad \mathbf{g}_1^H \mathbf{g}_1 = 1 \quad (4.13)$$

The weight vector $\mathbf{g}_1$ can be solved by applying Lagrange multipliers as

$$\mathbf{g}_1 = \frac{(\mathbf{I} - \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{C}^H) \mathbf{R}_{yy} \mathbf{w}_c}{\|(\mathbf{I} - \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{C}^H) \mathbf{R}_{yy} \mathbf{w}_c\|_2} \quad (4.14)$$

The scalar, u_1 , in Fig. 4.5 can be derived by using the MSE criteria.

$$u_1 = \frac{\mathbf{w}_c^H \mathbf{R}_{yy} \mathbf{g}_1}{\mathbf{g}_1^H \mathbf{R}_{yy} \mathbf{g}_1} \quad (4.15)$$

The output of the one dimensional weight vector PLIC detector in Fig. 4.5 can be obtained by

$$x = \mathbf{w}_c^H \mathbf{y} - u_1 \mathbf{g}_1^H \mathbf{y} \quad (4.16)$$

Substituting (4.15) into (4.16), the output of the filter can be derived as

$$x = \mathbf{w}_c^H (\mathbf{y} - \mathbf{U}_1^H \mathbf{g}_1^H \mathbf{y}) \quad (4.17)$$

$$\mathbf{y}_1 = \mathbf{y} - \mathbf{U}_1^H \mathbf{g}_1^H \mathbf{y} \quad (4.18)$$

$$\mathbf{U}_1 = \frac{\mathbf{g}_1^H \mathbf{R}_{yy}}{\mathbf{g}_1^H \mathbf{R}_{yy} \mathbf{g}_1} \quad (4.19)$$

$$(4.20)$$

From (4.17), we can get a new structure shown in Fig. 4.6, which is the one-stage cascaded modular structure interference cancellation detector. This design can remove most of interference and noise that exist in the input vector $\mathbf{y}$. In Fig. 4.6, $\mathbf{y}_1$ is the output vector of module one. By the same steps, we can design a multi-stage cascaded modular detector shown in Fig. 4.4. Each module can remove most of interference and noise remaining in the output of the preceding module.

For the ℓ th module of the D-stage cascaded modular structure detector shown in Fig. 4.4, $\mathbf{g}_\ell$, $\mathbf{U}_\ell$ is given by

$$\mathbf{g}_\ell = \frac{(\mathbf{I} - \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{C}^H) \mathbf{R}_{y_{\ell-1} y_{\ell-1}} \mathbf{w}_c}{\|(\mathbf{I} - \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{C}^H) \mathbf{R}_{y_{\ell-1} y_{\ell-1}} \mathbf{w}_c\|_2} \quad (4.21)$$

$$\mathbf{U}_\ell = \frac{\mathbf{g}_\ell^H \mathbf{R}_{y_{\ell-1} y_{\ell-1}}}{\mathbf{g}_\ell^H \mathbf{R}_{y_{\ell-1} y_{\ell-1}} \mathbf{g}_\ell} \quad (4.22)$$

where,

$$\mathbf{R}_{y_{\ell-1} y_{\ell-1}} = (\mathbf{I} - \mathbf{g}_{\ell-1} \mathbf{U}_{\ell-1})^H \mathbf{R}_{y_{\ell-2} y_{\ell-2}} (\mathbf{I} - \mathbf{g}_{\ell-1} \mathbf{U}_{\ell-1}) \quad (4.23)$$

Figure 4.6: One stage cascaded structure interference cancellation detector

The input data autocorrelation matrix $\mathbf{R}_{yy}$ can be estimated from L samples of the observation sequence,

$$\hat{\mathbf{R}}_{yy} = \frac{1}{L} \sum_{n=1}^L \mathbf{y}_n \mathbf{y}_n^T \quad (4.24)$$

The output of the ℓ th module is

$$\mathbf{y}_\ell = (\mathbf{I} - g_\ell \mathbf{U}_\ell)^H \mathbf{y}_{\ell-1} \quad (4.25)$$

The output of the D-stage cascaded modular structure detector is

$$x = \mathbf{w}_c^H \mathbf{y}_D \quad (4.26)$$

4.4.2 SINR Performance Analysis

The theoretical analysis of the convergence of the proposed cascaded modular structure detector is discussed in this section. We will show that the output SINR of the cascaded modular structure detector converges to the output SINR of the

blind MOE detector.

For a linear receiver, the output SINR is given by

$$SINR = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_i \mathbf{w}} = \frac{\mathbf{w}^H \mathbf{c}_1 \mathbf{c}_1^H \mathbf{w}}{\mathbf{w}^H (\mathbf{R}_y - \mathbf{c}_1 \mathbf{c}_1^H) \mathbf{w}} \quad (4.27)$$

where, $\mathbf{w}$ is the weight vector of the linear receiver and $\mathbf{c}_1$ is the signature vector of user one.

$$\mathbf{R}_s = \mathbf{c}_1 \mathbf{c}_1^H \quad (4.28)$$

$$\mathbf{R}_i = \mathbf{R}_y - \mathbf{c}_1 \mathbf{c}_1^H \quad (4.29)$$

For the MOE based receiver, the output SINR can be obtained by substituting (4.9) in (4.27)

$$SINR_{MOE} = \frac{\|\mathbf{f}^H (\mathbf{C}^H \mathbf{R}_y^{-1} \mathbf{C})^{-1} \mathbf{C}^H \mathbf{R}_y^{-1} \mathbf{c}_1\|^2}{\mathbf{f}^H (\mathbf{C}^H \mathbf{R}_y^{-1} \mathbf{C})^{-1} \mathbf{f} - \|\mathbf{f}^H (\mathbf{C}^H \mathbf{R}_y^{-1} \mathbf{C})^{-1} \mathbf{C}^H \mathbf{R}_y^{-1} \mathbf{c}_1\|^2} \quad (4.30)$$

Since the constrained optimization can be implemented with PLIC detector, the output SINR can also be obtained as

$$SINR_{MOE} = SINR_{PLIC} = \frac{\mathbf{w}_{PLIC}^H \mathbf{c}_1 \mathbf{c}_1^H \mathbf{w}_{PLIC}}{\mathbf{w}_{PLIC}^H (\mathbf{R}_y - \mathbf{c}_1 \mathbf{c}_1^H) \mathbf{w}_{PLIC}} \quad (4.31)$$

where

$$\mathbf{w}_{PLIC} = \mathbf{w}_c - \mathbf{M}^H \mathbf{w}_a = (\mathbf{I} - \mathbf{G}_{PLIC} \mathbf{U}_{PLIC}) \mathbf{w}_c \quad (4.32)$$

For the D stage cascaded modular structure detector shown in Fig. 4.4, the weight vector, $\mathbf{w}$, can be obtained as

$$\mathbf{w}_D = \left(\prod_{\ell=1}^D (\mathbf{I} - \mathbf{g}_\ell \mathbf{U}_\ell) \right) \mathbf{w}_c \quad (4.33)$$

It can be shown that (given in Appendix B)

$$\prod_{\ell=1}^D (\mathbf{I} - \mathbf{g}_\ell \mathbf{U}_\ell) = \mathbf{I} - \mathbf{G} \mathbf{U} \quad (4.34)$$

where

$$\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_D] \quad (4.35)$$

$$\mathbf{U} = (\mathbf{G}^H \mathbf{R}_{yy} \mathbf{G})^{-1} \mathbf{G}^H \mathbf{R}_{yy} \quad (4.36)$$

This means that the cascaded modular structure detector is equivalent to the PLIC detector implemented with the constrained optimization criteria discussed in Section 4.2 when the rank of $\mathbf{G}$ is equal to that of $\mathbf{G}_{PLIC}$. If the vectors $\mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_D$ with $D = N - 1$, are orthogonal to each other, then $\mathbf{G}$ composes of full rank orthogonal complementary space of the constraints $\mathbf{C}$. The performance of the cascaded modular structure detector converges to that of the optimal solution. If the vectors $\mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_D$ are not orthogonal to each other completely, more stages may be required to achieve the performance of the optimal solution.

It should be noted that, if the weight vectors for each module are determined by the proposed maximum magnitude cross correlation criteria, then the SINR performance of the cascaded modular structure detector converges to that of the optimal solution when $D \ll N - 1$, because the vectors, $\mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_D$ obtained based on the maximum magnitude cross correlation criteria can remove most of the interferences in the first few stages.

4.4.3 Adaptive Implementation

In this section, we propose two adaptive algorithms, the batch adaptive algorithm and the recursive adaptive algorithm, for the proposed blind cascaded modular structure detector. As the adaptive weight vector in each module depends only on the output of the preceding module in the cascaded structure, we only need to develop adaptive algorithm for one module.

Batch adaptive algorithm

Given a sequence of L input vectors for the ℓ th module

$$\mathbf{Y}_{\ell-1} = [\mathbf{y}_{\ell-1}(1), \mathbf{y}_{\ell-1}(2), \dots, \mathbf{y}_{\ell-1}(L)] \quad (4.37)$$

The batch adaptive algorithm for the ℓ th module can be derived as

$$\hat{\mathbf{R}}_{y_{\ell-1}y_{\ell-1}} = \frac{1}{L} \mathbf{Y} \mathbf{Y}^H \quad (4.38)$$

$$\mathbf{g}_\ell = \frac{(\mathbf{I} - \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{C}^H) \hat{\mathbf{R}}_{y_{\ell-1}y_{\ell-1}} \mathbf{w}_c}{\|(\mathbf{I} - \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{C}^H) \hat{\mathbf{R}}_{y_{\ell-1}y_{\ell-1}} \mathbf{w}_c\|_2} \quad (4.39)$$

$$\mathbf{U}_\ell = \frac{\mathbf{g}_\ell^H \hat{\mathbf{R}}_{y_{\ell-1}y_{\ell-1}}}{\mathbf{g}_\ell^H \hat{\mathbf{R}}_{y_{\ell-1}y_{\ell-1}} \mathbf{g}_\ell} \quad (4.40)$$

$$\mathbf{Y}_\ell = (\mathbf{I} - g_\ell \mathbf{U}_\ell)^H \mathbf{Y}_{\ell-1} \quad (4.41)$$

Recursive adaptive algorithm

Similarly, the recursive adaptive algorithm for each module can be derived as

$$\hat{\mathbf{R}}_{y_{\ell-1}y_{\ell-1}}(n) = (1 - \mu) \hat{\mathbf{R}}_{y_{\ell-1}y_{\ell-1}}(n-1) + \mu \mathbf{y}_{\ell-1}(n) \mathbf{y}_{\ell-1}^H(n) \quad (4.42)$$

where μ is a forgetting factor which discounts past data.

$$\mathbf{g}_\ell(n) = \frac{(1 - \mu)\mathbf{g}_\ell(n - 1) + \mu(\mathbf{I} - \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{C}^H) \mathbf{y}_{\ell-1}(n) \mathbf{y}_{\ell-1}^H(n) \mathbf{w}_c}{\|(1 - \mu)\mathbf{g}_\ell(n - 1) + \mu(\mathbf{I} - \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{C}^H) \mathbf{y}_{\ell-1}(n) \mathbf{y}_{\ell-1}^H(n) \mathbf{w}_c\|_2} \quad (4.43)$$

$$\mathbf{U}_\ell(n) = \frac{\mathbf{g}_\ell(n)^H \hat{\mathbf{R}}_{y_{\ell-1} y_{\ell-1}}(n)}{\mathbf{g}_\ell(n)^H \hat{\mathbf{R}}_{y_{\ell-1} y_{\ell-1}}(n) \mathbf{g}_\ell(n)} \quad (4.44)$$

$$\mathbf{y}_\ell(n) = (\mathbf{I} - g_\ell(n) \mathbf{U}_\ell(n))^H \mathbf{y}_{\ell-1}(n) \quad (4.45)$$

4.4.4 Complexity Analysis

The proposed detector partitions the blind MOE detector into several cascaded modules. Each module is one dimension weight vector adaptive filter. The computation of each module is very simple compared with the blind MOE detector. The simulation results in Section 4.5 show that the proposed detector only need few stages to converge to the performance of blind MOE detector. PC and CS reduced rank detectors have been discussed in Section 4.3.2. These reduce rank detectors need to compute the eigenvalue of matrix and the weight vector of adaptive filter is multiple dimensions. Thus, the proposed detector has better performance of complexity and convergence. Because each module of the detector can be processed at the same time, the proposed detector does not influence the data rate of the system compared with the existing methods.

4.5 Numerical Results

In this section, several experiments are conducted to examine the performance of the proposed cascaded modular structure detector. The convergence of the proposed detector and the batch adaptive algorithm is shown in experiment 1 and experiment 2. Experiment 3 compares the proposed detector with reduced-

rank detectors described in Section 4.3. In the simulation, K-user synchronous DS-CDMA BPSK modulation system is assumed and the spreading codes are chosen randomly. We use the averaged output signal-to-interference-and-noise-ratio (SINR) as performance measure. The results are based on averages over $M = 200$ simulation runs.

Experiment 1: Convergence of cascaded modular structure detector

This experiment compares the convergence of the SINR performance of the proposed cascaded modular structure interference cancellation detector with that of the full rank blind MOE detector at different system load and spreading code length. The average output SINR is used to compare the convergence performance of the proposed cascaded modular structure detector with that of the full rank blind MOE detector. All simulations assume that the length of the sample data L is enough for accurate estimation of the input data covariance matrix $\mathbf{R}_{yy}$ in practice ($L=500$ in the simulations). In the simulations, we choose different spreading gain N and the number of users K in order to examine the performance of the proposed detector in different system capacity and load.

Case 1: All users have equal received power.

Fig. 4.7-Fig. 4.10 show the average output SINR of the proposed cascaded modular detector when $SNR = 10\text{dB}$. The spreading gain $N = 128$ and the numbers of users $K = 16, 32, 64$ and 96 respectively. Fig. 4.11-Fig. 4.14 show the average output SINR of the proposed cascaded modular detector when $SNR = 10\text{dB}$. the spreading gain $N = 32$ and the numbers of users $K = 10, 15, 20$ and 25 respectively. The average output SINR of the blind MOE detector is also plotted in these Figures for comparison. These Figures show that when $D \leq 10$, the output SINR of the proposed detector can converge to that of the full rank blind MOE detector even in heavy system load situations as seen in Fig. 4.10 and Fig. 4.14. As a consequence, the full rank blind MOE detector's performance can be achieved by cascaded modular structure detector with stage $D \ll N$ for a wide range of

system loads and sizes.

Case 2: Near-far effect

The SNR of the desired user (user 1) is set to 10dB and the SNR of other users is set to 15dB. Fig. 4.15-Fig. 4.18 show the average output SINR of the proposed detector when the spreading gain $N = 128$ and the numbers of users $K = 16, 32, 64$ and 96 respectively. Fig. 4.19-Fig. 4.22 show the average output SINR of the proposed detector when the spreading gain $N = 32$ and the numbers of users $K = 10, 15, 20$ and 25 respectively. These simulation results show that with a few additional stages, the proposed detector is still able to achieve the same performance as that of the full rank blind MOE detector in the near-far situation.

The simulation results of experiment 1 are consistent with the analysis in Section 4.4.2. From these Figures, we can see that the proposed detector captures and reduces most interference by the first few stages by the maximum magnitude cross correlation criterion. The proposed detector can achieve good performance even in large system, high load and near-far situation.

Experiment 2: SINR of batch adaptive cascaded modular structure detector

This experiment investigates the effect of the observation data length L on the SINR performance of the batch adaptive cascaded modular structure detector. The average output SINR is plotted against different data lengths at $N = 32$ and $K = 15$. Two cases are also considered in this simulation.

Case 1: Equal received power

Fig. 4.23-Fig. 4.30 show the average output SINR of the batch adaptive cascaded modular detector at different D values ($D = 1$ to $D = 8$). When $D = 1$ to $D = 3$, Fig. 4.23-Fig. 4.25 show that the cascaded modular detector achieve the steady-state performance rapidly. However the steady-state performance at $D = 1, 2$ is very poor compared to that of the full rank blind MOE detector. The reason is that when $D = 1, 2$, the proposed detector only needs to determine one or

two weight vectors. The detector can converge fast to the steady-state with a few number of received data while the detector can't remove most of the interference.

When $D = 3$, the proposed detector has fast convergence speed, but a slightly poorer steady-state performance compared with that of the blind MOE detector. Fig. 4.26-Fig. 4.30 show that the cascaded modular detector with stage $D = 4$ to 8 can achieve steady-state performance close to that of the blind MOE detector and the convergence speed is faster than that of the full rank blind MOE detector. The reason is that the proposed cascaded modular structure detector is actually a reduced-rank detector for the blind MOE detector. The lower rank detector can converge faster than the full rank MOE detector.

Case 2: Near-far effect

Fig. 4.31-Fig. 4.38 show average output SINR of the batch adaptive cascaded modular detector for $D = 1$ to $D = 8$ when the desired user (user 1) has an input $SNR = 25\text{dB}$ and other users have an input $SNR = 30\text{dB}$. Fig. 4.31-Fig. 4.35 show the performance of the cascaded modular detector for $D = 1$ to $D = 5$. Although these detectors converge to the steady-state performance rapidly, they have severe performance loss in the steady-state compared with the full rank blind MOE detector. The steady-state performance improves when the number stages of the cascaded modular detector increases. Fig. 4.36-Fig. 4.38 show the cascaded modular detector with $D = 6$ to $D = 8$. For these cases, the proposed detector can achieve good steady-state performance and converge rapidly compared with the full rank blind MOE detector.

Experiment 3: Comparison of several reduced rank detectors

This experiment compares the SINR performance of the proposed cascaded modular structure detector with four existing reduced rank methods: the CS reduced rank method, the PC reduced rank method, the PC-PLIC method and the MSWF. The average output SINR is plotted against different data lengths and at $N = 32$, $K = 10$ and the $SNR = 20\text{dB}$ for all users.

Fig. 4.39-Fig. 4.44 show the average output SINR of the methods studied, at $D = 1$ to $D = 6$. The proposed cascaded modular structure detector achieves the optimal steady-state performance when $D = 3$. Although the three reduced rank detectors based on eigen-decomposition converge rapidly to the steady-state, the performance of the steady-state is inferior to that of the blind MOE detector and the proposed cascaded modular detector.

Fig. 4.45-Fig. 4.48 show the average output SINR when $D = 4$ for the proposed detector and $D = 7$ to $D = 10$ for PC, PC-PLIC and CS detectors. We can see from the Figures that the PC detector achieves the same steady-state output SINR as that of the proposed detector when $D = 10$, while the PC-PLIC and CS detectors achieve the same SINR when $D = 9$. The experiment shows that the proposed detector is able to achieve the same performance as that of the full rank MOE detector with much fewer stages than the PC, PC-PLIC and CS reduced rank detectors. When the four detectors all achieve the steady-state output SINR, they have nearly the same convergence speed and all have superior convergence performance compared with the full rank MOE detector.

Fig. 4.49-Fig. 4.57 give a comparison between the proposed detector and the MSWF when both have the same stages, $D = 1$ to $D = 9$. We can see from the Figures that the proposed detector has better performance than that of the MSWF and that the former can achieve the optimal output SINR when $D = 3$. When $D > 3$, the two detectors have nearly the same performance, although the proposed detector appears to converge a little slower than MSWF. When $D = 9$, both have similar convergence performance as that of the full rank MOE detector.

Experiment 4: BER performance of the proposed cascaded modular structure detector

This experiment investigates the effect of the observation data length L on the BER performance of the proposed cascaded modular structure detector. The average output BER is plotted against different data lengths at $N = 32$ and

$K = 15$. In the simulations, the output BER of blind MOE detector, CS-PLC detector and PC-PLC detector are also plotted for comparison.

Fig. 4.58 - Fig. 4.60 show the average BER performance of the proposed detector for $SNR = 10\text{dB}$, 15dB and 20dB . The simulation results show that the proposed detector converges faster than the blind MOE detector and the reduce rank detector such as CS-PLC detector and PC-PLC detector.

Figure 4.7: SINR of the proposed detector of different number of stages ($N = 128$, $K = 16$, $SNR = 10\text{dB}$)

Figure 4.8: SINR of the proposed detector of different number of stages ($N = 128$, $K = 32$, $SNR = 10\text{dB}$)

Figure 4.9: SINR of the proposed detector of different number of stages ($N = 128$, $K = 64$, $SNR = 10\text{dB}$)

Figure 4.10: SINR of the proposed detector of different number of stages ($N = 128$, $K = 96$, $SNR = 10\text{dB}$)

Figure 4.11: SINR of the proposed detector of different number of stages ($N = 32$, $K = 10$, $SNR = 10\text{dB}$)

Figure 4.12: SINR of the proposed detector of different number of stages ($N = 32$, $K = 15$, $SNR = 10\text{dB}$)

Figure 4.13: SINR of the proposed detector of different number of stages ($N = 32$, $K = 20$, $SNR = 10\text{dB}$)

Figure 4.14: SINR of the proposed detector of different number of stages ($N = 32$, $K = 25$, $SNR = 10\text{dB}$)

Figure 4.15: SINR of the proposed detector of different number of stages ($N = 128$, $K = 16$, $SNR_1 = 10\text{dB}$, $SNR_{others} = 15\text{dB}$)

Figure 4.16: SINR of the proposed detector of different number of stages ($N = 128$, $K = 32$, $SNR_1 = 10\text{dB}$, $SNR_{others} = 15\text{dB}$)

Figure 4.17: SINR of the proposed detector of different number of stages ($N = 128$, $K = 64$, $SNR_1 = 10\text{dB}$, $SNR_{others} = 15\text{dB}$)

Figure 4.18: SINR of the proposed detector of different number of stages ($N = 128$, $K = 96$, $SNR_1 = 10\text{dB}$, $SNR_{others} = 15\text{dB}$)

Figure 4.19: SINR of the proposed detector of different number of stages ($N = 32$, $K = 10$, $SNR_1 = 10\text{dB}$, $SNR_{others} = 15\text{dB}$)

Figure 4.20: SINR of the proposed detector of different number of stages ($N = 32$, $K = 15$, $SNR_1 = 10\text{dB}$, $SNR_{others} = 15\text{dB}$)

Figure 4.21: SINR of the proposed detector of different number of stages ($N = 32$, $K = 20$, $SNR_1 = 10\text{dB}$, $SNR_{others} = 15\text{dB}$)

Figure 4.22: SINR of the proposed detector of different number of stages ($N = 32$, $K = 25$, $SNR_1 = 10\text{dB}$, $SNR_{others} = 15\text{dB}$)

Figure 4.23: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR = 25\text{dB}$, $D = 1$)

Figure 4.24: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR = 25\text{dB}$, $D = 2$)

Figure 4.25: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR = 25\text{dB}$, $D = 3$)

Figure 4.26: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR = 25\text{dB}$, $D = 4$)

Figure 4.27: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR = 25\text{dB}$, $D = 5$)

Figure 4.28: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR = 25\text{dB}$, $D = 6$)

Figure 4.29: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR = 25\text{dB}$, $D = 7$)

Figure 4.30: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR = 25\text{dB}$, $D = 8$)

Figure 4.31: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR_1 = 25\text{dB}$, $SNR_{others} = 30\text{dB}$, $D = 1$)

Figure 4.32: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR_1 = 25\text{dB}$, $SNR_{others} = 30\text{dB}$, $D = 2$)

Figure 4.33: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR_1 = 25\text{dB}$, $SNR_{others} = 30\text{dB}$, $D = 3$)

Figure 4.34: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR_1 = 25\text{dB}$, $SNR_{others} = 30\text{dB}$, $D = 4$)

Figure 4.35: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR_1 = 25\text{dB}$, $SNR_{others} = 30\text{dB}$, $D = 5$)

Figure 4.36: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR_1 = 25\text{dB}$, $SNR_{others} = 30\text{dB}$, $D = 6$)

Figure 4.37: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR_1 = 25\text{dB}$, $SNR_{others} = 30\text{dB}$, $D = 7$)

Figure 4.38: SINR of batch adaptive cascaded modular structure detector for different data lengths ($N = 32$, $K = 15$, $SNR_1 = 25\text{dB}$, $SNR_{others} = 30\text{dB}$, $D = 8$)

Figure 4.39: Output SINR versus different data lengths for the comparison of cascaded modular detector with reduced-rank detectors based on eigen-decomposition

Figure 4.40: Output SINR versus different data lengths for the comparison of cascaded modular detector with reduced-rank detectors based on eigen-decomposition

Figure 4.41: Output SINR versus different data lengths for the comparison of cascaded modular detector with reduced-rank detectors based on eigen-decomposition

Figure 4.42: Output SINR versus different data lengths for the comparison of cascaded modular detector with reduced-rank detectors based on eigen-decomposition

Figure 4.43: Output SINR versus different data lengths for the comparison of cascaded modular detector with reduced-rank detectors based on eigen-decomposition

Figure 4.44: Output SINR versus different data lengths for the comparison of cascaded modular detector with reduced-rank detectors based on eigen-decomposition

Figure 4.45: Output SINR versus different data lengths for the comparison of cascaded modular detector with reduced-rank detectors based on eigen-decomposition

Figure 4.46: Output SINR versus different data lengths for the comparison of cascaded modular detector with reduced-rank detectors based on eigen-decomposition

Figure 4.47: Output SINR versus different data lengths for the comparison of cascaded modular detector with reduced-rank detectors based on eigen-decomposition

Figure 4.48: Output SINR versus different data lengths for the comparison of cascaded modular detector with reduced-rank detectors based on eigen-decomposition

Figure 4.49: Output SINR versus different data lengths for the comparison of cascaded modular detector with MSWF

Figure 4.50: Output SINR versus different data lengths for the comparison of cascaded modular detector with MSWF

Figure 4.51: Output SINR versus different data lengths for the comparison of cascaded modular detector with MSWF

Figure 4.52: Output SINR versus different data lengths for the comparison of cascaded modular detector with MSWF

Figure 4.53: Output SINR versus different data lengths for the comparison of cascaded modular detector with MSWF

Figure 4.54: Output SINR versus different data lengths for the comparison of cascaded modular detector with MSWF

Figure 4.55: Output SINR versus different data lengths for the comparison of cascaded modular detector with MSWF

Figure 4.56: Output SINR versus different data lengths for the comparison of cascaded modular detector with MSWF

Figure 4.57: Output SINR versus different data lengths for the comparison of cascaded modular detector with MSWF

Figure 4.58: Output BER versus different data lengths for the comparison of cascaded modular detector with reduced rank detectors (CS-PLC and PC-PLC)

Figure 4.59: Output BER versus different data lengths for the comparison of cascaded modular detector with reduced rank detectors (CS-PLC and PC-PLC)

Figure 4.60: Output BER versus different data lengths for the comparison of cascaded modular detector with reduced rank detectors (CS-PLC and PC-PLC)

4.6 Conclusions

In this chapter, we propose a cascaded modular detector to approximate the full rank blind MOE detector. The cascaded modular detector is composed of several modules and each module is a one dimensional vector adaptive filter. The weight vector is determined by the maximum magnitude cross correlation criteria which can cancel most of the interferences presented in the output of the preceding module. Simulation results show that the proposed cascaded modular detector is able to capture and reduce most interference by using only the first few stage weight vector filters and achieve a performance close to that of the conventional blind MOE detector even in large systems, high load and near-far situation. The proposed detector is able to achieve the same performance as that of the full rank MOE detector with fewer stages compared with PC, PC-PLIC and CS detectors. Simulation results show that both the proposed detector and MFWF detector have nearly the same performance when the number of stages is large, while the proposed detector has better performance than that of the MFWF detector when the number of stages is small. The simulation results also show that the proposed detector converge faster than other existing reduce rank detectors. The one dimension weight vector can be easily computed for each module. The proposed detector is very flexible for the implementation. Because the proposed detector is a cascading serially structure, we can easily choose the stages by adding a constrained weight vector at each module's output. Synchronization of the desired user's spreading code must be guaranteed for the implementation of the proposed detector.

Chapter 5

Blind RAKE Structure Cascaded Modular Detector

5.1 Background

In Chapter 4, we propose a cascaded modular detector to implement the blind MOE detector with reduced rank signal subspace. In AWGN channel, if the accurate timing acquisition of the desired user is available, the cascaded modular detector can achieve the blind MOE detector's performance when stage $D < 10$ as is shown by the simulation results in Chapter 4. Frequency selective fading channel often exists in the wireless mobile communications systems and the multipath effect will cause signature distortion. The constrained optimization approaches including the proposed cascaded modular detector are very sensitive to the possible signature mismatch and will have great performance degradation.

Sveral approaches have been proposed to overcome the channel dispersive problem. Subspace method was proposed to mitigate the problem by constraining the solution to the signal subspace to reduce signal cancellation [65], [66]. Another approach for the dispersive channel case was presented in [67], [68], where the receiver's response to all delayed copies of the desired user's signal is forced

to zero by adding more constraints. Although the additional constraints introduced in [67] alleviate the problem of signal cancellation, the method has inferior performance because it only exploits part of the signal energy. An improvement was proposed in [69]-[71] which jointly minimizes the interference and maximizes the signal component at the receiver's output. The performance of the improvement method has been shown to be close to that of the trained MMSE receiver at high signal to noise (SNR). However, the complexity is much high due to the need to perform eigenvalue decomposition. The adaptive algorithms for the max-min method are discussed in [72] in which two coupled least mean square recursions are implemented to reduce the computation complexity. In multipath fading environments, most of the detectors discussed above require the knowledge of the channel information which can be estimated blindly or by using pilot signals. However, the blind methods may be computationally expensive in practice and the use of pilot signals degrades the spectrum efficiency.

In this thesis, a RAKE structure cascaded modular detector is proposed to handle the signature mismatch problem. The receiver here only needs to know the desired user's spreading code and it does not need to estimate the channel characteristics. The proposed cascaded detector in Chapter 4 is used in each arm of the RAKE receiver to extract part of the desired signal and the output of all arms is coherently combined to detect the desired signal.

In the remaining parts of this chapter, Section 5.2 describes the multipath dispersive CDMA channel model, Section 5.3 gives a simple introduction of maximum-minimum method and the corresponding adaptive algorithms, Section 5.4 discusses the design of the blind RAKE structure cascaded modular detector, Section 5.5 provides some simulation results to show the performance of the proposed detector, A summary of the findings is given in the conclusions in Section 5.6.

5.2 Multipath Fading CDMA Channel Model

5.2.1 Channel Models

When signal $x(t)$ is transmitted through time-varying multipath fading channel of mobile wireless communications, the received signal $y(t)$ can be described by the following expression:

$$y(t) = \int_{-\infty}^{\infty} g(\tau, t)x(t - \tau)d\tau \quad (5.1)$$

where $g(\tau, t)$ is the fading channel impulse response. If the fading channel assumes a finite number (M) of the multipath components then the model in (5.1) reduces to

$$y(t) = \sum_{m=1}^M g(\tau_m, t)x(t - \tau_m) \quad (5.2)$$

The transmission model described by (5.1) identifies both the channel delay profile and the channel time variations. The channel delay profile determines the degree of frequency selectivity or memory of the channel and is represented by the parameter τ . On the other hand, the time variations of the fading are represented in the channel impulse response by the parameter t . The coefficients $g(\tau, t)$ are modelled as random process to reflect the randomness of the fading process time variations. In the following, we will give three channels as the special case of the model in (5.1). The first one is flat-flat fading channel which means memoryless time invariant. The channel is characterized by the input/output equation

$$y(t) = gx(t) \quad (5.3)$$

The second one is frequency-flat fading channel which means memoryless time varying. The channel satisfies the input/output relation described by

$$y(t) = g(t)x(t) \quad (5.4)$$

The third one is frequency-selective time-invariant fading channel characterized by the input/output equation

$$y(t) = \int_{-\infty}^{\infty} g(\tau)x(t - \tau)d\tau \quad (5.5)$$

5.2.2 Multipath Fading CDMA Channel Model

Consider a K -user synchronous DS-CDMA system. The received continuous time baseband signal can be represented as

$$y(t) = \sum_{n=0}^{\infty} \sum_{k=1}^K b_k(n)h_k(t - nT_s) + v(t) \quad (5.6)$$

where

$b_k(n)$ n th data symbol of the k th user;
 $h_k(t)$ effective signature waveform of the k th user;
 $v(t)$ additive white Gaussian noise (AWGN) with power spectral density σ_v^2 ;

T_s symbol period;

It is assumed that the data symbols are zero mean, independently and identically distributed (i.i.d) BPSK modulation.

The effective signature waveform is the convolution of the user's spreading code $\mathbf{c}_k$ and the multipath channel impulse response $g_{ck}(t)$,

$$h_k(t) = \sum_{l=0}^{N-1} c_k(l)g_{ck}(t - lT_c) \quad (5.7)$$

$g_{ck}(t)$ characterizes the channel propagation effects including the timing offset, delays, multipath reflections, etc. $h_k(t)$ can be realized as a time invariant finite impulse response (FIR) filter.

For convenience, we assume that user 1 is the desired user and that the spreading code $\mathbf{c}_1$ of user 1 and the timing of the first multipath component for user 1 are known. At the receiver, the received signal is sampled at the chip rate. The chip rate discrete time signal of (5.1) is given by

$$\mathbf{y}(n) = b_1(n)\mathbf{h}_1 + \sum_{k=2}^K b_k(n)\mathbf{h}_k + \mathbf{v}(n) \quad (5.8)$$

where

$$\mathbf{h}_k = [h_k(t)|_{t=0} \quad h_k(t)|_{t=T_c} \quad \cdots \quad h_k(t)|_{t=(N+M-1)T_c}]^T \quad (5.9)$$

$\mathbf{h}_k$ is the k th user's effective signature vector. M is the number of the multipath, and N is the length of the spreading sequence. The number of the multipath for all users is assumed to be the same.

The effective signature waveform $\mathbf{h}_1$ of user 1 can be determined from the known spreading code $\mathbf{c}_1$ and the unknown channel characteristic vector $\mathbf{g}_{c1}$ as

$$\mathbf{h}_1 = \mathbf{C}_1 \mathbf{g}_{c1} \quad (5.10)$$

where $\mathbf{C}_1$ is a $(N + M - 1) \times M$ matrix

$$\mathbf{C}_1 = \begin{pmatrix} c_1(0) & \mathbf{0} & & \\ \vdots & \ddots & c_1(0) & \\ c_1(N-1) & \ddots & \vdots & \\ \mathbf{0} & & c_1(N-1) & \end{pmatrix} \quad (5.11)$$

and $\mathbf{g}_{c1}$ is a $M \times 1$ vector

$$\mathbf{g}_{c1} = [g_{c1}(t)|_{t=0} \quad g_{c1}(t)|_{t=T_c} \quad g_{c1}(t)|_{t=(M-1)T_c}]^T \quad (5.12)$$

In flat-flat fading channel case, it means the $M = 1$. Then the effective signature waveform of user 1 is

$$\mathbf{h}_1 = g_1 \mathbf{c}_1 \quad (5.13)$$

where $\mathbf{c}_1$ is the spreading sequence of user 1.

5.3 Maximum-Minimum Method

In this section, we introduce a blind MOE detector based on the maximum-minimum method and the corresponding adaptive algorithm [69]-[72]. The max-min method combines all multipath components of the desired signal and jointly minimizes the interference and maximizes the signal component at the receiver output. The method exhibits superior performance that can approach that of the trained MMSE receiver under some circumstances.

If the effective signature sequence of the desired user (user 1) is known, the linear weight vector $\mathbf{w}_{MOE}$ of the blind MOE detector can be derived by solving the following constrained optimization problem

$$\min_{\mathbf{w}_{MOE}} \mathbf{w}_{MOE}^H \mathbf{R}_{yy} \mathbf{w}_{MOE} \quad \text{subject to} \quad \mathbf{w}_{MOE}^H \mathbf{h}_1 = 1 \quad (5.14)$$

The solution of the constrained optimization problem is given by

$$\mathbf{w}_{MOE} = (\mathbf{h}_1^H \mathbf{R}_{yy}^{-1} \mathbf{h}_1)^{-1} \mathbf{R}_{yy}^{-1} \mathbf{h}_1 \quad (5.15)$$

and the minimum output energy is

$$MOE(\mathbf{h}_1) = \frac{1}{\mathbf{h}_1^H \mathbf{R}_{yy}^{-1} \mathbf{h}_1} \quad (5.16)$$

From (5.10), we know that $\mathbf{h}_1$ depends on the multipath fading parameters $\mathbf{g}_1$ which is generally not known. To avoid the explicit estimation of $\mathbf{g}_1$, the maximum-minimum method is proposed to select $\mathbf{g}_1$ to maximize the $MOE(\mathbf{h}_1)$ in (5.16) subject to the constrained $\|\mathbf{g}_1\| = 1$. We can get the optimization problem by substituting $\mathbf{h}_1 = \mathbf{C}_1 \mathbf{g}_1$ in (5.16), the multipath fading parameters $\mathbf{g}_1$ can be obtained by

$$\mathbf{g}_1 = \arg \max_{\mathbf{g}_1} \frac{\mathbf{g}_1^H \mathbf{g}_1}{\mathbf{g}_1^H \mathbf{C}_1^H \mathbf{R}_{yy}^{-1} \mathbf{C}_1 \mathbf{g}_1} \quad (5.17)$$

Equation (5.17) represents a Rayleigh quotient. Hence, the solution corresponds to the principal eigenvector of the matrix $\mathbf{C}_1^H \mathbf{R}_{yy}^{-1} \mathbf{C}_1$. Because $\mathbf{R}_{yy}$ is a $(N + M - 1) \times (N + M - 1)$ matrix, the inverse of $\mathbf{R}_{yy}$ may be computationally too complex to the system with large spreading factor N . Therefore, the adaptive algorithms [72] are derived to estimate $\mathbf{w}_{MOE}$ and $\mathbf{g}_1$. Here we give a simple introduction of blind adaptive GSC algorithm for the maximum-minimum method.

By using the GSC structure, the weight vector $\mathbf{w}_{MOE}$ of constrained optimization problem in (5.15) can be solved by unconstrained problem and given by [63]

$$\mathbf{w}_{MOE} = \mathbf{C}_1 (\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{g}_1 - \bar{\mathbf{C}}_1 \mathbf{u} \quad (5.18)$$

where $\bar{\mathbf{C}}_1$ is the orthogonal complement of the space spanned by the columns of $\mathbf{C}_1$, while $\mathbf{u}$ is the non-constrained adaptive weight vector. Then the cost function J can be expressed as

$$\begin{aligned} J = & \mathbf{g}_1^H (\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{C}_1^H \mathbf{R}_{yy} \mathbf{C}_1 (\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{g}_1 + \mathbf{u}^H \bar{\mathbf{C}}_1^H \mathbf{R}_{yy} \bar{\mathbf{C}}_1 \mathbf{u} \\ & - \mathbf{g}_1^H (\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{C}_1^H \mathbf{R}_{yy} \bar{\mathbf{C}}_1 \mathbf{u} - \mathbf{u}^H \bar{\mathbf{C}}_1 \mathbf{R}_{yy} \mathbf{C}_1 (\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{g}_1 \end{aligned} \quad (5.19)$$

Minimizing J with respect to $\mathbf{u}$ and maximizing J with respect to $\mathbf{g}_1$, we can get two update equations for $\mathbf{u}$ and $\mathbf{g}_1$, respectively

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \mu_u \nabla_{\mathbf{u}} J \quad (5.20)$$

$$\mathbf{g}_1(n+1) = \mathbf{g}_1(n) - \mu_{g_1} \nabla_{\mathbf{g}_1} J \quad (5.21)$$

Then, we can get the adaptive algorithm as

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \mu_u \bar{\mathbf{C}}_1^H \mathbf{R}_{yy} \bar{\mathbf{C}}_1 \mathbf{u}(n) + \mu_u \bar{\mathbf{C}}_1^H \mathbf{R}_{yy} \mathbf{C}_1 (\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{g}_1(n) \quad (5.22)$$

$$\begin{aligned} \mathbf{g}_1(n+1) &= \mathbf{g}_1(n) + \mu_{g_1} (\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{C}_1^H \mathbf{R}_{yy} \mathbf{C}_1 (\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{g}_1(n) \\ &\quad - \mu_{g_1} (\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{C}_1^H \mathbf{R}_{yy} \bar{\mathbf{C}}_1 \mathbf{u}(n) \end{aligned} \quad (5.23)$$

$$\mathbf{g}_1(n+1) \leftarrow \frac{\mathbf{g}_1(n+1)}{\|\mathbf{g}_1(n+1)\|} \quad (5.24)$$

$$\mathbf{w}_{MOE}(n+1) = \mathbf{C}_1 (\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{g}_1(n+1) - \bar{\mathbf{C}}_1 \mathbf{u}(n+1) \quad (5.25)$$

5.4 RAKE Structure Cascaded Modular Detector

In this section, we design a novel blind receiver that combines cascaded modular structure detector with RAKE structure. The RAKE structure utilizes most of the signal energy by suitably combining multipath signals obtained from the output covariance matrix of the RAKE receiver. One advantage of this method is that the receiver does not need to explicitly estimate the effective signature waveform of the desired user and is thus suitable for implementation in practice.

The blind RAKE receiver shown in Fig. 5.1 processes the signals in two steps. In the first step, each arm of the RAKE receiver extracts the desired signal and

suppresses noise and interference from other users by using constrained cascaded modular interference cancellation detector. In the second step, the extracted multipath signals are combined to complete the detection process. Appropriate combination of the multipath component of the desired signal can improve the signal to interference and noise ratio (SINR) for the desired user, which will in turn improve the overall system performance.

Figure 5.1: The structure of the blind RAKE receiver

For the i th arm, the constraint for the blind MOE detector is given by

$$\mathbf{C}_1^H \mathbf{w}_{MOE}^i = \mathbf{f}^i \quad (5.26)$$

where $\mathbf{f}^i$ is the $M \times 1$ constrained vector with all elements equal to 0 except 1 at the i th position.

From the discussion in Chapter 4, we can implement the cascaded modular detector in i th arm in the following equation.

$$\mathbf{w}_c^i = \mathbf{C}_1 (\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{f}^i \quad (5.27)$$

$$\mathbf{g}_\ell^i = \frac{(\mathbf{I} - \mathbf{C}_1(\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{C}_1^H) \mathbf{R}_{y_{\ell-1}^i y_{\ell-1}^i} \mathbf{w}_c^i}{\|(\mathbf{I} - \mathbf{C}_1(\mathbf{C}_1^H \mathbf{C}_1)^{-1} \mathbf{C}_1^H) \mathbf{R}_{y_{\ell-1}^i y_{\ell-1}^i} \mathbf{w}_c^i\|_2} \quad (5.28)$$

$$\mathbf{U}_\ell^i = \frac{\mathbf{g}_\ell^{iH} \mathbf{R}_{y_{\ell-1}^i y_{\ell-1}^i}}{\mathbf{g}_\ell^{iH} \mathbf{R}_{y_{\ell-1}^i y_{\ell-1}^i} \mathbf{g}_\ell^i} \quad (5.29)$$

$$\mathbf{y}_\ell^i = (\mathbf{I} - g_\ell^i \mathbf{U}_\ell^i)^H \mathbf{y}_{\ell-1}^i \quad (5.30)$$

where

$$\mathbf{y}_0^i = \mathbf{y} \quad (5.31)$$

The output of the i th arm is

$$x^i = \mathbf{w}_c^{iH} \mathbf{y}_D^i \quad (5.32)$$

The output vector from all the arms is given in (5.33)

$$\mathbf{x} = \mathbf{h}_1 b_1 + \mathbf{I} \quad (5.33)$$

Thus, the correlation matrix $\mathbf{R}_{xx}$ can be obtained as

$$\mathbf{R}_{xx} = \mathbf{h}_1 \mathbf{h}_1^H + \mathbf{R}_{II} \quad (5.34)$$

After the first step, the interference and noise in the output signal from each arm is assumed to have been suppressed significantly. The correlation matrix can now be approximated as

$$\mathbf{R}_{xx} \approx \mathbf{h}_1 \mathbf{h}_1^H \quad (5.35)$$

This allows us to approximate the optimum combining vector using the principal eigenvector of the covariance matrix of the outputs of all the arms. The approximation gives a good tradeoff between optimality and complexity and is able to produce very good performance at high signal to noise ratio (SNR).

5.5 Numerical Results

In this section, some experiments are conducted to study the performance of the proposed RAKE structure cascaded modular detector. In all the experiments, frequency selective fading channels are simulated by using the Hoeher's method [74], where the discrete channel is modeled as tapped delay line with time varying coefficients. We assume the multipath fading channel has three paths for all users in the experiments so that the number of arm in the RAKE receiver, M , is 3. In the experiments, six group channel coefficients have been chosen to study the performance of the proposed detector in different channel situations. The first two group channel coefficients, $(\mathbf{g}_1 = [0.7201 \ 0.0044 \ 0.0046]^H$ and $\mathbf{g}_1 = [0.2378 \ 0.1317 \ 0.0029]^H)$, mean that the channel does not have strong fading. The other 4 group channel coefficients mean that the channel has strong fading. $(\mathbf{g}_1 = [0.076 \ 0.0772 \ 0.0015]^H$, $\mathbf{g}_1 = [0.1286 \ 0.0835 \ 0.0001]^H$, $\mathbf{g}_1 = [0.0808 \ 0.1208 \ 0.0033]^H$ and $\mathbf{g}_1 = [0.12 \ 0.0215 \ 0.0067]^H)$

Experiment 1: BER performance

This experiment examines the BER performance of the RAKE cascaded modular receiver. In the simulation, we compare the performance of four types of receivers, namely, the trained MMSE receiver, the single arm cascaded modular receiver (corresponding to the first path), the RAKE constrained MOE receiver and the RAKE cascaded modular receiver. We assume that all the receivers achieve the steady-state performance in the simulation. The number of users, $K = 20$ and the spreading code is randomly chosen with a spreading factor $N = 32$. The accurate timing of the first path of the desired user is assumed available. The stage D of the cascaded modular detector in each arm is 8. All users have the same input SNR.

In the experiment, six groups of channel coefficient are given to examine the performance of the RAKE cascaded modular receiver. Table 5.1-Table 5.6 show the estimated coherent combining vectors $\hat{\mathbf{g}}_{1MOE}$ obtained by the RAKE constrained

MOE receiver and $\hat{\mathbf{g}}_{1CM}$ obtained by the RAKE cascaded modular receiver for the six groups of channel coefficient. Fig. 5.2-Fig. 5.7 show the BER performance of the four receivers for the six groups of channel coefficient.

For channel coefficient $\mathbf{g}_1 = [0.7201 \ 0.0044 \ 0.0046]^H$, the main energy of the signal is in path one and no strong fading exists during the data transmission. Table 5.1 shows the estimated coherent combining vector for $SNR_1 = 2\text{dB}, 8\text{dB}, 14\text{dB}, 20\text{dB}$ respectively. We can see that the estimated coherent combining vectors of both receivers approach the normalized channel coefficient with the increase in SNR. From Fig. 5.2, the RAKE MOE receiver, the RAKE cascaded modular receiver and the single arm (path=1) cascaded modular receiver have nearly the same BER performance and all achieve the performance of the trained MMSE receiver for the group channel coefficient.

For channel coefficient $\mathbf{g}_1 = [0.2378 \ 0.1317 \ 0.0029]^H$, the energy of signal is mainly in path one and path two. The coefficient of path two is half of that of path one. No strong fading exists in the transmission for the group coefficient. Table 5.2 show that the coherent combining vector is not consistent with the channel coefficient for $SNR_1 = 2\text{dB}$ and 8dB , but can approach that for $SNR_1 = 14\text{dB}$ and 20dB . Therefore, BER performance in Fig. 5.3 shows that the RAKE MOE receiver and the RAKE cascaded modular receiver have nearly the same performance as the single arm (path=1) cascaded modular receiver in low SNR, and the RAKE MOE receiver and the RAKE cascaded modular receiver can achieve the trained MMSE receiver's performance while the single arm (path=1) cascaded modular receiver has great performance degradation in high SNR.

For channel coefficient $\mathbf{g}_1 = [0.076 \ 0.0772 \ 0.0015]^H$, a strong fading exists during the data transmission and the energy of signal is mainly in path one and path two. The coefficient of the two paths is nearly the same. In this case, from Table 5.3, we can see that the coherent combining vector can not be estimated accurately in low SNR and the vector only approaches the channel coefficient

for $SNR_1 = 20\text{dB}$. Fig. 5.4 shows that the RAKE MOE receiver, the RAKE cascaded modular receiver and the single arm (path=1) cascaded modular receiver have nearly the same performance and have performance degradation compared with the trained MMSE receiver in low SNR. With the increase of SNR, the RAKE MOE receiver and the RAKE cascaded modular receiver approach the performance of the trained MMSE receiver and have better performance than the single arm (path=1) cascaded modular receiver.

For channel coefficient $\mathbf{g}_1 = [0.1286 \quad 0.0835 \quad 0.0001]^H$, a strong fading exists during the data transmission and the energy of signal is mainly in path one and path two. The coefficient of path two is less than that of path one. We can also see that the coherent combining vector can only approach to the channel coefficient in high SNR from Table 5.4. Fig. 5.5 shows that the performance of the single arm (path=1) cascaded modular receiver can have better performance than the RAKE MOE receiver and the RAKE cascaded modular receiver in low SNR situation. With the increase of SNR, the RAKE MOE receiver and the RAKE cascaded modular receiver approach the performance of the trained MMSE receiver. While the single arm (path=1) cascaded modular receiver will have great performance loss compared with trained MMSE receiver.

For channel coefficient $\mathbf{g}_1 = [0.0808 \quad 0.1208 \quad 0.0033]^H$, a strong fading exists during the data transmission and the energy of signal is mainly in path one and path two. The coefficient of the path one is less than that of the path two. Fig. 5.6 shows that while the performance of the RAKE MOE receiver and the RAKE cascaded modular receiver approach to that of trained MMSE receiver with the increase in SNR, the single arm (path=1) cascaded modular receiver will have great performance loss.

For channel coefficient $\mathbf{g}_1 = [0.12 \quad 0.0215 \quad 0.0067]^H$, a strong fading exists during the data transmission and the energy of signal is mainly in path one. Fig. 5.7 shows that the RAKE MOE receiver, the RAKE cascaded modular receiver

and the single arm (path=1) cascaded modular receiver have nearly the same BER performance and all achieve the performance of the trained MMSE receiver. While the single arm (path=1) cascaded modular receiver has better performance to some extent in low SNR, but worse performance to some extent in high SNR compared with the RAKE optimal MOE receiver and the RAKE cascaded modular receiver.

The experiment shows that the RAKE cascaded modular receiver has nearly the same performance as the RAKE MOE receiver, but the cascaded modular detector in each arm has simpler structure and lower computation complexity than optimal MOE detector. The single arm cascaded modular receiver has nearly the same or even better performance compared with the RAKE MOE receiver and the RAKE cascaded modular receiver in low SNR situations, but will have great performance loss in high SNR situations if the energy of signal is not mainly in the chosen path.

Table 5.1: Coherent combining vector ($\mathbf{g}_1 = [0.7201 \ 0.0044 \ 0.0046]^H$, $\mathbf{g}_{1norm} = [1 \ 0.0061 \ 0.0064]^H$)

$SNR_1(dB)$	2	8	14	20
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.9694 \\ 0.2147 \\ 0.1191 \end{pmatrix}$	$\begin{pmatrix} 0.9984 \\ 0.0482 \\ 0.0305 \end{pmatrix}$	$\begin{pmatrix} 0.9998 \\ 0.0121 \\ 0.0165 \end{pmatrix}$	$\begin{pmatrix} 1.0000 \\ 0.0000 \\ 0.0087 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.9694 \\ 0.2147 \\ 0.1191 \end{pmatrix}$	$\begin{pmatrix} 0.9984 \\ 0.0480 \\ 0.0303 \end{pmatrix}$	$\begin{pmatrix} 0.9998 \\ 0.0118 \\ 0.0162 \end{pmatrix}$	$\begin{pmatrix} 1.0000 \\ 0.0015 \\ 0.0097 \end{pmatrix}$

Figure 5.2: BER of the RAKE cascaded modular detector ($N = 32$, $K = 20$)

Table 5.2: Coherent combining vector ($\mathbf{g}_1 = [0.2378 \ 0.1317 \ 0.0029]^H$, $\mathbf{g}_{1norm} = [0.8747 \ 0.4845 \ 0.0107]^H$)

$SNR_1(dB)$	2	8	14	20
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.7149 \\ 0.4935 \\ 0.4953 \end{pmatrix}$	$\begin{pmatrix} 0.9766 \\ 0.1800 \\ 0.1181 \end{pmatrix}$	$\begin{pmatrix} 0.8902 \\ 0.4552 \\ 0.0169 \end{pmatrix}$	$\begin{pmatrix} 0.8737 \\ 0.4865 \\ 0.0078 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.7149 \\ 0.4935 \\ 0.4953 \end{pmatrix}$	$\begin{pmatrix} 0.9766 \\ 0.1796 \\ 0.1181 \end{pmatrix}$	$\begin{pmatrix} 0.8894 \\ 0.4569 \\ 0.0162 \end{pmatrix}$	$\begin{pmatrix} 0.8721 \\ 0.4892 \\ 0.0079 \end{pmatrix}$

Figure 5.3: BER of the RAKE cascaded modular detector ($N = 32$, $K = 20$)

Table 5.3: Coherent combining vector ($\mathbf{g}_1 = [0.076 \ 0.0772 \ 0.0015]^H$, $\mathbf{g}_{1norm} = [0.7015 \ 0.7126 \ 0.0138]^H$)

$SNR_1(dB)$	2	8	14	20
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.8986 \\ 0.0313 \\ 0.4376 \end{pmatrix}$	$\begin{pmatrix} 0.9216 \\ 0.1304 \\ 0.3656 \end{pmatrix}$	$\begin{pmatrix} 0.8198 \\ 0.4189 \\ 0.3905 \end{pmatrix}$	$\begin{pmatrix} 0.7737 \\ 0.5969 \\ 0.2123 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.8986 \\ 0.0313 \\ 0.4376 \end{pmatrix}$	$\begin{pmatrix} 0.9215 \\ 0.1303 \\ 0.3658 \end{pmatrix}$	$\begin{pmatrix} 0.8193 \\ 0.4182 \\ 0.3921 \end{pmatrix}$	$\begin{pmatrix} 0.7771 \\ 0.5863 \\ 0.2289 \end{pmatrix}$

Figure 5.4: BER of the RAKE cascaded modular detector ($N = 32$, $K = 20$)

Table 5.4: Coherent combining vector ($\mathbf{g}_1 = [0.1286 \ 0.0835 \ 0.0001]^H$, $\mathbf{g}_{1norm} = [0.8387 \ 0.5446 \ 0.0007]^H$)

$SNR_1(dB)$	2	8	14	20
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.0965 \\ 0.6011 \\ 0.7934 \end{pmatrix}$	$\begin{pmatrix} 0.3165 \\ 0.4479 \\ 0.8362 \end{pmatrix}$	$\begin{pmatrix} 0.8150 \\ 0.5336 \\ 0.2258 \end{pmatrix}$	$\begin{pmatrix} 0.8363 \\ 0.5467 \\ 0.0415 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.0965 \\ 0.6011 \\ 0.7934 \end{pmatrix}$	$\begin{pmatrix} 0.3162 \\ 0.4479 \\ 0.8363 \end{pmatrix}$	$\begin{pmatrix} 0.8137 \\ 0.5348 \\ 0.2278 \end{pmatrix}$	$\begin{pmatrix} 0.8347 \\ 0.5489 \\ 0.0441 \end{pmatrix}$

Figure 5.5: BER of the RAKE cascaded modular detector ($N = 32$, $K = 20$)

Table 5.5: Coherent combining vector ($\mathbf{g}_1 = [0.0808 \ 0.1208 \ 0.0033]^H$, $\mathbf{g}_{1norm} = [0.5558 \ 0.8310 \ 0.0227]^H$)

$SNR_1(dB)$	2	8	14	20
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.4502 \\ 0.4547 \\ 0.7685 \end{pmatrix}$	$\begin{pmatrix} 0.5478 \\ 0.5708 \\ 0.6116 \end{pmatrix}$	$\begin{pmatrix} 0.5420 \\ 0.8031 \\ 0.2476 \end{pmatrix}$	$\begin{pmatrix} 0.5510 \\ 0.8340 \\ 0.0291 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.4502 \\ 0.4547 \\ 0.7685 \end{pmatrix}$	$\begin{pmatrix} 0.5480 \\ 0.5708 \\ 0.6115 \end{pmatrix}$	$\begin{pmatrix} 0.5407 \\ 0.8045 \\ 0.2459 \end{pmatrix}$	$\begin{pmatrix} 0.5575 \\ 0.8299 \\ 0.0199 \end{pmatrix}$

Figure 5.6: BER of the RAKE cascaded modular detector ($N = 32$, $K = 20$)

Table 5.6: Coherent combining vector ($\mathbf{g}_1 = [0.12 \ 0.0215 \ 0.0067]^H$, $\mathbf{g}_{1norm} = [0.9828 \ 0.1761 \ 0.0549]^H$)

$SNR_1(dB)$	2	8	14	20
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.6729 \\ 0.2910 \\ 0.6800 \end{pmatrix}$	$\begin{pmatrix} 0.7054 \\ 0.2168 \\ 0.6749 \end{pmatrix}$	$\begin{pmatrix} 0.8465 \\ 0.0472 \\ 0.5302 \end{pmatrix}$	$\begin{pmatrix} 0.9675 \\ 0.1085 \\ 0.2283 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.6729 \\ 0.2910 \\ 0.6800 \end{pmatrix}$	$\begin{pmatrix} 0.7053 \\ 0.2170 \\ 0.6748 \end{pmatrix}$	$\begin{pmatrix} 0.8502 \\ 0.0392 \\ 0.5250 \end{pmatrix}$	$\begin{pmatrix} 0.9627 \\ 0.1557 \\ 0.2214 \end{pmatrix}$

Figure 5.7: BER of the RAKE cascaded modular detector ($N = 32$, $K = 20$)

Experiment 2: Near-far effect

The experiment examines the influence of near-far effect to the performance of the RAKE cascaded modular receiver. In the simulation, the number of users, $K = 10$ and the spreading code is randomly chosen with a spreading factor $N = 32$. The SNR of the desired user 1 is 10dB and the interference user's SNR varies from 10dB below the desired user to 20dB above the desired user. We also compare the performance of four types of receivers, namely, the trained MMSE receiver, the single arm cascaded modular receiver (corresponding to the first path), the RAKE MOE receiver and the RAKE cascaded modular receiver. The stage D of the cascaded modular detector in each arm is 8.

Table 5.7-Table 5.12 give the estimated coherent combining vectors $\hat{\mathbf{g}}_{1MOE}$ and $\hat{\mathbf{g}}_{1CM}$ for the six groups of channel coefficient in the experiment 1. Fig. 5.8-Fig. 5.13 show the BER performance of the four receivers for the six groups of channel coefficient. From the Tables and Figures, we can see that all the receivers have good ability to resist the near-far effect. In strong interference and fading situations, the RAKE cascaded modular receiver has great performance loss compared with the RAKE MOE receiver because the cascaded modular detector in each arm need more stage to cancel the interference. If the interference can't be cancelled significantly, the estimate coherent combining vector will not be accurate to approach the channel coefficients.

Table 5.7: Coherent combining vector ($\mathbf{g}_1 = [0.7201 \ 0.0044 \ 0.0046]^H$, $\mathbf{g}_{1norm} = [1 \ 0.0061 \ 0.0064]^H$)

$ISR(dB)$	-10	0	10	20
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.9996 \\ 0.0260 \\ 0.0129 \end{pmatrix}$	$\begin{pmatrix} 0.9992 \\ 0.0288 \\ 0.0261 \end{pmatrix}$	$\begin{pmatrix} 0.9993 \\ 0.0288 \\ 0.0236 \end{pmatrix}$	$\begin{pmatrix} 0.9993 \\ 0.0306 \\ 0.0235 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.9996 \\ 0.0260 \\ 0.0130 \end{pmatrix}$	$\begin{pmatrix} 0.9992 \\ 0.0290 \\ 0.0264 \end{pmatrix}$	$\begin{pmatrix} 0.9993 \\ 0.0285 \\ 0.0242 \end{pmatrix}$	$\begin{pmatrix} 0.9992 \\ 0.0311 \\ 0.0241 \end{pmatrix}$

Figure 5.8: BER of the RAKE cascaded modular detector ($N = 32$, $K = 10$, $SNR_1 = 10dB$)

Table 5.8: Coherent combining vector ($\mathbf{g}_1 = [0.2378 \ 0.1317 \ 0.0029]^H$, $\mathbf{g}_{1norm} = [0.8747 \ 0.4845 \ 0.0107]^H$)

$ISR(dB)$	-10	0	10	20
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.9121 \\ 0.4092 \\ 0.0243 \end{pmatrix}$	$\begin{pmatrix} 0.9156 \\ 0.4015 \\ 0.0224 \end{pmatrix}$	$\begin{pmatrix} 0.9163 \\ 0.4000 \\ 0.0209 \end{pmatrix}$	$\begin{pmatrix} 0.9207 \\ 0.3895 \\ 0.0221 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.9120 \\ 0.4095 \\ 0.0241 \end{pmatrix}$	$\begin{pmatrix} 0.9157 \\ 0.4012 \\ 0.0230 \end{pmatrix}$	$\begin{pmatrix} 0.9163 \\ 0.3998 \\ 0.0211 \end{pmatrix}$	$\begin{pmatrix} 0.9226 \\ 0.3853 \\ 0.0188 \end{pmatrix}$

Figure 5.9: BER of the RAKE cascaded modular detector ($N = 32$, $K = 10$, $SNR1 = 10dB$)

Table 5.9: Coherent combining vector ($\mathbf{g}_1 = [0.076 \ 0.0772 \ 0.0015]^H$, $\mathbf{g}_{1norm} = [0.7015 \ 0.7126 \ 0.0138]^H$)

$ISR(dB)$	-10	0	10	20
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.7086 \\ 0.6651 \\ 0.2357 \end{pmatrix}$	$\begin{pmatrix} 0.6779 \\ 0.7233 \\ 0.1311 \end{pmatrix}$	$\begin{pmatrix} 0.6695 \\ 0.7323 \\ 0.1246 \end{pmatrix}$	$\begin{pmatrix} 0.6624 \\ 0.7383 \\ 0.1273 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.7086 \\ 0.6650 \\ 0.2357 \end{pmatrix}$	$\begin{pmatrix} 0.6782 \\ 0.7227 \\ 0.1334 \end{pmatrix}$	$\begin{pmatrix} 0.6683 \\ 0.7337 \\ 0.1226 \end{pmatrix}$	$\begin{pmatrix} 0.6392 \\ 0.7600 \\ 0.1173 \end{pmatrix}$

Figure 5.10: BER of the RAKE cascaded modular detector ($N = 32$, $K = 10$, $SNR_1 = 10dB$)

Table 5.10: Coherent combining vector ($\mathbf{g}_1 = [0.1286 \ 0.0835 \ 0.0001]^H$, $\mathbf{g}_{1norm} = [0.8387 \ 0.5446 \ 0.0007]^H$)

$ISR(dB)$	-10	0	10	20
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.8229 \\ 0.5660 \\ 0.0491 \end{pmatrix}$	$\begin{pmatrix} 0.8337 \\ 0.5449 \\ 0.0890 \end{pmatrix}$	$\begin{pmatrix} 0.7805 \\ 0.6251 \\ 0.0021 \end{pmatrix}$	$\begin{pmatrix} 0.7439 \\ 0.6676 \\ 0.0297 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.8229 \\ 0.5660 \\ 0.0491 \end{pmatrix}$	$\begin{pmatrix} 0.8337 \\ 0.5450 \\ 0.0889 \end{pmatrix}$	$\begin{pmatrix} 0.7819 \\ 0.6234 \\ 0.0008 \end{pmatrix}$	$\begin{pmatrix} 0.7441 \\ 0.6671 \\ 0.0356 \end{pmatrix}$

Figure 5.11: BER of the RAKE cascaded modular detector ($N = 32$, $K = 10$, $SNR1 = 10dB$)

Table 5.11: Coherent combining vector ($\mathbf{g}_1 = [0.0808 \ 0.1208 \ 0.0033]^H$, $\mathbf{g}_{1norm} = [0.5558 \ 0.8310 \ 0.0227]^H$)

$ISR(dB)$	-10	0	10	20
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.3903 \\ 0.9111 \\ 0.1325 \end{pmatrix}$	$\begin{pmatrix} 0.3040 \\ 0.9478 \\ 0.0962 \end{pmatrix}$	$\begin{pmatrix} 0.3407 \\ 0.9353 \\ 0.0959 \end{pmatrix}$	$\begin{pmatrix} 0.3712 \\ 0.9104 \\ 0.1827 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.3903 \\ 0.9111 \\ 0.1325 \end{pmatrix}$	$\begin{pmatrix} 0.3048 \\ 0.9477 \\ 0.0950 \end{pmatrix}$	$\begin{pmatrix} 0.3426 \\ 0.9344 \\ 0.0973 \end{pmatrix}$	$\begin{pmatrix} 0.4457 \\ 0.8378 \\ 0.3153 \end{pmatrix}$

Figure 5.12: BER of the RAKE cascaded modular detector ($N = 32$, $K = 10$, $SNR1 = 10dB$)

Table 5.12: Coherent combining vector ($\mathbf{g}_1 = [0.12 \ 0.0215 \ 0.0067]^H$, $\mathbf{g}_{1norm} = [0.9828 \ 0.1761 \ 0.0549]^H$)

$ISR(dB)$	-10	0	10	20
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.9942 \\ 0.1068 \\ 0.0120 \end{pmatrix}$	$\begin{pmatrix} 0.9928 \\ 0.1193 \\ 0.0034 \end{pmatrix}$	$\begin{pmatrix} 0.9922 \\ 0.1244 \\ 0.0103 \end{pmatrix}$	$\begin{pmatrix} 0.9916 \\ 0.1282 \\ 0.0192 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.9942 \\ 0.1068 \\ 0.0120 \end{pmatrix}$	$\begin{pmatrix} 0.9929 \\ 0.1191 \\ 0.0027 \end{pmatrix}$	$\begin{pmatrix} 0.9910 \\ 0.1329 \\ 0.0182 \end{pmatrix}$	$\begin{pmatrix} 0.9445 \\ 0.2412 \\ 0.2232 \end{pmatrix}$

Figure 5.13: BER of the RAKE cascaded modular detector ($N = 32$, $K = 10$, $SNR1 = 10dB$)

Experiment 3: Convergence performance

The experiment examines the convergence performance of the RAKE cascaded modular receiver based on the length of data. We will mainly compare the convergence of the RAKE cascaded modular receiver with the adaptive Max-Min GSC receiver, while the performance of the RAKE MOE receiver, trained MMSE receiver and the single path ($path=1$) cascaded modular receiver are also given in the Figures. In the simulation, the number of users, $K = 10$ and the spreading code is randomly chosen with a spreading factor $N = 32$. The SNR of the desired user 1 is 10dB for the first group of channel coefficient in experiment 1 and 20dB for the other five groups of channel coefficient. The stage D of the cascaded modular detector in each arm is 8. For the adaptive Max-Min GSC receiver, the initial values were set to be $\hat{\mathbf{g}}_{1MaxMin} = [1, 0, 0]^H$ and $\mathbf{u} = \mathbf{0}$. The step sizes for updating the vector were $\mu_u = 0.0009$ and $\mu_g = 0.014$.

Table 5.13-Table 5.18 give the estimated coherent combining vectors $\hat{\mathbf{g}}_{1MOE}$, $\hat{\mathbf{g}}_{1CM}$ and the estimated channel coefficient $\hat{\mathbf{g}}_{1MaxMin}$ for the six groups of channel coefficient in the experiment 1. Fig. 5.14-Fig. 5.19 show the BER performance versus the data length of the five receivers for the six groups of channel coefficient. From the simulation results, we can see that the adaptive Max-Min GSC receiver converges faster than the RAKE MOE receiver, the RAKE cascaded modular receiver and the single arm ($path = 1$) cascaded modular receiver if the energy of signal is mainly in the path 1 and does not have strong fading situations, such as the first group of channel coefficient. For other five groups of channel coefficient, the adaptive Max-Min GSC receiver converges slower than the three receivers. The convergence speed of the adaptive Max-Min GSC receiver is determined by the initial values of $\hat{\mathbf{g}}_{1MaxMin}$. The proposed RAKE cascaded modular receiver converges faster to the trained MMSE detector than the other three receivers.

Table 5.13: Coherent combining vector ($\mathbf{g}_1 = [0.7201 \quad 0.0044 \quad 0.0046]^H$, $\mathbf{g}_{1norm} = [1 \quad 0.0061 \quad 0.0064]^H$)

<i>Datalength</i>	100	500	900	1500
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.9999 \\ 0.0126 \\ 0.0056 \end{pmatrix}$	$\begin{pmatrix} 0.9996 \\ 0.0266 \\ 0.0067 \end{pmatrix}$	$\begin{pmatrix} 0.9996 \\ 0.0245 \\ 0.0107 \end{pmatrix}$	$\begin{pmatrix} 0.9997 \\ 0.0233 \\ 0.0061 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 1.0000 \\ 0.0061 \\ 0.0010 \end{pmatrix}$	$\begin{pmatrix} 0.9996 \\ 0.0263 \\ 0.0068 \end{pmatrix}$	$\begin{pmatrix} 0.9996 \\ 0.0247 \\ 0.0105 \end{pmatrix}$	$\begin{pmatrix} 0.9997 \\ 0.0233 \\ 0.0061 \end{pmatrix}$
$\hat{\mathbf{g}}_{1MaxMin}$	$\begin{pmatrix} 0.9951 \\ 0.0580 \\ 0.0800 \end{pmatrix}$	$\begin{pmatrix} 0.9979 \\ 0.0611 \\ 0.0212 \end{pmatrix}$	$\begin{pmatrix} 0.9996 \\ 0.0272 \\ 0.0093 \end{pmatrix}$	$\begin{pmatrix} 0.9992 \\ 0.0343 \\ 0.0199 \end{pmatrix}$

Figure 5.14: BER versus data length ($N = 32$, $K = 10$, $SNR1 = 10\text{dB}$)

Table 5.14: Coherent combining vector ($\mathbf{g}_1 = [0.2378 \ 0.1317 \ 0.0029]^H$, $\mathbf{g}_{1norm} = [0.8747 \ 0.4845 \ 0.0107]^H$)

<i>Datalength</i>	100	500	900	1500
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.8916 \\ 0.4524 \\ 0.0209 \end{pmatrix}$	$\begin{pmatrix} 0.8708 \\ 0.4914 \\ 0.0149 \end{pmatrix}$	$\begin{pmatrix} 0.8743 \\ 0.4851 \\ 0.0135 \end{pmatrix}$	$\begin{pmatrix} 0.8742 \\ 0.4855 \\ 0.0116 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.8859 \\ 0.4639 \\ 0.0021 \end{pmatrix}$	$\begin{pmatrix} 0.8715 \\ 0.4903 \\ 0.0139 \end{pmatrix}$	$\begin{pmatrix} 0.8742 \\ 0.4853 \\ 0.0131 \end{pmatrix}$	$\begin{pmatrix} 0.8745 \\ 0.4850 \\ 0.0122 \end{pmatrix}$
$\hat{\mathbf{g}}_{1MaxMin}$	$\begin{pmatrix} 0.9987 \\ 0.0494 \\ 0.0156 \end{pmatrix}$	$\begin{pmatrix} 0.9723 \\ 0.2330 \\ -0.0169 \end{pmatrix}$	$\begin{pmatrix} 0.9197 \\ 0.3883 \\ -0.0580 \end{pmatrix}$	$\begin{pmatrix} 0.8877 \\ 0.4585 \\ -0.0434 \end{pmatrix}$

Figure 5.15: BER versus data length ($N = 32$, $K = 10$, $SNR1 = 20\text{dB}$)

Table 5.15: Coherent combining vector ($\mathbf{g}_1 = [0.076 \ 0.0772 \ 0.0015]^H$, $\mathbf{g}_{1norm} = [0.7015 \ 0.7126 \ 0.0138]^H$)

<i>Datalength</i>	100	500	900	1500
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.6576 \\ 0.7530 \\ 0.0233 \end{pmatrix}$	$\begin{pmatrix} 0.6902 \\ 0.7228 \\ 0.0343 \end{pmatrix}$	$\begin{pmatrix} 0.6960 \\ 0.7165 \\ 0.0457 \end{pmatrix}$	$\begin{pmatrix} 0.6960 \\ 0.7167 \\ 0.0432 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.6774 \\ 0.7351 \\ 0.0275 \end{pmatrix}$	$\begin{pmatrix} 0.6896 \\ 0.7231 \\ 0.0392 \end{pmatrix}$	$\begin{pmatrix} 0.6964 \\ 0.7160 \\ 0.0481 \end{pmatrix}$	$\begin{pmatrix} 0.6956 \\ 0.7171 \\ 0.0427 \end{pmatrix}$
$\hat{\mathbf{g}}_{1MaxMin}$	$\begin{pmatrix} 0.9999 \\ 0.0041 \\ 0.0134 \end{pmatrix}$	$\begin{pmatrix} 0.9991 \\ 0.0417 \\ 0.0029 \end{pmatrix}$	$\begin{pmatrix} 0.9978 \\ 0.0646 \\ -0.0172 \end{pmatrix}$	$\begin{pmatrix} 0.9911 \\ 0.1279 \\ -0.0376 \end{pmatrix}$

Figure 5.16: BER versus data length ($N = 32$, $K = 10$, $SNR1 = 20\text{dB}$)

Table 5.16: Coherent combining vector ($\mathbf{g}_1 = [0.1286 \ 0.0835 \ 0.0001]^H$, $\mathbf{g}_{1norm} = [0.8387 \ 0.5446 \ 0.0007]^H$)

<i>Datalength</i>	100	500	900	1500
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.8691 \\ 0.4918 \\ 0.0520 \end{pmatrix}$	$\begin{pmatrix} 0.8666 \\ 0.4929 \\ 0.0779 \end{pmatrix}$	$\begin{pmatrix} 0.8653 \\ 0.4970 \\ 0.0647 \end{pmatrix}$	$\begin{pmatrix} 0.8641 \\ 0.4982 \\ 0.0714 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.8864 \\ 0.4537 \\ 0.0915 \end{pmatrix}$	$\begin{pmatrix} 0.8648 \\ 0.4974 \\ 0.0692 \end{pmatrix}$	$\begin{pmatrix} 0.8658 \\ 0.4964 \\ 0.0637 \end{pmatrix}$	$\begin{pmatrix} 0.8657 \\ 0.4954 \\ 0.0712 \end{pmatrix}$
$\hat{\mathbf{g}}_{1MaxMin}$	$\begin{pmatrix} 0.9974 \\ 0.0164 \\ 0.0702 \end{pmatrix}$	$\begin{pmatrix} 0.9832 \\ 0.1321 \\ 0.1258 \end{pmatrix}$	$\begin{pmatrix} 0.9619 \\ 0.2514 \\ 0.1077 \end{pmatrix}$	$\begin{pmatrix} 0.9071 \\ 0.4144 \\ 0.0740 \end{pmatrix}$

Figure 5.17: BER versus data length ($N = 32$, $K = 10$, $SNR1 = 20\text{dB}$)

Table 5.17: Coherent combining vector ($\mathbf{g}_1 = [0.0808 \ 0.1208 \ 0.0033]^H$, $\mathbf{g}_{1norm} = [0.5558 \ 0.8310 \ 0.0227]^H$)

<i>Datalength</i>	100	500	900	1500
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.5958 \\ 0.8005 \\ 0.0646 \end{pmatrix}$	$\begin{pmatrix} 0.5473 \\ 0.8356 \\ 0.0469 \end{pmatrix}$	$\begin{pmatrix} 0.5516 \\ 0.8321 \\ 0.0582 \end{pmatrix}$	$\begin{pmatrix} 0.5529 \\ 0.8322 \\ 0.0417 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.5807 \\ 0.8109 \\ 0.0724 \end{pmatrix}$	$\begin{pmatrix} 0.5485 \\ 0.8349 \\ 0.0464 \end{pmatrix}$	$\begin{pmatrix} 0.5520 \\ 0.8318 \\ 0.0582 \end{pmatrix}$	$\begin{pmatrix} 0.5527 \\ 0.8323 \\ 0.0427 \end{pmatrix}$
$\hat{\mathbf{g}}_{1MaxMin}$	$\begin{pmatrix} 0.9948 \\ 0.1009 \\ 0.0166 \end{pmatrix}$	$\begin{pmatrix} 0.9784 \\ 0.2064 \\ 0.0138 \end{pmatrix}$	$\begin{pmatrix} 0.9595 \\ 0.2817 \\ 0.0022 \end{pmatrix}$	$\begin{pmatrix} 0.9205 \\ 0.3893 \\ -0.0346 \end{pmatrix}$

Figure 5.18: BER versus data length ($N = 32$, $K = 10$, $SNR1 = 20\text{dB}$)

Table 5.18: Coherent combining vector ($\mathbf{g}_1 = [0.12 \ 0.0215 \ 0.0067]^H$, $\mathbf{g}_{1norm} = [0.9828 \ 0.1761 \ 0.0549]^H$)

<i>Datalength</i>	100	500	900	1500
$\hat{\mathbf{g}}_{1MOE}$	$\begin{pmatrix} 0.9910 \\ 0.1336 \\ 0.0096 \end{pmatrix}$	$\begin{pmatrix} 0.9757 \\ 0.1954 \\ 0.0988 \end{pmatrix}$	$\begin{pmatrix} 0.9761 \\ 0.1973 \\ 0.0916 \end{pmatrix}$	$\begin{pmatrix} 0.9760 \\ 0.1980 \\ 0.0912 \end{pmatrix}$
$\hat{\mathbf{g}}_{1CM}$	$\begin{pmatrix} 0.9880 \\ 0.1505 \\ 0.0347 \end{pmatrix}$	$\begin{pmatrix} 0.9757 \\ 0.1956 \\ 0.0988 \end{pmatrix}$	$\begin{pmatrix} 0.9761 \\ 0.1970 \\ 0.0915 \end{pmatrix}$	$\begin{pmatrix} 0.9760 \\ 0.1981 \\ 0.0909 \end{pmatrix}$
$\hat{\mathbf{g}}_{1MaxMin}$	$\begin{pmatrix} 0.9992 \\ 0.0091 \\ 0.0389 \end{pmatrix}$	$\begin{pmatrix} 0.9878 \\ 0.0618 \\ 0.1433 \end{pmatrix}$	$\begin{pmatrix} 0.9775 \\ 0.1245 \\ 0.1701 \end{pmatrix}$	$\begin{pmatrix} 0.9722 \\ 0.1531 \\ 0.1773 \end{pmatrix}$

Figure 5.19: BER versus data length ($N = 32$, $K = 10$, $SNR1 = 20\text{dB}$)

5.6 Conclusions

In summary, this chapter proposes a blind RAKE constrained cascaded modular receiver for the DS-CDMA system over multipath fading channels. The receiver does not need to have a prior knowledge of the fading channel and can achieve good performance with a simple structure compared to other blind receivers that need to estimate the channel information. Simulation results show that the proposed blind RAKE constrained cascaded modular receiver can achieve the performance of the MMSE receiver which requires the exact channel information in a few stages.

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

This thesis focuses on the study of low complexity multiuser detection algorithms for CDMA communications systems. Several new multiuser detection algorithms and interference cancellation algorithms have been proposed in this thesis and their performances have been tested through computer simulations.

A linear multistage detector, WPE detector, has been proposed to effectively approximate the decorrelating detector and MMSE detector. The proof of the convergence of the WPE detector to the decorrelating detector has been derived based on the Cayley-Hamilton theorem. The proof shows that the WPE detector can exactly implement the decorrelating detector for $L \geq K - 1$ (L denotes the number of stages of the WPE detector and K denotes the number of active users in the systems) and that the weights depend on the eigenvalues of the matrix $\mathbf{R}$ ($\mathbf{R}$ is a $K \times K$ correlation matrix). The optimal weight vector for the L -stage WPE detector has been derived based on the minimum MSE criterion. An adaptive WPE detector based on the least squares criteria and the RLS algorithm has been

proposed to adjust the weight vector iteratively until it becomes the exact optimal weight vector. From the complexity analysis in Chapter 3, we can see that the reduction in the implementation complexity for the multi-stage WPE detector increases significantly as the number of users increases.

Simulation results were given to show the performance of the optimal WPE detector and the adaptive WPE detector in Chapter 3. Fig. 3.3-Fig. 3.10 show that the optimal WPE detector can exactly implement MMSE detector in low system load situation. Fig. 3.23 and Fig. 3.24 show that, although there will be some performance loss on the WPE detectors compared with the decorrelating detector and MMSE detector in high system load, the optimal WPE detector and adaptive WPE detector can approximate the decorrelating detector and MMSE detector better than the asymptotic WPE detector and linear Chebyshev PIC detector. Simulation results show that the optimal WPE detector and adaptive WPE detector perform better than the linear Chebyshev PIC detector and asymptotic WPE detector but worse than the decorrelating detector and MMSE detector in the presence of the near-far effect (Fig. 3.31-Fig. 3.36).

A blind cascaded modular structure detector has been proposed to implement the blind MOE detector. The blind cascaded modular structure detector is a reduced rank detector which is relatively simple and has a low complexity compared with the conventional blind MOE detector. The proposed detector does not need to explicitly estimate the subspace. A vector filter is introduced for adaptive interference cancellation in each module of the proposed detector. The weight vector is derived based on maximum magnitude cross correlation criterion which maximizes the magnitude of the cross correlation between the output of the nonadaptive filter and that of the weight vector filter. Simulation results show that the proposed cascaded modular detector captures and reduces most interference by the first few stage weight vector filters and can achieve a performance of that of the conventional blind MOE detector even in large systems, high load and near-far situation

(Fig. 4.7-Fig. 4.22). Simulation results show that the proposed adaptive detector can achieve the steady-state performance close to that of the blind MOE detector and the convergence speed is faster than that of the full rank blind MOE detector when the number of stages is large enough. (Fig. 4.26-Fig. 4.30 and Fig. 4.36-Fig. 4.38). Simulation results show that, compared with the PC, PC-PLIC and CS detectors, the proposed detector is able to achieve the same performance as that of the full rank MOE detector with much few stages (Fig. 4.45-Fig. 4.48). Simulation results show that both the proposed detector and the MFWF have nearly the same performance when the number of stages, $D > 3$. The proposed detector has better performance than the MFWF when $D \leq 3$ (Fig. 4.49-Fig. 4.57).

A RAKE structure cascaded modular detector has also been proposed in this thesis to handle the signature mismatch problem in DS-CDMA system over multipath fading channels. The detector does not require the knowledge of the fading channel and can achieve good performance with a simple structure compared with other blind receivers that need to estimate the channel information. Simulation results on BER performance show that the RAKE cascaded modular receiver has nearly the same performance as that of the RAKE MOE receiver for six different channels (Fig. 5.2-Fig. 5.7). Fig. 5.8-Fig. 5.13 show the BER performance of the proposed RAKE cascaded modular detector for the six different channels in near-far situation. Simulation results show that although the RAKE cascaded modular receiver has large performance loss compared with the RAKE MOE receiver in strong interference and fading situations, the proposed detector has good ability to resist the near-far effect. Simulation results on convergence performance show that the proposed detector converges faster than the trained MMSE detector in most of the channels tested (Fig. 5.14-Fig. 5.19).

6.2 Recommendations for Further Research

This thesis has proposed several low complexity multiuser detectors for CDMA communications systems. Based on the work completed in this thesis, some research can be further investigated in future.

Chapter 3 has proposed a WPE detector to approximate the linear decorrelating detector and MMSE detector. Although the performance of the optimal WPE detector and the adaptive WPE detector has great improvement compared with that of the conventional matched filter, there is still a considerable performance loss compared with the optimal multiuser detector. For high system load, the WPE detector can not converge to the MMSE detector and will diverge when the number of stages increases. Thus, further research could be conducted to combine the decision feedback and the WPE detector for further performance improvement.

Chapter 4 has proposed a blind cascaded modular detector. The blind algorithm needs relatively more data and hence larger processing delay. Since there is a pilot channel in CDMA systems, it is advisable to modify the blind adaptive algorithm to a semi-blind algorithm to speed up the convergence using pilot symbols.

Chapter 5 has proposed a RAKE structure cascaded modular detector for interference cancellation and multiuser detection in CDMA multipath fading channels. The detector uses second order statistics, knowledge of the desired user's code and knowledge of the desired user's transmission delay for the blind detection. The detector works well at high SNRs and small multipath delays. The CDMA system can be formulated as a symbol rate MIMO system, taking different users' signals as the inputs and the chip-rate sampled data per symbol as the outputs. Some research work has thus focused on using higher order statistics to design blind multiuser detection algorithms for CDMA systems in multipath [79]-[83]. Further research could be considered to develop low complexity approaches and structures for the higher order statistics blind multiuser detection algorithms.

MIMO and Orthogonal Frequency Division Multiplexing (OFDM) are two notable advances that have demonstrated a high potential for employment in future high rate wireless communication systems. Various MUD algorithms, such as linear least squares (LS) and MMSE MUD, have been proposed for MIMO OFDM systems throughout these years. Based on the WPE detector proposed in Chapter 3, future research work could be considered to implement the WPE method to the multiuser detection in MIMO OFDM system. In MIMO OFDM systems, accurate channel estimation is required at the receiver for the sake of invoking both coherent demodulation and interference cancellation. In the literature, a number of blind channel estimation techniques have been proposed for MIMO OFDM systems. Since most of these approaches suffer from a slow convergence rate too, future research work could be considered to implement the cascaded modular detector proposed in Chapter 4 and 5 for the blind channel estimation in MIMO OFDM systems.

Author's Publications

Journal Papers

1. Zhongjie Li and Wee Ser, "An adaptive WPE multiuser detector," *IEEE Signal Processing Letters*, Vol.12, No.4, Apr. 2005, pp. 305 -308.
2. Zhongjie Li and Wee Ser, "A blind cascaded modular detector," *submitted, IEEE Signal Processing Letters*.
3. Zhongjie Li and Wee Ser, "RAKE Receiver with a cascaded detector structure for CDMA Communications," *submitted, IEEE Transaction on Wireless Communications*.

Conference Papers

1. Zhongjie Li and Wee Ser, "Weight optimum for weighted polynomial expansion multiuser detector," *The 14th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications Proceedings*, Beijing, China, Sept. 7-10, 2003, Vol.1, pp. 998-1002.
2. Zhongjie Li and Wee Ser, "A blind cascade structure interference cancellation RAKE receiver for CDMA communications," *International Wireless Communications and Mobile Computing Conference*, Vancouver, Canada, Jul. 3-6, 2006.

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Appendix A

Derivation of Equation (3.27)

Substitute (3.3) into (3.26), ε can be expressed as

$$\begin{aligned}
\varepsilon &= E\{(\mathbf{RAb} + \mathbf{S}^T \mathbf{v})^T L_{PE}^T L_{PE} (\mathbf{RAb} + \mathbf{S}^T \mathbf{v})\} \\
&\quad - E\{(\mathbf{RAb} + \mathbf{S}^T \mathbf{v})^T L_{PE} \mathbf{Ab}\} \\
&\quad - E\{\mathbf{b}^T \mathbf{A} L_{PE} (\mathbf{RAb} + \mathbf{S}^T \mathbf{v})\} \\
&= E\{\mathbf{b}^T \mathbf{A} \mathbf{R} L_{PE}^T L_{PE} \mathbf{RAb}\} + E\{\mathbf{b}^T \mathbf{A} \mathbf{R} L_{PE}^T L_{PE} \mathbf{S}^T \mathbf{v}\} \\
&\quad + E\{\mathbf{v}^T \mathbf{S} L_{PE}^T L_{PE} \mathbf{RAb}\} + E\{\mathbf{v}^T \mathbf{S} L_{PE}^T L_{PE} \mathbf{S}^T \mathbf{v}\} \\
&\quad - E\{\mathbf{b}^T \mathbf{A} \mathbf{R} L_{PE}^T \mathbf{Ab}\} - E\{\mathbf{v}^T \mathbf{S} L_{PE}^T \mathbf{Ab}\} \\
&\quad - E\{\mathbf{b}^T \mathbf{A} L_{PE} \mathbf{RAb}\} - E\{\mathbf{b}^T \mathbf{A} L_{PE} \mathbf{S}^T \mathbf{v}\}
\end{aligned} \tag{A.1}$$

Since ε is a scalar, we can apply trace operator to the right hand side of (A.1). Under the assumption in Chapter 3, $E\{\mathbf{v}\mathbf{v}^T\} = \sigma^2 \mathbf{I}$, $E\{\mathbf{v}\} = 0$, $E\{\mathbf{b}\mathbf{b}^T\} = \mathbf{I}$ and utilizing the property $tr(\mathbf{AB}) = tr(\mathbf{BA})$, we can obtain

$$\begin{aligned}
\varepsilon &= tr\{L_{PE}^T L_{PE} \mathbf{R}^2 \mathbf{A}^2\} + tr\{\sigma^2 \mathbf{I} \mathbf{R} L_{PE}^T L_{PE}\} \\
&\quad - tr\{\mathbf{A}^2 \mathbf{R} L_{PE}^T\} - tr\{\mathbf{A}^2 \mathbf{R} L_{PE}\}
\end{aligned} \tag{A.2}$$

where

$$L_{PE} = \sum_{\ell=0}^L w_{\ell}(\mathbf{R})^{\ell} \quad (\text{A.3})$$

Let $\mathbf{R} = \mathbf{T}\mathbf{\Lambda}\mathbf{T}^T$ be the eigenvalue decomposition of $\mathbf{R}$ and substitute L_{PE} in equation (A.2), we can obtain the equation (3.27)

$$\begin{aligned} \varepsilon &= tr\{(\mathbf{A}^2\mathbf{\Lambda}^2 + \sigma^2\mathbf{\Lambda})(\sum_{\ell=0}^L w_{\ell}\mathbf{\Lambda}^{\ell})^2\} - 2tr\{\mathbf{A}^2\mathbf{\Lambda}\sum_{\ell=0}^L w_{\ell}\mathbf{\Lambda}^{\ell}\} \\ &= \mathbf{w}_L^T \mathbf{C}^T (\mathbf{A}^2\mathbf{\Lambda}^2 + \sigma^2\mathbf{\Lambda}) \mathbf{C} \mathbf{w}_L - 2\mathbf{w}_L^T \mathbf{C}^T \mathbf{A}^2 \mathbf{\Lambda} \end{aligned} \quad (\text{A.4})$$

where,

$$\mathbf{w}_L = [w_0, w_1, \dots, w_L]^T$$

$$\lambda = [\lambda_1, \dots, \lambda_K]^T$$

$$\mathbf{C} = \begin{bmatrix} 1 & \lambda_1 & \lambda_1^2 & \cdots & \lambda_1^L \\ 1 & \lambda_2 & \lambda_2^2 & \cdots & \lambda_2^L \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \lambda_K & \lambda_K^2 & \cdots & \lambda_K^L \end{bmatrix}$$

Appendix B

Derivation of Equation (4.34)

Let $\mathbf{G} = [\mathbf{g}_1 \quad \mathbf{g}_2]$ and $\mathbf{y}_i = (\mathbf{I} - \mathbf{g}_{i-1}\mathbf{U}_{i-1})\mathbf{y}_{i-1}$, $1 \leq i \leq 2$, with $\mathbf{y}_0 = \mathbf{y}$, we will show

$$(\mathbf{I} - \mathbf{g}_1\mathbf{U}_1)(\mathbf{I} - \mathbf{g}_2\mathbf{U}_2) = \mathbf{I} - \mathbf{G}\mathbf{U} \quad (\text{B.1})$$

Proof: The solution for $\mathbf{U}_1$ and $\mathbf{U}_2$ in (B.1) is given by

$$\mathbf{U}_1 = (\mathbf{g}_1^H \mathbf{R}_y \mathbf{g}_1)^{-1} \mathbf{g}_1^H \mathbf{R}_y \quad (\text{B.2})$$

$$\mathbf{U}_2 = (\mathbf{g}_2^H \mathbf{R}_{y_1} \mathbf{g}_2)^{-1} \mathbf{g}_2^H \mathbf{R}_{y_1} \quad (\text{B.3})$$

where

$$\mathbf{R}_{y_1} = \mathbf{R}_y - \mathbf{R}_y \mathbf{g}_1 (\mathbf{g}_1^H \mathbf{R}_y \mathbf{g}_1)^{-1} \mathbf{g}_1^H \mathbf{R}_y \quad (\text{B.4})$$

Let

$$\mathbf{G}^H \mathbf{R}_y \mathbf{G} = \begin{pmatrix} \mathbf{g}_1^H \mathbf{R}_y \mathbf{g}_1 & \mathbf{g}_1^H \mathbf{R}_y \mathbf{g}_2 \\ \mathbf{g}_2^H \mathbf{R}_y \mathbf{g}_1 & \mathbf{g}_2^H \mathbf{R}_y \mathbf{g}_2 \end{pmatrix} \quad (\text{B.5})$$

$$= \begin{pmatrix} \mathbf{A} & \mathbf{D} \\ \mathbf{C} & \mathbf{B} \end{pmatrix} \quad (\text{B.6})$$

and define $\Delta = \mathbf{B} - \mathbf{C}\mathbf{A}^{-1}\mathbf{D}$. The left hand side (LHS) of (B.1) can be expressed as

$$\begin{aligned}
 LHS &= \mathbf{I} - \mathbf{g}_1\mathbf{U}_1 - \mathbf{g}_2\mathbf{U}_2 + \mathbf{g}_1\mathbf{U}_1\mathbf{g}_2\mathbf{U}_2 \\
 &= \mathbf{I} - \mathbf{g}_1(\mathbf{g}_1^H\mathbf{R}_y\mathbf{g}_1)^{-1}\mathbf{g}_1^H\mathbf{R}_y \\
 &\quad - \mathbf{g}_2(\mathbf{g}_2^H\mathbf{R}_y\mathbf{g}_2 - \mathbf{g}_2^H\mathbf{R}_y\mathbf{g}_1(\mathbf{g}_1^H\mathbf{R}_y\mathbf{g}_1)^{-1}\mathbf{g}_1^H\mathbf{R}_y\mathbf{g}_2)^{-1}\mathbf{g}_2^H\mathbf{R}_y \\
 &\quad + \mathbf{g}_2(\mathbf{g}_2^H\mathbf{R}_y\mathbf{g}_2 - \mathbf{g}_2^H\mathbf{R}_y\mathbf{g}_1(\mathbf{g}_1^H\mathbf{R}_y\mathbf{g}_1)^{-1}\mathbf{g}_1^H\mathbf{R}_y\mathbf{g}_2)^{-1}\mathbf{g}_2^H\mathbf{R}_y\mathbf{g}_1 \\
 &\quad (\mathbf{g}_1^H\mathbf{R}_y\mathbf{g}_1)^{-1}\mathbf{g}_1^H\mathbf{R}_y + \mathbf{g}_1(\mathbf{g}_1^H\mathbf{R}_y\mathbf{g}_1)^{-1}\mathbf{g}_1^H\mathbf{R}_y\mathbf{g}_2\Delta^{-1}\mathbf{g}_2^H\mathbf{R}_y \\
 &\quad - \mathbf{g}_1(\mathbf{g}_1^H\mathbf{R}_y\mathbf{g}_1)^{-1}\mathbf{g}_1^H\mathbf{R}_y\mathbf{g}_2\Delta^{-1}\mathbf{g}_2^H\mathbf{R}_y\mathbf{g}_1(\mathbf{g}_1^H\mathbf{R}_y\mathbf{g}_1)^{-1}\mathbf{g}_1^H\mathbf{R}_y \\
 &= \mathbf{I} - \mathbf{g}_1\mathbf{A}^{-1}\mathbf{g}_1^H\mathbf{R}_y - \mathbf{g}_2\Delta^{-1}\mathbf{g}_2^H\mathbf{R}_y + \mathbf{g}_2\Delta^{-1}\mathbf{C}\mathbf{A}^{-1}\mathbf{g}_1^H\mathbf{R}_y \\
 &\quad + \mathbf{g}_1\mathbf{A}^{-1}\mathbf{D}\Delta^{-1}\mathbf{g}_2^H\mathbf{R}_y - \mathbf{g}_1\mathbf{A}^{-1}\mathbf{D}\Delta^{-1}\mathbf{C}\mathbf{A}^{-1}\mathbf{g}_1^H\mathbf{R}_y \tag{B.7}
 \end{aligned}$$

By using the formula for the inverse of block matrices, the right hand side (RHS) of (B.1) is given by

$$\begin{aligned}
 RHS &= \mathbf{I} - [\mathbf{g}_1\mathbf{g}_2] \begin{pmatrix} \mathbf{A}^{-1} + \mathbf{A}^{-1}\mathbf{D}\Delta^{-1}\mathbf{C}\mathbf{A}^{-1} & -\mathbf{A}^{-1}\mathbf{D}\Delta^{-1} \\ -\Delta^{-1}\mathbf{C}\mathbf{A}^{-1} & \Delta^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{g}_1^H\mathbf{R}_y \\ \mathbf{g}_2^H\mathbf{R}_y \end{pmatrix} \\
 &= \mathbf{I} - \mathbf{g}_1\mathbf{A}^{-1}\mathbf{g}_1^H\mathbf{R}_y - \mathbf{g}_2\Delta^{-1}\mathbf{g}_2^H\mathbf{R}_y + \mathbf{g}_2\Delta^{-1}\mathbf{C}\mathbf{A}^{-1}\mathbf{g}_1^H\mathbf{R}_y \\
 &\quad + \mathbf{g}_1\mathbf{A}^{-1}\mathbf{D}\Delta^{-1}\mathbf{g}_2^H\mathbf{R}_y - \mathbf{g}_1\mathbf{A}^{-1}\mathbf{D}\Delta^{-1}\mathbf{C}\mathbf{A}^{-1}\mathbf{g}_1^H\mathbf{R}_y \tag{B.8}
 \end{aligned}$$

Term by term comparison of (B.7) and (B.8) establishes (B.1).

It is straightforward to see that if $\mathbf{G}$ is partitioned as $\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_D]$ and $\mathbf{y}_i = (\mathbf{I} - \mathbf{g}_{i-1}\mathbf{U}_{i-1})\mathbf{y}_{i-1}$, $1 \leq i \leq D$, with $\mathbf{y}_0 = \mathbf{y}$, we can obtain the following result.

$$(\mathbf{I} - \mathbf{g}_1\mathbf{U}_1)(\mathbf{I} - \mathbf{g}_2\mathbf{U}_2) \cdots (\mathbf{I} - \mathbf{g}_D\mathbf{U}_D) = \mathbf{I} - \mathbf{G}\mathbf{U} \tag{B.9}$$