Development of a pressure recovery unit for seawater reverse osmosis system

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**NANYANG
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UNIVERSITY**

**DEVELOPMENT OF A PRESSURE RECOVERY UNIT FOR
SEAWATER REVERSE OSMOSIS SYSTEM**

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SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

2008

DEVELOPMENT OF A PRESSURE RECOVERY UNIT FOR SEAWATER REVERSE OSMOSIS SYSTEM

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Abstract

As supplies of clean water diminish, increasing numbers of countries are turning to desalination to meet their water needs. Australia, for instant, is gearing up to build the largest desalination plant in the world after facing a seven-year drought from 2001-2007. Unfortunately, desalination is an energy intensive process that adds to their energy woes.

To lower its energy consumption, energy recovery devices (ERDs) were invented or adapted from other industries for use in desalination plants. In general, there are two types of ERDs. The first is a turbine type ERD and the second is a positive displacement type ERD. The latter has proven to be more effective in lowering energy consumption due to higher energy transfer efficiency (ETE). In fact, industrial observers even believed that the efficiency of these ERDs has already reached a plateau. So far, the most efficient desalination plants have achieved an energy consumption of 1.9 kWh/m^3 of clean water processed. This level of efficiency is more than 100% better than older designs. In the interest of design excellence, this project has identified two areas suited for further development. The first is to raise energy efficiency through the novel use of pressure exchange between the high pressure brine flow and the low pressure seawater flow. The second is to smooth out pressure fluctuations in the inlets and outlets of the ERD.

After the design of the ERD was finalized, a thermodynamic model was tailored to the pressure exchange process and the potential energy savings calculated. Savings of up to an additional $5 - 10 \text{ kJ/m}^3$ when compared to the newest ERD available on the market are achievable. Considering the net energy consumption of a modern desalination plant which is around $2 - 4 \text{ kWh/m}^3$, the expected improvement is only a small amount. The safety aspect of the ERD is not neglected either. As the ERD has to withstand enormous pressure, its structural integrity was first investigated theoretically and then verified using finite element method. Results show that the design of the ERD is robust enough to withstand pressure up to

8 MPa. Other aspects of the ERD which affected the energy efficiency of the ERD such as friction and leakage in the ERD were also investigated. Friction generated by the hydrodynamic bearing in the ERD was found using a relaxation method pioneered by Raimondi and Boyd. Results show that friction generated is extremely small as compared to the energy consumption in a desalination plant and is unlikely that it will upset overall energy consumption. The leakage within the ERD has a more significant impact on performance according to theoretical analysis. However, it is also more difficult to control as it is affected by the size of the clearances in the ERD which in turn, is affected by the machining technique used.

Experiments have been conducted to understand the pressure exchange process. Parameters such as the initial pressure used, the speed of valve opening, the size of the valve and the length of the pressure vessel were varied. Results are consistent with predictions from the thermodynamic model. An interesting result was also observed. In the last experiment when the pressure vessels were replaced with longer ones, pressure waves were observed to oscillate within the vessels. When the prototype ERD was completed, it was tested with various input pressures. The level of energy recovered is on par with existing designs with significant reduction of pressure fluctuations. Greater efficiency is possible with slight adjustments to the original design. Future work involving a second design is also proposed.

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Nomenclature

KE	kinetic energy	J
PE	potential energy	J
E	total energy	J
U	1. internal energy	J
	2. speed	m/s
w	1. work done per unit mass	J/kg
	2. deflection	m
q	1. heat transfer per unit mass	J/kg
	2. load	N
v	1. specific volume	m ³ /kg
	2. poisson ratio	
B	bulk modulus of elasticity (liquids)	Pa
V	1. volume	m ³
	2. velocity	m/s
t	1. time	s
	2. thickness	m
	3. temperature	°C
T	temperature	°C
A	area	m ²
Q	1. heat transferred	J
	2. flow rate	m ³ /h
	3. shear force	N
y	various properties of water	
m	mass	kg
W	1. work done	J
	2. weight	kg
$\angle$	angle	°
r	radius	m
h	height	m
R	recovery ratio	
c	1. velocity of pressure wave	m/s
	2. diametral clearance	m
p	1. pressure	N/m ²
	2. uniformly distributed loading	
M	bending moment	Nm
θ	angle	°
E	modulus of elasticity (solids)	Pa
D	diameter	m
g	gravity	m/s ²
P	pressure	Pa
h	1. height	m
	2. clearance	
f	frequency	Hz
G	modulus of elasticity in shear	Pa

Greek letters

ρ	density	kg/m^3
β_T	isothermal compressibility	Pa^{-1}
Δ	change	
ψ	slope	
δ	height of clearance	m
γ	specific weight	$\text{m/kg}\cdot\text{s}^2$
μ	viscosity	$\text{N}\cdot\text{s}\cdot\text{m}^2$
$\varnothing$	angle	°
ε	eccentricity	m
ω	angular velocity	rad/s

Subscripts

b	boundary
in	in-bound
LP	low pressure
HP	high pressure
compressed	compressed liquid
P	permeate
L	internal leakage
r	cylindrical coordinate
o	outer
i	inner
θ	cylindrical coordinate
ref	reference
s	1. squeeze 2. spacer
p	protrusion
ss316	stainless steel
min	minimum
f	film
n	natural
c	critical
m	mean
z	cylindrical coordinate

Introduction

Potable water supply is becoming a limited resource nowadays due to the increase in size and improved standard of living of communities. More countries are exploring alternative sources of clean water such as desalination. For many years multistage flash distillation (MSF) based desalination was the predominant process for treating large amounts of seawater. Since 1960, due to the development of reverse osmosis (RO) modules and membranes, RO has become an attractive process for treating seawater on a large scale. The interest in RO is justified by 1) low energy consumption, 2) high product recovery and quality, 3) simplicity in automation and control and 4) logistic considerations: membrane plants are often more compact (intensified process), can be scaled up easily and installed more quickly than alternative plants when compared to MSF process.

Of all the aforementioned factors, having a low energy consumption can be considered to be the most important criterion since energy is the largest single variable cost for a desalting plant, exceeding 50% of the total operating cost in most case. Consequently, lowering energy consumption translates directly into lower cost of producing desalted water. Although the RO process has significantly brought down the cost of producing desalted water, it can be reduced further.

In the RO process, seawater (feed) is pumped up to a high pressure and pushed through the RO membrane. The portion that passes through, which is called product or permeate, is desalted. The remaining portion, called reject or brine, is highly concentrated. The proportion of the two is dictated by the recovery of the system, for example a system designed with a 36% recovery will produce 36% of permeate and 64% of brine. Ordinarily, a RO plant would set the recovery as high as possible because of higher efficiency but is limited by considerations for permeate quality and longevity of RO membranes. Thus, typical recovery will be in the region of 40-60%. At the end of the process, the permeate exits the RO

membranes at 2-3 bar, while the brine remains at 60-80 bar, almost equal to the feed pressure minus friction losses. Considering the huge quantity of brine produced, there is tremendous hydraulic energy to be tapped in the brine stream.

1 Literature Review

1.1 Background

To harness the aforementioned hydraulic energy in the brine stream, an energy recovery device (ERD) can be incorporated into the brine stream. Currently, there are many types of ERD on the market but they can be broadly categorized into two classes according to Macharg [1]: the turbine type and the positive displacement (PD) type. The Pelton wheel, Francis turbine (reverse running pump) and hydraulic turbocharger can be considered turbine type ERD (ERT). In terms of energy transfer efficiency (ETE), PD type has proven to be better (above 90%) than turbine type (60-80% max). Beside the evaluation of the ERD in terms of system efficiency, other aspects have to be taken into consideration as well. They are: 1) specific energy consumption, 2) required degree of modifications (in case of retrofitting), 3) required additional capital investment, 4) ERD supplier's experience and 5) amount of regular maintenance needed. For example, both types of ERD require different plant layouts to work and this will have a direct impact on the capital investment cost of setting up the plant.

1.1.1 Turbine Type ERD

The problem of energy recovery from water streams is not new. One of the first inventions capable of harnessing this energy at high efficiency was the Pelton wheel. This device was patented by Lester Pelton [2] for gold mining operation in California (figure 1.1).

Figure 1.1 1883 Pelton wheel

5 The idea of using the momentum of fast moving water streams was not revolutionary, but Pelton's "peculiar-shaped" split buckets, which facilitated efficient transfer of the impinging flow from one bucket to the next during rotation, were novel and were what made the device work. Modern Pelton wheels utilize the split buckets and the flow-driven rotation of the original 1883 design. Their overall ETE is the product of the efficiencies of the nozzle, the
10 buckets and the directly coupled pump. Efficiencies as high as 81% have been reported for 90% efficient devices coupled to 90% efficient pumps, but typical Pelton efficiencies range from 40 to 60%. A Pelton wheel of 83% efficiency coupled to a typical multi-stage centrifugal pump of 67% efficiency would use up to 4.25 kWh/m^3 of permeate [5].

Another device using the centrifugal principle is the Francis turbine (figure 1.2). In actual
15 fact, a Francis turbine is a reverse running pump. Mechanically, there is no difference between a pump and a reverse running pump. A pump converts mechanical energy (shaft) into kinetic energy of the liquid pumped and finally into hydraulic energy, whereas a Francis turbine does the opposite.

Figure 1.2 Axial piston motor

5 However attractive the Francis turbine may appear due to its simple design and easy adaptability, its overall ETE is even lower than the Pelton wheel which can be attributed to its robust design and large sealing surfaces.

Another type of centrifugal ERD is the hydraulic turbocharger. It first appeared in early 1910's to capture the energy of exhaust gas exiting from internal combustion engine. It was
10 subsequently adapted for RO service in 1990. The patent for its application in RO was issued to Robert Oklejas [3]. The turbocharger efficiencies as high as 80% have been claimed, but the maximum possible ETE, which is the product of the efficiencies of the two impellers, typically ranges from 50 to 65%.

Silbernagel [4] evaluated such a device in 1991 at Port Hueneme, California. In this
15 experiment, the free running turbine driven centrifugal pump manufactured by Pump Engineering Inc. (PEI) was located along the brine stream of a RO module and transferred the recovered hydraulic energy to the feed stream in the form of pressure boosting. The RO module was a military 600 gallon per hour (gph) unit and the ERD was operated continuously over 3000 hours with periodic measurements of performance parameters and internal
20 inspections of the unit to characterize rates of wear.

The turbine/pump ERD was a self-contained unit incorporating its own turbine and pump section. The turbine section accepts the high pressure brine, passes the brine through a radial inflow turbine, and discharges the depressurized brine. The pump section accepts the feed

flow, passes the feed through a single stage centrifugal pump, and discharges the feed at a substantially higher pressure. The pump impeller was mounted on the turbine shaft and rotated with the turbine impeller as a unit.

Figure 1.3 shows the general arrangement of an RO system equipped with a turbine/pump ERD. Note that the ERD is placed between the feed pump discharge and the membrane feed inlet.

Figure 1.3 Pump/turbine ERD based RO system

Over the entire test period, the ETE of the ERD showed no long term decline and averaged 36.2% with fluctuations of less than $\pm 1.5\%$. Since ETE is the product of both the turbine and pump efficiencies, the component efficiency equals the square root of ETE or about 60% assuming both pump and turbine have the same efficiency. Considering the low flow rate, this is exceptionally high efficiency as there is a direct relationship between flow rate and efficiency of turbine type ERD.

In 2000, Oklejas Jr. [5] integrated advanced high pressure pumps with turbine type ERD to reduce capital and operating costs of RO systems. The integrated package boosted a feed pump and ERD developed specifically for RO service. The pump was equipped with a variable frequency drive (VFD) to regulate flow and pressure. So far, it is the most efficient way to do so. Typically, valves in a high-grade stainless steel alloy are used for flow control through an energy wasting throttling process. The ERD is a free-running, single stage turbine

driving a single-stage feed booster pump. It used the high pressure brine to energize the turbine which drove the single-stage feed booster pump. The feed pressure boost generated by the ERD reduced the required feed pump discharge pressure resulting in energy savings and a reduction in size of the feed pump and motor. Upon comparison, this ERD seems to be similar to the ERD used in the previous example [4] yet it is significantly improved through better fluid flow path design and full utilization of computer numerical control (CNC) manufacturing technique. Figure 1.4 shows the integrated ERD package as part of a RO system.

Figure 1.4 Integrated RO package (IROP)

The performance characteristics of the integrated ERD package were investigated at 120 m³/d and 500 m³/d permeate output RO plants. At the 120 m³/d RO plant, recovery was set at 36% and the average operating pressures were 60.0 bar and 58.0 bar for the feed and brine, respectively. The energy consumption of the package was 4.7 kWh/m³. If the ERD was coupled with a PD pump instead, the energy consumption was estimated to be slightly higher at 5.8 kWh/m³. At the 500 m³/d plant, recovery was set at 45% and the average operating pressures were 64.0 bar and 62.0 bar for the feed and brine, respectively. Due to the higher flow rate, the energy consumption was even lower at 4.18 kWh/m³.

The benefits of an energy recovery turbine (ERT) was confirmed by Kim [6] in 2002. A two stage pilot-scale RO plant with a capacity of 5 m³/d operated with 50% recovery and 66 to 75 bar of pressures. In these conditions, the energy consumption was 6.3 kWh/m³. This was a 12.5% reduction when the plant functioned without an ERT or any other ERDs. Had the plant operated at one stage mode and normal pressures for a recovery of 35%, the reduction would be 32.25%. On top of demonstrating the mechanical advantage of the ERT, the test also showed that energy saving by energy recovery decreases as recovery increases as an obvious result of reduction in brine flow rates.

1.1.2 Positive Displacement ERDs

PD based ERDs work by transferring pressure energy from the brine stream directly to the feed stream. Swaney [7] and Ross [8] were pioneers in PD ERDs used in recovery of energy in chemical processing. The first PD ERD intended solely for RO was patented by Cheng et al.[9]. Figure 1.5 shows the patent drawing by Cheng et al.

Figure 1.5 Cheng's patent drawing

Subsequently, a large number of PD ERDs surfaced, some became commercially available and found use in hand-pumped lifeboats units [10] or other seagoing vessels [11-

12]. Still they suffered from excessive flow and pressure fluctuations, prohibiting their use in large, critical applications. In 1975, a PD ERD patented by Taylor [13], believed to be the first, was installed in the first commercial seawater RO plant based on the mid-Atlantic island of Bermuda. However, the device, plagued by frequent breakdown, was soon removed from service. Although Taylor's device did not succeed, it proved the viability of PD ERD in large RO applications. Successful examples soon followed, spurred on by demands for Build-Own-Operate (BOO) seawater RO contracts in the Caribbean. Up to date, two different concepts have emerged as dominant designs. The 1st concept (Type 1) uses valves and static pressure vessels to facilitate the energy transfer while the 2nd (Type 2) uses only a single spinning cylindrical rotor to achieve the same thing. Devices that use the former include Aqualyng, DWEER (Dual Work Exchanger Energy Recovery), PES (Pressure Exchanger System) and DYPREX (Dynamic Pressure Exchanger). The PX (Pressure Exchanger) is the only device commercially available using the later.

A typical RO system has to be slightly modified before it can accept a PD ERD. Unlike an non-ERD or ERT based RO system, the low pressure (LP) feed is divided into two portions, one portion is directed into the main pump inlet while the other is pumped into the ERD's LP feed inlet. The proportion between the two is determined by the recovery of the system. In figure 1.6, the feed is split into flow A and B. Flow A will be equal in volume to the permeate, while flow B will be equal to the brine. In the PD ERD, flow B is pressurized by the hydraulic energy of the high pressure (HP) brine leaving the RO modules. Ideally, ETE should be 100% and flow B would receive the brine's entire hydraulic energy, but in actual fact the ETE is approximately 90~96%. To compensate for pressure losses incurred in the RO modules (0.5-1.5 bar), piping (0.4-0.6 bar) and the ERD (0.2-0.6 bar), flow B has to pass through a booster pump to bring its pressure up to the operating pressure. The separated flows will converge at the inlet of the RO modules.

Figure 1.6 PD ERD based RO system

The working principle of the PD ERD can be explained more easily with an illustration (figure 1.7). A cycle will start with the filling of the chamber with LP feed. When the chamber is full, valve V1 and V3 will open simultaneously. This action exposes the LP feed in the chamber to the HP brine allowing pressure to be directly transferred from the brine to the feed. At the same instant, the incoming brine starts pushing the feed out of the chamber and into the booster pump. When the chamber is completely filled with HP brine, V1 and V3 will shut off. V2 and V4 will then open causing the chamber to depressurize. The entry of LP feed will displace the LP brine and begin a new cycle. Evident from figure 1.7, a one-chamber PD ERD will pose intermittent flow disruption. Using two or more chambers in tandem would lessen this problem and steady the output.

Figure 1.7 Illustration of a one-chamber PD ERD

Drablos [14] investigated the performance characteristics of the Aqualyng system. The Aqualyng system is one of the large-scale ERD utilizing the 1st concept. To differentiate itself from other similar ERDs, the Aqualyng has vertical pressure vessels instead of horizontal ones. This has the advantage of a small footprint but present other packaging problems. To smooth variation in the flow rate, Aqualyng uses two pressure vessel rather than just one. Operating with two chambers is the same as having just one. The only difference is that one chamber is always one-half of a cycle apart. When one chamber is full of LP feed, the other will be filled with HP brine. As HP brine is diverted into the 1st chamber, the 2nd chamber is being depressurized. HP brine is slowly displacing the feed out of the 1st chamber at the same time when LP feed is driving out the brine in the 2nd. The end of these actions signals the completion of one cycle. To ensure that the chambers are always in sync, check valves are used at V1 and V2 while specially designed valves that open and close at predetermined timing are used for V3 and V4. Aqualyng makes no special mention for the valves used at V3 and V4 unlike other manufacturers who will, without fail, highlight theirs. To prevent mixing of the brine and feed, Aqualyng engage pistons to separate the flows. A drawing of the Aqualyng is presented in figure 1.8.

Figure 1.8 Aqualyng

The Aqualyng system was installed at the Tauro plant which had a capacity of 2000m³/d. Aqualyng performed admirably and helped lower energy consumption to only 1.93kWh/m³ when the plant operated with a recovery of 37%. It was also noted that energy consumption increase when recovery was either raised or lowered from 37%. When recovery was reduced, the pressure required at the inlet side of the RO membranes was also reduced. Although, the main pump continued to handle the same amount of fluid, it pumped them to a lower pressure and thus consumes less energy. On the other hand, the booster pump has to work harder since it needed to process a larger amount of fluid. This increased its energy consumption. The energy saving at the main pump brought on by the lowering of the recovery is negated by the increase at the booster pump. As recovery approached 37%, the positive effects outweighed the negative effects and vice versa. This configuration is not universal as each plant will have its own optimum conditions dependent on the number of membranes, the type of pressure vessels, the efficiency of the pumps etc.

Geisler [15-17] reported their operating experience with the PES produced by Siemag Transplan GmbH. The PES is proven equipment used in the field of mining for more than 20

years with volumetric flow rate up to 1400 m³/h and operating pressures up to 16 MPa. Operating experience was accumulated from 16 operational systems running unmanned 24 h/d. The PES bears many similarities to the Aqualyng. It works like the Aqualyng except it employs three pressure vessels instead of two. The 3rd vessel is used to further damp out fluctuations in the flow. To draft in the extra vessel and coordinate the flows to and fro the vessels, individual vessel operates at one-third of a cycle adrift. A combination of check valves and two-way valves are used to regulate the flow.

Mathematical simulation projected significant energy saving with the PES. A summary of the results is presented as a Sankey-diagram (figure 1.9) below. Assuming an efficiency of 69% for the motor-pump and a permeate recovery of 45%, as well as an efficiency of 98% for the PES, 36.8% of the input energy can be recovered from the energy contained in the brine that leaves the RO modules with a initial valve of 37.4%. Specific energy consumption per m³ also decreases by 35% in comparison to an ERT based system. The simulation was performed by a software developed by Siemag and the results showed good conformity with data collected from existing examples installed in desalinations plant.

Figure 1.9 Energy demand for PES based RO system

A pilot plant was subsequently built to verify the effectiveness of the PES and the results generated by the software. The results are:

1. Reduction of the specific energy demand=28% (Switching from ERT to PES)
- 5 2. ETE=98%
3. Specific energy savings=1.21 kWh/m³.

The Aqualyng, PES and various other similar ERDs were reviewed by Andrews [18]. Generally, this type of ERD utilizes 2 or 3 pressure vessels; check valves at the feed end, and differing valve systems at the brine end. A summary of these ERDs and their principal
10 differences are as follows:

1. Aqualyng, manufactured by LyngAgua S.L., utilizes two vessels and specially designed valves.
2. DWEER, manufactured by DesalCo Ltd., utilizes two vessels and a single patented multi-port valve, LinX [19].
- 15 3. DYPREX, manufactured by Ionics, Inc., developed by Aqua Design, Inc., utilizes three vessels and two-way valves.
4. PES, manufactured by Siemag Transplan GmbH, utilizes three vessels and two way valves.

The patented technology used on the DWEER, LinX, represents the latest innovation in
20 ERDs. This design replaced eight two-way valves with a single multi-port spool valve which simplified the system, improved reliability, reduced maintenance requirements and allowed faster valve switching. Singapore's RO desalination plant in Tuas is outfitted with DWEER to meet its energy recovery demands.

Another new innovation that has gained prominence is the PX manufactured by Energy
25 Recovery, Inc. (ERI). The PX was designed and developed to combine the high efficiency of isobaric chamber PD with the operational simplicity of passive, flow-driven rotation. The PX

is an evolution of a much earlier externally-driven rotary-type PD ERD-“An Apparatus Affecting Pressure Exchange”-patented by George Jendrassik of London in 1954 (figure 1.10). In adapting the concept of rotary PD ERD to SWRO, ERI’s founder Leif Hauge incorporated flow-driven rotation, a self-lubricating bearing and pressure transition control

5 features.

Figure 1.10 1954 rotary exchanger

A detailed description of how the PX works was presented by Stover [20]. The PX facilitates pressure transfer from the HP brine stream to the LP feed stream by putting the streams in direct, momentary contact. The transfer occurs in the rotor ducts. The rotor is fitted into a ceramic sleeve between two ceramic endcovers with precise clearances that, when filled with HP water, create an almost frictionless hydrodynamic bearing. At any given instant, half of the rotor ducts are exposed to the HP stream and half the ducts are exposed to the LP stream. As the rotor turns, ducts pass a sealing area that separates HP and LP. Thus,

25 the ducts that contain HP are separated from the adjacent ducts containing LP by the seal formed with the rotor’s ribs and the ceramic endcovers. A schematic representation of the ceramic PX is provided in figure 1.11.

30

Figure 1.11 PX flow schematic

- 5 The PX has condensed all the necessary hardware of Type 1 PD ERDs into one device with only one moving part. The rotor and its ducts function like the pressure vessels in Type 1 ERDs. Depending on the angular orientation of a duct, it is either filling up with LP feed or HP brine. The rotation of the rotor and strategically placed inlet/outlet on the endcovers behave like check valves and two-way valves timing and choreographing the filling process.
- 10 Requiring only one moving part to work, the PX can be more reliable and compact than its competitors.

- Stover also addressed a shortcoming in the PX. Because the PX has no pistons, there is no physical barrier between the brine and feed streams in the rotor ducts and mixing will increase. Mixing is undesirable because it has a detrimental effect on the RO process.
- 15 Increased salinity in the feed stream due to mixing in the PX increases its osmotic pressure and consequently, its energy consumption. For the PX, the degree of mixing between these streams is a function of the velocity of the streams and the duration of exposure. The duration

of exposure is controlled by the rotation speed, which is in turn a direct function of the velocity of the streams. The PX is designed so that the interface between the brine and feed streams never reaches the end of the rotor duct. Therefore, a quantity of liquid-“dead volume”-remains in the rotor duct at all times, traveling from one end of the rotor to the other and back again during a single rotation. The relationship between rotation speed, velocity of the brine-feed interface in the duct (dead volume), distance the interface travels and flow rate is illustrated in figure 1.12.

Figure 1.12 Rotor performance dynamics

As flow increases, the dead volume velocity increases, however, the rotor also speeds up which gives the dead volume less time to move in the duct. The variation of plug velocity and rotor speed with flow substantially cancel out such that the distance the brine-feed interface travels in the rotor duct varies little over the operating range of the PX. Accordingly, the degree of mixing between the streams varies little with flow.

Performance data gathered shows that mixing is approximately 6%. This value can be calculated using the following equation:

$$\text{Volumetric mixing} = \frac{\text{HP}_{\text{out}} \text{ salinity} - \text{LP}_{\text{in}} \text{ salinity}}{\text{HP}_{\text{in}} \text{ salinity} - \text{LP}_{\text{in}} \text{ salinity}} \times 100\% \quad (1.1)$$

The HP flow from the PX/booster pump blends with the feed from the HP pump before entering the membranes. The typical salinity increase at the membranes for a plant operating at 40% recovery is approximately 2.5% calculated with the following equation:

$$\text{Salinity increase} = \frac{\text{Membrane feed salinity} - \text{Seawater salinity}}{\text{Seawater salinity}} \times 100\% \quad (1.2)$$

A 2.5% increase in salinity at the membranes causes operating pressure to increase by approximately 1.3 bar. Expressed in terms of PX performance, the efficiency decrease caused by mixing in the PX at balanced flow is approximately 1%. Most plant operators and engineers choose to accept the increase in membrane pressure without changing process settings. As a comparison, the salinity increase caused by a 2°C temperature increase in the seawater feed-a daily occurrence in some plants-is the same as that caused by mixing in the PX.

However, if a plant operator or engineer chooses to compensate for the salinity increase to achieve the same amount of permeate without changing operating pressures; the following three options are available:

1. Increase feed by 5%.
2. Add 6% more membrane surface.
3. Increase LP feed to the PX by 5%.

Macharg [21] witnessed a third-party verification test on the PX. The test was conducted by the US Navy at the Seawater Desalination Test Facility at Port Hueneme, California. The PX was installed in a pilot plant designed to complement all the device's unique features and advantages. At the pilot plant, recovery was set at 36% and the average operating pressures were 700 psi. Pacific Ocean seawater was drawn in through an open intake and pumped up to the operating pressure by a CAT three-plunger, PD pump. Membranes are industry standard, off the shelf, Koch TFC spiral. Throughout the whole duration of the test, the ETE of the PX

averaged 95% and aided the pilot plant maintained a energy consumption of 2.0 kWh/m³. In plants equipped with a centrifugal main pump, the pump's lower efficiency would result in a higher consumption of 2.3 kWh/m³.

1.1.3 Turbine Type ERDs vs PD ERDs

5

The efficiencies of various ERDs and the design considerations of using them were discussed by Harris [22] and Wilf [23]. Turbine type ERDs are deemed to be less effective than PD ERDs because they do not directly transfer the hydraulic energy of the brine to the feed. Rather, this type of ERDs go through the process of converting hydraulic energy (brine) to mechanical energy (shaft) and then back to hydraulic energy (feed). This statement is backed by experimental results gathered by Silbernagel [4], Oklejas Jr. [5], Drablos [14] and Geisler [15-17].

Harris also noted that turbine type ERDs continue to be widely accepted in small and medium RO plants despite having a relatively lower ETE. He attributed this to the inherent simplicity of turbine type ERD equipped RO plants in terms of plant layout and automation and control. Figure 1.13 and figure 1.3 show two different arrangements of a RO plant equipped with turbine type ERDs.

Figure 1.13 Turbine type ERD based RO plant

Earlier generations of turbine type ERD used a configuration as simple as one depicted in the above figure. A turbine type ERD such as a Pelton wheel or a Francis turbine would be

located along the brine stream to harness its energy. The turbine would turn a common shaft joined to both the motor and the main pump. By contributing a share of the energy needs of the main pump, the ERD lessened the load on the motor and reduced energy consumption. In newer ideas such as those tested by Silbernagel [4] and Oklejas Jr. [5], a turbine type ERD would be mounted together with a smaller secondary pump (figure 1.3) while the main pump operated independently. Although it does not represent a significant improvement to the first design, this configuration helps to reduce the discharge pressure of the main pump by up to 20%. Also, the secondary pump needs no additional power input since the ERD supplies 100% of its requirement.

In contrast, a RO plant designed around a PD ERD has more parts. On top of the main pump, it also needs a motor-driven booster pump and numerous ancillary components (valves, controls, electrical equipment, pipeline manifolds, etc.) to ensure proper functioning of the ERD. However, the main pump is sized to handle only a volume equal to the permeate which is 40~60% of the entire flow. Likewise, the booster has to manage only a volume equal to the brine and supply a pressure boost of 3~5 bar. This primary and secondary pump sizing issue is the key to the reduced energy demands of the design and can only be accomplished with the use of a PD ERD.

One issue that is frequently overlooked is the ETE of both devices throughout their design flow range. The turbine type ERD has a wide variance of efficiency throughout its flow range. At very low flow rates, its ETE may hover around 35%, as flow rate is increased, ETE will also improve and peak at roughly 80% towards the upper limit of its design flow range. This issue will not pose a problem if the ERD is properly sized to work in its upper ranges of flow rate. However, it prevents the plant from having the liberty of operating at lower recoveries since the ERD – pump will have to pump more fluid for the same amount of permeate. PD ERDs, on the other hand, has a very constant ETE of around 90% over its

entire flow range. This gives PD ERDs based RO plants the flexibility to operate at any recoveries since the permeate flow and the brine are effectively decoupled. To adjust the system recovery, the booster pump can be throttled up or down without the penalty of higher energy consumption since the ETE of the PD ERD is almost constant. A lower recovery can
5 prolong membrane life as well as improve the quality of the permeate.

Sallangos [24] reported on the operating experience of the Dhekelia RO plant in using the Francis turbine and later, the PX when they wanted to decrease the overall plant energy consumption to the contracted figure of 5.3 kWh/m³.

Dhekelia RO plant in Cyprus had been in operation for 7 years. It included eight 5000
10 m³/d SWRO trains operating with Mediterranean seawater with a TDS content of 41800 ppm, with water temperature ranging from 17°C to 32°C. The ER system originally installed at the plant was the Francis turbine, which, at the time, was considered one of the most efficient and economical devices on the market. However, energy consumption for the whole plant reached as high as 6.0 kWh/m³ from the original start-up of the plant with current values approaching
15 6.3 kWh/m³. The high power consumption prompted a feasibility study, after which certain modifications were carried out to decrease consumption. Four different ERDs were considered for replacement for the existing system; Pelton wheel, DWEER, PX and PES. The latter was eventually counted out due to prohibitively high cost. The remaining three would roughly cost the same when the entire cost of retrofitting was considered so capital
20 investment ceases to be a selection criterion. In terms of required modifications, the DWEER would require the most and the PX the least. Amongst the three, the PX also promised the highest guaranteed efficiency of 94.0%. Consequently, the PX was selected and installed in phases. Following three years of continuous cooperation, actual operating data was reviewed and compared against projected values supplied by the manufacturer. The result was that the
25 PX has exceeded the guaranteed values on all counts. The actual ETE was 95.3%, up 1.3%

from the projected value. Salinity increased by 2.8% when the projected amount was 3.5%. Pressure drop across the PX was also lower than predicted. Up to date, the PX continues to operate with minimal maintenance and help Dhekelia RO plant meets its contractual agreements. Sallangos concluded by saying that perhaps efficiency in ERDs has reached a plateau.

1.2 Need for More Research

Experimental studies on dynamic process of PD ERDs conducted by Wang et al. [25-26] showed opportunities for further development. The experimental apparatus is shown in figure

1.14.

Figure 1.14 Flow diagram of the experimental apparatus

The system is typical of PD ERDs shown in figure 1.6. The primary part of the system consisted of two chambers made of organic glass with a length of 1.0 m, internal diameter of 50mm and working pressure up to 1.0 MPa. Tap water was used as the working fluid instead

of actual seawater and brine. A HP centrifugal pump and LP feed pump were included to supply HP brine from the hydro-pneumatic tank 1 and seawater from tank 2 to the ERD. Four magnet valves and four non-return valves were located respectively at the ends of the chambers, which could change the direction of the working fluid flowing through the control of a programmable controller. Four flow transmitters and eight pressure transmitters were installed in the corresponding positions. Standard current signals (4-20mA) from transmitters were collected and transferred to the data acquisition system.

Results showed that pressure difference between the incoming HP brine and outgoing HP feed was very small which confirmed the high ETE of PD ERD. Also, the orientation of the pressure vessels-horizontally or vertically, with and without pistons-had little effect on the experimental results. Additionally, pressure in the pressure vessels behaved like a rectangular wave function accompanying the opening and closing of the valves. The pressure in the vessels behaved like that because pressure inside remained high after the HP brine displaced the feed but plunged to around atmospheric pressure when the magnet valve opened. The opposite happened when the LP feed was about to be displaced by the HP brine. The pressure of the LP feed would shoot up when the HP brine entered the vessel. As a result, energy was discarded every time the pressure vessels undergo depressurization. The discarded energy is the pressure energy released by the HP brine whenever it is exposed to the LP feed. This can be seen easily in figure 1.15. Imagine that HP brine is contained in the piston-cylinder device. Ordinarily, the pressurized fluid in the device will expand and push the piston out, and thus accomplishing work. In current PD ERDs however, this is not allowed to happen. Instead, the energy is released into the air.

Figure 1.15 pV diagram of current ERDs

With reference to Cheng's patent drawing shown in figure 1.5, the ideal ERD he envisaged was being continually supplied with replacement fluid in sector A already pre-pressurized to the working pressure of the system. This is not permitted in current ERDs. Instead, a blow-down takes place when the HP brine is exposed to the LP feed entering the vessel. This causes a sudden release of pressure energy which may manifest as a water hammer. In the PES and the Aqualyng, pressure build-up valves and pressure relief valves work in cooperation with check valves at the brine end of the vessel to control the release of the pressure. The LinX of DWEER managed the release by gradually pressurizing and depressurizing the vessel. So far, no ERD on the market is geared to harness this energy. Had the deliberate release of this hydraulic energy been prevented and the energy meaningfully transferred to the incoming LP feed, the power consumption of the main pump would be reduced. In the experiment, the pressure level of the brine exhibited a momentary drop whenever a new cycle of pumping the LP feed begins, more precisely, when the HP brine front encounters the LP feed front. The drop in pressure was caused by the brief expansion of

the HP brine before the main pump recovered the drop in pressure. Such fluctuations can also be avoided by redirecting the hydraulic energy in the HP brine to the LP feed.

1.3 Objectives of Research

5 The objectives of this research are to design and develop an ERD incorporating all the proven technologies from PD ERDs while retrieving and transferring the pressure energy of the brine to the feed. Such a device has the potential to further lower the power consumption of RO process and make portable water even more readily available.

1.4 Scope of Research

10

1. A novel ERD will be designed and developed.
2. A functional prototype of the proposed design will be constructed and tested.
3. A pilot-scale plant designed to test the proposed ERD will be built.
4. Modifications to improve the design will be proposed for future studies.

2 Proposed ERD and experimental set-up

2.1 Concept

Consider a PD ERD such as the Aqualyng or PX. As one pressure vessel is being filled with HP brine, another is being topped up with LP feed. When both are completely filled, their roles will switch. As discussed in earlier sections, the switching process causes pressure energy to be lost from the HP brine. To overcome this problem, an additional step shall be introduced into their cycle. The HP brine and LP feed shall be allowed to briefly interact before they are pumped out. When the LP feed is exposed to the HP brine, hydraulic energy will be transferred to it almost instantaneously. A simplified version of the proposed concept is shown in figure 2.1. It consists of 2 pressure vessels which is the most common configuration among current PD ERDs. Additionally, the pressure vessels are joined by a connecting valve so that pressure transfer can occur between the vessels.

Figure 2.1 Proposed concept

After the interaction between the HP brine and LP feed, the liquids in both vessels would attain the same pressure. It will be higher than the LP feed but lower than the HP brine. The actual result can be estimated using conservation of mass.

2.1.1 Determining the final pressure in the pressure exchange process

To simplify the problem of finding the final pressure, several assumptions must be made and justified. No mass flow across the boundaries of the control volume so this must be a closed system. The system is stationary so ΔKE and ΔPE is equal to zero and ΔE is ΔU . The wall of the tank is rigid so w_b is zero. q_{in} is taken to be positive. The system is allowed to
 5 exchange heat with the surrounding. There is no electrical, shaft or any other kind of work involved. Lastly, both brine and seawater is treated as water as they differ mainly in their salt concentration.

Table 2.1 Properties of water

Density	ρ	999.1 kg/m ³
Specific volume @ atm pressure and 20 °C	v	0.001001 m ³ /kg
Bulk modulus of elasticity	B	214 x 10 ⁷ Pa
Isothermal compressibility	β_T	4.67 x 10 ⁻¹⁰ Pa ⁻¹

10 The volume, V of both vessels should be the same. Thus, the mass of the seawater and brine can be found by

$$m = \frac{V}{v} \quad (2.1)$$

The pressure in the vessels will continue to fluctuate until they are equalized. To find the final pressure, the combined mass of the seawater and brine must be determined

$$15 \quad m_{combined} = V \left(\frac{1}{v_{brine}} + \frac{1}{v_{seawater}} \right) \quad (2.2)$$

The specific volume of the combined fluids will be

$$v_{final} = \frac{2v_{brine}v_{seawater}}{v_{brine} + v_{seawater}} \quad (2.3)$$

The final pressure reached can be found by considering the generalized state equation for liquids and solids.

$$dv = \frac{\partial v}{\partial p_T} dp + \frac{\partial v}{\partial T_p} dT \quad (2.4)$$

Since temperature remains constant

$$dv = \frac{\partial v}{\partial p_T} dp \quad (2.5)$$

Introducing

$$5 \quad \beta_T = -\frac{1}{v} \frac{\partial v}{\partial p_T} \quad (2.6)$$

If (2.6) is substituted into (2.5)

$$dv = -v\beta_T dp \quad (2.7)$$

Rearranging

$$dp = -\frac{dv}{v\beta_T} \quad (2.8)$$

10 In finite form

$$P_{final} - P_{initial} = -\frac{\ln\left(\frac{v_{final}}{v_{initial}}\right)}{\beta_T} \quad (2.9)$$

As expected, the final pressure attained would roughly equal the mean of both high and low pressures. The design can still be improved. In order to raise the pressure even further, extra enhancements have to be added without increasing the complexity of the system.

15 Consider figure 2.2, the number of pressure vessels is doubled to permit a more progressive pressure transfer and potentially higher final pressure.

The working principle of this system is not dissimilar to the original design. The only significant difference is that the pressure exchange between the HP brine and LP feed takes place over three phases instead of just one. In stage one, the connecting valve between vessel
20 2 and 3 is opened, this cause the pressure in the HP brine to plummet to around half its previous level while the pressure of the LP feed increases to the same amount. In the 2nd

stage, the connecting valves between vessel 1 and 2 and vessel 3 and 4 are opened. As the LP feed in vessel 2 was already boosted in stage 1, the 2nd pressure transfer will further increased its pressure. The pressure reached is approximately 75% of the brine's original pressure. The pressure increase experienced by vessel 4 is more subtle since vessel 3's pressure is only half
5 its previous level. The pressure attained by vessel 4 will be around 25% of the brine's original pressure. In the 3rd and last stage, the valve joining vessel 1 and 4 opens, note that vessel's 1 pressure is 75% of its original while vessel 4 has around 25% of the brine's original pressure. At the end of the interaction, both will settle at the 50% level. When the contents of vessel 2 and 4 are combined, they will yield a pressure that is around 67.5 % of
10 brine's original pressure.

The improvement over the original idea is not salient, but it illustrates the potential of the idea. If more vessels can be built into the system, a higher final pressure is possible. While this relation is true for any number of vessels, it is likely that the change in final pressure attained will tail-off after a certain point. Adding more vessels after this point results in less
15 and less perceptible increase in the final pressure attained.

Figure 2.2 Layout of enhanced system

2.1.2 Mechanics of pressure transfer

The pressure energy of the HP brine is transferred to the LP feed by pressure waves. This concept can be illustrated with the help of a pressure vessel of cross section A built with a centrally-positioned ball valve (figure 2.3). Initially, one half is filled with LP feed and the other half is filled with HP brine with the ball valve completely shut. When the ball valve is suddenly opened, the HP brine will start to expand with a velocity V for time Δt . An increase in pressure of the LP feed results and manifests as a pressure wave with a velocity c through the liquid. As the pressure wave moves through the LP feed, the pressure, density and temperature will change by small amounts Δp_{LP} , $\Delta \rho_{LP}$ and ΔT_{LP} , respectively. There will also be an induced velocity V in the liquid immediately behind the pressure wave. At the same time, the expansion of the HP brine causes its pressure to decrease and results in the formation of another pressure wave with velocity $-c$ through the HP brine. Similarly, the HP brine is subjected to changes in the pressure, density and temperature by small amounts Δp_{HP} ,

$\Delta\rho_{HP}$ and ΔT_{HP} , respectively. As the pressure wave sweep through the HP brine, the liquid is accelerated to velocity V . As the HP brine continues to expand, the back and forth movement of the pressure waves will persist, making incremental changes to the liquid properties. The entire column of liquid will oscillate about its new equilibrium position before it comes to a rest. When it is completely stationary, the pressure in both columns will be equalized. The pressure wave travels at the speed of sound; therefore the pressure transfer is achieved in a very short time. This is particularly true for the short liquid column of less than a meter in length.

Figure 2.3 Elastic liquid subject to unsteady pressure force

The speed of the pressure wave can be derived by equating the mass displaced by an imaginary piston which form the boundary between the HP brine and the LP feed and the change in mass between position 1 and 2 due to increased in density [27-29].

$$\rho V A \Delta t = A c \Delta t \Delta \rho \quad (2.10)$$

Rearranging,

$$c = \frac{V \rho}{\Delta \rho} \quad (2.11)$$

Similarly, c can be calculated by equating the impulse produced by the expanding HP brine to the increase in momentum of the LP feed.

$$A\Delta p\Delta t = \rho cAV\Delta t \quad (2.12)$$

5 Rearranging,

$$c = \frac{\Delta p}{V\rho} \quad (2.13)$$

10 Multiplying (2.10) by (2.13) gives,

$$c = \sqrt{\frac{\Delta p}{\Delta \rho}} \quad (2.14)$$

If Δp and $\Delta \rho$ are small,

$$15 \quad \frac{\Delta p}{\Delta \rho} = \frac{dp}{d\rho} \quad (2.15)$$

The bulk modulus of elasticity is defined as

$$20 \quad B = \frac{\text{Increment in stress}}{\text{Increment in unit deformation}} = \frac{dp}{-dv/v} \quad (2.16)$$

Therefore (2.14) becomes

$$c = \sqrt{\frac{B}{\rho}} \quad (2.17)$$

25 Using (2.17) and properties of water in table 2.1, the speed of the pressure wave in water is estimated to be 1450 ms^{-1} . This can only be considered to be a very rough and initial estimate as many other factors such as radial expansion of vessel walls also contribute to the wave speed. However, it is not within the scope of this report to conduct an in-depth discussion on wave speed as it is more than suffice to show that the pressure transfer process
30 is extremely rapid provided that the vessel is not more than 100 m long and sufficiently rigid.

2.1.3 Energy balance of pressure exchange process

To understand the energy balance for the pressure exchange process, consider two rigid pressure vessels connected at the center by an on-off valve (figure 2.4). At the start, one pressure vessel contains a certain amount of liquid at p_{brine} Pa and room temperature T °C while the other vessel holds the same amount of liquid at the same temperature at p_{seawater} Pa.

The ball valve is then turned on and the high pressure liquid expands into the other tank. The liquid is allowed to exchange heat with its surrounding until its temperature returns to the initial value of T °C.

The low pressure liquid being compressed is regarded as the system. This is a closed system since no mass crosses the system boundary during the process.

Figure 2.4 Schematic and pV diagram for pressure exchange process

It is assumed that the system is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$ and $\Delta E = \Delta U$. Also the direction of heat transfer is into the system and the liquid temperature remains constant during the process. High pressure liquid (imaginary piston) in the first tank will push the enclosed liquid in the connecting valve into

the low pressure tank at decreasing pressure therefore boundary work is present. Under the stated assumptions and observations, the energy balance on the system can be expressed as

$$Q_{\text{net, in}} - W_{\text{net, out}} = \Delta U_{\text{system}} \quad (2.18)$$

Liquid properties are found from the compressed liquid table, otherwise it is taken from direct measurements made by Kell [35]. Thus,

$$y = y_{\text{compressed}} \quad (2.19)$$

The total boundary work done during the entire process as the imaginary piston moves is obtained by adding all the differential works from the initial state to the final state

$$W_b = m \int_a^b p dv \quad (2.20)$$

Substituting the infinitesimal form of equation (2.9) gives

$$W_b = m \int_a^b \left[-\frac{1}{\beta_T} \ln|v| + c_1 \right] dv \quad (2.21)$$

Integrating by parts yields

$$W_b = m \left\{ v \left[-\frac{1}{\beta_T} (\ln v - 1) + c_1 \right] \right\}_a^b \quad (2.22)$$

Depending on the initial pressures of both the brine and seawater, the amount of additional energy saved can range from 5 -10 kJ/m³ when compared to the newest and most efficiency ERDs on the market. The derivation of these figures can be found in appendix G.

2.2 ERD

After drawbacks of current ERD were identified and their solutions proposed, a long, tedious design and development process began. The final product must incorporate all the benefits available in current ERDs as well as enhancement features proposed in this report. At the same time, it must be economical, easy to machine and build and simple in design. The pipe and valve system used in section 2.1 as illustration was initially considered. To recover a

higher amount of pressure energy from brine, only more pipes and valves are needed. But as the number of pipe and valve combinations is increased, the system becomes more complicated too. To join all the pipes together, a sprawling network of adjoining pipes will be needed. Cost will go up, as will maintenance needs, especially when they sprang a leak. Its problems are too serious to ignore. However, its advantages must not be overlooked, pressure energy is 100% retained in the pipes.

The proposed ERD was the end result of a rigorous selection process. Multiple designs were prepared for each and every aspect of the ERD. More than ten different ways to seal the ducts in the rotor were suggested. The drivetrain (Appendix B) to turn the rotors was configured and re-configured so that the least number of components are needed. The selection process extends to the dimensions as well. Only 1st-choice sizes are used so that parts needed will be cheap and easily available.

To ensure that nothing was faulty, every calculation and every drawing were checked at least five times. It is a must because every mistake made on the drawing board will become costly errors in production and testing. In the end, every part worked.

2.2.1 Principle of operation

Figure 2.5 shows an exploded drawing of the proposed ERD. It combines all the proven advantages of a PD ERD and the aforementioned pressure transfer technique into one device. The ERD uses two rotors with longitudinal ducts parallel to its rotational axis to transfer the hydraulic energy from the brine to the feed through holes in the centerplate. The longitudinal ducts act like the pressure vessels while the holes act like the connecting valves illustrated in chapter 2.1. The rotors rotate in opposite directions in plain bearings supported by hydrodynamic action. HP brine and LP feed enter the rotor ducts through inlets on the centerplate while LP brine and HP feed leave via outlets on the endplates. Precise clearance between the rotors, endplates and centerplate form sealing surfaces for the ducts to lock in the

hydraulic energy of the HP brine and HP feed. While the centerplate forms a divider to separate the rotors, there are purposefully placed through-holes on it to allow pressure transfer between two opposite ducts.

Figure 2.5 Proposed ERD

By rotation of the rotors, HP brine would enter the ducts in both rotors while LP feed
 10 does the same through the other inlet on the centerplate (figure 2.6). Further rotation would
 bring opposite ducts in-line. Since one duct contains HP brine and the other LP feed, pressure
 would be transferred to the LP feed via the through-hole on the centerplate. The LP feed and
 HP brine will take on a pressure that is the mean of their respective initial values. The process
 will be repeated when the ducts coincide with another through-hole. As more exchanges take
 15 place, the pressure of the LP feed would be gradually raised whilst the pressure of the HP
 brine drops. There are a total of nine through-holes on each half of the centerplate. Towards

the end of one cycle, the LP feed would nearly reach the pressure of the HP brine minus hydraulic losses within the ducts and the HP brine would have relinquished almost all of its pressure energy. After 180° of rotation, the HP feed would be displaced by the HP brine at the HP brine inlet. At the same time, the LP brine would be discharged from its duct by the incoming LP feed. Regarding a query whether “the HP seawater outlets from the endplates would be more apt to be the brine streams and the LP brine outlets to be the seawater streams”, a more detail description of the mode of operation of the prototype can be found in appendix H.

Figure 2.6 Proposed ERD flow schematic

2.2.2 Determination of the number of ducts and through-holes in ERD

Very early in the design stage, through-holes were placed with the same interval separating the ducts. For example, if ten ducts are proposed, the through-holes will be positioned 36° apart. Later, it was found that pressure exchange does not take place

sequentially in the ducts as preferred but alternately (figure 2.7). As one duct rotates towards the next through-hole, another duct in the adjacent rotor is aligning itself with the 1st duct. Nothing happens, since there is no through-hole between the two of them to facilitate the pressure transfer. The number of through-holes must be doubled to maximize the frequency of pressure transfer. Now, there will be two through-holes for every duct.

Figure 2.7 Pressure exchange in ERD

The progressiveness of the pressure transfer and the eventual pressure attained by the LP feed are decided by the number of ducts and through-holes in the ERD. This was clearly explained in section 2.1.1. The more ducts and through-holes there are the more gradual the build-up or release of pressure in the ducts and the higher the hydraulic energy recovered.

This can be further elaborated by tracking the pressure changes in the ducts as the rotors rotated. Figure 2.8 compares the effectiveness of having three through-holes and nine through-holes. In order for the comparison to be valid, it was assumed that no losses occurred during the pressure transfer and opposite ducts attained a pressure that is the mean of their initial values. Also, the pressure readings are recorded only after the process has reached steady-state. Lastly, the HP brine was assumed to have an initial pressure of 8MPa. Microsoft Excel is used to perform the simulation.

Figure 2.8 Pressure attained by LP feed

The results showed that the pressure transfer process resembled a linearly increasing step function. On top of that, the comparison of these two configurations produced distinct results supporting the usage of more ducts and holes. The simulation of the pressure transfer using three holes produced more abrupt changes in pressure at each hole. More importantly, the final pressure reached by the LP feed was only 6 MPa for the three-hole design while the LP feed achieved 7.2 MPa using the nine holes configuration.

However, it is not possible to increase the numbers of ducts and holes without end due to heightened viscous drag and leakage problems. For example, given two ERD having the same capacity with different number of ducts, the one with more ducts have ducts that are smaller in diameter. The reduction in cross-sectional area means more drag will work against flow going in and out of the device. Consequently, the increase in frictional losses may negate any gains in efficiency by having more ducts. Therefore, it is important to make the ducts as wide as possible.

Leakage is another obstacle that limits the number of ducts. As the number of ducts is increased, the proximity between adjacent ducts will be reduced spelling the reduction in sealing area for each duct. Unlike the earlier pipe and ball valve design where each pipe is

sealed, the current design inevitably have some leak through the gaps that are necessary for rotation of its rotors. Such leakage must be minimized as well. The reason is obvious, if the leakage is overwhelming, the ducts would loose its capacity to preserve and transmit pressure.

5 Nevertheless, all these factors must be carefully analyzed and a compromise achieved for the ERD to work.

2.2.3 Mixing of brine and feed in the ERD

Because the ERD does not have any pistons like the PX, mixing of the feed and brine will
 10 occur. The same technique used in the PX to minimize mixing can also be employed here. The rotation speed of the rotor will be governed such that the interface of the feed and brine will never reach the end of the duct. Therefore, a quantity of liquid – “dead volume” – remains in the rotor duct at all times, moving back and forth with the rotation of the rotor. The rotors are rotated by an electric motor using a bespoke system of bevel gears and flexible
 15 couplings and their rotation speed can be change by adjusting the VFD on the motor.

The main pump used in the experimental setup has a maximum supply output of 0.5 m³/h or 8.33 x 10⁻³ m³/min. In order to determine the optimum rotation speed of the ERD, the volume of a duct in the rotor has to be calculated first.

$$V_{duct} = \pi r^2 h \quad (2.23)$$

20 Since $r = 0.005$ m and $h = 0.15$ m, $V_{duct} = 1.18 \times 10^{-5}$ m³. In one revolution of the rotor, the combined volume of 12 ducts will be 1.41×10^{-4} m³. In order to maintain a dead volume in the duct, the ERD must spin faster than

$$RPM = \frac{\dot{V}_{HPbrine}}{V_{rotor}} \quad (2.24)$$

this is 59.10 rpm.

2.2.4 Material used in the construction of the ERD

To keep corrosion at bay, the ERD was originally designed to be constructed from a combination of stainless steel SS316 for all wetted parts and aluminum T6061-0 for non-critical support structures. Due to budget constraints, carbon steel AISI 1045 was substituted for all stainless steel parts.

2.3 Experimental set-up

The pilot plant mentioned in the 1st year report “Design and develop a seawater reverse osmosis energy recovery device” did not materialize as the ACRP research fund application was rejected by the Ministry of Education. However, the design can still be found in appendix E should the need for such a plant arise.

A low cost substitute was used instead. The author bore all the cost arising from the project. The experimental setup is shown in figure 2.9 and a description of all the components in table 2.2.

Table 2.2 Description of items in experiment

Item no.	Description	
WT1	Reservoir	
V1,V2,V4	Ball valves PN 25	
V3, V5	Ball valves PN 64	
PU1	Centrifugal pump, 0.5 m ³ /h, 20 bar	20
PU2	Centrifugal pump, 0.5 m ³ /h, 3.5 bar	
PT1-5	Pressure transducers	
F1-2	Oil filters <10 µm	
ERD	Proposed energy recovery device	
		25

Figure 2.9 Experimental setup

The experiment rig was set up in the Energy System Lab in NTU with a production rate of 0.1-0.5 m³/h. Seawater and brine was substituted by engine oil. The rational behind this modification was to prevent corrosion of the ERD which was fabricated using a low-cost carbon steel material instead of stainless steel as preferred in the 1st year report. It would not influence the validity of the experiment significantly since neither viscosity nor other characteristics of the medium used affects the results adversely. This can be inferred from section 2.1. Even so, a few factors such as the compressibility of the liquid do affect the end results to a certain extent. For example, increased compressibility of the liquid means greater boundary movement and thus greater work done. Viscosity may affect the results as well since it directly affects the impedance against the liquid in its outward expansion. Naturally, the higher the viscosity, the higher the impedance and vice versa.

The substitution was made also with a long-term view towards the durability of individual components in the setup. It was anticipated that the experimental setup would operate intermittently over a period of three months, the frequent shut-down and inactivity of the components would increase the likelihood of breakdown due to corrosion and biological

growth. In addition, usage of seawater would require the provision of a flush-down system which is omitted due to cost considerations.

A 60 liter PVC tank served as a reservoir in the setup. A recirculatory system was the most economical way to test the prototype. Oil was directed back to the reservoir after
5 passing through the ERD. The oil level must be continually monitored to prevent dry running and damage to the equipment. To remove suspended solids and debris from the initial running-in, oil filters used in automobiles were installed at the inlets of the reservoir. Other solutions such as coffee bags and at a later stage, filter papers, were tried initially but could not provide effective high flow and high pressure filtering. To maintain optimum cleaning
10 performance, the oil filters were changed periodically.

A 3-phase, 415 V, high pressure centrifugal pump PU1, supplies 0.1-0.5 m³/h of oil from oil tank WT1 at 1 - 20 bar to the HP inlet of the ERD. This portion emulated the HP brine channeled out of the RO modules in desalination plants. As the pump was driven by a fixed speed motor, supply pressure and flow are inversely proportional to one another. To vary its
15 output, ball valve V5, was used. Output pressure was managed by manually throttling the valve. To determine the pump output pressure, the operator relied on the feedback from the pressure transducer PT1.

Another centrifugal pump, PU2, supplied an equal amount of oil at 0.1-0.2 bar to the LP inlet of the ERD. This quantity was supposed to be the LP seawater directed into the ERD. Its
20 pressure was not regulated but allowed to rise and fall according to the ERD. Its flow rate, on the other hand, would be regulated using ball valve V4 in order to match PU1's output.

Both channels of flow were measured using a large 1 liter measuring cylinder and a stopwatch. The cylinder will be inserted into the incoming flow at the oil tank and the time required to fill up the cylinder recorded. The results would be compared and balanced.
25 Pressure transducers PT2 and PT5 were used to measure the pressure difference at the ERD's

LP inlet and outlet while PT1 and PT4 were used to monitor the pressure difference at the ERD's HP inlet and outlet. An additional pressure transducer PT3 was positioned on one of the endplates sharing the same radial location as the ducts (figure 2.5) to detect pressure surges within the ERD. Pressure signals were emitted every time a duct skirted passes it.

- 5 Such surges provided evidence that pressure exchanges between adjacent ducts were taking place.

All pressure readings with the exception of PT4 were recorded using a 4-channel oscilloscope. PT4 produced only current output and not voltage output as required by the oscilloscope.

- 10 A breakdown of all the cost involved in the experiment is given in table 2.3.

Table 2.3 Cost breakdown of experimental setup

Category	Item no.	Description	Cost, S\$
Drivetrain	1	Chain & sprocket	85.00
	2	Ball bearings	80.00
	3	Spur gears	35.00
	4	Flexible coupling	200.00
	5	Raw materials for - Bearing housing - Shafts	200.00
	Sub-total		600.00
ERD	1	Raw materials for - Centerplate - Rotors - Endplates - Bearings - Shafts - Aluminum platform	790.00
	2	Rig	190.00
	3	Labour	Supported by NTU CNC Lab
	Sub-total		980.00
Experimental setup	1	High pressure centrifugal pump	On loan from Advance Water Technology Pte Ltd
	2	Low pressure centrifugal pump	On loan from Cleansea Pte Ltd

3	3 hp electric motor with VFD	Lab equipment
4	Piping and associated accessories	1150.00
5	4-channel oscilloscope	On loan
6	Pressure transducers x 4	On loan
7	Pressure transducers x 1	200.00
		(Heavily subsidized)
8	Infrared tachometer	Lab equipment
9	Infrared thermometer	Lab equipment
10	Labour	420.00
11	Expendable - Engine oil	150.00
	Sub-total	1920.00
	Total	3300.00

2.3.1 Calculating ETE

Using information collected by the sensors, the ETE of the ERD can be calculated using the following equation [20]:

$$ETE = \frac{\sum (P \times Q)_{out}}{\sum (P \times Q)_{in}} \times 100\% \quad (2.25)$$

Expanding it, the above equation becomes,

$$ETE = \frac{P_{HPfeed} Q_{HPfeed} + P_{LPbrine} Q_{LPbrine}}{P_{HPbrine} Q_{HPbrine} + P_{LPfeed} Q_{LPfeed}} \times 100\% \quad (2.26)$$

2.3.2 Balancing inputs and outputs of the ERD

The flows entering and leaving the ERD can be balanced by considering the following equations [30] together with figure 2.10. Firstly, a relationship with the main pump PU3 LP input, $Q_{LP\ feed}$, permeate flow, Q_P , and ERD internal leakage, Q_L , should be established. Since there are two outputs from the system, the 1st being the permeate and the 2nd, leakage from the ERD, we get:

$$Q_{LP\ feed} = Q_L + Q_P \quad (2.27)$$

The internal leakage occurs as a small amount of HP brine seeps through the clearances needed for rotation within the ERD. It leaves the system together with the LP brine. The internal leakage flow rate can be determined by operating the plant and temporarily shutting off the LP feed to the ERD. While the leakage flow rate is directly proportional to the operating pressure, it is usually below 5% of the feed flow. Recover ratio, R , of the system is defined as:

$$R = \frac{Q_{LP \text{ feed}} - Q_L}{Q_{HP \text{ feed}}} \quad \text{or} \quad \frac{Q_P}{Q_{HP \text{ feed}}} \quad (2.28)$$

In order to determine a balanced brine flow rate, the HP feed handled by the booster pump is calculated:

$$Q_{\text{booster}} = (1-R) \times Q_{HP \text{ feed}} \quad (2.29)$$

Knowing the booster pump flow rate, Q_{booster} , a simple relationship establishes the balanced brine flow rate, Q_{brine} .

$$Q_{\text{brine}} = Q_{\text{booster}} + Q_L \quad (2.30)$$

The HP brine flow rate and LP feed flow rate to the ERD must be match carefully, once the balance is upset, permeate salinity will increase.

Figure 2.10 Flow diagram

3 Bending in the circular endplate

3.1 Finite element analysis

Figure 2.5 shows an exploded view of the proposed ERD with the endplate. During operation, the rotors will spin at a high speed within the bearing while the endplate provides axial location and pressure sealing. This is achieved by ensuring there is a small clearance between the rotor and endplate. This clearance must be carefully controlled since an overly generous gap will prevent effective sealing and an unnecessarily narrow one will cause the rotor to seize during operation if particles larger than the gap enter the ERD. Although the clearance has been designed to be 10~20 μm , deformation of the endplate due to the pressure within the ERD may result in a bigger clearance. Thus, it is imperative that this deformation is considered. The ultimate aim of estimating the deformation is to determine the appropriate thickness of the endplate. An initial value of 20 mm is used. Figure 3.1 shows a sketch of the endplate in 2D view.

Figure 3.1 Endplate

The pressure region is the circular path of the rotor ducts. In previous sections, it was mentioned that the pressure within the ducts is progressively raised, so in actual fact, the pressure exerted is not uniform. However, to simplify the problem, the endplate is assumed to be subjected to uniformly applied pressure. This assumption will produce larger deformations

since a wholly loaded plate will deform more than one that is partially loaded at the same pressure.

The deflection of the endplate was investigated using a commercial finite element software ANSYS. The endplate was modeled as a solid cylinder with all the holes found on the actual part. Properties of stainless steel 316 were used. The element used was an eight-node quadrilateral 3-dimensional solid. The part was constrained by enforcing zero displacement in all degrees of freedom (DOF) around the outer circular surface to imitate clamping. The first round of simulation involved applying a uniformly applied pressure on the inner surface of the endplate. Deflections due to different pressures were recorded. In the next round of simulation, pressure was applied only to areas exposed to the ducts to increase the accuracy of the results.

The theoretical analysis done in the 1st report has been taken out and placed in the appendix for reference purposes.

3.2 Results and Discussion

Table 3.1 shows the major dimensions of the endplate as well as physical properties of stainless steel 316. Figure 3.1 shows the deflection of the endplate when subjected to a uniformly distributed load as predicted by ANSYS. The profile of the deflection did not change with variation in pressures. Therefore, it is representative of all the different pressure exerted. At 8MPa, the maximum designed internal pressure, the deflection at the center is estimated to be around 25 μm . At the region of interest ($r=55\text{ mm}$), deflection is only around 5 μm .

Table 3.1 Major dimensions of the endplate and physical properties of stainless steel 316

Diameter of endplate	D	150 mm
Thickness of endplate	t	20 mm
Modulus of elasticity of SS316	E	$193 \times 10^9 \text{ Pa}$
Poisson's ratio	ν	0.29

20

Figure 3.1 Graphical results of FEMA

25

When the FEM analysis was repeated using the more realistic pressure distribution, deflection of the endplate was greatly minimized (figure 3.2). As evident from figure 3.2, deflection caused by the actual pressure distribution is even lower than that caused by a uniformly distributed load. At $r=55$ mm, deflection is only $0.5\text{ }\mu\text{m}$. This result is expected as less surface area is exposed to the internal pressure.

Figure 3.2 Deflection of endplate due to variable pressure distribution

Based on these results, a thickness of 20 mm was specified for the endplates as the magnitude of deflection is not expected to affect the performance of the ERD adversely.

4 Leakage in the ERD

4.1 Identification of regions of leakage in the ERD

A 3-D drawing of a rotor can be found in figure 2.5. During operation, both HP brine and LP feed are injected into the rotor ducts. Through pressure transfer between these two liquids, the LP feed will acquire the hydraulic energy of the HP brine. As the pressure of the HP brine is many times higher than the surrounding (atmospheric pressure), leakage occurs. There are two main sources of leakage, the first comes from the HP brine inlet and the second from rotor ducts. Both spread through the gap between the endplate and rotor and the gap between the rotor and centerplate (figure 4.1).

Fig. 4.1a Leakage from HP brine inlet and HP feed outlet.

Figure 4.1

Fig. 4.1b Leakage from ducts.

Figure 4.1a illustrates the leakage flow as a result of the incoming HP brine flow. As HP brine is pumped into the ERD, the high pressure forces some brine to escape through the small gaps between the components.

In figure 4.1b, the leakage flow is caused by the expansion of the liquids in the ducts through the clearances. This flow will persist only when there is a difference in the liquid pressure and the surrounding pressure. As water is highly incompressible, unlike other gaseous substances, this leakage flow is expected to be very small and occurs only momentarily. In order to preserve the pressure energy in the liquid, the leakage must be minimized.

4.2 Analysis of leakage in ERD

Both leakage flows have a negative influence on the performance of the ERD. To minimize leakage and maintain a sufficiently large clearance for the smooth operation of the ERD, the relationship between the clearance, δ and the leakage flow, Q_L must be established. Although there are 12 eccentrically positioned ducts within a rotor, each duct is modeled as a hollow cylinder with thickness, (r_o-r_i) . Likewise, the HP brine inlet is also modeled as a hollow cylinder with thickness, (R_o-R_i) . Additionally, the rotor is assumed to be stationary as its sliding velocity is small compared to the velocity of the leakage flow, therefore its effect can be considered to be negligible.

4.2.1 Determination of R_o and r_o

As mentioned above, the HP brine inlet and rotor ducts are assumed to be hollow cylinders with thickness, (R_o-R_i) and (r_o-r_i) respectively. Figure 4.2 shows the determination of these dimensions.

Fig. 4.2a Determination of $(R_o - R_i)$. 15 Fig. 4.2b Determination of $(r_o - r_i)$.

Figure 4.2

In both figures, the shaded area refers to the thickness of the hollow cylinder. R_o is determined by matching the outer diameter of the cylinder with the circumference of the endplate. r_o is decided by intersecting the outer diameter of the cylinder with the circumferences of adjacent ducts. R_i and r_i are dimensions of the respective holes.

20 4.3 Stokes equation subject to a thin-film approximation

Figure 4.3 Leakage flow from a hollow cylinder

Figure 4.3 shows a sketch of homogeneous leakage flow moving outwards in a radial manner through the clearance δ between the cylinder and ground. To derive the velocity

distribution v of the leakage flow, consider the incompressible, Newtonian, steady, developed flow of a fluid between two stationary parallel plates. The Navier-Stokes equation for the r -direction is

$$\frac{Dv_r}{Dt} - \frac{v_\theta^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + g_r + v(\nabla^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta}) \quad (4.1)$$

5 where the total derivative is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r \frac{\partial}{\partial r} + \frac{v_\theta}{r} \frac{\partial}{\partial \theta} + v_z \frac{\partial}{\partial z} \quad (4.2)$$

and the Laplacian

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \quad (4.3)$$

The Navier-Stokes equation and other differential forms of the fundamental laws governing
10 the conservation of mass and energy can be obtained from most textbooks of fluid mechanics (see, e.g., Schlichting, 1979; White, 1991). Substituting equation (4.2) and (4.3) into equation (4.1), it becomes

$$\begin{aligned} & \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} \\ &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + g_r + v \left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} \right) \end{aligned} \quad (4.4)$$

For developed flow between two parallel plates the streamlines are parallel to the plates so

15 that $v_r = v_r(z)$ and $v_\theta = v_z = 0$. Therefore,

$$\begin{aligned} & \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} \\ &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + g_r + v \left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} \right) \end{aligned}$$

No swirl 0

Rewriting,

$$\rho v_r \frac{\partial v_r}{\partial r} = -\frac{\partial p}{\partial r} + \gamma_r + \mu \left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2} \right) \quad (4.5)$$

Using an order-of-magnitude analysis (appendix L), the relative importance of the different terms in equation (4.5) can be determined. Among all the terms, pressure and viscous forces are the two most important in a thin-film problem. Thus, the dominant terms in the r -component of the Navier-Stokes equation are:

$$5 \quad \frac{\partial p}{\partial r} = \mu \frac{\partial^2 v_r}{\partial z^2} \quad (4.6)$$

Which is referred to as the *Navier-Stokes equation subject to a thin-film approximation* [38].

4.4 Leakage from HP brine inlet and HP feed outlet – A theoretical analysis

To estimate the leakage from the HP inlet and outlet, viscosity of the fluid is assumed to be constant and Newtonian, other assumptions have also been made so that the Navier-Stokes equation can be applied to this problem. They can be found in chapter 4.3 and Appendix L in the derivation of the Navier-Stokes equation subject to a thin-film approximation. The fluid is assumed to be 1) incompressible, 2) Newtonian, 3) steady, 4) developed and moving between 5) two parallel plates which are 6) horizontal. Therefore equation (4.6) can be generalized to read:

$$15 \quad \frac{\partial p}{\partial r} = \frac{\partial}{\partial z} \left(\mu \frac{\partial v_r}{\partial z} \right) \quad (4.7)$$

Equation (4.7) is integrated twice to yield

$$v_r = \frac{1}{2\mu} \frac{dp}{dr} z^2 + C_1 z + C_2 \quad (4.8)$$

Applying the boundary conditions for a pair of stationary plates, $v_r=0$ at $z=0$ and $z=\delta$ to get

$$20 \quad v_r(z) = \frac{1}{2\mu} \frac{dp}{dr} (z^2 - \delta z) \quad (4.9)$$

Using this distribution, we can find the leakage flow rate per unit length to be

$$Q_L = \int v_r dA = \int_0^\delta \frac{1}{2\mu} \frac{dp}{dr} (z^2 - \delta z) dz = -\frac{\delta^3}{12\mu} \frac{dp}{dr} \quad (4.10)$$

To find the velocity distribution, $v_r(z)$ and the flow rate, Q_L the pressure gradient, $\frac{dp}{dr}$ must be found first. The pressure gradient can be obtained by solving the Reynolds equation in cylindrical coordinates.

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{\rho r h^3}{12\mu} \frac{\partial p}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{\rho h^3}{12\mu} \frac{\partial p}{\partial \theta} \right) = \frac{1}{2} U \frac{\partial(\rho h)}{\partial \theta} - r \rho V_s \quad (4.11)$$

5 Where

h - Clearance, m

V_s - Squeeze velocity, m/s

In determining the leakage between the rotor and endplate, the RHS of equation (4.11) $\rightarrow 0$

since there is an absence of wedge ($\frac{\partial h}{\partial r} = 0$), ultra-low-speed ($U \frac{\partial h}{\partial r} = 0$) and dynamic

10 squeeze action ($\frac{\partial h}{\partial t} = 0$). Also the 2nd term on the LHS is omitted as leakage flow is assumed

to be symmetrical. The remaining term is an ordinary differential equation since $p(r)$ only.

$$\frac{d}{dr} \left(\frac{r h^3}{12\mu} \frac{dp}{dr} \right) = 0 \quad (4.12)$$

The boundary conditions are $p=p_i$ (supply pressure) at $r=r_i$, $p=0$ (gauge pressure) at $r=r_o$. The solution to equation (4.12) is

$$15 \quad p = p_i \frac{\ln(r/r_o)}{\ln(r_i/r_o)} \quad (4.13)$$

And

$$\frac{dp}{dr} = \frac{p_i}{\ln(r_i/r_o)r} \quad (4.14)$$

Substituting equation (4.14) into (4.9) and (4.8) would result in

$$20 \quad v_r(z) = \frac{1}{2\mu \ln(r_i/r_o)r} p_i (z^2 - \delta z) \quad (4.15)$$

And

$$Q_L = -\frac{\delta^3}{12\mu \ln(r_i/r_o)} \frac{p_i}{r} \quad (4.16)$$

(4.15) and (4.16) were derived for calculating the leakage flow from the HP brine inlet or HP feed outlet.

5 4.5 Leakage from pressurized ducts – A theoretical analysis

For the determination of the leakage flow from the pressurized ducts, the liquids in the ducts were regarded as a compressible fluid. The bulk modulus of elasticity B is used to describe the resulting change in volume ΔV of the liquids as a consequent of a change in pressure:

$$10 \quad B = \lim_{\Delta V \rightarrow 0} - \frac{\Delta p}{\Delta V / V} \bigg|_T \quad (4.17)$$

B is also called the coefficient of compressibility. It is defined as the ratio of the change in pressure (Δp) to the relative change in volume ($\Delta V/V$) while the temperature remains constant. The bulk modulus of elasticity at standard conditions is approximately 214×10^7 Pa. Since the volume of the ducts is constant, the change in volume of the liquids is taken to be
15 equal to the leakage flow Q_L .

$$Q_L = \Delta V \quad (4.18)$$

4.6 Time for fluid in ducts to depressurize – A theoretical analysis

Having presented the solutions to both leakage problems, the time it takes for the liquids in the duct to depressurize shall be examined now. After rotating passed the HP brine inlet,
20 the ducts are filled with highly pressurized brine. At $t=0$, expansion of the brine forces a small amount to escape through the gaps between the components. The time it takes to completely depressurized can be found by applying the conservation of mass. This law states that the mass of a system is constant. Therefore,

$$\frac{Dm_{sys}}{Dt} = \frac{D}{Dt} \int_{sys} \rho dV = 0 \quad (4.19)$$

Where $\frac{D}{Dt}$ is used since a system is involved. According the Reynolds transport theorem

(4.19) can also be written as

$$0 = \frac{d}{dt} \int_{C.V.} \rho dV + \int_{C.S.} \rho \hat{n} \cdot \mathbf{V} dA \quad (4.20)$$

- 5 The control volume can be found in figure 4.1b. It is applied over one-half of a duct since the duct is symmetrical about its mid-point. Noted that the control surface coincides with the interior wall of the duct and endplate/centerplate so that the leakage flow cuts through the control surface. If the density ρ of the liquids is assumed to be uniform, (4.20) can be simplified to become

$$10 \quad \frac{d(\rho V)}{dt} + 2\pi r_i \rho \int_{C.S.} v_r \Big|_{r_i} dz = 0 \quad (4.21)$$

Where $\hat{n} = +1$ since the flow is moving outwards. Remember V is constant, so

$$V \frac{d\rho}{dt} + 2\pi r_i \rho \int_{C.S.} v_r \Big|_{r_i} dz = 0 \quad (4.22)$$

- 15 The velocity is given as a function of pressure p_i therefore density shall be expressed in terms of pressure using the bulk modulus of elasticity:

$$B = \lim_{\Delta\rho \rightarrow 0} \frac{\Delta p}{\Delta\rho / \rho} \Big|_T = \frac{p_i - p_{atm}}{\frac{\rho - \rho_{atm}}{\rho_{atm}}} \quad (4.23)$$

Rearranging,

$$\rho = \rho_{atm} \left(\frac{p_i}{B} + 1 \right) \quad (4.24)$$

- 20 Where $p_{atm}=0$ (gauge pressure). Substituting equation (4.24) and (4.15) into (4.22), we get

$$V \frac{\rho_{atm}}{B} \frac{dp_i}{dt} + 2\pi r_i \rho_{atm} \left(\frac{p_i}{B} + 1 \right) \int_{C.S.} \frac{1}{2\mu \ln(r_i / r_o)} r_i (z^2 - \delta z) dz = 0 \quad (4.25)$$

This is a 1st order differential equation with p_i as the unknown dependent variable. It can be written as

$$\frac{\pi\delta^3 B}{6V\mu\ln(r_i/r_o)} dt = \frac{1}{(\frac{p_i}{B} + 1)p_i} dp_i \quad (4.26)$$

5 Integrating (4.26) will produce the relationship of p_i with respect to t .

$$\frac{\pi\delta^3 B}{6V\mu\ln(r_i/r_o)} t = \ln\left(\frac{p_i}{B + p_i}\right) + c_1 \quad (4.27)$$

At $t=0$, p_i is $p_i(t=0)$ therefore

$$c_1 = -\ln\left(\frac{p_i(t=0)}{B + p_i(t=0)}\right) \quad (4.28)$$

So (4.27) becomes

$$10 \quad \frac{\pi\delta^3 B}{6V\mu\ln(r_i/r_o)} t = \ln\left(\frac{p_i}{B + p_i}\right) - \ln\left(\frac{p_i(t=0)}{B + p_i(t=0)}\right) \quad (4.29)$$

4.7 Time for fluid in ducts to depressurize – Experimental

4.7.1 Equipment

The time for the fluid in the ducts to depressurize was investigated experimentally. The apparatus used is shown in figure 4.4 and the experimental setup in figure 4.5. The apparatus is actually a mini pressure vessel. One end of the apparatus is connected to a centrifugal pump that supplies engine oil at a constant 3.5 bar to it. Engine oil is used as this experiment shares some of the equipment used to test the proposed ERD. A Perspex sight glass is bolted onto the other end of the apparatus but is separated from the body of the apparatus by a spacer. Pressurized fluid will seep through this gap like the pressurized fluid in the ERD. Between experiments, the spacer was grinded so that the gap size would be reduced. To

prevent pressure build-up beyond the investigated region, two holes were drilled into the sides of the spacer to allow fluid to freely pass through. Pressure variation within the vessel was picked up by a pressure transducer attached to the body of the pressure vessel and relayed to a 2-channel oscilloscope. Finally, a camera was used to capture the data on the
5 oscilloscope. A cross-sectional view of the apparatus showing the leakage region can be found in figure 4.6.

Figure 4.4 Observation apparatus for leakage flow studies

Figure 4.5 Experimental setup of leakage flow studies

4.7.2 Procedure

The size of the gap must be determined first. To do that, the height of the protrusion from the pressure vessel and thickness of the spacer must be measured using a depth micrometer and micrometer respectively. After that, the supply pump was started and pressure fluctuations within the vessel closely watched. When the reading was stabilized (steady state), the valve after the pump was turned off. At the same moment, the oscilloscope was switched to peak detect mode. As the pressurized fluid expanded through the gap between the sight glass and pressure vessel, its pressure dropped. Due to the setting of the oscilloscope, the drop would be captured on its screen.

After that, the apparatus was dismantled so that the spacer could be grinded. The experiment is then repeated with a new gap size.

Figure 4.6 Cross-sectional view of observation apparatus

5 4.8 Results and discussions

4.8.1 Leakage from HP brine inlet and HP feed outlet - Theoretical

Leakages from the HP inlet and outlet are presented here. Properties of water are used in the analysis. Clearance between the endplate and rotor and clearance between the rotor and centerplate were assumed to be the same. This is valid as hydrostatic pressure on both sides of the rotor will locate the rotor centrally. As such, only leakage from the inlet was measured. To obtain the total leakage flow rate, multiply the results by a factor of two. Figure 4.7 shows the leakage flow rate when water at standard atmospheric conditions was used.

Figure 4.7 Leakage flow rate of water

5 As the clearance, δ increases, Q_L increases rapidly. While results show that the leakage is an extremely small percentage of the total feed flow, it is important to minimize leakage by reducing the clearance to a bare minimum. Experiments or simulations could be performed to validate the results. One option is to model and solve the simplified problem described in chapter 4.2 using commercially available Computational Fluids Dynamics (CFD) software

10 such as Fluent. The problem will be treated as a static problem with liquid flowing radially outwards between two parallel plates from a hollow cylinder in a cylindrical coordinate system. The results obtained will be compared with the results from the theoretical analysis. However, it is obvious that doing so is overkill as the problem is so simple. Thus, the more appropriate option is to solve the actual problem. Naturally, more time and effort has to be

15 invested as the problem is more difficult, especially since the ERD involves rotating components. Otherwise, certain groundwork to use the software, such as the 3D data for the ERD, has already been prepared. Perhaps an empirical solution is the most accessible option since a physical prototype is available now. The results can be found in chapter 7.4.

4.8.2 Time for fluid in ducts to depressurize – Theoretical

The time taken for one duct to depressurize is presented here. Figure 4.8 shows the change of internal pressure p_i using various clearances δ with respect to time. The time to depressurize becomes decreasing small as the clearance increase. Only results for 1 μm , 2 μm and 4 μm are illustrated. Values for larger clearances are available but are omitted for clarity. They exhibited similar trends. Initial clearances for the ERD may be reconsidered (reduce) based on these results. Alternatively, the speed of the process may be increased by making the rotors rotate faster.

Lastly, it is noted that the clearance has a more adverse effect on the depressurization of the ducts than the leakage from the HP inlet and outlet. Therefore, it is a must to keep the clearance to the smallest permissible level. This is not an easily rectifiable problem since very precise machining will be required to produce the tolerances needed. In the earlier analysis of the deformation of the endplate, it was found that the internal pressure would cause the endplate to deflect by 0.5 μm . Though it is significant to the depressurization of the duct, it is less so for the leakage flow. Also, compared to the tolerance achievable in normal machining, 0.5 μm is very small. Properties of water were used.

Figure 4.8 Graph of p_i against t

5 4.8.3 Time for fluid in ducts to depressurize – Experimental

The measurements of the protrusion and spacers are recorded in table 4.2.

Table 4.1

Height of protrusion, h_p .

	Average	Angular position, °			
		0	90	180	270
h_p , mm	10.047	10.040	10.0475	10.0475	10.0525

10

Thickness of spacer, t_s

	$\delta = t_s - h_p$	Average	Angular position, °			
			0	90	180	270
t_s , mm	0.007	10.054	10.065	10.065	10.065	10.025

As the analysis in section 4.5.2 is based on the properties of water, a 2nd analysis using properties of oil (table 4.3) and gap size stipulated in table 4.2 is presented here.

15 Experimental results will be plotted together with the theoretical results for comparison and verification.

Table 4.2 Properties of oil and fluid film

L	0.0075 m
δ/L	10^{-3}

r_i	0.01 m
r_o	0.04 m
U	10 m/s
P_{ref}	Variable
B_{oil}	400×10^7 Pa
ρ_{oil}	880 kg/m^3
$\mu_{oil} @ 40 \text{ }^\circ\text{C}$	0.106 Ns/m^2
$\mu_{oil} @ 100 \text{ }^\circ\text{C}$	0.0144 Ns/m^2

Figure 4.9 compares the results obtained theoretically against the results obtained experimentally with an average gap size of $7 \text{ }\mu\text{m}$. The theoretical analysis was performed using the same mathematical model used in section 4.8.2 with the properties of oil and dimensions of the experimental equipment. They are not an exact match but display similar traits. Pressure falls rapidly in the initial stage of the experiment but flattens out towards the end. More than half the initial pressure is still present up to 2 sec. This compares favorably to the speed of rotation of the ERD. At its slowest rotational speed, one cycle of the pressure exchange process is completed every 0.8 sec, therefore a high percentage of pressure can be preserved.

The inaccuracy could be attributed to the inconsistent gap size. While the theoretical analysis was performed using a uniform gap size, the actual device was not so ideal. This problem could not be easily rectified unless the apparatus was lapped. Secondly, the pressure transducer, which was a loan unit, was more than 15 years old. It displayed some drift during calibration; this could also be one source of inconsistency. Its calibration chart can be found in appendix A.1. Lastly, the bulk modulus of elasticity of the oil used was not given, thus it was approximated using a value from commonly used engine oil. Certainly, this would also have a bearing on the accuracy of the analysis.

Figure 4.9 Graph of p_i vs t ($\delta = 7 \mu\text{m}$)

5 Frictional losses in the ERD

The frictional losses in the ERD has a direct impact on overall energy consumption of the RO plant, thus it must be examined. Frictional losses are present in the ERD because its
 5 rotors rotate during operation. The rotors are driven by an external motor through a drive-train but are not supported by it. Rather, the rotors rotate on a thin film of water supported by hydrodynamic action within its outer sleeve. The rational is simple; ball bearings must be installed within the ERD if the rotors were to rely on shafts for support. The bearings would be corroded and seized in hours by the seawater. They also increase the maintenance needs of
 10 the ERD. On the other hand, the journal bearing afforded by the outer sleeve is elegant and simple and largely maintenance free. The design also aids in the assembly and disassembly of the ERD since each part is modular and can be installed or removed individually. The duty of the bearing is simple – hold up the weight of the rotor. There are no dynamic longitudinal, latitudinal or axial forces acting on the rotor. A flexible coupling joining the rotor to the
 15 drivetrain isolate most, if not all of these forces. Table 5.1 shows a summary of the specification of the journal bearing.

Table 5.1 Specification of journal bearing

Width	0.150 m
Diameter	0.150 m
Width-to-diameter ratio, b/d	1
Angular velocity, ω	10~20 rad/s
Radial clearance-to-radius ratio, c/r	O (10^{-3} - 10^{-4})
Selection of fit	H7-d8
Extreme of fit	$+2.794 \times 10^{-4}_{\max}$ $+1.524 \times 10^{-4}_{\min}$ m
Viscosity of water @ 15°C, μ	1.140×10^{-3} Ns/m ²
Density of stainless steel, ρ_{ss316}	7250 kg/m ³
Load, W_r	160 N

5.1 Journal bearing with various slender ratios

Based on the bearing's b/d ratio, it cannot be treated as an infinitely wide or short width bearing which can be solved by assuming one-dimensional fluid flow. It has to be classified as a journal bearing with various slender ratios and must be solved with a two-dimensional Reynolds equation numerically. Solutions are presented as a function of bearing characteristic number or Sommerfeld number [33]. Sommerfeld found that

$$\mu\left(\frac{r}{c}\right) = \phi(S) \quad (5.1)$$

The solutions were obtained by A.A. Raimondi and J. Boyd using finite difference approximation. Charts for performance of a journal bearing are presented in figures 5.1-3. A side view of the journal bearing is shown in figure 5.4. The figure also shows the definition of attitude angle, ϕ (the angle between the load axis and minimum film thickness), eccentricity, ε (the offset between the centre of the shaft and the centre of the hole) and film thickness, h (self-explanatory).

The film stiffness of a journal bearing is given as

$$k_f = \frac{dW_r}{dh_{\min}} \quad (5.2)$$

The stiffness of the bearing is nonlinear. It may be taken as linear for small displacements about an equilibrium positions. In figure 5.1, a tangent can be drawn to the curve at the operating ε or $h_{\min}$ and its slope evaluated to get the stiffness in N/m at that point. The natural frequency of vibration in the vertical direction is

$$f_n = \frac{1}{2\pi} \left[\frac{k_f g}{W_r} \right]^{1/2} \quad (5.3)$$

This is known as the first critical speed (called synchronous whirl). The rotation of the shaft should always be smaller than this frequency.

Work is done on the fluid due to viscous shear, which results in an increase in the temperature of fluid when it leaves the contact. The temperature rise can be calculated by assuming that all of the work on the fluid is dissipated as heat. It is further assumed that all of the work on the fluid is dissipated as heat and this heat is carried away by the fluid flow.

5 Therefore,

$$\mu W_r r \omega = Q \rho_c \Delta t \quad (5.4)$$

Some oil also flows out from the side of the bearing before the hydrodynamic film is terminated. If the temperature of the side flow is assumed to be the mean of the inlet and outlet temperatures, the temperature rise of the side flow is $\Delta t/2$. This means that the flow $Q -$

10 Q_s by Δt , and Q_s by $\Delta t/2$. Therefore,

$$\mu W_r r \omega = \rho c_p \left[(Q - Q_s) + \frac{Q_s}{2} \right] \Delta t \quad (5.5)$$

The temperature rise for a given load can be calculated using equation 5.4 and 5.5 with data obtained from figure 5.1-3. Viscosity in the analysis of bearing performance should be based upon the mean of the inlet and outlet temperatures, t_m . If the value of t_m differs from the initial

15 value assumed, the bearing performance should be recalculated.

Figure 5.1 Effect of bearing characteristic number (S) on dimensionless minimum film thickness and attitude angle [31]

Figure 5.2 Effect of bearing characteristics number on dimensionless maximum film pressure and terminating position of the fluid film and the position of maximum film pressure [31]

Figure 5.3 Effect of bearing characteristic number on dimensionless coefficient of friction variable, dimensionless volumetric flow rate and volumetric side flow ratio [31]

10 where

ϕ - attitude angle, $^{\circ}$

ε - eccentricity of the journal, m

ω - angular velocity, rad/s

h - film thickness, m

15 W_r - weight, kg

Figure 5.4 Schematics of plain journal bearing geometry with journal rotating at angular speed ω

20 5.2 Results and discussion

It is customary to begin the solution by assuming an eccentricity ratio. Coupled with the b/d ratio, the Sommerfeld number and other performance parameters can be located on figures 5.1-3. However, the load imposed on the hydrodynamic bearing is already known. It is the weight of the rotor. Therefore, the Sommerfeld number would be derived from it instead.

Normally, the diametral clearance, c is fixed but it is not representative of a working journal bearing since theoretically exact dimensions are hard to achieve during machining. Thus, a loose running fit is used for the rotor and bearing combination as it can be easily achieved in manufacturing. As such, the solution obtained would encompass the total tolerance within the bearing.

The load capacity will be equal to the weight of the rotor, therefore

$$\bar{W}_r = \frac{W_r}{\eta_o \omega r b \left(\frac{r}{c}\right)^2} \quad (5.6)$$

As ω can vary from 10-20 rad/s, $\bar{W}_r$ will ranges from 0.322 – 2.16. As

$$S = \frac{1}{\pi \bar{W}_r} \quad (5.7)$$

5 S will range from 0.147 – 0.989. As a result, the dimensionless coefficient of friction which is

$$\frac{\mu r}{c} \quad (5.8)$$

will be 2.0 – 3.8. Consequently, the coefficient of friction is found to be $2.03 \times 10^{-3} - 7.08 \times 10^{-3}$. The viscous power loss which is

$$\mu W_r r \omega \quad (5.9)$$

10 can be found. It is shown in figure 5.5.

Figure 5.5 Viscous power loss over operating range of journal bearing

Figure 5.5 projects the power loss due to viscous drag in the bearing. The power loss
 15 experienced is enveloped by the boundaries of the shaded region. Its exact value depends on

the rotation speed of the rotor, the radial clearance of the bearing and coefficient of friction. More importantly, it is obvious that the power loss is very small compared to the energy consumption of a RO plant. Therefore, it can be considered negligible. It also implies that a self-driven design is viable and can be considered in the future.

6 Understanding the pressure exchange process

6.1 Testing of original concept

Understanding the pressure exchange concept as described in detail in chapter 2 section 1 through experiments is completely relevant to the project. The pressure exchange concept is the key innovation designed into the prototype ERD. To reiterate, the pressure exchange concept works by allowing a first high pressure liquid body to briefly interact with a second low pressure liquid body so that a portion of the pressure energy in the first body can be transmitted to the second. Outwardly, the ERD is totally different from the setups shown in figures 2.1 and 2.2 and also in figure 6.1 which only depicts a sprawling network of pipes and valves but they are identical in their working principle.

The setups shown in figures 2.1 and 6.1 are the simplest actualization of the concept which makes them ideal for illustration purposes. However, they are not practical as they are not able to effectively recoup the pressure energy in the high pressure liquid. Therefore, a second design was proposed in figure 2.2. According to chapter 2.1 page 28 line 15 – 16, “the number of pressure vessels is doubled to permit a more progressive pressure transfer and potentially higher final pressure.” But ¹ “the working principle of this system is not dissimilar to the original design. The only significant difference is that the pressure exchange between the HP brine and LP feed takes place over three phase instead of just one.” The setup in figure 2.2 illustrated that ² “if more vessels can be built into the system, a higher final pressure is possible”. This understanding led to the final design of the proposed ERD.

The proposed ERD is described in chapter 2.2.1. In essence, ³ “the ERD uses two rotors with longitudinal ducts parallel to its rotational axis to transfer the hydraulic energy from the brine to the feed through holes in the centerplate. The longitudinal ducts act like the pressure vessels while the holes act like the connecting valves illustrated in chapter 2.1. The rotors rotate in opposite directions in plain bearings supported by hydrodynamic action. HP brine

¹Chapter 2 section 1 page 28 line 17 – 19.

²Chapter 2 section 1 page 29 line 12.

³Chapter 2 section 2 page 35 line 17 – 22.

and LP feed enter the rotor ducts through inlets on the centerplate while LP brine and HP feed leave via outlets on the endplates.” Basically, the ERD functions like a multi-vessel system but without the complexity of connecting all the vessels together.

Experiments should be done to understand the pressure exchange process further since very little is known about it aside from 1) its final pressure, 2) the pressure wave’s speed, and 3) its energy balance which were all investigated in chapter 2.1.1 – 2.1.3. Four different parameters have been identified which may affect the performance of the pressure exchange process. They are

- 1) Initial pressure of the fluid
- 2) Rate at which the connecting valve is opened
- 3) Size of the valve and
- 4) Length of the pressure vessel

All these parameter are varied and their effects recorded.

6.1.1 Equipment

Table 6.1 Description of equipment

Item	Description
P	Hand pump
B1 – 3	Ball valves
PV1 – 2	Pressure vessels
WT	Oil tank
PT1 – 2	Pressure transducer

Figure 6.1 Test apparatus for pressure exchange process

Figure 6.1 shows a schematic of the setup used to investigate the pressure exchange process. The pressure vessels used in the experiment are just copper tubing. They are highlighted so they can be differentiated. A ball valve, B2 separates the 1st pressure vessel from the 2nd. A ball valve is selected as its opening and closing actions resemble what is happening inside the proposed ERD (figure 6.2). The 1st pressure vessel, PV1 nests a pressure transducer so that pressure fluctuation can be detected and recorded. Another two ball valves placed at the front and back of the pressure vessels isolate them during the pressure exchange process. A hand pump with a flow rate of 13 g/cycle and maximum supply pressure of 25 bar is used to raise the pressure in the vessels. Excess oil flows back to the supply tank. The actual setup is shown in figure 6.3.

Figure 6.2 Similarity between ball valve and proposed ERD

Figure 6.3 Experimental setup for understanding pressure transfer process

6.1.2 Procedure

5 All the ball valves were first opened and the pressure vessels gradually filled up with oil. When they were completely filled, B3 would be shut while pumping continued, causing pressure in both vessels to rise. When the pressure reached a pre-determined level, B2 would be shut as well. After that, excess oil from PV1 was released back to the oil tank via a valve on the hand pump so that the pressure in PV1 could equalize with the atmosphere. B1 was
 10 then shut. Now both pressure vessels were isolated with PV2 at a higher pressure than PV1. The oscilloscope was set to positive peak mode and B2 was opened. Pressure energy would be transmitted from PV2 to PV1 and captured by the oscilloscope.

The experiment could be conducted for pressure exchange from the opposite direction. To begin, both vessels were filled. Then B2 was shut to allow pressure in PV1 to rise. Since B3
 15 was still open, there was no pressure build-up in PV2. When pressure in PV1 had reached a pre-determined level, B1 and B3 were shut. Now, PV1 has a higher pressure than PV2, when B2 was opened, pressure was transmitted in the opposite direction.

6.1.3 Results and discussion

6.1.3.1 Effects of variation in initial pressure

The main objective of this experiment is to find out if variation in the initial pressure affects the amount of residual pressure p_{final} left after one exchange. According to equation 2.9, p_{final} should be the average of the initial pressures in both vessels. The 2nd objective is to plot how pressure varies with time over the pressure exchange period. Only fluid in one of the two vessels is pressurized, the other vessel is maintained at atmospheric pressure. The pressure of the pressurized vessel is varied from 2.5 – 15 bar.

All the experiments were conducted by the author. Typical p vs t graphs are shown in figure 6.4 while a summary of the results are given in table 6.2. Actual results can be found in appendix D. The graphs used are results from experiments conducted with an initial pressure of 5 bar in one vessel. The dash line represents the experiment where pressure energy is transferred from PV1 to PV2. The dotted line represents the experiment where pressure energy is transferred from PV2 to PV1. Generally, their graphs – including those derived from a different initial pressure – resemble one-half of a sine function. At the start of the process, pressure dropped/increased gradually since the ball valve was not fully opened; there was a limit on the maximum amount of fluid that could squeeze passes the slit. As the slit widened, more fluid could squeeze pass and pressure dropped/increased more rapidly. The rate of pressure drops/rises decreased again when equilibrium was nearly reached; this was understandable since the difference in pressure between the vessels had narrowed and fluid flows less quickly through the ball valve. In general, equilibrium in all the experiments was achieved in 30-60 ms.

When the direction of the pressure transfer was switched, the final pressure attained was different even though the starting pressure was the same. Apparently, PV1 (44±0.4 ml) was bigger than PV2 (39±0.4 ml). This was also why p_{final} occasionally exceeded 50%. When

pressure is transmitted from the vessel with more oil, the final pressure will be higher. This is in agreement with equation 2.9. With reference to table 6.2, the final pressure attained when pressure was transferred from PV1 to PV2 was roughly 43 – 62% of the initial pressure while the pressure attained was only 35% when the direction of the transfer was switched. The

5 unusually high p_{final} observed in sets 1 and 2 were likely to be erroneous which could be caused by the presence of air pockets in the vessels. When compressed, air has more stored energy than oil because of higher compressibility. Its pressure also changes less for a given change in volume. Presuming that air was present in PV1 at the onset of the 1st and 2nd experiments, p_{final} would be unrealistically high. This was exactly what was witnessed in both

10 sets of experiment. When the problem was discovered, the vessels were up-righted and tapped gently so that air trapped in the vessels could rise to the top. The oil was allowed to stand for 45 minutes. During this time, many bubbles were observed breaking through the oil surface.

15 Figure 6.4 Graph of p vs t ($p_{initial} = 5$ bar)

Table 6.2 Summary of results for different initial pressures

Set no.	P_{PV1} (bar)	P_{PV2} (bar)	P_{final} (bar)	Direction	$\frac{P_{final}}{P_{PV1}} \times 100\%$	$\frac{P_{final}}{P_{PV2}} \times 100\%$	Photo reference
1	2.40	atm	1.75	PV1→PV2	72.92	-	101-1741
2	2.46	atm	1.98	PV1→PV2	80.04	-	101-1742
3	atm	2.50	0.89	PV2→PV1	-	35.41	101-1743
4	atm	2.50	0.89	PV2→PV1	-	35.41	101-1744
5	atm	5.00	1.76	PV2→PV1	-	35.16	101-1745
6	atm	5.00	1.81	PV2→PV1	-	36.20	101-1746
7	4.86	atm	2.42	PV1→PV2	49.74	-	101-1747
8	4.86	atm	3.04	PV1→PV2	62.56	-	101-1748
9	9.11	atm	5.00	PV1→PV2	54.88	-	101-1749
10	9.58	atm	5.35	PV1→PV2	55.82	-	101-1750
11	atm	10.00	3.33	PV2→PV1	-	33.33	101-1751
12	atm	10.00	3.50	PV2→PV1	-	35.00	101-1752
13	atm	12.50	4.00	PV2→PV1	-	32.00	101-1753
14	atm	12.50	4.02	PV2→PV1	-	32.16	101-1754
15	12.57	atm	5.49	PV1→PV2	43.64	-	101-1755
16	13.92	atm	6.25	PV1→PV2	44.93	-	101-1756

6.1.3.2 Effects of variation in the rate that which the valve is opened and closed

The effect of the rate of opening of the ball valve B2 on the pressure transfer process was briefly investigated. This is relevant to the project as the proposed ERD is expected to operate at different rotation speed according to the flow rate processed. The main objective is to determine whether the opening speed of the valve affects p_{final} . If it does, the optimum speed

must also be determined. Based on equation 2.9, p_{final} is not dependent on how the valve is opened. The initial pressure was fixed at 8.5 bar. The direction of pressure transfer was from PV1 to PV2. The ball valve was opened and closed in one smooth action manually with four different speeds ranging from very slowly to fast. The usual speed used to open the valve is listed as *normal* for comparison.

In a preferred embodiment, a valve actuator would be used to open and close the valve mechanically and the valve's rotational speed measured with a rotational sensor so that all the experimental results can be quantified and duplicated. To repeat, there is no way to objectively assess the rate of opening of the valve as it is open and close by hand. To prevent results from overlapping, the valve was always rotated a few times faster than the previous test. A simpler experiment was first conducted to ascertain the significance of this parameter on the pressure exchange process before deciding to proceed with more thorough testing due to the lack of funds.

Results showed that the time taken by the pressure transfer process was affected by the speed used to open B2 although the appearance of the graphs for different speed remained similar. When B2 was opened very slowly, the slope of the graph was very gentle and the pressure took a longer time to stabilize. When the ball valve was rotated slowly, some oil started forcing through it as soon as a small slit appeared. Due to the size of the initial opening, only very little oil could be squeezed through per unit time. As the movement of the oil was actually much faster than the rotation of the ball valve, equilibrium was reached before the ball valve could open wide enough for pressure changes to happen more quickly. When the ball valve was abruptly opened, its graph became very steep. It did not resemble a sine function anymore. The top half of the graph when pressure normally fell gradually was truncated, leaving only the bottom half where pressure declined rapidly. When the ball valve

opened quickly, a much bigger cross-sectional area was available for the pressurized fluid to pass through straight away. As a result, pressure also fell quickly.

The opening speed of the valve has no significant effect on p_{final} (Table 6.3).

$\frac{P_{final}}{P_{PV1}} \times 100\%$ is in the region of 53 – 57% regardless of opening speed. This implies that the

- 5 proposed ERD can operate at any speed as long as the pressure transfer process has sufficient time to complete. The p vs t graphs of all four speeds are shown in figure 6.5. The graph becomes cluttered up when all the speeds are overlay together therefore they are separated into two graphs.

- 10 Graph of p vs t for different valve opening speed (Very slow, slow).

Graph of p vs t for different valve opening speed (Normal, fast).

Figure 6.5

Table 6.3 Summary of results for different valve opening speed.

Set no.	P_{PV1} (bar)	P_{PV2} (bar)	P_{final} (bar)	Opening speed	$\frac{P_{final}}{P_{PV1}} \times 100\%$	Photo reference
1	8.38	atm	4.83	Very slow	57.64	101-1757
2	8.32	atm	4.42	Slow	53.13	101-1758
3	8.80	atm	5.00	Normal	56.82	101-1759
4	9.13	atm	5.19	Fast	56.85	101-1760

5

6.1.3.3 Effects of variation in the size of the valve

B 2 is a full sized $\frac{1}{2}$ " ball valve while the pressure vessels are 15 mm copper tubing. The internal diameter of the ball valve and pressure vessel are almost the same. This is not so in the proposed ERD. The holes in the centerplate are 2 mm in diameter while the ducts in the rotors are 10 mm in diameter. The size of the holes is deliberately kept small to minimize mixing between the brine and the seawater. Therefore it is important to know what will happen when the diameter of the connecting valve is very small or is only a fraction of the diameter of the pressure vessel.

10

The objective of this experiment is to investigate how the size of the valve affects p_{final} . To achieve that, the original ball valve was replaced with a reduced bore ¼” ball valve. Although smaller sizes can produce more distinctive results they are also many times more expensive. The new ball valve was tested with initial pressures ranging from 5-15 bar.

- 5 The results show that the reduced bore ¼” ball valve has no significant impact on p_{final} . The p vs t graphs produced are almost identical to the ones produced with the larger ball valve. $\frac{p_{final}}{p_{PV}} \times 100\%$ is also consistent with earlier results. Unfortunately, these results may not be genuinely reflective of the actual problem as the small valve is still quite big. A pair of p vs t graphs based on the smaller ball valve is shown in figure 6.6 and a summary of the
- 10 results in table 6.4.

Figure 6.6 Graph of p vs t for reduced bore ¼” valve ($p_{initial} = 5$ bar)

15 Table 6.4 Summary of results for reduced bore ¼” valve.

Set no.	P_{PV1} (bar)	P_{PV2} (bar)	P_{final} (bar)	Direction	$\frac{p_{final}}{p_{PV1}} \times 100\%$	$\frac{p_{final}}{p_{PV2}} \times 100\%$	Photo reference
1	4.17	atm	2.00	PV1→PV2	47.96	-	101-1812

2	4.13	atm	1.98	PV1→PV2	47.94	-	101- 1813
3	atm	4.5	1.58	PV2→PV1	-	35.20	101- 1814
4	atm	4.5	1.58	PV2→PV1	-	35.20	101- 1815
5	9.96	atm	4.92	PV1→PV2	49.36	-	101- 1807
6	8.32	atm	4.17	PV1→PV2	50.08	-	101- 1808
7	atm	8.50	3.24	PV2→PV1	-	38.12	101- 1809
8	atm	8.50	3.24	PV2→PV1	-	38.12	101- 1810
9	14.96	atm	8.8	PV1→PV2	58.82	-	101- 1799
10	14.56	atm	7.74	PV1→PV2	53.15	-	101- 1803
11	14.08	atm	7.30	PV1→PV2	51.19	-	101- 1806
12	atm	14.50	5.09	PV2→PV1	-	35.14	101- 1800
13	atm	14.50	5.29	PV2→PV1	-	36.45	101- 1801
14	atm	14.50	5.67	PV2→PV1	-	39.08	101- 1802

6.1.3.4 Effects of variation in the length of the pressure vessels

The motive for this experiment sprung mainly from curiosity and also from the knowledge that some PD ERD on the market such as the DWEER and Aqualyng uses huge vessels that are several meters long for pressure recovery operations. These products can also benefit from the introduction of the pressure exchange technique. However, it is only feasible if the technology can blend seamlessly into their existing system. The cost of implementing the new technology is also a big concern. Hopefully, the tests on the longer vessels would yield some useful results. The main objective of this experiment is to find out if variations in the length of the pressure vessels affect the behavior of p over t . The original 0.15 m copper tubing used for each vessel was swapped for a 12 m one (figure 6.9). Due to their length, they were installed vertical using another rig as support. The pressure transducer was also

relocated to extreme end of PV2. Different $p_{initial}$ ranging from 5-25 bar were used. Pressure was transmitted from PV1 to PV2.

The experiment yielded interesting results (figure 6.7). P fluctuated over t like a wave function with diminishing strength. The pressurized liquid reacted like a tightly wound spring
 5 when the connecting valve was opened, extending and compressing several times before settling down. This behavior is not unlike what happens during a water hammer except that the initiating factor was different. Water hammers occur when there is a sudden valve closure as liquid is still being transported in the pipe. In this experiment, the liquid in both vessels was completely stationary at first. When the connecting valve was opened, liquid from the
 10 pressurized vessel accelerated into the other vessel due to pressure difference. Liquid immediately downstream of the connecting valve was compacted and its pressure increased. As more liquid from the pressurized vessel entered the other vessel, liquid further downstream became pressurized too. Due to momentum, more liquid would continue to enter PV2 until its pressure became much higher than in PV1. With the situation reversed, liquid
 15 would start flowing back to PV1 since it was at a lower pressure. The process was repeated until equilibrium was achieved in both vessels. All these happened in the span of a few micro-seconds.

When $p_{initial}$ was raised, the maximum pressure p_{max} attained during the pressure exchange process also increased. This is expected as the pressure difference between PV1 and PV2 was
 20 bigger, causing the pressurized liquid to be more forcefully accelerated and subsequently achieved a greater momentum. The momentum of the liquid is responsible for the surges in pressure. Therefore, an increase in momentum means an increase in p_{max} too.

The period of the graphs decreases as $p_{initial}$ increases. This is in agreement with equation 2.17 since ρ increases with p .

Figure 6.7 Graphs of p vs t for $L = 12$ m pressure vessel

- 5 An additional pressure transducer was later added and installed at the other extreme of PV1. The p vs t as seen by PT2 is plotted in figure 6.8 for greater clarity of the pressure exchange process. A summary of all the results is shown in table 6.5.

Figure 6.8 p vs t as observed at extreme ends of test apparatus

Figure 6.9 Test apparatus outfitted with $L = 12$ m pressure vessels

5 Table 6.5 Summary of results for $L = 12$ m pressure vessel.

Set no.	P_{PV1} (bar)	P_{PV2} (bar)	P_{final} (bar)	P_{max} (bar)	P_{min} (bar)	$\frac{P_{final}}{P_{PV1}} \times 100\%$	$\frac{P_{max}}{P_{PV1}} \times 100\%$	Photo reference
1	5.17	atm	2.36	4.08	-	45.65	78.92	101-1483
2	5.35	atm	2.40	4.32	-	44.86	80.75	-
3	4.84	atm	2.20	3.80	-	45.45	78.51	-
4	4.92	atm	2.16	4.16	-	43.90	84.55	-
5	5.39	atm	2.36	4.08	-	43.78	75.70	101-1497
6	10.20	atm	3.90	6.60	-	38.24	64.71	101-1479 – 101-1481
7	10.55	atm	4.08	7.44	-	38.67	70.52	101-1482
8	10.08	atm	4.00	6.32	-	39.68	62.70	-
9	11.40	atm	4.20	7.28	-	36.84	63.86	-
10	10.90	atm	4.40	6.72	-	40.37	61.65	-
11	14.30	atm	5.44	9.12	-	38.04	63.78	101-1487
12	15.00	atm	5.68	9.20	-	37.87	61.33	-
13	14.30	atm	5.76	9.20	-	40.28	64.34	101-1489
14	13.90	atm	5.28	7.52	-	37.99	54.10	-
15	14.40	atm	5.52	8.24	-	38.33	57.22	-
16	19.40	atm	7.28	13.60	-	37.52	70.10	101-1490
17	19.80	atm	7.52	12.40	-	37.98	62.63	-
18	19.80	atm	7.60	11.90	-	38.38	60.10	-
19	19.70	atm	7.68	10.70	-	38.98	54.31	-
20	20.60	atm	8.00	12.90	-	38.83	62.62	101-1494

21	22.50	atm	8.80	12.10	-	39.11	53.78	101-1491
22	22.40	atm	8.88	12.60	-	39.64	56.25	-
23	22.60	atm	8.72	12.40	-	38.58	54.87	-
24	22.70	atm	8.88	13.90	-	39.12	61.23	101-1492
25	24.00	atm	9.68	15.80	-	40.33	65.83	101-1493

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7 Testing of the ERD

7.1 Pre-test preparations

When the ERD was finally ready, each component was carefully deburred using hand files and various grades of sand paper. They were then cleaned in industrial strength detergent to remove machine oil, chips and other materials from their surfaces. After that, each part was inspected for dimensional and geometrical accuracy in NTU metrology lab.

With the exception of a few parts, all other components were within the tolerances stipulated. One part – a shaft in the drive-train – failed because of operator error, another two – bearings of the ERD – failed because appropriate machining techniques are not available in NTU. However, no remedy was performed as there was no fund to do so. After the inspection, the ERD and its drive-train were assembled on a rig. Some pictures of the completed ERD and experimental setup can be found in figure 7.1. More pictures can be found in the appendix F.

Figure 7.1 Pictures of the completed ERD and experimental setup

7.2 Equipment

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The equipment used in the testing of the ERD are clearly described in section 2.3.

7.3 Procedure

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WT was topped up with oil first. The oil level must at least reach above the height of the pumps so that the pumps could be primed. After that, the pumps were turned on and left to run until oil flowed back into the reservoir. This ensured that the entire pipeline and internals of the ERD were filled with oil. After that, the ERD could be started. With the ERD rotating slowly in the background, V5 was throttled to raise the outlet pressure of PU1 and the magnitude of the outlet pressure would be read off from PT1 and PT4. When the desired pressure was reached, the PU1 output flow rate would be measured and compared with PU2 output flow rate. V4 would be adjusted so that PU2 output matched PU1 output. Depending on the flow rate, the rotational speed of the ERD would be tuned as best as possible so that

15

ducts were properly filled. When all the pressures, flow rates and rotational speed of the ERD were set and steady state achieved, results were recorded. Measurement of the collective leakage flow rate was done by temporarily shutting off the LP seawater feed to the ERD.

7.4 Results and discussion

- 5 The experiments were conducted with different pressures ranging from 1-20 bar. Pressure higher than that was not possible because the HP flow rate would be reduced to a trickle. Moreover, newer RO systems are geared for lower operating pressure as compared to earlier designs as membrane technology improves. An operating pressure of 54 bar translates to a recovery ratio of 30% [34].
- 10 The results showed that energy recovery was achieved by the ERD. With reference to figure 7.2, low pressure liquid at atmospheric pressure enters the ERD through its LP inlet and leaves at 13 bar from the HP outlet, while high pressure liquid at 13.5 bar enters the ERD through the HP inlet and leaves at atmospheric pressure. The difference in the inlet and outlet pressure can be attributed to hydraulic losses in the ERD. Figure 7.2 was taken from one of
- 15 the results of the experiments; basically, other results shared the same characteristics. Both LP inlet and outlet would be around atmospheric pressure, HP outlet pressure would be slightly lower than HP inlet pressure and the pressure in the ERD would behave like a wave function. Details of some of the results can be found in appendix D.5.

Recall that the ETE of an ERD is given by

$$20 \quad ETE = \frac{P_{HPfeed} Q_{HPfeed} + P_{LPbrine} Q_{LPbrine}}{P_{HPbrine} Q_{HPbrine} + P_{LPfeed} Q_{LPfeed}} \times 100\% \quad (2.26)$$

However, flow rates of liquid entering the ERD, Q_{LPfeed} and $Q_{HPbrine}$, are not available unlike those (Q_{HPfeed} and $Q_{LPbrine}$) exiting from the ERD which can be determined with a measuring cylinder and a stopwatch. Q_{LPfeed} and $Q_{HPbrine}$ can only be determined with flow sensors which

were unavailable. Therefore, the author proposed using another indicator as a gauge for the performance of the ERD instead. It is given by

$$\frac{P_{out}}{P_{in}} \times 100\% \quad (7.1)$$

This is a logical substitute for ETE because the differences between inlet and outlet flow rates of rotary-type PD ERDs, like the prototype, are usually below 5% of the feed flow, according to Hauge [30]. Differences in the inlet and outlet flow rates were caused by leakage from the HP brine flow, HP feed flow and LP seawater flow. This is because the pressure of all three openings was higher than the LP brine outlet. The leakage can be attributed to the high pressure nature of all three flows and the presence of free play in the ERD. It is not possible to measure the leakage from each flow without flow meters. A comparison between ETE and the proposed indicator can be found in appendix J.

Pressure exchange between the LP liquid and HP liquid is also taking place even though its effect is not obvious. According to graph *ERD*, which is plotted by PT3, crests and troughs in pressure within the ERD are formed as the rotor rotates. The patterns are formed when the pressurized ducts skirt pass PT3. Obviously, the frequency of the pattern coincides with the rotational speed of the rotors. This is evidence that pressure exchange occurs in the ERD. The difference between the crests and troughs is roughly 1 bar which is lower than hoped. However, the difference can be increased by narrowing the gap between the rotating surfaces. Currently, the gap is between 16 – 79 μm , which is the minimum achievable by the CNC lab considering the size and bulk of the ERD. A smaller clearance is possible only if the parts are lapped.

Another encouraging result is observed as well, pressure fluctuations are absent from the inlets and outlets of the prototype. It seems that the pressure exchange process has done very

well at pre-pressurizing the LP liquid in the ERD so that pressure fluctuations common in other designs have been smoothed out.

Figure 7.2 Graph of p vs t around and inside the ERD (Table 7.1, No. 12)

5

Table 7.1 Results from testing of the ERD

No.	LP feed PT2 Ch 3 (bar)	LP brine PT5 Ch 4 (bar)	HP brine PT1 Ch 2 (bar)	HP feed PT4 Readout (bar)	LP brine flow rate (m ³ /h)	HP feed flow rate (m ³ /h)	ERD speed (rpm)	$\frac{p_{out}}{p_{in}} \times 100\%$
1*	atm	atm	2.50	0.94	0.54	1.08	95.65	37.60
2	atm	atm	3.85	3.45	0.55	0.83	60.23	89.63
3	atm	atm	4.70	4.12	0.55	0.68	63.24	87.65
4*	atm	atm	6.35	5.32	0.56	0.57	53.53	83.78
5	atm	atm	6.88	6.00	0.55	0.56	110.42	87.21
6	atm	atm	7.42	6.87	0.53	0.55	68.89	92.56
7*	atm	atm	8.64	7.65	0.55	0.54	54.56	98.82
8*	atm	atm	10.45	8.74	0.42	0.46	71.60	83.64
9*	atm	atm	10.47	9.00	0.42	0.40	105.10	85.96
10	atm	atm	10.44	9.51	0.41	0.39	67.21	91.08
11*	atm	atm	10.96	10.79	0.28	0.28	59.48	98.45
12*	atm	atm	13.09	11.54	0.28	0.27	81.03	88.13
13	atm	atm	13.22	12.31	0.25	0.27	57.98	93.10
14*	atm	atm	13.40	13.10	0.26	0.22	85.50	97.76
15*	atm	atm	13.44	13.02	0.27	0.27	47.89	96.88
16	atm	atm	14.03	13.45	0.35	0.40	70.90	95.87

17*	atm	atm	15.24	14.00	0.23	0.20	84.78	93.14
18*	atm	atm	18.10	17.00	0.13	0.10	89.77	93.92
19*	atm	atm	19.60	17.50	0.10	0.10	102.45	89.29
20*	atm	atm	19.60	18.00	0.09	0.05	95.43	91.84

* Details of this result can be found in the appendix D.5

The collective leakage flow rate ranged from 0.002 – 0.02 m³/h as inlet pressure was increased. Below 10 bar, the collective leakage was approximately 1.0 – 1.5 % of the feed rate and increased to about 20% above 15 bar. The high percentage could be attributed to the grossly reduced feed flow above 15 bar as the pump was running near its maximum output pressure. In normal operating circumstances, when the feed pump is operated within its performance envelop, the feed flow will be maintained within a specific range and the leakage flow as a percentage will be much lower. Results can be found in table 7.2 and figure 7.3.

All the objectives lay out in the initial phase of this project have been met.

Table 7.2 Collective leakage flow rate

No.	Supply pressure (bar)	Flow rate (m ³ /h)
1	0	0
2	2.5	0.00174
3	5.0	0.00390
4	7.5	0.00534
5	10.0	0.00660
6	12.5	0.01314
7	15.0	0.01920
8	17.5	0.02316

Figure 7.3 Leakage from the ERD

8 Future work

The design and development of the ERD in this project was sparked off by a keen interest to boost the efficiency of PD ERDs through the use of pressure exchange. The final prototype was a natural evolution of the simple concept introduced in section 2.1. What started as a two pressure vessel system morphed into a multi vessel system and finally into the contraption seen here.

The answer can be found in chapter 6. The wave-like behavior of the pressure in the 12 m vessel observed in Section 6.1.3.4 can be exploited. Recall that as pressure is transferred from PV1 to PV2, it has the tendency to overshoot the equilibrium level a few times before settling down. If B2 is timed to close when pressure peaks in PV2, additional pressure energy will be transferred and p_{PV2} will be higher without resorting to adding more pipes and valves. The only addition is the automatic actuator that is required to turn the valve on/off. As such, it is ideal for retrofitting some existing PD ERDs like the Aqualyng and DWEER. Consider the original concept again (figure 8.1).

Figure 8.1 Original concept

Valve closed

Valve opens

Figure 8.2 Pressure fluctuations in 12 m pressure vessels

When short vessels are used, equilibrium is reached almost instantly, but in longer vessels, pressure fluctuates periodically before settling down. In figure 8.2, if the center valve is shut at t_1 , more pressure energy can be transferred.

9 Conclusions

A literature review involving the state of the art in energy recovery devices for reverse osmosis based desalination has been done. The performance of an ERD is measured by its energy transfer efficiency (ETE) which refers to its effectiveness in transferring energy from the brine to the seawater. The review showed that positive displacement (PD) type ERDs have the highest ETE compared to other alternatives. However, it was discovered that a small amount of pressure energy is left in the high pressure brine and can be harnessed. Based on this understanding, a new ERD to recover this lost energy was proposed.

The new ERD is based closely on the one and only rotary PD ERD on the market. That device called the PX was in turn, based on an even older invention. The most important thing that differentiates the new ERD and the PX is a new feature called the pressure exchange concept which is able to recover some of the lost pressure energy. A detail description of how the pressure exchange concept works can be found in chapter 2.1. The description also includes calculations showing the speed of the pressure exchange process and its potential energy savings. Some experiments were also done to understand the pressure exchange process in chapter 6 since very little is known about it. No unexpected phenomenon was encountered.

Factors that may affect the performance of the ERD were also investigated.

Bending of the end piece of the ERD was investigated in chapter 3 as the pressure in the ERD is not insignificant. Based on the investigation, a thickness of 20 mm was specified for the component to maintain deflection of the part within an acceptable range.

Leakage within the ERD was also identified and investigated both experimentally and theoretically. The results were used to specify the size of the gap between the rotors and the end pieces. It also showed that leakage is a small percentage of the feed flow.

Frictional losses due to the hydrodynamic journal bearing in the ERD were also investigated. Results showed that friction losses are very small compared to the power consumption of the desalination plant.

Last but not least, a prototype of the new ERD was fabricated and connected to a test rig. It was tested to 20 bar. As the flow rates of the liquids entering the ERD is not available, another indicator was proposed. A comparison between ETE and this indicator was done and can be found in the appendix. The indicator is $\frac{P_{out}}{P_{in}} \times 100\%$. For the prototype, $\frac{P_{out}}{P_{in}} \times 100\%$ is approximately 85 – 90%.

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Appendix A

A.1 Calibration chart for PT 1

Input range: -1 – 20 bar

Output signal: 0 – 5 V

Excitation voltage: 24.3 V

Method of calibration: Dead-weight tester

Wiring sequence:

Excitation voltage (+): Red
(-): Brown

Signal output (+): Blue
(-): Brown

Pressure (bar)	Reading (V)				Average
	1	2	3	4	
Atm	0.241	0.242	0.241	0.240	0.241
1	0.497	0.496	0.494	0.495	0.496
2	0.728	0.725	0.728	0.729	0.727
3	0.963	0.964	0.964	0.964	0.964
4	1.20	1.20	1.20	1.20	1.20
5	1.44	1.44	1.44	1.44	1.44
6	1.65	1.64	1.64	1.64	1.64
7	1.89	1.89	1.89	1.89	1.89
8	2.12	2.13	2.12	2.12	2.12
9	2.37	2.37	2.37	2.37	2.37
10	2.61	2.61	2.60	2.60	2.61
11	2.82	2.82	2.83	2.84	2.83
12	3.08	3.08	3.08	3.08	3.08
13	3.33	3.33	3.33	3.33	3.33
14	3.58	3.58	3.58	3.58	3.58
15	3.83	3.83	3.83	3.83	3.83
16	4.07	4.07	4.08	4.07	4.07
17	4.32	4.32	4.33	4.32	4.32
18	4.48	4.49	4.49	4.49	4.49
19	4.74	4.74	4.74	4.74	4.74
20	4.98	4.99	4.97	4.98	4.98

A.2 Calibration chart for PT 2

Input range: -1 – 10 bar

Output signal: 0 – 100 mV

Excitation voltage: 9.8 V

Method of calibration: Dead-weight tester

Wiring sequence:

Excitation voltage (+): Red
(-): BlueSignal output (+): Green
(-): Yellow

Pressure (bar)	Reading (mV)				Average
	1	2	3	4	
Atm	0.10	0.60	0.60	0.60	0.48
1.0	10.2	10.4	10.3	10.5	10.25
1.5	15.8	15.7	15.7	15.8	15.8
2.0	20.7	20.6	20.7	20.6	20.7
2.5	25.5	25.5	25.5	25.5	25.5
3.0	30.4	30.4	30.5	30.4	30.4
3.5	35.4	35.3	35.3	35.2	35.3
4.0	40.3	40.3	40.3	40.2	40.3
4.5	45.2	45.2	45.1	45.3	45.2
5.0	50.1	50.1	50.1	50.1	50.1
6.0	59.6	59.4	59.5	59.5	59.5
7.0	69.3	69.2	69.9	69.3	69.5
8.0	79.3	79.2	79.2	79.2	79.2
9.0	89.0	89.1	89.1	89.0	89.1
10.0	98.8	98.8	98.8	98.8	98.8

A.3 Calibration chart for PT 5

Input range: -1 – 5 bar

Output signal: 0 – 125 mV

Excitation voltage: 10.0 V

Method of calibration: Dead-weight tester

Wiring sequence:

Excitation voltage (+): Red
(-): White

Signal output (+): Yellow
(-): Blue

Pressure (bar)	Reading (mV)				Average
	1	2	3	4	
Atm	23.6	23.7	23.7	23.7	23.7
1.0	44.5	44.6	44.5	44.6	44.6
1.5	54.9	54.8	54.8	54.9	54.9
2.0	65.0	65.1	65.1	65.1	65.1
2.5	75.4	75.4	75.5	75.4	75.4
3.0	85.3	85.6	85.6	85.7	85.6
3.5	95.9	95.8	95.9	96.0	95.9
4.0	106.0	106.0	106.0	106.0	106.0
4.5	115.0	115.0	115.0	115.0	115.0
5.0	123.0	123.0	123.0	123.0	123.0

Appendix B

B.1 Specification of motor

System: Electric motor

Date: 17.04.06

Revised: 05.05.06

The electric motor is bought 2nd hand and does not come with specifications. According to the vendor, the motor is

1. AC type.
2. Three phase.
3. Rated at 3 hp.
4. Equipped with variable frequency drive.

However, there is no information regarding

1. Whether it is an induction motor, wound-rotor motor, synchronous motor or universal motor.
2. Motor speed.
3. Type of enclosure, frame size or mounting details.

Pt 3 can be inferred by visual inspection.

In order to carry out the design for other parts of the drive train, let's assume that the motor supplied

1. Is an induction motor (NEMA Design B)
2. Has 4 poles

Using the above information, it is possible to calculate the speed of the motor according to the equation

$$n_s = \frac{120f}{p} \text{ rev/min}$$

Where

n_s - motor speed, rev/min

f - frequency of AC power, Hz

p - no of poles

Since the motor is equipped with an inverter, the speed of the motor can be regulated by altering the frequency using either the six-step method or the pulse-width modulation method (PWM). Nevertheless, the maximum speed permissible using 50-Hz power is calculated and presented in table B.1.

Table B.1 AC motor speed

No. of poles	Synchronous speed	Full load Speed (rpm)
4	1500	1425

Subsequently, a motor performance curve (figure B.1) has also been drawn.

Figure B.1 Motor performance curve

Since the rated power of the motor and its operating speed is known, the torque produced can be found using equation

$$T = \frac{P}{w}$$

Where

T – torque, Nm

w – angular velocity, rad/s

P – power, W

This produced a full-load torque equivalent to 14.24 Nm. Similarly, if the motor is operated at 50-200 rpm, the power produced is (figure B.2)

Figure B.2 Power produced from 0-200 rpm

Addition

Date: 05.05.06

Some information was gathered from the I.D. tag on the motor body. According to the tag, the motor is rated at 3 hp which is the same as the initial estimate and its no-load speed is 1700 rpm, which is 11.3% off the first guess. Nevertheless, the original result can still be used since the lower rotation used in the first estimate yields a higher torque. Despite the fact, attention will be paid when designing for the higher rotation speed.

Some measurements were also taken. As they are measured using a metal rule, they can only be used as rough guides.

All dimensions in mm unless otherwise stated.

Body size: L450 x W350 x H?

Base plate: L195 x W250

Shaft diameter: 28

Hole system: M12

Placement of holes: 12.5 from edge 1

10.0 from edge 2

Key size: L39.00 x W7.90 x H6.30

B.2 Specification for gear-train

System: Gear-train

Date: 17.04.06

A gear-train is used to rotate the ERD in opposite direction. As before, the shaft carrying both gears will rotate at 50 – 100 rpm and connected to an electric motor rated at 3 hp@1500 rpm. Accordingly, the motor produces 26.95 N/m of torque at all rpm. Based on this estimate, KG spur gears S1.5S 28A=1215 was selected for this task since it has been rated at 0.339 kW@100 rpm and 32.37 Nm@100 rpm.

Since the maximum torque transmitted is known, it is possible to find transmitted tangential force W_t using

$$W_t = \frac{T}{R}$$

Where

W_t – tangential force, N

T – torque, Nm

R – radius of pitch circle of gear, m

According to specifications of the gear, the diameter of its pitch circle is equal to 42 mm.

Correspondingly,

$$W_t = \frac{26.95}{0.021} = 1283.33 \text{ N}$$

Having found W_t , it is possible to proceed to finding the radial force W_r according to the equation

$$W_r = W_t \tan \phi$$

Where

ϕ is the pressure angle of the tooth form. Therefore,

$$W_r = 1283.33 \tan 20^\circ = 467.10 \text{ N}$$

Summary

Input speed: 50-100 rpm

Output speed: Directional change only

Pitch diameter: 42 mm

Tooth width: 12 mm

B.3 Specification of chain drive

System: Chain drive

Date: 08.04.06

Revised: 13.04.06, 05.05.06

Option 1 (Selected)

The chain drive system is driven by a 3 hp motor with a uniform load. The input shaft will rotate at 50-100 rpm. It will be step up to 100-200 rpm. The stepping up is performed by the chain system.

Analysis:

Power transmitted: 3 hp

Speed of input shaft: 50-100 rpm

Speed of output shaft: 100-200 rpm

Step 1: Determine the design power.

Load type: smooth

Type of driver: Electric motor

Service factor: 1.0

Design power: Service factor x rated power = $1.0 \times 3 = 3.0$ hp

Step 2: Compute the desired ratio.

Ratio = $50 \text{ rpm} / 100 \text{ rpm} = 0.5$

Step 3: Refer to tables for power capacity and select chain pitch.

To minimize cost, the no. 40 chain with $p = \frac{1}{2}$ is the best compromise. A 28-tooth sprocket is rated at 0.63 hp at 50 rpm. At this speed, type 1 lubrication is needed.

Step 4: Compute the no. of tooth needed on the smaller sprocket.

$$N_2 = N_1 \times \text{ratio} = 28 \times 0.5 = 14.$$

However, sprocket-14 cannot be used as the minimum no. of teeth should be 17.

No. of tooth on the smaller sprocket = 17.

Step 5: Compute the actual expected speed.

$$_{\text{low}} n_2 = n_1 (N_1 / N_2) = 50 \text{ rpm} (28 / 17) = 82.35 \text{ rpm.}$$

$$_{\text{high}} n_2 = n_1 (N_1 / N_2) = 100 \text{ rpm} (28 / 17) = 164.71 \text{ rpm.}$$

Step 6: Compute the pitch diameters of the sprockets:

$$D_1 = \frac{p}{\sin(180^\circ / N_1)} = \frac{0.5}{\sin(180^\circ / 28)} = 4.466 \text{ in}$$

$$D_2 = \frac{p}{\sin(180^\circ / N_2)} = \frac{0.5}{\sin(180^\circ / 17)} = 2.72 \text{ in}$$

Step 7: Specific the nominal center distance. Recommended range: $30p < C < 50p$.

$$30p = 30 \times 0.5 = 15 \text{ in}$$

For this project, let C be 220-230 mm = 9.0 in (figure B.1 and figure B.3).

Figure B.1 Schematic of chain drive

Step 8: Compute the required chain length in pitches.

$$C = 228.6 \text{ mm} = 9.0 \text{ in} = 18.0 \text{ pitches}$$

$$L = 2C + \frac{N_2 + N_1}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C}$$

$$L = 2(18.0) + \frac{17 + 28}{2} + \frac{(17 - 28)^2}{4\pi^2 (18.0)}$$

$$L = 58.670 \text{ pitches}$$

Step 9: Specify an integral no. of pitches for the chain length and compute the actual theoretical center distance. Lets use 60 pitches (**even no.**), then,

$$C = \frac{1}{4} \left[L - \frac{N_2 + N_1}{2} + \sqrt{\left[L - \frac{N_2 + N_1}{2} \right]^2 - \frac{8(N_2 - N_1)^2}{4\pi^2}} \right]$$

$$C = \frac{1}{4} \left[60 - \frac{17 + 28}{2} + \sqrt{\left[60 - \frac{17 + 28}{2} \right]^2 - \frac{8(17 - 28)^2}{4\pi^2}} \right]$$

$$C = 18.668 \text{ pitches} = 18.668(0.5 \text{ in}) = 9.334 \text{ in} = 237.082 \text{ mm}$$

Summary of design

Pitch No. 40 chain, ½ pitch.

Length = 60 pitches = 60 x 0.5 = 30 in.

Center distance = C = 18.668 pitches = 9.334 in (max).

Sprockets: Single strand, no. 40, ½ pitch, 17 tooth and 28 tooth.

Figure B.2 Summary of chain drive

Option 2

Date: 10.04.06

The problem is solved by using a set of spur gears to step up the speed of the input shaft.

Therefore there is no change in speed at the chain system.

Analysis:

Power transmitted: 3 hp

Speed of input shaft: 50-100 rpm

Speed of output shaft: no change

Step 1: Determine the design power.

Load type: smooth

Type of driver: Electric motor

Service factor: 1.0

Design power: Service factor x rated power = $1.0 \times 3 = 3.0$ hp

Step 2: Compute the desired ratio.

Ratio = 1

Step 3: Refer to tables for power capacity and select chain pitch.

To minimize cost, the no. 40 chain with $p = \frac{1}{2}$ is the best compromise. A 17-tooth sprocket is rated at 0.69 hp at 100 rpm. At this speed, type 1 lubrication is needed.

Step 4: Compute the no. of tooth needed on the smaller sprocket.

N.A.

Step 5: Compute the actual expected speed.

N.A

Step 6: Compute the pitch diameters of the sprockets:

$$D = \frac{p}{\sin(180^\circ / N)} = \frac{0.5}{\sin(180^\circ / 17)} = 2.721 \text{ in} = 69.116 \text{ mm}$$

Step 7: Specific the nominal center distance. Recommended range: $30p < C < 50p$.

$$30p = 30 \times 0.5 = 15 \text{ in}$$

$$50p = 50 \times 0.5 = 25 \text{ in}$$

Figure B.3 Center distance

For this project (figure B.3), let the nominal center distance be 230 mm.

Step 8: Compute the required chain length in pitches.

$$C = 230 \text{ mm} = 9.055 \text{ in} = 18.110 \text{ pitches}$$

$$L = 2C + \frac{2N}{2}$$

$$L = 2(18.110) + 17$$

$$L = 53.220 \text{ pitches}$$

Step 9: Specify an integral no. of pitches for the chain length and compute the actual theoretical center distance. Lets use 54 pitches (even no.), then,

$$C = \frac{1}{4} \left[L - \frac{2N}{2} + \sqrt{\left[L - \frac{2N}{2} \right]^2} \right]$$

$$C = \frac{1}{2} [54 - 17]$$

$$C = 18.5 \text{ pitches} = 18.5(0.5 \text{ in}) = 9.25 \text{ in} = 234.95 \text{ mm}$$

Summary of design

Pitch No. 40 chain, ½ pitch.

Length = 54 pitches = 54 x 0.5 = 27 in.

Center distance = C = 18.5 pitches = 9.25 in (max).

Sprockets: Single strand, no. 40, ½ pitch, 17 tooth x 2.

Figure B.5 Summary of option 2

Cn1, Cn2	Chains
ERD	Energy recovery device

Shaft 1

The 1st shaft to be designed is the input shaft connected to the motor (figure B.2). All dimensions in mm unless otherwise stated.

Figure B.2 Rough sketch of shaft 1

Based on the initial projection, a more detail drawing can be generated (figure B.3).

Figure B.3 Proposed geometry for Shaft 1

Power, speed and material for the shaft

Shaft 1 is will be machined from AISI 1045 steel. The shaft is part of a drive train for an energy recovery device. The flexible coupling is attached to a 3 hp motor rotating at 80-160 rpm. Sprocket 1 delivers power to sprocket 4 while gear 1 delivers power to gear 2. The power is equally distributed.

Solution:

First determine the properties of the steel for the shaft. As AISI 1045 is not available in the textbook, properties of AISI 1040 is used instead. From the appendix, $s_y = 290$ MPa, and $s_u = 496$ MPa. The percentage elongation is 18%, thus the material can be considered to have good ductility. Using figure 5-9, its endurance strength is estimated to be 150 MPa.

As the shaft is expected to carry 3 hp of load, it should be of a moderate size, therefore if the final average shaft diameter is 25 mm, a size factor of 0.92 will be specified. A reliability factor should also be specified. This is a design decision. For this problem, let's design for a reliability of 0.90 and use $C_R = 0.90$. Now, the modified endurance strength can be computed.

$$s'_n = s_n C_S C_R = (1.5 \times 10^8)(0.92)(0.90) = 1.24 \times 10^8$$

The design factor is taken to be $N = 3$, since the drive train is not expected to withstand any unusual shock or impact. The torque of in the shaft can be computed using equation 12-1.

$$T = \frac{P}{\omega} = \frac{(3)(746)}{(1700)(2\pi)/60} = 12.57 \text{ Nm}$$

Note that the full amount of torque is delivered into the shaft through point A and subsequently split into two equal portions at B and C. There is no torque in the remaining parts of the shaft.

Forces acting on the shaft

Gear

Gear 1 on the input shaft drives gear 2 on an intermediate shaft. Since half of the torque is sent through these gears, the tangential force can be found according to

$$W_t = T / (D/2) = (12.57/2) / (0.042/2) = 299.29 \text{ N}$$

Where

W_t – Tangential force, N

T – Torque, Nm

D – Pitch diameter, m

Similarly, the radial force can be found directly from

$$W_r = W_t \tan \phi = 299.29 \tan 20^\circ = 108.93 \text{ N}$$

Chain and sprockets

Sprocket 1 at B is used to drive one of the two output shafts in the design. As both sides of the ERD turns in opposite direction, it is not possible to design the tight side on both

sprocket drive to be on the top. In this case, the sprocket drive on the motor side will have its tight side on its bottom chain. Since the transmitted torque is known, the tension in the chain can be found,

$$F_c = T / (D/2) = (12.57/2) / (0.113/2) = 111.24N$$

Notice that the force, F_c , acts along the direction of the tight side of the belt. Because of the size difference between the two sprockets, that direction is at some angle from the centerline between the shaft centers. A precise analysis would call for the force, F_c , to be resolved into components parallel to the centerline and perpendicular to it. To simplify this exercise, the chain drive was design in such a way that the tight sides are almost horizontal. Thus, the error resulting from assuming that the forces acted only in the x -direction will be small.

Summary

Figure B.4 Pictorial view of forces on sprocket 1 and gear 1

Figure B.5 Forces on sprocket 1 and gear 1

Load, shear and moment diagrams

Figure B.6 collectively shows the forces acting on the shaft at each element, the reactions at the bearings, and the shearing force and bending moment diagrams for both the horizontal (x) and vertical (y) planes.

Let $\Sigma M_{xB1} = 0$,

$$R_{xB2} = (20F_c + 40W_{tG1})/60 = 237 \text{ N}$$

Let $\Sigma M_{xB2} = 0$,

$$R_{xB1} = (20W_{tG1} + 40F_c)/60 = 174 \text{ N}$$

Let $\Sigma M_{yB1} = 0$,

$$R_{yB2} = 40W_{rG1}/60 = 73 \text{ N}$$

Let $\Sigma F_y = 0$,

$$R_{yB1} = 37 \text{ N}$$

Figure B.6 Load, shear and moment diagrams for shaft 1

Equation 12-24

Equation 12-24 is reproduced here for convenience.

$$D = \left[\left(\frac{32N}{\pi} \right) \sqrt{\left(\frac{K_t M}{s'_n} \right)^2 + \frac{3}{4} \left(\frac{T}{s_y} \right)^2} \right]^{1/3}$$

1. Point C1

C1 is the location of the flexible coupling. It is located by retaining rings, therefore the computed diameter will be increased by 6% to compensated for the presence of a groove. As long as the motor shaft and shaft 1 is properly installed, there should be no residue bending moments or shear stresses. However, 100% of the torque is transmitted from C1 to S1 and G1. Since torque is assumed to be steady, stress concentrations can be neglected.

Using equation 12-24, $D_{C1} = 0.005$ m.

2. To the left of B1

B1 is the location of the bearing. To the left of the bearing, the shaft diameter is deliberately kept small to allow easy installation of the bearing, there is also a relief diameter leading up the bearing seat. A well-rounded radius will be specified for the place where D_{C1} joins D_{B1} . Similar to C1, there is no bending moments at B1, therefore the diameter is expected to be the same as well.

Left of B1, $D_{LB1} = D_{C1}$.

3. At point B1 and to the right of B1

This is the bearing seat with a shoulder fillet at the right, requiring a fairly sharp fillet. However, moment is still zero. Therefore,

$$D_{B1} = D_{C1}$$

4. Between B1 and S1

This region will be bigger than D_{B1} in order to provide for a shoulder for the bearing. Therefore, it will be safe. Its actual diameter will be specified after the shaft is designed and the bearing selected. It will be according to the minimum acceptable diameter as specified in the bearing manufacturer catalog.

5. S1

The sprocket S1 is position here. It is located by a left shoulder which could imply a sharp radius ($K_t = 2.5$). At point S1, there is a profile keyseat ($K_t = 2.0$) and to its right, there is a retaining ring groove ($K_t = 3.0$).

$$M_{S1} = \sqrt{M_{xS1}^2 + M_{yS1}^2} = 3.55 \text{ Nm}$$

At S1 and to the left of S1, $T = 12.57$ Nm and there is a sharp fillet ($K_t = 2.5$) and profile keyseat ($K_t = 2.0$). Then,

$$D_{LS1} = 0.013 \text{ m}$$

To the right of S1, $T = 6.285 \text{ Nm}$ and there is a retaining ring groove ($K_t = 3.0$). For retaining ring grooves, diameter of shaft will be increased by 6%.

$$D_{RS1} = 0.0146 \text{ m}$$

Since this is the larger value of the two, it will be the governing diameter for S1.

G1

The gear G1 is positioned here. It is located by two retaining rings placed on both sides of the gear. There is also a profile keyseat for transmitting the driving torque to the gear. The diameter of the shaft can be calculated by just considering 1 side of gear as conditions are similar on both sides of the gear except that the left side of the gear is subjected to the driving torque while there is no torque on the right side.

$$M_{G1} = \sqrt{M_{xG1}^2 + M_{yG1}^2} = 4.97 \text{ Nm}$$

$$D_{G1} = 0.0164 \text{ m}$$

To the left of B2 and B2

This is the location of the 2nd bearing. On its left, there is a shoulder with sharp fillet for positioning of the bearing. There is no torque or moment acting through this location, there is, however, a vertical shearing force equal to the reaction at the bearing. Using the resultant of the x - and y - plane reactions, the shearing force is

$$V_{B2} = \sqrt{V_{xB2}^2 + V_{yB2}^2} = 248 \text{ N}$$

The required diameter for the shaft at this point can be computed using equation 12-16

$$D = \sqrt{2.94 K_t (V) N / s'_n}$$

$$D_{B2} = 0.007 \text{ m}$$

As expected, the diameter at this point is very small. In reality, it will be much bigger because of the size of the bearing required to carry the radial load.

Summary

Table B.2 Minimum diameters at various key points on the shaft

Position	Minimum diameter (m)	Specified diameter (mm)
D_{C1}	0.005	14
D_{B1}	0.005	15
$D_{B1}-D_{S1}^*$	0.005	18.5
D_{S1}	0.0146	15.875
D_{G1}^*	0.0164	15
D_{B2}	0.007	12

* Denotes diameter of shoulder for bearings

Determining the ball bearing for shaft 1

The experimental setup is expected to be operated intermittently for a total of less than 200 hours. Bearing 1 is subjected to a load of 178 N while bearing 2 is subjected to 248 N.

The respectively dynamic load rating, C will be

$$C_{B1} = P_d (L_d / 10^6)^{1/k} = 178[(200)(160)(60) / 10^6]^{1/3} = 221.2N$$

$$C_{B2} = P_d (L_d / 10^6)^{1/k} = 248[(200)(160)(60) / 10^6]^{1/3} = 308N$$

Where

P_d – Design load, N

L_d – Design life, rev

k – 3.00 for ball bearing

Based on the above calculations and other important considerations such as shaft diameter around the bearing, bearing number 6002-2RSH from SKF is chosen for $B1$. Similarly, bearing number 16101-2RS1 is selected for $B2$. Using information gathered from the bearing supplier, Table B.2 can be completed.

Limits

Table B.3 Fits and tolerances for shaft 1

Position	Description	Fit	Clearance	Limits
C1	Coupling seat	Clearance fit (?)	-0.01 – 0.00	13.99 – 14.00
B1	Bearing seat	Transition fit (j5)	-0.003 – 0.005	14.997 – 15.005
S1	Sprocket seat	Clearance fit (?)	-0.008 – 0.000	14.992 – 15.000
G1	Gear seat	Clearance fit (?)	-0.008 – 0.000	14.992 – 15.000
B2	Bearing seat	Transition fit (j5)	-0.003 – 0.005	11.997 – 12.005

Shaft and housing shoulder diameter

Table B.4 Min. shaft shoulder diameter & max. housing shoulder diameter

Bearing	Shaft min.	Shaft max.	Actual	Housing min.	Housing max	Actual
B1	17.0	18.5	18.5	23.45	30.0	28.0
B2	14.4	16.5	15.0	19.60	27.6	24.5

Figure B.7 Dimensions of bearing 1

Figure B.8 Dimensions of bearing 2

Fillet radius

The diameter of the shaft is not constant throughout, at regions where the diameter steps up or steps down, a fillet must be incorporated to accommodate the change. To keep stresses resulting from sharp corners small, radii of the fillets must be as large as possible. This is not possible in all scenarios, consider for example, the ball bearings used to support the shaft. They are delivered with rounded corners albeit very small ones as produced by the ball bearing company. In order for the bearing to seat securely on the shoulder of the shaft, the fillet radius on the shaft must be smaller than that on the ball bearing. The well roundedness

or sharpness of a fillet can be determined by calculating the ratio of the fillet radius over the diameter of the shaft (figure B.9). According to the textbook, a ratio of 0.03 and below signals a sharp radius while a ratio of 0.15 and above indicates a well rounded one. The following table documents all the fillet radii on shaft 1 as well as the ratio, r/d .

Figure B.9 Fillets on shaft

Table B.4 Fillet radii on shaft 1

Position	Description	d (mm)	r (mm)	Ratio (r/d)	Remarks
C1 – B1	The shaft diameter is deliberately kept small to allow easy installation of the bearing.	14	2	0.14	This is a well rounded fillet.
Right of B1	This is a bearing seat. The fillet radius must be smaller than that of the bearing. It is common practice to maintain the tolerance of the radius to within 10% of the nominal radius.	15	<0.3	<0.02	Obviously, this is a sharp fillet.
Left of S1	S1 is seated against this shoulder. The fillet radius has to be smaller than the chamber on the bore of the sprocket.	15.875	0.50-0.55	>0.03	This is a fairly sharp fillet.
S1 – G1	There is a very small change in diameter at this point (>1mm). The change will take place at the left ring groove for the gear.	-	-	-	There is no fillet here.
G1 – B2	Like B1, this is a bearing seat. Therefore, similar conditions apply.	12	<0.3	<0.025	This is a sharp fillet like B1.

Dimensions of keys for coupling, sprocket and gear

Coupling

C1 is $\varnothing 14$. It transmits 12.67 Nm of torque. The material used for the shaft is AISI 1045. The max. insertion length of the shaft into the coupling is 21 mm. A suitable key for this application can be designed by considering the material selected for the key. Suppose the

material the same as the shaft and it has a lower yield strength than the coupling, equation 11-5 can be used to determine the min. length of the key.

$$L_{C1} = \frac{4TN}{DW_{S_y}} = \frac{4(12.67)(3)}{(0.014)(3.5 \times 10^{-3})(290 \times 10^6)} = 0.0107m$$

Where

N – Safety factor, typical industrial application

D – Diameter of shaft at that point, m

W – Width of key, m

Sprocket

S1 is $\varnothing 15.875$. Half the total torque is transmitted through the sprocket. The sprocket is 7.21 mm wide. Once again, let's assume that the key is the weakest part amount the other components. Therefore the min. length required is

$$L_{S1} = \frac{4TN}{DW_{S_y}} = \frac{4(6.335)(3)}{(0.016)(0.004)(290 \times 10^6)} = 0.0041m$$

Gear

G1 is $\varnothing 15$. Half the torque is sent through here. The gear is 12 mm wide. If it is once again assumed that the key is the weakest part, its min. length will be

$$L_{G1} = \frac{4TN}{DW_{S_y}} = \frac{4(6.335)(3)}{(0.015)(0.004)(290 \times 10^6)} = 0.0044m$$

Summary

Table B.5 Dimensions of keys

Position	Material	Dimension (L x W x H) (mm x mm x mm)	Tolerance on key	Tolerance on keyseat
C1	AISI 1045	15 x 3.5 x 3.5	0.000 – -0.050	0.000 – 0.050
S1	AISI 1045	3 x 4 x 4	0.000 – -0.050	0.000 – 0.050
G1	AISI 1045	6 x 4 x 4	0.000 – -0.050	0.000 – 0.050

Assembly drawingRaw materials needed

1. 1 x Rod, O.D. x Length: 18.5 + 10 = **28.5** x 111.5
+ 40 = **141.5**

Retaining rings: ext. $\varnothing 14 \times 1$, ext $\varnothing 15 \times 3$

A-A $D=14$ mm

B-B $D=15$ mm

C-C $D=15$ mm

$$Y = 0.429$$

$$Y = 0.429$$

$$S = 12.071$$

$$S = 12.071$$

$$S = D - Y - \frac{H}{2} = 11.038$$

$$Y = \frac{D - \sqrt{D^2 - W^2}}{2} = 0.462$$

ERD input

$D = 16$ mm, 24 mm

$Y = 0.401, 0.381$

$S = 13.099, 20.619$

Housing for shaft 1*Left*Raw material needed

1. 2 x plate, L x B x H: $171.25 + 5 = 176.25$ x 95
+ 5 = 100 x 14 + 10 = 24

Right

Retaining rings: int. Ø30 x 2

Shaft 4

Shaft 4 has to be design first before shaft 2 and shaft 3 can be design as the dimensions of shaft 4 determines the overall length of shaft 2. Shaft 4 is one of the two output shafts used to turn the ERD. Half the total torque of 12.67 Nm is transmitted through this shaft.

Figure B.10 Rough sketch of shaft 4

Based on the initial projection, a more detail drawing can be generated (figure B.11).

Figure B.11 Proposed geometry for shaft 4

Description of material, power and rpm for shaft 4

Shaft 4 is part of the drive train used to propel the ERD. Shaft 4 will use the same material shaft 1 used. As the input power is divided equally between shaft 3 and shaft 4, half the power will be transmitted through this shaft. It will rotate between 100-200 rpm.

Solution:

First determine the properties of the steel for the shaft. As AISI 1045 is not available in the textbook, properties of AISI 1040 is used instead. From the appendix, $s_y = 290$ MPa, and $s_u = 496$ MPa. The percentage elongation is 18%, thus the material can be considered to have good ductility. Using figure 5-9, its endurance strength is estimated to be 150 MPa.

As the shaft is expected to carry 3 hp of load, it should be of a moderate size, therefore if the final average shaft diameter is 25 mm, a size factor of 0.92 will be specified. A reliability factor should also be specified. This is a design decision. For this problem, let's design for a reliability of 0.90 and use $C_R = 0.90$. Now, the modified endurance strength can be computed.

$$s'_n = s_n C_S C_R = (1.5 \times 10^8)(0.92)(0.90) = 1.24 \times 10^8$$

The design factor is taken to be $N = 3$, since the drive train is not expected to withstand any unusual shock or impact. The torque transmitted can be calculated since F_c is known.

$$T = F_c r = (112)(0.069/2) = 3.870 Nm$$

The torque turns the shaft by $S4$ which rotates the ERD through $C3$. From the left side of $S4$ to the same end on the shaft, there is no torque.

Forces acting on the shaft

Chain & Sprockets

Sprocket 4 is rotated by sprocket 1 and shaft 1. The same force F_c that acted on shaft 1 also acts on shaft 4 but in the exact opposite direction. Although the tight side of the chain is not 100% horizontal, the vertical force is small and can be ignored. No other forces acted on the shaft.

Summary

Figure B.12 Pictorial view of forces on S4

Figure B.13 Forces on S4

Load, shear and moment diagrams

Figure B.14 shows all the load, shear and moments acting on shaft 4.

Let $M_{xB8} = 0$,

$$R_{xB7} = 20F_c / 40 = 55.5 \text{ N}$$

Let $M_{xB7} = 0$,

$$R_{xB8} = 20F_c / 40 = 55.5 \text{ N}$$

Figure B.14

B8

B8 is the location of one of the two bearings. No significant torque or bending moments exist in this region. Reversed vertical shearing forces govern the design of the shaft here. To the right of the bearing seat, there will be a shoulder to locate the bearing, therefore a sharp fillet at the base of the shoulder will be present. The stress concentration factor, K_t is expected to be 2.5. The diameter of the shaft can be solved by equation 12-16.

Eq 12-16,

$$D = \sqrt{2.94 K_t (V) N / s'_n}$$

$$D_{B8} = 0.0032 \text{ m}$$

S4

S4 is the location of the driven sprocket. It is responsible for the transmission of the torque from the motor to the ERD. To locate the sprocket, a retaining ring ($K_t = 3.0$) is used on its left while a shoulder with a sharp fillet ($K_t = 2.5$) machined on to the shaft is used on its right. To rotate the shaft, a square key ($K_t = 2.0$) is used.

$$M_{S4} = 1.11 \text{ Nm}$$

On the left side of S4, there is a retaining ring groove. No torque acts through this region though there is reversed bending.

$$D_{S4} = 0.001 \text{ m}$$

On the right side of S4, a shoulder with sharp fillet provides location for the sprocket. Also both torque and bending moment are present in this region.

$$D_{S4} = 0.001 \text{ m}$$

Since both approaches produce an almost identical result, there is no difference picking one result from the other.

Left of B7 and B7

B7 is the location of the 2nd bearing. Naturally, there will be a shoulder for positioning of the bearing ($K_t = 2.5$).

$$D_{B7} = 0.0003 \text{ m}$$

Right of B7

For ease of installation of the bearing, the portion of shaft leading the bearing seat will be deliberately smaller. Therefore, there will be a well rounded fillet ($K_t = 1.6$).

$$D_{LB7} = 0.0003 \text{ m}$$

C3

As a flexible coupling is used, there is supposedly no bending moment or shearing forces in this part of the shaft. Only torque is present. To transmit the torque, a square key is used ($K_t = 2.0$), also, a retaining ring ($K_t = 3.0$) is used to locate the coupling.

$$D_{C3} = 0.0003 \text{ m}$$

Summary

Table B.6 Minimum diameters at various key points on the shaft

Position	Minimum diameter (m)	Specified diameter (mm)
D_{B8}	0.003	12
D_{S4}^*	0.001	15.875
$D_{S4}-D_{B7}^*$	0.005	18.5
D_{B7}	0.0003	15
D_{C3}	0.0003	14

* Denotes diameter of shoulder for bearings

Determining the bearing for shaft 4

As shaft 4 rotates faster than shaft 1, it will complete more revolutions throughout its life cycle. Both bearings B7 and B8 are subjected to the same load of 55.5 N. Their dynamic load rating, C will be

$$C_{B7} = C_{B8} = P_d (L_d / 10^6)^{1/k} = 55.5[(200)(200)(60) / 10^6]^{1/3} = 74.3N$$

Where

P_d – Design load, N

L_d – Design life, rev

k – 3.00 for ball bearing

Based on the above calculations and other important considerations such as shaft diameter around the bearing, bearing number 6002-2RSH (figure 7) is chosen for duty at B7. Similarly, bearing number 16101-2RS1 (figure 8) is selected for duty at B8. Using information gathered from the bearing supplier, Table 6 can be confirmed.

Limits

Table B.7 Fits and tolerances for shaft 4

Position	Description	Fit	Clearance	Limits
B8	Bearing seat	Transition fit (js5)	± 0.004	11.996-12.004
S4	Sprocket seat	Clearance fit (?)	$-0.008 - 0.000$	14.992-15.000
B7	Bearing seat	Transition fit (js5)	± 0.004	14.996-15.004
C3	Coupling seat	Clearance fit (?)	$-0.01 - 0.00$	13.99-14.00

Shaft and housing shoulder diameter

Table B.8 Min. shaft shoulder diameter & max. housing shoulder diameter

Bearing	Shaft min.	Shaft max.	Actual
B1	17.0	18.5	18.5
B2	14.4	16.5	15.0

Fillet radius

Table B.9 Fillet radii on shaft 4.

Position	Description	d (mm)	r (mm)	Ratio (r/d)	Remarks
Right of B8	This is a bearing seat. The fillet radius must be smaller than that of the bearing. It is common practice to maintain the tolerance of the radius to within 10% of the nominal radius.	12	<0.3	<0.025	This is a sharp fillet.
Right of S4	S4 is seated against this shoulder. The fillet radius has to be smaller than the chamber on the bore of the sprocket.	15.875	0.50-0.55	>0.30	This is a fairly sharp fillet.
Left of B7	Like B8, this is a bearing seat. Therefore, similar conditions apply.	15	<0.3	<0.02	Obviously, this is a sharp fillet.
C3 – B7	The shaft diameter is deliberately kept small to allow easy installation of the bearing.	14	2	0.14	This is a well-rounded fillet.

Dimensions of keys for coupling and sprocket*Coupling*

C3 is $\varnothing 14$. It transmits 3.87 Nm of torque. The material used for the shaft is AISI 1045. The max. insertion length of the shaft into the coupling is 21 mm. A suitable key for this application can be designed by considering the material selected for the key. Suppose the material the same as the shaft and it has a lower yield strength than the coupling, equation 11-5 can be used to determine the min. length of the key.

$$L_{C3} = \frac{4TN}{DW_{s_y}} = \frac{4(3.87)(3)}{(0.014)(3.5 \times 10^{-3})(290 \times 10^6)} = 0.0033m$$

Where

N – Safety factor, typical industrial application

D – Diameter of shaft at that point, m

W – Width of key, m

Sprocket

S4 is $\varnothing 15.875$. Half the total torque is transmitted through the sprocket. The sprocket is 7.21 mm wide. Once again, let's assume that the key is the weakest part amount the other components. Therefore the min. length required is

$$L_{s4} = \frac{4TN}{DW_{s_y}} = \frac{4(3.87)(3)}{(0.016)(0.004)(290 \times 10^6)} = 0.0025m$$

Summary

Table B.10 Dimensions of key

Position	Material	Dimension (L x W x H) (mm x mm x mm)	Tolerance on key	Tolerance on keyseat
S4	AISI 1045	3 x 4 x 4	0.000 – -0.050	0.000 – 0.050
C3	AISI 1045	15 x 3.5 x 3.5	0.000 – -0.050	0.000 – 0.050

Retaining rings: ext. Ø14 x 2, ext. Ø15 x 2.

Raw materials needed
2 x rod, O.D. x length: $18.5 + 10 = 28.5 \times 91.5 + 40$
 $= 131.5$

Opposite coupling

Retaining ring: int. $\varnothing 30 \times 2$.

Technical drawing of a shaft assembly. The shaft has a total length of 560. On the left end, there is a spur gear with a pitch diameter $\varnothing_i = 15$. The gear is mounted on a shaft section with a diameter of 12. The distance from the gear to the first support (indicated by a cross) is 20. The distance between the two supports is 560. On the right end, there is a sprocket with a pitch diameter $\varnothing_i = 15.875$. The sprocket is mounted on a shaft section with a diameter of 7.21. The distance from the sprocket to the second support is 20. The distance between the two supports is 560. The shaft is supported by two bearings (indicated by crosses). The distance from the gear to the first support is 20. The distance between the two supports is 560. The distance from the sprocket to the second support is 20. The shaft has a diameter of 10 in the middle section.

As before, a more detailed design (figure B.15) can be presented once the major dimensions are confirmed.

Figure B.15 Proposed geometry for shaft 2

Description of material, power and rpm for shaft 2

Shaft 2 is part of the transmission that delivers power to shaft 3. Accordingly, it gets only half the maximum power input. It rotates at the same speed as shaft 1.

Solution:

First determine the properties of the steel for the shaft. As AISI 1045 is not available in the textbook, properties of AISI 1040 is used instead. From the appendix, $s_y = 290$ MPa, and $s_u = 496$ MPa. The percentage elongation is 18%, thus the material can be considered to have good ductility. Using figure 5-9, its endurance strength is estimated to be 150 MPa.

As the shaft is expected to carry 1.5 hp of load, it should be of a moderate size, therefore if the final average shaft diameter is 25 mm, a size factor of 0.92 will be specified. A reliability factor should also be specified. This is a design decision. For this problem, let's design for a reliability of 0.90 and use $C_R = 0.90$. Now, the modified endurance strength can be computed.

$$s'_n = s_n C_S C_R = (1.5 \times 10^8)(0.92)(0.90) = 1.24 \times 10^8$$

The design factor is taken to be $N = 3$, since the drive train is not expected to withstand any unusual shock or impact. The torque transmitted one half that calculated in shaft 1.

$$T_{S2} = T_{S1} / 2 = 12.76 / 2 = 6.38 Nm$$

The torque enters the shaft via G2 and leaves via S2. The entire shaft is subjected to the same torque.

Forces acting on the shaft

Gear

Using the principles of the 3rd law of Newton, it can be easily observed that G1 exerts an equal and opposite force on G2, therefore

$$W_{tS2} = W_{tS1} = 299.29 N$$

Similarly, the radial force can also be deduced

$$W_{rS2} = W_{rS1} = 108.93 N$$

Chain & sprockets

In the analysis for shaft 1, it was decided that the tight side for the chain & sprockets system on the motor would be on the bottom. Consequently, the tight side for this chain & sprockets system would have its tight side on the top. Nevertheless, the chain force, F_c , will stay the same.

$$F_{cS2} = F_{cS1} = 111.24 N$$

Summary

Figure B.16 Pictorial view of forces on G2 and S2

Figure B.17 Forces acting on G2 and S2

Load, shear and moment diagrams

Figure B.18 collectively shows the forces acting on the shaft at each element, the reactions at the bearings, and the shearing force and bending moment diagrams for both the horizontal (x) and vertical (y) planes.

Let $M_{xB3} = 0$,

$$20W_{tG2} - 560R_{xB4} + 580F_c = 0$$

$$R_{xB4} = 125.68 \text{ N}$$

Let $F_x = 0$

$$-R_{xB3} + W_{tG2} + R_{xB4} - F_c = 0$$

$$R_{xB3} = 314.68 \text{ N}$$

Let $M_{yB3} = 0$,

$$R_{yB4} = 20W_{rG2}/560 = 77.86 \text{ N}$$

Let $F_y = 0$,

$$R_{yB3} = W_{rG2} + R_{yB4} = 187 \text{ N}$$

x dir

Shearing
force, V (N)

Bending moment, M
(Nm)

y dir

Shearing
force, V (N)

Bending moment, M
(Nm)

Figure B.17 Diagram of load, shear and bending moment

G2

This is the location of gear 2. Although retaining rings grooves and a profile keyseat is present at this site, there is no need to consider their contribution to stress concentration as the torque is assumed to be steady.

$$D_{G2} = 0.004 \text{ m}$$

B3

Bearing number 3. The usual assumptions for a bearing seat will be used. Well-rounded fillet ($K_t = 1.5$) at the left of the seat and a sharp fillet ($K_t = 2.5$) at the bottom of the shoulder.

The bending moment endured by the shaft is

$$M_{B3} = \sqrt{M_{xB3}^2 + M_{yB3}^2} = 6.39 Nm$$

$$D_{LB3} = 0.013 \text{ m}$$

$$D_{RB3} = 0.016 \text{ m}$$

B4

B4 is a bearing identical to B3. Therefore the same procedure is followed here.

$$M_{B4} = \sqrt{M_{xB4}^2 + M_{yB4}^2} = 2.4 Nm$$

$$D_{LB4} = 0.011 \text{ m}$$

$$D_{RB4} = 0.0096 \text{ m}$$

S2

S2 resemble the conditions at G2 as they are both at the free end of the shaft as does not experience any bending moments. Still, they have to withstand the full amount of torque that is passing through the shaft. As before, there is no need to consider the effects of stress concentration arising from the discontinuities in the shaft.

$$D_{S3} = D_{G2} = 0.004 \text{ m}$$

Summary

Table B.11 Minimum diameters at various key points on the shaft

Position	Minimum diameter (m)	Specified diameter (mm)
D_{G2}	0.004	15
D_{B3}	0.016	17
$D_{B3}-D_{B4}^*$	0.016	19.15
D_{B4}	0.011	17
D_{S3}	0.004	15.875

* Denotes diameter of shoulder for bearings

Determining the ball bearing for shaft 2

The experimental setup is expected to be operated intermittently for a total of less than 200 hours. Bearing 3 is subjected to a load of 315 N while bearing 4 is subjected to 126 N.

The respectively dynamic load rating, C will be

$$C_{B3} = P_d (L_d / 10^6)^{1/k} = 315[(200)(160)(60) / 10^6]^{1/3} = 391.5N$$

$$C_{B4} = P_d (L_d / 10^6)^{1/k} = 126[(200)(160)(60) / 10^6]^{1/3} = 156.60N$$

Where

P_d – Design load, N

L_d – Design life, rev

k – 3.00 for ball bearing

Based on the above calculations and other important considerations such as shaft diameter around the bearing, bearing number 61903-2RS1 from SKF is chosen for $B3$ and $B4$. Using information gathered from the bearing supplier, Table B.11 can be completed.

Limits

Table B.12 Fits and tolerances for shaft 2

Position	Description	Fit	Clearance	Limits
G2	Gear seat	Clearance fit (?)	-0.008 – 0.000	14.992 – 15.000
B3	Bearing seat	Transition fit (j5)	-0.003 – 0.005	16.997 – 17.005
B4	Bearing seat	Transition fit (js5)	-0.004 – 0.004	16.996 – 17.004
S2	Sprocket seat	Clearance fit (?)	-0.008 – 0.000	14.992 – 15.000

Shaft and housing shoulder diameter

Table B.13 Min. shaft shoulder diameter & max. housing shoulder diameter

Bearing	Shaft min.	Shaft max.	Actual	Housing min.	Housing max.	Actual
B3	19.0	19.3	19.15	23.55	28	26.5
B4	19.0	19.3	19.15	23.55	28	26.5

Figure B.18 Dimensions of bearing 3 and 4

Fillet radius

Table B.14 Fillet radii on shaft 2

Position	Description	d (mm)	r (mm)	Ratio (r/d)	Remarks
Left of B3	The shaft diameter is kept small to allow easy installation	15	3	0.2	This is a well rounded fillet.
Right of B3	This is a bearing seat. The fillet radius must be smaller than that of the bearing. It is common practice to maintain the tolerance of the radius to within 10% of the nominal radius.	17	<0.3	<0.017	This is a sharp fillet.
Left of B4	Like B3, this is a bearing seat. Therefore, similar conditions apply.	17	<0.3	<0.017	Obviously, this is a sharp fillet.
Right of B4	The shaft diameter is kept smaller to allow easy installation of the bearing.	15.875	3	0.19	This is a well rounded fillet.

Dimensions of keys for coupling, sprocket and gear

Gear

G2 is $\varnothing 15$. Half the torque is sent through here. The gear is 12 mm wide. If it is once again assumed that the key is the weakest part, its min. length will be

$$L_{G2} = \frac{4TN}{DW_{s_y}} = \frac{4(6.335)(3)}{(0.015)(0.004)(290 \times 10^6)} = 0.0044m$$

Sprocket

S2 is $\varnothing 15.875$. Again, half the total torque is transmitted through the sprocket. The sprocket is 7.21 mm wide. Once again, let's assume that the key is the weakest part amount the other components. Therefore the min. length required is

$$L_{s2} = \frac{4TN}{DW_{s_y}} = \frac{4(6.335)(3)}{(0.016)(0.004)(290 \times 10^6)} = 0.0041m$$

Summary

Table B.15 Dimensions of keys

Position	Material	Dimension (L x W x H) (mm x mm x mm)	Tolerance on key	Tolerance on keyseat
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G2	AISI 1045	6 x 4 x 4	0.000 – -0.050	0.000 – 0.050
S2	AISI 1045	3 x 4 x 4	0.000 – -0.050	0.000 – 0.050

Raw material needed

1 x rod, O.D. x Length: $19.15 + 10 = 29.15$ x
 $684.295 + 40 = 724.295$

Left

1 x plate, L x B x H: $129.2 + 5 = 134.2$ x $80 + 5 = 85$
x $14 + 10 = 24$

Appendix C – Orthographic and 3D drawings for ERD

FOR ACTUAL DIMENSIONS REFER TO SHAFT 2 NO. 06-1
UNLESS OTHERWISE STATED, ALL DIMENSIONS IN MM

Title: SHAFT 2 MAGNIFIED

Scale: 0.700 No: 06-2

General view of ERD showing rotors inside

Appendix D

D.1 Results for different initial pressures

Figure D.1 Photo 101-1741

Figure D.4 Photo 101-1744

Figure D.2 Photo 101-1742

Figure D.5 Photo 101-1745

Figure D.3 Photo 101-1743

Figure D.6 Photo 101-1746

Figure D.7 Photo 101-1747

Figure D.10 Photo 101-1750

Figure D.8 Photo 101-1748

Figure D.11 Photo 101-1751

Figure D.9 Photo 101-1749

Figure D.12 Photo 101-1752

Figure D.13 Photo 101-1753

Figure D.16 Photo 101-1756

Figure D.14 Photo 101-1754

Figure D.15 101-1755

D.2 Results for different valve opening speed

Figure D.17 Photo 101-1757

Figure D.20 Photo 101-1760

Figure D.18 Photo 101-1758

Figure D.19 Photo 101-1759

D.3 Results for reduced bore 1/4" valve.

Figure D.21 Photo 101-1812

Figure D.24 Photo 101-1815

Figure D.22 Photo 101-1813

Figure D.25 Photo 101-1807

Figure D.23 Photo 101-1814

Figure D.26 Photo 101-1808

Figure D.27 Photo 101-1809

Figure D.30 Photo 101-1803

Figure D.28 Photo 101-1810

Figure D.31 Photo 101-1806

Figure D.29 Photo 101-1799

Figure D.32 Photo 101-1800

Figure D.33 Photo 101-1801

Figure D.34 Photo 101-1802

D.4 Results for $L = 12$ m pressure vessel

Figure D.35 Photo 101-1483

Figure D.38 Photo 101-1482

Figure D.36 Photo 101-1497

Figure D.39 Photo 101-1487

Figure D.37 Photo 101-1481

Figure D.40 Photo 101-1489

Figure D.41 Photo 101-1490

Figure D.44 Photo 101-1492

Figure D.42 Photo 101-1494

Figure D.45 Photo 101-1493

Figure D.43 Photo 101-1491

D.5 Results from the ERD

Please note that these are results as recorded by the oscilloscope. As each pressure transducers operates with a different input/output range, each voltage signal has to be converted into pressure first using the calibration chart in appendix 1 before it can be used. Figure 7.2 was plotted like this but it is very time consuming and tedious especially when plotting the ERD's internal pressure changes as it is time dependent. Another option is to include each signal respective Y-axis in the graph, as there are 4 different signals, this would entail the insertion of 4 different Y-axes, which is not feasible. More importantly, these results served as evidence of the functionality of the ERD. On top of that, the results are also consistent with one another throughout the ERD's operating range. Figure 7.2 is an excellent representation of these results. In general, the pressure of HP feed is 5-15% lower than the pressure of HP brine due to hydraulic losses in the ERD, the pressure of both LP feed and LP brine hover constantly around the atmospheric pressure and lastly, pressure fluctuations can be observed in the ERD.

Figure D.46 Raw data for Table 7.1, no. 11 ($P = 11$ bar, $Q = 0.28$ m³/h)

Figure D.47 Raw data for Table 7.1, no. 8 ($P = 8.5$ bar, $Q = 0.42$ m³/h)

Figure D.48 Raw data for Table 7.1, no. 14 ($P = 13.0$ bar, $Q = 0.26$ m³/h)

Figure D.49 Raw data for Table 7.1, no. 1 ($P = 1$ bar, $Q = 0.54$ m³/h)

Figure D.50 Raw data for Table 7.1, no. 7 ($P = 7.5$ bar, $Q = 0.55$ m³/h)

Figure D.51 Raw data for Table 7.1, no. 4 ($P = 5.5$ bar, $Q = 0.56$ m³/h)

Figure D.52 Raw data for Table 7.1, no. 12 ($P = 11.5$ bar, $Q = 0.28$ m³/h)

Figure D.53 Raw data for Table 7.1, no. 15 ($P = 14.0$ bar, $Q = 0.27$ m³/h)

Figure D.54 Raw data for Table 7.1, no. 9 ($P = 10.5$ bar, $Q = 0.42$ m³/h)

Figure D.55 Raw data for Table 7.1, no. 18 ($P = 19.0$ bar, $Q = 0.13$ m³/h)

Figure D.56 Raw data for Table 7.1, no. 20 ($P = 20.0$ bar, $Q = 0.09$ m³/h)

Figure D.57 Raw data for Table 7.1, no. 19 ($P = 20.0$ bar, $Q = 0.10$ m³/h)

Appendix E – Original experimental setup

Table E.1 List of equipments

Item no.	Description
F1 – 2	Filters
FS1 – 4	Flow sensors
LS1	Level switch
OP1	Orifice plate
PT1 – 5	Pressure transmitters
PU1	Feed pump
PU2	Booster pump
PU3	Main pump
V1, V3, V4	On-off valve
V2	Float valve
V5 – 7	Pump's outlets check valve
V8 – 9	Control valves
WT1	Water reservoir

Figure E.1 Original experimental setup

The proposed ERD is incorporated in a conventional RO plant like other PD ERDs. The feed is divided into two parts. One part, equal in volume to the permeate, would be diverted to the main pump. The main pump would raise the pressure of this portion to the operating pressure. The remaining portion, equal in volume to the brine, will be directed to the ERD where hydraulic energy from the outgoing HP brine is transferred to the LP feed. After passing through the ERD, the HP feed would pass through a booster pump to compensate for hydraulic losses in the system. The two flows would merge before entering the RO membranes. Such a layout allows the RO plant the benefit of operating at any recovery it requires without affecting power consumption since the permeate flow and brine flow is unrelated.

Following the design and fabrication of the ERD, testing of the prototype must be carried out. A pilot-scale RO plant will be set-up for this purpose. The experimental setup is shown in figure E.1.

The pilot plant will be built in the Energy Conversion Lab in NTU with a permeate production rate of 3 m³/h. Seawater shall be substituted by PUB tap water. The impact of this action will be minimal since the focus of this research is the ERD, which is unlikely to be affected. The substitution is made also with a long-term view on the durability of individual components in the plant. It is anticipated that the plant will operate intermittently, the frequent shut-down and inactivity of the components will increase the likelihood of breakdown due to corrosion and biological growth. In addition, usage of seawater will require the provision of a flush-down system, which is omitted due to cost considerations. A vented water tank is required because of the high flow rate in the system. Water level in the tank is regulated by a float valve and a level switch. The float valve will ensure there is sufficient water in the tank while the level switch serves as redundancy for the system. In the event of a leak, the level switch will be turn on when the water level falls below a minimum level. All

the pumps and ERD will be automatically shut off to prevent dry-running and damages to the equipment.

Two polyamide cartridge filters with absolute rating of 10 μ m and 1 μ m are installed in series at the outlet of water tank. They protect the high-precision pumps and ERD from suspended solids found in the water and piping. They are chosen on the basis of high filtration efficiency, low energy consumption and a low weighted average filtration rate. To ensure optimum filtration capability, pressure gauges are mounted on the filters to monitor their back-pressure. The LP feed pump, PU1, handles 100% of the system's flow or about 5-6 m³/h. It is equipped with a VFD instead of a throttling valve to control its output and improve the efficiency of the plant. Gravity fed by the water tank, the pump will raise the pressure of the feed to approximately 3 bar, the minimum inlet pressure for the HP main pump, PU3. The VFD relies on closed-loop feedback from pressure transmitter, PT1 and flow sensors, FS1 and FS2 to maintain a constant output. PU3 manages only the permeate portion of the feed flow. Once turned on, it will maintain a constant output of 3 m³/h at pressures of 60 to 80 bar.

LP feed and brine flow entering and leaving the ERD is controlled by control valve, V8, while HP feed and brine flow is adjusted using VFD booster pump, PU2. PU2 makes up for the pressure losses incurred within the RO module, connecting pipes and the ERD. The sum of all these losses is approximately 2 bar. These two flows would be balanced using flow sensors, FS1 and FS3. Pressure transmitters, PT1 and PT2, are used to monitor the pressure difference between the ERD's LP inlet and outlet. Likewise, PT4 and PT5 are used to measure the pressure difference between the ERD's HP inlet and outlet.

An orifice plate, OP1, and control valve, V9, is used to simulate a RO module since it is not economical to include a genuine one for this experiment. To inject realism, V9 is used to alter the permeate flow rate to imitate the actual permeate flow rate at a particular operating

pressure. At low operating pressure, permeate production will decrease, similarly, when operating pressure is high, permeate production will increase. In RO modules, pressure drop occurs due to hydraulic losses and is an inevitable process. The pressure of the brine will always be 1.5-2 bar lower than the pressure of the feed. OP1 is used to induce such a pressure drop. The magnitude of this pressure difference can be measured with PT3 and PT5. All permeate and brine will be recycled and dumped back into the water tank.

Finally, the ERD will be run continuously at various recoveries ratio and the energy consumption of the entire plant monitored by watt meters (not shown) attached to the various pumps. The optimum operating conditions will be noted. All data will be collected by a digital data-logger.

Appendix F – Photos of the prototype ERD

Figure F.1 ERD kept tightly under wraps awaiting installation

Figure F.2 45° view of ERD

Figure F.3 Alternate view of ERD

Figure F.4 Photo showing rotor

Figure F.5 Installation phase

Figure F.6 Close-up of reservoir

Figure F.7 Automobile engine oil filters

Figure F.8 Close-up of HP pump and LP pump

Figure F.9 Close-up of 3 hp electric motor and variable frequency drive

Figure F.10 4-channels oscilloscope and power supply of pressure transducer

Figure F.11 Overhead view of experimental setup

Appendix G – Calculation of potential energy savings with proposed ERD

G.1 Background

The energy balance of the pressure exchange process is investigated in section 2.1.3 using one pair of pressure vessels. The investigation shows that approximately 50% of the HP brine's initial pressure can be recovered by the LP seawater.

Table G.1 Density of water

	Pressure (bar)					
	1	5	10	25	50	75
Density (kg/m ³) @ 20 °C	998.158	998.387	998.615	999.300	1000.437	1001.567
Specific volume (m ³ /kg) @ 20 °C	1.0018 x 10 ⁻³	1.0016 x 10 ⁻³	1.0013 x 10 ⁻³	1.0007 x 10 ⁻³	0.99956 x 10 ⁻³	0.9984 4 x 10 ⁻³

The total boundary work done by the HP brine onto the LP seawater is obtained by adding all the differential works from the initial state to the final state

$$W_b = m \int_a^b p dv \quad (G.1)$$

The generalized equation of state for liquids and solids at a constant temperature gives

$$p = -\frac{1}{\beta_T} \ln|\nu| + c_1 \quad (G.2)$$

C_1 can be attained by substituting boundary conditions into equation (G.2). When p is 1×10^5 Pa, ν is 1.0018×10^{-3} m³/kg. Hence $c_1 = -1.47878 \times 10^{10}$. Substituting equation G.2 into G.1 gives

$$W_b = m \int_a^b -\frac{1}{\beta_T} \ln|\nu| + c_1 dv \quad (G.3)$$

Integrating by parts yields

$$W_b = m \left\{ \nu \left[-\frac{1}{\beta_T} (\ln \nu - 1) + c_1 \right] \right\}_a^b \quad (G.4)$$

G.2 Sample calculations and results

Assumption: Properties of seawater is assumed to be the same as those for water.

Table G.2 Specification of problem

Initial pressure of seawater, p_i	1×10^5 Pa
Final pressure of seawater, p_{final}	75×10^5 Pa
Mass of seawater, m_{seawater}	998.204 kg
Density of water @ 1 bar & 20°C	998.204 kg/m^3

$$\begin{aligned}
 w_b &= \left\{ v \left[-\frac{1}{\beta_T} (\ln v - 1) + c_1 \right] \right\}_a^b \\
 &= \left\{ 0.99844 \times 10^{-3} \left[-\frac{1}{4.67 \times 10^{-10}} (\ln |0.99844 \times 10^{-3}| - 1) - 1.47878 \times 10^{10} \right] \right\} \\
 &\quad \left\{ -1.0018 \times 10^{-3} \left[-\frac{1}{4.67 \times 10^{-10}} (\ln |1.0018 \times 10^{-3}| - 1) - 1.47878 \times 10^{10} \right] \right\} \\
 &= -12.470 \text{ J/kg}
 \end{aligned}$$

Conversely, for 1 m^3 of seawater at 1 bar and 20°C, the work done will be

$$\begin{aligned}
 W_b &= m_{\text{seawater}} w_b \\
 &= 998.204 \times -11.798 \\
 &= -12448 \text{ J/m}^3 \\
 &= -12.448 \text{ kJ/m}^3
 \end{aligned}$$

The result obtained is negative which means that work is performed on the seawater by the HP brine. The remainder of the results with p_{final} ranging from 5 – 75 bar are shown in the diagram below (figure G.1).

Figure G.1 Work done per cubic meter of seawater

Appendix H – Comparison of Hauge’s rotary PD ERD and proposed ERD

H.1 Background

The current invention is based closely upon the rotary PD ERD patented by Leif J. Hauge [36] of Energy Recovery Inc. The device is called the PX which stands for pressure exchanger. A description of the PX along with its schematic representation can be found in the thesis section 1.2.2. The PX itself is based on another design which was patented by George Jendrassik in 1954. A cross-sectional view of the device can also be found in the same section.

H.2 Objectives

1. To compare and demonstrate the similarity between the PX and the current prototype so as to answer examiner 1 query whether “the HP seawater outlets from the endplates would be more apt to be the brine streams and the LP brine outlets to be the seawater streams”.
2. To illustrate the rational behind the analysis done in chapter 4 and how it relates to the actual prototype.

H.3 Comparison

PX

As is evident from figure H.1, the PX comprises a tubular, mainly cylindrical housing, which at each end has a circular flange with a number of through-going holes. Two substantially identical end pieces both being provided with a circular flange with a diameter and through-going holes corresponding to the flanges of the housing, are sealingly fastened to respective end portions of the housing, the flanges of the housing being fastened to the

Figure H.2 Sectional view taken along the line II-II in figure H.1

Figure H.3 View of an end piece

The mode of operation of the PX according to Hauge's invention will in the following description be described in detail with reference to figure H.4. For the recovery of the pressure energy of a first liquid, for instance waste liquid in a process, whereby this liquid shall be used for raising the pressure of another liquid which is used in connection with another process, a supply tube which carries the waste liquid is connected to the duct 2 of the

PX, and a tube for supply of the other liquid is connected to duct 5. Further, a discharge tube for the waste liquid is connected to the duct 3, and a discharge tube for the other liquid is connected to the duct 4. In the following description, liquid pressure will be designated p , and for designation of the liquid pressure in the respective ducts this designation letter will be given a suffix corresponding to the designation number of the duct.

By way of introduction it is assumed that $p_2 > p_3 > p_4 > p_5$. For the description of the mode of operation the liquid flow for a particular rotor bore will be described, assuming that the rotor is driven by a motor. Figures H.4a to H.4f show successive positions of this bore during rotation of the rotor. Figure H.4a depicts the rotor in a position where the bore in question has just been brought to communication with the duct 3 and 5. As $p_5 > p_3$ the displacement of the waste liquid which is contained in the bore is thereby started. When the rotor has passed the position shown in figure H.4b and reached the position shown in figure H.4c, in which the bore is being closed by the partition walls, approximately all waste liquid has been displaced from the bore, and this has been filled with the other liquid. As the rotor reaches the position shown in figure H.4d, whereby the bore is opened for communication with the ducts 2 and 4, the pressure of the liquid is immediately raised to a level between the pressures p_2 and p_4 , and the high pressure p_2 of the waste liquid will cause initiation of a liquid flow into the duct 2 and displace the other liquid. So that this flows out of the duct 4. The pressure of the liquid in the duct 4 may hereby be controlled by means of a control valve (not shown) or a similar device.

When the rotor has passed the position shown in figure H.4e and reached the position shown in figure H.4f, where the bore again is being closed by the partition walls, approximately all the other liquid in the bore has been displaced by the waste liquid. When the rotor during continued rotation again reaches the position shown in figure H.4a, where the

duct is opened for communication with the duct 3 and 5, the above described cycle is started over again.

Figure a

Figure d

Figure b

Figure e

Figure c

Figure f

Figure H.4a – H.4f Sectional view depicting the mode of operation of the PX

Prototype

An exploded view of the current prototype is shown in figure H.5. The device is symmetrical about the plane shown in the same figure. If you split the prototype along its plane of symmetry and seal up the through-holes on the centerplate, you will obtain a device very similar to the PX.

Figure H.5 Exploded view of the prototype

For the pressure recovery of a first liquid – such as HP brine from the reverse osmosis modules – it will be channeled into duct 2 while a second liquid of lower pressure – such as LP seawater – will be channeled into duct 5. Further, a discharge pipe for the first liquid is connected to duct 4 while another discharge pipe for the second liquid is connected to duct 3. Similarly, the first liquid can also be channeled into duct 5 and the second liquid channeled into duct 2 instead with no detrimental effects as the device is symmetrical about its longitudinal plane. In fact, ducts 3 and 4 can also be used as inlets and ducts 2 and 5 as outlets. In the thesis, for example, the 1st liquid is channeled into duct 5 while the 2nd liquid is channeled into duct 2.

Once again, it is assumed that for the prototype $p_2 > p_3 > p_4 > p_5$. For the description of the mode of operation the liquid flow for just one rotor bore will be described, assuming that the rotor is driven by a motor. Figures H.6a to H.6d show successive positions of this bore during rotation of the rotor. Figure H.6a depicts the rotor in a position such that the bore is aligned with ducts 2 and 4. Assuming the bore is already filled with LP seawater, it will be displaced by the HP brine entering through duct 2 since $p_2 > p_4$. With further rotation, the rotor will reach a new position shown in figure H.6b in which the bore will be sealed off from all liquids entering the device. At the same time, the bore will be completely filled with HP brine. When the rotor reaches the position shown in figure H.6c, the bore will be aligned with ducts 3 and 5. When the HP brine is exposed to the LP seawater inlet, its pressure will plunge instantly to between p_3 and p_5 and the LP seawater will flow into the bore and displace the brine. Further rotation will bring the rotor to the next position shown in figure H.6d where the bore is again sealed off from any liquid entering the device. At the same time, the bore will be completely filled with LP seawater. Further rotation will return the rotor to its initial position for the next cycle. For the complete prototype, the above mentioned activity takes place separately in each half of the prototype while the through-holes in the centerplate allow pressure interactions to occur between the bores of the two rotors.

Figure H.6a – H.6d Sectional view depicting the mode of operation of the prototype

From the above description of both devices, it can be seen that the HP seawater outlets are not brine outlets and the LP brine outlets are not seawater outlets.

It is also obvious that they are similar in their design and their mode of operation. In fact, the current prototype can be regarded as two PX devices joined end-to-end together. The main difference with the current prototype lies in its use of through-holes in the centerplate to facilitate pressure transfer from one rotor to another.

More importantly, Hauge has determined the leakage from the PX to be less than 5% of the feed flow. If the PX can achieve a seal like this with a smaller sealing surface, there is no reason why the current prototype cannot.

The author investigated the leakage from the current prototype primarily to determine the optimum gap size between the rotors and the endplates. In order to minimize leakage, the most logical thing to do would be to reduce the gap size. However, deviations in the shapes and sizes of work pieces resulting from inaccuracy in machining force the author to

incorporate positive allowance into the rotating parts which have the effect of increasing the gap size. As Hauge's patent does not provide information on the optimum gap size, merely stating that "it (the sealing surface) forms a shoulder or gliding surface for the rotor which substantially prevents movement of the rotor in the longitudinal direction of the housing, while rotation is permitted and whereby a better sealing between the rotor and the housing" and not to unnecessarily impose very small dimensional and geometrical tolerances, a investigation on the leakage for engineering purpose, have to be carried out.

The leakage problem in the prototype can be divided into two problems as stated in the thesis. The first is leakage of the inlet of the HP brine and the second is leakage from the ducts in the rotors. Both leakages are unsteady flow between two rotating parallel (supposedly) plates.

In the first problem, leakage is affected by several factors including: 1) position of the rotor as the rotor is rotating and the rotor ducts are coming into alignment and out of alignment with the inlet all the time; 2) leakage from the ducts in the rotors since some of the ducts are highly pressurized; 3) rotation of the rotor as it drags across the inlet; and 4) symmetry of the problem as the brine inlet is high pressure while the seawater inlet is low pressure.

In the second problem, leakage is also affect by a multitude of factors: 1) leakage from other ducts; 2) pressure waves resulting from the pressure transfer process; 3) rotation of the rotor and 4) liquid flow from the inlet ducts to the outlet ducts.

The problem in both cases cannot be considered insignificant. Furthermore, the real intention behind the investigation is to determine a reasonable gap size so that leakage can be minimized without the need to specify for an unrealistic tolerance grade. To achieve the objective, the author simplified both problems by treating them as leakage flow from a hollow cylinder so that the Stokes equation subject to a thin-film approximation can be used.

Also, the author confined the sealing area – which is shown in the thesis – to less than a tenth of the entire surface of the endplate and rotor in order to produce a more conservative estimate. The author subsequently constructed an experiment in section 4.8 to verify the theoretical results obtained for the second leakage problem.

The author has also considered investigating the leakage of each opening empirically since a physical prototype is available. For the first problem, the leakage will exit through the LP brine outlet. Therefore, it is possible to estimate its amount by comparing the amount of liquid that comes out from the LP brine outlet and the HP seawater outlet. However, precision flow measuring devices are needed since the difference is small and the project does not have any. Only the collective leakage from the ERD can be obtained and can be found in table 7.2. For the second problem, leakage from the ducts can be estimated by measuring the pressure inside a particular duct during operation. However, measuring the pressure inside the rotating device requires a special pressure transducer which cost more than US\$10,000. Thus, no experiment was carried out.

Based on the theoretical analysis, the author specified a minimum clearance of 14 μm and a maximum clearance of 79 μm for the rotor with a parallelism tolerance of 5 μm for the face of the rotor.

Appendix J – Comparison between ETE and proposed indicator $\frac{P_{out}}{P_{in}} \times 100\%$

J.1 Background

For comparing the performance of different ERDs, the most commonly used indicator is the ETE. The definition and mathematical formula for ETE can be found in the thesis section 2.3 and 7.4. To determine ETE, two sets of data must be provided; they are 1) pressure at the inlets and outlets of the ERD and 2) all accompanying flow rates. As explained in the thesis, flow rates of the liquid entering the prototype are unavailable as there is no way of measuring them without a flow meter and the project does not have any. Therefore, there is a need to substitute ETE with another performance gauge. The author proposed using

$$\frac{P_{out}}{P_{in}} \times 100\% \quad (J.1)$$

J.2 Objectives

Compare ETE and proposed indicator with established data and published figures.

J.3 Comparison

Flow balance in a rotary ERD is upset by leakages. The main source of leakage comes from the HP brine inlet and the HP seawater outlet due to the presence of small gaps between the rotor and the endplates to facilitate rotation. Very little or no leakage is expected from the LP seawater inlet and LP brine outlet as their pressure is lower than the other two openings.

Suppose a plant operator begins using the ERD by injecting equal amounts of liquid into the ERD, the HP outlet will yield less output than the LP outlet due to leakages. In order to balance both outputs, the operator must increase HP brine by the amount leaked. This way, balance between both outputs can be restored. Flow equations and a flow diagram illustrating this point can be found in the thesis section 2.3.

Appendix J – Comparison between ETE and proposed indicator
Sallangos [24] obtained the following data from Dhekelia desalination plant.

Figure J.1 Experimental data from Dhekelia desalination plant

Therefore,

$$\begin{aligned} p_{HPbrine} &= 73.0 \text{ bar} \\ p_{LPbrine} &= 2.0 \text{ bar} \\ p_{HPfeed} &= 72.0 \text{ bar} \\ p_{LPfeed} &= 3.2 \text{ bar} \\ Q_{HPbrine} &= 487.2 \text{ m}^3/\text{h} \\ Q_{LPbrine} &= 487.2 \text{ m}^3/\text{h} \\ Q_{HPfeed} &= 482.3 \text{ m}^3/\text{h} \\ Q_{LPfeed} &= 482.3 \text{ m}^3/\text{h} \end{aligned}$$

Using these figures, ETE will be 94.0% and

$$\frac{p_{out}}{p_{in}} \times 100\% = \frac{72.0 + 2.0}{73.0 + 3.2} \times 100\% = 97.1\%$$

Although similar in value, it is easy to see that both are entirely different since Q can be different in separate cases. However, bear in mind that leakage flow in a rotary ERD is not more than 5% of the feed flow in normal circumstances according to Hauge [30]. Given more

data, it is possible to show that $\frac{p_{out}}{p_{in}} \times 100\%$ will be similar to ETE.

Appendix K – Bending in the circular endplate

Theoretical analysis

Figure 2.5 shows an exploded view of the proposed ERD, the endplate is clearly labelled. During operation, the rotors spin at a high speed within the bearing while the endplate provides axial location and pressure sealing. This is achieved by ensuring there is a small clearance between the rotor and endplate. This clearance must be carefully controlled since an overly generous gap will prevent effective sealing and an unnecessarily narrow one will cause the rotor to seize during operation if particles larger than the gap enter the ERD. Although the clearance has been designed to be 10~20 μm , deformation of the endplate due to the pressure within the ERD may result in a bigger clearance. Thus, it is imperative that this deformation is considered. The ultimate aim of estimating the deformation is to determine the appropriate thickness of the endplate. Figure K.1 shows a sketch of the endplate in 2D view.

Figure K.1 Endplate

The pressure region is also the circular path of the rotor ducts. In previous sections, it was mentioned that the pressure within the ducts is progressively raised, so in actual fact, the pressure exerted is not uniform. However, to simplify the problem and utilize solutions readily available in reference books [32], the problem is assumed to be a plate of radius r subjected to uniform pressure. This assumption is valid since a wholly loaded plate will deform more than one that is partially loaded. Figure K.2 shows an element taken from a

circular plate. It is in equilibrium under the action of uniformly distributed loading p per unit area and the resulting shear forces Q per unit length and bending moments M per unit length. Owing to symmetry, there are no shear forces on the radial sides of the element. For vertical equilibrium,

$$Qrd\theta + prdrd\theta - (Q + \frac{dQ}{dr}dr)(r + dr)d\theta = 0 \quad (\text{K.1})$$

Figure K.2 Elemental volume in a circular plate

From which

$$\frac{dQ}{dr} + \frac{Q}{r} = p \quad (\text{K.2})$$

For moment equilibrium,

$$(M_r + \frac{dM_r}{dr}dr)(r + dr)d\theta - M_r rd\theta - 2M_\theta dr \sin \frac{d\theta}{2} + Qrd\theta dr = 0 \quad (\text{K.3})$$

where the moment can be expressed in terms of slope ψ ,

$$M_r = \frac{Eh^3}{12(1-\nu^2)} \left(\frac{d\psi}{dr} + \nu \frac{\psi}{r} \right)$$

$$M_\theta = \frac{Eh^3}{12(1-\nu^2)} \left(\nu \frac{d\psi}{dr} + \frac{\psi}{r} \right) \quad (\text{K.4})$$

or in terms of deflection,

$$\begin{aligned}
 M_r &= -D \left(\frac{d^2 w}{dr^2} + \frac{v}{r} \frac{dw}{dr} \right) = D \left(\frac{1}{R_r} + \frac{v}{R_\theta} \right) \\
 M_\theta &= -D \left(\frac{1}{r} \frac{dw}{dr} + v \frac{d^2 w}{dr^2} \right) = D \left(\frac{1}{R_\theta} + \frac{v}{R_r} \right)
 \end{aligned}
 \tag{K.5}$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{K.6}$$

is termed the flexural rigidity. Equation (K.3) can be further simplified to

$$r \frac{dM_r}{dr} + M_r - M_\theta + Qr = 0 \tag{K.7}$$

If equation (K.4) is substituted into equation (K.7) the following equation is obtained which relates the slope at any radius to the shear force.

$$\frac{d^2 \psi}{dr^2} + \frac{1}{r} \frac{d\psi}{dr} - \frac{\psi}{r^2} = -\frac{Q}{D} \tag{K.8}$$

Similarly, if equation (K.5) is substituted into equation (K.7), an expression for the variation of deflection, w with radius is formed.

$$\frac{d^3 w}{dr^3} + \frac{1}{r} \frac{d^2 w}{dr^2} - \frac{1}{r^2} \frac{dw}{dr} = \frac{Q}{D} \tag{K.9}$$

Terms on the LHS of equation (K.8) and (K.9) can be combined to give

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (r\psi) \right] = -\frac{Q}{D} \tag{K.10}$$

and

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{dw}{dr} \right) \right] = \frac{Q}{D} \tag{K.11}$$

Equation (K.2) relates the shear force Q to the applied pressure p and is integrated to provide

$$Qr = \int_0^r p r dr \tag{K.12}$$

Substituting into equation (K.10)

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{dw}{dr} \right) \right] = \frac{1}{D} \int_0^r p r dr \quad (\text{K.13})$$

The above equation can be integrated to find the deflection at any radius if p is known. The constant of the integration are then evaluated from the boundary conditions for the particular problem being solved. For the endplate, the RHS of equation (K.13) reduces to $pr^2/2$, since p is constant; therefore

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{dw}{dr} \right) \right] = \frac{pr}{2D} \quad (\text{K.14})$$

Integrating,

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dw}{dr} \right) = \frac{pr^2}{4D} + C_1 \quad (\text{K.15})$$

Multiplying both sides by r and integrating again,

$$\begin{aligned} r \frac{dw}{dr} &= \frac{pr^4}{16D} + \frac{C_1 r^2}{2} + C_2 \\ \frac{dw}{dr} &= \frac{pr^3}{16D} + \frac{C_1 r}{2} + \frac{C_2}{r} \end{aligned} \quad (\text{K.16})$$

Finally,

$$w = \frac{pr^4}{64D} + \frac{C_1 r^2}{4} + C_2 \ln r + C_3 \quad (\text{K.17})$$

Since the endplate is mounted rigidly on the ERD, it can be considered a clamped periphery. The boundary conditions are $dw/dr=0$ at $r=0$ and $r=a$, and $w=0$ at $r=a$. (Figure K.3)

Figure K.3 Clamped periphery

Hence

$$C_2=0 \text{ and } \frac{pa^4}{16D} + \frac{C_1a^2}{4} = 0 \text{ so that } C_1 = -\frac{pa^2}{8D}$$

From equation (K.17),

$$0 = \frac{pa^4}{64D} + \frac{pa^4}{32D} + C_3$$

$$C_3 = -\frac{pa^4}{64D}$$

The deflection curve for the endplate is thus

$$w = \frac{P}{64D}(a^2 - r^2)^2 \quad (\text{K.18})$$

And the maximum deflection occurs at the centre and is given as

$$w = \frac{P}{64D}a^4 \quad (\text{K.19})$$

The author is acutely aware that the effect that shearing stresses and normal pressures on planes parallel to the surface of the plate has not been taken into account. Hence, the formulas obtained are only approximate, and their accuracy depends on the ratio of the thickness of the plate to its outer radius. The effect of this ratio can be found in most mechanics of materials textbooks (see, e.g., Timoshenko, 1959). More accurate formulas may be obtained by considering in an approximate manner the effect of shearing stresses and lateral pressures on deflections.

For a circular plate without a hole, supported along its edge and uniformly loaded. The shearing force Q per unit length of arc along a circle of radius r and load q is

$$Q = \frac{1}{2}qr \quad (\text{K.20})$$

For plates whose thickness is not assumed to be small, Timoshenko [39] proposed that the shearing stresses τ_{rz} vary across the thickness of the plate according to the parabolic law in the same way as in beams of narrow rectangular cross section. Hence the maximum shearing stress is at the middle surface of the plate and its magnitude is

$$(\tau_{rz})_{\max} = \frac{3qr}{4h} \quad (\text{K.21})$$

the corresponding shearing strain is

$$\frac{dw_1}{dr} = -\frac{3qr}{4Gh} \quad (\text{K.22})$$

where w_1 is the additional deflection of the middle surface of the plate due to the shearing stress. By integrating the deflections produced, the shearing stresses are found to be

$$w_1 = \frac{3q}{8Gh}(r_o^2 - r^2) \quad (\text{K.23})$$

When this deflection is added to equation K.18, a more accurate value of the deflection can be obtained. Interestingly, this is also the exact solution.

$$w = \frac{q}{64D} \left[(r_o^2 - r^2)^2 + \frac{4h^2}{1-\nu} (r_o^2 - r^2) \right] \quad (\text{K.24})$$

Appendix L – Order-of-magnitude analysis

L.1 Comparison of inertia terms and viscous terms

To determine the relative importance of the different terms in equation (4.5), an order-of-magnitude analysis is performed. Table L.1 provides some physical properties and dimensions of the fluid film.

$$v_r \sim U; r \sim L; z \sim \delta; p \sim P_{ref}$$

Where $\sim$ should be read as “order of”.

Table L.1 Properties of water and fluid film

L	0.0075 m
δ/L	10^{-3}
U	10 m/s
P_{ref}	8 MPa
ρ	1000 kg/m ³
μ	1.140×10^{-3} Ns/m ²

The viscous forces contain these terms:

$$\mu \frac{\partial^2 v_r}{\partial r^2}, \mu \frac{1}{r} \frac{\partial v_r}{\partial r}, \mu \frac{\partial^2 v_r}{\partial z^2}, \mu \frac{v_r}{r^2}$$

$\mu \frac{\partial^2 v_r}{\partial r^2}$ and $\mu \frac{\partial^2 v_r}{\partial z^2}$ are compared by evaluating their ratio:

$$\frac{\mu \frac{\partial^2 v_r}{\partial r^2}}{\mu \frac{\partial^2 v_r}{\partial z^2}} = \frac{\frac{\partial}{\partial r} \frac{\partial v_r}{\partial r}}{\frac{\partial}{\partial z} \frac{\partial v_r}{\partial z}} \sim \frac{\frac{1}{L} \frac{U}{L}}{\frac{1}{\delta} \frac{U}{\delta}} = \frac{\frac{U}{L^2}}{\frac{U}{\delta^2}} = \frac{\delta^2}{L^2} = \left(\frac{\delta}{L}\right)^2 \quad (\text{L.1})$$

Since $\frac{\delta}{L}$ is approximately 1×10^{-3} , $\mu \frac{\partial^2 v_r}{\partial r^2} \gg \mu \frac{\partial^2 v_r}{\partial z^2}$. Next, $\mu \frac{1}{r} \frac{\partial v_r}{\partial r}$ and $\mu \frac{\partial^2 v_r}{\partial z^2}$ are

compared.

$$\frac{\mu \frac{1}{r} \frac{\partial v_r}{\partial r}}{\mu \frac{\partial^2 v_r}{\partial z^2}} = \frac{\frac{1}{r} \frac{\partial v_r}{\partial r}}{\frac{\partial}{\partial z} \frac{\partial v_r}{\partial z}} \sim \frac{\frac{1}{L} \frac{U}{L}}{\frac{1}{\delta} \frac{U}{\delta}} = \frac{\frac{U}{L^2}}{\frac{U}{\delta^2}} = \frac{\delta^2}{L^2} = \left(\frac{\delta}{L}\right)^2 \quad (\text{L.2})$$

Similarly, $\mu \frac{\partial^2 v_r}{\partial z^2}$ is the dominant term. Finally, $\mu \frac{\partial^2 v_r}{\partial z^2}$ and $\mu \frac{v_r}{r^2}$ are evaluated.

$$\frac{\mu \frac{v_r}{r^2}}{\mu \frac{\partial^2 v_r}{\partial z^2}} = \frac{\frac{v_r}{r^2}}{\frac{\partial}{\partial z} \frac{\partial v_r}{\partial z}} \sim \frac{\frac{U}{L^2}}{\frac{1}{\delta} \frac{U}{\delta}} = \frac{\frac{U}{L^2}}{\frac{U}{\delta^2}} = \frac{\delta^2}{L^2} = \left(\frac{\delta}{L}\right)^2 \quad (\text{L.3})$$

From the comparison of the viscous terms, $\mu \frac{\partial^2 v_r}{\partial z^2}$ is the most dominant term. This is an important finding which is generally true in most thin-film problems: the viscous forces with gradient across the film gap are the dominant terms. Viscous forces and inertia forces are now compared.

$$\frac{\rho v_r \frac{\partial v_r}{\partial r}}{\mu \frac{\partial^2 v_r}{\partial z^2}} \sim \frac{\rho U \left(\frac{U}{L}\right)}{\mu \left(\frac{U}{\delta^2}\right)} = \frac{\rho U L}{\mu} \left(\frac{\delta}{L}\right)^2 = \text{Re} \left(\frac{\delta}{L}\right)^2 = \text{Re}_m \quad (\text{L.4})$$

Where Re_m is a modified Reynolds number. Once again, $(\delta/L)^2$ - a quantity of order 10^{-6} - shows up as a multiplier. Clearly, unless Re becomes very large, the viscous forces tend to dominate the inertia forces. To illustrate, Re_m shall be calculated using the values from Table L.1.

$$\text{Re}_m = \frac{\rho U L}{\mu} \left(\frac{\delta}{L}\right)^2 = \frac{1000(\text{kg}/\text{m}^3)10(\text{m}/\text{s})0.0075(\text{m})}{1.14(\text{Ns}/\text{m}^2)} (10^{-3})^2 \cong 0.0065$$

Therefore, the inertia forces are negligible compared to the viscous forces unless Re exceeds the 1000~2000 range, with inertia effects coming into predominance.

L.2 Contribution of gravity

Next, the role of gravity in a thin-film problem is assessed.

$$\gamma_r = \rho g_r \quad (\text{L.5})$$

If the parallel plates are horizontal, $g_r = g$ (acceleration due to gravity). Comparing,

$$\frac{\rho v_r \frac{\partial v_r}{\partial r}}{\rho g} \sim \frac{U(\frac{U}{L})}{g} = \frac{U(U/L)}{g} = \frac{U^2}{Lg} = Fr \quad (\text{L.6})$$

where Fr is a dimensionless quantity referred to as Froude number. Evaluating Fr using values taken from Table A11.1 yields

$$Fr = \frac{U^2}{Lg} = \frac{10^2}{(0.0075)(9.8)} \cong 1360$$

This implies that body forces due to gravity are much smaller than inertia forces and therefore are negligible compared to viscous forces.

L.3 Contribution of the pressure term

Here the effects of the inertia forces and pressure are compared.

$$\frac{\frac{\partial p}{\partial r}}{\rho v_r \frac{\partial v_r}{\partial r}} \sim \frac{(\frac{P_{ref}}{L})}{\rho(\frac{U^2}{L})} = \frac{P_{ref}}{\rho U^2} = Eu \quad (\text{L.7})$$

Where Eu is a dimensionless parameter known as the Euler number. If the average pressure inside the rotor duct is in the order of 5MPa, then

$$Eu = \frac{5 \times 10^6 (N/m^2)}{(1000)(10)^2} \cong 50$$

This implies that the contribution of pressure is much greater than the inertia effect and must be considered in conjunction with viscous forces.

L.4 Comparison of pressure and viscous forces

Finally, it is useful to compare viscous forces with pressure:

$$\frac{\mu \frac{\partial^2 v_r}{\partial z^2}}{\frac{\partial p}{\partial r}} \sim \frac{\mu(\frac{U}{\delta^2})}{\frac{P_{ref}}{L}} = \frac{\mu UL}{P_{ref} \delta^2} = \Lambda_B \quad (\text{L.8})$$

where Λ_B is referred to the bearing number. Using the same typical numerical values, the bearing number is:

$$\Lambda_B = \frac{(1.14 \times 10^{-3})(10)(0.0075)}{5 \times 10^6 (0.01 \times 10^{-3})^2} \approx 0.171$$

Hence pressure and viscous forces are both important in a thin-film problem. The above analysis indicates that the dominant terms in the r -component of the Navier-Stokes equation are:

$$\frac{\partial p}{\partial r} = \mu \frac{\partial^2 v_r}{\partial z^2} \quad (\text{L.9})$$

Which is referred to as the *Navier-Stokes equation subject to a thin-film approximation* [38].