Numerical simulation and topology optimization of slider air bearing

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**NUMERICAL SIMULATION AND TOPOLOGY
OPTIMIZATION OF SLIDER AIR BEARING**

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A dissertation submitted to Nanyang Technological University
in fulfillment of the requirements for degree of Doctor of Philosophy

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Abstract

Since the first commercial hard disk drive was developed in 1956, the stability of the flying attitude is always an important factor to be considered in the slider air bearing design. The objective of this dissertation is to develop an efficient computational modeling technique for simulating the air bearing slider (ABS) in hard disk drives and ultimately develop an effective topology optimization methodology to systematically and automatically design the air bearing slider with a target of achieving a near-constant flying attitude when the slider moves over the surface of the hard disk drive. Although sizing and shape optimization approaches have been explored for optimal design of the slider air bearing, those previous works were mainly restricted to variation of simple geometric parameters. This dissertation is attempted to propose a true and complete topology optimization approach for the slider air bearing design for potential practical applications.

As the foundation of the air bearing slider, the modified Reynolds equation for Fukui slip model is derived from the momentum equation, the continuity equation and the boundary conditions. To solve the partial differential Reynolds equations, the Least Square Finite Difference (LSFD) algorithm is presented to numerically discretize the differential equations for Fukui slip model. Two iterative algorithms, Gauss-Seidel SOR and Newton-SOR iterative methods, are employed to solve the numerically discretized equations and are compared with one another by simulating a Plate Slider.

To further increase the computational efficiency, the full approximation multigrid algorithm is subsequently applied to solve the numerically discretized Reynolds equations from the LSFD method. The Least Square Finite Difference Multigrid algorithm makes use of the three-level v-cycle strategy calculated from the finest grid. The numerical simulations of the Infinitely-Long Plane-Wedge Slider, the Plane Slider, the Two-Rail Slider and the Advanced Step Slider are carried out using this procedure. By comparing the results obtained with those from the published papers or from the finite volume method, it is proven that the Least Square Finite Difference Multigrid algorithm is accurate and efficient in the numerical simulation of the slider air bearing.

As a necessary step to complete the process for topology optimization of slider air bearing, the flying attitude determination of the slider air bearing is carried out first. The optimization algorithm is required for both the flying attitude determination and the topology optimization later. From the flying attitude determination of the Plane Slider and the Two-Rail Slider, it verified the effectiveness of the procedure for determining the flying attitude of the slider air bearing.

Finally, a simple parametric optimization for the flying attitude of the slider air bearing is studied to verify the theoretical assumption that the flying attitudes of the slider over the whole disk surface are near-constant as long as the flying attitudes of the slider at the most outer track and at the most inner track are near-constant. Based on this theoretical assumption, the topology optimization of the slider air bearing is implemented and carried out. A multi-objective function is proposed to simultaneously minimize the air bearing force difference, the air bearing X-moment

difference and the air bearing Y-moment difference of the slider at the most outer track and at the most inner track of the disk surface. To realize the ultimate topology optimization, the slider height at every grid point in the meshed geometry domain is set as a design variable. With the use of multigrid algorithm in calculating the first-order sensitivities of the multi-objective function with respect to the design variables, an efficient topology optimization methodology to systematically and automatically design the slider air bearing is developed for achieving a near-constant flying attitude when the slider moves over the whole disk surface. Applying the topology optimization methodology developed, a novel optimal slider is designed. From the flying attitude determinations along different tracks of the disk surface, it is proven that the novel slider is able to achieve a near-constant flying attitude when it moves along different tracks of the disk surface.

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List of Abbreviations

ABS	Air Bearing Surface
CML	Computer Mechanics Laboratory
CAD	Computer Aided Design
CAE	Computer Aided Engineering
DEM	Diffuse Element Method
DSMC	Direct Simulation Monte Carlo
EFG	Element Free Galerkin method
FAM	Full Approximation Multigrid
FAS	Full Approximation Scheme
FDM	Finite Difference Method
FEM	Finite Element Method
FVM	Finite Volume Method
GFD	General Finite Difference method
HDD	Hard Disk Drive
HDI	Head-Disk Interface
LSFD	Least Square Finite Difference method
LSFDM	Least Square Finite Difference Multigrid method
MLPG	Meshless Local Petro-Galerkin method
MLS	Moving Least Square method
MMA	Method of Moving Asymptotes
PIM	Point Interpolation Method
RKP	Reproducing Kernel Particle method

SAB	Slider Air Bearing
SCP	Sequential Convex Programming
SLP	Sequential Linear Programming
SOR	Successive Over Relaxation
SPH	Smoothed Particle Hydrodynamics method
SQP	Sequential Quadratic Programming
SUPG	Streamline Upwind Petrov-Galerkin method

List of Symbols

Chapter 3

K_n	Knudsen number
λ	Mean free path
h_m	Minimum flying height
h	Flying height
x	X-coordinate
y	Y-coordinate
z	Z-coordinate
H	Normalized flying height
X	Normalized X-coordinate
Y	Normalized Y-coordinate
Z	Normalized Z-coordinate
u	Airflow velocity in X-direction
v	Airflow velocity in Y-direction
U	Constant airflow velocity in X-direction
V	Constant airflow velocity in Y-direction
p	Pressure
P	Normalized pressure
p_a	Ambient pressure
ρ	Airflow density
μ	Dynamic viscosity

Λ_x	Air bearing number in X-direction
Λ_y	Air bearing number in Y-direction
σ	Reflection coefficient
Q	Flow factor
Q_p	Poiseuille flow rate coefficient
Q_{con}	Continuum flow rate coefficient
D_0	Characteristic inverse Knudsen number
K_{1-4}	Corresponding coefficients
L	Length of slider
W	Width of slider
α	Pitch angle
θ	Roll angle
φ	Sway angle
i	Sequence number of reference point in X-direction
j	Sequence number of reference point in Y-direction
X_{ij}	X-coordinate of i, j reference point
Y_{ij}	Y-coordinate of i, j reference point
H_{ij}	Height of i, j reference point
F_{ij}	Residual of i, j reference point
R_{ij}	Residual of i, j reference point
P_{ij}	Pressure of i, j reference point
P_{ij}^n	Pressure of nth supporting point of i, j reference point
d_0	Maximum distance between supporting points and i, j reference point
SN	Maximum number of supporting points
a_{ij}^n	Coefficients decided by grid points

b_{ij}^n	Coefficients decided by grid points
c_{ij}^n	Coefficients decided by grid points
d_{ij}^n	Coefficients decided by grid points
ω	Successive over relaxation factor

Chapter 4

Ω^h	First-level computation domain
Ω^{2h}	Second-level computation domain
Ω^{4h}	Third-level computation domain
A	Parametric matrix
E	Error approximation matrix
U	Solution matrix
V	Variable approximation matrix
F	Constant matrix
R	Residual matrix
$A^{2^{(n-1)}h}$	Parametric matrix
$E^{2^{(n-1)}h}$	Error approximation matrix in nth-level computation domain
$U^{2^{(n-1)}h}$	Solution matrix in nth-level computation domain
$V^{2^{(n-1)}h}$	Variable approximation matrix in nth-level computation domain
$F^{2^{(n-1)}h}$	Constant matrix in nth-level computation domain
$R^{2^{(n-1)}h}$	Residual matrix in nth-level computation domain
$I_{2^{(n-1)}h}^{2^nh}$	Restriction operator from nth-level to (n+1)th-level computation domain
$I_{2^nh}^{2^{(n-1)}h}$	Prolongation operator from (n+1)th-level to nth-level computation domain

Chapter 5

h_l	Leading edge height of slider
h_t	Trailing edge height of slider
$\bar{h}$	Film thickness ratio
W_{Ra}	Rail width of slider
H_{Ra}	Rail height of slider
W_{Pr}	Wall profile width of slider
L_{Tan}	Nth taper length of slider
W_{Tan}	Nth taper width of slider
ω	Disk rotating speed
R_d	Disk rotating radius

Chapter 6

$\mathbf{x}$	Vector of optimization variables
$f_0(\mathbf{x})$	Objective function
$f_i(\mathbf{x})$	Constraints
$f_i^{\max}$	Upper bounds on constraints
$x_j^{\min}$	Lower bounds on optimization variables
$x_j^{\max}$	Upper bounds on optimization variables
$L_j^{(k)}$	Lower asymptote in kth iteration
$U_j^{(k)}$	Upper asymptote in kth iteration
$L(\mathbf{x}, \mathbf{y})$	Lagrangian function
y_i	Lagrange multiplier

F	Calculated air bearing force
M_x	Calculated air bearing X-moment
M_y	Calculated air bearing Y-moment
F_0	Applied pre-load force
L_{x0}	Applied pre-load X-moment arm
L_{y0}	Applied pre-load Y-moment arm
M_{x0}	Applied pre-load X-moment
M_{y0}	Applied pre-load Y-moment
O	Coordinate origin
ΔF	Force difference
ΔM_x	X-moment difference
ΔM_y	Y-moment difference
ψ_1	Multi-objective function for attitude determination of slider air bearing
ΔF^2	Sub-objective function for force difference ΔF
ΔM_x^2	Sub-objective function for X-moment difference ΔM_x
ΔM_y^2	Sub-objective function for Y-moment difference ΔM_y
F_U	Calculated air bearing force at airflow velocity U
M_{xU}	Calculated air bearing X-moment at airflow velocity U
M_{yU}	Calculated air bearing Y-moment at airflow velocity U
w_{1-3}	Weighting factors
m_{1-3}	Magnification coefficients
$h_{\min}$	Minimum constraint of minimum flying height h_m
$h_{\max}$	Maximum constraint of minimum flying height h_m
$\alpha_{\min}$	Minimum constraint of pitch angle α
$\alpha_{\max}$	Maximum constraint of pitch angle α

$\theta_{\min}$	Minimum constraint of roll angle θ
$\theta_{\max}$	Maximum constraint of roll angle θ
v_i	Design variables h_m, α, θ
E_F	Calculated force error
E_{M_x}	Calculated X-moment error
E_{M_y}	Calculated Y-moment error

Chapter 7

R_{out}	Radius of most outer track of disk surface
R_{inn}	Radius of most inner track of disk surface
ψ_2	Multi-objective function for attitude optimization of slider air bearing
F_{out}	Calculated air bearing force in most outer track of disk surface
M_{xout}	Calculated air bearing X-moment in most outer track of hard disk
M_{yout}	Calculated air bearing Y-moment in most outer track of disk surface
F_{inn}	Calculated air bearing force in most inner track of disk surface
M_{xinn}	Calculated air bearing X-moment in most inner track of disk surface
M_{yinn}	Calculated air bearing Y-moment in most inner track of disk surface
ψ_3	Multi-objective function for topology optimization of slider air bearing
h_{ij}	Flying height of i, j reference point
$(h_{ij})_{\min}$	Minimum constraint of flying height h_{ij}
$(h_{ij})_{\max}$	Maximum constraint of flying height h_{ij}
Δh_m	Variation of minimum flying height h_m
$\Delta \alpha$	Variation of pitch angle α
$\Delta \theta$	Variation of roll angle θ

List of Publications

Hejun Du, Qiang Li, Jor Huat Ong and Chang Shu, 2005, “A Newton-Multigrid Method for Numerical Simulation of Slider Air Bearing”, *Modern Physics Letters B*, Vol. 19, pp. 1647-1650.

Chapter 1

Introduction

1.1 Background

Since the first commercial hard disk drive, IBM product RAMAC (Random Access Method of Accounting and Control), was introduced in 1956, the hard disk storage has become the important member of the storage hierarchy due to its low cost per megabyte, high data access rate and short access time compared with other alternative storage hierarchies, e.g. optical storage and solid state memory. With the development of the hard disk drive industry, the areal density has increased from 2000bits/in² to 100Gb/in². Recently the hard disk industry has begun to increase the areal density to 1Tb/in². No matter how the areal density increases, to date the main operation principle of the traditional magnetic-based hard disk storage remains invariant.

With the rapid increase of areal density, reducing the spacing between the magnetic recording head and the recording medium becomes one important issue. In the hard disk drive design, magnetic bits are written and stored on the stacks of a disk, which is mounted on a rotating spindle. The spindle usually rotates at a speed from a few thousands to tens of thousands of revolutions per minute. A slider is mounted on the tip of a suspension arm through a gimbal which is so flexible that the slider is capable to move freely within very small range. Compared with the low stiffness of the gimbal, the suspension arm has a relatively high stiffness. By a voice coil motor,

the suspension arm can locate in an arbitrary radial position on the hard disk. The configuration of a hard disk drive is referred to Figure 1.1.

Fig. 1.1 Configuration of hard disk drive

On the tip of the slider, a read/write transducer is carried to read/write the magnetic information in circular tracks on the hard disk, as shown in Figure 1.2.

Fig. 1.2 Overview of head-disk interface

In the figure above, the left side of the slider is named as the leading edge where the air flows in. The right side of the slider is named as the trailing edge where the air flows out. The edge rotates at the most outer track is named as the outer edge while the edge rotates at the most inner track is named as the inner edge. The slider in Figure 1.2 is a typical positive pressure slider. Two rails are located at the outer edge and inner edge of the slider, respectively, and two tapers are located at the leading edge of the two rails. Between these two rails, a large cavity is generated to release the inputted pressure compressed by two rails. All surfaces of rails, tapers and cavity are manufactured by the etch technology.

Some geometric characteristics of the slider air bearing are shown in Figure 1.3.

Fig. 1.3 Geometric parameters of slider air bearing

In the left plot, the angle between the suspension axis and the reference line connecting the hard disk origin and the position motor origin is named as the sway angle, which defines the radial position of the slider in the disk surface. In the right plot, the angle between the air bearing surface and the disk surface around the Y-axis is named as the pitch angle. The angle between the air bearing surface and the disk

surface around the X-axis is named as the roll angle. The minimum one of all flying heights is named as the minimum flying height. These three parameters define the flying attitude of the slider air bearing. On the top of the slider is attached a suspension where the applied pre-load force, the applied pre-load X-moment and the applied pre-load Y-moment with respect to the coordinate origin are exerted. When the hard disk rotates at a steady speed, the air bearing and air shear forces, together with the any present contact and friction forces lift the slider from the disk surface by a small distance. Due to the pre-load force, the applied pre-load X-moment and the applied pre-load Y-moment provided by the suspension, the slider will fly at some equilibrium location by balancing all relevant forces and moments. In the dynamic state of the slider flying, the time varying air bearing, air shear, contact and friction forces are dependent on the flying height. At each time step, these forces are applied on the nodes of air bearing surface of the slider model, causing the motion of the suspension assembly. In this thesis, the situations of load/unload, contact and shock are not studied, hence the forces and moments acting on the slider from the slider mechanical support are simplified as constant values in the normal operation state of the slider air bearing.

1.2 Objective

Developments in slider manufacturing techniques, e.g. photolithography and plasma etching, have made it possible to produce arbitrary air bearing surface geometries. With this technology breakthrough, the topology optimization of the slider air bearing is made feasible.

One of the important issues in designing a high-performance magnetic slider is to achieve a stable spacing between the slider and the disk surface when the slider moves along different tracks of the disk surface. Therefore the main objectives of the research are to develop an efficient numerical simulation technique and tool for modeling the slider air bearing in hard disk drives and ultimately develop an efficient topology optimization methodology to systematically and automatically design the slider air bearing with a target of achieving a near-constant flying attitude when the slider moves over the whole disk surface. With the verification of the theoretical assumption that the flying attitudes of the slider over the whole disk surface are near-constant as long as the flying attitudes of the slider at the most outer track and at the most inner track are near-constant, the multi-objective function for optimization is set to simultaneously minimize the calculated air bearing force difference, the calculated air bearing X-moment difference and the calculated air bearing Y-moment difference at the most outer track and at the most inner track.

In general, most previous topology optimizations of the slider air bearing published in the literature assume that the main parts of the air bearing surface have been pre-defined and only a small quantity of design variables are perturbed in the design domain. Some of those works divided the design domain into several sub-domains where all geometric parameters remain invariant so as to increase the efficiency of the optimization procedures. However, in this dissertation, all grid points in the geometry configuration of the slider air bearing are set as the design variables. Compared with previous works, the present approach is a true and complete topology optimization for the slider air bearing in the practical applications. To realize the true and complete topology optimization, uniform grid distribution has to be

adopted to ensure that the grid points are evenly distributed in the design domain. Hence, most case studies in this dissertation are based on the uniform grid distribution.

1.3 Organization

The ultimate goal of this dissertation is to develop a topology optimization methodology for slider air bearing design. However, as the fundamental work for topology optimization, the numerical simulation of the slider air bearing is first introduced and solved. Since there are a large quantity of design variables in the topology optimization, the computational efficiency is critical and has to be enhanced. This is done through the full approximation multigrid algorithm. Then the flying attitude determination of the slider air bearing is presented and solved, which is basically to find the equilibrium state under the various applied forces and moments. Thereafter the flying attitude optimization of the slider air bearing is presented to verify the theoretical assumption that the flying attitudes of the slider over the whole disk surface are near-constant as long as the flying attitudes of the slider at the most outer track and at the most inner track are near-constant. This theoretical assumption is the mathematical fundamental of the topology optimization. Finally the topology optimization of the slider air bearing is carried out. As a result, a novel slider air bearing is designed.

In Chapter 2, the numerical techniques applicable to simulate the slider air bearing are first reviewed. Thereafter the development of the multigrid algorithm and its implementation into the numerical simulation of the slider air bearing are described.

Finally, a review on the history of structural optimization and its application to the slider air bearing design is presented.

In Chapter 3, the classical Reynolds equation is derived by integrating the momentum equation and the continuity equation in fluid mechanics. On the basis of different boundary conditions, the first-order slip model, the second-order slip model and the Fukui slip model are introduced. Considering the extensive application in the wide range of Knudsen numbers, the modified Reynolds equation for Fukui slip model is adopted to simulate the slider air bearing. To solve the partial differential equation numerically, the least square finite difference method is presented to numerically discretize the normalized Reynolds equation by approximating the pressure derivatives with respect to the coordinates for every calculated grid point. Two possible iterative methods for solving numerically the discretized equation, the Gauss-Seidel and the Newton algorithms, are illustrated in detail. On the consideration of the effect of the SOR factor, the Gauss-Seidel SOR and the Newton-SOR iterative methods are compared for solving the discretized Reynolds equations for Fukui slip model. By the numerical simulation of the Plate Slider, the Newton-SOR iterative algorithm is found to be more efficient than Gauss-Seidel SOR iterative algorithm.

In Chapter 4, the basic multigrid principle is presented by introducing the fine and coarse grids in the iterative step. As a further introduction, two iterative strategies are demonstrated to show the historical evolution of the multigrid algorithm. Two transfer mechanisms are illustrated to show the vector intergrid relations between the fine and coarse grids. Two classical cycle strategies are presented to show the

different transfer ways in various grid levels. Two implementations of the multigrid method are proposed for the linear equations and the nonlinear equations. To accelerate the convergence rate of the numerical simulation of the slider air bearing, the full approximation multigrid scheme is introduced and discussed in detail. Based on the normalized Reynolds equation discretized by the least square finite difference method, the full weighting restriction operator and the linear interpolation prolongation operator are presented, and the flowchart of the full approximation multigrid algorithm is illustrated based on the three-level v-cycle strategy calculated from the finest grid.

In Chapter 5, to verify the developed methodology and computer code, the numerical simulations of the Infinitely-Long Plane-Wedge Slider, the Plane Slider, the Two-Rail Slider and the Advanced Step Slider are solved with the least square finite difference multigrid procedures. It is found that less computational time is required with the increase of the number of grid levels. By comparing the results with the solutions from the other published papers or from the finite volume method, the least square finite difference multigrid method is proven efficient and accurate.

In Chapter 6, the method of moving asymptotes algorithm is introduced to deal with the optimization problems. Therefore the formulation of the objective function, the constraint definition of the design variables, and the solution of the optimal problem are illustrated in detail. As one practical application of the optimization procedure, the flying attitude determination of the slider air bearing is studied to demonstrate and verify the optimization algorithm. Some case studies are presented

to verify the effectiveness of the procedure for determining the flying attitude of the slider air bearing.

In Chapter 7, as a case of parametric optimization, the flying attitude optimization of the slider air bearing is studied to verify the theoretical assumption that the flying attitudes of the slider over the whole disk surface are near-constant as long as the flying attitudes of the slider at the most outer track and at the most inner track are near-constant. Based on this theoretical assumption, an efficient topology optimization methodology is developed with a target of achieving a near-constant flying attitude when the slider moves over the disk surface. The multi-objective function is set to simultaneously minimize the calculated air bearing force difference, the calculated air bearing X-moment difference and the calculated air bearing Y-moment difference at the most outer track and at the most inner track. As an implementation, a novel optimal slider is systematically and automatically designed with a target of achieving a near-constant flying attitude when it moves over the disk surface.

Finally, Chapter 8 summarizes the achievements of this dissertation, and presents some recommendations for future work.

Chapter 2

Literature Review

2.1 Introduction

In designing a high-performance magnetic slider, it is very important to achieve a stable spacing between the slider and the hard disk when the slider moves over the whole disk surface. Therefore, this dissertation focuses on the numerical modeling and simulation of slider air bearing and ultimately the development of an efficient optimization methodology to systematically and automatically design the slider air bearing with a target of achieving a near-constant flying attitude when the slider moves over the disk surface. To accomplish the ultimate task of topology optimization for slider air bearing design, there are two important integrated fundamental issues to be addressed. These are: the numerical modeling and simulation of the slider air bearing and the topology optimization of the slider air bearing. As an effective method to improve the computational efficiency of the numerical simulation, the multigrid algorithm also has to be adopted.

2.2 Numerical Simulation of Slider Air Bearing

As the fundamental element for topology optimization of the slider air bearing, the numerical simulation of the slider air bearing has to be performed to calculate the pressure distribution in the static condition. As the governing equation generally

applied in the computational fluid mechanics, the Reynolds equation is usually adopted in the simulation.

2.2.1 Various Methods for Slider Air Bearing Simulation

To solve the modified Reynolds equations for different slip models, analytical method can not be employed due to its complex nonlinear properties. Hence some numerical methods are developed to solve these generalized lubrication equations. These methods can be classified into the finite element method, the finite volume method and the finite difference method.

2.2.1.1 Finite Element Method

The finite element method is a powerful numerical solution tool for a wide range of engineering problems, e.g. the structural analysis, the solid mechanics and the fluid mechanics. With the advances in computer technology and computer-aided engineering systems, complex problems can be modeled with relative ease. Due to its extensive application, the finite element method was applied in the solution of slider air bearing problems recently.

By the implementation of the finite element method, many studies have been performed to further quantify the characteristics of the slider air bearing. Nguyen (1991) employed the p-version finite element formulation in conjunction with the Newton-Raphson iteration procedure to analyze the steady-state compressible isothermal lubrication problems. Peng and Hardie (1995) developed a rapid convergence finite element approach for determining the air bearing slider flying

attitude. By comparison with experimental measurements, the accuracy of the numerical solution was verified. Wu and Bogy (2001) presented a finite element method based on the streamline upwind Petrov-Galerkin (SUPG) approach to solve the convection part of the generalized Reynolds equation over unstructured triangular meshes. The results predicted by the code agreed well with experimental data. Zhang and Talke (2003) applied a finite element based air bearing simulator to study the flying height sensitivity of pico and femto sliders on air bearing contour design, operation conditions and manufacturing tolerances.

Although the finite element method is successful in the numerical simulation of the air bearing surface, the requirements for large computer memory and much computational time may hamper its further development in this field.

2.2.1.2 Finite Volume Method

When the geometry of the slider air bearing becomes complicated, it is difficult to generate good structured meshes. As a consequence, the finite volume method is preferred. Due to its local conservative quality, the finite volume method is good at capturing sharp gradients in the numerical simulation of the slider air bearing.

In past years, the finite volume method is applied to solve the slider simulation problems. Wu and Bogy (1999) employed a fully implicit finite volume scheme to locally integrate the generalized Reynolds equation. From the verification of the results, this scheme was highly efficient and unconditionally stable. One year later, Wu and Bogy (2000a, 2000b) presented an upwind higher order finite volume numerical scheme over unstructured triangular mesh to solve the slider air bearing

problem of hard disk drives. In this scheme, a non-nested full approximation multigrid algorithm was employed to speed up the convergence rate. In 2005, Hua and Liu (2005) studied the mechanism of the multiple flying states of the air bearing slider with the finite volume method, instead of the previous finite element method, to improve the numerical convergence at very low flying heights. The numerical results agreed well with the experimental data.

Due to the unstructured nature of the finite volume method, the additional information about connectivity between meshes and the complicated data structure are required. It will results in the increase of the computer memory and the computational time.

2.2.1.3 Finite Difference Method

The finite difference method is known for its relatively small memory and computational time required if the smooth and adequate meshes can be generated. At present, the potential of the finite difference method is desired to be further explored.

Some researches have been performed to employ the finite difference method into the solution of the slider simulation problems. White et al. (1986) used a variable grid implicit finite difference scheme to provide steady-state solutions of the Reynolds equation over a bearing number range of five orders of magnitude. Cha and Bogy (1995) developed a new numerical scheme based on the finite difference method to solve for the pressure distribution under subambient pressure sliders with arbitrary rail shapes. This scheme can be easily used for the static analysis of various air bearing surface designs. In the optimal design of HDD air lubricated slider

bearings for improving dynamic characteristics and operating performance, Kang et al. (2001) adopted the finite difference technique and the control volume formulation to suppress the numerical difficulties caused by the clearance discontinuities, which existed in arbitrary shaped air bearing surface.

However, in the numerical simulation of the complex slider configuration, the finite difference method is not effective. It is because that the construction of body-fitting meshes and the transformation of governing equations introduce the additional geometrical error into the scheme and degrade the accuracy of the solution. Therefore other methods are better choices to solve the numerical simulation of the complex air bearing surface.

2.2.2 Development of Meshless methods

It is well known that the finite element method, the finite volume method and the finite difference method are the most popular numerical techniques in the simulation of computational fluid mechanics. However, when complex geometry is involved in the numerical simulation of the slider air bearing, the generation of the mesh will become an expensive and time-consuming task. It means that a more general method is required to remove the traditional meshing. As a consequence, the meshless methods are developed to replace the mesh generation with the node generation. From the view of the computational efforts, the node generation is an easier and faster process.

As described in the name, the meshless methods are deliberately designed to eliminate the dependence on the mesh. The term of meshless refers to the ability of this method to construct functional approximation or interpolation entirely from the information at a set of nodes, without any pre-specified connectivity or relationships among these nodes. In recent years, considerable efforts have been devoted to the development of meshless methods. As a meshless Lagrangian particle method, the smoothed particle hydrodynamics (SPH) method, proposed by Lucy (1977), is comprehensively investigated and widely applied to different areas. In this method, the collocation is employed at the nodes to approximate the solution in the corresponding subdomain. The element free Galerkin (EFG) method, proposed by Belytschko et al. (1994), is a viable method developed from the diffuse element method (DEM), proposed by Nayroles et al. (1992). In this method, the moving least square (MLS) approach is applied to construct the meshless shape functions in a Galerkin method. The reproducing kernel particle (RKP) method, proposed by Liu et al. (1995), further extends the SPH method with improved continuity and multiple scales. The meshless local Petrov-Galerkin (MLPG) method, proposed by Atluri and Zhu (1998), is based on a local weak form over a local subdomain. In this method, both interpolation and integration are performed without using a global mesh, which makes it easier in implementation. The point interpolation method (PIM), proposed by Liu and Gu (2001a, 2001b), is a meshless method based on the Galerkin or Petrov-Galerkin formulations. In this method, the field value of a point of interest is obtained by interpolation of the field values at nodes in the support domain.

As another advantage of its implementation, the meshless methods can facilitate the construction of the higher-order discretization. By application of the meshless

methods, it is easy to add or subtract nodes from the pre-existing nodes. Hence, these methods are applicable for the numerical simulation of the complex slider air bearing.

2.2.3 Least Square Finite Difference Method

As one type of meshless methods, the general finite difference (GFD) method, proposed by Liszka and Orkisz (1980), is derived from the finite difference method. In the GFD method, the least square technique is employed to avoid the possible ill-conditioned configurations of the coefficient matrix which arise from the arbitrary point distribution. At this point, it is not the same as the finite difference method. To further develop an efficient general finite difference method, a weighted least-square-based finite-difference scheme was proposed by Ding et al. (2004). In this scheme, the approximation of partial derivatives is formulated from the Taylor series expansions and the weighted least square technique. Compared with the conventional finite difference methods, the least square finite difference method is more flexible in the practical applications.

Considering the superior ability in easy node generation without element connectivity, the least square finite difference is implemented into the numerical simulation of the slider air bearing by Li et al. (2004). By the solution of the pressure distribution for IBM3380 slider with two rails, the least square finite difference method is proven effective to solve the lubrication problem in the hard disk drive. Based on the least square finite difference method, Du et al. (2005) presented a Newton-multigrid method to simulate the slider air bearing. By using the Newton method to solve the generalized Reynolds equation and applying the full

approximation multigrid method to accelerate the convergence rate, the efficiency of the least square finite difference method in the numerical simulation of the slider air bearing is greatly increased.

2.3 Implementation of Multigrid Method

As an effective method to increase the computational efficiency of the topology optimization for the slider air bearing, the multigrid algorithm is required to be employed in this dissertation.

The multigrid method introduces coarser meshes to accelerate the convergence in the fine mesh. It naturally leads to a decomposition of the solution error into rough error components, which can not be solved in a coarser mesh, and the complementary smooth error components, which can be solved in the coarser grid. Therefore a multigrid scheme begins by eliminating high frequency errors in the fine mesh. The smoothed solution is then transferred to a coarser mesh and corrections are obtained in this coarser mesh. Thereafter, these corrections will be interpolated back to the fine mesh to update the solution. Since the multigrid method is a convergence acceleration technique used in any existing discretization, its application has no effect on the accuracy of the computed solution, but improves the computational efficiency.

2.3.1 Development of Multigrid Method

The multigrid method was first applied successfully on elliptic partial differential equations by Federenko (1964). Later Bakhvalov (1966) applied this method on the

convergence of a relaxation method with natural constraints on the elliptic operator. The nonlinear version of multigrid method was developed by Brandt (1977) to solve boundary value problems. As two practical applications of this version of multigrid method, it was employed to solve the Euler equations by Jameson (1986) and solve the Navier-Stokes equations by Martinelli (1987). As the most popular multigrid scheme for the numerical solution of the high-Reynolds flow equations, the full approximation scheme (FAS) is proposed by Brandt (1984). In the full approximation scheme, the multigrid equations on the coarser grids are cast in terms of the solution variables, in conformity with the corresponding formulation for the fine grid. The full approximation scheme is considered to be a suitable multigrid technique for the Reynolds equations, compared with other methods, which apply correction schemes to the linearized flow equations on the coarser grids.

With the development of the multigrid method, many publications are presented in these two decades. Wesseling (1992) published a book to comprehensively introduce the multigrid method. After one year, Bramble (1993) also summarized the achievements and applications of the multigrid method. In the monograph, Stewart (1995) described the implementation of the multigrid methods for the finite elements. As a classical book to present the essential ideas and applications of the multigrid method, a multigrid tutorial was published by Briggs (2000). The goal of this tutorial is to give readers enough familiarity with the multigrid method so that they can apply it in their own research fields.

2.3.2 Multigrid Application to Slider Air Bearing Simulation

Since the numerical simulation of slider air bearing is the basic solver for the ultimate topology optimization of the slider air bearing, the multigrid method is applied to accelerate the convergence rate in the iterative process in the solution of the slider air bearing problem.

Hu and Bogoy (1998) developed a computationally efficient multigrid control volume method for the solution of the very high bearing number and shaped rail air bearing problems. In detail, an additive correction base multigrid method is used to accelerate the solution of the resulting discretization equations. To improve the efficiency of the scheme, Wu and Bogoy (1999) applied a non-nested multigrid technique in the numerical simulation of the slider air bearing with complex shaped rails, based on the unstructured triangular mesh generation techniques and the finite volume numerical scheme. Du et al. (2005) employed a Newton-multigrid method for the numerical simulation of the slider air bearing. Between the different sizes of grids, the full approximation multigrid method is adopted to accelerate the convergence rate by eliminating the dominating low frequency error mode in the late stages of convergence. As a popular program for the solution of the slider air bearing problem, the CMLAir32, developed by the Computer Mechanics Laboratory at the University of California at Berkeley (O'Hara and Bogoy, 1997), adopts the multi-level multigrid method to increase the computational efficiency in the numerical simulation. Recently, the multigrid method has been implemented by the Data Storage Institute of ASTAR in Singapore to accelerate the convergence rate of its in-house-developed slider air bearing simulation code (Hua and Liu, 2005). In this code, the unstructured triangular

mesh is applied to capture the rail boundaries accurately, as well as the finite volume method is employed to solve the Reynolds equation at a high Knudsen number.

2.4 Structural Optimization of Slider Air Bearing

Developments in slider manufacture techniques, e.g. photolithography and plasma etching, have made it possible to produce arbitrary air bearing surface geometries. With this advancement, the structural optimization of the slider air bearing becomes feasible.

2.4.1 History of Structural Optimization

One of the first problems in optimal design was proposed by Lagrange in 1770 when the optimal shape of a column subject to an axial load was obtained. Later the first optimization problem was solved by Venant (1856), who found the stiffest cross section of a prismatic beam subject to torsion, i.e. the annular cross section.

The concept of structural optimization was first set up by Maxwell (1872). After some years, Mitchell (1904) reported the generalization of the theory for minimum weight structures. As a further development, more modern structural optimization techniques were introduced by Schmit (1960). In his report, the nonlinear programming method is first implemented to solve structural optimization problems. Later, Fox (1965) presented the concept of sensitivity analysis. Sensitivity analysis is a procedure to obtain the derivatives of the objective function and the constraints with respect to design variables. In the 1970's, the structural optimization had been widely

utilized in many engineering fields. The structural design methodology mostly relied on intuitive heuristic approaches from the designers' experience and practice. Simultaneous failure model design (Spunt, 1971) and fully stressed design (Gallagher and Zienkiewicz, 1973) were two popular design techniques at that time. However, the development of optimal design was so primitive that it could be only applied to simple structures. Until the early 1980's, a general-purpose structural optimization system went on optimizing the sizes of structures after its association with the finite element method. Meanwhile, the concept of the Computer Aided Engineering (CAE) was introduced to the structural design field and made a great impact on the development of the structural optimization system. Generally such systems are made up of three modules: structural analysis, sensitivity analysis, and the optimization process.

In recent years, coupled with desktop 3D Computer Aided Design (CAD), the structural optimization has been widely applied for various practical applications (Salitou et al., 2005). Even in some product segments, the structural optimization has completely replaced physical tests. With its acceptance as a design tool, the structural optimization seems to gain the mainstream popularity.

2.4.2 Classification of Structural Optimization

According to the design variables of the structural optimization problem, the development of the structural optimization can be classified into three categories: sizing optimization, shape optimization and topology optimization.

2.4.2.1 Sizing Optimization

The goal of sizing optimization is to find optimal sizes of structures, e.g. cross-sectional dimensions and thicknesses of the plate. In the size optimization problem, the configuration of a structure is pre-fixed throughout the optimization process. Hence, there is no change in the geometry of the discretized model. The optimal solution can be found by obtaining the sensitivities of the objective function and constraint functions of the formulated optimization problem.

Since Schmit (1960) implemented non-linear programming techniques to design a fully stressed elastic three-bar truss, the subject of sizing optimization for truss-like structures has been mature enough to produce commercial systems incorporating the finite element method. In truss-like structures, the design domain is discretized and the number of structural elements is finite. Hence, functions representing the objective and constraints of the optimization problem are continuously differentiable, and optimal layout can be obtained with the numerical method.

2.4.2.2 Shape Optimization

The shape optimization became the focus of many studies after finite element method had been widely accepted by many researchers. The shape optimization pursues the determination of the dimensional variables, which maximize the ability of the structure or part to perform the intended function under the prescribed operating conditions.

In the 1970's, Ramakrishnan and Francavilla (1974) minimized the weight of the structure. In their case, the locations of the boundary nodes are set as design variables, and during the course of calculation, enormous number of variables are required to avoid boundary distortion caused by the boundary nodes. To reduce the number of variables, Kristensen and Madsen (1976) defined the boundary using a linear combination of some shape functions. Later, Bhavikatti and Ramakrishnan (1977) divided the boundary of the structure into few regions, over which the boundary was prescribed with a polynomial function. Braibant and Fleury (1984, 1985) employed a blending function with Bezier and B-spline to describe the design element boundaries. To avoid the unstable boundary shapes resulting from utilization of high-order polynomials to describe the boundary, Yang and Choi (1985) introduced the spline function to represent the boundary.

2.4.2.3 Topology Optimization

The topology optimization, also named as the layout optimization, is a general design method to find the best configuration of structural systems. This method is regarded as a significant breakthrough in the field of structural optimization because it results in greater material savings than sizing optimization or shape optimization. In the topology optimization, the optimal configuration is performed by treating infinite design variables, which make this problem far more difficult than the conventional methods.

At the beginning, Bendsøe and Kikuchi (1988) put together a research method based on structural optimization and homogenization techniques in order to solve the layout topology optimization problem in plane elasticity. They used homogenization

techniques plus an optimality criterion approach to solve state equation by the finite element method. The similar work was done by Zochowski (1988) at the same time. Since then, many researches on homogenization based topology had been published. Bendsøe (1989) developed the homogenization idea for the topology design of a linear elastic structure. Suzuki and Kikuchi (1991) implemented the minimum compliance problem, and obtained clear and innovative optimal solutions. Ma et al. (1993) constructed a multi-eigenvalue optimization method with the convex approach for an eigenvalue problem. Neves et al. (1995) investigated the buckling in topology design. Min (1997) implemented this approach to a multi-criteria problem incorporating the minimum compliance and the maximum eigenvalue, and a transient problem. Rodrigues and Fernandes (1995) applied to the thermo-elastic structure designs under thermal loads. Ou and Kikuchi (1996) reported an application to controlled structures.

2.4.3 Optimization Approaches for Slider Air Bearing

Hard disk drives continue to increase in the areal density. Densities as high as 1 Tbit/in² are targeted at present. The extremely high areal density requires a slider air bearing design with lower flying height. At such low flying height, the designs of the air bearing surface must satisfy very strict performance requirements, e.g. the stable flying attitude. By using the modern optimization techniques, it is possible to optimize the slider air bearing according to different objectives or requirements.

Several methods have been presented to obtain the optimal configuration subject to different objective functions. O'Hara and Boggy (1995) employed genetic

algorithms and simulated annealing to develop global optima in the presence of many local optima. They presumed an original design of a transverse pressure contour slider and some key points in the slider were varied in a minimal constraint domain. Yoon and Choi (1995, 1997) used the sequential quadratic programming method for the determination of the optimal configuration of a traverse pressure contour slider. In the optimization, only the steady state flying performance of the slider is considered. Kang et al. (2001) introduced a new design of the air bearing surface with a target of achieving the stable flying attitude when the slider moved on the disk surface. They employed the method of modified feasible directions and adopted the weighting method to solve the multi-objective function. Zhu and Bogy (2004) applied the modified DIRECT algorithm into the optimization of the slider air bearing. However, the optimization design of the slider air bearing had been defined first. Zhang and Talke (2005) presented the combined genetic algorithm-subregion method, which combines the advantage of the genetic algorithm and the subregion approach to design the air bearing surface. In this method, the design domain is divided into many rectangular subregions and a binary string composed of '1's and '0's that stands for the height level of a particular subregion. Hwang and Khan (2005) used the conjugate-gradient method and the perturbation method in the static analysis of the slider. The steady state flying attitude is studied for various radial positions. It is found that the perturbation method is an efficient method for gradient evaluation.

The applied methods above for determining the optimization configuration of the slider air bearing have a common feature. It is the existence of an original design of the air bearing surface before the initialization of the optimization procedure. Hence some important design variables, e.g. the vertices of the rail, the recess depth and the

pivot position are specified first. As a consequence, no matter how the design variables are changed, a rail can not break into two disjoint rails and no new cavities can emerge from the optimization. In another word, these optimization techniques only change the shape and size of the initial design of the slider air bearing. In general, they are still the sizing optimization and shape optimization. Therefore there is a significant necessity to develop an efficient optimization methodology to systematically and automatically find the best feasible topology at a very early stage of the design process.

As primitive attempts to the topology optimization of the slider air bearing, some researches have been done in recent years. Yoon and Choi (2003, 2004) proposed an analytical design sensitivity analysis to topological parameters of the molecular gas film lubrication sliders by introducing an adjoint variable method. By the adjoint variable method, the computing time for the design sensitivity is greatly reduced. Lu and Wang (2004) employed the genetic algorithms for the determination of the optimal configuration of sliders with a specified taper and recess depth. In the optimization, a binary string composed of '1's and '0's is applied to stand for the height of a grid point.

So far the popular program for the air bearing optimization was developed at the Computer Mechanics Laboratory (CML) at the University of California at Berkeley (O'Hara, 1997). The program provides the necessary tools for the optimization of air bearing designs. The algorithm implemented to optimize the slider air bearing designs is a variation of the simulated annealing optimization algorithm, in which a stochastic optimization technique is modeled after the physical process of annealing. The

algorithm does not make use of function derivative information, neither analytical nor numerical. Instead, it uses a neighborhood operator to find the global optimum. However, this method should be classified into the sizing optimization for its optimal results are greatly dependent on the initial designs.

Chapter 3

Least Square Finite Difference Method for Slider Air Bearing Simulation

3.1 Introduction

To study the lubrication film of the slider air bearing, the derivation of Reynolds equation, which governs the lubrication principle in fluid, is introduced in this chapter. As an accurate and efficient meshless method, the least square finite difference (LSFD) method, proposed by Ding et al. (2004), is applied to numerically discretize the modified Reynolds equation.

In the numerical simulation of the slider air bearing, due to the extremely narrow space between the air bearing surface and the disk surface, the extremely rarefied gas in the clearance will slip on the film surface. Accordingly the classical lubrication theory is not applicable because the gas can not be assumed as a continuum flow in such condition. Therefore several slip correction models are studied to simulate the practical situation of the air bearing films for different Knudsen numbers.

Knudsen number $K_n = \lambda / h_m$, defined as the ratio of the molecular mean free path λ to the minimum flying height h_m , is an important parameter to categorize the various slip models. The micro-flow can be classified into four flow regimes based on Knudsen numbers. When $K_n \leq 10^{-3}$, the micro-flow is assumed as a continuum flow and the

Navier-Stokes equation is adopted. When $10^{-3} < K_n \leq 0.1$, the micro-flow is assumed as a slip flow. When $0.1 < K_n < 10$, the micro-flow is assumed as a transition flow. When $K_n \geq 10$, the micro-flow is in the free molecular region and the collisionless Boltzmann equation is used.

Many attempts have been made to develop the slip correction models since Maxwell discovered the ‘velocity slip’ effect near a moving wall. Burgdorfer (1959) introduced a first-order correction to the continuum non-slip boundary condition. Hsia and Domoto (1983) presented a second-order correction to the slip-flow boundary condition. Fukui and Kaneko (1990) proposed a polynomial fitting procedure to explicitly express Poiseuille flow rate as a function of an inverse Knudsen number. Among them, the Fukui slip model can be used for a wide range of Knudsen numbers. The modified Reynolds equation for Fukui slip model has the most extensive application in the numerical simulation of the slider air bearing.

Traditionally, the finite element method, the finite volume method and the finite difference method are the most popular mesh-based procedures to discretized complex Reynolds equations. Compared with the finite element method and the finite volume method, the finite difference method requires less computational time and storage memory for its simple data structure to locate the information of the grid points. Although the conventional finite difference scheme has good performance characteristics for one-dimensional problems, it confronts many difficulties when it is directly extended to multi-dimensional problems, especially those with complex geometries. Therefore the LSFD method is presented to solve the two-dimensional numerical simulation problems.

3.2 Reynolds Equation for Slider Air Bearing

Reynolds equation is the classical differential equation, derived by the English scientist Osborne Reynolds in 1886, to govern the hydrodynamic lubrication theory. Reynolds equation is derived by integrating the momentum equation and the continuity equation in fluid mechanics.

3.2.1 Momentum Equation and Continuity Equation

Gas film lubrication provides non-contact support by the pressurized gas, which is generated by hydrodynamic bearings as shown in Figure 3.1.

Fig. 3.1 Pressurized gas in slider air bearing

It describes the macro-channel flow between the stationary and inclined air bearing surface and the horizontal disk surface which moves with velocity u in the X-direction. The classical lubrication equation for the calculation of gas pressure generated in the lubrication region is the well-known Reynolds equation.

The momentum equation for laminar-continuum fluid flow from the Stokes idealization is called the Navies-Stokes equation. Assuming that the flow is steady, the momentum equation combined with the continuity equation yields the Reynolds equation. The followings are the derivation of the Reynolds equation for a compressible fluid.

For a Newtonian fluid with uniform viscosity, which is generally valid for gas flow, the macroscopic equations of motion are

$$\begin{cases} \rho(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z})u = -\frac{\partial p}{\partial x} + \mu(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})u + (\frac{1}{3}\mu)\frac{\partial}{\partial x}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}) \\ \rho(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z})v = -\frac{\partial p}{\partial y} + \mu(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})v + (\frac{1}{3}\mu)\frac{\partial}{\partial y}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}) \\ \rho(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z})w = -\frac{\partial p}{\partial z} + \mu(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})w + (\frac{1}{3}\mu)\frac{\partial}{\partial z}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}) \end{cases} \quad (3.1)$$

where x, y, z are Cartesian coordinates. u, v, w are gas velocities in the X-direction, Y-direction and Z-direction respectively. p is the pressure. μ is the dynamic viscosity. ρ is the density of the fluid. In Equation (3.1), the left side denotes the inertial force of the fluid, and the right side denotes the force from the pressure gradient and the viscous force.

The continuity equation is expressed as

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \quad (3.2)$$

The full Reynolds equation is derived from the momentum equation and the continuity equation based on the following assumptions:

1. Body forces are neglected, i.e. there are no extra fields where forces are exerted on the fluid.
2. The pressure is constant throughout the thickness of the film.
3. The radius of curvature of surface is large compared with film thickness, and the surface velocity change need not be considered in the direction.
4. There is no slip on the boundaries.
5. The lubricant is Newtonian, i.e. the stress is proportional to the rate of shear strain.
6. The flow is laminar flow.
7. Fluid inertia is neglected.
8. The viscosity is constant throughout the film thickness. This is an approximation but leads the great simplification.

Consequently, Equation (3.1) can be simplified. The microscopic equations for two-dimensional flow are

$$\begin{cases} \frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu} \frac{\partial p}{\partial x} \\ \frac{\partial^2 v}{\partial z^2} = \frac{1}{\mu} \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} = 0 \end{cases} \quad (3.3)$$

The continuity Equation (3.2) can be simplified as

$$\frac{\partial}{\partial x} \int_0^h \rho u dz + \frac{\partial}{\partial y} \int_0^h \rho v dz + \frac{\partial}{\partial t} (\rho h) = 0 \quad (3.4)$$

For ideal fluids, the density ρ is proportional to the pressure p . As a result, under the time-independent condition the equation above can be transformed into

$$\frac{\partial}{\partial x} \int_0^h p u dz + \frac{\partial}{\partial y} \int_0^h p v dz = 0 \quad (3.5)$$

3.2.2 Classical Reynolds Equation for Continuum Flow

The classical Reynolds equation is limited. However, it is applicable for flows where the molecular mean free path of the gas is negligible compared with the dimensions of the air bearing clearance.

The boundary conditions for continuum flow are

$$\left\{ \begin{array}{l} u|_{z=0} = U \\ u|_{z=h} = 0 \\ v|_{z=0} = V \\ v|_{z=h} = 0 \end{array} \right. \quad (3.6)$$

The solution of Equation (3.3) under such boundary conditions is

$$\begin{cases} u = -\frac{1}{2\mu} \frac{\partial p}{\partial x} (h-z)z + U(1-\frac{z}{h}) \\ v = -\frac{1}{2\mu} \frac{\partial p}{\partial y} (h-z)z + V(1-\frac{z}{h}) \end{cases} \quad (3.7)$$

The flow velocity is a linear superposition of a pressure-driven Poiseuille flow and a disk velocity-driven Couette flow. Substituting Equation (3.7) into Equation (3.5) gives the classical Reynolds equation for continuum flow, as below:

$$\frac{\partial}{\partial x} \left(ph^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(ph^3 \frac{\partial p}{\partial y} \right) = 6U\mu \frac{\partial}{\partial x} (ph) + 6V\mu \frac{\partial}{\partial y} (ph) \quad (3.8)$$

Under most cases, the normalized Reynolds equation is used. The normalizations are

$$H = \frac{h}{h_m}, \quad X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad P = \frac{p}{p_a}, \quad \Lambda_x = \frac{6\mu UL}{h_m^2 P_a}, \quad \Lambda_y = \frac{6\mu VL}{h_m^2 P_a} \quad (3.9)$$

where h_m is the minimum flying height of the slider air bearing. L is the length of the slider air bearing. μ is the fluid dynamic viscosity. p_a is the ambient pressure. Λ is the air bearing number, which can be understood as a ratio of the Couette flow rate to the Poiseuille flow rate.

Therefore the classical Reynolds equation is normalized as

$$\frac{\partial}{\partial X} \left(PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right) + \frac{\partial}{\partial Y} \left(PH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right) = 0 \quad (3.10)$$

3.2.3 Modified Reynolds Equations for Various Slip Models

Since the present flying height in the hard disk drive are only a fraction of the gas molecular mean free path, the slip between the gas and the walls occurs, resulting in the change of the boundary condition. Therefore the classical Reynolds equation based on the continuum assumption with non-slip boundary condition is no longer valid. According to different boundary conditions, modified Reynolds equations are derived for various slip models.

3.2.3.1 Modified Reynolds Equation for First-Order Slip Model

The first correction of the Reynolds equation was derived by Burgdorfer in 1959. In this first-order slip flow model, the boundary conditions at the walls are

$$\left\{ \begin{array}{l} u|_{z=0} = U + \frac{2-\sigma}{\sigma} \lambda \frac{\partial u}{\partial x} \Big|_{z=0} \\ u|_{z=h} = -\frac{2-\sigma}{\sigma} \lambda \frac{\partial u}{\partial x} \Big|_{z=h} \\ v|_{z=0} = V + \frac{2-\sigma}{\sigma} \lambda \frac{\partial v}{\partial y} \Big|_{z=0} \\ v|_{z=h} = -\frac{2-\sigma}{\sigma} \lambda \frac{\partial v}{\partial y} \Big|_{z=h} \end{array} \right. \quad (3.11)$$

Here σ is the reflection coefficient, which is normally set as unity for diffuse reflection.

After substituting the equation above into Equations (3.3) and (3.5), the modified Reynolds equation for first-order slip model is derived as

$$\begin{cases} \frac{\partial}{\partial X} \left(QPH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right) + \frac{\partial}{\partial Y} \left(QPH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right) = 0 \\ Q = 1 + 6 \frac{K_n}{PH} \end{cases} \quad (3.12)$$

Here Q is the flow factor for different slip models. This equation is applicable as long as the minimum flying height of the slider air bearing is greater than 100nm.

3.2.3.2 Modified Reynolds Equation for Second-Order Slip Model

For lower minimum flying height where the Knudsen number approximately approaches unity, Hsia proposed a second-order slip flow model in 1983. Its slip boundary conditions are given by

$$\begin{cases} u|_{z=0} = U + \frac{2-\sigma}{\sigma} \left(\lambda \frac{\partial u}{\partial x} \Big|_{z=0} - \frac{\lambda^2}{2} \frac{\partial^2 u}{\partial x^2} \Big|_{z=0} \right) \\ u|_{z=h} = \frac{2-\sigma}{\sigma} \left(-\lambda \frac{\partial u}{\partial x} \Big|_{z=h} - \frac{\lambda^2}{2} \frac{\partial^2 u}{\partial x^2} \Big|_{z=h} \right) \\ v|_{z=0} = V + \frac{2-\sigma}{\sigma} \left(\lambda \frac{\partial v}{\partial y} \Big|_{z=0} - \frac{\lambda^2}{2} \frac{\partial^2 v}{\partial y^2} \Big|_{z=0} \right) \\ v|_{z=h} = \frac{2-\sigma}{\sigma} \left(-\lambda \frac{\partial v}{\partial y} \Big|_{z=h} - \frac{\lambda^2}{2} \frac{\partial^2 v}{\partial y^2} \Big|_{z=h} \right) \end{cases} \quad (3.13)$$

After substituting the equation above into Equations (3.3) and (3.5), the modified Reynolds equation for second-order slip model is derived as

$$\begin{cases} \frac{\partial}{\partial X} \left(QPH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right) + \frac{\partial}{\partial Y} \left(QPH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right) = 0 \\ Q = 1 + 6 \frac{K_n}{PH} + 6 \left(\frac{K_n}{PH} \right)^2 \end{cases} \quad (3.14)$$

3.2.3.3 Modified Reynolds Equation for Fukui Slip Model

For extremely low minimum flying height where the Knudsen number is greater than unity, the Boltzmann equation from kinetic gas theory gives the best slip flow correction. Gans (1985) was the first to derive a lubrication equation based on a linearized Boltzmann equation. Fukui and Kaneko (1988) presented a generalized Reynolds equation suitable for the first-order and second-order slip flow corrections as well as the Boltzmann correction. The modified Reynolds equation for Fukui slip model is

$$\begin{cases} \frac{\partial}{\partial X} \left(QPH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right) + \frac{\partial}{\partial Y} \left(QPH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right) = 0 \\ Q = \frac{Q_p}{Q_{con}} \end{cases} \quad (3.15)$$

where Q_p is a Poiseuille flow rate coefficient and Q_{con} is a continuum flow rate coefficient, shown as the followings:

$$\begin{cases} Q_p = -2.22919D + 2.10673 + 0.01653/D - 0.0000694/D^2 & (0.01 \leq D < 0.15) \\ Q_p = 0.13852D + 1.25087 + 0.15653/D - 0.00969/D^2 & (0.15 \leq D < 5) \\ Q_p = D/6 + 1.0162 + 1.0653/D - 2.1354/D^2 & (5 \leq D) \end{cases} \quad (3.16)$$

$$Q_{con} = D/6 \quad (3.17)$$

Here D is the function of the characteristic inverse Knudsen number D_0 , shown as below:

$$D = D_0 PH \quad (3.18)$$

$$D_0 = \sqrt{\pi}/(2K_n) \quad (3.19)$$

3.3 Numerical Discretization of Reynolds Equation

Due to the complexity of the Reynolds equations for various slip models, it is difficult to solve them analytically. Therefore the numerical method is applied to approximate the pressure derivatives with respect to the coordinates. On the consideration of extensive application for various Knudsen numbers, the normalized Reynolds equation for Fukui slip model is implemented in this dissertation.

3.3.1 Normalized Reynolds Equation for Fukui Slip Model

Equation (3.15) can be transformed into

$$F = \frac{\partial}{\partial X} \left\{ \left[K_1 + K_2(PH)^{-1} + K_3(PH)^{-2} - K_4(PH)^{-3} \right] PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right\} + \frac{\partial}{\partial Y} \left\{ \left[K_1 + K_2(PH)^{-1} + K_3(PH)^{-2} - K_4(PH)^{-3} \right] PH^3 \frac{\partial P}{\partial X} - \Lambda_y PH \right\} = 0 \quad (3.20)$$

Here $K_i (i=1, \dots, 4)$ are corresponding coefficients, as shown below:

$$\begin{cases} K_1 = -13.37514 & K_2 = 12.64038D_0^{-1} & K_3 = 0.09918D_0^{-2} & K_4 = 0.0004164D_0^{-3} & (0.01 \leq D < 0.15) \\ K_1 = 0.83112 & K_2 = 7.50522D_0^{-1} & K_3 = 0.93918D_0^{-2} & K_4 = 0.05814D_0^{-3} & (0.15 \leq D < 5) \\ K_1 = 1 & K_2 = 6.0972D_0^{-1} & K_3 = 6.3918D_0^{-2} & K_4 = 12.8124D_0^{-3} & (5 \leq D) \end{cases} \quad (3.21)$$

After expanding Equation (3.20), the normalized Reynolds equation for Fukui slip model based on the two-dimensional grid is obtained as

$$\begin{aligned}
F_{ij} = & \left[\left(\frac{\partial P}{\partial X} \right)_{ij}^2 + \left(\frac{\partial P}{\partial Y} \right)_{ij}^2 \right] (K_1 H_{ij}^3 - K_3 P_{ij}^{-2} + 2K_4 P_{ij}^{-3}) \\
& + \left(\frac{\partial H}{\partial X} \right)_{ij} \left(\frac{\partial P}{\partial X} \right)_{ij} + \left(\frac{\partial H}{\partial Y} \right)_{ij} \left(\frac{\partial P}{\partial Y} \right)_{ij} (3K_1 P_{ij} H_{ij}^2 + 2K_2 H_{ij} + K_3 P_{ij}^{-1}) \\
& + \left(\frac{\partial^2 P}{\partial X^2} \right)_{ij} + \left(\frac{\partial^2 P}{\partial Y^2} \right)_{ij} (K_1 P_{ij} H_{ij}^3 + K_2 H_{ij}^2 + K_3 P_{ij}^{-1} H_{ij} - K_4 P_{ij}^{-2}) \\
& - P_{ij} \left(\Lambda_{xij} \frac{\partial H}{\partial X} \right)_{ij} + \Lambda_{yij} \left(\frac{\partial H}{\partial Y} \right)_{ij} - H_{ij} \left(\Lambda_{xij} \frac{\partial P}{\partial X} \right)_{ij} + \Lambda_{yij} \left(\frac{\partial P}{\partial Y} \right)_{ij} = 0 \quad (3.22)
\end{aligned}$$

where i and j denote the sequence numbers of the reference point in the grid. F_{ij} and P_{ij} denote the residual and pressure of the reference point. X_{ij} , Y_{ij} and H_{ij} denote X-coordinate, Y-coordinate and the height of the reference point. Λ_{xij} and Λ_{yij} denote the air bearing numbers in X-direction and Y-direction.

3.3.2 Derivative Approximation by Least Square Finite Difference Method

The configuration of the slider air bearing is meshed into the grid as shown in Figure 3.2.

Fig. 3.2 Mesh configuration of slider air bearing

The pressure of the n th supporting point P_{ij}^n can be approximated by the 2-D Taylor series expansion of the pressure of reference point P_{ij} , as the following:

$$\begin{aligned}
 P_{ij}^n = & P_{ij} + \Delta X_{ij}^n \frac{\partial P}{\partial X} \Big|_{ij} + \Delta Y_{ij}^n \frac{\partial P}{\partial Y} \Big|_{ij} + \frac{1}{2} (\Delta X_{ij}^n)^2 \frac{\partial^2 P}{\partial X^2} \Big|_{ij} + \frac{1}{2} (\Delta Y_{ij}^n)^2 \frac{\partial^2 P}{\partial Y^2} \Big|_{ij} \\
 & + \Delta X_{ij}^n \Delta Y_{ij}^n \frac{\partial^2 P}{\partial X \partial Y} \Big|_{ij} + \frac{1}{6} (\Delta X_{ij}^n)^3 \frac{\partial^3 P}{\partial X^3} \Big|_{ij} + \frac{1}{6} (\Delta Y_{ij}^n)^3 \frac{\partial^3 P}{\partial Y^3} \Big|_{ij} \\
 & + \frac{1}{2} (\Delta X_{ij}^n)^2 \Delta Y_{ij}^n \frac{\partial^3 P}{\partial X^2 \partial Y} \Big|_{ij} + \frac{1}{2} \Delta X_{ij}^n (\Delta Y_{ij}^n)^2 \frac{\partial^3 P}{\partial X \partial Y^2} \Big|_{ij}
 \end{aligned} \quad (3.23)$$

The matrix form of the equation above can be written as

$$\begin{bmatrix} P_{ij}^1 - P_{ij} \\ \vdots \\ P_{ij}^n - P_{ij} \end{bmatrix} = \begin{bmatrix} \frac{\Delta X_{ij}^1}{d_0} & \cdot & \cdot & \cdot & \frac{\Delta X_{ij}^n}{d_0} \\ \frac{\Delta Y_{ij}^1}{d_0} & \cdot & \cdot & \cdot & \frac{\Delta Y_{ij}^n}{d_0} \\ \frac{1}{2} \left(\frac{\Delta X_{ij}^1}{d_0} \right)^2 & \cdot & \cdot & \cdot & \frac{1}{2} \left(\frac{\Delta X_{ij}^n}{d_0} \right)^2 \\ \frac{1}{2} \left(\frac{\Delta Y_{ij}^1}{d_0} \right)^2 & \cdot & \cdot & \cdot & \frac{1}{2} \left(\frac{\Delta Y_{ij}^n}{d_0} \right)^2 \\ \frac{\Delta X_{ij}^1}{d_0} \frac{\Delta Y_{ij}^1}{d_0} & \cdot & \cdot & \cdot & \frac{\Delta X_{ij}^n}{d_0} \frac{\Delta Y_{ij}^n}{d_0} \\ \frac{1}{6} \left(\frac{\Delta X_{ij}^1}{d_0} \right)^3 & \cdot & \cdot & \cdot & \frac{\Delta X_{ij}^n}{d_0} \frac{\Delta Y_{ij}^n}{d_0} \\ \frac{1}{6} \left(\frac{\Delta Y_{ij}^1}{d_0} \right)^3 & \cdot & \cdot & \cdot & \frac{1}{6} \left(\frac{\Delta Y_{ij}^n}{d_0} \right)^3 \\ \frac{1}{2} \left(\frac{\Delta X_{ij}^1}{d_0} \right)^2 \frac{\Delta Y_{ij}^1}{d_0} & \cdot & \cdot & \cdot & \frac{1}{2} \left(\frac{\Delta X_{ij}^n}{d_0} \right)^2 \frac{\Delta Y_{ij}^n}{d_0} \\ \frac{1}{2} \frac{\Delta X_{ij}^1}{d_0} \left(\frac{\Delta Y_{ij}^1}{d_0} \right)^2 & \cdot & \cdot & \cdot & \frac{1}{2} \frac{\Delta X_{ij}^n}{d_0} \left(\frac{\Delta Y_{ij}^n}{d_0} \right)^2 \end{bmatrix}^T D \begin{bmatrix} \frac{\partial P}{\partial X} \Big|_{ij} \\ \frac{\partial P}{\partial Y} \Big|_{ij} \\ \frac{\partial^2 P}{\partial X^2} \Big|_{ij} \\ \frac{\partial^2 P}{\partial Y^2} \Big|_{ij} \\ \frac{\partial^2 P}{\partial X \partial Y} \Big|_{ij} \\ \frac{\partial^3 P}{\partial X^3} \Big|_{ij} \\ \frac{\partial^3 P}{\partial Y^3} \Big|_{ij} \\ \frac{\partial^3 P}{\partial X^2 \partial Y} \Big|_{ij} \\ \frac{\partial^3 P}{\partial X \partial Y^2} \Big|_{ij} \end{bmatrix} \quad (3.24)$$

where D is a diagonal matrix to prevent the ill-condition of the dimensional matrix for inversion. Its diagonal entries are $[d_0 \ d_0 \ d_0^2 \ d_0^2 \ d_0^2 \ d_0^3 \ d_0^3 \ d_0^3 \ d_0^3]$. Here d_0 is the maximum distance between the supporting points and the reference point.

After simplifying the expression, Equation (3.24) becomes

$$[\Delta P] = \bar{S}^T D [\partial P] \quad (3.25)$$

After multiplying W matrix on the both sides of the equation above, it is

$$W[\Delta P] = W\bar{S}^T D [\partial P] \quad (3.26)$$

Here W is a diagonal matrix to weigh the contribution of the supporting points. Its diagonal entries are $[d_0 / \sqrt{(\Delta X_{ij}^1)^2 + (\Delta Y_{ij}^1)^2} \ \cdot \ \cdot \ \cdot \ d_0 / \sqrt{(\Delta X_{ij}^n)^2 + (\Delta Y_{ij}^n)^2}]$. According to the fundamental principle of the LSFD method, $\bar{S}$ is multiplied on the both sides of Equation (3.26). It is

$$\bar{S}W[\Delta P] = \bar{S}W\bar{S}^T D [\partial P] \quad (3.27)$$

Then the equation above can be solved as

$$[\partial P] = D^{-1} \left(\bar{S}W\bar{S}^T \right)^{-1} \bar{S}W[\Delta P] \quad (3.28)$$

Therefore all pressure derivatives with respect to the coordinates at the reference point i, j can be expressed as the weighting summation of pressure differences between the supporting points and the reference point, as the following:

$$\begin{cases} \left. \frac{\partial P}{\partial X} \right|_{ij} = \sum_{n=1}^{SN} a_{ij}^n (P_{ij}^n - P_{ij}) & \left. \frac{\partial P}{\partial Y} \right|_{ij} = \sum_{n=1}^{SN} b_{ij}^n (P_{ij}^n - P_{ij}) \\ \left. \frac{\partial^2 P}{\partial X^2} \right|_{ij} = \sum_{n=1}^{SN} c_{ij}^n (P_{ij}^n - P_{ij}) & \left. \frac{\partial^2 P}{\partial Y^2} \right|_{ij} = \sum_{n=1}^{SN} d_{ij}^n (P_{ij}^n - P_{ij}) \end{cases} \quad (3.29)$$

where a_{ij}^n , b_{ij}^n , c_{ij}^n and d_{ij}^n are the coefficients decided only by the grid point coordinates. SN denotes the maximum number of the supporting points.

3.3.3 Discretized Reynolds Equation by Least Square Finite Difference Method

After all pressure derivatives with respect to the coordinates have been approximated by the LSFD scheme, the discretized Reynolds equations for Fukui slip model can be obtained by substituting Equation (3.29) into Equation (3.22). It is

$$\begin{aligned} F_{ij} = & \left\{ \left[\sum_{n=1}^{SN} a_{ij}^n (P_{ij}^n - P_{ij}) \right]^2 + \left[\sum_{n=1}^{SN} b_{ij}^n (P_{ij}^n - P_{ij}) \right]^2 \right\} (K_1 H_{ij}^3 - K_3 P_{ij}^{-2} + 2K_4 P_{ij}^{-3}) \\ & + \left\{ \left. \frac{\partial H}{\partial X} \right|_{ij} \left[\sum_{n=1}^{SN} a_{ij}^n (P_{ij}^n - P_{ij}) \right] + \left. \frac{\partial H}{\partial Y} \right|_{ij} \left[\sum_{n=1}^{SN} b_{ij}^n (P_{ij}^n - P_{ij}) \right] \right\} (3K_1 P_{ij} H_{ij}^2 + 2K_2 H_{ij} + K_3 P_{ij}^{-1}) \\ & + \left[\sum_{n=1}^{SN} c_{ij}^n (P_{ij}^n - P_{ij}) + \sum_{n=1}^{SN} d_{ij}^n (P_{ij}^n - P_{ij}) \right] (K_1 P_{ij} H_{ij}^3 + K_2 H_{ij}^2 + K_3 P_{ij}^{-1} H_{ij} - K_4 P_{ij}^{-2}) \\ & - P_{ij} \left(\Lambda_{xij} \left. \frac{\partial H}{\partial X} \right|_{ij} + \Lambda_{yij} \left. \frac{\partial H}{\partial Y} \right|_{ij} \right) - H_{ij} \left\{ \Lambda_{xij} \left[\sum_{n=1}^{SN} a_{ij}^n (P_{ij}^n - P_{ij}) \right] + \Lambda_{yij} \left[\sum_{n=1}^{SN} b_{ij}^n (P_{ij}^n - P_{ij}) \right] \right\} = 0 \quad (3.30) \end{aligned}$$

3.4 Numerical Solution of Discretized Reynolds Equation

The normalized Reynolds equation has been discretized by the LSFD method. Consequently some iterative method is needed to solve the discretized Reynolds equations.

3.4.1 Implementation of Iterative Methods

In general, there are two types of iterative methods, i.e. the stationary method and the nonstationary method. The stationary method is conventional in the numerical solution, while the nonstationary method is a relatively recent development with the idea of sequences of orthogonal vectors (Barrett et al., 1994). Due to the flexibility and popularity, two stationary methods, Gauss-Seidel and Newton iterative algorithms, are adopted to solve the discretized Reynolds equations.

3.4.1.1 Gauss-Seidel Iterative Method

For a linear system as shown below:

$$\sum_{j=1}^n a_{ij}x_j = b_i, i = 1, 2, \dots, n \quad (3.31)$$

Gauss-Seidel iterative solution is obtained as the following:

$$\Delta x_i^k = \left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^k - \sum_{j=i+1}^n a_{ij}x_j^{(k-1)} \right) / a_{ii}, i = 1, 2, \dots, n \quad (3.32)$$

Although Equation (3.30) is a nonlinear equation about P_{ij} , it can be transformed into a linear equation if some items of P_{ij} are assumed as constants. It is

$$\begin{aligned}
F_{ij} = & \left\{ \left[\left(\sum_{n=1}^{SN} a_{ij}^n (P_{ij}^n - P_{ij}) \right) \left(-\sum_{n=1}^{SN} a_{ij}^n \right) + \left(\sum_{n=1}^{SN} b_{ij}^n (P_{ij}^n - P_{ij}) \right) \left(-\sum_{n=1}^{SN} b_{ij}^n \right) \right] (K_1 H_{ij}^3 - K_3 P_{ij}^{-2} + 2K_4 P_{ij}^{-3}) \right. \\
& + \left[\frac{\partial H}{\partial X} \Big|_{ij} \left(-\sum_{n=1}^{SN} a_{ij}^n \right) + \frac{\partial H}{\partial Y} \Big|_{ij} \left(-\sum_{n=1}^{SN} b_{ij}^n \right) \right] (3K_1 P_{ij} H_{ij}^2 + 2K_2 H_{ij} + K_3 P_{ij}^{-1}) \\
& + \left[\left(-\sum_{n=1}^{SN} c_{ij}^n \right) + \left(-\sum_{n=1}^{SN} d_{ij}^n \right) \right] (K_1 P_{ij} H_{ij}^3 + K_2 H_{ij}^2 + K_3 P_{ij}^{-1} H_{ij} - K_4 P_{ij}^{-2}) \\
& - \left(\Lambda_{xij} \frac{\partial H}{\partial X} \Big|_{ij} + \Lambda_{yij} \frac{\partial H}{\partial Y} \Big|_{ij} \right) - H_{ij} \left[\Lambda_{xij} \left(-\sum_{n=1}^{SN} a_{ij}^n \right) + \Lambda_{yij} \left(-\sum_{n=1}^{SN} b_{ij}^n \right) \right] \Big\} P_{ij} \\
& + \left[\left(\sum_{n=1}^{SN} a_{ij}^n (P_{ij}^n - P_{ij}) \right) \sum_{n=1}^{SN} a_{ij}^n P_{ij}^n + \left(\sum_{n=1}^{SN} b_{ij}^n (P_{ij}^n - P_{ij}) \right) \sum_{n=1}^{SN} b_{ij}^n P_{ij}^n \right] (K_1 H_{ij}^3 - K_3 P_{ij}^{-2} + 2K_4 P_{ij}^{-3}) \\
& + \left(\frac{\partial H}{\partial X} \Big|_{ij} \sum_{n=1}^{SN} a_{ij}^n P_{ij}^n + \frac{\partial H}{\partial Y} \Big|_{ij} \sum_{n=1}^{SN} b_{ij}^n P_{ij}^n \right) (3K_1 P_{ij} H_{ij}^2 + 2K_2 H_{ij} + K_3 P_{ij}^{-1}) \\
& + \left(\sum_{n=1}^{SN} c_{ij}^n P_{ij}^n + \sum_{n=1}^{SN} d_{ij}^n P_{ij}^n \right) (K_1 P_{ij} H_{ij}^3 + K_2 H_{ij}^2 + K_3 P_{ij}^{-1} H_{ij} - K_4 P_{ij}^{-2}) \\
& - H_{ij} \left(\Lambda_{xij} \sum_{n=1}^{SN} a_{ij}^n P_{ij}^n + \Lambda_{yij} \sum_{n=1}^{SN} b_{ij}^n P_{ij}^n \right) = 0
\end{aligned} \tag{3.33}$$

Therefore the iterative step is obtained by

$$\Delta P_{ij} = -F_{ij} / \psi_{ij} \tag{3.34}$$

where

$$\begin{aligned}
\psi_{ij} = & \left[\left(\sum_{n=1}^{SN} a_{ij}^n (P_{ij}^n - P_{ij}) \right) \left(-\sum_{n=1}^{SN} a_{ij}^n \right) + \left(\sum_{n=1}^{SN} b_{ij}^n (P_{ij}^n - P_{ij}) \right) \left(-\sum_{n=1}^{SN} b_{ij}^n \right) \right] (K_1 H_{ij}^3 - K_3 P_{ij}^{-2} + 2K_4 P_{ij}^{-3}) \\
& + \left[\frac{\partial H}{\partial X} \Big|_{ij} \left(-\sum_{n=1}^{SN} a_{ij}^n \right) + \frac{\partial H}{\partial Y} \Big|_{ij} \left(-\sum_{n=1}^{SN} b_{ij}^n \right) \right] (3K_1 P_{ij} H_{ij}^2 + 2K_2 H_{ij} + K_3 P_{ij}^{-1}) \\
& + \left[\left(-\sum_{n=1}^{SN} c_{ij}^n \right) + \left(-\sum_{n=1}^{SN} d_{ij}^n \right) \right] (K_1 P_{ij} H_{ij}^3 + K_2 H_{ij}^2 + K_3 P_{ij}^{-1} H_{ij} - K_4 P_{ij}^{-2}) \\
& - \left(\Lambda_{xij} \frac{\partial H}{\partial X} \Big|_{ij} + \Lambda_{yij} \frac{\partial H}{\partial Y} \Big|_{ij} \right) - H_{ij} \left[\Lambda_{xij} \left(-\sum_{n=1}^{SN} a_{ij}^n \right) + \Lambda_{yij} \left(-\sum_{n=1}^{SN} b_{ij}^n \right) \right]
\end{aligned} \tag{3.35}$$

3.4.1.2 Newton Iterative Method

For a nonlinear system as shown below:

$$f_i(x_1, x_2, \dots, x_n) = 0, i = 1, 2, \dots, n \quad (3.36)$$

Newton iterative solution is obtained as the following:

$$\Delta x_i^k = -\frac{f_i}{\partial f_i / \partial x_i}, i = 1, 2, \dots, n \quad (3.37)$$

According to Equation (3.30), the derivative of F_{ij} with respect to P_{ij} is expressed as

$$\begin{aligned} \frac{\partial F_{ij}}{\partial P_{ij}} = & 2 \left[\sum_{n=1}^{SN} a_{ij}^n (P_{ij}^n - P_{ij}) \right] \left[- \left(\sum_{n=1}^{SN} a_{ij}^n \right) \right] (K_1 H_{ij}^3 - K_3 P_{ij}^{-2} + 2K_4 P_{ij}^{-3}) \\ & + 2 \left[\sum_{n=1}^{SN} b_{ij}^n (P_{ij}^n - P_{ij}) \right] \left[- \left(\sum_{n=1}^{SN} b_{ij}^n \right) \right] (K_1 H_{ij}^3 - K_3 P_{ij}^{-2} + 2K_4 P_{ij}^{-3}) \\ & + \left\{ \left[\sum_{n=1}^{SN} a_{ij}^n (P_{ij}^n - P_{ij}) \right]^2 + \left[\sum_{n=1}^{SN} b_{ij}^n (P_{ij}^n - P_{ij}) \right]^2 \right\} (2K_3 P_{ij}^{-3} - 6K_4 P_{ij}^{-4}) \\ & + \left\{ \frac{\partial H}{\partial X} \bigg|_{ij} \left[- \left(\sum_{n=1}^{SN} a_{ij}^n \right) \right] + \frac{\partial H}{\partial Y} \bigg|_{ij} \left[- \left(\sum_{n=1}^{SN} b_{ij}^n \right) \right] \right\} (3K_1 P_{ij} H_{ij}^2 + 2K_2 H_{ij} + K_3 P_{ij}^{-1}) \\ & + \left\{ \frac{\partial H}{\partial X} \bigg|_{ij} \left[\sum_{n=1}^{SN} a_{ij}^n (P_{ij}^n - P_{ij}) \right] + \frac{\partial H}{\partial Y} \bigg|_{ij} \left[\sum_{n=1}^{SN} b_{ij}^n (P_{ij}^n - P_{ij}) \right] \right\} (3K_1 H_{ij}^2 - K_3 P_{ij}^{-2}) \\ & + \left[- \left(\sum_{n=1}^{SN} c_{ij}^n \right) - \left(\sum_{n=1}^{SN} d_{ij}^n \right) \right] (K_1 P_{ij} H_{ij}^3 + K_2 H_{ij}^2 + K_3 P_{ij}^{-1} H_{ij} - K_4 P_{ij}^{-2}) \\ & + \left[\sum_{n=1}^{SN} c_{ij}^n (P_{ij}^n - P_{ij}) + \sum_{n=1}^{SN} d_{ij}^n (P_{ij}^n - P_{ij}) \right] (K_1 H_{ij}^3 - K_3 P_{ij}^{-2} H_{ij} + 2K_4 P_{ij}^{-3}) \\ & - \left(\Lambda_{xij} \frac{\partial H}{\partial X} \bigg|_{ij} + \Lambda_{yij} \frac{\partial H}{\partial Y} \bigg|_{ij} \right) - H_{ij} \left\{ \Lambda_{xij} \left[- \left(\sum_{n=1}^{SN} a_{ij}^n \right) \right] + \Lambda_{yij} \left[- \left(\sum_{n=1}^{SN} b_{ij}^n \right) \right] \right\} \end{aligned} \quad (3.38)$$

Therefore the iterative step can be obtained as

$$\Delta P_{ij} = -\frac{F_{ij}}{\partial F_{ij} / \partial P_{ij}} \quad (3.39)$$

3.4.2 Effect of Successive Over Relaxation Factor

Although the Gauss-Seidel and Newton iterative methods are implemented to solve the discretized Reynolds equations, sometimes a successive over relaxation (SOR) factor ω is needed to reduce computational time or ensure the global convergence in the iterative process. A one-dimensional root-finding study will be presented to demonstrate the effect of the SOR factor.

As shown in Figure 3.3, the general procedure of one-dimensional root-finding consists geometrically of extending the tangent line at a current point $x_i^{(k)}$ until it crosses zero, then setting the next guess $x_i^{(k+1)}$ to the abscissa of that zero-crossing, and repeating this process until the intersection point of the curve with the abscissa is found. It is obvious that a large value of SOR factor can be used to magnify the iterative step $\Delta x_i^{(k)}$ as $\omega \Delta x_i^{(k)}$ ($\omega > 1$) and accelerate the convergence rate.

Fig. 3.3 1D root-finding procedure in good condition

As shown in Figure 3.4, the curve shape is too oscillatory in the computation domain. It follows that no solution exists in the abscissa because the zero-crossing of the extended tangent line is outside of the root of the equation. It means that the iterative process will not converge in the numerical computation. Hence, another set of initial guesses for variable $x_i^{(0)}$ will be used to solve this problem. However, in most conditions, this phenomenon is generated from the natural property of the geometry. Therefore a small value of SOR factor is needed to diminish the iterative step $\Delta x_i^{(k)}$ as $\omega \Delta x_i^{(k)}$ ($\omega < 1$) to ensure the global convergence in the iterative process.

Fig. 3.4 1D root-finding procedure in ill condition

In general, it is not possible to calculate in advance ω that is optimal with respect to the size of the mesh. If the size of the mesh is large for a complicated geometry, the tangents of the grid points are continuous enough to adopt a large value of SOR factor. Inversely SOR factor should be reduced. This is why there is a limitation for the least square finite difference multigrid method to be discussed in Chapter 5, which needs to apply the small-scale grid in the numerical simulation of the slider air bearing.

With the implementation of the SOR factor, the Gauss-Seidel iterative method and the Newton iterative method are known as the Gauss-Seidel SOR method and the Newton-SOR method. Hence the new pressure values in the numerical simulation of the slider air bearing are updated by

$$P_{ij}^{(k+1)} = P_{ij}^{(k)} + \omega \Delta P_{ij} \quad (3.40)$$

3.4.3 Plate Slider Simulation with Different Iterative Methods

In this section, both Gauss-Seidel SOR and Newton-SOR are used to simulate the pressure distribution of a Plate Slider in the static condition. The parameters of the Plate Slider are shown as below:

Table 3.1 Parameters of Plate Slider

Minimum flying height	$h_m = 25\text{nm}$	Dynamic viscosity	$\mu = 2.08 \times 10^{-5} \text{Pa/sec}$
Total length	$L = 4.0\mu\text{m}$	Disk velocity	$U = 25\text{m/s}$
Total width	$W = 3.3\mu\text{m}$	Pitch angle	$\alpha = 1.0 \times 10^{-2} \text{rad}$
Ambient pressure	$P_a = 1.01325 \times 10^5 \text{Pa}$	Roll angle	$\theta = 0\text{rad}$
Molecular mean free path	$\lambda = 63.5\text{nm}$	Sway angle	$\varphi = 0\text{rad}$

Figures 3.5 & 3.6 show the geometry profile and pressure profile of the Plate Slider meshed by 193×193 uniform grids.

Fig. 3.5 Geometry profile of Plate Slider

Fig. 3.6 Pressure profile of Plate Slider

As shown in Figure 3.6, the pressure profile in the leading edge slowly increases from the ambient pressure till the maximum pressure near to the trailing edge, because the airflow clearance between the slider and the disk gradually becomes smaller and compresses the airflow within the boundary. After the pressure peaks, the pressure profile sharply decreases to the ambient pressure due to the constraint of the boundary condition. In addition, if the minimum flying height becomes smaller, the pressure peak will increase and the position of the pressure peak will move toward the trailing edge.

In the computation, the order of two-dimensional Taylor series expansion for the pressure derivative approximation is 9. The number of the supporting points around the reference point is 15. The initial pressure values at all grid points are set as one standard atmosphere pressure, and the boundary pressure values are fixed as one standard atmosphere pressure in the whole iterative process. The convergence criterion is set as $|R_{ij}|_{\max} \leq 1.0 \times 10^{-3}$ where $|R_{ij}|$ is the absolute residual value of the Reynolds equation for every calculated reference point.

The computed results by using either the Gauss-Seidel or the Newton method with or without the SOR factor are exactly the same. When the computed results are compared with the solution of the Direct Simulation Monte Carlo (DSMC) method as shown in Figure 3.7 (Huang and Bogy, 1997), the pressure distributions as shown in Figure 3.8 are similar and accurate. In Figures 3.7 & 3.8, the plots on the left-top, the right-top, the left-down and the right-down depict the pressure profiles viewed from the front, the left, the top and the oblique.

Fig. 3.7 Pressure profile of Plate Slider by DSMC method

Fig. 3.8 Pressure profile of Plate Slider by LSFD method

The comparisons of the computational time and the maximum SOR factor for the different iterative methods are shown in Table 3.2 and Table 3.3 respectively.

Table 3.2 Comparison of computational time for different iterative methods

	97×97	145×145	193×193
Gauss-Seidel SOR	1min 6secs	3mins 40secs	14mins 18secs
Newton-SOR	56secs	2mins 39secs	5mins 38secs

Table 3.3 Comparison of SOR factor for different iterative methods

	97×97	145×145	193×193
Gauss-Seidel SOR	1.55	1.70	1.65
Newton-SOR	1.60	1.75	1.80

The comparisons are based on uniform grid distribution and the computations are completed on the Dell 530 workstation with 2.2GHz CPU and 512MB RAM. As shown in Table 3.2, The Newton-SOR iterative method is more efficient than the Gauss-Seidel SOR iterative method. This is because the Newton-SOR iterative method can adopt the higher SOR factor number, which will accelerate the convergence rate, as shown in Table 3.3. Comparing the formulae in Equation (3.35) and Equation (3.38), more information is given in Equation (3.38). It means that the iterative step in Equation (3.39) is more accurate than that in Equation (3.34). Hence the Newton-SOR iterative method is prone to apply a higher SOR factor to increase the iterative step.

3.5 Summary

In this chapter, the classical Reynolds equation, which is suitable for the numerical simulation of the slider air bearing in hard disk drives, is derived by integrating the momentum equation and the continuity equation in fluid mechanics. After introducing the first-order slip model, the second-order slip model and the Fukui slip model, the modified Reynolds equation for Fukui slip model is chosen to simulate the slider air bearing for its extensive application in the wide range of Knudsen numbers. Then the LSFD method is introduced to numerically discretize the modified Reynolds equation by approximating the pressure derivatives with respect to the coordinates for every calculated grid point. As the iterative methods to solve the numerically discretized equation, the Gauss-Seidel and the Newton algorithms are illustrated in detail. On the consideration of the effect of the SOR factor, the Gauss-Seidel SOR and the Newton-SOR iterative methods are compared to solve the discretized Reynolds equations for Fukui model. By the numerical simulation of the Plate Slider, the Newton-SOR iterative method is proven more efficient than Gauss-Seidel SOR.

Chapter 4

Full Approximation Multigrid Method for Slider Air Bearing Simulation

4.1 Introduction

The numerical simulation of slider air bearing is carried out in the previous chapter. The LSFD method together with the Newton-SOR iterative method is applied to simulate the complex slider air bearing configuration where a large number of grid points is needed. This requires a large amount of computational time. To further accelerate the convergence rate and increase the efficiency of the procedure, this chapter will apply the full approximation multigrid (FAM) method, proposed by Brandt (1977), is introduced for the numerical simulation of slider air bearing.

The Fourier analysis of the convergence of various iterative methods showed that although short wavelength Fourier modes are reduced rapidly, long wavelength Fourier modes decay slowly (Brandt, 1984). However, the multigrid method was found to be able to accelerate the convergence of traditional iterative methods in both Fourier modes.

By introducing the coarse grid in the iterative step, it leads to a decomposition of the solution error into rough error components, which are difficult for the solution in the fine grid, but easy for the solution in the coarse grid. In the fine grid, the short

wavelength components of the error modes are reduced with a small number of iterations by a traditional iterative method, as well as the long wavelength components of the error modes are transferred to the short wavelength components in the coarse grid, which can be reduced by a sufficient number of iterations. As a result, the efficiency of the procedure is greatly enhanced.

As one scheme of the multigrid method, the full approximation algorithm is the most commonly used multigrid algorithm for solving nonlinear equations.

4.2 Multigrid Principle

In general, the traditional iterative methods are effective at eliminating the high frequency or oscillatory components of the error, while leaving the low frequency or smooth components relatively unchanged. Hence, the principle of the multigrid method is to first smooth the high frequency errors in a fine grid and then transfer the design variables and residuals to a series of coarser grids. Therefore, the low frequency errors in the fine grid are transferred to the high frequency errors in the coarser grids. Since the coarser grid has a small number of grid points, less computational time is needed to smooth these errors. Thereafter they are transferred back to the fine grid to correct the solutions and reduce the low frequency errors in the fine grid. This process is repeated over a sufficient number of times until satisfactory convergence in the fine grid is obtained.

For the point above, it can be referred to a wave study, as shown in Figure 4.1. It is obvious that a smooth wave with $k = 4$ in a grid Ω^h with $n = 13$ grid points has been

projected directly to the grid Ω^{2h} with $n = 7$ grid points. In this coarse grid, the projected wave still has a wave number of $k = 4$. However, the smooth wave in Ω^h looks more oscillatory in Ω^{2h} . This suggests that when relaxation begins to cease in the fine grid, highlighting the predominance of smooth error modes, it is advisable to move to a coarser grid where the smooth error modes appear more oscillatory and relaxation will be more effective.

Fig. 4.1 Wave configuration using different grids

4.2.1 Iterative Strategies

Herein the error equations are introduced to illustrate two iterative strategies of the multigrid method, as shown below:

$$AE = R = F - AV \quad (4.1)$$

where A is the parametric matrix already known. E is the error approximation matrix between the exact solution matrix U and the variable approximation matrix V , shown as $E = U - V$. R is the residual matrix calculated in the iterative step. F is the constant matrix on the right side of the original equation $AU = F$.

Equation (4.1) shows that the error approximations can be directly relaxed by using the error equations. Relaxation in the original equations $AU = F$ with the arbitrary initial guesses V_0 is equivalent to relaxing in the error equations $AE = R$ with the specific initial guess E_0 . This intimate connection between the original and the error equations further motivates the application of the error equations. As a result, two different iterative strategies of the multigrid method are proposed.

4.2.1.1 Nested Iteration

The nested iteration strategy is prone to obtain the better initial guesses by using the coarse grids. Its flowchart is described as below:

1. Relax the original equations $A^{2^n h} U^{2^n h} = F^{2^n h}$ with initial guesses $V_0^{2^n h}$ in $\Omega^{2^n h}$ to obtain initial guesses $V_0^{2^{(n-1)} h}$ for $\Omega^{2^{(n-1)} h}$,
 -
 -
 -
- N-2. Relax the original equations $A^{4h} U^{4h} = F^{4h}$ with initial guesses V_0^{4h} in Ω^{4h} to obtain initial guesses V_0^{2h} for Ω^{2h} ,

- N-1. Relax the original equations $A^{2h}U^{2h} = F^{2h}$ with initial guesses V_0^{2h} in Ω^{2h} to obtain initial guesses V_0^h for Ω^h ,
- N. Relax the original equations $A^hU^h = F^h$ with initial guesses V_0^h in Ω^h to obtain variable approximations to the solution.

Although some improvement is developed by the nested iteration strategy, the final iteration will cease if smooth components in the fine grid still remain. Hence, it is not commonly applicable in practical application.

4.2.1.2 Coarse Grid Correction

The coarse grid correction strategy incorporates the idea of using the error equations to relax the errors. Its flowchart is described as below:

1. Relax the original equations $A^hU^h = F^h$ with initial guesses V_0^h in Ω^h to obtain variable approximations V_1^h ,
2. Compute the residuals R^h by $R^h = F^h - A^hV_1^h$ and transfer R^h in Ω^h to R^{2h} in Ω^{2h} ,
3. Relax the error equations $A^{2h}E^{2h} = R^{2h}$ with initial guesses E_0^{2h} in Ω^{2h} to obtain error approximations E_1^{2h} , and transfer E_1^{2h} in Ω^{2h} to E^h in Ω^h ,
4. Correct the variable approximations V_1^h obtained in Ω^h with the error approximations E^h to obtain new variable approximations V_2^h by $V_2^h \leftarrow V_1^h + E^h$.

Here subscripts 0~2 means the initial guesses, the first-calculated values and the second-calculated values respectively. Due to the stability and flexibility of the coarse grid correction, it is extensively applicable in many practical applications.

4.2.2 Transfer Mechanisms

In the discussion of intergrid transfer mechanisms, the coarse grid is assumed to have twice the grid spacing of the fine grid for it seems useless in using grid spacing with ratios other than 2. For the coarse grid correction scheme, it is required to transfer the variable approximations V^h and the residuals R^h from the fine grid to the coarse grid and transfer the error approximations E^{2h} from the coarse grid to the fine grid. These two steps are basic procedures in the FAM method, generally known as restriction and prolongation.

To simplify the illustration of the transfer mechanisms, the restriction and prolongation mechanisms are presented for the one-dimensional geometry. The algorithms for the two-dimensional geometry will be introduced later when the FAM is applied into the numerical simulation of the slider air bearing.

4.2.2.1 Restriction Mechanism

As one part of intergrid transfer mechanisms, the restriction is to move vectors from a fine grid to a coarse grid. It is defined by $V^{2h} = I_h^{2h} V^h$, where I_h^{2h} denotes the fine-to-coarse restriction operator. The most obvious restriction operator is the injection, which means that the coarse grid obtains its values directly from the fine grid. Therefore the V^{2h} is obtained by $v_i^{2h} = v_{2i-1}^h$, $1 \leq i \leq (N+1)/2$, where N is the number of the fine grid points. Figure 4.2 illustrates how the variable approximations V^h are transferred from the fine grid to the coarse grid.

Fig. 4.2 Illustration of restriction mechanism

4.2.2.2 Prolongation Mechanism

As the other part of intergrid transfer mechanisms, the prolongation is to move vectors from a coarse grid to a fine grid. It is defined by $E^h = I_{2h}^h E^{2h}$, where I_{2h}^h denotes the coarse-to-fine prolongation operator. The most obvious prolongation operator is the linear interpolation, which means that the fine grid obtains or produces its values from the coarse grid. Therefore the E^h are obtained by $e_{2i-1}^h = e_i^{2h}$ and produced by $e_{2i}^h = \frac{1}{2}(e_i^{2h} + e_{i+1}^{2h})$ $1 \leq i \leq M$, where M is the number of the coarse grid points. Figure 4.3 illustrates how the error approximations E^{2h} are transferred from the coarse grid to the fine grid.

Fig. 4.3 Illustration of prolongation mechanism

4.2.3 Classical Cycle Strategies

For the multigrid method, each time that the system moves from the finest grid down to some coarser grids and back again is known as a cycle. There are a number of different types of cycles available. Herein two classical cycle strategies, i.e. the v-cycle and the w-cycle, are introduced on the basis of three-level grids. In general, the nested iteration strategy can be used to obtain better initial guesses.

4.2.3.1 V-Cycle Strategy

The v-cycle strategy means that the restriction and prolongation steps are completed in a straight way, from the first-level grid to the third-level grid and then back to the first-level grid, as shown in Figure 4.4.

Fig. 4.4 Illustration of classical v-cycle strategy

Here a filled circle denotes a relaxation operation and a filled box denotes the exact solutions obtained by a relaxation operation in the third-level grid. A descending line $\searrow$ denotes that a fine grid is restricted down to a coarse grid. An ascending line $\nearrow$ denotes a coarse grid is interpolated up to a fine grid.

The pattern on the left is the classical v-cycle calculated from the finest grid, while the pattern on the right is the classical v-cycle calculated from the coarsest grid whose function is to obtain the better initial guesses by the nested iteration strategy.

4.2.3.2 W-Cycle Strategy

Compared with the v-cycle strategy, the w-cycle strategy means the presence of the repeated prolongation and restriction steps between the coarse grid and the fine grid, as shown in Figure 4.5.

Fig. 4.5 Illustration of classical w-cycle strategy

The pattern on the left is the classical w-cycle calculated from the finest grid, while the pattern on the right is the classical w-cycle calculated from the coarsest grid whose function is to obtain the better initial guesses by the nested iteration strategy.

There are many different diagrams for these cycle strategies, e.g. the μ -cycle and the full multigrid v-cycle. However, the v-cycle and the w-cycle are the fundamentals of all diagrams. It is obvious that these diagrams can be extended to the multi-level grids.

4.2.4 Application for Linear and Nonlinear Equations

So far, all multigrid principles are introduced based on the linear equations. However an extension of the multigrid method is able to solve the nonlinear equations with the high efficiency. Herein the classical v-cycle strategy calculated from the finest grid is presented to demonstrate the applications of the multigrid algorithm for linear and nonlinear equations.

4.2.4.1 Application for Linear Equations

In the application of multigrid algorithm for linear equations, the calculated vectors in the transfer operation are the error approximations because the linear relationship exists between the variable approximations and the equation residuals. Hence, the flowchart of the multigrid algorithm for linear equations is simplified as

1. Relax the original equations $A^h U^h = F^h$ with initial guesses V_0^h in the first-level grid to obtain variable approximations V_1^h and residuals R_1^h by $R_1^h = F^h - A^h V_1^h$,
2. Restrict R_1^h from the first-level grid to the second-level grid to obtain residuals R_1^{2h} by $R_1^{2h} = I_h^{2h} R_1^h$,
3. Relax the error equations $A^{2h} E^{2h} = F^{2h} = R_1^{2h}$ with initial guesses E_0^{2h} in the second-level grid to obtain error approximations E_1^{2h} and residuals R_2^{2h} by $R_2^{2h} = F^{2h} - A^{2h} E_1^{2h}$,
4. Restrict R_2^{2h} from the second-level grid to the third-level grid to obtain residuals R_1^{4h} by $R_1^{4h} = I_{2h}^{4h} R_2^{2h}$,
5. Relax the error equations $A^{4h} E^{4h} = F^{4h} = R_1^{4h}$ with initial guesses E_0^{4h} in the third-level grid to obtain error approximations E_1^{4h} ,
6. Prolong E_1^{4h} back to the second-level grid to obtain error approximations E_2^{2h} by $E_2^{2h} \leftarrow E_1^{2h} + I_{4h}^{2h} E_1^{4h}$,
7. Relax the error equations $A^{2h} E^{2h} = F^{2h} = R_1^{2h}$ with initial guesses E_2^{2h} in the second-level grid to obtain error approximations E_3^{2h} ,
8. Prolong E_3^{2h} back to the first-level grid to obtain variable approximations V_2^h by $V_2^h \leftarrow V_1^h + I_{2h}^h E_3^{2h}$.

4.2.4.2 Application for Nonlinear Equations

In the application of multigrid algorithm for nonlinear equations, the calculated vectors in the transfer operation are the variable approximations because there is not the obvious relationship between the variable approximations and the equation residuals. As one of multigrid algorithms for nonlinear equations, the full approximation scheme is most commonly adopted in many implementations with high efficiency, as shown below:

1. Relax the original equations $A^h U^h = F^h$ with initial guesses V_0^h in the first-level grid to obtain variable approximations V_1^h and residuals R_1^h by $R_1^h = F^h - A^h V_1^h$,
2. Restrict V_1^h and R_1^h from the first-level grid to the second-level grid to obtain variable approximations V_1^{2h} and residuals R_1^{2h} by $V_1^{2h} = I_h^{2h} V_1^h$ and $R_1^{2h} = I_h^{2h} R_1^h$,
3. Relax the modified equations $A^{2h} U^{2h} = F^{2h} = A^{2h} V_1^{2h} + R_1^{2h}$ with initial guesses V_1^{2h} in the second-level grid to obtain variable approximations V_2^{2h} and residuals R_2^{2h} by $R_2^{2h} = F^{2h} - A^{2h} V_2^{2h}$,
4. Restrict V_2^{2h} and R_2^{2h} from the second-level grid to the third-level grid to obtain variable approximations V_1^{4h} and residuals R_1^{4h} by $V_1^{4h} = I_{2h}^{4h} V_2^{2h}$ and $R_1^{4h} = I_{2h}^{4h} R_2^{2h}$,
5. Relax the modified equations $A^{4h} U^{4h} = F^{4h} = A^{4h} V_1^{4h} + R_1^{4h}$ with initial guesses V_1^{4h} in the third-level grid to obtain error approximations E^{4h} by $E^{4h} = V_2^{4h} - V_1^{4h}$,
6. Prolong E^{4h} back to the second-level grid to obtain variable approximations V_3^{2h} by $V_3^{2h} \leftarrow V_2^{2h} + I_{4h}^{2h} E^{4h}$,
7. Relax the modified equations $A^{2h} U^{2h} = F^{2h} = A^{2h} V_1^{2h} + R_1^{2h}$ with initial guesses V_3^{2h} in the second-level grid to obtain error matrix E^{2h} by $E^{2h} = V_4^{2h} - V_1^{2h}$,

8. Prolong E^{2h} back to the first-level grid to obtain variable approximations V_2^h

$$\text{by } V_2^h \leftarrow V_1^h + I_{2h}^h E^{2h}.$$

4.3 Full Approximation Scheme for Slider Air Bearing Simulation

After the discretized Reynolds equations were solved by the Newton-SOR iterative method mentioned in Section 3.4, the numerical simulation of the slider air bearing has been developed. To further accelerate the convergence rate of the procedure, the FAM algorithm is applied by transferring the vectors between the fine grid and the coarse grid. As shown in Figure 4.6, the number sequence relationship exists between the fine grid and the coarse grid. If i, j is the number sequence of the vector in the coarse grid, then $2i-1, 2j-1$ is the number sequence of the same vector in the fine grid.

Fig. 4.6 Intergrid configuration of slider air bearing mesh

Based on the above intergrid configuration of the grids for the slider air bearing, the restriction and prolongation operators for the numerical simulation of the slider air bearing are specifically introduced in the followings.

4.3.1 Restriction Operator

There are many restriction operators applied in the implementation, e.g. the straight injection, the 7-point restriction, the half weighting restriction and the full weighting restriction. Considering the high accuracy and efficiency, the full weighting restriction operator is adopted in this dissertation. It is the weighted average of the values at the surrounding 9 grid points. If the restriction operation is expressed as $V^{2h} = I_h^{2h} V^h$, then V can represent many transferred vectors in the slider air bearing simulation, e.g. the pressure approximations P , the residuals R , the X-coordinates X , the Y-coordinates Y , the heights H , the gradients of height against X-coordinate $\partial H / \partial X$, the gradients of height against Y-coordinate $\partial H / \partial Y$, the air bearing numbers in X-direction Λ_x and the air bearing numbers in Y-direction Λ_y . All of these design variables can be transferred from the fine grid to the coarse grid to further increase the computational efficiency.

Based on the full weighting restriction scheme, the components of the vector V^{2h} can be obtained by

$$v_{ij}^{2h} = \frac{1}{16} [v_{2i-1,2j-1}^h + v_{2i-1,2j+1}^h + v_{2i+1,2j-1}^h + v_{2i+1,2j+1}^h + 2(v_{2i,2j-1}^h + v_{2i,2j+1}^h + v_{2i-1,2j}^h + v_{2i+1,2j}^h) + 4v_{2i,2j}^h] \quad (4.2)$$

4.3.2 Prolongation Operator

As the inverse step of the restriction operator, many different prolongation schemes can be implemented, e.g. the linear interpolation, the first-order interpolation and the second-order interpolation. Due to the high efficiency of the linear interpolation prolongation, it is adopted in this dissertation. If the prolongation operation is expressed as $V^h = I_{2h}^h V^{2h}$, then V represents the errors E , which are transferred from the coarse grid to the fine grid.

Based on the linear interpolation prolongation scheme, the components of the vector V^h can be obtained by

$$\begin{cases} v_{2i,2j}^h = v_{i,j}^{2h} \\ v_{2i+1,2j}^h = \frac{1}{2}(v_{i,j}^{2h} + v_{i+1,j}^{2h}) \\ v_{2i,2j+1}^h = \frac{1}{2}(v_{i,j}^{2h} + v_{i,j+1}^{2h}) \\ v_{2i+1,2j+1}^h = \frac{1}{4}(v_{i,j}^{2h} + v_{i+1,j}^{2h} + v_{i,j+1}^{2h} + v_{i+1,j+1}^{2h}) \end{cases} \quad (4.3)$$

4.3.3 Full Approximation Scheme in V-Cycle Strategy

The full approximation scheme, the nonlinear version of the multigrid method, is applied to accelerate the convergence of the slider air bearing simulation.

Based on the discretized Reynolds equations, the flowchart of the FAM algorithm in the three-level v-cycle strategy calculated from the finest grid is shown as the following:

1. Relax three times on the discretized Reynolds equations $F^h(P^h) = 0$ with initial guesses $P_0^h = P_a$ in the first-level grid with Newton-SOR iterative method and estimate whether the convergence criterion in the first-level grid is satisfied, to obtain pressure approximations P_1^h and residuals R_1^h by $R_1^h = 0 - F^h(P_1^h)$,
2. Restrict P_1^h and R_1^h from the first-level grid to the second-level grid with the full weighting scheme to obtain pressure approximations P_1^{2h} and residuals R_1^{2h} by $P_1^{2h} = I_h^{2h} P_1^h$ and $R_1^{2h} = I_h^{2h} R_1^h$,
3. Relax three times on the discretized Reynolds equations $F^{2h}(P^{2h}) = F^{2h}(P_1^{2h}) + R_1^{2h}$ with initial guesses $P_0^{2h} = P_1^{2h}$ in the second-level grid with Newton-SOR iterative method to obtain pressure approximations P_2^{2h} and residuals R_2^{2h} by $R_2^{2h} = F^{2h}(P_1^{2h}) + R_1^{2h} - F^{2h}(P_2^{2h})$,
4. Restrict P_2^{2h} and R_2^{2h} from the second-level grid to the third-level grid with the full weighting scheme to obtain pressure approximations P_1^{4h} and residuals R_1^{4h} by $P_1^{4h} = I_{2h}^{4h} P_2^{2h}$ and $R_1^{4h} = I_{2h}^{4h} R_2^{2h}$,
5. Relax many times on the discretized Reynolds equations $F^{4h}(P^{4h}) = F^{4h}(P_1^{4h}) + R_1^{4h}$ with initial guesses $P_0^{4h} = P_1^{4h}$ in the third-level grid with Newton-SOR iterative method until the convergence criterion in the third-level grid is satisfied, to obtain error approximations E^{4h} by $E^{4h} = P_2^{4h} - P_1^{4h}$,
6. Prolong E^{4h} back to the second-level grid with the linear interpolation scheme to obtain pressure approximations P_3^{2h} by $P_3^{2h} \leftarrow P_2^{2h} + I_{4h}^{2h} E^{4h}$,
7. Relax many times on the discretized Reynolds equations $F^{2h}(P^{2h}) = F^{2h}(P_1^{2h}) + R_1^{2h}$ with initial guesses $P_0^{2h} = P_3^{2h}$ in the second-level grid with Newton-SOR iterative method until the convergence criterion in the

second-level grid is satisfied, to obtain error approximations E^{2h} by

$$E^{2h} = P_4^{2h} - P_1^{2h},$$

8. Prolong E^{2h} back to the first-level grid with the linear interpolation scheme to obtain pressure approximations P_2^h by $P_2^h \leftarrow P_1^h + I_{2h}^h E^{2h}$.

The whole procedure will cease as soon as the convergence criterion in the first-level grid is satisfied. In general, the convergence criterion of the first-level grid is more relaxing than that of the second-level grid, while the convergence criterion of the second-level grid is more relaxing than that of the third-level grid. Therefore, computational efficiency can be enhanced more reasonably.

4.3.4 Full Approximation Scheme in W-Cycle Strategy

As another cycle strategy of the full approximation scheme in conjunction with the discretized Reynolds equations, the three-level w-cycle strategy calculated from the finest grid is shown as the following:

1. Relax three times on the discretized Reynolds equations $F^h(P^h) = 0$ with initial guesses $P_0^h = P_a$ in the first-level grid with Newton-SOR iterative method and estimate whether the convergence criterion in the first-level grid is satisfied, to obtain pressure approximations P_1^h and residuals R_1^h by $R_1^h = 0 - F^h(P_1^h)$,
2. Restrict P_1^h and R_1^h from the first-level grid to the second-level grid with the full weighting scheme to obtain pressure approximations P_1^{2h} and residuals R_1^{2h} by $P_1^{2h} = I_h^{2h} P_1^h$ and $R_1^{2h} = I_h^{2h} R_1^h$,

3. Relax three times on the discretized Reynolds equations $F^{2h}(P^{2h}) = F^{2h}(P_1^{2h}) + R_1^{2h}$ with initial guesses $P_0^{2h} = P_1^{2h}$ in the second-level grid with Newton-SOR iterative method to obtain pressure approximations P_2^{2h} and residuals R_2^{2h} by $R_2^{2h} = F^{2h}(P_1^{2h}) + R_1^{2h} - F^{2h}(P_2^{2h})$,
4. Restrict P_2^{2h} and R_2^{2h} from the second-level grid to the third-level grid with the full weighting scheme to obtain pressure approximations P_1^{4h} and residuals R_1^{4h} by $P_1^{4h} = I_{2h}^{4h} P_2^{2h}$ and $R_1^{4h} = I_{2h}^{4h} R_2^{2h}$,
5. Relax many times on the discretized Reynolds equations $F^{4h}(P^{4h}) = F^{4h}(P_1^{4h}) + R_1^{4h}$ with initial guesses $P_0^{4h} = P_1^{4h}$ in the third-level grid with Newton-SOR iterative method until the convergence criterion in the third-level grid is satisfied, to obtain error approximations E_1^{4h} by $E_1^{4h} = P_2^{4h} - P_1^{4h}$,
6. Prolong E_1^{4h} back to the second-level grid with the linear interpolation scheme to obtain pressure approximations P_3^{2h} by $P_3^{2h} \leftarrow P_2^{2h} + I_{4h}^{2h} E_1^{4h}$,
7. Relax three times on the discretized Reynolds equations $F^{2h}(P^{2h}) = F^{2h}(P_1^{2h}) + R_1^{2h}$ with initial guesses $P_0^{2h} = P_3^{2h}$ in the second-level grid with Newton-SOR iterative method to obtain pressure approximations P_4^{2h} and residuals R_3^{2h} by $R_3^{2h} = F^{2h}(P_1^{2h}) + R_1^{2h} - F^{2h}(P_4^{2h})$,
8. Restrict P_4^{2h} and R_3^{2h} from the second-level grid to the third-level grid with the full weighting scheme to obtain pressure approximations P_3^{4h} and residuals R_2^{4h} by $P_3^{4h} = I_{2h}^{4h} P_4^{2h}$ and $R_2^{4h} = I_{2h}^{4h} R_3^{2h}$,
9. Relax many times on the discretized Reynolds equations $F^{4h}(P^{4h}) = F^{4h}(P_3^{4h}) + R_2^{4h}$ with initial guesses $P_0^{4h} = P_3^{4h}$ in the third-level grid with Newton-SOR iterative method until the convergence criterion in the third-level

- grid is satisfied, to obtain error approximations E_2^{4h} by $E_2^{4h} = P_4^{4h} - P_3^{4h}$,
10. Prolong E_2^{4h} back to the second-level grid with the linear interpolation scheme to obtain pressure approximations P_5^{2h} by $P_5^{2h} \leftarrow P_4^{2h} + I_{4h}^{2h} E_2^{4h}$,
 11. Relax many times on the discretized Reynolds equations $F^{2h}(P^{2h}) = F^{2h}(P_1^{2h}) + R_1^{2h}$ with initial guesses $P_0^{2h} = P_5^{2h}$ in the second-level grid with Newton-SOR iterative method until the convergence criterion in the second-level grid is satisfied, to obtain error approximations E^{2h} by $E^{2h} = P_6^{2h} - P_1^{2h}$,
 12. Prolong E^{2h} back to the first-level grid with the linear interpolation scheme to obtain pressure approximations P_2^h by $P_2^h \leftarrow P_1^h + I_{2h}^h E^{2h}$.

The whole procedure will cease as soon as the convergence criterion in the first-level grid is satisfied. Although the convergence rate of the w-cycle strategy is faster than that of the v-cycle strategy in one cycle, the computational time of one w-cycle computation is higher than that of one v-cycle computation. Overall, the computational times resulted from these two cycle strategies are very similar in the practical application. As a consequence, only the v-cycle strategy is presented in the FAM method for the numerical simulation of the slider air bearing.

4.4 Summary

In this chapter, the basic multigrid principle is presented by introducing the fine and coarse grids in the iterative step. First, two iterative strategies are presented to show the historical evolution of the multigrid algorithm. The nested iteration strategy

is to obtain better initial guesses by using the coarse grids, while the coarse grid correction strategy is to apply the error equations to relax the errors from the fine grid to the coarse grid. Then, two important transfer mechanisms are illustrated to show the vector intergrid relations in the multigrid method. The restriction mechanism is to transfer the vectors from the fine grid to the coarse grid, while the prolongation mechanism is to transfer the vectors from the coarse grid to the fine grid. Next, two classical cycle strategies are demonstrated to show the different transfer ways at various grid levels. The v-cycle strategy is to transfer the vectors from the finest grid to the coarsest grid and then back to the finest grid in a straight way, while the w-cycle strategy is to repeat some sub transfer steps in the whole transfer process. Last, two implementations of the multigrid method are presented for different types of numerical equations. The implementation for linear equations is to transfer the error approximations from the fine grid to the coarse grid, while the implementation for nonlinear equations is to transfer the variable approximations from the fine grid to the coarse grid.

To accelerate the convergence rate of the numerical simulation of the slider air bearing solved by the Newton-SOR iterative algorithm, the FAM method is specifically introduced. Two important schemes, the full weighting restriction operator and the linear interpolation prolongation operator are derived for the two-dimensional slider air bearing simulation. In conjunction with the discretized Reynolds equation, the FAM algorithms in three-level v-cycle and w-cycle strategies calculated from the finest grid are shown in the flowcharts, respectively.

Chapter 5

Least Square Finite Difference Multigrid Method for Slider Air Bearing Simulation

5.1 Introduction

After the numerical simulation of the slider air bearing has been solved by the LSFD method in Chapter 3, the full approximation multigrid algorithm was presented to accelerate the convergence rate in the simulation in the previous chapter. In this chapter, the Least Square Finite Different Multigrid (LSFDM) method is implemented to simulate slider air bearing problems and analyze the characteristics of various film configurations. To evaluate the effectiveness of the LSFDM method, some one-dimensional and two-dimensional slider air bearing problems are solved and discussed.

5.2 Numerical Simulation of Infinitely-Long Plane-Wedge Slider

In order to compare the numerical results from the LSFDM method with the analytical solutions from the early paper published by Gross (1980), the Infinitely-Long Plane-Wedge Slider is presented to calculate the one-dimensional pressure profiles for the continuum flow and the first-order slip model.

Figure 5.1 shows the geometry profile of the Infinitely-Long Plane-Wedge Slider. Here the ratio of the leading edge height h_l to the trail edge height h_t is named as the film thickness ratio $\bar{h}$.

Fig. 5.1 Geometry profile of Infinitely-Long Plane-Wedge Slider

Due to the high efficiency and the simplicity in the one-dimensional numerical simulation, the one-level LSFDM procedure is adopted only. The computation domain is meshed into 100 uniform grid points. The order of one-dimensional Taylor series expansion for the pressure derivative approximation is 4. The number of the supporting points used in the LSFDM approximation around a reference point is 6. The initial pressure values at all grid points are set as one standard atmosphere pressure, as well as the boundary pressure values are fixed as one standard atmosphere pressure in the whole iterative process. The convergence criterion is set as

$$|R_{ij}|_{\max} \leq 1.0 \times 10^{-3}.$$

5.2.1 Numerical and Analytical Results for Continuum Flow

Figure 5.2 illustrates the numerical result calculated from the LSFDM method and the analytical solution obtained from the published paper (Gross, 1980). It is shown that the numerical simulation is accurate with the maximum relative error at 0.38% only.

Fig. 5.2 Pressure comparison of Infinite-Long Plane-Wedge Slider for continuum flow

Here the discretized Reynolds equations are based on the continuum flow. The film thickness ratio h is set as 1.5 and 2.0, respectively. The air bearing number Λ_x is set as 10.

5.2.2 Numerical and Analytical Results for First-Order Slip Model

Figure 5.3 illustrates the numerical result calculated from the LSFDM method for the first-order slip model and the analytical solution obtained from the published paper (Gross, 1980). It is shown that the numerical simulation is accurate with the maximum relative error at 1.06% only.

Fig. 5.3 Pressure comparison of Infinite-Long Plane-Wedge Slider for first-order slip model

Here the discretized Reynolds equations are based on the first-order slip model flow.

The film thickness ratio h is 2.0. The air bearing number Λ_x is 30.

5.3 Numerical Simulation of Plane Slider

To illustrate the high efficiency and accuracy of the LSFDM method, the numerical simulation of a Plane Slider is carried out and presented for the comparison

with the solution from the finite volume method. The parameters of the Plane Slider are listed in Table 5.1. In the computation, the order of two-dimensional Taylor series expansion for the pressure derivative approximation is 9. The number of the supporting points used in the LSFDM approximation around a reference point is 15. The initial pressure values at all grid points are set as one standard atmosphere pressure, as well as the boundary pressure values are fixed as one standard atmosphere pressure in the whole iterative process. The convergence criterion in the first-level grid is set as $|R_{ij}|_{\max} \leq 1.0 \times 10^{-3}$.

Table 5.1 Parameters of Plane Slider

Minimum flying height	$h_m = 50\text{nm}$	Disk rotating speed	$\omega = 5400\text{RPM}$
Total length	$L = 1.0\text{mm}$	Disk rotating radius	$R_d = 18\text{mm}$
Total width	$W = 0.8\text{mm}$	Pitch angle	$\alpha = 1.0 \times 10^{-4}\text{rad}$
Molecular mean free path	$\lambda = 63.5\text{nm}$	Roll angle	$\theta = 0\text{rad}$
Ambient pressure	$P_a = 1.01325 \times 10^5\text{Pa}$	Sway angle	$\varphi = 0\text{rad}$
Dynamic viscosity	$\mu = 1.806 \times 10^{-5}\text{Pa/sec}$		

5.3.1 Simulation Result Using Uniform Grids

Due to its superiority in the grid generation, The ANSYS program is implemented to mesh the computation domain of the Plane Slider into 193×193 uniform grids, as shown in Figure 5.4.

Fig. 5.4 Mesh configuration of Plane Slider in uniform grids

Figures 5.5 & 5.6 show the geometry configuration and the pressure profile of the Plane Slider meshed by 193×193 uniform grids, respectively.

Fig. 5.5 Geometry profile of Plane Slider in uniform grids

Fig. 5.6 Pressure profile of Plane Slider in uniform grids

5.3.2 Simulation Result Using Nonuniform Grids

The mesh configuration of 129×129 nonuniform grids is shown in Figure 5.7.

Fig. 5.7 Mesh configuration of Plane Slider in nonuniform grids

Figures 5.8 & 5.9 show the geometry configuration and the pressure profile of the Plane Slider meshed by 129×129 nonuniform grids, respectively.

Fig. 5.8 Geometry profile of Plane Slider in nonuniform grids

Fig. 5.9 Pressure profile of Plane Slider in nonuniform grids

5.3.3 Comparison of Results from Different Grids

Comparing Figure 5.7 with Figure 5.4, it is clear that more grid points are distributed in the region of the trailing edge in Figure 5.7. As a result, the 129×129 nonuniform grids are able to produce more accurate pressure profile of the Plane Slider in comparison to the 193×193 uniform grids.

As shown in Figures 5.5 & 5.8, a plane is inclined from the leading edge to the trailing edge. However, compared with the grids in Figure 5.5, more grid points are distributed near to the trailing edge in Figure 5.8 so as to suit the sharp pressure variations of the Plane Slider near the trailing edge.

As shown in Figures 5.6 & 5.9, the same pressure profiles of the Plane Slider are obtained using different grids. The pressure in the leading edge slowly increases from the ambient pressure till the maximum pressure near to the trailing edge, because the airflow clearance between the slider and the disk gradually becomes smaller and compresses the airflow within the boundary. After the pressure peak, the pressure sharply decreases to the ambient pressure due to the constraint of the boundary condition. Generally speaking, if the minimum flying height becomes smaller, the pressure peak will increase and the position of the pressure peak will move toward the trailing edge.

Figures 5.10 & 5.11 show the comparisons of the pressure profiles from the LSFDM method and the solutions from the finite volume method, respectively. Here CMLAir32 from UC. Berkeley, known as the classical software for the slider air

bearing simulation with the finite volume method, is adopted to compute the reference pressure profile.

Fig. 5.10 Pressure comparison of Plane Slider along $Y=0.4125$

Fig. 5.11 Pressure comparison of Plane Slider along $X=0.98$

As shown in the figures above, the 385×385 adaptive grids, which are the maximum grids in CMLAir32 with the finite volume method, are applied to produce the accurate reference pressure profile. It is clear that the pressure distributions obtained from various grids are similar and accurate. With the increase of the number of the uniform grids, more accurate pressure profiles are obtained.

Table 5.2 shows the computational time resulted from the Plane Slider simulation using different levels of uniform and nonuniform grids. In the numerical simulation, all computations are completed by in-house developed Fortran 90 program on the Dell 530 workstation with 2.2GHz CPU and 512MB RAM.

Table 5.2 Computational time for Plane Slider in different levels of grids

	Non-multigrid	Two levels	Three levels	Four levels
Uniform grids 129×129	1min 35secs	17secs	12secs	10secs
Nonuniform grids 193×193	24secs	10secs	9secs	N/A

It is obvious that the computational efficiency in the numerical simulation of the Plane Slider is greatly enhanced by using the multi-level LSFDM method, and more time is saved by using the nonuniform grid distribution than by using the uniform grid distribution. In the nonuniform simulation, due to the relatively small number of grids in the first-level computation domain, the number of the grid points in the fourth-level computation domain is too small to reflect the sharp pressure derivatives of the Plane Slider. Hence it results in the divergence of the Newton-SOR iterative step, which was illustrated in Chapter 3. As a consequence, a limitation on the grid levels exists in the practical application of the LSFDM method.

5.4 Numerical Simulation of Two-Rail Slider

The parameters of a Two-Rail Slider, a typical positive pressure slider, are listed in Table 5.3.

Table 5.3 Parameters of Two-Rail Slider

Minimum flying height	$h_m = 50\text{nm}$	Ambient pressure	$P_a = 1.01325 \times 10^5 \text{Pa}$
Total length	$L = 4.05\text{mm}$	Dynamic viscosity	$\mu = 1.806 \times 10^{-5} \text{Pa/sec}$
Total width	$W = 3.0\text{mm}$	Disk rotating speed	$\omega = 5400\text{RPM}$
Width of rail	$W_{Ra} = 0.5\text{mm}$	Disk rotating radius	$R_d = 18\text{mm}$
Height of rail	$H_{Ra} = 0.022\text{mm}$	Pitch angle	$\alpha = 8.0 \times 10^{-5} \text{rad}$
Width of profile	$W_{Pr} = 0.28\text{mm}$	Roll angle	$\theta = 0\text{rad}$
Molecular mean free path	$\lambda = 63.5\text{nm}$	Sway angle	$\varphi = 0\text{rad}$

Here the order of two-dimensional Taylor series expansion for the pressure derivative approximation is 9. The number of the supporting points used in the LSFDM approximation around a reference point is 15. The initial pressure values at all grid points are set as one standard atmosphere pressure, as well as the boundary pressure values are fixed as one standard atmosphere pressure in the whole iterative process. The convergence criterion in the first-level grid is set as $|R_{ij}|_{\max} \leq 1.0 \times 10^{-3}$. The convergence criterion in the second-level grid is set as $|R_{ij}|_{\max} \leq 2.0 \times 10^{-4}$. The convergence criterion in the third-level grid is set as $|R_{ij}|_{\max} \leq 4.0 \times 10^{-5}$. The same principle can be applied to the convergence criterion for coarser grids.

5.4.1 Simulation Result Using Uniform Grids

Here the computation domain of the Two-Rail Slider is meshed into 225×225 uniform grids, as shown in Figure 5.12.

Fig. 5.12 Mesh configuration of Two-Rail Slider in uniform grids

Figures 5.13 & 5.14 show the geometry configuration and the pressure profile of the Two-Rail Slider meshed by 225×225 uniform grids, respectively.

Fig. 5.13 Geometry profile of Two-Rail Slider in uniform grids

Fig. 5.14 Pressure profile of Two-Rail Slider in uniform grids

5.4.2 Simulation Result Using Nonuniform Grids

The mesh configuration of 129×129 nonuniform grids is shown in Figure 5.15. Figures 5.16 & 5.17 show the geometry and pressure profiles of the Two-Rail Slider, respectively.

Fig. 5.15 Mesh configuration of Two-Rail Slider in nonuniform grids

Fig. 5.16 Geometry profile of Two-Rail Slider in nonuniform grids

Fig. 5.17 Pressure profile of Two-Rail Slider in nonuniform grids

5.4.3 Comparison of Results from Different Grids

Comparing Figure 5.15 with Figure 5.12, more grid points are distributed in the regions of the wall profiles and two rails in Figure 5.15. Hence the 129×129 nonuniform grids are able to produce more accurate pressure profile.

As shown in Figures 5.13 & 5.16, two rails are located in the inner and outer edges, respectively. A large cavity exists between them. However, compared with the grid in Figure 5.13, more grid points are distributed in the wall profiles in Figure 5.16 to suit the regional geometry of the Two-Rail Slider.

As shown in Figures 5.14 & 5.17, the same pressure profiles are obtained by different grids. The positive pressures are generated in the rail regions but the initial pressures are invariant in the cavity region. This is because the input airflow is compressed in the higher rail regions and released in the lower cavity region.

Figures 5.18~5.21 show the comparisons of the pressure profiles from the LSFDM method and the solutions from the finite volume method, respectively.

Fig. 5.18 Pressure comparison of Two-Rail Slider along $Y=0.04$

Fig. 5.19 Pressure comparison of Two-Rail Slider along $Y=0.66$ Fig. 5.20 Pressure comparison of Two-Rail Slider along $X=0.03$

Fig. 5.21 Pressure comparison of Two-Rail Slider along $X=0.99$

As shown in the figures above, it is clear that the pressure distributions are similar and accurate. With the increase of the number of uniform grids, more accurate pressure profiles are obtained.

Table 5.4 shows the computational time resulted from the Two-Rail Slider simulation using different levels of uniform and nonuniform grids. In the numerical simulation, all computations are completed by in-house developed Fortran 90 program on the Dell 530 workstation with 2.2GHz CPU and 512MB RAM.

Table 5.4 Computational time for Two-Rail Slider in different levels of grids

	Non-multigrid	Two levels	Three levels	Four levels
Uniform grids 225x225	19hs 6mins	1h 42mins	4mins 52secs	2mins 19secs
Nonuniform grids 129x129	2hs 57mins	8mins 5secs	1mins 35secs	N/A

It is obvious that the computational efficiency of the numerical simulation of the Two-Rail Slider is greatly enhanced by using the multi-level LSFDM method, and more time is saved by using the nonuniform grid distribution than by using the uniform grid distribution. Same as in the numerical simulation of the Plane Slider, a limitation on the grid levels exists in the practical application of the LSFDM method.

5.5 Numerical Simulation of Advanced Step Slider

To further illustrate the high efficiency and accuracy of the LSFDM method, the numerical simulation of an Advanced Step Slider, a typical negative pressure slider, is presented for the comparison with the solution from the CMLAir32 procedure. Considering the complexity of the geometry configuration, some important parameters of the Advanced Step Slider are listed in Table 5.5. The convergence criterion can be referred to the numerical simulations of the Two-Rail Slider.

Table 5.5 Parameters of Advanced Step Slider

Minimum flying height	$h_m = 50\text{nm}$	Width of third taper	$W_{Ta3} = 0.28\text{mm}$
Total length	$L = 1.25\text{mm}$	Ambient pressure	$P_a = 1.01325 \times 10^5 \text{Pa}$
Total width	$W = 1.0\text{mm}$	Dynamic viscosity	$\mu = 1.806 \times 10^{-5} \text{Pa/sec}$
Length of first taper	$L_{Ta1} = 0.18\text{mm}$	Molecular mean free path	$\lambda = 63.5\text{nm}$
Width of first taper	$W_{Ta1} = 0.68\text{mm}$	Disk rotating speed	$\omega = 7200\text{RPM}$
Length of second taper	$L_{Ta2} = 0.24\text{mm}$	Disk rotating radius	$R_d = 16\text{mm}$
Width of second taper	$W_{Ta2} = 0.68\text{mm}$	Pitch angle	$\alpha = 8.0 \times 10^{-5} \text{rad}$
Length of third taper	$L_{Ta3} = 0.22\text{mm}$	Roll angle	$\theta = 0\text{rad}$

5.5.1 Simulation Result Using Uniform Grids

For clearer demonstration, the mesh configuration in the third-level computation domain with 129×129 uniform grids is shown in Figure 5.22. The geometry and pressure profiles are shown in Figures 5.23 & 5.24 in the second-level computation domain with 257×257 uniform grids, respectively.

Fig. 5.22 Mesh configuration of Advanced Step Slider in uniform grids

Fig. 5.23 Geometry profile of Advanced Step Slider in uniform grids

Fig. 5.24 Pressure profile of Advanced Step Slider in uniform grids

For the actual LSFDM computation, 513×513 uniform grids are used in the first-level computation domain.

5.5.2 Simulation Result Using Nonuniform Grids

The mesh configuration of 257×257 nonuniform grids is shown in Figure 5.25.

Fig. 5.25 Mesh configuration of Advanced Step Slider in nonuniform grids

Figures 5.26 & 5.27 show the geometry and pressure profiles of the Advanced Step Slider, respectively.

Fig. 5.26 Geometry profile of Advanced Step Slider in nonuniform grids

Fig. 5.27 Pressure profile of Advanced Step Slider in nonuniform grids

5.5.3 Comparison of Results from Different Grids

Comparing Figure 5.25 with Figure 5.22, more grid points are distributed in the regions of the wall profiles and the three pads in Figure 5.25. Hence the 257×257 nonuniform grids are able to produce more accurate pressure profile.

As shown in Figures 5.23 & 5.26, three tapers are located in the leading and trailing edges, respectively. A large cavity exists between two tapers. Compared with the grid in Figure 5.23, more grid points are distributed in the region of the wall profiles in Figure 5.26 to suit the regional geometry of the Advanced Step Slider.

As shown in Figures 5.24 & 5.27, the same pressure profiles are obtained using different grids. Positive pressures are generated in the taper regions, but negative pressures are generated in the cavity region. This is because the input airflow first compressed by the two tapers is suddenly released into a large cavity and then compressed by another taper again.

Figures 5.28~5.31 show the comparisons of the pressure profiles obtained from the LSFDM method and the solutions from the finite volume method, respectively.

Fig. 5.28 Pressure comparison of Advanced Step Slider along $Y=0.4$

Fig. 5.29 Pressure comparison of Advanced Step Slider along $X=0.3$ Fig. 5.30 Pressure comparison of Advanced Step Slider along $X=0.4$

Fig. 5.31 Pressure comparison of Advanced Step Slider along X=0.9

As shown in the figures above, it is clear that the pressure distributions are similar and accurate except the cases using relatively small number of uniform grids. With the increase of the number of the uniform grids, more accurate pressure profiles are obtained.

Table 5.6 shows the computational time resulted from the Advanced Step Slider simulation using different levels of uniform and nonuniform grids. In the numerical simulation, all computations are completed by in-house developed Fortran 90 program on the Dell 530 workstation with 2.2GHz CPU and 512MB RAM.

Table 5.6 Computational time for Advanced Step Slider in different levels of grids

	Non-multigrid	Two levels	Three levels	Four levels
Uniform grids 513x513	5days 5mins	8hs 37mins	42mins 55secs	15mins 46secs
Nonuniform grids 257x257	16hs 44mins	1hs 36mins	8mins 40secs	N/A

It is obvious that the computational efficiency of the numerical simulation of the Advanced Step Slider is greatly enhanced by using the multi-level LSFDM method, and more time is saved by using the nonuniform grid distribution than by using the uniform grid distribution. Same as in the numerical simulation of the Plane Slider, a limitation on the grid levels exists in the practical application of the LSFDM method.

5.6 Summary

In this chapter, the numerical simulations of the Infinitely-Long Plane-Wedge Slider, the Plane Slider, the Two-Rail Slider and the Advanced Step Slider are performed by the LSFDM method. It is found that much less computational time is required with the use of more levels of multigrids. By comparing the results with the solutions from the published papers or from the finite volume method, it is proven that the LSFDM is an efficient and robust computational method in the numerical simulation of the slider air bearing.

Chapter 6

Flying Attitude Determination of Slider Air Bearing

6.1 Introduction

After the previous chapters have applied LSFDM method to solve and accelerate the numerical simulation of the slider air bearing, this chapter will apply the developed solver for the topology optimization of slider air bearing. However, as the final part of the complete process for topology optimization of slider air bearing, the flying attitude determination of the slider air bearing has to be carried out first. This is required later for verifying the topology optimization results. The flying attitude determination of the slider air bearing is basically a procedure of parametric optimization. Hence, some optimization algorithm is to be selected and adopted here.

In general, an optimization problem is to minimize a defined objective function subject to some equality and inequality constraints. Many simple types of constrained optimization problems incorporating a small number of design variables can be solved in closed-form using optimality conditions. For complex and large-scale problems, computer-based numerical techniques are more applicable. Mathematical programming approaches, which apply gradient-based search algorithms to solve the optimization problem, can easily be implemented for nearly any type of optimization problem. As the most efficient techniques for solving problems formulated by continuous design variables, these mathematical programming algorithms typically determine the direction of ascent from an initial starting point (Nagendra et al., 1992).

Given a starting point, the algorithm may sequentially move towards a solution by some update scheme. There are many types of sequential optimization algorithms available. However, the Sequential Linear Programming (SLP), the Sequential Quadratic Programming (SQP), and the Sequential Convex Programming (SCP) are the most applied algorithms to solve large-scale problems.

The SLP method approximates a nonlinear constrained problem using a linearized objective and linear constraints. Then the linearized subproblem is solved by linear programming. After solving the linear subproblem starting at the optimum point of previous iteration, the exact optimum point for the original problem can be obtained with certain accuracy (Vanderplaats, 1993). Since linear programming is used as a solver, the SLP can inherently handle thousand of variables. In addition, large number of inequality or equality constraints can be also handled. To avoid bad estimates, move limitations are imposed to restrict the change of design variables in each subproblem. Selection of move limitation is critical to the performance of the SLP method. If not properly selected, it is easy to generate unboundness, infeasible solution, and divergence as a result of oscillation among vertexes of linear constraints. This method is simple and robust in the structural optimization. However, since linear approximation is less accurate, it requires large number of iterative steps to converge to the optimum solution.

The SQP method approximates a nonlinear constrained problem using a quadratic objective and linear constraints. The subproblem is solved by quadratic programming and gives a search direction, which in turn is used to optimize an unconstrained one-dimensional problem with respect to a merit function. It has been proven that the SQP can converge globally and superlinearly near the optimal point.

In addition, this method can handle both equality and inequality constraints. Usually the SQP is limited to the medium-scale problem (Schittkowski et al., 1994). However, in the latest development, the large-scale application is reported (Gill et al., 1997).

Instead of linear or quadratic approximation, the SCP method estimates the nonlinear problem using convex approximation (Schittkowski and Zillober, 1995). The convex approximation is based on semi-empirical rules, which are derived from structural optimization. It requires first-order sensitivities of objective function and constraint functions with respect to indirect variables. The indirect variable can either be reciprocal variable, e.g. $1/x_j$ as proposed in convex linearization or CONLIN scheme (Fleury, 1989), or substitute variables, i.e. $1/(x_j - L_j)$ and $1/(U_j - x_j)$ as proposed in the Method of Moving Asymptotes (MMA) scheme (Svanberg, 1984). As a result of convex approximation, a separable and convex subproblem is obtained. It is further converted into dual subproblem. The dual problem can be efficiently solved by unconstrained minimization technique, e.g. Newton method or conjugate direction method. When the number of design variables is very much larger than the number of constraints, this method can effectively reduce the computational cost. In general, the convex approximation is more conservative and requires less computation.

6.2 Method of Moving Asymptotes

As one of the SCP algorithms, the MMA has proven itself to be versatile and well suited for large-scale structural optimization problems (Bendsøe and Sigmund, 2003). This method generally outperforms most other algorithms in terms of speed and accuracy. It is able to handle a large number of design variables and all types of

constraints, provided only that the derivatives of the objective function and constraint functions with respect to the design variables could be calculated. Further, the present MMA algorithm is written in Fortran program. Therefore, it is easy to be implemented and utilized in this dissertation

6.2.1 General Description of Method of Moving Asymptotes

The original optimization problem is written as the followings:

$$\begin{aligned}
 T: \quad & \text{minimize} && f_0(\mathbf{x}) && (\mathbf{x} \in \mathbf{R}) \\
 \\
 C: \quad & \text{subject to} && f_i(\mathbf{x}) \leq f_i^{\max} && i = 1, \dots, m \\
 & && x_j^{\min} \leq x_j \leq x_j^{\max} && j = 1, \dots, n
 \end{aligned}$$

where T denotes the original problem, C denotes the original constraints. $\mathbf{x} = (x_1, \dots, x_n)^T$ denotes the vector of optimization variables, $f_0(\mathbf{x})$ denotes the objective function, $f_i(\mathbf{x})$ denotes the constraints, $f_i^{\max}$ denotes the upper bounds on the constraints, and $x_j^{\min}$ and $x_j^{\max}$ denote given lower and upper bounds on the optimization variables.

A well established general procedure for solving such problem is to generate and solve a sequence of explicit subproblems, as the followings:

1. Choose a starting point $\mathbf{x}^{(0)}$, and set the iteration index $k = 0$,
2. Given an iteration point $\mathbf{x}^{(k)}$, calculate $f_i(\mathbf{x}^{(k)})$ and the gradients $\nabla f_i(\mathbf{x}^{(k)})$ for $i = 0, 1, \dots, m$,
3. Generate a subproblem $T^{(k)}$ by replacing T , and explicit functions $f_i^{(k)}$ by replacing implicit functions f_i , based on the calculations from the previous step,
4. Solve $T^{(k)}$ and let the optimal solution of this subproblem be the next iteration point $\mathbf{x}^{(k+1)}$.

The whole procedure will cease as soon as some convergence criterion is satisfied.

In the MMA, the approximating function $f_i^{(k)}$ is obtained by a linearization of f_i based on the variables $1/(x_j^{(k)} - L_j^{(k)})$ or $1/(U_j^{(k)} - x_j^{(k)})$, which depends on the derivative signs of f_i at $\mathbf{x}^{(k)}$. Here $L_j^{(k)}$ and $U_j^{(k)}$ are named as ‘moving asymptotes’, whose values are changed between the iterations. Therefore, by setting values of $L_j^{(k)}$ and $U_j^{(k)}$, a more flexible and powerful method is obtained. The decent parameters can efficiently stabilize the general procedure. As an valued aspect, the MMA is sensitive to neither translation nor scaling of the variables.

6.2.2 Technical Description of Method of Moving Asymptotes

If the iteration point $\mathbf{x}^{(k)}$ and the lower and upper bounds $x_j^{\min}$ and $x_j^{\max}$ are defined, the parameters $L_j^{(k)}$ and $U_j^{(k)}$ can be obtained by a simple formula, as below:

$$\begin{cases} L_j^{(k)} = x_j^{(k)} - \lambda(x_j^{\max} - x_j^{\min}) \\ U_j^{(k)} = x_j^{(k)} + \lambda(x_j^{\max} - x_j^{\min}) \end{cases} \quad (6.1)$$

where λ is a real number. Since $L_j^{(k)}$ and $U_j^{(k)}$ do not depend on the iterative number k , they are named as the fixed asymptotes rather than moving.

Although the fixed asymptotes can be applied in the practical application, so as to increase the procedural efficiency of the MMA, $L_j^{(k)}$ and $U_j^{(k)}$ are required to vary in some smart way in iterative steps. A general principle to change these parameters is

- I. If the procedure tends to oscillate, it requires the stabilization, which can be accomplished by moving the asymptotes closer to the current iteration point.
- II. If the procedure is monotone and slow, it requires the relaxation, which can be accomplished by moving the asymptotes away from the current iteration point.

Based on this principle, a simple implementation is shown as the followings:

- I. If $k = 0, 1$

$$\begin{cases} L_j^{(k)} = x_j^{(k)} - (x_j^{\max} - x_j^{\min}) \\ U_j^{(k)} = x_j^{(k)} + (x_j^{\max} - x_j^{\min}) \end{cases} \quad (6.2)$$

- II. If $k \geq 2$

- i. If the signs of $x_j^{(k)} - x_j^{(k-1)}$ and $x_j^{(k-1)} - x_j^{(k-2)}$ are different, indicating an oscillation in the variable $x_j^{(k)}$, then

$$\begin{cases} L_j^{(k)} = x_j^{(k)} - \lambda(x_j^{(k-1)} - L_j^{(k-1)}) \\ U_j^{(k)} = x_j^{(k)} + \lambda(U_j^{(k-1)} - x_j^{(k-1)}) \end{cases} \quad (6.3)$$

- ii. If the signs of $x_j^{(k)} - x_j^{(k-1)}$ and $x_j^{(k-1)} - x_j^{(k-2)}$ are same, indicating an monotone in the variable $x_j^{(k)}$, then

$$\begin{cases} L_j^{(k)} = x_j^{(k)} - (x_j^{(k-1)} - L_j^{(k-1)})/\lambda \\ U_j^{(k)} = x_j^{(k)} + (U_j^{(k-1)} - x_j^{(k-1)})/\lambda \end{cases} \quad (6.4)$$

Here λ is set as a given real number less than unity. Although there are many extensions to this implementation, this procedure works well in different practical problems.

Hence, the iteration point $x^{(k)}$ is restrained between the modified lower and upper bounds $L_j^{(k)}$ and $U_j^{(k)}$, as below:

$$L_j^{(k)} < x_j^{(k)} < U_j^{(k)} \quad (6.5)$$

Therefore $f_i^{(k)}$ is defined as

$$f_i^{(k)}(x) = f_i(x^{(k)}) - \sum_{j=1}^n \left(\frac{p_{ij}^{(k)}}{U_j^{(k)} - x_j^{(k)}} + \frac{q_{ij}^{(k)}}{x_j^{(k)} - L_j^{(k)}} \right) + \sum_{j=1}^n \left(\frac{p_{ij}^{(k)}}{U_j^{(k)} - x_j} + \frac{q_{ij}^{(k)}}{x_j - L_j^{(k)}} \right) \quad (6.6)$$

where

$$p_{ij}^{(k)} = \begin{cases} (U_j^{(k)} - x_j^{(k)})^2 \cdot \frac{\partial f_i}{\partial x_j}(x^{(k)}) & \frac{\partial f_i}{\partial x_j}(x^{(k)}) > 0 \\ 0 & \frac{\partial f_i}{\partial x_j}(x^{(k)}) \leq 0 \end{cases} \quad (6.7)$$

$$q_{ij}^{(k)} = \begin{cases} 0 & \frac{\partial f_i}{\partial x_j}(\mathbf{x}^{(k)}) \geq 0 \\ -\left(x_j^{(k)} - L_j^{(k)}\right)^2 \cdot \frac{\partial f_i}{\partial x_j}(\mathbf{x}^{(k)}) & \frac{\partial f_i}{\partial x_j}(\mathbf{x}^{(k)}) < 0 \end{cases} \quad (6.8)$$

It is easily verified that $f_i^{(k)}$ is a first-order approximation of f_i at $\mathbf{x}^{(k)}$, as shown below:

$$\begin{cases} f_i^{(k)}(\mathbf{x}^{(k)}) = f_i(\mathbf{x}^{(k)}) \\ \frac{\partial f_i^{(k)}}{\partial x_j}(\mathbf{x}^{(k)}) = \frac{\partial f_i}{\partial x_j}(\mathbf{x}^{(k)}) \end{cases} \quad (6.9)$$

Therefore the second-order derivatives of $f_i^{(k)}$ at any point x_j should be

$$\begin{cases} \frac{\partial^2 f_i^{(k)}}{\partial x_j^2} = \frac{2p_{ij}^{(k)}}{(U_j^{(k)} - x_j)^3} + \frac{q_{ij}^{(k)}}{(x_j - L_j^{(k)})^3} \\ \frac{\partial^2 f_i^{(k)}}{\partial x_j \partial x_l} = 0 & j \neq l \end{cases} \quad (6.10)$$

Since $p_{ij}^{(k)} \geq 0$ and $q_{ij}^{(k)} \geq 0$, $f_i^{(k)}$ is a convex function. After substituting $p_{ij}^{(k)}$ and $q_{ij}^{(k)}$ into the equations above, it is

$$\frac{\partial^2 f_i^{(k)}}{\partial x_j^2}(\mathbf{x}^{(k)}) = \begin{cases} \frac{2 \frac{\partial f_i}{\partial x_j}(\mathbf{x}^{(k)})}{U_j^{(k)} - x_j^{(k)}} & \frac{\partial f_i}{\partial x_j}(\mathbf{x}^{(k)}) > 0 \\ -\frac{2 \frac{\partial f_i}{\partial x_j}(\mathbf{x}^{(k)})}{x_j^{(k)} - L_j^{(k)}} & \frac{\partial f_i}{\partial x_j}(\mathbf{x}^{(k)}) < 0 \end{cases} \quad (6.11)$$

Hence, the closer $L_j^{(k)}$ and $U_j^{(k)}$ are chosen to $x_j^{(k)}$, the larger become the second-order derivatives. It means that the approximating functions $f_i^{(k)}$ have more curvatures and the approximation of the original problem becomes more conservative.

After the approximating functions $f_i^{(k)}$ has been defined, the original optimization problem can be transformed to

$$\begin{aligned}
 O^{(k)}: \text{minimize} \quad & f_0(x^{(k)}) - \sum_{j=1}^n \left(\frac{p_{0j}^{(k)}}{U_j^{(k)} - x_j^{(k)}} + \frac{q_{0j}^{(k)}}{x_j^{(k)} - L_j^{(k)}} \right) + \sum_{j=1}^n \left(\frac{p_{0j}^{(k)}}{U_j^{(k)} - x_j} + \frac{q_{0j}^{(k)}}{x_j - L_j^{(k)}} \right) \\
 C: \text{subject to} \quad & f_i(x^{(k)}) - \sum_{j=1}^n \left(\frac{p_{ij}^{(k)}}{U_j^{(k)} - x_j^{(k)}} + \frac{q_{ij}^{(k)}}{x_j^{(k)} - L_j^{(k)}} \right) + \sum_{j=1}^n \left(\frac{p_{ij}^{(k)}}{U_j^{(k)} - x_j} + \frac{q_{ij}^{(k)}}{x_j - L_j^{(k)}} \right) - f_i^{\max} \leq 0 \\
 & \max\{x_j^{\min}, \alpha_j^{(k)}\} \leq x_j \leq \min\{x_j^{\max}, \beta_j^{(k)}\}
 \end{aligned}$$

where

$$\begin{cases} \alpha_j^{(k)} = 0.9L_j^{(k)} + 0.1x_j^{(k)} \\ \beta_j^{(k)} = 0.9U_j^{(k)} + 0.1x_j^{(k)} \end{cases} \quad (6.12)$$

Here the parameters $\alpha_j^{(k)}$ and $\beta_j^{(k)}$ are move limitations to avoid the possibility of any unexpected division by zero while solving the subproblem $T^{(k)}$.

6.2.3 Mathematical Method to Solve Subproblems

To simplify the expression, r_0 and b_i are adopted as below:

$$r_0 = f_0(\mathbf{x}^{(k)}) - \sum_{j=1}^n \left(\frac{p_{0j}^{(k)}}{U_j^{(k)} - x_j^{(k)}} + \frac{q_{0j}^{(k)}}{x_j^{(k)} - L_j^{(k)}} \right) \quad (6.13)$$

$$b_i = f_i(\mathbf{x}^{(k)}) - \sum_{j=1}^n \left(\frac{p_{ij}^{(k)}}{U_j^{(k)} - x_j^{(k)}} + \frac{q_{ij}^{(k)}}{x_j^{(k)} - L_j^{(k)}} \right) - f_i^{\max} \quad (6.14)$$

To simplify the notation, the iteration index k is neglected on the coefficients in the subproblem $T^{(k)}$. And $\max\{x_j^{\min}, \alpha_j^{(k)}\}$ is replaced by α_j as well as $\min\{x_j^{\max}, \beta_j^{(k)}\}$ is replaced by β_j . Therefore the subproblem $T^{(k)}$ defined in the previous section can be modified as

$$\begin{aligned} T^{(k)} : & \text{minimize} && r_0 + \sum_{j=1}^n \left(\frac{p_{0j}}{U_j - x_j} + \frac{q_{0j}}{x_j - L_j} \right) \\ C: & \text{subject to} && \sum_{j=1}^n \left(\frac{p_{ij}}{U_j - x_j} + \frac{q_{ij}}{x_j - L_j} \right) - b_i \leq 0 \quad i = 1, \dots, m \\ & && \alpha_j \leq x_j \leq \beta_j \quad j = 1, \dots, n \end{aligned}$$

where

$$p_{ij} \geq 0 \quad q_{ij} \geq 0 \quad L_j < \alpha_j < \beta_j < U_j$$

Since $T^{(k)}$ is a convex and separable problem, a mathematical method can be used to solve it. The Lagrangian function corresponding to $T^{(k)}$ is given by

$$L(\mathbf{x}, \mathbf{y}) = f_0^{(k)}(\mathbf{x}) + \sum_{i=1}^m y_i \left(f_i^{(k)}(\mathbf{x}) - f_i^{\max} \right) \quad y_i \geq 0 \quad (6.15)$$

Here y_i is the Lagrange multiplier. Based on the modified subproblem $T^{(k)}$, Equation (6.15) can be transformed to

$$L(x, y) = r_0 + \sum_{i=1}^m y_i b_i + \sum_{j=1}^n \left(\frac{p_{0j} + \sum_{i=1}^m y_i p_{ij}}{U_j - x_j} + \frac{q_{0j} + \sum_{i=1}^m y_i q_{ij}}{x_j - L_j} \right) \quad (6.16)$$

To further simplify the expressions, $P(y)$, $Q(y)$ and $l_j(x_j, y)$ are adopted as below:

$$P(y) = p_{0j} + \sum_{i=1}^m y_i p_{ij} \quad (6.17)$$

$$Q(y) = q_{0j} + \sum_{i=1}^m y_i q_{ij} \quad (6.18)$$

$$l_j(x_j, y) = \frac{P(y)}{U_j - x_j} + \frac{Q(y)}{x_j - L_j} \quad (6.19)$$

Hence, the equation above is transformed to

$$L(x, y) = r_0 + \sum_{i=1}^m y_i b_i + \sum_{j=1}^n l_j(x_j, y) \quad (6.20)$$

Hence the derivative of $l_j(x_j, y)$ with respect to x_j is derived as

$$\frac{\partial l_j(x_j, y)}{\partial x_j} = \frac{P(y)}{(U_j - x_j)^2} - \frac{Q(y)}{(x_j - L_j)^2} \quad (6.21)$$

and the second-order derivative of $l_j(x_j, y)$ with respect to x_j is derived as

$$\frac{\partial^2 l_j(x_j, y)}{\partial x_j^2} = \frac{2P(y)}{(U_j - x_j)^3} + \frac{2Q(y)}{(x_j - L_j)^3} \quad (6.22)$$

It can be proven that $\partial^2 l_j(x_j, y)/\partial x_j^2$ is strictly positive and $\partial l_j(x_j, y)/\partial x_j$ is strictly increasing against x_j . It results that $l_j(x_j, y)$ is a convex function of x_j . Therefore the following conclusions can be obtained as

I. If $\partial l_j(\alpha_j, y)/\partial x_j \geq 0$

$$x_j(y) = \alpha_j \quad (6.23)$$

II. If $\partial l_j(\beta_j, y)/\partial x_j \leq 0$

$$x_j(y) = \beta_j \quad (6.24)$$

III. If $\partial l_j(\alpha_j, y)/\partial x_j < 0$ and $\partial l_j(\beta_j, y)/\partial x_j > 0$

$$x_j(y) = \frac{P^{0.5}(y)L_j + Q^{0.5}(y)U_j}{P^{0.5}(y) + Q^{0.5}(y)} \quad (6.25)$$

Since there is an explicit expression for $x_j(y)$, there is also an explicit expression for Equation (6.16), which can be transformed to

$$L(y) = r_0 + \sum_{i=1}^m y_i b_i + \sum_{j=1}^n \left(\frac{P(y)}{U_j - x_j(y)} + \frac{Q(y)}{x_j(y) - L_j} \right) \quad (6.26)$$

Hence the derivative of $L(y)$ with respect to y_i is derived as

$$\frac{\partial L(y)}{\partial y_i} = b_i + \sum_{j=1}^n \left[\frac{p_{ij}}{U_j - x_j(y)} + \frac{q_{ij}}{x_j(y) - L_j} + \frac{P(y) \cdot \partial x_j(y) / \partial y_i}{(U_j - x_j(y))^2} - \frac{Q(y) \cdot \partial x_j(y) / \partial y_i}{(x_j(y) - L_j)^2} \right] \quad (6.27)$$

$$\frac{\partial x_j(y)}{\partial y_i} = \frac{(U_j - L_j)(P^{0.5}(y)Q^{-0.5}(y)q_{ij} - P^{-0.5}(y)Q^{0.5}(y)p_{ij})}{2(P^{0.5}(y) + Q^{0.5}(y))^2} \quad (6.28)$$

and the second-order derivative of $L(y)$ with respect to y_i is derived as

$$\begin{aligned} \frac{\partial^2 L(y)}{\partial y_i^2} = \sum_{j=1}^n & \left[\frac{2p_{ij} \cdot \partial x_j(y) / \partial y_i + P(y) \cdot \partial^2 x_j(y) / \partial y_i^2}{(U_j - x_j(y))^2} - \frac{2q_{ij} \cdot \partial x_j(y) / \partial y_i + Q(y) \cdot \partial^2 x_j(y) / \partial y_i^2}{(x_j(y) - L_j)^2} \right. \\ & \left. + \frac{2P(y) \cdot (\partial x_j(y) / \partial y_i)^2}{(U_j - x_j(y))^3} + \frac{2Q(y) \cdot (\partial x_j(y) / \partial y_i)^2}{(x_j(y) - L_j)^3} \right] \end{aligned} \quad (6.29)$$

$$\begin{aligned} \frac{\partial^2 x_j(y)}{\partial y_i^2} = & (U_j - L_j) \frac{(3P^{-1}(y)Q^{0.5}(y) + P^{-1.5}(y)Q(y))p_{ij}^2 - (3P^{0.5}(y)Q^{-1}(y) + P(y)Q^{-1.5}(y))p_{ij}^2}{4(P^{0.5}(y) + Q^{0.5}(y))^3} \\ & + (U_j - L_j) \frac{(P^{-0.5}(y) - Q^{-0.5}(y))p_{ij}q_{ij}}{2(P^{0.5}(y) + Q^{0.5}(y))^3} \end{aligned} \quad (6.30)$$

It can be proven that $\partial^2 L(y) / \partial y_i^2$ is strictly positive and $\partial L(y) / \partial y_i$ is strictly increasing against y_i . It results that $L(y)$ is a convex function of y_i . Therefore the former subproblem $T^{(k)}$ can be transformed to another subproblem D , as the followings:

$$D: \text{ minimize } r_0 + \sum_{i=1}^m y_i b_i + \sum_{j=1}^n \left(\frac{P(y)}{U_j - x_j(y)} + \frac{Q(y)}{x_j(y) - L_j} \right)$$

$$C: \text{ subject to } y_i \geq 0$$

To obtain the solution y for subproblem D , the equations $\partial L(y)/\partial y_i$ shown in Equation (6.27) can be solved by the traditional iterative methods. Thereafter the optimal solution of the subproblem $T^{(k)}$ is directly obtained by substituting the solution y in the expression for $x_j(y)$ shown in Equation (6.25).

So far, the original optimization problem T is successfully solved in the MMA procedure. Generally speaking, this method can be extensively applied in many research fields, e.g. the parametric optimization, the shape optimization and the topology optimization.

6.3 Problem Formulation

As one application of the optimization technology, the flying attitude determination of the slider air bearing is to be addressed in this chapter.

When a slider is fabricated out, a pre-load or gram-load is simultaneously applied to the slider by its structural flexibility. Since the closed loop position control of the slider is not considered in this thesis, the forces and moments acting on the slider from the slider mechanical support are simplified as constant values in the normal operation of the slider air bearing, and the flying attitude of slider air bearing at every track is assumed as a static problem, which is strictly governed by force equilibrium. By balancing the calculated air bearing force/moments with the applied pre-load force/moments, some certain flying attitude can be obtained for the slider air bearing. This process is named as the flying attitude determination, which is required later for theoretical verification of the topology optimization results.

If a known pre-load is applied to the slider, it means that the applied pre-load force F_0 , the applied pre-load X-moment arm L_{x0} and the applied pre-load Y-moment L_{y0} against the coordinate origin O are obtained. To find the flying attitude of the slider air bearing, the objective is defined to simultaneously minimize the force difference ΔF , the X-moment difference ΔM_x and the Y-moment difference ΔM_y between the calculated air bearing force/moments and the applied pre-load force/moments. Hence the objective function is defined as

$$\psi_1 = w_1 \cdot m_1 \cdot \Delta F^2 + w_2 \cdot m_2 \cdot \Delta M_x^2 + w_3 \cdot m_3 \cdot \Delta M_y^2 \quad (6.31)$$

$$\begin{cases} \Delta F = F_U - F_0 \\ \Delta M_x = M_{xU} - M_{x0} \\ \Delta M_y = M_{yU} - M_{y0} \end{cases} \quad (6.32)$$

$$\begin{cases} M_{x0} = F_0 \cdot L_{x0} \\ M_{y0} = F_0 \cdot L_{y0} \end{cases} \quad (6.33)$$

$$\begin{cases} F_U = \int_0^W \int_0^L (p_{ij} - p_a) dx dy \\ M_{xU} = \int_0^W \int_0^L x (p_{ij} - p_a) dx dy \\ M_{yU} = \int_0^W \int_0^L y (p_{ij} - p_a) dx dy \end{cases} \quad (6.34)$$

$$\begin{cases} w_1 = 1.0 \\ w_2 = 1.0 \\ w_3 = 1.0 \end{cases} \quad (6.35)$$

$$\begin{cases} m_1 = 1.0 \\ m_2^{(k+1)} = (\Delta F^2 / \Delta M_x^2)^{(k)} \\ m_3^{(k+1)} = (\Delta F^2 / \Delta M_y^2)^{(k)} \end{cases} \quad (6.36)$$

Here ψ_1 is the multi-objective function for the flying attitude determination of the slider air bearing. ΔF^2 , ΔM_x^2 and ΔM_y^2 are sub-objective functions for the force difference ΔF , the X-moment difference ΔM_x and the Y-moment difference ΔM_y , respectively. M_{x0} and M_{y0} are the applied pre-load X-moment and the applied pre-load Y-moment against the coordinate origin O . F_U , M_{xU} and M_{yU} are the calculated air bearing force, the calculated air bearing X-moment and the calculated air bearing Y-moment against the coordinate origin O when the slider moves at certain track of the disk surface. p_{ij} is the air bearing pressure. p_a is the ambient pressure. w_{1-3} are weighting factors, which control the contribution of every sub-objective function to the multi-objective function ψ_1 . m_{1-3} are magnification coefficients, which prevent the multi-objective function ψ_1 from being dominated by a very large value of the three sub-objective functions.

Since the flying attitude of the slider is defined by the minimum flying height h_m , the pitch angle α and the roll angle θ , these three parameters are set as design variables and constrained as below:

$$\begin{cases} h_{\min} < h_m < h_{\max} \\ \alpha_{\min} < \alpha < \alpha_{\max} \\ \theta_{\min} < \theta < \theta_{\max} \end{cases} \quad (6.37)$$

where $h_{\min}$ and $h_{\max}$ are the minimum and maximum constraints of the minimum flying height h_m . $\alpha_{\min}$ and $\alpha_{\max}$ are the minimum and maximum constraints of the pitch angle α . $\theta_{\min}$ and $\theta_{\max}$ are the minimum and maximum constraints of the roll angle θ .

The first-order sensitivity of the multi-objective function ψ_1 with respect to the design variables can be derived as

$$\frac{\partial \psi_1}{\partial v_i} = 2w_1 \cdot m_1 \cdot \Delta F \cdot \frac{\partial F_U}{\partial v_i} + 2w_2 \cdot m_2 \cdot \Delta M_x \cdot \frac{\partial M_{xU}}{\partial v_i} + 2w_3 \cdot m_3 \cdot \Delta M_y \cdot \frac{\partial M_{yU}}{\partial v_i} \quad (6.38)$$

$$\begin{cases} \frac{\partial F_U}{\partial v_i} = \frac{F_U(1.005v_i) - F_U(0.995v_i)}{0.01v_i} \\ \frac{\partial M_{xU}}{\partial v_i} = \frac{M_{xU}(1.005v_i) - M_{xU}(0.995v_i)}{0.01v_i} \\ \frac{\partial M_{yU}}{\partial v_i} = \frac{M_{yU}(1.005v_i) - M_{yU}(0.995v_i)}{0.01v_i} \end{cases} \quad (6.39)$$

Here v_i denotes the design variables h_m , α and θ . $F_U(1.005v_i)$, $M_{xU}(1.005v_i)$ and $M_{yU}(1.005v_i)$ denote the calculated air bearing force value F_U , the calculated air bearing X-moment value M_{xU} and the calculated air bearing Y-moment value M_{yU} after the design variable v_i is perturbed up by 0.5%. $F_U(0.995v_i)$, $M_{xU}(0.995v_i)$ and $M_{yU}(0.995v_i)$ denote the calculated air bearing force value F_U , the calculated air bearing X-moment value M_{xU} and the calculated air bearing Y-moment value M_{yU} after the design variable v_i is perturbed down by 0.5%.

By integrating the pressure in the computation domain, the values of F_U , M_{xU} and M_{yU} can be obtained. Then the multi-objective function ψ_1 can be solved by the MMA procedure in the constraint domain. Although it is common to use the multi-objective ψ_1 as the standard to investigate the convergence history of the flying attitude determination of the slider air bearing, it is not suitable for this practical

application because the iterative multi-objective values are greatly affected by the magnification coefficients m_2 and m_3 . Therefore the convergence criterion for the multi-objective function ψ_1 is set as

$$\begin{cases} E_F = |F_U - F_0| / F_0 < 0.1 \times 10^{-1} \\ E_{M_x} = |M_{xU} - M_{x0}| / M_{x0} < 0.1 \times 10^{-1} \\ E_{M_y} = |M_{yU} - M_{y0}| / M_{y0} < 0.1 \times 10^{-1} \end{cases} \quad (6.40)$$

where E_F denotes the relative error between the calculated air bearing force and the applied pre-load force. E_{M_x} denotes the relative error between the calculated air bearing X-moment and the applied pre-load X-moment. E_{M_y} denotes the relative error between the calculated air bearing Y-moment and the applied pre-load Y-moment.

6.4 Flying Attitude Determination of Plane Slider

To verify the effectiveness of the flying attitude determination of the slider air bearing with MMA procedure, the flying attitude determination of a Plane Slider is illustrated.

6.4.1 Initialization of Model

In this case study, the model is the same as the Plane Slider in Chapter 5. The minimum flying height h_m , the pitch angle α , and the roll angle θ are set as the

design variables v_i . The initial guess and the constraint domain of the design variables v_i are shown in Table 6.1.

Table 6.1 Initial guess and constraint domain for flying attitude determination of Plane Slider

	Minimum constraint	Initial guess	Maximum constraint
Minimum flying height	$h_{\min} = 50\text{nm}$	$h_m = 50\text{nm}$	$h_{\max} = 150\text{nm}$
Pitch angle	$\alpha_{\min} = 1.0 \times 10^{-5} \text{ rad}$	$\alpha = 1.0 \times 10^{-3} \text{ rad}$	$\alpha_{\max} = 1.0 \times 10^{-3} \text{ rad}$
Roll angle	$\theta_{\min} = -1.0 \times 10^{-5} \text{ rad}$	$\theta = -1.0 \times 10^{-5} \text{ rad}$	$\theta_{\max} = 1.0 \times 10^{-5} \text{ rad}$

Meshed by the 193×193 uniform grids and calculated by the three-level LSFDM procedure, the flying attitude and pressure profile of the Plane Slider with initial guess can be presented, as shown in Figures 6.1 & 6.2, respectively.

Fig. 6.1 Flying attitude of Plane Slider with initial guess

Fig. 6.2 Pressure profile of Plane Slider with initial guess

6.4.2 Optimal Results and Verifications

To demonstrate and determine the flying attitude of a slider, a pre-load is applied to the Plane Slider as the following:

$$\begin{cases} F_0 = 1.0 \times 10^{-1} \text{ N} \\ L_{x0} = 0.8 \times 10^{-3} \text{ m} \\ L_{y0} = 0.4 \times 10^{-3} \text{ m} \\ M_{x0} = 0.8 \times 10^{-4} \text{ Nm} \\ M_{y0} = 0.4 \times 10^{-4} \text{ Nm} \end{cases}$$

After some optimal iterative steps in the LSFDM procedure and the MMA procedure, the flying attitude of the Plane Slider can be obtained as

$$\begin{cases} h_m = 0.584660 \times 10^{-7} \text{ m} \\ \alpha = 0.720337 \times 10^{-3} \text{ rad} \\ \theta = -0.645829 \times 10^{-5} \text{ rad} \end{cases}$$

In this situation, the values of F_U , M_{xU} and M_{yU} are calculated at the radius 18mm of the hard disk drive, as shown below:

$$\begin{cases} F_U = 1.00022357 \times 10^{-1} \text{ N} \\ M_{xU} = 0.79933437 \times 10^{-4} \text{ Nm} \\ M_{yU} = 0.39648781 \times 10^{-4} \text{ Nm} \end{cases}$$

It is shown that the errors between the calculated air bearing force/moments and the applied pre-load force/moments are very small, as the following:

$$\begin{cases} E_F = 0.22356524\% \\ E_{M_x} = 0.08320317\% \\ E_{M_y} = 0.87804812\% \end{cases}$$

To evaluate the convergence history of the flying attitude determination of the Plane Slider, the variations of F_U , M_{xU} and M_{yU} against the number of iterations are shown in Figures 6.3~6.5, respectively.

Fig. 6.3 Force variations in flying attitude determination of Plane Slider

Fig. 6.4 X-moment variations in flying attitude determination of Plane Slider

Fig. 6.5 Y-moment variations in flying attitude determination of Plane Slider

From the three figures above, it is shown that the values of F_U , M_{xU} and M_{yU} become closer to the applied ones after 27 iterative steps.

After the flying attitude determination, the flying attitude and pressure profile of the Plane Slider are shown in Figures 6.6 & 6.7, respectively.

Fig. 6.6 Flying attitude of Plane Slider after flying attitude determination

Fig. 6.7 Pressure profile of Plane Slider after flying attitude determination

Comparing Figures 6.6 & 6.7 with Figures 6.1 & 6.2, it is clear that the flying attitudes and the pressure profiles of the Plane Slider have great changes after the flying attitude determination.

6.5 Flying Attitude Determination of Two-Rail Slider

To further verify the effectiveness of the flying attitude determination of the slider air bearing with MMA procedure, the flying attitude determination of a Two-Rail Slider is illustrated in this section.

6.5.1 Initialization of Model

In this case study, the model is the same as the Two-Rail Slider in Chapter 5. The minimum flying height h_m , the pitch angle α , and the roll angle θ are set as the design variables v_i . The initial guess and the constraint domain of the design variables v_i are shown in Table 6.2.

Table 6.2 Initial guess and constraint domain for flying attitude determination of Two-Rail Slider

	Minimum constraint	Initial guess	Maximum constraint
Minimum flying height	$h_{\min} = 50\text{nm}$	$h_m = 50\text{nm}$	$h_{\max} = 150\text{nm}$
Pitch angle	$\alpha_{\min} = 1.0 \times 10^{-5} \text{rad}$	$\alpha = 1.0 \times 10^{-3} \text{rad}$	$\alpha_{\max} = 1.0 \times 10^{-3} \text{rad}$
Roll angle	$\theta_{\min} = 0\text{rad}$	$\theta = 1.0 \times 10^{-5} \text{rad}$	$\theta_{\max} = 1.0 \times 10^{-5} \text{rad}$

Meshed by the 225×225 uniform grids and calculated by the three-level LSFDM procedure, the pressure profile of the Two-Rail Slider in certain flying attitude are presented, as shown in Figures 6.8 & 6.9, respectively.

Fig. 6.8 Flying attitude of Two-Rail Slider with initial guess

Fig. 6.9 Pressure profile of Two-Rail Slider with initial guess

As shown in Figure 6.9, the pressure peaks on two rails are different due to the existence of the roll angle θ .

6.5.2 Optimal Results and Verifications

To demonstrate and determine the flying attitude of a slider, a pre-load is applied to the Two-Rail Slider as the following:

$$\begin{cases} F_0 = 0.15\text{N} \\ L_{x0} = 0.30 \times 10^{-2}\text{m} \\ L_{y0} = 0.15 \times 10^{-2}\text{m} \\ M_{x0} = 0.45 \times 10^{-3}\text{Nm} \\ M_{y0} = 0.23 \times 10^{-3}\text{Nm} \end{cases}$$

After some optimal iterative steps in the LSFDM procedure and the MMA procedure, the flying attitude of the Two-Rail Slider can be obtained as

$$\begin{cases} h_m = 0.703246 \times 10^{-7}\text{m} \\ \alpha = 0.568575 \times 10^{-4}\text{rad} \\ \theta = 0.663893 \times 10^{-5}\text{rad} \end{cases}$$

In this situation, the values of F_U , M_{xU} and M_{yU} are calculated at the radius 18mm of the hard disk drive, as shown below:

$$\begin{cases} F_U = 0.14908818\text{N} \\ M_{xU} = 0.45287897 \times 10^{-3}\text{Nm} \\ M_{yU} = 0.23191472 \times 10^{-3}\text{Nm} \end{cases}$$

It is shown that the errors between the calculated air bearing force/moments and the applied pre-load force/moments are very small, as the following:

$$\begin{cases} E_F = 0.60787857\% \\ E_{M_x} = 0.63977143\% \\ E_{M_y} = 0.83248571\% \end{cases}$$

To evaluate the convergence history of the flying attitude determination of the Two-Rail Slider, the variations of F_U , M_{xU} and M_{yU} against the number of iterations are shown in Figures 6.10~6.12, respectively. From these figures, it is shown that the values of F_U , M_{xU} and M_{yU} become closer to the applied ones after 22 iterative steps.

Fig. 6.10 Force variations in flying attitude determination of Two-Rail Slider

Fig. 6.11 X-moment variations in flying attitude determination of Two-Rail Slider

Fig. 6.12 Y-moment variations in flying attitude determination of Two-Rail Slider

After the flying attitude determination, the flying attitude and pressure profile of the Two-Rail Slider are shown in Figures 6.13 & 6.14, respectively.

Fig. 6.13 Flying attitude of Two-Rail Slider after flying attitude determination

Fig. 6.14 Pressure profile of Two-Rail Slider after flying attitude determination

Comparing Figures 6.13 & 6.14 with Figures 6.8 & 6.9, it is clear that the flying attitudes and the pressure profiles of the Two-Rail Slider have great changes after the flying attitude determination.

6.6 Summary

In this chapter, the flying attitude determination of the slider air bearing is presented. The determination of flying attitude is required for the theoretical verification of the topology optimization results of the slider air bearing.

The MMA algorithm is first introduced to deal with the optimization problems for its global convergence and high flexibility. The formulation of the objective function, the constraint definition of the design variables, and how to transform the original problem into the modified subproblem are described in detail. Then the mathematical method is applied to solve the modified subproblem to obtain the optimal result.

The flying attitude determination of the slider air bearing is basically a parametric optimization procedure. When a known pre-load is applied to the slider air bearing, the applied pre-load force, the applied pre-load X-moment and the applied pre-load Y-moment are obtained. By simultaneously minimizing the force difference, the X-moment difference and the Y-moment difference between the calculated air bearing force/moments and the applied pre-load force/moments, the flying attitude of the slider air bearing can be determined. From the flying attitude determinations of the

Plane Slider and the Two-Rail Slider, it proves the effectiveness of the MMA procedure in the flying attitude determination of the slider air bearing.

In most practical applications of the flying attitude determination of the slider air bearing, the computation time will be affected by the selection of the initial guess and the constraint domain for the design variables. Therefore it is important to adopt a suitable initial guess and the constraint domain according to theoretical and experimental studies.

Chapter 7

Topology Optimization of Slider Air Bearing

7.1 Introduction

In Chapter 6, the equilibrium flying attitude of the slider air bearing was solved by the flying attitude determination procedure, which is required to verify the topology optimization results in this chapter. Since the topology optimization of slider air bearing is based on the theoretical assumption that the flying attitudes of the slider over the whole disk surface are near-constant as long as the flying attitudes of the slider at the most outer track and at the most inner track are near-constant, one flying attitude optimization procedure is demonstrated to verify it. Based on this theoretical assumption, an efficient topology optimization methodology is developed to systematically and automatically design a novel slider according to the requirements.

7.2 Verification of Theoretical Assumption

One important issue in designing a high-performance magnetic slider is to achieve a stable spacing or flying height between the air bearing surface and the disk surface when the slider moves along different tracks of the disk surface. To realize this objective, the flying attitude of the slider should be near-constant when it moves over the whole disk surface. Hence, it can be assumed that the flying attitudes of the slider over the whole disk surface are near-constant as long as the flying attitudes of

the slider at some selected extreme tracks are near-constant. Since the flying attitude of the slider air bearing is determined by the air bearing force and air bearing moments, the air bearing force F , the air bearing X-moment M_x and the air bearing Y-moment M_y are used to estimate the flying attitude of the slider air bearing. If the air bearing force difference ΔF , the air bearing X-moment difference ΔM_x and the air bearing Y-moment difference ΔM_y at the selected extreme tracks are simultaneously minimized, then the flying attitude of the slider should be maximum stabilized when it moves over the whole disk surface.

7.2.1 Problem Formulation

To ensure the flying attitude of the slider at some selected extreme tracks of the disk surface near-constant, it means that the values of the calculated air bearing force F , the calculated air bearing X-moment M_x and the calculated air bearing Y-moment M_y at these tracks should as close to each other as possible. Hence two extreme tracks are selected for the flying attitude optimization, as the followings:

$$R_{out} = 30 \times 10^{-3} \text{ m} \quad R_{inn} = 10 \times 10^{-3} \text{ m}$$

Here R_{out} is the radius of the most outer track and R_{inn} is the radius of the most inner track. Therefore the objective is to simultaneously minimize the air bearing force difference ΔF , the air bearing X-moment difference ΔM_x and the air bearing Y-moment difference ΔM_y at the two tracks R_{out} and R_{inn} , as the followings:

$$\psi_2 = w_1 \cdot m_1 \cdot \Delta F^2 + w_2 \cdot m_2 \cdot \Delta M_x^2 + w_3 \cdot m_3 \cdot \Delta M_y^2 \quad (7.1)$$

$$\begin{cases} \Delta F = F_{out} - F_{inn} \\ \Delta M_x = M_{xout} - M_{xinn} \\ \Delta M_y = M_{yout} - M_{yinn} \end{cases} \quad (7.2)$$

where ψ_2 is the multi-objective function for the flying attitude optimization of the slider air bearing. F_{out} , M_{xout} and M_{yout} are the calculated air bearing force, the calculated air bearing X-moment and the calculated air bearing Y-moment at the most outer track. F_{inn} , M_{xinn} and M_{yinn} are the calculated air bearing force, the calculated air bearing X-moment and the calculated air bearing Y-moment at the most inner track. The first-order sensitivity of the multi-objective function ψ_2 can be derived as

$$\frac{\partial \psi_2}{\partial v_i} = 2w_1 \cdot m_1 \cdot \Delta F \cdot \Delta \frac{\partial F}{\partial v_i} + 2w_2 \cdot m_2 \cdot \Delta M_x \cdot \Delta \frac{\partial M_x}{\partial v_i} + 2w_3 \cdot m_3 \cdot \Delta M_y \cdot \Delta \frac{\partial M_y}{\partial v_i} \quad (7.3)$$

$$\begin{cases} \Delta \frac{\partial F}{\partial v_i} = \frac{\partial F_{out}}{\partial v_i} - \frac{\partial F_{inn}}{\partial v_i} \\ \Delta \frac{\partial M_x}{\partial v_i} = \frac{\partial M_{xout}}{\partial v_i} - \frac{\partial M_{xinn}}{\partial v_i} \\ \Delta \frac{\partial M_y}{\partial v_i} = \frac{\partial M_{yout}}{\partial v_i} - \frac{\partial M_{yinn}}{\partial v_i} \end{cases} \quad (7.4)$$

$$\begin{cases} \frac{\partial F_{out}}{\partial v_i} = \frac{F_{out}(1.005v_i) - F_{out}(0.995v_i)}{0.01v_i} \\ \frac{\partial M_{xout}}{\partial v_i} = \frac{M_{xout}(1.005v_i) - M_{xout}(0.995v_i)}{0.01v_i} \\ \frac{\partial M_{yout}}{\partial v_i} = \frac{M_{yout}(1.005v_i) - M_{yout}(0.995v_i)}{0.01v_i} \end{cases} \quad (7.5)$$

$$\begin{cases} \frac{\partial F_{inn}}{\partial v_i} = \frac{F_{inn}(1.005v_i) - F_{inn}(0.995v_i)}{0.01v_i} \\ \frac{\partial M_{xinn}}{\partial v_i} = \frac{M_{xinn}(1.005v_i) - M_{xinn}(0.995v_i)}{0.01v_i} \\ \frac{\partial M_{yinn}}{\partial v_i} = \frac{M_{yinn}(1.005v_i) - M_{yinn}(0.995v_i)}{0.01v_i} \end{cases} \quad (7.6)$$

7.2.2 Initialization of Model

In this case study, the model used is a Plane Slider in Chapter 5. The minimum flying height h_m , the pitch angle α , and the roll angle θ are set as the design variables v_i , as shown in Table 7.1.

Table 7.1 Initial guess and constraint domain for flying attitude optimization of Plane Slider

	Minimum constraint	Initial guess	Maximum constraint
Minimum flying height	$h_{\min} = 50\text{nm}$	$h_m = 50\text{nm}$	$h_{\max} = 200\text{nm}$
Pitch angle	$\alpha_{\min} = 1.0 \times 10^{-5} \text{rad}$	$\alpha = 1.0 \times 10^{-3} \text{rad}$	$\alpha_{\max} = 1.0 \times 10^{-3} \text{rad}$
Roll angle	$\theta_{\min} = 1.0 \times 10^{-5} \text{rad}$	$\theta = 1.0 \times 10^{-3} \text{rad}$	$\theta_{\max} = 1.0 \times 10^{-3} \text{rad}$

Meshed by the 193×193 uniform grids, the flying attitude of the Plane Slider is shown in Figure 7.1. The pressure profiles of the Plane Slider with the initial guess are shown in Figures 7.2 & 7.3 for the positions at most outer track & at most inner track, respectively.

Fig. 7.1 Flying attitude of Plane Slider with initial guess

Fig. 7.2 Pressure profile of Plane Slider with initial guess at most outer track

Fig. 7.3 Pressure profile of Plane Slider with initial guess at most inner track

As shown in Figures 7.2 & 7.3, the pressure profiles of the Plane Slider at the most outer track and at the most inner track are similar. Due to the effect of the positive pitch angle α and the positive roll angle θ , the pressure peaks occur at the right-top corners. However, the peak value at the most outer track is far larger than that at the most inner track. In the computation, the three-level LSFDM procedure is used to obtain the values of F_{out} , F_{inn} , M_{xout} , M_{xinn} , M_{yout} and M_{yinn} . In the MMA procedure, the convergence criterion for the multi-objective function ψ_2 is set as

$$\begin{cases} E_F = |F_{out} - F_{inn}| / (F_{out} + F_{inn}) < 0.5 \times 10^{-1} \\ E_{M_x} = |M_{xout} - M_{xinn}| / (M_{xout} + M_{xinn}) < 0.5 \times 10^{-1} \\ E_{M_y} = |M_{yout} - M_{yinn}| / (M_{yout} + M_{yinn}) < 0.5 \times 10^{-1} \end{cases} \quad (7.7)$$

7.2.3 Optimal Results and Verifications

After some optimal iterative steps in the LSFDM procedure and the MMA procedure, the performance characteristics of the Plane Slider after the flying attitude optimization are shown in Table 7.2.

Table 7.2 Performance characteristics for flying attitude optimization of Plane Slider

	Initial Guess	Optimal Result
Force in most outer track F_{out} (N)	0.32001×10^{-1}	0.17906×10^{-2}
Force in most inner track F_{inn} (N)	0.14414×10^{-1}	0.19683×10^{-2}
Force error E_F	0.37901	0.47265×10^{-1}
X-moment in most outer track M_{xout} (Nm)	0.23058×10^{-4}	0.10630×10^{-5}
X-moment in most inner track M_{xinn} (Nm)	0.99813×10^{-5}	0.96186×10^{-6}
X-moment error E_{M_x}	0.39580	0.49887×10^{-1}
Y-moment in most outer track M_{yout} (Nm)	0.15817×10^{-4}	0.98315×10^{-6}
Y-moment in most inner track M_{yinn} (Nm)	0.71695×10^{-5}	0.10829×10^{-5}
Y-moment error E_{M_y}	0.37620	0.48300×10^{-1}

To evaluate the convergence history of the flying attitude optimization of the Plane Slider, the variations of F_{out} and F_{inn} , M_{xout} and M_{xinn} , and M_{yout} and M_{yinn} against the number of iterations are shown in Figures 7.4~7.6, respectively.

Fig. 7.4 Force variations in flying attitude optimization of Plane Slider

Fig. 7.5 X-moment variations in flying attitude optimization of Plane Slider

Fig. 7.6 Y-moment variations in flying attitude optimization of Plane Slider

From the three figures above, it is clear that the values of ΔF , ΔM_x and ΔM_y become very small after 6 iterative steps.

Based on the optimal results obtained from the flying attitude optimization of the Plane Slider, the applied pre-load is determined by the formulae, as shown below:

$$\begin{cases} F_0 = \frac{F_{out} + F_{inn}}{2} \\ L_{x0} = \frac{M_{xout} + M_{xinn}}{F_{out} + F_{inn}} \\ L_{y0} = \frac{M_{yout} + M_{yinn}}{F_{out} + F_{inn}} \end{cases} \quad (7.8)$$

Hence, the magnitude and location of the applied preload F_0 are obtained as

$$F_0 = 0.18794637 \times 10^{-2} \text{ N} \quad L_{x0} = 0.53864099 \times 10^{-3} \text{ m} \quad L_{y0} = 0.54965148 \times 10^{-3} \text{ m}$$

The procedure of the flying attitude determination of the slider air bearing in Chapter 6 is applied to obtain the equilibrium flying attitude along different tracks of the disk surface based on the applied pre-load force above. The calculated flying attitudes along different tracks are shown in Table 7.3.

Table 7.3 Flying attitudes along different tracks for flying attitude optimization of Plane Slider

Radius of track R_d (mm)	Minimal flying height h_m (nm)	Pitch angle α (rad)	Roll angle θ (rad)
5	51	0.10819×10^{-4}	0.75722×10^{-3}
10	49	0.98095×10^{-5}	0.75778×10^{-3}
15	51	0.97892×10^{-5}	0.75787×10^{-3}
20	49	0.10080×10^{-4}	0.75746×10^{-3}
25	49	0.10288×10^{-4}	0.75720×10^{-3}
30	49	0.10451×10^{-4}	0.75536×10^{-3}
35	49	0.10142×10^{-4}	0.75902×10^{-3}

From the table above, it can be found that the variations of h_m , α and θ (i.e. the flying attitude of the slider air bearing) are very small along different tracks of the disk surface. It means that the Plane Slider is able to achieve a near-constant flying attitude when it moves over the disk surface.

The pressure profiles of the Plane Slider after the flying attitude optimization are shown in Figures 7.7 & 7.8 for the positions at most outer track & at most inner track, respectively.

Fig. 7.7 Pressure profile of Plane Slider after flying attitude optimization at most outer track

Fig. 7.8 Pressure profile of Plane Slider after flying attitude optimization at most inner track

As shown in Figures 7.7 & 7.8, all pressure peaks occur at the outer edges due to the effect of the larger roll angle θ . It is clear that the pressure peak value and the

position of the pressure peak at the most outer track are very close to those at the most inner track. It makes it possible that the values of F_{out} , M_{xout} and M_{yout} are almost equal to those of F_{inn} , M_{xinn} and M_{yinn} . By the computation with the flying attitude determination procedure, the similar pressure profiles of the Plane Slider also occur at the other tracks. As a consequence, the Plane Slider can keep a stable spacing between the air bearing surface and the disk surface when it moves along different tracks.

As a necessary preparation for the topology optimization of the slider air bearing, the optimization for flying attitude of the slider air bearing has shown the effectiveness of the theoretical assumption that the flying attitudes of the slider over the whole disk surface are near-constant as long as the flying attitudes of the slider at the most outer track and at the most inner track are near-constant. It is the theoretical foundation of the topology optimization of the slider air bearing proposed here.

7.3 Topology Optimization of Slider Air Bearing

Based on the theoretical assumption stated above, an efficient topology optimization methodology is developed to systematically and automatically design a novel slider.

7.3.1 Problem Formulation

To ensure a near-constant flying attitude of the slider along different tracks of the disk surface, it means that the calculated air bearing force F , the calculated air bearing X-moment M_x and the calculated air bearing Y-moment M_y at those tracks

should be almost a constant. Hence, two extreme tracks are selected for the topology optimization, as the followings:

$$R_{out} = 30 \times 10^{-3} \text{ m} \quad R_{inn} = 10 \times 10^{-3} \text{ m}$$

Therefore, the multi-objective function ψ_3 is set to simultaneously minimize the air bearing force difference ΔF , the air bearing X-moment difference ΔM_x and the air bearing Y-moment difference ΔM_y at the two extreme tracks R_{out} and R_{inn} , as the following:

$$\psi_3 = w_1 \cdot m_1 \cdot \Delta F^2 + w_2 \cdot m_2 \cdot \Delta M_x^2 + w_3 \cdot m_3 \cdot \Delta M_y^2 \quad (7.9)$$

Here the height of the slider air bearing h_{ij} at the reference point i, j in the LSFDM procedure is set as the design variable. It means that the slider height at every grid point in the meshed geometry configuration of the slider will vary within the constraint domain, as the following:

$$(h_{ij})_{\min} < h_{ij} < (h_{ij})_{\max} \quad (7.10)$$

The first-order sensitivity of the multi-objective function ψ_3 with respect to the design variables h_{ij} can be derived as

$$\frac{\partial \psi_3}{\partial h_{ij}} = 2w_1 \cdot m_1 \cdot \Delta F \cdot \Delta \frac{\partial F}{\partial h_{ij}} + 2w_2 \cdot m_2 \cdot \Delta M_x \cdot \Delta \frac{\partial M_x}{\partial h_{ij}} + 2w_3 \cdot m_3 \cdot \Delta M_y \cdot \Delta \frac{\partial M_y}{\partial h_{ij}} \quad (7.11)$$

$$\begin{cases} \Delta \frac{\partial F}{\partial h_{ij}} = \frac{\partial F_{out}}{\partial h_{ij}} - \frac{\partial F_{inn}}{\partial h_{ij}} \\ \Delta \frac{\partial M_x}{\partial h_{ij}} = \frac{\partial M_{xout}}{\partial h_{ij}} - \frac{\partial M_{xinn}}{\partial h_{ij}} \\ \Delta \frac{\partial M_y}{\partial h_{ij}} = \frac{\partial M_{yout}}{\partial h_{ij}} - \frac{\partial M_{yinn}}{\partial h_{ij}} \end{cases} \quad (7.12)$$

To save the computational time of the computation in the topology optimization of the slider air bearing, the calculations of the first-order sensitivities of F , M_x and M_y with respect to h_{ij} are simplified numerically as

$$\begin{cases} \frac{\partial F_{out}}{\partial h_{ij}} = \frac{F_{out}(1.01h_{ij}) - F_{out}}{0.01h_{ij}} \\ \frac{\partial M_{xout}}{\partial h_{ij}} = \frac{M_{xout}(1.01h_{ij}) - M_{xout}}{0.01h_{ij}} \\ \frac{\partial M_{yout}}{\partial h_{ij}} = \frac{M_{yout}(1.01h_{ij}) - M_{yout}}{0.01h_{ij}} \end{cases} \quad (7.13)$$

$$\begin{cases} \frac{\partial F_{inn}}{\partial h_{ij}} = \frac{F_{inn}(1.01h_{ij}) - F_{inn}}{0.01h_{ij}} \\ \frac{\partial M_{xinn}}{\partial h_{ij}} = \frac{M_{xinn}(1.01h_{ij}) - M_{xinn}}{0.01h_{ij}} \\ \frac{\partial M_{yinn}}{\partial h_{ij}} = \frac{M_{yinn}(1.01h_{ij}) - M_{yinn}}{0.01h_{ij}} \end{cases} \quad (7.14)$$

To further increase the efficiency of the topology optimization of the slider air bearing, the first-order sensitivities $(\partial\psi_3/\partial h_{ij})^{4h}$ in the third-level multigrid computation domain Ω^{4h} are first calculated in the LSFDM procedure, referring to Chapter 4. Then the first-order sensitivities $(\partial\psi_3/\partial h_{ij})^h$ in the first-level multigrid computation domain Ω^h can be prolonged from the first-order sensitivities $(\partial\psi_3/\partial h_{ij})^{4h}$ in the third-level multigrid computation domain Ω^{4h} , as shown below:

$$\left(\frac{\partial \psi_3}{\partial h_{ij}}\right)^h = I_{2h}^h \left(\frac{\partial \psi_3}{\partial h_{ij}}\right)^{2h} = I_{2h}^h \left[I_{4h}^{2h} \left(\frac{\partial \psi_3}{\partial h_{ij}}\right)^{4h} \right] \quad (7.15)$$

7.3.2 Initialization of Prototype Slider

To verify the practical application of the topology optimization methodology of the slider air bearing, a so-called Prototype Slider is adopted to initialize the design variables h_{ij} in the first-level multigrid computation domain Ω^h , as shown below:

Table 7.4 Parameters of Prototype Slider

Total length	$L = 1.0\text{mm}$	Ambient pressure	$P_a = 1.01325 \times 10^5 \text{ Pa}$
Total width	$W = 0.8\text{mm}$	Dynamic viscosity	$\mu = 1.806 \times 10^{-5} \text{ Pa/sec}$
Minimal flying height	$h_m = 100\text{nm}$	Molecular mean free path	$\lambda = 63.5\text{nm}$
Pitch angle	$\alpha = 1.0 \times 10^{-4} \text{ rad}$	Disk rotating speed	$\omega = 5400\text{RPM}$
Roll angle	$\theta = 0\text{rad}$		

Here the constraint domain of the design variable h_{ij} in the first-level multigrid computation domain Ω^h is set as the following:

$$\begin{cases} (h_{ij})_{\min} = h_{ij} - 10\text{nm} \\ (h_{ij})_{\max} = h_{ij} + 10\text{nm} \end{cases} \quad (7.16)$$

To precisely weigh the contribution of every grid point in the topology optimization of the Prototype Slider, the uniform grid distribution is used to ensure that the grid points are evenly distributed in the computation domain. Herein 97×97 uniform grids

are used to initialize the design variables h_{ij} in the first-level multigrid computation domain Ω^h , as shown in Figure 7.9. The pressure profiles of the Prototype Slider with the initial design variables h_{ij} are shown in Figures 7.10 & 7.11 for the positions at most outer track & at most inner track, respectively.

Fig. 7.9 Initial design variables in first-level multigrid computation domain

Fig. 7.10 Pressure profile of Prototype Slider with initial guess at most outer track

Fig. 7.11 Pressure profile of Prototype Slider with initial guess at most inner track

As shown in Figures 7.10 & 7.11, the pressure profiles obtained with the initial design variables h_{ij} at the most outer track and at the most inner track are similar. Due to the effect of the positive pitch angle α , the pressure peaks occur at the trailing edges. However the peak value at the most outer track is larger than that at the most inner track. It means that the values of F_{out} , M_{xout} and M_{yout} will be greater than those of F_{inn} , M_{xinn} and M_{yinn} .

Since the three-level LSFDM procedure is used in the topology optimization of the Prototype Slider, it can improve the computational efficiency by calculating the first-order sensitivities of the multi-objective function ψ_3 with respect to the design variables h_{ij} in the third-level multigrid computation domain Ω^{4h} by 25×25 uniform grids, as shown in Figure 7.12. Therefore, most computations in the numerical simulation and the topology optimization of the Prototype Slider are performed in the third-level multigrid computation domain Ω^{4h} .

Fig. 7.12 Design variables for first-order sensitivities in
third-level multigrid computation domain

In the MMA procedure, the convergence criterion for the multi-objective function ψ_3 is set as

$$\begin{cases} E_F = |F_{out} - F_{inn}| / (F_{out} + F_{inn}) < 0.1 \times 10^{-1} \\ E_{M_x} = |M_{xout} - M_{xinn}| / (M_{xout} + M_{xinn}) < 0.1 \times 10^{-1} \\ E_{M_y} = |M_{yout} - M_{yinn}| / (M_{yout} + M_{yinn}) < 0.1 \times 10^{-1} \end{cases} \quad (7.17)$$

7.3.3 Optimal Results and Verifications

After some optimal iterative steps in the LSFDM procedure and the MMA procedure, the optimal configuration of the optimized slider and the performance characteristics of the Prototype Slider after the topology optimization are shown in Figures 7.13~7.15 and Table 7.5, respectively.

Fig. 7.13 Optimal configuration of topology optimization of Prototype Slider (side view)

Fig. 7.14 Optimal configuration of topology optimization of Prototype Slider (downward view)

Fig. 7.15 Optimal configuration of topology optimization of Prototype Slider (upward view)

Table 7.5 Performance characteristics for topology optimization of Prototype Slider

	Initial Guess	Optimal Result
Force in most outer track F_{out} (N)	0.27036×10^{-1}	0.21600×10^{-1}
Force in most inner track F_{inn} (N)	0.23643×10^{-1}	0.21547×10^{-1}
Force error E_F	0.66944×10^{-1}	0.12280×10^{-2}
X-moment in most outer track M_{xout} (Nm)	0.18820×10^{-4}	0.16071×10^{-4}
X-moment in most inner track M_{xinn} (Nm)	0.16248×10^{-4}	0.15442×10^{-4}
X-moment error E_{M_x}	0.73322×10^{-1}	0.19945×10^{-1}
Y-moment in most outer track M_{yout} (Nm)	0.10794×10^{-4}	0.86222×10^{-5}
Y-moment in most inner track M_{yinn} (Nm)	0.93940×10^{-5}	0.85677×10^{-5}
Y-moment error E_{M_y}	0.69353×10^{-1}	0.31676×10^{-2}

To evaluate the convergence history of the topology optimization of the Prototype Slider, the variations of F_{out} and F_{inn} , M_{xout} and M_{xinn} , and M_{yout} and M_{yinn} against the number of iterations are shown in Figures 7.16~7.18, respectively.

Fig. 7.16 Force variations in topology optimization of Prototype Slider

Fig. 7.17 X-moment variations in topology optimization of Prototype Slider

Fig. 7.18 Y-moment variations in topology optimization of Prototype Slider

From the three figures above, it is clear that the values of ΔF , ΔM_x and ΔM_y become small after 19 iterative steps.

Based on the optimal results obtained from the topology optimization of the Prototype Slider, the applied pre-load is determined by Equation (7.8), as shown below:

$$F_0 = 0.21573812 \times 10^{-1} \text{ N} \quad L_{x0} = 0.73034452 \times 10^{-3} \text{ m} \quad L_{y0} = 0.39839686 \times 10^{-3} \text{ m}$$

Subsequently, the procedure of the flying attitude determination of the slider air bearing is carried out to obtain the equilibrium flying attitude of the optimal slider design along different tracks of the disk surface based on the applied pre-load obtained above. Since the optimal configuration of the topology optimization of the Prototype Slider is too complex to define the initial flying attitude, Δh_m , $\Delta \alpha$ and $\Delta \theta$ along different tracks, in comparison with the flying attitude of the Prototype Slider with the initial guess, are used as a relative indicator to reflect flying attitude variations, as shown in Table 7.6.

Table 7.6 Flying attitudes along different tracks for topology optimization of Prototype Slider

Radius of track R_d (mm)	Minimal flying height variation Δh_m (nm)	Pitch angle variation $\Delta\alpha$ (rad)	Roll angle variation $\Delta\theta$ (rad)
5	2	0.13348×10^{-6}	-0.46731×10^{-6}
10	1	0.26338×10^{-6}	-0.34766×10^{-6}
15	0	-0.10000×10^{-5}	-0.79186×10^{-6}
20	0	-0.10000×10^{-5}	0.53955×10^{-7}
25	0	-0.10000×10^{-5}	0.10000×10^{-5}
30	2	-0.94966×10^{-6}	0.98026×10^{-6}
35	2	0.41016×10^{-6}	-0.344557×10^{-7}

From the table above, it is clear that the values of flying attitude variations Δh_m , $\Delta\alpha$ and $\Delta\theta$ are very small. The pressure profiles of the optimal slider air bearing after the topology optimization are shown in Figures 7.19 & 7.20 for the positions at most outer track & at most inner track, respectively.

Fig. 7.19 Pressure profile of optimal configuration after topology optimization at most outer track

Fig. 7.20 Pressure profile of optimal configuration after topology optimization at most inner track

As shown above, the negative pressure peaks occur at the leading edges and the positive pressure peaks occur at the trailing edges due to the special configuration of the optimal slider shown in Figures 7.13~7.15. Comparing the pressure profiles in Figures 7.19 & 7.20, it is found that the values of the positive pressure peaks and the negative pressure peaks will simultaneously increase at the most outer track, while the values of the positive pressure peaks and the negative pressure peaks will simultaneously reduce at the most inner track. Hence, it is possible to maintain the near-constant values of F_U , M_{xU} and M_{yU} when the optimal slider air bearing moves along different tracks of the disk surface.

As a consequence, a novel slider air bearing is designed by using the topology optimization methodology with a target of achieving a near-constant flying attitude when it moves over the whole disk surface. From the very small variations of the calculated air bearing force, the calculated air bearing X-moment and the calculated

air bearing Y-moment along the different tracks, it is proven that the proposed topology optimization methodology is effective for designing optimal slider air bearing.

7.4 Summary

In this chapter, a flying attitude optimization procedure is obtained to verify the theoretical assumption that the flying attitudes of the slider over the whole disk surface are near-constant as long as the flying attitudes of the slider at the most outer track and at the most inner track are near-constant. Based on this theoretical assumption, an efficient topology optimization methodology is developed to systematically and automatically design the slider air bearing with a target of achieving a near-constant flying attitude when the slider moves over the whole disk surface. The multi-objective function is defined to simultaneously minimize the air bearing force difference, the air bearing X-moment difference and the air bearing Y-moment difference of the slider at the most outer track and at the most inner track. The slider height at every grid point in the meshed geometry configuration is set as a design variable. To save the computational time, the first-order sensitivities of the multi-objective function with respect to the design variables are calculated in the third-level multigrid computation domain. Then the first-order sensitivities of the multi-objective function with respect to all design variables can be prolonged from those in the third-level multigrid computation. Applying the developed topology optimization methodology, a novel optimal slider is designed. From the flying attitude determinations along different tracks, it is proven that the novel slider is able to achieve a near-constant flying attitude along different tracks of the disk surface.

Chapter 8

Conclusions and Recommendations

Two main objectives have been achieved in this research. An efficient numerical simulation technique and tool was developed for modeling the slider air bearing in hard disk drives and an efficient topology optimization methodology was ultimately developed to systematically and automatically design the slider air bearing with a target of achieving a near-constant flying attitude when the slider moves over the whole disk surface. In future work, more studies need to be performed to further increase the efficiency of the numerical simulation for more complex slider air bearing, to testify the stability of the topology optimization algorithm for the global optimum, and to extend the application of the topology optimization solver for different governing equations or different design domains.

8.1 Conclusions

Firstly, to solve the partial differentiate Reynolds equations in the numerical simulation of the slider air bearing, the Gauss-Seidel SOR and the Newton-SOR iterative methods, two iterative algorithms to solve the numerically discretized equations, are compared by simulating the Plate Slider. From the numerical results, the Newton-SOR iterative algorithm is found to be more efficient than the Gauss-Seidel SOR iterative algorithm.

Secondly, based on the numerically discretized Reynolds equations by the Least Square Finite Difference method, the Least Square Finite Difference Multigrid algorithm using the three-level v-cycle strategy calculated from the finest grid is applied to further accelerate the convergence rate and increase the efficiency of the simulation procedure. Thereafter the numerical simulations of the Infinitely-Long Plane-Wedge Slider, the Plane Slider, the Two-Rail Slider and the Advanced Step Slider were carried out by the in-house developed solver. By comparing the results obtained with those from the published papers or from the finite volume method, it is found that the Least Square Finite Difference Multigrid algorithm is accurate and efficient in the numerical simulation of the slider air bearing.

Thirdly, after the efficiency of the numerical simulation of the slider air bearing was enhanced by the Least Square Finite Difference Multigrid procedure, the in-house developed solver combined with the Method of Moving Asymptotes program is applied for the flying attitude determination of the slider air. From the flying attitude determination of the Plane Slider and the Two-Rail Slider, it is found the procedure is effective for determining the flying attitude of the slider air bearing.

Finally, a theoretical assumption, that the flying attitudes of the slider over the whole disk surface are near-constant as long as the flying attitudes of the slider at the most outer track and at the most inner track, was verified by a simple parametric optimization for the flying attitude of the slider air bearing are near-constant. Based on this theoretical assumption, an efficient topology optimization methodology to systematically and automatically design the slider air bearing was developed to achieve a near-constant flying attitude when the slider moves over the whole disk

surface. Applying the topology optimization methodology developed, a novel optimal slider was designed. From the flying attitude determinations along different tracks of the disk surface, it is found that the novel slider is able to achieve a near-constant flying attitude when it moves along different tracks of the disk surface.

8.2 Recommendations

The research work conducted so far has led to an efficient topology optimization methodology to systematically and automatically design the slider air bearing by using the Least Square Finite Difference Multigrid algorithm and the Method of Moving Asymptotes algorithm. To make the procedure mature enough to be applied to more practical applications, several aspects should be improved in the future work.

Firstly, since the advantage of the Least Square Finite Difference Multigrid algorithm lies in the flexibility of the grid generation without element connectivity. The multilevel adaptive scheme can be adopted to cluster a high quality fine mesh in the recess wall region based on geometrical considerations. In air bearing surfaces, the majority regions with smoothly varying pressure are surrounded by small regions with extremely high pressure gradients. Therefore the multilevel adaptive scheme, which applies the undivided pressure difference as the sensor, is needed to refine a good quality mesh in the high pressure gradient regions so as to save the computational time.

Secondly, to ensure that the grid points are evenly distributed in the topology optimization of the slider air bearing, all studies in the dissertation were performed

based on the rectangular grid distribution. Actually, as an effective method to increase the efficiency of the procedure, the triangular grid distribution should be adopted to carry out the numerical simulation of the slider air bearing. Compared with the rectangular grid distribution, the triangular grid distribution is applicable for all types of sliders with complex shapes due to its superior ability in the optimal distribution generation of the existing grid. In conjunction with the Least Square Finite Difference Multigrid algorithm, the advantage of the triangular grid distribution can be fully demonstrated in the future work.

Thirdly, although an effective topology optimization methodology has been developed in this thesis to systematically and automatically design the air bearing slider with a target of achieving a near-constant flying attitude when the slider moves over the surface of the hard disk drive, more study cases are required to verify its global optimum by adopting different initial guesses and different computational domains. Thus this topology optimization method is possible to be extended to other practical application where the governing equations or design domains are different.

Fourthly, even more powerful and efficient mathematical programming algorithms are required to for the topology optimization of the slider air bearing. In the Method of Moving Asymptotes algorithm, the design variables can be optimized in a small range of constraint domain. However, with the increase of the constraint domain, it is difficult to ensure the continuity of the neighboring design variables. In another word, the geometry configuration of the slider air bearing will oscillate substantially. Hence more recently developed optimization algorithms should be explored for the topology optimization of the slider air bearing. As another possible

aspect of the following investigation, a more profound understanding of the weighting factors in the multi-objective function should be studied in more detail. It is known that the choice of the weighting factors is dictated by experience or some initial trial searches. How to automatically define the weighting factors is also an important step to make the topology optimization process more systematical and practical.

Finally, since much computation time is wasted for numerical calculating the sensitivities of every grid node in the topology optimization of the slider air bearing, an analytical method is required to be applied to simplify the calculation of the sensitivities of the objective function in the next study. Moreover, since the slider dynamics play a very important role in the slider stability, it is necessary to incorporate the suspension dynamics in the future work with the consideration of the mechanics model of the slider suspension.

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