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2018

Li, L., Lin, Z., Chai, Y., & Cai, J. (2019). Output feedback stabilization of two-dimensional fuzzy systems. *Multidimensional Systems and Signal Processing*, 30(4), 1731-1748.
doi:10.1007/s11045-018-0625-x

<https://hdl.handle.net/10356/136677>

<https://doi.org/10.1007/s11045-018-0625-x>

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Output Feedback Stabilization of Two-dimensional Fuzzy Systems

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Received: date / Accepted: date

Abstract This paper is concerned with the output feedback stabilization of two-dimensional (2D) discrete fuzzy systems described by the Fornasini-Marchesini (FM) second model. Based on the fuzzy-basis-dependent Lyapunov function, a new criterion is proposed for the fuzzy static output feedback (SOF) controller, which is expressed as strict linear matrix inequalities and hence numerically tractable. The main advantage of the developed SOF control scheme is that no constraints are imposed on system matrices, which is expected to have a wider range of applications. The applicability and the advantage of the proposed results are shown through two numerical examples.

Keywords Two-dimensional fuzzy systems · output feedback stabilization · static output feedback control · linear matrix inequality (LMI) approach

The work described in this paper was substantially supported by the National Nature Science Foundation of China under Grant 11502139 and 11701359.

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1 Introduction

Two-dimensional (2D) systems have been extensively studied due to its wide applications in practical systems, such as thermal processes in chemical reactors, heat exchangers and pipe furnaces, river pollution modeling, 2D digital control systems (Kaczorek, 1985). In recent years, 2D Takagi-Sugeno (T-S) fuzzy system has attracted considerable attention due to the fact that, as an universal function approximator, it can describe a majority of 2D nonlinear systems. The distinguished feature of the T-S fuzzy model is that it is able to approximate any smooth nonlinear function to any degree of accuracy in any convex compact region (Cao et al, 1997). So far, some efforts have been made to utilize fuzzy-model-based control for 2D nonlinear and other complex systems. Sufficient conditions of stabilizability are presented for 2D fuzzy systems described with Roesser and Fornasini-Marchesini models (Benzaouia and El Hajjaji, 2014; Li and Wang, 2012; Chen et al, 2013; Hien and Trinh, 2017). However, the above controllers considered are state feedback controllers. To the best of our knowledge, few works have been considered on output feedback controllers of 2D fuzzy systems.

In practical applications, due to the fact that a full knowledge of the state vector is rarely available while the system output signals are easy to access, output feedback control is widely adopted for time-varying (1D) fuzzy systems, including the static output feedback (SOF) case (Yu and Mok, 2003; Tseng and Chen, 2010; Zhang et al, 2014; Chang et al, 2015a; Chang and Yang, 2014; Chang et al, 2015b) and the dynamic output feedback (DOF) case (Peng and Guan, 2009; Su et al, 2014; Hua and Guan, 2016). It is known that the 1D output feedback control problem can be formulated in terms of a bilinear matrix inequality (BMI), which is nonconvex and NP-hard (Blondel and Tsitsiklis, 1997). During the past decades, various kinds of methods based on the linear matrix inequality (LMI) have been applied to deal with the difficult BMI problem (Kau et al, 2007; Dong and Yang, 2009; Jiang and Fang, 2013; Shen and Lam, 2016; Yong et al, 2008; Bara and Boutayeb, 2005; Chadli and Guerra, 2012). In Yong et al (2008), a modified cone complementarity linearization (CCL) algorithm is used to solve the nonconvex feasibility SOF control problem. Based on a numerical optimization algorithm, an LMI-based condition is proposed for static output feedback controller design in Chang and Yang (2014); Chang et al (2015b), without the need to impose any constraints on system matrices. In Dong and Yang (2013), sufficient conditions for designing static output feedback controllers are given in terms of solutions to a set of linear matrix inequalities with a line search over a scalar variable.

Similar to the 1D case, the 2D output feedback control problem can also be represented as a BMI problem. In Xu et al (2005), the SOF control problem is discussed for 2D singular systems described by the Roesser model. In Feng and Xu (2010), an iterative algorithm is proposed based on the cone complementarity linearization method to solve the nonconvex SOF control problem for 2D Roesser model. In Feng et al (2012); Peng and Hua (2014), by assuming the input matrix has full-column rank, sufficient LMI conditions are proposed for

the SOF control problem of the 2D Fornasini-Marchesini second model. However, the above results are for 2D linear systems. In contrast to the 2D linear case, to the best of authors' knowledge, the output feedback control problem has been rarely investigated for 2D fuzzy systems except in the authors' work in Li et al (2013a). However, the controlled matrices in Li et al (2013a) are assumed to be row full rank. Thus the proposed methods have limited range of applications.

Motivated by the above discussion, in this paper, we focus on the fuzzy-model-based SOF control for 2D nonlinear systems. **The T-S fuzzy model is formed by weighting each local linear model by its respective membership function. The local linear model is described by the Fornasini-Marchesini second model. In this way, the SOF controller design can be studied for 2D nonlinear systems by virtue of the conventional control theories and techniques.** The rest of the paper is organized as follows. The problem under consideration is formulated in Sect. 2. SOF controller designs are presented in Sect. 3. Illustrative examples are given in Sect. 4 to demonstrate the effectiveness and the advantage of the results. Finally, the paper is concluded in Sect. 5.

Throughout this paper, the superscript 'T' denotes matrix transposition. In symmetric block matrices or long matrix expressions, we use an asterisk (*) to represent a term induced by symmetry and $\text{diag}\{\dots\}$ denotes a block-diagonal matrix. $\mathbb{R}^n$ denotes n -dimensional Euclidean space; $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices; $P > 0$ means that the matrix P is positive definite; I is an appropriately dimensional identity matrix.

2 Problem formulation and preliminaries

Since the system information is propagated along two independent directions for 2D systems, a 2D fuzzy model is established by connecting the local linear model from two directions. Consider the 2D second Fornasini-Marchesini (FM) fuzzy model which is a fuzzy blending of the well known 2D discrete **Fornasini-Marchesini local state space** (FMLSS) model (Fornasini and Marchesini, 1978). The l th rule is of the following format.

$$\begin{aligned} &\text{If } \theta_1(i, j+1) \text{ is } M_{1l}, \text{ and } \dots \text{ and } \theta_\rho(i, j+1) \text{ is } M_{\rho l}; \\ &\quad \theta_1(i+1, j) \text{ is } M_{1k}, \text{ and } \dots \text{ and } \theta_\rho(i+1, j) \text{ is } M_{\rho k}, \\ &\text{then } x_{i+1, j+1} = A_{1l}x_{i, j+1} + A_{2k}x_{i+1, j} + B_{1l}u_{i, j+1} + B_{2k}u_{i+1, j}, \\ &\quad y_{i, j} = C_l x_{i, j}. \end{aligned} \tag{1}$$

where $\theta_1(i, j), \dots, \theta_\rho(i, j)$ are premise variables and measurable. It is assumed that the premise variables do not depend on control variables. **$M_{\beta l}$ ($\beta = 1, \dots, \rho$) is the fuzzy set.** $x_{i, j}$ is the state vector; $y_{i, j}$ is the measured output; $u_{i, j}$ is the control input. $A_{1l}, A_{2l}, B_{1l}, B_{2l}, C_l$ are system matrices with appropriate dimensions.

Remark 1 A fuzzy control algorithm consists of a set of heuristic control rules. Fuzzy sets and fuzzy logic are used, respectively, to represent linguistic terms and to evaluate the rules.

Remark 2 It has been proven that T-S fuzzy models are able to approximate any smooth nonlinear functions to any degree of accuracy in any convex compact region (Cao et al, 1997). Therefore, the above fuzzy modeling method offers an alternative approach to describing complex nonlinear systems. Since each local model in (1) is linear, linear control theories and techniques can be used to its stability analysis and synthesis.

Then the overall fuzzy model achieved by fuzzy blending of each individual plant rule is given by

$$\begin{aligned} x_{i+1,j+1} &= A_1(h_{i,j+1})x_{i,j+1} + A_2(h_{i+1,j})x_{i+1,j} \\ &\quad + B_1(h_{i,j+1})u_{i,j+1} + B_2(h_{i+1,j})u_{i+1,j} \\ y_{i,j} &= C(h_{i,j})x_{i,j} \end{aligned} \quad (2)$$

where

$$\begin{aligned} \begin{bmatrix} A_1(h_{i,j+1}) & B_1(h_{i,j+1}) \\ A_2(h_{i+1,j}) & B_2(h_{i+1,j}) \\ C(h_{i,j}) \end{bmatrix} &= \sum_{l=1}^r \begin{bmatrix} h_l(i,j+1)A_{1l} & h_l(i,j+1)B_{1l} \\ h_l(i+1,j)A_{2l} & h_l(i+1,j)B_{2l} \\ h_l(i,j)C_l \end{bmatrix} \\ h_l(i,j) &= \frac{\varphi_l(i,j)}{\sum_{l=1}^r \varphi_l(i,j)}, \varphi_l(i,j) = \prod_{\beta=1}^p f_{M_{\beta l}}(\theta_{\beta}(i,j)). \end{aligned} \quad (3)$$

and r is the number of the If-then rules. Here $f_{M_{\beta l}}(\theta_{\beta}(i,j))$ denotes the grade of membership of $\theta_{\beta}(i,j)$ in $M_{\beta l}$. Note that $h_l(i,j)$ is the normalized weight, that is, $h_l(i,j) \geq 0$ and $\sum_{l=1}^r h_l(i,j) = 1$.

In terms of the parallel-distributed compensation (PDC) proposed in (Tanaka et al, 1998; Ma et al, 1998), we will use the following SOF controller. That is

$$\begin{aligned} &\text{If } \theta_1(i,j) \text{ is } M_{1l}, \text{ and } \cdots \text{ and } \theta_p(i,j) \text{ is } M_{pl} \\ &\text{then } u_{i,j} = F_l y_{i,j}. \end{aligned} \quad (4)$$

where F_l is the controller gain to be determined.

The defuzzified output of the SOF controller (4) is inferred as

$$u_{i,j} = F(h_{i,j})y_{i,j} \quad (5)$$

where $F(h_{i,j}) = \sum_{l=1}^r h_l(i,j)F_l$.

The aim of this paper is to find an SOF controller of the form (5) such that the resulting closed-loop system is asymptotically stable.

The corresponding closed-loop system by combining the above controller in (5) to the original system in (2) is presented by

$$x_{i+1,j+1} = \bar{A}_1(h_{i,j+1})x_{i,j+1} + \bar{A}_2(h_{i+1,j})x_{i+1,j} \quad (6)$$

where

$$\begin{aligned}\bar{A}_1(h_{i,j+1}) &= A_1(h_{i,j+1}) + B_1(h_{i,j+1})F(h_{i,j+1})C(h_{i,j+1}) \\ \bar{A}_2(h_{i+1,j}) &= A_2(h_{i+1,j}) + B_2(h_{i+1,j})F(h_{i+1,j})C(h_{i+1,j}).\end{aligned}$$

To solve for the SOF controller, the following preliminary lemma is needed.

Lemma 1 *The 2D T-S fuzzy system (6) is globally asymptotically stable if there exist fuzzy-basis-dependent matrices $P(h_{i,j})$ and $Q(h_{i,j})$ such that*

$$\begin{bmatrix} -P(h_{i,j+1}) + Q(h_{i,j+1}) & 0 & \bar{A}_1^T(h_{i,j+1})P(h_{i+1,j+1}) \\ * & -Q(h_{i+1,j}) & \bar{A}_2^T(h_{i+1,j})P(h_{i+1,j+1}) \\ * & * & -P(h_{i+1,j+1}) \end{bmatrix} < 0. \quad (7)$$

Proof Following a similar line to that in Theorem 1 in Kar and Singh (2003), which established a criterion for the stability of linear 2D systems described by the first Fornasini–Marchesini model, it is evident that this lemma guarantees that the system considered is globally asymptotically stable. Thus, the proof is omitted here.

Lemma 2 (*Khargonekar et al, 1987*) *Let $\mathcal{X}$ and $\mathcal{Y}$ be any given real matrices of appropriate dimensions. Then, for any scalar $\epsilon > 0$,*

$$\mathcal{X}^T \mathcal{Y} + \mathcal{Y}^T \mathcal{X} \leq \epsilon^{-1} \mathcal{X}^T \mathcal{X} + \epsilon \mathcal{Y}^T \mathcal{Y} \quad (8)$$

3 Design of SOF Stabilization Controller

In the following, a sufficient condition is presented for the SOF stabilizing problem.

Theorem 1 *If there exist scalars $\lambda > 0, \epsilon > 0$, and matrices $F(h_{i,j})$, $N(h_{i,j})$, $S(h_{i,j})$, $T(h_{i,j})$, $\Gamma_{11}, \Gamma_{12}, \Gamma_{13}, \Gamma_{14}, \Gamma_{22}, \Gamma_{23}, \Gamma_{24}, \Gamma_{33}, \Gamma_{34}, \Gamma_{44}, \Gamma_{15}(h_{i,j+1}, h_{i+1,j+1})$, $\Gamma_{25}(h_{i+1,j}, h_{i+1,j+1})$, $\Gamma_{35}(h_{i,j+1}, h_{i+1,j+1})$, and $\Gamma_{45}(h_{i+1,j}, h_{i+1,j+1})$ such that*

$$\Sigma = \begin{bmatrix} \Sigma_1(h_{i,j+1}) & 0 & 0 & 0 & \Sigma_3(h_{i,j+1}) & 0 & 0 \\ * & \Sigma_2(h_{i+1,j}) & 0 & 0 & 0 & \Sigma_3(h_{i+1,j}) & 0 \\ * & * & -\epsilon I & 0 & \Sigma_4(h_{i,j+1}) & 0 & -I \\ * & * & * & -\epsilon I & 0 & \Sigma_4(h_{i+1,j}) & 0 \\ * & * & * & * & \Sigma_5(h_{i,j+1}) & 0 & N^T(h_{i,j+1}) \\ * & * & * & * & * & \Sigma_5(h_{i+1,j}) & 0 \\ * & * & * & * & * & * & \Gamma_{11} \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \end{bmatrix}$$

$$\begin{bmatrix}
0 & C^T(h_{i,j+1}) & 0 & A_1^T(h_{i,j+1})S(h_{i+1,j+1}) \\
0 & 0 & C^T(h_{i+1,j}) & A_2^T(h_{i+1,j})S(h_{i+1,j+1}) \\
0 & 0 & 0 & B_1^T(h_{i,j+1})S(h_{i+1,j+1}) \\
-I & 0 & 0 & B_2^T(h_{i+1,j})S(h_{i+1,j+1}) \\
0 & -I & 0 & 0 \\
N^T(h_{i+1,j}) & 0 & -I & 0 \\
\Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15}(h_{i,j+1}, h_{i+1,j+1}) \\
\Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25}(h_{i+1,j}, h_{i+1,j+1}) \\
* & \Gamma_{33} & \Gamma_{34} & \Gamma_{35}(h_{i,j+1}, h_{i+1,j+1}) \\
* & * & \Gamma_{44} & \Gamma_{45}(h_{i+1,j}, h_{i+1,j+1}) \\
* & * & * & -S(h_{i+1,j+1})
\end{bmatrix} < 0 \quad (9)$$

where $\Sigma_1(h_{i,j}) = -S(h_{i,j}) + T(h_{i,j}) - \lambda C^T(h_{i,j})C(h_{i,j})$, $\Sigma_2(h_{i,j}) = -T(h_{i,j}) - \lambda C^T(h_{i,j})C(h_{i,j})$, $\Sigma_3(h_{i,j}) = \lambda C^T(h_{i,j})$, $\Sigma_4(h_{i,j}) = \varepsilon F(h_{i,j})$, $\Sigma_5(h_{i,j}) = -\lambda I - \varepsilon F^T(h_{i,j})F(h_{i,j})$, then the 2D T-S fuzzy system (6) is asymptotically stable.

Proof In the following, we will show the equivalence of Theorem 1 and Lemma 1, which implies the asymptotic stability for the 2D T-S fuzzy system (6).

(Sufficiency). Suppose (9) holds, then define Θ as

$$\Theta = \begin{bmatrix} \hat{\Theta} & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}, \quad (10)$$

where

$$\hat{\Theta} = \begin{bmatrix} I & 0 & \hat{\Theta}_{13} & 0 & C^T(h_{i,j+1}) & 0 \\ 0 & I & 0 & \hat{\Theta}_{24} & 0 & C^T(h_{i+1,j}) \end{bmatrix}, \quad (11)$$

and $\hat{\Theta}_{13} = C^T(h_{i,j+1})F^T(h_{i,j+1})$, and $\hat{\Theta}_{24} = C^T(h_{i+1,j})F^T(h_{i+1,j})$.

Suppose (9) holds, then pre- and post-multiplying both sides of the inequality matrix (9) by row full rank matrix Θ and its transpose, respectively, it results in

$$\begin{bmatrix}
-S(h_{i,j+1}) + T(h_{i,j+1}) & 0 & \Sigma_6(h_{i,j+1}) & 0 & 0 \\
* & -T(h_{i+1,j}) & 0 & \Sigma_6(h_{i+1,j}) & 0 \\
* & * & \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\
* & * & * & \Gamma_{22} & \Gamma_{23} \\
* & * & * & * & \Gamma_{33} \\
* & * & * & * & * \\
* & * & * & * & *
\end{bmatrix}$$

$$\begin{bmatrix} 0 & \bar{A}_1^T(h_{i,j+1})S(h_{i+1,j+1}) \\ 0 & \bar{A}_2^T(h_{i+1,j})S(h_{i+1,j+1}) \\ \Gamma_{14} & \Gamma_{15}(h_{i,j+1}, h_{i+1,j+1}) \\ \Gamma_{24} & \Gamma_{25}(h_{i+1,j}, h_{i+1,j+1}) \\ \Gamma_{34} & \Gamma_{35}(h_{i,j+1}, h_{i+1,j+1}) \\ \Gamma_{44} & \Gamma_{45}(h_{i+1,j}, h_{i+1,j+1}) \\ * & -S(h_{i+1,j+1}) \end{bmatrix} < 0, \quad (12)$$

where $\Sigma_6(h_{i,j}) = C^T(h_{i,j})(F^T(h_{i,j}) + N^T(h_{i,j}))$.

It can be seen that (7) is a main submatrix of (12). Hence, (7) holds where $P(h_{i,j})$ and $Q(h_{i,j})$ in (7) can be chosen as $S(h_{i,j})$ and $T(h_{i,j})$, respectively. Hence, the 2D T-S fuzzy system (6) is asymptotically stable.

(Necessity). Suppose (7) holds, then the following inequality holds for any positive scalar $\varepsilon' > 0, \lambda' > 0$, and $\tilde{P}(h_{i,j}) > P(h_{i,j})$, that is,

$$\begin{aligned} \hat{A} = & \begin{bmatrix} \bar{A}_1^T(h_{i,j+1}) \\ \bar{A}_2^T(h_{i+1,j}) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} P(h_{i+1,j+1}) \begin{bmatrix} \bar{A}_1^T(h_{i,j+1}) \\ \bar{A}_2^T(h_{i+1,j}) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \\ & + \begin{bmatrix} -\tilde{P}(h_{i,j+1}) + Q(h_{i,j+1}) & 0 & 0 & 0 & 0 & 0 \\ * & -Q(h_{i+1,j}) & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix} \\ & - \begin{bmatrix} \hat{A}_{1,1} & 0 & 0 & 0 & -\lambda' C^T(h_{i,j+1}) & 0 \\ * & \hat{A}_{2,2} & 0 & 0 & 0 & -\lambda' C^T(h_{i+1,j}) \\ * & * & \varepsilon' I & 0 & -\varepsilon' F^T(h_{i,j+1}) & 0 \\ * & * & * & \varepsilon' I & 0 & -\varepsilon' F^T(h_{i,j+1}) \\ * & * & * & * & \hat{A}_{5,5} & 0 \\ * & * & * & * & * & \hat{A}_{6,6} \end{bmatrix} < 0 \quad (13) \end{aligned}$$

with

$$\begin{aligned} \hat{A}_{1,1} &= \lambda' C^T(h_{i,j+1})C(h_{i,j+1}) \\ \hat{A}_{2,2} &= \lambda' C^T(h_{i+1,j})C(h_{i+1,j}) \\ \hat{A}_{5,5} &= \varepsilon' F^T(h_{i,j+1})F(h_{i,j+1}) + \lambda' I \\ \hat{A}_{6,6} &= \varepsilon' F^T(h_{i+1,j})F(h_{i+1,j}) + \lambda' I \end{aligned}$$

Furthermore, there exist sufficiently large scalars $\tilde{\varepsilon} > 0$ and $\tilde{\lambda} > 0$ such that

$$\begin{aligned}
\Lambda = & \hat{\Lambda} + \begin{bmatrix} \bar{A}_1^T(h_{i,j+1}) \\ \bar{A}_2^T(h_{i+1,j}) \\ 0 \\ 0 \\ 0 \end{bmatrix} \Lambda_5(h_{i+1,j+1}) \begin{bmatrix} \bar{A}_1^T(h_{i,j+1}) \\ \bar{A}_2^T(h_{i+1,j}) \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \\
& + \begin{bmatrix} A_1^T(h_{i,j+1}) \\ A_2^T(h_{i+1,j}) \\ B_1^T(h_{i,j+1}) \\ B_2^T(h_{i+1,j}) \\ 0 \\ 0 \end{bmatrix} \Lambda_5(h_{i+1,j+1}) \begin{bmatrix} A_1^T(h_{i,j+1}) \\ A_2^T(h_{i+1,j}) \\ B_1^T(h_{i,j+1}) \\ B_2^T(h_{i+1,j}) \\ 0 \\ 0 \end{bmatrix}^T \\
& - \begin{bmatrix} \Lambda_6 \\ \Lambda_7 \\ -B_1^T(h_{i,j+1}) \\ -B_2^T(h_{i+1,j}) \\ 0 \\ 0 \end{bmatrix} \Lambda_5(h_{i+1,j+1}) \begin{bmatrix} \Lambda_6 \\ \Lambda_7 \\ -B_1^T(h_{i,j+1}) \\ -B_2^T(h_{i+1,j}) \\ 0 \\ 0 \end{bmatrix}^T \\
& + \begin{bmatrix} A_1^T(h_{i,j+1}) \\ A_2^T(h_{i+1,j}) \\ B_1^T(h_{i,j+1}) \\ B_2^T(h_{i+1,j}) \\ 0 \\ 0 \end{bmatrix} \Lambda_8(h_{i+1,j+1}) \begin{bmatrix} A_1^T(h_{i,j+1}) \\ A_2^T(h_{i+1,j}) \\ B_1^T(h_{i,j+1}) \\ B_2^T(h_{i+1,j}) \\ 0 \\ 0 \end{bmatrix}^T < 0, \quad (14)
\end{aligned}$$

where

$$\begin{aligned}
\Lambda_5(h_{i+1,j+1}) &= P(h_{i+1,j+1})\Delta P(h_{i+1,j+1}), \\
\Delta &= \frac{1}{\tilde{\varepsilon}}B_1(h_{i,j+1})B_1^T(h_{i,j+1}) + \frac{1}{\tilde{\varepsilon}}B_2(h_{i+1,j})B_2^T(h_{i+1,j}) \\
&+ \frac{1}{\tilde{\lambda}}B_1(h_{i,j+1})F(h_{i,j+1})F^T(h_{i,j+1})B_1^T(h_{i,j+1}) \\
&+ \frac{1}{\tilde{\lambda}}B_2(h_{i+1,j})F(h_{i+1,j})F^T(h_{i+1,j})B_2^T(h_{i+1,j}), \\
\tilde{P}(h_{i+1,j+1}) &= P(h_{i+1,j+1}) + \Lambda_5(h_{i+1,j+1}), \\
\Lambda_6 &= C^T(h_{i,j+1})F^T(h_{i,j+1})B_1^T(h_{i,j+1}), \\
\Lambda_7 &= C^T(h_{i+1,j})F^T(h_{i+1,j})B_2^T(h_{i+1,j}), \\
\Lambda_8(h_{i+1,j+1}) &= \Lambda_5(h_{i+1,j+1})P^{-1}(h_{i+1,j+1})\Lambda_5(h_{i+1,j+1}). \quad (15)
\end{aligned}$$

In fact, $\Lambda_5(h_{i+1,j+1})$ and $\Lambda_8(h_{i+1,j+1})$ are dependent on $\tilde{\varepsilon}$ and $\tilde{\lambda}$. Hence when $\tilde{\varepsilon}$ is sufficiently large, (14) can be guaranteed.

In the following, we will prove that by choosing appropriate matrices, $\Sigma < 0$ can be guaranteed in (9).

Note that $\Sigma < 0$ is equivalent to $\Phi - \Psi\Gamma^{-1}\Psi^T < 0$, where

$$\Phi = \begin{bmatrix} \Sigma_1(h_{i,j+1}) & 0 & 0 & 0 & \Sigma_3(h_{i,j+1}) & 0 \\ * & \Sigma_2(h_{i+1,j}) & 0 & 0 & 0 & \Sigma_3(h_{i+1,j}) \\ * & * & -\varepsilon I & 0 & \Sigma_4(h_{i,j+1}) & 0 \\ * & * & * & -\varepsilon I & 0 & \Sigma_4(h_{i+1,j}) \\ * & * & * & * & \Sigma_5(h_{i,j+1}) & 0 \\ * & * & * & * & * & \Sigma_5(h_{i+1,j}) \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15}(h_{i,j+1}, h_{i+1,j+1}) \\ 0 & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25}(h_{i+1,j}, h_{i+1,j+1}) \\ * & * & \Gamma_{33} & \Gamma_{34} & \Gamma_{35}(h_{i,j+1}, h_{i+1,j+1}) \\ * & * & * & \Gamma_{44} & \Gamma_{45}(h_{i+1,j}, h_{i+1,j+1}) \\ * & * & * & * & -S(h_{i+1,j+1}) \end{bmatrix},$$

$$\Psi = \begin{bmatrix} 0 & 0 & C^T(h_{i,j+1}) & 0 & A_1^T(h_{i,j+1})S(h_{i+1,j+1}) \\ 0 & 0 & 0 & C^T(h_{i+1,j}) & A_2^T(h_{i+1,j})S(h_{i+1,j+1}) \\ -I & 0 & 0 & 0 & B_1^T(h_{i,j+1})S(h_{i+1,j+1}) \\ 0 & -I & 0 & 0 & B_2^T(h_{i+1,j})S(h_{i+1,j+1}) \\ N^T(h_{i,j+1}) & 0 & -I & 0 & 0 \\ 0 & N^T(h_{i+1,j}) & 0 & -I & 0 \end{bmatrix}.$$

Then by choosing

$$\begin{aligned} \Gamma_{11} &= \Gamma_{22} = -I/\tilde{\varepsilon}, \\ \Gamma_{12} &= \Gamma_{13} = \Gamma_{14} = 0, \\ \Gamma_{23} &= \Gamma_{24} = \Gamma_{34} = 0, \\ \Gamma_{33} &= \Gamma_{44} = -I/\tilde{\lambda}, \\ \Gamma_{15} &= B_1^T(h_{i,j+1})P(h_{i+1,j+1})/\tilde{\varepsilon}, \\ \Gamma_{25} &= B_2^T(h_{i+1,j})P(h_{i+1,j+1})/\tilde{\varepsilon}, \\ \Gamma_{35} &= F^T(h_{i,j+1})B_1^T(h_{i,j+1})P(h_{i+1,j+1})/\tilde{\lambda}, \\ \Gamma_{45} &= F^T(h_{i+1,j})B_2^T(h_{i+1,j})P(h_{i+1,j+1})/\tilde{\lambda}, \\ S(h_{i,j}) &= P(h_{i,j}) + \Lambda_5(h_{i,j}), \\ T(h_{i,j}) &= Q(h_{i,j}), \\ \varepsilon &= \varepsilon' + \tilde{\varepsilon}, \quad \lambda = \lambda' + \tilde{\lambda} \end{aligned}$$

one gets

$$\Gamma^{-1} = - \begin{bmatrix} B_1(h_{i,j+1}) \\ B_2(h_{i+1,j}) \\ F^T(h_{i,j+1})B_1^T(h_{i,j+1}) \\ F^T(h_{i+1,j})B_2^T(h_{i+1,j}) \\ P^{-1}(h_{i+1,j+1}) \end{bmatrix} P(h_{i+1,j+1}) \times$$

$$\begin{bmatrix} B_1(h_{i,j+1}) \\ B_2(h_{i+1,j}) \\ F^T(h_{i,j+1})B_1^T(h_{i,j+1}) \\ F^T(h_{i+1,j})B_2^T(h_{i+1,j}) \\ P^{-1}(h_{i+1,j+1}) \end{bmatrix}^T - \begin{bmatrix} \tilde{\varepsilon}I & 0 & 0 & 0 & 0 \\ * & \tilde{\varepsilon}I & 0 & 0 & 0 \\ * & * & \tilde{\lambda}I & 0 & 0 \\ * & * & * & \tilde{\lambda}I & 0 \\ * & * & * & * & 0 \end{bmatrix}.$$

Then after some calculations, we have $\Phi - \Psi\Gamma^{-1}\Psi^T = \Lambda$. From (14), one gets $\Lambda < 0$, implying (9) holds. This completes the proof.

From the above proof, one can see that there is no multiplication of feedback gains with other system matrices, which overcomes the traditional obstacle in the SOF controller design as shown in Lemma 1. However, due to the fact that matrices $F^T(h_{i,j})F(h_{i,j})$ and $F(h_{i,j})$ are both involved in (9), we encounter new challenges. To convert (9) into a set of LMI constraints, Lemma 2 is utilized, which yields

$$-\varepsilon F^T(h_{i,j})F(h_{i,j}) - \lambda I \leq \varepsilon F^T(h_{i,j}) + \varepsilon F(h_{i,j}) + \varepsilon I - \lambda I. \quad (16)$$

By solving the following LMI constraints, the SOF controller of the 2D fuzzy system can be obtained.

Theorem 2 *If there exist scalars $\varepsilon > 0, \lambda > 0$ and matrices $S_l, T_l, N_l, \Sigma_{4l}, \Gamma_{11}, \Gamma_{12}, \Gamma_{13}, \Gamma_{14}, \Gamma_{22}, \Gamma_{23}, \Gamma_{24}, \Gamma_{33}, \Gamma_{34}, \Gamma_{44}, \Gamma_{15}^{l\tau}, \Gamma_{25}^{l\tau}, \Gamma_{35}^{l\tau}$, and $\Gamma_{45}^{l\tau}$ such that*

$$\Sigma_{kl\tau}^{\alpha\beta} = \begin{bmatrix} \Sigma_{1k\alpha} & 0 & 0 & 0 & \Sigma_{3k} & 0 & 0 & 0 & C_k^T & 0 & A_{1k}^T S_\tau \\ * & \Sigma_{2l\beta} & 0 & 0 & 0 & \Sigma_{3l} & 0 & 0 & 0 & C_l^T & A_{2l}^T S_\tau \\ * & * & -\varepsilon I & 0 & \Sigma_{4k} & 0 & -I & 0 & 0 & 0 & B_{1k}^T S_\tau \\ * & * & * & -\varepsilon I & 0 & \Sigma_{4l} & 0 & -I & 0 & 0 & B_{2l}^T S_\tau \\ * & * & * & * & \tilde{\Sigma}_{5k} & 0 & N_k^T & 0 & -I & 0 & 0 \\ * & * & * & * & * & \tilde{\Sigma}_{5l} & 0 & N_l^T & 0 & -I & 0 \\ * & * & * & * & * & * & \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15}^{k\tau} \\ * & * & * & * & * & * & * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25}^{l\tau} \\ * & * & * & * & * & * & * & * & \Gamma_{33} & \Gamma_{34} & \Gamma_{35}^{k\tau} \\ * & * & * & * & * & * & * & * & * & \Gamma_{44} & \Gamma_{45}^{l\tau} \\ * & * & * & * & * & * & * & * & * & * & -S_\tau \end{bmatrix} < 0 \quad (l, k, \alpha, \beta, \tau = 1, 2 \dots r) \quad (17)$$

where $\Sigma_{1k\alpha} = -S_k + T_k - \lambda C_k^T C_\alpha$, $\Sigma_{2l\beta} = -T_l - \lambda C_l^T C_\beta$, $\Sigma_{3l} = \lambda C_l^T$, $\tilde{\Sigma}_{5l} = \Sigma_{4l} + \Sigma_{4l}^T + \varepsilon I - \lambda I$, then the 2D T-S fuzzy system (6) is asymptotically stable, and

$$u_{i,j} = \varepsilon^{-1} \sum_{l=1}^r h_l(i,j) \Sigma_{4l} x_{i,j}$$

is the SOF controller for this 2D T-S fuzzy system.

Proof Suppose that $\varepsilon, \lambda > 0$ Σ_{4l} , S_l, T_l, N_l , Γ_{11} , Γ_{12} , Γ_{13} , Γ_{14} , Γ_{22} , Γ_{23} , Γ_{24} , Γ_{33} , Γ_{34} , Γ_{44} , $\Gamma_{15}^{l\tau}$, $\Gamma_{25}^{l\tau}$, $\Gamma_{35}^{l\tau}$, and $\Gamma_{45}^{l\tau}$ form a set of feasible solutions of inequalities in (20). We use these matrices and the fuzzy function $h_{i,j}$ to define the following functions:

$$\begin{aligned}
S(h_{i,j}) &= \sum_{l=1}^r h_l(i,j) S_l, \quad T(h_{i,j}) = \sum_{l=1}^r h_l(i,j) T_l \\
\Sigma_4(h_{i,j}) &= \sum_{l=1}^r h_l(i,j) \Sigma_{4l}, \quad N(h_{i,j}) = \sum_{l=1}^r h_l(i,j) N_l \\
\Gamma_{15} &= \sum_{l=1}^r \sum_{\tau=1}^r h_l(i,j+1) h_v(i+1,j+1) \Gamma_{15}^{l\tau} \\
\Gamma_{25} &= \sum_{l=1}^r \sum_{\tau=1}^r h_l(i+1,j) h_v(i+1,j+1) \Gamma_{25}^{l\tau} \\
\Gamma_{35} &= \sum_{l=1}^r \sum_{\tau=1}^r h_l(i,j+1) h_v(i+1,j+1) \Gamma_{35}^{l\tau} \\
\Gamma_{45} &= \sum_{l=1}^r \sum_{\tau=1}^r h_l(i+1,j) h_v(i+1,j+1) \Gamma_{45}^{l\tau}
\end{aligned} \tag{18}$$

It is clear by (3), (16) and (18) that

$$\begin{aligned}
\tilde{\Sigma} &= \sum_{k=1}^r \sum_{l=1}^r \sum_{\alpha=1}^r \sum_{\beta=1}^r \sum_{v=1}^r h_k(i,j+1) h_\alpha(i,j+1) \times \\
&\quad h_l(i+1,j) h_\beta(i+1,j) h_\tau(i+1,j+1) \Sigma_{kl\tau}^{\alpha\beta} \\
&= \begin{bmatrix}
\Sigma_1(h_{i,j+1}) & 0 & 0 & 0 & \Sigma_3(h_{i,j+1}) & 0 & 0 \\
* & \Sigma_2(h_{i+1,j}) & 0 & 0 & 0 & \Sigma_3(h_{i+1,j}) & 0 \\
* & * & -\varepsilon I & 0 & \Sigma_4(h_{i,j+1}) & 0 & -I \\
* & * & * & -\varepsilon I & 0 & \Sigma_4(h_{i+1,j}) & 0 \\
* & * & * & * & \tilde{\Sigma}_5(h_{i,j+1}) & 0 & N^T(h_{i,j+1}) \\
* & * & * & * & * & \tilde{\Sigma}_5(h_{i+1,j}) & 0 \\
* & * & * & * & * & * & \Gamma_{11} \\
* & * & * & * & * & * & * \\
* & * & * & * & * & * & * \\
* & * & * & * & * & * & * \\
* & * & * & * & * & * & *
\end{bmatrix}
\end{aligned}$$

$$\begin{bmatrix}
0 & C^T(h_{i,j+1}) & 0 & A_1^T(h_{i,j+1})S(h_{i+1,j+1}) \\
0 & 0 & C^T(h_{i+1,j}) & A_2^T(h_{i+1,j})S(h_{i+1,j+1}) \\
0 & 0 & 0 & B_1^T(h_{i,j+1})S(h_{i+1,j+1}) \\
-I & 0 & 0 & B_2^T(h_{i+1,j})S(h_{i+1,j+1}) \\
0 & -I & 0 & 0 \\
N^T(h_{i+1,j}) & 0 & -I & 0 \\
\Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15}(h_{i,j+1}, h_{i+1,j+1}) \\
\Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25}(h_{i+1,j}, h_{i+1,j+1}) \\
* & \Gamma_{33} & \Gamma_{34} & \Gamma_{35}(h_{i,j+1}, h_{i+1,j+1}) \\
* & * & \Gamma_{44} & \Gamma_{45}(h_{i+1,j}, h_{i+1,j+1}) \\
* & * & * & -S(h_{i+1,j+1})
\end{bmatrix} < 0, \quad (19)$$

where $\tilde{\Sigma}_5(h_{i,j}) = \Sigma_4(h_{i,j})^T + \Sigma_4(h_{i,j}) + \varepsilon I - \lambda I$. By considering (16), one can conclude that $\Sigma < 0$ and hence the result of Theorem 1 follows immediately.

Remark 3 It is worth mentioning that, by following a similar line of the proof as Theorem 2, one can obtain the SOF stabilization controller design method for 2D discrete linear systems. Comparing to existing methods on SOF stabilization controller design for 2D discrete linear system, the main advantage of the developed SOF control scheme is that no constraints are imposed on system matrices.

The following corollary gives a methodology for 1D fuzzy SOF stabilization controller design.

Corollary 1 *If there exist scalars $\varepsilon > 0, \lambda > 0$ and matrices $S_l, N_l, \Sigma_{4l}, \Gamma_{11}, \Gamma_{13}, \Gamma_{33}, \Gamma_{15}^{l\tau}, \Gamma_{35}^{l\tau}$, such that*

$$\Sigma_{k\alpha\tau} = \begin{bmatrix}
\Sigma_{1k\alpha} & 0 & \Sigma_{3k} & 0 & C_k^T & A_k^T S_\tau \\
* & -\varepsilon I & \Sigma_{4k} & -I & 0 & B_k^T S_\tau \\
* & * & \tilde{\Sigma}_{5k} & N_k^T & -I & 0 \\
* & * & * & \Gamma_{11} & \Gamma_{13} & \Gamma_{15}^{k\tau} \\
* & * & * & * & \Gamma_{33} & \Gamma_{35}^{k\tau} \\
* & * & * & * & * & -S_\tau
\end{bmatrix} < 0 \quad (k, \alpha, \tau = 1, 2 \dots r) \quad (20)$$

where $\{A_k, B_k, C_k\}$ is the k -th local linear model of a 1D T-S fuzzy system, $\Sigma_{1k\alpha} = -S_k - \lambda C_k^T C_\alpha$, $\Sigma_{3k} = \lambda C_k^T$, $\tilde{\Sigma}_{5k} = \Sigma_{4k} + \Sigma_{4k}^T + \varepsilon I - \lambda I$, then the 1D T-S fuzzy system is asymptotically stable, and

$$u(k) = \varepsilon^{-1} \sum_{l=1}^r h_l(k) \Sigma_{4l} x(k)$$

is the SOF stabilization controller for this 1D T-S fuzzy system.

Remark 4 Since Theorem 2 and Corollary 1 do not require system matrices satisfy any constraints, it provides an alternative and effective approach to fuzzy SOF controller design. The effectiveness is illustrated through two examples in the next section.

4 Simulation Examples

Two simulation examples are given in this section to illustrate the effectiveness of the proposed method.

Example 1:

A dimensionless model which describes some thermal processes is considered.

$$\frac{\partial T(x, t)}{\partial x} = a(x, t) \frac{\partial T(x, t)}{\partial t} + b(x, t) T(x, t)$$

where $T(x, t)$ is usually temperature, and $a(x, t)$ and $b(x, t)$ are usually nonlinear functions at x (space) $\in [0, x_f]$ and t (time) $\in [0, \infty]$.

Taking

$$\begin{aligned} T(i, j) &= T(i\Delta x, j\Delta t) \\ \frac{\partial T(x, t)}{\partial x} &= \frac{T(i+1, j) - T(i, j)}{\Delta x}, \quad \frac{\partial T(x, t)}{\partial t} = \frac{T(i, j) - T(i, j-1)}{\Delta t}, \end{aligned}$$

we can obtain the following 2D nonlinear model:

$$T(i+1, j) = (1 + a(i, j) \frac{\Delta x}{\Delta t} + b(i, j) \Delta x) T(i, j) - a(i, j) \frac{\Delta x}{\Delta t} T(i, j-1).$$

Denoting $x_{i,j}^T = [T^T(i, j-1) \ T^T(i, j)]$, one can easily obtain the following 2-D FMII model:

$$x(i+1, j+1) = A_1 x_{i,j+1} + A_2 x_{i+1,j}, \quad (21)$$

where

$$A_1 = \begin{bmatrix} 0 & 0 \\ -a(i, j+1) \frac{\Delta x}{\Delta t} & 1 + a(i, j+1) \frac{\Delta x}{\Delta t} + b(i, j+1) \Delta x \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Let $\Delta t = \Delta x = 1$, $a(i, j) = -0.7 + 0.3 \sin^2(T(i, j))$, $b(i, j) = 0.5 - \cos^2(T(i, j))$. To obtain a 2D T-S fuzzy representation for 2D nonlinear system (21), consider the following two rules obtained for $\sin^2(T(i, j))$

$$\begin{aligned} R_1 : & \text{If } \sin^2(T(i, j)) \text{ is about } 0, \\ & \text{then } x(i+1, j+1) = A_{11} x_{i,j+1} + A_{21} x_{i+1,j}. \\ R_2 : & \text{If } \sin^2(T(i, j)) \text{ is about } 1, \\ & \text{then } x(i+1, j+1) = A_{12} x_{i,j+1} + A_{22} x_{i+1,j}. \end{aligned}$$

where

$$A_{11} = \begin{bmatrix} 0 & 0 \\ 0.7 & 0.295 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 & 0 \\ 0.4 & 0.605 \end{bmatrix}, \quad A_{21} = A_{22} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

The membership functions are given by

$$\begin{aligned} h_1(i, j) &= 1 - \sin^2(T(i, j)), \\ h_2(i, j) &= \sin^2(T(i, j)). \end{aligned}$$

Moreover, to consider the SOF controller design problem, the process is modelled in the form of (2) with:

$$\begin{aligned} B_{11} &= \begin{bmatrix} 0.1 & -0.25 \\ -0.2 & -0.3 \end{bmatrix}, B_{12} = \begin{bmatrix} 0.2 & -0.3 \\ -0.3 & -0.1 \end{bmatrix} \\ B_{21} &= \begin{bmatrix} -0.4 & -0.5 \\ -0.3 & -0.5 \end{bmatrix}, B_{22} = \begin{bmatrix} -0.4 & 0.5 \\ 0.3 & -0.5 \end{bmatrix} \\ C_1 &= C_2 = \begin{bmatrix} -0.5 & -0.2 \\ -0.1 & 0.1 \end{bmatrix} \end{aligned}$$

By using Theorem 2, a fuzzy SOF controller was found to be the following.

$$\begin{aligned} F_1 &= \begin{bmatrix} -0.9801 & -0.0503 \\ -0.0356 & -0.9244 \end{bmatrix} \\ F_2 &= \begin{bmatrix} -0.9891 & 0.0026 \\ -0.0031 & -1.0149 \end{bmatrix} \end{aligned} \quad (22)$$

Response of the open-loop and closed-loop states $x_{i,j}$ are shown in [Figure 1](#) and [Figure 2](#), respectively, showing the effectiveness of the proposed results.

Example 2:

It is worth to mention that the proposed SOF controller design method is also applicable to 1D discrete fuzzy systems after some modification. Consider the backing up control of a truck-trailer. We use the following truck-trailer model, taken from Dong and Yang (2009) and Tanaka and Sano (1994):

$$\begin{aligned} x_1(k+1) &= (1 - \frac{vt}{L})x_1(k) + \frac{vt}{l}u(k) \\ x_2(k+1) &= \frac{vt}{L}x_1(k) + x_2(k) \\ x_3(k+1) &= x_3(k) + vtsin(\theta_k) \\ y(k) &= ax_1(k) + bx_2(k) - 0.1552x_3(k) \end{aligned} \quad (23)$$

where $x_1(k)$ is the angle difference between truck and trailer, $x_2(k)$ is the angle of trailer, $x_3(k)$ is the vertical position of rear end of trailer, $u(k)$ is the steering angle. L is the length of trailer, l is the length of truck, t is sampling time and v is the constant speed of backing up. For simulation purpose, in this paper, $-4 \leq a \leq -2$, $3.5 \leq b \leq 5.5$, $\theta_k = x_2(k) + \frac{vt}{2L}x_1(k)$, $L = 5.5[m]$, $l = 2.8[m]$, $v = -1[m/s]$.

A fuzzy system model for the above system is given as:

$$\begin{aligned} R_1 : &\text{If } \theta(k) \text{ is } M_1, \text{ then} \\ &x(k+1) = A_1x(k) + B_1u(k), \\ &y(k) = Cx(k) \\ R_2 : &\text{If } \theta(k) \text{ is } M_2, \text{ then} \\ &x(k+1) = A_2x(k) + B_2u(k), \\ &y(k) = Cx(k) \end{aligned} \quad (24)$$

Fig. 1 State response of $x_1(i, j)$ and $x_2(i, j)$ for the open-loop system.

where

$$A_1 = \begin{bmatrix} 1 - \frac{vt}{L} & 0 & 0 \\ \frac{vt}{L} & 1 & 0 \\ \frac{v^2 t^2}{(2L)} & vt & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 - \frac{vt}{L} & 0 & 0 \\ \frac{vt}{L} & 1 & 0 \\ g \frac{v^2 t^2}{(2L)} & gvt & 1 \end{bmatrix}$$

$$C = [a \ b \ -0.1552]$$

$$B_1 = B_2 = \begin{bmatrix} \frac{vt}{L} \\ 0 \\ 0 \end{bmatrix}$$

Fig. 2 State response of $x_1(i, j)$ and $x_2(i, j)$ for the closed-loop system.

and $x(k) = [x_1^T(k) \ x_2^T(k) \ x_3^T(k)]^T$, $g = 0.01/\pi$, the membership functions of M_1 and M_2 are denoted as $h_1(\theta(k))$ and $h_2(\theta(k))$, respectively:

$$\begin{aligned} h_1(\theta(k)) &= \frac{\sin(\theta(k))}{\theta(k)} \\ h_2(\theta(k)) &= 1 - h_1(\theta(k)) \end{aligned} \quad (25)$$

For comparison purpose, Corollary 1 in this paper, Corollary 1 in Chang et al (2015b) and Theorem 7 in Dong and Yang (2009) are all applied to design SOF controller. The feasible regions according to Corollary 1 (indicated by “o”), Corollary 1 in Chang et al (2015b) (indicated by “x”) and Theorem 7 in Dong and Yang (2009) (indicated by “+”) are shown in Fig. 3. It is

Fig. 3 Feasible regions based on the conditions in Theorem 3 (indicated by “o”), corollary 1 in Chang et al (2015b) (indicated by “x”) and Theorem 7 in Dong and Yang (2009) (indicated by “+”).

clear that Corollary 1 is feasible for regions shown in Fig. 3, while the other two methods are infeasible. Moreover, when the output matrix C is singular, Theorem 7 in Dong and Yang (2009) is invalid, while the proposed results in this paper is still available.

Particularly, when $C = [-3.88 \ 3.88 \ -0.1552]$, the SOF gain matrices can be given as:

$$F_1 = -0.9617, \quad F_2 = -0.9569. \quad (26)$$

The state response and the control signal are shown in Fig. 4 and Fig. 5, respectively. From Figs. 4 and 5, it can be seen that the fuzzy system (23) is asymptotically stable with the designed SOF controller.

5 Conclusion

This paper deals with the static output feedback controller design of 2D discrete fuzzy systems described by the FM second model. New design criterions for SOF controllers are formulated, which are expressed in terms of LMIs. Since the conditions imposed no constraints on system matrices, the proposed method can be of great interest to a wide variety of control systems. The numerical examples demonstrate the effectiveness of this method. In the future, we plan to extend the proposed method to the 3D and higher-dimensional counterparts since output feedback control for 3D and higher-dimensional systems is a very challenging topic (Lin, 2000; Li et al, 2013b). Another future topic is to try to reduce the conservatism of the proposed method in this paper by incorporating with some recent stability criterion for 2D linear systems.

Fig. 4 State trajectories of the closed-loop systems.

Fig. 5 System control input $u(k)$

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