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**From Cellular Automata to Agent-based Models:
Addressing Urban Issues**

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School of Physical and Mathematical Sciences

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2019

From Cellular Automata to Agent-based Models: Addressing Urban Issues

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2019

From Cellular Automata to Agent-based Models:
Addressing Urban Issues

by

Wei Liang QUEK

Abstract

The continual urbanization of our modern world brings forth problems which are increasingly difficult to solve. This difficulty is mainly due to the complexity of the systems involved. Comprehensive strategies to solve these problems have to be based on rigorous treatments to understand the key interactions governing the systems' complexities. Using various forms of microscopic modelling, we seek to address two issues in urban systems: congestion when processing a large number of vehicles, and the persistence of bus bunching.

By means of the Nagel-Schreckenberg model, we investigated the maximum vehicular flow rate achieved by different solution strategies undertaken to process large amount of traffic. The evaluated analytical form of this flow rate found that for large-scale expansion, parallel expansion - or lane-expansion - is less efficient than serial expansion - which takes the form of a multi-point tollbooth - in the absence of human driving behavior. However when considering human reaction time, it diminishes the efficacy of the serial expansion such that it is no longer tenable for traffic processing. By proposing a novel combination of serial and parallel expansions, the analytical flow rate shows that optimal efficiencies are achieved via configurations with few (many) lanes of a large (small) number of serial units when the processing time is short (long).

The problem of bus bunching can be found in every bus system. Using a discrete model, the quantitative dynamics of bus bunching was evaluated analytically. It was found that passenger arrival rate and the difference in buses' velocities are essential factors in the dynamics of bus bunching. Based on this, a bus-bunching agent-based model known as the Empirically-based Monte-Carlo Bus-network (EMB) model is proposed. A case study on NTU's Shuttle Bus System demonstrates that the EMB model accurately captures the bus bunching dynamics of the empirical system. Using the EMB model, three classes of intervention strategies - holding, no-boarding and centralized-pulsing - were studied. It was found that the holding and no-boarding strategies are only effective at specific scenarios, whereas the centralized-pulsing seems to be a potentially comprehensive strategy. Through that, we suggest a series of future works to ascertain its practicability as an implementable strategy to solve bus bunching.

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Zhongyan, Solomon, Weizhen, Hemma, ZL, Clement, Swee Sien, Teck Wee, Gu gu, Jeray, Jemay. I thank you all.

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research done by me except where otherwise stated in this thesis. The thesis work has not been submitted for a degree or professional qualification to any other university or institution. I declare that this thesis is written by myself and is free of plagiarism and of sufficient grammatical clarity to be examined. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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.....
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The contributions of the co-authors are as follows:

- Assoc. Prof Chew provided the initial project direction and edited the manuscript drafts.
- I co-designed the study with Assoc. Prof Chew and Dr. Chung and performed all the simulation and analytical work at the School of Physical and Mathematical Sciences
- I prepared the manuscript drafts. The manuscript was revised by Assoc. Prof Chew and Dr. Chung

31 Jul 2019

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Date

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Chapter 1

Introduction

In the past 100 years, there has been a steady increase in urbanization around the world [2]. Developing cities have seen staggering growth rates in a very short amount of time and by 2040, there will be an estimated two billion more people living in urban cities. With urbanization, people's quality of life has improved drastically. The life expectancy of people has almost doubled and global infant mortality has dropped from 1.3% to 0.2% [3].

However, together with the positive effects of urbanization, urban areas also face problems which are unheard of in rural counterparts. For example, every year during a week-long public holiday, millions of Chinese city dwellers get caught in a gridlock traffic jam which lasts for days. In another instance, bus operators struggle to keep buses arriving on time and tens of people die from human stampede every year. To make matters worse, traditional solutions do nothing in alleviating these problems. Adding an extra lane does not necessarily ease congestion in a 10-lane highway. Using extra buses does not ensure an increase the frequency of bus arrivals. It is clear that our traditional understanding of the world is becoming increasingly irrelevant in the face of many urban issues; the issue becomes more pressing with the unhalting urbanization.

In fact, these phenomena exist not only in urban system, but also in socio-ecological [4], financial [5] and even biological systems [6]. Together, these systems fall under the broad classification of complex systems. While there is no strict definition for complex systems, it is well-agreed that a complex system is one that is made up of many components that interact strongly with one another, resulting in the emergent behavior [7]. The difficulty in studying such systems lies in the fact that the macroscopic emergent behavior cannot be inferred from the individual behavior of the agents.

For over 50 years, computer-based mathematical modelling has been an attractive approach for studying complex systems. In general, modelling approaches can be classified

based on the way reality is reflected in the model. Macroscopic models describe reality by modelling the evolution of a macroscopic variable. For example, the Lighthill-Whitham-Richards (LWR) model [8] describes vehicular traffic by modelling the spatial density of vehicles and the flow rate by a series of partial differential equations which are formulated based on fluid-dynamic equations. In contrast, a microscopic traffic model describes traffic flow by modelling the specific behavior of individual cars and their interaction with each other [9]. In at least three separate fields of complex systems [10–12], studies have shown that the accuracy of the models are agnostic to its modelling approach. However, there are some common characteristic in each of the modelling approaches. As the microscopic approach involves modelling each individual component of the system, it is typically more computationally intensive than macroscopic approaches. Yet, the individualistic modelling facilitates qualitative and quantitative analysis on the interactions in the systems which is - based on our earlier discussion - quintessential in understanding the complex system. Due to this, most academic studies of complex systems are based on microscopic modelling [13,14]. Conversely, due to its strength in computational efficiency, macroscopic models are usually used as part of a control system which requires real-time prediction of the system [15].

Based on current modelling techniques, there exist a number of subclasses of microscopic models. The subclass which continues to bear the name *microscopic models* typically have non-autonomous particles evolving continuously based on a series of differential equations [16–18]. As a result, it is mostly used in studying a wide range of different circumstances which have potential theoretical or practical value. An example of a microscopic model for vehicular traffic is the optimal velocity (OV) model [19]. The OV model captures the dynamics of traffic flow by modelling the instantaneous velocity of each vehicle as the 'optimal' velocity based on the velocity of the preceding vehicle. The resulting model is found to capture various macroscopic dynamics of traffic flow such as the stability of traffic jam clusters [20]. At the same time, the OV model has also been widely used for studying flow intervention techniques, designed to alleviate traffic congestion [21]. While microscopic modelling does not have the computational efficiency for real-time flow prediction, its strength lies in its comprehensibility, making a useful tool for theoretical research.

The challenge of the named 'microscopic modelling' is that not all behaviors can be easily formulated mathematically. Some interactions are easier to model algorithmically, in the style of cellular automata. For that, there exists a subclass of microscopic models which model state variables as discrete values on a fixed n-dimensional (usually 1 or 2-D)

array. The Nagel-Schreckenberg Cellular Automata (NaSch) model [22] is an example of a discrete traffic flow model. In this model, vehicles occupy a single site of an array and its evolution involves a strict 4-step prescription which models vehicles' natural acceleration and deceleration. As with the OV model, the NaSch model is also able to capture a wide range of traffic phenomenon such as the existence of a free-flowing and a congested phase. The simplicity of the rule-based evolution results in the NaSch model being highly popular in the traffic modelling community, from theoretically motivated studies [23, 24] to studies with high practical applicability [25–31]. Most notably, the simplicity of the NaSch model - or discrete models in general - facilitates analytical predictions [32, 33] on various macroscopic variables, which can provide valuable insights in understanding a complex system. This characteristic of discrete models will be featured in Chaps. 2 and 3 of this thesis.

While the earlier two classes of microscopic models features non-autonomous particles, agent-based models involve autonomous particles - also known as agents - which exhibit sophisticated behaviors such as decision making and adaptation. More often than not, agent-based models are formulated to uncover the underlying mechanism which determine the macroscopic dynamics of a highly intricate complex system [34]. Ref. [35] formulated an opioid-use agent-based model to study intervention strategies to reduce the number of opioid-related deaths. Opioid-related deaths are typically caused by an addiction to over-prescribed opioid-based painkiller. A widely-used intervention is the Prescription Drug Monitoring Program (PDMP) which monitors the amount of drugs prescribed to each patient. Interestingly, the study found that the PDMP to have little impact on the number of opioid-related deaths as patients who are already addicted to the drugs would seek out other means of getting them. In fact, a more effective strategy would be to lower the initial doses of opioid-based painkiller such that patients will be less inclined to develop an addiction. The modelled system is made up of multiple classes of agents - physicians, drug users, drug dealers, law-enforcers - exhibiting a wide range of behaviors and interacting with each other. Without an agent-based model, it would have been almost impossible to pinpoint the key contributor of opioid-related deaths.

With a plethora of models available for complexity scientists, it is important to not neglect a crucial aspect of modelling: Empiricism. In fact, the approach of modelling must always take into account empirical observations and use it to continually improve the models. While the simple NaSch model is able to capture the dynamics of jam formation in homogeneous traffic flow, it lacks mechanisms that capture key features of empirical traffic flow. Examples of missed features are: (1) meta-stable high-flow states;

(2) reduced outflow from jams; and (3) wide-scattering of flow-density states [9]. In that regard, Ref. [36], Ref. [37] and Ref. [38] build upon the NaSch model to incorporate mechanisms in a bid to capture the empirically observed phenomena. Likewise, in the field of agent-based modelling, empiricism is also widely preached with Refs. [34, 39, 40] separately proposing frameworks for incorporating empirically-driven concepts into the model formulation.

With the ever increasing demands on urban infrastructure, urban issues will only get worse unless there are comprehensive solutions to the problems. The problems faced by urban systems are complex, and addressing it requires a thorough understanding of its many interacting elements. In this doctoral thesis, we will attempt to study two of the problems brought forth by urbanization, with the aim of proposing solution strategies to alleviate these problems. In Chapter 2, we will discuss a problem similar to that of the 50-Lane Traffic Jam on China’s G4 Beijing-Hong Kong-Macau Expressway. To process the traffic - in the form of toll collection - a 50-lane toll station was built on the highway. However, this lane-expansion strategy frequently presents a large traffic jam downstream of toll stations. While macroscopic models [41] and continuous microscopic models [42, 43] have been utilized in similar studies, a discrete microscopic model - the NaSch model - will be employed to facilitate analytical tractability of the maximum flow rate through a modelled lane-expansion strategy at a generic highway bottleneck. This lane-expansion can be viewed as a parallel expansion of the bottleneck. At the same time, we also study the flow rate through a lesser-used expansion known as a multi-point tollbooth, which conversely can be viewed as a serial expansion of the bottleneck. Together, we propose an analytical theory which demonstrates how a combination between the two expansion would serve as an improved strategy to process large amount of traffic. In Chapters 3, 4 and 5 we address a longstanding problem of the bus system: the pervasiveness of bus bunching. While there have been numerous studies on solving the bus bunching problem [44–60], most of which does not consider the specific bus dynamics. In a bus system, the effects of bus dynamics and passenger boarding are not independent of each other. Ignoring this complexity can lead to solutions which are not effective - or could even backfire - in specific scenarios. If such breakdowns happen in an operational bus system, the effect will be catastrophic. By means of a discrete model, we will present in Chapter 3 a quantitative theory which describes the dynamics of bus bunching. In Chapter 4, these dynamics will facilitate the formulation of an agent-based model, which will be parameterized and validated using empirical data from a case study of Nanyang Technological University’s Shuttle Bus System. Lastly in Chapter 5, the agent-based

model will be used to test-bed various intervention strategies which are designed to reduce the bunching of buses. With that, a summary of the findings and concluding statements on this thesis will be presented in Chapter 6.

Chapter 2

Highway traffic flow and bottlenecks [1]

2.1 Background

The Nagel-Schreckenberg (NaSch) model [22] is a cellular automaton that has been used extensively for theoretical traffic modelling. While it is simplistic and does not describe the full spectrum of real-world traffic phenomenology, it does capture the emergence of congestion - in the form of stop-and-go waves - when vehicular density increases past a critical value [9, 61, 62]. At present, the nature of this transition in empirical traffic flow remains highly controversial, with the existence of a particular critical point - i.e. a phase transition or a crossover - as the main point of contention between different empirically-based traffic theories [14, 63–67]. Nonetheless, a good number of active research continue to employ the concept of a critical density in (typically two-phase) traffic models as an analogue to the critical transition observed in empirical traffic [24, 33, 68–74]. In fact, one example of such is the study of alleviating congestion at traffic bottlenecks, of which active research invariably employs the critical density observed in the NaSch model for its analytical tractability [75, 76]. Traffic states at the plateau are observed to display the characteristics of phase-segregation in the spatiotemporal trajectories of the vehicles, with a continuous change from the free flow to the congestion state as the traffic density increases. Interestingly, this change is similar to the separation of liquid phase and vapor phase in a van der Waals fluid, giving the study of NaSch model with bottleneck a direct relevance to statistical physics [32, 77–81].

In real-world traffic system, the presence of traffic bottlenecks is fundamental and they are inevitable sources of congestion. Traffic bottlenecks appear in various forms: as

accident sites, merging of lanes, or even slow moving vehicles. In general, traffic bottleneck is defined as a localized section of the road or highway at which vehicles experience reduced speeds, and vehicles may require more time to transverse through these sites. This reduction in flow rate results in an increase in vehicle density, which in turn leads to the instability of the free flow phase due to the collective vehicular interactions, eventually leading to traffic congestions at the bottlenecks [37, 64, 82, 83].

While traffic bottlenecks can arise spontaneously and randomly such as in the case of a traffic accident, a vast majority of bottlenecks are fixed in their locations. Examples of these latter bottlenecks include tollbooths, highway's on/off-ramps, pick-up/drop-off points for public transport, and many more. Nowadays, such fixed-site bottlenecks are common sights in the traffic landscape of modern societies. Their purpose is to process the mobility of traffic flow, which is of interest in this thesis. Since bottlenecks serve only to increase vehicular density which in turn destabilizes the free flow phase, an important conclusion is that the effects of a bottleneck on traffic flow is independent of the specific bottleneck mechanism [9, 84–86]. The consequence is a maximum outflow through the bottleneck, which is known as the bottleneck capacity. Bottleneck capacity corresponds to the maximum value of the macroscopic traffic flow captured by the fundamental diagram in a closed, or sufficiently long stretch of road. Its universal feature enables a general study of bottlenecks by solving the maximal outflow through it, or by the time-headway approach adopted in this thesis.

At present, there are numerous studies on traffic bottlenecks using NaSch model, such as the effects of time-delay bottlenecks [84], on- and off-ramps [87], and lane expansion at bottlenecks [75, 88]. These studies have been further facilitated by full analytical solutions of the bottleneck capacity, and compared against the flow-density relation of the homogeneous NaSch model in the deterministic limit, or in the $v_{\max} = 1$ limit [61, 62]. Solutions based on mean-field approximation have also been explored for general parameter range [89, 90]. The analytical solutions from these studies allow for greater quantitative insights into the physical mechanisms underlying traffic bottlenecks, which are not tenable from empirically consistent traffic models since the complex parameterisation and stochasticity within these models render such solutions almost impossible [9]. The analytical consequences of NaSch model as exhibited in these research give strength to the approach, and explains its continued relevance in current traffic studies even when there exist traffic models that are more empirically consistent [38, 68, 81, 86, 91, 92].

In this thesis, we examine into a new type of bottleneck whose function is to process the mobility and transportation needs of modern society. In order to sustain the rapid growth

Fig 2.1. Schematic illustration of 3 types of bottleneck expansions: serial ($m = 2$); parallel ($n = 5$); and combination ($m = 2, n = 5$). Note that direction of travel is upwards

of traffic demands, these *processing* bottlenecks are required to function efficiently. Our idea is a bottleneck that comprises multiple processing units that serve the same function, with both serial and parallel processing occurring at the same time. In other words, our proposed processing bottleneck combines two configurations: processing in parallel; and in serial (see Fig. 2.1).

Parallel expansion of traffic bottleneck involves splitting a lane into multiple lanes in the vicinity of a processing site, with each lane supporting a single processing unit. Akin to a fork-join queue in queuing theory, parallel expansion is a typical method used to increase the capacity by simultaneously processing two or more vehicles at tollbooths and immigration counters. Several studies were performed to probe the effect of parallel expansions of bottlenecks on traffic flow [69, 88, 93]. Notably, the capacity of a highway is found to be fully recoverable through parallel expansion of bottleneck [88]. On the other hand, serial expansion of traffic bottleneck allows multiple processing units to be co-located along a lane such that two or more vehicles are simultaneously processed. As an example, a bus bay that extends over a length of two buses allows passengers from two buses to alight and board at the same time. This decreases the overall processing time of the two buses. Such an expansion of traffic bottleneck was explored in Ref. [69] to investigate the influence of manual versus electronic toll collection on traffic flow.

In the following sections, we first measure and compare the flow-density relation of

a simplified NaSch model, modified with a parallel and a serial expansion of traffic bottlenecks. These values will be compared against the processing capacity of the different expansions, which would be solved analytically. Note that such characterization has yet to be achieved in the literature. Interestingly, the two expansions perform differently with each outperforming the other under certain conditions. One key advantage of a serial expansion is that it requires no expansion of lane to function. This raises the possibility of trading processing efficiency with the cost of lane expansion. For this, we examine into the mechanism underlying the discrepancy in efficiency of the two expansions. In addition, we quantify the difference between them in terms of various bottleneck parameters. After which, we proceed to analyze the performance of a traffic bottleneck that combines parallel and serial processing into a single multi-processing unit. In particular, we investigate into the conditions under which this combination bottleneck outperforms both the parallel and serial expansions in terms of traffic processing capacity.

2.2 Model and Results

In this thesis, a bottleneck is defined as a section of road which comprises one or more processing units. The processing unit represents various functions and is modeled as a generic time-delay site. When a vehicle passes through the processing site, it would stop and be delayed for a time of t_d before moving off. Traffic flow, on the other hand, is modeled based on NaSch cellular automata model [22] on a road of length L cells, single-lane with periodic boundary condition. Each vehicle occupies a single site, and moves with a speed which is less than or equal to the maximum velocity v_m . Vehicles with speed less than v_m accelerate at a rate of $1 \text{ site}/s^2$ as long as there is sufficient unblocked cells ahead. To simplify mathematical analysis, we have set the random deceleration probability to $p = 0$. In all of our simulations, we let $L = 1000$ and $v_m = 4$. The flow-density measurements are determined after ignoring transients that occur within the first $T_0 = 5L$ time steps. Note that each bottleneck site is assumed to be equivalent.

2.2.1 Serial Bottleneck

In a serial bottleneck, a traffic bottleneck is expanded into m equivalent processing bottlenecks arranged in a series along a single lane (see Fig. 2.1 for the illustration of a serial bottleneck with 2 processing sites). Vehicles that pass into this expanded unit have to undergo processing by one (and only one) of the m processing bottlenecks. The processing

takes a time of t_d . The serial expansion is facilitated with a series of gantries to control the movement of vehicles passing by the bottleneck. A car will travel through a site if its gantry is open and will stop in front of a closed gantry. In addition, the gantry system is able to distinguish between processed and unprocessed cars and stops only unprocessed cars.

For efficient processing, an unprocessed car will always be directed to the furthest unoccupied and unblocked site before it is stopped by the gantry system. Specifically, if site 1 to site k are empty while site $k + 1$ is occupied, the car will be directed to the k th site and occupy it. Here, site 1 denotes the first processing site which is nearest to the bottleneck entrance while site m denotes the last processing site. A second car which follows the previous car will then be directed to the $(k - 1)$ th site. Note that sites further down the bottleneck (from $k + 2$ onwards) could be occupied or unoccupied. In this case, any unoccupied processing site further down the bottleneck cannot be utilized as it is blocked. In other words, if the first site is occupied while all the other processing sites are empty, vehicles arriving at the bottleneck will have to wait for the car at the first site to leave before they can enter the bottleneck. Once the car leaves, vehicles that are waiting to enter the bottleneck are directed to the m th, $(m - 1)$ th, $(m - 2)$ th sites consecutively. While this delay is a disadvantage of the serial expansion over the parallel expansion, serial expansion can nonetheless be a better choice as it requires no expansion of lane to function. An illustration of vehicle trajectories resulting from the serial bottleneck is presented in the Appendix.

We first examine the relation between traffic flow rate (J) and traffic density (c) for serial bottlenecks with one, three, and eight gantries. The fundamental diagrams are shown in Fig. 2.2a, where plateaus of different heights are observed. Essentially, the fundamental diagram is made up of three parts: (i) the free-flow phase on the left; (ii) the fully-congested phase on the right; and (iii) the plateau phase which indicates the maximum allowable flows through the bottlenecks. As shown in Fig. 2.2a, the slopes for the free-flow and the fully-congested phases are given by v_m and -1 respectively for all the three serial bottlenecks.

This qualitative shape of the fundamental diagram is consistent with studies of traffic bottleneck on two-phase traffic models. Under the assumption of a long time limit, the fundamental diagram plateaus in a range of densities which is determined by the flow-density relation of a homogenous system and the outflow of the bottleneck, such that the net flow through the system remains the same. In other words, the serial expansion has minimal effect on the flow-density relation in either the free-flow or the fully-congested

phases. Thus, we shall focus our study on the maximum flow rate at which the fundamental diagram plateaus. Indeed, we observe an increase in the maximum flow rate through the bottleneck as the number of processing bottlenecks in the serial expansion increases.

Next, we study analytically the maximum flow rate for serial bottlenecks with m number of processing sites. Specifically, the maximum flow rate is determined through the time-headway approach (see appendix for details) with the following results:

$$J_{max} = \frac{m}{m + \Delta + t_d}, \quad (2.1)$$

where

$$\Delta = \begin{cases} \left\lceil \frac{\sqrt{1+8m}-1}{2} \right\rceil & m < \frac{v_m(v_m+1)}{2} \\ \left\lceil \frac{2m-v_m(v_m+1)}{2v_m} \right\rceil + v_m & m \geq \frac{v_m(v_m+1)}{2}. \end{cases} \quad (2.2)$$

Note that the above equation gives $J_{max} = 1/(2 + t_d)$ for $m = 1$. This corresponds to the maximum flow rate for a single bottleneck [22]. On the other hand, we have $J_{max} = v_m/(v_m + 1)$ as m approaches infinity. By considering the flow-density relation of a homogenous NaSch model, we determine that the fundamental diagram plateaus at densities that satisfies $J_{max}/v_m < \rho < 1 - J_{max}$, where J_{max} is defined as in Eq. (2.1). This is consistent with results of our numerical simulations presented in Fig. 2.2a.

2.2.2 Parallel Bottleneck

Vehicular processing can also be expanded in parallel by splitting the road into n lanes. In our implementation, the splitting starts at the point $L_{1 \rightarrow n}(x = 451)$; and it merges into a single lane at the point $L_{n \rightarrow 1}(x = 550)$. Here, each split lane is facilitated by one processing bottleneck, with the n bottlenecks forming a parallel bottleneck as a whole. Upon arriving at the point $L_{1 \rightarrow n}$, a vehicle has a probability of $1/n$ to move into any of the n lanes. The vehicle would then experience a time-delay of t_d in the processing bottleneck before moving off. After being processed, cars at different splitted lanes may arrive at the merging point simultaneously, upon which they move into the main lane with equal probability [75, 94]. Similar to the serial bottleneck, we examine the relation between traffic flow rate and traffic density for parallel bottlenecks with one, three and eight processing sites based on the fundamental diagrams as shown in Fig. 2.2b. Once again, plateaus of different heights are observed. Interestingly, a parallel expansion does not always increase the maximum flow rate through the bottleneck. For the example

(a) Serial bottlenecks

(b) Parallel bottlenecks

Fig 2.2. Fundamental diagrams of the (a) serial and (b) parallel bottleneck configuration for $t_d = 3$. The results presented are for the mean flow rate J at each traffic density c . Note that the serial bottleneck has consistently higher bottleneck capacity as m increases. However, the bottleneck capacity for parallel bottleneck increases when n goes from 1 to 3, but reduces as n becomes 8. Note that for both serial and parallel bottleneck, the free flow phase and congestion phase exhibit the expected slope of v_m and -1 respectively, in the fundamental diagram.

shown in Fig. 2.2b, while the maximum flow rate through a three-laned bottleneck is higher than the maximum flow rate through a single bottleneck, an expansion into an eight-laned bottleneck reduces this enhancement, resulting in a maximum flow rate which is lower than that of the three-laned bottleneck.

For a bottleneck with n parallel processing units, the maximum flow rate is solved

analytically as follows:

$$J_{max} = \min\left(\frac{n}{3 + t_d - 1/n}, \frac{nv_m}{1 + 2nv_m - v_m}\right). \quad (2.3)$$

Note that the first term is the combined outflow from the n processing sites and is limited by processing delays. The second term, on the other hand, quantifies the maximum allowable flow when traffic merges back into a single lane at $L_{n \rightarrow 1}$. Thus, in a parallel bottleneck, the traffic system essentially contains two different bottlenecks. When there is more than one bottleneck, the stronger one dominates [84]. For small value of n , the delay of the processing site dominates. At higher values of n , flows are limited mainly by the merging of traffic. In Fig. 2.3, we compare the maximum flow rate through a parallel bottleneck to that of a pure merging lane (no processing site). Indeed, J_{max} is the same for the two cases when $n \geq 3$.

Fig 2.3. A comparison between maximum flow rate for parallel expansion without processing site and with processing sites for $t_d = 3$. While analytical plots are presented as a continuous line, only the results at integer values are relevant.

For $n = 1$, we again obtain the expected result of $J_{max} = 1/(2 + t_d)$ from Eq. (2.3). Meanwhile, with the probability of entering the merged lane being $1/n$, a car in the splitted lane can hardly move into the merged lane if n is large. In this scenario, the average velocity for cars arriving at the merging point $L_{n \rightarrow 1}$ is approximately zero. Due to finite acceleration of the NaSch model, cars moving out of $L_{n \rightarrow 1}$ typically have a velocity of $v = 1$. Under the parallel updating procedure of the NaSch model, with a car at $v = 1$ on $L_{n \rightarrow 1}$ at time t , the next car can only reach $L_{n \rightarrow 1}$ at time $t + 2$. This would give an average flow rate of 0.5 at $L_{n \rightarrow 1}$. In other words, we expect $J_{max} \rightarrow 0.5$ as $n \rightarrow \infty$.

Moreover, we observe that the range of densities where the fundamental diagram plateaus satisfies $J_{max}/v_m < \rho < 1 - J_{max}$, which is similar to the serial expansion. However, J_{max} is defined according to Eq. (2.3) in this case.

Note that due to the random interactions between vehicles that occur at the merging bottleneck, there is a difference in the numerical and analytical results for parallel expansion as shown in Fig. 2.4 (as well as figures related to the parallel bottleneck shown later). This is a consequence of the statistical approximation we have made in deriving the analytical expression (see Appendix A.2 for details), with our approximation not taking into full account of the intricate vehicular interactions that happen at the merging point.

2.2.3 Comparison between Serial and Parallel Bottleneck

We first examine the flow rates given by Eqs. (2.1) and (2.3). As expected, the flow through a bottleneck is limited mainly by the processing time, and J_{max} is inversely proportional to t_d . Expanding the bottleneck introduces a multiplicative effect when two or more vehicles are processed simultaneously. With it, J_{max} becomes proportional to m (or n). These relationships illustrate the similarities between the serial and parallel bottlenecks in their response to traffic processing. Nonetheless, there are differences in their mechanisms which causes a difference in outflow. These differences are discussed in this section.

Under serial expansion, cars cannot move concurrently into or out of the processing sites. Processing sites are lined up along a single lane, and each car is required to enter sequentially into it. In particular, two cars that are waiting immediately upstream of the bottleneck do not move simultaneously into the bottleneck after the processing sites are cleared for entry. Instead, the second car moves only after the first car has moved off. Similarly, processed cars exit the bottleneck in tandem with the movement of a car being dependent on whether the car directly in front of it has vacated the site. Underlying this motion is the mechanism that stationary vehicles moves off consecutively due to the parallel updating rule of the NaSch model. This leads to a bottleneck mechanism that constrains the collective motion of the neighboring vehicles, suppressing the flow rate as the number of bottleneck sites increases. This mechanistic feature is captured by the $m + \Delta$ term in the denominator of Eq. (2.1).

In contrast, lane separation in parallel bottleneck has allowed vehicles to traverse the different processing sites simultaneously. However, the effect of a parallel bottleneck depends on whether it is dominated by time delay processing or lane merging. In the former case, bottleneck effect takes the form $3 + 1/n$; and for the latter case, it is $2n +$

$1/v_m - 1$ (see Eq. (2.3)).

A comparison on the processing efficiency of the serial and parallel bottleneck is shown in Fig. 2.4. At small-scale expansion, the parallel bottleneck functions well and is dominated by time delay processing. Under this circumstance, $m + \Delta > 3 + 1/n$ which implies that the outflow from parallel bottleneck is greater than that of the serial bottleneck. Hence, parallel bottleneck performs better. On the other hand, as the number of expansion site increases, the bottleneck due to merging dominates over time delay for the parallel bottleneck. At this regime of large-scale expansion, the flow rate under parallel bottleneck possesses an inflection point that arises from a competition between the multiplicative effect and the flow limiting effect of lane merging, whereupon the flow saturates at 0.5 as n becomes large. Conversely, the processing capacity of a serial bottleneck rises steadily as the scale of expansion increases. Beyond a certain number of processing sites, when interaction accrued during serial processing becomes less significant than that from lane-merging: $2n + 1/v_m - 1 > m + \Delta$, the relative effectiveness between serial and parallel bottleneck interchanges. Serial bottleneck now outperforms the parallel expansion in terms of processing efficiency, with a J_{max} that is larger than 0.5.

Fig 2.4. A plot of J_{max} versus parameters m and n for serial and parallel expansion respectively for $v_m = 4$ and $t_d = 3$. Serial bottleneck is observed to perform better than parallel bottleneck when the number of processing sites is greater than 7. Note that the data points are the mean numerical results of 100 realizations, while the lines are from the analytical prediction of Eqs. (2.1) and (2.3). Error bars of numerical results are smaller than the marker size. While analytical plots are presented as a continuous line, only the results at integer values are valid. Note that this condition is applicable to all analytical plots in the chapter.

Finally, it is important to determine the optimum choice of either the serial or parallel bottlenecks as characterized by different parameters for traffic processing purposes. Here, we present the performance between the two expansions for general values of v_m and t_d .

Specifically, with the same number of processing units, we found that the serial expansion performs better than the parallel expansion for

$$m \geq \frac{2v_m^2 + v_m + 4v_m t_d}{4v_m - 4}. \quad (2.4)$$

Note that the term on the right of Eq. (2.4) represents a critical point, at which a serial expansion outperforms a parallel expansion in terms of flow rate. Furthermore, it highlights that this critical point is a function of t_d .

2.2.4 Combination Bottleneck

To provide further expansion and efficiency, we propose to combine the strength of both the serial and parallel bottlenecks. This creates a combination bottleneck with n lanes, each of which is facilitated by m processing units. In other words, there are a total of mn processing bottlenecks as illustrated in Fig. 2.1. Figure 2.5 shows the fundamental diagram of a combination bottleneck with 4 two-gantry lanes and a combination bottleneck with 2 four-gantry lanes. Interestingly, the two configurations perform differently under the same conditions.

The maximum flow rate for a combination bottleneck with n number of m -gantry lanes is expressed as follows:

$$J_{max} = \min \left(\frac{n^2 m}{(2m + \Delta + t_d)n - m}, \frac{nv_m}{1 + 2nv_m - v_m} \right), \quad (2.5)$$

with Δ being defined in Eq. (2.2). The derivation of Eq. (2.5) is given in the Appendix.

In Figs. 2.6a and 2.6b, maximum traffic flow rate are shown for combination bottlenecks with 1, 2, 3, and 4 lanes. In general, the flow rate increases with an increase in the number of gantries, before it reaches the limit imposed by lane merging which is indicated by the appearance of a horizontal section of the curve. Note that an increase in the number of m -gantry lanes does not always leads to an increase in J_{max} .

Interestingly, at the regime of large-scale expansion, it is possible to optimize the outflow by combining the strengths of the serial and parallel bottleneck. In other words, there is an optimum configuration for a fixed number of processing units that gives the most efficient traffic flow. Such a combination bottleneck has a collective vehicular effect on the maximal flow ascribed by $2u/n - m/n + \Delta$, with $u = mn$ being the total number of sites (see Eq. (2.5)). It is significant that this collective effect of the combination

(a) $t_d = 3$

(b) $t_d = 13$

Fig 2.5. Fundamental diagrams of combination bottleneck for $t_d = 3$ and $t_d = 13$ for two different configurations with a total of 8 processing sites. The results are displayed for mean flow rate J against traffic density c based on an ensemble of 100 realizations.

bottleneck can be made smaller than that of the parallel or serial bottleneck with the same u , thus achieving a more optimal traffic processing efficiency than either of them. Let us illustrate this fact by considering bottlenecks with 12 processing units which are organized among: (1) 12 one-gantry lanes; (2) 6 two-gantry lanes; (3) 4 three-gantry lanes; (4) 3 four-gantry lanes; (5) 2 six-gantry lanes; and (6) 1 twelve-gantry lane (see Fig. 2.7a). Depending on the processing time t_d , one of these configurations outperforms the others. While configuration (5) is a better choice when t_d is small, configuration (2) performs better than the other configurations when t_d is large. In particular, a more serial processing gives better performance for a smaller t_d . This observation is clearly borne out in Fig. 2.7a. As t_d reduces, we notice that the successive optimal configuration goes from

(a) $t_d = 3$

(b) $t_d = 13$

Fig 2.6. A plot of J_{max} versus m for combination bottleneck. Note that the data points are from numerical results and represent the mean of 100 realizations, while lines are plots from the analytical prediction of Eq. (2.5). Error bars of numerical results are smaller than the marker size. Only the integer values of the analytical plots, which are presented as continuous line, are relevant.

configuration (2) to configuration (3), then to configuration (4), to configuration (5), and finally to configuration (6). Similar results are shown in Fig. 2.7b for bottlenecks with 30 processing units. A common mechanistic feature that can be gleaned from Figs. 2.7a and 2.7b is that J_{max} is rate-limited by lane merging for small t_d , where J_{max} displays a constant value above 0.5. Beyond a critical t_d whose value depends on the configuration of the combination bottleneck, the delay bottlenecks due to the processing sites start to dominate, and we observe a monotonic decrease in J_{max} as t_d increases.

(a) 12 time-delay sites

(b) 30 time-delay sites

Fig 2.7. A plot of J_{max} versus t_d for a combination expansion of 12 and 30 processing sites in various combinations of m and n values.

2.2.5 Relation to current real-world traffic

It has been suggested that a realistic traffic flow model must account for the effects of (1) velocity-dependent randomization (braking), (2) slow-to-start and (3) velocity anticipation. Thus far, our study had been based on an elementary traffic model, which does not capture any of these effects. While the simplicity of the employed model has enabled an analytical study and the discernment of the underlying physics of different bottlenecks, it lacks mechanisms that capture key features of empirical traffic flow. In addition, NaSch model has also been found to not capture the numerous spatio-temporal traffic patterns observed in empirical traffic. Notably, these traffic patterns correspond to the existence of a third traffic phase - apart from free-flow and congested state - known as synchronized traffic flow, which occurs in moderately high vehicle density. As drivers in moderately high vehicle density tend to avoid sudden deceleration (or acceleration) by observing and anticipating the behavior of the surrounding traffic, the existence of synchronized traffic - and consequently the empirically-observed traffic patterns - is largely predicated on velocity anticipation effects in the traffic model.

To account for velocity anticipation effects, the brake-light model [95] modifies the NaSch model with an added feature which gradually slows down a vehicle as it approaches a jam. By including slow-to-start (STS) [36] and velocity-dependent randomization (VDR) [37] rules in the model, the brake-light model is able to more accurately reproduce the interaction between vehicles at moderately high densities. While the spatio-temporal patterns resulting from the brake-light model was found to be in good agreement with empirical data, it does not model the stylized wide-scattering of flow-density states found in synchronised traffic flow. Based on Three-phase theory [96], the velocity anticipation behavior is more accurately captured by the existence of a “synchronized gap”, in which cars that are traveling with the same velocity on a single lane can take on an arbitrary space gap between each other. The Kerner-Klenov-Wolf (KKW) model [38] incorporated this mechanism with both the VDR and STS effects to gives rise to a model comparable to real traffic. To put this study into a more realistic context, we have extended our numerical studies using the KKW model.

Figure 2.8a presents the equivalent of Fig. 2.4 using the KKW model. Note that bottleneck capacity in empirically-based traffic models spans a certain range [67], which we illustrate here in terms of the mean and standard error. While the relationship between J_{max} and m is qualitatively similar to those of the NaSch model, the resultant bottleneck capacities shown in Fig. 2.8a are much lower. This is a consequence of the incorporation of additional mechanisms which have further limited the flow of vehicle through the

bottleneck. We deduce that the STS rule is the largest contributor to this reduction in flow rate among the mechanisms introduced into the KKW model. Stop-and-start motion is very common in processing bottlenecks and when it is coupled with the STS rule, becomes the main mechanism in enhancing the stochastic deceleration as the vehicle moves off from rest. It models and attributes the larger delay in moving off a vehicle to that of human reaction time [36]. In consequence, J_{max} for the parallel bottleneck has become more heavily limited, changing from 0.5 to 0.34.

From these empirically-based models, parallel expansion is found to consistently outperform the serial expansion in terms of processing efficiency. More importantly, the increased human reaction time captured by the STS rule have enlarged the performance differences between the serial and parallel expansion. This can be understood from the fact that serial bottleneck tends to cause cars to stop and wait for the preceding vehicle to complete processing before moving off. Since the STS rule affects cars that are stopped, there is a higher frequency that this rule is applied in the serial bottleneck than the parallel bottleneck. As a result, the cumulative delay due to slow human response in starting off the vehicle from rest accounts for the relative inefficiency of the serial processing bottleneck, justifying the expenditure of lane-expansion cost for the implementation of parallel bottleneck. It also explains the observed abundance of parallel bottleneck, and a scarcity of serial bottleneck, in current traffic systems.

2.3 Discussion and Concluding Remarks

In order to improve the efficiency of the processing bottleneck, our analysis has revealed the need to optimize the collective motions of the vehicle. In particular, human reaction time and characteristics in mobilizing a vehicle, which are modeled by the STS rule in the last section, have caused a restriction in outflow which drastically decreases the efficiency of all processing bottlenecks as depicted in Figs. 2.8. By removing the human factor in the bottleneck system, as exemplified by the simplified NaSch model, we observe the feasibility of achieving a processing efficiency that goes beyond current technology and capabilities. Thus, our results point to the potential benefits of autonomous or self-driving vehicles in enhancing traffic capacity through optimizing the collective motions of vehicle by putting human drivers out of the loop. This is particular pertinent in the current era of urbanization, where more than half of the world population is living in cities. It was projected that by 2050, 66% of the world population will reside in urban areas [7]. With the increasing human density comes greater demand on our road systems. In order for

Fig 2.8. A plot of $J_{\max}$ versus m and n for serial, parallel, and combination expansion of bottlenecks for the KKW-1 model. The parameters for the simulation are: $p = 0.04$, $p_0 = 0.425$, $p_{a1} = 0.2$, $p_{a2} = 0.052$, $k = 2.55$, $v_p = 3$, $t_d = 3$ and 13 . Note that computations were made only at the data points with the lines between the points serve the sole purpose of illustration.

the cities to be liveable, there is a need to enhance the capacity of current road networks and to improve the efficiency of traffic systems in handling the heavier vehicular flow. Ultimately, the purpose of the various technological solutions is to prevent the occurrence of severe traffic congestion.

In this respect, our proposed combination processing bottlenecks is directly applicable in addressing these traffic woes of the future. We have seen that for a sufficiently large number of processing units (see Eq. (2.4)), the new configuration of serial bottleneck becomes tenable for traffic processing. While this is true for small processing time, our results have demonstrated that as the processing time increases, a set of combination bottlenecks with progressively more lanes and less serial processing (assuming a fixed total number of processing units) is optimal for traffic processing. In fact, for large

processing time, a multi-lane bottleneck with at least two serial processing site is found to be optimal. These results show that in the processing of dense traffic flow, serial bottleneck is an essential component that can serve to produce greater efficiency in the system as long as delay due to human reaction time (as illustrated by the STS rule) has been amply minimized.

In summary, we have studied the efficiencies of processing bottlenecks expanded in different configurations. Taking human reaction into account, parallel expansion is the most effective strategy of expanding a bottleneck. In the absence of delayed human reactions, parallel bottleneck remains an efficient strategy in increasing roadway capacity. On the other hand, serial expansion is a better choice when the number of processing units grows beyond a critical point, if the slow-to-start effect can be minimized. More significantly, by incorporating serial bottleneck into parallel bottleneck during large-scale expansion, we uncover an even more optimal alternative solution where a combination of these bottlenecks is able to process and manage more efficiently the heavy traffic flow of urban cities of the future.

Chapter 3

The phenomenon of bus bunching

In recent years, to alleviate growing traffic and congestion problems in urban cities, road transit services have been emphasized over private cars. Traditionally, road transit services have been divided into two broad categories: fixed-route (buses) and demand responsive (taxis) transits. Between having a predetermined schedule and a large loading capacity which consolidates many passenger trips onto a single vehicle, the cost efficiency of using a fixed-route transit makes it a popular choice in many urban cities. In 2015, Singapore’s bus system served over 3.7 million bus rides, with a staggering 16 million man-kilometers covered every day [97].

Since the 1960s, the scientific community has noted a longstanding problem with buses: the pervasiveness of bus bunching [44–59, 98–100]. Bus bunching occurs when the headway of two or more buses reduces to zero, where in traffic studies, headway is defined as the forward distance or time of vehicle to its leading vehicle. When bunching persists, the involved buses arrive almost simultaneously at the bus stop. Excluding the rare occurrence of bunched buses arriving exactly where there is a sudden influx of passengers, this typically leads to the under-utilisation of the trailing bus’s capacity. Furthermore, with two or more buses bunching and moving together as a single bus, it increases the inter-arrival time of the buses, which in turn leads to large variations in passengers’ waiting time. Regardless of specific route traits or vehicle specifications, the pervasive tendency of bus bunching seems to be universal [52, 56, 98].

In literature, there exists a number of analytical theories which describe the phenomenon of bus bunching. The Bus Route Model by Ref. [101] describes the onset of bus bunching analogously to the onset of vehicular traffic congestion. Using the framework of a modified circle map, Refs. [102–107] attribute the fluctuation of bus headway - and in turn bus bunching - to a deterministic chaos based on the loading rate of the buses. In this

chapter, we present an analytical theory which describes the dynamics of bus bunching in a framework analogous to the synchronization of self-oscillators. With that, we will discuss the pervasiveness of bus bunching as an implication of this novel correspondence.

3.1 Physical mechanism of bus bunching

Qualitatively, the physical mechanism of bunching of buses goes as follows. Consider two buses operating in a loop of length L , with two bus stops in antipodal positions of $L/2$ and L respectively (Without loss of generality, this argument ignores passenger alighting). For simplicity, we will also consider that passengers arrive at a constant rate λ , board at a loading rate of l and each bus moves with a constant velocity v such that it takes $T_0 = L/v$ unit time to travel the loop without picking up any passengers. Note that loading rate has to be greater than the arrival rate such that $l/\lambda > 1$. Otherwise, buses will theoretically be stuck at the bus stop indefinitely.

Consider the case illustrated by Fig. 3.1a, where each bus starts in a different bus stop, without any passengers initially. Each bus departs and travels at a constant speed to the next bus stop. There will be the same number of passengers at each bus stop, since each bus has travelled for the same amount of time. This results in the same dwell time τ for each bus before departing for the next bus stop. All in all, both buses move with a constant period of $T_0/2 + \tau$, allowing the buses to maintain a constant phase difference. We define this equilibrium state as the equal headway state.

Now, consider a general case where the buses start with unequal headway such that Bus 1 is x away from the Bus Stop b - as illustrated in Fig. 3.1b. Bus 2 takes $T_0/2$ to reach Bus Stop b and stops for τ for passenger boarding. Concurrently as Bus 1 started with a smaller headway, it only travels for a time $T' < T_0/2$ before reaching Bus Stop a . This also results in a smaller dwell time $\tau' < \tau$ before it departs the bus stop. Fig. 3.1c plots the state of the two buses after $T_0/2 + \tau$, in which case Bus 1 has moved a distance x' ahead of Bus Stop a . Mathematically, x' can be expressed as $x + \frac{x\lambda}{l}$, it is greater than x for all values of $x > 0$. In fact, as x' depends linearly on x , any deviation from the equal headway state will result in a larger deviation. This positive feedback of smaller-than-equal headway minimizing with time qualitatively describes the propensity of buses bunching.

In the presence of natural fluctuations found in a real bus system, the equal-headway state described in Fig. 3.1a does not remain in equilibrium as the fluctuations inevitably result in a difference in bus velocities which in turn places the buses in an unequal headway

Fig 3.1. Schematic diagram illustrating the different headway configuration taken by a bus system. Figures (b) and (c) are presented in chronological order

state. If the magnitude of the fluctuation are small (in the order of $\frac{x\lambda}{l}$), the dynamics described earlier remains statistically the same in the unequal headway state. Under that assumption, the described mechanism can also describe the propensity of buses bunching in a real bus system.

3.2 Analytical derivation of the dynamics of a two-bus and two-bus-stop system

The physical mechanism of bus bunching presented in the earlier section describes a relationship between the time taken to complete the route and the initial distance headway h_d of a bus (note that the subscript d is a descriptor of the variable h , describing it as a distance-headway). In that regard, the distance-headway of Bus i in Fig.3.1b is $h_{d,1} = \frac{L}{2} - x$ and $h_{d,2} = \frac{L}{2} + x$. Alternatively, this relationship can be expressed as a dependence of the effective angular velocity ω_i on the initial time-headway h_t of each bus such that $h_{t,i}$ is the time-headway of Bus i . In the absence of passengers, each bus takes $T_0 = L/v$ to complete one revolution. We define this angular velocity as $\omega_0 = \frac{2\pi}{T_0}$. With passengers, the buses stop at each bus stop for boarding and alighting. We define the amount of time Bus i stops at its preceding bus stop as stoppage time τ_i . With that the effective angular velocity aggregated for half a loop can be expressed as $\frac{\pi}{\frac{T_0}{2} + \tau_i}$.

With that:

$$\begin{aligned}\frac{\omega_i}{\omega_0} &= \frac{\frac{\pi}{\frac{T_0}{2} + \tau_i}}{\frac{2\pi}{T_0}} \\ \frac{\omega_i}{\omega_0} &= 1 - \frac{\tau_i}{\frac{\pi}{\omega_0} + \tau_i} \\ \omega_i &= \omega_0 - \omega_0 \left(\frac{\tau_i}{\frac{\pi}{\omega_0} + \tau_i} \right)\end{aligned}\tag{3.1}$$

defines the general dynamics of the buses in a two-bus system.

3.2.1 Problem formulation

With Eq. 3.1, the effective angular velocity of the buses in a two-bus system depends on the stoppage time at the Bus Stop τ_i , which is to be expressed as a function of its initial headway $h_{t,i}$. On closer inspection for different $h_{t,i}$, the dynamics of the system are different and have to be formulated individually. We will now go through the different dynamical cases before proceeding to solve for each case.

In Sect. 3.1, we have shown that the equal headway state (Fig. 3.1a) is an equilibrium state as both buses will have the same effective angular velocity. We define the effective angular velocity, stoppage time and initial headway in this state as ω_e , τ_e and $h_{t,e}$ respec-

tively, with the subscript e describing it as the equal-headway state. Note that in the two-bus system, there is another two time-headway states where $\omega_i = \omega_e$. It occurs in the configuration of the buses completely bunched such that the headway of the trailing bus $h_{t,i} = 0$ and that of the leading bus is $h_{t,i} = 2h_{t,e}$. In this state, the two buses move as a single bus, resulting in twice the number of passengers each time the buses arrive at a bus stop. However, as both buses share the loading of passengers, the stoppage time of the buses remain the same. This results in the same effective angular velocity ω_e . Note that the completely bunched state is also an equilibrium state.

Fig 3.2. Schematic diagram describing two initial headway states which can be taken by the system. The configuration in (a) will lead to Bus 2 departing Bus Stop b before Bus 1 arrives. Conversely, configuration in (b) will lead to Bus 1 arriving at Bus Stop b before Bus 2 has finished loading passengers.

As illustrated by Fig. 3.2, now consider two buses starting with a general phase difference $\Delta\theta_i = h_{t,i}v$ such that $h_{t,1} < h_{t,e}$. Comparing two different initial states, there exists a critical headway, $h_{t,c1}$ such that if $h_{t,1} > h_{t,c1}$, Bus 1 would arrive at Bus Stop b after Bus 2 has boarded all passengers and departed (Fig. 3.2a). However, if $h_{t,1} < h_{t,c1}$, Bus 1 would arrive at Bus Stop b before Bus 2 boarded all passengers (Fig. 3.2b). In this case, Bus 1 will share the loading of remaining passengers.

The existence of a critical headway splits the dynamics of the system into four different cases. Considering Bus 1 of Figs. 3.2a and 3.2b:

1. Headway ranging from zero to $h_{t,c1}$ results in buses overlapping - and shared loading - at the bus stop. In this case, the dynamics is described by the stoppage time $\tau_{o,1}$ - where o denotes overlap - and will be solved as a function of $h_{t,1}$.
2. If the headway ranges from $h_{t,c1}$ to $h_{t,e}$, the buses do not overlap and the trailing bus will only arrive at the bus stop after the leading bus has left. The dynamics

is described by the stoppage time $\tau_{n,1}$ - where n denotes no-overlap - and will be solved as a function of $h_{t,1}$.

With reference to Bus 2:

3. If headway is larger than $h_{t,e}$ but smaller than $h_{t,c2}$, the bus will leave the stop before the trailing bus arrives. In this case, the dynamics is described by the stoppage time $\tau_{n,2}$ and will be solved as a function of $h_{t,2}$.
4. Lastly for $h_{t,2} > h_{t,c2}$, the trailing bus will catch up at the bus stop, and share loading with Bus 1. In this case, the dynamics is described by the stoppage time $\tau_{o,2}$ and will be solved as a function of $h_{t,2}$.

As this analytical approach aggregated the effective headway every half a loop, $h_{t,2}$ will not be exactly measurable as the headway of the buses changes every half a cycle. As such, we assume the relation between $h_{t,2}$ as $h_{t,1} + h_{t,2} = 2h_{t,e}$.

3.2.2 Equal headway state

In this configuration, the time headway between each bus is:

$$\begin{aligned} h_{t,e} &= \frac{T_0}{2} + \tau_e \\ &= \frac{\pi}{\omega_0} + \tau_e \end{aligned}$$

The number of passengers picked up by a bus each time it stops is

$$\tau_e l = \left(\frac{\pi}{\omega_0} + \tau_e \right) \lambda \tag{3.2}$$

then, the dwell time is:

$$\tau_e = \frac{\pi}{\omega_0(r - 1)} \tag{3.3}$$

where $r = \frac{l}{\lambda}$. From Eq. 3.1, the effective angular velocity will be:

$$\omega_e = \omega_0 - \omega_0/r \tag{3.4}$$

and the time-headway between each bus is

$$h_{t,e} = \frac{\pi r}{\omega_0(r-1)} \quad (3.5)$$

3.2.3 Without overlapping

Fig 3.3. Schematic diagram illustrating the evolution of the bus system where buses do not overlap after the half-round. Figures presented in chronological order, with Fig (c) being the initial headway state.

Here we consider the no-overlap cases, solving for $\tau_{n,i}$ for both buses. The evolution of this case is illustrated chronologically in Fig. 3.3, with Fig. 3.3c being the state where the initial time-headway $h_{t,i}$ is considered.

To compute $\tau_{n,1}$, we must consider the total number of passengers at Bus Stop b . Bus Stop b was empty when Bus 1 left the stop, which is illustrated in Fig.3.3a. The total number of passenger boarding bus 2 would a product of the arrival rate with time elapsed between Fig.3.3a and Fig.3.3d. After leaving Bus Stop b , Bus 1 takes $T_0/2$ to reach Bus Stop a (Fig.3.3b) and dwells at bus stop a to for boarding passengers. The number of passengers picked up by Bus 1 is equal to $\frac{h_{t,1}}{r}$, illustrated in Fig.3.3c and from Fig.3.3c to Fig.3.3d, it takes $\frac{T_0}{2} - h_{t,1} + \tau_2$ time steps. With that:

$$\begin{aligned} \tau_{n,2}l &= \left(\frac{T_0}{2} + \frac{h_{t,1}}{r} + \frac{T_0}{2} - h_{t,1} + \tau_2 \right) \lambda \\ \tau_{n,2} &= \frac{2\pi r + (1-r)\omega_0 h_{t,1}}{\omega_0 r(r-1)} \end{aligned}$$

Given that $h_{t,1} + h_{t,2} = 2h_{t,e}$,

$$\tau_{n,2} = \frac{h_{t,2}}{r} \quad (3.6)$$

The stoppage time of Bus 1 is given by the product between λ and the *eventual* time-headway of Bus 1, after it leaves Bus Stop b . The headway change is brought forth by the different dwell times at Bus Stop b and hence the eventual headway can be expressed as $(h_{t,1} - \tau_2 + \tau_1)$. Therefore:

$$\begin{aligned}\tau_{n,1}l &= (h_{t,1} - \tau_2 + \tau_1)\lambda \\ \tau_{n,1} &= \frac{h_{t,1}(r+1)}{r(r-1)} - \frac{2\pi}{\omega_0(r-1)^2}\end{aligned}\quad (3.7)$$

If $h_{t,1} = h_{t,c}$, Bus 1 will arrive at Bus Stop b at the time Bus 2 is leaving the bus stop. As such, there will be no passengers for pick up. This results in $\tau_{n,1} = 0$ and:

$$\begin{aligned}\frac{h_{t,c1}(r+1)}{r(r-1)} &= \frac{2\pi}{\omega_0(r-1)^2} \\ h_{t,c1} &= \frac{2\pi r}{\omega_0(r^2-1)}\end{aligned}\quad (3.8)$$

$$h_{t,c2} = \frac{2\pi r^2}{\omega_0(r^2-1)}\quad (3.9)$$

3.2.4 With overlapping

Fig 3.4. Schematic diagram illustrating the evolution of the bus system where buses overlap after the half-round. Figures presented in chronological order, with Fig (a) being the initial headway state.

We consider the case of $h_{t,1} < h_{t,c}$. Given that the two buses will overlap at Bus Stop 2, Bus 2 will arrive $h_{t,1}$ before Bus 2 and they will leave together. Therefore the different in their dwell time can be expressed as

$$h_{t,1} = \tau_{o,2} - \tau_{o,1}$$

Given that the linear relationship between $\tau_{o,1}$ and $h_{t,1}$, together with two fixed points at $h_{t,1} = 0$, $\tau_{o,1} = \tau_{o,2} = \frac{\pi}{\omega_0(r-1)}$ and $h_{t,1} = h_{t,c1}$, $\tau_{o,1} = 0$, $\tau_{o,1}$ can be solved as:

$$\tau_{o,1} = \frac{\pi}{\omega_0(r-1)} - h_{t,1} \frac{r+1}{2r} \quad (3.10)$$

With that:

$$\tau_{o,2} = \frac{\pi r}{\omega_0(r-1)} - h_{t,2} \frac{r-1}{2r} \quad (3.11)$$

3.2.5 Corrections due to unloading

The analytical formulation thus far considers only the number of boarding passengers. Since loading and alighting takes place concurrently, the formulation is accurate only when the number of passengers boarding is greater than the number of passengers alighting. However, when the number of passenger boarding is less than that of alighting, the stoppage time will be based on the number of alighting passengers. In this case, our formulation under-estimates the stoppage time.

Let us consider which cases would have more alighting than boarding passengers. In the two-bus model, the number of passengers alighting at one bus stop is equal to the number of passengers boarding at the previous bus stop. Considering the positive feedback described in Sect. 3.1, only the bus with the headway smaller than $h_{t,c}$, i.e. Bus 1, will consistently be alighting more passengers than it boards. Without considering passengers unloading, if Bus 1 starts at $h_{t,c1}$, it will leave immediately as it arrives at Bus Stop b as Bus 2 would have finished boarding all the passengers. However considering unloading, Bus 1 would not leave immediately due to passengers alighting. Based on this, we see that the critical behavior at $h_{t,c1}$ is replaced by the behavior which would occur at $h_{t,cu}$ (u as it is due to unloading passengers) such that if Bus 1's initial headway is smaller than $h_{t,cu}$ Bus 1 will alight less passengers than *half* the amount of boarding passengers such that the buses overlaps and shares loading. Below $h_{t,cu}$, the stoppage time will not be determined by the alighting, and instead is correctly described by Eq. 3.10. Therefore considering passenger unloading, the true stoppage time will be $\tau_{un,1}$ instead of $\tau_{n,1}$ (Eq. 3.7) and the critical transition would take place at $h_{t,cu}$ instead of $h_{t,c1}$ (Eq. 3.8).

As the number of passengers alighting at a bus stop is dependent on the number of

passengers boarding at the previous bus stop, Eq. 3.7 would describe the stoppage:

$$\tau_{un,1} = \frac{h'_{t,1}(r+1)}{r(r-1)} - \frac{2\pi}{\omega_0(r-1)^2} \quad (3.12)$$

where $h'_{t,1}$ is the initial headway of Bus 1 for its previous half-lap. As the other derivations, our aim is to express $\tau_{un,1}$ as a function of $h_{t,1}$. Therefore, we will first derive the relationship between $h'_{t,1}$ and $h_{t,1}$.

Fig 3.5. Schematic diagram describing the evolution of buses' time-headway in a two-bus system

Fig. 3.5 illustrates the relationship between $h'_{t,1}$ and $h_{t,1}$. The relationship between h and h' can be solved by considering stoppages times - $\tau_{u,2}$ for Bus 2 at Bus Stop b and $\tau_{u,1}$ for Bus 1 at Bus Stop a . For Bus 1, the stoppage time is:

$$\begin{aligned} \tau_{u,1}l &= \left(\frac{T_0}{2} - (h_{t,e} - h'_{t,1}) + \tau_{u,1} \right) \lambda \\ \tau_{u,1} &= \frac{\frac{T_0}{2} - (h_{t,e} - h'_{t,1})}{r-1} \end{aligned} \quad (3.13)$$

Likewise for Bus 2:

$$\begin{aligned} \tau_{u,2}l &= \left(\frac{T_0}{2} + (h_{t,e} - h'_{t,1}) + \tau_{u,2} \right) \lambda \\ \tau_{u,2} &= \frac{\frac{T_0}{2} + (h_{t,e} - h'_{t,1})}{r-1} \end{aligned} \quad (3.14)$$

The evolution of the headway i.e. $h - h'$ is equal to the difference of $\tau_{u,1}$ and $\tau_{u,2}$. Using Eqs. 3.13 and 3.14, we get:

$$\begin{aligned} h_{t,1} &= \frac{h'_{t,1} - 2(h_{t,e} - h'_{t,1})}{r - 1} \\ h'_{t,1} &= \frac{r - 1}{r + 1} h_{t,1} + 2 \frac{h_{t,e}}{r + 1} \end{aligned}$$

With that,

$$\begin{aligned} \tau_{un,1} &= \frac{h'_{t,1}(r + 1)}{r(r - 1)} - \frac{2\pi}{\omega_0(r - 1)^2} \\ &= \frac{h_{t,1}}{r} \end{aligned} \tag{3.15}$$

The critical point where the dynamics change from alighting to load sharing is found by equating $\tau_{un,1}$ and $\tau_{o,1}$:

$$\begin{aligned} \frac{h_{t,cu}}{r} &= \frac{\pi}{\omega_0(r - 1)} - h_{t,cu} \frac{r + 1}{2r} \\ h_{t,cu} &= \frac{2\pi r}{\omega_0(r + 3)(r - 1)} \end{aligned} \tag{3.16}$$

3.2.6 Quantitative dynamics of two-bus system

To recap, the main aim of the previous sections was to describe the effective angular velocity of the buses in a two-bus system as a function of its initial headway h_t . Eq. 3.1 presents the effective angular velocity as a function of the stoppage time τ , which was found to exhibit different dynamics in different ranges of h_t . By considering the different dynamical states, we obtain:

1. $\tau = \tau_e$ for $h_t = 0, h_{t,c}$ and $2h_{t,c}$
2. $\tau = \tau_{o,1}$ for $h_t \in \{0, h_{t,cu}\}$
3. $\tau = \frac{h_t}{r}$ for $h_t \in \{h_{t,cu}, h_{t,c}\}$
4. $\tau = \frac{h_t}{r}$ for $h_t \in \{h_{t,c}, h_{t,c2}\}$
5. $\tau = \tau_{o,2}$ for $h_t \in \{h_{t,c2}, 2h_{t,e}\}$

With that, we present the analytical dynamics of the two bus system:

$$\omega_i = \omega_0 - \omega_0 \left(\frac{\tau}{\pi/\omega_0 + \tau} \right) \quad (3.17)$$

$$\tau \text{ is given by: } \tau = \begin{cases} \frac{\pi}{\omega_0(r-1)} - h_t \frac{r+1}{2r} & \text{if } h_t \in \{0, \frac{2\pi r}{\omega_0(r+3)(r-1)}\} \\ \frac{h_t}{r} & \text{if } h_t \in \{\frac{2\pi r}{\omega_0(r+3)(r-1)}, \frac{2\pi r^2}{\omega_0(r+1)(r-1)}\} \\ \frac{\pi r}{\omega_0(r-1)} - h_t \frac{r-1}{2r} & \text{if } h_t \in \{\frac{2\pi r^2}{\omega_0(r+1)(r-1)}, \frac{2\pi r}{\omega_0(r-1)}\} \end{cases}$$

where $\omega_0 = \frac{2\pi v}{L}$ and $r = \frac{L}{\lambda}$.

A simple discrete model based on a cellular automaton was proposed to verify Eq. 3.17. It models a two-bus and two-bus-stop system, where passengers injected at one bus stop are destined for the other. Bus stops are placed in antipodal positions along road of length $L = 1000$, passengers arrive at each bus stop at a uniform rate λ and both buses move at velocity v . For any $\lambda > 0$, it was found that for all initial conditions (excluding the equal headway state), the two buses will bunch and remains in a bunched state at long time limit. This demonstrates the pervasiveness of bus bunching even in a simplified model. It is important to note that these dynamics are only valid in the specific scenario where both buses have the same velocity. This point will be elaborated in section 3.3.

In this model, based on the analytical formulation, initial time-headways are measured each time a bus leaves one bus stop, and the effective angular velocities are averaged from a bus leaving one stop and until it leaves the other. 50 realizations of varying initial positions for the buses were simulated and each realization was run for $T = 1000$ time steps. For illustration, Fig. 3.6 plots the simulated dynamics as well as the analytical prediction for three different velocities: $v = 3$, $v = 7$, $v = 12$.

3.3 Comparison with the Kuramoto synchronisation mechanism

The analytical dynamics of the two-bus system elucidates a novel correspondence between the bunching of two buses and the phenomenon of synchronization described by a Kuramoto model.

Fig 3.6. Plot of simulated bus dynamics with analytical predictions from Eq. 3.17. 3 different velocities - $v = 3$, $v = 7$, $v = 12$ - are presented.

The dynamics of an N-oscillator Kuramoto model is described by the equation:

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i), i \neq j \quad (3.18)$$

Fundamentally, both systems contain a coupling term that are functions of the phase difference and the coupling reflects a positive feedback which causing phase-locking between independent oscillators. The analytical dynamics of the two-bus system also shows an apparent 2π -periodicity, which is characteristic of the Kuramoto model. It is noted that the coupling in the Kuramoto model is globally contributed by every self oscillator and is continuous. However, the 'coupling' in a bus system is discrete as it only acts at the bus stops. In addition, when expanded to a n-bus system, the 'coupling' between buses would be local (only from the leading bus) and uni-directional (the trailing bus is unaffected by the leading bus), which is different from the global effect of a Kuramoto Model. Nonetheless,

there is notably no difference between global coupling and local-uni-directional coupling when considering a two-bus/oscillators system.

The most intriguing similarity between the two systems is in the rate of change of phase difference. Consider a case where the phase difference between two oscillators is $\Delta\theta \lesssim \pi$. The interaction term in a two-oscillator Kuramoto model results in the slowing down of the leading oscillator and the speeding up of the trailing. As the phase difference continues to reduce, there is an increase in the rate at which the leading oscillator slows down and the trailing oscillator speeds up. This increase peaks at $\Delta\theta = \frac{\pi}{2}$. As $\Delta\theta \lesssim \frac{\pi}{2}$, the rate at which this slowing down/speeding up progressively reduces such that it approaches zero when $\Delta\theta \rightarrow 0$. This reduction in coupling as the system approaches phase-locked state is quintessential in the stability of the synchronized state. Interestingly, the analytical dynamics of the two-bus system exhibits the same behavior. This results in the stability of the bunched state and consequently the propensity of bus bunching.

From literature, there have been multiple studies - in areas ranging from electrical circuits to social behavior - that draw parallels to the Kuramoto model by formulating its coupling term in the characteristic form of $K \sum \sin \Delta\theta$ [108–111]. While our analytical formulation does not do so, the common key features imply the correspondence between the phenomenon of bus bunching and synchronization in a Kuramoto model.

While the qualitative similarities between bus bunching and the synchronization of a Kuramoto model describes the former as a natural tendency of the system, it is noted that the two-bus system studied here is a gross simplification of a real bus system. Notably, buses in the current study travels with a constant velocity and passenger injections are modelled by a uniform process. To substantiate our findings, we have extended our numerical simulations by incorporating fluctuations in the bus velocities, and by injecting passenger using a Poisson process. As expected, the presence of the added stochasticity results in an absence of the equal headway state (Fig. 3.1a) as an equilibrium state. In contrast, the dynamics of the buses in all the other states - unequal headway and completely bunched - remains the same with the added stochasticity. As discussed earlier, the former observation is expected as the stochasticity would perturb the equal headway state into an unequal headway state. The latter observation suggests that the natural tendency described in our analytical formulation is invariant to the added stochasticity in the system.

Other than the absence of stochasticity, the analytical formulation is also based on all buses having the same velocity. Due to the inherent behavior of human drivers in a real bus system, the average speed of one bus is almost certainly different from another.

Additionally, while most bus systems have more than two buses, the analytical study in this section only focuses on a system with two buses. A comprehensive study which relates the phenomenon of bus bunching to a general synchronisation phenomenon was presented in a recent paper by [112]. This paper presents a theory which describes the dynamics of N buses of different velocities serving a loop of M bus stops. In the framework of synchronisation, these buses are regarded as coupled self-oscillators with different angular velocities. The coupling strength is quantified by the ratio of the arrival and loading rates, i.e, $k = \frac{\lambda}{l}$. Considering that loading rate is typically constant throughout the day, the coupling strength is primarily a function of the arrival rate λ . Bus bunching is described as complete phase-locking, of which its onset is described by a phase transition when the coupling strength exceeds a critical threshold. For general bus speeds, complete phase-locking appears when λ exceeds:

$$\lambda_c(N) = \frac{l}{M} \sum_{i=1}^{N-1} \left(1 - \frac{\omega_N}{\omega_i} \right) \quad (3.19)$$

where ω_N is the angular velocity of the slowest travelling bus. With two buses of the same velocity, the critical coupling strength will be $\lambda_c = 0$. This corresponds to our observation of bus bunching occurring for any $\lambda > 0$ (excluding the very-specific equilibrium case) when the buses have the same speeds.

A potentially important finding involving different bus velocities is that when the coupling strength is below the threshold, buses exhibit another set of dynamics: periodic bunching. At a low number of passengers, the bunching mechanism has little effect on minimizing the headway. At the same time, the difference in bus velocities result in the repetitive dynamics of the fast bus overtaking a slow bus, escaping and catching up with it again. Below the threshold, the detuning effect outweighs the bunching dynamics. Instead of a persistent bunching, the buses only bunch periodically, under the detuning effect. Above the threshold, the bunching mechanism outweighs the detuning effect such that after overtaking the slower bus, it leaves a small backward headway such that the slower trailing bus starts picking up minimal amount of passengers. Without stopping much for passenger boarding, the effective velocity of the slower bus increases. With a particular headway, the speed of the ‘effectively-spiced-up’ slower bus matches the speed of faster bus such that they becomes phase-locked. This results in the same sustained bunching as with buses having the same velocities. Hitherto, the existence of both a periodically and sustained bunched state has not been explored in most literature of bus bunching. Crucially, it also demonstrates how the general dynamics of a bus system is

established by the passenger arrival rate (strictly speaking, the ratio k) and the difference in velocity between the buses. In the agent-based model, these essential features form the basis of our model formulation and the simulation study in Chapter 5 will demonstrate that the importance of these dynamics as a periodically bunched state and sustained bunched states would interact very differently with the intervention strategies designed to reduce the effects of bus bunching.

Chapter 4

Empirically-based Monte-Carlo Bus-Network Simulation Model

As discussed in the earlier chapter, the phenomenon of bus bunching is attributed to the natural dynamics of the buses and its interaction with the boarding passengers. To solve this problem, researchers and traffic planners have been proposing intervention methods to alleviate bus bunching. However, to evaluate the effectiveness of proposed interventions, experimentation on a full bus system is potentially very costly and a scaled-down experimentation might not capture the actual dynamics of the system. As such, simulation modelling has been the primary approach in the study of bus bunching phenomenon. Specifically, there have been numerous studies modelling intervention strategies to curb the effects of bus bunching [44–55, 57–60, 99, 100]. In these literature, the two main modelling approaches fall under macroscopic modelling, in the subclasses of analytical modelling and mathematical optimization [113].

Analytical modelling applies an event-based formulation, which typically uses closed-form functions to describe macroscopic states of the system, such as the arrival times of buses. [102–105] used such formulation to present various dynamical states (periodic, quasi-periodic, chaotic etc.) present in bus dynamics. [44, 45, 51, 52, 54, 98, 113] studied methods to reduce bus bunching by modelling the effects of intervention strategies on the analytical formulation of bus arrival times. Due to the simplistic perspective taken by analytical modeling, the findings have high academic value as they are highly generalizable in most idealized systems, as exemplified by [98]. On the other hand, this simplicity also results in it being inaccurate for real-world predictions. To mitigate this issue, recent studies of analytical modelling couple their formulation with a separate predictive model [51, 52, 113, 114]. By using the outputs of the predictive model as inputs to the analytical

model, these analytical-based simulations are able to have fairly accurate predictions of empirical observations.

Optimization approaches [46, 49, 50, 53, 55–59, 99, 100] formulate weighted functions of macroscopic objective functions; typically, passenger waiting time and optimisation tools are used at each decision state to make a decision based on maximising the objectives. As the simulation is continually updated with real-time information of the buses at each decision state, these models are highly accurate [58] when validated against empirical data. However, due to the intricacy in an optimisation process coupled with the dependency of the data inputs, optimization models typically do not produce results which are applicable to a range of other instances.

As discussed in Chapter 1, microscopic modelling - specifically, agent-based modelling - has been a popular approach in the study of complex systems [35, 115–117]. Agent-based modelling focuses on modelling the microscopic behaviors of the agents. Agents are modelled as entities with the ability to execute different actions in the simulation environment, which either alter their own state or the state of other agents. With simulation parameters as inputs, outputs of agent-based models are typically macroscopic observations which emerge as the aggregation of the microscopic behaviors. As such, not only do agent-based models pale in comparison in terms of accuracy when compared against analytical-based or optimisation simulations, agent-based models do not provide any guarantee of producing output that reflect empirically-observed phenomenon. However, its ground-up approach facilitates logical analysis between different parts of the model. Specifically, it elucidates the relationship between the modelled microscopic behaviors and the measured macroscopic outputs.

In this chapter, we propose a novel framework for an agent-based model - using a Monte-Carlo approach - to model a bus network. By modelling the interactions between buses and passengers, the aim of the model is to capture the pervasiveness of sustained bus bunching. However, instead of a generalisitic agent-based model designed to study the general dynamics of bus bunching, this model falls under the category of an empirical agent-based model designed to capture the bus bunching dynamics of a *specific empirical system*. While the connection between most empirical agent-based models and their target system is the parameter values measured from the real system, the stochastic behavior of a bus system is better captured by a series of empirical distributions. As such, the novel approach of the *empirically-based Monte-Carlo* model is by parameterizing the model using empirical distributions measured from a real bus system. In this chapter, the model formulation will first be presented, followed by a series of data analytics to derive

the required distributions from our case study of the Nanyang Technological University’s (NTU) campus shuttle service. Finally, the parameterized model will be compared against empirical data as a model validation. In Chapter 5, the fully parameterized model, known as the Empirically-based Monte-Carlo Bus-Network (EMB) model, will be used to study potential intervention strategies which can be taken by the operators of the campus shuttle service to alleviate the effects of bus bunching.

4.1 EMB model formulation

The basis of the Empirically-based Monte-Carlo Bus-Network (EMB) model is an agent-based model formulated to simulate the dynamics of buses in a bus system by modelling the detailed interactions between the buses and passengers. In this section, we present the process-driven method taken to model these interactions, i.e., the detailed interactions proposed based on heuristic understanding of a bus system.

It is noted that the primary purpose of the EMB model is to simulate the dynamics of a *specific* bus system by also modelling the microscopic behaviour of buses and passengers. This is done by parameterizing the simulation using empirical probability distribution. However, this parameterization process is exogenous to the simulation dynamics. As such, the model formulation will first be presented as a generic bus-network simulation, using probability distributions in general.

Environment: The simulation environment models a bus route, which can be represented by a 1-dimensional space-discretized array of length L . Based on the nature of the modelled system, the boundary conditions could either be open or closed. Closed boundary conditions model a continuous shuttle service, where a set of buses service a closed loop route. Open boundary conditions model buses which start and end their service at two different bus terminals. Regardless of boundary conditions, bus stops are placed along the route and passengers are continually injected at the bus stops.

Agent - Buses: There are two sets of behaviors that the buses in the EMB model have to exhibit: moving along the bus route and stopping at each bus stop to pick up passengers. For the former, the EMB model formulation is designed to capture three major factors of the empirical buses’ motion. They are its stochasticity, heterogeneity between buses and an overall realistic trajectory. This is done by a Monte-Carlo process. While there is no clear definition of Monte-Carlo methods [118], the general consensus is that any algorithm bearing the name *Monte-Carlo* involves repeated random sampling to obtain a probabilistic interpretation of an expected value. In this study, we follow

the definition laid down by [119] which defines a Monte-Carlo simulation as one which calculates the average behaviour - in this case the average bus trajectory - by drawing a large number of (pseudo) random numbers. For each time step t , Bus i samples a probability distribution for a velocity $v'_{i,t}$. In general, each position on the route has a unique distribution, which captures the difference in average velocity allowed at different segments of a road. However, this process does not capture the natural speed difference between each human-driven bus. As such, $v'_{i,t}$ is multiplied by a dimensionless constant K_i which describes the average speed of Bus i . With that, for each time step t , Bus i moves with an instantaneous velocity of $v_{i,t} = K_i v'_{i,t}$. This repeated sampling and scaling by K_i captures the heterogeneity and stochasticity of real world buses, but ensures that each bus takes a realistic average trajectory.

As it is predicated that the bus dynamics are a direct consequence of the bus-passenger interactions, buses are modelled independent from one another. As such, buses can occupy the same position on the route, allowing them to overtake freely. When there are passengers at a bus stop, passing buses will stop at the bus stop to allow passenger boarding. More than one bus is allowed to board passengers at a single bus stop simultaneously and when there are two or more buses, the passenger load is shared equally. Each passenger is initialized with a destination bus stop. When passengers on-board a bus have reached its destination, the bus will stop to allow for alighting. If there is nobody on the bus who wishes to alight and no passengers at the bus stop waiting to board, the bus will continue travelling towards the next stop.

Agent - Passengers: Passengers are injected to each Bus stop j by a Poisson process of mean λ_j . Each passenger injected at Bus stop j will have a randomly assigned destination Bus stop i such that $i \neq j$. After injection, each passenger remains at their respective bus stops until the arrival of a bus. When a bus arrives, passengers at a bus stop board on a first-come-first-serve basis. When the approaching bus stop is the destination of any on-board passengers, the bus will stop at the bus stop for passenger alighting. Alighting completes the journey for the passengers. The rate at which boarding and alighting takes place is determined by the parameter l . Boarding and alighting are set to take place concurrently, which models how most buses have different doors for boarding and alighting.

4.2 EMB simulation methodology

With the model inputs, the primary outputs of the EMB model are the trajectories of the N buses. Each realization begins with N buses at their respective initial positions, with no passengers on-board. There are also no passengers at any of the M bus stops at this point. Sensitivity analysis with various initial conditions have found that for all studies presented here, the initial conditions do not affect the results significantly.

As the model evolves, the simulated buses move along the bus route and passengers continually arrive at the bus stops. As buses reach the bus stops, they stop to allow passenger boarding and passengers on-board the buses alight when the buses reach their destination bus stops. By setting the initial positions as the equal headway state, Fig. 4.1 plots the bus trajectories simulated by the EMB model. When $\bar{\lambda}$ is greater than zero, the theory described in the previous chapter predicts that the buses will bunch and remain bunched over time. This corresponds to the observations of Fig 4.1a, which plots the simulated trajectories of two buses with the same velocities. When the bus velocities are different, the analytical theory predicts that the buses will exhibit periodic bunching. Likewise, this corresponds to the observations of Fig 4.1b, which plots the simulated trajectories of two buses of different velocities. All in all, these preliminary results show that by modeling the detailed interactions between buses and passengers, the EMB model is capable of capturing the basic dynamics of sustained and periodic bus bunching.

Two additional measurables from the EMB simulations are the waiting times and the overall trip times of the individual passengers. While numerous studies have found the equal headway state as the most efficient configuration, the efficiency of this simulated system can be quantified directly from the waiting and trip times of the passengers. For each passenger that has completed their journey, their waiting time is calculated as the time difference between the passenger's arrival time and boarding time. Similarly, their total trip time is calculated based on the difference between the arrival time and alighting time. We summarize the EMB model formulation as follows. The input parameters of the model are:

- Empirical probability distributions of bus velocities
- Dimensionless constant describing the relative velocity of each bus, K_i
- Passenger arrival rate at each bus stop, λ_j

With that, the model outputs are:

Fig 4.1. Simulated trajectories of two different bus systems with an average passenger arrival rate of $\bar{\lambda} = 0.03$.

- Simulated trajectories of each bus
- Waiting time and trip time of each passenger

In the following chapter, we present a series of data analytical methods in bid to obtain the required input parameters from our selected system: NTU's shuttle bus system. As the methods used are based on the available data, it is to be noted that they are not prescribed to the EMB model. With sufficient statistics as model inputs, the EMB model can be used to simulate any system without these data analytical methods.

4.3 Data analytics of NTU shuttle bus system

Nanyang Technological University (NTU) runs an internal shuttle bus service which is designed to help people get around campus. NTU shuttle bus service (NTU-SBS) is made up of two bus routes - the Blue and Red routes. Both routes traverse very similar paths and run in periodic loops; the main difference being that the Blue route operates in a clockwise direction, while the Red route operates in an anti-clockwise direction. Each bus route has 12 unevenly-spaced bus stops, and the buses operate daily from approximately 0730h to 2230h. The number of buses on each of these routes range from 1 to 7, depending

on the time of the day. This study only focuses on buses from the blue route. Nonetheless, the methods of this study are applicable to both routes, or any other bus system in general.

A web portal (<https://baseride.com/maps/public/ntu/>) maintains an applet which plots the live positions of the buses in GPS coordinates. These coordinates were recorded daily during its operational hours from 23 Aug 2018 to 11 Nov 2018, which was the first semester of NTU’s academic year of 2018-2019. These raw data were filtered and transformed into 1-D spatio-temporal trajectories i.e. $x_{i,t}$ are integers within $[1, 688]$ for Bus i at time t which are used in this study. Based on the resolution of the data, spatial and temporal resolutions are set as $\Delta x = 7.5m$ and $\Delta t = 12s$ respectively. Details of the data processing methodology and the spatio-temporal resolution can be found in Appendix B. Data from weekends and public holidays were excluded as they are not reflective of an average day of shuttle bus operation.

4.3.1 Velocity statistics

Velocity is defined as $v_{i,t} = x_{i,t+1} - x_{i,t}$. Fig. 4.2a plots the average velocities on different positions along the route. The decrease of average speeds near the bus stops and road junctions are consistent with heuristic understandings of the system. Figs. 4.2b and 4.2c plot the empirical velocity distribution $P_x(v)$ at the road position where $x = 542$ and 61, which reflect road segments with the highest and lowest average velocities respectively. Based on our analysis, the average velocity of buses at position $x = 542$ is 31km/h, which is a reasonable average speed for a bus travelling within a campus.

4.3.2 Passenger statistics

As the data collected from the web portal does not include passenger boarding or alighting information, these information had to be estimated from the bus trajectories. It is hypothesized that passengers arrive at each bus stop infrequently ($\approx 60s$ per passenger). As such, passenger arrival would follow a Poisson process, of which the inter-arrival time (t^{inj}) of passengers would follow an exponential distribution of rate λ which describes the rate of passenger arrival.

As Bus i passes each stop, t_i^{arr} is measured as the arrival time of the bus (while t_{i-1}^{arr} denotes the arrival time of the preceding bus at the same bus stop). The stoppage time τ_i is measured as the total duration Bus i spends stationary at the bus stop. By assuming that there is always the same number of passengers alighting and boarding, the stoppage time at a bus stop will be the number of boarding passengers divided by the loading

(a) Average velocity taken by buses at different positions on the route. The color bar describes the average velocity in units of $[\Delta x]/[\Delta t]$.

(b) Empirical distribution of velocities taken by buses at position $x = 542$, which is the middle of a straight stretch of road.

(c) Empirical distribution of velocities taken by buses at position $x = 61$, which is upstream of a T-junction.

Fig 4.2. Velocities statistics of the NTU-SBS from 23 Aug 2018 to 11 Nov 2018

rate, which is assumed to be $l = 1s$. It is noted that this assumption leads to an over-estimation of the number of boarding passengers when there are more alighting than boarding passengers. However without additional information on passenger boarding, the

accuracy cannot be improved.

The total time it takes for τ_i passengers to arrive at the bus stop can be given by Δt_i^{arr} , which is the difference between the departure time of the current bus $t_i^{\text{arr}} + \tau_i$ and the departure time of the previous bus, which is given by $t_{i-1}^{\text{arr}} + \tau_{i-1}$. With that,

$$\begin{aligned} t^{inj} &= \frac{\Delta t_{arr,i}}{\tau_i l} \\ &= \frac{t_i^{\text{arr}} + \tau_i - t_{i-1}^{\text{arr}} - \tau_{i-1}}{\tau_i l} \end{aligned} \quad (4.1)$$

Fig. 4.3 plots the probability of $t_{inj} > t$ at each bus stop. The fitted parameter λ_j describes the rate of passenger arrival per second. However, heuristically, a single λ for each bus stop does not fully describe the passenger arrival rates for NTU-SBS. This is because the number of passengers varies significantly across different times of the day. For better clarity, the same analysis should be done separately for peak and off-peak hours of a day or even for every hour of the day. To facilitate our simulation methodology presented in the next section, each day is split into 4 segments $\bar{t}$ with breakpoints at 1000h, 1400h and 2020h. The choice of these segments and breakpoints will be elaborated in the next section. With that, table 4.1 tabulates $\lambda_{j,\bar{t}}$ measured from each Bus stop j at each of the four time periods $\bar{t}$.

Table 4.1. Passenger arrival rates, $\lambda_{j,\bar{t}}$ at Bus stop j , at different times of the day $\bar{t} \in \{1, 2, 3, 4\}$. The units are given by number of passenger per second.

Bus stop index	<1000h	1000h - 1400h	1400h - 2020h	> 2020h
1	0.028	0.025	0.027	0.018
2	0.022	0.019	0.015	0.009
3	0.016	0.013	0.014	0.009
4	0.029	0.025	0.028	0.017
5	0.016	0.012	0.010	0.006
6	0.021	0.012	0.011	0.008
7	0.025	0.017	0.012	0.008
8	0.046	0.027	0.018	0.009
9	0.032	0.014	0.011	0.012
10	0.023	0.015	0.014	0.008
11	0.028	0.015	0.012	0.006
12	0.044	0.025	0.022	0.013

The derived passenger arrival rates were analyzed against heuristic understanding of NTU-SBS. Temporally, the decrease in passenger arrival rate after 2020h is consistent with

Fig 4.3. Plot of $P(t^{\text{inj}} > t)$ distributions of the passengers at each of the bus stops. The fitted parameter λ_j describes the daily average rate of passenger arrivals at each bus stop.

empirical observations as the majority of students and staff do not reside on campus. Spatially, Bus stop 4 seems to consistently have higher injection rates than other bus stops. This bus stop is located at the *North Spine Plaza*, where there is an aggregation of a variety of amenities. Notably, this bus stop is also located in the vicinity of the biggest library in NTU, and there is usually a significant number of library users up till its closing

time at 2200h. This could lead to a relatively high arrival rate, even after 2020h. The one highest recorded $\lambda_{j,\bar{t}_i}$ occurs at Bus stop 12 in the morning. This bus stop serves one of the largest residential halls on campus, which could explain the large number of passenger arrival rates in the morning.

4.4 EMB model using empirical data

4.4.1 Parameterizing EMB model

To use the EMB model to simulate the dynamics of NTU-SBS, results from the data analytics done in the previous section will be used as parameters in the model. The bus route is made up of $L = 688$ unique sites, under periodic boundary conditions. As illustrated in Fig. 4.2, the data analytics have also found 688 unique empirical distributions which describe the velocities taken by buses at the different positions of the route. These distributions are used in the Monte-Carlo process of determining the instantaneous velocity of the buses.

Based on the positions of the bus stops in NTU-SBS, positions [62, 99, 152, 205, 256, 307, 353, 416, 479, 548, 606, 667] are marked as Bus stops 1-12 respectively. In general, passengers' arrival rates at the bus stops are determined by the results in Table. 4.1; however the exact parameters used - i.e., which of the time segments - will be discussed in the next section. As with our derivation, the rate of passengers loading is assumed to be 1 passenger per second.

4.4.2 Simulation methodology using empirical data

While the EMB model is formulated to model the dynamics of a real bus system, it is unable to model every specific situation that could happen in the real world. Here we highlight two specific situations which will not be considered: (1) The dynamical change(s) brought forth by bus injection or ejection from the system, and (2) the dynamical change(s) brought forth by a sudden change in passenger arrival rate. To circumvent these premises, the simulation methodology is based on specific scenarios where both the number of buses and the passenger arrival rate remain constant.

Qualitative analysis of collected data finds that buses in NTU-SBS operate in overlapping shifts, which result in the number of buses fluctuating frequently throughout the day as buses enter and leave the system. From the 13-weeks of bus trajectories, 256 sets of trajectories which start and end when there is either a bus starting or ending its shift,

were selected. The sets of trajectories range from 30 minutes to 84 minutes long and involve a fixed number of buses, ranging from 2 to 7. While the operating shifts of each bus varies daily, it was found that 1000h, 1400h and 2020h are typical times at which bus drivers start and end their shifts. This hence motivates the methodology in Sect. 4.3.2 of splitting each day into 4 different segments based on these three breakpoints. Incidentally, none of the 256 sets of trajectories cross any of the breakpoints. By utilizing each set of trajectories as a simulation scenario for the EMB model, the passenger arrival rate can be fixed throughout the simulation.

An example of a selected scenario is 11 Sept, 14:04h - 15:16h, there are three buses of average velocities 8.65, 9.47, 8.77 (in units per time). The average velocities V_i are used to calculate the required K_i by $K_i = \frac{V_i}{V}$ where V is the average velocity from the all empirical distributions across the entire route. The set of λ_j is selected using the ‘start time’, t_{start} of the scenario (in this example, 14:04h). This ‘start-time’ is matched against 4.1 to determine the corresponding passenger arrival rates, $\lambda_{j, \bar{t}_i}$.

4.5 Model validation

Based on the framework of Kuramoto model, the degree of synchronisation can be quantified by r^2 , which is given by:

$$r^2(t) = \frac{1}{N^2} \left[\left(\sum_i \cos \theta_i(t) \right)^2 + \left(\sum_i \sin \theta_i(t) \right)^2 \right] \quad (4.2)$$

where $\theta_i(t)$ is the angular position of oscillator i , $r^2 = 0$ describes a state where all oscillators are randomly spread out in phase and $r^2 = 1$ describes a state where all oscillators are moving together in-phase as a single platoon.

When applied on a bus system where $\theta_i(t)$ is the angular position of Bus i , the equal headway state and a completely bunched-up state can be described by $r^2 = 0$ and $r^2 = 1$ respectively. However, it is noted that r^2 does not accurately describe the state of bunching for $N > 2$.

Consider a case where there are 4 buses and the buses bunch up in pairs. If each pair occupies antipodal positions, it will also result in the $r^2 = 0$. However by our earlier definition of bus bunching, this does qualify as a case where two of the buses are bunched. Strictly, this means that r^2 is not a measure capturing the occurrence of bus bunching. Instead, it is a measure of *complete* bunching (analogous to complete phase-locking in the framework of synchronization) where all the buses are bunched as a single platoon. Even

as a measure of complete bunching, r^2 is still effective in simplifying N trajectories into a single trajectory, which facilitates the comparison between empirical and simulation trajectories in the model validation process. Yet, when interpreting r^2 , must one exercise care in its actual definition in the context of bus bunching.

Fig 4.4. Empirical trajectories of the three buses operating on 11 Sept 18, from 1404h - 1516h. The r^2 time series is derived from the trajectories and Eq. 4.2.

Each of the 256 scenarios comprises of N trajectories which can be expressed as a r^2 trajectory. Using the scenario parameters, the EMB simulation also produces a simulated set of trajectories of the N buses, which can also be quantified by $r^2(t)$. The model validation process involves comparing between the empirical and simulated r^2 trajectories. This determines the accuracy of the model by examining how well the simulation captures the evolution of complete bunching between the buses, based on their specific behaviors.

To minimize the effects of noise in empirical data, the comparison process requires an aggregation of the empirical r^2 . Averaging across all 256 scenarios results in over-aggregation. Instead, a selective aggregation method is adopted in this process. This involves averaging the empirical $r^2(t)$ based on their initial r^2 , i.e., $r^2(1)$. In this study, the r^2 trajectories are aggregated into 10 different bins, based on the initial r^2 , i.e., $r^2(1) \in [0.0, 0.1), \dots, r^2(1) \in [0.9, 1.0]$. While such an aggregation reduces statistical fluctuations, it preserves the general trend of how different states of bunching evolve over time based on their initial values.

It is noted that the common practice [10, 35, 120] of validating a predictive model involves two mutually exclusive sets of data, namely one for model parameterization (or training) and another for model testing. As the EMB model is designed as a framework to simulate intervention strategies within general scenarios, it is not subjected to the rigor of a typical predictive model designed for real-time forecasting. Indeed, the EMB model is unable to satisfactorily capture the full range of behavior exhibited by the empirical

system as the specific dynamics of a specific scenario can be dependent on the factors which were not considered in the model. Therefore in our study, we have employed the full set of 13-week data set for both the model parameterization as well as its validation to minimize the statistical variance in both the parameterization and in the validation process.

Fig 4.5. Plot of empirical and simulated trajectories. Each graph is selectively aggregated based on the initial r^2 of the trajectories from $[0.0, 0.1)$ to $[0.9, 1.0]$. The statistical similarity between the empirical and simulated trajectories is tabulated in Table 4.2

Fig. 4.5j compares between the empirical and simulated r^2 trajectories. From empirical r^2 , there is a general trend of r^2 increasing over time, especially for low initial r^2 . This demonstrates the propensity of the system to bunch over time. This trend breaks down for large initial r^2 . On average, r^2 trajectories which starts with the buses in a bunched state tend to unbunch over time. From an informal interview with one of the bus drivers, we learnt that drivers would start to drive slower when drivers see that their bus is bunching with the bus in front. Without any in-built monitoring system, they are only able to find out that they are bunching when they are sufficiently close to the bus in front. At such proximity, the r^2 of the two buses would be very close to 1. As such, this effect is only observed in very high initial r^2 s.

Table 4.2. Average euclidean distance between the aggregated simulated and empirical trajectories, tabulated with the standard deviation of both trajectories.

Range of initial r^2	Average euclidian distance	Average SD (data)	Average SD (simulation)
0.0 - 0.1	0.01	0.14	0.11
0.1 - 0.2	0.03	0.14	0.14
0.2 - 0.3	0.03	0.19	0.16
0.3 - 0.4	0.05	0.22	0.16
0.4 - 0.5	0.14	0.25	0.22
0.5 - 0.6	0.05	0.22	0.21
0.6 - 0.7	0.03	0.20	0.16
0.7 - 0.8	0.03	0.19	0.18
0.8 - 0.9	0.16	0.14	0.09
0.9 - 1.0	0.14	0.22	0.21

Tables 4.2 and 4.3 tabulate the statistical similarity of the empirical and simulated r^2 trajectories. Correspondence between empirical and simulated trajectories are typically quantified by the euclidean distance between the two trajectories within the empirical horizon [57, 121]. This is then compared against the standard deviation of both the empirical and the simulated trajectories. As seen, most of the trajectories have a significant lower distance than the standard deviations.

As one of the intended outputs of the EMB model, passenger waiting time can also be used to study the similarity between the simulated and empirical trajectories. Passenger waiting time in the simulation model is a measurable quantity. However, we do not have information regarding the passenger waiting time in the empirical cases. Hence, passenger waiting time for the empirical trajectories are simulated by injecting exactly

Table 4.3. Maximum euclidean distance between the aggregated simulated and empirical trajectories, tabulated with the standard deviation of both trajectories.

Range of initial r^2	Max euclidian distance	Max SD (data)	Max SD (simulation)
0.0 - 0.1	0.05	0.23	0.22
0.1 - 0.2	0.05	0.21	0.21
0.2 - 0.3	0.07	0.28	0.25
0.3 - 0.4	0.07	0.31	0.22
0.4 - 0.5	0.26	0.34	0.30
0.5 - 0.6	0.15	0.28	0.28
0.6 - 0.7	0.11	0.30	0.21
0.7 - 0.8	0.09	0.25	0.25
0.8 - 0.9	0.25	0.22	0.16
0.9 - 1.0	0.25	0.35	0.35

the same set of passenger as their corresponding simulation on the empirical trajectories. For each passenger i that is injected in Bus stop j at time $t_{\text{inj},i,j}$, the waiting time is defined as the minimum time taken for a bus to arrive at Bus stop j after $t_{\text{inj},i,j}$. For each of the empirical and simulated scenarios, the average waiting time of the passengers are calculated. Additionally, the deviation between the average waiting time of empirical trajectories and their corresponding simulated ones are calculated. It was found that in 226 out of the 256 scenarios, the empirical deviations are less than 1 standard deviation, and in all cases they are less than 2 standard deviations.

Chapter 5

Test-bedding intervention strategies for NTU Shuttle Bus System

As discussed in the earlier chapters, it is well studied that the most efficient state of a bus system is when all buses keep an equal headway with their leading buses [56, 98]. However, due to the mechanism of bus bunching as discussed in Chapter 3 - as well as numerous studies [52, 98] - buses do not naturally keep in this state and tend to bunch up over time. As such, the primary objective of most studies on bus bunching is to design intervention strategies for buses to maintain in the equal-headway state [44–60].

In the 70s, Ref. [44, 45, 98] formed the first theoretical framework of an intervention strategy for alleviating bus bunching: a holding strategy. This strategy has been adapted in numerous studies [46, 47, 49, 53], even very recent ones [100]. Other solutions such as stop skipping [50, 99], expressing [48] and priority signalling [47] have also been explored. Including the earliest works, the main approach taken for such studies has been macroscopic modelling, in the form of either analytical modelling or optimization models [113].

As mentioned, studying bus bunching by means of an empirical agent-based model is a novel approach in the field. Having formulated and tested the EMB model - an agent-based model for bus bunching - in the previous chapter, this chapter presents the usage of the EMB model as a test-bed for potential intervention strategies. After discussing the general methodology, three classes of intervention strategies will be proposed and test-bedded in the EMB model, with the aim of gaining insights into the effectiveness of the different intervention strategies. While the state-of-the-art strategies involve highly intricate control systems, a simplified strategy would allow for better elucidation of its effect on the natural bus dynamics. As such, the scope of the study is limited to simplified intervention strategies of each class.

Consistent with the theory of bus bunching, the majority of trajectories presented in Fig. 4.5 show buses bunching over time. Theoretically, this results in a higher passenger waiting time and thus a less efficient system [52, 56, 98]. Using the EMB simulation, we are able to propose and model different technology-based intervention strategies which can be taken by the buses in NTU-SBS to mitigate this phenomenon. While the proposed interventions all have the direct effect of minimising bus bunching, their effectiveness are evaluated based on how it affects the waiting and trip time of the passengers.

5.1 Modelling methodology

To study the intervention strategies, the simulation will be based on the empirical scenarios described in Sect. 4.5. Each intervention strategy is studied by varying its key parameter and examining the subsequent effects on the system. Simulations are initialized with the buses at the equal-headway state and all simulations are only measured at steady state, where the statistics of the system remains the same in long time limit. The measurements include the $r^2(t)$ of the buses, the average waiting time and the average trip time of the passengers. These measurements are compared against a control, which is a similar system that does not involve any interventions. To minimize the effects of statistical variance, the same set of passenger arrival rates are used across all the models regardless of strategies or parameter values, as well as for the control. This is analogous to experimenting on the same reality in parallel, which is a feat only possible through modelling.

As detailed in Sect. 3.3, the dynamical state of a bus system depends on the passenger arrival rates and the bus velocities. Notably, when the passenger arrival rates are low and velocity difference of the buses are high, the system exhibits periodic bunching. As we expect the different intervention strategies to work differently in different dynamical states, the simulation study will study four different scenarios. These test scenarios are chosen based on the passenger arrival rate λ - which we define a low λ as the lull phase and high λ as busy phase - and the velocity difference between the buses. They are described as:

Lull-same Scenario with low passenger arrival rate and buses having the same intrinsic velocity.

Lull-different Scenario with low passenger arrival rate and buses having different intrinsic velocities.

Busy-same Scenario with high passenger arrival rate and buses having the same intrinsic velocity.

Busy-different Scenario with high passenger arrival rate and buses having different intrinsic velocities.

Out of the 256 empirical scenarios available, we identified 3 scenarios which match the descriptions above:

Lull-same 01 Oct 2018, from 21:41h - 22:33h, 2 buses with velocities 6.94 and 6.93 respectively, $\bar{\lambda} = 0.011$.

Lull-different 10 Oct 2018, from 20:22h - 20:59h, 2 buses with velocities 6.66 and 8.65 respectively, $\bar{\lambda} = 0.011$.

Busy-different 17 Sept 2018, from 08:56h - 09:32h, 7 buses with velocities 5.93, 7.07, 8.00, 7.49, 7.62, 7.13, 9.08 respectively, $\bar{\lambda} = 0.040$.

Regretably, there is no empirical scenario of a busy phase with buses travelling with the same intrinsic velocity. This is because NTU-SBS always deploys a large number of buses during the busy phase and it is statistically improbable for a large number of buses to have similar velocities. Nonetheless, the EMB simulation can easily be used to model a hypothetical variant of the busy-different scenario - which will be referred to as the busy-same scenario - which uses all the parameters of busy-different scenario, but with all the buses having the same intrinsic velocity. Note that this modification does not reflect an unrealistic scenario. In the near future where traditional buses are replaced by autonomous buses - an initiative that has been on trial since 2019 [122] - a large number of buses can be easily kept at the same intrinsic velocity.

It is noted that a fair theoretical study would fix the number of buses across the four scenarios. Furthermore, our measurement r^2 is potentially misleading when comparing between the lull and busy scenarios. This is because $r^2 = 0$ in the two lull phase scenarios - that involve two buses - describes the buses in completely staggered configuration, but $r^2 = 0$ in the busy scenarios - that have seven buses - might not. For example, in a scenario with 4 buses, a staggered configuration of two units of two bunched buses also has $r^2 = 0$. In addition, the hypothetical scenarios of 2-buses serving in a busy phase or 7-buses serving during the lull phase does not reflect a realistic bus system. Hence, as the general approach of the EMB model is rooted in empiricism, the scope of our study will be based on these empirical scenarios.

Preliminary studies found that scenarios with buses of the different velocities - lull-different and busy-different exhibit more complex dynamics under the different interventions. As such, Sect. 5.2 will first present the strategy formulation followed by the simulation results on lull-same and busy-same scenarios. In Sect. 5.3, the simulation studies will be extended to the lull-different and busy-different scenarios, in which the effects of the intrinsic velocity variations of the buses on the intervention strategies will be discussed.

5.2 Strategy formulation and its effects on the same-velocities scenarios

Intervention: Stop-based Holding

Summarized by [113], a simple holding strategy involves holding buses at a specified control point for a set amount of time, based on its current time-headway. Simply put, as a trailing bus tends towards bunching with its leading bus, this strategy involves holding the trailing bus such that it extends its headway with the leading bus. This produces a negative feedback which opposes the positive feedback mechanism of bus bunching. In a naive headway holding strategy, the holding time of Bus i at time t given by:

$$t_{i,t}^{h'} = \begin{cases} h_t^* - h_t, & \text{if } h_t < h_t^* \\ 0, & \text{otherwise} \end{cases}$$

where h_t is the current time-headway of a bus, and h_t^* is a predetermined scheduled headway. For the purposes of minimizing bus bunching in our closed-loop system, the equal time-headway state is chosen as the scheduled headway in our study. This is given by $h_t^* = H_t/N$ - where H_t is the average period of a bus around the loop, and N is the number of buses. It is noted that scheduling buses for holding strategy is an ongoing research area [121, 123] and using a fixed time interval of H_t/N is a simplification.

In most holding strategies, control points are set at some - if not all - of the bus stops [52, 58, 60, 113]. In these strategies, h_t is calculated by the time difference between each bus, based on their departure time, such that buses only depart the stop when $h_t \geq h_t^*$. Mathematically, it can be expressed as $h_t = t^{\text{curr}} - \max(t^{\text{dep},j})$ where t^{curr} is the current time and $t^{\text{dep},j}$ lists all departure times from Bus Stop j before t such that $\max(t^{\text{dep},j})$ is the time of the last departure. By applying this measurement in the naive

holding strategy, buses only leave the bus stops when its headway is equal or greater than H_t/N . As this manner of measuring headway is based on when a bus leaves the bus stops, Ref. [57] refers to it as stop-based headway. In that regard, we refer to this holding strategy as stop-based holding strategy or s-holding.

Ref. [51] modified the naive headway approach by multiplying the holding time by a constant α to compensate for unstable headway dynamics found in the real world. With that, holding time of Bus i at time t would be:

$$t_{i,t}^h = \begin{cases} \alpha(h_t^* - h_t), & \text{if } h_t < h_t^* \\ 0, & \text{otherwise} \end{cases} \quad (5.1)$$

where α is a dimensionless constant. While [51] employs this parameter as part of a real-time control strategy, of which α is a dynamical variable which changes based on the current demand, we will limit our scope of study to static values of α . A holding strategy with $\alpha > 1$ over-reacts to its current headway e.g. if $\alpha = 2$, a bus which is 2 minutes behind its scheduled headway will hold for 4 minutes. Conversely, $\alpha < 1$ models an under-reaction to its headway. At the $\alpha = 0$ limit, the bus have *no reaction* to its headway and does not hold for any amount of time. In this case, the system behaves as the control, where there is no intervention in place. On the individual level, holding strategies increase the travelling time of passengers on-board the held bus. However on a system level, the negative feedback would tend the bus system close to the equal-headway state, which would give rise to a lower average travelling time.

Lull phase

Results of lull-same scenario in Fig. 5.1a shows that the s-holding strategy is able to reduce the average waiting and trip time of the passengers. In the absence of interventions ($\alpha = 0$), the r^2 shows the buses sustaining a completely bunched state, resulting in a long waiting time of around 12 minutes per passenger. As α increases, the state of complete bunching decreases with both the waiting and the trip time, with waiting time decreasing up to 50% and trip time decreasing up to 20%. The difference in savings between waiting time and trip time can be attributed to the mechanism of the holding strategy. When a bus is being held, the time passengers spend on-board the bus increases. However, this mechanism does not directly increase the waiting time of the passengers at any of the bus stops. In fact, it only decreases them, on average, as the buses are more evenly spread out.

(a) S-holding strategy in lull-same

(b) S-holding strategy in busy-same

Fig 5.1. Results of the s-holding strategy in the two same-velocity scenarios.

As the s-holding strategy increases the trip time of some of the passengers, one might expect that excessive holding - with large α - might result in a decrease in the system efficiency. Interestingly, s-holding performs well over a large range of α . By investigating the dynamics of the system at large α s, we found that excessive holding time also results in a large extension of the bus headway. As s-holding strategy only triggers on buses with small headway, buses which are held with large α s gain a large headway allowance, which leads to more infrequent holding. This trade-off between longer but more infrequent holding results in the system maintaining the average trip time over a range of α . This relationship however, fails at extreme values of α , where one bus is held for a time longer than the effective period of the other bus. As this value represents a completely impractical strategy, it is not considered in our study. While larger values of α do produce an improvement in the average trip time, traffic planners do need to consider the practicality of holding a bus for an extended period, especially when it affects the sentiments of the passengers. With that consideration, we recommend the smallest α in which buses do not bunch.

Busy phase

Fig. 5.1b plots the results of the s-holding strategy in the busy phase. From the control cases of $\alpha = 0$, buses with the same velocity exhibit complete bunching at equilibrium, similar to the lull phase. However for $\alpha > 0$, the waiting time of the passengers increases linearly with α , representing a complete failure of the s-holding strategy. Furthermore, the comparison between the uncertainty of the r^2 between $\alpha = 0$ and $\alpha > 0$ highlights a marginal increase in the degree of bunching for $\alpha > 0$. This suggests that instead of alleviating bus bunching, the s-holding strategy in the busy phase could induce buses to bunch even more. This phenomenon will be further discussed in Sect. 5.3, when we look at the effects of s-holding in the busy-different scenario.

Intervention: Continuous-time Holding

In the previous stop-based holding strategy, the time-headway was calculated by the time difference between successive departure times from a bus stop. Arguably, it is not the only way to define the time-headway of the vehicle. In fact, the concept of time-headway - or any vehicular headway in general - is not well-defined in the literature. In this regard, Ref. [57] proposed another definition of time-headway as the time taken for a vehicle to reach the current position of the leading vehicle. This was referred to as *continuous-time* time-headway.

The intricacy of using continuous-time time-headway is that it requires prediction to be made of a vehicle's trajectory. As such, it has not been used as a viable measurement for most studies on holding strategy. However, this can be done using the empirical input parameters of the EMB model. Excluding the bus stops, the average time taken for a bus to traverse a site can be given by the inverse of the average velocity at the site. This can be easily calculated from the empirical distributions. At the bus stops, the time spent would be equal to the stoppage time due to passenger boarding. As the number of passengers at a bus stop is dependent on its headway, by assuming all buses are in an equal time-headway state, i.e., $h_t = H_t/N$, the stoppage time at bus stop j can be given

by:

$$\begin{aligned}
\tau_j &= \frac{\lambda_j}{l} h_t \\
&= \frac{\lambda_j}{l} \frac{T_0 + T_0 \frac{\sum \lambda_j}{l}}{N} \\
&= \frac{\lambda_j}{l} \frac{T_0(l + \sum \lambda_j)}{lN} \\
&= \frac{\lambda_j L(l + \sum \lambda_j)}{l^2 N \bar{v}_i}
\end{aligned}$$

where $\bar{v}_i$ is the average velocity of Bus i .

Fig 5.2. Graph plotting the average time buses take to traverse each site.

With that, Fig. 5.2 plots the time taken to traverse each site for different number of buses in the system. Based on the current position of both the departing bus and its leading bus, the time-headway map is used to determine the time taken for the departing bus to reach the current position of the leading bus. As with s-holding, the resultant headway is used to compute the holding time of the bus based on Eq. 5.1. As this strategy uses the continuous-time time-headway, it will be referred to as the continuous-time holding strategy, or c-holding for short.

Fig 5.3 plots the effects of the c-holding strategy on the lull-same and busy-same scenarios. In both scenarios, we see that c-holding performs better than the s-holding strategy. Notably, the biggest improvement over the s-holding strategy is that the c-holding strategy is able to keep the buses unbunched in the busy phase. The major difference between the two holding strategies is that the headway measurement in c-

(a) C-holding strategy in lull-same

(b) C-holding strategy in busy-same

Fig 5.3. Results of the c-holding strategy in the two same-velocity scenarios.

holding factors in the specific dynamics of the bus system (passenger arrival and bus velocity). This suggests that an effective holding strategy has to account for the specific dynamics of the buses, which is absent in the s-holding strategy. This hypothesis will be further analyzed after comparing the effects of both holding strategies in the busy-different scenario in Sect. 5.3

Intervention: No-boarding

Unlike holding strategy, which effectively slows down a bus when it draws too near to the leading bus, a no-boarding strategy serves to speed up buses which have their trailing buses too near, i.e, the backward headway $h_d^\dagger$ (or the trailing bus's h_d) is small. As the act of boarding passengers requires buses to stop and dwell at the bus stops, they can be ‘sped up’ by not boarding the passengers at the bus stops. While these buses will still stop for passengers alighting, this strategy reduces the additional dwell time due to passenger boarding, thereby allowing the bus to pull away from the trailing bus. In the ensuing analysis, the concept of ‘speeding up’ the buses by the no-boarding strategy and

‘slowing down’ buses by the holding strategy will be important.

In a recent work, Ref. [124] studied the effects of no-boarding in a simplified simulation, and provided analytical estimates for the waiting time under the effects of no-boarding. In this paper, the no-boarding strategy is parameterized by a threshold distance, such that buses stop boarding passengers when their backward distance headway $h_d^\dagger$ falls below a particular threshold $h_{d,c}^\dagger$. For $h_{d,c}^\dagger = 0$, no-boarding never gets activated, allowing it to be used as a control. An upper-bound in the threshold is found such that when $h_{d,c}^\dagger$ is greater than the threshold, the waiting and trip time of the passengers increases drastically. Consider the no-boarding strategy in a system of 2 buses. If $h_{d,c}^\dagger \geq \frac{L}{2}$, boarding will be permanently disabled on at least one of the buses regardless of their configuration. In fact if the buses are in an equal-headway state, both buses will not be boarding passengers. In general, the upper-bound of the threshold is a non-trivial function of N , which can be simplified to $h_{d,c}^\dagger < \frac{L}{N}$. Therefore, our simulation study will be limited to $h_{d,c}^\dagger \in [0, \frac{L}{N}]$.

As the distance-headway does not take into account the time taken to traverse different segments of the roads (as described by Fig. 4.2a of Sect. 4.3.1), using distance-headway might introduce significant inaccuracy to the no-boarding strategy. A more accurate approach could be to use the time-headway of the buses. Similar to the distance-based approach, the time-based no-boarding strategy will disable boarding on buses that have a backward time-headway $h_t^\dagger$ smaller than $h_{t,c}^\dagger$. The upper-bound is set at $h_{t,c}^\dagger = \frac{H_t}{N}$ where H_t is the period of an average bus. With that, Figs. 5.4 and 5.5 plot the simulation results of both the distance-based and time-based no-boarding strategies respectively.

By comparing Figs. 5.4 and 5.5, no significant difference was found between the distance-based and time-based no-boarding strategy. This refutes our hypothesis that there will be significant inaccuracy brought forth by using distance-headway. In our simulation methodology, the parameter $h_{d,c}^\dagger$ was studied by varying in step sizes of $0.05 \frac{L}{N}$. Based on the relationship between distance and time-headway from Fig. 5.2, the inaccuracy generated by the road heterogeneity accounts for $0.015 \frac{L}{N}$, which is less than the simulation step size. This represents insignificant inaccuracy as compared to the parameter step-size, which leads to the resulting similarity between the two strategies. Based on this, further study of the no-boarding strategy will focus only on the distance-based no-boarding.

(a) Distance-based no-boarding strategy in lull-same

(b) Distance-based no-boarding strategy in busy-same

Fig 5.4. Results of the distance-based no-boarding strategy on the two same-velocity scenarios

Lull phase

Fig. 5.4a plots the effects of the no-boarding strategy in the lull-same scenario. Fundamentally, no-boarding is able to keep the buses in a staggered configuration (described by $r^2 \simeq 0$ in Fig. 5.4a) using a large parameter value of $h_{d,c}^\dagger = 1$. However based on the large waiting time, a high amount of no-boarding is required to keep the buses in this configuration. The minimum waiting time occurs with a threshold headway of $h_{d,c}^\dagger = 0.82$, with a 37% savings in average waiting time and a 20% savings in average trip time.

Busy phase

The busy-same phase in Fig 5.4b shows similar dynamics to that of in the lull phase, such that the no-boarding strategy is able to improve the waiting and trip time of the passengers. One interesting feature of the results is that the upper-bound of the threshold seems to be larger than $\frac{L}{N}$. This is an artifact of our simplification in determining the upper-bound of the threshold, described in the earlier part of the section.

Fig 5.5. Results of the time-based no-boarding strategy on the two same-velocity scenarios

Intervention: Centralized-pulsing

As discussed in Ref. [112], a significant takeaway from theorizing bus bunching as a synchronization phenomenon is the potential to ‘entrain’ the buses by a periodic force. In the theory of synchronization, self-oscillators can be synchronized by a periodic pulse. Similarly, we propose a novel approach of keeping the buses staggered by a system of periodic driving forces. While the two previous strategies deal with the buses’ interaction with passengers, this intervention involves the intrinsic behaviors of the buses. This strategy involves actuating each bus periodically (every Δt_p time steps) to either speed up or to slow down, based on their current headway. Specifically, each bus with time-headway greater than the equal time-headway state of $h_t^* = H_t/N$ will be forced to slow down, and buses with $h_t < H_t/N$ will be sped up. As with the previous two strategies, this effect acts as a negative feedback for the buses to maintain an equal time-headway state.

Typically, our Monte-Carlo method of the EMB model involves sampling velocities from the empirical velocity distribution of the positions the buses are on. Speeding up

of the buses is modelled by restricting the sampling to velocities from the upper median of the distribution. Conversely, slowing a bus down involves sampling only velocities below the median of the distribution. By restricting the sampling to the upper and lower medians, it ensures that the buses move with a velocity above or below the average velocity, but conserves the largest possible set of empirical velocities to retain a certain degree of stochasticity in the bus dynamics.

By varying the parameter Δt_p at which this mechanism is applied, we are able to vary the effect of the intervention strategy. To be consistent with the other strategies, the zero-parameter case represents a control, in which the buses are never nudged to move faster or slower. The maximum $\Delta t_p = 100$ represents actuating the buses to move above/below its average speed only 1% of the time. Conversely, $\Delta t_p = 1$ models buses being sped up/slowed down *all the time*. The practicality of this will be discussed below.

Fig 5.6. Results of the centralized-pulsing strategy on the two same-velocity scenarios.

The results presented in Fig. 5.6 agrees with our intuition that with a higher actuating frequency, the buses stay unbunched. Correspondingly, the interventions with large Δt_p have no effect on either the r^2 or the waiting/trip times. As there are similar dynamics

between the same-velocity and the different-velocity scenarios, the detailed analysis of the centralized-pulsing strategy will be presented with the results in the next section.

5.3 Effects of intervention strategies on different-velocities scenarios

Before analyzing the effects of the intervention strategies, we will first discuss the dynamics of the buses in the lull-different and busy-different scenarios, in the absence of any interventions.

From Sect. 3.3, we discussed that in the lull-different scenario, the buses exhibit periodic bunching. This is the case where the fast bus continuously overtakes the slower bus and, as such, the buses are unable to maintain sustained bunching. A preliminary analysis of the periodic-bunching state found that a bus system exhibiting periodic bunching is a relatively efficient system. This can be seen in the zero-parameter of the simulation results of this section (e.g. in Fig. 5.7a) where the average waiting time of passengers is 7.4 minutes compared to the 12 minutes when the bus velocities are the same (in Fig. 5.1a).

Unlike the lull-different scenario, the high passenger arrival rate in this busy-different scenario does result in some degree of sustained bunching. However due to the velocity differences, the dynamics are neither completely sustained bunching nor periodic bunching. The system takes on a complex dynamical state of buses bunching locally in smaller platoons, with the size and number of platoons fluctuating constantly. An important feature is that these ‘bus platoons’ are scattered randomly along the bus route. The lower value of r^2 ($\alpha = 0$ of Fig. 5.7b) is a result of platoons of bunched buses spanning the bus route and does not imply a complete absence of bus bunching. This is an example of when $r^2 = 0$ cannot be interpreted as a equal-headway state when $N > 2$. Nonetheless, this complex dynamical state serves as a more efficient state than the completely bunched state, with waiting time averaging at 4.9 minutes instead of 10.

Intervention: S-holding

Lull phase

By invoking the holding strategy on a bus system in the periodic-bunching state, the strategy backfires with the control performing better than any $\alpha > 0$. At steady state,

Fig 5.7. Results of simulating the s-holding strategy in the two different-velocity scenarios.

only the faster moving bus will have a small headway as it closes towards a slower bus. As such, only the faster buses will be held by the holding strategy. At equilibrium, the holding strategy effectively slows the faster bus by a fixed amount every round, with the amount depending on α . With small α , the holding strategy slows down the faster buses by a little each round, but not enough to prevent periodic bunching. As there is no effect on the state of bunching, the sole effect of the holding strategy is in increasing the passengers' waiting and trip time. The least efficient system exists when α is large enough such that the effective velocity of the faster buses match that of the slower buses. With the buses having the same effective velocity, the bunching mechanism takes over and causes the buses to approach a bunched state ($r^2 \approx 0.8$). This results in the point where waiting and trip time is at the maximum. The average of 11 minutes is comparable to the completely bunched state found in $\alpha = 0$ of Fig. 5.1a.

By studying both the r^2 and the steady state dynamics, the holding strategy is able to completely nullify the mechanism of periodic bunching with large α , resulting in the buses maintaining a staggered configuration. The waiting time in this state is comparable to that

of the control, suggesting that the periodic-bunching state and the staggered configuration are both equally efficient in keeping the waiting time of the passengers low. However, the trip time at large α is marginally worse than the control. This can be attributed to the increase in trip time of the passengers on-board the held buses.

Busy phase

As with the results shown in Fig. 5.1b, the results of the busy-different scenario also show a linear increase in waiting time, which represents the complete failure of the system. With the no-intervention dynamics of the busy-different scenario averaging an $r^2 = 0.37$, it is apparent that the s-holding strategy with the increased passenger arrival results in a greater degree of bus bunching. While the mechanism of the onset *holding-induced* bunching is unclear, the linear relationship between α and waiting time can be explained by the steady state dynamics of the bunched buses.

The bunching dynamics starts with all 7 buses completely bunched at a single bus stop (also described by the $r^2 \approx 1$). When there are no more boarding passengers, the leading bus - which is not held - leaves the bus stop while the other buses are held by the holding strategy. The leading bus travels towards and stops at the second bus stop to pick up passengers. As all the 6 remaining buses are all bunched into a single platoon which trails the leading bus, the time between this leading bus's arrival and the previous bus's departure is very large. This results in a large number of boarding passengers at this second bus stop. As such, the leading bus takes a long time to board the passengers. In the range of α s, the holding time of the other buses is lower than the time taken for the leading bus to board all the passengers. Therefore, the other 6 buses would have finished holding at the previous bus stop and would then arrive at this second bus stop before the leading bus finished boarding all passengers. With 7 buses boarding concurrently, all the passengers would have boarded a bus and the leading bus would then depart for the next stop. This dynamics of one bus moving to the next stop and the remaining buses following after repeats at every single bus stop. As the speed of the platoon is dependent on the holding time, which is dependent on α , the waiting and trip time of the passengers increases linearly with α . The same set of dynamics occurs in both busy-same and busy-different.

While the rules of any holding strategy would provide a negative feedback to the bus bunching behavior, the s-holding strategy in the busy phase results in a peculiar *holding-induced* bunching. Between the governing equation of the s-holding strategy and the simplistic scheduled headway, we speculate that the holding-induced' bunching can

be attributed to its over-simplistic nature which does not take into account important attributes of the bus system. An in-depth analysis into the mechanism of this phenomenon could highlight a previously-unknown limitation to a stop-based holding strategy. Nonetheless in the next subsection, we would see how - by taking passenger arrival rate and bus velocity into consideration - a more dynamical holding strategy could achieve better results. Through that, we will also elaborate on how the attributes of the bus system - passenger arrival rate and velocities - would interact with a holding strategy such that a failure to account for them could result in the holding-induced bunching.

Intervention: C-holding

Fig 5.8. Results of the c-holding strategy in the two different-velocity scenarios.

In the lull-different case, the dynamics at low values of $\alpha < 1$ is the same for the s-holding, in which the c-holding strategy does not keep the buses in the staggered configuration. Unlike the s-holding strategy, Fig. 5.8a shows a marginal improvement in the efficiency at the range of $\alpha > 2$, with a 14% reduction in waiting time.

As discussed earlier, the no-intervention dynamics exhibited by buses in the busy-different scenario results in an improvement over the busy-same scenario as buses are kept far from complete bunching. With the c-holding strategy, the r^2 in Fig. 5.8b suggest that the buses are kept at a lesser degree of complete bunching. However, this slight im-

provement in the state of bunching does not result in an improvement in the system, with average trip time increasing slightly as α increases. While this implies an improvement over the s-holding strategy, the drop in efficiency suggests that the c-holding strategy does not quite achieve its function when the buses have different intrinsic velocities.

Improvement over the s-holding strategy

The improvement of the c-holding strategy over s-holding can be attributed to the oversimplistic approach of the former. As the stop-based time-headway is measured as the difference of two successive buses' departure time, it only ensures that the buses leave each bus stop at or after the scheduled headway. However, it does not consider how headway changes after the holding time. There are two mechanisms which can cause the headway of the buses to change: one involving the bus velocity and the other involving passenger arrival.

As discussed, when there is a difference in bus velocities, the holding strategy only affects the faster bus. Being the faster bus, it will always catch up with its leading bus after holding. The rate at which it catches up depends on the velocity difference between the two buses. To keep the buses exactly at the equal-headway state, a large velocity difference requires a longer holding time than a small velocity difference. By considering the velocities of the buses, the c-holding strategy is able to keep the buses very close to the equal-headway state, suggested by the r^2 at large α in Fig. 5.8a. In contrast, the simplistic s-holding strategy in Fig. 5.8a cannot.

At the same time, holding strategy possess an inherent self-accentuating effect which is dependent on the passenger arrival rate. Holding a bus extends its distance from its leading bus, effectively increasing its time-headway. An increased time-headway results in additional passenger arrival at the next stop, which in turn leads to a longer stoppage time. As with the velocity considerations, to keep the buses exactly at the equal-headway state requires a shorter holding time when the passenger arrival rate is high and a longer holding time when the arrival rate is low.

As the continuous-time measurement of the c-holding strategy takes into account the velocities of the buses and the different arrival rates at different bus stops, it is able to consistently give an optimal holding time for the buses to keep in the equal-headway state. Furthermore, as the approximation taken by Eq. A.1 assumes that the buses are in a equal-headway state, the deviation between the assumption and reality decreases as the buses maintain in that headway state. Conversely, as s-holding does not account for the effects of bus velocity and arrival rates, it is ineffective in maintaining the buses at an

equal-headway state effectively, which ultimately results in the buses bunching together.

Intervention: No-boarding

Fig 5.9. Results of the no-boarding strategy in the two different-velocity scenarios.

Lull phase

As with the holding strategies, the no-boarding strategy backfires in the lull-different scenario. However, there is a fundamental difference between the holding and no-boarding strategies. In the holding strategy, the headway of the buses are modulated by ‘slowing-down’ the buses through holding. The parameter α determines how long each bus is held, which effectively scales the amount the buses is slowed down by the strategy. In contrast, the parameter $h_{d,c}^\dagger$ only affects the point at which no-boarding is invoked, but does not affect how much a bus is ‘sped-up’. In the no-boarding strategy, the amount buses are sped-up is dependent on the passenger arrival rate, i.e, the higher number of passengers *not picked up*, the faster a bus will be. Therefore, across the parameter range, the no-boarding strategy behaves like a holding strategy of low α such that the strategy increases the period of the periodic bunching, but does not modulate the buses enough to achieve a staggered configuration. This is seen from the equilibrium dynamics of the slower bus being periodically overtaken by the faster bus, with no-boarding triggering

when the latter is within its threshold headway. As the only change in the dynamics is the frequency of no-boarding taken by the slower bus, the waiting and trip time of the passengers increase linearly with $h_{d,i}^\dagger$.

Busy phase

The results of no-boarding in the busy-different scenario in Fig. 5.9b show marginal improvement in the waiting time of the passengers. This is consistent with the hypothesis presented earlier that a higher number of passenger arrival would result in greater degree of speed modulation required to keep the buses unbunched. While our simulation study only captures a marginal improvement in waiting time, a similar work by [124] shows significant savings in waiting time when the passenger arrival rates are even higher (average $k = 0.063$). This result is consistent with our analysis and supports that no-boarding is a viable strategy for a very busy bus system, even with buses travelling at different velocities.

Intervention: Centralized-pulsing

Fig 5.10. Results of the centralized-pulsing strategy in the two different-velocity scenarios.

Comparing Figs. 5.6 and 5.10, it can be seen that the effects of centralized-pulsing across all four scenarios are very similar. A potentially important finding - and an in-

triguing path for further investigation - is the existence of a sharp transition between a completely unbunched and the completely bunched/periodically bunched state, which takes place consistently at $\Delta t_p \approx 30$.

A further analysis was done by studying the system for $\Delta t_p = 30^+$. As discussed, our methodology involves initializing the system in the equal-headway state and measuring the system only in steady state. By investigating the transient state in the lull-same scenario, it is found that centralized-pulsing with $\Delta t_p = 30^+$ maintains the initial equal-headway state for a long time before a sharp transition to the bunched state. This bunching persists in long time limit. This set of dynamics resembles physical systems which exhibit 1st order phase transition, where the system exhibits metastability within a particular range of parameter values. At this range, the system could exist in the metastable phase for a long time. However, any perturbations would result in the system undergoing a phase transition into the stable phase. In this case, the equal-headway state is analogous to the metastable state and the bunched state is the stable state.

While centralized-pulsing appears to be a very promising intervention strategy, the practicality of implementation has to be considered carefully. As Δt_p is analogous to the ‘inter-actuation time’, $\Delta t_p = 1$ models buses consistently moving strictly above or below its average speed based on its current headway. For a bus with slow intrinsic velocity, it has to move above its average velocity *at all times*. Based on the road or traffic conditions, this may not always be possible. In fact, the issue with actuating frequency relates back to the theory of the entrainment of self-oscillators. Discussed in [125], the onset of synchronisation by a periodic driving force is dependent on both the period and the magnitude of the driving force. In this simulation study, the frequency is set by Δt_p and the ‘magnitude of the force’ is modelled by the velocity selection rule. However, these values might not correspond to the real world. In fact, a reasonable nudging frequency and ‘magnitude of the force’ would not only be driver-dependent, but also spatially dependent, i.e., there will be segments of the road that the driver can choose to drive a little faster or slower. To gather sufficient information to refine our implementation into a realistic and practical centralized-pulsing strategy, an experimental study measuring the bus velocities when prompting drivers to speed up and slow down should be carried out

5.4 Comparison of the various intervention strategies

Having test-bedded four different intervention strategies, Table 5.1 tabulates the effectiveness of each strategy by the fractional savings in the waiting time and trip time over

Table 5.1. Fractional savings in waiting time and trip times (respectively in two columns) taken in the four test scenarios, under the four different strategies. The waiting and trip time of each strategy is represented by its best performing parameter and a null entry represents the control case outperforming the strategy at all parameter values. The fractional savings is given by the fractional difference between the average waiting/trip time of the strategy and the control case.

Strategy	Lull-same (%)		Busy-same (%)		Lull-diff (%)		Busy-diff (%)	
S-hold	45.8	23.2	-	-	-	-	-	-
C-hold	48.3	24.0	82.5	34.5	14.9	0.0	-	-
No-board	36.7	18.8	67.0	30.5	-	-	4.1	0.5
Pulsing	53.4	59.6	86.4	46.8	28.4	10.3	63.3	26.3

a control. Similar works in literature have found savings in waiting times ranging from 5% to 80% [46–49, 55, 59], which are in agreement with our findings. Among the three classes of strategies, a recurring finding is that centralized-pulsing stands out as the best-performing strategy. With this recurring over all scenarios, the comparison between the holding and no-boarding strategies will first be presented.

In the lull phase, both strategies generally only work when the buses have similar intrinsic velocities. While both holding strategies outperform the no-boarding strategy in the lull-same scenario, this marginal out-performance (compared against the uncertainty of the measurement) does not represent any significant benefit in choosing holding strategies over no-boarding for this scenario. In the lull-different scenario, none of the strategies (except centralized-pulsing) provide significant improvement in the system, with c-holding strategy performing marginally better than the control case. By comparing the control case of the lull-same and lull-different scenarios, having buses with different velocities produce an average waiting and trip time comparable to the no-boarding strategy. This implies that, unlike sustained bunching, periodic bunching in bus systems result in it being relatively efficient. Conveniently, forcing buses to travel with different velocities during lull phase could be another intervention strategy which can be undertaken by the bus operators. Alternatively, as smaller buses have a tendency to move faster than larger buses, bus operators could also employ buses of different sizes to increase the tendency of periodic bunching.

In the busy phase, the holding strategies backfire in most cases. While c-holding could reduce bunching in the busy-same scenario, it requires a relatively long holding time (implied by the large α), which would affect its practicality. In contrast, the effectiveness

of the no-boarding strategy increases with the number of passengers; in the busy-same scenario, waiting time is reduced by 67%. While we find the no-boarding strategy to be less effective where there is a large variation in buses' velocity, it was discussed that similar works find an improvement in efficiency in a busy-different scenario where the passenger arrival rate is much higher. This shows that the no-boarding strategy is potentially an effective strategy in the busy phases.

By considering the different scenarios of a bus system, it is shown that neither the holding nor no-boarding strategy are comprehensive solutions, as their effectiveness in reducing passenger waiting and trip times are scenario-specific. In contrast, our novel solution of a bus system controlled by a centralized pulse seems to be able to reduce bus bunching successfully across the different dynamical states of the bus system. With that, we propose a future study to ascertain its practicality as a bus bunching solution. This includes an experimental study to determine an effective actuating frequency for buses, as well as the amount of speed variation which can be taken by real buses under the actuation. With that, the simulation study should then be refined by using parameters obtained from this experimental study, before an actual prototyping of the required technologies to implement the strategy is carried out.

Concluding remarks on interventions

In this chapter, we have proposed and studied three classes of intervention strategies. It was found that traditional intervention strategies - holding and no-boarding - only perform well under specific scenarios. Nonetheless, the no-boarding strategy seems to be viable when the passenger arrival rate is high. All in all, the centralized-pulsing strategy seems to be the most promising strategy as its effectiveness is scenario-agnostic. Further studies are proposed to ascertain its practicality as an implementable strategy.

In this study, we have employed simplified strategies on fixed scenarios to study its effectiveness to alleviate bus bunching. Simulation results found the existence of a different range of parameter values which allows a strategy to work optimally for a different scenarios. However, fixed scenarios is an over-simplification of a real bus system, which typically finds passenger arrival rates and bus velocities fluctuating non-stationarily. In that regard, our simplified strategy would be insufficient in resolving the issue of bus bunching faced in the real world. To account for this, most state-of-the-art intervention strategies [51, 54, 57–60, 113] involve a real-time control system which adjusts the strategies' effect dynamically. In similar fashion, the current study can be generalized using adaptive strategies onto dynamical scenarios, where passenger arrival rates and bus veloc-

ities varies. These adaptive strategies can be explored using reinforcement learning and can include a mixture of holding, no-boarding and travelling speed variation. This could lead to a potentially implementable solution which can effectively alleviate the problem of bus bunching faced by real-world bus systems

Chapter 6

Conclusion

As we move into the third decade of the 21st century, the problems faced by our rapidly urbanizing world are becoming increasingly complex. Combating these problems require clear understanding of different interactions governing the systems. In that regard, microscopic modelling - which is formulated based on the behavior of the individual elements of the system - is an effective approach in capturing the complexity of the modelled system. This allows the elucidating, and even quantification of the key interactions in a system, which facilitates the model's use to design scalable and adaptable solutions to the urban problems.

The study presented in Chapter 2 is based on a widely-used solution strategy of lane-expansion to reduce congestion at traffic bottlenecks. By modifying the Nagel-Schreckenberg (NaSch) Cellular Automata model with a parallel expansion of a time-delay bottleneck, it was found that with a short time-delay, the increased inter-vehicle interaction during lane-merger outweighs the increase in flow rate due to the lane expansion. This describes the congestion which occurs with excessive lane-expansion. The NaSch model was also used to analyze the flow rate from a serial expansion of bottlenecks which would take the form of a multi-point tollbooth. Through that, we propose a novel combination of serial and parallel expansion at traffic bottlenecks which can be used as an effective solution for processing a large number of vehicles.

The study presented in Chapters 3, 4 and 5 deal with the longstanding problem of bus bunching. While the physical mechanism of bus bunching is a well-known theoretical concept, its dynamics have not been described quantitatively. Facilitated by a discrete model, an analytical derivation of a two-bus, two-bus-stop system is formulated in Chapter 3. The quantitative dynamics of bus bunching shed light on a novel correspondence with the synchronization phenomenon described by the Kuramoto model. As with the

Kuramoto model, the quantitative dynamics of buses demonstrates a positive feedback mechanism which shrinks small headways and extends long headways between buses. With that, it illustrates the two important factors of the bus bunching phenomenon: passenger arrival rate and bus velocities.

In Chapter 4, we formulate the Empirically-based, Monte-carlo Bus-network (EMB) model, which is an empirical agent-based model designed for testbedding potential intervention strategies for reducing bus bunching. The novel approach of parameterization by empirical distributions allows the EMB model to not only capture the essential interactions between the agents, but also the microscopic behavior of the specific bus system. This allows the EMB model to simulate the dynamics of any empirical system given proper parameterization. The potential of the model is demonstrated by a case study of the NTU Shuttle Bus System (NTU-SBS). A series of data analytics was performed on empirical bus trajectories to obtain the necessary parameters. Bus trajectories were then simulated using the EMB model and it was found that the trajectories simulated by EMB model captures similar bunching dynamics to the NTU-SBS.

In Chapter 5, the fully parameterized EMB model is used to study three classes of intervention strategies: holding strategy, no-boarding strategy and a novel centralized-pulsing strategy. It was found that the holding and no-boarding strategies are only effective in specific scenarios, primarily when the velocity of the buses are similar. In contrast, the simulation study found that the centralized-pulsing strategy seems to be effective across a wide range of scenarios, suggesting it as a comprehensive solution for the bus bunching problem.

As our theoretical studies have found potential solution strategies which would improve their respective systems, future works should strive to bring the solution from a simulation environment to reality. To illustrate with a concrete example, in order to encourage a healthy supply of taxis at Singapore’s Changi Airport, drivers earns a surcharge for pick-ups at the airport. Due to this, there is a perpetual queue *of* taxis heading towards the airport. Interestingly, the queue of passengers waiting to board a taxi in the airport is also relatively substantial during busy times of the day. This highlights a weakness of the system at the passengers-to-taxi process. As passenger boarding falls under the category of *long* processing time, many lanes of a few serial sites would give the optimal efficiency of the collective dynamics. Physical experimentation on this specific system is also highly practical. Unlike in toll-collection, there is no need for additional protocols to ensure that every vehicle leaving the site have been processed. Furthermore, the combination expansion can also be implemented using temporary fixtures such as cones and barricades

which would keep the cost of experimentation to a minimum.

While applications of the EMB model have been presented exclusively on the NTU-SBS, the formulation is generic and can be applied to any other bus system. There are two benefits of applying the modelling framework to other empirical systems. As we have shown, modelling an empirical system using the EMB model allows the users to study and gain insights about the system, facilitating the development of intervention strategies. Additionally - and in fact crucially - applying the modelling framework to other empirical bus systems can facilitate the continual development of the EMB model. As an example, the interaction between buses and the other cars on the road have not been considered in the current formulation, due to the apparent infrequency of vehicular traffic in NTU's campus. However, if the EMB model is used to model a busy bus route - e.g. one which runs through the city center - the interactions between buses and vehicles become highly significant in determining the dynamics of the bus system. With added features, the EMB model might yield new insights into the modelling framework and even the phenomenon of bus bunching in general.

While centralized-pulsing was found to be effective across the different scenarios, future work is required to validate the results using experimental data. Here, we propose a future experimental direction. By determining the highest possible frequency of actuating the buses and also the amount of velocity changes when the buses are actuated, additional simulation studies should be performed to ensure the efficacy of the strategy in real-world scenarios. Following which, the technology required for a small-scale implementation in the NTU-SBS is easily available: an algorithm which uses the real-time positions of the buses to determine if a bus should speed up or slow down at the actuation time. With this algorithm built in a mobile app, the cost of even a full-scale implementation would be minimal.

"We spend billions of dollars trying to understand the origins of the universe, while we still don't understand the conditions for a stable society, a functioning economy, or peace." - Dirk Helbing, Professor of Sociology at ETH Zürich. Unlike traditional physical systems, solving urban issues involves a keen understanding of the relationship between the microscopic behavior and the macroscopic phenomenon. By the approach of microscopic modeling - from cellular automata to agent-based modeling - we have taken a small step in addressing two issues in urban systems: congestion when processing high volume of vehicular traffic and the tendency for buses to bunch.

Appendix A

The time-headway approach

We adopt the time-headway approach to determine the maximum flow rate through a bottleneck. The underlying assumption of this approach is that of a two-phase traffic, where at the critical point between the free flow and the state where the flow maximizes, vehicles flow in an organized way on the verge of interacting. At this point, the spatial-temporal profile of the vehicles assumes an average pattern in the steady state, allowing us to extract a mean period $\bar{h}_{tc}$ from the pattern. The maximum flow rate J_{max} is then determined as $J_{max} = 1/\bar{h}_{tc}$.

A.1 Serial bottleneck

For serial expansion, the m vehicles are observed to move in a periodic configuration under critical density due to the deterministic nature of the flow (see Fig. A.1). In this configuration, it is easy to see that the total time-headway is given by $t_d + m + \Delta$. This implies an average time-headway of

$$\bar{h}_{tc}^s = \frac{m + \Delta + t_d}{m}, \quad (\text{A.1})$$

between each vehicle. The maximum flow rate J_{max} for the serial expansion is then given by $J_{max} = 1/\bar{h}_{tc}^s$, which is Eq. (2.1).

From Fig. A.1, Δ is observed to be the duration requires for the vehicle delayed at the left-most site to leave the m sites. Since a delayed vehicle starts from rest and accelerates at $1 \text{ site}/s^2$, the distance covered by this vehicle would increase systematically at each time step. In fact, the successive distance covered follows an arithmetic progression which indicates a total distance traversed of $d = (t/2)(t + 1)$. By letting $d = m$ and $t = \Delta$,

Fig A.1. A plot of the space-time trajectories of vehicles moving through a serial bottleneck at critical density. The parameters of the simulation are: $m = 7$, $t_d = 3$, and $v_m = 4$. Note that the different colors are used to differentiate each vehicle, and the repeated colors do not represent the same vehicle. The dotted line indicates the trajectory of one of these vehicles.

we solve for $\Delta = \left\lceil (\sqrt{1 + 8m} - 1) / 2 \right\rceil$, where the ceiling function $\lceil \dots \rceil$ is a consequence of the discretized time in the NaSch model and that this vehicle must travel at least m sites. Because vehicles in the NaSch Model accelerates to a finite velocity, this equation is applicable only for vehicle velocity $v \leq v_m$. In terms of m , this condition translates to $m \leq v_m(v_m + 1)/2$.

When $m > v_m(v_m + 1)/2$, the vehicle reaches v_m and maintains at this velocity in its subsequent motion. As a result, once past the first $v_m(v_m + 1)/2$ sites (which takes v_m time steps to traverse), the vehicle covers the remaining sites in $\left\lceil [2m - v_m(v_m + 1)] / [2v_m] \right\rceil$ time steps. Combining these results on Δ , Eq. (2.2) is obtained.

A.2 Parallel bottleneck

As mention in section 2.2.2, there are two regimes of maximum flow rate under parallel expansion, of which their time-headway have to be determined separately. As with most

Fig A.2. A plot of the space-time trajectories of vehicles moving through a parallel bottleneck. Note that the first 5 and the last 5 cells of each row in each subfigures are common between the two subfigures, as they illustrate the single-laned road. The middle 5 cells show the 2 expanded lanes with individual processing bottlenecks. The parameters of the simulation are: $n = 2$, $t_d = 3$, and $v_m = 4$. We use different colors to differentiate between vehicles, while the dotted line indicates the trajectory of one of these vehicles. The dashed line serves to highlight the difference between the two lanes at a single point in time.

fork-join queue models, the stochastic behavior in the parallel expansion hinders the formulation of an exact analytical solution of the maximum flow rate. Nonetheless, by considering the aggregate behavior of a large number of vehicles, we can ascertain its average behavior from which we obtain the mean time-headway for the vehicular flow.

The multiplicative effects of parallel expansion have already been explored in [94], where the overall flow rate was estimated to be the combined flow rate of each of the lanes. The analytical results of [84] state that the maximum flow rate at the processing site is $1/(2 + t_d)$, which is equivalent to a time-headway of $(2 + t_d)$. The total flow rate of the n parallel lanes is then given approximately as $n/(2 + t_d)$. However, this evaluation assumes no interaction between the vehicles at the point of merging which is a gross over-estimation.

In the following, we improve this approximation by including the effect of interaction at the merged flow in a statistical way. The consequence is a flow rate that is less than the

sum of the flow rate of each lane. When vehicles at different lanes approach the merging point, only one vehicle passes through unobstructed per unit time, while the rest of the $n - 1$ vehicles have to wait one extra time step relative to the unobstructed vehicle to get through. This gives a probability of $1/n$ for a time-headway of $(2 + t_d)$ for outflow without obstruction; and a probability of $1 - 1/n$ for time-headway of $(3 + t_d)$ for outflow with vehicular interaction. Thus, the average total time-headway is given by $3 + t_d - 1/n$, which leads to an average time-headway for parallel bottleneck due to multiplicative effect to be

$$\bar{h}_{tc}^{pm} = \frac{3 + t_d - \frac{1}{n}}{n}. \quad (\text{A.2})$$

In the regime where the traffic flow is limited by lane merging, the effect of the processing bottleneck is negligible. This is analogous to ignoring the presence of processing site in each lane. According to Ref. [62], vehicles moving with no bottleneck has a maximum flow rate of $v_m / (v_m + 1)$. Thus, we expect the outflow without obstruction to have a time-headway of $(v_m + 1) / v_m$ for one vehicle. For the rest of the $n - 1$ vehicles, vehicular interactions cause them to stop at the merging point. They later move off with a velocity of $v = 1$ due to unit acceleration of the NaSch model, which gives each a time-headway of 2 after considering an extra time step as before. Putting these n time-headway together lead to an average total time-headway of $(v_m + 1) / v_m + 2(n - 1)$. The average time-headway in consequence of flow limitation by lane-merging is therefore

$$\bar{h}_{tc}^{pf} = \frac{1 + 2nv_m - v_m}{nv_m}. \quad (\text{A.3})$$

Finally, the maximum flow rate through the parallel bottleneck depends on the dominance of the processing bottleneck or the lane-merging bottleneck. The dominance of the former leads to $J_{max} = 1 / \bar{h}_{tc}^{pm}$, while the latter $J_{max} = 1 / \bar{h}_{tc}^{pf}$. These results together then gives Eq. (2.3) of the parallel bottleneck. Note that a space-time trajectories of vehicles moving through a parallel bottleneck is illustrated in Fig. A.2.

A.3 Combination bottleneck

The derivation of the maximum flow rate of the combination bottleneck employs both the arguments of the serial bottleneck and the parallel bottleneck. Similar to the parallel bottleneck, the combination bottleneck possesses two regimes: one dominated by the

processing bottleneck, while the other by lane-merging bottleneck.

For the former case, we again consider the outflow without obstruction. This gives a time-headway of $\bar{h}_{tc}^s$ resulting from the serial bottleneck, and happens with a probability of $1/n$. On the other hand, we have a time-headway of $\bar{h}_{tc}^s + 1$ which occurs with a probability of $1 - 1/n$ for the rest of the outflow. Note an extra time unit has been added to account for the effect of interactions. The average time-headway of the combination bottleneck for multiplicative effect is then

$$\bar{h}_{tc}^{cm} = \frac{(2m + \Delta + t_d)n - m}{n^2m}. \quad (\text{A.4})$$

In the case of the lane-merging bottleneck, we again ignore the processing sites. In consequence, the result here is exactly the same as that for the parallel bottleneck above, i.e. $\bar{h}_{tc}^{cf} = \bar{h}_{tc}^{pf}$, when lane-merging dominates.

The maximum flow rate through the combination bottleneck is then given by $J_{max} = 1/\bar{h}_{tc}^{cm}$ when processing bottleneck dominates, and $J_{max} = 1/\bar{h}_{tc}^{cf}$ when lane-merging bottleneck dominates. This proves Eq. (2.5).

Appendix B

Data filtering

Date/time	Vehicle registration number	Latitude	Longitude
Sep 14 07:54:06 2018	PC4964L	1.353176	103.681695
Sep 14 07:54:06 2018	PC3087A	1.336900	103.677067
Sep 14 07:54:09 2018	PC4964L	1.353176	103.681695
Sep 14 07:54:09 2018	PC3087A	1.335191	103.676586
...	...	...	...

Table B.1. Raw data collected from <https://baseride.com/maps/public/ntu/>. The latitude and longitude makes up $\tilde{x}_{i,t}$.

Table. B.1 illustrates the tabulated raw data collected from <https://baseride.com/maps/public/ntu/>. This includes a date/time stamp, vehicle registration number, and the longitude/ latitude of each bus as it moves along a designated route. While the data only updates every 9-12s, the script was set to record the data in 3s intervals. This redundancy results in about 60% of repeated data. This will be dealt with in later parts of the report. The 2-D lat-lon of bus i at time t is defined as $\tilde{x}_{i,t}$. As the buses only move along a fixed periodic route, the bus positions will be mapped onto a discretized 1-D array, where $x_{i,t}$ is an integer which describes the position of the bus along the route. The following paragraphs will describe the methodology to transform $\tilde{x}_{i,t} \rightarrow x_{i,t}$, and the subsequent filtering applied to the data.

The lower limit of spatial resolution was first determined by calculating minimum $\delta\tilde{x}_{i,t+1}$ - where $\delta\tilde{x}_{i,t+1} = \tilde{x}_{i,t+1} - \tilde{x}_{i,t}$, for $\tilde{x}_{i,t+1} \neq \tilde{x}_{i,t}$ - from all the individual trajectories throughout the sampling period. It was found that the minimum $\tilde{x}_{i,t+1}$ is equivalent to approximately 7.9m. While this value could be used as the lower limit of our spatial resolution, it is to be noted that it is in close approximation to the preset spatial resolution

used in the NaSch model of 7.5m, which reflects the length of an average vehicle, and the interaction distance between vehicles. Setting a lower spatial resolution as 7.5m, instead of 7.9m allows for future modularity of the simulation, specifically with regards to the interactions with other vehicles. As such, the lower limit of the spatial resolution would be set as 7.5m.

Fig B.1. Scatter plot of all $\tilde{x}_{i,t+1}$, with a fitted line representing the average route taken by the buses.

All the spatial coordinates collected in the sample period are plot in a scatter, as illustrated in fig B.1. A smooth line fitted along the scattered coordinates traces out the bus route. This line is then split at regular intervals equivalent to 7.5m, resulting in a path made up of 688 evenly distributed points ($\tilde{X}_j$, where $j \in 1, 688$). These points are

used to transform the collected 2-D $\tilde{x}_{i,t}$ into a 1-D $x_{i,t}$ by defining $x_{i,t} = j_0$ if $|\tilde{X}_{j_0} - \tilde{x}_{i,t}| \leq |\tilde{X}_j - \tilde{x}_{i,t}|, j \in 1, 688$.

Fig B.2. Graphs of normalized stopping frequency i.e. probability of bus stopping at a each position along the bus route, and velocity distributions of two different positions on the road. These graph are derived from the raw data collected over 13 weeks. Red lines on the graph are the sites where there are bus stops.

Preliminary studies on the collected data found inaccuracy in the tick-by-tick update of the bus positions, even when taken at 12s intervals. Fig. B.2 plots the occurrence of stoppages, i.e, $\delta x = 0$, where $\delta x_{i,t} = x_{i,t+1} - x_{i,t}$, along the bus route. While the probability of stoppage peaking at road junctions and bus stops are as expected, the median probability of stoppages is 0.2. This value corresponds to buses being stationary 20% of the time, even when traversing a seemingly uninterrupted road. Empirical observation during field work found that this does not reflect the average motion of buses. Further examination on the velocity distribution at single sites (also in Fig. B.2) also shows the unrealistic propensity of zero-velocities occurring on the middle of a long stretch of road. A plausible explanation for this artefact is that the positioning data sent by the buses was interrupted for a significant period of time while the bus was in motion. As such, there is a possibility that the collected bus positions may not reflect the actual tick-by-tick positions of the buses.

Fig B.3 illustrates the process of removing these false occurrences of zero velocities from the data. The data points in a single bus trajectory contains large amount of repeated position data (Fig.B.3a). This results from (1) buses being stationary, (2) buses

(a) Trajectory, $x(t)$ of a single bus, collected at a time interval of 3 seconds.

(b) Removing repeated data points which are not at bus stops. Red circles indicates the remaining points.

(c) Black dots is the linear interpolation of the remaining points.

(d) Black circles indicates the filtered data in a intended time resolution of $\delta t = 12s$

Fig B.3. Illustration of the interpolation process done to filter out false occurrence of zero velocities.

experiencing interruption in updating their live positions and (3) sampling the web portal at a higher frequency than its refresh rate. Regardless, all repeated position data are removed (Fig. B.3b), keeping only data which reflects when buses moved. Note that this removal process is not done when the buses are at the bus stops. Due to the higher propensity of stoppages at the bus stops, we assume all stoppages are collected accurately. The remaining position data are linearly interpolated in time interval of $\delta t = 1$, based on the assumption that buses move with a uniform speed between each position update (Fig. B.3c).¹ With that, updated trajectories are taken at the intended time resolution of $\delta t = 12s$ (Fig. B.3d). After undergoing this process, plots in Fig. B.4 show a revised stopping frequency and velocity distribution at the two sites.

¹It was found that by interpolating the data with the collection interval of $\delta t = 3$ results in a reduced occurrence of stopping at traffic junctions, which is not realistic.

Fig B.4. Graphs of normalized stopping frequency along the bus route, and velocity distributions of two different positions on the road. These graph are derived after the filtering process as described. Red lines on the graph are the sites where there are bus stops.

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