Cyclostationary beamforming for multiple cycle frequencies estimation and detection

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**NANYANG
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SINGAPORE

**CYCLOSTATIONARY BEAMFORMING FOR MULTIPLE
CYCLE FREQUENCIES ESTIMATION AND DETECTION**

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SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING

2019

CYCLOSTATIONARY BEAMFORMING FOR MULTIPLE CYCLE FREQUENCIES ESTIMATION AND DETECTION

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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John Yang Xingguang

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Date

WS

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Assoc Prof Ser Wee

Authorship Attribution Statement

This thesis contains material from 1 paper published in the following peer-reviewed journal where I was the first corresponding author.

Chapter 3 is published as J. X. Yang, W. Ser and L. Lei, "Multiple Cycle Frequencies Estimation Under Cochannel Interference," in *IEEE Signal Processing Letters*, vol. 25, no. 11, pp. 1645-1649, Nov. 2018.
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The contributions of the co-authors are as follows:

- A/Prof Ser provided the initial research direction as well as in-depth discussions and advice on the technical rigor of some of my proposed algorithms.
- I prepared the manuscript drafts.
- I discussed with Dr. Lei Lei the formulation of dominant eigenvalue algorithm.
- I proposed the eigenanalysis cyclic null spectrum algorithm.
- I incorporated existing methods for comparison with the proposed methods.
- I wrote the MATLAB script for all the simulation result.

09/08/2019

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Summary

The research work reported in this thesis focuses on the investigation of the design and performances of the cyclostationary beamforming technique for multiple cycle frequencies estimation and detection.

Many communications, radar and sonar systems have statistical parameters which vary in time. As a result, a significant amount of research work has been reported on the cyclostationarity property of such signals. Cyclostationary beamforming algorithms, such as the cyclic adaptive beamforming (CAB), are sensitive to the accuracy of the presumed cycle frequency (CF) of the signal-of-interest (SOI) and their performance degrades significantly in the presence of mismatch between the presumed and actual CF. In cognitive radio networks, the main challenge for spectrum sensing (SS) is to develop blind sensing techniques that can detect primary user (PU) with low signal-to-noise ratio (SNR). For SS, cycle frequency of the PU signal can be used as a feature for detection. Furthermore, due to the hidden primary user problem, SS performance degrades significantly when the received signal-to-interference ratio (SIR) at the receiver is small. Hence, this thesis studies the application of using cyclostationary beamforming for estimating and detecting multiple cycle frequencies under co-channel interference.

Firstly, the thesis proposes two novel multiple cycle frequencies estimation algorithms for the estimation of the number of weak cyclostationary signals and their

cycle frequencies in the presence of interfering signals. Both cycle frequencies estimation algorithms are eigen analysis-based methods which use the cyclic autocorrelation function of the received signals. A new eigen analysis cyclic null spectrum (ECNS) is introduced to determine the number of cyclostationary signals and estimate their cycle frequencies.

Secondly, the thesis addresses the signal detection problem in SS using the novel methods proposed. Unlike conventional signal detection, now the decision is to decide if the SOI is present in strong co-channel interference and results are demonstrated through the Receiver Operating Characteristic (ROC) curve and peak-to-average power ratio (PAPR).

Thirdly, the thesis proposes another three novel methods called the cyclic adaptive beamforming multiple signal classification (CAB-MUSIC) algorithm, the dominant cyclic eigensubspace (DCE) and non-dominant cyclic eigensubspace (N-DCE) algorithms which could detect and estimate multiple cycle frequencies. The proposed DCE and N-DCE algorithms use the eigenstructure of the cyclic autocorrelation function (CAF) for eigensubspace projection and signal reconstruction. Unlike conventional projection method, the DCE and N-DCE algorithms do not require *a priori* information on the number of sources in the received data and could work in spatially correlated noise.

In summary, this thesis contributes five novel methods for the estimation and detection of multiple cycle frequencies under the presence of strong co-channel interference: (i) the dominant eigenvalue method; (ii) the eigen-analysis cyclic null spectrum (ECNS) method; (iii) the cyclic adaptive beamforming multiple signal classification (CAB-MUSIC) algorithm; (iv) the dominant cyclic eigensubspace (DCE) method; (v) the non-dominant cyclic eigensubspace (N-DCE) method. The proposed

algorithms have been evaluated via numerical simulations and the results obtained show that they outperform other existing algorithms in estimation accuracy.

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List of Abbreviations

AIC	Akaike Information Criterion
BPSK	Binary Phase Shift Keying
CA	Canonical Analysis
CAB	Cyclic Adaptive Beamforming
CAF	Cyclic Autocorrelation Function
CC	Cyclic Correlation
CCC	Cyclic Conjugate Correlation
CF	Cycle Frequency
CFAR	Constant False Alarm Rate
CR	Cognitive Radio
CSI	Channel State Information
DCE	Dominant Cyclic Eigensubspace
DOA	Direction-of-Arrival
ECNS	Eigen-analysis Cyclic Null Spectrum
EV	Eigenvalue

EV-CSS	Eigenvalue-based Cyclostationary Spectrum Sensing
FFT	Fast Fourier Transform
IID	Independent Identically Distributed
INR	Interference-to-Noise Ratio
MDL	Minimum Description Length
MUSIC	Multiple Signal Classification
MVDR	Minimum Variance Distortionless Response
N-DCE	Non-Dominant Cyclic Eigensubspace
PAPR	Peak-to-Average Ratio (PAPR)
PSD	Power Spectral Density
PU	Primary User
ROC	Receiver Operating Characteristic
SC	Spectral Coherence
SCD	Spectral Correlation Density
SCF	Spectral Correlation Function
SIR	Signal-to-Interference Ratio
SINR	Signal-to-Interference-Plus-Noise Ratio
SNR	Signal-to-Noise Ratio
SOI	Signal-of-Interest
SRR	Spectrum Resolution Ratio

SS	Spectrum Sensing
SU	Secondary User
SVD	Singular Value Decomposition
ULA	Uniform Linear Array
WSS	Wide-sense Stationary

Notations

In this thesis, the following conventions hold.

- Matrices and vectors are denoted by boldface capital and small letters, e.g., $\mathbf{X}$ and $\mathbf{x}$, respectively.
- Scalars are denoted by non-boldface italics.
- $\Sigma(\cdot)$ is the summation operator.
- $(\cdot)^*$ is the complex conjugate operator.
- $(\cdot)^{(*)}$ is the optional complex conjugate operator.
- $\langle \cdot \rangle_\infty$ denotes the average over the time interval $[0, \infty]$.
- $\int_{-\infty}^{\infty} (\cdot) dx$ denotes the integral over the time interval $[-\infty, \infty]$.
- $e^{(\cdot)}$ denotes the exponential function.
- $|\cdot|$ is the absolute power operator.
- $E\{\cdot\}$ Is the statistical expectation operator.
- $\min\{\cdot\}$ obtains the minimum value of the argument(s).
- $\max\{\cdot\}$ obtains the maximum value of the argument(s).
- $(\cdot)^T$ is transpose operator.

- $(.)^H$ is the Hermitian transpose operator.
- $(.)^{-1}$ is the inverse operator.
- $x \in X$ means “x is in the set X”.
- $\rightarrow \infty$ means “tends to infinity”.
- $\|\mathbf{X}\|$ is the Frobenius norm of a complex matrix $\mathbf{X}$.
- $P\{.\}$ obtains the principal eigenvector of a matrix.

Symbols

In this thesis, the following conventions hold.

t, t_0, u, q	time index
T_0	period of a time domain process
$M_x(t)$	mean function of a random time process in time t
$R_x(t, u)$	autocorrelation function of a random time process at time t and time u
τ	time delay
α	cycle frequency
n	positive integer
$R_{xx(*)}^\alpha(\tau)$	cyclic (or cyclic conjugate) autocorrelation function
$\hat{R}_{xx}^\alpha(\tau)$	sample cyclic autocorrelation function
N	number of array snapshots collected
T_s	symbol period
f_s, f_c	sampling frequency and carrier frequency, respectively
$S_{xx}^\alpha(f)$	spectral correlation function

$S_{x_T}^\alpha(f)$	spectral correlation density
f	frequency index
X_T	time-variant Fourier transform
$C_X^\alpha(f)$	spectral coherence
φ	azimuth angle of the propagating wave with respect to the x-axis
θ	polar angle of the propagating wave with respect to the z-axis
D	distance between the antenna elements
M	number of antenna elements
c	speed of light
Δt_m	time difference of the wavefront at the m th antenna element relative to the array origin
$x_m(t)$	baseband received signal at the m th antenna element
T	sampling period
λ	wavelength
K	total number of signals
$s_k[n]$	k th signal in the array
θ_k	Direction-of-Arrival of the k th signal in the array
$a_m(\theta_k)$	Direction-of-Arrival of the k th signal in the array at the m th antenna element

$v_m[n]$	additive noise added to the m th antenna element
$\mathbf{x}_n$	snapshot array output vector
$\mathbf{A}$	$M \times K$ matrix whose columns are the steering vectors of the array
$\mathbf{s}_n$	snapshot signal vector
$\mathbf{v}_n$	snapshot noise vector
$\mathbf{R}_{xx}$	theoretical array covariance matrix
$\mathbf{R}_{ss}$	covariance matrix of signal $\mathbf{s}_n$
σ_v^2, E_n	white noise variance
$\mathbf{I}$	identity matrix
μ_i	eigenvalue
ξ_i	eigenvector
$\mathbf{U}_s, \mathbf{\Lambda}_s$	matrix that contains the eigenvectors which span the signal-plus-interference subspace and diagonal matrix that contains the corresponding eigenvalues, respectively
$\mathbf{U}_v, \mathbf{\Lambda}_v$	matrix that contains the eigenvectors which span the noise subspace and diagonal matrix that contains the corresponding eigenvalues, respectively
$\hat{\mathbf{R}}_{xx}$	sample array covariance matrix
$y[n]$	beamformer array output
w_m	complex weights of the beamformer

$\mathbf{w}$	$M \times 1$ complex vector which contains the beamformer weights
J	total number of interferences
$s_0[n], s_i[n]$	waveforms of the SOI and the i th interference, respectively
$\mathbf{a}(\theta_0), \mathbf{a}(\theta_i)$	true steering vector of the SOI and the i th interference, respectively
θ_0, θ_i	DOAs of the SOI and the i th interference, respectively
$\mathbf{R}_{\text{in}}$	covariance matrix of the interference-plus-noise components
$\mathbf{a}(\theta)$	$M \times 1$ array steering vector
σ_s^2, E_s	SOI power
$L(g), L(g')$	Lagrange functions
g, g'	Lagrange multipliers
$\hat{\mathbf{R}}_{\text{xx}}^{-1}$	inverse sample covariance matrix
$\hat{\rho}_{\text{MVDR}}(\theta)$	estimated PSD of MVDR beamformer
$\hat{\rho}_{\text{MUSIC}}(\theta)$	estimated PSD of MUSIC beamformer
α_k	cycle frequency of the k th signal
$\mathbf{w}_k, \mathbf{c}_k$	complex weight vector for scalar signals

$\hat{s}_k[n], \hat{q}_k[n]$	scalar signals of $\mathbf{x}[n]$ and $\mathbf{u}[n]$, respectively
$\mathbf{w}_k, \mathbf{c}_k$	complex weight vectors for scalar signals $\hat{s}_k[n]$ and $\hat{q}_k[n]$, respectively
$\hat{\mathbf{R}}_{\mathbf{xu}}$	sample CAF matrix
ψ	positive constant
$\mathbf{U}, \mathbf{V}$	matrix that contains the left-singular and right singular vectors of $\hat{\mathbf{R}}_{\mathbf{xu}}$, respectively
Σ	diagonal matrix and the diagonal entries are the singular values of $\hat{\mathbf{R}}_{\mathbf{xu}}$
ξ_m	left-singular vector
ζ_m	right-singular vector
$\mathbf{w}_{\text{CAB}}$	CAB weight vector
$\mathbf{w}_{\text{C-CAB}}$	C-CAB weight vector
$\mathbf{R}_{\text{CCA}}$	matrix for CCA
$\mathbf{R}_{\mathbf{xy}}, \mathbf{R}_{\mathbf{yx}}$	covariance matrix of two random vectors $\mathbf{x}[n]$ and $\mathbf{y}[n]$, respectively
$\mathbf{R}_{\mathbf{yy}}$	covariance matrix of random vector $\mathbf{y}[n]$
$\hat{\mathbf{R}}_{\mathbf{xx}}(\tau)$	sample covariance matrix with time delay
$\hat{\mathbf{R}}_{\mathbf{xx}}^\alpha(\tau)$	sample CAF matrix with time delay τ and cycle frequency α
ϱ	combined eigenvalue

$\hat{\rho}_{\text{EV-CSS}}$	output of EV-CSS algorithm
$\hat{\alpha}_k$	estimated cycle frequency of the k th signal
$\hat{\rho}_{\text{CAF}}(\alpha, \tau)$	output of the summation CAF algorithm
$\hat{\mathbf{R}}_{\text{xx}^*}^{\alpha}(\tau)$	sample conjugate CAF matrix with time delay τ and cycle frequency α
$\hat{\rho}_{\text{EV}}(\alpha, \tau)$	output of dominant EV algorithm
$\hat{\rho}_{\text{UC}}(\alpha, \tau)$	unconstrained output power of the CAB beamformer
$\hat{\rho}_{\text{C}}(\alpha, \tau)$	constrained output power of the CAB beamformer
$\hat{\rho}_{\text{ECNS}}(\alpha, \tau)$	output of ECNS algorithm
S_{ECNS}	difference of unconstrained and constrained output power
I_1, I_2	interfering signals
α_{I1}, α_{I2}	cycle frequencies of interfering signals
$v(t)$	additive Gaussian noise
h	amplitude gain of the channel
$\mathcal{H}_0$	null hypothesis
$\mathcal{H}_1$	alternative hypothesis
θ	a set of data for the detection problem
θ_0, θ_1	partitioned data for null and alternative hypothesis, respectively
$\phi(x)$	decision rule function

χ_0, χ_1	data space for the decision rule function
P_{FA}	probability of false alarm
P_D	probability of detection
$T_{EV}(\alpha_k, \tau)$	test statistic of dominant EV detector
γ	detection threshold
$PAPR_{EV}$	power-to-average power ratio of dominant EV detector
$T_{ECNS}(\alpha_k, \tau)$	test statistic of ECNS detector
$PAPR_{ECNS}$	power-to-average power ratio of ECNS detector
χ^2	chi-square distribution
$\mathbf{U}_s, \mathbf{V}_s$	signal matrix that contains the left-singular and right singular vectors of $\hat{\mathbf{R}}_{xu}$, respectively
$\mathbf{U}_n, \mathbf{V}_n$	noise matrix that contains the left-singular and right singular vectors of $\hat{\mathbf{R}}_{xu}$, respectively
$\sum s$	summation of all signal eigenvalues
$\sum n$	summation of all noise eigenvalues
$\mathbf{U}_{n1}, \mathbf{U}_{n2}$	noise matrix that contains the left-singular singular vectors of sample CAF matrix and conjugate CAF matrix, respectively
$\rho_{C-MUSIC}$	output of C-MUSIC algorithm
$\hat{\rho}_{CAB-MUSIC}(\alpha, \tau)$	output of CAB-MUSIC algorithm
$T_{CAB-MUSIC}(\alpha_k, \tau)$	test statistic of CAB-MUSIC detector

$PAPR_{CAB-MUSIC}$	power-to-average power ratio of CAB-MUSIC detector
$\hat{\mathbf{R}}_{DCE}$	estimated DCE covariance matrix
$\hat{\mathbf{R}}_{N-DCE}$	estimated N-DCE covariance matrix
$\mathbf{w}_{opt}$	optimal weight
$\hat{\rho}_{DCE}(\alpha, \tau)$	output of DCE algorithm
$T_{DCE}(\alpha_k, \tau)$	test statistic of DCE detector
$PAPR_{DCE}$	power-to-average power ratio of DCE detector
$\hat{\rho}_{N-DCE}(\alpha, \tau)$	output of N-DCE algorithm
$T_{N-DCE}(\alpha_k, \tau)$	test statistic of N-DCE detector
$PAPR_{N-DCE}$	power-to-average power ratio of N-DCE detector
$\mathbf{R}_{vv}$	noise covariance matrix
ρ_s	noise correlation factor

Chapter 1

Introduction

1.1 Motivation

Most man-made signals used in wireless communications, radar and sonar systems have statistical parameters which vary with respect to time. As a result, significant research has been made to exploit the cyclostationarity of such signals [1], [2] and more recently in [3], [4]. Thus, the study of cyclostationary signal has major impact in applications involving adaptive array beamforming [5-7], blind source separation [8], [9], spectrum sensing (SS) [10-13] and carrier frequency estimation [14], [15].

In [6], blind cyclic adaptive beamforming (CAB) algorithms are proposed. The key difference between CAB and conventional array beamforming methods [16], [17] is that it uses the signal of interest (SOI) frequency-domain property instead of its spatial signature. These cyclostationary beamforming algorithms require only the knowledge of the cycle frequency (CF) of the SOI in implementation. A cycle frequency is calculated from a baud rate or the carrier frequency. Therefore, such methods relax the

requirements of the conventional beamforming which include the knowledge of training data sequence, spatial signature of the SOI at the receiving sensory array, and the knowledge of sensory array geometry and gain/phase response [17].

Cyclostationary-based beamforming algorithms are sensitive to the accuracy of the presumed CF of the SOI and their performance degrades substantially in the presence of mismatch between the presumed and actual CF. Robust CAB algorithms have been proposed to handle small CF mismatch problem due to doppler frequency offset [6], [18-20]. However, under significant CF mismatch condition, the performance of these robust CAB algorithms would degrade significantly as well. In most blind CAB papers, the CF is assumed to be perfectly known, or if not, CF error is assumed to be very small. However, in practice, the CF of the SOI is not precisely known and there might be a large CF error. Therefore, one of the motivations in this thesis is to develop algorithms which could estimate cycle frequencies without any *a priori* knowledge on the CF of the SOI.

In cyclostationary-based sensing applications, cyclostationarity feature detection has been used to detect primary users (PUs) transmissions by exploiting the cyclostationarity features of the received signals [21]. The spectral correlation density (SCD) function which is the Fourier transform of the cyclic autocorrelation function (CAF) [1], [2] outputs peak values when the CF matches the fundamental frequencies of the transmitted signal. As the noise is wide-sense stationary (WSS) with no correlation, cyclostationarity-based detection algorithms can differentiate noise from the PUs' modulated signals as they are cyclostationary with some spectral correlation. Thus, CAF can be used to formulate hypothesis testing problem to detect the presence of cyclostationary signals using multiple antenna receivers. The distinct feature of cyclostationary signals can be extracted using SCD function; examples of cyclic spectral

analysis being used to classify typical digitally modulated signals in high signal-to-noise ratio (SNR) conditions is shown in [22]. It is shown in [22] that under low SNR condition, cyclostationarity feature detection using cyclic spectral analysis still performs well when there is training data available. However, in practice, time constraints limit the amount of training data available. Therefore, another motivation in this thesis is to develop algorithms which could estimate cycle frequencies with small sample size. Cyclostationarity feature detection is further extended to multivariate processes using multiple antenna receiver. The detection uses nonconjugate and conjugate CAFs [23] which is the cyclic correlation (CC) function and the cyclic conjugate correlation (CCC) function used to formulate CAB in [6].

One of the main problems for spectrum sensing (SS) is the hidden primary user problem where the primary user receiver cannot detect the primary transmitter's signal due to unwanted interference from nearby cognitive radio devices. Cooperative sensing [24-25] is proposed in the literature for handling the hidden primary user problem but faces difficulties in developing efficient information sharing algorithms among the cognitive radios and increased complexity. Therefore, one of the main challenges for SS is to develop blind sensing techniques that can detect PUs in the presence of strong co-channel interferences. In [10], an adaptive cyclostationary beamforming has been designed and proposed as a SS method to detect cyclostationary. It is reported that the probability of detection drops far below 1 when the signal-to-interference ratio (SIR) at the receiver is small. In [11], cyclic correlation significance test is used to detect cyclostationary signal. Similar to [10], the probability of detection in [11] drops to 0.5 in the presence of co-channel interference (SIR = -20 dB). Many existing SS algorithms only work well with a large sample size and degrade significantly under co-channel

interference or when the SIR at the receiver becomes small due to the presence of multiple PUs with different signal powers.

1.2 Objectives

The objective of the research in this thesis is to develop cycle frequencies estimation and detection algorithms using cyclostationary beamforming.

In most blind CAB papers, the CF is assumed to be perfectly known, or if not, CF error is assumed to be very small. However, in practice, the CF of the SOI is not precisely known and there might be large CF error as well. Therefore, the main objective in this thesis is to develop algorithms which could estimate the number of unknown cyclostationary signals and their cycle frequencies in the received data. As the SIR in the receiver might be very small, multiple cycle frequencies estimation algorithms should be able to work under strong co-channel interference. In addition to cycle frequencies estimation, cycle frequencies detection is also one of the main objectives in this thesis. Unlike conventional signal detection problem in SS, now the decision is to decide between if the SOI is present in strong co-channel interference. Detection performance results are demonstrated through the Receiver Operating Characteristic (ROC) curve and PAPR in this thesis.

The subspace projection method in [26] has been widely used to determine the direction-of-arrival (DOA) of the SOI in the received data. However, this method needs *a priori* information on the number of sources in the received data. The performance of this method degrades significantly if the number of sources is estimated inaccurately.

Therefore, one of the objectives in this thesis is to develop algorithms to detect and estimate multiple cyclostationary signals without the need of any *a priori* knowledge on the received data under co-channel interference.

Lastly, cycle frequencies estimation and detection algorithms should be implementable at a low cost. Another challenge for SS is to develop fast blind sensing techniques that can detect very weak PU signals with low implementation cost. The spectral correlation function (SCF) requires high computational complexity, making implementation costly. For example, in the multitaper method described in [27], each sample size N requires N singular value decompositions (SVD) to be computed. This potentially results in high the computational complexity, even for a small sample size. Therefore, this thesis investigates the computational complexity of all the novel methods introduced which make use of the CAF instead of SCF.

1.3 Major contributions of the Thesis

The major contributions of the thesis are as follows:

1. In Chapter 3, a dominant eigenvalue estimation algorithm is proposed to estimate the number of cyclostationary signals and their respective cycle frequencies in the received data. In [6], the dominant eigenvector corresponding to the largest singular eigenvalue of CAF matrix is used to estimate the DOA of the SOI. Hence, large singular eigenvalue of the CAF is a clear indication of a high correlation value due to a cyclostationary signal associated with a distinct cycle frequency in the received data. The performance of the dominant eigenvalue estimation algorithm is demonstrated in Chapter 3 against existing cycle frequencies estimation methods.
2. The dominant eigenvalue can be used to infer the presence of a cyclostationary signal. In [28], the eigenvector corresponding to the maximum eigenvalue is used to estimate the channel state information and then, the SCF based on the eigenvector is computed for cyclostationary detection. Such method has high computational complexity due to the calculation of SCF. Hence, the proposed dominant eigenvalue detection algorithm in Chapter 4 reduces the computational cost significantly without compromising the detection performance. The proposed dominant eigenvalue detection algorithm only needs to perform a SVD on the CAF to obtain the maximum eigenvalue, eliminating the need for taking a high frequency resolution using SCF. In Chapter 4, the detection performance of the dominant eigenvalue detection algorithm is evaluated.

3. It is found that the performance of the dominant eigenvalue algorithm proposed in Chapter 3 degrades significantly when the number of cyclostationary signals increases in the received data. Therefore, an Eigen-analysis Cyclic Null Spectrum (ECNS) is proposed in Chapter 3 to estimate the number of cyclostationary signals and their respective cycle frequencies in the received data. The weighting vector from CAB is used to calculate the unconstrained and constrained beamformers' outputs. The difference of the two beamformers output powers produces a cyclic null spectrum which could estimate the cycle frequencies of the cyclostationary signals. The performance of the ECNS algorithm is demonstrated in Chapter 3 against existing approaches to cycle frequencies estimation. The use of beamformer in array signal processing [16-17] allows the extraction of the SOI with different chosen criterions. Hence, with the criterions of the unconstrained and constrained beamformers, the eigen-analysis cyclic null spectrum could detect multiple cyclostationary signals with different SNRs. This overcomes one of the main challenges in SS which is the hidden primary user problem. In Chapter 4, the detection performance of ECNS detection algorithm is evaluated.
4. In [26], a subspace method for DOA estimation called multiple signal classification (MUSIC) "spectrum" is proposed. Although the MUSIC algorithm has superior resolution performance and is flexible in array geometry, it has several disadvantages. The main disadvantage is that it requires the number of signals that impinge on the array to be known in advance. If the number of signals is unknown, information theoretic criteria such as the Akaike information criterion (AIC) or minimum description length (MDL), proposed in [29], could be used to estimate the number of signals. However, these methods would not work in non-ideal

scenarios such as model mismatch or closely located sources [30]. Another disadvantage of MUSIC is that the sources in the data need to be uncorrelated. However, the noise in the receiver might be spatially correlated. Therefore, CF estimation and detection algorithm should be able to perform without *a priori* information on the number of sources in the received data and in spatially correlated noise. In Chapter 5, three algorithms are proposed to detect the number of cyclostationary signals and estimate their respective cycle frequencies. Two of the proposed algorithms do not need *a priori* information on the number of sources in the received data. They only rely on the eigenstructure of the CAF for eigensubspace projection and signal reconstruction. This eliminates the use of source estimation algorithms which increases complexity and only works in ideal scenarios. The performance of the proposed algorithms is demonstrated in Chapter 5 against existing cycle frequencies estimation methods and the detection performance of the proposed algorithms are evaluated as well.

5. In addition, all the algorithms proposed in this thesis were studied with a small sample size. Fast implementation is important in SS algorithms to prevent interference to and from the primary users. The SS algorithms should be able to identify the presence of primary users as quickly as possible to vacate the band immediately. Therefore, all the multiple cyclostationary frequencies estimation and detection algorithms proposed in this thesis should work well within a small sample size for a shorter sensing duration.

1.4 Organization of the Thesis

This thesis is organized as follows. Chapter 2 gives an introduction on cyclostationarity with the formulation of cyclic autocorrelation function (CAF) and spectral correlation density (SCD) function. An overview of the array system model used in this thesis is given together with the coordinate system, the beamforming array geometry, the spatial and temporal characteristics of a signal, and the important array steering vector model used in beamforming. The concept of beamforming and the beamformer's performance evaluation used in this thesis are given. Well-known beamformers which have been used in adaptive beamforming and cyclostationary beamforming are described. Finally, the problem formulation for signal detection using hypothesis test is explained.

Chapter 3 to 5 contain the main contributions of this thesis. In Chapter 3, two cycle frequencies estimation algorithms are proposed to estimate the number of weak cyclostationary signals and their cycle frequencies in the presence of high power interfering signal. Both cycle frequencies estimation algorithms are eigen analysis-based methods that use the cyclic autocorrelation function of the received signals. A new eigen-analysis cyclic null spectrum (ECNS) is introduced to determine the number of cyclostationary signals and estimate their cycle frequencies. The performance of the proposed cycle frequency estimation algorithms is demonstrated with numerical examples and compared with the existing cycle frequency estimation algorithms. It is shown that the proposed cycle frequencies estimation algorithms have better accuracy than the existing methods and could work with a small sample size for faster estimation.

In Chapter 4, multiple cyclostationary frequencies detection is investigated using the algorithms proposed in Chapter 3. The dominant eigenvalue and ECNS algorithms are used to detect multiple cyclostationary signals with different SNR. The signal

detection problem is formulated using a hypothesis test that is different from the conventional signal detection hypothesis test procedure. The null hypothesis contains noise only and the alternative hypothesis contains the SOI, the co-channel interference and noise. The objective is to detect weaker cyclostationary signal in the presence of strong interfering signal. The performances of the cycle frequency detection algorithms are demonstrated with numerical examples and compared with the existing cycle frequency detection algorithms. It is shown that the proposed cycle frequencies detection algorithms have better detection performance in terms of receiver operating characteristic curve (ROC) and PAPR than the existing methods and could work with small sample size for faster estimation.

In Chapter 5, cyclic adaptive beamforming multiple signal classification (CAB-MUSIC) algorithm, Dominant Cyclic Eigensubspace (DCE) and Non- Dominant Cyclic Eigensubspace (N-DCE) are proposed to detect the number of cyclostationary signals and estimate their respective cycle frequencies. In particular, the proposed DCE and N-DCE algorithms do not need *a priori* information on the number of sources in the received data. They only rely on the eigenstructure of the CAF for eigensubspace projection and signal reconstruction. This eliminates the use of source estimation algorithms which increases complexity and only works in an ideal scenario. In Chapter 5, the performance of the proposed algorithms is demonstrated against existing cycle frequencies estimation approaches and the detection performance of the proposed algorithms is evaluated. Finally, Chapter 6 concludes this thesis with recommendations for future work.

Chapter 2

Cyclostationary Beamforming

2.1 Introduction

In this chapter, signal cyclostationarity and the array signal model used in this thesis are first described. Next, the beamforming concept and more importantly, the CAB algorithm, is discussed. Lastly, the performance metric used for signal detection is illustrated. This chapter is organized as follows. Section 2.2 explains the two important cyclostationary functions which are the spectral correlation density (SCD) function and cyclic autocorrelation function (CAF). Section 2.3 introduces the signal system model which encompasses the array geometry, the spatial and temporal characteristics of the signal and most importantly, the array steering vector model. Section 2.4 discuss several adaptive beamforming methods which motivate the investigation of multiple cycle frequencies estimation and detection algorithms in this thesis. Section 2.5 describes the performance measures important for signal detection and hypothesis test under strong co-channel interference.

2.2 Signal Cyclostationarity

2.2.1 Definition

Many man-made stochastic processes exhibit cyclostationarity. Communication signal is one such example due to modulation and/or channel coding. A second-order wide-sense cyclostationary process is a random time domain process for which its mean and autocorrelation periodically change as a function of time, with period T_0 [1] and can be described in [31] for all t and u as

$$M_x(t + T_0) = M_x(t) \quad (2.1)$$

$$R_x(t + T_0, u + T_0) = R_x(t, u) \quad (2.2)$$

where $M_x(t)$ is the mean function of a random time process in time t and $R_x(t, u)$ is the autocorrelation function of a random time process at time t and time u . Hence, the cyclic autocorrelation in terms of t and a time delay τ , $R_x\left(t + \frac{\tau}{2}, t - \frac{\tau}{2}\right)$, can be expressed as a Fourier series [1]

$$R_x\left(t + \frac{\tau}{2}, t - \frac{\tau}{2}\right) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{j2\pi\alpha t} \quad (2.3)$$

where $\alpha = n/T_0$ and n is a positive integer.

Cyclic autocorrelation of a scalar waveform $x(t)$ would be self-coherent (or conjugate self-coherent) spectrally at a frequency α if the cyclic (or cyclic conjugate) autocorrelation function (CAF) shifted in frequency α is nonzero for some delay τ [5], [32]:

$$R_{xx^{(*)}}^{\alpha}(\tau) = \langle x(t)[x^{(*)}]^{(*)}(t - \tau)e^{-j2\pi\alpha t} \rangle_{\infty} \quad (2.4)$$

where $*$ is the complex conjugate operation of a complex quantity, the optional complex conjugation ($*$) is applied in the conjugate self-coherence case. Only treatment of complex conjugate CAF is shown in this thesis and similar comments apply to the nonconjugate CAF. $\langle \cdot \rangle_\infty$ denotes the average over the time interval $[0, \infty]$ α is known as the cycle (or conjugate cycle) frequency. The sample CAF is defined as

$$\hat{R}_{xx}^\alpha(\tau) = \langle x(t) x^T(t - \tau) e^{-j2\pi\alpha t} \rangle_N \quad (2.5)$$

where N is the number of array snapshots collected. The conventional autocorrelation occurs when $\alpha = 0$ and $\tau = 0$. Therefore, the cyclic autocorrelation of a signal has a component at $\alpha = \frac{1}{T_0}$ if it is cyclostationary with period T_0 . For stationary signals, $R_{xx}^\alpha = 0$ for $\alpha \neq 0$.

A communication signal usually processes nontrivial cycle frequencies which are determined by its features, such as the baud rate, the carrier frequency, etc. Table 2-1 [5] lists the cycle and conjugate cycle frequencies for a number of common modulation formats. For example, in binary phase shift keying (BPSK), cyclostationary features exist at $\alpha = \frac{n}{T_s}$ and at $\alpha = \pm 2f_c + \frac{n}{T_s}$, where f_c is the carrier frequency, T_s is the symbol period. Detailed analysis of the cyclostationary features for different digital modulations can be found in [33]. For BPSK modulation, the maximum self-coherence occurs at $\tau = 0$ [5] and hence $\tau = 0$ is used in all experiments unless otherwise stated.

Table 2-1: Examples of spectrally self-coherent signal

Complex Modulation Format	Self-Coherence Frequencies	Conj. Self-Coherence Freq. (@ 2 x Carrier)
ASK, BPSK	Baud-rate mult.	Baud-rate mult.
QPSK	Symbol-rate mult.	None
MSK, SQPSK	Baud-rate mult.	$\pm \frac{1}{2}$ baud-rate

CPFSK	Symbol-rate mult.	2 x symbol frequencies*
FDM-FM	Pilot-tone mult.	None
DSB-AM	None	2 x carrier
SSB-AM	None	None

*frequencies deviation = multiple of $\frac{1}{2}$ only

The magnitude of SC ranges from 0 to 1 with $C_X^\alpha(f) = 1$ for all f . Examples of SCFs of typically digitally modulated signals can be found in [22].

2.2.2 Representation of Cyclostationary Process

The spectral correlation function (SCF), infer from the cyclic Wiener relation, can be obtained from the Fourier transform of the cyclic autocorrelation in (2.4)

$$S_{xx}^\alpha(f) = \int_{-\infty}^{\infty} R_{xx(*)}^\alpha(\tau) e^{-j2\pi f\tau} d\tau \quad (2.6)$$

However, spectral correlation needs to be estimated from a finite set of sample size as the number of samples at the received antenna element is limited. The spectral correlation density (SCD) can be defined using the cyclic periodogram by [1]

$$S_{X_T}^\alpha(f)_{\Delta t} = \frac{1}{\Delta t} \int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} \frac{1}{T} X_T \left(t, f + \frac{\alpha}{2} \right)^* X_T^* \left(t, f - \frac{\alpha}{2} \right) dt \quad (2.7)$$

with frequency components $f + \frac{\alpha}{2}$, $f - \frac{\alpha}{2}$, f , spectral location (center of shift), α spectral separation (amount of shift). Therefore, a cyclostationary process can be represented by a stationary narrowband process with width α . X_T is the time-invariant Fourier transform defined as

$$X_T(t, v) = \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x(v)^* e^{-j2\pi f v} dv \quad (2.8)$$

The spectral correlation function (SCF) is a cross-correlation between frequency components separated by $f + \frac{\alpha}{2}$ and $f - \frac{\alpha}{2}$. The correlation coefficient of SCF is commonly known as spectral coherence (SC) and can be defined as

$$C_X^\alpha(f) = \frac{S_X^\alpha(f)}{\left| S_X^0\left(f + \frac{\alpha}{2}\right)^* S_X^0\left(f - \frac{\alpha}{2}\right) \right|^{\frac{1}{2}}} \quad (2.9)$$

Cyclostationary processes can also be represented in discrete time processes in multirate processing. Let $n_0 = nP + i$ and $x[n_0] = x[nP + i]$, where the remainder $i = 0, 1, \dots, P - 1$. Thus, subprocesses $x_i[n] = x_i[nP + i]$ can be formulated. In multirate processing, the $P \times 1$ vector $x[n] = [x_0[n] \dots x_{P-1}[n]]^T$ forms the polyphase decomposition of $x[n]$ [87]. Hence, cyclostationary signals with period P can be analyzed as stationary $P \times 1$ multichannel processes and vice versa. Therefore, cyclostationary process can also be represented as a τ -variate stationary multichannel discrete time process $x[n]$ with decimated components $x_i[n] = x_i[nP + i]$, $i = 0, 1, \dots, P - 1$ [88].

For stationary processes, sample correlations and spectral density estimators converge to their ensembles when the number of sample size goes to infinity. Therefore, a large number of sample size is needed for such estimators. Whereas for nonstationary processes, building a reliable estimator is challenging as the statistics vary with time and is generally impossible. Fortunately, as shown in this section, cyclostationary signals belong to a class of finitely parameterized nonstationary processes. The key is to note

that although R_{xx}^α is time-varying, it can be expressed in terms of cyclic quantity $S_{xx}^\alpha(f)$ which are time-invariant.

Cyclostationary processes can be viewed as the most common class of nonstationary signals used in engineering and time series application. Cyclostationary properties can be found in signals and systems which process repetitive variations. This allows the separation of components using their cycle frequencies. This spectral diversity can be used to suppress stationary noise using cyclic signal processing algorithms.

2.3 System Model

The radiation of the electromagnetic field in the vicinity of an antenna can be near field (Fresnel zone) or far field (Frauenhoffer zone). In this thesis, the focus is on the sources of signals that are at the far field of the array.

The direction of arrival (DOA) of a signal impinging on an antenna element plane is shown in Figure 2-1. All the antenna elements in the array are in one plane. The DOA of the incoming signal is simply defined by two angles, the azimuth (φ) and the elevation (θ). In this thesis, φ is 0 for all cases.

Figure 2-1: Impinging signal on antenna plane with the azimuth (φ) angle and the elevation (θ) angle.

2.3.1 Array Geometry

The array geometries used in beamforming applications include the linear array, rectangular planar array and the circular planar array. Less commonly used array geometries such as the hexagonal planar array and the cylindrical non-planar array are described in [17]. In this thesis, the linear array is studied.

Figure 2-2 shows a uniform linear array (ULA) of sensors. The array is uniform because the distance D between the antenna elements is kept constant. Consider a uniform linear array geometry consisting of M antenna elements and with half-a-wavelength spacing between them. The signals received by the different elements would be correlated as the array elements are closely spaced.

Figure 2-2: Uniform linear array with M antenna elements. A plane wave impinges at θ angle from the array axis.

If a propagating wave carries a baseband signal, $s(t)$, impinge on the antenna array, each array antenna element would receive $s(t)$ at a different phase and time different from other antenna elements. The phase of $s(t)$ received at each of the antenna element will be measured with respect to the phase of the signal received by the 1st element in Figure 2-2. The time difference of the wavefront at the m th antenna element relative to the array origin at x_0 is

$$\Delta t_m = \frac{mD \sin \theta}{c} \quad (2.10)$$

where c is the speed of light. If $s(t)$ is a digitally modulated narrowband signal, the baseband received signal at the m th antenna element would be

$$x(t) = s(t - \Delta t_m) e^{-j2\pi f_c \Delta t_m} \quad (2.11)$$

The sampled received baseband signal with sampling period T seconds therefore is

$$x(nT) = s(nT - \Delta t_m) e^{-j2\pi f_c \Delta t_m} \quad (2.12)$$

2.3.2 Narrowband Approximation

In wireless digital communication systems, the maximum time delay for a signal to traverse the entire array is much smaller than its correlation time. Therefore, the symbol period is much larger than each of the time delays at the array

$$T_s \gg \Delta t_m, m = 1, 2, \dots, M \quad (2.13)$$

and the received signal at the m th antenna element could be approximated [34] as

$$x(nT) \approx s(nT) e^{-j2\pi f_c \Delta t_m} \quad (2.14)$$

Substituting Δt_m in (2.14) with (2.10) with $c = \lambda f_c$

$$x(nT) \approx s(nT) e^{-j2\pi \frac{D}{\lambda} m \sin \theta} \quad (2.14a)$$

The distance between the antenna elements D must be $\frac{\lambda}{2}$ to avoid aliasing [1], [35].

Therefore, $D = \frac{\lambda}{2}$ is used throughout the thesis and (2.14a) simplifies to

$$x(nT) \approx s(nT) e^{-j\pi m \sin \theta} \approx s(nT) e^{-jm\omega} \quad (2.14b)$$

where

$$\omega = \pi \sin \theta \quad (2.14c)$$

The n th sample of the baseband signal at the m th antenna element can be discretely denoted as $x_m[n]$. The baseband, sampled signal at the m th element is

$$x_m[n] = \sum_{k=1}^K s_k[n] a_m(\theta_k) \quad (2.15)$$

where K is the total number of signals present and

$$a_m(\theta_k) = e^{-jm\omega_k} \quad (2.15b)$$

2.3.3 Array Steering Vector Model

The array output (2.15) can be written in a matrix form

$$\begin{bmatrix} x_1[n] \\ x_2[n] \\ \vdots \\ x_M[n] \end{bmatrix} = \begin{bmatrix} a_1(\theta_1) & a_1(\theta_2) & \cdots & \cdots & a_1(\theta_K) \\ a_2(\theta_1) & a_2(\theta_2) & \cdots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ a_M(\theta_1) & \vdots & \vdots & \vdots & a_M(\theta_K) \end{bmatrix} \begin{bmatrix} s_1[n] \\ s_2[n] \\ \vdots \\ s_K[n] \end{bmatrix} + \begin{bmatrix} v_1[n] \\ v_2[n] \\ \vdots \\ v_M[n] \end{bmatrix} \quad (2.16a)$$

where additive noise, $v_m[n]$, is added at each antenna element. (2.16a) can be written in a compact matrix notation

$$\mathbf{x}_n = \mathbf{A}\mathbf{s}_n + \mathbf{v}_n \quad (2.16b)$$

where $\mathbf{x}_n$ is $M \times 1$ vector, $\mathbf{A}$ is an $M \times K$ matrix. $\mathbf{s}_n$ is the signal vector and $\mathbf{v}_n$ is the noise vector. Each signal $s_k(t)$ is associated with a steering vector represented by the columns of the matrix $\mathbf{A}$. The column vectors are linearly independent given that the DOA of each k signal is different. The noise present at each antenna element is represented by $\mathbf{v}_n$ and they are assumed to be uncorrelated.

The set of all possible steering vectors is known as the *array manifold* [35]. The methods presented in this thesis all require that $K < M$ [35]. In the case where noise is absent, the array output is spanned by the array steering vectors restricted to an M -dimensional subspace of the complex K -space. This constitutes the signal subspace and forms the basis of subspace-based methods which are studied in the subsequent sections

of this thesis. Subspace projection method is used in many well-known applications such as DOA [26], frequency estimation [36] and low-rank filtering [37].

2.3.4 Eigenstructure of the Spatial Covariance Matrix

Signals $\mathbf{s}_n$ can be of zero mean, stochastic and Gaussian as well as independent from the Independent and Identically Distributed (IID) Gaussian noise $\mathbf{v}_n$. This resulted with a received array output $\mathbf{x}_n$ which are also zero mean and IID Gaussian with a covariance as follows

$$\mathbf{R}_{xx} = E\{\mathbf{x}_n \mathbf{x}_n^H\} = E\{(\mathbf{A}\mathbf{s}_n + \mathbf{v}_n)(\mathbf{A}\mathbf{s}_n + \mathbf{v}_n)^H\} = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma_v^2 \mathbf{I} \quad (2.17)$$

where $\mathbf{R}_{ss}$ is the covariance matrix of signal $\mathbf{s}_n$ and $\sigma_v^2 \mathbf{I}$ is the noise covariance matrix. $E\{\cdot\}$ is the statistical expectation operator and $(\cdot)^H$ is the Hermitian transpose operator. Assume that $K < M$, then the eigen decomposition of the covariance matrix (2.17) is

$$\mathbf{R}_{xx} = \sum_{i=1}^M \mu_i \boldsymbol{\xi}_i \boldsymbol{\xi}_i^H \quad (2.18)$$

where μ_i and $\boldsymbol{\xi}_i$, $i = 1, 2, \dots, M$ are the eigenvalues and eigenvectors of $\mathbf{R}_{xx}$, respectively. Hence, the eigenvalues of $\mathbf{R}_{xx}$ can be ranked as $\mu_1 \geq \mu_2 \geq \dots \geq \mu_K \geq \mu_{K+1} = \mu_{K+2} = \dots = \mu_M = \sigma_v^2$. The $M-K$ smallest eigenvalues are identical and equal to the noise variance σ_v^2 . It is also useful to express $\mathbf{R}_{xx}$ as

$$\mathbf{R}_{xx} = \mathbf{U}_s \boldsymbol{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_v \boldsymbol{\Lambda}_v \mathbf{U}_v^H \quad (2.19)$$

where $\boldsymbol{\Lambda}_s$ is a $K \times K$ diagonal matrix whose diagonal elements are the K largest eigenvalues and the associated eigenvectors corresponding to these eigenvalues are in the columns of $\mathbf{U}_s$ ($M \times K$ matrix). As for $\mathbf{U}_v$ ($M \times (M-K)$ matrix), the columns are the eigenvectors corresponding to the $M-K$ smallest eigenvalues and $\boldsymbol{\Lambda}_v$ (an $(M-K) \times (M-K)$ matrix) contains the corresponding eigenvalues.

K) diagonal matrix) contains the $M - K$ smallest eigenvalues which are on the diagonal elements of the matrix. Hence, the signal subspace can be constructed from the column of $\mathbf{U}_s$ and the noise space from the columns of $\mathbf{U}_v$.

In practical applications, neither $\mathbf{R}_{ss}$ nor $\mathbf{R}_{xx}$ is available. Thus, the sample array covariance matrix

$$\hat{\mathbf{R}}_{xx} = \frac{1}{N} \sum_{n=1}^N \mathbf{x}[n] \mathbf{x}^H[n] \quad (2.20)$$

is used to estimate $\mathbf{R}_{xx}$ and N is the number of sample(s) collected from the receiver antenna element.

2.4 Beamforming

Figure 2-3: A narrowband beamforming structure with M antenna elements. The array output at the m th antenna at the n th snapshot is $x_m[n]$ which is then multiplied by w_m and summed up for $m = 1, \dots, M$ to form the beamformer output $y[n]$.

An array output can be a weighted linear combination of the output of each antenna element

$$y[n] = \sum_{m=1}^M w_m^* x_m[n] \quad (2.21)$$

where w_m are the complex weights of the beamformer shown in Figure 2-3. In vector notation, the output can be written as

$$y[n] = \mathbf{w}^H \mathbf{x}[n] \quad (2.22)$$

where the $M \times 1$ complex vector $\mathbf{w}$ contains the beamformer weights. The array's response to the receiving signals can be controlled by adjusting the elements of the weight vector. Such adjustment can be referred to as *spatial filtering* or *beamforming*.

The received array output $\mathbf{x}[n]$ consists of the signal-of-interest (SOI), J interferences, and spatially white Gaussian noise

$$\mathbf{x}[n] = s_0[n]\mathbf{a}(\theta_0) + \sum_{i=1}^J s_i[n]\mathbf{a}(\theta_i) + \mathbf{v}[n] \quad (2.23)$$

where θ_0 and θ_i are the DOAs of the SOI and the i th interference, respectively. s_0 and s_i refer to the waveforms of the SOI and i th interference, respectively. $\mathbf{v}[n]$ represents the spatially white Gaussian noise. The array steering vector $\mathbf{a}(\theta_i)$ corresponds to the i th signal source impinging from an angle θ_i on the array. The SOI and J interferences are assumed to be stationary, zero mean and mutually independent [35]. The array output power is $E\{|y[n]|^2\} = E\{|\mathbf{w}^H \mathbf{x}[n]|^2\} = \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w}$ where $|\cdot|$ is the absolute power operator and $\mathbf{R}_{xx}$ is the array covariance matrix in (2.17).

The most important performance measure of beamforming systems is the output SINR which is the ratio between the output power of SOI and the output power of the interference-plus-noise components. It can be expressed as

$$\text{SINR} = \frac{E\{|s_0[n]\mathbf{w}^H \mathbf{a}(\theta_0)|^2\}}{E\left\{\left|\sum_{i=1}^J s_i[n]\mathbf{w}^H \mathbf{a}(\theta_i) + \mathbf{w}^H \mathbf{v}[n]\right|^2\right\}} = \frac{\sigma_0^2 |\mathbf{w}^H \mathbf{a}(\theta_0)|^2}{\mathbf{w}^H \mathbf{R}_{in} \mathbf{w}} \quad (2.24)$$

where $\sigma_0^2 = E\{|s_0[n]|^2\}$ is the power of SOI and

$$\mathbf{R}_{in} = E\left\{\left[\sum_{i=1}^J s_i[n]\mathbf{a}(\theta_i) + \mathbf{v}[n]\right]\left[\sum_{i=1}^J s_i[n]\mathbf{a}(\theta_i) + \mathbf{v}[n]\right]^H\right\} \quad (2.25)$$

2.4.1 The Conventional Beamformer

For conventional beamformer, all the weights in the array are assigned with a magnitude of $\frac{1}{M}$ and a different phase

$$\mathbf{w} = \frac{1}{M} \mathbf{a}(\theta) = \frac{1}{M} [1, e^{-j\omega}, e^{-j2\omega} \dots e^{-jM\omega}] \quad (2.26)$$

Assuming only one signal is present without any noise, the conventional beamformer output would be

$$y[n] = \mathbf{w}^H \mathbf{x}[n] = s_0[n] \mathbf{w}^H \mathbf{a}(\theta) = s_0[n] \left(\frac{1}{M} \right) \mathbf{a}^H(\theta) \mathbf{a}(\theta) = s_0[n] \quad (2.27)$$

In this case, the beamformer is phase-aligned with the SOI and is undistorted at the output. Assuming that $E\{\mathbf{v}[n] \mathbf{v}^H[n]\} = \sigma_v^2 \mathbf{I}$ and knowing $\mathbf{a}^H(\theta) \mathbf{a}(\theta) = N$. The beamformer output is

$$y[n] = \mathbf{w}^H (\mathbf{a}(\theta) s_0[n] + \mathbf{v}[n]) = s_0[n] + \mathbf{w}^H \mathbf{v}[n] \quad (2.28)$$

The signal power is

$$E_s = E\{|s_0[n]|^2\} = \sigma_s^2 \quad (2.29)$$

and hence the noise power is

$$E_n = E\{|\mathbf{w}^H \mathbf{v}[n]|^2\} = \frac{1}{M^2} E\{\mathbf{a}(\theta)^H \mathbf{v}[n] \mathbf{v}^H[n] \mathbf{a}(\theta)\} = \frac{M \sigma_v^2}{M^2} \quad (2.30)$$

Therefore, the SNR for the overall beamformer output is $\frac{E_s}{E_n} = \frac{M \sigma_s^2}{\sigma_v^2}$. The SNR of a conventional beamformer increased by a factor of M over the SNR of a single antenna element.

Clearly, the conventional method has limitations. One of the main disadvantages is that the conventional beamformer is only optimal for one impinging signal in the received data.

2.4.2 The Minimum Variance Distortionless Response Beamformer

The most widely used spectral estimation algorithm is the minimum variance distortionless response (MVDR) [38], also known as the Capon's approach. MVDR is extensively studied in the literature and can be considered as a high-resolution method [39]. In filter design, MVDR is similar to an output of a bank of filters with individual filter centered at a particular frequency (or pointing to a specific DOA). The SOI with a given DOA passed through the filter undistorted but attenuate the ones with different DOAs as much as possible. The average periodogram-based approach has bandpass filters that are discretized Fourier matrix [40] and both data and frequency independent. MVDR is different from such an approach as the bandpass filters are both data and frequency dependent.

The MVDR beamformer is designed by minimizing the output power of the beamformer while constraining the gain to be one in the direction of SOI. The criterion of the MVDR algorithm can be written as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \hat{\mathbf{R}}_{xx} \mathbf{w} \\ \text{s. t.} \quad & \mathbf{w}^H \mathbf{a}(\theta) = 1 \end{aligned} \quad (2.31)$$

Using the Lagrange multiplier method, the Lagrangian for criterion (2.31) is given as

$$L(g) = \mathbf{w}^H \hat{\mathbf{R}}_{xx} \mathbf{w} - g(1 - \mathbf{w}^H \mathbf{a}(\theta)) \quad (2.32)$$

where g is the Lagrange multiplier and the solution to (2.31) is equating the gradient of (2.32) to zero

$$\nabla_{\mathbf{w}} L(g) = \hat{\mathbf{R}}_{xx} \mathbf{w} - g \mathbf{a}(\theta) = 0 \quad (2.33)$$

and thus, the optimal weight is

$$\mathbf{w}_{\text{opt}} = g \hat{\mathbf{R}}_{xx}^{-1} \mathbf{a}(\theta) \quad (2.34)$$

The Lagrange multiplier g is found by substituting (2.34) in the constraint of (2.31)

$$g = \frac{1}{\mathbf{a}^H(\theta) \hat{\mathbf{R}}_{xx}^{-1} \mathbf{a}(\theta)} \quad (2.35)$$

and by substituting (2.35) into (2.34), the weights of the MVDR are given by

$$\mathbf{w}_{\text{MVDR}} = \frac{\hat{\mathbf{R}}_{xx}^{-1} \mathbf{a}(\theta)}{\mathbf{a}^H(\theta) \hat{\mathbf{R}}_{xx}^{-1} \mathbf{a}(\theta)} \quad (2.36)$$

where $\mathbf{a}(\theta)$ is the steering vector corresponding to the SOI and $\mathbf{w}$ is the vector of complex weights. And hence the power spectral density (PSD) of MVDR beamformer is

$$\hat{\rho}_{\text{MVDR}}(\theta) = \frac{1}{\mathbf{a}^H(\theta) \hat{\mathbf{R}}_{xx}^{-1} \mathbf{a}(\theta)} \quad (2.37)$$

The MVDR beamformer represents a significant improvement over the conventional beamformer because, for a given DOA, it constrained the beamformer gain to be 1 in a particular direction and the output power from other signals coming from all other directions is being minimized. However, the MVDR beamformer is very sensitive to slight mismatches between the assumed and actual array response of the SOI ($\hat{\mathbf{a}}(\theta) \neq \mathbf{a}(\theta)$) and this caused the beamformer to treat the SOI as an interfering source and

therefore, suppressed it instead of maintaining a distortionless response on it. This is called signal cancellation or signal self-nulling.

2.4.3 Multiple Signal Classification Algorithm

A well-known subspace-based method named multiple signal classification (MUSIC) [26] is derived by enforcing the solution for the signal to be orthogonal to the noise subspace. Earlier in Section 2.3.4, the property of eigenvalues and eigenvectors of the spatial covariance matrix which consists of uncorrelated plane waves in the presence of white noise is discussed. For convenience, the eigenstructure of the covariance matrix is reproduced here

$$\mathbf{R}_{xx} = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{U}_n^H \quad (2.18)$$

MUSIC utilized the following subspace projection

$$\mathbf{A}^H \mathbf{U}_n = 0 \quad (2.38)$$

The columns of matrix $\mathbf{A}$, as defined by (2.16a) are orthogonal to the noise subspace and span the signal subspace. Therefore, the DOAs of the signal in the received data are the only solution to the equation

$$\mathbf{a}^H(\theta) \mathbf{U}_n \mathbf{U}_n^H \mathbf{a}(\theta) = 0 \quad (2.39)$$

when $M > K$. In practice, although $\mathbf{a}(\theta)$ is in the signal subspace, it will not achieve true orthogonality to the noise subspace due to errors in estimating $\mathbf{R}_{xx}$ (using $\hat{\mathbf{R}}_{xx}$ for (2.18) instead). However, the PSD of MUSIC,

$$\hat{p}_{\text{MUSIC}}(\theta) = \frac{1}{\mathbf{a}^H(\theta) \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}(\theta)} \quad (2.40)$$

will still produce a very large output when θ is equal to the DOAs of the signals in the received data as the noise eigenvalues are very small compared to the signal eigenvalues when using $\hat{\mathbf{R}}_{xx}$.

The MUSIC algorithm basically exploits (2.39) to find a set of signals that are orthogonal to the noise subspace as defined by the columns of $\mathbf{U}_n$. Note that to determine the noise space, the number of signals present in the received data needs to be known. This is the main disadvantage of MUSIC algorithm. The number of signals can be determined using information-theoretic approach such as Akaike information criterion (AIC) or minimum description length (MDL) proposed in [29]. However, these methods could not work in non-ideal scenarios such as model mismatch or closely located sources [30].

2.4.4 Cyclic Adaptive Beamformer

The conventional array beamforming methods mentioned in section 2.4.1 to 2.4.3 depend on the knowledge of the DOA in well-calibrated array. A training signal can be transmitted first so that the knowledge of the SOI's DOA is obtained. However, the interferences from the channel might disrupt the training signal. As a result, more bandwidth must be catered to adjust the duration of the training signal for interference mitigation. Thus, the estimation of the DOA itself requires a relatively high computational complexity. Significant research has been carried out exploiting the cyclostationarity of signals [1], [2] to combat the above-mentioned shortcomings of the conventional beamforming methods. Three self-coherent restoral (SCORE) blind adaptive array beamforming algorithms has been developed in [5] to maximize the

degree of cyclostationary at a particular frequency and time lag of the output signal. The main advantages of SCORE algorithms are:

1. No reference signal is required.
2. No advance knowledge of the correlation properties of noise and interference is needed.
3. No antenna array calibration is necessary.
4. Signal selectivity is achieved by using the knowledge of the cycle frequencies of the SOI.

In [6], three new cyclic adaptive beamforming (CAB) algorithms for extracting the SOI have been developed. The key advantages of the algorithms proposed are the faster convergence rates due to simpler implementation and higher SINR output compared to the SCORE algorithms. A class of CAB algorithms has been summarized in [32] as well. This thesis will focus on the CAB developed in [6].

The concept of the CAB algorithm is as follows: A data vector $\mathbf{x}[n]$ and its frequency and time shifted version $\mathbf{u}[n]$ would contain the signal components $s_k[n]$ and $s_k[n + n_0]e^{j2\pi\alpha_k n}$, respectively, and have high correlation value at the cycle frequency α_k of the signal. Therefore, the scalar signal $\hat{q}_k[n] = \mathbf{c}_k^H \mathbf{u}[n]$ must exist $\mathbf{w}_k$ and $\mathbf{c}_k$ such that the scalars $\hat{s}_k[n] = \mathbf{w}_k^H \mathbf{x}[n]$ and $\hat{q}_k[n]$ have high correlation value at α_k . More detailed explanation can be found in [6]. When the weighting $\mathbf{w}_k$ and $\mathbf{c}_k$ are properly chosen, the resulting signal $\hat{s}_k[n]$ can be taken as an estimate of the SOI $s_k[n]$. Therefore, continue from (2.4), the CAF is

$$R_{xx^{(*)}}^\alpha(\tau) = \langle x(t)[x^*]^{(*)}(t - \tau)e^{-j2\pi\alpha t} \rangle_\infty \quad (2.4)$$

if we let $u(t) = x(t - \tau)e^{-j2\pi\alpha t}$ then the CAF can be written as

$$R_{xu(*)} = \langle x(t)[u^*]^{(*)}(t) \rangle_{\infty} \quad (2.41)$$

In practice, an estimate of R_{xu} is obtained by taking an average over a finite number of N samples

$$\widehat{\mathbf{R}}_{xu} = \langle \mathbf{x}[n]\mathbf{u}^H[n] \rangle_N \quad (2.42)$$

where $\mathbf{x}_n$ is the array output in (2.16b). The criterion of the CAB algorithm [6] can be written as

$$\begin{aligned} \max_{\mathbf{w}, \mathbf{c}} & |\langle \mathbf{w}^H \mathbf{x}[n] \mathbf{u}^H[n] \mathbf{c} \rangle_N|^2 \\ &= \max_{\mathbf{w}, \mathbf{c}} |\mathbf{w}^H \widehat{\mathbf{R}}_{xu} \mathbf{c}|^2 \\ &= \max_{\mathbf{w}, \mathbf{c}} \mathbf{w}^H \widehat{\mathbf{R}}_{xu} \mathbf{c} \mathbf{c}^H \widehat{\mathbf{R}}_{xu}^H \mathbf{w} \\ \text{s. t. } & \mathbf{w}^H \mathbf{w} = \mathbf{c}^H \mathbf{c} = 1 \end{aligned} \quad (2.43)$$

where

$$\hat{s}_k[n] = \mathbf{w}_k^H \mathbf{x}[n] \quad (2.43a)$$

$$\hat{q}_k[n] = \mathbf{c}_k^H \mathbf{u}[n] \quad (2.43b)$$

Using the Lagrange multiplier method, the Lagrangian for criterion (2.43) is given as

$$L(g, g') = \mathbf{w}^H \widehat{\mathbf{R}}_{xu} \mathbf{c} \mathbf{c}^H \widehat{\mathbf{R}}_{xu}^H \mathbf{w} - g(\mathbf{w}^H \mathbf{w} - 1) - g'(\mathbf{c}^H \mathbf{c} - 1) \quad (2.44)$$

where g and g' are the Lagrange multipliers and the solution to (2.43) is equating the gradient of (2.44) to zero and therefore, obtaining the following conditions

$$\widehat{\mathbf{R}}_{xu} \mathbf{c} \mathbf{c}^H \widehat{\mathbf{R}}_{xu}^H \mathbf{w} = g \mathbf{w} \quad (2.45a)$$

$$\hat{\mathbf{R}}_{xu} \mathbf{w} \mathbf{w}^H \hat{\mathbf{R}}_{xu}^H \mathbf{c} = g' \mathbf{c} \quad (2.45b)$$

Combining (2.45a) and (2.45b),

$$\hat{\mathbf{R}}_{xu} \hat{\mathbf{R}}_{xu}^H \mathbf{w} = \psi \mathbf{w} \quad (2.46a)$$

$$\hat{\mathbf{R}}_{xu} \hat{\mathbf{R}}_{xu}^H \mathbf{c} = \psi \mathbf{c} \quad (2.46b)$$

where ψ is a positive constant. (2.46a) and (2.46b) indicate that $\mathbf{w}$ and $\mathbf{c}$ are, respectively, left and right singular vectors of the matrix $\hat{\mathbf{R}}_{xu}$. The optimal choices of $\mathbf{w}$ and $\mathbf{c}$ are given by

$$\mathbf{w}^H \hat{\mathbf{R}}_{xu} \mathbf{c} = \psi_{\max} \quad (2.47)$$

And hence, the weighting of CAB can be directly obtained by finding its left singular vector corresponding to the largest singular value of $\hat{\mathbf{R}}_{xu}$. Therefore, using singular vector decomposition (SVD),

$$\hat{\mathbf{R}}_{xu} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H \quad (2.48)$$

where $\mathbf{U} = [\xi_1, \xi_2, \dots, \xi_m]$ is a $M \times M$ matrix of left-singular vectors, $\mathbf{\Sigma} = \text{diag}[\mu_1, \mu_2, \dots, \mu_m]$ is a rectangular diagonal matrix and the diagonal entries are the singular values of $\hat{\mathbf{R}}_{xu}$, $\mathbf{V} = [\zeta_1, \zeta_2, \dots, \zeta_m]$ is a $M \times M$ matrix of right-singular vectors. The beamformer weight vector of CAB is

$$\mathbf{w}_{\text{CAB}} = \mathbf{U}(:, 1) \quad (2.49)$$

Note that vector $\mathbf{c}$ has no significant part in the algorithm implementation because $\mathbf{w}_{\text{CAB}}$ can be directly obtained by finding its left singular vector.

However, this method is designed to direct the main beam to the SOI's DOA without the suppression of interfering signals which resulted in performance degradation

under strong interference case. This leads to an improved CAB algorithm named constrained cyclic adaptive beamforming (C-CAB) [6]. C-CAB adopts the principle of linearly constrained MVDR in 2.4.2 and together with the weighting vector $\mathbf{w}_{\text{CAB}}$ to minimize the beamformer output power by constraining the distortionless response towards this vector. The beamformer weight vector of C-CAB is

$$\mathbf{w}_{\text{C-CAB}} = \hat{\mathbf{R}}_{\text{xx}}^{-1} \mathbf{w}_{\text{CAB}} \quad (2.50)$$

Chapter 3

Multiple Cycle Frequencies

Estimation Under Cochannel

Interference

3.1 Objectives

The main objective in this chapter is to develop algorithms which could estimate the number of unknown cyclostationary signals and their cycle frequencies in the received data under strong co-channel interference.

Signal parameter estimation such as cycle frequency (CF) estimation has been widely used in adaptive array beamforming, blind source separation, SS and carrier frequency estimation. Numerous cycle frequency estimators have been proposed in literature [10-15], [41-47]. In several applications, the SIR at the receiver might be very

small and therefore, one very important criterion for cycle frequencies estimation algorithms is the ability to work under strong co-channel interference. However, only a few CF estimators reported the performance with co-channel interference. In particular, [11] reported the effect of a co-channel BPSK interference on several existing methods [45-47]. The proposed method in [11] reported superior performance compared to the existing methods [45-47]. Hence, the proposed method is chosen as the representative of comparison in our study.

In this chapter, two cycle frequencies estimation algorithms were proposed. The first uses the dominant eigenvalue of the CAF to estimate the cycle frequency of SOI. The second method uses weight vectors derived from CAB to develop the unconstrained and constrained output powers. A new eigen-analysis cyclic null spectrum (ECNS) is produced to determine the number of cyclostationary signals and estimate their cycle frequencies. The performances of the proposed methods are demonstrated in 3.4 with numerical examples and compared with [11].

3.2 Numerical Study of the Multiple Cycle Frequency Estimation of Urriza *et al.*

The key idea of the proposed method in [11] relies on the canonical correlation analysis (CCA) [48] and the concept of factors. A signal component, which is common to two data sets, can be estimated from either one of each data set from a linear transformation is called a common factor [49]. The number of common factors is equal to the rank of the cross-correlation matrix of the two data sets since they are independent of each other. The number of common factors between two $M \times 1$ time series vectors $\mathbf{x}[n]$ and $\mathbf{y}[n]$ is the rank of the matrix [50]

$$\mathbf{R}_{CCA} = \mathbf{R}_{xx}^{-1} \mathbf{R}_{xy} \mathbf{R}_{yy}^{-1} \mathbf{R}_{yx} \quad (3.1)$$

where $\mathbf{R}_{xy} = \text{cov}(x, y)$ of the two random vectors $\mathbf{x}[n]$ and $\mathbf{y}[n]$.

The main objective of canonical analysis [48] is to find the relationships between two groups of variables in a data set. It searches for the most $\min\{u, q\}$ pairs of $(\mathbf{u}_i, \mathbf{q}_i)$, where u and q are the length of two random vectors $\mathbf{x}$ and $\mathbf{y}$ respectively, such that the correlation between the linear combinations, $U_i \triangleq \mathbf{u}_i^H \mathbf{x}[n]$ and $Q_i \triangleq \mathbf{q}_i^H \mathbf{y}[n]$, is maximized. This draws similarity on the rationale of the CAB algorithm mentioned in 2.4.4. U_i and Q_i must be uncorrelated with U_j and Q_j for $i \neq j$. The linear combinations are sorted in descending order of correlation such that U_1 and Q_1 have the highest correlation. The correlation coefficient, $\rho_i \triangleq \text{cov}(U_i, Q_i) / (\sigma_{U_i} \sigma_{Q_i})$, between U_i and Q_i is referred to as the i th canonical correlation. The procedure of finding $\mathbf{u}_i$ and $\mathbf{q}_i$ can be performed using SVD similar to (2.48) and the square of the canonical correlations can be found by finding the eigenvalues of (3.1).

For array beamforming, canonical analysis allows the combinations of all signals from the array of M antennas. Thus, the canonical correlations, ρ_i from up to M mutually uncorrelated cyclostationary signals in the received data can be tabulated. By performing canonical analysis on $x[n]$ and $x[n - \tau]e^{-j2\pi\alpha nT_s}$ for a given lag τ and CF α , the canonical correlations indicated the degree of cyclic correlation for all possible linear combinations of the received signals from the M antennas. Therefore, similar to the CAB algorithms, high canonical correlations occur at the CF of the signals.

The procedure to estimate the presence of a cyclostationary signal with CF, α_k , using the Eigenvalue-based Cyclostationary Spectrum Sensing (EV-CSS) is summarized as follows:

1. Estimate the covariance matrix of size $M \times M$ at time delay τ

$$\hat{\mathbf{R}}_{\text{xx}}(\tau) = \frac{1}{N} \sum_{n=1}^N \mathbf{x}[n] \mathbf{x}^T[n - \tau] \quad (3.2)$$

2. Estimate the cyclic correlation matrix using a cyclic-correlogram at CF α and time delay τ

$$\hat{\mathbf{R}}_{\text{xx}}^{\alpha}(\tau) = \frac{1}{N} \sum_{n=1}^N \mathbf{x}[n] \mathbf{x}^T[n - \tau] e^{-j2\pi\alpha n} \quad (3.3)$$

Let the conventional and CAF matrices be $\hat{\mathbf{R}}_{\text{xx}}$ and $\hat{\mathbf{R}}_{\text{xx}}^{\alpha}$, respectively

$$\hat{\mathbf{R}}_{\text{CCA}} = \hat{\mathbf{R}}_{\text{xx}}^{-1} \hat{\mathbf{R}}_{\text{xx}}^{\alpha} \hat{\mathbf{R}}_{\text{xx}}^{-1} (\hat{\mathbf{R}}_{\text{xx}}^{\alpha})^H \quad (3.4)$$

3. Find the eigenvalues, $\mu = [\mu_1^2, \mu_2^2, \dots, \mu_m^2]^T$, of $\hat{\mathbf{R}}_{\text{CCA}}$.
4. Combine the eigenvalues as

$$\varrho = \prod_{i=1}^M (1 - \mu_i^2) \quad (3.5)$$

and finally, the canonical correlation:

$$\hat{\rho}_{\text{EV-CSS}} = -(N - M - 1) \ln \varrho \quad (3.6)$$

5. Select the estimated CF

$$\hat{\alpha}_k = \max_{\alpha} \hat{\rho}_{\text{EV-CSS}}(\alpha, \tau) \quad (3.7)$$

where $\hat{\alpha}_k$ is the estimated CF.

3.2.1 Comparison with CAF CF estimation Algorithm

By exploiting the use of an antenna array, CAF for each antenna in the array can be computed. The CF, α_k , of SOI can be estimated by summing up the CAF of all M channels. This will decrease the cyclic correlation when $\alpha \neq \alpha_k$ and more importantly, increase the cyclic correlation when $\alpha = \alpha_k$. The summation CAF CF estimation algorithm is

$$\hat{\rho}_{\text{CAF}}(\alpha, \tau) = \sum_{m=1}^M \sum_{n=1}^N |x_m[n] x_m^T[n - \tau] e^{-j2\pi\alpha n}| \quad (3.8)$$

$$\hat{\alpha}_k = \max_{\alpha} \hat{\rho}_{\text{CAF}}(\alpha, \tau) \quad (3.9)$$

where $\hat{\alpha}_k$ is the estimated CF. Note that for $K > 1$, the k th peak in $\hat{\rho}_{\text{CAF}}$ represents the k th signal. For BPSK signal, cyclostationary features exist at $\alpha = \pm 2f_c$. Figure 3-1 shows the summation CAF CF estimation algorithm using (3.8) versus the CAF of the k th antenna element of the array. The summation CAF algorithm estimated a peak at the correct CF of $0.1f_s$ for a simulated BPSK signal with a carrier frequency of $0.05f_s$. However, the CAF output from the k th antenna element computes a different maximum peak's location which is not at the correct CF of the simulated BPSK signal.

Next, the comparison of the EV-CSS CF estimation algorithm and the summation CAF algorithm is studied in two scenarios in Figure 3-2 and Figure 3-3. A ULA of $m = 6$ omnidirectional sensors spaced half-wavelength apart is considered. In both scenarios, one interference arrives with a DOA of 20° in spatially white Gaussian noise of unit variance. The interference is a simulated BPSK signal with a carrier frequency of

Figure 3-1: k th antenna element CAF output versus summation of M antenna elements CAF output ($M = 4$, SNR = -10 dB, $N = 500$)

$0.2f_s$. Hence, the CF of the interference signal is located at $0.4f_s$. The DOA of the SOI is 20° with a carrier frequency of $0.05f_s$ and hence, the CF of the SOI is located at $0.1f_s$. In each scenario, there are two cases. In the first scenario shown in Figure 3-2, both cases have the same sample size $N = 1000$. The INR is 10 dB and 15 dB in Case 1 and Case 2, respectively. The EV-CSS algorithm can locate the SOI's CF accurately as reported in [11] with $N = 1000$. The SUM. CAF algorithm has similar performance as the EV-CSS algorithm. However, in Case 2 when the INR is increased from 10 dB to 15 dB, both algorithms could not locate the SOI's CF correctly.

In the second scenario shown in Figure 3-3, both cases have the same INR of 10 dB. The sample size is $N = 1000$ and $N = 500$ in Case 1 and Case 2, respectively. The EV-CSS algorithm can locate the SOI's CF accurately again as the simulated condition is the same as Case 1 of scenario 1. However, in Case 2 when the sample size is reduced from 1000 to 500, both algorithms could not locate the SOI's CF correctly.

In conclusion, both the EV-CSS algorithm and SUM. CAF algorithm is not able to estimate the CF of the SOI when the INR increases (practical case as the SIR at the receiver will be small) or when the sample size reduces (practical case as sensing duration must be fast). Thus, this motivates the need in this thesis to develop CF estimation algorithm that could estimate with small sample size in the presence of strong interfering signal.

Figure 3-2: Comparison of EV-CSS algorithm [11] with summation CAF algorithm in Case 1 with INR = 10 dB and Case 2 with INR = 15 dB.

Figure 3-3: Comparison of EV-CSS algorithm [11] with summation CAF algorithm in Case 1 with $N = 1000$ and Case 2 with $N = 500$.

3.3 Proposed Multiple Cycle Frequencies Estimation Algorithms Under Cochannel Interference

In this section, the proposed methods are described. The proposed dominant eigenvalue (EV) method is first presented and extended to the proposed eigen-analysis cyclic null spectrum (ECNS) method. The ECNS method is based on the unconstrained and constrained cyclic adaptive beamformer weights. The combination of the beamformer outputs with the respective weights produce a new cyclic null spectrum which could estimate the cyclostationary features in the array's received data. The processing block diagram of the proposed algorithms are shown in Figure 3-4.

3.3.1 Dominant Eigenvalue (EV) Algorithm

In 2.4.4, the weighting of CAB can be directly obtained by finding its left singular vector corresponding to the largest singular value of $\hat{\mathbf{R}}_{xu}$ in (2.47). Therefore, the largest singular value of $\hat{\mathbf{R}}_{xu}$ is a clear indication of high correlation at the CF of the SOI ($\alpha = \alpha_k$).

The procedure to estimate the presence of a cyclostationary signal with CF, α_k , using the dominant eigenvalue (EV) algorithm is summarized as follows:

1. Estimate the sample conjugate CAF matrix for a certain α and τ

$$\hat{\mathbf{R}}_{xx^*}^{\alpha}(\tau) = \frac{1}{N} \sum_{n=1}^N \mathbf{x}[n] \mathbf{x}^T[n - \tau] e^{-j2\pi\alpha n} \quad (3.10)$$

If we let $\mathbf{u}[n] = \mathbf{x}[n - \tau]e^{-j2\pi\alpha n}$ then the sample conjugate CAF matrix can be written as $\hat{\mathbf{R}}_{xu}$ (2.42).

2. Perform SVD on $\hat{\mathbf{R}}_{xu}$

$$\hat{\mathbf{R}}_{xu} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad (3.11)$$

where $\mathbf{U} = [\xi_1, \xi_2, \dots, \xi_m]$ is a $M \times M$ matrix of left-singular vectors, $\mathbf{\Sigma} = \text{diag}[\mu_1, \mu_2, \dots, \mu_m]$ is a rectangular diagonal matrix and the diagonal entries are the singular values of $\hat{\mathbf{R}}_{xu}$, $\mathbf{V} = [\zeta_1, \zeta_2, \dots, \zeta_m]$ is a $M \times M$ matrix of right-singular vectors.

3. Compute the dominant eigenvalue spectrum

$$\hat{\rho}_{EV}(\alpha, \tau) = \mathbf{\Sigma}(1,1) \quad (3.12)$$

4. Select the estimated CF

$$\hat{\alpha}_k = \max_{\alpha} \hat{\rho}_{EV}(\alpha, \tau) \quad (3.13)$$

where $\hat{\alpha}_k$ is the estimated CF.

Similar eigenanalysis-based method to the proposed dominant EV method is reported in [51]. Instead of using the dominant eigenvalue, the dominant eigenvector is used to estimate the channel state information (CSI) under the proposed blind channel estimation based on CAF in [51]. A combining output is computed using the CSI and finally, cyclic SCF is performed on the combining output. Therefore, the complexity of the method in [51] is much higher than the proposed dominant EV method. A study on the complexity of the algorithms is found in the subsequent Section 3.3.3.

Figure 3-4: Processing block diagram for the proposed methods in Chapter 3: Dominant Eigenvalue Spectrum and Eigen-analysis Cyclic Null Spectrum (ECNS).

3.3.2 Eigen-analysis Cyclic Null Spectrum (ECNS)

The key difference between the beamformers mentioned in section 2.4 and the eigen-analysis cyclic null spectrum (ECNS) is that the former algorithm output is an estimated signal, while the latter algorithm output is an estimated cycle frequency. A two-stage cycle frequencies estimation framework is proposed in Figure 3-4. At the first stage, a cycle frequency is used as an input for the cyclostationary beamformer to compute the cyclostationary beamforming weight vector. At the second stage, the concept of DOA estimation is adopted. For DOA estimation, the weight vector of the beamformer is used to plot the beampattern with the DOA (x-axis) against the gain of the beamformer (y-axis). Therefore, at the second stage of the proposed cycle frequencies estimation framework, the cyclostationary beamforming weight vector is used to compute the unconstrained and constrained output power. Thus, the unconstrained and constrained output power (y-axis) can be plotted against the input cycle frequency (x-axis). Note that there are several blind cyclostationary beamforming algorithms in the literature [32] which use different computation of weighting vectors. In this thesis, the weighting vector of CAB [6] is chosen.

The unconstrained output power of the CAB beamformer follows the implementation of the conventional beamformer in 2.4.1 except that the CAB weight vector (2.49) is used instead. The unconstrained output power with a given α and τ is

$$\hat{\rho}_{UC}(\alpha, \tau) = E\{\mathbf{w}_{CAB}^H \mathbf{x}[n] \mathbf{x}^H[n] \mathbf{w}_{CAB}\} = \mathbf{w}_{CAB}^H \hat{\mathbf{R}}_{xx} \mathbf{w}_{CAB} \quad (3.14)$$

The constrained output power of the CAB beamformer follows the implementation of the MVDR beamformer in 2.4.2. The criterion of the constrained output power can be written as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \hat{\mathbf{R}}_{xx} \mathbf{w} \\ \text{s. t.} \quad & \mathbf{w}^H \mathbf{w}_{\text{CAB}} = 1 \end{aligned} \quad (3.15)$$

Using the Lagrange multiplier method, the optimal weight is

$$\mathbf{w}_{\text{C-CAB}} = \frac{\hat{\mathbf{R}}_{xx}^{-1} \mathbf{w}_{\text{CAB}}}{\mathbf{w}_{\text{CAB}}^H \hat{\mathbf{R}}_{xx}^{-1} \mathbf{w}_{\text{CAB}}} \quad (3.16)$$

And hence, the constrained output power is

$$\hat{\rho}_C(\alpha, \tau) = \mathbf{w}_{\text{C-CAB}}^H \hat{\mathbf{R}}_{xx} \mathbf{w}_{\text{C-CAB}} = \frac{1}{\mathbf{w}_{\text{CAB}}^H \hat{\mathbf{R}}_{xx}^{-1} \mathbf{w}_{\text{CAB}}} \quad (3.17)$$

A new eigen-analysis based cyclic null spectrum (ECNS) to estimate the CF is obtained by taking the difference between the unconstrained and constrained output powers in (3.14) and (3.17), respectively. Therefore, the ECNS is

$$\hat{\rho}_{ECNS}(\alpha, \tau) = \left| \frac{1}{\hat{\rho}_{UC}(\alpha, \tau) - \hat{\rho}_C(\alpha, \tau)} \right| \quad (3.18)$$

Let $|\hat{\rho}_{UC}(\alpha, \tau) - \hat{\rho}_C(\alpha, \tau)|$ be S_{ECNS}

$$S_{ECNS} = \left| \mathbf{w}_{\text{CAB}}^H \hat{\mathbf{R}}_{xx} \mathbf{w}_{\text{CAB}} - \frac{1}{\mathbf{w}_{\text{CAB}}^H \hat{\mathbf{R}}_{xx}^{-1} \mathbf{w}_{\text{CAB}}} \right| \quad (3.19)$$

$$= \left| \frac{(\mathbf{w}_{\text{CAB}}^H \hat{\mathbf{R}}_{xx} \mathbf{w}_{\text{CAB}}) \cdot (\mathbf{w}_{\text{CAB}}^H \hat{\mathbf{R}}_{xx}^{-1} \mathbf{w}_{\text{CAB}}) - 1}{\mathbf{w}_{\text{CAB}}^H \hat{\mathbf{R}}_{xx}^{-1} \mathbf{w}_{\text{CAB}}} \right| \quad (3.20)$$

And let $\mathbf{w}_{\text{CAB}}^H \hat{\mathbf{R}}_{xx} \mathbf{w}_{\text{CAB}}$ be p and $\mathbf{w}_{\text{CAB}}^H \hat{\mathbf{R}}_{xx}^{-1} \mathbf{w}_{\text{CAB}}$ be h

$$S_{ECNS} = \left| \frac{ph - 1}{h} \right| \quad (3.21)$$

when $\alpha = \hat{\alpha}_k$

$$p \approx \sigma_s^2 \quad (3.22)$$

$$h \approx 1/\sigma_s^2 \quad (3.23)$$

$$\lim_{N \rightarrow \infty} S_{ECNS} = 0 \quad (3.24)$$

And the estimated CF would be

$$\hat{\alpha}_k = \max_{\alpha} \hat{\rho}_{ECNS}(\alpha, \tau) \quad (3.25)$$

Note that in (3.21), when the term ph is near to 1, the ECNS output high peaks which are possible solutions for (3.25). When the weighting vector $\mathbf{w}_{CAB}$ is a consistent estimate of $a(\theta)$ in (2.15), ph approaches a near unity value. Likewise, a mismatch of weighting vector will result in having a higher ph value than 1 and hence, a smaller $\hat{\rho}_{ECNS}$ output.

The procedure to estimate the presence of a cyclostationary signal with CF, α_k , using the eigen-analysis cyclic null spectrum (ECNS) algorithm is summarized as follows:

1. Estimate the sample conjugate CAF matrix for a certain α and τ

$$\hat{\mathbf{R}}_{xx^*}^{\alpha}(\tau) = \frac{1}{N} \sum_{n=1}^N \mathbf{x}[n] \mathbf{x}^T[n - \tau] e^{-j2\pi\alpha n} \quad (3.26)$$

If we let $\mathbf{u}[n] = \mathbf{x}[n - \tau] e^{-j2\pi\alpha n}$ then the sample conjugate CAF matrix can be written as $\hat{\mathbf{R}}_{xu}$ (2.42).

2. Perform SVD on $\hat{\mathbf{R}}_{xu}$ and compute the weighting vector $\mathbf{w}_{CAB}$

$$\hat{\mathbf{R}}_{xu} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T \quad (3.27)$$

$$\mathbf{w}_{CAB} = \mathbf{U}(:, 1) \quad (3.28)$$

where $\mathbf{U} = [\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \dots, \boldsymbol{\xi}_m]$ is a $M \times M$ matrix of left-singular vectors, $\mathbf{\Sigma} = \text{diag}[\mu_1, \mu_2, \dots, \mu_m]$ is a rectangular diagonal matrix and the diagonal entries are the singular values of $\hat{\mathbf{R}}_{xu}$, $\mathbf{V} = [\boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2, \dots, \boldsymbol{\zeta}_m]$ is a $M \times M$ matrix of right-singular vectors.

3. Compute $\hat{\rho}_{UC}(\alpha, \tau)$ using (3.18)
4. Compute $\hat{\rho}_C(\alpha, \tau)$ using (3.18)
5. Compute $\hat{\rho}_{ECNS}(\alpha, \tau)$ using (3.18)
6. Select the estimated CF

$$\hat{\alpha}_k = \max_{\alpha} \hat{\rho}_{ECNS}(\alpha, \tau) \quad (3.29)$$

where $\hat{\alpha}_k$ is the estimated CF.

3.3.3 Complexity Study on the Proposed Algorithms

An approximate complexity comparison of the proposed methods is made against the best performing existing method (EV-CSS) studied in this thesis by taking the number of complex multiplications required for each under the same number of sample size N . The CAF operation and SVD are common to the EV-CSS and the proposed methods and hence, are not included in the analysis. EV-CSS approach requires multiplications as stated in [11] in addition to the common operation. For the $NM^2 + 2M^3/3$ proposed ECNS method, the number of operations required to find the $\mathbf{w}_{C-CAB}$ weights in (3.16) is $2M^3/3$ using Gaussian elimination [52]. The weights, $\mathbf{w}_{CAB}$, in (3.14) is computed in the SVD common operation. Therefore, the subtraction of the output powers in (3.14) and (3.16) requires $M^2 + 2M^3/3 + N$ operations. There is overall a significant decrease in complexity in the proposed ECNS method as SVD is not calculated twice here, as opposed to EV-CSS. For example, if we take $N = 1000$ and $M = 6$, the EV-CSS requires $\sim 36K$ multiplications while ECNS requires only $\sim 1K$ multiplications, without adding the common operations. The proposed dominant eigenvalue (EV) method requires the least operations as it only used the common operations to obtain the dominant eigenvalue.

The proposed methods in this thesis has lower computational complexity than existing methods in literature using SCD function [31], [46], [51]- [53]. The proposed methods only compute the cyclic covariance matrix once in time domain and do not use a high frequency resolution FFT method like the SCD with large sample size N . The computation complexity of SCD is $N^2 + \left(\frac{N}{2}\right) \log_2 N$. Table 3-1 provides a summary of algorithmic complexity.

Furthermore, for a given $\hat{\mathbf{R}}_{xu}$ in (2.48), a fast algorithm has been developed in [6] of calculating $\mathbf{w}_{CAB}$ which reduces the computational complexity significantly from M^3

multiplications to only $M(M - 1)$ summations. Therefore, using CAF in general has a huge reduction in computational complexity compared to the SCF operation.

Table 3-1: Summary of algorithmic complexity

Algorithm	Complexity	$M = 6, N = 1000$
ECNS	$M^2 + 2M^3/3 + N$	1180
EV-CSS	$NM^2 + 2M^3/3$	36144
SCD	$N^2 + \left(\frac{N}{2}\right) \log_2 N$	1.005×10^6

3.4 Simulation Results and Discussion

In this section, the comparison of the conventional DOA estimation power spectrums and the proposed eigen-analysis cyclic null spectrum is studied in two scenarios. A ULA of 10 isotropic antenna elements sensors spaced half-wavelength apart is considered. In both scenarios, three BPSK signals arrives with DOAs of 0° , 60° and -60° , respectively and with carrier frequency of $0.05f_s$, $0.15f_s$ and $0.2f_s$, respectively. Hence the cycle frequencies of the three BPSK signals are $0.1f_s$, $0.3f_s$ and $0.4f_s$, respectively. All the scenarios are simulated in spatially white Gaussian noise of unit variance. The sample size is $N = 1000$.

Two different regions, A and B, are used to illustrate the performance of the conventional DOA estimation spectrums and the proposed ECNS. For the conventional DOA estimation methods, region A is defined as the region where the estimated DOA of the SOI matches the true DOA of the SOI. Region B is defined as the region where the estimated DOA of the SOI does not match the true DOA of the SOI. For the proposed

ECNS method, region A is defined as the region where the cycle frequency of the SOI matches the true cycle frequency of the SOI. Region B is defined as the region where the cycle frequency of the SOI does not match the true cycle frequency of the SOI. Table 3-2 described the conditions of the region A and region B for both the simulated scenarios.

In the first scenario, all the simulated signals have SNRs of 0 dB. For the signal parameter estimation on DOA, both the conventional beamformer and the MVDR can locate three distinct high peaks at region A and flat low output are observed at region B in Figure 3-5. For the signal parameter estimation on CF, the unconstrained output power (3.14) did not output any distinct peaks in regions A and B and the constrained output power (3.17) indicates peaks in region A and numerous smaller peaks in region B in Figure 3-6. However, in Figure 3-7, the proposed ECNS algorithm outputs three distinct peaks with respect to the cycle frequencies of the simulated signals.

In the second scenario, one of the simulated signals ($\text{DOA} = 60^\circ$, $\text{CF} = 0.4f_s$) has a SNR of 10 dB and the rest of the signals have SNRs of 0 dB. For the signal parameter estimation on DOA, both the conventional beamformer and the MVDR in Figure 3-8 can locate peaks at region A. Flat low output is observed as well at region B in Figure 3-8. For the signal parameter estimation on CF in Figure 3-9, the unconstrained output power (3.14) output a distinct peak for the higher powered signal ($\text{SNR} = 10 \text{ dB}$) and two “inverted peaks” for the lower powered signals ($\text{SNR} = 0 \text{ dB}$) in region A and a consistently flat output in region B. The constrained output power (3.17) in Figure 3-9 indicates a high distinct peak for the higher powered signal ($\text{SNR} = 10 \text{ dB}$) and did not output any peaks for the lower power signals ($\text{SNRs} = 0\text{dB}$) in region A and has similar low output in region B. However, in Figure 3-10, the proposed ECNS algorithm outputs three distinct peaks with respect to the cycle frequencies of the simulated signals. In

conclusion, the proposed ECNS algorithm consistently output three distinct peaks even if the SNRs of the received signal varies. This is critical for signal parameter estimation algorithms as the SIR at the receiver might be small.

Table 3-2: Conditions of region A and region B on different signal parameters estimation algorithms.

Signal Parameter For Estimation	Algorithm	Region A	Region B
DOA	Conventional BF	$\hat{\theta}_k = \theta_k$	$\hat{\theta}_k \neq \theta_k$
DOA	MVDR	$\hat{\theta}_k = \theta_k$	$\hat{\theta}_k \neq \theta_k$
CF	Unconstrained Power	$\hat{\alpha}_k = \alpha_k$	$\hat{\alpha}_k \neq \alpha_k$
CF	Constrained Power	$\hat{\alpha}_k = \alpha_k$	$\hat{\alpha}_k \neq \alpha_k$

Figure 3-5: Power spectrum of the conventional beamformer and the MVDR in Scenario 1

Figure 3-6: Unconstrained and constrained output powers CAB beamformer in scenario 1.

Figure 3-7: Proposed eigen-analysis cyclic null spectrum (ECNS) in scenario 1 (Difference of the unconstrained and constrained output powers in Figure 3-6.)

Figure 3-8: Power spectrum of the conventional beamformer and the MVDR in Scenario 2

Figure 3-9: Unconstrained and constrained output powers of the proposed ECNS algorithm in scenario 2.

Figure 3-10: Proposed eigen-analysis cyclic null spectrum (ECNS) in scenario 2 (Difference of unconstrained and constrained output powers in Figure 3-9).

3.4.1 RMSE Comparisons of Dominant EV and ECNS Versus INR under Co-channel Interference

In this section, the RMSE comparison of dominant EV and ECNS is studied in two scenarios. A ULA of $m = 6$ omnidirectional sensors spaced half-wavelength apart is considered. There is one SOI and two interfering signals in all simulations. All signals are BPSK. The DOA of the SOI is 10° . The DOA of interfering signal I_1 is 30° and I_2 is 50° . The SOI carrier frequency is $0.05f_s$. The interfering signal I_1 carrier frequency is $0.15f_s$ and I_2 is $0.2f_s$. Therefore, the SOI cycle frequency is at $\alpha_0 = 0.1f_s$ and the interfering signal I_1 cycle frequency is at $\alpha_{I1} = 0.3f_s$ and I_2 is at $\alpha_{I2} = 0.4f_s$. These cyclic features are only present in the conjugate CAF. For BPSK modulation, the maximum self-coherence occurs at $\tau = 0$ [5] and hence $\tau = 0$ is used in all experiments unless otherwise stated. The noise variance is assumed to be equal to one in each sensor.

In the first scenario, the SNR of all interfering signals and SOI is 0 dB. Therefore, the SIR is 0 dB as well. The proposed methods (EV and ECNS) are compared with the existing methods (SUM, CAF and EV-CSS) methods in Figure 3-11 at a single Monte Carlo run. In Figure 3-11, all the methods estimate three distinct peaks at $\alpha_0 = 0.1f_s$, $\alpha_{I1} = 0.3f_s$ and $\alpha_{I2} = 0.4f_s$.

In the second scenario, one of the interfering signals (I_1) is stronger than the SOI (SIR = -15 dB). The proposed methods (EV and ECNS) are compared with the existing methods (SUM, CAF and EV-CSS) methods in Figure 3-12 at a single Monte Carlo run. In Figure 3-12, only the eigen-analysis cyclic null spectrum could estimate three distinct peaks at $\alpha_0 = 0.1f_s$, $\alpha_{I1} = 0.3f_s$ and $\alpha_{I2} = 0.4f_s$. Both the summation CAF and EV-CSS methods could only estimate accurately the strong interfering signal at $\alpha_{I1} = 0.4f_s$.

The dominant EV method could estimate the weaker signals but the estimated peaks are not as distinct compared to the ECNS method.

Figure 3-13 shows the root mean square errors (RMSE) comparisons of different CF estimation methods with varying SNR (SIR = -15 dB) by averaging 10000 independent Monte Carlo runs. The RMSE computation is as follows:

$$RMSE = \sqrt{E\{(\hat{\alpha}_k - \alpha_k)^2\}} \quad (3.30)$$

The proposed method ECNS exhibits the best performance compared to all other methods.

Figure 3-11: Comparisons of different CF estimation methods (SIR = 0 dB, $N = 500$, $\alpha_0 = 0.1f_s$, $\alpha_{I1} = 0.3f_s$, $\alpha_{I2} = 0.4f_s$).

Figure 3-12: Comparisons of different CF estimation methods (SIR = -15 dB, $N = 500$, $\alpha_0 = 0.1f_s$, $\alpha_{I1} = 0.3f_s$, $\alpha_{I2} = 0.4f_s$).

Figure 3-13: RMSE comparisons of different CF estimation methods with varying SNR (SIR = -15 dB, $N = 500$).

3.4.2 RMSE Comparisons of Dominant EV and ECNS Versus Sample Size under Co-channel Interference

In this section, a ULA of $m = 6$ omnidirectional sensors spaced half-wavelength apart is considered. There is one SOI and two interfering signals in all simulations. All signals are BPSK. The DOA of the SOI is 10° . The DOA of interfering signal I_1 is 30° and I_2 is 50° . The SOI carrier frequency is $0.05f_s$. The interfering signal I_1 carrier frequency is $0.15f_s$ and I_2 is $0.2f_s$. Therefore, the SOI cycle frequency is at $\alpha_0 = 0.1f_s$ and the interfering signal I_1 cycle frequency is at $\alpha_{I1} = 0.3f_s$ and I_2 is at $\alpha_{I2} = 0.4f_s$. These cyclic features are only present in the conjugate CAF. For BPSK modulation, the maximum self-coherence occurs at $\tau = 0$ [5] and hence $\tau = 0$ is used in all experiments unless otherwise stated. The noise variance is assumed to be equal to one in each sensor. The simulation results are obtained by averaging 10000 independent Monte Carlo runs.

Figure 3-14 shows the root mean square errors (RMSE) comparisons of different CF estimation methods with varying sample size. The SNR of the SOI is fixed at 0 dB. The RMSE of eigen-analysis cyclic null spectrum method improve significantly when the number of sample size increases.

Figure 3-14: RMSE comparisons of different CF estimation methods with varying sample size N (SIR = -15 dB).

3.5 Summary

In this chapter, two fast multiple antenna cyclostationary-based CF estimation methods namely dominant EV and ECNS are proposed. The first proposed algorithm, named the dominant eigenvalue (EV) method, uses the dominant eigenvalue of the CAF matrix to estimate the cycle frequencies of cyclostationary signals. The largest singular value of the CAF matrix is a clear indication of high correlation at the CF of the SOI. It has the least complexity from the algorithmic complexity study.

The second proposed algorithm, name ECNS, consists of a novel two-stage CF estimation framework. At the first stage, it computes the weight vector from the cyclostationary beamforming algorithm named CAB. At the second stage, it uses the CAB weight vectors to compute the unconstrained and constrained output powers to form an eigen-analysis cyclic null spectrum (ECNS) which could estimate the cycle frequencies of cyclostationary signals. Due to the criterions of the unconstrained and constrained output powers, the ECNS algorithm could perform under co-channel interference.

The proposed methods have been investigated by numerical experiments. The proposed methods not only can achieve higher estimation accuracy under strong co-channel interference condition, they can also estimate faster using fewer samples compared to existing cyclostationary cycle frequency estimation methods.

Chapter 4

Multiple Cycle Frequencies Detection Under Cochannel Interference

4.1 Objectives

The main objective in this chapter is to modify the algorithms proposed in Chapter 3 to detect the number of unknown cyclostationary signals in the received data under strong co-channel interference.

Two main characteristics of cognitive radio (CR) are cognitive capability and reconfigurability [54]. The cognitive cycle includes three major components [54], [55]: SS, cognition/management, and control action. In this chapter, developing algorithms for SS is the main objective. The schemes SS is used to search for unused frequency

bands can be either proactive or reactive. A detector is developed to identify if a sensed false alarm corresponds to just noise, or more importantly, a primary user (PU) or a secondary user. The individual CR should also be able to cooperate with other CRs for the differentiation between CRs and SUs. Features can be extracted from the received data using time or frequency domain signal processing techniques to identify the type of PUs presence. The features can also be used for classification known as signal detection. Usually, there are available on the PUs transmission properties and hence, the PUs can be identify through these associated properties. Cyclostationary feature detection improves detection performance by exploiting the associated cyclostationary properties of the PUs signal. In [59]-[60], low SNR signal with strong cyclostationary properties can be detected with *a priori* information of the modulation format.

One of the major challenges of SS in CR is the hidden terminal problem [21]. Cooperative SS, [56]-[58], is applied to solve the hidden-terminal problem by allowing several CRs cooperating together. Hence, cooperative detection can help to overcome the multipath fading and shadowing effect. However, due to the resources used for cooperating the CRs, there is an increase in complexity, extra traffic overhead, and the need for a control channel when using cooperative SS. In some practical applications, the availability of several CRs is not available. Thus, this chapter focuses on only using non-cooperative SS to solve the hidden-terminal problem.

Beside the hidden terminal problem, another challenge for SS is to develop fast blind sensing techniques that can detect very weak PU signals with low implementation cost. In this chapter, the main objective is to use blind adaptive cyclostationary beamforming to perform SS for multiple-antenna CR, which is assumed to have limited *a priori* knowledge of the PU's signal characteristics.

4.2 Background

4.2.1 Signal Detection and Classification

As stated in the earlier section, SS is the most important function of cognitive radio. If CR wrongly decides that the PU is not in the band and starts to transmit, it destroys the PU signal. For conventional SS, the problem is to decide between two hypotheses: A null hypothesis $\mathcal{H}_0$ where the signal is absent and a hypothesis $\mathcal{H}_1$ where the signal is present. Therefore, the signal detection problem can be modeled by a simple binary hypothesis testing problem as

$$x(t) = \begin{cases} v(t) & \mathcal{H}_0 \\ hs(t) + v(t) & \mathcal{H}_1 \end{cases} \quad (4.1)$$

where $x(t)$ is the received signal, $s(t)$ is the PU signal, h is the amplitude gain of the channel, and $v(t)$ is the additive Gaussian noise (AWGN).

In detection theory [61], there is a need to identify which *hypothesis* is true in (4.1)

$$\mathcal{H}_0: \alpha \in \theta_0 \quad (4.2)$$

$$\mathcal{H}_1: \alpha \in \theta_1$$

where $\mathcal{H}_0$ is a null hypothesis where PU signal is absent and $\mathcal{H}_1$ is an alternative hypothesis which PU signal is present. α is estimated parameter $\alpha \in \theta$ given a set of data such as the received array output. Hence the decision rule shown in Figure 4-1 is

$$\phi(x) = \begin{cases} 1, & \text{decide } \mathcal{H}_0 \\ 0, & \text{decide } \mathcal{H}_1 \end{cases} \quad (4.3)$$

which partitions the data space χ into two:

$$\chi_0 = \{x: \phi(x) = 0\} \quad (4.4)$$

$$\chi_1 = \{x: \phi(x) = 1\}$$

The probability of false alarm can be defined as

$$P_{FA} = \int_{\chi_1} p(x|\theta) dx \quad \text{where } \theta \in \theta_0 \quad (4.5)$$

And the probability of detection (correctly deciding $\mathcal{H}_1$) is

$$P_D = \int_{\chi_1} p(x|\theta) dx \quad \text{where } \theta \in \theta_1 \quad (4.6)$$

Figure 4-1 illustrates the probabilities of false alarm and detection with the test statistic.

Figure 4-1: Probabilities of false alarm (P_{FA}) and detection (P_D) for simple hypothesis.

However, to better demonstrate the detector performance under co-channel interference, a new signal detection problem is used in this chapter. In order to detect spectrum holes, each SU decides between two following hypothesis that represent the presence or absence of the PU signal

$$x(t) = \begin{cases} v(t) & \mathcal{H}_0 \\ \sum_{k=1}^P h_k s_k(t) + \sum_{k=1}^I h_k s_k(t) + v(t) & \mathcal{H}_1 \end{cases} \quad (4.7)$$

where the received signal is the superposition of P signal sources (PU transmissions) and I signal sources (interfering signals). In SS, the CRs can represent the interfering signals. Figure 4-2 illustrate the hidden primary user problem where the dashed circles show the operating ranges of the PU and the CR device. The CR device causes unwanted strong interference to the PU (receiver) as the PU transmitter's signal could not be detected because of the locations of devices. Hence, the two hypotheses in (4.7) illustrate the hidden primary user problem in CR where there is an important need to detect primary user under strong co-channel interference.

Figure 4-2: Illustration of hidden primary user problem in cognitive radio systems [21].

4.2.2 Receiver Operating Characteristic (ROC) Curve

Receiver operating characteristic (ROC) curves [74] can be used to characterize the performance of a detector. The detection probability (P_D) and false alarm probability (P_{FA}) in Section 4.2.1 can be used to generate the ROC curves. Well established ROC curves and analysis of the energy detector is reported in the literature [75]-[80]. ROC analysis is also used in several other areas of application, like in the healthcare field for diagnostic tests, drug testing and others uses [81], and in machine learning algorithms [82].

The detection threshold (γ) of the detector which output the detection probability P_D and false alarm probability P_{FA} relies mainly on the number sample size (N) taken for the decision statistic, and the average SNR. In Figure 4-3, when the threshold γ increases from $0 \rightarrow \infty$, the ROC curve starts at the upper right point (1, 1) and moves down to the lower-left point (0,0). When γ increases, the P_{FA} goes down but so does P_D . Different ROC curves can be generated by different combinations of parameters of interest chosen. For example, one parameter which is the P_{FA} can be fixed while varying other parameters like the number of sample size or the SNR. ROC curve should be above the 45° line otherwise the detector's performance is no better than 50%. Therefore, the ROC curve can be a measure of the detection capability.

Figure 4-3: Parameters of the receiver operating characteristic (ROC) curve [87].

4.2.3 Peak to Average Power Ratio (PAPR)

Beside using the ROC, using peak-to-average power ratio (PAPR) is another way to characterize the performance of a detector. A higher PAPR means it is easier to distinguish signals from noise. PAPR can be used as a detection threshold in SS or to judge if a channel is a spectrum hole [10]. The PAPR calculation is affected by the step size of the cycle frequency which can be referred to as the spectrum resolution ratio (SRR). In practical application, the computational complexity of the PAPR analysis is directly affected by the SRR [63].

Increasing the sampling rate can also yield a better PAPR result. However, increasing sampling rate means a larger amount of data for calculation and hence, a longer duration for detection is needed. Therefore, CRs should consider the trade-off between detection performance and detection duration. Depending on the condition of the channel, the detectors can decrease the sampling rate or increase it to get better detection performance.

4.3 Proposed Multiple Cycle Frequencies Detection Algorithms Under Cochannel Interference

4.3.1 Dominant Eigenvalue (EV) Detector

The procedure to detect the presence of a cyclostationary signal with CF, α_k , using the dominant eigenvalue (EV) detector is summarized as follows:

1. Estimate the sample CAF matrix for a certain α and τ

$$\hat{\mathbf{R}}_{\mathbf{x}\mathbf{x}^*}^{\alpha}(\tau) = \frac{1}{N} \sum_{n=1}^N \mathbf{x}[n] \mathbf{x}^T[n - \tau] e^{-j2\pi\alpha n} \quad (4.8)$$

If we let $\mathbf{u}[n] = \mathbf{x}[n - \tau] e^{-j2\pi\alpha n}$ then the sample CAF matrix can be written as $\hat{\mathbf{R}}_{\mathbf{x}\mathbf{u}}$ (2.42).

5. Perform SVD on $\hat{\mathbf{R}}_{\mathbf{x}\mathbf{u}}$

$$\hat{\mathbf{R}}_{\mathbf{x}\mathbf{u}} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H \quad (4.9)$$

where $\mathbf{U} = [\xi_1, \xi_2, \dots, \xi_m]$ is a $M \times M$ matrix of left-singular vectors, $\mathbf{\Sigma} = \text{diag}[\mu_1, \mu_2, \dots, \mu_m]$ is a rectangular diagonal matrix and the diagonal entries are the singular values of $\hat{\mathbf{R}}_{\mathbf{x}\mathbf{u}}$, $\mathbf{V} = [\zeta_1, \zeta_2, \dots, \zeta_m]$ is a $M \times M$ matrix of right-singular vectors.

2. Compute the dominant eigenvalue spectrum

$$\hat{\rho}_{\text{EV}}(\alpha, \tau) = \mathbf{\Sigma}(1,1) \quad (4.10)$$

3. Using the dominant EV output as the test statistic

$$T_{EV}(\alpha_k, \tau) = \hat{\rho}_{EV}(\alpha, \tau) \quad (4.11)$$

4. Decide if

$$T_{EV}(\alpha_k, \tau) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{>}} \gamma \quad (4.12)$$

where $\gamma > 0$ is a threshold chosen to achieve constant false alarm rate (CFAR) which will be discussed in 4.3.3.

The PAPR calculation of dominant eigenvalue (EV) detector is

$$PAPR_{EV} = \frac{\max(\hat{\rho}_{EV}(\alpha, \tau))}{E\{|\hat{\rho}_{EV}(\alpha, \tau)|^2\}} \quad (4.13)$$

4.3.2 Eigen-analysis Cyclic Null Spectrum (ECNS) Detector

The procedure to estimate the presence of a cyclostationary signal with CF, α_k , using the eigen-analysis cyclic null spectrum (ECNS) detector is summarized as follows:

1. Estimate the sample conjugate CAF matrix for a certain α and τ

$$\hat{\mathbf{R}}_{xx^*}^{\alpha}(\tau) = \frac{1}{N} \sum_{n=1}^N \mathbf{x}[n] \mathbf{x}^T[n - \tau] e^{-j2\pi\alpha n} \quad (4.14)$$

If we let $\mathbf{u}[n] = \mathbf{x}[n - \tau] e^{-j2\pi\alpha n}$ then the sample conjugate CAF matrix can be written as $\hat{\mathbf{R}}_{xu}$ (2.42).

2. Perform SVD on $\hat{\mathbf{R}}_{xu}$ and compute the weighting vector $\mathbf{w}_{CAB}$

$$\hat{\mathbf{R}}_{xu} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T \quad (4.15)$$

$$\mathbf{w}_{\text{CAB}} = \mathbf{U}(:,1) \quad (4.16)$$

where $\mathbf{U} = [\xi_1, \xi_2, \dots, \xi_m]$ is a $M \times M$ matrix of left-singular vectors, $\mathbf{\Sigma} = \text{diag}[\mu_1, \mu_2, \dots, \mu_m]$ is a rectangular diagonal matrix and the diagonal entries are the singular values of $\hat{\mathbf{R}}_{\text{xu}}$, $\mathbf{V} = [\zeta_1, \zeta_2, \dots, \zeta_m]$ is a $M \times M$ matrix of right-singular vectors.

3. Compute $\hat{\rho}_{UC}(\alpha, \tau)$ using (3.18)
4. Compute $\hat{\rho}_C(\alpha, \tau)$ using (3.18)
5. Compute $\hat{\rho}_{ECNS}(\alpha, \tau)$ using (3.18)
6. Using the ECNS output as the test statistic

$$T_{ECNS}(\alpha_k, \tau) = \hat{\rho}_{ECNS}(\alpha, \tau) \quad (4.17)$$

7. Decide if

$$T_{ECNS}(\alpha_k, \tau) \begin{matrix} >_{\mathcal{H}_1} \\ <_{\mathcal{H}_0} \end{matrix} \gamma \quad (4.18)$$

where $\gamma > 0$ is a threshold chosen to achieve constant false alarm rate (CFAR) which will be discussed in 4.3.3.

The PAPR calculation of eigen-analysis cyclic null spectrum (ECNS) detector is

$$PAPR_{ECNS} = \frac{\max(\hat{\rho}_{ECNS}(\alpha, \tau))}{E \left\{ |\hat{\rho}_{ECNS}(\alpha, \tau)|^2 \right\}} \quad (4.19)$$

4.3.3 Detection Threshold

The probability of detection, (P_D), and the probability of false alarm, (P_{FA}), are two key parameters used to evaluate the performance of spectrum sensing algorithms. Figure 4-1 illustrates the probabilities of false alarm and detection with the test statistic as follows.

$$P_D = Pr(\text{teststatistic} > \gamma \mid \mathcal{H}_1) \quad (4.20)$$

$$P_{FA} = Pr(\text{teststatistic} > \gamma \mid \mathcal{H}_0) \quad (4.21)$$

P_D is the probability of being at $\mathcal{H}_1$ and accurately detecting the PU and P_{FA} is the probability of being at $\mathcal{H}_0$ and mistakenly detecting at PU.

It has been shown in [45], [62] that the limiting distribution, when $N \rightarrow \infty$, of the test statistics in both (4.12) and (4.18) for a real and normally distributed random vectors approaches χ^2 distribution.

Based on the distribution of T_{EV} and T_{ECNS} under $\mathcal{H}_0$, the detection threshold γ can be set to achieve a desired P_{FA} by satisfying

$$\int_{\gamma}^{\infty} f_{\chi_k^2}(x) dx = P_{FA} \quad (4.22)$$

Hence, a detection threshold γ can be set for a given P_{FA} .

4.4 Simulation Results and Discussion

In Section 4.4.1 and Section 4.4.2, a ULA of $m = 6$ omnidirectional sensors spaced half-wavelength apart is considered. There is one SOI and one interfering signal in all experiments. All signals are BPSK. The DOA of the SOI is 10° . The DOA of interfering signal I_1 is 70° . The SOI carrier frequency is $0.05f_s$. The interfering signal I_1 carrier frequency is $0.2f_s$. Therefore, the SOI cycle frequency is at $\alpha_0 = 0.1f_s$ and the interfering signal I_1 cycle frequency is at $\alpha_{I1} = 0.4f_s$. These cyclic features are only present in the conjugate CAF. For BPSK modulation, the maximum self-coherence occurs at $\tau = 0$ [5] and hence $\tau = 0$ is used in all experiments unless otherwise stated. The noise variance is assumed to be equal to one in each sensor. All simulation results are obtained by averaging 10000 independent Monte Carlo runs.

4.4.1 Peak to Average Ratio

In the first example, the peak-to-average ratio (PAPR) of the proposed methods against the existing methods are investigated. The algorithms output is normalized to [0,1] where each value among them is divided by the maximum value. Figure 4-4 shows the performance of the proposed methods against the existing methods with varying INR. The sample size is kept small at 100 samples. The ECNS detector exhibits the best PAPR performance. Even when $INR = 15$, the ECNS detector could reach a high PAPR (≈ 9) compared to the low PAPRs (< 2) computed by the rest of the detectors. Figure 4-5 shows the performance of the proposed methods against the existing methods with varying the number of sample size. The ECNS detector exhibits the best PAPR performance with consistently the highest PAPR when the number of sample size

increases. This simulation further illustrated the better detection performance when sampling rate is increased.

Figure 4-4: Peak to average ratio comparisons of the proposed methods against existing methods with varying interference-to-noise ratio ($N = 100$, $SRR = 200$ kHz).

Figure 4-5: Peak to average ratio comparisons of the proposed methods against existing methods with varying interference-to-noise ratio ($\text{INR} = 20 \text{ dB}$, $\text{SRR} = 200 \text{ kHz}$).

4.4.2 Detector Performance Analysis using Receiver Operating Characteristic (ROC) Curves

The ROC curves of the proposed methods (dominant eigenvalue (EV) algorithm and eigenanalysis cyclic null spectrum (ECNS) algorithm) are compared with the existing methods (summation CAF algorithm and the EV-CSS algorithm) in this section. The detection probability (P_D) of the various detection algorithms are analyzed under three key detection parameters: (i) Probability of false alarm (P_{FA}); (ii) Interference-to-noise ratio (INR); (iii) Number of sample size (N). In all the simulations in this section, the constant false alarm rate (CFAR) is set to 0.1 unless otherwise stated.

In the first example, the ROC curves under $\text{INR} = 15$ dB, $\text{SNR} = 0$ dB and $N = 300$ with the effect of varying probability of false alarm on probability of detection for the proposed methods and other existing methods are shown in Figure 4-6. The best detector is the proposed ECNS algorithm. This is consistent with the PAPR result in Figure 4-4. The proposed dominant EV algorithm slightly outperformed the EV-CSS algorithm but it has a much lower computational complexity (as shown in 3.3.3) than the EV-CSS algorithm. The summation CAF algorithm is the worst performing detector as it could not suppress the interference.

In the second example, the ROC curves under $P_{FA} = 0.1$, $\text{SNR} = 0$ dB and $N = 300$ with the effect of varying INR on probability of detection for the proposed methods and other existing methods are shown in Figure 4-7. The best detector is the proposed ECNS algorithm consistently as it could suppress the interference effectively using both the unconstrained and unconstrained outputs. Even with the presence of a higher-powered interference signal at 20 dB, it could still reach a high detection probability ($P_D \approx 1$) compared to the low detection probability ($P_D < 0.3$) from the rest of the detectors.

In the third example, the ROC curves under $INR = 15$ dB, $SNR = 0$ dB and $P_{FA} = 0.1$ with the effect of varying the number of samples on probability of detection for the proposed methods and other existing methods are shown in Figure 4-8. Significant improvements in probability of detection can be seen with increasing sample size across all algorithms. The slope of the curve of ECNS algorithms increased rapidly and is the best performing algorithm. The required sensing time can be cut down drastically by utilizing the ECNS algorithm.

Figure 4-6: Receiver operating characteristic comparisons of the proposed methods against existing methods with varying probability of false alarm. (INR = 15 dB, SNR = 0 dB, $N = 300$)

Figure 4-7: Probability of detection comparisons of the proposed methods against existing methods with varying INR. ($P_{FA} = 0.1$, $SNR = 0$ dB, $N = 300$).

Figure 4-8: Probability of detection comparisons of the proposed methods against existing methods with varying sample size. ($P_{FA} = 0.1$, $INR = 20$ dB, $SNR = 0$ dB).

4.5 Summary

In this chapter, two fast multiple antenna cyclostationary-based CF detection methods - dominant EV and ECNS - are proposed. The first proposed algorithm, named the dominant eigenvalue (EV) method, uses the dominant eigenvalue of the CAF matrix to detect the cycle frequencies of cyclostationary signals. The largest singular value of the CAF matrix is a clear indication of high correlation at the CF of the SOI. Therefore, the dominant EV output can be used as the test statistic by the detector for signal detection.

The second proposed algorithm, named ECNS, consists of a novel two-stage CF detection framework. At the first stage, it computes the weight vector from the cyclostationary beamforming algorithm named CAB. At the second stage, it uses the CAB weight vectors to compute the unconstrained and constrained output powers to form an eigen-analysis cyclic null spectrum (ECNS) which could detect the cycle frequencies of cyclostationary signals. Due to the criterions of the unconstrained and constrained output powers, the ECNS detector could perform under co-channel interference using the ECNS output as the test statistic.

The test statistics and detection thresholds for both the dominant EV and ECNS are formulated in this chapter. The PAPRs of the proposed algorithm are also computed.

The proposed methods have been investigated by numerical experiments. The proposed methods not only can achieve higher probability of detection under strong co-channel interference condition, they can also detect faster using fewer samples compared to existing cyclostationary cycle frequency detection methods. The proposed algorithms also exhibit higher probability of detection for a given probability of false alarm, and

also a higher PAPRs compared to existing cyclostationary cycle frequency detection methods.

Chapter 5

Multiple Cycle Frequencies Estimation and Detection Using Subspace Method

5.1 Objectives

The main objective in this chapter is to develop algorithms which could detect and estimate the number of unknown cyclostationary signals and their cycle frequencies in the received data under strong co-channel interference using subspace projection.

Conventional DOA estimation techniques have been widely used in many applications such as radar, sonar, or telecommunications. DOA estimators that make use of signal subspace has been proposed [64]-[68]. These DOA estimators have high-resolution estimation capabilities. Signal subspace methods originated in spectral

estimation [64] research. The autocorrelation of a signal plus noise model is estimated and used to form a matrix whose eigenstructure gives rise to the signal and noise subspaces (2.3.4). Among these algorithms, the MUSIC algorithm in 2.4.3 has been extensively studied and widely applied in many fields. A cyclic MUSIC algorithm has been proposed in [69] and later an extended cyclic MUSIC algorithm in [70]. Instead of using the correlation matrix as in conventional methods, these algorithms require the estimation of the cyclic correlation matrix that reflects the cyclostationarity of the incoming signals in the received data.

The main objective of this chapter is to detect and estimate multiple cycle frequencies under cochannel interference using subspace method. The subspace method in [70] is chosen as the representative in our study. In this chapter, two algorithms are proposed which make use of the cyclic eigensubspace. The two key advantages of the proposed algorithms over the extended cyclic MUSIC is that no *a priori* information on signal and noise subspace is needed, and the ability to handle spatially correlated noise environments. The performances of the proposed methods are demonstrated in 5.4 with numerical examples and compared with [70].

5.2 Numerical Study of Subspace Method using Cyclic MUSIC

A cyclic MUSIC algorithm used for signal-selective DOA estimation has been proposed in [69]. The cyclic MUSIC algorithm can be implemented as

1. Estimate the sample CAF matrix for a certain α and τ

$$\hat{\mathbf{R}}_{\mathbf{x}\mathbf{x}}^{\alpha}(\tau) = \frac{1}{N} \sum_{n=1}^N \mathbf{x}[n] \mathbf{x}^H[n - \tau] e^{-j2\pi\alpha n} \quad (5.1)$$

If we let $\mathbf{u}[n] = \mathbf{x}[n - \tau] e^{-j2\pi\alpha n}$ then the sample CAF matrix can be written as $\hat{\mathbf{R}}_{\mathbf{x}\mathbf{u}}$ (2.42).

2. Perform SVD on $\hat{\mathbf{R}}_{\mathbf{x}\mathbf{u}}$ and find the number of signals and compute the noise subspace $\mathbf{E}_n$

$$[\mathbf{U}_s \quad \mathbf{U}_n] \begin{bmatrix} \sum s & 0 \\ 0 & \sum n \end{bmatrix} [\mathbf{V}_s \quad \mathbf{V}_n]^H \quad (5.2)$$

Where $[\mathbf{U}_s \quad \mathbf{U}_n]$ and $[\mathbf{V}_s \quad \mathbf{V}_n]$ are unitary matrices, and the diagonal elements $\sum s$ and $\sum n$ are sorted in decreasing order. $\sum s$ represents the summation of all signal eigenvalues and $\sum n$ represents the summation of all noise eigenvalues. $\sum n$ converges to zero as the integration time used in the estimate tends to infinity.

3. Find the minima of $\|\mathbf{U}_n^H \mathbf{a}(\theta)\|^2$ or maxima of $\|\mathbf{U}_s^H \mathbf{a}(\theta)\|^2$

$$\begin{aligned} \min & \|\mathbf{U}_n^H \mathbf{a}(\theta)\|^2 \\ \max & \|\mathbf{U}_s^H \mathbf{a}(\theta)\|^2 \end{aligned} \quad (5.3)$$

The key difference between the cyclic MUSIC [69] and the extended cyclic MUSIC in [70] is that the latter exploits the information of both the cyclic correlation matrix and the cyclic conjugate correlation matrix. The spatial spectrum of the extended cyclic MUSIC method is given by

$$\rho_{C-MUSIC} = \frac{1}{\mathbf{a}^H(\theta) \mathbf{U}_{n1} \mathbf{U}_{n1}^H \mathbf{a}(\theta) - \|\mathbf{a}^T(\theta) \mathbf{U}_{n2} \mathbf{U}_{n1}^H \mathbf{a}(\theta)\|} \quad (5.4)$$

where $\mathbf{U}_{n1}$ and $\mathbf{U}_{n2}$ are two submatrices of the same dimension computed by the SVD of sample CAF matrix and sample conjugate CAF matrix, respectively. They are defined by

$$\mathbf{U}_n = \begin{bmatrix} \mathbf{U}_{n1} \\ \mathbf{U}_{n2} \end{bmatrix} \quad (5.5)$$

It is emphasized in [70] that the extended cyclic MUSIC method has better estimation performance than the cyclic MUSIC when both the cyclic correlation and the cyclic conjugate correlation matrices are nonzero. This occurs for many digitally modulated signals. If one of these two correlation matrices equals to zero, the method is strictly equivalent to the cyclic MUSIC method. The extended cyclic MUSIC method does not require the noise to be uncorrelated from one sensor to another, but it requires a long integration time to ensure that the contribution of the noise as well as that of the interfering signals in the cyclic correlation matrices are sufficiently small. This is a main disadvantage in spectrum sensing as the contribution from the interfering signals would be high and a long integration time would impact the sensing duration. Note that the dimension of the signal subspace (or noise subspace) in (5.9a) need to be known or can be estimated by the MDL criterion [29].

Although cyclic MUSIC has proven to be a better DOA estimation algorithm [69], [70], it could not work for cyclostationary feature estimation or detection using $\mathbf{w}_{\text{CAB}}$. The orthogonality condition in (5.3) work for $\mathbf{a}(\theta)$ when $\hat{a}_k(\theta) = a_k(\theta)$. However the orthogonality of $\|\mathbf{E}_n^H \mathbf{w}_{\text{CAB}}\|^2$ is always true even if $\hat{\mathbf{w}}_k \neq \mathbf{w}_k$ as $\mathbf{E}_n^H$ and $\mathbf{w}_{\text{CAB}}$ are decomposed from the same eigenstructure in (5.1). Therefore, the noise subspace must be chosen carefully. One natural choice would be using the noise subspace from the sample covariance matrix (2.20) instead of (5.1). This would be equivalent to the noise subspace used in MUSIC (2.40) and *a priori* information of signal and noise subspace

is needed. Therefore, a proposed CAB-MUSIC algorithm is shown in the following section and the comparison made with cyclic MUSIC can be found in Figure 5-1.

5.3 Proposed Multiple Cycle Frequencies Detection and Estimation Algorithms Under Cochannel Interference using Subspace Method

5.3.1 Proposed Cyclic Adaptive Beamforming Multiple Signal Classification (CAB-MUSIC) Algorithm

The procedure to detect and estimate the presence of a cyclostationary signal with CF, α_k , using the CAB-MUSIC algorithm is summarized as follows:

1. Estimate the sample conjugate CAF matrix for a certain α and τ

$$\hat{\mathbf{R}}_{\mathbf{x}\mathbf{x}^*}^{\alpha}(\tau) = \frac{1}{N} \sum_{n=1}^N \mathbf{x}[n] \mathbf{x}^T[n - \tau] e^{-j2\pi\alpha n} \quad (5.6)$$

If we let $\mathbf{u}[n] = \mathbf{x}[n - \tau] e^{-j2\pi\alpha n}$ then the sample conjugate CAF matrix can be written as $\hat{\mathbf{R}}_{\mathbf{x}\mathbf{u}}$ (2.42).

2. Perform SVD on $\hat{\mathbf{R}}_{\mathbf{x}\mathbf{u}}$ and compute the weighting vector $\mathbf{w}_{\text{CAB}}$

$$\hat{\mathbf{R}}_{\mathbf{x}\mathbf{u}} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad (5.7)$$

$$\mathbf{w}_{\text{CAB}} = \mathbf{U}(:, 1) \quad (5.8)$$

where $\mathbf{U} = [\xi_1, \xi_2, \dots, \xi_m]$ is a $M \times M$ matrix of left-singular vectors, $\mathbf{\Sigma} = \text{diag}[\mu_1, \mu_2, \dots, \mu_m]$ is a rectangular diagonal matrix and the diagonal entries are the singular values of $\hat{\mathbf{R}}_{\mathbf{x}\mathbf{u}}$, $\mathbf{V} = [\zeta_1, \zeta_2, \dots, \zeta_m]$ is a $M \times M$ matrix of right-singular vectors.

3. Compute $\hat{\rho}_{\text{CAB-MUSIC}}(\alpha, \tau)$ using

$$\hat{\rho}_{CAB-MUSIC} = \frac{1}{\mathbf{w}_{CAB}^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{w}_{CAB}} \quad (5.9a)$$

where $\mathbf{U}_n$ is the noise subspace from (5.7)

$$\mathbf{U} = [\mathbf{U}_s \quad \mathbf{U}_n] \quad (5.9b)$$

4. Using the CAB-MUSIC output as the test statistic

$$T_{CAB-MUSIC}(\alpha_k, \tau) = \hat{\rho}_{CAB-MUSIC}(\alpha, \tau) \quad (5.10)$$

5. Decide if

$$T_{CAB-MUSIC}(\alpha_k, \tau) \begin{matrix} >_{\mathcal{H}_1} \\ <_{\mathcal{H}_0} \end{matrix} \gamma \quad (5.11)$$

where $\gamma > 0$ is a threshold chosen to achieve constant false alarm rate (CFAR) which will be discussed in 4.3.3.

6. Select the estimated CF

$$\hat{\alpha}_k = \max_{\alpha} \hat{\rho}_{CAB-MUSIC}(\alpha, \tau) \quad (5.12)$$

where $\hat{\alpha}_k$ is the estimated CF.

The PAPR calculation of CAB-MUSIC detector is

$$PAPR_{CAB-MUSIC} = \frac{\max(\hat{\rho}_{CAB-MUSIC}(\alpha, \tau))}{E \left\{ \left| \hat{\rho}_{CAB-MUSIC}(\alpha, \tau) \right|^2 \right\}} \quad (5.13)$$

Note that the number of sources is assumed to be known in order to estimate $\mathbf{U}_n$ (5.9b). Figure 5-1 shows the comparison of the proposed CAB-MUSIC algorithm with the existing cyclic MUSIC algorithm with two cyclostationary signals with cycle

frequencies of $0.1f_s$ and $0.4f_s$ respectively. As mentioned in section 5.2, cyclic MUSIC could not work for cyclostationary feature estimation or detection using $\mathbf{w}_{CAB}$.

Figure 5-1: Comparisons of the proposed CAB-MUSIC and the cyclic-MUSIC algorithms. ($N = 500$, $\text{SNR} = 0$ dB, $\rho_s = 0$, $\alpha_0 = 0.1f_s, 0.4f_s$).

The two key disadvantages of the CAB-MUSIC are:

1. Need *a priori* information on the signal subspace (or noise subspace).
2. Sources need to be uncorrelated (including the noise).

To develop an algorithm that does not need any *a priori* information on the signal subspace (or noise subspace), the noise subspace in (5.9a) needs to be independent to the number of sources in the received data. Hence, the eigenstructure of the CAF is shown here again to investigate the eigensubspaces in more detail

$$\mathbf{R}_{xu}^\alpha = \mathbf{R}_{ss}^\alpha(\tau)e^{-j2\pi\alpha\tau} \mathbf{A}\mathbf{A}^H + \sigma_v^2 \mathbf{I}e^{-j2\pi\alpha\tau} \quad (5.14)$$

And it can be further decomposed to sample CAF matrix in the form of (2.18)

$$\hat{\mathbf{R}}_{xu} = \sum_{i=1}^K \mu_i \boldsymbol{\xi}_i \boldsymbol{\xi}_i^T + \sum_{i=1}^{M-K} \mu_i \boldsymbol{\xi}_i \boldsymbol{\xi}_i^T \quad (5.15)$$

where μ_i and $\boldsymbol{\xi}_i$, $i=1,2,\dots, M$ are the eigenvalues and eigenvectors of $\hat{\mathbf{R}}_{xu}$. (5.15) provides a general case where $K \geq 2$ uncorrelated SOI having the same cycle frequency (such cases are rare in practical applications). In practical applications, various signals are usually deliberately assigned slightly different carrier frequencies and/or baud rates so that each would have a different cycle frequency [71]. Therefore, the decomposition of the CAF eigenstructure becomes

$$\hat{\mathbf{R}}_{xu} = \mu_D \boldsymbol{\xi}_D \boldsymbol{\xi}_D^T + \sum_{i=1}^{M-1} \mu_i \boldsymbol{\xi}_i \boldsymbol{\xi}_i^T \quad (5.16)$$

where μ_D is the solution in (4.10) which is the dominant eigenvalue and $\boldsymbol{\xi}_D$ is the CAB beamformer weight $\mathbf{w}_{CAB}$ in (2.49). Thus, the decomposition can be categorized as

$$\hat{\mathbf{R}}_{xu} = \hat{\mathbf{R}}_{DCE} + \hat{\mathbf{R}}_{N-DCE} \quad (5.17)$$

dominant cyclic eigensubspace where $\hat{\mathbf{R}}_{\text{DCE}}$ is the estimated (DCE) covariance matrix and $\hat{\mathbf{R}}_{\text{N-DCE}}$ is the estimated non-dominant cyclic eigensubspace (N-DCE) covariance matrix.

5.3.2 Proposed Dominant Cyclic Eigensubspace (DCE) Algorithm

In the general case, the optimal solution for weight vector (which maximizes the SINR in (2.24)) by maintaining a distortionless array response to the desired signal while minimizing the output interference-plus-noise power [72]:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{R}_{\text{in}} \mathbf{w} \\ \text{s. t.} \quad & \mathbf{w}^H \mathbf{R}_{\text{ss}} \mathbf{w} = 1 \end{aligned} \quad (5.18)$$

Using the Lagrange multiplier method, the Lagrangian for criterion (5.18) is given as

$$L(g) = \mathbf{w}^H \mathbf{R}_{\text{in}} \mathbf{w} - g(1 - \mathbf{w}^H \mathbf{R}_{\text{ss}} \mathbf{w}) \quad (5.19)$$

where g is the Lagrange multiplier and the solution to (5.18) is equating the gradient of (5.19) to zero

$$\nabla_{\mathbf{w}} L(g) = \mathbf{R}_{\text{in}} \mathbf{w} - g \mathbf{R}_{\text{ss}} \mathbf{w} = 0 \quad (5.20)$$

and thus the optimal weight is

$$\mathbf{w}_{\text{opt}} = P\{\mathbf{R}_{\text{in}}^{-1} \mathbf{R}_{\text{ss}}\} \quad (5.21)$$

where $P\{\cdot\}$ is the operator that yields the principal eigenvector of a matrix, i.e., the eigenvector that corresponds to the maximal eigenvalue. The solution (5.21) is of limited practical use because in most applications, the matrix $\mathbf{R}_{\text{ss}}$ is unknown, and often no reasonable estimate of it is available. However, if the estimate of $\mathbf{R}_{\text{ss}}$ and $\mathbf{R}_{\text{in}}$ is available, (5.21) provides a simple solution to the adaptive beamforming problem for

the general-rank source. Therefore, a dominant cyclic eigensubspace (DCE) algorithm to detect and estimate multiple cycle frequencies under cochannel interference is proposed by estimating the $\mathbf{R}_{ss}$ and $\mathbf{R}_{in}$ in (5.21) with

$$\mathbf{w}_{DCE} = P\{\hat{\mathbf{R}}_{xx}^{-1}\hat{\mathbf{R}}_{DCE}\} \quad (5.22)$$

where $\hat{\mathbf{R}}_{DCE}$ is the DCE covariance matrix in (5.17) and $\hat{\mathbf{R}}_{xx}^{-1}$ sample covariance matrix. Therefore, the output of the DCE algorithm is

$$\hat{\rho}_{DCE}(\alpha, \tau) = \mathbf{w}_{DCE}^H \hat{\mathbf{R}}_{xx} \mathbf{w}_{DCE} \quad (5.23)$$

The procedure to detect and estimate the presence of a cyclostationary signal with CF, α_k , using the DCE algorithm is summarized as follows:

1. Estimate the sample conjugate CAF matrix for a certain α and τ

$$\hat{\mathbf{R}}_{xx^*}^\alpha(\tau) = \frac{1}{N} \sum_{n=1}^N \mathbf{x}[n] \mathbf{x}^T[n - \tau] e^{-j2\pi\alpha n} \quad (5.24)$$

If we let $\mathbf{u}[n] = \mathbf{x}[n - \tau] e^{-j2\pi\alpha n}$ then the sample conjugate CAF matrix can be written as $\hat{\mathbf{R}}_{xu}$ (2.42).

2. Perform SVD on $\hat{\mathbf{R}}_{xu}$ and compute the weighting vector $\mathbf{w}_{CAB}$

$$\hat{\mathbf{R}}_{xu} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T \quad (5.25a)$$

$$\mathbf{w}_{CAB} = \mathbf{U}(:, 1) \quad (5.25b)$$

where $\mathbf{U} = [\xi_1, \xi_2, \dots, \xi_m]$ is a $M \times M$ matrix of left-singular vectors, $\mathbf{\Sigma} = \text{diag}[\mu_1, \mu_2, \dots, \mu_m]$ is a rectangular diagonal matrix and the diagonal entries are the singular values of $\hat{\mathbf{R}}_{xu}$, $\mathbf{V} = [\zeta_1, \zeta_2, \dots, \zeta_m]$ is a $M \times M$ matrix of right-singular vectors.

3. Compute $\hat{\rho}_{DCE}(\alpha, \tau)$ using (5.21)
4. Using the DCE algorithm output as the test statistic

$$T_{DCE}(\alpha_k, \tau) = \hat{\rho}_{DCE}(\alpha, \tau) \quad (5.26)$$

5. Decide if

$$T_{DCE}(\alpha_k, \tau) \begin{matrix} >_{\mathcal{H}_1} \\ <_{\mathcal{H}_0} \end{matrix} \gamma \quad (5.27)$$

where $\gamma > 0$ is a threshold chosen to achieve constant false alarm rate (CFAR) which will be discussed in 4.3.3.

6. Select the estimated CF

$$\hat{\alpha}_k = \max_{\alpha} \hat{\rho}_{DCE}(\alpha, \tau) \quad (5.28)$$

where $\hat{\alpha}_k$ is the estimated CF.

The PAPR calculation of DCE detector is

$$PAPR_{DCE} = \frac{\max(\hat{\rho}_{DCE}(\alpha, \tau))}{E \left\{ |\hat{\rho}_{DCE}(\alpha, \tau)|^2 \right\}} \quad (5.29)$$

5.3.3 Proposed Non-Dominant Cyclic Eigensubspace (N-DCE) Algorithm

Note that in (5.9a) the noise subspace is dependent on the *a priori* knowledge of the signal subspace. Therefore, in order to be independent of the signal subspace *a priori* knowledge, the noise subspace can be replaced by the non-dominant cyclic eigensubspace in (5.17). And hence, a non-dominant cyclic eigensubspace (N-DCE) algorithm to detect and estimate multiple cycle frequencies under cochannel interference is proposed as follow

$$\hat{\rho}_{N-DCE}(\alpha, \tau) = \frac{1}{\mathbf{w}_{C-CAB}^H \hat{\mathbf{U}}_{N-DCE} \hat{\mathbf{U}}_{N-DCE}^H \mathbf{w}_{C-CAB}} \quad (5.30)$$

where $\hat{\mathbf{R}}_{N-DCE} = \hat{\mathbf{U}}_{N-DCE} \hat{\mathbf{U}}_{N-DCE}^H$. Note that $\mathbf{w}_{C-CAB}$ is used here instead of $\mathbf{w}_{CAB}$ because $\hat{\mathbf{U}}_{N-DCE}$ is not defined as a noise subspace. Hence, only a constrained weighting towards the same CF as the N-DCE covariance matrix will result in orthogonal projection.

The procedure to detect and estimate the presence of a cyclostationary signal with CF, α_k , using the N-DCE algorithm is summarized as follows:

1. Estimate the sample conjugate CAF matrix for a certain α and τ

$$\hat{\mathbf{R}}_{xx^*}^\alpha(\tau) = \frac{1}{N} \sum_{n=1}^N \mathbf{x}[n] \mathbf{x}^T[n - \tau] e^{-j2\pi\alpha n} \quad (5.31)$$

If we let $\mathbf{u}[n] = \mathbf{x}[n - \tau] e^{-j2\pi\alpha n}$ then the sample conjugate CAF matrix can be written as $\hat{\mathbf{R}}_{xu}$ (2.42).

2. Perform SVD on $\hat{\mathbf{R}}_{xu}$ and compute the weighting vector $\mathbf{w}_{CAB}$

$$\hat{\mathbf{R}}_{xu} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T \quad (5.32a)$$

$$\mathbf{w}_{\text{CAB}} = \mathbf{U}(:,1) \quad (5.32b)$$

where $\mathbf{U} = [\xi_1, \xi_2, \dots, \xi_m]$ is a $M \times M$ matrix of left-singular vectors, $\mathbf{\Sigma} = \text{diag}[\mu_1, \mu_2, \dots, \mu_m]$ is a rectangular diagonal matrix and the diagonal entries are the singular values of $\hat{\mathbf{R}}_{\text{xu}}$, $\mathbf{V} = [\zeta_1, \zeta_2, \dots, \zeta_m]$ is a $M \times M$ matrix of right-singular vectors.

3. Compute $\hat{\rho}_{N-DCE}(\alpha, \tau)$ using (5.30)
4. Using the N-DCE algorithm output as the test statistic

$$T_{N-DCE}(\alpha_k, \tau) = \hat{\rho}_{N-DCE}(\alpha, \tau) \quad (5.33)$$

5. Decide if

$$T_{N-DCE}(\alpha_k, \tau) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{>}} \gamma \quad (5.34)$$

where $\gamma > 0$ is a threshold chosen to achieve constant false alarm rate (CFAR) which will be discussed in 4.3.3.

6. Select the estimated CF

$$\hat{\alpha}_k = \max_{\alpha} \hat{\rho}_{N-DCE}(\alpha, \tau) \quad (5.35)$$

where $\hat{\alpha}_k$ is the estimated CF.

The PAPR calculation of N-DCE detector is

$$PAPR_{N-DCE} = \frac{\max(\hat{\rho}_{N-DCE}(\alpha, \tau))}{E \left\{ |\hat{\rho}_{N-DCE}(\alpha, \tau)|^2 \right\}} \quad (5.36)$$

5.4 Simulation Results and Discussion

In this section, a ULA of $m = 6$ omnidirectional sensors spaced half-wavelength apart is considered. There is one SOI and one interfering signals in all experiments. All signals are BPSK. The DOA of the SOI is 10° . The DOA of interfering signal I is 70° . The SOI carrier frequency is $0.05f_s$. The interfering signal I carrier frequency is $0.2f_s$. Therefore, the SOI cycle frequency is at $\alpha_0 = 0.1f_s$ and the interfering signal I cycle frequency is at $\alpha_I = 0.4f_s$. These cyclic features are only present in the conjugate CAF. For BPSK modulation, the maximum self-coherence occurs at $\tau = 0$ [5]; hence, $\tau = 0$ is used in all experiments unless otherwise stated. The noise variance is assumed to be equal to one in each sensor. All simulation results are obtained by averaging 10000 independent Monte Carlo runs.

In this section, both the estimation and detection performance of the proposed dominant cyclic eigensubspace (DCE) and the non-dominant cyclic eigensubspace (N-DCE) algorithms are analyzed and compared with the existing methods (summation CAF and EV-CSS in Section 3.2). The impact of spatially correlated noise is also analyzed in all simulations in this section.

5.4.1 Spatially Correlated Noise

Many proposed estimators assume spatially white noise such that the array noise covariance matrix is proportional to an identity matrix. However, this assumption is not realistic in many practical applications [89]-[95] where the noise fields are spatially colored. The spatial correlation or nonuniformity in the colored noise may significantly degrade the performance of the estimators/detectors assuming spatially white noise only. Therefore, it is important to take into consideration the effect of spatially correlated noise on the proposed algorithms in this thesis. The presence of correlated noise is generally due to crosstalk between channels, random radiation from distributed sources, reverberation, undesired interference, etc. [96].

In the case of spatially correlated noise, the noise can be modeled with a covariance matrix given by $\mathbf{R}_{vv}$. The spatial covariance matrix of the noise, which describe the cross-correlation between noise received by antenna i and j , is [73], [97]

$$\{\mathbf{R}_{vv}\}_{ij} = E\{\mathbf{v}_i \mathbf{v}_j^H\} = \begin{cases} \sigma_v^2 \mathbf{I}, & i = j \\ \sigma_v^2 \rho_s^{|i-j|}, & i \neq j \end{cases} \quad (5.37)$$

Thus with $\rho_s = 0$, the covariance matrix simplifies to $\sigma_v^2 \mathbf{I}$ giving spatially white noise, while $\rho_s = 1$ gives fully correlated noise over all antennas. Varying degrees of partial correlation can be achieved by setting $0 < \rho_s < 1$.

In this section, both the estimation and detection performance of the proposed dominant cyclic eigensubspace (DCE) and the non-dominant cyclic eigensubspace (N-DCE) algorithms are analyzed and compared with the existing methods (summation CAF and EV-CSS in Section 3.2). The impact of spatially correlated noise is also analyzed in all simulations in this section.

Figure 5-2 shows the comparison of the proposed DCE and N-DCE algorithms with

the CAB-MUSIC algorithm with zero noise correlation ($\rho_s = 0$). All the algorithms output the correct peaks of the simulated cyclostationary signals. Both the N-DCE and CAB-MUSIC algorithms have a better noise suppression compared to the DCE algorithms. This would be further illustrated in the peak-to-average ratio (PAPR) example in Section 5.4.3. Figure 5-3 shows the comparison of the proposed DCE and N-DCE algorithms with the CAB-MUSIC algorithm with 50% noise correlation ($\rho_s = 0.5$). Only the proposed DCE and N-DCE algorithms output the correct peaks of the simulated cyclostationary signals. The CAB-MUSIC could not work in correlated noise environment.

Figure 5-2: Comparisons of the proposed DCE and N-DCE algorithms with the CAB-MUSIC algorithm ($N = 500$, $\text{SNR} = 0$ dB, $\rho_s = 0$, $\alpha_0 = 0.1f_s$, $\alpha_l = 0.4f_s$).

Figure 5-3: Comparisons of the proposed DCE and N-DCE algorithms with the CAB-MUSIC algorithm ($N = 500$, $\text{SNR} = 0$ dB, $\rho_s = 0.5$, $\alpha_0 = 0.1f_s$, $\alpha_l = 0.4f_s$).

5.4.2 RMSE Comparisons of DCE and N-DCE Algorithms Versus INR under Co-channel Interference

The proposed methods (CAB-MUSIC, DCE and N-DCE algorithms) are compared with the existing methods (SUM, CAF and EV-CSS algorithms) methods in Sections 5.4.2, 5.4.3 and 5.4.4. In Figure 5-2, when the interfering signals have the same SNR as the SOI, all the methods estimated two distinct peaks at $\alpha_0 = 0.1f_s$ and $\alpha_I = 0.4f_s$. In Figure 5-4, the SNR of the SOI is 0 dB and the interfering signal (I_1) is stronger than the SOI (INR = 10 dB); all the methods estimated two distinct peaks at $\alpha_0 = 0.1f_s$ and $\alpha_I = 0.4f_s$. Both the N-DCE and CAB-MUSIC algorithms have a better noise suppression compared to the DCE algorithms. The amplitude of the SOI is 10 dB lower for the DCE algorithm whereas the amplitude of the SOI are 20 dB lower for the N-DCE and CAB-MUSIC algorithms under co-channel interference. This is due to the interference suppression capability of the DCE algorithm using (5.18). Both the N-DCE and CAB-MUSIC use orthogonality projection and do not provide any interference suppression.

Figure 5-5 shows the root mean square errors (RMSE) comparisons of different CF estimation methods with varying SNR (SIR = -15 dB). The RMSE is computed using (3.30). All the proposed algorithms (CAB-MUSIC, DCE and N-DCE) have lower RMSE compared to the existing methods. The CAB-MUSIC algorithm is the worst among the proposed methods and it needed *a priori* information on the signal subspace. In Figure 5-6, the effectiveness of all methods to spatially correlated noise ($\rho_s = 0.5$) is investigated. To generate colored noise samples, the white noise samples pass through a spatially correlation filter and the correlation of the noise covariance matrix is illustrated in 5.4.1. CAB-MUSIC algorithm could not work in correlated noise environment. The proposed DCE and N-DCE algorithms are able to handle correlated

noise as only slight degradation in performance is observed. In summary, the proposed algorithms have superior estimation performance over the existing methods and the DCE algorithm is the best performing CF estimation algorithm.

Figure 5-4: Comparisons of the proposed DCE and N-DCE algorithms with the CAB-MUSIC algorithm ($N = 500$, $\text{SNR} = 0$ dB, $\text{INR} = 10$ dB, $\rho_s = 0$, $\alpha_0 = 0.1f_s$, $\alpha_{I1} = 0.4f_s$).

Figure 5-5: RMSE comparisons of different CF estimation methods with varying SNR ($SIR = -15$ dB, $N = 300$, $\rho_s = 0$).

Figure 5-6: RMSE comparisons of different CF estimation methods with varying SNR ($SIR = -15$ dB, $N = 300$, $\rho_s = 0.5$)

5.4.3 Peak to Average Ratio (PAPR)

In this section, the peak-to-average ratio (PAPR) of the proposed methods against the existing methods are investigated. The values of the test statistic are normalized to $[0,1]$ where each value among them is divided by the maximum value. The PAPR calculation is affected by the step size of the cycle frequency which can be referred to as the spectrum resolution ratio (SRR). In practical application, the computational complexity of the PAPR analysis is directly affected by the SRR.

A higher PAPR means it is easier to distinguish signals from noise. Figure 5-7 shows the performance of the proposed methods against the existing methods with varying INR and zero noise correlation ($\rho_s = 0$). The sample size is kept small at 100 samples. The proposed CAB-MUSIC and N-DCE detectors exhibit the best PAPR performance. Even when $\text{INR} = 15$ dB, for the proposed methods, the CAB-MUSIC and N-DCE detectors could reach a PAPR of 18 and the DCE detector could reach a PAPR of 11. For the existing methods, the EV-CSS could only reach a PAPR of 4 and SUM. CAF has the worst PAPR value of 2 at $\text{INR} = 15$ dB. Note that DCE algorithm has consistent PAPR ($8 \leq \text{PAPR} \leq 12$) from 0 dB INR to 15 dB INR and a consistent linear downslope in PAPR after 15 dB INR. This further illustrates the interference suppression capability of the DCE algorithm using (5.18).

Figure 5-8 shows the performance of the proposed methods against the existing methods with varying INR and 50% noise correlation ($\rho_s = 0.5$). Although the existing EV-CSS detector has the highest PAPR at 0 INR. It's PAPR performance degrades sharply under co-channel interference. The proposed DCE and N-DCE detectors exhibit the best PAPR performance. Even when $\text{INR} = 15$ dB, for the proposed methods, the DCE and N-DCE detectors could reach a PAPR of around 8 in correlated noise

environment. Although there is a drop in PAPR due to the higher mean power level of the correlated noise, the PAPR of DCE and N-DCE detectors still outperform all the existing methods by almost 4 times. For the existing methods, the EV-CSS and SUM SAF could only reach a PAPR of 2 at $\text{INR} = 15$ dB. CAB-MUSIC algorithm could not work in correlated noise environment.

Figure 5-7: Peak to average ratio comparisons of the proposed methods against existing methods with varying interference-to-noise ratio ($N = 100$, $\text{SRR} = 200$ kHz, $\rho_s = 0$).

Figure 5-8: Peak to average ratio comparisons of the proposed methods against existing methods with varying interference-to-noise ratio ($N = 100$, $SRR = 200$ kHz, $\rho_s = 0.5$).

5.4.4 Detector Performance Analysis using Receiver Operating Characteristic (ROC) Curves

The ROC curves of the proposed methods (CAB-MUSIC, DCE and N-DCE algorithms) are compared with the existing methods (SUM, CAF and EV-CSS algorithms) methods in this section. The detection probability (P_D) of the various detection algorithms are analyzed under three key detection parameters: (i) Probability of false alarm (P_{FA}); (ii) Interference-to-noise ratio (INR); (iii) Number of sample size (N). The impact of spatially correlated noise is also analyzed. In all the simulations in this section, the constant false alarm rate (CFAR) is set to 0.1 unless otherwise stated.

In the first example, the ROC curves under $INR = 20 \text{ dB}$, $SNR = 0 \text{ dB}$, $N = 300$ and $\rho_s = 0$ with the effect of varying probability of false alarm on probability of detection for the proposed methods and the existing methods are shown in Figure 5-9. All the proposed detectors have comparable performance and are superior to the existing methods. This is consistent with the PAPR result in Figure 5-7. Note that while CAB-MUSIC has comparable good performance, it requires the signal and noise subspaces to be known. The summation CAF algorithm is the worst performing detector as it could not suppress the interference. In Figure 5-10, the effectiveness of all methods to spatially correlated noise ($\rho_s = 0.5$) is investigated. CAB-MUSIC algorithm could not work in correlated noise environment. Note that in Figure 5-8, EV-CSS might has the best PAPR at $INR = 0 \text{ dB}$ but when $INR = 20 \text{ dB}$, its detection probability degrades sharply in a correlated noise environment.

The best performing algorithm is the proposed DCE algorithm which uses the principle eigenvector derived from the maximum signal-to-interference plus noise ratio (SINR). The array gain with correlated noise [83] - [85] is as follows:

$$\beta = \frac{E\{\hat{\gamma}_{max}\}}{\Gamma \frac{1}{(1 + \rho_s)} \frac{1}{(1 - \rho_s)} \frac{2}{1 - \rho_s^2}} \quad (5.38)$$

where $E\{\hat{\gamma}_{max}\}$ is the maximum average SNR in the case of correlated noise and Γ denotes the per-receive antenna average SNR. Therefore, the detection probability of the proposed DCE algorithm increased from below 0.8 in Figure 5-9 to near 1 in Figure 5-10 as correlated noise ($\rho_s = 0.5$) increased the array gain which is larger than that of independent and identically distributed (IID) noise ($\rho_s = 0$).

Figure 5-9: Receiver operating characteristic comparisons of the proposed methods against existing methods with varying probability of false alarm. ($INR = 20\text{ dB}$, $SNR = 0\text{ dB}$, $N = 300$, $\rho_s = 0$).

Figure 5-10: Receiver operating characteristic comparisons of the proposed methods against existing methods with varying probability of false alarm. (INR = 15 dB, SNR = 0 dB, $N = 300$, $\rho_s = 0.5$).

In the second example, the ROC curves under $P_{FA} = 0.1$, $SNR = 0$ dB, $N = 300$ and $\rho_s = 0$ with the effect of varying INR on probability of detection for the proposed methods and other existing methods are shown in Figure 5-11. All the proposed detectors have comparable performance and are superior to the existing methods. The CAB-MUSIC has a slightly better performance among the proposed detectors, but it requires the signal and noise subspaces to be known. Even with the present of a high powered interference signal at 20 dB, the proposed detectors could still reach a high detection probability ($P_D \approx 0.8$) compared to the low detection probability ($P_D < 0.4$) of the existing detectors.

In Figure 5-12, the effectiveness of all methods to spatially correlated noise ($\rho_s = 0.5$) is investigated. CAB-MUSIC algorithm clearly could not work in correlated noise environment as its detection probability is less than 50% ($P_D < 0.5$). The detection probability of the existing methods reduced under correlated noise environment. The DCE algorithm exhibits the best detection performance under correlated noise environment with varying INR.

Figure 5-11: Receiver operating characteristic comparisons of the proposed methods against existing methods with varying INR. ($P_{FA} = 0.1$, $SNR = 0$ dB, $N = 300$, $\rho_s = 0$).

Figure 5-12: Receiver operating characteristic comparisons of the proposed methods against existing methods with varying INR. ($P_{FA} = 0.1$, $SNR = 0$ dB, $N = 300$, $\rho_s = 0.5$).

In the third example, the ROC curves under $\text{INR} = 15 \text{ dB}$, $\text{SNR} = 0 \text{ dB}$, $P_{FA} = 0.1$ and $\rho_s = 0$ with the effect of varying the number of sample size on probability of detection for the proposed methods and other existing methods are shown in Figure 5-13. Significant improvements in detection probability of detection can be seen with increasing sample size across all algorithms. The slope of the curve of the proposed algorithms increased rapidly and are superiors to the existing methods. The required sensing time can be cut down drastically by utilizing the proposed algorithms.

In Figure 5-14, the effectiveness of all methods to spatially correlated noise ($\rho_s = 0.5$) is investigated. CAB-MUSIC algorithm clearly could not work in correlated noise environment as its detection probability is less than 50% ($P_D < 0.5$). The detection probability of the existing reduced significantly under correlated noise environment. The DCE algorithm exhibits the best detection performance with increased sample size.

Figure 5-13: Receiver operating characteristic comparisons of the proposed methods against existing methods with varying number of sample size. ($\text{INR} = 20 \text{ dB}$, $\text{SNR} = 0 \text{ dB}$, $\rho_s = 0$).

Figure 5-14: Receiver operating characteristic comparisons of the proposed methods against existing methods with varying number of sample size. ($\text{INR} = 20 \text{ dB}$, $\text{SNR} = 0 \text{ dB}$, $\rho_s = 0.5$).

5.5 Summary

In this chapter, three multiple antenna cyclostationary-based CF estimation and detection methods - CAB-MUSIC, DCE and N-DCE - are proposed. The first proposed algorithm, named the CAB-MUSIC, uses the weight vector from the cyclostationary beamforming algorithm named CAB to form a projection onto the noise subspace derived from the sample covariance matrix. The output of the algorithm could estimate and detect the cycle frequencies of cyclostationary signals under co-channel interference. However, the number of signals that impinge on the array must be known in advance in order to find the noise subspace. Moreover, CAB-MUSIC could not work under spatially correlated noise environment.

The second proposed algorithm, named dominant cyclic eigensubspace (DCE), uses the dominant cyclic eigensubspace for signal reconstruction to obtain the optimal weight. The weight vector of DCE is then used to compute the output power which could estimate and detect the cycle frequencies of cyclostationary signals. DCE algorithm can work in spatially correlated noise and without *a priori* information on the number of sources in the received data.

The third proposed algorithm, named non-dominant cyclic eigensubspace (N-DCE), uses the non-dominant cyclic eigensubspace for projection onto the constrained weight vector from the cyclostationary beamforming algorithm named CAB. The output of the algorithm could estimate and detect the cycle frequencies of cyclostationary signals under co-channel interference. N-DCE algorithm can work in spatially correlated noise and without *a priori* information on the number of sources in the received data.

The proposed methods have been investigated by numerical experiments. The proposed methods not only can achieve higher estimation accuracy and higher probability of detection under strong co-channel interference condition, they can also estimate and detect faster using fewer samples compared to existing cyclostationary cycle frequency estimation and detection methods.

Chapter 6

Conclusion and Future Work

The research studies reported in this thesis focused on the investigation of multiple cycle frequencies estimation and detection using cyclostationary beamforming under co-channel interferences. The main objective is to develop algorithms that could estimate and detect the weaker cyclostationary signals in the presence of strong cyclostationary signals. Furthermore, this thesis also studies the effect of spatially correlated noise on the proposed methods. The main conclusions and future work are summarized as follows.

6.1 Conclusions

To understand the effect of co-channel interference on existing algorithms, numerical studies have been conducted on the detection method in [11] which is based on the cyclic correlation significance test and the summation CAF algorithm in 3.2.1. It is found that under co-channel interference, both existing methods could not estimate the cycle frequency of the SOI correctly. From the two scenarios of the numerical studies shown,

either the SNR of the SOI or the number of sample size must be increased. However, based on the hidden user problem or the limitation of the sensing duration, such solutions might not be feasible.

Based on the investigations of the existing eigenvalue-based algorithm of Chapter 3, a technique which is also eigenvalue-based is proposed for multiple cycle frequencies estimation under co-channel interference. It is found that the proposed dominant eigenvalue algorithm, which is based on the CAF matrix, has a better estimation performance than the existing methods based on the RMSE. Not only does it offer better accuracy, it has the least computational complexity as reported in Chapter 3. The use of the CAF instead of the SCF for CF estimation has reduced the computational complexity significantly.

Although the dominant eigenvalue algorithm has a better performance than the existing methods, findings show that the dominant EV algorithm degrades when the co-channel interference increases in power. Therefore, a technique, which offers interference suppression, is proposed in Chapter 3. The proposed eigenanalysis cyclic null spectrum uses both the unconstrained and constrained beamformer outputs to compute a cyclic null spectrum which could detect weaker cyclostationary signals in the presence of strong cyclostationary signals. The effectiveness of the proposed ECNS algorithm against the existing methods are shown in Chapter 3.

In cyclostationary spectrum sensing, it is important that the spectrum sensing algorithm can detect the primary users under co-channel interference environment. Therefore, besides reporting on the estimation performance of the proposed algorithms in Chapter 3, the proposed dominant EV and ECNS algorithms are modified and their detection thresholds are included. The detection performance results from the receiver

operating characteristic (ROC) of the proposed algorithms in Chapter 3 are demonstrated in Chapter 4.

Subspace projection method in [26], [69] and [70] has been widely used to determine the direction-of-arrival (DOA) of the SOI in the received data. However, these methods need *a priori* information on the number of sources in the received data. The performance of these methods degrades significantly if the number of sources is estimated inaccurately. Therefore, in Chapter 5, a numerical study has been conducted on the method in [70] which is based on the subspace projection on the cyclic covariance matrix. It is found that the cyclic covariance matrix could not be used for subspace projection on the weighting vector of CAB as they are decomposed from the same eigenstructure. Therefore, based on such findings, a technique is proposed to detect and estimate multiple cycle frequencies under co-channel interference using subspace projection. The proposed CAB-MUSIC algorithm allows orthogonality projection using the weighting vector of CAB and the MUSIC noise subspace.

Two key disadvantages of the proposed CAB-MUSIC algorithm are that: (i) *a priori* information of the signal and noise subspace is needed; (ii) the sources could not be correlated. Therefore, investigation is made on the eigenstructure of the cyclic covariance matrix to decompose the eigenspaces to dominant eigensubspace and non-dominant eigensubspace in Chapter 5. Two techniques are proposed which could estimate multiple cycle frequencies under co-channel interference without *a priori* information of the signal and noise subspace. As the proposed algorithms use the cyclic covariance matrix, they worked well under spatially correlation noise environment. Estimation and detection performances of the proposed algorithms are shown in Chapter 5.

6.2 Future Work

Based on studies and investigations conducted, the findings obtained, and the techniques proposed in this thesis, two main suggestions for future research are as follows.

In this thesis, the weighting vector of the CAB [6] is chosen. Note that there are several other blind cyclostationary beamforming algorithms in the literature [32] with different computation of weighting vectors. Investigation can be carried out to study the weighting vectors of such CAB algorithms which could potentially be suitable for multiple cycle frequencies estimation and detection under co-channel interference.

Another potential future work would be the study of joint cycle frequencies and lags utilization [86] for multiple cycle frequencies estimation and detection. The choice of the lag affects the accuracy of the CF estimation and detection. Further investigations could be done regarding the use of multiple delays (lags) and cyclic frequencies to improve CF estimation and detection under co-channel interference.

Author's Publications

Journal Papers

- [1] J. X. Yang, W. Ser and L. Lei, "Multiple Cycle Frequencies Estimation Under Cochannel Interference," in *IEEE Signal Processing Letters*, vol. 25, no. 11, pp. 1645-1649, Nov. 2018.
- [2] J. X. Yang, W. Ser and L. Lei, " Multiple Antenna Cyclostationary-Based Spectrum Sensing under Co-Channel Interference for Cognitive Radio," in *IEEE Transactions on Vehicular Technology*, (Status: Under Review with Associate Editor)
- [3] J. X. Yang, W. Ser and L. Lei, " Multiple Cyclostationary Cycle Frequencies Estimation and Detection using Subspace Method," in *IEEE Transactions on Signal Processing*, (Status: In Preparation for Submission)

Conference Papers

- [1] X. G. J. Yang, Wee Ser, S. G. Razul and C. M. S. See, "Impulsive noise mitigation using an adaptive WMA-MVDR estimator for array based wireless communications," *2013 9th International Conference on Information, Communications & Signal Processing*, Tainan, 2013, pp. 1-4.
- [2] J. X. Yang, W. Ser and L. Lei, "Multiple Cycle Frequencies Estimation Under Cochannel Interference," *ICASSP 2019 - 2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Brighton, United Kingdom, 2019.

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