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A sequence model for air traffic flow management rerouting problem

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ABSTRACT

With the continuous growth of air transportation industry, the current Air Traffic Management systems can hardly meet the demand recently. In this work, an integer optimization model for Air Traffic Flow Management (ATFM) which integrates 4D trajectory-based operations is proposed to improve the situation. Besides addressing the issues of rerouting, ground-holding delay, fuel consumption and flight cancellation on a flight-by-flight basis, the model also employs sector-less airspace configurations. In order to solve the model efficiently, Dantzig-Wolfe decomposition and a heuristic approach via column generation are developed to generate 4D conflict-free trajectories for each flight. By applying commercial optimization software, solutions could be obtained in 20 min for the ATFM rerouting problems of the whole Southeast Asia region.

1. Introduction

With the recovering global economies, air travel demand has been increasing rapidly. The [Federal Aviation Administration \(2013\)](#) (FAA) predicted that the U.S. carrier passenger growth rate is approximated to be 2.2% annually in the next 20 years, and the passenger demand over Asia-Pacific area is estimated to increase 4.3% per year. This continuous growth of air transportation industry has put tremendous pressure on the aviation system. Airspace congestion frequently happens because adverse weather always makes airspace unflyable. The U.S. Dept. of Transportation reported that approximately 21.6% of the flights within the United States were delayed in 2014 and 2.25% were canceled ([Bureau of Transportation Statistics, 2014](#)). The cancellation ratio has hit record highs during 2014 ([Fig. 1](#)). In 2004, the European Commission launched the Single European Sky ATM Research (SESAR) program to restructure the European airspace, create additional capacity and increase the overall efficiency of the ATM system. Whereas in 2007, the Joint Planning and Development Office of the U.S. initiated a long-term program which is called the Next Generation Air Transportation System (NextGen). The Motivation of NextGen is to improve ATM systems in various aspects such as terminal operations, coordination between all ATM subsystems, tracking on the airway and aircraft monitoring technology. SESAR and NextGen have different implementation frameworks, but they are all developing state-of-the-art technologies to enable aircraft fly their preferred 4D trajectories without limitations.

The motivation of this paper is to propose a type of trajectory-based Air Traffic Flow Management (ATFM) system to support SESAR and NextGen. In this work, we embed 4D trajectory-based operations into ATFM to minimize total delay cost and improve the capacity of airspace, while achieving energy-efficiency. ATFM is a strategic planning methodology introduced by [Odoni \(1987\)](#) which is employed to smooth the air traffic flow at an optimal rate while not exceeding the capacities of airports and other facilities. Nowadays, ATFM plays a crucial role in minimizing air traffic delays with the continuous growth of air transportation industry. In the framework of ATFM, there are two stakeholders: Air Navigation Service Providers and Airlines. Air navigation

Fig. 1. On-time performance. (Data Source: Bureau of Transportation Statistics, 2014.)

service providers take charge of the functioning of the whole ATFM system. In the U.S., the principal air navigation service provider is the FAA, and in Europe, EUROCONTROL provides service to all users. Airlines are the primary users of the navigation service. Navigation service providers have several types of practical ATFM tools which are depicted in Fig. 2 to control all phases of each aircraft from taking-off to landing, and these approaches include Ground Delay Program (GDP), Airborne Holding Delay, Rerouting and Speed Control.

The initial ATFM models are proposed to solve the Ground-Holding Problem (GHP) which concentrates on how to assign ground-holding delays. Terrab and Odoni (1993) and Richetta and Odoni (1993, 1994) develop different approaches to address the problem of assigning ground-holding delays for a single destination airport (Single-Airport Ground-Holding Problem). Multiple-Airport Ground-Holding Problem (MAGHP) is studied by Vranas et al. (1994). In recent years, ATFM models start to address the sector capacity constraints. Agustin et al. (2010) classify these problems as Air Traffic Flow Management Problem (ATFMP) and Air Traffic Flow Management Rerouting Problem (ATFMRP). The only difference between the two problems is that rerouting operations are allowed in ATFMRP. Bertsimas and Patterson (1998) prove ATFMP is NP-hard and propose an integer programming model for the deterministic multi-airport ATFM problem. The ATFM situations of the U.S. and Europe are different: in the U.S., the restriction of ATFM system is the Airport Acceptance Rate, whereas, in Europe, sector capacity acts as the bottleneck. Lulli and Odoni (2007) provide an integer optimization approach to address the peculiarities of European ATFM problems, and they obtain a counter-intuitive conclusion that airborne holding delay sometimes can reduce the total delay cost in Europe. Bertsimas and Patterson (2000) solve the multi-airport ATFMRP by applying a dynamic network flow approach. Bertsimas et al. (2011) propose an ATFMRP model on a flight-by-flight basis and employ strong valid inequalities to solve large-scale applications efficiently. Churchill et al. (2009) point out that ATFMRP is hard to solve because too many sectors must be included, and they introduce a revised airspace structure based on Bertsimas et al. (2011)'s work to improve computational performance. Mukherjee and Hansen (2009) present a model for ATFMRP to reroute flights inbound to airports dynamically. Dell'Olmo and Lulli (2003) develop a two hierarchical ATFMRP model with airway capacity constraints, and conflict-free trajectories can be generated for each flight. Alonso-Ayuso et al. (2012a,b) propose two ATFMRP models based on the airway to address both deterministic and stochastic scenarios. The issue of energy-efficiency has drawn lots of academic attention, but the above works have not considered it.

There has been significant research effort related to sector-based ATFMRP, but the systems still cannot meet the huge demand in the future. One primary reason is that the sector-based airspace configuration imposes restrictions on airspace capacity. In some situations that sector capacity is low, conflicts could still happen, and the workload of air traffic controller is very high. The concept of "Free Flight" intends to transform the air traffic control system from a centralized system controlled by air traffic controller to a distributed system controlled by each pilot to improve system capacity and flexibility. Unfortunately, it is far from legal because no state-of-the-art technologies and procedures could ensure safety in the framework of "Free Flight". With these practical issues, there is

Fig. 2. ATFM tools.

Fig. 3. Abstract airway structure.

an urgent need to propose an implementation framework of 4D trajectory-based ATM system to support the long-term goals of SESAR and NextGen.

The objective of this work is to propose a new sequence model for ATFM which incorporates 4D trajectory-based operations based on the proposals of SESAR and NextGen. In this paper, the constraints from the sector-based airspace configuration are dropped to increase airspace capacity, and trajectory-based operations in ATFM become implementable by introducing a sector-less airway configuration. Even though the concept of “Free Flight” fails in execution now, conflict-free could be put into practice by applying 4D trajectory-based operations. This model assigns each flight a conflict-free 4D trajectory which would make a substantial change in improving airspace capacity and the ATM system predictability. The issues of energy-efficiency and safe separation could also be addressed by the approach.

This paper is organized as follows: in Section 2 we describe the ATFM model with the corresponding mathematical formulation. Section 3 introduces the Dantzig-Wolfe decomposition and heuristic method. Section 4 is the computation analysis and numerical test concerning the ATFM applications of Southeast Asia region. Section 5 contains the conclusion and future works.

2. Mathematical formulation

The mathematical formulation of ATFM model proposed in this section is to generate 4D trajectories to minimize total delay, cancellation cost, and fuel burn. As discussed in the introduction section, some configurations and assumptions of this model are different from other existing ATFM works, and we give a brief introduction in the first step.

The airspace configuration in our work is changed from sector-based structure to a sector-less configuration called abstract airway structure. A demonstration of abstract airway structure is shown in Fig. 3. There are three types of representations of airways in the illustration. The one on the top is strategic airway graph, in which there is only one arc to indicate that there are airways between waypoints C and D. This type of configuration is employed in the traditional sector-based ATFM models by [Alonso-Ayuso et al. \(2012a,b\)](#), but it is hard to track the flight levels and locations of the aircraft. The second one is tactical airway structure, and there are lots of details to demonstrate practical airways of different directions and flight levels. However, it is hard to reflect tactical structure by planar graphs since their entrances are separated. The third one is abstract airway configuration which applies planar graphs preserve the topological structure of tactical airway by assuming their entrances are identical.

In the model, we also assume that aircraft must traverse the airway if it enters that airway, and they are allowed to enter other airways only at the waypoints. This assumption is based on practical situations that aircraft is not allowed to arbitrarily change flight levels or enter other airways without ATC permissions. We may consider each airway is on a specific flight level.

The third assumption is that each airway prescribes a specific constant speed for aircraft to traverse, so overtakes are impossible on airways. In fact, the optimal cruising speed for all types of aircraft is around 850 km per hour. Since each airway has its fixed altitude, the fuel consumption costs for aircraft to traverse a different airway would be different. The reason is that jet engines are designed for specific optimal altitudes on which the fuel consumption cost and emission are minimized. Fuel consumption and emission will increase if aircraft enter airways at other altitudes. Constant speed could ensure safe separation between aircraft on the same airway just by allocating different entering time slots. Without loss of generality, we assume that various types of aircraft have their own fuel consumption costs in traversing the same airway.

Fig. 4. Abstract airway graph.

2.1. The mathematical model

In this part, a **Sequence Model for Air Traffic Flow Management Rerouting Problem (SATFMRP)** is developed based the above assumptions. In SATFMRP, the abstract airway structure includes all information of flight levels, directions, traversing time, and fuel consumption costs. The nodes in the graph represent airports or waypoints. Specifically, there are some modifications in the abstract airway graph to represent landing airports as shown in Fig. 4. We add a dummy node sa_f for the node of landing airport, and between the two nodes, a dummy arc called arrival airway is employed to track the arrival time. The arrival airway has no entrance capacity limitation since it is a virtual airway.

Parameters:

$\mathcal{F}$	Set of flights
$\mathcal{A}$	Set of airways in the system
$\mathcal{A}^f \subseteq \mathcal{A}$	Set of airways for flight f
$\mathcal{T}$	Set of time intervals
$[T_j^f, \bar{T}_j^f] \equiv T_j^f$	Set of feasible time periods for flight f to enter airway j
T_j^f	First time period of the set T_j^f
$\bar{T}_j^f$	Last time period of the set T_j^f
$\mathcal{S}$	Set of waypoints and airports
$\mathcal{S}_f \subseteq \mathcal{S}$	Set of waypoints and airports for flight f
$\mathcal{A}_{out}^f(s)$	Set of airways (arcs) outbound from waypoint s
$\mathcal{A}_{in}^f(s)$	Set of airways (arcs) inbound to waypoint s
$I_a(t)$	Entrance indicator of airway a at time t
d_f	Scheduled departure time for flight f
a_f	Scheduled arrival time for flight f
dep_f	Set of departure airways for flight f
arr_f	Arrival airway for flight f
sd_f	Departure waypoint for flight f
sa_f	Arrival waypoint for flight f
l_a	Traversing time of airway a
C_a^f	Fuel consumption cost to traverse airway a for flight f
$C_{id}^f(t)$	Total delay cost for flight f to land at time t
D^f	Cancellation cost for flight f

Airway entrance indicator:

$$I_a(t) = \begin{cases} 1 & \text{If airway } a \text{ is not closed at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Decision variables:

Airway enter variable:

$$e_{a,t}^f = \begin{cases} 1 & \text{If flight } f \text{ enters airway } a \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

For the objective function, we minimize the summation of fuel consumption cost $\sum_{f \in \mathcal{F}} \sum_{a \in \mathcal{A}_f} \sum_{t \in \mathcal{T}} C_a^f e_{a,t}^f$, the total delay cost $\sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}} C_{td}^f(t) e_{arrf,t}^f$ and the flight cancellation cost $\sum_{f \in \mathcal{F}} (1 - \sum_{t \in \mathcal{T}} e_{arrf,t}^f) D^f$.

Model:

$$\text{Min} \quad \sum_{f \in \mathcal{F}} \sum_{a \in \mathcal{A}_f} \sum_{t \in \mathcal{T}} C_a^f e_{a,t}^f + \sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}} C_{td}^f(t) e_{arrf,t}^f + \sum_{f \in \mathcal{F}} \left(1 - \sum_{t \in \mathcal{T}} e_{arrf,t}^f \right) D^f$$

The Constraints:

$$\sum_{t \in [T_{arrf}^f, T_{depf}^f]} e_{arrf,t}^f = \sum_{t \in [T_{depf}^f, T_{depf}^f]} e_{depf,t}^f \leq 1, \quad \forall f \in \mathcal{F}, \quad (1)$$

$$\sum_{f \in \mathcal{F}} e_{a,t}^f \leq I_a(t) \quad \forall t \in \mathcal{T}, \quad \forall a \in \mathcal{A}_f, \quad (2)$$

$$\sum_{b \in \mathcal{A}_{out}^f(s)} \sum_{t \in \mathcal{T}} e_{b,t}^f = \sum_{b \in \mathcal{A}_{in}^f(s)} \sum_{t \in \mathcal{T}} e_{b,t}^f \quad \forall f \in \mathcal{F}, \quad \forall s \in \mathcal{S}_f: s \neq sa_f, sd_f, \quad (3)$$

$$\sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}_{out}^f(s)} e_{a,t}^f \leq 1 \quad \forall f \in \mathcal{F}, \quad \forall s \in \mathcal{S}_f: s \neq sa_f, \quad (4)$$

$$\sum_{b \in \mathcal{A}_{out}^f(s)} e_{b,t}^f = \sum_{a \in \mathcal{A}_{in}^f(s)} e_{a,t-l_a}^f \quad \forall f \in \mathcal{F}, \quad \forall t \in \mathcal{T}, \quad \forall s \in \mathcal{S}_f: s \neq sa_f, sd_f, \quad (5)$$

$$e_{a,t}^f \in \{0,1\}, \quad \forall f \in \mathcal{F}, \quad \forall t \in \mathcal{T}, \quad \forall a \in \mathcal{A}_f. \quad (6)$$

Constraint set (1) states that any flight could take off or be cancelled. Constraint set (2) ensures that only one flight could enter one airway in a time period when the entrance indicator of that airway is 1. Constraint set (3) guarantees the network flow balance at each waypoint which means that if a flight enters one airway inbound to a waypoint, it will enter one subsequent airway outbound from that waypoint. Constraints (4) state that at each waypoint, each flight could enter at most one subsequent airway. Constraints (5) are the connectivity constraints which ensure after traversing an airway inbound to a waypoint, aircraft enters the subsequent airway outbound from that waypoint immediately. The above formulation of SATFMRP drops the constraints from the sector-based airspace configuration. Thus the airspace could accommodate more aircraft theoretically.

2.2. Complexity of the SATFMRP

In this section, we prove that the SATFMRP is NP-hard.

Theorem 1. *The SATFMRP with all airway traversing times equal to 1 is NP-hard.*

Proof 1. Job-Shop Scheduling Problem (JSP) can be reduced to SATFMRP.

JSP consists of m processors and a job set J . In each job $j \in J$, there is an ordered collection of tasks $t_k[j]$, $1 \leq k \leq n_j$ (n_j is the number of tasks in job j). For each task t , there is a processing time $l(t) \in \mathbb{Z}_0^+$ ($l(t) = 1$ in this problem) and a processor $p(t) \in \{1, 2, \dots, m\}$. In JSP, $p(t_k[j]) \neq p(t_{k+1}[j]) \quad \forall j \in J, 1 \leq k < n_j$, which means that two consecutive tasks must be processed by different processors. The objective of JSP is to minimize the deadline D which is a positive integer. JSP subjects to the following 3 restrictions: (a) once a machine starts processing one task, it must finish this task before processing other ones. (b) At any given time period, each machine can only process at most one task. (c) The tasks of a job must be processed in the required order.

The decision problem for JSP is that whether there is a possible schedule meet the overall deadline.

For each job in the set, we create a flight and each airway of our proposed structure corresponds to a processor. For each task $t_k[j]$ of job j , we associate a time segment for flight j to traverse an airway. Furthermore, the starting time of task $t_k[j]$ corresponds to the airway enter time for flight j . Then, we obtain an ordered collection of airways, $(A_j^1, A_j^2, \dots, A_j^k, \dots, A_j^{n_j})$, and an ordered sequence of time segments accordingly, $(t_{A_j^1}^1, t_{A_j^2}^2, \dots, t_{A_j^{n_j}}^{n_j})$. For each flight j , the relationship could be described as follows:

$$A_j^1 = p(t_j[1]), t_{A_j^1}^1 = l(t_j[1]), \dots, A_j^{n_j} = p(t_j[n_j]), t_{A_j^{n_j}}^{n_j} = l(t_j[n_j]).$$

If we can find a feasible solution for the SATFMRP in which each flight is assigned a conflict-free trajectory and arrives before the deadline D , a corresponding job-shop schedule that meets the required conditions would be obtained.

The starting time of the subsequent task is

$$\sigma_{i'}(t_{k+1}[j]) = \sigma_i(t_k[j]) + l(t_k[j]) = \sigma_i(t_k[j]) + 1, \quad (i' = p(t_{k+1}[j]), \quad i = p(t_k[j])).$$

According to the above relationship, no two tasks would be processed simultaneously on one processor, which is equivalent to limit the entrance capacity of airways to one and no two flights could enter the same airway at the same time period. The relationship also requires that a task is not qualified to be processed unless the previous one has been processed and this ensures the connectivity between airways. Thus, if there is a feasible solution to the SATFMRP, a feasible job-shop schedule could be found. $\square$

Since the traversing time of each airway is not necessarily equal to one in practice, we could convert each long airway into a sequence of short airways with traversing time of one. Thus, we could show that SATFMRP is NP-hard.

3. Solution approach

We have proved SATFMRP is NP-hard, and it is impossible to handle large-scale instances by solving the original model directly with commercial software. The reason is that the scale of original formulation proliferates with the number of flights, airways and time periods. The out of memory cases happen even if the instances include only 50 flights in a practical setting which have 105 airways, 60 time periods and seven airports. In order to properly handle practical applications, it is necessary to design some heuristic approaches. In this section, we propose an efficient heuristic methodology based on column generation. Before introducing the algorithm, we must decompose the original formulation into pricing subproblems and a restricted master problem by applying Dantzig-Wolfe decomposition (George and Dantzig, 1960). Each pricing subproblem is responsible for generating trajectories for a specific flight. The master problem takes charges of flight cancellation and conflict resolution. Based on the decomposed formulation, a heuristic column generation algorithm is proposed to obtain the integer solutions for large-scale applications.

3.1. Dantzig-Wolfe decomposition

In the original problem, there is an exponentially large number of possible 4D trajectories, and we resort to Dantzig-Wolfe decomposition algorithm to reformulate the original problem. The integer restricted master problem which addresses flight cancellation and conflict resolution could be formulated as follows:

Restricted Master Problem

Parameters:

$\mathcal{T}$	Set of time intervals
$\mathcal{F}$	Set of flights
$\mathcal{A}$	Set of airways in the system
$\mathcal{A}^f \subseteq \mathcal{A}$	Set of airways can be chosen by flight f
$\mathcal{R}$	Set of 4D trajectories
$\mathcal{R}_f \subseteq \mathcal{R}$	Set of 4D trajectories can be chosen by flight f
$\mathcal{R}_a^f(t) \subseteq \mathcal{R}_f$	Set of 4D trajectories that flight f enters airway a at time t
c_r^f	Delay and fuel consumption cost for flight f to choose trajectory r
D^f	Cancellation cost for flight f

Airway entrance indicator:

$$I_a(t) = \begin{cases} 1 & \text{If airway } a \text{ is not closed at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Decision variable:

Trajectory selection variable:

$$x_r^f = \begin{cases} 1 & \text{if trajectory } r \text{ is selected by flight } f, \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{Min} \quad \sum_{f \in \mathcal{F}} \sum_{r \in \mathcal{R}_f} c_r^f x_r^f + \sum_{f \in \mathcal{F}} \left(1 - \sum_{r \in \mathcal{R}_f} x_r^f \right) D^f$$

The Constraints:

$$\sum_{r \in \mathcal{R}_f} x_r^f \leq 1 \quad \forall f \in \mathcal{F}, \quad (7)$$

$$\sum_{f \in \mathcal{F}} \sum_{r \in \mathcal{R}_a^f(t)} x_r^f \leq I_a(t) \quad \forall t \in \mathcal{T}, \quad \forall a \in \mathcal{A}, \quad (8)$$

$$x_r^f \in \{0, 1\}. \quad (9)$$

The objective function could be reformulated and the mathematical formulation of the whole model can be expressed as follows,

$$\text{Max} \quad \sum_{f \in \mathcal{F}} \sum_{r \in \mathcal{R}_f} (D^f - c_r^f) x_r^f - \sum_{f \in \mathcal{F}} D^f$$

The Constraints:

Table 1
Matrix representation of the restricted master problem.

	$x_{r1}^{f_1}$ $Df_1 - c_{r1}^{f_1}$	$x_{r2}^{f_2}$ $Df_2 - c_{r2}^{f_2}$	RHS	Dual	Objective coefficients
Flight f_1	1	0	≤ 1	ϕ_{f_1}	Each flight selects at most one trajectory
Flight f_2	0	1	≤ 1	ϕ_{f_2}	
a_1 Time period 1	1	0	≤ 1	$\psi_{a_1}(t_1)$	Airway entrance capacity
a_1 Time period 2	0	1	≤ 1	$\psi_{a_1}(t_2)$	
a_1 Time period 3	0	0	≤ 1	$\psi_{a_1}(t_3)$	
a_1 Time period 4	0	0	≤ 1	$\psi_{a_1}(t_4)$	
a_2 Time period 1	0	0	≤ 1	$\psi_{a_2}(t_1)$	
a_2 Time period 2	0	0	≤ 1	$\psi_{a_2}(t_2)$	
a_2 Time period 3	1	0	≤ 1	$\psi_{a_2}(t_3)$	
a_2 Time period 4	0	1	≤ 1	$\psi_{a_2}(t_4)$	
a_3					Dummy airway

Primal problem:

$$\begin{aligned}
 & \text{Max} \quad \sum_i c_i x_i \\
 & \text{s. t.} \quad \sum_i a_{ij} x_i \leq 1, \quad \forall j \\
 & \quad \quad x_i \geq 0, \quad \forall i
 \end{aligned}$$

from the duality theory in linear programming, we can obtain the dual problem from the primal problem.

Dual problem:

$$\begin{aligned}
 & \text{Min} \quad \sum_j y_j \\
 & \text{s. t.} \quad \sum_i a_{ij} y_j \geq c_i, \quad \forall i \\
 & \quad \quad y_j \geq 0, \quad \forall j
 \end{aligned}$$

The values of y_j in the dual problem correspond to that of ϕ_f and $\psi_a(t)$. $\square$

3.2. Subproblem

In this section, we present the formulation of the subproblem. In the framework of column generation, each flight associates a subproblem to generate 4D trajectories for itself. After the linear programming relaxation of the restricted master problem is solved, the dual values are employed in the subproblems as the coefficients of the objective function. Then, we could solve the subproblems to generate good columns and add them to the column set of the restricted master problem. The subproblem can be formulated as follows:

Parameters:

$\mathcal{T}$	Set of time intervals
$\mathcal{A}^f \subseteq \mathcal{A}$	Set of airways can be chosen by flight f
$\mathcal{P}_f \subseteq \mathcal{P}$	Set of waypoints by flight f
$\mathcal{A}_{out(s)}^f$	Set of airways (arcs) outbound from waypoint s
$\mathcal{A}_{in(s)}^f$	Set of airways (arcs) inbound to waypoint s
$[T_j^f, T_j^f] \equiv T_j^f$	Set of feasible time periods for flight f to enter airway j
$\underline{T}_j^f$	First time period in the set T_j^f
$\bar{T}_j^f$	Last time period in the set T_j^f
$I_a(t)$	Entrance indicator of airway a at time t
	Scheduled departure time of flight f

d_f	
a_f	Scheduled arrival time of flight f
dep_f	Set of departure airways for flight f
arr_f	Arrival airway for flight f
sd_f	Departure waypoint for flight f
l_a	Traversing time of airway a
C_a^f	Fuel consumption cost of traversing airway a for flight f
$C_{id}^f(t)$	Total delay cost for flight f to land at time t
D^f	Cancellation cost for flight f
ϕ_f	Dual variables for constraints (13)
$\psi_a(t)$	Dual variables for constraints (14)

Decision variable:

Airway enter variable:

$$e_{a,t}^f = \begin{cases} 1 & \text{flight } f \text{ enters airway } a \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

After the restricted master problem is solved, we get the values of dual variables ϕ_f and $\psi_a(t)$. The increased profit of choosing a 4D trajectory by flight f can be calculated by **Flight cancellation cost – Fuel consumption cost – Arrival delay cost – Dual of constraints (13) – Dual of constraint (14)**.

The mathematical expression is

$$D_f - \sum_{a \in \mathcal{A}_f} \sum_{t \in \mathcal{T}} C_a^f e_{a,t}^f - \sum_{t \in \mathcal{T}} C_{id}^f(t) e_{arr_f,t}^f - \phi_f - \sum_{a \in \mathcal{A}_f} \sum_{t \in \mathcal{T}} \psi_a(t) e_{a,t}^f.$$

The objective of subproblems is to search a 4D trajectory that could maximize the increased profit for each flight. The formulation of subproblem for each flight can be formulated as follows:

$$\text{Max } D_f - \sum_{a \in \mathcal{A}_f} \sum_{t \in \mathcal{T}} C_a^f e_{a,t}^f - \sum_{t \in \mathcal{T}} C_{id}^f(t) e_{arr_f,t}^f - \phi_f - \sum_{a \in \mathcal{A}_f} \sum_{t \in \mathcal{T}} \psi_a(t) e_{a,t}^f$$

The Constraints:

$$\sum_{t \in \mathcal{T}} e_{dep_f,t}^f = 1 \quad \forall t \in [T_{dep_f}^f, \bar{T}_{dep_f}^f], \quad (16)$$

$$\sum_{t \in \mathcal{T}} e_{arr_f,t}^f = 1 \quad \forall t \in [T_{arr_f}^f, \bar{T}_{arr_f}^f], \quad (17)$$

$$\sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}_{out}^f(s)} e_{a,t}^f \leq 1 \quad \forall t \in \mathcal{T}, \quad \forall s \in \mathcal{S}_f: s \neq sa_f, \quad (18)$$

$$\sum_{b \in \mathcal{A}_{out}^f(s)} e_{b,t}^f = \sum_{a \in \mathcal{A}_{in}^f(s)} e_{a,t-l_a}^f \quad \forall t \in \mathcal{T}, \quad \forall s \in \mathcal{S}_f: s \neq sa_f, sd_f, \quad (19)$$

$$\sum_{b \in \mathcal{A}_{out}^f(s)} \sum_{t \in \mathcal{T}} e_{b,t}^f = \sum_{b \in \mathcal{A}_{in}^f(s)} \sum_{t \in \mathcal{T}} e_{b,t}^f \quad \forall s \in \mathcal{S}_f: s \neq sa_f, sd_f, \quad (20)$$

$$e_{a,t}^f \in \{0,1\}, \quad \forall f \in \mathcal{F}, \quad \forall t \in \mathcal{T}, \quad \forall a \in \mathcal{A}_f. \quad (21)$$

The first and second constraint sets require that the flight must select feasible time slots for departure and landing. Constraint set (18) ensures that at each waypoint, the flight could enter at most one subsequent airway. Constraints (19) are the connectivity constraints which ensure that after traversing an airway inbound to a waypoint, the flight would enter one subsequent airway outbound from that waypoint immediately. Constraint set (20) guarantees the network flow balance at each waypoint which means that if the flight enters one airway inbound to a waypoint, it will enter one subsequent airway outbound from that waypoint.

3.3. Heuristic methodology

In this section, we present a heuristic method to handle large-scale instances. In the beginning, the default set of columns (4D trajectories) for the restricted master problem is empty, and we first develop a preprocessing algorithm as shown in Algorithm 1 to generate one trajectory for each flight. In the algorithm, we apply dynamical search method 12.4 (CPLEX-MIP) to solve the integer subproblem since it is already fast.

Algorithm 1. Preprocessing Algorithm

Initialization: Sorting the scheduled departure times of all flights in ascending order, and labeling each flight with a number n from 1 to $|F|$ according to the departure order;
Set $n = 1$, $\phi_f = 0$, $\forall f \in F$ and $\psi_a(t) = 0$, $\forall a \in A$, $\forall t \in T$. θ is a large positive integer;
while $n \leq |F|$ **do**
 Solve the integer subproblem for flight n by CPLEX and obtain the solution $e_{a,t}^n$;
 Pick the solution to the column set of the restricted master problem;
 Set $\psi_a(t) = \theta * e_{a,t}^n + \psi_a(t)$;
 $n = n + 1$;
end

We employ a simple demonstration to show how the preprocessing algorithm works on the problem in Fig. 5. The demonstration is in Tables 2 and 3. Since two flights have identical departure times, we break the tie arbitrarily and label f_1 and f_2 with numbers 1 and 2, respectively. Then we set $\phi_f = 0$, $\forall f \in F$; $\psi_a(t) = 0$, $\forall a \in A$, $\forall t \in T$; $\theta = 1$. After that, we apply $\psi_a(t)$ as the coefficients of the subproblem to get a trajectory for flight 1 which is in the second column of Table 2, and we update the values of $\psi_a(t)$ by $\psi_a(t) = \theta * e_{a,t}^1 + \psi_a(t)$. The new dual values are in the fifth column of Table 2. In the next step, the updated values of $\psi_a(t)$ are applied in the subproblem of flight 2. A trajectory for flight 2 is obtained which is listed in the third column of Table 3 after solving the subproblem. The final values of $\psi_a(t)$ are updated again which are listed in the fifth column of Table 3.

Table 2
Restricted master problem after iteration 1.

	$x_{r_1}^{f_1}$ $D^{f_1} - c_{r_1}^{f_1}$	$x_{r_2}^{f_2}$ $D^{f_2} - c_{r_2}^{f_2}$	RHS	Pseudo dual
Flight f_1	1	0	≤ 1	$\phi_{f_1} = 0$
Flight f_2	0	0	≤ 1	$\phi_{f_2} = 0$
a_1 Time period 1	1	0	≤ 1	$\psi_{a_1}(t_1) = 1$
a_1 Time period 2	0	0	≤ 1	$\psi_{a_1}(t_2) = 0$
a_1 Time period 3	0	0	≤ 1	$\psi_{a_1}(t_3) = 0$
a_1 Time period 4	0	0	≤ 1	$\psi_{a_1}(t_4) = 0$
a_2 Time period 1	0	0	≤ 1	$\psi_{a_2}(t_1) = 0$
a_2 Time period 2	0	0	≤ 1	$\psi_{a_2}(t_2) = 0$
a_2 Time period 3	1	0	≤ 1	$\psi_{a_2}(t_3) = 1$
a_2 Time period 4	0	0	≤ 1	$\psi_{a_2}(t_4) = 0$

Table 3
Restricted master problem after iteration 2.

	$x_{r_1}^{f_1}$ $D^{f_1} - c_{r_1}^{f_1}$	$x_{r_2}^{f_2}$ $D^{f_2} - c_{r_2}^{f_2}$	RHS	Pseudo dual
Flight f_1	1	0	≤ 1	$\phi_{f_1} = 0$
Flight f_2	0	1	≤ 1	$\phi_{f_2} = 0$
a_1 Time period 1	1	0	≤ 1	$\psi_{a_1}(t_1) = 1$
a_1 Time period 2	0	1	≤ 1	$\psi_{a_1}(t_2) = 1$
a_1 Time period 3	0	0	≤ 1	$\psi_{a_1}(t_3) = 0$
a_1 Time period 4	0	0	≤ 1	$\psi_{a_1}(t_4) = 0$
a_2 Time period 1	0	0	≤ 1	$\psi_{a_2}(t_1) = 0$
a_2 Time period 2	0	0	≤ 1	$\psi_{a_2}(t_2) = 0$
a_2 Time period 3	1	0	≤ 1	$\psi_{a_2}(t_3) = 1$
a_2 Time period 4	0	1	≤ 1	$\psi_{a_2}(t_4) = 1$

Fig. 6. Flow chart of the heuristic method.

The preprocessing algorithm is based on the insight of First-Schedule-First-Departure (FSFD), and according to Corollary 4.1, all dual variables must be non-negative. With the growth of $\psi_a(t)$, the flights that take off later would avoid entering airway a at time t if $\psi_a(t) > 0$, and that's because it is a positive penalty. If θ is large enough, the preprocessing algorithm is FSFD, which means that the initial feasible solution is provided by FSFD. We could implement the preprocessing algorithm again and again under a random flight sequence to add more columns to the initial solution set. The flow chart of the whole algorithm is shown in Fig. 6. After the preprocessing algorithm, the column set of the restricted master problem is nonempty. Linear programming relaxation algorithm can be applied to the restricted master problem in calculating the values of dual variables. Then, all subproblems update the coefficient according to the dual values, and we solve them one by one to search good columns. In the searching step, we set a threshold, and if the objective value of the generated column is greater than the threshold, we send the column to the column set of the restricted master problem for the next iteration. When the optimal value of the linear programming relaxation cannot be improved, we stop the iterations and apply CPLEX to solve the integer version of the restricted master problem by limiting the decision variables to be 0–1.

4. Numerical results

In this section, we report the computational results of the heuristic algorithm proposed in the above section. In the solving process, we employ CPLEX 12.4 as the principle solver, implemented under CPLEX IDE with Optimization Programming Language. All instances are solved on a laptop with an Intel I7-3630 QM processor, 2.40 GHz, 8 GB RAM with a platform of Windows 7. In the solving process, when we encounter integer subproblems and integer master problem, we apply CPLEX dynamical search method 12.4 (CPLEX-MIP) to handle the problem with the MIP gap 0.0001%. In the experiments, we assume that all flights are homogeneous. For the values of aircraft speed, we set them at 850 km per hour. The values of total delay are calculated by hours, and we set a large positive integer for cancellation cost.

The first computation test is to show the gaps between the heuristic algorithm's solution and the exact optimal solution. The test instances include 15 airways that connect 7 airports, 15 time periods and 30–50 flights. The results are plotted in Fig. 7, and the solution gap is less than 10% which is acceptable.

The second test in this section is to demonstrate the computational performance with respect to large-scale instances which are capable of reflecting the ATFM system of the Southeast Asia. The size of the tested instance depends on the number of flights, time periods, airways and airports. We use simulated data, and all parameters of the instances in this part are based on practical situations

Fig. 7. Solution gap.

Fig. 8. Snapshot of flights. (Data source: Flightradar24.com.)

of the Southeast Asia region. Fig. 8 is a snapshot at 4 pm in the Southeast Asia region. We consider a five-hour horizon and divide it equally into 60 time periods with 5 min each. We also assume all flights must take off and land within the 60 periods if they are not cancelled. These instances consist of 300–500 flights, 105 abstract airways, 60 time periods and seven main international airports of the Southeast Asia region. These airports include Singapore Changi Airport, Hong Kong International Airport and etc. All these airports are located according to their positions in the test. Flights of the same origin-destination (o-d) pairs adopt same airway networks, and the computational results for instances with different numbers of flights are reported in Fig. 9.

It is evident in Fig. 9 that CPLEX could solve large-scale instances which include 500 flights in 20 min with the heuristic algorithm. So the methodology could be applied to practical applications. The model provides conflict-free flight plans that minimize the combination of fuel consumption, delay and cancellation cost for 5 h. One noticeable trend in the figure is that the solution time grows linearly with the number of flights, and it becomes tractable under the framework of the heuristic algorithm. One reason is that the preprocessing algorithm could provide initial solutions of high quality, and the linear programming relaxation of the restricted

Fig. 9. Solution time.

Fig. 10. Airway configurations.

Fig. 11. Conflict between aircraft.

Fig. 12. Time period 1.

Fig. 13. Time period 2.

Fig. 14. Time period 3.

Fig. 15. Time period 4.

Fig. 16. Total delay.

Fig. 17. Canceled aircraft.

master problem could be solved to optimality quickly. Another reason is that the solution time mainly depends on the number and solution time of the subproblems. We need to solve all integer subproblems and pick good columns into the master problem, so the solution time grows linearly with the number of flights since each flight associates an integer subproblem. When the optimal value of the linear relaxed restricted master problem can be no longer improved, the number of columns for each flight is not very large. CPLEX could solve the integer master problem quickly and the solution time could be ignored compared with that of all subproblems. Thus, we can get a conclusion that the solution time mainly depends on the initial solution, solution time of each subproblem and the number of subproblems.

5. Conclusion

In this paper, we propose a new sequence model for ATFMRP which has three distinguishing features. The first one is that the model drops the constraints due to the sector-based airspace configuration, so the airspace capacity could be improved theoretically while safe separation can be ensured. By introducing airway configurations, each flight could be assigned a conflict-free 4D trajectory to cover all phases from departure to landing. The second feature is that an efficient heuristic is proposed to handle large-scale instances for our ATFM model. The algorithm could guarantee the quality of solutions by the initial solutions and the computation time grows linearly with the number of flights. The solution time could be controlled within 20 min for instances which are capable of reflecting the entire Southeast Asia ATFM system. The solution includes the details of all aircraft during a 5-h duration so that the model can be applied only 4–5 times in a day with the updated weather information. The third one is that the model in this work could address the issue of fuel consumption, and it minimizes the total fuel consumption cost of the whole system to achieve energy-efficiency which is also the goal of SESAR and NextGen.

The NextGen and SESAR motivate our future research in ATFM. The first challenging work is how to define and implement 4D trajectory in ATM systems. Since today's technologies or methodologies still cannot address the safety issues on the notion of "Free-Flight", the implementation framework of 4D trajectory-based tools should be built first as a foundation. The second one is to collect more realistic data, and the data includes time periods, airway and airports which reflect the whole ATM system. Then, we can test the model and refine it based on the real situations. Thirdly, computational performance needs to be improved further to handle large-scale applications with more information.

Appendix A

The formulation of SATFMRP drops the constraints from the sector-based airspace configuration, thus the airspace could accommodate more aircraft theoretically. A simple demonstration (consisting of two homogeneous aircraft, 4 time periods) is employed here to show SATFMRP may improve the system capacity.

Parameters are listed as follows:

$$\begin{aligned}\mathcal{T} &= \{1;2;3;4\}, \\ \mathcal{F} &= \{f_1;f_2\}, \\ \mathcal{A} &= \{(\text{origin};\text{destination})\}, \\ l_a &= 2.\end{aligned}$$

The scheduled departure time is 1 and arrival time is 3. The airway configuration is shown in Fig. 10. The length of the airway is 2 and its prescribed constant speed is 1, so the traversing time is 2. Without loss of generality, we assume that the whole airway in one sector.

The sector-based ATFM models only consider the limitations on sector capacity, and if we set the sector capacities greater than or equal to 2, two flights would take off and enter the airway at the same time period. A conflict between them occurred as shown in Fig. 11, so the sector capacity must be less than 2. Since the capacity should be a positive integer, it can only be 1.

We set the sector capacity to 1 for the sector-based ATFM model in the following comparison.

We demonstrate and compare the optimal solutions of both model period by period. A and B are employed to represent traditional sector-based ATFM model and SATFMRP, respectively. At the beginning of time period 1, we demonstrate the optimal solution in Fig. 12, and there's no difference between optimal solutions of both model. Only one aircraft could depart because of safe separation and the other one must be delayed on the ground.

In Fig. 13, one obvious difference is that the delayed aircraft in SATFMRP is taking off at the beginning of time period 2 while keeping safe separation, but the traditional ATFM model still holds the delayed aircraft on the ground.

In Fig. 14, it is apparent that the delayed aircraft in traditional ATFM model is taking off at the beginning of time period 3.

From optimal strategy of the traditional ATFM model, we can find that even though the aircraft could take off safely, it must be delayed because of the limitations of sector capacity. SATFMRP could overcome the limitation and increase system capacity (see Fig. 15).

The numerical test for the next case study is to analyze the total delay and the number of aircraft canceled. The test instance has 10 time periods, 11 aircraft with random take-off time from time period 1–5. Based on the above configuration, the duration of each flight is 2 and aircraft would be canceled if they cannot take off or arrive within the time domain. In Fig. 16, it is obvious that the total delay for SATFMRP grows slower when the total number of aircraft is less than 8. But when the total number of flights is greater than 8, the total delay of SATFMRP grows faster. The reason is that SATFMRP could accommodate more delayed aircraft which would be canceled by ATFMRP. The numerical result in Fig. 17 depicts that when the number of aircraft greater than 4, ATFMRP must cancel flights because there are insufficient time slots for take-off or landing, but SATFMRP canceled its first flight when there are 11 aircraft. The numerical results in both figures demonstrate that SATFMRP outperforms the existing ATFMRP in total delay and cancellation. SATFMRP has a smaller total delay value when the time slots are sufficient, and if congestion happens, SATFMRP cancels fewer flights than that of ATFMRP.

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