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2017

Zhang, Y., & Huang, W. (2018). Impact of strategy switching on wealth accumulation. *Journal of Evolutionary Economics*, 28(4), 961-983. doi:10.1007/s00191-017-0543-3

<https://hdl.handle.net/10356/137028>

<https://doi.org/10.1007/s00191-017-0543-3>

This is a post-peer-review, pre-copyedit version of an article published in *Journal of Evolutionary Economics*. The final authenticated version is available online at:
<https://doi.org/10.1007/s00191-017-0543-3>

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Impact of Strategy Switching on Wealth Accumulation

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Abstract

Various investment strategies coexist in financial markets. Fluctuations in the profitability of strategies rationalize investors' strategy switching behaviors. Under bounded rationality and limited information, such behavior is usually driven by comparing the past performance of different strategies. But at what pace should investors change their strategies? Does a frequent strategy switching lead to a higher wealth in the end? To answer these questions, a discrete dynamic heterogeneous agent model is proposed, in which agents follow heuristic rules and a market maker adjusts the price of a risky asset. Agents are classified by their propensity of strategy switching. It is found that agents with a higher propensity adopt the better strategy more often, but end up with less final wealth. This counter-intuitive phenomenon is caused by the inconsistency between short-run profit and long-run wealth accumulation.

Keywords: heterogeneous agent model; intensity of choice; propensity of strategy switching; adaptive learning; wealth accumulation

JEL classification codes: C60; G10

Acknowledgements

We thank Shu-Heng Chen, Xue-Zhong He, Mikhail Anufriev and the participants of the 19th Workshop on Economic Science with Heterogeneous Interacting Agents (WEHIA 2014) at Tianjin University and the 21st International Conference on Computing in Economics and Finance (CEF 2015) in Taipei for valuable comments and suggestions. The usual caveat applies.

1. Introduction

To seize opportunities in a fast-changing world, people often need to make quick decisions. With bounded rationality and limited information, people tend to imitate the behavior of individuals who are more intelligent or who are believed to possess more information. Such imitation in financial markets is usually referred to as adaptive learning (Brock and Hommes, 1997, 1998) or herding (Lux, 1995). Investors observe the performance of different strategies and switch to strategies that produce higher profit than the ones they initially use. In a world of uncertainty and imperfect information, it is almost impossible to find a strictly dominant strategy. A more realistic scenario is that sometimes one strategy performs the best and another strategy performs better in another time. Therefore, investors have incentives to change their strategies from time to time.

But at what pace should investors change their strategies, and how will the frequency of change affect their wealth accumulation? These questions will be explored with a heterogeneous agent model (hereafter HAM) following the classical "fundamentalist-chartist-market maker" framework (Day and Huang, 1990)¹. Investors' tendency to switch between fundamental strategy and technical strategy will be formally referred to as *the propensity of strategy switching*². Investors with the same propensity, regardless whether they are fundamentalists or chartists, will be classified into the same group. The whole population is therefore divided into N different groups, ranging from the lowest propensity to the highest propensity. Such classification enables us to investigate, with identical initial endowment and updated information, how investors' strategy switching affects their wealth accumulation. The numerical simulation indicates that the groups with higher propensities always end up with relatively less final wealth. Such counterintuitive phenomenon reflects the inconsistency between short-run profit pursuing and long-run wealth accumulation frequently observed in reality.

Most HAMs on wealth dynamics focus on the wealth distribution among fundamentalists and chartists. Hommes and Wagener (2009) track wealth accumulated by stubborn fundamentalists and chartists.

¹ Following the tradition of behavioral economics, we choose the combination of agents' heuristic rule and a market-maker price adjustment, instead of assuming agents to maximize a specific objective such as myopic mean-variance or expected utility of wealth. Franke (2008) suggested that the story of maximization of myopic mean-variance or expected utility should be dropped, if the price adjustment is done under the market-maker framework. The similar difficulty was encountered while we tried to combine the mean-variance maximization and the market-maker framework. See Chiarella and He (2005), Chiarella et al. (2009), Hommes (2006), Hommes and Wagener (2009), LeBaron (2006) and Lux (2009) for a comprehensive review of HAM.

² The introduction of a propensity parameter in HAMs is not new at all. The parameter, commonly referred to as the intensity of choice, plays a crucial role in HAMs with adaptive learning. Most models investigate its bifurcation role to induce complex dynamics (Brock and Hommes, 1997, 1998; Chiarella et al., 2006). Recently, the estimation of this parameters caused great interest (Amilon, 2008; Anufriev et al., 2016; Boswijk et al., 2007; Chen et al., 2012; de Jong et al., 2009).

Chiarella and He (2005) and Brianzoni et al. (2010, 2012) bypass the identification problem³ by assuming that all agents of the same group agree to share their wealth and when an agent switches from one group to another, he/she brings his/her share of wealth to the second group. Another way to resolve investors' ambiguous identity is provided in Anufriev and Dindo (2010), where investors are endowed with various degrees of risk aversion. It was found that when the general growth rate of the risky asset is larger than the risk-free rate, agents with the lowest risk aversion seize most wealth and dominate the market.

In our treatment, however, such difficulty does not exist since both fundamentalists and chartists coexist in each group. No agent jumps between groups while implementing strategy switching. In this case, a group can represent an institutional investor, a household, or even an individual investor who integrates fundamental and technical analyses into his/her financial investment.

The rest of the paper is organized as follows. In Section 2, a discrete financial market with heterogeneous agents is built. Section 3 analyzes the profitability of strategies under a single propensity of strategy switching. Section 4 presents our main findings, robustness check, and further discussion. Section 5 concludes this paper.

2. A discrete dynamic HAM model

2.1. A baseline model

Assume that a financial market consists of heterogeneous investors who invest in a risk-free asset and a risky asset. Assume the return rate of the risk-free asset is r , then the gross return is $R = 1 + r$. Let P_t denote the price per share of the risky asset at time t , which includes all kinds of return from investing in this asset⁴.

There are two kinds of strategies: fundamental strategy and technical strategy. Investors adopting fundamental strategy are named as fundamentalists⁵ and denoted by superscript f , while investors adopting technical strategy are named as chartists and denoted by superscript c . At period t , type h investors' expectations about the price of risky asset in the next period $t + 1$ are $E_t^h(P_{t+1})$, where $h = f$ or c .

³ The so-called identification problem is the logic flaw that agents are identified by their strategies while strategy switching is allowed.

⁴ Our model shows no explicit dividend. Stock price P_t is a cum-dividend price.

⁵ In our model, fundamentalists or chartists are only investors' temporary roles. After we introduce investors' propensity for strategy switching as another source of heterogeneity, we identify investors by their propensities rather than their strategies.

Fundamentalists believe that there is a fundamental value underneath the fluctuating price series, to which the current price will eventually converge. P_0^f is the initial fundamental value. Later the fundamental value grows at the same rate of the risk-free asset.

$$E_t^f(P_{t+1}) = P_0^f R^t \quad (1)$$

Chartists form their expectations of future price by observing historical prices. Various technical strategies are developed in practice. Some are naive, and some are sophisticated. Here, we assume that chartists are *trend followers* who believe that the current price trend will continue in the short future. Their expectation of future price follows one period trend extrapolation, as shown in Eq. (2), in which the parameter a displays the extent of chartists' trend following

$$E_t^c(P_{t+1}) = P_t + a \cdot (P_t - P_{t-1}). \quad (2)$$

Following Day and Huang (1990), investors derive their demands for risky asset z_t^h from their expectations on future price $E_t^h(P_{t+1})$. They buy shares when they predict an excess profit from investing risky asset and sell when they foresee a loss. Their excess demand for the risky asset is of the same amount as the expected excess return on the risky asset

$$z_t^h = E_t^h(P_{t+1}) - R \cdot P_t. \quad (3)$$

Alternatively, Eq. (3) can be interpreted as investors' optimal demand to maximize the expectation of their utility, which is an exponential function of wealth showing constant absolute risk aversion (Chiarella et al., 2009), with the assumption of a constant second moment of investors' price expectation.

Under a market-maker framework, the price of the risky asset is adjusted by the market maker according to the aggregate demand for the risky asset D_t . The aggregate demand is the sum of fundamentalists' and chartists' demand weighted by their market fraction n_t^h

$$D_t = n_t^f z_t^f + n_t^c z_t^c. \quad (4)$$

Deterministic and stochastic price dynamics are respectively defined as

$$P_{t+1} = P_t + \gamma D_t, \quad (5)$$

$$P_{t+1} = P_t + \gamma D_t + \varepsilon_t, \quad (6)$$

where γ is the market maker's price-adjustment speed, and ε_t is a white noise that represents the perturbation caused by noise traders or the arrival of trivial new information.

After the asset price has been updated, the realized profit of each strategy can be calculated

$$\pi_t^h = (P_{t+1} - P_t R) \cdot z_t^h = (P_{t+1} - P_t R) [E_t^h(P_{t+1}) - P_t R]. \quad (7)$$

The realized profit assesses the performance of strategies. The comparison of the realized profits of different strategies drives investors' strategy switching. From investors' viewpoint, it is reasonable to think that the recent performance of strategies reveals the latest market condition and the strategy which performed better is more likely to maintain its advantage in the short future. This kind of thought gives investors the incentive to adjust their strategies.

When investors adjust their strategies, the market fraction of fundamentalists and chartists is updated according to the discrete choice model with multinomial logit probabilities in Brock and Hommes (1998):

$$n_{t+1}^h = \frac{e^{\beta\pi_t^h}}{\sum_{k=f,c} e^{\beta\pi_t^k}}, \quad (8)$$

where β is the intensity of choice measuring how sensitive agents are to the past profitability of different strategies. In most HAMs, the intensity of choice is an aggregate-level parameter depicting the whole population's sensitivity. In our model, β is treated as an individual-level behavior parameter (Weisbuch et al., 2000) whose value varies with agents. It is formally referred to as *the propensity of strategy switching*.

In reality, there are flexible people and stubborn people. Even when the past performance of different strategies is public information observed by all investors, investors behave differently. Flexible investors abandon current strategy as long as the other strategy works better while stubborn investors do not easily change their strategy unless a significant disadvantage unfolds.

Assume there are N levels of β so that whole the population is divided into N groups, which will be distinguished by the subscript i , $i = 1, 2, \dots, N$. Eq. (4) and Eq. (8) are then modified respectively

$$D_t = \sum_{i=1}^N z_{i,t}, \quad (9)$$

$$z_{i,t} = n_{i,t}^f z_t^f + n_{i,t}^c z_t^c, \quad (10)$$

$$n_{i,t+1}^h = \frac{e^{\beta_i \pi_t^h}}{\sum_{k=f,c} e^{\beta_i \pi_t^k}}. \quad (11)$$

Now the aggregate demand for the risky asset is the sum of N groups' demand. Inside any group i , there are fundamentalists and chartists. A group's demand for the risky asset is a weighted sum of fundamentalists' and chartists' demand. The fraction of fundamentalists in a group follows Eq. (11). Every group observes the same information on the performance of strategies, but the heterogeneous β values lead dynamically to different fractions of fundamentalist and chartists in different groups. A group with a higher β is more sensitive to the past strategies performance than a group with a lower β and, therefore, reacts stronger to the past performance.

2.2. Wealth accumulation

The wealth of a group of investors is the market value of the portfolio held by the group. Numerical simulation makes it possible to track a group's portfolio, trading, and wealth. At the beginning of period t , the value of group i 's investment in the risk-free asset is $S_{i,t}$. Group i holds $Z_{i,t}$ shares of risky asset in total. Group i 's total wealth is $W_{i,t}$. In period t , investors trade stock shares at the current price P_t and then the market maker adjusts the stock price to P_{t+1} . Before the end of period t , every unit of risk-free asset earns a risk-free return R . Group i 's wealth is updated to $W_{i,t+1}$ at the beginning of period $t + 1$ as follows:

$$W_{i,t+1} = S_{i,t+1} + P_{t+1}Z_{i,t+1}, \quad (12)$$

$$S_{i,t+1} = R \cdot (S_{i,t} - z_{i,t}P_t), \quad (13)$$

$$Z_{i,t+1} = Z_{i,t} + z_{i,t}. \quad (14)$$

Substituting Eq. (13) and Eq. (14) into Eq. (12) gives us

$$W_{i,t+1} = R \cdot W_{i,t} + (P_{t+1} - R \cdot P_t)Z_{i,t+1}, \quad (15)$$

that is, group i 's wealth at any period is the risk-free return earned by its latest wealth plus the excess return earned by its total holding of the risky asset.

3. Profitability of strategies

This section justifies investors' strategy switching behavior. In the situation of homogenous propensity of strategy switching and no noise term, the relative advantage of one strategy is determined by the price expectations of all strategies. No strategy can dominate the other. This motivates investors to adjust their strategies. Huang (2010) studies the strategy switching dynamics in a discrete model and finds rich dynamic phenomenon. Huang (2011) provides a dynamic analysis of profitability similar to ours in a quantity-competition oligopoly case.

For any strategy h , $h = f$ or c , its *absolute profit* π_t^h is the excess profit over the opportunity cost of investing in risk-free asset. It can be derived from substituting Eq.s (3-5) into Eq. (7)

$$\pi_t^h = [\gamma \sum_{k=f,c} n_t^k E_t^k(P_{t+1}) - (r + \gamma R)P_t][E_t^h(P_{t+1}) - P_t R]. \quad (16)$$

When $E_t^h(P_{t+1}) = P_t R$, the absolute profit of strategy h is zero as there is no trading at all. In all other cases, the absolute profit of a strategy is determined by the balance of the price expectations of both strategies, as summarized in Proposition 1.

Proposition 1. *The absolute profit of a strategy can be positive, negative or zero. Let $\bar{h}$ denote the opposite strategy of strategy h . Define $\tilde{P}_{t+1}^h \triangleq [(r/\gamma + R)P_t - n_t^{\bar{h}} E_t^{\bar{h}}(P_{t+1})]/n_t^h$.*

Case 1: $\pi_t^h > 0$ if and only if $E_t^h(P_{t+1}) > \max(P_t R, \tilde{P}_{t+1}^h)$ or $E_t^h(P_{t+1}) < \min(P_t R, \tilde{P}_{t+1}^h)$.

Case 2: $\pi_t^h < 0$ if and only if $\min(P_t R, \tilde{P}_{t+1}^h) < E_t^h(P_{t+1}) < \max(P_t R, \tilde{P}_{t+1}^h)$.

Case 3: $\pi_t^h = 0$ if and only if $E_t^h(P_{t+1}) = P_t R$ or $E_t^h(P_{t+1}) = \tilde{P}_{t+1}^h$. When $E_t^h(P_{t+1}) = P_t R$, there is no trading and thus no profit. When $E_t^h(P_{t+1}) = \tilde{P}_{t+1}^h$, fundamentalists' and chartists' expectations are perfectly balanced so that $P_{t+1} = P_t R$ and $\pi_t^h = 0$.

Based on Proposition 1 and more calculation, the signs of the absolute profits of strategies are shown in Fig. 1. To facilitate the demonstration, coordinate axes are divided by the current price. Fig. 1 shows that the profitability of a strategy is determined simultaneously by both strategies. The balance of expectations affects future price via the market maker's adjustment, and further influences the profitability of strategies.

The *relative profit* $\Delta\pi_t$ is defined as the difference of fundamentalists' and chartists' absolute profits

$$\begin{aligned} \Delta\pi_t &= \pi_t^f - \pi_t^c \\ &= [\gamma \sum_{k=f,c} n_t^k E_t^k(P_{t+1}) - (r + \gamma R)P_t][E_t^f(P_{t+1}) - E_t^c(P_{t+1})] \\ &= \gamma n_t^h [E_t^h(P_{t+1}) - \tilde{P}_{t+1}^h][E_t^f(P_{t+1}) - E_t^c(P_{t+1})]. \end{aligned} \tag{17}$$

Proposition 2. *The relative profit of the fundamental strategy can be positive, negative or zero.*

Case 1: $\Delta\pi_t > 0$ if and only if $E_t^f(P_{t+1}) > \max(E_t^c(P_{t+1}), \tilde{P}_{t+1}^f)$ or $E_t^f(P_{t+1}) < \min(E_t^c(P_{t+1}), \tilde{P}_{t+1}^f)$.

Case 2: $\Delta\pi_t < 0$ if and only if $\min(E_t^c(P_{t+1}), \tilde{P}_{t+1}^f) < E_t^f(P_{t+1}) < \max(E_t^c(P_{t+1}), \tilde{P}_{t+1}^f)$.

Case 3: $\Delta\pi_t = 0$ when both strategies share the same expectation, or when the expectations of strategies are perfectly balanced, i.e. $E_t^h(P_{t+1}) = \tilde{P}_{t+1}^h$.

Figure 1 Absolute profit of the fundamental strategy (top) and the trend-following strategy (bottom). In the first quadrant, the profit of a strategy is positive in the gray shaded region, zero along two lines and negative in the other area.

Figure 2 The relative profit. Top panel: The value of relative profit in a 3D space. Bottom panel: Details on signs and comparisons of strategies' absolute profits. The shaded regions indicate areas where the relative profit is positive.

Fig. 2 shows the relative profitability of two strategies. Depending on the combination of fundamentalists' and chartists' expectations, any strategy can be more profitable than the other. Under some expectation combination, one strategy is better; while under some other combinations, the other strategy is better. Therefore, to pursue a higher profit, it is reasonable for investors to switch between different strategies. This provides a motivation for investors' strategy switching behavior.

4. Simulations and discussions

4.1. Numerical results

With the stochastic HAM model given by Eq.s (1-3, 6, 7, and 9-14), we assume there are 100 propensity of strategy switching levels, following an arithmetic sequence from 0.02 to 2, so that propensity of strategy switching of the i th group is $0.02 * i$. Simulations are conducted under a default set of parameters and initial conditions: $r = 0.0001$, $\gamma = 0.001$, $a = 2$, $\varepsilon_t \sim N(0,1)$ and $S_{i,0} = 5000$, $Z_{i,0} = 50$, $P_0 = 100$, $P_0^f = 100$, $E_0^c(P_1) = 100$, $n_{i,0}^f = 0.5$. Every simulation lasts 200 periods.

Fig. 3 shows results of the price series, cumulative relative profit, groups' historical average fraction of fundamental strategy and groups' final wealth in a typical simulation.

The cumulative relative profit of fundamental strategy is shown in Fig. 3(b). Investors adjust their strategies according to the relative profit of fundamental strategy. The relative profit of fundamental strategy varies a lot. So we accumulate the relative profit to get a better view on the comparison of the profitability of strategies. Generally speaking, the fundamental strategy performs better than the trend-following strategy, but its advantage becomes weak after 140 periods.

In Fig. 3(c) and Fig. 3(d), groups of investors are sorted by their propensity of strategy switching from small to large. Fig. 3(c) shows that groups with a higher propensity have larger historical average fractions of fundamental strategy. As for the wealth held by each group by the end of the simulation, in Fig. 3(d), groups with a higher propensity have less final wealth. Fig. 3(c) and Fig. 3(d) show that groups with a higher propensity of strategy switching adopt the better strategy more often, but end up with less final wealth.

The Appendix shows more figures and discussions on what happens within this simulation, for example, fundamentalists' and chartists' demand for the stock, the evolvement of the market fraction of fundamental strategy, and the behavior of groups with lowest and highest propensity of strategy switching.

Figure 3 A typical simulation. (a) Time series of stock price and fundamental expectation. (b) Cumulative relative profit of fundamental strategy. (c) 100 groups' historical average fraction of fundamental strategy over 200 periods. (d) 100 groups' final wealth. In (c) and (d), groups of agents are sorted by their propensity of strategy switching from low to high.

(a)

(b)

(c)

(d)

Figure 4 Average of 1000 simulations. Blue dashed lines indicate 95% confidential intervals. (a) Time series of stock price and fundamental expectation. (b) Cumulative relative profit of fundamental strategy. (c) 100 groups' historical average fraction of fundamental strategy. (d) 100 groups' final wealth. In (c) and (d), groups of agents are sorted by their propensity of strategy switching from low to high.

Because of the noise term in the stochastic price dynamics, the trajectory of stock price varies from simulation to simulation. To eliminate the influence of the noise term, we average the results of 1000 independent simulations under the same default set of parameters and initial conditions.

Fig. 4 shows the average result. In Fig. 4(a), the average stock price is below the fundamental expectation because the risk-free interest rate is positive. With a negative risk-free interest rate, the average stock price will be higher than the fundamental expectation.

Fig. 4(b) shows that, in general, the fundamental strategy is better than the trend-following strategy in the sense of producing a higher cumulative relative profit⁶. The advantage of the trend-following strategy is more likely to appear in the first few periods, and it depends on the initial condition of our simulation. At the beginning of our simulation, fundamentalists' expectation is equal to the stock price. In this case, the market power of fundamental strategy is weak, and the trend-following strategy performs better than the fundamental strategy. If the initial stock price deviates from the fundamental expectation, the early advantage of the trend-following strategy will become weak or even disappear.

In Fig. 4(c) and Fig. 4(d), agent groups with a higher propensity of strategy switching adopt the fundamental strategy more often but end up with less final wealth. That is to say, frequently adopting a more profitable strategy does not guarantee a higher final wealth.

In Fig. 4, blue dashed lines indicate 95% confidential intervals. In Fig. 4(a) and Fig. 4(b), the confidential intervals start from zero because the 1000 independent simulations start with identical initial condition and parameter setting. The next subsection will explain why the confidential interval narrows with the lower propensity of strategy switching in Fig. 4(c).

4.2. Main findings and underlying mechanisms

Why adopting a better strategy more often does not guarantee a higher final wealth? In this subsection, we summarize our main findings and show that the key to our question lies in the inconsistency between short-run profit and long-run wealth accumulation.

⁶ This is an *ex post* observation. As shown in many studies, fundamental strategy stabilizes the market while technical strategy destabilizes it. As long as the fundamental strategy performs better, stock prices do not diverge. In periods when technical strategy performs better, like in the first 20 periods in Fig. 4(b), stock prices may deviate from the fundamental values. But such periods do not last long enough for the stock prices to diverge. Before the divergence happens, fundamental strategy regains relative advantage, stabilizes the market and prevents stock prices from divergence.

4.2.1. Fraction of fundamental strategy

Analytically, from Eq. (11) and Eq. (17), we get each group's fraction of fundamental strategy in period $t + 1$ as

$$n_{i,t+1}^f = \frac{1}{1 + e^{-\beta_i \Delta \pi_t}}. \quad (18)$$

Eq. (18) shows that a group's fraction of fundamental strategy is a strictly concave increasing function of groups' propensity of strategy switching. For given propensity of strategy switching β_i , the corresponding group's fraction of fundamental strategy $n_{i,t+1}^f$ is higher (lower) than 0.5 when the relative profit $\Delta \pi_t$ is positive (negative), and it converges to 1 (0) as the absolute value of the relative profit increases. For given positive (negative) relative profit $\Delta \pi_t$, different groups' fractions of fundamental strategy $n_{i,t+1}^f$ start from 0.5 and converge to 1 (0) while groups' propensity of strategy switching increases.

Fig. 3(c) and Fig. 4(c) show groups' historical average fraction of fundamental strategy over 200 periods in one simulation and in 1000 simulations. As we focus on the average historical fraction, we ignore the time subscript⁷. Similar to Eq. (18), $n_i^f = 1/(1 + e^{-\beta_i \Delta \pi})$ is a strictly concave increasing function of β_i when $\Delta \pi > 0$. However, curves in Fig. 3(c) and Fig. 4(c) do not converge to 1 because we average the fractions $n_{i,t+1}^f$ over all values of $\Delta \pi_t$. Therefore, the shapes of these curves simply reflect the fact that the fundamental strategy performs better, in the sense that it generated positive relative profits more than negative profits. In Fig. 4(c), the 95% confidential interval narrows with lower propensity of strategy switching because n_i^f converges to 0.5 when groups' propensity of strategy switching drops, and this convergence dominates random factors in the simulations.

Fig. 3(c) and Fig. 4(c) echo the cumulative relative profit shown in Fig. 3(b) and Fig. 4(b). Intuitively, because the fundamental strategy is generally more profitable, investors prefer it to the other strategy. So the average fraction of fundamental strategy in every group is always larger than 0.5. Comparing different groups, groups with a higher propensity of strategy switching have stronger adaptive learning and adopt the fundamental strategy more often. As a result, groups with a higher propensity have a larger average fraction of fundamental strategy.

⁷ Here, we do this just to show a neat economic intuition rather than a strict proof. The time subscript cannot be easily ignored in a strict proof. A group's historical average fraction of fundamental strategy is actually $n_i^f = \sum_{t=1}^T n_{i,t}^f / T$, and the average relative profit is $\Delta \pi = \sum_{t=1}^T \Delta \pi_t / T$.

4.2.2. Final wealth and wealth decomposition

Fig. 3(d) and Fig. 4(d) present the most important finding of our work. That is, investors with a higher propensity of strategy switching end up with less final wealth.

When the fundamental strategy is generally more profitable, the coexistence of a positive relation between groups' propensity of strategy switching and their average fraction of fundamental strategy and a negative relation between groups' propensity and their final wealth seems counterintuitive. That is to say, a group with stronger adaptive learning adopts the better strategy more often but ends up with less final wealth. How could this happen? To answer this question, we decompose investors' wealth change in every period. By manipulating Eq.s (7, 10, 12-14), the wealth change in one period is formulated as follows:

$$\begin{aligned}
 \Delta W_{i,t+1} &= W_{i,t+1} - W_{i,t} \\
 &= S_{i,t+1} + P_{t+1}Z_{i,t+1} - S_{i,t} - P_tZ_{i,t} \\
 &= S_{i,t}R - z_{i,t}P_tR + P_{t+1}Z_{i,t} + P_{t+1}z_{i,t} - S_{i,t} - P_tZ_{i,t} \\
 &= S_{i,t}r + (P_{t+1} - P_t)Z_{i,t} + (P_{t+1} - P_tR)z_{i,t} \\
 &= S_{i,t}r + (P_{t+1} - P_t)Z_{i,t} + (n_{i,t}^f\pi_t^f + n_{i,t}^c\pi_t^c).
 \end{aligned} \tag{19}$$

There are three parts in the fourth line of the right-hand side of Eq. (19). The first part is the interest from investing in the risk-free asset; the second part is the change of the market value of all risky asset a group has already held, and the third part is the excess profit from the latest purchase or sale of the risky asset. The excess profit per share equals the change in market value of a share minus the opportunity cost of buying a share. It is a sum of the profits of strategies weighted by the fractions of strategies, i.e. $n_{i,t}^f\pi_t^f + n_{i,t}^c\pi_t^c$, as shown in the last line of Eq. (19).

Fig. 5 decomposes the wealth change of the group with the lowest propensity of strategy switching, i.e. $\beta_1 = 0.02$, in two simulations. Fig. 5(a) is the exact same simulation shown in Fig. 3. Fig. 5(b) is another simulation under the same parameter set but with zero initial endowments $S_{i,0} = 0$ and $Z_{i,0} = 0$. In both cases, it is obvious that the change of market value of risky asset this group has already held dominates the wealth change in almost all periods. Though only one group is demonstrated, the analysis of other groups looks similar. On average, the change of market value of risky asset a group has already held, i.e. $\frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T (P_{t+1} - P_t)Z_{i,t} / \Delta W_{i,t+1}$, contributes to more than 90% of a group's wealth change in a period.

The explanation of our main findings becomes clear now. Investors' strategy switching behavior is driven by the comparison of the short-run profitability of strategies. However, the sum of short-run profit of strategies weighted by the fraction of strategies only contributes to a small part of the wealth change in every period. If investors' ultimate goal is to accumulate wealth as much as possible, then their strategy switching behavior is inconsistent with their goal. This inconsistency is an inconsistency between short-run profit and long-run wealth accumulation.

Figure 5 The decomposition of the wealth change of the group with the lowest propensity. (a) The same simulation shown in Fig. 3. (b) A new simulation with zero initial endowment.

When investors adjust their strategies, they may pay too much attention to the recent performance of strategies that they overlook the lasting effect of their strategy switching. A large position in the risky

asset⁸ is achieved by continuously buying or selling the risky asset. Agents who stick to the fundamental strategy are more likely to accumulate a large position. Agents with frequent strategy switching do not stick to the fundamental strategy for long. When they switch to chartists, the process of risky asset accumulation is interrupted. Therefore, they do not hold a large position in the risky asset.

The change of the market value of investors' risky asset position contributes to the majority of their wealth change. When an investor adjusts his strategy or executes a purchase, the immediate effect on one-period wealth change is visible. However, there is no way to tell what will happen in the long run because that requires information on future price, which is obviously unavailable to him. This is how the inconsistency between short-run profit and long-run wealth accumulation emerges in our model. This inconsistency eventually causes the interesting phenomenon that investors with stronger adaptive learning adopt the better strategy more often but end up with less final wealth⁹.

4.3. Robustness

In this section, robustness checks are carried out to confirm that the main findings are stable when parameters vary within a reasonable range. The risk-free interest rate r , the market maker's price-adjustment speed γ and chartists' trend-following extent α are tested. We check one parameter at a time and keep others fixed at their default values. Under every parameter setting, we run 1000 simulations and average the results. Each simulation lasts 1000 periods.

Fig. 6 shows the robustness check results. Our main findings are robust when the risk-free interest rate varies from -0.001 to 0.001 (top), or when the market maker's price-adjustment speed ranges from 0.00005 to 0.002 (middle), or when chartists' trend-following extent ranges from 0.05 to 3 (bottom).

Moreover, Fig. 6 reveals more information on parameters' influence on the main findings. As shown in Fig. 6(a) and Fig. 6(b), the risk-free interest rate barely affects the results. Even when the interest rate is negative, our main findings are stable, except that the general levels of groups' final wealth decrease with the risk-free interest rate.

⁸ A large position in the risky asset is either a positive position in a bull market or a negative one in a bear market, from an *ex post* viewpoint.

⁹ Please note that our discussion does not mention how agents' strategy switching behavior affects strategies' profitability. If this issue would be covered, our finding on final wealth is likely to be strengthened as Massari (2007) shows that strategies' profit drops with strategies' wide adoption.

Figure 6 Robustness check. Investors with a higher propensity of strategy switching have a higher average fraction of fundamental strategy (left column) and lower final wealth (right column) under different risk-free interest rates (a, b), market maker's price-adjustment speeds (c, d) and chartists' trend-following extents (e, f).

As shown in Fig. 6(c), when the market maker increases the price-adjustment speed, the positive relation between groups' propensity of strategy switching and their fraction of fundamental strategy becomes more pronounced at first and then less obvious. Groups' final wealth first increases and then decreases with the price-adjustment speed in Fig. 6(d). Unfortunately, we do not have an explanation for this so far.

Fig. 6(e) and Fig. 6(f) show that the negative relation of investors' propensity and their final wealth does not depend on a strong relative advantage of the fundamental strategy. When chartists' trend-following becomes stronger, the positive relation between investors' propensity and their fraction of fundamental strategy weakens, but the negative relation between their propensity and their final wealth are enhanced. Based on the discussion of the profitability of strategies in Section 3, the strategy with larger impact is more likely to drive the price moving toward the direction benefit to that strategy. As chartists' trend following becomes stronger, chartists have larger influence on the price formation. Though chartists' strategy still cannot surpass the fundamental strategy, the relative advantage of the fundamental strategy is weakened. So every group's average fraction of fundamental strategy decreases with chartists' trend-following extent. Meanwhile, the negative relation between strategy switching propensity and final wealth becomes more remarkable. This detaches the success of groups with lower propensity from the relative advantage of the fundamental strategy.

4.4. Noise term

In HAMs, a few works focus on the role that noise term plays in model dynamics. He and Li (2007) study how the simulated price, especially the autocorrelation of returns, are affected by different noise processes. Amilon (2008) finds that the way noise added to models shows a large influence on models' ability to generate some stylized facts.

This section runs a robustness check on the standard deviation of noise term and reveals that the noise term plays a critical role in our model. The noise term in Eq. (6) represents the uncertainty in price formation, e.g., noise traders' random transaction or the arrival of trivial new information. In Fig. 7, without the uncertainty, i.e., the standard deviation of noise term is 0, the main findings of our work disappear. The main findings appear when the uncertainty exists, and further become more pronounced when the uncertainty increases.

In other words, the main finding of investors' wealth accumulation depends on the existence of uncertainty. Therefore, to be precise, the main finding should be restated as when uncertainty exists, frequently adopting a more profitable strategy does not guarantee a higher final wealth.

Figure 7 The effect of the standard deviation of the noise term.

From another viewpoint, the average of 1000 simulations in Fig. 4 partially cancels out the influence of the noise term. As for the case without uncertainty, a plausible guess is that the stock price converges to its fundamental and grows at the risk-free interest rate. As long as the stock price grows at the risk-free interest rate, according to Eq. (7), both fundamental and trend-following strategies have zero absolute profit. Therefore, all groups' fractions of fundamental strategy are 0.5, which is shown as a straight line parallel to the axis of the propensity of strategy switching in the left panel of Fig. 7. Moreover, since all groups start from the same endowment and share the same fraction of fundamental strategy, they behave in the same way. Therefore, all groups' wealth at any period is equal, and this is shown as a straight line parallel to the axis of the propensity of strategy switching in the right panel of Fig. 7.

5. Conclusion

In this paper, a heterogeneous agent model is built to study whether a stronger adaptive learning helps investors accumulate more wealth in the end. It is found that, when the fundamental strategy is more profitable than the trend-following strategy, investors with a higher propensity of strategy switching adopt the fundamental strategy more often but end up with less final wealth. This counterintuitive phenomenon is robust under various parameter settings. After decomposing the wealth change in every period, this interesting phenomenon is attributed to the inconsistency between short-run profit and long-run wealth accumulation. The comparison of the short-run profits of different strategies drives investors' strategy switching. However, the short-run profit only takes a small proportion of the wealth change in every period. Investors' wealth accumulation relies heavily on the market value of risky asset possessed by investors rather than the short-run profit from latest trading. Therefore, investors' strategy switching behavior is inconsistent with their wealth accumulation.

This paper raises some questions for future study. Because of the inconsistency, we do not expect that a series of investment actions driven by short-run profit eventually lead to a long-run success, but why to a long-run failure instead of a mediocre ending? The key to this problem may lie in the timing of investors' strategy switching. Even the fundamental strategy is superior on average; investors still have to distinguish its well-performing time from its poor-performing time. The timing of investors' strategy switching is valuable in theory and practice. In this paper, we rephrase the intensity of choice parameter as the propensity of strategy switching for convenient terminological usage. In fact, our model with heterogeneous intensity of choice is an extension and generalization of previous models with homogeneous intensity of choice. It will be meaningful to study how stylized facts and other results produced by models with homogeneous intensity of choice would be affected by such generalization. The findings of our study are based on the basic assumption that the artificial stock market in our model is filled with fundamentalists and trend-followers. Whether these findings remain in markets with other two or more kinds of investors? Should old findings be generalized or refined? Will new findings be discovered? These questions show us another direction of future research. Last but not the least, to what extent the conclusions of our model are subject to the model structure? Major revisions to the model structure are likely to make a difference. Further investigation is required to get a better understanding of these questions.

Appendix: More details of the typical simulation

Because of the white noise in the price dynamics, the trajectory of price varies every simulation. Fig. 8 shows the stock price series, fundamentalists' and chartists' demands, relative profit and average fraction of fundamental strategy in a typical simulation. In Fig. 8(a), the stock price deviates from its fundamental from time to time. Fig. 8(b) shows fundamentalists' and chartists' demand of stock shares weighted by their market fraction.

In our model, the market maker satisfies all investors' demand, so investors' demand equals their trading volume. Fundamentalists' and chartists' demand for stock shares are comparable in absolute value. Whenever the price deviates far from its fundamental, fundamentalists increase the absolute value of their demand and drive the stock price towards its fundamental. After the price moves closer to its fundamental, fundamentalists reduce their demand. When fundamentalists' demand is lower than chartists', the stock price may start to deviate from its fundamental value. The competition between fundamental strategy and trend-following strategy continues. Any strategy can gain a temporary advantage, but no one can dominate the other for long.

Figure 8 Time series of a typical simulation. (a) Stock price and fundamentalists' expectation of the fundamental value. (b) Fundamentalists' and chartists' demand weighted by their market fraction. (c) Relative profit of fundamental strategy comparing with chartists' strategy. (d) The average market fraction of fundamental analysis within the whole population.

The relative profit drives investors' strategy switching. In Fig. 8, when the relative profit of fundamental strategy is positive, some investors who have adopted the trend-following strategy switch to fundamental strategy in the next period, and the market fraction of fundamental strategy increases in the next period. When the relative profit of fundamental strategy is negative, some investors who initially adopt the fundamental strategy switch to the trend-following strategy, and the market fraction of fundamental strategy decreases in the next period. Because of this cause-effect relation between the relative profit and market fraction of fundamental strategy, similar sequences of ups and downs appear in Fig. 8(c) and Fig. 8(d), but with different fluctuation ranges. The market fraction of fundamental strategy lags one period behind the relative profit time series.

Within a group, there are fundamentalists and chartists. The behavior of a group is a convex combination of fundamentalists' and chartists' behavior. Moreover, because the fraction of fundamental strategy in a group is a strictly increasing function of groups' strategy-switching propensity, the behavior of any group is a convex combination of the behavior of the two groups with minimum and maximum strategy-switching propensity¹⁰.

Therefore, instead of showing the fraction of fundamental strategy in every group, we only need to demonstrate the fraction in the two groups with minimum and maximum strategy-switching propensity, and all other groups are located in between. As for the evolvement of groups' wealth, a demonstration of these two groups gives us a sketch of other groups, though strictly speaking the wealth of a group in between may go beyond the band defined by these two groups with extreme propensity.

Fig. 9 shows the fraction of fundamental strategy and the process of wealth accumulation of these two groups. In Fig. 9(a), the group with minimum propensity holds a stable combination of fundamental and technical strategies, and the exact proportion fluctuates slightly under the influence of the previous profitability of strategies. The group with maximum propensity adjusts its proportion of fundamental strategy intensively, showing a stronger adaptive learning.

Fig. 9(b) illustrates the evolvement of the wealth of these two groups. Most of the time, the group with minimum propensity enjoys a higher wealth. However, there are some periods when the group with maximum propensity is wealthier, for example around the 122^{ed} period. But as time passes by, the group with minimum propensity becomes more and more wealthy, and the gap of wealth between these two groups gradually widens.

¹⁰ To be clear, this statement is limited to variables determined by the information one or two periods ago, e.g. price prediction, demand, and fractions of strategies. It cannot be extended to variables involving distant historical information like historical average fraction of strategies and wealth accumulation.

Figure 9 Comparison of the group with the maximum strategy-switching propensity and the group with the minimum strategy-switching propensity. (a) The fraction of fundamental analysis within a group. (b) Wealth accumulation of a group.

Compliance with Ethical Standards

The authors declare that they have no conflict of interest.

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