Jetting from cavitation bubbles in viscous and confined liquids

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**NANYANG
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SINGAPORE

**JETTING FROM CAVITATION
BUBBLES IN VISCOUS AND
CONFINED LIQUIDS**

ZENG QINGYUN

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

2020

JETTING FROM CAVITATION BUBBLES IN VISCOUS AND CONFINED LIQUIDS

ZENG QINGYUN

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

A thesis submitted to the Nanyang Technological
University in partial fulfillment of the requirements for the
degree of Doctor of Philosophy

2020

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research done by me except where otherwise stated in this thesis. The thesis work has not been submitted for a degree or professional qualification to any other university or institution. I declare that this thesis is written by myself and is free of plagiarism and of sufficient grammatical clarity to be examined. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Asst. Prof. YONG Ee Hou

Authorship Attribution Statement

This thesis contains material from 3 papers published in the following peer-reviewed journals where I was the main author.

Chapter 2 is published as [Zeng, Q., Gonzalez-Avila, S.R., Dijkink, R., Koukouvini, P., Gavaises, M. and Ohl, C.D., 2018. Wall shear stress from jetting cavitation bubbles. Journal of Fluid Mechanics, 846, pp.341-355. DOI:10.1017/jfm.2018.286](#)

The contributions of the co-authors are as follows:

- I performed the numerical simulation.
- Dr. Gonzalez-Avila and Dr. Dijkink conducted the experiment.
- Prof. Ohl and I analyzed the data and wrote the manuscript draft.
- All authors discussed the results and revised the paper.
- Prof. Ohl supervised the project.

Chapter 4 is published as [Zeng, Q., Gonzalez-Avila, S.R., Ten Voorde, S. and Ohl, C.D., 2018. Jetting of viscous droplets from cavitation-induced Rayleigh–Taylor instability. Journal of Fluid Mechanics, 846, pp.916-943. DOI:10.1017/jfm.2018.284](#)

The contributions of the co-authors are as follows:

- Dr. Gonzalez-Avila and Miss Ten Voorde and I conducted the experiment.
- I performed the numerical simulation and derived the analytic model.
- Prof. Ohl and I analyzed the data.
- I wrote the manuscript draft.
- All authors discussed the results and revised the paper.
- Prof. Ohl supervised the project.

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The contributions of the co-authors are as follows:

- I performed the numerical modeling and involved in analyzing the data, I wrote part of the manuscript related to numerical work.

- Dr. Krivokorytov, Dr. Lakatosh, Dr. Medvedev and Prof. Ohl designed the experiment.
- Dr. Krivokorytov, Dr. Vinokhodov, Dr. Sidelnikov and Dr. Krivtsun designed and developed the experimental setup.
- Dr. Krivokorytov, Dr. Lakatosh, Dr. Kompanets conducted the experiment.
- Dr. Krivokorytov, Dr. Lakatosh, Dr. Medvedev, Prof. Ohl and I wrote the manuscript draft.
- All authors analyzed the results and revised the paper.
- Dr. Medvedev and Prof. Ohl supervised the project.

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ZENG QINGYUN

Abstract

Historically, the jetting of cavitation bubbles was notoriously considered unwanted because of its destructive effect on rigid surfaces. This has changed and nowadays the jetting dynamics has attracted attention for medical and industrial applications due to its ability to deliver liquid matter and apply tangential stress. This thesis researches the jetting dynamics from non-spherically collapsing bubble in confined geometries where the boundaries and boundary layer flow together with viscosity of fluids are affecting the flow. While most of the studies are done through simulations, we provide an analytic model and comparison of many of the results to experiments.

This thesis begins with determining the wall shear stress on a no-slip rigid wall induced by a bubble collapsing close to it. We solve the dynamics of a laser-induced bubble and the resulting boundary layer flow numerically with a viscous volume of fluid (VoF) solver accounting for compressibility and surface tension. The computed bubble dynamics is validated with experimental result from high-speed photograph while the shear stress calculation is confirmed once the boundary layer is fully resolved. We conclude the complex stress distribution on the boundary with a spatio-temporal map and explain it with the computed boundary flow detail.

Next we look at the jetting from single bubbles confined in a liquid gap bounded by two rigid walls. This study gives an example how to manipulate the jet dynamics with geometric confinement. Depending on the gap height and bubble position three jetting scenarios are found: *transferred jet* impacting on the distant wall, *double jet* as a result of a bubble splitting and impacting onto both walls, and the *directed jet* from a conically-shaped bubble impacting on the nearest wall. The impact velocity of the liquid jets onto the walls can reach more than ~ 200 m/s and is a strong function of the gap height and bubble position. We also conduct VoF simulations to study the flow details, which reveal that the viscous boundary layers and flow inertia affect the bubble deformation and therefore the directions of jetting.

In the last part, we study the jetting from the surface of a droplet. There the viscous jets are driven by an oscillating cavitation bubble from within the droplet.

This type of jetting is due to a Rayleigh-Taylor (RT) unstable surface of the droplet. Experimentally we achieve jetting by introducing a laser-induced bubble into an acoustically levitated droplet. We also conduct numerical simulations that reproduce the experiments with remarkable agreement. The computed results reveals that RT instabilities are a precondition before pressure transients emitted upon bubble collapse eject fine jets from the undulated surface. Viscosity is found to suppress the jetting and also reduce the amount of jets. We also establish an analytic model for the RT instability in an spherical oscillating flow which can be used to predict the onset of the jetting. The transition between jetting and no-jetting shows a strong dependency on viscosity. To validate the model further we apply it for the jetting dynamics observed from laser-induce bubbles in metal droplets. The model predicts that the number of jets from a liquid metal droplet is reduced and increases with the energy of the bubble/laser, which is well reproduced in the experiments.

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Dedicated to my parents.

Chapter 1

Introduction

Introducing tensile strength or depositing energy into liquid can rupture it and thus generates cavitation bubble, which is a volume of gas and vapor mixture that oscillates violently in both shape and volume ([Lauterborn and Kurz, 2010](#)). Cavitation bubbles are microscopic agents that once after conversion provide kinetic energy to the fluid. While a spherical bubble (if existent) would focus the energy to small spherical volume, non-spherically collapsing bubbles, e.g., near a boundary can accelerate liquid into a fast and transient jet.

The history of discovering the jetting and the non-spherical collapse of cavitation bubbles will be briefly visited in below section [1.1](#). The development of the early research was strongly linked to cavitation erosion of hydraulic equipment in complex hydrodynamic cavitation environments. With the invention of controllable bubble nucleated techniques such as laser-induced and thermal-generated bubbles, research on jetting bubbles has moved on to study beneficial applications for example in medicine, microfluidics, and cleaning, where the bubbles interact with viscous fluids and various kinds of boundaries. We will introduce the dynamics of bubble jetting next to flat rigid boundaries in section [1.2](#). The challenges met to resolve the viscous boundary layer flow are described. Then we present how a cavitation bubble can produce a fast and thin liquid jet into air from a free surface and describe the difficulties on achieving the jetting of single viscous droplets where the free surface is convex in section [1.3](#). The jetting dynamics near more complex boundaries where the bubble is influenced by multiple boundaries with

various shapes will be reviewed in section 1.4. We show the importance of viscosity when the bubble is strongly confined by rigid boundaries.

1.1 Jetting from non-spherically collapsing cavitation bubbles

Leonhard Euler firstly proposed the concept of cavitation bubble in 1756 when he studied the water flow dynamics in a pipe. While considering Bernoulli's principle in hydrodynamics, Leonhard Euler suggested that the pressure in a pipe flow may become negative when the water flows through a constricted channel. As a result, the water attached to the pipe walls will leave the boundary thus a void would be formed (Euler, 1756). One century later, another mathematician William Henry Besant continued the problem and dealt with the later dynamics of the void. He simplified the problem as the dynamics of a spherical cavity shrinking inside an incompressible, inviscid and unbounded fluid. Mathematically he gave a solution on the velocity field evolution and the time needed to fill up the cavity (Besant, 1859). Yet the related physical phenomenon was firstly published in 1894 by Osborne Reynolds, who observed the generation and collapse of "boiling bubble" under room temperature when the water flowed through the constricted section of a tube (Reynolds, 1894). In the same year, Thornycroft and Barnaby also discovered the bubble cloud formation on the back sides of ship propeller blades where the hydraulic pressure was low, this was mentioned by Dryden et al. (1932). Thornycroft and Barnaby (1895) firstly named this type of bubble from the rupture of liquids as *cavitation bubble*.

However, cavitation bubble became a nightmare for engineers since they were found to cause severe damage to hydraulic equipment, such as propellers and propeller screws (Silberrad, 1912; Kornfeld and Suvorov, 1944). To find out this hydraulic destructive mechanism, Lord Rayleigh solved the shrinkage of a spherical vapor bubble due to an ambient pressure in an incompressible fluid based on Besant's work (Besant, 1859). He obtained a very high pressure of 1260 atmospheres when the bubble shrank to a minimum size (Rayleigh, 1917). This high pressure was considered as the truth for the cavitation bubble erosion for over three decades until Kornfeld and Suvorov proposed another damage mechanism related to the

non-spherical cavitation bubble. They captured the experimental phenomenon with a camera and noticed that bubbles were actually non-spherical when they oscillated near rigid walls. Moreover, the bubble surfaces produced jets towards the wall. These jets were found incredible fast to capture and could be the main reason for the erosion ([Kornfeld and Suvorov, 1944](#)).

On the other hand, theoretical analysis on the stability of a spherically oscillating bubble showed that any surface distortion of the bubble would be amplified during the collapse, i.e. the bubble surface is unstable ([Plesset, 1954](#); [Plesset and Mitchell, 1956](#)). A disturbance is always introduced into the collapsing flow when the bubble oscillates next to a rigid boundary since the pressure distribution is no longer spherical-symmetric. As the disturbance grows during the collapse, it is possible to perturb the bubble's top surface towards the boundary that forms a re-entrant jet according to the analysis of [Rattray \(1951\)](#).

Capturing the jet requires single cavitation bubble nucleated to avoid the extra disturbance and high-speed cameras to record the rapid jetting process. This was done first in 1961 by [Naude and Ellis \(1961\)](#), who used a spark to rupture the liquid to nucleate bubbles next to a solid plane boundary and filmed the bubble with the Ellis-Kerr cell camera ([Ellis, 1953](#)) at 100,000 frames per second (fps). They captured that the wall distant bubble surface shrank towards the wall and pierced through the bubble, leading to form a liquid jet impacting directly on the wall. However, they were not able to resolve the details of the jet development due to an insufficient temporal resolution. In order to capture the jet, [Benjamin and Ellis \(1966\)](#) scaled-down the bubble dynamics by reducing the ambient pressure to 0.04 atmospheres to make the jet dynamics sufficiently slow to be recorded. They recorded a jet speed greater than 35 m/s, corresponding to a velocity of 175 m/s in normal atmospheric pressure. They claimed such a fast jet was likely to cause damage on the substrate through impact.

On the theory side, [Plesset and Chapman \(1971\)](#) firstly solved the potential flow to simulate the vapor bubble collapse near a plane solid boundary. Their result showed that the bubble's top surface developed into a jet towards the plane due to a non-uniformly distributed flow potential along the bubble surface. The computed result was later confirmed by [Lauterborn and Bolle \(1975\)](#), who performed the experiments with a high-speed camera and a new technique for nucleating cavitation bubbles: laser-induced cavitation.

With the evidence from both experimental and theoretical investigations, it was confirmed that the bubble collapses non-spherically next to a rigid boundary. And the bubble's top surface develops a fast liquid jet impacting on the boundary during its collapse, which is destructive and may cause damage on boundaries.

The laws of Bjerknes on bubble migration indicates that for bubbles oscillating with minor deformation, their movement is affected by the boundary, i.g. bubbles move towards a rigid wall while away from a free surface (Birkhoff and Zarantonello, 1957). Interestingly jetting bubbles were found following similar laws. Gison (1968) experimentally showed that the jet direction was dependent on the boundary properties. The bubble and the induced liquid jet were found to direct outwards the boundary when the bubble was next to a free surface or a flexible boundary. Moreover, a free surface can be deformed as a liquid "spike" jetting into the air by the expanding and collapsing bubbles, the phenomenon was reported by Chahine (1977), Longuet-Higgins (1983), Blake et al. (1987) and Robinson et al. (2001) to name a few.

The *Rayleigh-Plesset* equation firstly derived by Rayleigh (1917) and later improved by Plesset (1949) describes excellently the spherical bubble dynamics. Unfortunately, it fails to be applied to non-spherical bubble dynamics due to a large amount of high order terms introduced when involving liquid jet development (Plesset and Prosperetti, 1977). To advance with better models for non-spherical cavitation bubbles Benjamin and Ellis (1966) introduced the *Kelvin impulse* concept to characterize the jet dynamics. The *Kelvin impulse* describes the local momentum of an unsteady flow. Its sign indicates the direction of the jetting. This method was further developed by Blake and coworkers who derived succinct expressions for the *Kelvin impulse* for cavitation bubbles near various boundaries, such as free surface (Blake and Gibson, 1981), rigid boundary (Blake et al., 1986), liquid surface and inertial boundary (Blake, 1988) in the consideration of a perfect system (inviscid and incompressible fluid) and zero surface tension. These results are useful to predict the gross motion of the jetting bubble in practical applications (Supponen et al., 2016). Yet the *Kelvin impulse* does not provide inside into details of the flow, such as bubble and free surface deformation, jet speed, and particular flow configurations.

Nowadays the significance of jetting from non-spherical cavitation bubbles encompasses much more than just the erosion of rigid walls. Jetting has very broad

applications because of its diverse dynamics from bubble interactions with boundaries in various configurations. These configurations include boundary stiffness and geometric shape. Application examples are cleaning of rigid surface and membranes (Ohl et al., 2006a; Reuter et al., 2017), drug delivery into cells (Prentice et al., 2005; Ohl et al., 2006b), mixing (Hellman et al., 2007) and pumping (Dijkink and Ohl, 2008a) in microfluidics, lithotripsy and histotripsy (Coleman et al., 1987; Xu et al., 2007), laser-induced forward transfer (Willis and Grosu, 2005; Kuznetsov et al., 2012), and needle free injection (Tagawa et al., 2013; Berrospe-Rodriguez et al., 2016) to name a few.

Many past experimental studies focused on capturing and measuring the jet parameters related to a stand-off distance to the boundary with high-speed photography, while numerical simulations complemented experiments to give additional details on the bubble deformation and pressure and velocity distribution to understand the mechanisms. Yet many issues remain unsolved especially when jetting happens in more complex geometries where near boundary flow and viscosity become significant.

1.2 Jetting very close to a rigid boundary

A cavitation bubble at maximum volume converts its potential energy into the kinetic energy of the fluid attached to its surface during collapse. For a spherical collapse, the kinetic energy distributes equally along the spherical bubble surface. Yet the kinetic energy is no longer uniformly distributed when a bubble shrinks very close to a plane rigid boundary. Indeed it is concentrated on the fluid near the upper surface of the bubble (i.e. the wall distant bubble surface) due to a pressure gradient (Plesset and Chapman, 1971) with the higher pressure further away from the boundary. This gradient drives the top bubble surface towards the boundary and is responsible to form a liquid jet that travels through the bubble's center and eventually through the bubble's lower surface. In many cases (when the bubble is further from the boundary) a gas/vapour layer remains between the jet flow and the lower bubble surface thus the jet technically does not pierce through the bubble wall. Eventually, the liquid jet flow impacts on the almost stagnant liquid layer underneath the bubble. After impacting the jet spreads along the surface. A typical sequence of non-spherical collapse is sketched in figure 1.1.

FIGURE 1.1: Sketch of the jetting from cavitation bubble near rigid boundary.

The details of the bubble dynamics and jet formation near boundaries are strongly dependent on the bubble's stand-off distance $\gamma = h/R_{max}$, where R_{max} is bubble's equivalent radius at its maximum expansion and h is the distance from the bubble nucleation site to the boundary. An interesting regime is $\gamma < 2.0$ where the damage and/or cleaning of the surface from the impacting jet is found (Philipp and Lauterborn, 1998; Reuter and Mettin, 2016).

The jet velocity upon impact is considered a key parameter to evaluate the disruptive and cleaning effect of the bubble, as it reflects the water hammer pressure on the boundary from the abrupt deceleration of the jet and the resulting shear flow radially spreading along the boundary later on. Many studies have measured directly the jet velocity with high-speed photography (Benjamin and Ellis, 1966; Lauterborn and Bolle, 1975; Philipp and Lauterborn, 1998). Among these authors, Benjamin and Ellis (1966) were the first ones to measure the jet velocity of a spark-induced bubble approximately 10 m/s for $\gamma = 1.3$ while 35 m/s for a very small γ value (for this the γ value was not provided). The measurements were conducted in a low-pressure system where the jet dynamics was slowed down by a factor of about 5. The recording rate was up to 5,000 fps. However, a jet measurement under normal atmospheric pressure requires a much higher frame rate. Using laser-induced bubbles and a camera with capture rate at 300,000 fps, Lauterborn and Bolle (1975) obtained a jet speed of 120 m/s for $\gamma = 3.08$ which they considered as an underestimated value due to an insufficient frame rate. Philipp and Lauterborn (1998) experimentally gave a parametric study of the jet tip velocity and the impact velocity on a rigid boundary as a function of the γ -value. The jet tip velocity was found to increase with γ for $\gamma > 1.0$ because the wall effect on slowing down the collapse becomes weaker along γ (Philipp and Lauterborn, 1998). However, the jet tip velocity fluctuates when $\gamma < 1.0$ due to a complex interaction between the bubble's lower surface and the liquid layer above the rigid boundary.

The jet impact velocity on the rigid boundary is affected by this cushioning fluid and was found smaller than 25 m/s for $\gamma > 1.0$ when the water layer is thick but can be more than 83 m/s for $\gamma < 0.7$ where the bubble is about to touch the boundary.

The calculation of the jet velocity using numerical simulation started with the ground-breaking work from [Plesset and Chapman \(1971\)](#). The initial configuration is a vapour filled spherical bubble expanded to maximum size, at some distance from a boundary and whole fluid body at rest. They simulated its collapse similar to the Rayleigh collapse in spherical geometry. The computed jet velocities were 170 m/s for $\gamma = 1.5$ and 130 m/s for $\gamma = 1.0$. The later experiment of [Lauterborn and Bolle \(1975\)](#) confirmed the calculation from [Plesset and Chapman \(1971\)](#) for $\gamma = 1.5$ as reliable. Using a boundary element method (BEM) for potential flow, [Blake et al. \(1986\)](#) gave three scaling laws on the jet velocity U_{jet} depending on the driving pressure gradient base on the potential flow theory which are $U_{jet} = 8.6(\Delta p/\rho)^{1/2}$ for $\gamma = 1.0$, $U_{jet} = 11.0(\Delta p/\rho)^{1/2}$ for $\gamma = 1.5$ and $U_{jet} = 16.1(\Delta p/\rho)^{1/2}$ for $\gamma = 2.0$, where Δp is the difference between ambient and vapor pressure, ρ is the liquid density, respectively. [Supponen et al. \(2016\)](#) summarized the jet speed with a universal scaling law base on the dimensionless form of the *Kelvin impulse* and the stand-off distance γ . However, the scaling law has large discrepancy with experimental data in the low gamma regime, especially for $\gamma \leq 1.0$. The result also shows a strong dependence of the jet velocity when a viscous liquid layer is involved for low γ -values.

A liquid layer exists between the rigid wall and the expanding bubble. This liquid layer can prevent the bubble from touching the wall and flatten its lower surface. Thus the bubble grows into a non-spherical shape at its maximum expansion. This bubble deformation from the expansion phase has great importance on the jet formation when γ is small. [Blake et al. \(1986\)](#) noticed the issue when they compared their simulations starting from a small bubble that expanded and collapsed with the configuration used in [Plesset and Chapman \(1971\)](#) where the expansion phase was omitted (the Rayleigh collapse case, where the initial bubble is assumed as a spherical vapor bubble at its maximum radius). Recently [Lauterborn et al. \(2018\)](#) revisited the problem with numerical simulations and discussed the difference. The results demonstrated that the Rayleigh collapse is not suitable for studying the

bubble dynamics near a rigid boundary as it overestimates the jet velocity and differs in distinct bubble shapes.

Very complex fluid dynamics happens at the late collapse stage when the jet strikes the bubble's lower surface and impacts on the liquid layer. The bubble while shrinking turns into a toroidal shape. Toroidal bubble generation, shock-wave emission, and liquid splash are involved simultaneously (Zhang et al., 1993; Brujan et al., 2002). The processes occur over a very short time interval, e.g. within $2\ \mu\text{s}$ for a bubble with a maximum radius of 1.5 mm, thus require recording of a million and more frames per second. A successful capture was done by Brujan et al. (2002) with 5 millions fps camera (temporal resolution is $0.2\ \mu\text{s}$), who also used a BEM simulation to calculate the bubble dynamics. Both experimental and computed results showed that the jet impacting the stagnant liquid layer results in a splash, which ejects liquid into the toroidal bubble. The then distorted toroidal bubble together with the inward flow and the radial spreading jet generates a vortex ring. This vortex ring lifts up and slows down the flow spreading on the boundary.

The jet spreads along the surface radially after the impact and generates shear stress onto a no-slip wall. This flow can be causing sonoporation (Ohl et al., 2006b) and is important for cleaning on rigid surfaces (Ohl et al., 2006a). To understand the acting forces on particles or cells on the substrate, researchers were interested in determining the wall shear stress from the flow spreading on the substrate. Maisonhaute et al. (2002a) applied a nanosecond electrochemical method to determine the viscous liquid layer to be 40 to 80 nm height for bubbles jetting in acoustic cavitation. Base on the measurement, they estimated the maximum spreading velocity to be in the order of 200 m/s thus calculated the peak wall shear stress about 25 to 50 bars (Maisonhaute et al., 2002b). Directed measurement on the shear flow is non-trivial and only a few reports are available in literature. Dijkink and Ohl (2008b) implemented a hot-film sensor to directly measure the temporal shear stress from a laser-induced cavitation bubble based on a temperature sensing principle. Their measurements showed a maximum stress of 3.5 kPa for a $\gamma = 1.0$ bubble. The peak value was found to occur after the first collapse however decayed slowly due to a vortex ring formed by the interaction of the spreading flow and the toroidal bubble. The limitation of their method is the low temporal bandwidth and the pointwise measurement. Recently Reuter

and Mettin (2018) developed a shear stress microscopy using a fast electrochemical sampling method. The device allows to scan the substrate to obtain a spatial distribution of the measured wall shear stress from laser-induced cavitation bubbles by repeating the same experiment many times. They recorded a maximum stress value of 6.2 kPa. The spatial and temporal resolution are 50 μm and 3.3 μs , which are considered rough especially for small bubbles. Yet both measurements ignored the particular boundary layer structure, and assumed a laminar constant shear. Assuming a laminar boundary flow in a gap of 100 μm , for a spreading flow about 100 m/s, the boundary layer thickness could be within the order of 1 to 10 μm (Schlichting and Gersten, 2016), which is extremely difficult to address in an experiment. Also, the complexity of the transient shear flow involving toroidal bubbles creating a vortex ring results to complex wall shear stresses and may need faster sampling.

Numerical simulations can provide abundant details on the jet formation and related fluid dynamics. The difficulties in simulating the bubble dynamics lay on solving the non-linearity in the governing equations involving viscosity, the bubble-liquid interface, discontinues cross the two phase interface, compressibility and surface tension. The first simulations from Plesset and Chapman (1971) simplified the problem with a potential flow model, i.e. an incompressible and inviscid fluid. They solved the potential flow and tracked the bubble motion using the so-called Marker-and-Cell method (Harlow and Welch, 1965). The bubble dynamics near the rigid wall and jet formation details were produced.

To deal with the difficulty in bubble interface tracking, Blake and his coworkers introduced the boundary integral/element method (BIM/BEM) to the research field of cavitation bubbles (Blake and Gibson, 1987). This numerical technique can handle even complex bubble deformations with rather simple numerical implementations. BIM/BEM has been applied widely to study cavitation bubble dynamics, such as the formation of toroidal bubbles (Blake et al., 1997; Zhang et al., 2015), their velocity fields and pressure distributions (Li et al., 2016; Wang et al., 2016), the dynamics during the late stage of collapse (Zhang et al., 1993; Brujan et al., 2002), and the rebounding of the bubble after the first collapse (Lee et al., 2007). Over the time the BEM technique has experienced many improvements such as the weak compressible model (Wang and Blake, 2010; Wang, 2014), a weak viscous model for the bubble boundary layer via the viscous correction pressure (Zhang

and Ni, 2014), coupling with material pitting model (Hsiao et al., 2014; Chahine, 2014), and particle interaction model (Chahine et al., 2016). Yet limited to the potential flow model, the boundary element method is not able to resolve the viscous liquid layer flow near rigid boundaries nor capture the shock-wave. As a result, BEM calculations are only appropriate for $\gamma > 0.9$ because of the neglect of the viscous layer. A contact with the rigid boundary occurs for $\gamma < 0.9$ (Wang et al., 2015).

In contrast, domain-based numerical solvers for the Euler equations (Johnsen and Colonius, 2009) or the Navier-Stokes (NS) equations (Popinet and Zaleski, 2002; Minsier et al., 2009) were used for simulating single bubble dynamics. These methods were coupled to an interface capturing approach such as the volume-of-fluid (VoF) and the phase-field method, or interface tracing techniques like level-set (LS) methods or front-tracking methods. The Euler method provides a simple method to implement compressible models for the liquid and gas and high order numerical schemes for interface and shock-wave capturing. Yet it limits the applicability to inviscid fluids and the Rayleigh collapse near a rigid boundary (Johnsen and Colonius, 2009) because it requires an initial condition to describe the unknown non-equilibrium thermodynamics to solve the bubble growth. In contrast, the implementation of the Navier-Stokes equations is much harder for compressible flows, yet very suitable for implementing surface forces like viscous forces and surface tension. Popinet and Zaleski (2002) solved the incompressible NS equations for the liquid phase with a viscous model while controlling the bubble with a polytropic equation of state. They found viscosity suppressed the jet formation and its influence reduced when γ decreased. Yet they did not solve the jetting dynamics for $\gamma < 1.2$ nor the spreading flow after impact. Minsier et al. (2009) used a commercial VoF solver in ANSYS Fluent with a viscous model to solve the incompressible NS equations, in which the bubble was compressible following an ideal gas law. They found a much smaller jet velocity for the viscous fluid. The bubble dynamics for $\gamma \leq 1.0$ was well simulated but limited to the time before the jet penetrated the bubble as the model could not handle compressible effects.

In cavitation studies, compressibility becomes important at the moment when the jet impacts the lower bubble wall or the last stage of bubble compression and rebound. To solve the compressible effect in both phases, Miller et al. (2013) introduced a novel pressure-based numerical technique to solve the compressible

Navier-Stokes equations based on the OpenFOAM (Weller et al., 1998) framework. The method embedded a VoF method for capturing the bubble interface and models for viscous stress and surface tension. The model for compressibility was introduced by the equation of state for each phase. Although the bubble mass was not strictly conserved, the method showed an excellent stability on problems related to bubble dynamics. Han et al. (2015) reproduced the jet formation from bubble pair interaction using a same solver but with a linear model for compressibility, the results showed qualitative agreements with experimental results. Koch et al. (2016) improved the solver with a mass conservation algorithm for the bubble and validated the code with the Gilmore model (Gilmore, 1952). They also simulated the laser-induced bubble dynamics near a rigid wall for a stand-off distance $\gamma \approx 1.4$ and achieved good agreement with the experimental result from Lindau and Lauterborn (2003). Lechner et al. (2017) extended the solver to study the pressure field during the jet formation, in which the shock-waves from the jet impact and bubble collapse were well captured.

The important fluid mechanics occurring during jetting near a rigid boundary are usually fast (on the order of microseconds) and small-scale (in the order of $10 \mu\text{m}$ for 1 mm diameter bubbles), which requires high temporal and spatial resolutions for the simulations. In experiments, it is nearly impossible to address both requirements simultaneously. The VoF solver in OpenFOAM presents as a promising tool to understand the jet dynamics and the accompanying spreading flow; for this the viscous boundary layer flow and the complex bubble dynamics close to rigid boundary need to be resolved.

1.3 Jetting near a free surface

Chahine (1977) firstly reported with extraordinary images that a cavitation bubble oscillating beneath a free surface can transform the planar air-liquid interface into a liquid "spike" jetting while the bubble moves and jets away from the free boundary due to the Bjerknes force (Blake et al., 1987). Over the last 40 years, this bubble dynamics has been explored further through experiments (Blake et al., 1987; Robinson et al., 2001) and simulations (Wang et al., 1996; Robinson et al., 2001) due to its importance for underwater explosions. Yet nowadays the liquid jet ejection from the air-liquid surface is attracting great interest in applications

FIGURE 1.2: Sketch of the jetting from cavitation bubble near free surface.

on delivering liquid matter into other mediums in the form of either thin focused jets (Tagawa et al., 2012) or mono-disperse droplets that develop through inherent instabilities from the jet (Visser et al., 2012).

To initiate free surface jetting, the bubble must be close to the liquid surface. The process is sketched in figure 1.2. Firstly the liquid surface perturbs as a crest as the expanding bubble approaches. This surface perturbation keeps growing into a jet outwards during the collapse while the bubble itself jets towards the opposite direction. The jetting mechanism here is different from the one of a bursting bubble near a free surface, where the jetting is triggered by the geometric singularity and the bubble wall becomes part of the free surface (Duchemin et al., 2002).

Surprisingly, the jetting dynamics was well modeled with a Dirichlet hyperbolic flow (Longuet-Higgins, 1983), which revealed that a flow focusing phenomenon leads to the jetting. Here the oscillating cavitation bubble drives the flow, which can also be produced with other sources such as a Faraday wave (Longuet-Higgins, 1983). In comparison to the wave method, the advantage of the cavitation bubble method in driving the flow is that it can be accomplished with focused laser pulse or electrode or chemical reaction without interrupting other parts of the domain.

Fundamentally, one can customize the geometry to utilize the focusing effect to achieve desirable jetting behaviour. For example, Fletcher and Palanker (2001) used a micro-nozzle with a $30\text{ }\mu\text{m}$ in diameter outlet to confine the free surface and obtained a pulsed microjet of 90 m/s in tip velocity and $30\text{ }\mu\text{m}$ in diameter for microsurgery using an electric discharge to generate the bubble. Inspired by the cavitation based micro-pump (Dijkink and Ohl, 2008a), Karri et al. (2012) designed a similar configuration that the free surface was covered by a plate opening a small hole of approximately $25\text{ }\mu\text{m}$ in diameter to generate microjets. The bubble was nucleated by a spark beneath the hole. Such a design can focus the kinetic

energy from the liquid of the oscillating bubble towards the hole. Two fast jets, the first from the initial expansion and the second from the subsequent collapse were observed. The first jet had a velocity of approximately 10 m/s while the collapse induced a considerably faster jet of ~ 50 m/s. After ejection, the second jet collided with the first jet in air and formed a liquid sheet, creating additional fragmentation. This design was later improved by [Gonzalez-Avila et al. \(2015\)](#), who put two electrodes on a plate submerged under water to generate a single hemispherical bubble with its geometric center close to the hole. The oscillation of the hemispherical bubble can produce transient microjets with a jet tip velocity of up to 400 m/s. The device can also generate microjets in viscous liquids of up to 100 cSt. In this type of design, the generation of thin jets requires a small hole for the focusing effect while a configuration with a large hole produces thicker and non-smooth jets with reduced velocities ([Liu et al., 2016b](#)). By fabricating the air-liquid surface as a concave meniscus with a capillary tube, [Tagawa et al. \(2012\)](#) reported about a focused microjets with a speed of 850 m/s using laser-induced cavitation bubble, where the pressure wave from the bubble expansion drove the inwards curved air-liquid interface into a focused jet with a diameter about 10% of the tube diameter. Later this design was tested for the application of needle-free injection into soft tissue ([Tagawa et al., 2013](#)). In applications, the jet velocity was found to depend strongly on the curvature of the meniscus, which can be modified by the contact angle and the tube diameter. In general, a concave surface enhances while a convex surface suppresses the flow focusing effect.

Jetting can also be generated from a convex surface using cavitation bubble although the flow focusing is suppressed. Examples are found for liquid cylinders ([Robert et al., 2007](#)), sessile hemispherical droplets ([Heijnen et al., 2009](#)), and for single spherical droplets ([Obreschkow et al., 2006](#); [Gonzalez-Avila and Ohl, 2016](#)). In this series of free surfaces, the jet formation usually requires many cycles of bubble oscillations. The initial convex air-liquid surface is not able to focus enough flow energy to develop jets, however it becomes unstable under a radial oscillating flow ([Plesset, 1954](#)) and develops surface waves which contain many concave surfaces. The pressure wave from bubble re-expansion is thus then able to focus the concave surfaces into jets.

Jetting from a single droplet initiated by a cavitation bubble is of particular interest for the formation of small scale fragments or mist from inertial droplets. This

may be achieved by thermal or chemical control in the absence of high injection velocities or pressures that are commonly used in spray and atomization applications. Yet a fundamental study on the complex process was lacking. [Obreschkow et al. \(2006\)](#) used a microgravity environment to produce a large spherical droplet and generated a cavitation bubble inside the droplet through electrode discharge. The bubble oscillation induced two jets that ejected liquids from opposite sides of the droplet surface. However only a few experiments were reported and parameters leading to the particular shape were not addressed in detail. Recently, [Gonzalez-Avila and Ohl \(2016\)](#) levitated nearly spherical droplets acoustically and created laser-induced bubbles at the droplet center. They reported a coarse fragmentation of the droplet where many fast jets emerged from the droplet surface while a cavitation bubble oscillated inside the droplet. This jetting was related to a spherical Rayleigh-Taylor instability caused by a spherical oscillating flow between a dense fluid and the surrounding air.

The difficulty in producing jetting from single droplets lays in the suppression of the flow focusing effect from the globally convex surface. The Rayleigh-Taylor instability provides a solution to manipulate the convex surface by amplifying surface disturbances of the droplet into small scale undulations that convert the globally convex surface into locally concave topologies ([Gonzalez-Avila and Ohl, 2016](#)). Yet a fundamental understanding is lacking on how the surface disturbances grow from the internal bubble oscillations, neither on the dynamics flow field detail nor through the stability analysis. Because the bubble now is bounded by a finite liquid domain, it follows a modified *Rayleigh-Plesset* equation ([Obreschkow et al., 2006](#)), whose dynamics is affected by surface tension and viscosity through the normal stresses on both the bubble-droplet and the droplet-air interface. Fluid properties are also important factors in instability growth. Firstly it is predicted that viscosity stabilizes the spherical Rayleigh–Taylor instability ([Prosperetti, 1977](#)). In single jet dynamics from a concave surface, viscosity is also found to slow down the jet formation by dissipating the kinetic energy through vorticity motion ([Onuki et al., 2018](#)). The viscous effect may be pronounced at the outer droplet surface since the surface wave with high mode generates more vorticities due to the baroclinic instability. Surface tension on the other side stabilizes the surface against small scale and highly curved surface modes ([Plesset, 1954](#)).

1.4 Jetting near complex boundaries

In industrial and medical applications cavitation bubbles are usually confined by complex geometries. These geometries often contain multiple boundaries of various shapes, causing complex bubble dynamics and rich jetting dynamics, very different from simple boundaries discussed above. Research has already addressed multi-rigid walls systems and free surface coupling rigid boundaries, such as a gap consisting of two parallel rigid walls submerged in a liquid ([Chahine, 1982](#); [Ishida et al., 2001](#)) or closed with a rigid wall at one exit ([Brujan et al., 2019](#)), two perpendicular rigid boundaries ([Brujan et al., 2018](#); [Jayaprakash et al., 2012](#)), two inclined rigid walls with various angle ([Tagawa and Peters, 2018](#)), between a flat rigid wall and a curved rigid boundary ([Li et al., 2017](#)), a free surface with a perpendicular ([Jayaprakash et al., 2012](#)) or an inclined ([Liu et al., 2016a](#)) rigid wall, between a free surface and a curved ([Li et al., 2018a](#)) or a flat ([Zhang et al., 2013](#)) rigid boundary, floating structure on a free surface ([Klaseboer et al., 2005](#); [Cui et al., 2018](#)).

One notable influence on the jet dynamics is that the jet direction is determined by the combined effect of individual stand-off distances. The *Kelvin impulse* and potential flow theory neglecting viscosity predicts that the jetting direction of a bubble next to a simple boundary aligns with the pressure gradient at the maximum expansion ([Obreschkow et al., 2011](#); [Supponen et al., 2016](#)). The pressure gradient directs towards rigid wall while outwards free surface, whose strength can be seen as a function of the stand-off distance ([Obreschkow et al., 2011](#)). In contrast pressure gradient of a bubble oscillating close to a rigid boundary that is curved can be either towards or away from the boundary. Its sign depends on the curvature and the distance as both affect the bubble shape ([Tomita et al., 2002](#)). In geometries with multiple boundaries, the pressure gradient can be decomposed into several pressure gradients associated with each individual boundary, thus the jet direction is determined by the competition of all pressure gradients, which is a function of individual stand-off distances ([Brujan et al., 2018, 2019](#)). Applying this method [Tagawa and Peters \(2018\)](#) used the potential flow theory to derive a theoretical model to predict the jet direction near two inclined rigid walls. They obtained a quantitative agreement with their experiments on the jet directions of bubbles with significantly large stand-off distances.

Novel bubble jetting dynamics can be introduced by multiple boundaries. [Zhang et al. \(2013\)](#) showed that the induced jet from bubble collapsing between a free surface and a rigid wall is much faster than near a free surface, indicating that two boundaries with the same direction of pressure gradient can strengthen the jetting. Interestingly for other geometries the bubble may split into two toroidal bubbles when it is close to two boundaries with pressure gradients acting in opposite directions. These can be either two rigid walls forming a gap ([Chahine, 1982](#); [Ishida et al., 2001](#)) or a system with one curved rigid wall and a free surface ([Li et al., 2018a](#)). The splitting of the bubble is then caused by an axial flow towards the bubble center and upon impact this flow is redirected in two opposite directions.

Besides splitting the bubble, a narrow gap bounded by two rigid walls can also make the bubble collapses onto the distant wall ([Gonzalez-Avila et al., 2011](#); [Quah et al., 2018](#)). This phenomenon happens when a bubble is nucleated very close to one wall with a maximum radius comparable to the gap height. Yet the mechanism leading to this unexpected bubble dynamics is not confirmed. Nevertheless, the special dynamics shows great potential in applications on cleaning in confined spaces and transporting large molecules into cells ([Gonzalez-Avila et al., 2018](#)). Reducing the gap height slows down the jet dynamics due to a decreasing inertial effect on the collapsing flow ([Quinto-Su et al., 2009](#)). When the gap height is sufficiently small the flow domain can be viewed as two-dimensional, e.g. the microfluidics channel. The 2D *Rayleigh-Plesset* equation describes well the bubble dynamics of small bubbles however fails to apply to the collapse of big bubbles because viscous effect becomes important ([Zwaan et al., 2007](#)).

Boundaries can influence the shape of jets. [Brujan et al. \(2019\)](#) observed planar jets from collapsing bubbles when putting an additional rigid wall at one outlet of the gap. The planar jets were due to the competition of three individual pressure gradients from three rigid walls. Once the two pressure gradients from the upper and lower wall of the gap have relevant strength, the jet developed towards the third wall and was parallel to the gap's upper and lower walls. Owing to the strong wall effect from the three walls, the bubble and jet were not able to hold the axisymmetry thus a planar jet was formed. Moreover, if the bubble has a comparable diameter with the gap height, its upper and lower surface still stuck to the nearby walls when the jet penetrated the bubble, leading to form a jet with a large cross-sectional area.

Elastic boundary is another important configuration regarding its significance in medical and biological applications (Dollet et al., 2019). The elastic boundary deforms based on its own material properties and the acting force from nearby flow including hydraulic pressure and shear stresses, this flow-structure coupling increase the complexity of the jetting dynamics. Early, the elastic boundary was proposed to prevent the damage from jetting bubble because it can repel the bubble away from the boundary thus mitigate or even hinder the jet from impacting (Gibson and Blake, 1980; Gibson and Blake, 1982). However it was experimentally proved by Brujan et al. (2001a,b) that although the bubble jetted away from the elastic surface it still caused damages. Indeed a low hydraulic pressure was generated next to the boundary when the bubble moved away, which forced the elastic surface to deform into the surrounding liquid (Brujan et al., 2001a). Therefore the damage remains and becomes even more pronounced since the soft material is fragile compared to rigid boundary. These experiments reveal the complexity as well as the importance of the interaction between jetting bubbles and elastic boundaries. To solve the jetting dynamics one must consider the two ways coupling between the bubble and elastic boundary (Shima et al., 1989; Gong and Klaseboer, 2016). The coupling flow becomes more complicated under complex geometries. Considerable research has been conducted on bubble dynamics involving complex geometries, such as in elastic tubes (Ye and Bull, 2006), inside a flexible vessel (Coralic and Colonius, 2013), near a multi-layered synthetic beam (Gong et al., 2018), between an elastic membrane and a solid wall (Horvat et al., 2018), next to an elastic ball (Li et al., 2018b), and between two deformable planar boundaries (Doinikov et al., 2016). This list is not exhaustive and many more geometries have been reported in the literature. As emphasized by Brujan et al. (2001a), the pressure and velocity distribution around the bubble and near the elastic boundary are essential to understand the complicated dynamics. Yet they are not easy to access through experiments only and numerical simulations with reliable fluid structure interaction models are needed to fill this gap (Turangan et al., 2017).

Given the above examples (also see figure 1.3 for some sketches), complex boundaries can influence the direction, topology, and velocity of the cavitation induced jet. In contrast, this inspires controlling the jet dynamics with designed boundaries. Therefore the jetting mechanism and related scaling laws are desired to be understood, which may be different case by case due to the nonlinearity brought by viscosity and the complex coupling between bubble, boundaries and liquid medium.

FIGURE 1.3: Examples of boundary influence on jet dynamics. The solid and dashed curve represent the bubble shape at maximum expansion and jet formation, respectively. (a) Bubble splitting in a gap bounded by two rigid walls. (b) Influence on jet direction from two inclined rigid walls. (c) Planar shape jet generated in a three walls confined system. (d) Interaction of a bubble with an elastic boundary.

1.5 Guide through the thesis

This thesis focuses on the fundamental dynamics of liquid jetting from laser induced cavitation bubbles near rigid boundaries and free surface with the emphasis of viscous effects and complex geometries.

A numerical model for simulating the cavitation bubble dynamics in viscous liquid is described in great detail in chapter 2. The same numerical model is also applied to chapter 3 and 4.

To address tangential force on a no-slip solid wall by an oscillating bubble nearby, we study the near boundary flow with a compressible VoF solver numerically in chapter 2. A spatio-temporal distribution on the shear stress is presented and explained in detail and with respect to the overall bubble dynamics.

With the constriction of an additional solid boundary, a bubble can experience even richer dynamics. In chapter 3 we study the jetting of cavitation bubbles in a more complex geometry which is a thin gap bounded by two rigid plates with experiment and simulation. VoF simulation reveals that the viscous flow near the rigid boundaries plays a crucial role on the formation of the rich dynamics such as bubble splitting, redirecting the jetting direction, and translation of the bubble centroid.

In chapter 4& 5 we deal with the jetting from the surface of single droplets, i.e. convex free surfaces. The generation of fast liquid jets from single and viscous droplets by an internal oscillating bubble is presented in chapter 4. We combine experiments and VoF simulation and a stability analysis together to explore the jetting mechanism. Viscous effect will be included in a parameter map showing the jetting regimes.

In chapter 5 we apply the analytic model derived in chapter 4 to investigate the jetting and fragmentation of liquid metal droplet from a near-infrared laser. Here the ambient pressure is close to zero, surface tension becomes important in driving the collapse.

I conclude the thesis in chapter 6 and propose further studies in the outlook of chapter 6.

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Chapter 2

Wall shear stress from jetting cavitation bubbles near no-slip rigid boundary¹

This chapter studies the wall shear stress generated by a collapsing bubble near rigid wall. The collapse of a cavitation bubble near a rigid boundary induces a high-speed transient jet accelerating liquid onto the boundary. The shear flow produced by this event has many applications, examples of which are surface cleaning, cell membrane poration, and enhanced cooling. Yet the magnitude and spatio-temporal distribution of the wall shear stress are not well understood, neither experimentally nor by simulations. Here we solve the flow in the boundary layer using an axisymmetric compressible volume-of-fluid (VOF) solver from the OpenFOAM framework and discuss the resulting wall shear stress generated for a non-dimensional distance, $\gamma = 1.0$ ($\gamma = h/R_{max}$, where h is the distance of the initial bubble centre to the boundary, and R_{max} is the maximum spherical equivalent radius of the bubble). The calculation of the wall shear stress is found to be reliable once the flow region with constant shear rate in the boundary layer is determined. Very high wall shear stresses of 100 kPa are found during the early spreading of the jet, followed by complex flows composed of annular stagnation rings and secondary vortices. Although the simulated bubble dynamics agrees very well with experiments, we obtain only

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qualitative agreement with experiments on the cavitation bubble induced wall shear stress due to inherent experimental challenges.

2.1 Introduction

Cavitation bubbles expanding and shrinking near rigid boundaries move towards the boundary due to the action of the secondary Bjerknes force (Blake and Gibson, 1987; Best and Kucera, 1992). The oscillating flow near the wall creates naturally a strong and unsteady boundary layer flow with high shear rates. Additionally, the radial flow is accompanied by a jetting flow accelerating liquid to impinge on and spread along the wall.

Historically, the shear stress generated from steady impinging jets on solid walls has been solved by Glauert (1956), and confirmed through simulations (Deshpande and Vaishnav, 1982; Phares et al., 2000), and experiments (Narayanan et al., 2004; Visser et al., 2015). However, the shear stress produced by transient jets from near-wall cavitation has recently received more attention, in particular for its importance for surface cleaning (Ohl et al., 2006a; Kim et al., 2009; Gonzalez-Avila et al., 2011) and cell membrane poration cells (Ohl et al., 2006b; Le Gac et al., 2007). Very few experiments have tried to measure the wall shear stress. Dijkink and Ohl (2008) combined high-speed photography with a hot film anemometer and found short peaks with stress amplitudes of up to 3.5 kPa occurring right after the jet impact. Given the limited temporal bandwidth of their device the maximum shear stress reported may only be a lower bound. Their sensor can only measure at a single location. Thus, to map spatial stress requires many measurements with identical bubbles, which is difficult to achieve. To fill this knowledge gap, we present here a numerical approach to estimate the wall shear stress. It is based on the finite-volume method (FVM) to discretize the fluid domain and the volume-of-fluid (VOF) method to capture the bubble-water interface. We can reliably simulate the bubble dynamics through the first and second collapse and obtain a spatio-temporal map of the wall shear stress.

This chapter is organized as follows. First, we start with a description of the experimental and numerical methodologies. The numerical models for simulating single cavitation bubble in viscous and confined liquid is described in details. Next, we

verify the numerical model with experiments and previous shear stress measurement from [Dijkink and Ohl \(2008\)](#). We provide a test of our numerical method against previous works and an analytic solution to confirm the calculation of wall shear stress. Then we study the flow field details near the boundary to clarify the flow features that lead to the wall shear stress. After that a spatio-temporal map of the wall shear stress distribution is presented and discussed.

2.2 Methodology

Here we resolve the shear flow produced by a cavitation bubble collapsing near a solid boundary in water. The flow field is obtained by solving the compressible Navier-Stokes (NS) equations and capturing the gas-water interface with the VOF method based on the Finite Volume framework of OpenFOAM ([Weller et al., 1998](#)). We verify the numerical method by comparing the simulation results with experiments. Here the validation consists of two parts, the bubble dynamics and the wall shear stress. For the bubble dynamics, we conduct a high-speed photographic experiment on laser induced bubble to validate the gas-phase dynamics and thus the flow field in the numerical simulations. For the validation of the wall shear stress from the simulations, we benchmarked the model with earlier simulations of a submerged steady wall jet impinging on a surface from [Deshpande and Vaishnav \(1982\)](#), see below. We also compare the simulation result with experiments of [Dijkink and Ohl \(2008\)](#) of cavitation bubble induced wall shear stress.

2.2.1 Experiment

Figure [2.1 \(a\)](#) shows the top view of the experimental set-up. In the experiment, single cavitation bubbles are created with a laser pulse from a Q-switched Nd:YAG laser pulse (532 nm wavelength, 6 ns duration), which is focused at a distance h close to a wall. The bubble is generated by the dielectric breakdown of the liquid where the laser energy is focused on. A detail description on laser induced cavitation dynamics can be referred to [Philipp and Lauterborn \(1998\)](#); [Lauterborn and Kurz \(2010\)](#). To nucleate single and spherical initial bubble, the laser pulse needs to be focus into a small spot. For these the laser beam is expanded and collimated with one plano-concave and one plano-convex lens and then focused

FIGURE 2.1: (a) Experiment set up. (b) The simulation domain. The region in red is the computational domains used in the simulations considering the symmetry.

into the test section with a $10\times$ microscope objective (Olympus UPlanFL N10X 0.30 NA). The test section is an acrylic cuvette ($D = 10\text{ mm}$ is the length of the square base of the cuvette). To ensure spherical bubble induced, the wall where laser beam is introduced through needs to be thin enough to avoid dispersing the laser beam, here we use a glass cover slip with a thickness of $170\text{ }\mu\text{m}$. From the experimental image figure 2.3 we can see the initial bubble is nearly spherical. The aspect ratio of the horizontal and vertical radii of the plasma is about 1.18 at $t = 0$ while this ratio later becomes about 1 at $t = 5\text{ }\mu\text{m}$. Therefore it is reasonable to initialize the bubble as a sphere in the simulation. The bubble dynamics is recorded with a high-speed camera (Photron SAX-2, Japan) operating at 200,000 frames per second (293 ns exposure time). The illumination light source is an LED powered fiber optic (REVOX SLG 150V). The recorded image resolution is $20\text{ }\mu\text{m}$ per pixel. A delay pulse generator (BNC, Berkeley Scientific) is used to synchronize the camera, pulsed laser and light source.

2.2.2 CFD simulation

2.2.2.1 Governing equations and numerical models

The flow is dominated mostly by inertia of the liquid and the compressibility of the gas; the viscosity is only important close to the surface. For simplicity we model the initial (laser-created) cavitation bubble consisting of non-condensable gas starting from a small volume at high pressure. The gas and liquid phases are treated as compressible and immiscible Newtonian fluids while neglecting heat and mass transfer. This assumption has been widely used for cavitation bubble simulations (Brujan et al., 2002; Tiwari et al., 2013; Koch et al., 2016; Fuster and Popinet, 2018). Thus, we solve the equation of continuity together with the compressible Navier-Stokes equations (Miller et al., 2013; Koch et al., 2016):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad , \quad (2.1)$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{T} + \mathbf{f}_\delta \quad , \quad (2.2)$$

where ρ is the density of the fluid, $\mathbf{u}$ the velocity field, p the pressure and $\mathbf{T}$ the viscous stress tensor which is calculated as $\mathbf{T} = \mu[\nabla \mathbf{u} + \nabla \mathbf{u}^T - \frac{2}{3}(\nabla \cdot \mathbf{u})\mathbf{I}]$. $\mathbf{I}$ is the identity tensor and $\mathbf{f}_\delta$ the source term due to surface tension which is modeled according to the so-called Continuous-Surface-Force (CSF) method (Brackbill et al., 1992). We assume an adiabatic process and relate the gas density as a function of pressure with the Noble-Abel equation of state (EOS), see Koch et al. (2016).

$$p \left(\frac{1}{\rho_g} - \frac{\beta}{\rho_n} \right)^\kappa = p_n \left(\frac{1}{\rho_n} - \frac{\beta}{\rho_n} \right)^\kappa \quad , \quad (2.3)$$

where ρ_g is the density of the gas, ρ_n and p_n are the density and pressure for a small bubble of radius R_n in an equilibrium state, β is the co-volume coefficient, which accounts for the available volume for molecular motion, and κ is the specific heat ratio or adiabatic exponent.

For the liquid phase the Tait EOS is applied:

$$p = (p_0 + B) \left(\frac{\rho_l}{\rho_0} \right)^K - B \quad , \quad (2.4)$$

Liquid Parameters	Value	Gas Parameters	Value/formula
Tait exponent K	7.15	specific heat ratio κ	1.4
reference pressure p_0	1.01315 bar	equilibrium pressure p_n	$p_0 + 2\sigma/R_n$
Tait pressure B	3046 bar	normalized co-volume β	0.0015
reference density ρ_0	$1000 \text{ kg} \cdot \text{m}^{-3}$	reference density ρ_n	$1 \text{ kg} \cdot \text{m}^{-3}$

TABLE 2.1: The parameter used in the EOS for the gas and the liquid.

where ρ_l is the liquid density, p_0 and ρ_0 are the reference density and reference pressure, respectively. K is the Tait exponent and B the Tait pressure. The parameters are chosen the same as in Koch et al. (2016) and are listed in table 2.1.

The governing equations are solved in a single-field formulation for both phases including the interface cells. In order to capture the interface, the VOF method is used. The indicator function α is defined as the volume fraction of one of the phases, which is solved with a transport equation. For convenience, we introduce instead two volume fraction functions, α_l for the liquid phase

$$\alpha_l = \begin{cases} 1, & \text{liquid} \\ 0, & \text{gas} \\ 0 < \alpha_l < 1, & \text{interface} \end{cases} \quad (2.5)$$

and α_g for the gas phase

$$\alpha_g = 1 - \alpha_l \quad . \quad (2.6)$$

Considering the mass conservation for each phase we obtain

$$\begin{cases} \frac{\partial \rho_l \alpha_l}{\partial t} + \nabla \cdot (\rho_l \mathbf{u} \alpha_l) = 0 \\ \frac{\partial \rho_g \alpha_g}{\partial t} + \nabla \cdot (\rho_g \mathbf{u} \alpha_g) = 0 \end{cases} \quad . \quad (2.7)$$

Rewriting equations 2.7 and introducing the compressibility $\psi = D\rho/Dp$ from the EOS we obtain an evolution equation for the liquid volume fraction

$$\frac{\partial \alpha_l}{\partial t} + \nabla \cdot (\alpha_l \mathbf{u}) + \nabla \cdot (\alpha_l \alpha_g \mathbf{U}_r) = \alpha_l \alpha_g \left(\frac{\psi_g}{\rho_g} - \frac{\psi_l}{\rho_l} \right) \frac{Dp}{Dt} + \alpha_l \nabla \cdot \mathbf{u} \quad , \quad (2.8)$$

where $\mathbf{U}_r$ is the relative velocity between the two phases. From equation 2.8 we see that $\mathbf{U}_r$ acts as an artificial compressibility which helps to ensure a sharp

interface (Rusche, 2003). This relative velocity is determined from the normal vector of the cell surface $\mathbf{n}_f$, the cell surface area S_f , the volume flux ϕ_f , and an adjustable compression coefficient C_f between 0 and 1

$$\mathbf{U}_r = \mathbf{n}_f C_f \frac{|\phi_f|}{|S_f|} . \quad (2.9)$$

Typical VOF method uses compressive differencing scheme such as the CICSAM (Compressive Interface Capturing Scheme for Arbitrary Meshes) by Ubbink and Issa (1999) and THINC (Tangent of Hyperbola for Interface Capturing) by Xiao et al. (2005) to introduce necessary compression to reconstruct a sharp interface. In OpenFOAM, the compression term $\nabla \cdot (\alpha_l \alpha_g \mathbf{U}_r)$ in equation 2.8 contributes similarly to a compressive differencing scheme. As seen in equation 2.8, the compression is only active in a narrow interface region where $0 < \alpha_l < 1$ and does not affect the solution outside the interface. The coefficient C_f in equation 2.9 is used to adjust the compression as well as the interface thickness. The advantage of this term is its simplicity in the implementation and its independence on computational mesh. Detailed numerical test on this method can be found in Rusche (2003) and Deshpande et al. (2012).

Once equation 2.8 is solved, the volume fraction α_l and α_g are obtained. The interface is then chosen as $\alpha_l = 0.5$.

The surface tension source term $\mathbf{f}_\delta$ is obtained from

$$\mathbf{f}_\delta = \sigma \left[-\nabla \cdot \left(\frac{\nabla \alpha_l}{|\nabla \alpha_l|} \right) \right] \nabla \alpha_l , \quad (2.10)$$

where σ is the coefficient of surface tension. Physical parameters depending on the void fraction, j such as viscosity or density are calculated through

$$j = j_l \alpha_l + j_g \alpha_g , \quad (2.11)$$

where j_l and j_g are the parameters for liquid and gas phases respectively. The viscosities are constant in each of the phase while densities are updated based on the equations of state.

Considering the compressibility, the continuity equation 2.7 can be re-written as

$$\left(\frac{\alpha_l}{\rho_l} \psi_l + \frac{\alpha_g}{\rho_g} \psi_g \right) \frac{Dp}{Dt} + \nabla \cdot \mathbf{u} = 0 \quad . \quad (2.12)$$

Here the PIMPLE algorithm solves the pressure-velocity coupling in the momentum equation (Greenshields, 2015), which is a combination of the PISO (pressure-implicit split-operator) and the SIMPLE (semi-implicit method for pressure-linked equations) algorithms. The volume fraction equation in the VOF method is solved using a MULES (multidimensional universal limiter with explicit solution) algorithm (Rusche, 2003; Greenshields, 2015) and an AIC (artificial interface compression) algorithm to ensure the sharp interface without geometry reconstruction (Ubbink, 1997). The time step is adaptively changed depending on the Courant-Number to ensure the stability of the solution. In short the solution is obtained through the following steps:

1. Solve VOF transport equation 2.8 to get α_l and α_g ;
2. Update the material properties according to α_l for the first correction of one PIMPLE loop;
3. Update density according to equation of continuity 2.1;
4. Solve momentum equation 2.2 and the pressure equation 2.12;
5. Compute compressibility and densities from state equations 2.3 & 2.4;

Every time step is treated as a PIMPLE algorithm loop, all the above steps are repeated for each loop until the simulation end-time is reached. The temporal derivatives are discretized using the implicit Euler scheme. A second order van Leer scheme and a second order upwind scheme are applied to discretize the convective terms in the volume fraction equation and in the momentum equation, respectively. For the other terms a second order Gauss linear difference scheme is used.

2.2.2.2 Computational set-up

Simulation domain and mesh. As the experimental maximum bubble radius is approximately $R_{max} \approx 600 \mu\text{m}$, i.e. $D \gg R_{max}$, we simplify the flow domain as a cylinder with a radius of 5 mm and height of 10 mm and take advantage of the axisymmetry. The simulation domain is shown in figure 2.1 with a size of 5 mm in radial and 10 mm in axial directions. In OpenFOAM, the 2D axisymmetric domain is a wedge with an angle less than 5° of the 3D cylindrical geometry. We use a static hexahedral mesh and make sure individual cells have same length in radial

and vertical direction. The computational mesh contains 101 grid points in radial direction and 201 grid points in axis direction initially. These are refined recursively four times using the OpenFOAM utility "refineMesh", in each refinement we reduce the size of the region to be refined, like the method used in [Popinet and Zaleski \(2002\)](#). In the area of interest, i.e. where the bubble is located the cell size is approximately $3\mu\text{m}$ wide. In the near-boundary region $y < 10\mu\text{m}$, the mesh is further refined recursively down to a spacing $\Delta x = 0.05\mu\text{m}$, to resolve a linear increasing velocity within the thin boundary layer. To ensure the stability of the solution when the flow passes through different size of grids, the adaptive time step is used to control the time step which satisfies $Courant - Number < 0.1$ for the whole domain.

Boundary conditions. If not specified, the boundary condition is set to "zero-Gradient", which applies a zero normal gradient for the field. In the implementation, this boundary condition simply extrapolates the quantity to the boundary patch from the nearest cell value. For the solid boundary a no-slip condition is used for the velocity field. The pressure at the outer rim of the simulation domain is set to atmospheric pressure $p_\infty = 1.01325\text{ bar}$.

Initial conditions. To reproduce the experiment with simulation, while keeping a constant initial bubble radius $R_0 = 50\mu\text{m}$, we set the nucleated position h as the same as the experiment and then adjust the initial bubble pressure to meet the experimental period of oscillation. By trial and error we set the initial bubble pressure as $p(t = 0) = 1300\text{ bar}$. The pressure in water is set to 1.01315 bar initially. The dynamic viscosity is $1 \times 10^{-3}\text{ kg} \cdot \text{m}^{-1}\text{s}^{-1}$ for water and $1.84 \times 10^{-5}\text{ kg} \cdot \text{m}^{-1}\text{s}^{-1}$ for the bubble and the coefficient of surface tension is $\sigma = 0.0728\text{ N} \cdot \text{m}^{-1}$.

2.3 Results

2.3.1 Comparison with experiments

Before we have a detailed look into the flow near to the boundary and the wall shear stress generated we compare the overall bubble dynamics together with the snapshots of the bubble shape. Figure 2.2(a) shows the sketch of the problem and introduces the coordinates of the axisymmetric problem. Figure 2.2 compares the

FIGURE 2.2: (a) Sketch of the problem and parameters. The dashed circle is the bubble nucleation and the solid circle is the bubble at maximum size; (b) Comparison of the equivalent spherical bubble radius from simulations (lines) with the experiment (filled symbols). The dashed line shows the result using a constant polytropic exponent of $\kappa = 1.4$; and the red solid line represents the result using a polytropic exponent of $\kappa = 1.4$ from $t = 0 \mu\text{s}$ to $t = 136.5 \mu\text{s}$ and a higher polytropic exponent of $\kappa_2 = 2.4$ from $t = 136.5 \mu\text{s}$. The error bar presents the uncertainty from the recorded images.

volume-equivalent spherical bubble radius between the experiment (filled symbols, $R_{\text{max}} = 0.6 \text{ mm}$, $\gamma = 1.1$) and the simulations (lines). In the simulations and the experiments the bubble due to the high pressure inside expands in the initially stagnant liquid. By adjusting this pressure while keeping the radius, $R(t = 0) = 50 \mu\text{m}$, constant we can achieve good agreement for the expansion stage to an almost spherical bubble and the later shrinkage. Yet, after re-expansion we find a considerable deviation of the equivalent bubble radius between the experiment and a straightforward simulation (dashed line); the bubble after collapse expands much larger in the simulations. We attribute this difference to the cushioning effect of the gas during the collapse: the non-condensable gas slows down the last stage of the collapse resulting in less energy being lost from acoustic radiation and viscous damping, hence allowing for a larger re-expansion. To deal with this discrepancy, while not modelling the mass transfer [Lee et al. \(2007\)](#) implemented an experimental correlation accounting for the energy loss during the collapse into their boundary integral method based on data from [Cole \(1965\)](#). [Wang \(2014\)](#) also considered the energy loss due to shock wave emission during the early expansion stage and the late bubble collapse and uses the bubble wall Mach number as a criterion. Both methods achieved reasonable agreement with experimental results, however they are difficult to implement consistently in the present VOF framework. Here we provide an alternative approach by increasing the polytropic exponent

FIGURE 2.3: Comparison of bubble shape evolution for standoff distance $\gamma = 1.1$. The left part in a single frame is the simulation result, while the right part is the experimental result. Time of the first frame in each row is indicated in μs , inter-frame time is $5 \mu\text{s}$, and the scale bar represents 0.5 mm . The boundary is below each frame.

of the gas after the collapse. This effectively removes internal energy from the bubble and the bubble expansion is reduced although the mass transfer is not modeled. The new polytropic exponent is found by fitting the experimental second re-expansion to the simulation, see solid line in figure 2.2. Here we use for the polytropic exponent during the initial expansion and collapse $\kappa = 1.4$ and later $\kappa = 2.4$. The increase of κ at the bubble rebound indicates the energy loss of the bubble at the collapse, i.e. less energy of the bubble is converted to kinetic energy of the flow after its rebound. This energy loss is γ dependent (Vogel and Lauterborn, 1988; Vogel et al., 1989). By comparing the simulations and experimental results, we find that $\kappa = 2.4$ after the first collapse model sufficiently well the energy loss for $\gamma \approx 1$. According to the energy loss analysis by Vogel et al. (1989), κ of rebounding bubble decreases with γ for $\gamma < 0.9$ but increases with γ for $\gamma > 0.9$. However the accurate value of the κ need to be determined by trial and error for different γ comparing experimental and computed results.

Figure 2.3 compares the computed bubbles dynamics (left part of each frame) with the experiments (right part of each frame). The rigid boundary for the experiment and the simulation is located at the bottom of each frame. At time $t = 0$ the bubble is nucleated, in the simulation the bubble starts with a spherical nucleus $R(t = 0) = 50 \mu\text{m}$ containing gas at a pressure of $p_g(t = 0) = 1300 \text{ bar}$ while the bright spot in the experiment is due to the plasma emission from the dielectric breakdown. Although the complex plasma and fluid dynamics of the nucleation is ignored already at frame $t = 5 \mu\text{s}$ very good agreement of the expansion dynamics is obtained. After an expansion of the bubble to a maximum volume of an approximate sphere with radius $R(t = 70 \mu\text{s}) = 0.6 \text{ mm}$ the bubble shrinks and develops a jet flow towards the boundary before reaching its minimum volume at $t \approx 135 \mu\text{s}$. During re-expansion a rather complex bubble shape is observed; the bubble separates into a pancake shape close to the boundary and small fragments higher up along the axis of symmetry. The simulations reveal a second splitting at $t = 145 \mu\text{s}$ and re-connection of the flat bottom part when the bubble volume shrinks again, e.g. from $t = 210 \mu\text{s}$. Till the second collapse we find good agreement of the outer shape of the bubble between simulation and experiment. Please keep in mind that the simulation shows a cut through the fluid domain while the high-speed recording are a photographic projection of the bubble.

2.3.2 Determination of the wall shear stress

Next we present a comparison of the wall shear stress at one position on the boundary with a measurement of [Dijkink and Ohl \(2008\)](#) for the cavitation bubble at a stand-off $\gamma = 1.0$. For this we have to approximate the wall shear stress as:

$$\tau = \mu \left. \frac{d\mathbf{u}_r}{dy} \right|_{y=0} \approx \mu \left. \frac{\mathbf{u}_r(y)}{y} \right|_{y \leq \epsilon}, \quad (2.13)$$

where $\mathbf{u}_r$ is the flow velocity component parallel to the wall and y is the distance to the boundary, and ϵ is the thickness of the region with constant shear rate which is located within the boundary layer ([Schlichting et al., 1955](#); [Visser et al., 2015](#)). According to the boundary layer theory, the thickness of the boundary layer and ϵ decrease with the flow velocity ([Schlichting et al., 1955](#); [Visser et al., 2015](#)). Therefore to apply equation 2.13 we need to determine a minimum ϵ value where

FIGURE 2.4: Comparison on wall shear stress of the submerged jet impingement. (a) the normalized stress distribution on wall for $Re = 100$; the blue line is the result of Deshpande and Vaishnav (1982), the orange line and green dotted line represent the result calculated at $y = 0.1 \mu\text{m}$ and $y = 0.2 \mu\text{m}$ from the simulation, while the cyan dash line is the Glauert solution. The normalized stress and normalized distance are also plotted in log scale in the inset. (b) the maximum normalized wall shear stress of various Re Number from Deshpande and Vaishnav (1982) and the current simulations.

the highest velocity occurs, as a result the chosen y will be always located within the constant shear rate region.

To validate the calculation by equation 2.13, we perform a simulation of the steady submerged wall jet simulation and compare the results with Glauert (1956)'s analytical solution and the classical simulation simulation from Deshpande and Vaishnav (1982), see below. The comparisons indicate that the calculation of the wall shear stress with the present model is reliable once the region with constant shear rate within the boundary layer is resolved.

To validate the calculation of wall shear stress from the code, we now reproduce the steady wall jet impingement simulation by Deshpande and Vaishnav (1982): a cylindrical submerged jet impacting on to a no-slip flat surface. The jet has a radius of $R_{jet} = 50 \mu\text{m}$ and a flat velocity of U_{jet} , whose inlet is located $d = 4R_{jet} = 200 \mu\text{m}$ above the boundary. The domain is axisymmetric with both width and height of $10R_{jet}$. The mesh has a spacing $\Delta x = 2.5 \mu\text{m}$ in the bulk flow while is refined to $\Delta x = 0.05 \mu\text{m}$ in the near boundary region. Glauert (1956) gave an analytic similarity solution of the wall shear stress for this problem:

$$\tau = \mu \left. \frac{d\mathbf{u}_r}{dy} \right|_{y=0} = \rho \left(\frac{125F^3\rho}{216\mu x^{11}} \right)^{1/4}, \quad (2.14)$$

FIGURE 2.5: Flow and wall shear stress at $t = 133 \mu\text{s}$ for the bubble of $\gamma = 1.0$. (a) the radial velocity distribution along vertical direction within the boundary layer at $x=70 \mu\text{m}$. U_x is linear with a slope of 10^8 when $y < 0.4 \mu\text{m}$; the crosses represent the grid points within the boundary layer. (b) the wall shear stress distribution along boundary calculated at $y = 0.1 \mu\text{m}$ and $y = 0.2 \mu\text{m}$.

where ρ is the flow density and F the momentum flux which is defined as $F = (1/8)U_{jet}^3 R_{jet}^4$ for a jet with a flat velocity profile.

Figure 2.4(a) shows the comparison on the wall shear stress distribution along the boundary of our current calculation and the numerical result of [Deshpande and Vaishnav \(1982\)](#) and the Glauert solution (equation 2.14) for the case $Re = 100$, where $Re = 2\rho U_{jet} R_{jet}/\mu$ is the defined Reynolds number. The current calculation of the wall shear stress from equation 2.13 at $y = 0.1 \mu\text{m}$ and $y = 0.2 \mu\text{m}$ is indistinguishable, indicating that the stress is converged in the near boundary cells and the flow field in the region with constant shear rate is resolved. The stress distribution along the boundary from the current simulation matches very well with [Deshpande and Vaishnav \(1982\)](#) as well as the Glauert solution for $x/R_{jet} > 3$. The normalized maximum shear stress $\tau_{max}/(4\rho U_{jet}^2)$ is also compared to the numerical work of [Deshpande and Vaishnav \(1982, 1983\)](#), good agreement is found for a wide range of Re numbers. Thus the calculation of the wall shear stress using our current model is reliable once the region with constant shear rate in the boundary layer is determined.

Before we compare the wall shear stress reproduced by this model to the measurement of [Dijkink and Ohl \(2008\)](#) for the cavitation bubble at a distance of $\gamma = 1.0$, we look for the boundary layer formed by the spreading of the liquid jet induced by collapsing bubble to determine the minimum ϵ . Figure 2.5(a) shows the vertical distribution of the radial velocity U_x at $t = 133 \mu\text{s}$ and $x = 70 \mu\text{m}$, where the largest shear occurs (see figure 2.8). The crosses indicate the positions of the resolved cells close to the boundary. We obtain a constant slope greater than 5 and more cells

FIGURE 2.6: Comparison of measured and simulated wall shear stress at a non-dimensional distance of $\eta = 0.33$ from the boundary for the bubble at a distance of $\gamma = 1.0$. The horizontal axis is the dimensionless time normalized by the collapse time T_c , the vertical axis is the wall shear stress in kPa. The dashed lines in each plot represent the instances of jet impact, first collapse, and the second collapse, respectively. The jet impact occurs before the bubble reaches its minimum volume. Also note in experiment $R_{max} \approx 750 \mu\text{m}$, $T_c \approx 80 \mu\text{s}$; in simulation $R_{max} \approx 600 \mu\text{m}$, $T_c \approx 69 \mu\text{s}$. Top: the experimental result from [Dijkink and Ohl \(2008\)](#); Bottom: calculation from current study.

demonstrating sufficient resolution to resolve the constant shear near the boundary with a thickness of approximately $1.5 \mu\text{m}$. For $y < 0.4 \mu\text{m}$, U_x is linear with y with a shear rate of 10^8s^{-1} , i.e. the minimum $\epsilon \approx 0.4 \mu\text{m}$. As a proof that this holds for all the calculations we plot in figure 2.5(b) the wall shear stress distribution as a function of x using equation 2.13 with $y = 0.1 \mu\text{m}$ and $y = 0.2 \mu\text{m}$. The agreement demonstrates that the stresses are fully resolved even for at locations of highest shear. In the remaining calculations we use $y = 0.1 \mu\text{m}$.

Next we compares the simulated wall shear stress with the measurement of [Dijkink and Ohl \(2008\)](#). In the experiment of [Dijkink and Ohl \(2008\)](#), a pulsed laser (Solo PIV, New Wave, Nd:YAG laser, wavelength 532 nm, pulse duration 6 ns) is focused 0.8 mm above the boundary to induced a bubble with maximum radius 0.75 mm ($\gamma \approx 1.0$). The shear stress is measured with a resistive thin film sensor with

active area of $750 \times 200 \mu\text{m}^2$ (Dantec Dynamics, model 55R46), which is mounted on the rigid boundary and located at a horizontal distance of 0.25 mm to the bubble nucleation. The probe of the sensor is connected to a constant temperature anemometer (CTA, Dantec Dynamics). Since the probe operates at a constant temperature mode, once the sensor is cooled down by advection from the flow, the CAT can measure the current that used to stabilize the sensor temperature. Therefore the flow velocity (or shear stress) can be determined through a relation resembling King's law $E^2 = E_0^2 + k\tau^n$, where E is the measured voltage, E_0 and k are the coefficients to be confirmed with calibration. The calibration measurements are conducted in micro-channel flows, where the wall shear stress can be calculated from the driven flow velocity analytically.

Figure 2.6 compares the measured and simulated wall shear stresses recorded at $\eta = 0.33$ ($\eta = s/R_{max}$, where s is the distance to the axis of symmetry) at a stand-off parameter of $\gamma = 1.0$. The horizontal axis is the time normalized by the collapse time T_c , while the vertical axis is the measured and calculated wall shear stresses in kPa. In the simulations we have accounted for the imperfections of the stress sensor, i.e. its finite size and non-directivity by taking the average of the absolute of the wall shear stress over the radial distance of $200 \mu\text{m}$. Overall the simulations predict much higher peak wall shear stress than the measurements. We explain this discrepancy with the limited bandwidth of the experimental sensor from [Dijkink and Ohl \(2008\)](#) and the limits of the calibration method (see below). Let us compare the features qualitatively: the highest value in experiment and simulation is found right after the liquid jet impinges and spreads along the boundary. During the decay of the stress a second peak forms caused by the liquid accelerated by the re-expanding bubble. Moreover the decay of this peak is slow because the flow is stabilized by a vortex ring forming close to the surface. A third peak is also captured in the simulations and found in the measurement. This is due to the re-expansion of the bubble after its second collapse. The features of a prominent peak, a second maximum, slow decay of shear stress during the re-expansion till 2nd collapse, and second peak during 2nd re-expansion are consistently found in experiment and simulation.

The simulation shows stresses an order of magnitude greater than the experiment on the maximum stress, this discrepancy mainly comes from three factors. First, the upper limit of the sampling frequency in the experiment is 250 kHz which

gives a bandwidth of $4 \mu\text{s}$. As showed in simulation figure 2.6,2.7 and 2.8, the maximum shear stress increases from 0 to 100 kPa in about $2 \mu\text{s}$ after jet impact, this fast rising is not fully captured by the low bandwidth measurement. Second, the calibration of Dijkink and Ohl (2008) is conducted for shear stress over a range from 0.05 Pa to 10 Pa, while the measurement operates for a much larger range from 0 to 3.5 kPa. Third, in the experiment the shear stress actually is calculated using the measured flow velocity according to equation 2.13. To apply equation 2.13, the position of the measured flow velocity (or the sensor) should be located within the constant shear region $y < \epsilon$. From figure 2.5 we see $\epsilon \approx 0.4 \mu\text{m}$ for the maximum shear stress in simulation. This location is extremely difficult to achieve in experiment since the vertical position can only be adjusted within $\pm 10 \mu\text{m}$ in the set-up of Dijkink and Ohl (2008). Let's assume the sensor is located at $y = 10 \mu\text{m}$ and measures a flow velocity of 60 m/s (obtained from figure 2.5), the measured shear stress is about 6 kPa. If we further consider a 50% loss due to a limited bandwidth and insufficient calibration, we obtain a maximum measured wall shear stress of 3 kPa, which is very close to the result from Dijkink and Ohl (2008).

2.3.3 Details of the flow and wall shear stress

We'll now explore the flow within and near the gap formed between the bubble and the rigid boundary. Consecutive snapshots of 6 interesting points in time of the flow ($\gamma = 1.0$, $R_{\text{max}} = 0.6 \text{ mm}$) are shown in Figure 2.7. The figure depicts the approach of the lower bubble wall towards the rigid boundary during the early bubble expansion ($t = 10 \mu\text{s}$), the early stage of jetting ($t = 130 \mu\text{s}$) and just after jet impact ($t = 133 \mu\text{s}$), then close to minimum volume ($t = 140 \mu\text{s}$) and during the torus collapse ($t = 215 \mu\text{s}$). The last frame in figure 2.7 shows the early third re-expansion ($t = 245 \mu\text{s}$). Below each frame in figure 2.7 the wall shear stress is plotted as a function of radial distance in units of 10^3 Pa . Please note that zero crossings of the wall shear stress are stagnation points (or rings). The magnitude of the velocity is color coded. Owing to symmetry the origin $x = 0$ remains a stagnation point. At time $t = 10 \mu\text{s}$ the wall shear stress follows the expected shape similar to a source position near an impermeable wall (Ye and Bull, 2006). During early collapse the interface closest to the wall flattens into an approximately $50 \mu\text{m}$ thin gap, see $t = 130 \mu\text{s}$. In this gap the shear stress increases

FIGURE 2.7: Selected flow maps for 6 different times: (a) the early expansion ($t = 10 \mu\text{s}$); (b) during jet formation ($t = 130 \mu\text{s}$); (c) on jet impact ($t = 133 \mu\text{s}$); (d) early re-expansion ($t = 140 \mu\text{s}$); (e) second collapse ($t = 215 \mu\text{s}$); (f) early third expansion ($t = 245 \mu\text{s}$). The plot below the flow fields is the wall shear stress distribution along the radial direction x , the units are mm in along the horizontal and kPa on the vertical axis. Negative values of the wall shear stress depict flow towards the axis of symmetry while positive value in positive radial direction. The velocity is color coded with a unit of m/s.

with radial distance x and reaches a mild maximum because the radial incoming flow decelerates this gap flow and reflects it upwards. There a stagnation ring is created separating the positive and negative wall shear stresses. Up to this point in time the magnitude of the wall shear stress is comparable with the ones during the spherical bubble expansion, i.e. around 1.0 kPa. A rapid increase of the wall shear stress is seen once the jet pierces with approximately 80 m/s through the lower bubble wall and impacts on the boundary, $t = 133 \mu\text{s}$ in figure 2.7. The shear stress increases from 0 at the stagnation point to the rim of the jet at $x = 80 \mu\text{m}$ to $\tau \approx 100 \text{ kPa}$. Shortly afterwards at $t = 140 \mu\text{s}$ the spreading jet flow dominates and creates a nearly purely outward directed flow with positive wall shear stresses within the field of view. The maximum τ has shifted downstream with a velocity of 19 m/s. Away from the boundary the jet flow is connected to the expanding flow forming two counter clockwise rotating vortex rings, a large scale encompassing a smaller scale vortex ring. The low pressure at their core stabilizes the expanding bubble fragments into an overall toroidal shaped bubble. The last two frames in figure 2.7 depict the second collapse and re-expansion at $t = 215 \mu\text{s}$ and $t = 245 \mu\text{s}$, respectively. During collapse we have a similar competition of the flow along the axis of symmetry (however at a much lower velocity than the previous jet flow) with the inward flow due to the bubble acting as a sink. The stagnation ring is now located further away at $x \approx 640 \mu\text{m}$ at $t = 215 \mu\text{s}$. Yet the toroidal bubble fragments into many pieces, which leads to an even more complex flow pattern at $t = 245 \mu\text{s}$.

2.3.4 Spatio-temporal wall shear stress

Figure 2.8 provides a detailed view on the spatio-temporal dynamics of the wall shear stress for $\gamma = 1.0$. Positive values of the wall shear stress (directed away from the axis of symmetry) are plotted in red, while the stress towards $x = 0$ is color-coded in blue. Because of the large range of values the logarithm to the base 10 is taken of the wall shear stress values before plotting. It is evident that the highest stress occurs during the time of first minimum bubble volume $t \approx 135 \mu\text{s}$ near to the axis of symmetry $x \approx 0.15 \text{ mm}$. During most of the expansion (from $t = 0 \mu\text{s}$ to $t \approx 50 \mu\text{s}$) the stress is positive for the full domain, i.e. when the bubble expands rapidly. However at a later stage of expansion ($t \approx 50 \mu\text{s}$) the stress for large x becomes negative because the expansion is slowed down and the

FIGURE 2.8: The time-space map on the wall shear stress for a standoff distance $\gamma = 1.0$. The first color bar represents the positive wall shear stress (away from the axis of symmetry), while the second one represents the negative wall shear stress. Overlaid is the bubble dynamics as a solid line and the instances shown in figure 2.7. The unit for τ is Pa.

upper bubble wall starts to move towards the boundary driving the liquid at larger distances inwards. Near the axis of symmetry ($x < 0.2$ mm), the stresses decrease with time but remain positive for the whole expansion and collapse cycle. High stresses occur after $t \approx 132 \mu\text{s}$ in the small x region, this is the instant of time when the liquid jet impacts on the boundary and forms a boundary layer while spreading along the surface. The location of this outward spreading flow is nicely captured in figure 2.8 by following the location of maximum stress as a function of time. Initially the velocity is about 19 m/s and drops to 1 m/s at $t = 200 \mu\text{s}$. Moreover negative stress is found ahead of the spreading flow (starts from $x \approx 170 \mu\text{m}$ at $t \approx 135 \mu\text{s}$, moves to $x \approx 420 \mu\text{m}$ at $t \approx 146 \mu\text{s}$), which is due to a reversed flow driven by a high adverse pressure gradient. This flow separation is detailed below with figure 2.9. The stress distribution during the re-expansion and later second collapse of the toroidal bubble differs greatly from the first oscillation cycle. Now a vortex ring has developed which continuously accelerates fluid counter clockwise creating a positive shear stress for the entire second oscillation cycle (also see

FIGURE 2.9: Flow separations during the jet spreading and bubble re-expansion. The radial velocity U_x is color coded, the blue color indicates the reversed flow (flow to the axis of symmetry); white line represents the streamline; the spatial units are in μm . Please note that the offset of the x-axis differs in the frames.

figure 2.7 $t = 140 \mu\text{s}$). During the second bubble collapse at time $\approx 240 \mu\text{s}$ the wall shear stress distribution is more complex and the maximum of the shear stress occurs further away from the axis of symmetry ($x \approx 0.45 \text{ mm}$) due to the toroidal bubble collapse.

Figure 2.9 shows the boundary flow during the spreading of the impacting liquid jet, from $t = 135 \mu\text{s}$ to $t = 146 \mu\text{s}$. Flow separations and the reversed flow is nicely captured by the streamline and the radial velocity contour. In figure 2.9, the radial velocity U_x is color coded, the red color represents the flow directed away from the axis of symmetry while blue color indicates the reversed flow (flow

to the axis of symmetry). White lines are the streamlines. At $t = 135 \mu\text{s}$, a vortex ring is nucleated at $x \approx 180 \mu\text{m}$ due to an adverse pressure caused by the fast flow spreading over the stagnant liquid. As a result the flow close to the boundary is opposite to the main flow resulting in a shear stress direction opposite to the driving jet flow. The vortex ring expands in size and moves outwards with a velocity of around 25 m/s between $t = 135 \mu\text{s}$ to $t = 138 \mu\text{s}$ inducing a maximum negative velocity at $t = 138 \mu\text{s}$ of 10 m/s before it slows down at $t = 139 \mu\text{s}$. The flow from the re-expanding bubble triggers two new vortex rings at $t = 141 \mu\text{s}$ which coalesce into a single ring at $t = 142 \mu\text{s}$ and is transported with a velocity of approx. 20 m/s before it decays at $t = 146 \mu\text{s}$.

2.4 Discussions and conclusion

The present simulations predict a complex spatiotemporal distribution of the wall shear stress during the non-spherical collapse of a bubble near a rigid wall. We have compared the temporal dynamics of the amplitude on the wall shear stress and found qualitative agreement with measurements. In particular that a short lived stress of maximum amplitude is generated just after impact and during the early spreading of the jet, while a vortex ring which is formed during the re-expansion contributes for a longer duration stress at a lower amplitude. The simulations predict stresses of the order of 50 kPa while the experimental results are an order of magnitude lower. The difference may be explained with the limited bandwidth and the calibrations conducted for the shear stress measurements. These were conducted over a range from 0.05 Pa to 10 Pa ([Dijkink and Ohl, 2008](#)). [Maisonhaute et al. \(2002b\)](#) provided a rough yet considerably higher estimate based on their earlier electrochemical measurements of the flow velocity of around 200 m/s at the distance of 40 to 80 nm to the boundary ([Maisonhaute et al., 2002a](#)). They predict that shear stress from oscillating and jetting bubbles driven by ultrasound could provide $25 - 50 \text{ bar}$. In contrast bubbles oscillating only mildly with no or little deformation of their shape may create shear stresses of the order of a few kPa , according to simulations from [Krasovitski and Kimmel \(2004\)](#).

[Chahine et al. \(2016\)](#) have simulated the (inviscid) collapse of a bubble near a movable dirt particle with a focus of the forces generated by the pressure drag on the particle. Already during expansion of the bubble the particle may be

pushed along the surface, while the complex recirculating flow after the bubble collapse may attract or push away the particle depending on the specific configuration. However to understand the particle dynamics better, a more complex fluid-structure-interaction model should be included.

The importance of the flow following the collapse of the bubble has also been acknowledged in the experimental study by [Reuter and Mettin \(2016\)](#). There the strongest cleaning effect was observed for $\gamma \leq 1.1$ and was explained with the interaction of the jet spreading flow with the inward flow during the collapse. This results in a wall bound vortex creating a long lasting exposure of the surface to shear. The present simulations confirm this scenario and are revealing the complex interaction leading to multiple stagnation rings and opposing directions of the shear stress.

Already during the expansion and early collapse of the bubble, the shear stresses are of the order of 1 kPa while their increase to around 100 kPa occurs after the liquid jet impact; a system of vortex ring is formed under the combined effect of the main shear flow and the re-expansion of the toroidal bubble, which stabilizes the flow and thereby slows down the decay of the shear stress.

Starting from the present results it will be interesting to conduct a detailed parameter study to evaluate the effect of bubble size, viscosity, and density. For example, in ultrasound cleaning baths smaller bubbles are found which may not generate strong re-entrant jets ([Kim and Kim, 2014](#)) yet they translate along the surface ([Kim et al., 2009](#); [Zijlstra et al., 2009](#)). Simulations with suitable measurement techniques, which may still have to be developed, will help to understand how bubbles clean.

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Chapter 3

Splitting and jetting of cavitation bubbles in thin gaps

In chapter 2, a bubble *always* jets to the nearby rigid wall and creates shear stress on it. In this chapter, we will present distinct bubble dynamics due to the constriction of an additional rigid wall.

We will study the dynamics of cavitation bubbles and the induced jets in a thin liquid gap bounded by two rigid walls. The bubbles are generated experimentally with a focused laser pulse and are compared to simulations. The gap height H and the distance of the position of bubble nucleation, h , with respect to the nearest wall are varied. The bubble dynamics is recorded at 500,000 frames per second and is compared to simulation results from the compressible volume of fluid solver in OpenFOAM that takes into account viscosity and surface tension. Good agreement of the spatio-temporal bubble dynamics between the experiments and simulations are obtained. The findings are that the parameter space separates three regions with distinct jetting dynamics that are characterized by two dimensionless parameters: The normalized gap height, $\eta = H/R_{max}$, where R_{max} is the spherical equivalent radius of the bubble at maximum expansion and the normalized stand-off distance of the bubble measured from the center of the gap, $\zeta = (H/2 - h)/R_{max}$. The three qualitative distinct jetting behaviours are the *transferred jet* impacting on the distant wall, *double jet* as a result of a bubble splitting and impacting on both walls, and the *directed jet* from a conically-shaped bubble impacting on the nearest wall. The impact velocity of the liquid jets onto

the walls can reach more than ~ 200 m/s and is strong function of the gap height and bubble position. The simulations reveal that the viscous boundary layers effect the bubble splitting and therefore the directions of jetting. Additionally, we found that with increasing length of the thin gap up the bubble oscillation period increases, however only up to a maximum duration.

3.1 Introduction

Cavitation bubble and the induced jet dynamics near a single solid boundary have been studied in great detail over the last 5 decades (Plesset and Chapman, 1971; Lauterborn and Bolle, 1975; Zhang et al., 1993; Supponen et al., 2016), however the dynamics is still unclear when it comes to a narrow gap, a configuration now is attracting increasing attention in cleaning (Gonzalez-Avila et al., 2018), electrical discharge machining (Shervani-Tabar et al., 2006) and biological applications (Hsiao et al., 2013; Yuan et al., 2015; Mohammadzadeh et al., 2017). With the constriction of an additional solid boundary, a bubble oscillating inside a narrow gap with two rigid walls experiences a more complex dynamics. The bubble is found to be split into two daughter bubbles with two jets impacting on both walls when the gap is sufficiently small and the bubble is nucleated near the gap center (Chahine, 1982; Kucherenko and Shamko, 1986; Ishida et al., 2001; Azam et al., 2013), this phenomenon is further reproduced by simulations using boundary integral method (BIM) (Zhang et al., 2010) and volume of fluid (VOF) method in a finite volume method (FVM) framework (Han et al., 2018). For a bubble created very close to one wall in a gap with a height comparable to the bubble's maximum radius, it may translate to and induce a jet impacting on the other wall during the collapse. To the best of our knowledge, this phenomenon was first observed by Gonzalez-Avila et al. (2011) and later reproduced by Quah et al. (2018). Yet a thorough explanation is still lacking as the boundary flow details leading to the bubble deformation is difficult to capture in the experiments of Gonzalez-Avila et al. (2011) and Quah et al. (2018). In a large gap, an off-centered nucleated bubble acquires a quasi-conical shape and collapses on the closer wall (Kucherenko and Shamko, 1986; Gonzalez-Avila et al., 2011). Gonzalez-Avila et al. (2011) demonstrated all three types of jetting in a single study. With laser induced cavitation bubbles nucleated *always* very close to one of the walls, they found the collapse

scenario depends on a dimensionless gap height, which is the gap height normalized by the maximum horizontal bubble radius. However, the details at the final stage was not resolved due to spatial and temporal limitations in the experiments. The effect of the initial position of the bubble nucleation was not considered, too. Here, we complete this first study that additionally varies the position and accounts in the simulations for viscosity.

Due to a very fast dynamics and microscopic scale of the bubble when splitting and pinch off occurs, numerical studies are carried out to obtain these details of the flow that are difficult to obtain experimentally. Previously many of the simulations used the numerically efficient boundary element method ([Ishida et al., 2001](#); [Shervani-Tabar et al., 2006](#); [Zhang et al., 2010](#); [Gonzalez-Avila et al., 2011](#); [Hsiao et al., 2013](#)), which is able to predict the splitting of the bubble and its conical shape while neglecting viscosity on the walls. Eventually, viscosity becomes important in the unsteady boundary layer that affects the bubble interface once it is close to the wall. Viscosity had been considered in an earlier work dealing with the bubble oscillations in a very narrow gap where [Mohammadzadeh et al. \(2017\)](#) used a commercial VOF to solve the shear flow induced by a pancake shaped bubble oscillating from the center of the gap. They found even for non-jetting cavitation bubbles boundary layer separation caused by an adverse pressure gradient and strong shear at cross-over region between the boundary layer and the bulk. [Han et al. \(2018\)](#) simulated a smaller cavitation bubble between two parallel solid plates using OpenFOAM. Here, the bubble creates wall normal and annular jets and eventually splits. Their work focuses on a few cases where the bubble position with respect to the wall is varied. In contrast [Gonzalez-Avila et al. \(2018\)](#) conducted a systematic study on the channel height, where the bubble position was fixed and close to one of the two walls. That latter study used a compressible VOF solver again with the open source package OpenFOAM. Simulation and experiments show excellent agreement for the bubble shape, center of mass motion, jet direction and jet speed.

The present chapter connects the previous two works from [Gonzalez-Avila et al. \(2018\)](#) and [Han et al. \(2018\)](#) in a parametric study that accounts for the effect of the gap height and the bubble position within the gap. It does not repeat previous work on on mostly planar in very narrow gaps such as in [Zwaan et al. \(2007\)](#) and [Quinto-Su and Ohl \(2009\)](#). Additionally we account for a parameter which has not received much attention, namely the length of the gap. The work compares

FIGURE 3.1: (a) Experimental set-up (top view). (b) 3D sketch of the geometry, the bubble is nucleated at the center of yz -plane. (c) Sketch of the problem and parameters. The small circle at the axis is the nucleated bubble, the hat shape curve represents the bubble at maximum expansion.

the bubble shapes obtained from the experiment using high speed photography at 500,000 frames per second with axisymmetric simulations that account for viscosity, nonlinear compressibility and surface tension. In this chapter we start with a description of the experimental and numerical methodologies, then present their results in section 3.3, and conclude with a discussion.

3.2 Methodology

Here we combine the experimental results from high speed photography and numerical simulation to investigate the bubble and induced jet dynamics. Individual bubbles are generated by focusing a laser pulse into a narrow gap whose height is adjustable, see figure 3.1(a) for the top view of the experimental set-up and figure 3.1(b) for a 3D view from the camera side. To nucleate the bubble, a Q-switched Nd:YAG laser pulse (532 nm wavelength, 6 ns duration) is expanded and collimated with one plano-concave and one plano-convex lens and then focused into the gap with a $10\times$ microscope objective (Olympus UPlanFL N10X 0.30 NA). The narrow gap is formed by the left boundary of an acrylic cuvette and a movable glass

wall parallel to it, which is used to vary the gap height. The position of the bubble nucleation site can be controlled by moving the whole gap along the laser beam direction, thus we can adjust the parameter h shown in figure 3.1(c). A high speed camera (Shimadzu Hypervision 1 Mfps maximum) is used to record the bubble dynamics from the side view at 500,000 frames per second. The scene is recorded with mildly diffused illumination from a flash lamp. The resolution is $18\text{ }\mu\text{m}$ per pixel. The camera, pulsed laser and flash are synchronized with a delay pulse generator (Quantum, 9520 series).

As shown in figure 3.1(b), in the experiment the flow domain is a rectangle with a size of $H \times D \times D$ (here $D = 10\text{ mm}$ is the length of the square base of the cuvette, H is varied between 0.6 mm to 2.7 mm), its top and bottom rigid boundaries form the gap. Other boundaries are two rigid walls of the cuvette (front and back walls along y direction) and two submerged outlets (left and right boundaries along the z direction). In each test we ensure the bubble is nucleated at the center of the yz -plane. The two rigid walls along y -direction are far away from the bubble, which is expected to have negligible wall effect on the bubble dynamics when the gaps are presented. Thus we can neglect wall effects from the far away boundaries and by simplifying the domain as a cylinder as axisymmetric. The view presented in the simulations and experiments is that of the xz -plane, see figure 3.1(c). The simulations start with a spherical bubble with an initial radius R_0 at an uniform internal overpressure P_0 positioned at a distance h to the lower boundary of a submerged liquid gap, which has a height of H and a radius (or length) of L ($L = D/2$). The bubble then expands to a maximum spherical equivalent radius R_{max} , which is the radius of a sphere with the same volume. For a bubble near a single rigid boundary, its dynamics can be characterized by the stand-off distance $\gamma = h/R_{max}$ considering the secondary Bjerknes force (Plesset and Chapman, 1971; Lauterborn and Bolle, 1975; Blake and Gibson, 1987). Similarly, for a bubble inside a gap, the combined secondary Bjerknes force from two opposite walls can be simply described with the difference of two standoff distances $\gamma_1 = h/R_{max}$ and $\gamma_2 = (H - h)/R_{max}$. For convenience here we introduce the normalized off-center distance $\zeta = (H/2 - h)/R_{max}$, which is also the half of the difference of the two standoff distances. The bubble collapse dynamics is also affected by the normalized gap height $\eta = H/R_{max}$ and the normalized gap radius $l = L/R_{max}$, which together describe the geometric confinement and inertia. The stand-off distance here refers to the closest wall $\gamma = h/R_{max} = \eta/2 - \zeta$.

To simulate the bubble dynamics and induced liquid jet, we assume the bubble as non-condensable gas following a polytropic equation of state (EOS) and consider the surrounding liquid as a Tait-compressible liquid (Macdonald, 1969), similar to Koukouvini et al. (2018). The nonlinear compressibility is computed as $\psi = D\rho/Dp$ based on the EOS, where ρ and p are the density and pressure, respectively. Using the volume of fluid (VOF) method for a compressible two-phase flow problem, the volume fraction of the liquid phase α can be solved with the transport equation (Miller et al., 2013; Koch et al., 2016; Zeng et al., 2018b):

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) + \nabla \cdot (\alpha(1 - \alpha) \mathbf{U}_r) = \alpha(1 - \alpha) \left(\frac{\psi_g}{\rho_g} - \frac{\psi_l}{\rho_l} \right) \frac{Dp}{Dt} + \alpha \nabla \cdot \mathbf{u} \quad , \quad (3.1)$$

where $\mathbf{u}$ is the velocity field, t is time, the subscript l and g represent the liquid and the gas phase respectively. $\mathbf{U}_r$ is the relative velocity between two phases and acts as an artificial compressible term ensuring a sharp interface (Rusche, 2003).

The compressible Navier–Stokes (NS) equations and the above transport equation are solved together in an axisymmetric domain to reproduce the flow field in OpenFOAM (Weller et al., 1998). Compressibility of both gas and liquid phases, surface tension and viscosity are included in the model. In particular the viscosity is essential for resolving the flow in the vicinity of the boundary layer. A locally and recursively refined mesh is used, which has a cell size of approximately $1.5 \mu\text{m}$ wide in the area where the bubble is located. The boundary conditions are no-slip for both walls and the ambient pressure $P_\infty = 101325 \text{ Pa}$ at the exit of the gap. The "zeroGradient" boundary condition is applied for the rest boundaries. More details about the simulations can be found in section 2.2.2 and Zeng et al. (2018a,b).

The thin liquid layer formed between the bubble and wall becomes thinner as the nucleation moves closer to the wall. The bubble will dewet the lower wall if the liquid layer has drained. However, the physics of the process is still unclear and not implemented in the current model. The current numerical set-up can capture a liquid film with a minimum thickness of approximately $1.5 \mu\text{m}$, i.e. the cell height. Once the film thickness is smaller than the cell height, the VOF method leads to a numerical dewetting. And the boundary condition "zerogradient" for α_l will set a static contact angle $\theta_0 = 90^\circ$.

FIGURE 3.2: Bubble dynamics under a condition of $H = 840 \mu\text{m}$ and $h = 228 \mu\text{m}$. In the experiment (a)&(c), $R_{max}=553 \mu\text{m}$, $\eta = 1.52 \pm 0.08$, $\zeta = 0.35 \pm 0.04$, and $\gamma = 0.41 \pm 0.03$. In the simulation (b)&(d), $R_{max}=526 \mu\text{m}$, $\eta=1.60$, $\zeta=0.37$, $\gamma = 0.43$, and the initial pressure $P_0=1000$ bar. The scale bar represents $500 \mu\text{m}$. Time in μs .

3.3 Results

3.3.1 Bubble dynamics

In this section, we present different scenarios of the bubble dynamics from both experiment and simulation and analyze the flow details during jet formation from simulations. In both experiment and simulation the radius of the gap is kept to be $L = 5 \text{ mm}$ while the height of the gap and the bubble position are varied. To reproduce the experiment with simulation, while keeping a constant initial bubble radius $R_0 = 50 \mu\text{m}$, we set the gap height and nucleated position as the same as the experiment and then adjust the initial bubble pressure to meet the experimental period of oscillation.

Figure 3.2 shows a comparison between experimental results and numerical simulations for selected frames of the bubble dynamics. Here, $H = 840 \mu\text{m}$ and $h = 228 \mu\text{m}$. The images from experiment are the photographic projection of the bubble while the simulation result shows the cut plane of the fluid domain. The top and bottom of each frame are the rigid boundaries of the gap. In the experiment, the nucleation is not perfectly spherical but elongated and the bubble generated may initially be composed of multiple merging bubbles. However, this process has little effect on the later bubble dynamics. The computed bubble has a slightly smaller maximum radius $R_{max}=526 \mu\text{m}$ than the experimental bubble

$R_{max}=553\text{ }\mu\text{m}$. Yet, we obtain good agreement between simulation and experiment on the bubble dynamics.

The bubble is nucleated at $t = 0$, in the experiment the initial state is a plasma created by the focused pulsed laser. The complexity is ignored by starting the simulation the bubble with an already present spherical nucleus of radius $R_0 = 50\text{ }\mu\text{m}$ and internal pressure $p_0 = 1000\text{ bar}$. As the bubble grows and approaches the walls, the lower interface is hindered by the immobile viscous boundary layer fluid thus becomes flat, which makes the bubble grow horizontally from $t = 10\text{ }\mu\text{s}$ and turns into a top-hat shape at the maximum expansion around $t = 100\text{ }\mu\text{s}$. The bubble almost occupies the whole gap in the vertical direction with its lower interface seemingly in contact with the lower wall, it shrinks only radially during the collapse since there is less freedom to let the liquid flow from the top and the bottom. During the collapse, accelerated by a curved surface, the bottom part of the bubble shrinks faster and moves upwards later, developing a fast jet impinging the upper boundary. We name this jet *transferred jet*, as it jets towards the distant wall. From figure 3.2, we estimate an average impact velocity of 210 m/s in the experiment and 280 m/s in the simulation. Moreover, before the jet is formed, we observe a complex annular undulation on the bubble surface both in experiment and simulation.

Next, we look at the flow field details to study the jet formation process. The computed flow field and pressure distribution during the jet formation are depicted in figure 3.3. The left part of each frame is the pressure field coded in colour while the right part shows the velocity field. For this the velocity magnitude is indicated by the colour and the flow direction by arrows. During the entire oscillation cycle the pressure within the bubble is homogeneous.

Already at the late stage of expansion ($t = 87\text{ }\mu\text{s}$), the liquid far from the bubble is accelerated back towards the axis of symmetry by the adverse pressure gradient. Moreover, at the same time the bubble is still expanding and pushing the nearby liquid outwards. Near the walls this expanding flow is deflected by the incoming flow, resulting in a downward flow from the upper wall and an upward flow from the lower wall. The downward flow and the upward flow collide and together with the incoming flow an annular jet flow towards the bubble is formed. This flow forms a dimpled annular ring on the bubble. The dimple serves as an initial perturbation and during the collapse develops into a 'neck' on the bubble, see the

FIGURE 3.3: Pressure distribution (left part in each frame) and velocity field (right part in each frame) during the jet formation for the case in figure 3.2. The arrows in the velocity field only indicate the directions. The colour bar range is updated every frame.

central part of the bubble at $t = 170 \mu s$, $t = 174 \mu s$, and $t = 174.5 \mu s$ in figure 3.3. During the expansion, as the lower interface of the bubble approaches the wall, it flattens into a thin gap (the minimum gap height is predicted from the simulations as $5 \mu m$). The liquid inside the gap flows outwards, lifting up the lower part of the bubble with a pronounced curvature (see $t = 87 \mu s$ the bubble edge). During the early stage of collapse, this part is exposed to the highest pressure gradient and therefore experiencing the highest acceleration. Very close to the wall, the bubble is hindered by viscosity. As a result, a wavy shape with high curvature close to the wall becomes visible at $t = 150 \mu s$ that shrinks inwards with an velocity approximately $10 m/s$. Later an annular jet at the edge and develops up to $90 m/s$ at $t = 174 \mu s$. Upon the collision of this annular jet, an upward jet pushing the bubble upwards is released (see $t = 174.5 \mu s$). The upward jet keeps developing, penetrates the bubble and impacts on the upper wall.

Figure 3.4 compares the experimental and simulated bubble dynamics for $H =$

FIGURE 3.4: Bubble dynamics under a condition of $H=1100 \mu\text{m}$ and $h=513 \mu\text{m}$. In the experiment (a)&(c), $R_{max}=566 \mu\text{m}$, $\eta = 1.94 \pm 0.09$, $\zeta = 0.07 \pm 0.03$, $\gamma = 0.90 \pm 0.03$; in the simulation (b)&(d), $R_{max}=545 \mu\text{m}$, $\eta=2.02$, $\zeta=0.07$, $\gamma = 0.94$, and the initial pressure $P_0=1100 \text{ bar}$. The scale bar represents $500 \mu\text{m}$. Time in μs .

$1100 \mu\text{m}$ and $h = 513 \mu\text{m}$. In the simulation the bubble expands to a maximum radius $R_{max} = 545 \mu\text{m}$ with an initial pressure of 1100 bar while in the experiment the bubble reaches a maximum radius $R_{max} = 566 \mu\text{m}$. The computed dimensionless parameters are $\eta = 2.02$ and $\zeta = 0.07$, which are very close to the experimental parameter $\eta = 1.94$ and $\zeta = 0.07$. Nevertheless, the simulation matches the experiment very well.

In this case the bubble is nucleated close to the center of the gap and expands spherically at the early stage. The upper and lower interface flatten as they approach the walls due to the cushioning effect from the liquid boundary layer. The bubble acquires a cylindrical shape at maximum expansion and later during collapse a dimple is formed near the bubble's equator due to the faster flow here, see $t = 120 \mu\text{s}$. The dimple ring turns into an annular jet flowing inwards during the collapse and produces a pinch off when the annular jet impacts the axis of symmetry. The bubble splits into two daughter bubbles at $t = 158 \mu\text{s}$, resulting in two jets impinging on both walls at $t = 162 \mu\text{s}$. We name this kind of jet *double jet* as there are two jets generated which impact on both walls.

The details of the jet formation for the case presented in figure 3.4 are shown in figure 3.5. Similar to figure 3.3, the flow from the walls is reflected by the incoming center flow during the late expansion phase of the bubble (see $t = 85 \mu\text{s}$), forming

FIGURE 3.5: Pressure distribution (left part in each frame) and velocity field (right part in each frame) during the jet formation for case in figure 4. The arrows in the velocity field only indicate the directions. The colour bar range is updated every frame.

a pair of counter-rotating vortices near the upper wall and the lower wall. These two vortex rings which are of similar strengths collide and focus an annular jet impacting the central part of the bubble during the later collapse, see $t = 100 \mu\text{s}$, $t = 120 \mu\text{s}$, $t = 150 \mu\text{s}$, and $t = 157.5 \mu\text{s}$ in figure 3.5. Also since the bubble expands very close to both walls, the viscous boundary layers obviate the bubble shrinking from the top and low interface. As a result, only one annular jet is generated resulting into a single split of the bubble into two similar sized shapes. Upon closure of the central bubble, an approximately spherical pressure wave is transmitted into the liquid.

In figure 3.6 we present a comparison of the bubble dynamics for $H = 1055 \mu\text{m}$ and $h = 295 \mu\text{m}$. The bubble reaches a maximum radius $R_{max} = 536 \mu\text{m}$ in the

FIGURE 3.6: Bubble dynamics at a condition of $H=1055 \mu\text{m}$ and $h=295 \mu\text{m}$. In the experiment (a)&(c), $R_{max}=536 \mu\text{m}$, $\eta = 1.97 \pm 0.09$, $\zeta = 0.43 \pm 0.05$, and $\gamma = 0.55 \pm 0.03$; in the simulation (b)&(d), $R_{max}=517 \mu\text{m}$, $\eta=2.04$, $\zeta=0.45$, $\gamma = 0.57$, and the initial pressure $P_0=1100$ bar. The scale bar represents $500 \mu\text{m}$. Time in μs .

experiment thus we obtain $\eta=1.97$, and $\zeta=0.43$ while the computed bubble has a maximum radius $R_{max} = 517 \mu\text{m}$ with an initial pressure of $P_0=1100$ bar, leading to $\eta=2.04$ and $\zeta=0.45$. Overall we see again good agreement between simulation and experiment.

The bubble expands to a hemispherical shape at its maximum expansion $t = 80 \mu\text{s}$ with its bottom interface seemingly touching the lower wall; when zoomed in we find a thin liquid layer of about $5 \mu\text{m}$ between the bubble and the wall. The shrinkage of the bubble starts from the side where has a higher pressure gradient at the maximum expansion. Until the incoming annular flow shapes the bubble into an elongated head at $t = 140 \mu\text{s}$, the top interface start to move down. The bubble experiences a similar shape as the conical shape bubble mentioned in the literature (Tomita and Shima, 1986; Chahine, 1982). The annular and downwards flow meet on top of the bubble tip where has a small radius of curvature (see $t = 152 \mu\text{s}$), leading to a fast jet impacting on the lower wall. We estimate from figure 3.6 the average velocities are approximately 100 m/s in the experiment and 140 m/s in the simulation, which are higher than the impacting jet from the bubble near single solid boundary which are less than 100 m/s for $0.4 \leq \gamma \leq 1.0$ (Philipp and Lauterborn, 1998). Yet the tip velocity of the jet in figure 3.6 can be even higher. We name this induced jet as *directed jet* since it impacts directly on the closer boundary to its nucleation.

FIGURE 3.7: pressure distribution (left part in each frame) and velocity field (right part in each frame) during the jet formation for case in figure 6. The arrows in the velocity field only indicate the directions. The colour bar range is updated every frame.

FIGURE 3.8: Zoom in details of figure 3.7 on the tip development.

FIGURE 3.9: Movement of the jet tip under various mesh spacing during the jet formation. The black dash line represents a slop of 580 m/s .

Let's look at the flow detail leading to the jet development in figure 3.7. Again, at the late expansion stage $t = 75 \mu\text{s}$ we see the flow in the middle of the channel and close to the bubble is reflected as an inward flow near the middle of the gap, which is also the position where the shrinkage of the bubble starts (see $t = 100 \mu\text{s}$). During the early collapse, the bubble shrinks fastest by the annular flow while the upper part moves inwards much slower, see $t = 120 \mu\text{s}$ in figure 3.7. The top interface starts to move downwards with an appreciable velocity only after the bubble obtains the elongated tip. The part acts as a spherical sink and amplifies the pressure.

From this fast and high pressure build up at the top of the tip a fast liquid jet emerges. The curved interface and rapid pressure increase resembles the situation in Antkowiak et al. (2007) where a jet is formed by a pressure impulse acting on curved liquid gas interface. What is the cause of the sudden pressure build-up? The flow from the top and the side meet on the axis of symmetry. Upon impact a high pressure region is created resembling the findings in Lechner et al. (2019). The liquid-liquid impact results in a water hammer pressure between the frames $t = 152.4 \mu\text{s}$ and $t = 154 \mu\text{s}$ of figure 3.7. This situation resembles the sudden acceleration of a curved interface in Antkowiak et al. (2007). Yet the pressure on the tip of the bubble is radiated into the whole liquid volume in particular along and downwards the liquid-gas interface of the cylindrical upper bubble with a conical

tip. This geometry is similar to a shaped charge (Birkhoff et al., 1948), where instead of a detonation wave a pressure wave travels over a conical boundary between the explosive/liner and surrounding air. To analyze the situation in closer detail we enlarge the temporal scale between $t = 152.4 \mu\text{s}$ and $t = 154 \mu\text{s}$ of figure 3.7 in figure 3.8.

The flow details in figure 3.8 clearly shows that the top surface acquires a wavy shape after the impact of the annular flow and the vertical flow at $t = 152.6 \mu\text{s}$. High pressure up to 300 bar arises above the top surface and accelerate the concave surface to develop into a fast jet.

To capture the thin jet and demonstrate its robustness to the specific mesh, we locally refine the mesh from $1.5 \mu\text{m}$ to a grid spacing of $0.8 \mu\text{m}$, $0.4 \mu\text{m}$ and $0.2 \mu\text{m}$. A comparison of the jet tip development is shown in figure 3.9. The tip movements in all mesh resolutions show a transition in slope, i.e. the tip velocity, around $t \approx 152.6 \mu\text{s}$, when the conical shape surface occurs (see figure 3.8). The tip velocity for $t < 152.6 \mu\text{s}$ is found approximately 160 m/s caused by the shrinking bubble while the tip velocities converge to approximately 580 m/s for $t > 152.6 \mu\text{s}$, showing a remarkable increase in jet velocity due to the topological enhancement. This fast thin jet is found to occur near the transition from a *double jet* to a *directed jet*, where the vertical flow velocity and the annular flow velocity are comparable and thus lead to this unexpected phenomenon.

3.3.2 Mechanism of the jet formation and jet regimes

Based on the above discussion, we here conclude the jet formation is affected by three main flow features. Firstly, due to the no-slip gap walls and viscosity, the expanding flow near the boundaries is reflected by the pressure gradient guided recoil flow at the late expansion. The flow coming from the lower and the upper walls merges and develops as an annular flow impacting the middle part of the bubble during the late stage of the expansion. The bubble deforms slightly under this flow. Yet this minor deformation grows during the collapse and helps to develop an annular flow towards the axis of symmetry. We refer to this flow as "splitting flow" since it tends to split the bubble. Once formed, the flow is driven by inertia. This flow is seen in all three scenarios and is determined by the dimensionless gap height η and the off-center distance of the bubble ζ . Secondly, when the bubble bottom is very

FIGURE 3.10: Sketches of different jet regimes. (a) The transferred jet. (b) The double jet. (c) The directed jet. (d) Dewetting, the bubble touches the lower wall. The small circle, solid curve, and dash curve represent the bubble nucleation, bubble at maximum expansion, and jet formation, respectively. The arrow shows the dominated flow direction.

close to the boundary, e.g. figure 3.2 and figure 3.6, the lower edge of the bubble becomes sharp and develops into an annular jet near the boundary as the bubble shrinks. This edge annular jet leads to a pinch off from the boundary layer and drives the bubble upwards, generating a transferred jet as seen in figure 3.2. The third one is the vertical collapsing flow from the top interface of the bubble, which happens when the top interface is sufficiently far from the upper wall to have less viscous effect on the fluid near the top surface. The formation of the transferred jet, double jet, and the directed jet are dominated by the edge annular flow, splitting flow and the vertical collapsing flow, respectively. The jetting regimes and the respective dominated flows are sketched in figure 3.10. Theoretically, the co-effect of these three types of flow can be described with a function combining η and ζ . However, due to the high non-linearity involving viscosity and complex geometry, we intend not to give a detailed expression on the flows but rather summarize the jet distributions with a phase diagram consisting of η and ζ based on the experimental data and simulation results.

Next, we look at how the two dimensionless parameters η and ζ affect the formation of the various jets with both experimental and numerical data. Figure 3.11 summarizes the occurrence of various induced jets on a parameter map covered by η and ζ . Filled symbols are the computed results while the open symbols indicates the results from the experiment. The square, circle, and diamond symbols represent the formation of transferred jet, double jet, and directed jet, respectively. In the experiment the equivalent bubble radius varies from $R_{max} \approx 520 \mu\text{m}$ to

FIGURE 3.11: Phase diagram for various induced jets for a gap length of $L = 5 \text{ mm}$. Experimental data and computed results are plotted as fill and open symbols respectively. Under each catalogue the square, circle, and diamond symbols represents the transferred jet, the double jet, and the directed jet, respectively. The dewetting cases from simulation are plotted as filled cyan triangles. Black line is $\zeta = 0$ while the magenta dashed and dotted lines are the standoff distance from the closer wall $\gamma = 0.3$ and $\gamma = 0.4$. The black dash lines are drawn for guiding the transitions.

$R_{max} \approx 580 \mu\text{m}$ while in the simulation we keep the nucleated radius $R_0 = 50 \mu\text{m}$ and pressure $P_0 = 1250 \text{ bar}$ of the bubble to be the same and obtain the maximum bubble radius between $550 \mu\text{m}$ to $580 \mu\text{m}$. An artificial dewetting of the lower wall by the bubble is found in the simulation when the bubble is nucleated very close to the wall. It is the result of insufficient numerical resolution as the physics of the process is not implemented in the simulations. These unreliable points are shown as triangle symbols in the map. The experimental setup does not allow distinguishing between a wetted and dewetted boundaries either. For $\eta < 2.5$, the dewetting occurs when the standoff distance of the bubble to a nearer wall $\gamma < 0.3$. Also, it is seen from figure 3.11 that the gap with higher height η has a larger transition for the dewetting due to more inertia. We only consider the non-dewetting bubble dynamics in this study due to the unsolved complicated contact line dynamics problem in our simulation.

The phase diagram in figure 3.11 is seen nicely divided into three regions, see the black dashed lines for reference. The transferred jets happen in a finite condition when $\eta < 1.8$, while the double jet and the directed jets occur for $1.3 < \eta < 2.8$ and $\eta > 1.8$, respectively. We now discuss the transitions between the regimes. These transitions mark the dependence of the off-center distance ζ on the dimensionless gap height η . The first transition ζ_{TD} is between the transferred jet and the double jet. Both experimental and simulated results show that ζ_{TD} increases along with η with a slope about 1, from $\eta \approx 1.3$ to $\eta \approx 1.8$. The reason for the changing of ζ_{TD} is that, as the gap height increases, the bubble needs to be nucleated closer to the wall so that its lower interface can be still highly confined by the viscous liquid layer to produce a sharp edge and a strong edge flow during the collapse, leading to a transferred jet. The second transition ζ_{DD} indicates the boundary between the double jet and the directed jet, which decreases along with η with a gradient about 0.4 from $\eta \approx 1.8$ to $\eta \approx 2.8$. It can be simply explained that, as η increases the splitting flow becomes weaker, to ensure the occurrences of the double jet the bubble nucleation site needs to be moved closer to the center of the gap, where the splitting flow is concentrated mostly by inertia. For $\eta > 3.0$, the splitting flow becomes less important and the vertically collapsing flow dominates the jet formation, thus only the directed jet is seen.

3.3.3 Influence of η and ζ on jet dynamics

In this section we focus on the impact of the bubble position and gap height on the jet dynamics especially on the jet velocity based on the simulation results. The computed jet velocities are also compared with experimental data. For the jet velocity, we account for the average velocity from the formation until a continuous jet with a radius larger than the computed cell length ($1.5, \mu\text{m}$) hits the wall. Here, we define the jet formation to start at the detachment for the transferred jet and the splitting moment for the double jet, and when the top interface reaches a constant velocity before it penetrates the lower interface for the directed jet. Although the average velocity underestimates the tip velocity, it shows a overall strength of the jet. The uncertainty in the experiment is calculated as $\Delta V/V = ((\Delta S/S)^2 + (\Delta T/T)^2)^{1/2}$, where the V is the calculated jet velocity, S and T are the travel distance and the travel time interval of the jet from the formation to the impact, $\Delta S = 18 \mu\text{m}$ and $\Delta T = 1 \mu\text{s}$ are the spatial and temporal resolutions,

FIGURE 3.12: Comparison of the jet dynamics between $\zeta = 0.03$ ((a)&(c)) and $\zeta = 0.14$ ((b)&(d)) under $\eta=1.26$. The gap height $H=700 \mu\text{m}$. Time in μs .

respectively. From figure 3.12 to figure 3.15, we show how the jet dynamics varies with nucleated position for four selected gap heights η with simulations under the same initial bubble radius $R_0 = 50 \mu\text{m}$ and pressure $P_0 = 1250 \text{ bar}$. The bubbles under the same gap height have nearly undistinguished difference on the temporal equivalent radius evolution. A summery of the jet velocity for four η is plotted in figure 3.16, in which the impact velocity on the lower wall is defined as positive and plotted in red while the impact velocity on the upper wall is negative and in blue. The experimental data are shown in filled symbols while the computed result are plotted with open symbols.

Figure 3.12 shows the comparison on jet dynamics between $\zeta = 0.03$ and $\zeta = 0.14$ under a gap height of $\eta = 1.26$, where only a transferred jet is produced referring to figure 3.11. At $\zeta = 0.03$, the bubble expands to an approximately rectangle at $t = 100 \mu\text{s}$ and deforms to an elongate shape before it detaches from the lower wall at $t = 201 \mu\text{s}$. While for the bubble at $\zeta = 0.14$, it undergoes a more asymmetric shape at the maximum expansion and becomes an inverted cone before the detachment at $t = 195 \mu\text{s}$. The induced jet velocities are about 250 m/s for $\zeta = 0.03$ and 140 m/s for $\zeta = 0.14$. This is due to two main reasons: for $\zeta = 0.03$, the jet is starting at $t = 201 \mu\text{s}$ which is at a later stage in the collapse comparing to $t = 195 \mu\text{s}$ for $\zeta = 0.14$ thus the collapsing bubble is under a higher acceleration; also for $\zeta = 0.03$ a shape with higher curvature at the jet formation helps to develop faster jet. These also explain the impact velocity on the upper wall decrease to a limit as ζ increases in figure 3.16(a).

With the gap height increasing to $\eta = 1.43$, figure 3.13 compares the jet dynamics at $\zeta = 0.09, 0.18$, and 0.31 . For $\zeta = 0.09$, the bubble splits at $190 \mu\text{s}$ and produces

FIGURE 3.13: Comparison of the jet dynamics among $\zeta = 0.09$ ((a)&(d)), $\zeta = 0.18$ ((b)&(e)), and $\zeta = 0.31$ ((c)&(f)) under $\eta=1.43$. The gap height $H=800\mu\text{m}$. Time in μs .

a downward jet impacting the lower wall with a velocity about 190 m/s and an impacting upward jet at about 160 m/s. As ζ increases to 0.18, the bubble splits at $t = 191\mu\text{s}$ however with both daughter bubbles all move to the upper wall. The velocity of this transferred jet can be up to 260 m/s due to the splitting enhancement. With an even higher ζ to 0.31, a transferred jet with a lower velocity about 200 m/s is found meanwhile the bubble collapses without splitting. The velocity versus ζ is plotted in figure 3.16(b). The double jet are formed at $0 < \zeta < 0.15$, in this regime the jet velocities (absolute values) on both walls increase along with ζ . The transferred jet occurs when $\zeta > 1.8$ and its evolution is similar to figure 3.16(a).

For a higher gap height $\eta = 1.93$, the bubble moves from the double jet regime to the directed jet regime as ζ increases. We compare the jet dynamics at $\zeta = 0.04$, 0.31, and 0.57 in figure 3.14 and plot the velocity in figure 3.16(c). For $\zeta = 0.04$, the bubble attaches to both walls and splits into two daughter bubbles with approximately equal size. This is also seen in figure 3.16(c) that the impacting jet velocity on both walls are comparable at $0 < \zeta < 0.22$ with little variation. For $\zeta = 0.31$, the bubble only attaches to the lower wall and releases a smaller daughter bubble moving upwards at the splitting. This bubble impacts the upper wall at a later time with a lower velocity compared to the attached daughter bubble. Similar dynamics happens at $0.27 < \zeta < 0.45$, as seen in figure 3.16(c). For $\zeta = 0.57$, the

FIGURE 3.14: Comparison of the jet dynamics among $\zeta = 0.04$ ((a)&(d)), $\zeta = 0.31$ ((b)&(e)), and $\zeta = 0.57$ ((c)&(f)) under $\eta=1.93$. The gap height $H=1100\mu\text{m}$. Time in μs .

bubble collapses onto the lower wall and induces a directed jet, which decreases as ζ increases for $\zeta > 0.45$.

Increasing the gap height η to 3.10 makes the bubble collapses only onto the lower wall with a directed jet. Two selected induced jets dynamics at $\zeta = 0.34$ and $\zeta = 0.86$ are compared in figure 3.15. At $\zeta = 0.34$ the bubble expands to a sphere at $t = 75\mu\text{s}$ and shrinks acquiring a dripping water droplet shape before the jet formation at $t = 142\mu\text{s}$. The sharp top interface pierces through the bubble and develops a directed jet impacting the wall with a velocity up to 220 m/s at $t = 146\mu\text{s}$. At a higher ζ to 0.86, the bubble expands to a hemisphere at $t = 75\mu\text{s}$ and collapses to a cone at $t = 135\mu\text{s}$, developing a jet with velocity about 90 m/s impacting at $t = 145\mu\text{s}$. In figure 3.16(d), the impact velocity for $0 < \zeta < 0.3$ is from the re-entrant vapor jet where bubble is far away from the boundary. For $\zeta > 0.34$, the jet velocity decreases with ζ as the bubble is closer to the wall.

FIGURE 3.15: Comparison of the jet dynamics between $\zeta = 0.34$ ((a)&(c)) and $\zeta = 0.86$ ((b)&(d)) under $\eta=3.10$. The gap height $H=1800 \mu\text{m}$. Time in μs .

FIGURE 3.16: Jet impact velocities as a function of ζ for selected η -values. (a) $\eta = 1.26$; (b) $\eta = 1.43$; (c) $\eta = 1.93$; (d) $\eta = 3.10$. The positive direction is downwards to the closest boundary. The blue symbols indicate an upward jet impacting onto the upper and distant wall, thus negative; the red symbols represent the downward jets impacting on the lower wall (positive value). Open symbols are from experiments and the filled symbols from simulations.

FIGURE 3.17: (a) Comparison of the equivalent spherical bubble radius under various gap radius from simulations. The gap height $H = 0.7\text{ mm}$ and the distance to the closer wall $h = 250\text{ }\mu\text{m}$ is kept constant. The bubble dynamics converges at $L = 180\text{ mm}$. (b) The expansion time and collapse time versus gap radius. Note that the time is normalized by the Rayleigh collapse time.

FIGURE 3.18: The bubble interface. (a) The late expansion $t = 100\text{ }\mu\text{s}$, the bubble shape converges at $L = 80\text{ mm}$. (b) The late collapse $t = 200\text{ }\mu\text{s}$, the bubble shape converges at $L = 180\text{ mm}$.

3.3.4 Influence of the gap radius

In this section we investigate the influence of the gap radius on the bubble dynamics. Figure 3.17(a) compares the temporal evolution of the equivalent bubble radius and figure 3.17(b) shows the expansion and collapse time of the bubble from $L = 5\text{ mm}$ to $L = 400\text{ mm}$ for the bubbles under the same initial condition and gap height. The constant parameters are $H = 0.7\text{ mm}$, $h = 250\text{ }\mu\text{m}$, $R_0 = 50\text{ }\mu\text{m}$ and $P_0 = 1250\text{ bar}$. The expansion and collapse time are defined as the interval from the nucleation to the maximum size and from the maximum size to the minimum size. The time shown in figure 3.17(b) is normalized by the Rayleigh collapse time

$t_r = 0.915R_{max}(\rho/\Delta P)^{1/2}$ (Brennen, 2014). In figure 3.17(a) we can see the R_{max} decreases along with the gap length as more viscous liquid in gap suppresses the bubble expanding. Thus the bubble pressure at the maximum volume increases according to the polytropic EOS. Also, we observe a fluctuation at the late expansion for $L = 80\text{ mm}$, this is the result from a local oscillation since the bubble pressure is still sufficiently high to expand part of the bubble in the low liquid pressure region. Due to this fluctuation, the bubble undergoes a longer expansion than the collapse (see $50\text{ mm} < L < 90\text{ mm}$ in figure 3.17(b)). This fluctuation vanishes when the local oscillation is fully suppressed by the inertia and viscosity, see figure 3.17(a) $L = 160\text{ mm}$. More accurately, the local oscillation disappears for $L > 90\text{ mm}$, where the expansion time drops and becomes smaller than the collapse time in figure 3.17(b). The viscous and inertial effect stops increasing when $L > 180\text{ mm}$, here we see the bubble dynamics for $L = 180\text{ mm}$, $L = 200\text{ mm}$ and $L = 400\text{ mm}$ are nearly indistinguishable, indicating that the increasing gap radius has negligible effect on bubble dynamics when $L \geq 180\text{ mm}$. This convergence of the bubble is also further proved in figure 3.18, where we show the right part of the bubble interface for various L at a late expansion and a late collapse time when the bubble is greatly deformed. Overall, the oscillating time in one bubble cycle is much higher than the Rayleigh oscillation time and increases with L but drops down a small value to a constant value after the bubble reaches a convergence.

Next, we study the effect of the gap radius on the induced jet dynamics with the phase diagram for a gap length of $L = 13\text{ mm}$. The data plotted in figure 3.19 represents the same physical denotations as in figure 3.11. Overall, the phase diagram is similar to that of figure 3.11, it is also divided into three regimes. Yet, there are some difference compared to figure 3.11. First, the dewetting transition is found increasing to $\gamma = 0.4$ for the whole parameter range in the plot because of the increasing inertia. Second, the transferred jet regime becomes narrower, this is mainly limited by a higher dewetting transition. Overall, a large gap radius increases the effect of inertia; this enhances the splitting flow and makes the bubble easier to produce double jet than the transferred jet in the small η regime, where the induced jet dynamics depends on the competition of the viscous affected edge annular flow and the inertia driven splitting flow. In the large η domain, it is easier for the bubble to collapse on the near wall than split into two bubble as the fluid in a gap with longer radius reduces the confined effect of the upper wall, which makes the fluid easier to flow from the top interface.

FIGURE 3.19: Phase diagram for the induced jet for gap radius of $L = 13\text{ mm}$. Experimental data and computed results are plotted as fill and open symbols respectively. Under each catalogue the square, circle, and diamond symbols represents the transferred jet, the double jet, and the directed jet, respectively. The dewetting cases from simulation are plotted as filled cyan triangles. Black line is $\zeta = 0$ while the magenta dot line is the standoff distance from the closer wall $\gamma = 0.4$. The black dash lines are drawn for guiding the transitions.

3.4 Discussions and conclusion

With the confinement of two parallel rigid walls, the bubble experiences distinct scenarios of jetting. Here, the importance is laid on the effect of the gap height and location of the bubble nucleation within that gap. The results from experiment and simulation are collected and summarized over a large range of parameters. The axisymmetric simulations that account for the liquid's viscosity, compressibility and surface tension are in good agreement with the experiments, in particular they predict well the complex deformation of the near-wall bubble interface.

The flow fields and pressure distribution obtained from the axisymmetric simulations reveal the mechanism of jetting: it is the competition between the splitting flow, the annular edge flow, and the vertically inward flow. Viscosity is crucial to develop the fast annular edge flow from very close to the wall. For example

this flow feature was absent in the the earlier simulations ([Gonzalez-Avila et al., 2011](#); [Hsiao et al., 2013](#)) using the inviscid boundary element method. The two dimensionless variables ζ and η allows to construct a phase diagram of the jetting scenarios based from experiments and simulations.

Fast thin jets are the consequences of bubble-liquid interface driving by large axial pressure gradients ([Gekle and Gordillo, 2010](#)). Herein the high-pressure build-up is caused by flow converging due to bubble deformations. In the transferred jet and double jet regimes, the annular flow impacting on the axis leads to a pinch-off of the bubble, while in the directed jet regime flows are converged on the elongated apex of the bubble. The first mechanism is similar to those Worthington jets due to solid object impact ([Gekle et al., 2009](#)), where pinch-off of the cavity builds up a pressure gradient. In contrast, a bursting bubble generates the pressure gradient through the focusing of capillary wave ([Duchemin et al., 2002](#); [Deike et al., 2018](#)). The later dynamics after jet formation also has some similarity with Worthington jets ([Gekle et al., 2009](#)) that the high-pressure is created continuously and accelerates the jet while the bubble is collapsing. Nevertheless, a notable feature of the thin jets studied here is that the bubble deformation guiding flow converging is the result of bubble-boundaries interaction, which is not seen in other jet formation systems like [Duchemin et al. \(2002\)](#), [Antkowiak et al. \(2007\)](#), and [Gekle et al. \(2009\)](#).

The bubble can jet towards the distant wall in a sufficient narrow gap. This was first reported by [Gonzalez-Avila et al. \(2011\)](#) where the bubble was nucleated close to one wall. There for small η the bubble translated to the distant wall, while for even smaller η the bubble became disk-shaped and collapsed between the two walls. Recently, [Quah et al. \(2018\)](#) reproduced the experiment with larger spark generated bubbles in the small η regime and observed the dynamics from the side and from below (through the transparent boundary). They pointed out that once the gap height decreases below a certain value, the jet will impact on the closer wall instead. They claimed this regime is where ([Gonzalez-Avila et al., 2011](#)) observed the disk-shaped bubble collapsing between the two walls. Base on our current results, the discrepancy between these two studies may come from the pinning effect of the walls in the dewetting regime. As shown in figure 3.11, the range of ζ allowing a transferred jet becomes smaller as η decrease, meaning it is easier for the bubble to touch the near wall in a gap with smaller η . Once pinned, the bubble dynamics is greatly affected by the substrate properties. Our current model uses a

static contact angle $\theta_0 = 90^\circ$ for the solid wall, which may not be sufficient for an accurate contact line dynamics. Therefore we cannot provide predictions for the jet dynamics in the dewetted regime. Also we would expect a time dependency on the dewetting such that the scaling by the bubble radius alone may not be sufficient, i.e. η and ζ are not suitable parameters. Thus a further study with appropriate model for contact line dynamics will be interesting to supplement a new phase diagram with suitable parameters for the dewetting regime.

For intermediate η , a bubble may split into two daughter bubbles impacting on both walls. If the bubble is nucleated near the gap center, e.g. ζ is small the bubble splits almost symmetric around the center-line of the gap. Then two daughter bubbles are created with similar size and accompanying fast jet impacting with up to 200 m/s. When the bubble is obviously off-centered, it undergoes an asymmetric splitting, releasing a small bubble with low speed jetting towards the upper wall and a big bubble remain impacting onto the lower wall. Theoretically, the impact speed on the upper wall can be adjusted with the off-center position; this allows to design the geometry for specific impact velocity.

The directed jet develops from a bubble that takes the shape of a cone, an elongated tip, or water drop detaching from a drip. The velocity at impact decreases with ζ , yet can be still above 100 m/s and even up to 200 m/s. A high impact velocity is found when the bubble obtains a high surface curvature. Recently [Lechner et al. \(2019\)](#) reported simulation results of a bubble collapsing near a single boundary. For very small stand-off distances a novel collapse scenario was reported that demonstrated an order of magnitude increase in the jet velocity as compared to slightly larger distances. Their case with $\gamma = 0.048$ and a jet velocity of 1000 m/s has some similarity with the shown in figures 3.6-3.9. There also region with a high pressure is formed due to the rapid deceleration of the flow from where a pressure wave is emitted which drives a curved interface to high speeds. While there are some striking similarities, the present case (figures 3.6-3.9) reaches a considerably lower tip velocity of 580 m/s. A plausible reason may be that the jet from [Lechner et al. \(2019\)](#) is formed at a later stage of the collapse, where the pressure at the top of the tip is higher due to the faster velocities. Also the particular shape of the conical tip may affect the resulting tip velocity.

The lateral extend of the gap affects the bubble dynamics, too. In a previous study, [Gonzalez-Avila et al. \(2011\)](#) using boundary element methods commented

that the collapse time increases with the radius of the axisymmetric gap. The present work confirms their conclusion and also shows that the bubble dynamics with increasing extend gap converges. In a long gap, the bubble collapse is longer than its expansion due to the energy dissipation in the viscous boundary layer. Moreover, a longer gap leads to a narrower transferred jet regime.

An interesting finding in this chapter is that a strong jet generated by a bubble can impact onto a bubble-distant wall of a gap, the jet can be the transferred jet or double jet shown in figure 3.11 and 3.19. This is particularly attractive in cleaning applications and cell manipulation. In the related applications using bubbles near single boundaries, bubbles are required to be nucleated very close to the target surface to achieve high efficiency (Reuter and Mettin, 2016). High energy densities and temperatures that used to nucleate bubbles can be harmful to temperature-sensitive surfaces or delicate biological cells due to heat radiation in short distance. The current result seems to provide a solution for the above problem by generating the bubble at a distance. For demonstration, we recently conducted two experiments to test the efficiency of particle cleaning and molecular delivery into cells using a thin gap device (Gonzalez-Avila et al., 2020). In the experiment, the bubble is generated using a spark on a movable rigid wall, which together with the target surface form a thin gap. Varying the gap distance, strong shear flow accompanied by the impacting jet on the distant target surface is found and sufficient to remove particles and deliver molecular into cells. The working conditions there fall into the transferred jet and double jet regimes. The jet velocities in figure 3.16 can be referenced to optimize the shear flow in applications.

Overall, the present work covered the bubble dynamics in a narrow gap using the location and bubble size as parameters. The important results is the phase diagrams in figure 3.11 & 3.19 that depict the regimes of scenarios. While the dynamics of a cavitation bubble near a single rigid boundary has been studied carefully for many years, the present study provides a rather complete picture for the next more complex geometry. While only recently the importance of the viscous boundary layers for the formation of very high-speed jets were understood in the single wall case Lechner et al. (2019). Here with two boundaries a similar mechanism is found, too. Additionally, the jet dynamics (direction and strength) is adjustable with the thickness of the gap and the location of bubble inception.

Thus for applications these two parameters may be chosen to obtain a suitable flow pattern while limiting erosion.

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Chapter 4

Jetting of viscous droplets from cavitation induced Rayleigh-Taylor instability¹

The following two chapters study the jetting of droplets by an internal oscillating bubble. The bubble in this chapter is generated due to the dielectric breakdown of liquid by a focused pulsed laser while in chapter 5 the cavitation is nucleated by tensile stress generated by pulse laser loading on a metal droplet surface.

Liquid jetting and fragmentation are important in many industrial and medical applications. Here, we study the jetting from the surface of single liquid droplets undergoing internal volume oscillations. This is accomplished by an explosively expanding and collapsing vapor bubble within the droplet. We observe jets emerging from the droplet surface, which pinch off into finer secondary droplets. The jetting is excited by the spherical Rayleigh-Taylor instability where the radial acceleration is due to the oscillation of an internal bubble. We study this jetting and the effect of fluid viscosity experimentally and numerically. Experiments are carried out by levitating the droplet in an acoustic trap and generating a laser-induced cavitation bubble near the center of the droplet. On the simulation side, the volume of fluid method (OpenFOAM) solves the compressible Navier-Stokes equations while accounting for surface tension and viscosity. Both 2D axisymmetric and 3D

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simulations are performed and show good agreement with each other and the experimental observation. While the axisymmetric simulation reveals how the bubble dynamics results destabilizes the interface, only the 3D simulation computes the geometrically correct slender jets. Overall experiments and simulations show good agreement for the bubble dynamics, the onset of disturbances and the rapid ejection of filaments after bubble collapse. Additionally, an analytic model for the droplet surface perturbation growth is developed based on the spherical Rayleigh-Taylor (RT) instability analysis, which allows to evaluate the surface stability over a large parameter space. The analytic model predicts correctly the onset of jetting as a function of Reynolds number and normalized internal bubble energy.

4.1 Introduction

Liquid jets are of great significance in many industrial and medical applications as they deliver liquid matter with high velocity (Tagawa et al., 2012; Gonzalez-Avila et al., 2015) from which fine droplets develop through inherent instabilities (Eggers, 1997; Villiermaux, 2007; Eggers and Villiermaux, 2008). Jetting is frequently found in liquid phenomena involving surface singularities such as bursting of bubbles (Boulton-Stone and Blake, 1993; Duchemin et al., 2002), collapsing of bubbles near a free surface (Chahine, 1977; Blake and Gibson, 1981), impact of droplets or solid objects onto a deep liquid pool (Bergmann et al., 2009; Agbaglah et al., 2015), focusing pressure pulses on curve surfaces (Antkowiak et al., 2007; Kiyama et al., 2016), nonlinear Faraday waves (Longuet-Higgins, 1983; Zeff et al., 2000), focusing laser pulses inside partially liquid filled capillary tube (Tagawa et al., 2012; Peters et al., 2013), and focusing a laser pulse inside liquid marbles (Marston and Thoroddsen, 2015) to name a few. Recently another occurrence of jetting from single droplets have been reported by Gonzalez-Avila and Ohl (2016). These jets were induced from an explosively expanding spherical liquid surface driven by a cavitation bubble at its center.

A typical sequence of such jetting is illustrated in figure 4.1(a): a cavitation bubble generated with a focused laser pulse oscillates inside a millimeter-sized water droplet. We have selected 4 frames during the course of jetting depicting the stages of bubble nucleation ($t = 0 \mu\text{s}$), bubble expansion ($t = 50 \mu\text{s}$), bubble collapse ($t = 125 \mu\text{s}$), and its re-expansion ($t = 150 \mu\text{s}$). Due to the rapidly expanding

FIGURE 4.1: (a) An example of the jetting of a single water droplet under an internal cavitation bubble oscillation; (b) sketch of the problem. Time in μs ; the scale bar represents 1 mm.

bubble the outer air-water interface is destabilized with ripples being formed at the upper and lower pole of the droplet, see figure 4.1(a) $t = 50 \mu\text{s}$. During bubble shrinkage these ripples increase in size and are the sources of fast jets observed during the re-expansion of the bubble. These jets then pinch off into fine daughter droplets. The origin of the jets were related to a Rayleigh-Taylor unstable air-water interface caused by the spherical geometry undergoing rapid radial acceleration (Gonzalez-Avila and Ohl, 2016).

The geometry of the problem is sketched in figure 4.1(b). After the laser has nucleated a vapor bubble we assume that at time $t = 0$ we have a bubble with an initial radius $R_b(t = 0)$ at uniform high pressure P_0 . The bubble is located at the center of the droplet, $r = 0$, and we assume for simplicity a spherical droplet with an initial radius of $R_d(t = 0) = [R_{d0}^3 + R_b^3(t = 0)]^{1/3}$ being surrounded with gas. Far from the droplet the pressure is P_∞ . The bubble oscillation within the droplet leads to a time-dependent radial acceleration. Both interfaces, the inner interface between the bubble and the droplet and the outer interface between the droplet and the surrounding gas are susceptible to Rayleigh-Taylor instabilities (Taylor, 1950). In this work we focus on the outer interface; here the density decreases with increasing r . The stability of the interface is quantified from the growth dynamics of the disturbances. For this we take into account the spherical geometry, the time dependence of the acceleration, surface tension, and viscosity. Viscosity (Kull,

1991; Elgowainy and Ashgriz, 1997) and surface tension (Daly, 1969; Plesset, 1954) in general stabilize the interface.

The spherical Rayleigh-Taylor instability is also known as the Bell-Plesset instability, for the pioneering work of Bell (1951) and fundamental research of Plesset (1954). Bell (1951) was the first to consider the Rayleigh-Taylor instability in the spherical and cylindrical geometry. Later Binnie (1953) proposed a solution on the spherical Rayleigh-Taylor problem for the stability analysis of a cavitation bubble surface by extending the exponential growth rate calculation of Taylor (1950) from a planar surface to a spherical surface. Plesset (1954) later improved the model and formulated an ordinary differential equation (ODE) for small perturbations. The ODE allows calculating the stability criteria of any spherical surface separated by two incompressible, immiscible, and inviscid fluids with time-dependent radial acceleration. Prosperetti (1977) extended the formulation by accounting for viscous effects through vorticity induced by viscous stresses in the boundary layer. Prosperetti's method has been widely applied to understand the shape stability of sonoluminescing bubbles (Brenner et al., 1995; Hilgenfeldt et al., 1996; Brenner et al., 2002). Here, we extend the solution of Prosperetti (1977) to our problem to analyze the stability of the droplet surface, and couple the volume oscillation of the droplet, which is controlled by the internal bubble dynamics.

The present chapter is focused on the mechanism leading to jetting from droplets undergoing volume oscillation. We particularly account for the stabilizing effect of viscosity. The study encompassed experiments, CFD simulations in axisymmetry and 3 dimensions, and an analytic model for surface stability. The methodologies for all three approaches are described in section 4.2, results and discussions are given in section 4.3, and the work is summarized in section 4.4.

4.2 Methodology

4.2.1 Experiment

In the experiments, a single millimeter sized droplet is levitated with a standing-wave acoustic field. The expansion of the droplet is induced by nucleating a cavitation bubble inside the droplet with a pulsed and focused laser, see figure 4.2.

FIGURE 4.2: Experimental setup

Levitating single droplets. A Langevin-type transducer at the top is mounted to an acoustic horn, which together with the spherical reflector form a levitation device. The transducer is driven at its resonant frequency $27.4 \pm 0.1 \text{ kHz}$ by an input sinusoidal wave, which interacts the reflect wave to create a standing wave at a same frequency. The droplet is introduced with a syringe at one of the nodes of the standing wave. These nodes have the largest upward acoustic pressure that can balance gravity. By adjusting the amplitude of the input sinusoidal wave, we can control the acoustic pressure to an appropriate value to levitate the droplet. Usually larger droplet requires higher input amplitude, however once the acoustic pressure is too high the droplet will be acoustically fragmented into smaller size. With the current set-up we can levitate liquid droplets of size range from $100 \mu\text{m}$ to a few millimeter in radius. The position that droplet is levitated at may shift a bit for different droplet size and liquid material. Also in the experiment the droplet wanders a bit along the horizontal levitation plane, therefore more effort is needed to adjust the experimental parameters ensuring the bubble is generated at the center of the droplet. For this the whole levitator is designed to be movable in three directions. More details about acoustic levitation can be found in (Shi and Apfel, 1996; Yarin et al., 1998).

Droplet shape. The balance of gravity, acoustic forces, and surface tension shapes the droplet into an approximate ellipsoid (Shi and Apfel, 1996; Yarin et al., 1998). The equivalent radius $R_{d,eq}$ is the radius of a sphere with the same volume as this ellipsoid. Considering the axisymmetric shape of the droplet ($R_{d,x} = R_{d,z}$), we have

$R_{d,eq} = (R_{d,x}^2 R_{d,y})^{1/3}$, where $R_{d,x}$, $R_{d,y}$, and $R_{d,z}$ are the semi-principal axis in the x , y , and z direction, respectively. Their ratio can be modified within range by varying the amplitude of the acoustic field. In general the ellipsoidal shape will affects the jetting dynamics as curvature suppresses the Rayleigh-Taylor instability (Trinh et al., 2014). Yet the jetting mechanism is still the same. In the experiment we intend to control the droplet to have not too large aspect ratio of the principal axes, which is more closer to our assumption in the following stability analysis.

Laser-induced cavitation bubble. A Nd:YAG laser pulse (532 nm wavelength, 6 ns duration) is first collimated with a telescope consisting of a plano-concave and a plano-convex lens (f_1 and f_2 in figure 4.2) and then focused with a $4\times$ microscope objective (Olympus UPlanFL N, NA=0.13) into the droplet. The laser energy deposited inside the droplet is estimated by sampling about part of the laser pulse energy into the energy meter using a glass slide, as shown in figure 4.2. This sampling energy is calibrated to be 4 % of the original beam energy and the energy at the focus point is about 50 % of the original beam energy due to the losses from all optical components. The bubble is generated by the dielectric breakdown of the liquid where the laser energy is focused on, if the bubble is generated off-centered it will jet away from the closest free surface thus not able to provide a radial acceleration to the droplet surface. Therefore we need to ensure the focus point located at droplet center. To achieve this we find out the focus point of the laser in air first and mark using camera imaging. Then we levitate the droplet and bring the it to marked point. Now the focus point shifts a very short distance to the left due to the refraction of droplet. To correct this we move the whole levitation device slowly towards the incoming direction of the laser beam to ensure the focus point at the droplet center by trial and error experiment. For different size of droplet, the above process is repeated as they have different refracted effect on the laser beam. The radius of the focus spot in air varies from approximately $30\ \mu\text{m}$ at laser energy of 0.3 mJ to approximately $60\ \mu\text{m}$ at 4 mJ, which is much smaller than the droplets.

High speed photography. A high-speed camera records the experiment while the illumination is provided with an LED powered fiber optic (REVOX SLG 150V). The camera and laser are triggered and synchronized with a signal delay generator.

4.2.2 CFD simulation

Once the bubble formed after optical breakdown, it converts part of its internal energy into droplet's kinetic energy by expanding the inner droplet wall. This process is inertia dominated. For the computed bubble dynamics, heat and mass transfer are usually neglected because the oscillation process is too fast to have effective heat and mass transfer. The important effects governing the bubble dynamics are compressibility and inertia therefore it is usually considered as non-condensable gas bubble. Without the implementation of heat and mass transfer one can still obtain reasonably accurate results (Brujan et al., 2002; Tiwari et al., 2013; Koch et al., 2016; Fuster and Popinet, 2018).

The CFD simulations of the droplet interface are done based on the OpenFOAM framework (Weller et al., 1998) by solving the compressible Navier-Stokes equations with the Finite Volume Method (FVM) and using the Volume of Fluid (VOF) based method for interface capturing. The laser-induced cavitation bubble is modeled as non-condensable gas starting from a small volume at a high pressure. The gas and liquid phases are treated as compressible and immiscible Newtonian fluids while neglecting heat and mass transfer. The numerical detail of the VOF solver is written in chapter 2 section 2.2.2.1.

4.2.3 Stability analysis

In this section we propose a stability analysis for the droplet surface, this is done by assuming an initial perturbation on the droplet surface and calculating its amplitude growth under the radial acceleration. We derive an ODE for the perturbation growth by adopting the analysis of Prosperetti (1977) and introducing necessary modifications. The radial acceleration is calculated based on the bubble dynamics within a droplet, which is solved with a modified Rayleigh-Plesset equation (Obreschkow et al., 2006; Gonzalez-Avila and Ohl, 2016).

Here we model the Rayleigh-Taylor stability of the droplet's spherical surface, i.e., the stability of the surface from which the jets are ejected from. As sketched in figure 1(b), the droplet is a sphere of radius R , density ρ_1 and dynamic viscosity μ_1 ; the outer interface is surrounded by a lighter fluid with density ρ_2 and dynamic viscosity μ_2 , i.e., the surrounding gas. The droplet and the surrounding gas are

treated as immiscible and incompressible fluids. The interface position, i.e., the droplet radius R is a function of time, in particular the surface is susceptible to a radial acceleration driven by the internal bubble. This may lead to the growth of surface perturbations.

The surface perturbations are modelled as a spherical harmonic Y_n with mode n , i.e. $r_s(t) = R(t) + a(t)Y_n$, $a(t)$ is the amplitude of the perturbations, starting from some small initial value $|a(t=0)| \ll R(t=0)$. The evolution equation of this perturbation amplitude to the first order has been derived by Prosperetti (1977) for incompressible and viscous spherical flows:

$$\begin{aligned}
& \left(\frac{\rho_2}{n+1} + \frac{\rho_1}{n} \right) R\ddot{a} + \left[3 \left(\frac{\rho_2}{n+1} + \frac{\rho_1}{n} \right) \dot{R} - \frac{2}{R}(n+2)(n-1)(\mu_2 - \mu_1) \right] \dot{a} \\
& + \left[\left(\frac{n+2}{n}\rho_1 - \frac{n-1}{n+1}\rho_2 \right) \ddot{R} + 2(n-1)(n+2)(\mu_2 - \mu_1)\frac{\dot{R}}{R^2} + (n-1)(n+2)\frac{\sigma}{R^2} \right] a \\
& + n(n+2)\mu_2\frac{T_2(R,t)}{R} - (n+1)(n-1)\mu_1\frac{T_1(R,t)}{R} \\
& + (n+1)\rho_1\frac{\dot{R}}{R}\int_0^R \left[1 - \left(\frac{s}{R} \right)^3 \right] \left(\frac{s}{R} \right)^{n-2} T_1(s,t) ds \\
& - n\rho_2\frac{\dot{R}}{R}\int_R^\infty \left[1 - \left(\frac{R}{s} \right)^3 \right] \left(\frac{R}{s} \right)^n T_2(s,t) ds = 0 \quad .
\end{aligned} \tag{4.1}$$

Here σ is the coefficient of surface tension, $T_1(r,t)$ and $T_2(r,t)$ are the terms related to viscous effects for the liquid region and the surrounding gas region respectively, which we discuss later. For convenience and due to the large ratio of density and viscosity i.e. $\rho_1 \gg \rho_2$ and $\mu_1 \gg \mu_2$, we neglect the air density and viscosity and use $\rho = \rho_1$, $\mu = \mu_1$ and $T(r,t) = T_1(r,t)$ to denote the only relevant liquid properties. The perturbation equation 4.1 can then be simplified to

$$\begin{aligned}
& \ddot{a} + B_n(t)\dot{a} - A_n(t)a - (n-1)n(n+1)\frac{\mu}{\rho R^2}T(R,t) \\
& + n(n+1)\frac{\dot{R}}{R}\int_0^R \left[1 - \left(\frac{s}{R} \right)^3 \right] \left(\frac{s}{R} \right)^{n-2} T(s,t) ds = 0 \quad ,
\end{aligned} \tag{4.2}$$

with

$$A_n(t) = -(n+2)\frac{\ddot{R}}{R} - (n-1)n(n+2)\frac{\sigma}{\rho R^3} + (n-1)n(n+2)\frac{2\mu\dot{R}}{\rho R^3} \quad (4.3)$$

and

$$B_n(t) = \frac{3\dot{R}}{R} + \frac{2(n-1)n(n+2)\mu}{\rho R^2} \quad . \quad (4.4)$$

The $T(r, t)$ is a toroidal component of the vorticity ω produced by the viscous stress near the surface. The vorticity ω is decomposed into a poloidal scalar field $S(r, t)$ and a toroidal scalar field $T(r, t)$:

$$\omega = \nabla \times \nabla \times [S(r, t)Y_n^m(\theta, \phi)\mathbf{e}_r] + \nabla \times [T(r, t)Y_n^m(\theta, \phi)\mathbf{e}_r] \quad . \quad (4.5)$$

The poloidal part $S(r, t)$ is needed only to satisfy the particular initial conditions during the brief initial time of the dynamic problem, which is neglected in our case. In contrast the toroidal part $T(r, t)$ is the main contribution to the vorticity. Its evolution according to Prosperetti (1977) and Hilgenfeldt et al. (1996) is

$$\frac{\mu}{\rho} \frac{\partial^2 T(r, t)}{\partial r^2} - \frac{\partial T}{\partial t} - \frac{\partial}{\partial r} \left[\left(\frac{R}{r} \right)^2 \dot{R} T(r, t) - \frac{\mu n(n+1)}{\rho r^2} T(r, t) \right] = 0 \quad . \quad (4.6)$$

The boundary condition for $T(r, t)$ is the continuity of tangential stresses to first order in a (Prosperetti, 1977; Hilgenfeldt et al., 1996), i.e.

$$T(R, t) - 2R^{-n-2} \int_0^R s^{n+1} T(s, t) ds = \frac{2}{n} \left[(n+1)\dot{a} - (n+2)\frac{\dot{R}}{R}a \right] \quad . \quad (4.7)$$

The second boundary condition is $T(R=0, t) = 0$.

Equations 4.2- 4.7 are a complete set of equations for the stability analysis, yet it contains an integral $T(R, t)$. The vorticity is only considered to be generated within a small boundary layer of thickness δ near the droplet-gas interface. We therefore can approximate the integral term in equation 4.2 with

$$\int_0^R \left[1 - \left(\frac{s}{R} \right)^3 \right] \left(\frac{s}{R} \right)^{n-2} T(s, t) ds \approx \delta \left[1 - \left(\frac{s}{R} \right)^3 \right] \left(\frac{s}{R} \right)^{n-2} T(s, t)|_{s=R} = 0 \quad . \quad (4.8)$$

and in equation 4.7 with

$$R^{-n-2} \int_0^R s^{n+1} T(s, t) ds \approx R^{-n-2} [R^{n+1} T(R, t) \delta] = \frac{\delta}{R} T(R, t) \quad . \quad (4.9)$$

Combining equations 4.7 and 4.9 we obtain an expression for

$$T(R, t) = \frac{1}{1 - \frac{2\delta}{R}} \frac{2}{n} \left[(n+1)\dot{a} - (n+2)\frac{\dot{R}}{R}a \right] \quad . \quad (4.10)$$

Since the droplet radius $R(t)$ is much larger than the boundary layer δ i.e. $R \gg \delta$, $T(R, t)$ is approximated as

$$T(R, t) \approx \frac{2}{n} \left[(n+1)\dot{a} - (n+2)\frac{\dot{R}}{R}a \right] \quad . \quad (4.11)$$

Combining the above equations we obtain

$$\ddot{a} + D_n(t)\dot{a} - C_n(t)a = 0 \quad , \quad (4.12)$$

with

$$C_n(t) = -(n+2)\frac{\ddot{R}}{R} - (n-1)n(n+2)\frac{\sigma}{\rho R^3} - (n-1)(n+2)\frac{2\mu\dot{R}}{\rho R^3} \quad , \quad (4.13)$$

and

$$D_n(t) = \frac{3\dot{R}}{R} + \frac{2(n-1)(2n+1)\mu}{\rho R^2} \quad . \quad (4.14)$$

To solve the perturbation equation 4.12, the droplet radius R , velocity $\dot{R}$, and acceleration $\ddot{R}$ are needed. The droplet radius and velocity are related through

$$R^3 - R_b^3 = R_0^3 - R_{b0}^3 \quad , \quad (4.15)$$

thus in an incompressible liquid the droplet size and the interface velocity

$$R^2 \dot{R} = R_b^2 \dot{R}_b \quad , \quad (4.16)$$

are uniquely define through the bubble radius and the constant volume of the droplet. Here R and R_b are the time varying radii of the droplet and the bubble, R_0 and R_{b0} are the initial droplet and bubble radius, and $\dot{R}$ and $\dot{R}_b$ are the radial velocities of the droplet and the bubble, respectively.

The bubble dynamics is obtained from the unsteady Bernoulli equation similar to Obreschkow et al. (2006):

$$\frac{P_b - P_\infty}{\rho} = \frac{3}{2} \dot{R}_b^2 + R_b \ddot{R}_b + \frac{4\mu}{R_b \rho} \dot{R}_b + \frac{2\sigma}{\rho R_b} + \frac{2\sigma}{\rho R} - 2 \dot{R}_b^2 \lambda - R_b \ddot{R}_b \lambda + \frac{1}{2} \dot{R}_b^2 \lambda^4 \quad (4.17)$$

with

$$\lambda = R_b(t)/R(t) \quad (4.18)$$

and assuming an adiabatic bubble surface:

$$P_b R_b^{3\gamma} = P_0 R_{b0}^{3\gamma} \quad , \quad (4.19)$$

where P_b is the pressure of the gas in the bubble starting from an initial value of P_0 while P_∞ is the pressure far from the bubble and γ is the ratio of specific heats.

There are necessary assumptions made to ensure an analytic solution for the perturbation analysis, which may have an impact on the accuracy of the solution. Yet these assumptions are reasonable because in stability analysis the most amplified

terms represent the global stability. The similar analysis has been successfully used in predicting the bubble stability in single-bubble sonoluminescence (Brenner et al., 1995; Hilgenfeldt et al., 1996; Brenner et al., 2002). To reproduce the acceleration, the internal bubble dynamics is calculated through a modified Rayleigh-Plesset equation 4.17. This bubble model has been evaluated suitable for small and medium λ (Obreschkow et al., 2006; Gonzalez-Avila and Ohl, 2016).

4.3 Results

4.3.1 Experiment

We start with a description of the experiments with droplets having different viscosities and undergoing accelerated growth due to internal cavitation. Selected snapshots from high-speed videos recorded at 100,000 frames/s are displayed in figure 4.3 for three droplets of increasing viscosity: water (1 cSt), 10 cSt silicone oil, and 100 cSt silicone oil. They have similar size and are exposed to a laser pulse of similar energy. The laser-induced cavitation bubbles are nucleated very close to the droplet center, which safeguards the bubble oscillates within an intact droplet while instabilities are excited on the outer surface. We see jets develop in water and in 10 cSt oil droplets; no jets are generated in 100 cSt oil droplets.

During bubble nucleation a shock wave is emitted which reflects at the droplet surface as a rarefaction wave. This reflection may lead to a short lived Richtmyer-Meshkov instability (Krechetnikov, 2009). Additionally, we can not rule out secondary cavitation generated on an annular ring close to the droplet equator, explanation see Gonzalez-Avila and Ohl (2016). Thus the amplitude of the initial disturbance may be governed by these two processes. The growth of the amplitude occurs only much later and we therefore see the main contribution for jet formation from the Rayleigh-Taylor instability driven by the unsteady flow of the expanding and shrinking bubble.

When jetting occurs surface perturbations on the droplet are already noticeable at maximum bubble expansion (around $40 \mu\text{s}$); they grow during the bubble collapse (around $100 \mu\text{s}$) into thin filaments or jets. The velocity of the jets increases during the second bubble expansion (around $170 \mu\text{s}$) and increase even more during the

FIGURE 4.3: Experimental records of different fluids under similar laser energy: (a) water droplet, laser energy is 2.9 mJ, $R_{d,eq} = 884 \mu\text{m}$; (b) 10 cSt silicone oil, laser energy is 2.6 mJ, $R_{d,eq} = 839 \mu\text{m}$; (c) 100 cSt silicone oil, laser energy is 2.6 mJ, $R_{d,eq} = 839 \mu\text{m}$. Time in μs ; the scale bar represents 1 mm.

FIGURE 4.4: The computational domains. The region in red is the computational domains used in the simulations considering the symmetry. Left is the domain for 2D axisymmetric simulation, right is the domain for 3D simulation.

third expansion of the bubble (around $300 \mu\text{s}$). The 10 cSt oil droplet exhibits a lower number of jets on an overall smoother surface as compared to the water droplet. Moreover, the water jets break up earlier than the oil jets. At a later time, $t = 4 \text{ ms}$ the droplet has stretched vertically in the direction of the filamentous jets, again the more viscous droplet in 10 cSt extends more than the water case. At higher viscosities, e.g. 100 cSt surface instabilities are noticeably damped.

4.3.2 Simulation

We now turn to the CFD results: figure 4.4 shows the domains and figure 4.5 compares the experiment of a bubble expanding in a water droplet with simulations done in axisymmetric symmetry and in 3D. Initially, the droplet starts as an ellipsoid with semi-principal axes of 1.58 mm, 1.17 mm, and 1.58 mm. The laser energy is approximated 2.2 mJ, $7.1 \pm 4.0 \%$ of this energy is converted in mechanical energy of the cavitation bubble (Gonzalez-Avila and Ohl, 2016). In the simulations the complex physics of bubble nucleation is avoided by starting with a spherical bubble with a radius of $100 \mu\text{m}$ centered in the droplet. In the axisymmetric simulations, figure 4.5b, the domain size is $6 \text{ mm} \times 6 \text{ mm}$ (see figure 4.4 left), and the computational mesh contains 801 grid points in both radial and vertical directions. For the 3D simulations only $1/8^{\text{th}}$ of the ellipsoid is computed to reduce computational time and resources. The domain is $4 \times 4 \times 4 \text{ mm}^3$ (see figure 4.4 right), with an unequally distributed mesh with 401 grid points in all directions. The boundary

FIGURE 4.5: Comparison of experiment, 2D axisymmetric, and 3D simulation results of water droplet: (a) Experiment, $R_{d,eq} = 1.429$ mm, laser energy is 2.2 mJ; (b) 2D axisymmetric, initial bubble size is $100 \mu\text{m}$, initial bubble pressure is 400 bar; (c) 3D simulation, same condition as 2D axisymmetric; Droplets are the same in experiment, 2D axis axisymmetric simulation, and 3D simulation. Time in μs ; the scale bar represents 1 mm.

FIGURE 4.6: Comparison of the equivalent radius for the water droplet in figure 4.5. Left: bubble; right: droplet. The blue line, orange dashed line and red dash-dot line represent the results from axisymmetric simulation, 3D simulation and analytic solution (the modified Rayleigh-Plesset equation 4.17), respectively. Filled symbols are the experimental data. The error from the recorded image is $\pm 20 \mu\text{m}$, which is not taken into account the optical distortion.

condition of all boundaries except the symmetry boundaries is a vanishing gradient of velocities and stresses that allows for in and out flux. The initial conditions at $t = 0$ are for the pressure of the surrounding air $p = p_\infty = 1.01315$ bar and $p = 400$ bar for the gas in the bubble. This initial bubble pressure and initial bubble radius together reflect the energy in the experiment and we get similar maximum size of the bubbles. The dynamic viscosity is $1 \times 10^{-3} \text{ kg} \cdot \text{m}^{-1}\text{s}^{-1}$ for water and $1.84 \times 10^{-5} \text{ kg} \cdot \text{m}^{-1}\text{s}^{-1}$ for the surrounding air and the coefficient of surface tension is $\sigma = 0.0728 \text{ N} \cdot \text{m}^{-1}$.

A comparison of the bubble and droplet radius for the axisymmetric and 3D CFD simulations, the Rayleigh-Plesset solution, and the experiment for the water droplet as shown in figure 4.5 is provided in figure 4.6. The initial radius of the cavitation bubble is set to $100 \mu\text{m}$, which is about the size of the plasma spot. By adjusting the initial pressure in the simulations we can achieve a similar maximum size of the bubbles: these are $916 \mu\text{m}$, $939 \mu\text{m}$, $881 \mu\text{m}$, and $976 \mu\text{m}$ in experiment, axisymmetric simulation, 3D simulation, and the analytic solution, respectively. The pressure at maximum expansion in the CFD simulations is around 3.9 kPa . The 3D simulation results provide the best agreement with the experiments while the results from the axisymmetric simulation and analytic solution are very close.

In figure 4.5 we focus on the first $300 \mu\text{s}$ of the bubble oscillation covering three oscillation cycles with two bubble rebounds. The axisymmetric simulation in figure 4.5b is presented as a cut through the center of the droplet, showing clearly the shape of the droplet and bubble. In axisymmetry these perturbations are not column-like jets but annular surfaces. To compare more closely the shape of the jets with experiments 3D simulations are shown in figure 4.5c. Overall, we find very good agreement between the simulations and experiments, in particular the onset of disturbances and their velocity. Also, the bubble dynamics is covered up to the first collapse around $t \approx 117 \mu\text{s}$, thereafter the corrugated droplet's surface does not allow a closer look on the cavitation bubble in the experiments.

Let's first compare the experiment in figure 4.5a, with the cut plane of the 2D simulation as shown in figure 4.5b. The cavitation bubble expands and reaches a maximum volume at $t \approx 50 \mu\text{s}$ and collapses around $117 \mu\text{s}$. As the bubble expands and shrinks the droplet volume changes mostly along the vertical direction which is the direction of lowest inertia. Around $t \approx 117 \mu\text{s}$ the bubble collapses into an approximate toroidal shape (figure 4.5a); in the simulations also a torus is

found forming during the re-expansion, figure 4.5b. It is the result of the bubble growing while two jets from the top and bottom of the droplet are accelerating to the droplet center. Already before the bubble collapses some roughness on the droplet surface is visible, starting at the upper and lower poles at $t = 50 \mu\text{s}$ in the experiment and simulations. The ripples develop into thin ligaments during the second expansion. During this process the crests on the surface convert into troughs which we explain below with the specific time varying radial acceleration. The ligaments are formed at $t = 300 \mu\text{s}$ all over the droplet surface both in experiments and in the simulations, yet they are fastest at both poles, at places closest to the bubble which are also the places of highest acceleration. Overall, the growth of the perturbations on the droplet surface during the bubble expansion possesses some similarity to the formation of jets under impulsive acceleration of a curved free surface (Antkowiak et al., 2007). The obvious difference in our present experiment is that the initial interface is convex. Yet the initial perturbation generated by the Rayleigh-Taylor instability transform the globally convex (ellipsoidal) interface into a locally concave interface.

The 3D-simulation in figure 4.5c depicts overall a very similar scenario as the 2d-simulations and excellent agreement with the experiment. The bubble collapses slightly faster showing a smaller volume at $t = 117 \mu\text{s}$ than the 2d-simulations. The biggest advantage of the 3d-simulation is that the jets are thin ligaments as observed in the experiments.

A second comparison of the simulations with an experiment is shown in figure 4.7 for a droplet with 10-times higher viscosity, i.e. $\mu_l = 10^{-2} \text{ Pa}\cdot\text{s}$, $\rho_l = 10^3 \text{ kg m}^{-3}$, and $\sigma = 0.02 \text{ N/m}$. We find overall agreement and a dynamics resembling the water case as shown in figure 4.5. Yet on a closer look we notice that the axisymmetric simulation, figure 4.7b, predicts a faster jet along the center of symmetry. Starting from $t = 140 \mu\text{s}$ after the first collapse there are some differences in the overall shape of the droplet. The droplet is stretched more in the horizontal direction than in the simulation. This may be caused by the dominance of the acoustic forces (Foresti et al., 2012) which are not accounted for in the simulations. Interestingly, the jetting in the experiment is more closely predicted with the 3D simulations, figure 4.7c. Overall, the effect of viscosity is the reduction of surface ripples during the first bubble expansion and collapse leading to jets closer to the poles and a smoother interface along the droplet's equator.

FIGURE 4.7: Comparison of experiment, 2D axisymmetric, and 3D simulation results of 10 cSt silicone oil droplet; (a) Experiment, $R_{d,eq} = 1.02$ mm, laser energy is 1.4 mJ; (b) 2D axisymmetric, initial bubble size is $100 \mu\text{m}$, initial bubble pressure is 250 bar; (c) 3D simulation, same condition as 2D axisymmetric. Droplets are the same in experiment, 2D axis axisymmetric simulation, and 3D simulation. Time in μs ; the scale bar represents 1 mm.

The initial acceleration of the droplet affects the amplitude and coverage of surface ripples. Viscosity is opposing the fluid acceleration and stabilizes the interface. Next we'll compare axisymmetric simulations for different initial accelerations and viscosities to understand their effect on the jetting seen after the first bubble collapse. The initial acceleration is difficult to control experimentally and numerically, the simpler approach is to vary the initial potential energy of the bubble, accelerating the fluid and liquid-gas interface. Figure 4.8 summarizes results from three 2D-axisymmetric simulations with increasing bubble energy (from 4.8a to c) while keeping the liquid properties and geometry constant, i.e. oil droplet at 10 cSt with initial bubble radius $R = 100 \mu\text{m}$. For an initial bubble pressure of 50 bar and 100 bar only minute jets are seen after the first bubble collapse and lengthen after the second collapse $t = 200 \mu\text{s}$. Most prominent is the increase in bubble volume

FIGURE 4.8: Comparison of 2D axisymmetric simulation results under different initial internal pressure for 10 cSt oil droplet: (a) 50 bar; (b) 100 bar; (c) 250 bar. Time in μs ; the scale bar represents 1 mm; $R_{d,eq} = 1.02$ mm

with increasing pressure. For an initial pressure of 250 bar, figure 4.8c, fast jets develop after the first bubble collapse. The surface instability modes and the position of the jets are similar for all the cases, yet the amplitudes of the perturbations increase with initial bubble energy.

Figures 4.9a to c illustrate axisymmetric simulations of the flows with increasing viscosity from top to bottom. Interestingly, although the bubble dynamics for these three cases is very similar the jetting from the droplet is largely different; i.e. no jetting at $\mu = 50$ cSt while fast jets are seen at $\mu = 20$ cSt and $\mu = 10$ cSt. Fluid viscosity in this range affects the bubble dynamics only mildly yet it has a pronounced effect on the stability of the surface, particularly after the first bubble collapse.

The internal cavitation bubble will move away from the nearest free surface due to the action of the secondary Bjerknes force. In semi-infinite geometry the bubble

FIGURE 4.9: Comparison of 2D axisymmetric simulation results under different viscosity and same initial internal pressure 250 bar: (a) 10 cSt oil droplet; (b) 20 cSt oil droplet; (c) 50 cSt oil droplet. Time in μs ; the scale bar represents 1 mm; $R_{d,eq} = 1.02$ mm

translates away from the free surface and develops a jet in the direction of translation. Within a droplet two jets are formed from the locations closest the free surface. The jets move towards the center of the droplet and impact onto each other re-directing the jet along the equatorial plane. The result of a bubble split into a torus with a liquid sheet in the equatorial plane is found in the simulations in figure 4.5, 4.7, 4.8, 4.9. The strength of the jet is affected by viscosity and can be fully suppressed by viscosity. This is demonstrated in in figure 4.10. Figure 4.10a presents the bubble dynamics in a 50 cst oil droplet with an initial bubble pressure

FIGURE 4.10: Suppression of the jet formation of the bubble. (a) 50 cst oil droplet with initial pressure of 20 bar, the maximum bubble radius is about $330 \mu\text{m}$; (b) 500 cst oil droplet with initial pressure of 250 bar, the maximum bubble radius is about $690 \mu\text{m}$. Time in μs ; the scale bar represents 1 mm; droplets size are the same as in figure 4.9.

of 20 bar. The bubble expands and collapses spherically. In this case, the instability on the bubble surface is hindered by the viscosity. Figure 4.10b shows a bubble with a higher internal pressure of 250 bar. The development of jets within and on the surface of the droplet is hindered again with a viscosity of 500 cst. The bubble expands and shrinks almost spherically, yet takes an ellipsoidal shape during collapse. While the viscosity suppresses jetting it does not guarantee a spherical shape during collapse.

4.3.3 Mechanism of jetting

In this section we investigate the mechanism of the ejection of thin jets by analyzing the velocity and pressure fields near the droplet's outer surface. Figure 4.11 shows the surface evolution and the flow details of the water droplet from the axisymmetric simulation. We pay attention to how the velocity and pressure affect the shape of the droplet surface at later times. The droplet surface is smooth during the early expansion of the bubble ($t = 20 \mu\text{s}$) while the velocity and pressure is nonuniform along the surface resulting into an undulated droplet later during bubble expansion, e.g. at $t = 50 \mu\text{s}$. During the shrinkage of the bubble ($t = 70 \mu\text{s}$), the liquid flows towards the droplet center. The pressure field shows a wavy structure as it aligns to the surface corrugation. Troughs once formed on the surface are inherently unstable during the bubble collapse and extend in depth. This can be simply explained with the sink flow having a higher velocity for smaller r which results in a growth of the amplitude of the surface perturbations during the bubble collapse ($t = 110 \mu\text{s}$). The bubble expands at $t = 126 \mu\text{s}$ creating a pressure impulse. This

FIGURE 4.11: Flow details of a single water droplet near the surface in selected time. Time in μs , and the scale bar represents 1 mm. The image size is 0.95 in length and 0.75mm in height for (b), (d), and (f). The figures in first, third and fifth rows are the overviews of the droplets surface shape, whose flow details in the black rectangle region are shown below. In the flow fields images, the black curve represents the surface, the white curve represents the contour line of pressure, the color fields is the contour face of velocity magnitude and the arrow represents the direction of velocity. Noted that the color fields are in different range.

FIGURE 4.12: Zoomed view of the surface evolution of the water droplet during shrinkage (left) and expansion (right). Left: $t = 60 \mu\text{s}$ to $t = 118 \mu\text{s}$; Right: $t = 120 \mu\text{s}$ to $t = 178 \mu\text{s}$. Time increases from blue to red at intervals of $\Delta t = 4 \mu\text{s}$.

accelerates the fluid surface such that troughs transform into crests (Antkowiak et al., 2007) visible at $t = 136 \mu\text{s}$ where jets form at locations of previous troughs. The central retracting jet on the axis of symmetry is an exception, it merges with the newly created neighbouring jet and entraps a cavity, see figure 4.11 $t = 200 \mu\text{s}$. The velocities of the pressure impulse induced jets are considerably higher than the initial Rayleigh-Taylor instability created ones, leading to more filaments during the second bubble collapse at time $t = 240 \mu\text{s}$. This we explain that between two jets a new jet is formed from the troughs through the very same mechanism, see $t = 258 \mu\text{s}$.

Let us now have a closer look on the shape of the droplet interface: Figure 4.12 depicts the conversion of troughs into crests during the bubble collapse and expansion. The left frame in figure 4.12 covers the bubble shrinkage during the time $t = 60 \mu\text{s}$ (from top blue contour) until $t = 118 \mu\text{s}$ (bottom red contour) spaced $\Delta t = 4 \mu\text{s}$ in time. The surface disturbances are amplified during the bubble shrinkage. The bubble reaches minimum size at $t = 124 \mu\text{s}$ which is covered in the right frame of figure 4.12. Here the contours are staggered from the bottom (in blue, $t = 120 \mu\text{s}$) in time intervals of $\Delta t = 4 \mu\text{s}$ to the top (red, $t = 178 \mu\text{s}$). The third contour $t = 128 \mu\text{s}$ depicts the droplet shape just after re-expansion. At this time all troughs or concave features are flipped into crests or convex features. With time they develop into thin jets; most pronounced is the jet closest to the axis of symmetry.

FIGURE 4.13: Flow details from the axisymmetric simulation. (a) The baroclinity; (b) the vorticity. Thick lines represent the surface, thin lines are the pressure contour with a spacing of 2 kPa, the vector shows the flow direction. The color fields show the component perpendicular to the slice plane, positive value means outwards while negative value means inwards. Time in μs , from $t = 60 \mu s$ to $t = 66 \mu s$ is within the collapse of the bubble, $t = 120 \mu s$ and $t = 122 \mu s$ are the late stage of the bubble shrinkage, $t = 124 \mu s$ and $t = 126 \mu s$ are the re-expansion. The units of the axes are mm and the origin is the droplet center.

FIGURE 4.14: Flow details from the 3D simulation. (a) The baroclinicity; (b) the vorticity. Thick lines represent the surface, thin lines are the pressure contour with a spacing of 2 kPa, the vector shows the flow direction. The color fields show the component perpendicular to the slice plane, positive value means outwards while negative value means inwards. Time in μs , from $t = 56 \mu\text{s}$ to $t = 62 \mu\text{s}$ is within the collapse of the bubble, $t = 116 \mu\text{s}$ is the late stage of the bubble shrinkage, from $t = 118 \mu\text{s}$ to $t = 122 \mu\text{s}$ are the re-expansion. The units of the axes are mm and the origin is the droplet center.

In general the Rayleigh-Taylor instability can be viewed as the result of baroclinity (Roberts and Jacobs, 2016). It is proportional to $\nabla\rho \times \nabla p$, thus formed by the misalignment of the density and the pressure gradients at the perturbed interface. Looking at the vorticity equation for compressible fluids

$$\frac{D\boldsymbol{\omega}}{Dt} = \frac{\partial\boldsymbol{\omega}}{\partial t} + (\mathbf{u} \cdot \nabla)\boldsymbol{\omega} = \boldsymbol{\omega} \cdot \nabla\mathbf{u} - \boldsymbol{\omega}(\nabla \cdot \mathbf{u}) + \frac{1}{\rho^2} \nabla\rho \times \nabla p \quad , \quad (4.20)$$

we see that baroclinity generates vorticity feeding liquid into the jets. To document this mechanism we measure baroclinity and vorticity at the droplet surface and correlate it spatially and temporally with the formation of troughs (during bubble shrinkage) and jets (during re-expansion) in axisymmetry (figure 4.13) and 3D (figure 4.14).

Figure 4.13 displays the evolution of baroclinity and vorticity from the axisymmetric simulation. The time $t = 60 \mu\text{s}$ to $t = 66 \mu\text{s}$ covers the early stage of bubble shrinkage, that is when the lighter fluid is accelerated into the heavier fluid. The pressure gradient in the direction towards the center of the droplet and the density gradients are not parallel resulting into a clockwise baroclinic torque on the surface created at the left side of the troughs and a counterclockwise baroclinic torque at the right side. The resulting vorticity amplifies the surface perturbation. At $t = 120 \mu\text{s}$, the late stage of the collapse, the direction of the pressure gradient is now away from the droplet center. Thus the baroclinic torques change directions: counterclockwise at the left side and clockwise at the right side of the trough. Accordingly, the vorticity also changes directions which results into a flipping of the troughs into crests. Because of the abrupt increase of the pressure gradient at the rebound $t = 124 \mu\text{s}$ a large increase in the vorticity at $t = 126 \mu\text{s}$ is observed.

Figure 4.14 presents the dynamics of the baroclinity and vorticity from the 3D simulation. They confirm that the mechanism at work are the same as in the axisymmetric simulation. Please note the slightly different timing of the two simulations.

In summary we can explain the pronounced jetting of low viscosity droplets with a two-step mechanism. First the R.T. instability prepares a corrugated surface upon which the pressure impulse accelerates fine ligaments out of its troughs. Vorticity created through baroclinity feeds the ligaments. Later during second bubble collapse the flow field of the ligaments "detaches" from the main droplet body and

they do not retract but continue to move away from. As a result the droplet develops and partly disintegrates into the filamentary structure seen in the experiments, e.g. figure 4.1a $t = 150 \mu\text{s}$.

4.3.4 Stability analysis

Next we'll discuss the stability of the droplet surface with a spherical droplet model derived in section 2.3. The parameters affecting jetting are bubble energy, droplet size, liquid viscosity, surface tension, and the ambient pressure. We can formulate three dimensionless quantities: normalized energy E , Reynolds number Re , and Weber number We :

$$E = \frac{E_{b0}}{V_{d0}P_\infty} = \frac{V_{b0}P_{b0}}{V_{d0}P_\infty} = \frac{R_{b0}^3 P_{b0}}{R_{d0}^3 P_\infty} \quad (4.21)$$

$$Re = \frac{\rho U_{ref} R_{d0}}{\mu} \quad (4.22)$$

$$We = \frac{\rho U_{ref}^2 R_{d0}}{\sigma}, \quad (4.23)$$

where E_b is the initial bubble energy, V is the volume, R is the radius, ρ is the droplet density, and the subscripts $b, d, 0$ relate to the bubble, droplet, and initial state, respectively. The reference velocity $U_{ref} = \sqrt{P_\infty/\rho}$ is obtained from the closure time of a spherical void in an infinite medium.

The stability of the droplet surface is quantified by calculating the growth of an initial small yet finite surface perturbation. The results are only weakly affected by the particular choice of a_0 , which is here set to $a_0 = 0.001 R_{d0}$. Thus for a millimeter sized droplet $a_0 = 1 \mu\text{m}$. Their amplitude is from now on stated by the non-dimensional disturbance $|a(t)|/R_d$. Figure 4.15 depicts in the upper graphs the radius-time curve of the bubble and the droplet surfaces, in the middle graph the non-dimensional disturbance, and in the lower graph the mean acceleration of the droplet surface. In figure 4.15(a) the viscosity is varied while the energy remains constant, $E = 0.4$, and in figure 4.15(b) the normalized energy is varied for a fixed dynamic viscosity of 10 cSt. We chose mode 16 for the disturbance, see later discussion.

FIGURE 4.15: The radius and normalized perturbation amplitude. (a) radius time traces (top), non-dimensional perturbation amplitude (middle), and the mean surface acceleration (bottom) of droplets with varying viscosity and constant normalized energy $E = 0.4$: for water $Re = 10000$, for 10 cSt oil $Re = 1000$, for 100 cSt oil $Re = 100$. (b) comparison of droplets with different normalized energy but same viscosity: $Re = 1000$, the normalized energies are $E = 0.1, 0.2$, and 0.4 . The mode is 16 for all the cases in this figure. The dashed-dot line is the initial perturbation.

Let us first look at the results for the water droplet in figure 4.15(a) solid line and the mode $n = 16$. The disturbance during the early growth phase drops from 0.1 % (see inset) before it grows 60-fold during the late expansion and early collapse to 6 %. During the late collapse and early rebound the disturbance on the droplet shrinks such that it returns back to its initial value around $t = 110 \mu\text{s}$. Naively, this would predict that the droplet surface is stabilized during the collapse and no jetting should be occurring. Yet, the incompressible model does not account for acoustic wave emitted and their interaction with the surface. It is this wave leading to the pressure impulse accelerating the droplet surface into thin jets. Thus, the important quantity to consider is the relative amplitude $|a(t = t_c)|/R_d$ of the droplet surface at the time of bubble collapse, $t = t_c$.

Let us now look at the effect of viscosity in figure 4.15(a) while keeping the initial energy constant. Viscosity affects the model through the normal stresses in the Rayleigh-Plesset equation 4.17 and the evolution equation for the disturbance 4.14. The first droplet/bubble oscillation cycle for a $\mu = 10$ cSt viscosity liquid is nearly indistinguishable from the water case, yet the amplitude at bubble collapse has

FIGURE 4.16: (a) The influence of the mode number n on the perturbation growth for a water droplet of $Re = 10000$, $We = 1428$, and $E = 0.4$. On top is the bubble radius time trace, on the bottom the normalized amplitude of the surface perturbations of the different mode number. The horizontal axis is the time normalized by the collapse time of the bubble T_c . The most unstable mode is found between 24 to 32; (b) The most unstable mode versus $1/Re$ for different We numbers. The red square represents the water droplet in figure 4.5, the bubble circles from left to right represent the oil droplets of 10, 20, 50 cSt viscosity in figure 4.9.

markedly decreased to 4 %. A ten-times higher viscosity $\mu = 100$ cSt although it does effect mildly the bubble dynamics, it fully prevents the growth of the instability. This is very similar to the observation in the experiment, see figure 4.3.

The second easily accessible parameter in experiments is the internal energy of the bubble. Here, we fix the viscosity to 10 cSt and report 3 different energies E in figure 4.15(b). In the upper graphs we see the expected effect that the bubble with larger energy expand to a larger maximum radius and therefore collapse later. Similarly, the amplitude of the disturbance as well as the mean acceleration of the droplet surface are largest for the highest energy. Yet, we find a clear threshold process: while the amplitude $|a(t = t_c)|/R_d$ reaches 4 % for $E = 0.4$, it remains below and around 0.1 % for $E = 0.1$. This predicts that below a certain bubble/laser energy, the droplet will not result into jetting during the first collapse.

At last we discuss the effect of surface modes n on the perturbation growth by studying the amplitudes of the surface modes n as a function of time, see equation 4.12. Once the parameters Re , We , and E are fixed, their growth is a function of the mode number n only. For figure 4.16(a) we have chosen a specific triple of $Re = 10000$, $We = 1428$, and $E = 0.4$. We are specifically interested in the dominant mode at the collapse when the pressure impulse drives the jets from the

troughs of the surface perturbations. A higher mode number would be a larger number of jets paired with a smaller distance between neighbouring jets. The time in figure 4.16(a) is normalized by the Rayleigh collapse time T_c of a bubble and droplet of the specific size. For the specific configuration we observe that the amplitude growth occurs during the early stage of the bubble collapses and is a strong function on the mode- n . Here an intermediate mode of $n = 28$ has the highest amplitude at time $t = 2T_c$. Interestingly, the amplitude of all modes shrink just before the bubble collapse and modes of highest amplitude over the whole cycle may not be dominant at time of collapse, e.g. here $n = 32$ versus $n = 28$.

Figure 4.16(b) reveals the dependency of the most unstable mode on Re and We numbers, i.e., it is a combined effect of droplet size, surface tension, and viscosity. The mode with the highest normalized amplitude at minimum bubble radius is plotted in figure 4.16(b). For low viscosity or small $1/Re$ number the most unstable mode is approximately constant for a fixed We number, thus it is then only a function of surface tension. The mode number n decreases with higher surface tension. In the high $1/Re$ number regime, the viscous regime, the most unstable mode ceases to depend on the surface tension and viscosity dominates. The modes are considerably smaller than in the high Re -number regime.

Our experiments and simulations both show that with increasing viscosity the number of jets reduces and the droplet surface is smoother. This is in quantitative agreement with figure 4.16 (also see the cases in figure 4.16(b) symbols).

Next, we try to use the simple model for surface perturbations to predict the jetting regime. The axisymmetric, 3-d simulations, and the experiments are compared with the model. The underlying assumption is that jetting occurs once the surface perturbations are larger than some threshold needed for pressure impulse driven jetting.

Two parameter plots for the surface stability regimes are shown in figure 4.17 for $We = 2400$ and $We = 7500$, respectively. Again the inverse Re -number is plotted on the horizontal and the normalized energy E along the vertical axis. Filled symbols indicate none-jetting droplets, while open symbols cases of jetting. Additionally, lines of constant normalized amplitude of the surface perturbation at minimum bubble volume are plotted. The value of $|a|/R_d$ is varied between 0.001 and 0.1 for 11 lines. We find that for $|a|/R_d \approx 0.63\%$ all but one measurement

FIGURE 4.17: Jetting regimes. The parameter map consists of $1/Re$ in horizontal axis and the normalized internal energy E in vertical axis. The contours represent the normalized amplitude of surface perturbations $|a|/R_d$ at the bubble collapse in logarithm scale, from the RT stability analysis. The circles represent experimental data; square symbols are 2D axisymmetric simulation results; diamond symbol are 3D simulation results. The filled symbols are the cases of jetting while the empty symbols are the cases of no jetting.

falls into the regime of jetting (above) or no jetting (below). Additionally, the most unstable mode is shown in the inset in figure 4.17 for the same inverse Re -number regime as in the main plot.

The energy of the transition level is increasing from small $1/Re$ to large values. Thus as expected less energy is needed for low viscosity droplets to induce jetting. Interestingly, for not too large $1/Re$ the energy remains approximately constant, indicating that surface tension and not viscosity is the limiting parameter stabilizing the droplet. Yet a further increase, i.e., from $1/Re \approx 10^{-4}$ the energy grows approximately linearly in the double logarithmic plot in figure 4.17. A second parameter plot at a higher $We = 7500$ is shown in figure 4.17(b) using the same values for the contours. The higher We number leads to a lower energy threshold for jetting as compared to figure 4.17(a) for the low viscosity regime. The high viscosity regime reveals very similar boundaries of the normalized perturbation amplitude. Here again the threshold of $|a|/R_d = 10^{-2.2} \approx 0.63\%$ seems to provide a good criteria between jetting and no-jetting.

In the experiments the droplet due to the acoustic trapping obtains not a spherical shape but is an oblate spheroid. In the stability analysis however we assumed a sphere and the results indicate that this is a reasonable approximation for not too large aspect ratio of the principal axes. In general, surface tension is a stabilizing force limiting the Rayleigh Taylor instability (Trinh et al., 2014), therefore one expects that the surface closer to the long axes of the oblate spheroid are more stable. Although this seems to be confirmed in the experiments, there the acceleration is also lower than at the upper and lower poles. A detailed evaluation of the effect of the shape of the droplet on the growth of the instability and jetting remains to be done.

4.4 Conclusions

We have investigated a particular kind of jetting from single droplets caused by Rayleigh-Taylor instabilities in spherical geometry. The instabilities lead to a corrugated liquid-air interface; upon collapse of the bubble a pressure impulse focuses the troughs of the perturbed surface into thin jets. Most prominently, the number of jets decreases with viscosity. Experiments were reproduced with axisymmetric

and 3d CFD simulations. Good agreement of the bubble dynamics, the timing of jet formation, and the location as well as the number of jets are found between experiments and simulations. Only the 3D simulation reveals the cylindrical shape of the ejected shape, while the axisymmetric model would predict annular ring-shaped jets. An analytic model of the spherical Rayleigh-Taylor instability including viscosity based on Prosperetti's (Prosperetti, 1977) work has been re-derived.

The number of jets in the experiments and simulations are described well with the most unstable mode in the stability analysis. They depend only on We -number and Re -number. In the low Re -number regime the droplet surface is mostly smooth with a small number of jets, while the high Re -number regime displays many dimples resulting into more jets.

Although the analytic model is strictly valid only to first order in the perturbations and for incompressible flows, nevertheless it is able to predict the onset of jetting as a function of We number. This is shown by comparing the analytic model with simulation and experiments in the parameter map using Re number and the normalized energy. The transition from jetting to no jetting regimes seems to be close to $|a|/R_d = 10^{-2.2} \approx 0.63\%$. The minimum energy required for jetting increases approximately linearly in a double logarithmic plot of $1/Re$ vs. We . The jetting is dominated by We number for low $1/Re$ number and independent of it at high $1/Re$ value.

The study has not explored the particular shape of the droplet. Although we have assumed for the analytic model that the droplet in the experiment is a sphere, their ellipsoidal shape may be more susceptible to the Rayleigh Taylor instability as compare to a spherical droplet (Trinh et al., 2014). It may be therefore interesting to extend the present study by utilizing the acoustic pressure to deliberately shape the bubble and to explore the importance of curvature on jetting.

The results may provide a pathway to explore flashing sprays. These are sprays consisting initially of large droplets governed by inertia. Before impact the large droplets are activated by some physical or chemical mechanism resulting into a surface instability. The mechanism present in this study could help to design this process such that the droplets eject a suitable fine spray for a good surface coverage. We also would like to point out that this problem has some similarities to a

branching firework with their resulting spectacular fragmentation dynamics ([Inoue et al., 2017](#)).

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Chapter 5

Shaping and controlled fragmentation of liquid metal droplets through cavitation¹

This chapter studies the jetting of metal droplets in vacuum. The jetting mechanism is the same as the one in chapter 4 therefore the same instability analysis can be applied. Yet the cavitation in this chapter is nucleated by tensile stress within the droplet not the dielectric breakdown of liquid.

Targeting micrometer sized metal droplets with near-infrared sub-picosecond laser pulses generates intense stress-confined acoustic waves within the droplet. Spherical focusing amplifies their pressures. The rarefaction wave nucleates cavitation at the center of the droplet, which explosively expands with a repeatable fragmentation scenario resulting into high-speed jetting. We predict the number of jets as a function of the laser energy by coupling the cavitation bubble dynamics with Rayleigh-Taylor instabilities. This provides a path to control cavitation and droplet shaping of liquid metals in particular for their use as targets in extreme-UV light sources.

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5.1 Introduction

Cavitation and bubble dynamics in liquid metals are a challenge to experimental studies as conventional tools such as high-speed photography or acoustic detection with hydrophones are not possible. In order to directly observe cavitation bubbles in bulk liquid metals complex experiments employing pulsed X-ray imaging using e.g. synchrotron radiation sources are required (Huang et al., 2014). The properties of metals such as their high densities at low vapour pressure combined with a large surface tension coefficient makes them particularly interesting for cavitation bubble dynamics studies. For example it was revealed (Kuttruff, 1962; Troia and Madonna Ripa, 2013) that sonoluminescence can be induced in liquid metals by creating cavitation through acoustic excitation. The dynamics of cavitation bubbles particularly in mercury has received attention after large scale cavitation erosion of structural elements were observed in spallation neutron sources (Naoe et al., 2016; Haines et al., 2005). While cavitation plays an important role in various technological processes of modern metallurgy (Eskin, 1998, 2001) it also allows to probe for tensile strength thresholds (De Ressaiguier et al., 2007; Kanel et al., 2015; Zaretsky, 2016).

In the present work we reveal for the first time the mechanism by which a laser pulse transforms an initial spherical liquid metal droplet into an expanding droplet decorated with multiple radial jets. The key finding is that the laser pulse nucleates by acoustic means a cavitation bubble at the center of the droplet. During the volume expansion of this cavitation bubble the droplet surface becomes unstable and develops thin jets. We support this observation with a model based on a spherical expanding bubble able to predict this instability and the number of jets created.

5.2 Methods

Figure 5.1 shows the experimental set-up top view (a) and side view (b). The test section is a vacuum chamber at a residual pressure of $< 10^{-4}$ mbar.

FIGURE 5.1: Experimental set-up. Experiments are conducted in a vacuum chamber at a residual pressure of $< 10^{-4}$ mbar, shown as a circle in the sketch. (a) Top view. LS: light source for illumination. CCD: CCD camera. (b) Side view. CG: metal droplet generator.

Droplet generation. Liquid metal droplets of Sn-In eutectic (48% - 52% mass stoichiometry) alloy were generated with an in-house-made droplet generator utilizing stimulated jet breakup. The temperature of the alloy in the system was maintained at 140°C , which is 20°C higher than the melting temperature of the Sn-In eutectic. The droplets remained liquid state during the flight inside the vacuum chamber due to weak radiative cooling at the working temperatures. All droplets studied in this paper had a diameter of $49.0 \pm 1.6 \mu\text{m}$. Their vertical velocity was 9 m/s . In each test, multiply free-flying droplets are generated and released at the nozzle. The distance between each droplet is adjustable with the droplet generator. In this work, a laser beam impacts on one droplet at a sufficient long distance from the droplet generator nozzle where the droplet is more stable as its natural oscillations are damped. Also the distance between droplets is larger enough to avoid influence on the fragmentation dynamics from neighbour droplets. More details of the droplet generator can be found in [Vinokhodov et al. \(2016\)](#).

Impacting laser. To initiate the complex flow inside the droplets, they were irradiated by pulses from a mode-locked Ti:sapphire laser (Spectra Physics, wave length $780\text{-}820 \text{ nm}$). The pulse duration was fixed to 0.8 ps (full width at half maximum, FWHM). The laser beam diameter at the focal plane was $150 \mu\text{m}$ (full width at half amplitude, FWHA). The laser pulse energy was varied between 0.33 mJ and

1.83 mJ. The laser was directed under a small angle of 15° to the horizontal, see figure 5.1(b) and 5.2(a). The experimental target is to introduce a laser pulse impacting on the surface of a metal droplet. The laser beam diameter made 3 times size as the droplet diameter is to ensure half of the droplet surface is totally irradiated by the laser. The total impact energy is determined by the pulse duration and the laser beam power. In the experiment to vary laser energy, we keep the duration to a constant value and adjust the laser power.

Shadow photography and synchronization. The fragmentation scenarios are highly repeatable which allows stroboscopic shadow photography for visualization of the dynamics. 30-ns-long pulses from diode laser operating at the wavelength of 850 nm were used for the instantaneous back-lighting for the shadow photography, see LS in figure 5.1(a). The shadow images were recorded using two cameras equipped with long distance microscopes and CCD arrays. The cameras under nearly perpendicular view observed the resulting droplet dynamics. The signal delay generator synchronize the droplet generation, impacting laser, camera and illumination laser.

5.3 Results and discussion

5.3.1 Droplet jetting dynamics

Figure 5.2 depicts a typical experimental result following the exposure of the metal droplet with a short laser pulse with an energy of 0.83 mJ ($5.8 \cdot 10^{12}$ W/cm²). Snapshots of the initial droplet (first frame) and its shape transformation after the laser impact are shown in figure 5.2(b) (view of camera 1 is shown in the upper row, camera 2 below). The drop expands for the first $2 \mu\text{s}$ following the laser pulse. Assuming that the density of the liquid remains unchanged, conservation of mass dictates that the expansion is due to void creation within the droplet, e.g. the expansion of the droplet into a thin shelled object. Interestingly, the initial smooth surface of the expanding droplet displays humps already visible at $t = 2 \mu\text{s}$. The droplet reaches its maximum size around $t = 6 \mu\text{s}$; here the humps have transformed into slender jets which detach from the core droplet during the retraction of the shell visible from $t = 12 \mu\text{s}$ in figure 5.2(b). The central shell collapses back into a small droplet while the jets move radially away and fragment further due

FIGURE 5.2: (a) The geometry of the experiment. (b) The shadow-graphs (top-camera 1 (side-view), bottom-camera 2 (front-view)) of deformed droplets taken at different time delays t after the laser pulse impact; the laser pulse energy equals 0.83 mJ; each frame corresponds to a separate experimental realization. (c) Analysis of the fragments' positions.

to Rayleigh-Plateau instabilities (Yu et al., 2014, 2016). The estimated size of the largest fragments from jets observed on the shadow-graphs is $20\ \mu\text{m}$. The measured velocities of these fragments are of the order of 10 m/s.

We want to emphasize that the described evolution of the droplet shape upon the laser impact as well as the numbers of jets and their directions is very stable from pulse-to-pulse. By analyzing the perpendicular camera views at the same instant of time it is possible to obtain the position of the jets. Figure 5.2(c) depicts such an analysis for $t = 12\ \mu\text{s}$ for the upper and lower row, respectively. In total we find 6 jets which are numbered from j_1 to j_6 . Two jets j_1, j_2 move in opposite directions along the laser beam axis. The remaining four jets can be separated in two jets each (j_3, j_4) and (j_5, j_6) propagating in perpendicular planes, which cross each other on the laser beam axis.

5.3.2 Cavitation nucleation

Next we discuss what causes the nucleation of cavitation leading to the droplet expansion. Although we are not able provide direct experimental evidences like x-ray images of the cavitation, we herein propose a nucleation mechanism base on the propagation of shockwaves generated by impacting laser, as sketched in figure 5.3.

Shockwave generation. Pulsed laser loading is a well-known method of generation of stress waves in condensed matter (Eliezer et al., 1990; De Rességuier et al.,

FIGURE 5.3: Sketch of shockwave propagation after laser impact. The laser impact direction is from left to right. The dark and white color represent the positive and negative pressure respectively. (a) A hemispherical shockwave coded in dark is formed after laser impact, trailed by a rarefaction wave with a negative pressure. (b) Both shockwaves focus in droplet center, the pressure magnitudes are enhanced. Cavitation is nucleated once the negative pressure (the white spot) reaches the value of the tensile strength of the liquid metal. (c) The shock wave propagates to the right side. (d) Reflection of the pressure wave at the droplets free interface.

1997; Fortov et al., 2002; Krasnyuk et al., 2016). The directed laser energy deposition first produces a compression pulse or a shockwave. Diffraction waves at the edges of the illuminated region trail the compressive wave as a rarefaction wave and subjects the droplet to tensile stress. This process is sketched in figure 5.3 (a). Once the tensile stress is above the yield strength of the liquid it ruptures and a vapor cavity is formed. These phenomena were extensively studied for planar target geometries for various laser parameters and target materials. Typically liquids have yield strength of thousand bar of negative pressures. The question arises what causes such large amplitudes. In the present experiments laser intensities of $10^{12} - 10^{13} \text{ W/cm}^2$ in the focal plane are achieved. These are well above the ablation threshold for tin (Gamaly, 2011). Due to the ablation process the laser pulse launches a pressure pulse from the irradiated side of the droplet. According to the numerical simulations by Eidmann et al. (2000), pressure in the ablation front can reach magnitudes of Mbar-scale at our laser intensities. Note that the pressure is applied within a time scale, which is four orders of magnitude faster than the sonic time for the droplet, i.e. $t_s \approx d/c_s \approx 25 \text{ ns}$. Such ultra-short and ultra-strong pressure kick inevitably ignites a shockwave at the laser-irradiated surface of the droplet target which then propagates through the target's body. The significant factor that distinguishes our experiments from the classical studies with planar targets is the shape of the shockwave front. In this study, the target (droplet) geometry implies formation of a hemispherical shockwave.

Shockwave focusing. Furthermore, the shockwave converges, i.e. focuses, during its propagation through the droplet, see schematic diagram figure 5.3(b). There are two competing processes that determine the pressure magnitude at the front of the shockwave. The first process is the energy dissipation which leads to the decrease of pressure. The second process is the shockwave focusing, which acts in the opposite direction, i.e. enhancing the pressure. Let's take the analogy with acoustic waves as an estimation. The pressure enhancement factor due to the focusing can be estimated as $kR\alpha$ (Baac et al., 2012), where k is the acoustic wave number, R is the radius of the focusing geometry, and $\alpha(f_N) = 1 - \sqrt{1 - 1/(4f_N^2)}$ a geometric factor depending on the acoustic f -number; $\alpha \approx 0.1$ for the present geometry. With an estimate of the acoustic wavelength of the $c_s\tau \approx 2$ nm we obtain an acoustic gain of the pressure within the droplet of $G \approx 1.5 \cdot 10^4$. Such a high gain suggests that the focusing can compensate effect of the energy dissipation and can deliver the ultra-strong shockwave to the center of the droplet. The same considerations are applicable for the rarefaction wave that follows the shockwave. The rarefaction wave focused to the center of the droplet. There the magnitude of the negative pressure (tensile stress) becomes enhanced. When the magnitude of the negative pressure reaches the value of the tensile strength of the liquid a cavitation bubble in the metal droplet is nucleated. Note that the rupture of liquid may also occur from the reflection of the pressure wave at the droplets free interface (its back) (Stan et al., 2016; Basko et al., 2017), as seen in figure 5.3(d). However, in this study we operate the laser in an energy regime to avoid the latter phenomenon.

5.3.3 Influence of laser energy

Figure 5.4 reveals the importance of the laser energy on controlling the size and shape of the expanded droplets. For the smallest laser pulse energy used -0.33 mJ ($I = 2.3 \cdot 10^{12}$ W/cm²) - we see only a mild expansion yet already here the surface is decorated with some humps. The diameter increases approximately linearly with the laser energy, see figure 5.4(a). Interestingly, not only the volume of the droplet but also the number of humps increases, as shown in figure 5.4(b); and so does the number of jets. The number of jets is counted in both camera views for 100 experiments, averaged and plotted in figure 5.5(b) as a function of the laser energy. Typical snapshots showing these jets are depicted in figure 5.5(a). The pictures reveal that the directions of the jets are a function of the laser energy: for laser pulse

FIGURE 5.4: Experimental results. (a) Maximum expansion of the droplet in the direction (black symbols) and vertical (red symbols) to the laser beam. (b) Shadow-graphs of the droplet at maximum expansion.

FIGURE 5.5: Experimental results. (a) Shadow-graphs of the droplet taken at $8 \mu\text{s}$ after laser pulse. (b) Mean number of jets as a function of pulse energy for camera 1 view (diamond symbols) and camera 2 view (round symbols) both with the numerical result on the peak number of the most unstable surface wave.

energies below 1.33 mJ we find jets always along the six main jetting directions $j1$ - $j6$. Above 1 mJ more jets form while their direction remains reproducible from shot-to-shot. Interestingly, the jets along the axis of the laser $j1$ and $j2$ become less pronounced while the jets in the perpendicular directions ($j3$ - $j4$ and $j5$ - $j6$) remain for all studied energies. From 1.49 mJ additional jets appear which vary from shot-to-shot. Thus for laser energy of 1.49 mJ and above the trajectories of the fragments are unpredictable.

5.3.4 Stability analysis

Detailed understanding and quantitative analysis of the presented experimental studies require a multi-physics model, which describes the processes of laser ablation, formation and propagation of the shockwave with account for its energy dissipation, thermodynamics of the meta-stable liquid and nucleation of cavitation bubbles, and the fluid dynamics of free-surface flow with fragmentation and heat transfer. Fortunately, to understand the mechanism destabilization of the interface already a greatly simplified model can be applied. Because the experiments revealed that the humps are formed during the early time of the bubble expansion, i.e. during times of high acceleration, we investigate next the stability of the outer droplet surface to this radial acceleration.

The model starts with a bubble of initial radius R_{b0} and uniform high pressure P_0 being nucleated by the convergent rarefaction wave. It is located at the center of the droplet with an initial radius of R_0 . The droplet is placed in vacuum, thus the retraction force leading to the shrinkage of the droplet is surface tension only. The bubble $R_b(t)$ and droplet $R(t)$ radii are related through conservation of volume: $R(t)^3 - R_b(t)^3 = R_0^3 - R_{b0}^3$. The spherical bubble dynamics is modeled with a modified Rayleigh-Plesset equation similar to [Obreschkow et al. \(2006\)](#), two surface tension terms have been added, which are causing the bubble collapse:

$$\frac{P_0 R_{b0}^{3\gamma}}{R_b^{3\gamma} \rho} = \frac{3}{2} \dot{R}_b^2 + R_b \ddot{R}_b + \frac{4\mu}{R_b \rho} \dot{R}_b + \frac{2\sigma}{\rho R_b} + \frac{2\sigma}{\rho R} - 2\dot{R}_b^2 \lambda - R_b \ddot{R}_b \lambda + \frac{1}{2} \dot{R}_b^2 \lambda^4 \quad , \quad (5.1)$$

with $\lambda = R_b(t)/R(t)$. Here $\rho = 7.3 \text{ g} \cdot \text{cm}^{-3}$ and $\mu = 1.75 \times 10^{-3} \text{ kg} \cdot \text{m}^{-1} \text{s}^{-1}$ are the density and dynamic viscosity of the liquid tin alloy, $\sigma = 0.534 \text{ N/m}$ is the coefficient of surface tension. The model assumes that the droplet remains a sphere, this allows to model the initial part of the bubble expansion, yet once the jets are formed the model may only provide a qualitative description of the gross dynamics of the droplet. Yet, the humps are formed during the early expansion phase which is unstable to Rayleigh-Taylor instabilities, see [Gonzalez-Avila and Ohl \(2016\)](#).

To compare the simulation with the experiment we model the number of jets as a function of laser energy by identifying the humps with the fastest growing modes

FIGURE 5.6: Numerical results. Top: the comparison of experimental and numerical results, the dots represent the droplet radius in experiment under pulse energy of 0.83 mJ, the solid line and the dash line are the numerical result of the droplet and bubble radius evolution with the initial condition of $R_0 = 25 \mu m$, $R_{b0} = 5 \mu m$, $P_0 = 900 bar$. Bottom: the normalized amplitude of the perturbations of different modes.

of the spherical surface perturbations. These surface waves can be described with spherical harmonics Y_n of mode n , i.e. $A(t) = \sum_{n=2}^{\infty} a_n(t)Y_n$. Here n is the mode number and also the peak number of each surface wave; a_n is the amplitude of the spherical harmonic of mode n , starting from small initial value $|a_n(t=0)| \ll R_0$, its evolution equation to the first order for incompressible and viscous spherical flows can be obtained from Prosperetti (1977):

$$\ddot{a}_n + B_n(t)\dot{a}_n - A_n(t)a_n = 0 \quad (5.2)$$

with

$$A_n(t) = -(n+2)\frac{\ddot{R}}{R} - (n-1)n(n+2)\frac{\sigma}{\rho R^3} - (n-1)(n+2)\frac{2\mu\dot{R}}{\rho R^3} \quad (5.3)$$

and

$$B_n(t) = \frac{3\dot{R}}{R} + \frac{2(n-1)(2n+1)\mu}{\rho R^2} \quad (5.4)$$

Firstly we calculated the time dependent droplet radius and bubble radius with

Eq. (5.1). The initial conditions are $R_0 = 25 \mu m$, $R_{b0} = 5 \mu m$, $P_0 = 900 \text{ bar}$. The numerical result is displayed in the top of figure 5.6 together with the experimental droplet radius for a pulse energy of 0.83 mJ . Good agreement is achieved, even the collapse time in the experiment, $\sim 12 \mu s$, is reproduced indicating the importance of surface tension driving the droplet shrinkage. Based on this comparison we can estimate the converted fraction of the laser energy deposited on the droplet to the initial bubble energy $\eta = P_0 V_{b0} / E \approx 10^{-4}$. The amplitude of each surface perturbations $a_n(t)$ can be obtained by combining Eqs. 5.1 and 5.2 to obtain the most unstable surface wave and its number of positive peaks. This is plotted in figure 5.5(b) as a function of laser energy. We find excellent agreement with the experimental observation considering the simplicity of the model and complex acoustics and fluid mechanics leading to jetting.

However, the peak number is slightly larger than the experimental jet number at low energy regime while slightly underestimated the experiment at high pulse energy. The reasons for the discrepancy are different at these two regimes. We first discuss the low energy regime. In the model we assume that the initial bubble energy is linearly increasing with the pulse energy. Since the ambient pressure is negligible and most of the bubble energy is used to expand the droplet against surface tension, we can expect the maximum expansion of droplet has a strong linear relation with the initial bubble energy. However, in the experiment result figure 5.4(a) we find that the data points for $< 0.8 \text{ mJ}$ are below the linear fitting line, which means the numerical model overestimates the initial bubble energy at this regime. Therefore the calculated peak number is larger than the peak number in the experiment. Next, we discuss the high energy regime. The comparison in figure 5.5(b) is carried between the number of jets from the experiment and the peak number of the most unstable surface wave from the model. As mentioned before, the initial surface perturbation contains many surface waves. These surface waves with different mode can be amplified under Rayleigh-Taylor instability (see mode 4, 5, 6, 7 in figure 5.6(b)), i.e. more than one surface wave is excited. This will be more remarkable for high pulse energy as the surface wave is more amplified due to more laser energy converted to the kinetic energy. Therefore the peak number in the experiment should be considered from a combination of many individual surface waves, which is expected higher than the individual peak number.

5.4 Conclusion

Dynamics of the cavitation bubbles inside liquid metal droplets was experimentally studied. Cavitation was incepted using the rarefaction waves trailing laser-generated shockwaves rather than the tensile waves from the reflection of shockwaves at a free surface. This finding is of interest for studies of the interaction of shockwaves with condensed matter. Usually in this topic, shocks with plane wavefront are used to propagate in planar samples, e.g. plates. Due to the free surface (boundary) of the sample, the shockwave is then reflected into an expansion wave that creates tensile stresses to cause rupture of matter, i.e. spallation occurs. In the present study we limit the experimental conditions that the spallation is suppressed and rupture occurs within the bulk of the liquid only due to the laser-generated rarefaction waves.

Expansion and collapse of the droplet with bubble inside was observed. During the expansion the outer droplet surface becomes Rayleigh-Taylor unstable which leads to the ejection of jets. These instabilities were analyzed using the adopted Rayleigh-Plesset model. The model simulations were found to be in good agreement with the experimental observations.

The reported results provide useful tests for numerical models of strongly compressible multi-phase flows accompanied by cavitation and fragmentation ([Reijers et al., 2017](#); [Saurel and Lemetayer, 2001](#); [Saurel et al., 2008](#)) and for equation of states of liquid metals at extreme conditions ([Fortov et al., 1998](#); [Zein et al., 2010](#); [Lomonosov and Fortova, 2017](#)). Besides the fundamental interest, the findings may help for the optimization of industrial sources for extreme ultraviolet radiation. The hollow droplets as they are created by the expanding cavitation bubble may be an alternate to currently used target morphologies for extreme ultraviolet radiation ([Fujimoto et al., 2012](#); [Kurilovich et al., 2016](#)), in particular as the fragmentation scenario is highly repeatable and predictable.

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Chapter 6

Conclusion and outlook

This thesis focused on the jetting dynamics from cavitation bubbles in confined liquids with emphasis on the effects of viscosity. In particular, we have studied single collapsing bubbles in viscous fluids near single rigid boundaries, bubbles confined within a narrow gap filled with a viscous liquid, and bubbles oscillating inside a viscous droplet. The simulations are all compared to experiments and overall obtain favorable agreement. For the jetting from a Rayleigh-Taylor unstable droplet surface, we develop additionally an analytic growth model for the surface modes.

6.1 Conclusion

6.1.1 Wall shear stress from jetting cavitation bubbles

The shear flow and accompanying tangential stresses produced by a jetting bubble on a flat rigid boundary is notoriously difficult to determine experimentally due to requirements of high temporal and spatial resolutions. We performed numerical simulations that can agree very well with measured bubble dynamics and reproduce the thin boundary layer flow in chapter 2. Detailed flow fields and spatio-temporal stress distributions are presented for a stand-off distance of $\gamma = 1.0$. The shear stress is strongly dependent on the details of the bubble dynamics. The initially expanding flow generates a positive wall shear stress while the early sink flow from the shrinking bubbles induces inward-directed (negative) stresses. The magnitudes

of both stresses are up to about 1 kPa. Much higher stresses occur after the jet strikes the bubble and spreads onto the wall. The speed of the spreading flow is about 80 m/s, creating a boundary layer flow with a thickness of approximately $2\ \mu\text{m}$. Currently, this can't be resolved with off the shelf experimental equipment. A maximum shear stress of 100 kPa from the spreading jet is found near the time of the collapse. Moreover, this fast-spreading flow creates a reversed flow through a high adverse pressure gradient when it flows over the stagnant liquid. The reversed flow induces shear stress directing opposite to the main spreading flow. During the re-expansion, the reversed flow and the expanding toroidal bubble together generate a system of vortex rings that deflect the spreading flow thus slows down the decline of the stresses. For the first time, we present a spatio-temporal wall shear stress map that reveals the complex flow details occurring at the boundary. We hope that the finding provides insight for the applications based on cavitation, e.g. cleaning and biological applications of cavitation.

6.1.2 Jet dynamics inside a thin gap

We explore the jetting dynamics of bubbles in a gap consisting of two rigid plates in chapter 3. Here the gap height and location of the bubble nucleation within that gap are important in the resulting jet direction and velocity. Three jetting scenarios named *transferred jet*, *double jet* and *directed jet* are found. We summarize results from experiment and simulation over a large parameter range into a phase diagram that shows the jetting regimes. The *transferred jet* impacting on the distant wall happens in a narrow gap with sufficiently small height. We find viscosity forming a thin boundary liquid layer is of great importance in producing the *transferred jet* as it can flatten the bubble surface and slow down the near boundary motion of the bubble and help to develop a curved edge. The high pressure gradient near the curved edge enhances the shrinkage nearby and develop a fast annular edge flow in the later collapse. This annular edge flow impacts the axis and detaches the bubble from the boundary. As a result, the bubble collapses and induces a *transferred jet* onto the distant wall. The *double jet* occurs in a gap with intermediate height, where the bubble collapses from the side and develops an axial jet near the middle part of the bubble that splits the bubble into two toroidal daughter bubbles impacting on both walls. The *directed jet* is induced by a conically-shaped bubble and impacts on the nearest wall. In all scenarios, the jet

velocity is adjustable with the gap height and the bubble inception site. We find average impact velocities up to 200 m/s in all three jetting scenarios. Moreover, a jet tip velocity of ~ 580 m/s is found near the transition from a *double jet* to a *directed jet*, which is the result of a high pressure driving a concave bubble surface. This high pressure is generated by the impact of a vertical flow and an annular flow on the bubble tip. The result from chapter 3 provides a rather complete picture of the cavitation bubble dynamics in a narrow gap, which will benefit some potential applications on surface cleaning and biological manipulations.

6.1.3 Jetting of single droplets

The processes needed to fragment an isolated free droplet are rather complex. In chapter 4 we achieve the ejection of liquids through thin jets of acoustically levitated single droplets by creating a cavitation bubble with a focused laser pulse. The computed results reproduce the experiments and thus give a deep insight of the flow and help to reveal the destabilizing and the jetting mechanism of the droplet surface: the rapidly expanding bubble induces a Rayleigh–Taylor (RT) instability, i.e. a droplet with a wavy surface. This is the key observation. During the bubble collapse, a pressure wave is reflected from the surface. A droplet with purely convex surface would not produce thin jets as flow focusing is inhibited, yet the concave parts of the wavy surface support the jetting. While the bubble collapses the surface deformations grow. Jets are formed from crests once the pressure impulse from the collapsing and re-expanding bubble reflects at the interface.

An analytic model for the spherical RT instability is derived to quantitatively describe the instability growth on the droplet surface under the acceleration from the internal bubble. For the analytic model, a most unstable mode exists and indicates the number of jets that eject out from the droplet surface. The model predicts the most unstable mode decreases with viscosity while depends on surface tension in low viscosity regimes, which are in qualitative agreement with the number of jets in experiments and simulations. The results combining experiments, simulations and analytic solutions give a prediction on the onset of jetting with a parameter map containing the dimensionless energy and Re number. The transition between jetting and no-jetting shows a strong dependency on viscosity.

The analytic model for spherical RT instability also works well for predicting the number of jets from jetting metal liquid droplets in an extreme working condition of vacuum, which we study in chapter 5. There are some differences from the jetting of viscous droplets discussed in chapter 4. Firstly the bubble is incepted by the tensile stress from the refraction of the laser-generated stress waves and shrunk by the surface tension not the ambient pressure. Secondly, the most unstable mode of a metal droplet is typically low but increases with laser energy while that of viscous liquid droplet is usually one order of magnitude larger but is considered independent with the bubble/laser energy. The result from chapter 5 is useful in manipulating the metal droplets which are used as a source for extreme UV production in the semiconductor industry.

6.2 Outlook

The wall shear stress studied in chapter 2 is limited to a bubble with a stand-off distance $\gamma \approx 1.0$, nevertheless, the validated numerical model is applicable for jetting bubbles of various γ and minor deformed oscillating bubbles near rigid boundary. For jetting bubbles, fundamentally fluid viscosity and the stand-off distance γ influence the generated shear stress on the boundary because they affect the jet velocity, the gap formation between the bubble and the wall, and the boundary layer thickness. Experimentally, Reuter and Mettin (2016) showed that the strongest cleaning effect was found for $\gamma < 1.8$, indicating high stress generated was due to a strong spreading flow for specific stand-off distance. Therefore, a parametric study on the distance and viscosity would be interesting to evaluate the scaling dependence of the generated shear stress. Furthermore, in some ultrasonic cleaning applications, small bubble attached to substrate may not create strong jet, they turn out to oscillate with surface modes and migrate along the boundary. Yet the shear stress generated by these bubbles may be efficient to remove particles (Kim et al., 2009). Our model together with appropriate experimental measurements which may still have to be developed, can give a comprehensive understanding on the cleaning effect of different bubbles.

Besides, the liquid-solid surfaces sometimes are slip. Although the slip lengths are typically less than 100 nm (Lauga et al., 2007), the slip boundary affects the boundary layer formation for small bubble therefore it may have a pronounced

effect on the shear stress. A further study on the effect of the partial-slip condition can help to understand the cleaning on slip boundaries.

The jet dynamics (direction and strength) in a submerged gap consisting of two rigid plates is adjustable with the thickness of the gap and the bubble inception site as shown in chapter 3. This allows to customize specific designs to choose the right parameters to obtain desirable impact velocities while limiting erosion on the material. One example is to use the transferred jet to deliver drugs into cells and remove surface particle on a distant substrate, taking advantage of the high impact jet while avoiding the damage to the nucleated device ([Gonzalez-Avila et al., 2018](#)). Yet the spreading flow dynamics is not explored in chapter 3, a completed wall shear stress study will benefit the applications. Viscous effect is also interesting to investigate because viscosity can slow down the dynamics and also affect the boundary layer formation and the bubble deformation.

Chapters 4& 5 give a pathway on how to generate fast liquid jet from single droplets using an internal laser-created bubble. In chapter 4 the droplet is levitated as an ellipsoid while the analytic model is limited to spherical droplet. It can complement the study to further consider the particular droplet shape effect on the jetting as the Rayleigh-Taylor instability is susceptible to the surface curvature. Tuning the acoustic pressure can obtain a deliberate shape of the droplet for the experiments. In these studies, the internal bubbles were laser-generated, however from the aspect of applications, a more convenient and controllable nucleated method by chemical reaction or through thermal control should be proposed. One possible way is to generate droplets containing smaller size droplet of low boiling point matter such as perfluorocarbon (PFC), a material is widely used to generate microbubbles in medical applications ([Kripfgans et al., 2004](#)). Under acoustic excitation, bubbles can be generated from the evaporation of the internal PFC droplet then it oscillates inside the main droplet. Moreover, to achieve a sustaining jetting, one can make the bubble oscillates mildly for many cycles while maintaining a spherical shape. An external acoustic wave working at the bubble's resonant frequency could be a solution.

The droplet contains more than two components if PFC or other functional liquid is added. These multi-components droplets can be generated using microfluidics techniques such as coflow and flow focusing ([Anna, 2016](#)). More parameters are

involved when studying these droplets. Among them surface tension plays an important role in stabilizing the interface between every two components. Therefore the most challenging difficulty is the identification of the interfaces. In experiment fluid fluorescence techniques are required to identify the interface movement while in simulation numerical methods for capturing multi-phase (more than two) flow are needed to be developed.

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List of Author's Publications

.1 Journal Articles

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