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**DEVELOPMENT AND VALIDATION OF NEW
SIMULATION METHOD FOR WAKE-VORTEX
DECAY IN GROUND PROXIMITY WITH
ARTIFICIAL ENHANCEMENTS**

Sindhu Paramasivam

School of Mechanical and Aerospace Engineering

A thesis submitted in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

Supervisor: Retd. Associate Prof. Chua Leok Poh

January 2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Associate Prof. Chua Leok Poh

Authorship Attribution Statement

This thesis contains material from 3 paper(s) published in the following conferences where I was the first and/or corresponding author.

Chapter 4 is published as Paramasivam, S., Zhao, D., Skote, M. & Schlüter, J. U. (2016). Detailed study of effects of crosswind and turbulence intensity on Aircraft wake-vortex in ground proximity. In 34th AIAA Applied Aerodynamics Conference (p. 4184).

The contributions of the co-authors are as follows:

- Dr. Jorg Uwe Schlüter provided the initial project direction.
- I performed changes to the simulation software to suit the needs of crosswind and turbulence study.
- All simulations were executed by myself using NTU HPC cluster. I wrote the post-processing codes to analyse the results.
- Dr. Jorg Uwe Schlüter reviewed the final results.
- I prepared the manuscript draft. The manuscript was revised by Prof. Martin Skote and Prof. Zhao Dan.

Parts of Chapter 5 and Chapter 6 is published as Paramasivam, S., Chua, L. P. & Schlüter, J. U. (2018). Study of Multiple Wake Vortex System Behind Aircraft Near Ground Proximity using Prandtl-Lifting-Line Theory. In Tenth International Conference on Computational Fluid Dynamics (no. 10-269).

The contributions of the co-authors are as follows:

- I performed the simulations and wrote the manuscript. Dr. Jorg Uwe Schlüter and Associate Prof. Chua Leok Poh reviewed the results and the manuscript.

24/ 01/ 2019

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Date

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Sindhu Paramasivam

Dedication

To my Grandparents

T. S. Krishnamurthy and K. Kamala

Abstract

The aviation industry is undergoing a tremendous growth and is expected to continue in the decades ahead. One of the main factors affecting this growth is the airport capacity, which is limited by the frequency of landings and take-offs. Aircrafts need to be separated since each aircraft is producing a pair of vortices in its wake that pose a danger to following aircraft. Even though a few concepts for reduced separation under certain conditions are being implemented at selected airports worldwide, it still persists to be a hurdle due to the limited knowledge of aircraft wake vortex decay. A clear understanding of wake-vortices and a precise prediction and avoidance system is required to establish an efficient operational method without jeopardising the safety of the aircrafts.

The goal of this project is to study aircraft wake vortex decay and to reduce the impact of wake-vortices on the runway throughput. Hence, the wake-vortex dynamics at various atmospheric and wing span-loading conditions, in ground proximity is of primary focus for the current research. In this dissertation, the simulation software Jetcode, which was developed for combustion research at Stanford University, is adapted and validated for the wake-vortex research. Throughout the research, Lamb-Oseen vortex model is used to initialise the velocity fields of the shed wake-vortices. Large Eddy Simulations(LES) with dynamic Smagorinsky model is used to solve the unsteady, incompressible and viscous Navier-Stokes equation.

The Temporal LES methodology is used for studying the atmospheric effects on wake-vortices. The effect of crosswind and turbulence intensity on the formation of secondary vortical structures in ground proximity are analysed in detail. The lateral transport of the primary vortex pair is investigated with two new parameters. Postprocessing codes to track the vortices and to determine the circulation of the vortices individually are developed. After performing the preliminary analysis on the wake vortex evolution, it is concluded that enhancing the secondary vortices interaction with the primary vortices result in an accelerated decay of the primary vortex pair.

A new Prandtl Vorticity Distribution (PVD) method is introduced to initialise vortices, including those originating from the flaps, for any given lift distribution using Temporal LES methodology. This method is based on Prandtl Lifting-Line Theory and is effective in simulating the vortices shedding behind any type of aircraft with any high-lift configuration. The available measurements of B747 aircraft is considered for rest of the study.

LIDAR measurements of landing ‘Heavy’ category aircrafts are used to validate the results of the new method. It has been found that the landing configuration of B747 results in a two pair wake-vortex system. The additional pair of vortices is due to the extension inboard flap. These inboard flap vortices greatly reduce the strength of the primary vortex pair.

Using this method, the possibility of reducing the strength of the vortices by means of different modified span loading and roll oscillations are investigated. One of the modified span loadings resulted in an enhanced dissipation of the wake-vortices. Roll oscillations considered in this research did not provide the expected increase in the dissipation. Also, the vortex dynamics of parallel flights with lateral and vertical separation distances are analysed as one of the temporary solutions for enhancing the wake-vortex dissipation. In both the cases, the upwind vortex of one of the aircrafts remains in the domain for longer period of time.

From this research, it has been observed that the initial parameters of the wake-vortices in the near-field greatly depend on the wing span-loading. Hence, it is proposed to be the key parameter in enhancing the wake-vortices dissipation.

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- [1] Paramasivam, S., Zhao, D., Skote, M. & Schlüter, J. U. (2016). Detailed study of effects of crosswind and turbulence intensity on Aircraft wake-vortex in ground proximity. In 34th AIAA Applied Aerodynamics Conference (p. 4184).
- [2] Wang, C. H. J., Paramasivam, S., Zhao, D., Schlüter, J. U., Stephan, A. & Holzäpfel, F. N. (2017). Optimization of Single Obstacle Pair for Aircraft Wake Dissipation under Crosswind Condition. In 9th AIAA Atmospheric and Space Environments Conference (p. 4238).
- [3] Paramasivam, S., Chua, L. P. & Schlüter, J. U. (2018). Study of Multiple Wake Vortex System Behind Aircraft Near Ground Proximity using Prandtl-Lifting-Line Theory. In Tenth International Conference on Computational Fluid Dynamics (no. 10-269).
- [4] Schlüter, J. U. & Paramasivam, S. (2019). Hazard Assessment of Wind Turbine Wakes Turbulence: Initial Results. In 18th Australian International Aerospace Conference:): HUMS-11th Defence Science and Technology (DST) International Conference on Health and Usage Monitoring (HUMS 2019): ISSFD-27th International Symposium on Space Flight Dynamics (ISSFD) (p. 99).

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Nomenclature

Alphabets:

b_o, b	wing span
c	chord of the airfoil (m)
cl	lift coefficient
Di	induced drag
$G(r, x)$	spatial filter function
l_s	length scale
L	lift force
L'	lift force per unit span
p	pressure
Q	Q-criteria
$ Q $	volume integrated Q-criteria in the secondary vortices regime
r	radial separation distance
r^*	non-dimensionalised radial separation distance
r_c	radius of the vortex core
Re_Γ	Reynolds number based on circulation
S	turbulent shear stress,
S_{ij}	turbulent shear stress
T_{ij}	subtest-scale stress
S	rate of strain tensor
u_i	i th velocity component
u_{inflow}	inflow velocity
$u(x, t)$	three-dimensional velocity field

u'	residual velocity field in three-dimension
$\bar{u}$	resolved velocity field in three-dimension
U, V	induced velocity components in x and y direction
v^*	non-dimensionalised inflow velocity
$v(x,y,z)$	x-component of the vorticity field
$\vec{V}$	three-dimensional velocity field in vector form
V_∞	freestream velocity
V_o	descent speed
w	downwash velocity (m/s)
$\mathbf{x}$	three-dimensional coordinate
x_i	i th spatial coordinate
x	axial direction along the vortex pair
y	lateral direction along the wingspan
z	altitude from ground
x^*, y^*, z^*, t^*	non-dimensionalised Spatial coordinates and time
z_{min}, z_{max}	minimum and maximum height of one of the two vortices

Greek symbols:

α	angle of attack (degrees)
β	Ground linking factor
γ_{init}	initial free vortex strength
Γ	circulation
Γ_o	non-dimensionalised circulation
Γ^*	Non-dimensionalised circulation
$\hat{\Delta}$	coarser test filter

θ	Relative angle of vortex pair
λ	wavelength
μ	viscosity
ν	kinematic viscosity of the fluid
ν_{SGS}	the eddy viscosity of the residual stress
ρ	density
ρ_∞	Density of freestream
τ_{ij}^R, τ_{ij}	residual stress
Ψ	Gravitational potential
Ω	vorticity tensor

Mathematical

operator:

$(\cdot)$	Dot product of vectors
∂	gradient operator

Others:

$\mathcal{L}$	Leonard stress
---------------	----------------

Abbreviations

ABL	Atmospheric Boundary Layer
AVOSS	Aircraft Vortex Spacing System
B747	Boeing 747
BL	Boundary Layer
CAAS	Civil Aviation Authority of Singapore
CREDOS	Crosswind - Reduced Separations for Departure Operations
CROPS	CRosswind OPerationS
CW	Crosswind
DLR	Deutsches Zentrum für Luft- und Raumfahrt
DNS	Direct Numerical Simulation
EASA	European Union Aviation Safety Agency
EU	European Union
FAA	Federal Aviation Administration (FAA)
ICAO	International Civil Aviation Organization
IGE	In-Ground Effect
LDG	LanDinG
LES	Large Eddy Simulation
LIDAR	Light Detection and Ranging
LLT	Lifting-Line Theory
MAD	Mean Avergae Deviation
MLDG	Modified LanDinG

MTOW	Maximum Take-Off Weight
OGE	Out-of-Ground Effect
PIV	Particle Image Velocimetry
PVD	Prandtl Vorticity Distribution
RANS	Reynolds Averaged Navier-Stokes
RECAT	RE CATegorisation
Ri	Richardson number
SESAR	Single European Sky ATM Research
SGS	SubGrid Scale
SPV	Single Pair Vortex
TBS	Time Based Separation
WVDSS	Wake Vortex Decision Support System
WVPMS	Wake Vortex Prediction and Monitoring System
WWWS	Wake Vortex Warning System
WSG	Wasser Schleppkanal Göttingen

1 Introduction

1.1 Background

Air traffic congestion is a major issue that challenges the growth of air travel. International Air Transport Association (IATA) represents 83% of global air traffic. The Association has predicted that there will be 7.2 billion air travellers in 2035 which is almost double that of the number of air travellers in 2014 [1]. Worldwide some of the airports are classified as Level 3 in which every aircraft requires authorization for its landing and take-off slots from the corresponding authorities. As of 26th October 2018, IATA has reported that there are over 200 Level 3 airports worldwide facing scheduling constraints [2]. For example, Changi Airport in Singapore is one of the world's best and busiest airports. It is categorised under Level 3 airport for its traffic record. There are over 7200 aircrafts flying in and out of Changi Airport in a week [3]. It acts as an air hub connecting Eastern and Western parts of the globe. Similarly, many airports of developing and emerging countries in Asia-Pacific Region are gaining importance for their contribution to the air-transport industry.

With such a heavy traffic inflow record, it is essential to study the factors that limit the traffic growth. It is not advisable for any busy airport to keep increasing the number of runways but to increase the runway throughput. One of the main constraints that hinders the inflow rate is the minimum wake separation distance between landing/taking-off aircrafts apart from the usual minimum RADio Detection And Ranging (RADAR) separation distance.

Any aircraft in flight produces a pair of strong counter-rotating vortices in its wake: one shed at the portside wing-tip and another shed at the starboard-side wing-tip of the aircraft. Wake-vortices are a by-product of lift generation and are unavoidable. Figure 1.1 shows the portside wake vortex shed behind a 'Heavy' category aircraft during its take-off. The figure gives an idea on the magnitude of the wake-vortices shed behind a 'Heavy' category aircraft [4]. These intensely rotating structures shed behind an aircraft poses threat to the following aircraft if their separation distance is not long enough. The rolling moment

imposed by these strong counter-rotating wake-vortex pair on the following aircrafts might cause structural damage and even loss of control of the aircraft itself. The National Transportation Safety Board (NTSB) of United States, counted 130 incidents between 1983 and 2000 in their country where a trailing aircraft flew into the wake of a leading aircraft, with 14 of these incidents resulting in fatalities [5]. The majority of incidents (74 out of 130) occurred during approach and landing as the aircrafts have limited control in ground proximity.

Figure 1.1 Portside wake-Vortex - Visualization by German Aerospace Research Centre [4]

1.2 Need for the current research

The aircraft wake-vortices are the main reason for the landing and take-off of aircrafts to be staggered. A large time interval is necessary in between any two-consecutive landing/taking-off aircrafts for the vortices of the leading aircraft to vanish. This poses a problem to the airport operators and the aviation regulatory bodies, as these time-intervals reduce the capacity of the runway, increase cost and operational delays. The increase in the capacity of an airport by reducing the wake turbulence-based separation distance should not cost the safety of the passengers and crew. Hence, the easement of air traffic congestion

requires a good understanding of the lifespan of the wake-vortices. The wake turbulence-based separation standard is highly based on the size of the wake-generating leading aircraft and the wake-encountering following aircraft. Now that a large mixture of aircraft sizes is in operation, it is high time to revise the existing separation standards in airport vicinity. Otherwise, the air-transport industry may soon reach a stagnant situation where the aircrafts manufactured exceed the handling capacity of the airports. There are various reduced separation concepts under consideration, research and implementation. For an efficient planning of an aircraft scheduling, a detailed guideline is needed for a wake-turbulence avoidance and encounter. With a better understanding of the physics of wake-vortices, a precise monitoring, prediction and avoidance system can be devised and a more reliable operational concept can be developed and implemented.

1.3 Dissertation hypothesis

The aim of this study is to reduce the lifespan of an aircraft wake vortex thereby paving way for a tighter and safer staggering of aircrafts in the vicinity of the airport. The main source of energy for the wake-vortices is the induced drag. Induced drag in turn depends on the *lift distribution* over an aircraft wing. After a careful review, it is concluded that manipulation of lift distribution is the best way to reduce the intensity of the wake-vortices. Reducing the intensity of the aircraft wake-vortices will eventually result in a short lifespan of vortices in the atmosphere.

In this dissertation, the close relationship between lift distribution and wake-vortices is explored. To facilitate this investigation, an effective initialisation method based on Prandtl Lifting-Line Theory is proposed for the Temporal Large Eddy Simulation (LES) methodology. This methodology can be categorised as *Quasi-temporal LES* as it involves the roll-up phase of the multiple wake-vortices shed behind an aircraft and their evolution into a single pair of counter-rotating vortices.

Figure 1.2 shows the flowchart of the approach followed in this research to find a way to enhance the dissipation of the wake vortices in ground proximity using the lift distribution as a key parameter with the support of most influencing atmospheric parameter.

Figure 1.2 Flow chart of the approach

1.4 Outline of the report

Table 1.1 gives an overview of the structure of the dissertation.

Table 1.1 Outline of the dissertation

Chapter 1	Introduction of wake vortices
	Need for the current research
	Dissertation hypothesis
Chapter 2	Literature survey of current separation standards, methods and methodologies to simulate wake vortex evolution
	Research gap
Chapter 3	Description of numerical methods used in this study
	Introduction of Jetcode, a new software tool
	Validation of the Jetcode with experimental methods
Chapter 4	Influence of crosswind and turbulence on the wake vortex evolution using Temporal LES
Chapter 5	Introduction of PVD method
	Application of PVD method and its validation
Chapter 6	Influence of wing-loading on wake vortex evolution
	Influence of roll oscillations on wake vortex evolution
	Influence of parallel landing aircrafts on wake vortex evolution
Chapter 7	Conclusion and future works

- Chapter 1 provides an introduction of the problem statement, objective and the hypothesis of the present study
- An extensive literature review is presented in Chapter 2. Starting from an inviscid flow around a 2D airfoil, the literature extends to three dimensional flow over a finite wing. An overview of current wake-separation standards followed in various parts of the world is presented. Improved operational concepts with reduced separation standards that are currently under implementation and research are listed. The evolution of wake-vortices behind an aircraft at different distances are detailed. Since

wake-vortices are more dangerous during landing and take-off, main focus is drawn on the wake-vortex decay and its instabilities in ground proximity. State of the art computational methods such as Hybrid RANS-LES, to simulate the wake-vortices are briefed and their advantages and disadvantages are discussed.

- Chapter 3 explains the methodologies used in this study. An in-house code, Jetcode is used to perform the LES. It was originally developed by Dr. Charles Pierce, Stanford University, and later modified to the needs of the wake-vortex research. The numerical methodologies used in Jetcode and its validation with water tunnel experiments by Deutsches Zentrum für Luft- und Raumfahrt (DLR), Germany, are described in detail.
- The results of parametric study is presented in Chapter 4. As a preliminary study, the vortices are assumed to be fully rolled-up as mentioned in many leading journals and the influence of crosswind and turbulence are studied in depth. The generation of secondary vortices and its interaction with the primary vortices are analysed for various background flow conditions. The asymmetric behaviour of upwind and downwind vortices in the presence of crosswind is investigated and the results are elaborated.
- In Chapter 5, the new initialisation method, that is, the Prandtl Vorticity Distribution method (PVD method) is proposed as part of this research to study the effect of span loading on the wake-vortices. The study continues with the validation of the new tool with the real-time LIght Detection And Ranging (LIDAR) measurements conducted at Frankfurt Airport in 2004. Interaction of multiple vortex pairs resulting from a span-loading of high-lift configuration of B747 is analysed.
- Chapter 6 focusses on finding ways to artificially enhance the decay of the wake-vortices. Using the proposed new method, wake-vortex shedding behind different span loadings for B747 aircraft are initialised and their evolution is compared. The results with the details of the vortex dynamics of different wing-loadings are presented. Wake-vortices of roll oscillations and parallel landing aircrafts are also investigated.

- Chapter 7 presents the summary of the findings and the recommendations of the future works.

1.5 Contribution of the current study

This study is an attempt to combine the near-field and far-field simulation of wake-vortices. The success of this method lies in the versatility of its application. It not only applies to aircraft wake-vortices but can also be extended to simulate the wake of wind turbines. Also, it is the first attempt to perform LES study of the effect of span loading on the evolution mechanism of the wake-vortices. Additional novelty of this research is that the flow simulations are performed at a Reynolds number as high as 10^6 .

2 Literature Review

An overview of the practical and theoretical concepts related to aircraft wake vortices and their simulation methods are presented in this chapter. The literature review starts off with an explanation for the formation of aircraft wake vortices and then continues to provide an overview of current Wake Turbulence Separation Standards and Reduced Separation Standards. Then, the chapter outlines the evolution phases of the wake vortices, their instabilities and ground effect. The atmospheric crosswind and turbulence effects on the wake vortex decay are discussed as they play a vital role in the evolution of wake vortices. A brief description of researches on active and passive methods for artificially enhancing the dissipation of wake vortices are also presented as it is relevant to the main objective of the current work. State-of-the-art flow simulation methods available for the analysis of aircraft wake vortices, for example, Hybrid RANS-LES, are also reviewed to identify the research gap.

2.1 *Formation of wing-tip vortices*

Figure 2.1 shows the formation of wing-tip vortices behind a three-dimensional wing. As it is well known, the lift force on an aircraft is the resultant of the pressure difference between upper and lower surfaces of the wing on left and right sides of the aircraft. Wings are of finite wing span. Therefore, when the high-pressure and low-pressure region meets at the wing-tip on both sides of the aircraft, the flow sweeping the lower surface of the wing tends to curl up at the tips as shown in Figure 2.1. This curled-up flow at the tip is shed downstream as the aircraft moves forward. This trailing rotary flow behind the wing, eventually develops into a pair of strong counter-rotating vortices. Hereafter, these counter-rotating vortices will be referred as *wake vortices* (or *wing-tip vortices* or *primary vortices*). The turbulence caused by this wake vortices are referred as wake turbulence.

These vortices are counter-rotating flow features suspending in the atmosphere. Therefore, they suck the outside air from the atmosphere and pushes it between them. The region between the vortex pair experience a downwash due to the induced downward force

exerted by the rotating vortex flow while the region outside experiences an upwash. Since the wing is between the vortex pair just upstream, they cause a small downwash component in the flow over the wing towards the root .

Figure 2.1 Flow over a finite wing and formation of wake vortices [6]

Figure 2.2 shows the velocity components induced by the wake vortices [6]. In the figure, V_∞ represents the forward velocity of the aircraft but in the opposite direction. The induced downward velocity component (w) reduces the total angle of attack (α) by an angle, α_i , thereby displacing the lift vector by the same angle. Due to the displacement of the resultant lift force vector (L), there is an additional horizontal component of force called induced drag (D_i). It is to be noted that the induced drag is linked to the downwash of the wake vortices whose formation is in turn related to the pressure difference over the surfaces

of the wing. Therefore, it can be concluded that the induced drag is a by-product of lift generation and is unavoidable.

From the above discussion, it can be inferred that *the kinetic energy that is fed into the vortex systems comes from the energy spent by the aircraft engine to recover this drag due to lift*. This conclusion is essential as it reveals the dependency of wake vortices on the lift distribution.

Figure 2.2 Effect of downwash over a local airfoil section of a finite wing [6]

2.2 Finite wing analysis – Prandtl Lifting-Line Theory

In the thin airfoil theory, airfoil is represented by a vortex sheet along its camber line. This theoretical representation is justified by the starting vortex theory and its experimental results. When this context was extended to the three-dimensional lifting surfaces, the concept of bound vortex was introduced. A lifting surface can be represented by a vortex filament and this vortex filament is assumed to be bound to the lifting surface spanning across its length. The vorticity of this vortex can also be used to understand the flow around the lifting surface and the cause of the difference in pressure that produces lift. A detailed discussion of thin airfoil theory, Kelvin's circulation theorem and starting vortex are considered to be out of the scope for this current report. Since these theories form the

basics in the understanding of the vortex theory, it is recommended to refer the extensive discussion of the two-dimensional flow around the airfoil found in [6]. However, the necessary theory for understanding the bound and free vortex in detail is discussed in this section using Prandtl Lifting-Line theory.

Prandtl Lifting-Line Theory is popular in finite wing theory and can be found in many aerodynamics textbooks. Prandtl proposed that any finite wing can be replaced by a vortex filament of circulation Γ . This vortex filament is named as bound vortex. This strength of this vortex is proportional to the lift produced by the corresponding lifting surface, which is the lift produced by the wing. This vortex is of higher importance as it determines the flow around the wing and also the pressure difference between the upper and lower surfaces of the wing. Until the aircraft touches down the ground, that is until the lift production is stopped, the bound vortex is assumed to be moving along with the wing. The strength of the bound vortex (Γ) is given by Kutta-Juokowski theorem as follows,

$$\Gamma = L' / \rho_{\infty} V_{\infty} \quad (2.1)$$

where, L' - Lift per unit span, ρ_{∞} - Density of freestream, V_{∞} - Velocity of freestream.

According to Helmholtz theorem, a vortex filament cannot end in the fluid domain. Hence, the bound vortex is continued by the semi-infinite vortex at $-b/2$ and $b/2$. These vortices are known as free trailing vortices. The bound vortex together with the two free trailing vortices are called as horseshoe vortex. Figure 2.3 shows the schematic diagram of a horseshoe vortex replacing a finite wing of wingspan b [6]. In Figure 2.3, V_{∞} is the freestream velocity. The wingspan length presented in Figure 2.3 is b .

Figure 2.3 Horseshoe vortex replacing a finite wing of wing span, b [6]

Each vortex filament exhibits an induced force on the other. The induced force exhibited by the free vortex pair on the bound vortex is acting downwards and so there is a downward component of induced velocity, named downwash, acting over bound vortex/wing. This downwash velocity distributed over the wing induced by these free vortices along the spanwise direction is given by Biot-Savart law as follows,

$$w(y) = -\frac{\Gamma}{4\pi} \frac{b}{(b/2)^2 - y^2} \quad (2.2)$$

where w – induced downwash velocity at a given spanwise location (y). Downwash goes to infinity as $y = b/2$ at the wingtips. In reality, downwash cannot be infinite hence the wing is replaced by a large number of horseshoe vortices superimposed along a single line called *lifting line* as shown in Figure 2.4.

For a given lift distribution, circulation along the wingspan is calculated using Kutta-Juokowski theorem as follows,

$$\Gamma(y) = \frac{1}{2} V_{\infty} c(y) c_l(y) \quad (2.3)$$

where, Γ – spanwise circulation (m^2/s), V_{∞} – freestream velocity (m/s), c – spanwise chord distribution in spanwise direction (m), c_l – spanwise lift coefficient, y – a given position in spanwise direction. The continuous spanwise circulation distribution over the wing is

discretized as shown in Figure 2.4 to three points (A, B, C – D, E, F) on each half of the wing.

Figure 2.4 Superposition of three horseshoe vortices along lifting line [6]

The change of circulation at each spanwise location (A, B, C – F, E, D) is named as $d\Gamma_1$, $d\Gamma_2$ and $d\Gamma_3$ on each side of the wing respectively. A horseshoe vortex with strength $d\Gamma_1$ is placed spanning from the point A to point F. The next horseshoe vortex with a strength equals to the sum of previous horseshoe vortex strength ($d\Gamma_1$) and the increment, $d\Gamma_2$ is placed between the points B and E. Similarly, another horseshoe vortex is placed between the points C and D. It is to be noted that the strength and length of the superimposed bound vortices changes along the spanwise direction. Upon superposition of all bound vortices, the circulation distribution over the lifting line should be the same as spanwise circulation distribution over the wingspan.

When an infinite number of such horseshoe vortices are superimposed, a more realistic continuous curve for the spanwise circulation distribution over the lifting line is obtained. Figure 2.5 shows the superposition of infinite number of horseshoe vortices along the lifting line forming a vortex sheet downstream [6].

Figure 2.5 Superposition of infinite number of horseshoe vortices along lifting line [6]

Consider an infinitesimally small element in the lifting line, the change in circulation over this element is $d\Gamma = (d\Gamma/dy)dy$ and so is the strength of the bound vortex. In a horseshoe vortex, the strength of the bound vortex ($d\Gamma$) is the strength of the free vortex trailing this element dy .

The free vortices trailing downstream of the lifting line forms a continuous vortex sheet as shown in Figure 2.5. This vortex sheet eventually rolls up into a single strong counter-rotating vortex pair in the far-field wake of an aircraft. The strength of these free vortices is linked to the strength of the bound vortex which in turn depends on the spanwise lift distribution. This essential conclusion from the Lifting-Line Theory (LLT) will be used as the basic idea in our initialization method proposed in Chapter 5, to build a relationship between the span loading and the vortex shed downstream. Since the bound vortices move along the wing, it is not necessary to introduce them in the computational domain.

2.3 Overview of Current Wake Turbulence Separation Standards

The strength of these wing-tip vortices is proportional to the lift force generated by the wing and so proportional to the weight of the aircraft. The light weighted aircrafts are at risk as it can lose control if flown into the wake of a heavy aircraft due to the heavy rolling moment imposed by these rotating vortices. This risk factor is higher if the wake is encountered near ground proximity that is during landing/take-off. Hence, Wake

Turbulence Separation Standards are established for take-off and landing to ensure safety. Current Wake Turbulence Separation Standards are still highly based on the wake vortices research conducted between 1970s-1990s.

According to International Civil Aviation Organization (ICAO) PANS-ATM doc.4444, to ensure safety from wake turbulence, the aircrafts are classified based on their maximum take-off weights (MTOW) [7]. The minimum separation distances for each class of aircrafts are given in Table 2.1. It is to be noted that the Airbus 380 (A380) aircraft falls under the category of Super (S) as per the provisional State Letter published later by ICAO in 2008.

ICAO Aircraft Classification

Category	Definition
Heavy	MTOW>136 tons
Medium	7>MTOW<136 tons
Light	MTOW<7 tons
MRS – Minimum Radar Separation = 3 NM or 2.5 NM under given conditions described in ICAO PANS-ATM doc.4444	
NM – Nautical miles (unit of measure for distance)	
MTOW – Minimum Take-Off Weight	

Table 2.1 ICAO Wake Turbulence Separation Standards [7]

Leader aircraft	Follower aircraft			
	Super	Heavy	Medium	Light
Super	MRS	6 NM	7 NM	8 NM
Heavy	MRS	4 NM	5 NM	6 NM
Medium	MRS	MRS	MRS	5 NM
Light	MRS	MRS	MRS	MRS

The aircrafts were recategorized by FAA and EASA separately. The new separation standards namely, RECAT 1 FAA and RECAT 1 EU were established in USA and some parts of Europe respectively. RECAT 1 EU classifies the aircraft fleet into 6 different categories based on the leader aircraft's vortex strength and the follower aircraft's resistance capability. The classification and the separation standards of RECAT 1 EU are given in Table 2.2 [7].

RECAT 1 EU Aircraft Classification [7]

Category	Definition
A - Super Heavy	Includes A380 and An124.
B - Upper Heavy	MTOW above 100 tons and wingspan between 52 m and 72 m.
C - Lower Heavy	MTOW above 100 tons and wingspan below 52 m
D - Upper Medium	MTOW between 15 and 100 tons and wingspan above 32 m.
E - Lower Medium	MTOW between 15 and 100 tons and wing span below 32 m
F - Light	MTOW below 15 tons.

Table 2.2 RECAT 1 EU Wake Turbulence Separation Standards [7]

		Follower aircraft					
		Super Heavy	Upper Heavy	Lower Heavy	Upper Medium	Lower Medium	Light
Leader aircraft	Super Heavy	3 NM	4 NM	5 NM	5 NM	6 NM	8 NM
	Upper Heavy	MRS	3 NM	4 NM	4 NM	5 NM	7 NM
	Lower Heavy	MRS	MRS	3 NM	3 NM	4 NM	6 NM
	Upper Medium	MRS	MRS	MRS	MRS	MRS	5 NM
	Lower Medium	MRS	MRS	MRS	MRS	MRS	4 NM
	Light	MRS	MRS	MRS	MRS	MRS	3 NM

Further developments are currently in progress to reduce the distance of this separation standard while ensuring the safety. RECAT 2 classifies more than 95% of the common global aircrafts into 115 categories and provides 115 by 115 separation matrix for each pair of aircrafts. RECAT 3 will have more flexibility as it will include the weather data.

Cruise:

There are no separation standards for aircraft at cruise altitude. Due to increase in the air traffic, decrease in the vertical separation distance and the wide class of aircrafts ('Light – Super Heavy'), the wake vortex encounters at cruise altitudes should also be studied for future implementations.

2.4 Operational concepts and Advisory systems

As it is already known that current separation standards are overly protective and affects the air traffic movements in and around the airports. Air traffic control researchers are proposing new operational concepts which can act as temporary or permanent solutions in the future. In general, all the concepts are based on reducing the separation distance which in turn depends on the aircraft information, wake data, weather data and the information on the airport itself. RECAT versions are one such example.

Other concepts under operation/research are as follows,

- CREDOS, CROPS – operations that include crosswind information [8]
- Time Based Separation (TBS) – includes some of the weather information. Figure 2.6 shows the difference between the Distance Based System (such as ICAO standards) and Time-Based System (TBS). It can be seen that in the presence of strong headwind, the number of landing aircrafts are increased by 2 – 4 in number [9].

Figure 2.6 TBS Operational Concept example [9]

SESAR P6.8.1 – Flexible and dynamic use of Wake Turbulence Separations and NextGen have an objective of devising and implementing a Dynamic Pairwise Separation Standard (D-PWS) to increase the runway throughput [9].

Some of the wake vortex advisory systems in place and in research are as follows,

- Aircraft Vortex Spacing System (AVOSS) [10]
- Wake Vortex Warning System (WVWS) [11]
- SESAR P12.2.2 has an objective of developing Wake Vortex Decision Support System (WVDSS) and Wake Vortex Prediction and Monitoring System(WVPMS)
- SESAR P9.11 & P9.30 have an objective of developing Wake-Encounter Prevention System (Predication and Control). [12]

It can be easily concluded that the study of wake vortices is necessary to implement a reduced separation standard and to develop a new prediction and support system. The following sections cover the physics of wake vortices, influencing parameters and the modelling methods to aid the investigation.

2.5 Phases of wake vortices evolution

Figure 2.7 shows the different phases of evolution of wake vortices [13]. The evolution of wake vortices downstream of wing can be classified into four different regions as marked in Figure 2.7 and are described in the following sections [13, 14]. The x -coordinate is parallel to the freestream direction (U_∞) and is used to refer the approximate length of different regions. Other coordinates are not considered as it is irrelevant to the current discussion. The wingspan of the aircraft is denoted as b and the effective span between the rolled-up vortex pair after extended near field is b_0 .

Figure 2.7 Phases of wake vortices evolution [13]

2.5.1 Near field

Any discontinuity in the geometry of the surface above and below the aircraft lifting surfaces (wing and tail) creates concentrated vortices in the flow field downstream. If port-side of the wing is considered, six dominant vortices can be identified in the near field namely the wing tip vortex, the outboard flap vortex, the outer and inner engine nacelle vortices, the wing-fuselage vortex and the horizontal tailplane vortex. The concentrated vortices are shed from wing-tip, edges of control surfaces, tail, wing-fuselage junction and engine pylons and start to roll-up in this region. This region is marked in Figure 2.7 as 'Near field'. This region extends from the wing trailing edge to the rear end of the fuselage.

Figure 2.8 Non-dimensional axial vorticity distribution at $x^* = 0.37$ for reference configuration 1 (E403 model) and shedding locations of dominant near field vortices:

(a) vortex topology and (b) qualitative circulation distribution [13]

Figure 2.8 shows the non-dimensional axial vorticity distribution for reference configuration (E403 model). The locations of the dominant vortices in the near field regions are also marked in the figure. The circular arrows presented in the figure represents the direction of the vorticity shed. It can be noted from the Figure 2.8 (b) that for every change in the circulation distribution on the half wing span, there is a concentrated vortices shed. This is an important observation for the initialisation technique proposed in Chapter 5. A wing vortex sheet is emanated at the trailing edge due to the difference in the lateral velocities of the upper and lower side of the wing. This vortex sheet links the dominating vortices in the near field region. It is to be noted that the dominant vortices mentioned here

are specific to a particular aircraft type and configuration and does not represent the generic vortex system shed behind any aircraft.

2.5.2 Roll-up phase, Extended near field wake

At this phase, different vortices start to roll-up and merge with the nearby co-rotating vortices. The outboard flap vortex and the outboard nacelle vortex roll-up to form a single strong main vortex. The wing tip vortex and the inner nacelle vortex revolve around the main vortex (rolled up outboard flap-nacelle vortices). While the wing tip vortex merges with the main vortex, the vorticity of the inner nacelle vortex is redistributed and fed into the development of strong main vortex. The wing-fuselage vortex is dissipated at faster rate due to the higher turbulence shed downstream from the fuselage region. The Horizontal tailplane vortex remains concentrated until the formation of the stronger main vortex. Although it remains intact, it is subjected to the induced force of the main vortex system. It should be noted that the roll-up process is also highly dependent on the aircraft type and configuration.

As a result of this redistribution, there will be two distinct counter rotating vortices formed downstream. This region extends up to ten times the aircraft wingspan (b) and is marked right next to the near field region in Figure 2.7. In clean configuration, Extended near-field is shorter, as stronger vorticity is shed mostly at the wing-tips and so the vortices roll-up faster.

2.5.3 Vortex phase, Mid/far field wake

The vortex pair is stable and steady in this region as indicated in Figure 2.7. On facing the direction of the motion of the aircraft, the left vortex rotates in clockwise direction while the right vortex in anti-clockwise direction. They descend through the atmosphere due to mutual induction. As they descend, the vortices diffuse and so there will be a gradual decrease in their strength.

2.5.4 *Decay region*

Decay region can be otherwise called as wake breakdown zone. In cruise condition, Crow instability is induced in the vortices due to the presences of fluctuations in the atmosphere. Further information on Crow instability of the aircraft wake vortices can be found in Section 2.7. A more turbulent environment may even help in accelerating the decay. However, in the presence of ground, the vortices in this phase behave distinctly different at this phase. As it evolves, the vortices interacts with the boundary layer formed near the ground and induces the formation of complex secondary vortices. A detailed decay mechanism of vortex pair in ground proximity and their interaction with the secondary vortices will be explained in the Section 2.6.

2.6 *In-ground effect*

Figure 2.9 (a) Induced crossflow and formation of separation zone, (b) Formation of secondary vortices from the separation zone [15]

When the airplane is in ground proximity i.e. about one wingspan height from the ground, the wings are said to be in-ground effect. Harvey and Perry in 1971 [15] had clearly described the vortex-ground interaction with the help of a schematic diagram as given in Figure 2.9. It should be noted that the schematic represents only one of the two wing-tip vortices for clarity. The vortices as they descend through the atmosphere, induces a cross flow over the ground beneath them as shown in Figure 2.9 (a). The boundary layer of this cross flow possesses an opposite sign of vorticity as compared to the nearby primary wing-tip vortex. As crossflow moves outward beneath the vortex, an adverse pressure gradient is experienced by its boundary layer. As vortices move closer to the ground, the pressure gradient increases leading to a separation zone marked as ‘bubble’ in Figure 2.9 (a). This separation zone starts off as a bubble. Gradually, the separation zone results in the detachment of vortex sheet from the boundary layer and interacts with primary vortex. These detached vortex sheets from the boundary layer in ground proximity that approaches the primary vortex pair are referred as secondary vortices as shown in Figure 2.9 (b). These ground proximity effects were also found in the experimental observations and in-situ measurements [15-17].

Generally, the wake vortices shed by the aircrafts flying in higher altitudes, descend through the atmosphere away from each other, due to mutual induction until they decay. Contrastingly, in ground proximity, due to the presence of the induced cross flow and its boundary layer over the ground, the primary vortices cannot descend after a certain altitude and are forced to move only in lateral direction without much difference in their altitude. This phenomenon is known as vortex rebound and the height attained by the vortices after the rebound is known as rebound altitude. However, it should be noted that the lateral motion due to the mutual induction remains unchanged and they move farther apart from each other. Because of this rebound phenomena, the vortices stay in the landing path for a longer time causing safety issues to the following landing aircraft. Hence, it is mandatory to follow a separation standard between any two landing aircrafts.

Few notable papers published in the last 80 years are presented. The pioneers for the study of induced drag were Wieselsberger [18], Prandtl [19] and Betz [20]. Pistolesi [21] in

1937 published a review paper consisting of theories and experimental results on ground effect since 1912. It is concluded from all of these researches [18-21] that the outcomes are not practically usable due to limited numerical and experimental facilities. Widnall and Barrows [22] in 1970 proposed an analytical solution to derive the lift coefficient for a flat wing with straight trailing edge in ground proximity using the method of matched asymptotic expansion. The lift coefficient reflects the effect of downwash induced by the wake vortices in ground proximity. In the olden days, predicting the induced drag in ground proximity was considered to be the first step towards quantifying the effect of ground on the wake vortices.

Gradually, as the numerical and experimental resources are improved, researchers started to work on solutions closer to reality. Zheng and Ash [23] in 1996 modelled a two dimensional rolled-up vortex in ground proximity and studied its evolution in the presence of different surface weather conditions. Fischenberg [24] in 1999 devised a two-point aerodynamic model to study the important parameters of the ground effects such as induced drag, slope of the lift curve and downwash angle. Proctor and Han [25] in 1999 simulated three-dimensional wake vortices shedding behind a landing L-1011 and also investigated the sensitivity of the wake vortices in ground proximity. Daeninck et al. [26] in 2006 had studied the in-ground effect on the span-loading of the aircraft. A two-dimensional flow simulation of wake roll-up behind an airfoil was also performed for wings with four aspect ratios at five different altitudes from ground. Real-time measurement of 288 pairs of wake vortex in Frankfurt was executed and results were analyzed by Holzäpfel & Steen [27]. The important parameters to assess the vortex probabilistic prediction models were obtained. Stephan et al. [28] in 2013 performed a LES of a rolled-up wake vortex pair in ground proximity with a high-resolution mesh and a reasonably high Reynolds number to describe the physical mechanism in detail.

Ground linking factor is proposed by Proctor et al. [29] as follows,

$$\beta(t) = \frac{z_{max} - z_{min}}{z_{max} + z_{min}} \quad (2.4)$$

where $z_{\min}$ and $z_{\max}$ are minimum and maximum height of one of the two vortices above the ground. If β exceeds 0.85, then the vortex is said to be linked with its ground image.

2.7 *Instabilities of wake vortices*

The knowledge of instabilities of wake vortex pair is necessary to find a way to induce a faster decay of the wing-tip vortices. Researchers have found that the onset of these instabilities plays an essential role in the decay process of the wake vortices. There are three types of instabilities associated with the wake vortex pair: 1. Long-wave Crow instability; 2. Short-wave elliptic instability; 3. Secondary vortex instability.

The photographs of the three types of instability are presented in Figure 2.10 [30]. The first two instabilities are proven to be invoked only farther away from the ground, that is, when the aircraft is Out of Ground Effect (OGE). *Crow instability* is a long wave instability with the wavelength that is several times larger than the wing span [31]. It shows a bending displacement mode where the whole vortex tube is bent in sinusoidal waves. As it evolves, the vortex displacement amplitude increases and there will be zones of reconnection which eventually leads to the formation of vortex rings. *Short-wave elliptic instability* has a wavelength that is comparable to the vortex core size [32, 33]. This involves a more complex and detailed deformation within the vortex tube. When the aircraft is in ground proximity, the secondary vortices that are detached from the ground, induced by the primary vortex pair are unstable. They exhibit a long wave instability in the low Reynolds number regime [34] and an elliptic instability in high Reynolds number regime (Re of the order of $10^3 - 10^5$) [35]. Both together are referred as secondary vortex instabilities. It is also suggested that the omega-like vortical structures that loops around the primary vortices in the presence of crosswind and ground, are formed by the outer layer of these secondary vortex instabilities. These structures are of higher importance to this study as they aid in the reduction of the strength of the primary vortices.

It is to be noted that the vortices shed from the junctions of flaps also exhibit long wave instabilities and eventually evolve into omega-shaped structures around the wing-tip vortices. The evolution of these flap vortices is further discussed in Chapter 5.

Figure 2.10 (a) Crow instability, (b) Elliptic instability in the vortex pair, (c) Secondary vortex instability [30]*

*Images are not to scale.

2.8 Atmospheric influence on the evolution of wake vortices

It is unanimously accepted by wake vortices researchers around the world that the meteorological conditions have heavy impact on the evolution of wake vortices. Decades of

research were performed, and the following parameters are proven to be the most influential: Crosswind shear, turbulence and stratification. Other parameters include atmospheric humidity, turbulent kinetic energy and headwind. In this section, a mixture of early and recent findings will be presented to give an idea of progress of the parametric study. Current Wake Turbulence Separation Standards are still highly based on the research conducted between 1970s and 1990s.

Ambient crosswind shear

Figure 2.11 Schematic of wake vortex with crosswind [28]

Note: The x-axis direction is perpendicularly out of the plane of the paper.

A more comprehensive description with a schematic diagram as shown in Figure 2.11 can be found in the recent paper published by Stephan et al. [28]. Assume that the vortices extend in and out of the planar view presented in the figure. The circular arrow that is marked as BL, is a representation of vorticity sign of the induced shear layer due to the primary vortex pair in ground proximity and should not be mistaken for a vortex. These flow features are different from a vortex as these are shear dominant vorticity features. As shown in Figure 2.11, presence of crosswind flow develops an additional vorticity at the boundary layer (BL) beneath the upwind and downwind vortices. This additional vorticity supports the formation of same signed vorticity layer and attenuates the other. Thus, the flow separation from the boundary layer at the ground (as described in Section 2.6), occurs

earlier near downwind vortex in the presence of crosswind and the formation of secondary vortices are enhanced. Due to this flow phenomenon, evolution of the two primary vortices are asymmetric.

Figure 2.12 shows an image at time $t = 46$ seconds, from the Temporal LES of aircraft wake vortices in ground proximity in the presence of crosswind [28]. It can be seen from the figure that the secondary structures emerge out of the boundary layer on the ground in the presence of crosswind. It is also evident from the figure that the formation of secondary vortices around the downwind vortex is enhanced as compared to that of the upwind vortex in the presence of crosswind. Due to the prolonged interaction with the secondary vortices. That is, the downwind vortex decays faster as compared to the upwind vortex.

*Figure 2.12 Formation of secondary vortices in the presence of crosswind at $t = 46s$ * [28]*

Isosurfaces of $\|\omega^\| = 39.4$ coloured by ω_y^* in the spanwise direction. Note that x-direction is parallel to the longitudinal axis of the upwind and downwind vortex. y-direction is the lateral direction, that is, the direction parallel to the horizontal distance between the two vortices and z-direction is perpendicular to the xy-plane, out of the paper. Note that the image is reproduced from Ref. [28] and is a snapshot taken from a rotated three dimensional domain. Author could not add the reference axis that is aligned with the rotation since the rotated angle is not mentioned.

It is to be noted that the colour code given in Figure 2.12 is based on the non-dimensionalised vorticity strength (ω_y^*) in the spanwise direction. Vorticity is non-

dimensionalised using wing span and descent speed. Further explanation on the non-dimensionalisation is out of the scope for this particular section. The choice of vorticity in y-direction to visualise the vortices is made as the vorticity of secondary vortices dominates in this direction than the primary vortex pair. The primary vortex pair has higher vorticity values in the z-direction. Hence, the maximum and minimum values of the colour code in Figure 2.12 corresponds to the secondary vortices rather than the primary vortex pair.

A brief history of research findings on crosswind effects on wake vortices are presented below.

Tombac [36] showed experimentally that one of the two vortices decay faster under certain atmospheric conditions. Rossow [37] in 1977 through his point-vortex simulations found that the vortex with opposite vorticity sign to the crosswind shear decays rapidly while the other remains intact for longer time. This particular physical observation is named as Solitary vortex phenomenon. From here on, the vortex with same sign as crosswind will be referred as *upwind vortex* and the one with opposite sign as *downwind vortex*. Bilanin et al. [38] in 1978 and Ash et al. [39] in 1994 used turbulent transport equations to examine the influence of crosswind on the wake vortices. In both the studies, rolling moment on the follower aircraft is used as a parameter to quantify the effect. It is commonly concluded that one of the two vortices induced lesser rolling moment on the follower as compared to the other. In Bilanin et al. [38] study, passive tracers were used to visualize the vortex evolution.

Robins and Delisi [40] studied the crosswind effects on the wake vortex behavior using two dimensional incompressible Navier-Stokes equations. From their study, the vortices in ground proximity with the presence of crosswind were proven to be hazardous even after 3 minutes of evolution time. Zheng and Baek [41] examined the crosswind shear effect on the descend history of the vortices. Mokry [42] claims that in the case of a strong interaction between the crosswind shear and wake vortex pair, there is a possibility of temporary intensification of the wake vortices. Proctor et al. [43] suggested that the combination of crosswind shear and its shear gradient has profound effect on the vortex movement and decay. But in their studies, the effect of turbulence, stratification and ground

on the wake vortices were not considered. Proctor [44] in 2014 presented numerical result showing that the solitary vortex phenomenon in ground proximity is related to the derivatives of the crosswind.

Ambient turbulence and stratification

Due to the adiabatic compression of the vortex core, the center of the vortex gets warmer and lighter and when it starts to descend, the static pressure increases. If the atmospheric density also varies adiabatically with pressure, then the density of the core and the ambient air will be the same. If the lapse rate is not adiabatic then the difference will result in a buoyant force on the wake. This is referred as atmospheric stratification effect on the wake vortices, and it is effective only in the cruise altitudes. Ambient turbulence and stratification play a combinational role in the decay and descent of the vortices. Once again, there are countless simulations, experiments and in-situ measurements were published in the last four decades.

The impact of ambient turbulence with no crosswind and stratification on the wake vortices is low. In a non-stratified atmosphere, the vortices are initially disturbed by the short-wave instability (elliptic instabilities) due to the aircraft boundary layer induced turbulence. These instabilities grow in amplitude and result in Crow instability at later stages. The amplitude of these instabilities is observed to be increased by the presence of stratification. The induced baroclinic vorticity, owing to the stratified atmosphere, forms counter-rotating vertical streaks between the primary vortices and aids in effective turbulent vorticity exchange across the center plane [45]. The lifespan is shortened through core-bursting and generation of counter-sign vorticity [46].

Strong atmospheric stratification not only affects the strength of the vortices but also slows the descent rate of the vortex pair. Hecht et al. [47] showed that the vortex pair comes to a halt while descending in a stable density stratified atmosphere. A strongly stratified atmosphere can even cause a rebound to the flight level [45, 48]. Even though there is a rebound, the vortices decay significantly before reaching the flight altitude [45]. The wake behind a ‘Medium’ and ‘Light’ category aircrafts is less susceptible to the atmospheric stratification influence [49].

Robins and Delisi [40] performed a study on wake vortex pair in a stratified and sheared background flow. In this study, vortex evolution for various Richardson number (Ri), which is a ratio of buoyant force to shear force, are analyzed. For $Ri \leq 1$, that is, for shear flow dominated cases, asymmetric vortex descent and decay resulting in solitary vortex phenomenon occurred. For $Ri \geq 2$, that is, for stratification dominated flows, the vortices decay and descent symmetrically. In lower altitude, within the Atmospheric Boundary Layer (ABL), the vortices descent is reduced by 5m and lifetime increased by 5s. In ground effect, sensitivity analysis of behavior of wake vortices revealed that the decay is mostly independent of stratification and ambient turbulence [48].

Over time, an extensive literature was developed on the influence of the atmosphere on wake vortices. The core concepts and conclusions are the same as explained earlier in this section but was arrived at using different numerical and experimental techniques. Some of the noteworthy research papers examining the effect of atmospheric parameters on the decay of wake vortices are published by the following authors in chronological order: Scorer & Davenport [50] in 1970; Saffman [51] in 1972; Hill [52] in 1975; Greene [53] in 1986; Garten et al. [54] in 1998; Darracq et al. [55] in 1999; Han et al. [56] in 2000; Switzer and Proctor [57] in 2000; Gerz et al. [58] in 2002; Hofbauer and Holzäpfel [59] in 2003; Gerz and Baumann [60] in 2006; Dengler et al. [61] in 2012; De Vescher et al.[62] in 2013.

2.9 Artificial enhancements

One of the major ongoing researches is to find a way to artificially increase the dissipation rate of the wake vortices during landing/take-off. There are active and passive methods to artificially enhance the decay of the wake vortices or to reduce the induced drag.

2.9.1 *Passive methods*

The prominent passive methods include modification of wingtips [63] and ground. The review papers published by Breitsamter [13] and Hallock and Holzäpfel [64] present a brief review of passive vortex alleviation methods.

Wingtip modification:

Wing-tip devices are mainly used to reduce the induced drag [65] thereby indirectly affecting the energy fed into the wake vortices. NASA assessment on drag-reduction devices reported that wingtip devices increase the induced drag efficiency by 10-15% [66]. In an aerodynamic perspective, wingtip devices impact the flow pattern over the wing, modifies the spanwise loading, resulting in a significant change in the position, strength and shape of the wake [67].

Figure 2.13 Wingtip devices [67]

Figure 2.13 shows several wingtip shapes proposed and tested for artificial enhancement of wake vortex dissipation [67]. Hoerner and Küchemann wingtips are the simplest in geometry compared to the rest of the designs presented in the figure. Raked wingtip and Winglet are widely used in commercial aircrafts [68-70]. As part of the Modelling and Design of Advanced Wingtip devices (M-DAW) project, a variety of wingtip devices are tested and validated against the baseline Küchemann wingtip [71]. Complicated winglet shapes such as Spiroid winglets, wingtip sails and grids may increase the wave drag and are still under experimentation [67,72]. Rakelet series [70] exhibit mid characteristics between wingtip extensions (for example, Raked wingtip) and winglets. Reverse delta type wingtip modification is also found to be effective [73].

Ground modification:

Different vortex dissipation devices such as suction/blowing device or plate type barriers were installed on ground and experimented for its effect on wake vortex evolution

by NASA Langley Research Centre [74]. Barriers, parallel to the runway resulted in an increased dissipation rate of the vortices. On the other hand, installation of wall-like obstacles on the ground, perpendicular to the flight direction behind the touch down point of the aircraft is found to enhance the formation of secondary vortices locally. Figure 2.14 shows the wake vortices in proximity to an artificial obstacle. It can be observed from the figure that the obstacle is introduced in the direction perpendicular to the landing direction of the aircraft. The induced boundary layer is formed over the obstacle and so they are closer to the primary vortex pair as compared to the boundary layer formed at the ground. Hence, there is an early onset of elliptic instability and the secondary vortices are formed earlier at locations where the vortices cross the obstacle [28].

Figure 2.14 Wake vortices evolution with obstacle in proximity [28]*

*Isosurfaces of vorticity strength = 39.4 coloured by vorticity strength in the spanwise direction. Colour scale is given at the top of the figure. Note that the coordinate system is same as Fig. 2.12

These secondary vortices approach the primary vortices near the obstacle quicker and the disturbance is transported along the axial direction of the vortices [75-77]. Both experiments and simulations confirm this physical phenomenon and together conclude that the interaction of primary and secondary vortices are intensified thus leading to a rapid decay of the primary vortex. Wang and Schlüter [78] and Wang et al. [79-82] studied the effect of obstacle shape, aspect ratio and position on the wake vortices. Instead of mounting a single long obstacle, a line of plates has also proven to enhance the dissipation rate [83]. There is a 25% reduction in the wake vortices strength due to the installation of plate lines

[84]. Even though, an optimum size of these plate lines is aircraft-size-dependent, it is shown that a minimum aspect ratio of 0.25 and a length of 10m provides reasonable performance [82, 85]. Although this passive method seems to be effective, in general, airport authorities may not prefer to have any objects installed in/near the runway due to safety issues.

In addition, there have been patents filed for using jet blasts, water fountains or trees to enhance the formation of secondary vortices but none of them were effective. Other approaches include “turbulence injection” by means of splines, fins, vortex generators, spoilers, and Gurney flaps [86-90]. Stuff and Vollmers [91] induced the Raleigh Ludwig instability to attenuate the wake vortices of transport aircraft but it works only with certain conditions for control surface vortex characteristics.

2.9.2 *Active methods*

Most of the active methods include hastening the vortex breakup or inducing the instabilities at earlier stages of evolution. Modulation of lift distribution is a noteworthy active/passive vortex alleviation technique. This particular method is of higher relevance to this research and will be explained in detail in Chapter 5. In a nutshell, it was found through simulations and experiments that inboard loading of wing sheds multiple vortices closer to each other resulting in a faster decay. Oscillating flaps, spoilers, winglets, and ailerons were also tested as potential active control methods to manipulate vortices shed downstream [92-109]. All of these researches come to the same conclusion conceptually that the time-dependent control inputs are proven to be effective and can serve as a temporary alternative to enhance the dissipation of the vortices. However, passenger comfort and structural stress on the wings have to be studied in detail to realize these methods in practice. In this dissertation, altering the lift distribution over the wings, using flaps, will be one of the three main methods considered, to artificially enhance the dissipation rate of the vortices.

Vortex Leveraging Tabs installed at wingtips and horizontal tails as patented by Bilanin and Quackenbush [98], have safety issues despite its effectiveness. Lessen [96] has a patent on injecting jet flow through the vortex core to induce hydrodynamic instability.

Synthetic jet actuations were used to invoke the long wave instabilities earlier. However, implementation of such flow devices at the wing-tip is difficult and also not advisable for structural reasons. Most of these methods reduce the peak tangential velocity and diffuses the vortex over a wider region comparatively. However, the rolling moment will still be experienced by the small category aircrafts as it depends on the circulation rather than the velocity. Kranepuhl et al. [107] conducted experiments by passing spanwise alternating jets and found that it indeed reduces the maximum circulation strength in the downstream.

Many of these methods were only experimented in labs and a real-time implementation in an aircraft might make them obsolete. Important point to be noted from this section is that only a few of these methods were focused on attenuating the vortices in ground proximity. While finding an alleviation method in OGE itself is a difficult task, the ground effect adds complexity by imposing stringent restrictions on the usage of control surfaces and comfort of the passengers.

2.10 History of CFD methods

Betz [20] formulated theorems for an inviscid analytical method of studying the behavior of the vortices. It predicted the position of the rolled-up vortices and its movement downstream. Uniformly distributed vorticity method [110] is another method that described the wake well outside the core of the vortices. After almost 40 years, Donaldson [111] rediscovered the Betz method in his review paper. Following that, Mason [112] extended Betz method to any wing planforms that can be represented by lifting-line method. Discrete vortex approximation method was also implemented to model the roll-up and study the stratification effects.

With the introduction of finite different schemes and turbulence closure models, the unsteady two-dimensional simulations were performed. Since the vortex decay depends mainly on the three-dimensional instabilities, all of the above methods did not provide a better agreement with the experimental results. First three-dimensional analysis was published in 1996 [113]. Followed by many researchers studying the environmental effects (Section 2.8) in three dimensional simulations. Corjon and Poinso [114] performed a three-

dimensional Direct Numerical Simulation (DNS) of wake vortices. Reynolds Averaged Navier-Stokes (RANS) simulations are mainly used for flow over the wing studies. To study the evolution of wake vortices downstream at distances of multiple wingspans, RANS is used with LES as a hybrid method will be discussed in Section 2.11. Till date, successful three-dimensional simulations of far-field decay that agrees well with the experiments were resulted from LES [62,115-118]. These methods are still state-of-the-art.

Quite a few researches on wake vortices are performed using the vortex methods. These vortex methods can be used with LES methodology, solving for the Navier-Stokes vorticity equations instead of momentum equations [119]. Similar to the SubGrid Scale (SGS) models for modelling the momentum-based sub-grid scale eddies, there are models for vorticity-based LES technique too. Mansfield et al. [120] proposed a dynamic model similar to Dynamic Smagorinsky model to solve the vorticity-based Navier-Stokes equation.

Cho and Han [121] simulated the wing-tip vortices of an elliptical loaded wing and of fuselage/flap-wing configurations, using Discrete Vortex Methods. Liu [122] has performed analysis of wake vortex encounters using Vortex Lattice method. Wincklemans et al. [123] had presented the application of vortex particle method and vortex filament method in detail for simulating the trailing vortices. Vortex particle methods are combined with viscous schemes and particle redistribution for accuracy and convergence. One of the main advantages of using vortex methods is that the dispersion error is negligible. For cases with solid boundaries, these methods can be used with Boundary Element Method (BEM). Cogle et al. [124] had devised a new method of using vortex-in-cell methods in combination with parallel fast multipole methods for studying of instability of the aircraft trailing vortices .

The ground effect in these vortex methods are incorporated using image technique and are not as straightforward as their competitors (RANS and LES). Also, the vortex methods cannot capture the detachment of the flow from the ground and some of the three dimensional characteristics of the flow. These methods are computationally expensive too.

2.11 State-of-the-art simulation technique

The simulation methods evolved from an over-simplified 2D analytical calculation to three dimensional highly sophisticated DNS, RANS and LES. Most of the researches have treated the vortex roll-up process, development and decay as separate problems. RANS is preferred for the flow around the aircraft and the roll-up phase while LES is preferred for studying the decay of fully rolled-up vortices in the far-field [125, 126, 127]. In LES simulations, mostly the counter-rotating vortex pair out of ground effect is initialised as a two-independent vortex filament using various analytical vortex models [45, 128, 129]. This simulation approach is known as Temporal LES methodology or just Temporal simulation. In this methodology, the strength of the vortices are assumed to be constant along its axial direction and is proportional to the weight of the aircraft. Moreover, the path of the aircraft is not considered as it determines the location of the vortices. This method is also used in ground proximity and might help in understanding the general vortex behaviour in ground proximity. But, a more appropriate method is to consider the different phases of an aircraft for the results to be of practical significance.

Misaka et al. [130-132] used Hybrid LES/RANS method in which the LES domain is swept by the RANS flow field data. The aircraft is assumed to be landing in a designated path from one end of the computational domain. The region around the aircraft model is simulated using RANS method and the region in the far wake of the aircraft are simulated using LES. This approach is named as Spatial LES and includes all phases of the wake from roll-up to decay. This hybrid method is validated with experiments for cruising configuration [132] and high-lift configuration during landing in ground proximity [133].

As of today, this is most advanced method for simulating the vortices shed behind an accurate computer aided design of an aircraft, modelled from roll-up to decay, through all phases of a landing fight including engine exhaust. But in this case, RANS does take into account of the influence of ground in this method. Only, LES region take into account of the presence of ground using a wall distance parameter. Major concern in this coupled method is that, the length scale of the vorticity sheet roll-up is larger than the vortex core size. Thus,

the coupling of RANS and LES poses serious constraint for every change in the study parameters. For example, to study the jet-wake interaction, the RANS domain was increased from one chord length to 3.33 wingspans downstream [134]. Meshing is also a concern in this method as RANS requires finer mesh near the aircraft boundary layer while LES requires finer mesh near the ground.

2.12 Summary

Over six to seven decades, there are numerous papers published by authors from diverse backgrounds. There were studies on wake vortex dynamics, ground effects, atmospheric effects, artificial wake vortex decay enhancement methods using different numerical methods such as DNS, vortex methods, Hybrid RANS-LES and LES approaches. To sum up, Temporal LES methodology is too generic while the Spatial LES methodology is too complicated.

Wake vortex evolution involves a wide range of length and velocity scales. Simulating its reaction to any change in the aircraft or surrounding system is not an easy task and requires constant improvements. Hence, there is a need of a simpler methodology that is capable of simulating the vortices throughout its evolution phases. This is considered as one of the research focus in this dissertation. Adding to it, none of the proposed artificial enhancement methods so far has proven to work in practical and in large-scale, either they are aircraft specific or airport specific. Therefore, potential ways to enhance the wake vortex dissipation are also investigated as part of this research.

3 Methodology

Wake-vortices possess high turbulent kinetic energy involving a wide range of energy scales. The Reynolds number associated with such vortices are of a minimum order of 10^5 . The in-house code used to simulate this chaotic massively scaled flow is presented in this chapter. The methods used in the code to solve an unsteady incompressible viscous Navier-Stokes equation in the turbulent regime is discussed in this chapter. In addition to the methods, an overview of the simulation set-up used across all of the test cases is presented.

3.1 Turbulent shear Stress

Figure 3.1 Effect of turbulent eddies on a shear flow [135]

Turbulent flow visualization reveals that there are rotational flow structures named turbulent eddies. Consider a control volume in a two-dimensional shear flow as shown in Figure 3.1 [135]. The circular motions depicted in the figure are representation of turbulent eddies in the flow. In the presence of turbulent eddies, there will be a transport of energy and momentum into and out of the control volume. Because of this additional momentum exchange, there is a velocity gradient within the same layer of the shear flow. The schematics in the figure clearly show fluctuations with negative and positive y-velocities, v' within the same layer of the shear flow due to the presence of turbulent eddies. This

velocity gradient results in an additional turbulent shear stress known as **Reynolds stress**. Thus, the turbulent flow computations are entirely different from that of the laminar flows.

3.2 Governing equation

The governing equation in vector notation, under consideration are,

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j) = 0 \quad (3.1)$$

Momentum equation (Navier-Stokes equation):

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} (2\mu S_{ij}) - \frac{\partial \Psi}{\partial x_j} \quad (3.2)$$

where, $i, j = 1, 2, 3$ ρ – density, u_i – i th velocity component, x_i – i th spatial coordinate, p – pressure, μ – viscosity, S – turbulent shear stress, and Ψ - Gravitational potential. The flow is now assumed incompressible and viscous and has no external body forces acting on it.

Hence, continuity equation becomes,

$$\frac{\partial u_j}{\partial x_j} = 0 \quad (3.3)$$

and the momentum equation becomes,

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} (2\nu S_{ij}) \quad (3.4)$$

where ν – kinematic viscosity of the fluid. In the in-house code used for this research, the above-mentioned governing equations are solved using the Large Eddy Simulation (LES) technique. The principle of this technique along with its turbulence model is described in the subsequent sections.

3.3 Large Eddy Simulation (LES)

The principle of LES is as follows: The flow field is filtered using a spatial filter to separate large eddies from the small ones in the dissipation scale. The large scales of the flow are then solved through Navier-Stokes equation in time and space while a model is introduced to account for small scales. One of the main hypotheses in this method is that the small scales exhibit local-isotropy behaviour and are assumed to be statistically identical. The cut-off frequency that draws line between large-scale and small-scale is determined the spatial filter.

The three-dimensional velocity field, $\mathbf{u}(\mathbf{x}, t)$ is decomposed to a sum of resolved (filtered) and residual component by a filtering operation of a particular filter width (Δ) as follows,

$$\mathbf{u}(\mathbf{x}, t) = \bar{\mathbf{u}}(\mathbf{x}, t) + \mathbf{u}'(\mathbf{x}, t) \quad (3.5)$$

where $\bar{\mathbf{u}}$ – resolved velocity field in three-dimension, $\mathbf{x}$ – three-dimensional coordinate and $\mathbf{u}'$ – residual velocity field in three-dimension. Residual velocity field represents the smaller scales in the energy spectrum.

Resolved velocity field is given by, $\bar{\mathbf{u}}(\mathbf{x}, t) = \int G(\mathbf{r}, \mathbf{x}) \mathbf{u}(\mathbf{x} - \mathbf{r}, t) d\mathbf{r}$, where $G(\mathbf{r}, \mathbf{x})$ is the Spatial filter function. Substituting the equation for $\mathbf{u}(\mathbf{x}, t)$ into the governing equation gives the following,

Continuity equation (3.3) becomes,

$$\frac{\partial \bar{u}_j}{\partial x_j} = 0 \quad (3.6)$$

Momentum equation (3.4) becomes,

$$\frac{\partial \bar{u}_j}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_i \bar{u}_j) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} (2\nu \bar{S}_{ij}) \quad (3.7)$$

The second term on the left-hand side of the equation (3.7) is non-linear and is known as sub-grid-scale stresses or Residual stresses. It is expressed in terms of unresolved velocity components which add to the complexity as the equations are solved only for the filtered flow field variables. Hence, to solve the filtered continuity and momentum equations, the residual stress needs a closure model. This model should be able to capture the statistics of the unresolved scales and its effect on the evolution of resolved scales. The energy transfer between the resolved eddies of frequency that are close to the cut-off frequency, and the subgrid scales are assumed to resemble the energy dissipation due to viscosity. The only difference is that the viscosity used to represent the energy transfer is not a fluid property, but flow property represented by ν_{SGS} where SGS in the subscript denotes Sub-Grid Scale.

3.3.1 *Dynamic Smagorinsky model*

Smagorinsky [136] in 1963 proposed a simple linear model for the residual stress as follows,

$$\tau_{ij}^R = -2\nu_{SGS}\overline{S_{ij}}, \quad (3.8)$$

where τ_{ij}^R – residual stress, otherwise denoted as τ_{ij} , ν_{SGS} is the eddy viscosity of the residual stress and S_{ij} – turbulent shear stress, $S_{ij} = \partial\overline{u_i}/\partial x_j - \partial\overline{u_j}/\partial x_i$.

Eddy viscosity of the residual stress is given by an expression similar to that of mixing-length hypothesis of RANS closure model [136]. The length scale in this model is assumed to be proportional to the filter width.

$$\begin{aligned} \nu_{SGS} &= l_s^2 \overline{S_{ij}} = (c\Delta)^2 \overline{S_{ij}} \\ \tau_{ij} &= c\Delta^2 |\overline{S_{ij}}| \overline{S_{ij}} \end{aligned} \quad (3.9)$$

where, l_s is length scale, c is a constant of proportionality, otherwise known as *Smagorinsky coefficient*. For this model, ν_{SGS} is always positive and there is no backscatter;

that is, the energy is transferred only from larger eddies to the smaller and not the vice versa.

In the present study of wake-vortices, the modified Germano's method [137] of calculating the coefficient as proposed by Lilly [138] is used. Consider the non-linear term in the filtered momentum equation,

$$\overline{u_i u_j} = \overline{u_i} \overline{u_j} + \tau_{ij} \quad (3.10)$$

The first term on the right-hand side is the resolved part and the second term is the modelled part, which represents the sub-grid scales, otherwise known as Subgrid Residual Stress. Now a coarser test filter of filter width $\hat{\Delta} = 2\Delta$, is applied to the previously filtered momentum equation. The variables after applying test filter are denoted with the hat $\hat{\cdot}$ symbol and the corresponding non-linear term is given as,

$$\widehat{\overline{u_i u_j}} = \widehat{\overline{u_i}} \widehat{\overline{u_j}} + T_{ij} \quad (3.11)$$

where T_{ij} – subtest-scale stress. The next step is to subtract the test-scale average of Eq. 3.10 and Eq. 3.11 and the result is termed as Leonard stress ($\mathcal{L}$).

$$\mathcal{L} = T_{ij} - \widehat{\tau_{ij}} \quad (3.12)$$

This term represents the residual stress of the lowest resolved frequencies between the two filters. Assume the relationship proposed by Smagorinsky (Eq. 3.9) for the residual stresses,

$$\tau_{ij} = -c\Delta^2 |\overline{S_{ij}}| \overline{S_{ij}} = -cS(\overline{u}, \Delta), \quad T_{ij} = -cS(\widehat{\overline{u}}, \hat{\Delta}) \quad (3.13)$$

Substituting Eq. 3.13 into the Leonard stress term ($\mathcal{L}$) (Eq.3.12),

$$\mathcal{L} = c [S(\widehat{\overline{u}}, \hat{\Delta}) - S(\overline{u}, \Delta)] \quad (3.14)$$

$$\text{Considering, } \mathcal{M} = S(\widehat{\overline{u}}, \hat{\Delta}) - S(\overline{u}, \Delta), \quad \mathcal{L} = c\mathcal{M}. \quad (3.15)$$

$$c = \frac{\langle \mathcal{L} \cdot \mathcal{M} \rangle}{2\langle \mathcal{M} \cdot \mathcal{M} \rangle} \quad (3.16)$$

Least Square method is used to solve for the Dynamic Smagorinsky co-efficient (c). The dot products in the numerator and denominator are averaged over the homogenous direction of the given flow.

3.4 Numerical methods

The numerical methods used to solve the filtered Navier-Stokes equation are of second order accuracy. The methods and models used in the code are explained in the following sections. It is to be noted that *Second Order Finite Volume Method* is used to represent all the spatial derivatives of velocity field. At the boundaries, first-order one sided approximation is used.

3.4.1 Velocity-Pressure coupling

First and foremost problem in solving the filtered Navier-Stokes equation is the velocity-pressure coupling. Since the flow is incompressible, the momentum and continuity equations are decoupled and the pressure equation is solved implicitly.

The central idea is as follows [139],

- i. The pressure field is initially assumed to be known and an intermediate velocity field is obtained by solving the discretised momentum equation.
- ii. A pseudo pressure equation is formulated from the continuity equation. The intermediate velocities are used to solve the pseudo pressure equation.
- iii. This pseudo pressure is used to correct the intermediate velocity and obtain the final velocity and associated pressure field for the next time step.

Figure 3.2 Staggered grid [140]

While it is sufficient to perform two iterations for each time step, in order to maintain accuracy in time, a considerable number of iterations are performed.

Also, this method is used along with a special grid called Staggered grid as shown in Figure 3.2 [140]. To capture the influence of pressure gradient accurately, the pressure field information are to be stored at the staggered grid centred around the cell faces of the momentum cell. It is then convenient to explain that the flow with horizontal velocity w from the point P to point E is due to the pressure difference between the points P and E.

3.4.2 *Semi-implicit time scheme*

The convection and diffusion terms in the wall-normal direction poses a severe restriction for time-step. Hence, it is advisable to treat these terms in the wall normal direction *implicitly*. A modified third order Runge-Kutta scheme is used for terms treated explicitly and second order Crank-Nicholson is used for terms treated implicitly [141, 142].

Using the semi-implicit time advancement technique will be a problem because of the absence of time derivative in the continuity equation. To preserve the second-order accuracy of the implicit scheme and also avoid the coupling problem in time advancement, Fractional Step Method in conjunction with the approximate factorisation is used [143]. *Semi-implicit time* advancement technique is adopted as it requires significantly lesser storage space and computational time compared to fully implicit scheme.

3.4.3 *Courant–Friedrichs–Lewy (CFL) condition*

Since the method is semi-implicit, it is necessary to have a check on the time advancement. Due to the small time step inherently imposed by the CFL condition for LES of turbulent flows, the semi-implicit time advancement is sufficient for numerical stability. The maximum allowable CFL number is set to be 0.5 and the maximum incremental time is set to be 0.025. The code automatically reduces the incremental time advancement below the maximum allowed if there is any divergence detected in the solution. Hence, the actual time advancement used in the simulation is up to 0.0027.

3.4.4 *Poisson equation*

Poisson pressure equation is solved by combination of Fourier transform method and iterative solver. By setting an uniform computational grid and periodic boundary conditions in z-direction, Fourier transform can be used to convert the three dimensional Poisson solver to two dimensional uncoupled equation (Helmholtz equation). Second order finite volume discretisation is employed for the spatial derivatives. The resulting discretised equation involves the values from all four neighbouring nodes. Hence, a modified version of the iterative solver, Alternate Direction Implicit (ADI) is applied. When using a non-uniform grid in the wall-normal direction, it is beneficial to treat the corresponding terms implicitly.

The finer the mesh size, the lesser is the discretisation error, as well as the *lower* is the convergence rate. After a careful analysis of the error behaviour in different mesh resolution, it is concluded that the short wavelength errors are attenuated rapidly by the iterative solvers. Hence, if the mesh is fine, the longer wavelength errors persist in the solution for many iterations thus affecting the convergence rate. Multigrid solver technique as described in the next sub-section is adopted along with the Poisson iterative solver to improve the convergence rate.

3.4.5 Multigrid method

The basic concept is that the longer wavelength errors can be eliminated faster in a coarse mesh while short wavelength in a finer mesh resolutions.

Figure 3.3 Example : Two level multigrid method schematics [144]

A simplest two-level multigrid solver as shown in Figure 3.3 [144] can be explained as follows,

- i. The iterations are performed on the finest grid and the intermediate solutions are generated. Error and residual vectors are calculated at this mesh resolution.
- ii. *Restriction ($\mathcal{R}$)*: The solution and the residual vectors are then interpolated to a coarser grid and the iterations are performed once again. This time the iterations are performed for the error equation formulated as per multigrid method. As proceed with the iteration the longer wavelength errors are eliminated as it is shorter for the coarse mesh.
- iii. *Prolongation ($\mathcal{P}$)*: The final error and residual vector are transferred back to the fine mesh (through linear interpolation) to correct the intermediate solution. The iterations are performed once again to arrive at the final solution for the governing equation.

In the in-house code, a **W-cycle Multigrid solver** with **five levels** is used for solving the Poisson equation. For a simple illustration of a higher order W-cycle, a 3-level W-cycle is presented in Figure 3.4. For a 3-level W-cycle, three types of mesh sizes are considered. In the figure, the downward arrows represent the Restriction step and the upward arrows represent the Prolongation step of the solver algorithm. Similarly, for a 5-level W-cycle,

five mesh sizes from fine to coarse are considered and the Restriction and Prolongation are performed accordingly.

Figure 3.4 Example: Three level W-cycle multigrid method [144]

3.5 Initial conditions

3.5.1 Vortex initialisation

The absolute value of circulation depends highly on the approach speed, approach mass and lift distribution. A circulation of 530m²/s and core radius of 3m which are representative of ICAO category ‘Heavy’ aircraft, are chosen as the strength of the initial counter rotating vortex pair and is imposed in the computational domain using Lamb-Oseen vortex model [145].

The velocity components of the Lamb-Oseen vortex model are given as follows,

$$U = \frac{-y\Gamma_o}{2\pi\sqrt{x^2 + y^2}} \left[1 + e^{\frac{-(x^2+y^2)}{r_c}} \right], \quad (3.17)$$

$$V = \frac{-x\Gamma_o}{2\pi\sqrt{x^2 + y^2}} \left[1 + e^{\frac{-(x^2+y^2)}{r_c}} \right] \quad (3.18)$$

where Γ_o is the initial strength of the vortex pair, U and V are the induced velocity components in x and y direction respectively, r_c is the radius of the vortex core. The radius of the vortex core is considered as 3m, same as the one used in the experimentally validated computational method of DLR.

Table 3.1 Non-dimensionalisation of spatial and time coordinates

Non-dimensionalised time, t^*	t/t_o
Non-dimensionalised Spatial coordinates, x^*, y^*, z^*	$x/b_o, y/b_o, z/b_o$

The speciality of this model is that it blends the viscous inner vortex core with the outer potential flow. Separation distance of one wingspan ($b_o = 47.1\text{m}$) is considered. The vortices are initialised at a height of 47.1m . The aircrafts are said to be in ground effect only when they are initialised at a height of one wing span. Initial descent speed of the vortex pair (V_o), is calculated to be 1.79m/s using Biot-Savart law as described in Chapter 2. The time scale, t_o is given by b_o / V_o which is 26.3s .

3.5.2 *Inflow initialization*

The inflow initialisation adopted in this research is a well-established inflow technique used for LES of wide variety of flows in the literatures [146, 147]. Schlüter et al.[147] named this method as Matching Database method. This method involves two steps as follows:

1. A separate periodic LES is performed to reflect the specific mean velocity profile and a turbulent statistics, chosen from the experimental/DNS data. This is referred as precursor flow simulation. A virtual body force is used to drive the flow inside the domain. Since this simulation is periodic, a fully developed flow will be established sooner or later. It is essential for the background flow to have realistic structures to ensure that the onset of wake-vortex decay is simulated accurately. Hence, it is necessary to perform this precursor simulation to obtain realistic turbulent eddies. This method also ensures that the turbulent kinetic energy distribution with respect to the wave number follows the decay pattern. For the current study, the DNS results of pipe flow by Moser et al.[148] is considered as the baseline.

2. The velocity information at a cross-sectional plane in the fully developed region is stored in a database for every few time steps. The mean velocity and the turbulent statistics from the inflow database are rescaled to obtain the desired values using body forces in the x-momentum equation. The boundary layer is imposed to the mean flow profile of the precursor flow simulation with a specific boundary layer thickness. Finally the resulting velocity field is rewritten as a new inflow database and is fed to the main simulation domain at the inlet nodes. Linear interpolation is used for any mesh or time mismatch. Once the end of database is reached, the inflow conditions are recycled for the rest of the main simulation.

The advantage of this method is that for any change in the desired velocity profile, it is not necessary to perform the channel flow simulation every time. Since the inflow here represents the atmospheric crosswind, in all the simulations presented in the Chapters 4, 5 and 6, the flow is allowed to develop through the computational domain before the vortex initialisation. This ensures that the turbulent structures and wind information mimic the ambient atmosphere as close as possible.

3.5.3 *Boundary conditions*

For all **outflow boundaries**, *convective condition* is used since it is best suited for the advection turbulent structures outside the domain. The convective velocity at every time step is assumed to be constant and equal to the maximum outflow velocity in the outflow boundary. Top and bottom planes in y-direction are given no-slip Dirichlet boundary condition for velocity. The top plane is placed at a considerable distance from the vortex so that the wall-condition will not affect the vortices. Extremes planes in z-direction is defined as periodic to facilitate the Fourier transform in Poisson equation solver.

3.5.4 *Jetcode*

In this research, a new simulation package named Jetcode from hereon, is used. It is a set of FORTRAN codes developed at the Centre for Turbulence Research, Stanford University. The code was proven in simulating simple turbulent flow like backward step

turbulent flow and coaxial annular flows with *second order accuracy* in *time* and *space* [149]. The solver and its variations have been validated on a large variety of turbulent flow [150-159]. Pierce [159] in 2001 revised the Jetcode for simulating the combustion flow and also updated few of the underlying methods to improve its efficiency. The version of the code published by Pierce in 2001 is adopted for the current research. However, it is modified to solve only the momentum and continuity equations through LES technique. The code is validated with the DLR experimental results and the outcomes are presented in Section 3.6.

Jetcode consists of two main folders: *jet_forcing* and *setup* as well as two auxiliary folders: *library* and *inflow*. *Setup* folder has the routines to mention the computation domain, mesh size, and to initialise the velocity flow fields. These routines are executed only at time, $t = 0$. *Library* folder has auxiliary mathematical functions used in the models to solve the governing equations. For example, a function for solving penta-diagonal and tri-diagonal algebraic equations can be found in this folder. *Jet_forcing* folders control over the entire simulation and time loop. It includes: the description of boundary conditions, inflow/outflow conditions, LES filtering and SGS model, routines for parallel-processing, procedures to solve the governing, definitions for meshes of staggered grid in Cartesian co-ordinate, data handling routines and the records of flow statistics for all time steps. It is capable of running in multiple processors (up to 128 cores).

Some of the *advantages* of Jetcode over available commercial software are,

- i. Faster execution time, 1-2 order of magnitude faster than the commercial software.
- ii. Second order accurate in both time and space - sufficient for the current research.
- iii. Flexibility in editing the software routines to incorporate any other methods.
- iv. The codes can handle a Reynolds number with the order of 10^5 .
- v. It is stable and robust.
- vi. Parallel processing enabled

The only *disadvantages* of using this software tool is the user interface. Since it comprises of 20,000 lines of code, careful understanding of every function is mandatory to make any changes.

3.5.5 *Computational grid*

Figure 3.5 Computational domain

Figure 3.5 shows the computational domain considered for the simulations presented. The dimension of the computational domain is $8b_o \times 5b_o \times 8b_o$ as shown in Figure 3.5, where b_o is the wing span of the aircraft considered. The origin is located at the centre of the bottom plane. Hence, the domain extends from -4 to 4 in the x - and z -direction and from 0 to 5 in the y -direction. For all the simulations presented in this report, the mesh is stretched in the wall-normal direction (y -direction) so that the mesh size is small enough to resolve the boundary layer near the wall. A combination of hyperbolic tangent functions are used in the stretching function of the mesh size. In the streamwise direction (z -direction), a uniform meshing is prescribed due to the nature of the vortex movement. Also, uniform mesh is a suitable choice as it is simple and help to improve the accuracy. In the spanwise direction (x -direction), uniform mesh is implemented in order to solve the pressure Poisson equation using Fourier transform method. The computational grid defines the LES filter width implicitly. Hence, mesh size is very critical for the simulation efficiency.

3.6 Validation and verification

DLR towing tank experiment were conducted at Wasser Schleppkanal Göttingen (WSG), in Göttingen and the results are used as reference for validation purposes [28]. The tank is 18m long with a test section of $1.1\text{m} \times 1.1\text{m}$. The tank is equipped with a carriage to tow the model and can traverse through the tank with a speed of 5m/s. Instead of a recirculating water tunnel, this type of towing tank experiment enables measurements of older age vortices. Before every run, water is left to rest for at least 20 minutes to minimize the turbulence levels. One wingspan distance between the wall and the test model is necessary to minimize the side wall influence.

A F13 model aircraft with a rectangular wing of wingspan 175 mm and a chord length of 35 mm is propelled across the tank. The airfoil profile considered is Wortman FX63-137B-PT. Contrast agents are set to be released from the wingtips to track the center of the vortices. The carriage and the holder on the model allow for vertical position adjustment. The flow field is measured using a time-resolved stereo Particle Image Velocimetry (PIV) system. The initial measured vortex parameters are given in Table 3.2.

The vortex parameters mentioned in Table 3.2 is used for setting up the LES of wake-vortex pair.

Table 3.2 Initial measured vortex parameters

Circulation	0.052 m ² /s
Descent speed	459 mm/s
Separation distance	153 mm
Reference time	3.1 s
Reynolds number	52,000
Towing speed	2.44 m/s

Reynolds number similarity is followed for direct comparison of results in this section. Circulation of the wake-vortices is chosen to be the parameter of study for validating the simulated results as it represents the strength of the evolving vortex pair. Figure 3.6 shows the comparison of the circulation strength of the wake-vortices in ground proximity between the Jetcode and DLR water tunnel experiment. It can be clearly seen from Figure 3.6 that the simulated data has a good correlation with the experimental data set. Both exhibits a diffusion phase where the vortices diffuse followed by a rapid decay phase. In the diffusion phase, the vortices gradually diffuse and the circulation strength do not vary much. While, in the rapid decay phase, the secondary vortices are formed and so the primary vortices strength drops rapidly.

Figure 3.6 Validation of Jetcode with DLR water tunnel experiment [28]

From $t^* = 0.5 - 1.0$, there is a slight difference between the two results. One of the possible reasons for the mismatch in the diffusion phase is due to the difference in the method to calculate and non-dimensionalise the circulation strength from experimental and numerical data. Since the vortices possess higher strength in the diffusion phase, it is unavoidable to have these minor errors at the earlier time instances. However, Jetcode provides a decent correlation with the experimental data thereafter which is more important for studying the far-field wake decay characteristics.

Different mesh sizes are compared against the experimental result for the mesh refinement study and the circulation is chosen to be the parameter of comparison. The simulation results which match well with the experimental results is considered as the criteria for the convergence test. Figure 3.7 presents the results of convergence test for various mesh sizes. 128x64x64, 256x128x128, 33.5 million and 67.1 million grid points are used for the convergence test as indicated in Figure 3.7. Since the accuracy of 33.5 million nodes is acceptable and further reduction in mesh size did not give an equivalent increase in accuracy, a mesh resolution of 33.5 million cells is chosen for first half of the simulation. As the simulations were transferred from NTU HPC to NSCC, a convergence test was once again performed and a mesh resolution of 27 million grid points are found to have same accuracy. Hence, 27 million grid points are adopted for the simulations presented in Chapters 5 and 6.

Figure 3.7 Convergence test

3.7 Post-processing algorithm

Two important parameters used to study the characteristics of the wake-vortices are vortex position and circulation strength. In Section 3.7.1, an algorithm used to find the vortex position in lateral and vertical directions and its circulation strength is described.

3.7.1 Characteristics of Vortex

Aim: To find the non-dimensionalised circulation, lateral and vertical position of the centre of the vortex pair

Circulation is given by the mathematical formulation, $\Gamma = - \oint \vec{V} \cdot d\vec{s}$ with line integral taken around a closed loop represented by $\vec{s}$ in a counter-clockwise direction. Stokes' theorem is used to convert the line integral to surface integral.

Algorithm:

1. The two-dimensional results are generally exported in a '.csv' format. The name of the files reflect the computed time step. For example, if the results are stored for every time step of 0.05 up to $t^* = 5$. Then, there will be 100 files named 0001.csv, 0002.csv,...0100.csv. Every grid point in the mesh is referred as node and possess a specific coordinate in x, y and z direction. The flow field information are stored in this '.csv' files for each of the nodes. Hence, for a domain with 33.5 million grid points, there will be 33.5 million data for pressure, vorticity and velocity fields in the file.
2. Import all the results files of format '.csv' in the source directory. The order of the imported files will be different in MATLAB from that it looks in the folder view of the computer.
3. Instead of sorting the files, the csv files names are read and the numbers are extracted in array called tint, as they are named after the time steps.
4. The '.csv' files are accessed one by one using the tint variable in the sequential order of time. The pressure and vorticity component values are imported.

5. The two-dimensional domain is divided into two segments in the x-direction with each segment containing one of the two vortices. Since the vortices are initialised one on the positive x-direction and the other on the negative x-direction, the line of separation can initially be assumed to be at $x^* = 0$.
6. The coordinates in the x- and y- directions corresponding to the minimum pressure denotes the centre of the vortex in each of the segment. Vortex position is also cross verified using maximum vorticity criteria.
7. Since the vortices move in the positive x-direction with time, the line of separation is also moved in the same direction. A distance of $\Delta x^* = 0.3$ is maintained from the position of the centre of the upwind vortex for all time steps. This is to ensure that there is no influence of the vorticity distribution of one vortex in the calculation of the strength of the other.
8. Due to the presence of crosswind in most of the simulated cases, the downwind vortex moves out of the domain after a time period. Hence, a condition is set to stop finding the vortex centre of the downwind vortex after it reaches $x^* = 3.7$.
9. Next step is to calculate the vortex strength. The calculation of circulation needs a minimum and maximum radius within which the maximum amount of vorticity of the vortices is distributed. Majority of the simulations use a minimum radius of 5m and a maximum of 15m. It is the radius limits within which laser measurements of vortices are available for validation purposes. The simulations in which multiple vortex pairs such as wing-tip vortex, outboard and inboard flap vortices are present in the flow, the maximum radius is reduced to 12m. This radius is carefully chosen by trial and error process so that the presence of other flap vortices do not affect the calculation of the strength of the wing-tip vortex.
10. Once the radius is chosen, the vorticity and position information of the nodes that lie within this radius are stored in a separate matrix.
11. Circulation is surface integral of vorticity ($v(x, y, z)$). Since the domain for each nodal value is a simple rectangle, the surface integral reduces to area integration as follows.

$$\iint_S v(x, y, z) ds = \iint_A v(x, y) dx dy h \quad (3.19)$$

where v – z-component of the vorticity, dx, dy – size of the mesh element corresponding to the calculated node in x- and y- directions.

12. Circulation is found for all the time steps and is stored in an array called `gama1` and `gama2` for the two vortices respectively.
13. The lateral and vertical positions of vortices are also saved for plotting with respect to time.
14. The vortex centre calculation for upwind and downwind vortices is performed until they reach a distance of $x^* = 3.7$.

The area integration is performed up to a radius of 15m from the center of the vortex. A special condition is imposed to avoid the influence of boundary layer turbulence. Also, in practice, the strength of the wake-vortices from LIDAR measurements were calculated between the radii of 5m and 15m.

3.7.2 *Flap vortex*

The flap vortices considered in this research work belong to inboard tip of the inboard flap. They have opposite vorticity sign as compared to their closest primary vortex. Hence, it is easier to find the opposite maximum vorticity following the same steps in Section 3.7.1.

3.8 *Measure for secondary vortices*

Q-criteria is chosen to *visualize* the turbulent structure. In an incompressible flow, vortices are assumed to be a connected fluid region with a positive second invariant of velocity vector [160].

$$Q = -\frac{1}{2}u_{i,j}u_{j,i} = \frac{1}{2}(\|\boldsymbol{\Omega}\|^2 - \|\boldsymbol{S}\|^2) > 0 \quad (3.20)$$

where Q – Q-criteria, u – velocity, i,j – gradient in vector notation, Ω – vorticity tensor and S – rate of strain tensor. It should be noted that in tensor notation the subscript comma denotes differentiation. That is, $u_{1,1}$ represents the $\frac{\partial u}{\partial x}$.

In general, gradient of the velocity field can be high due to the presence of high shear or turbulent eddies. Velocity field tensor comprises of two parts: Rate of strain tensor and Vorticity tensor. Rate of strain tensor is the symmetric part of the velocity gradient tensor and is higher in the shear dominated flows. Vorticity tensor is the asymmetric part of the velocity gradient tensor and is higher in the eddies dominated flows. However, for extreme cases of shear flow with eddies, the velocity tensor to define the flow topology is insufficient. Hence, a higher order invariant of the velocity field became necessary. The second invariant of the velocity field tensor is referred as Q and is found to be helpful in devising a criterion to distinguish the turbulent eddies from the shear layers in a flow. In practical terms, Q represents a balance between the shear strain rate and the vorticity magnitude. The regions where the vorticity tensor is higher than the magnitude of the strain-rate tensor is defined as vortices. Hence the fluid region with positive Q value connected to each other is identified as eddies/vortices. This condition is referred as Q-criteria. In addition to this, there is a secondary condition by which the pressure within this region has to be lower than the ambient pressure.

Table 3.3 Summary of Q-criteria range for different flow features

Flow features	Range of Q-criteria value
Primary vortex pair	1000 – 10,000
Secondary vortices	1 – 100
Crosswind turbulent structures	≤ 1

The primary vortex pair is the strongest as it is proportional to the weight of the aircraft. The secondary vortices are induced by the primary vortices in ground proximity and their circulation is significantly less than that of primary vortices. Lastly, the crosswind

turbulent structures are numerically added fluctuations approximating the disturbances caused by the crosswind flow of 1.7 m/s around the buildings and other structures in the vicinity of the runway. It possesses the least strength out of the three.

Table 3.3 summarizes the Q-criteria values for different flow features. Based on the knowledge on the strengths of different vortices and an extensive visual analysis of the Q-criteria ranges obtained from simulation cases considered within the scope of this research. From the table, it can be observed that the strength of the primary vortex is equal to or greater than 10 orders of magnitude when compared to the strength of the secondary vortices generated from the induced shear layer. The Q-criteria values of primary and secondary vortices are compared for all the simulation cases considered in this research. It is commonly found in all of the simulated cases that the Q-criteria value corresponding to the primary vortex pair is of the order of 1000-10,000 while that of secondary vortices are less than 100 for the considered duration of the simulation time. The Q-criteria values of turbulent structures found in the background flow is less than or equal to 1.

A new quantifying parameter $|Q|$, is proposed based on this distinct category of Q-criteria values corresponding to different flow features in the domain. The main objective of this new parameter is to quantify the amount of secondary vortices generated. Circulation cannot be calculated for the secondary vortices due to the presence of strong primary vortex pair in proximity. A threshold of 1-100 is set for the Q-criteria value. This threshold will remove the primary vortex pair from the domain leaving only the secondary vortex structures as shown in Figure 3.8. The coordinates x^* and y^* mentioned in Figure 3.8 are non-dimensionalised x and y coordinates by the wingspan (b_0). A volume integration of the threshold imposed Q-criteria is represented as $|Q|$. $|Q|$ that belongs to the secondary vortices regime is calculated for every time step. Although turbulent structures from background flow is visible in the Figure 3.8, they belong to Q-criteria value equals to 1 and do not affect the final values of $|Q|$. A careful investigation was performed to ensure that $|Q|$ represents only the secondary vortices within the duration of the simulation.

Figure 3.8 *Q-criteria of secondary vortices**

*Note that the axis presented in the top left corner is the coordinate direction for reference.

In general, primary vortex pair decay comprises of two phases: *Diffusion Phase* and *Rapid Decay Phase*. The formation of secondary vortices marks the rapid decay phase. $|Q|$ represents the quantity of vorticity magnitude within the secondary vortices. A typical $|Q|$ versus time curve is presented in Figure 3.9. It starts with zero and has a value closer to one during the diffusion phase of the primary vortex. As the flow gradually evolves and secondary vortices are shed from the induced shear layer in ground proximity, the value of $|Q|$ takes-off and reaches a maximum.

Figure 3.9 Typical $|Q|$ versus time plot

Due to the presence of crosswind, the downwind vortex pair exits the domain after a certain amount of time. This exit time is based on the crosswind speed. Here in the simulation case presented in Figure 3.9, the downwind vortex exits the domain just around $t^* = 3.9$. When the downwind vortex exits the domain, it also takes the secondary vortices along with it. Hence, there is a sudden drop in the $|Q|$ value after $t^* = 3.9$. This parameter is used in Chapters 5 and 6 to justify the effectiveness of certain modified span loading configurations in producing higher degree of secondary vortices.

4 Parametric Study – Temporal Simulation

The important focus of this dissertation is to propose new ways to alleviate the danger posed by the wake-vortices in ground proximity. It is well established in the literature that the atmospheric parameters influence the position and decay rate of the primary vortex pair to a great extent. In this dissertation, influence of atmospheric parameters and lift distribution on the wake vortices are studied. Since the computational capacity is limited, it is essential to find an optimum atmospheric condition that will be effective in decaying the vortices at a faster rate. This atmospheric conditions will then be a fixed parameter out of the two and will be then used in the simulations to study the influence of lift distribution on the wake vortices presented in Chapter 5 and 6.

Out of the three main atmospheric parameters listed in Section 2.8, crosswind and turbulence level are chosen for the current parametric study. The third parameter, atmospheric stratification affects the vortices only at higher altitudes and so it is ignored in the current study. This chapter provides a basic understanding of the influence of these two parameters on the formation of secondary vortices and on the temporal evolution of primary vortices.

Since it is a straightforward parametric study, Temporal simulation methodology is used due to its simplicity. Also, it is to be noted that the Temporal simulation methodology forms the basis of the new methodology which will be proposed in Chapter 5. Although there have been many crosswind studies performed earlier, current study takes the analysis one step deeper. A wide range of crosswind velocities are tested as part of this research. Five different turbulent intensity levels are considered. The vortex centreline in the three dimensional computational domain is tracked to understand the effect of atmospheric parameters in detail. Crosswise velocity and its exit time are introduced as new parameters to predict the vortex position for various crosswind velocities. Change in relative angle and radial separation distance of the wake-vortices for different background flow conditions are also analysed.

4.1 Initial conditions for Temporal simulation

Temporal simulation involves initialising the vortices as a fully developed cylindrical counter-rotating vortex pair using Lamb-Oseen vortex model. The vortex initialization parameters chosen for the current study corresponds to that of ‘Heavy’ category aircrafts. The details of the initial parameters of the wake-vortices are given in Table 4.1.

Table 4.1 Vortex initial parameters - Temporal Simulation

Circulation, Γ_o	530 m ² /s
Wing span, b_o	47.1 m
Descent speed of the vortex, V_o	1.79 m/s
Height of the vortex core from ground, b_o	47.1 m
Distance between the two vortices, b_o	47.1 m
Reynolds number, Re_Γ	23120
Characteristic time scale, t_o	26.3 s

Table 4.2 Non-dimensionalised variables

Non-dimensionalised time, t^*	t/t_o
Non-dimensionalised spatial coordinates, x^*, y^*, z^*	$x/b_o, y/b_o, z/b_o$
Non-dimensionalised circulation, Γ^*	Γ/Γ_o
Non-dimensionalised inflow velocity, v^*	u_{inflow}/V_o

Circulation and radius of the vortex core in Table 4.1, are input parameters for the Lamb-Oseen vortex model. Descent speed is calculated using the Biot-Savart law and is used as the characteristic velocity scale. Characteristic time scale and wing span are used for non-dimensionalisation of circulation, time, length and velocity as listed in Table 4.2.

4.2 Inflow profile

4.2.1 CAAS – *Manual of Aerodrome Standards*:

The choice of maximum permissible crosswind component in runway from the manual of Aerodrome Standards by Civil Aviation Authority of Singapore (CAAS) [161], under Section 7.2.1.3 is given as follows,

“— 37 km/h (20 knots) in the case of aeroplanes whose reference field length is 1,500 m or over, except that when poor runway braking action owing to an insufficient longitudinal coefficient of friction is experienced with some frequency, a cross-wind component not exceeding 24 km/h (13 knots) should be assumed;

— 24 km/h (13 knots) in the case of aeroplanes whose reference field length is 1,200 m or up to but not including 1,500 m; and

— 19 km/h (10 knots) in the case of aeroplanes whose reference field length is less than 1,200 m.”

4.2.2 Crosswind velocity limits

Generally, in the presence of crosswind low velocities, the vortices stay longer in the runway and poses threat to the follow aircraft. The maximum allowable crosswind velocity limit mentioned in the CAAS manual is 5.14 m/s (10kt) for aeroplanes with reference field length less than 1200m. Hence, the crosswind with velocity limits from greater than 0m/s – 5.14m/s are ideal for studying their influence on the wake vortices. Due to the requirement of bigger computational domain for higher crosswinds, in this research work, a crosswind speed of up to 4.8m/s which is 220% of the descent speed (V_o) is initially considered. While examining the results, it is found that the primary vortices of the case with crosswind

velocity 4.8m/s moves laterally outside the computational domain even before the formation of secondary vortices, in less than 1minute. Since the main purpose of this study is to examine the interaction between the primary and secondary vortices under different crosswind speeds, the case with 4.8m/s is omitted for the study and the crosswinds upto approximately of 3m/s are considered. Since the descent speed ($V_o = 1.79\text{m/s}$) is used for the non-dimensionalisation for the velocity scale, the crosswind velocities are considered in multiples of the descent speed for the ease of computational initialization.

Since the simulations are performed in the low altitudes, there is no significant difference in the pressure, temperature and density of the atmospheric wind. Hence, the atmospheric boundary layer flow is considered as a fully developed pipe flow with turbulent boundary layer. Due to the comparatively smaller computational domain considered, this assumption will hold true for all of the simulation cases considered in this scope of work. It is also to be noted that this is one of the best practices used in the wake vortex field to mimic the background atmospheric flow.

To generate the background inflow database, a Matching database method [147] as described earlier in Section 3.5.2 is used. A separate periodic LES of the pipe flow (Precursor simulation) is performed. This is to ensure that the flow has realistic turbulent structures that are responsible for inducing the vortex instabilities. The resulting flow information are then scaled up and down by a percentage, to obtain the necessary crosswind speeds and turbulence levels. The advantage of this method is that it is not necessary to perform the precursor simulation for every change in the desired inflow profile. It saves time and computational costs at the same time maintains the accuracy.

Table 4.3 provides the list of different crosswind velocities considered for the current study. The mean inflow velocity of the turbulent pipe flow is scaled up for each crosswind case by the corresponding percentage mentioned in the table, so that it is equal to the required crosswind speed.

Table 4.3 Crosswind flow velocities

Case no.	Crosswind speed (% of V_o)	Non-dimensionalised crosswind velocity (v^*)	Required Crosswind speed (U_{inflow}), in m/s
1	20	0.2	0.36
2	40	0.4	0.72
3	60	0.6	1.07
4	80	0.8	1.43
5	100	1.0	1.79
6	120	1.2	2.15
7	170	1.7	3.04

Figure 4.1 Crosswind velocity profile for Case no. 5 (as listed in Table 4.3)

The simulations are performed closer to ground and so the atmospheric boundary layer are ignored. Instead, shear layer due to ground vicinity are considered. To get the initial shear near the ground, we are employing a boundary layer profile from ground ($z = 0$)

to b. The boundary length is chosen such that the vortices stay in ground effect (in the shear layers) during their lifespan.

Figure 4.1 represents a typical inflow velocity profile for Case no. 5. The non-dimensionalised boundary layer thickness is fixed for all the cases. It extends till $y^* = 1$ where the flow velocity reaches 99% of the required freestream crosswind velocity. The turbulence level is neither reduced nor increased for the crosswind study (Cases no. 1-7). Hence, the inflow turbulence is equal to the turbulence generated in the pipe flow. In reality, the fluctuations in the inflow profile symbolises the atmospheric turbulent flow and disturbances due to the presence of buildings and other structures around the runway. It should be noted that the pipe flow is the simplest alternate to model the atmospheric flow. Since the focus is mainly on the evolution wake vortices in ground proximity, it is sufficient if the boundary layers and turbulence levels are similar to that of a typical turbulent pipe flow.

Table 4.4 shows the different turbulent levels considered for studying the effect of turbulent intensity. The maximum turbulence level for the baseline (pipeflow) case is 30%. The turbulent fluctuations of the pipe flow resulting from the precursor simulation are then scaled up by a percentage of 3-50. The TI values of Cases 11 and 12 are almost twice as their previous case values. The reason for such choice is to directly compare the influence of higher turbulence levels and the lower turbulence levels on the wake vortices. The maximum and minimum velocity for each turbulent fluctuation is presented in Table 4.4 for clarity.

The turbulent intensities, listed in Table 4.4, represent the amplification percentage of the turbulence fluctuations in the pipe flow. It has to be noted that it does not represent the absolute atmospheric turbulence level. For example, $TI = '0\%'$ do not mean that there is no turbulence but just that there is no amplification of the turbulence level from the turbulent pipe flow data. That is, the Case no. REF has a turbulence level equal to that of the turbulent pipe flow simulation. Hence, there is a fluctuation in the velocity field and the maximum and minimum velocity for the Case no. REF is equal to that of the pipe flow. Rest of the cases, from Case no. 8-12 are direct amplification of the velocity fluctuations

from the baseline of Case no. REF. The Cases no. 11 and 12 are considered to be high turbulence levels. Also, the mean freestream crosswind velocity in all cases is 100% of V_o .

Table 4.4 Velocity maxima and minima for various turbulent intensities

Case no.	Turbulence intensity (in %)	Maximum velocity (m/s)	Minimum velocity (m/s)
REF (Baseline)	0	1.0991	0.8686
8	3	1.1048	0.8341
9	6	1.1169	0.7746
10	9	1.1499	0.7196
11	20	1.2083	0.4658
12	50	1.4595	-0.1489

Figure 4.2 Inflow velocity profile for high turbulent intensities

Figure 4.2 shows a sample velocity profile of the inflow with higher turbulence levels. As the turbulence amplification increases to 50%, it can be clearly seen from the figure that the fluctuations are amplified accordingly. With this inflow profile, the

background flow is allowed to develop in the computational domain before the vortices are initialized. This is to ensure that the mathematically amplified turbulence result in a physically turbulent flow. It has to be noted that the velocity reaches a negative value for the case of 50% and so further amplification of turbulent fluctuations are not considered.

4.3 Influence of Crosswind

The influence of crosswind on the wake vortex characteristics are described in this section. Position and circulation/strength of the vortices are the parameters chosen for the present work. The results presented under this section are published as part of the paper presented at 34th AIAA Applied Aerodynamics conference [162]. In this section, the crosswind velocities are presented as percentage of the descent speed (V_o) of the vortex as mentioned in Table 4.3. For example, the label ‘20%’ listed in the legends of the graphs presented in this section corresponds to the Case no. 1, 20% of V_o in Table 4.3. Figure 4.3 shows the direction of the background flow with respect to the primary vortex pair. The vortex that is closer to the crosswind inlet is referred as upwind vortex and the farther vortex is referred to as downwind vortex as shown in Figure 4.3.

Figure 4.3 Example of vortex initialised computational domain

Axial direction: Direction parallel to the axis of the vortices (z^*)

Lateral direction: The direction perpendicular to the axis of the vortices (x^*)

Vertical direction: Direction parallel to the altitude of the vortices from ground (y^*)

4.3.1 Circulation decay characteristics

Figures 4.4 (a) and (b) present the circulation decay of upwind and downwind vortices pair respectively.

(a) upwind vortex

(b) downwind vortex

Figure 4.4 (a) Evolution of circulation of upwind and (b) Evolution of circulation downwind vortices for various crosswind velocities

The circulation curve follows a typical two-phase decay characteristic for all crosswind velocities. In the diffusion phase, the wake-vortices gradually diffuse and so the circulation strength is almost constant for all the crosswind cases. During this phase, the vortices descend through the atmosphere due to mutual induction. As they descend, a crossflow is induced at the ground beneath the vortices. This crossflow experiences an adverse pressure gradient beneath the primary vortices leading to a separation zone. This separation zone consists of omega-shaped secondary structures. These secondary structures detach from the boundary layer and loop around the primary vortices due to self-induction. Once the secondary vortices start to interact with the primary vortices, rapid decay phase sets in and there is a considerable reduction in their strength. In Figures 4.4 (a), the phases are marked for a clear understanding.

In Figures 4.4 (a) and (b), the sudden drop in the circulation strength marks the onset of rapid decay phase in the primary vortices. The onset of rapid decay phase is earlier for higher crosswind velocities for both the vortex pair. Also, the decay rate at this phase increases with increase in crosswind velocities. This trend is similar for both upwind and downwind vortices. However, compared to the upwind vortex, the decay rate is higher and the onset of rapid decay phase happens earlier for the downwind vortex. By comparing Figures 4.4 (a) and (b), it can be inferred that the rapid decay phase sets in around $t^* = 1.5$ - 2.0 for the upwind vortex and around $t^* = 1.0$ - 1.5 for downwind vortex. This asymmetric behavior of the vortices is due to the presence of crosswind. The presence of crosswind flow favors the formation of secondary structures near the downwind vortex while delays them near upwind vortex due to their vorticity signs. This has been discussed earlier in detail in Section 2.9 with the help of a schematic diagram.

Figure 4.5 shows a comparison of circulation strength of the upwind and downwind vortices at $t^* = 2.6$ for all of the crosswind velocities considered. It can be inferred from the figure that stronger crosswinds result in lower strengths for both the vortices. From Figure 4.5, it is also clear that gradient of the upwind and downwind vortical curves decreases and becomes closer to zero after a crosswind velocity of 80% of V_o . This implies that there is a

limit to which crosswind can enhance the dissipation of the primary vortex pair. This is an important conclusion that has to be taken into account by the wake-advisory systems.

Figure 4.5 Non-dimensionalised circulation of upwind and downwind vortices at $t^* = 2.6$

Since the crosswind favors the secondary vortex formation near the downwind vortex but delays it near upwind vortex, the rapid decay phase sets in downwind vortex much earlier as compared to the upwind vortex. Hence, the strength of downwind vortex is lower than that of upwind vortex for all the cases as shown in Figure 4.5. In practice, upwind vortex is more dangerous than the downwind vortex as it stays in the domain longer with a considerably larger strength. For example, the upwind vortex of Case no. 5 stays within the domain with a circulation strength of $350 \text{ m}^2/\text{s}$ at time $t = 1.14$ minutes. This circulation value is almost equal to or higher than the landing circulation strength of aircrafts like B737-500, A320-200, B757-200. When these aircrafts of comparable strength fly into the upwind vortex of the leading aircraft, they lose control and may even crash.

Figure 4.6. (a) – (g) Comparison of vortex evolution at $t^*=2.6$ for various crosswinds

Note: Q -criteria isosurfaces are coloured according to the z -component of vorticity scale.

Figures 4.6 (a) to (g) present a comparison of evolved vortices at time, $t^* = 2.6$ under different crosswind velocities. It can be confirmed from these figures that increase in crosswind velocity enhances the formation of omega-shaped secondary vortices and its interaction with the primary vortices. In addition, the upwind vortex is not surrounded by as many secondary vortices as those of the downwind vortex for the same time step, proving that the presence crosswind favors the formation of secondary vortices near downwind vortex. This favouring of crosswind results in asymmetric temporal evolution of the primary vortex pair.

The significance of choosing this particular time is that the downwind vortex is in its rapid decay phase and has significant difference in the circulation values as presented in Figure 4.5. It can also be inferred from Figures 4.6 (a) – (g), that the downwind vortex is pushed out of the domain along with its secondary vortices earlier for higher crosswind velocities. At time, $t^* = 2.6$, for the case of crosswind with 170% of descent speed, it can be seen from Figure 4.6 (g) that the downwind vortex is completely out of the considered computational domain.

The position of upwind and downwind vortices varies distinctly for different crosswind velocities. Although a detailed investigation with the aid of graphs is presented in the subsequent sections, it can be seen clearly from Figures 4.6 (a)-(g) that the upwind and downwind vortices are moving further in the positive lateral direction with the increasing crosswind speeds. The speed of the lateral motion and the exit time of the primary vortex for various crosswind velocities will be presented in detail in Sections 4.3.3 and 4.3.4 respectively.

4.3.2 Position of the vortices

(a) upwind vortex

(b) downwind vortex

Figure 4.7 Centreline of (a) upwind and (b) downwind vortices in 3D domain for Case no. 5

Figures 4.7(a) and (b) show the centerline of the upwind and downwind vortices respectively at various times for the Case no. 5, 100% of V_0 . Comparing the two figures, it is confirmed once again that the downwind vortex moves in the lateral direction at a higher speed than the upwind vortex. It can be seen from Figures 4.7 (a) and (b) that the centerlines are starting to get distorted in the spanwise direction, around the time, $t^* = 2.0$ for upwind vortex and $t^* = 1.5$ for downwind vortex. It is reminded that the upwind and downwind vortices undergo rapid decay phase right after the above-mentioned time respectively.

In general, this distortion of the centerlines of the primary vortex pair are due to its interaction with the secondary vortices. Since there is an early detachment of secondary vortices from the ground around downwind vortex as compared to the upwind vortex, the distortion of vortex core can be seen in the downwind vortex at earlier time. At $t^* = 2.45$, the vortex corelines of upwind and downwind vortices shows significant difference in the degree of distortion.

Figures 4.8 (a) and (b) show the vortex center on a x^*-y^* plane with $z^* = 0$ for time $t^* = 0, 1, 1.5, 2, 2.45$. In ground proximity, it can be seen from the figure that the primary vortex pair rebounds after reaching a minimum altitude due to the presence of oppositely signed boundary layer of the induced crossflow. Due to the presence of crosswind boundary layer in addition to the vortex-induced boundary layer of the same sign, the downwind vortex rebounds earlier and also attains a higher altitude as compared to the upwind vortex. It can be clearly seen from Figures 4.8 (a) and (b) that the maximum altitude obtained by upwind vortex for the five-time steps considered is $y^* \leq 1$ while that of downwind vortex is $y^* \geq 1$.

(a) upwind vortex

(b) downwind vortex

Figure 4.8 Vortex centre of (a) upwind and (b) downwind vortex in the midplane perpendicular to the axis of the vortex for time, $t^* = 0, 1, 1.5, 2.0, 2.45$.

The position and transport of vortices are investigated further by introducing two variables: radial separation distance and relative angle of vortex pair. Figure 4.9 is a schematic diagram explaining the radial separation distance and relative angle between the vortex pair. Radial separation distance (r) is the axially averaged distance between the

centrelines of the two vortices in polar coordinates with upwind vortex centre as origin. It is non-dimensionalised by b_0 and denoted as r^* . Relative angle (θ) of vortex pair is the axially averaged angle between the centreline of the vortices and given in degrees. A positive relative angle implies that the downwind vortex is at a higher altitude compared to the upwind vortex, that is, the anti-clockwise is taken as positive by convention.

In order to quantify the distortion due to the primary-secondary vortex interactions, the deviations from the mean, Mean Average Deviation (MAD) are calculated for the two introduced variables. The lower the value of MAD, the lower is the fluctuation in the axial direction and the lower is the distortion of the primary vortex centerline, caused by its interaction with the secondary vortices.

Figure 4.9 Non-dimensionalised radial separation distance (r^) and relative angle (θ) between the primary vortex pair*

For the crosswind cases above 1.79m/s (100% of V_0), the downwind vortex moves out of the domain within half of the considered computational time and so they are not considered for the analysis presented in this section. An additional simulation with zero crosswind is performed exclusively for the study in this section.

Figure 4.10 Non-dimensionalised radial separation distance vs time for various crosswind velocities

Figure 4.10 shows the non-dimensionalised radial separation distance of the vortex pair with respect to time for various crosswind velocities. From Figure 4.10, it can be inferred that the vortex separation distance does not vary with crosswind until $t^* = 1.5$. It increases from its initial value due to mutual induction of the primary vortex pair, for all crosswind velocities. Once the rapid decay phase sets in, there is a reduction in the distance between the two vortices. This may be due to their interaction with secondary vortices causing energy losses, thus reducing the effect of mutual induction. When there is an increase in the crosswind velocity, the motion of downwind vortex due to mutual induction is favoured and so the distance between the vortices starts to increase despite the energy losses. The rate of change depends on the magnitude of the crosswind velocity as it is the prime factor promoting the lateral motion.

Figure 4.11 Relative angle of vortex pair vs time for various crosswind velocities

Figure 4.11 shows the relative angle between the two vortices. In other words, it shows the relative altitude difference between the two vortices. When there is no crosswind velocity, the vortex centreline of both the vortices are approximately in the same height and so the value of relative angle is almost zero. Introduction of crosswind velocity results in the increment of the altitude difference between the vortices centreline. The reason for such higher bounce back of downwind vortex is due to the growing boundary layer that are aided by the presence of crosswind velocities. The relative angle is positive through all times, implying that the downwind vortex stays above the upwind vortex throughout the simulated time. Furthermore, the relative angle increases with the increase in crosswind velocity.

Figures 4.12 (a) and 4.12 (b) show the relative angle of the vortices and the radial separation distance between the vortices in axial direction for different time steps respectively. The considered time steps are two from the diffusion phase ($t^* = 1$ and 1.5) and two from the rapid decay phase ($t^* = 2.0$ and 2.5) for the Case no. 5. The maximum of MAD of the relative angle is found to be less than 0.6% in the diffusion phase and is between 0.6% to 1% in the rapid decay phase for all crosswind velocities.

(a)

(b)

Figure 4.12 (a) Variation of relative angle of vortex pair (in degrees) and (b) Non-dimensionalised radial separation distance between the two vortex centrelines in axial direction (z^*) for Case no.5 for $t^* = 1, 1.5, 2.0, 2.5$

In the diffusion phase, the variation of the relative angle is limited to $\pm 0.6^\circ$ and it can also be seen from both the figures that the two parameters are almost constant for $t^* = 1$

and 1.5 especially for the radial separation distance. The MAD of rapid decay phase implies that there is $\pm 0.6\%$ to 1% fluctuations in the relative angle along the axial direction between the vortex centrelines. It means that the interaction of secondary vortices with the primary vortex pair has a tilting and twisting effect on the centreline of the vortices in axial direction. However, MAD for non-dimensionalised radial separation distance is only of the order of 10^{-2} for all time steps. That is, there is only a change of ± 0.01 in the non-dimensionalised radial distance along the axial direction. Change in the relative angle without much change in the radial distance is possible only when there is changes in the altitude between the vortex pair.

In general, these relatively low MAD values compared to the large length scales of the vortices, ensure that the axially averaged values can be considered for the investigation of vortex pair movement.

4.3.3 *Crosswise velocity of the vortices*

The rate at which the vortex moves in lateral direction increases is named as **crosswise velocity** of the vortex in the present work. The crosswise velocity of the vortices in the rapid decay phase for both vortices is plotted against the crosswind velocity in Figure 4.13. The curve of upwind vortex can be approximated to a linear curve while that of downwind vortex follows a quadratic curve.

As it can be seen from Figure 4.13, the crosswise velocity at zero crosswind velocity is positive for downwind and negative for upwind, this explains the mutual induction behavior of the vortices. They tend to move away from each other. Since the transport of downwind vortex due to mutual induction is in the same direction as the crosswind, the motion is favored. Hence, at any given crosswind condition, Crosswise velocity is higher for the downwind vortex than that of upwind vortex.

Figure 4.13 Crosswise velocity vs crosswind velocity for upwind and downwind vortices

For crosswinds as low as 20% of descent speed (V_o), the crosswise velocity for upwind vortex is nearly zero and that for downwind vortex is a small positive value. This indicates that the vortices stay in the domain until it dissipates. The upwind vortex moves against the crosswind and so it is hard to get rid of it from the domain, that is from the runway. Since the formation of secondary vortices is also delayed for upwind vortex, it stays longer in the domain with a higher strength and pose a real challenge to the researchers. It is evident from the discussion that a crosswind greater than 4m/s ($\approx 230\%$ of V_o) is required for the runway to be clear of both the vortices. CREDOS, one of the reduced separation standards [8] based on crosswind velocity, also has the same conclusion.

4.3.4 Exit time of the vortices

Exit time is the time at which the vortex moves out of the considered computational domain in the direction of the crosswind flow, that is, when the vortex centerline crosses a lateral location, $x^* = 3.8$. The downwind vortex travels $4b_o$ of lateral distance and the upwind vortex travels $5b_o$ of lateral distance from its initial position during the course of the

exit time mentioned in this section. The exit time of the upwind vortex is higher than that of the downwind vortex for all crosswind cases. This is due to three reasons. The first two reasons are mutual induction and presence of crosswind which resulted that the downwind vortex possess higher crosswise velocity than the upwind vortex. Thirdly, the upwind vortex has to travel an extra distance of 47.1m to reach the end of the domain in the lateral direction.

Total distance travelled by the upwind vortex to exit the domain = $5 b_o$

Total distance travelled by the downwind vortex to exit the domain = $4b_o$

Extra distance travelled by the upwind vortex to exit the domain = $1b_o = 47.1\text{m}$

Figure 4.14 Non-dimensionalised exit time of the upwind and downwind vortices

Figure 4.14 shows the variation of time at which the upwind and downwind vortices exits the computational domain with various crosswind velocities. The exit time and the crosswind velocity are related empirically by means curve fitting. The exit time of upwind vortex follows $1/(v^*)^{1.006}$ curve and that of downwind vortex follows $1/(v^*)^{0.752}$ curve approximately. The vertical asymptote of the two empirical curves that belongs to upwind

and downwind vortices, is the y-axis itself. Theoretically, this states that both the vortices take infinite time to move out of the domain for crosswinds closer to zero. In reality, infinite times is not practically possible, and the vortices dissipate even before they exit the domain.

Figure 4.15 Exit time (in minutes) for the upwind vortex

Figure 4.16 Exit time (in minutes) for the downwind vortex

The relationships established in Figure 4.14 can be used to predict the exit times of the primary vortex pair for crosswinds higher and lower than the values considered in the

present study. The practical significance of this empirical relationship between the exit time and the crosswind velocity can be explained through Figures 4.15 and 4.16.

Figures 4.15 and 4.16 show the exit time of the upwind vortex and downwind vortex respectively in minutes for various crosswind velocities in m/s. The black line labelled 'empirical data' represents the calculated exit time using the empirical relation for various crosswind speeds. Using the empirical relation in Figure 4.14, the exit time for both upwind and downwind vortices for a crosswind velocity of around 5m/s, are predicted and presented in Figures 4.5 and 4.6 respectively, as black circle. The upwind vortex has to travel a distance of $5b_0$, that is, 235m from its initial position, in the lateral direction to exit the domain while the downwind vortex has to travel a distance of $4b_0$, that is, 188m from its initial position. It is predicted through the empirical relations that the upwind vortex takes approximately 75 seconds while the downwind vortex takes only 35 seconds.

This prediction of exit time of primary vortex pair can be used for devising a preliminary crosswind-based wake vortex advisory system. There are minor deviations from the predicted values for downwind vortex but still it is good enough for a preliminary analysis. It will give an approximate idea on whether the advisory system has to find both the vortices in the runway or just one of the two. This preliminary analysis can help the systems of the currently developing operational method named Time Based Separation (TBS) standards to provide a more practical and useful parameter.

4.4 Influence of Turbulence Intensity

Effect of atmospheric turbulent intensity on the strength and position of the vortices are described in this section. The results presented under this section are presented at the 34th AIAA Applied Aerodynamics Conference [162].

4.4.1 Circulation decay characteristics

Figures 4.17 and 4.18 show the non-dimensionalised circulation decay of upwind and downwind vortices respectively with time under all the turbulent intensity levels investigated.

Figure 4.17 Evolution of circulation of upwind vortex for various turbulent intensities

Figure 4.18 Evolution of circulation of downwind vortex for various turbulent intensities

The evolution of circulation of upwind and downwind vortices for all turbulent intensity levels follow the two-phase decay characteristics. The evolution of circulation does not show distinct difference for low turbulence levels (3% - 9%) but shows a

comparatively higher difference for higher turbulent levels (20% and 50%) in the later stages of the rapid decay phase.

Figure 4.19 (a) – (e) Comparison of vortex evolution at $t^*=2.6$ for various TI levels

Note: Q-criteria isosurfaces are coloured according to the z-component of vorticity scale.

Figures 4.19 (a) – (e) show a comparison of change in turbulent intensity from 3% to 50% respectively, barely affects the dynamics of the vortex and this is evident from Figures 4.19 (a) to (c). Also, this confirms that the low turbulent intensity levels have no profound influence on the circulation characteristics of both upwind and downwind vortices as shown in Figures 4.17 and 4.18 respectively. It should also be noted from Figures 4.19 (a) to (c) that the local secondary turbulent structures formed in the flow field do not vary distinctly.

From Figures 4.19 (d) and (e), it can be seen that larger secondary vortical structures are locally formed, thereby enhancing the interaction of secondary vortices with the primary vortices. This local variation explains a relatively lower circulation values for the cases with 20% and 50% background turbulent intensities, as compared to other cases in Figures 4.17 and 4.18.

4.4.2 Position of the vortices

It has to be reminded that there is a presence of background crosswind flow of velocity 1.72m/s (100% of V_o) along with the addition of different levels of turbulence. As the presence of crosswind aids the motion of the downwind vortex, it moves downstream at a faster rate than the upwind vortex. This phenomena is explained in Section 4.3.3. Addition of any level of turbulence to the background flow, did not affect the motion of the vortices. However, there is difference in the distortion of the centreline of the two vortices.

Figures 4.20 and 4.21 show the centreline of the vortex core for upwind and downwind vortices respectively in the three dimensional computational domain at five different times: one at $t^* = 0$, one in the diffusion phase ($t^*=1$), one at the onset of rapid decay phase ($t^* = 1.5$) and two ($t^* = 2.0, 2.45$) after the formation of secondary vortices.

Figure 4.20 Centreline of upwind vortex for Case no. 12

Figure 4.21 Centreline of downwind vortex for Case no. 12

When Figures 4.20 and 4.21 are compared with Figures 4.7 (a) and (b) respectively, it can be inferred that, for a highly fluctuating crosswind flow (say Case no. 12), the centreline for both upwind and downwind vortices after the onset of rapid decay phase is highly distorted. This confirms the previous argument that the interaction of locally formed secondary vortical structures with the primary vortex pair is more for the simulation Cases no. 12 and 13 than that of the other cases. The rebound altitude attained by the upwind vortex is $y^* \leq 1$ while that reached by the downwind vortex is $y^* \geq 1$. This can be confirmed with the centreline positions of the two vortices at various time as shown in Figures 4.20 and 4.21.

Figure 4.22 Non-dimensionalised radial separation distance vs time for various turbulent intensity levels

Figure 4.23 Relative angle between the vortex pair vs time for various turbulent intensities

Figures 4.22 and 4.23 represent the axially averaged non-dimensionalised radial distance and angle (in degrees) between the primary vortex pair in polar co-ordinates with the centre of upwind vortex core as origin at each time step (note that for a detailed definition Figure 4.9 can be referred). Due to mutual induction, the radial distance between the centres of the two vortices steadily increases with respect to time for all crosswinds. From Figure 4.22, it can be inferred that within the considered turbulence levels, there is no significant difference in the radial separation distance. It follows the same curve as the baseline case of 0% turbulent intensity. The relative angle of the vortex pair is positive and increases with respect to time with a peak at $t^* = 2.1$, 1.95 and 2.36 for 0%-9%, 20% and 50% turbulence intensity respectively. Then, they decrease thereafter as shown in Figure 4.23. Hence, it can be concluded that the downwind vortex stays at a higher altitude at all times as compared to the upwind vortex.

The upwind vortex enters the rapid decay phase after $t^* = 1.5$. After this time, the upwind vortex also crosses its minimum altitude to the ground and rebound to a higher altitude due to the presence of induced boundary layer near ground as presented in Figure

4.21 at $t^* = 1.5, 2.0, 2.45$. Hence, the relative angle between the two primary vortices decreases after the time $t^* \approx 1.75$ as shown in Figure 4.23. In the same figure, it can be inferred that the relative angle for Cases no. 11 (TI = 20%) and 12 (TI = 50%) deviates from the reference case which is TI = 0%, after a time of $t^* = 2.0$ as larger vortical structures are formed locally in highly turbulent scenarios as compared to others.

Figure 4.24 Non-dimensionalised radial separation distance in the axial direction for Case no. 12

Figure 4.24 shows the non-dimensionalised radial separation distance between the center of the vortices at different planes perpendicular to the axial direction. From the data presented in Figure 4.24, it can be calculated that the maximum MAD of radial separation is 0.01, that is, the values in the axial direction vary only by a maximum value of 0.01. The lower the value of MAD, the higher is the accuracy of the representation of axially averaged radial distance between the center of the vortices at various time steps. Also, this ensures that the local secondary structures, due to the higher background turbulence, does not affect the values of the non-dimensionalised radial separation distance in the axial direction.

Figure 4.25 Relative angle in the axial direction for Case no. 12 ($TI = 50\%$)

Figure 4.25 shows the relative angle between the centers of the vortices at different planes perpendicular to the axial direction. In other words, relative angle represents the relative twist between the centerline of the two vortices with respect to each other. The fluctuations in Figure 4.25 is due to the interaction of secondary structures with the primary vortex pair. The higher the turbulent intensity, the larger the local secondary vortices which are formed and the greater is the twisting of the vortex centerline. Twisting of the centerline of the vortices results in altitude difference between the two vortices along the axial direction. From the simulation results presented in Figure 4.25, it can be concluded that the MAD of the relative angle for low TI (3%-9%) is less than 1° while that of higher TI is up to 1.6° . Since the MAD is of the same order as the averaged value itself as presented in Figure 4.25, it can be concluded that the highly turbulent background flow is capable of causing an uneven effect on the centerline of the vortices in the axial direction. Also, it is surprising to note that when there is a high fluctuating turbulent background flow, the angle between the vortices can change to even a negative value locally due to their local interaction with the secondary vortices.

4.5 *Summary*

Temporal LES of aircraft wake vortices in ground proximity for different crosswind velocities and turbulence levels were discussed in this chapter. The initial conditions of the wake vortices for the simulation were described before discussing the results. Circulation strength of the wake vortices was considered as the primary parameter in understanding the effect of different crosswind speeds and turbulence levels on the wake vortices. In addition to the strength parameter, the study of motion of wake vortices were also focused as it is of concern in the vicinity of the airport. Exit time and crosswise velocity were proposed as additional measures for the motion of wake vortices in the influence of different crosswind velocities. Exit time refers to the time at which the wake vortices are exiting the computational domain. Crosswise velocity refers to the rate at which the wake vortices move in the lateral direction. It is concluded from this study that the crosswind has profound effect on the dissipation and convection of the wake vortices. As the crosswind velocity increases, the formation of secondary vortices is enhanced and so is the dissipation of the primary vortices. The crosswise velocity of the primary vortices increases with increase in the crosswind velocity. On the other hand, the turbulence intensity levels of the crosswind flow considered in this study do not have much influence on the dissipation and convection of the vortices.

5 Prandtl Distributed Vorticity Method

5.1 *Motivation*

To have a better understanding of the aircraft wake vortices, it is a must to simulate them from the roll-up phase to decay phase. Different phases of wake vortices are discussed earlier in Section 2.5. In the roll-up phase, the wake vortices comprise of multiple-vortex pairs interacting with each other. All of these vortices eventually roll-up into a single counter-rotating vortex pair called primary vortex pair in the far-field. But, the evolution and decay mechanism of this primary vortex pair is greatly affected by the presence of other vortices during the roll-up phase. The number of vortices present in the roll-up phase is determined by the spanwise lift distribution. This conclusion is derived from the works earlier in 1970s and 1990s [163-171]. In all of these studies, B747 was considered as the vortex generating aircraft. Numerous experimental and numerical studies were performed to analyse its wake vortices. Rossow [170] had also studied the effect of roll oscillations on the wake vortices formed behind B747 and L-1011 aircrafts. For a same rolling angle, the vortices in the wake of the two considered aircrafts are distinctly different. This is due to the difference in the position and deflection of the high-lift devices on the wing, resulting in a different spanwise lift distribution profile for the two aircrafts. The effect of different flap settings and fins on a rectangular wing were also found to be profound on the far-field development of the wake vortices [172]. These studies provide a solid evidence for the significance of the effect of lift distribution on the aircraft wake structure, strength and decay phenomenon.

5.2 *Need for a new method*

Modelling different aircraft types with different high-lift device configurations using Spatial LES methods may be cumbersome and may give rise to a lot of concerns during simulation. For example, meshing around the lifting surfaces is different for each aircraft type and has to be carefully done as there is high chance of flow separation in this region.

On the other hand, Temporal LES method does not take into account of the lift configurations of an aircraft. A fully developed counter-rotating vortex pair is assumed to be in the wake of an aircraft and the simulations are performed to study its evolution. The initial parameters of these vortices in Temporal LES method are determined primarily by the total weight of the aircraft.

Carefully considering the fact that the span loading predominantly affects the initial vortex parameters, in this chapter, a new initialisation method is proposed for the Temporal LES [173]. This new method provides a relationship between the initial parameters of the wake vortices in the near-field of the wake and spanwise lift distribution over the wing. This method is based on Prandtl Lifting-Line Theory and so is named as *Prandtl Vorticity Distribution method (PVD method)*. Once the vortices are initialized, it is followed by the Temporal LES method for studying the evolution of the vortices. Inclusion of this initialisation method perfectly finds a mid-spot between the Temporal and Spatial simulations and so it belongs to a new category called *Quasi-Temporal LES*.

5.3 *Prandtl Vorticity Distribution (PVD) method*

Simulation of rollup phase of the vortices involves lot of computational complexity. In order to avoid the excess computational costs, the general practice is to assume that the vortices are already rolled-up so that simulation starts from $1b_0$ to $2b_0$ behind the wing. The vorticity distribution at the trailing edge of the wing is calculated based on the lift distribution using Kutta-Juokowski theorem and Prandtl Lifting-Line Theory, which is then used to calculate the induced velocity distribution downstream. It is important to note that for an ideal symmetric lift distribution, the resulting induced velocity field would be same as that of a vortex sheet which confirms the method is a valid representation of the vorticity distribution downstream.

The PVD method depends on the Prandtl Lifting-Line Theory explained in Chapter 2 under Section 2.2.

The vorticity initialization method can be described in steps as follows,

- i. Consider the spanwise lift distribution profile of any aircraft.
- ii. Calculate the spanwise circulation distribution $\Gamma(y)$, using Kutta-Juokowski theorem (Eq. 2.3).
- iii. The continuous curve is then discretized to a data set of N values similar to the six points (A, B, C, D, E, F) in Figure 2.4.

Note: The gradient of the spanwise circulation distribution plays an important role as it determines the strength of the free vortices downstream. Hence, a careful consideration has to be taken when choosing the number of data points for the discretization. Although it is advisable to have at least 20 data points for a smooth curve, there is no other restriction to the number of discrete data points. As long as it accounts for any sudden changes in the spanwise circulation gradient, the method will work better.

- iv. Based on Prandtl Lifting-Line Theory, for every infinitesimally small change in the spanwise circulation, a free vortex is assumed to be shed downstream. The strength of n^{th} free vortex (γ_n) is given by

$$\gamma_n = \Gamma_n - \Gamma_{n-1} \quad (5.1)$$

where Γ_n is the circulation at n^{th} location in the spanwise direction, $n = 1, 2, \dots, N$. For example, consider $\Gamma = 600, 650, 620 \text{ m}^2/\text{s}$ at $y = 0, 4.71, 9.42\text{m}$ respectively. Then the free vortex strength, $\gamma_2 = 600 - 650 = -50 \text{ m}^2/\text{s}$ at $y = 4.71\text{m}$.

- v. Lamb-Oseen vortex model is used to initialize the free vortex in the computational domain with the calculated circulation strength. At any given point in computational domain, the induced tangential velocity by each of the free vortices are summed up. For example, assume V_1 as the tangential velocity at point A in the computational domain induced by the free vortex of strength γ_1 , V_2 induced by 2nd free vortex and V_n induced by the n^{th} free vortex.

Then, the total velocity at point A is given by,

$$V_A = V_1 + V_2 + \dots + V_n \quad (5.2)$$

- vi. Once the initialization is done, LES is performed to study the evolution of these free vortices.

The free vortex strength parameter is of prime importance to this study as it determines the shape, size and rotational direction of the vortices shed downstream of any aircraft. It is to be noted that this parameter is indirectly proportional to the change in spanwise circulation distribution over the wing. In other words, if there is an increase in the circulation along the spanwise direction over the wing, there will be a decrease in the free vortex strength parameter and vice versa.

As a demonstration of the newly proposed method, simulation of wake vortices shed behind a landing B747 aircraft was performed using this method. This particular aircraft falls under the category of ‘Heavy’ by ICAO separation standards and will be a perfect example to validate the method with the LIDAR measurements conducted at the Frankfurt Airport.

In reality, the vortices are at different phases of evolution along their axial direction. That is, the vortices near the aircraft wing will be in roll-up phase while those at the far-field from the aircraft will be in decay phase at the same instant. Thus, the age of the vortices varies in the direction parallel to the aircraft motion. Since the proposed method is an upgraded version of Temporal simulation methodology, the vortices are assumed to be in the same phase along its axial direction. The entire vortex system is initialized using PVD method at the same time and then, their roll-up is investigated. Also, the results of the PVD method is compared with the most commonly used Temporal simulation in the subsequent section.

To conclude, with this initialisation method, the initial vortex roll-up process is simulated from the vortex sheet at the trailing edge of the wing until its decay. Since the presence of the flaps generates a large patches of vorticity, the vortex sheet is obscured and so the resulting velocity field looks as if the vortices are already rolled-up.

5.4 Wake-vortex system of B747 LDG configuration

5.4.1 PVD method initialization

Figure 5.1 shows the specifications of wing and high-lift devices of a typical B747 aircraft [166]. Note that the B747 aircraft has an inboard and outboard flap on each wing. The flaps can be deployed at different angles allowing for different lift distributions along the wing. The PVD method can now account for different flap settings of inboard and outboard flaps.

It is important to note that the coordinate system used in the discussion of this chapter and the Chapter 6 follows a different notation. The direction parallel to the axis of vortices are considered as x -axis, the direction perpendicular to the axis of the vortices are y -axis and the distance from the ground in vertical direction is denoted as z -axis.

Landing configuration (LDG):

Inboard (IB) flap = 46° ,

Outboard (OB) flap = 46°

Figure 5.1 B747 specifications [166]

Table 5.1 Wake vortex parameters of B747 [174]

Parameters of B747	Landing
Initial circulation (m^2/s)	554.6
Aircraft speed (m/s)	80
Characteristic velocity scale (m/s)	1.75
Characteristic time scale	29 s
Wing span	64.4 m

Table 5.1 presents the wake vortex parameters of B747 during landing and take-off. The landing configuration of B747 is considered for the demonstration of the working of the newly proposed initialization method. For the current configuration, the inboard and outboard flap are deflected by 46° . The presence of crosswind generally enhances the secondary vortex formation. Earlier onset of secondary vortices formation will aid in analyzing the vortex dynamics in depth and also mimic the atmospheric turbulent flow. Hence, a crosswind of 1.75m/s is considered for the simulated cases presented under this section. The choice of this 100% of V_0 as crosswind velocity, out of the all cases discussed in Chapter 4 is that it enables us to study the evolution of the vortices equivalent to three wingspan distances. Although a slight lesser or higher crosswind velocity can also be used, the usage of 100% of V_0 provides a computational ease as descent speed is used to non-dimensionalise the velocity scale. If the crosswind is too high, the exit time of the downwind vortex is shorter as described in Section 4.3.4. The wingspan, characteristic time and velocity in Table 5.2 are used to non-dimensionalise the circulation, position and time. Note that Table 5.2 is a repetition of Table 4.2 for ease of reference.

Figure 5.2 shows the spanwise lift coefficient obtained from the experimental results of Corsiglia et al. [166] on the left and the calculated spanwise circulation distribution on the right. Kutta-Juokowski theorem is used to calculate the spanwise circulation distribution

as described in Eq. 2.3. Since the lift is proportional to lift-coefficient, the distribution is similar.

Table 5.2 Non-dimensionalised variables

Non-dimensionalised time, t^*	t/t_o
Non-dimensionalised spatial coordinates, x^*, y^*, z^*	$x/b_o, y/b_o, z/o$
Non-dimensionalised circulation, Γ^*	Γ/Γ_o
Non-dimensionalised crosswind velocity, v^*	u_{inflow}/V_o

Figure 5.2 Predicted spanwise lift coefficient [166] and calculated circulation distribution for a landing B747 aircraft

The strength of each free vortex along the spanwise direction as demonstrated in Figure 5.3 is determined by the gradient of the spanwise circulation distribution (*step iii of Section 5.3*). The stronger the gradient at a spanwise location, the stronger will be the strength of the free vortex with opposite sign. Comparing the spanwise lift distribution in Figure 5.2 and the spanwise free vortex in Figure 5.3, it can be deduced that if there is an upslope in the lift distribution, there will be a downslope in the free vortex strength distribution. If the slope of the lift distribution over a span is almost constant, then the free vortex strength is also constant over that span of the wing. The vortices will look diffused over this span due to the superposition of the velocity fields induced by the free vortices with the same circulation strength.

Figure 5.3 Spanwise free vortex strength distribution

Spanwise free vortex strength distribution will give an approximate idea on how many vortices can be expected downstream of an aircraft and their comparative circulation strengths. Here, from Figure 5.3, it can be concluded that the positive and negative peaks of the free vortex strength will result in two distinct vortices of opposite sign. It is to be noted

that if the positive peak is followed by a comparatively high positive free vortex strength values, there are high chances that they all merge into single vortices during the evolution of the flow. This deduction can be confirmed from the following figures. Figure 5.4 shows the resulting free vortex sheet in the three-dimensional computational domain for the considered lift distribution at time, $t^* = 0$.

Figure 5.4 Free vortex sheet in the three dimensional computational domain at $t^* = 0$

a – wing-tip vortex; Anti-clockwise direction, b – outboard flap vortex; Anti-clockwise direction

c – inboard flap vortex ; Clockwise direction

Figure 5.5 Initial Tangential vorticity distribution

Figure 5.5 presents a closer look at the initial vorticity distribution corresponding to the considered lift distribution. The port-side and starboard-side of the wing is marked in Figure 5.5 for clarity. Comparing Figures 5.6 and 5.8, it can be observed that the vorticity is diffused near the wing-tip as the free vortex strength is almost constant from $y^* = 0.3$ to 0.5 . This wing-tip vortex on the starboard-side is marked as ‘a’ in Figure 5.5. Additionally, there are two more vortices seen in the wake. One is a comparatively weaker outboard flap vortex ‘b’ as shown in Figure 5.5, shed at the junction of the two flaps due to the lift gradient change between the flaps. Another is a comparatively strong, oppositely signed inboard flap vortex ‘c’ as shown in Figure 5.5, shed from the inboard tip of the inboard flap.

Although the absolute values of the free vortex strength of the positive peak is higher than the negative peak, the difference in the vorticity distribution between the corresponding vortices is due to the presence of wing-tip vortex. This is a solid evidence on how the initial vorticity distribution is affected by the presence of multiple wake vortices in the wake of an aircraft. It can be clearly seen that the resulting wake is complex with multiple pairs of vortices and their structure in the two-dimensional plane is not exactly circular as it is commonly assumed in Temporal LES simulations.

Since this is a numerical initialization, LES simulation is conducted to analyze the vortices roll-up and the interaction of multiple vortices during evolution. Figures 5.6 and 5.7 respectively present the three pair vortex system at $t^* = 0.05$ and the rolled-up two pair vortex system at $t^* = 0.1$ resulting from a lift distribution of landing B747 aircraft. It can be observed from the figures that the vortices are descending through the atmosphere due to mutual induction. The flap vortices start to revolve around the primary vortices due to its lower strength. Sooner, the outboard flap vortex gets absorbed into the primary vortices (upwind and downwind vortices) forming two-pair vortex system (primary vortex pair and the inboard flap vortex pair). From here on, the inboard flap vortices are denoted as flap vortices for the ease of referencing as shown in Figure 5.5. The inboard flap vortex in the port-side of the wing is referred as upwind-flap vortex and the one on the starboard-side is referred as downwind-flap vortex. Similarly, the wing-tip vortex in the port-side of the wing

is referred as upwind vortex and the one on the starboard-side is referred as downwind vortex.

Figure 5.6 Tangential vorticity distribution at $t^* = 0.05$

Figure 5.7 Tangential vorticity distribution at $t^* = 0.1$

Figure 5.8 Schematics of multiple wake vortices and their vorticity signs

Figure 5.8 shows the schematics of the two pair vortex system equivalent to the tangential vorticity distribution presented in Figure 5.7 at $t^* = 0.1$ and their corresponding vorticity signs for the ease of understanding. From here on, the interaction between the flap vortices and primary vortices are given therein focus in order to understand how the presence of flap vortices affects the decay mechanisms of the primary vortex pair.

The circular arrows are only a representation of the direction of the strain due to shear and rotation and do not reflect their strength. It is to be noted that the circular arrow of BL represents the vorticity shear layer and not a vortex.

5.4.2 *Interaction of flap and wing-tip vortex*

In this sub-section, the interaction between the flap vortices and the primary vortices are studied. Although the flow field is dominated by a single pair vortex after $t^* = 0.75$, the interaction of flap vortices with the primary vortices until then is highly dynamic. Figures 5.9 and 5.10 demonstrate the development of flap vortices into secondary structures from $t^* = 0.5$ to 0.75 or from $t = 14.25s$ to $21.75s$, and their interaction with the upwind and downwind vortices respectively. Firstly, the general behavior of the flap vortices is explained and then their interaction with the downwind and upwind vortices are discussed.

The upwind and downwind flap vortices exhibit long wave instabilities soon after initialization. The wiggles found in the flap vortices at $t^* = 0.5$ in Figure 5.9 and Figure 5.10 are due to this long wave instabilities. Since the flap vortices are lesser in strength as compared to the primary vortex pair, they are more prone to the instabilities induced by the background turbulence. As the flow evolves, the flap vortices start to revolve around the primary vortices due to the induced force of the primary vortex pair. Along with the atmospheric air that is present in between the two wing-tip vortices, the flap vortices are also sucked beneath the primary vortex structures due to induced force of the primary vortex pair. At this point of time, both flap vortices undergo an unfavorable pressure gradient along with the background flow. Therefore, the unstable flap vortices are further distorted.

Figure 5.9 Interaction between upwind vortex and upwind flap vortex (Port-side of the wing)

Note: Q -criteria isosurfaces are coloured according to the z -component of vorticity scale.

Figure 5.10 Interaction of downwind Flap - tip-Vortex (Starboard-side of the wing)

Note: Q -criteria isosurfaces are coloured according to the z -component of vorticity scale.

Meanwhile, it is to be noted that the upwind and downwind vortices are also moving downwards due to mutual induction. The vortices as they descend through the atmosphere,

induces a crossflow over the ground beneath them. The boundary layer of this induced crossflow possesses an opposite sign of vorticity as compared to the nearby primary wing-tip vortex but same sign as the nearby flap vortices. This is clear from Figure 5.8 which shows the vorticity signs of the crosswind, primary vortices, flap vortices and the induced boundary layer (BL).

Until a time of $t^* = 0.5$, that is, $t = 14.5s$, both upwind and downwind flap vortices behave the same way. When the flap vortices are between the primary vortices and the ground, the downwind vortex reaches a minimum altitude with the ground. Thereafter, the vortex interaction between the flap vortices and the two primary vortices differ significantly. For a better understanding, the starboard-side wing-tip-flap vortex pair interaction, as given in Figure 5.10, is discussed first and then moved onto the upwind wing-flap vortex pair as given in Figure 5.9.

As the downwind vortex reaches its minimum height, the adverse pressure gradient is stronger, and the vortex-induced flow beneath is more likely to separate. The additional vorticity due to the presence of crosswind enhances the flow separation as discussed in Section 2.8. As the downwind-flap vortex passes through this unfavorable pressure gradient along with the same-signed induced flow, the long wave instabilities are amplified. This can be observed from $t^* = 0.5$ to $t^* = 0.75$ in Figure 5.10 as the wiggles continue to grow. As downwind-flap vortex is in the strong induced flow field of the downwind vortex, further stretching and tilting occurs and results in the formation of omega-shaped structure as indicated in Figure 5.10 at $t^* = 0.5$. The head and tail of these secondary structures are marked in Figure 5.10 at $t^* = 0.6$ for reference. The head of the secondary structure loop around the primary vortices due to self-induction as shown in Figure 5.10 at $t^* = 0.6$. The head of the structures widens along the axial direction of the downwind vortex, while their tail attaches to the boundary layer of induced flow on the ground at $t^* = 0.7$. Thus, the energy for the secondary vortices are continuously fed by the induced boundary layer.

In the case of port-side vortices as shown in Figure 5.9, the upwind-flap and upwind vortex are still at considerable height from the ground. Comparison of altitude (vertical distance of the vortices from the ground) between the two vortices will be further discussed

in Section 5.5.3. Crosswind induced vorticity layer attenuates the tip-vortex induced vorticity layer as they are opposite in sign in port-side. This results in no visible linking of flap vortices with the induced boundary layer through all the times as shown in in Figure 5.9 from $t^* = 0.5$ to 0.75. Comparing Figures 5.9 and 5.10, it can be inferred that the secondary structures formed from the upwind-flap vortex and its evolution are much slower as compared to the downwind-flap vortex. This is due to the lack of kinetic energy provided by the boundary layer in comparison with the downwind vortices.

In both the cases, the primary vortex pair is heavily strained by the presence of the secondary structures leading to its rapid decay. Since the formation of the secondary vortices are enhanced for the downwind vortex, the circulation of downwind vortex is considerably lower than that of upwind vortex.

5.5 *Comparison of Temporal and Quasi-temporal simulations*

Since the Quasi-temporal simulation method is an upgraded version of Temporal simulation, a comparison study on the evolution of vortices between the two cases are studied and presented in this section. Parameters of quasi-temporal simulation are hereafter referred as LDG since the landing B747 is considered for the comparison and that of Temporal simulations as SPV (Single Pair Vortex). SPV uses the same crosswind, meshing and other simulation setups as that of the Quasi-temporal case except for the initial velocity field. It is reminded here that in Temporal simulation method, the wake is assumed to be a fully developed counter-rotating vortex pair of fixed strength irrespective of the lift distribution. The initial strength of the fully developed vortex pair is considered from the B747 vortex data in Table 5.1 and the velocity field is imposed in the computational domain using Lamb-Oseen vortex model for the Temporal simulation method.

5.5.1 *Circulation and vortex dynamics*

Figure 5.11 shows the comparison of the circulation of the primary vortex pair for the two types of simulation methods. It is clear that there is a striking difference between them. For every given time, the circulations of LDG are much lower than those of SPV.

Figure 5.11 Comparison of non-dimensionalised circulation of wake vortices between LDG and SPV cases

A closer look at the multiple-vortex dynamics will help to understand the results. Figure 5.12 shows the Q-criteria isosurfaces of the flap and tip vortices of both methods colored with tangential vorticity scale for the time, $t^* = 0.45$ and $t^* = 0.6$. The results presented in the left-hand side column corresponds to the LDG case while the right-hand side corresponds to the SPV case. The reason for these two choices of time is because both lies in the rapid decay phase for the LDG case. The first look on the comparison gives a clear picture on the difference in vortex dynamics between the two methods. There are multiple vortices present in the LDG case while there are only two primary vortices present in the SPV case.

$$t^* = 0.45$$

$$t^* = 0.6$$

LDG

SPV

Figure 5.12 Comparison of vortex dynamics between LDG and SPV cases.

As described in the Section 5.4.2, the presence of flap vortices in the LDG case enhances the secondary vortices formation and their interaction with the primary vortices. This is also evident from the images presented in Figure 5.12 at both times, for LDG, as the secondary structures around the primary vortices are visibly seen. This increased vortex interaction results in a sudden drop in the circulation of the primary vortex pair as presented

in Figure 5.11, for the LDG case after $t^* = 0.4$. On the other hand, it is clear from right-hand side images of Figure 5.12, that the secondary structures are yet to form, and the primary vortices are still in diffusion phase. Also, the secondary vortices interaction with the primary vortex structure are much longer for LDG case as compared to the SPV case. In conclusion, due to the presence of multiple vortices in LDG case, the interaction of secondary vortices with primary vortex pair is enhanced as compared to the single pair vortex system.

5.5.2 *Intensity of secondary vortices*

The parameter $|Q|$ represents the quantity of vorticity contained within the secondary vortices. It is a quantitative measure of intensity of secondary vorticity distribution as described in Section 3.8. Table 3.2 in Section 3.8 is presented here as Table 5.3 for the ease of reference. The values of Q-criteria belonging to the range of secondary vortices are integrated over the spatial domain and presented for LDG and SPV cases in Figure 5.13. Since the Q-criteria values associated with different flow features are defined as shown in Table 5.3, any sudden increase in volume integrated Q-criteria ($|Q|$) is a definite representative of increase in the formation of secondary structures.

Table 5.3 Summary of Q-criteria range for different flow features

<i>Flow features</i>	<i>Range of Q-criteria value</i>
Primary vortex pair	$1000 > Q < 10,000$
Secondary vortices	$1 > Q < 100$
Crosswind turbulent structures	$0 > Q \leq 1$

For the LDG case, until $t^* = 0.4$, the primary vortex pair undergoes diffusion phase and the flap vortices gradually revolve around the primary vortices. The secondary vortices are not formed yet and so the value of $|Q|$ is closer to zero until $t^* = 0.4$. As the flap vortices gradually transform to secondary vortices, the $|Q|$ value gradually increases. From $t^* \geq 0.75$,

the induced omega-shaped secondary vortices are formed and so the value of $|Q|$ starts to increase rapidly. After $t^* = 1$, both the vortices are completely in rapid decay phase and there is a large number of secondary vortices formed. Hence, there is a higher value of $|Q|$.

On the other hand, the primary vortices of SPV case undergo a longer diffusion phase than that of the LDG case. This is evident from the figure as the values of $|Q|$ are closer to zero until $t^* = 1$ and the onset of the formation of large number of secondary vortices is delayed for the case of SPV as the increase in $|Q|$ value is seen only after $t^* = 1.5$. It can also be inferred from the figure that the rate of increment of the $|Q|$ value is higher for LDG case than those of the SPV case. The higher the value of the $|Q|$, the higher is the amount of vorticity contained within the secondary vortical structures.

Figure 5.13 Comparison of volume integrated Q -criteria between LDG and SPV cases.

After $t^* = 3$, the downwind vortex along with its secondary vortices exit the domain. The absence of the secondary vortices of downwind vortex causes sudden drop in the $|Q|$ value for both cases.

5.5.3 *Position of the vortices*

Figures 5.14 and 5.15 show the change in position at different t^* in lateral and vertical direction of the primary vortex pair for LDG and SPV cases respectively in time. Note that the lateral direction is the same as the spanwise direction over which the circulation distribution was considered earlier. The word ‘spanwise’ is usually used in wing aerodynamics while the word ‘lateral’ is used in the vortex dynamics. Due to the presence of crosswind, in both LDG and SPV cases, the primary vortices move laterally in the positive y -direction. There is not much difference in the lateral motion of the vortex cores between the two methods as shown in Figure 5.14. However, from Figure 5.15, it can be inferred that the presence of the oppositely signed flap vortices and the early onset of secondary vortices causes significant altitude difference between the two methods, until $t^* = 2.5$ for upwind vortices and $t^* = 1.5$ for downwind vortices.

Figure 5.14 Comparison of lateral position of the vortex core between LDG and SPV cases

Figure 5.15 Comparison of vertical position of the vortex core between LDG and SPV cases

In general, if there are no flap vortices, as in the case of SPV, crosswind favors the formation of the induced boundary layer beneath the downwind vortex. Hence it rebounds at a higher altitude while the upwind reaches the lowest altitude as the induced boundary layer formed below is attenuated. Due to the presence of flap vortices in the LDG case, the induced boundary layer beneath both the vortices are favored and exhibit an almost similar vertical motion. The downwind vortex of LDG case hits a lower altitude between $t^* = 0.5 - 1.0$, as there is an early onset of secondary vortices compared to the upwind vortex as discussed in Section 5.4.2.

5.5.4 *CPU time consumed*

Table 5.4 compares the resources of the high performance cluster of National Supercomputing Centre, utilised by the two methods. CPU time is the addition of total number of hours each nodes have spent to perform the simulation. Memory used is the

temporary memory to perform the calculations. Wall-time is the total number of hours the simulation occupied all of the requested nodes.

With only one extra hour of wall-time, Quasi-temporal simulation was able to deliver results that are specific to aircraft and its lift configurations. The CPU time usage difference between the two methods is around 27 hours. The memory used for the computation is almost the same with a difference of a few hundred Megabytes (Mb).

Table 5.4 Comparison of time and memory consumption

Parameters	Temporal simulation	Quasi-Temporal
CPU time used	80hrs 27min 53s	107hrs 49min 27s
Memory used	11.761 Gigabytes	11.520 Gigabytes
Wall-time used	03hrs 21min 47s	04hrs 30min 18s

5.6 Validation

The proposed Quasi-temporal method is validated with the real-time LIDAR measurements from the WakeFRA campaign conducted at Frankfurt Airport in 2004 over five months. The LIDAR measurement data of 288 vortex pairs in ground proximity of ‘Heavy’ category aircrafts such as A340-300, A340-600, and B747-400 are considered for validation [27]. The flow field is scanned using a 2- μ m pulsed LIDAR system. The measured wake-vortex properties such as circulation and position were derived from the estimated tangential velocity profiles using an interactive four-stage data processing algorithm [175]. The background flow for the simulation is considered such that it mimics the measured atmospheric parameters of the experiments [27]. It should be noted that there is no crosswind in the measured atmospheric parameters [27]. The measured LIDAR data corresponds to a mixture of approaching ‘Heavy’ category aircrafts in ground proximity. So

the landing configuration of B747 is considered once again for the initialization using PVD method. Hence, both upwind and downwind vortices exhibit similar characteristics. In Figures 5.16 and 5.17, Prandtl LDG model in the legend refers to the results of newly proposed Quasi-temporal LES methodology.

Figure 5.16 Evolution of non-dimensionalised circulation: LIDAR measurements [27] vs Quasi temporal simulation results

Non-dimensionalised circulation of radii from 5m to 15m is plotted against the LIDAR data in Figure 5.16. From the figure it is evident that the results of the Quasi-temporal methodology agree well with the real-time measured circulation data. Non-dimensionalised circulation being an important parameter in the vortex study, its correlation with the measured data marks the success for the newly proposed method. The vertical position of the vortex core is plotted in Figure 5.17. It is clear from the figure that the vertical position oscillates around the measured data and provides a better match after the initial roll-up process ($t^* = 1.5$). The simulated wake vortices rebound height is more or less the same as the measured data. From another perspective, it can be inferred that the vortices vertical position matches well with the measured data after the vortex rebound. The

deviations may also be due to the generalized measured LIDAR data representing a mixture of ‘Heavy’ aircraft types.

Figure 5.17 Non-dimensionalised vertical position: LIDAR measurements [27] vs Quasi-temporal simulation results

Figure 5.18 shows the Q-criteria isosurfaces of the wake vortices before and after roll-up. Initially, there are wing-tip and flap vortices which then roll-up into a distinct counter-rotating vortex pair as shown in Figure 5.18 at $t^* = 1.7$. The additional vorticity sheets found in the domain are the induced shear layer by the primary vortex pair and should not be confused with the flap vortices. As the flow evolves, the flap vortices gradually merge with these induced shear layers.

Figure 5.18 Wake vortices before ($t^ = 1.2$) and after roll-up ($t^* = 1.7$)*

5.7 Advantages of PVD method

The most important advantages of the PVD method presented in this thesis, are as follows:

- Universally adaptable for all type of aircrafts and its lift configurations. The required input to perform the wake vortex analysis is the spanwise lift distribution profile of the aircraft.

- This method consumes only one extra hour of computational wall-time as compared to Temporal LES but provides a solution with an accuracy equivalent to real-time LIDAR measurements.
- The whole simulation of roll-up to decay phases of wake vortices is performed only in LES. There is no swapping of velocity information between RANS and LES taking place during the simulation. Therefore, this method consumes lesser computational resources than Spatial LES.
- It can be extended to do preliminary study of wake vortices shed behind any novel wing designs like Blended-Wing Body. This preliminary study can be used to optimise the wing design so that the resulting wake vortices dissipate faster.
- It is not restricted to simulation of wake vortices in ground proximity but can also be extended to study wake vortices in cruise altitudes.
- During a roll motion, the lift and circulation distributions will be asymmetric on either side of the aircraft. This difference in circulation is assumed to be the circulation of the rolling aircraft structure. With this assumption, the proposed method is valid to investigate the effects of pilot control input such as roll maneuvers on the wake vortices.

5.8 *Limitations*

- The wake of aircrafts with high angle of attack cannot be simulated due to the possibility of flow separation.
- The jet exhaust from the engine and the turbulence due to landing gear extension is not included.
- The lift force vanishes once the aircraft touches down in the runway. End effect is the vortex bursting phenomena that occurs in the wake behind an aircraft at the touchdown point due to this sudden change in lift. The method does not account for the end effect. However, this limitation can be overcome by including the lift-producing bound vortex as detailed in the Section 7.2.

5.9 *Summary*

Recalling the fundamental fact about wake vortices, they are by-products of the lift. Hence, the study of influence of lift distribution on the shed wake vortices downstream is of prime importance. Quasi-temporal LES methodology was introduced in this chapter primarily for the study of effect of different lift distribution on the trailing wake vortices. The initialization method for the Quasi-temporal LES based on Prandtl Lifting-Line theory was discussed in detail and was referred as Prandtl Vorticity Distribution (PVD) method. The proposed PVD method is simple and effective. The only requirement for this method is a known distribution of lift/coefficient of lift and the aircraft wing geometry. The validation of the proposed method against the experimental landing of B747 aircraft was presented in this chapter. One of the main advantage of this method is that, it does not assume the number of vortices present downstream of an aircraft. Rather, it allows the vortices to be initialised based on the vorticity distribution calculated from the lift distribution of any given aircraft. Hence, for a landing B747 aircraft, the resulting trailing wake vortices consist of two pairs, one is the primary/wing-tip vortices and the other is the flap vortices formed due to the extended flaps. The interaction of flap and primary vortices were also discussed in this chapter. A detailed comparison between Temporal and Quasi-temporal simulations were also presented to demonstrate the superiority of the proposed new method.

6 Artificial Enhancement of Wake-Vortex Dissipation

In this chapter, four methods to artificially enhance the dissipation rate of the aircraft wake-vortices are examined. It is to be noted that the higher the interaction of secondary vortices with the primary vortices, the higher is the reduction in their strength and more effective is the method of dissipation. Out of the four methods, firstly, the evolution of flap vortices in the presence and absence of crosswind is discussed. This study is essential as the long wave instabilities in the flap vortices (as discussed in Section 5.4.2) are caused by the atmospheric conditions and are responsible for the increase in the formation of secondary vortices and its interaction with the primary vortex pair. Secondly, the span loading of the B747 aircraft is modified and its effect on dispersing the primary vortex structure is studied. It is proven in early researches that the wake vortex dynamics is highly dependent on the lift distribution over the aircraft wing [163-172]. Most of them are quantitative experimental and real-time measurement data, which are obtained between 1970s and 2000s. A detailed quantitative numerical study was not performed then, possibly due to the lack of the computational resources and numerical methodologies.

The Quasi-temporal methodology, proposed as part of this dissertation, is capable of accounting for any change in the lift distribution and is more efficient in simulating the corresponding wake-vortices. Hence, the idea of spanloading modification is revisited and its effect on the wake vortex characteristics is studied in detail. Strength and position of the primary vortex pair and flap vortices along with $|Q|$ plots are used as tools to aid the understanding of the vortex dynamics.

Jordan [176] and Rossow [170] and in 1980s had presented experimental results to prove that roll manoeuvres alleviate wake-vortices. Inspired by their work, as a third artificial alleviation method, a small amplitude roll oscillation case with two wavelengths are considered in order to see its effectiveness in dissipating the primary vortex pair. However, the idea of using roll manoeuvre in ground proximity is still in its nascent stage and needs an extensive study to become a feasible solution. Lastly, the dynamics of wake-vortices behind two configurations of formation flights are investigated: one with a 400ft lateral separation distance while the other with 500ft vertical separation distance between

the centreline of two ‘Heavy’ category aircrafts. These cases are explored to examine if parallel flights could be a temporary solution to relieve the air traffic congestion.

6.1 Enhancement of flap vortex instability

6.1.1 Circulation with/without crosswind

The enhancement of wake vortex decay due to the flap vortex instability is described earlier in Section 5.4. It is proven that the additional vortices introduced in the wake due to the flap deployment, can reduce the life span of the vortices. Hence, it is necessary to study the parameters which induce this flap vortex instability.

Based on the results presented in Sections 4.3 and 5.4.2, it is speculated that the presence of crosswind induces the long wave instability in the flap vortices and also causes an asymmetric evolution of the primary vortex pair and their corresponding flap vortex. Hence, to verify this observation, the evolution of wake-vortices behind a standard landing configuration of B747 is studied with and without the presence of crosswind. It is to be noted that the crosswind used for the simulations are based on turbulent pipe flow and they possess small-scale turbulent eddies mimicking the atmospheric crosswinds.

Figure 6.1 Schematics of multiple wake vortices and their vorticity signs

Figure 6.1 is a recap of the schematics presented earlier as Figure 5.8 in Section 5.4.2 as Figure 5.8 for ease of reference. The flap vortex in the port-side of the wing is referred as upwind-flap vortex and the one on the starboard-side is referred as downwind-flap vortex. Similarly, the wing-tip vortex in the port-side of the wing is referred as upwind

vortex and the one on the starboard-side is referred as downwind vortex. The direction and sign of the vortices and crosswind are as presented in the Figure 6.1. The vorticity sign of the flap vortex is opposite to the nearby primary vortex while same as the nearby induced shear layer of the Boundary Layer (BL). The induced shear layer in the figure is represented as a circular arrow to represent its vorticity sign and should not be mistaken for a rotating vortex. Hereafter, the case with crosswind is denoted as CW and the case without crosswind is denoted as No CW in the upcoming figures and discussions.

Figure 6.2 Evolution of circulation of wake-vortices behind landing B747 (with and without crosswind)

The circulation plot over time for the primary vortices of both cases is presented in Figure 6.2. From this figure, it can be inferred that when there is crosswind, the presence of flap vortices have profound effect on the evolution of primary vortices. This can be verified from Figure 6.2 as the primary vortices of CW case possess lower circulation strength at every time step when compared to that of No CW case. Hence, it is concluded that the effectiveness of using flap vortices to induce rapid decay in the primary vortex pair is greatly depending on the atmospheric crosswind.

Figure 6.3 Position of the centre of the upwind vortex and upwind-flap vortex from $t^* = 0$ to $t^* = 1.25$

Figure 6.4 Top-view of flap and wing tip vortices of a landing B747 aircraft in the presence of crosswind at $t^* = 1.2$

Note: Q-criteria isosurfaces are coloured according to the z-component of vorticity scale.

Figure 6.3 shows the position of upwind flap and wing-tip vortices in the port-side from $t^* = 0$ to $t^* = 1.25$, that is, $t = 36.25$ s. After $t^* = 1.25$, the flap vortices are completely

deformed. It can be seen that the flap vortex complete a 360° revolution around the upwind vortex with the time period of $t^* = 1.2$. As they revolve around the primary vortices, the vorticity diffuses and its strength gradually decreases.

Figure 6.4 presents the top view of the Q-criteria isosurfaces of the flap and wing-tip vortices in the presence of crosswind at $t^* = 1.2$ coloured by tangential vorticity. In the presence of crosswind, the flap vortices exhibit long wave instability at early stages resulting in an earlier onset of formation of secondary vortices around the primary vortex pair. The evolution mechanism is detailed in Section 5.4.2 in Figures 5.9 and 5.10. The flap vortices in the presence of crosswind are distorted significantly even before it revolves half way around the primary vortex pair. It is clearly seen from Figure 6.4 that the flap vortices are eventually evolved into omega-shaped secondary structures and interacting with the primary vortex pair along its axial direction.

Figure 6.5 Evolution of flap and tip-vortex without crosswind

Note: Q-criteria isosurfaces are coloured according to the z-component of vorticity scale.

On contrast, in the absence of crosswind, there is no long wave instability (wiggles) in the flap vortices along its axial direction. This is evident from the Q-criteria isosurfaces

presented in Figure 6.5. Thus, the flap vortices do not get distorted with time. Once the flap vortices reach the minimum altitude from ground after one complete revolution, they merge with the induced vorticity shear layer beneath the primary vortices. It is important to note that the merging of the flap and vorticity layer in the absence of crosswind do not result in the formation of secondary vortices. Due to the absence of crosswind, the induced secondary vorticity layer on the ground by the primary vortices are also not strong enough to initiate the detachment of flow from the ground. Hence, there is no visible detachment of secondary vortices structure from the ground until $t^* = 1.7$ as shown in Figure 6.5.

In conclusion, the crosswind flow plays an important role in inducing the instability of the flap vortex structure thereby resulting in an early onset of rapid decay phase. Further, it provides flap vortices with the kinetic energy to interact with the primary vortex pair through the enhanced induced vorticity layer formation on ground. The evolution of upwind and downwind vortices are symmetrical for the case without crosswind.

6.1.2 Position of the primary vortex pair

In the presence and the absence of the crosswind, the lateral motion of the two-pair vortex system resulting from the LDG configuration of B747 aircraft behaves the same way as the single pair vortex system as described in Section 4.3.2. Figure 6.6 shows the lateral position of the two-pair vortex system with and without crosswind. Although, the computational domain extends up to $y^* = 4.0$, when the center of the primary vortex pair crosses $y^* \approx 3.0$, the secondary vortices surrounding them already reaches $y^* = 4.0$. Hence, the dashed line is considered as the end of the domain for the study of the lateral motion of the vortices. From the figure, it can be inferred that in the presence of crosswind, the primary vortex pair behaves asymmetrically. That is, the upwind vortex pair stays longer in the domain with a comparatively higher strength while the downwind vortex pair leaves the domain earlier. This phenomenon of the upwind vortex evolving alone in the domain at later times is called Solitary vortex phenomenon and is detailed in Section 2.8. The crosswise speed of the upwind vortex is lower than that of the downwind vortex.

In the absence of crosswind, the flap and wing-tip vortex pairs revolve around each other and stay within the domain until they decay. The upwind vortex stays in the port-side and the downwind vortex stays in the starboard-side. Due to mutual induction, initially the two vortices move away from each other. As they move, they gradually lose their strength because of diffusion and so the induced force on each other reduces. After $t^* = 3.0$, the vortices stay almost in the same lateral location, i.e., upwind and downwind vortices at $y^* = 1.2$ and $y^* = 1.4$ respectively.

Figure 6.6 Lateral position of wake-vortices behind landing B747 aircraft (with and without crosswind)

Figure 6.7 shows the vertical motion of the two pair vortex system with and without background crosswind flow. In both cases, initially the upwind and downwind vortices move downwards due to mutual induction and then they rebound (or regain altitude) due to the presence of induced boundary layer on the ground beneath them. In the absence of crosswind, the vortices show symmetric behaviour. That is, both the vortices move in the vertical direction in a similar way from its original position. Due to the longer revolution of

flap vortex around the primary vortex pair in the absence of the crosswind, there is an oscillation in vertical position of the primary vortex pair until $t^* = 2.0$ as indicated in Figure 6.7.

When the circulation plot of CW case presented in Figure 6.2 is closely examined, the downwind vortex starts to decay faster than the upwind vortex from $t^* = 0.75$. This is due to the favouring of formation of the secondary vortices around the downwind vortex in the presence of crosswind. This is the same reason for the downwind vortex to reach a lower altitude as compared to the upwind vortex around this time as shown in Figure 6.7. The interaction of the downwind vortex with the secondary vortices pulls them towards the ground.

Figure 6.7 Vertical position of wake-vortices behind a landing B747 aircraft (with and without crosswind)

6.2 *Spanloading modification study*

The aim of this section is to prove that modifying the lift distribution will result in redistribution of the vorticity, thereby changing the vortex dynamics of the shed vortex. Current investigation is based on a water tunnel experimental study conducted by Corsiglia et al. in 1976 [166]. They used a scaled down B747 model as the wake-vortex generator. Corsiglia et al. [166] used two unconventional span loadings that were theoretically found to yield larger vortex cores and multiple vortex systems. These span loadings were achieved by independent deflection of the trailing edge inner and outer flaps. They concluded from their experiments that one of the two span loadings were effective in dissipating the vortices. Inspired by their research, in this section, the same span loadings are considered for initialization through the new PVD method as it can account for different flap settings. Their vortex dynamics are studied in detail to investigate why one of the two configurations resulted in better dispersion of the vortices. The vortex characteristics of the modified span loadings are compared with the results of the conventional landing configurations for a better understanding.

6.2.1 *Recap of B747 specifications*

Figure 6.8 [166] shows the specifications of wing and high-lift devices for a typical B747 aircraft. Table 6.1 [166] provides the information on the position of the high-lift devices that corresponds to the original landing configuration (LDG) and two modified landing configurations (MLDG-1 and MLDG-2). Table 6.2 [174] presents various wake-vortex parameters of B747. The wingspan, characteristic time and velocity in the table are used to non-dimensionalise the circulation, position and time in the upcoming simulated results as presented in earlier Table 4.2. A crosswind of 1.75m/s is considered for the simulated cases presented under this section as it mimics the atmospheric conditions. The presence of crosswind also enhances the secondary vortex formation. Earlier onset of secondary vortices formation will aid in analyzing the vortex dynamics in depth.

Table 6.1 Position of high-lift devices for B747 aircraft [166]

	<i>Landing configuration (LDG)</i>	<i>Modified LDG – 1 (MLDG – 1)</i>	<i>Modified LDG -2 (MLDG – 2)_r</i>
<i>Inboard flap deflection angle</i>	46°	46°	0°
<i>Outboard flap deflection angle</i>	46°	0°	46°

Figure 6.8 B747 specifications [166]

Table 6.2 Wake vortex parameters of B747 [174]

Parameters of B747	Landing
Initial circulation (m^2/s)	554.6
Aircraft speed (m/s)	80
Characteristic velocity scale (m/s)	1.75
Characteristic time scale	29 s
Wing span	64.4 m
Crosswind speed (m/s)	1.75

6.2.2 Modified landing configuration -1 (MLDG-1)

Figure 6.9 [166] shows the predicted spanwise lift and calculated circulation distribution for the first modified landing configuration of B747 aircraft. Unlike the conventional landing configuration, here, the outboard flap is retracted and the inboard flap is extended. As described in Section 5.3, there will be a free vortex shed for every change in the lift distribution over the aircraft wing. The strength of these free vortices downstream are indirectly proportional to the gradient of lift distribution. Figure 6.10 shows the free vortex strength distribution over the wing and Figure 6.11 represents the initial vorticity distribution using PVD method.

Figure 6.9 Predicted spanwise lift [166] and calculated circulation distribution for a MLDG - 1

From Figures 6.9, 6.10 and 6.11, it can be seen that the first upslope in the lift distribution corresponds to the inboard edge of the inboard flap and results in an inboard flap vortex with a considerable free vortex strength. Hereafter this vortex is referred as flap

vortex). It should be noted that the free vortex strength of this vortex is of opposite sign to that of the tip-vortex as demonstrated in Figures 6.10 and 6.11. The downward slope from the outboard tip of the inboard flap and over the outboard flap is higher than the slope near the tip. Hence, the tip-vortex is split into two concentrated vorticity distributions. It is important to note that the one located between the two flaps is stronger than the tip-vortex. The tip-vortex finally merges with this inner stronger vortex in an infinitesimally short time and is referred as primary vortex pair in the following discussions. Thus, the MLDG-1 configuration results in a two-pair vortex system with the stronger counter-rotating vortex pair closer to the inboard flap vortices. It should be noted that this two pair vortex system was also seen in the experimental study by Corsiglia et al.[166]. This also serves as a further support of the LES simulation using the newly proposed initialisation technique.

Figure 6.10 Spanwise free vortex strength distribution for MLDG-1 configuration

In Figure 6.11, the two end vortices of same-sign on each side of the aircraft wing constitute the wing-tip vortices. After a time of 2.5 seconds, these two like-signed vortices merges into single vortex on each side of the wing. The vortex rotating in clockwise direction in the port-side of the aircraft is referred as upwind vortex and the one rotating in

the anti-clockwise direction in the starboard-side of the aircraft is referred as downwind vortex. The flap vortex in the port-side is referred as upwind-flap vortex and the one in the starboard-side is referred as downwind-flap vortex.

Figure 6.11 Initial vorticity distribution using PVD method for MLDG – 1

6.2.3 Modified landing configuration – 2 (MLDG – 2)

In MLDG – 2 configuration, the outboard flap is extended and the inboard flap is retracted. Hence, there is an increase in the lift over the span of the extended outboard flap as shown in Figure 6.12. This sudden change in lift results in the formation of the flap vortices downstream in the wake of the aircraft. Unlike MLDG-1 configuration, here, there is only one pair of flap vortex and one pair wing-tip vortex from the beginning of the time, $t^* = 0$. Another significant difference is that the oppositely signed flap vortices correspond to the change in lift distribution over the inboard edge of the outboard flap.

The change in the circulation at the inboard edge of the outboard flap is lower as compared to the change in circulation of the inboard edge of the inboard flap in the MLDG-1 configuration. This will be clear when Figure 6.12 is compared with Figure 6.9. Hence, the resulting flap vortex pair for the current configuration will have a lower free vortex

strength as compared to that of MLDG-1. Figure 6.13 shows the spanwise variation of free vortex strength which can be compared with Figure 6.10 to confirm that the free vortex strength of the oppositely signed flap vortex of MLDG-2 is lower than that of MLDG-1. Also, the absolute maximum free vortex strength of wing-tip vortex is higher for the MLDG-1 as compared to MLDG – 2.

The initial vorticity distribution based on the spanwise free vortex strength is presented in Figure 6.14. The downslope from outboard edge of the outboard flap to wing-tip is gradual as shown in Figure 6.12 and so it results in a diffused wing-tip vortex spanning from the outboard tip of outboard flap to the wing tip as shown in Figure 6.14.

Figure 6.12 Predicted spanwise lift [166] and calculated circulation distribution for a MLDG – 2 configuration

Figure 6.13 Spanwise free vortex strength distribution for MLDG - 2 configuration

Figure 6.14 Initial vorticity distribution using PVD method for MLDG - 2

6.2.4 Evolution of circulation

Figures 6.15 and 6.16 show comparison of evolution of upwind and downwind vortices' strength respectively behind three different landing configurations. It is clear from the figures that the MLDG – 2 lift configuration results in higher circulation strength for both upwind and downwind vortices at all time steps. When the evolution of circulation of LDG and MLDG – 1 configurations are compared, the upwind vortex strength shows considerable reduction after $t^* = 1$ while the downwind vortex do not have significant difference. This can be explained using Figures 6.17 and 6.18.

Figure 6.15 Evolution of circulation of upwind vortex for various landing configurations

Figure 6.16 Evolution of circulation of downwind vortex for various landing configurations

Figures 6.17 and 6.18 show the flap and wing-tip vortex interactions at various time step for MLDG-1 and MLDG-2 configurations respectively. As the vortex evolution is closely monitored with the help of Figure 6.17, for the case of MLDG-1, the flap and wing-tip vortices are closer in proximity to each other as compared to the other case. This is also evident from Figure 6.11 in which the initial vorticity distribution and locations are presented for MLDG-1 case. Due to the presence of high velocity field of the primary vortex pair in close proximity, there is a higher degree of distortion in the flap vortices resulting in the formation of large number of strong secondary structures. These secondary structures approach the primary vortex at many points along the axial direction of the primary vortex pair due to self-induction. While for the case of LDG case, the distorted flap vortex initially approach the primary vortex pair only at one point along the axial direction at $t^* = 0.5$ as can be seen in Figure 5.10 of Section 5.4.2.

Figure 6.17 Flap and wing-tip vortex interaction for MLDG – 1 configuration
Q-criteria isosurfaces. coloured according to the tangential vorticity.

Figure 6.18 Flap and wing-tip vortex interaction for MLDG – 2 configuration. Q -criteria isosurfaces. coloured according to the tangential vorticity.

Also, it should be noted from Figure 6.14 that the wing-tip vortices separation distance is smaller for the case of MLDG-1. This reduced separation distance enables the secondary structures of both the vortices to dynamically interact with each other. This combined effect of difference in the location and strength of the vortices resulted in a lower circulation value as compared to the LDG case.

Figure 6.18 shows the top view of the evolution of vortex system shed behind a MLDG-2 lift configuration. It is clear from Figure 6.18 that the oppositely signed flap vortices are feeble compared to the primary vortex pair. Hence, the secondary structures that are arising out of these flap vortices are also lower in strength. Although it may seem like more number of secondary structures are formed from the flap vortices at the earlier time steps, these weak structures do not last long.

As the vortices evolve in time, the secondary structures formed are no more defined structures like the omega-shaped structures formed in other configurations. In addition, their interaction were also not strong enough to tilt and distort the primary vortex structure. Hence, a higher strength is found for both the upwind and downwind vortices of this case at all time steps compared to the other two cases.

6.2.5 *Intensity of secondary vortices*

Volume integration of Q-criteria corresponding to the secondary vortices regime is presented in Figure 6.19. The values stay closer to zero until $t^* = 0.75$ as only the flap vortices are distorted and looping around the primary vortices and the secondary vortical structures are not exactly formed. The minor peak in the MLDG-1 curve before $t^* = 0.75$ is due to the induced vortices in region around the flap vortices. It can be ignored as they are not those vorticity regions disappear hinder the calculation of $|Q|$ as the vortices evolve. The value of $|Q|$ after $t^* = 1$ is the main focus as that is when the flap vortices are completely transformed to secondary structures for both upwind and downwind vortices in all of the cases. It can be clearly seen from the graph that the amount of vorticity corresponding to the secondary vortical structures is higher for MLDG-1 case followed by the LDG case and then the MLDG-2. This supplements the argument that MLDG-1 configuration results in a

formation of stronger secondary vortices and thus leading to a lower strength of the primary vortex pair. The higher the interaction of secondary structures with the primary vortices, the higher is the dispersion of the primary vortex strength.

Figure 6.19 Intensity of secondary vortices for various landing configurations

At $t^* = 3.5$, the downwind vortex for all cases exits the domain along with its secondary vortex structures resulting in a drop of $|Q|$ value for all the cases.

6.2.6 Position of the vortices

Figures 6.20 and 6.21 present the lateral movement of the upwind and downwind vortices respectively. In the presence of crosswind, both vortices move in the positive y -direction with time. As discussed in Section 4.3.3, the upwind vortex of all three cases possess lower crosswise velocity than the downwind vortex. The same conclusion is drawn for wake-vortices of all three landing configurations. For all of the landing configuration cases, the upwind vortex stays longer in the domain compared to the downwind vortex.

Figure 6.20 Lateral movement of upwind vortex for various landing configurations

Figure 6.21 Lateral movement of downwind vortex for various landing configurations

From around $t^* = 1.0$ to 2.2 , upwind vortex has small variation in the y -direction for all three landing configurations respectively for LDG which is nearly stationary as shown in Figure 6.20. The halting location of upwind vortex of MLDG-2 configuration is closer to the mid-plane. The upwind vortex of conventional landing configuration stays still around their initial y^* location within this time period. The halting position of upwind vortex of MLDG-1 case vary between the positions of upwind vortex of MLDG-2 and LDG configurations. Its lateral position is not as stationary as it is for the other two cases. The lateral motion of the downwind vortex traces a linear curve as shown in Figure 6.21. The crosswise velocity, that is the slope of the linear curve of y^* with respect to t^* , is slightly different for each cases. The crosswise velocity of LDG case marks the highest value followed by MLDG-1 and MLDG-2. Therefore, the time of the downwind vortex to reach $y^* = 3.0$ differs slightly among the three cases as it can be seen from Figure 6.21 that the gradient of LDG is the highest while that of MLDG-2 is the lowest.

Figures 6.22 and 6.23 show the altitude of the upwind and downwind vortex respectively over time. Generally both vortices descend through the atmosphere due to mutual induction. After reaching a minimum altitude to the ground, due to the presence of the induced shear layer at the ground, the vortices rebound after $t^* = 1.5$ and start to ascend through the atmosphere.

From Figure 6.22, it can be inferred that the minimum vortex height of upwind vortex from the ground is the same for both modified span loading cases and is lower than that of conventional LDG case. After vortex rebound, that is after crossing the minimum altitude from the ground, the upwind vortex of both modified configurations reach almost the same altitude at all time. The rebound altitude for the upwind vortex of the LDG case is the highest among the three throughout all the time investigated.

Figure 6.22 Vertical movement of upwind vortex for various landing configurations

Figure 6.23 Vertical movement of downwind vortex for various landing configurations

Figure 6.23 shows that the downwind vortex of LDG configurations descends to an altitude of almost 50% of its original height at $t^* = 0.5$. It stays around this altitude for a duration of $\Delta t^* = 0.5$. The downwind vortex of the MLDG-1 configuration, as it descends through the atmosphere, it reaches an altitude slightly higher than that of the LDG case. This is due to the presence of the stronger flap vortices around the downwind vortex of the MLDG-1 configuration, which is additionally favoured by the crosswind. The vortex rebound of downwind vortex of MLDG-2 configuration happens at a later time compared to other two cases. The vortex rebound height of the downwind vortex of the MLDG-1 is the highest among three cases. The weaker the secondary vortices, the weaker is their pulling effect on the primary vortex pair in the downward direction during its interaction in later times.

Figure 6.24 Position of upwind vortex core

Figure 6.25 Position of downwind vortex core

Figures 6.24 and 6.25 show the movement of upwind and downwind vortices in the y^*-z^* plane respectively. It can be clearly seen that initially both the vortices are pushed towards ground for all cases due to mutual induction. Then, the upwind vortex stays in a particular y -location while it descends and ascends through the atmosphere. This can be cross-verified with the help of the curves presented in Figure 6.20 where there is nearly a horizontal straight line (i.e. a constant y^* value) for LDG configuration, while MLDG-1 and MLDG-2 configurations with small variations especially for MLDG-2 with only two straight lines with very small y^* difference between $t^* = 1.0 - 2.0$. From Figure 6.25, it can be inferred that when the rebound of downwind vortex happens, there is a constant lateral motion unlike in the case of upwind vortex. This asymmetric behaviour of the upwind and downwind vortices is due to the presence of crosswind which enhances the crosswise velocity of the downwind vortex while initially inhibiting that of the upwind vortex.

6.3 Roll oscillation

The effect of roll oscillations on the wake-vortices for B747 landing configurations is investigated in this section. Assume that the landing aircraft is slowly rolled to its right and then back to its original position. Figure 6.26 shows the direction of flight and the direction of the roll motion. It is essential to note that the roll motion introduces changes in the lift and circulation distribution in the aircraft's landing direction (z^*). The lift and circulation over the right-wing will be higher and the left-wing will be lower due to the rolling motion. This differential circulation will result in a change in the wake-vortices shed behind the aircraft.

Figure 6.26 Roll motion of the aircraft

In this section, this change in the wake-vortices shed behind the aircraft in landing direction is examined as one of the methods to introduce instability into the system. Since the aircraft is in ground proximity, a minimum change in the lift distribution is assumed to ensure safety and passenger comfort. It is important to note that the roll motion is assumed to follow only a half wavelength sine curve ($\theta = 0 - \pi$) and the roll of the aircraft to the

left is not considered. This is due to the limited time available for the aircraft to perform the maneuver before landing.

6.3.1 PVD method initialization

To ensure that a smooth rolling motion is simulated, it is assumed that the aircraft rolls following a sine curve and there is no flow separation over the wing during the rolling motion. The rolling manoeuvre of aircrafts is generally in time. Since time and spatial coordinates are interchangeable in wake vortices study, the motion is assumed to be happened in the landing direction (z^*) of the aircraft. Hence, the PVD method initialisation technique is still valid.

Figure 6.27 Spanwise circulation distribution over left and right wing during roll motion.

For simplicity, the outboard flaps of the B747 aircraft are assumed to perform the function of ailerons. In Figure 6.27, the spanwise circulation distribution over the left-wing and right-wing due to the roll motion is compared with the conventional landing configuration (baseline). The curve with blue 'o' marking represents the baseline spanwise circulation distribution of an unmodified landing configuration where the aircraft flies

through the domain with no roll motion. The change in circulation on the left and right wing due to roll maneuver is marked by orange ‘*’ and yellow ‘□’ respectively in Figure 6.27. The circulation over the outboard flap is increased by 5% in the right-wing, while it is reduced by 5% over the left-wing. This differential spanwise circulation distribution represents the roll of an aircraft. Since the deflection is small, the vertical and lateral displacements of the aircraft and the resulting wake vortices due to roll is neglected.

The smooth transition of circulation due to roll at a specific y-location in the z-direction is given as Equation 6.1.

$$\gamma_{init}(z^*) = a + b * \sin\theta \text{ at } y^* = j \quad (6.1)$$

where, γ_{init} is free vortex strength function used for PVD method initialization at specific y-location, j, z^* is non-dimensionalised location in the landing direction of the aircraft, y^* is non-dimensionalised spanwise location, a is baseline free vortex strength value at $y^* = j$, b is amplitude of the oscillation and θ is function of z^* .

Before the aircraft starts to roll, the lift distribution has to be the values of conventional landing configuration, Hence, θ is assumed to be zero and the constant ‘a’ takes the free vortex strength values of the baseline. Since aircraft rolls only to the left, the aircraft is assumed to be in the left most rolled position at $\theta = \pi/2$. At this moment, the left and right wings have to have the maximum difference in the circulation. Hence, the values of ‘b’ is the values of the increase/decrease in the free vortex strengths at the corresponding y^* -location. When the aircraft is back to its original position, that is when the roll maneuver is completed at $\theta = \pi$, the lift distribution is back to the conventional landing configuration and so are the free vortex strength values. Since the change cannot be sudden pulse but a continuous motion, a sine wave is assumed and the angle for the sine curve and the free vortex strength is given by the following equation,

$$\begin{aligned} \theta &= \pi(z^* + \lambda) \\ \gamma_{init}(z^*) &= a + b * \sin \pi(z^* + \lambda) \text{ at } y^* = j \end{aligned} \quad (6.2)$$

where λ – wavelength of the sine wave.

This parameter determines the duration within which the rolling motion is completed. If the wavelength, $\lambda = 1b_0$ (or $\lambda = 64.4\text{m}$), it implies that the aircraft completes its roll motion within this distance. Two wavelengths $1b_0$ and $2b_0$ are considered for the current study while the amplitude is kept constant.

Figure 6.28 shows the corresponding free vortex strength for the baseline, left-wing and right-wing during roll. It can be concluded from the figure that the main difference comes into picture only at $y^* = 0.2247$ and $y^* = 0.3538$, that is where the slope of the circulation distribution is either increased or decreased due to the rolling motion of the aircraft. Since $b = 0$ in the rest of the y^* -locations, the free vortex strength (γ_{init}) at these locations during the roll maneuver is the same as the baseline case.

Figure 6.28 Spanwise free vortex strength distribution over left and right wing during roll motion

The change in the values of free vortex strength at $y^* = 0.2247$ and $y^* = 0.3538$ for both sides of the wing from the baseline are given as follows,

Left wing: (6.3)

$$\gamma_{init}(z^*) = -0.5686 + 0.2913 \sin \pi(z^* + \lambda) \text{ at } y^* = 0.2247$$

$$\gamma_{init}(z^*) = 0.7179 - 0.2259 \sin \pi(z^* + \lambda) \text{ at } y^* = 0.3538$$

Right wing: (6.4)

$$\gamma_{init}(z^*) = -0.5686 - 0.2913 \sin \pi(z^* + \lambda) \text{ at } y^* = 0.2247$$

$$\gamma_{init}(z^*) = 0.7179 + 0.2259 \sin \pi(z^* + \lambda) \text{ at } y^* = 0.3538$$

The constant in the Equations 6.3 and 6.4 are the constant, γ_0 mentioned in the Equation 6.1 which represents the free vortex strength of the conventional landing configuration at the corresponding y^* location. Taking a closer look at the Equations 6.3 and 6.4, it can be inferred that an increase in the circulation of the right wing resulted in an increase in the absolute value of the free vortex strength of the wake vortices shed downstream while a decrease in the circulation distribution over the left wing resulted in decrease in the absolute values of free vortex strength.

It is inevitable to note that using the PVD initialisation method, an entire roll maneuver is simplified into changing the values of the free vortex parameter. Note that in the subsequent sections, ‘LDG’ refers to the baseline case, ‘Roll-1b’ refers to the first case where the wavelength is equal to $1b_0$ and ‘Roll-2b’ refers to the second case where the wavelength is equal to $2b_0$. It is reminded that all the other parameters in Equation 6.1 are unchanged except for the θ .

6.3.2 *Evolution of circulation*

Figures 6.29 and 6.30 compare the evolution of upwind and downwind vortices respectively with and without roll oscillations.

Figure 6.29 Evolution of circulation of upwind vortex for roll oscillations

Figure 6.30 Evolution of circulation of downwind vortex for roll oscillations

It can be inferred from Figure 6.29 that between $t^* = 2.75$ and 4.5, there is a slight difference in the strength of the upwind vortex when the aircraft performs roll motion. But the vortices are already 30% of their original strength and so it does not have any practical

significance. From Figure 6.30, it can be concluded that the downwind vortex of all three cases do not show much difference until $t^* = 2.0$. The rolling motion with a wavelength of $1b_0$ results in a slightly higher circulation than that of the baseline while the other rolling motion results in a slightly lower value after $t^* = 2.0$. The strength of the downwind vortex at this point of time is already 30% of its initial strength just like in the case of upwind vortex. Hence, the effect of roll oscillation of both wavelengths are concluded to be not that effective in dispersing the wake-vortices.

6.3.3 *Position of the vortices*

Figures 6.31 and 6.32 represent the lateral motion of the upwind and downwind vortices in ground proximity respectively for the baseline and the two rolling motion cases. The upwind vortex of the aircraft performing a rolling manoeuvre traverses the same path as the one in the LDG case until time $t^* = 3$. Thereafter, the upwind vortex of the Roll-1b case deviates from the position of the upwind vortex of the LDG case. The lateral movement of the upwind vortex of Roll-2b case deviates from the baseline (LDG) case after time $t^* = 4.0$.

Figure 6.31 Lateral movement of upwind vortex for roll oscillations

Figure 6.32 Lateral movement of downwind vortex for roll oscillations

From Figure 6.32, the lateral motion of the downwind vortex for both rolling cases are same as the LDG case. Figures 6.33 and 6.34 show the vertical motion of the upwind and downwind vortices respectively for the considered three cases. Figures 6.33 and 6.34 confirm that the roll oscillation does not affect the altitude of the vortex.

Figure 6.33 Vertical position of upwind vortex for roll oscillations

Figure 6.34 Vertical position of downwind vortex for roll oscillations

To state a closing remark, roll oscillation do not seem to have profound effect on the wake vortices. However, in the past literatures, roll oscillation has proven to be effective in dissipating the vortices. One possible explanation for this behaviour is that the amplification of the sine wave considered and the wavelength may not be sufficient. Since the aircraft is flying in ground proximity, there will always be restrictions in the amount of rolling moment an aircraft and passenger can handle. Hence, a detailed numerical and experimental studies have to be performed to prove that this theory is effective in practice.

6.4 Formation flight

Generally, when an aircraft approaches for landing in the presence of crosswind, the upwind vortex of the landing aircraft stays longer in the flight path of the follower aircraft, as discussed in Chapters 4 and 5. This is described as solitary vortex phenomenon in the literature. A considerable time is allowed between two consecutive landing aircrafts so that the strength of this upwind vortex reduces before the next aircraft coming into the wake of the previous aircraft. This time difference between the two landing aircrafts is generally

referred to as the Wake Turbulence Separation Standard, as explained in Chapter 2. As discussed earlier, there are multiple ways to reduce the separation time between two landing aircrafts. One of the more innovative ways to reduce the separation time and increase the runway throughput is to have parallel landings of aircrafts. Although they have some operational constraints such as lack of technologies and runways to support parallel landing, it is an interesting idea to explore, from a research perspective.

Two types of parallel formation flights are studied in this section:

1. Parallel flight with 400 ft lateral separation and
2. Parallel flight with 500ft vertical separation.

Currently, these types of formation flights are not in operation in the commercial aircraft sector. This is a hypothetical study to understand how the vortices would interact if the aircrafts were flown in parallel and whether it would be of any benefit to consider such formation flights for the future wake vortices research. The study is still at its nascent stage and needs further numerical and experimental investigation to quantify its effectiveness. Since the vortex dynamics of the two considered cases are complex, the usual wake parameters like the strength and the position of the vortex cores cannot be used for this analysis. Developing a new parameter to quantify the effects of formation flight on the wake vortices will be considered as one possible avenue of research in the future, as it requires the support of experiments and real-time measurements for validation purposes. Hence, in the current study, the interaction of different vortices shed behind the two B747 aircrafts landing in parallel, is discussed with the aid of images at different time steps. This study helps to understand the basic dynamics upon which future wake vortices research objectives can be built.

Note that in the wake vortices research, it is common for not considering the flight path angle for a landing aircraft, for simplicity. However, to represent the aircraft's approach to the runway, the landing lift configuration of the B747 is considered and ground proximity is taken into account.

6.4.1 Parallel flight - 400 ft lateral separation

Under this section, the concept of very closely spaced runways is investigated as a potential way of enhancing the vortex dissipation. Proctor [177] has studied parallel flight landings of B747 aircraft in a four different runway spacings. In his works, vortices are considered to be fully developed and did not account for near-field roll-up of the wake vortices. Therefore, the wake-vortices of aircrafts with minimum lateral separation distance accounting for the lift distribution effect using the newly proposed initialization method is reinvestigated as part of this dissertation.

Figure 6.35 Schematic of wake-vortices behind parallelly flown aircrafts [177]

Figure 6.36 Initial vorticity distribution behind two parallel flow aircrafts with a lateral separation distance of 400ft

The schematics of the wake-vortices behind a parallelly flown aircraft pair is demonstrated in Figure 6.35. Two B747 aircraft in their landing configurations are considered to be flying in parallel with a separation distance of 400ft as indicated in Figure 6.35. The simulation is performed with crosswind of 1.79m/s. The aircrafts are named as Aircraft-1 and Aircraft-2 as shown in both Figures 6.35 and 6.36. The port-side wake vortex, that is, the upwind vortex of the aircraft-1 and the starboard-side wake vortex, that is, the downwind vortex of the aircraft-2 are referred to as Outer vortices. The starboard-side wake vortex (or downwind vortex) of aircraft-1 and the port-side wake vortex (or upwind vortex) of the aircraft-2 are referred as inner vortices.

Figure 6.37 Schematics of location, direction and labels of the multiple vortices

Note: Please refer to the caption in the next page.

Figure 6.38 Vortex dynamics of wake-vortices of laterally separated parallel flights at time, $t^ = 0.5, 0.75, 1.0, 1.5, 3.0$ and 5.0*

Figure 6.36 presents the initial vorticity distribution behind the two aircrafts. In Figure 6.36, there are inboard flap, outboard flap and wing-tip vortices shed behind each of the aircraft. If the wake of one of the two aircrafts is considered, it can be clearly seen that they resemble the schematics presented in Figure 5.8 of Section 5.4.1. Shortly after initialization, outboard flap vortex merges with the wing-tip vortex leaving only two vortex pairs (primary vortex pair and flap vortex pair) for each of the aircrafts. The schematics after merging of the vortices is presented in Figure 6.37.

Figure 6.37 represents the location, direction and labels of the inboard flap and primary vortices of the two aircrafts at the initial time steps. Figure 6.38 is the pictorial representation of vortex interaction in the wake behind the parallelly flown aircrafts at times, $t^* = 0.25, 0.75, 1.0, 1.5, 3.0$ and 5.0 . The time steps are chosen in a way that it represents every new development in the vortex interaction. At $t^* = 0.25$ as presented in Figure 6.38, the flap vortices of upwind and downwind vortices of both the aircrafts exhibit long wave instability. Then, the flap vortices start to distort as they revolve around the primary vortex pair.

If there is a single aircraft, then the flap vortices usually evolve into secondary structures after sometime as discussed in Section 5.4.2. The evolved upwind-flap vortex approach their nearby upwind vortex to its left side and the evolved downwind-flap vortex approach to the right side of the downwind vortex. In parallel flights, due to the presence of two aircrafts, the downwind vortex of aircraft-1 is in close proximity to the upwind vortex of aircraft-2 as can be seen in Figure 6.38 at $t^* = 0.75$. Hence, the evolution of downwind-flap vortex around this downwind vortex into secondary structures are disturbed. In the contrast, the secondary structures formed out of the upwind-flap vortices of the aircraft-2 approaches the downwind vortex of aircraft-1 as shown in Figure 6.38, $t^* = 0.75$. After this time step, there is an interesting interaction of the secondary structures of these inner vortices resulting in a complex vortex dynamics as presented in Figure 6.38 at $t^* = 1.0, 1.5$ and 3.0 .

Coming over to the outer vortices, the upwind vortex of aircraft-1 and downwind vortex of aircraft-2 evolves as usual in the presence of crosswind. This can be seen from

Figures 6.38 through all time steps. The secondary vortices of upwind vortex of aircraft-1 is closer to the secondary vortices of inner vortices at $t^* = 1.5$ as shown in Figure 6.38. Due to the complex vortex dynamics of the inner vortices, the secondary vortices of upwind vortex of aircraft-1 undergo considerable change in their shape between the times from $t^* = 1.5$ to $t^* = 3.0$. At $t^* = 3.0$ in Figure 6.38, it can be seen that the secondary vortices of upwind vortex of aircraft-1 is farther away from the inner vortices and they are no more omega-shaped structures. Until $t^* = 3.0$, the downwind vortex of aircraft-2 moves in the positive y-direction and is pushed out of the computational domain due to the presence of crosswind.

The complex vortex structures formed by the inner vortices interaction also moves in lateral direction and exits the domain after $t^* = 4.0$. The longevity of inner vortices were higher in the results presented by Proctor [177]. In the current study, the inner vortex pair shows significant difference in the decay. This is due to the presence of flap vortices and crosswind which play significant role in the onset of secondary structures. In conclusion, it can be observed from the figures presented in this section that out of the two pairs of primary vortices, only one vortex remains in the domain after $t^* = 5.0$ ($t = 145s$).

If two aircrafts are flown in parallel, the primary vortex pairs of two aircrafts are shed at the same time and the upwind vortex of only one of the aircraft remains the domain for longer time while the other vortices are dissipated due to complex vortex interactions. Thus, it enables approximately four landing aircrafts with a time gap required to dissipate one upwind vortex unlike the conventional landing where two aircrafts are landed with a time gap required to dissipate one upwind vortex. From a wake turbulence perspective, parallel landing can increase the runway throughput. It may be considered as one of the temporary alternates to reduce the airport congestion. However, the analysis performed here is only preliminary. A detailed comparison of time taken to dissipate the remaining upwind vortex of the parallel landing and single landing has to be performed to implement this in reality.

6.4.2 Parallel flight - 500 ft vertical separation

Parallel flights with a vertical separation distance of 500ft is considered as one of the formation flight configurations. Two landing B747 aircrafts are considered flying in parallel with a vertical separation distance of 500ft. The aircraft in ground proximity is referred as Aircraft-1 and the aircraft that is flying above is referred as Aircraft-2. Figure 6.39 shows the initial vorticity distribution in the wake of the two landing B747 aircrafts in a parallel course with a vertical separation distance of 500ft. On each side of the wing, there are a set of inboard flap, outboard flap and wing-tip vortices that are shed for the two aircrafts as shown in Figure 6.39. The outboard flap vortices eventually merge with the wing-tip vortices in less than 2.5 seconds resulting in a two pair vortex system for each of the aircraft. The inboard flap vortices are referred as flap vortices.

Figure 6.39 Initial vorticity distribution behind the two parallel flights with a vertical separation of 500ft.

Figure 6.40 Schematics of location, direction and labels of the multiple vortices

Note: Please refer to the caption in the next page.

$t^* = 1.0$

$t^* = 1.25$

Note: from here on front view: $t^ = 1.5$*

$t^* = 2.0$

$t^* = 2.5$

Note: Please refer to the caption in the next page

Figure 6.41 Vortex dynamics of wake-vortices behind vertically separated parallel flights

The flap vortex in the port-side of the wing is referred as upwind-flap vortex and the one on the starboard-side is referred as downwind-flap vortex. Similarly, the wing-tip vortex in the port-side of the wing is referred as upwind vortex and the one on the starboard-side is referred as downwind vortex. Figure 6.40 shows the location, direction and labels of all the vortices for clarity.

The characteristic length and velocity scales are the same as those listed in Table 5.1. Since two set of wake-vortices are initialized with vertical separation distance, only one of them is actually in ground proximity. The wake-vortices that are closer to ground starts to decay well before the vortex pair present above. Figure 6.41 shows the vortex dynamics of the aircraft wake from time $t^* = 0.25$ to $t^* = 4.0$ at various time intervals. In Figure 6.41, at $t^* = 0.25$, the flap vortices of aircraft show long wave instability while that aircraft-2 is comparatively stable. It can be seen that around $t^* = 0.5$, secondary vortex structures are formed around the primary vortex pair of aircraft-1 as it is in close proximity to the ground. They are unaffected by the vortices present above until this time. As the flow evolves, the number of secondary vortices increases around the vortex pairs of the aircraft-1 through time, $t^* = 0.5 - 1.5$ as can be observed in Figure 6.41.

The primary vortex pair of both aircrafts show downward motion due to mutual induction. Because of the presence of the strong vortex pair beneath the vortex pair of aircraft-2, it experiences an additional downward velocity. The upwind vortex and

downwind vortex of aircraft-2 are brought closer to each other by the induced force of the primary vortex pair of aircraft-1. As vortex pair of aircraft-2 moves closer to ground, the secondary vortex structures of aircraft-1, starts to tilt and strain the primary and flap vortex pair of aircraft-2 as shown in Figure 6.41 at time, $t^* = 1.0$, 1.25 and $t^* = 1.5$. At $t^* = 2.0$, the primary vortex pair of the aircraft-2 is completely surround by secondary vortex structures of the downwind vortex of the aircraft-1, marking a rapid interaction as presented in Figure 6.41. This leads to a reduced strength for the vortex pair of aircraft-2 even though they are not in ground proximity.

Meanwhile the complete vortex system moves in lateral direction along the direction of crosswind. At $t^* = 4$, the primary vortex pair of aircraft-2 and the downwind vortex pair of aircraft-1 exits the domain along with its secondary vortex structures. The upwind vortex of aircraft-1 stays in the domain for longer time. The secondary structures around this upwind vortex is affected by the complex interaction of the other three primary vortices. Hence, the secondary structures of upwind vortex of aircraft-1 are no more omega-shaped. This may lead to an increase in the strength of the upwind vortex of aircraft-1 as compared to the upwind vortex of the aircrafts landing conventionally. This effect has to be quantified with a new parameter to represent the strength of vortices and the parameter has to be validated with real-time measurements from airports and laboratory experiments.

It should be noted that in both cases of the parallel flights considered, the upwind vortex of one of the two aircrafts remain in the domain while all other vortices exhibit a complex interaction and exit the domain earlier in lateral direction.

6.4.3 *Formation flights – is it a feasible solution?*

It is concluded from studies under Sections 6.4.1 and 6.4.2 that the vortex dynamics in the wake of a parallel landing aircraft pair, is not as straightforward as it is for the conventional one-by-one landing of aircrafts. In general, it is well established that the upwind vortex stays longer in the runway and poses the most hazard for following landing aircrafts in single runway. Two aircrafts flying in parallel could be a better solution to reduce the initial strength of the one of the two upwind vortices that is shed behind the

aircraft pair. But, this type of formation flight results in a highly complex vortex interaction for a comparatively longer duration of time. There is even a possibility that this complex interaction may actually result in regions of high local turbulence. The limitation for using this method as temporary alternative may also arise from the air-traffic controllers, as planning a parallel flight landing needs highly sophisticated navigation and guidance systems.

Although this preliminary analysis shed lights on what is expected in the wake of a formation flight, it is to be noted that detailed experimental studies have to be performed to further study the vortical structures and their strengths. Also the vortex dynamics may be slightly different if landing angle and touch down point of each aircraft is considered.

6.5 Summary

In this chapter, Quasi-temporal simulation of two modified lift distributions of a landing B747 aircraft were discussed and the potential impact on the corresponding trailing wake vortices were presented. To provide a better understanding, a comparison of the results of the modified configurations with the conventional were also incorporated in the discussion. It is evident from this study that the lift distribution has profound impact on the initial position, strength and number of trailing wake vortices formed. Due to significant difference in the initial vorticity distribution, there is a significant difference in the dynamics of the vortices as they evolve in time. The influence of a roll oscillation of B747 aircraft on the wake vortices is also studied. It is the first attempt in this field to study the rolling motion impact on the wake vortices and so the author has considered a hypothetical rolling motion which in future can be extended to be as realistic as possible. In addition to the rolling motion study, wake vortex dynamics of formation flight were also discussed in this chapter as a potential way to increase the runway throughput. The formation flight landing operations are not as straight forward as in the case of single flight landings. It should be noted that it is also the first attempt in this field to study the inter-wake vortex dynamics between two aircrafts landing in parallel.

7 Conclusion and Recommendations

7.1 Conclusion

The research on the wake vortex evolution is essential for the revision of the separation standards in airport vicinity. After performing an elaborate study of wake vortices on the existing literature, it is concluded that the interaction of secondary vortices with the primary vortices determines the dissipation rate of the primary vortex pair in ground proximity. The characteristics of these secondary vortices are greatly influenced by the atmospheric parameters and the number of vortices shed behind the port-side and starboard-side wings of an aircraft.

The current thesis is extending the knowledge by performing LES on various scenarios and by studying physical processes as well vortex dynamics of wake vortex decay. Jetcode, a software originally developed at Stanford University for combustion research, is then modified to perform the wake vortex study. The code uses LES technique with Dynamic Sub-Grid Scale Model to solve the Navier-Stokes equation. New post-processing codes were written as part of this study, in order to find the strength and position of the primary vortex pair in any 2D plane and 3D domain. Vorticity and Q-criteria isosurfaces are used for visualization of the vortex dynamics. Lamb-Oseen vortex model is used to initialize the flow field for a given initial circulation strength in the computational domain.

Firstly, the influence of the atmospheric crosswind and turbulence intensity on the formation of secondary vortices and the decay of primary vortices were investigated. For this investigation, the wake-vortices are assumed to be a single fully rolled-up counter-rotating vortex pair system. The characteristics of primary vortex pair are observed and compared for crosswinds of speed up to 4.8 m/s. It is inferred that the primary vortex pair behaves asymmetrically in the presence of crosswind. The formation of secondary vortices around the downwind vortex is enhanced due to its identical sign of vorticity as the crosswind induced vorticity. Hence, at all times, the strength of downwind vortex is less than that of the upwind vortex. The degree of interaction of secondary vortices determines

the degree of distortion in the vortex core line. The motion of the downwind vortex due to mutual induction aligns with the direction of the crosswind and so their lateral transport velocity (crosswise velocity) is higher. An empirical relationship for crosswise velocity and exit time, as a function of crosswind velocity is proposed for each of the vortices. This is important, as it allows estimating the timespan required for the crosswind to push the aircraft wake-vortices out of the flight path of a following aircraft. This relationship will enable us to predict the motion of the upwind and downwind vortices under various atmospheric crosswind velocities. This information can be used as a preliminary data for the crosswind-based wake prediction and advisory systems.

For studying the influence of turbulence intensity, a crosswind speed of 1.79m/s is maintained while the background turbulence is amplified from 3% to 50%. It is found that low turbulence amplifications, say, 3% - 9%, do not affect any of the vortex characteristics while higher turbulence amplification of 20% and 50% have profound effects on the locally formed large-scale secondary structures. The distortion of the centerline of the primary vortices are higher for the high turbulence intensity levels, due to the locally enhanced interaction of primary vortex pair with the secondary vortices.

In the next step of investigation, the relationship between the number of wake-vortices shed behind each side of the aircraft wing and the characteristics of secondary vortices are examined. Hence, it is necessary to include the near-field roll-up phase of the vortices into the Temporal simulation methodology. In order to facilitate this research, the current study introduces a new velocity initialization method named as Prandtl Vorticity Distribution (PVD method) based on Prandtl Lifting-Line Theory is proposed. With this method, the detailed lift distribution over a wing including the location and the deflection of the flaps can be included to the LES computation of the wake-vortices. The success of this method lies in the formulation of a parameter called free vortex strength. This parameter forms a relationship between the characteristics of aircraft wake-vortices and the gradient of the spanwise loading on the generating aircraft. This parameter will be of immense help to form a preliminary idea of wake-vortices behind a wing with any type of lift configuration,

which includes even an entirely new wing design/lifting device (such as Blended Wing Body).

This new initialization method is used to simulate the wake-vortices behind a landing B747 aircraft. It is found that the results are in agreement with the real-time LIDAR measurement data of ‘Heavy’ category aircrafts. For a landing configuration of B747 aircraft, the wake-vortices shed behind the wing initially consist of three vortices: wing-tip vortex, outboard and inboard flap vortices. In less than time, $t = 2.5$ seconds, there are only two distinct vortices seen in the computational domain with opposite signs on each side of the wing. One belongs to the high gradient of lift near the wing-tip while the other belongs to the lift gradient at the inboard tip of the inboard flap. The most interesting conclusion drawn from this study is that the inboard flap stretches, tilts and finally deforms into omega-shaped secondary vortex structures around the primary vortex pair. The tail part of this structure links with the boundary layer at the ground thereby paving way for a continuous shedding of secondary vortex structures from the surface. This early onset of interaction of secondary vortex structures with the primary vortices result in a higher dissipation rate for the primary vortices as compared to the single vortex pair system. It is important to note that the effectiveness of flap vortices in reducing the strength of the primary vortex pair depends greatly on the presence of crosswind. Q-criteria isosurfaces are used for visualizing the interaction of wing-tip and flap vortices. Even though, the LES is performed for a multi-vortex wake system, it is to be noted that computational time and memory consumed by the new methodology is only slightly higher than the conventional one.

The last section of the dissertation focusses on three ways to artificially enhance the dissipation rate of the primary vortex pair, as follows,

1. *Modification of lift distribution:* Influence of modified landing lift distributions by changing the flap settings on the dynamics of the wake-vortices behind a B747 aircraft is investigated. Two different flap setting configurations, apart from the conventional landing configuration are considered as part of this analysis. It is concluded that changing the flap setting results in a different free vortex strength profile in spanwise direction. This leads to a difference in the shape, position and strength of the wing-tip

and flap vortices, eventually, affecting the evolution of flap vortices and the characteristics of secondary vortices. Apart from the understanding of physics, a more practical conclusion from this study is that, one of the two modified configurations for a landing B747 aircraft resulted in a more efficient way dissipating the primary vortices.

2. *Roll oscillations:* The aircraft is assumed to be rolling to its left and then back to its original position along the axial direction. This rolling motion results in redistribution of lift and circulation over the left and right sides of the wing. The circulation over left and right sides of the wing is assumed to be decreased and increased by 5%, respectively. To make the change in circulation smoother, a sine wave is assumed with the amplitude of circulation that corresponds to the change. Two wavelengths are considered for the study. It is deduced that the wavelength and amplitude considered in this research are not sufficient to cause significant impact on the decay of the wake-vortices. Since the aircrafts have limitations on the maneuvering capability in ground proximity and also to ensure passenger comfort and safety, a rolling motion with higher amplitude and wavelength may not be advisable. However, this conclusion has to be further verified with concrete experimental results.
3. *Formation of flight:* Lastly, two different formation flight configurations of commercial aircrafts are examined. One of the cases involves two landing B747 aircrafts in parallel with a lateral separation of 400ft. This case is considered as it will be useful for airports with very closely spaced runways. The other case involves two flights in parallel with a vertical separation of 500ft. In both cases, the resulting flow field consists of a pair of counter-rotating flap and wing-tip vortices for each of the aircrafts. For both configurations of formation flight, the port-side vortex of the one of the aircrafts remains in the domain for longer duration as compared to other vortices. Its evolution is found to be similar to that of the port-side vortex of a single landing B747 aircraft. A very complex vortex interaction takes place between the rest of the flap and wing-tip vortices of the two aircrafts resulting in a highly localized turbulent region, which remain in the flight path of the aircraft following behind the vortex generating aircraft.

To sum up, the new methodology provides a reliable, accurate and effective way to study the evolution of wake-vortices behind any type of aircraft with any high-lift device

configuration. Also, modifying the spanwise loading is proposed to be an effective parameter in enhancing the dissipation of the wake-vortices. Using the insights gained in this research and the methodology proposed, researchers can analyze the wake-vortices shed behind any novel wing designs and use it to optimize the wing design even during the early preliminary design stage.

7.2 Recommendations and future work

For a more accurate simulation of wake-vortices, following recommendations are proposed for the future work. These improvements will increase the application of the PVD initialization method.

- Landing angle of the aircraft flight path in airport vicinity can be introduced as a new parameter. This will make the study more realistic as it will introduce difference in the vortex altitude from ground thereby changing the secondary vortex formation in the spanwise direction.
- Pilot control input – The study of pilot control input can be extended to test the effectiveness of pitch oscillations on wake vortex decay. Roll oscillations can be reinvestigated for various functions, amplitude and wavelength. Corresponding passenger comfort study is necessary to be performed to ensure the proposed solution is practically viable.
- Wake-vortices behind modern aircrafts with revolutionary wing designs like blended-wing body can be studied with the help of the proposed method.
- In Prandtl Lifting-Line Theory, the aircraft wing is assumed to be an overlap of infinite number of horseshoe vortices. As long as there is lift force, the bound vortex travels along the wing and is not shed downstream. Only the free vortices on either side of the horseshoe vortices evolves into wake-vortices behind an aircraft wing. However, once the landing aircraft touches the ground, the lift force vanishes, and the bound vortex is shed downstream. This sudden change in the lift force introduces a disturbance into the wake-vortices and is called as end effects. The present method

can be easily extended to study this end effect by initializing the bound vortices along with the free vortices at the touchdown point.

- Jetcode has the provision to initialize the velocity field using body forces. Hence, the method proposed in this dissertation can also be extended to introduce the different vortex ages along its axial direction and its effect on the parameters.
- The application of the PVD method for simulating the wind turbine wakes is in progress and the initial results are already presented by Schlüter and Paramasivam [178]. Figure 7.1 shows the wake behind a wind turbine simulated based on Lifting-Line Theory (LLT). This work can be extended to study effects of wind turbine wakes on aviation and the surrounding structures.

Figure 7.1 Wake behind a wind turbine simulated based on Lifting-Line Theory (LLT) [178]

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Appendix

MATLAB Program 1 – To find the position and circulation of the vortex pair

```
clear all;
clc;
r1=0.1/47.1;
r2=15/47.1;
% TO import all the files into the
source_dir = 'G:\JETCODE
FILES\Crosswind\512_256_256\Later_CW\220_Umean\CSV';
source_files = dir(fullfile(source_dir, '*.csv'));
k=1;
k1=1;
N=length(source_files);
% To extract time from file name
tint=zeros(N,1);
% Extracting time
for i=1:N
    name=source_files(i,1).name;
    e=(length(name))-4;
    tsum=0;
    for q=4:e
        t=str2num(name(q));
        tsum=(tsum*10)+t;
    end
    div=10^(numel(num2str(tsum)));
    tint(i)=round((tsum/div)*div);
end
%-----Time ends-----
%---Sort-Time-----
for i=1:N
    fd= find(tint==i);
    time(i)=fd;
end
%-----
temp=0;
```

```

t2=1;
for ij=1:N
    i = time(ij);
    tint(i)
    %-- To import U,V,W, P and vorticity values in x,y and z --
    rc=zeros(1,2);
    data = csvread(fullfile(source_dir, source_files(i).name),1,0);
    rc(1)=length(data);
    rc(2)=numel(data)/length(data);
    wx=data(:,rc(2)-3);
    wz=data(:,rc(2)-4);
    wy=data(:,rc(2)-5);
    p=data(:,rc(2)-9);
    y=data(:,rc(2)-2);
    z=data(:,rc(2)-1);
    x=data(1,rc(2));
    %magw=sqrt((wx.^2)+(wy.^2)+(wz.^2));
    magw=wx;
    %-----Imported-----
    ref=1;
    r=round(rc(1)/2);
    %-----Centre-1-----
    for j=1:rc(1)
        if y(j)<temp && y(j)>-3 && z(j)>0.4
            p1(ref)=p(j);
            mag1(ref)=magw(j);
            y_ar1(ref)=y(j);
            z_ar1(ref)=z(j);
            ref=ref+1;
        end
    end
    minimum=0;
    for inc=1:ref-1
        if p1(inc)<minimum
            minimum=p1(inc);
            indxl=inc;

```

```

        end
    end
    ymin1(k)=y_ar1(indx1);
    zmin1(k)=z_ar1(indx1);
    %-----Vortex centre is found-----
    %---Circulation for first vortex-----
    t1=ymin1(k)+0.4;
    if t1>temp
        temp=temp+0.5;
    end
    if ymin1(k)>3.7
        break;
    end
    tstar(k)=k;
    ki=1;
    for ind=1:1:ref-1        % CHANGE HERE FOR SECOND VORTEX
        rad=sqrt(((y_ar1(ind)-ymin1(k))^2)+((z_ar1(ind)-zmin1(k))^2));
% CHANGE
        if rad > r1 && rad < r2
            ny(ki)=y_ar1(ind);
            nz(ki)=z_ar1(ind);
            nmagw(ki)=mag1(ind);
            ki=ki+1;
        end
    end
    oy(1)=ny(1);
    flag=0;
    ind2=1;
    for ind=1:(ki-1)
        for j=1: numel(oy)
            if ny(ind)== oy(j)
                flag=1;
            end
        end
        if flag==0
            ind2=ind2+1;
        end
    end
end

```

```

        oy(ind2)=ny(ind);
    end
    flag=0;
end
oy=sort(oy);
oz(1)=nz(1);
flag=0;
ind2=1;
for ind=1:(ki-1)
    for j=1:numel(oz)
        if nz(ind)== oz(j)
            flag=1;
        end
    end
    if flag==0
        ind2=ind2+1;
        oz(ind2)=nz(ind);
    end
    flag=0;
end
oz=sort(oz);
omagw=zeros(numel(oy),numel(oz));
for ind=1:(ki-1)
    for j=1:numel(oy)
        if oy(j)==ny(ind)
            index1=j;
        end
    end
    for j=1:numel(oz)
        if oz(j)==nz(ind)
            index2=j;
        end
    end
    omagw(index1,index2)=nmagw(ind);
end
sum=0;

```

```

for row=1:(numel(oy)-1)
    for col=1:(numel(oz)-1)
        dy=oy(row+1)-oy(row);
        dz=oz(col+1)-oz(col);

mat=[omagw(row,col),omagw(row,col+1),omagw(row+1,col),omagw(row+1,col+1)];
        avg=mean(mat);
        sum=sum+(avg*dy*dz);
    end
end
gama1(k)=-sum;
ny=[]; nz=[]; oy=[]; oz=[]; nmagw=[];
%---Circulation for second vortex-----
if t2==1
    ref1=1;
    ki=1;
    for j=1:1:rc(1)
        if y(j)>=temp && z(j)>0.4 && z(j)<3.5
            p2(ref1)=p(j);
            mag2(ref1)=magw(j);
            y_ar2(ref1)=y(j);
            z_ar2(ref1)=z(j);
            ref1=ref1+1;
        end
    end
    minimum=0;
    for inc=1:ref1-1
        if p2(inc)<minimum
            minimum=p2(inc);
            indx2=inc;
        end
    end
    ymin2(k)=y_ar2(indx2);
    zmin2(k)=z_ar2(indx2);
    for ind=1:1:ref1-1

```

```

        rad(ind)=sqrt(((y_ar2(ind)-ymin2(k))^2)+((z_ar2(ind)-
zmin2(k))^2));
        if rad(ind) > r1 && rad(ind) < r2
            ny(ki)=y_ar2(ind);
            nz(ki)=z_ar2(ind);
            nmagw(ki)=mag2(ind);
            ki=ki+1;
        end
    end
    oy(1)=ny(1);
    flag=0;
    ind2=1;
    for ind=1:(ki-1)
        for j=1:numel(oy)
            if ny(ind)== oy(j)
                flag=1;
            end
        end
        if flag==0
            ind2=ind2+1;
            oy(ind2)=ny(ind);
        end
        flag=0;
    end
    oy=sort(oy);
    oz(1)=nz(1);
    flag=0;
    ind2=1;
    for ind=1:(ki-1)
        for j=1:numel(oz)
            if nz(ind)== oz(j)
                flag=1;
            end
        end
        if flag==0
            ind2=ind2+1;

```

```

        oz(ind2)=nz(ind);
    end
    flag=0;
end
oz=sort(oz);
omagw=zeros(numel(oy),numel(oz));
for ind=1:(ki-1)
    for j=1:numel(oy)
        if oy(j)==ny(ind)
            index1=j;
        end
    end
    for j=1:numel(oz)
        if oz(j)==nz(ind)
            index2=j;
        end
    end
    omagw(index1,index2)=nmagw(ind);
end
sum=0;
for row=1:(numel(oy)-1)
    for col=1:(numel(oz)-1)
        dy=oy(row+1)-oy(row);
        dz=oz(col+1)-oz(col);

mat=[omagw(row,col),omagw(row,col+1),omagw(row+1,col),omagw(row+1,col+1)];
        avg=mean(mat);
        sum=sum+(avg*dy*dz);
    end
end
gama2(k1)=sum;
if ymin2(k1)>3.7
    t2=0;
end
tstar1(k1)=k1;
k1=k1+1;

```



```

end
k=k+1;
end
%  %--Circulation--Non-dimensionalised-----gama/Vo*bo-----
-----
gama1(1)
gama2(1)
gama1=gama1./gama1(1);
gama2=gama2./gama2(1);
%-- Circulation vs Time and Vortex centre plot-----
----
%tstar=1:N;
tstar=tstar*0.05;
tstar1=tstar1*0.05;
figure;
plot(tstar,abs(gama1),'*b');
xlabel('Time (s)');
ylabel('Circulation')
title('Circulation of LHS vortex vs time');
axis([0 9 0 1.2]);
%---RHS--plots-----
figure;
plot(tstar1,abs(gama2),'*b');
xlabel('Time (s)');
ylabel('Non-dimensionalised Circulation')
title('Circulation of RHS vortex vs time');
%-----RHS-End-----
figure;
plot(ymin1,zmin1,'go',ymin2,zmin2,'*r');
xlabel('y*');
ylabel('z*');
title('Vortex centre');
axis([-4 4 0 1.5]);
%-----Ymin--Plots-----
figure;
plot(tstar,ymin1,'*b',tstar1, ymin2,'*r');

```

```

xlabel('t*');
ylabel('y*');
%-----Zmin--Plots-----
figure;
plot(tstar,zmin1,'*b',tstar1, zmin2,'*r');
xlabel('t*');
ylabel('z*');

```

MATLAB Program – 2 – To plot the inflow velocity profile

```

clear all;
clc;
data = csvread('220.0.csv',1,0);
rc(1)=length(data);
rc(2)=numel(data)/length(data);
y=data(:,rc(2)-2);
z=data(:,rc(2)-1);
x=data(1,rc(2));
u=data(:,rc(2)-8);
j=1;
for i=1:rc(1)
    if y(i)==min(y)
        nu(j)=u(i);
        nz(j)=z(i);
        j=j+1;
    end
end
j=1;
for i=1:129
    if nz(i)>2 && nz(i)<5
        ou(j)=nu(i);
        j=j+1;
    end
end
max=max(ou)

```

```

min=min(ou)
figure;
plot(nu,nz);
ylabel('z*');
xlabel('Non-Dimensionalised inflow velocity')
axis([0 2.5 0 5]);

```

FORTRAN Program – 3 – Sampling down the data files

```

program setup
integer iunit, ierr, nx, ny, nz, i, j, k
real*4, allocatable :: x(:,:,:), y(:,:,:), z(:,:,:)
real*4, allocatable :: x2(:,:,:), y2(:,:,:), z2(:,:,:)
iunit1 = 11
open (iunit1, file="grid_z_formatted.xyz", form="formatted", status="old", iostat=ierr)
if (ierr .ne. 0) stop "A data file is required"
open (iunit2, file="small/grid_z_formatted.xyz", form="formatted", iostat=ierr)
if (ierr .ne. 0) stop "Cannot write data file"
read (iunit1, *) nx, ny, nz
write (iunit2, *) nx/2, ny/2, nz
allocate (x(nx,ny,nz), y(nx, ny,nz), z(nx, ny, nz))
allocate (x2(nx/2,ny/2,nz), y2(nx/2, ny/2,nz), z2(nx/2, ny/2, nz))
write(*,*) "Reading coordinates..."
read (iunit1, *) x, y, z
write(*,*) "Sampling down..."
do i=1, nx
  if ((mod(i,2))==0) then
    do j=1,ny
      if ((mod(j,2))==0) then
        do k=1, nz
          !write(*,*) i, j, k
          x2(i/2,j/2,k)=x(i,j,k)
          y2(i/2,j/2,k)=y(i,j,k)
          z2(i/2,j/2,k)=z(i,j,k)
        end do
      end if
    end do
  end if
end do

```

```

        end if
    end do
end if
end do
write(*,*) "Writing coordinates..."
write (iunit2, *) x2, y2, z2
close (iunit1)

```

FORTRAN Program – 4 – To average data in axial direction

```

program setup
integer iunit, ierr, nx, ny, nz, i, j, k, ifile, nfile, ivar, z
integer ifilestart, ifileend, ifileincr
real*8, allocatable :: R(:, :, :)
real*8, allocatable :: avg(:, :)
real*8 dummy1, dummy2, dummy3, time
character*10 fname
character*16 fnameout
data fname / "Qdata.xxxx" /
data fnameout / "small/QData.xxxx" /
nfile=10
iunit1 = 11
iunit2 = 10
write (*, *) "Start number : "
read (*, *) ifilestart
write (*, *) "End number : "
read (*, *) ifileend
write (*, *) "Increment : "
read (*, *) ifileincr
ifile = ifilestart
do while (ifile<=ifileend)
    write(fname(7:10), "(i4.4)") ifile
    write(fnameout(13:16), "(i4.4)") ifile
    write(*, *) "Reading ", fname, " ..."

```

```

open (iunit1, file=fname, form="formatted",status="old", iostat=ierr)
if (ierr .ne. 0) stop "A data file is required"
open (iunit2, file=fnameout, form="formatted", iostat=ierr)
if (ierr .ne. 0) stop "Cannot write data file"
read (iunit1, *) nx, ny, nz
write (iunit2, *) nx,ny, 1
if (ifile==ifilestart) then
    allocate (R(nx,ny,nz))
    allocate (avg(nx,ny))
end if
read (iunit1, *) dummy1, dummy2, dummy3, time
write (iunit2, *) dummy1, dummy2, dummy3, time
do ivar=1,5
    !write(*,*) "Reading data..."
    read (iunit1, *) R
    if (ivar/=1) then
        avg = R(:, :, 1)
        do z=2,nz
            avg=avg+R(:, :, z)
        end do
        avg=avg/nz
!    print*,minval(avg),maxval(avg),nx,ny,nz
        write (iunit2, *) avg
    end if
    if (ivar==1) then
        write (iunit2, *) R(:, :, 1)
    end if
end do
close (iunit1)
close (iunit2)
ifile = ifile + ifileincr
end do
stop "Exited normally"
end

```

```

stop "Exited normally"
end

```

FORTRAN Program – 5 – To track the vortices in 3D domain

```

!-- 3D vortex tracking -----
program setup
  integer iunit, nx, ny, nz, ifile, nfile, ivar, i, j, k, minimum1, minumum2, tstar
  integer ifilestart, ifilened, ifileincr
  real*4, allocatable :: radius(:,,:), theta(:,,:), b0(:,,:)
  real*4, allocatable :: x(:,,:), y(:,,:), z(:,,:)
  real*4, allocatable :: z_plane(:,,:), ymin1(:,,:), xmin1(:,,:), ymin2(:,,:), xmin2(:,,:), temp(:,,:)
  real*8, allocatable :: R(:,,:)
  character*10 fname
  data fname / "Qdata.xxxx" /
  character*15 fnameout1
  character*15 fnameout2
  character*15 fnameout3
  character*15 fnameout4
  character*15 fnameout5
  character*15 fnameout6
  character*15 fnameout7
  character*15 fnameout8
  iunit1 = 11
  iunit2 = 12
  iunit3 = 10
  !-----To read geometry file-----
  open (iunit2, file="grid_formatted.xyz", form="formatted",status="old",iostat=ierr)
  if (ierr .ne. 0) stop "A data file is required"
  read (iunit2, *) nx, ny, nz
  allocate (x(nx,ny,nz), y(nx, ny,nz), z(nx, ny, nz))
  write(*,*) "Reading coordinates..."
  read (iunit2, *) x, y, z
  write(*,*) nx, ny, nz

```

```

allocate (z_plane(nz,1))
z_plane(:,1) = z(1,1,:)
!-----data extraction-----
write (*,*) "Start number : "
read (*,*) ifilestart
write (*,*) "End number : "
read (*,*) ifileend
write (*,*) "Increment : "
read (*,*) ifileincr
ifile = ifilestart
tstar = 1
do while (ifile<=ifileend)
    write(fname(7:10),"(i4.4)") ifile
    write(*,*) "Reading ",fname," ..."
    open (iunit1, file=fname, form="formatted",status="old", iostat=ierr)
    if (ierr .ne. 0) stop "A data file is required"
    read (iunit1, *) nx, ny, nz
    write(*,*) nx, ny, nz
    if (ifile==ifilestart) then
        allocate (R(nx,ny,nz))
        allocate (ymin1(nz,ifileend))
        allocate (xmin1(nz,ifileend))
        allocate (ymin2(nz,ifileend))
        allocate (xmin2(nz,ifileend))
        allocate (radius(nz,ifileend))
        allocate (theta(nz,ifileend))
        allocate (b0(nz,ifileend))
        allocate (temp(nz,1))
        temp = 0
    end if
    read (iunit1, *) dummy1, dummy2, dummy3, time
    do ivar = 1,5
        read(iunit1,*) R
        if (ivar == 5) then

```

```

do k = 1, nz
  minimum1 = 0
  minimum2 = 0
  do j = 1, ny
    do i = 1, nx
      if (y(i,j,k) > 0.3 .AND. x(i,j,k)<temp(k,1) .AND. x(i,j,k)>-3 .AND. y(i,j,k)<4.5) then
        if (R(i,j,k) < minimum1) then
          minimum1 = R(i,j,k)
          ymin1(k,tstar) = y(i,j,k)
          xmin1(k,tstar) = x(i,j,k)
        end if
      end if
    end do
  end do
  if (xmin1(k,tstar)+0.4> temp(k,1)) then
    temp(k,1) = temp(k,1) + 0.5
  end if
  do j = 1, ny
    do i = 1, nx
      if (y(i,j,k) > 0.3 .AND. x(i,j,k) > temp(k,1) .AND. y(i,j,k) < 4.5) then
        if (R(i,j,k) < minimum2) then
          minimum2 = R(i,j,k)
          ymin2(k,tstar) = y(i,j,k)
          xmin2(k,tstar) = x(i,j,k)
        end if
      end if
    end do
  end do
end do
tstar = tstar + 1
close (iunit1)
ifile = ifile + ifileincr

```



```

        end do
write(*,*) temp
radius = sqrt((xmin2-xmin1)**2 + (ymin2-ymin1)**2)
theta = atan ((ymin2-ymin1)/(xmin2-xmin1))
b0 = xmin2 - xmin1
!write(*,*) theta
!write(*,*) ymin1
!--- To write csv file-----
data fnameout1 / "track/xmin1.csv" /
open (iunit1,file=fnameout1,action="write",status="replace")
do i=1,nz
    write (iunit1,"(100(f0.6,'',:))" ) xmin1(i,:)
end do
close (iunit1)
data fnameout2 / "track/ymin1.csv" /
open (iunit1,file=fnameout2,action="write",status="replace")
do i=1,nz
    write (iunit1,"(100(f0.6,'',:))" ) ymin1(i,:)
end do
close (iunit1)
data fnameout3 / "track/xmin2.csv" /
open (iunit1,file=fnameout3,action="write",status="replace")
do i=1,nz
    write (iunit1,"(100(f0.6,'',:))" ) xmin2(i,:)
end do
close (iunit1)
data fnameout4/ "track/ymin2.csv" /
open (iunit1,file=fnameout4,action="write",status="replace")
do i=1,nz
    write (iunit1,"(100(f0.6,'',:))" ) ymin2(i,:)
end do
close (iunit1)
data fnameout5 / "track/radii.csv" /
open (iunit1,file=fnameout5,action="write",status="replace")

```

```

do i=1,nz
    write (iunit1,"(100(f0.6,'',:))") radius(i,:)
end do
close (iunit1)
data fnameout6 / "track/theta.csv" /
open (iunit1,file=fnameout6,action="write",status="replace")
do i=1,nz
    write (iunit1,"(100(f0.6,'',:))") theta(i,:)
end do
close (iunit1)
data fnameout7 / "track/sep0.csv" /
open (iunit1,file=fnameout7,action="write",status="replace")
do i=1,nz
    write (iunit1,"(100(f0.6,'',:))") b0(i,:)
end do
close (iunit1)
data fnameout8 / "track/z_pln.csv" /
open (iunit1,file=fnameout8,action="write",status="replace")
do i=1,nz
    write (iunit1,"(100(f0.6,'',:))") z_plane(i,:)
end do
close (iunit1)
stop "Exited normally"
end

```