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Unconditionally Stable Multiple One-Dimensional ADI-FDTD Method for Coupled Transmission Lines

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Abstract—The unconditionally stable multiple one-dimensional alternating direction implicit finite-difference time-domain (M1-D ADI-FDTD) method for coupled transmission lines is presented. The differential equations for coupled transmission lines are formulated in compact matrix form. Proper 4×4 split matrices are introduced, which feature unconditional stability for the ADI procedures. These matrices also lead to update equations with left-hand sides involving tridiagonal matrices that can be solved efficiently. The update equations of M1-D ADI-FDTD method are given, which comprise one implicit and three explicit updates in each procedure. The analytical proof of unconditional stability is also provided based on von Neumann method. The M1-D ADI-FDTD method allows quick simulation and visualization of electromagnetic fields everywhere along the coupled transmission lines. To demonstrate the usefulness of M1-D ADI-FDTD method, various coupled line structures for filter and filtering antenna are analyzed using the method.

Index Terms—ADI-FDTD, coupled transmission lines, filtering antenna, multiple 1-D method, unconditional stability, FDTD

I. INTRODUCTION

Over the years, filtering antennas [1]–[6] have become more popular in wireless communication systems. They involve direct integration of antennas and filters, which offer compactness and flexibility in their frequency responses. For the filters, they are commonly designed and synthesized using microwave transmission line structures such as stepped impedance lines, stubs, as well as coupled transmission lines [7]–[9]. To analyze the filters and antennas, finite-difference time-domain (FDTD) method [10], [11] can be employed to solve for electromagnetic (EM) waves. Nevertheless, full 3-D FDTD method generally requires large memory storage and long computation time. There is also severe time step restriction imposed by the Courant-Friedrichs-Lewy (CFL) stability constraint. Such limitation is even more acute with the presence of coupled transmission lines, whereby the line width and spacing often call for very fine meshes that further inhibit its computational efficiency.

To alleviate the inefficiency of full 3-D FDTD method, the multiple one-dimensional (M1-D) FDTD method proposed in [12] can be used to simulate EM wave propagation along transmission lines and stubs. Meanwhile, to overcome the CFL stability constraint, the unconditionally stable M1-D method is further proposed in [13] for transmission lines. In [14], an alternative unconditionally stable FDTD method is used for analyzing composite right/left-handed transmission line. It should be noted that the methods in [12]–[14] are only able to treat uncoupled transmission lines and are not applicable for

Fig. 1. Schematic of coupled transmission lines including the fields and coordinate system. Also shown is the time snapshot of electric fields E_{x1} and E_{x2} along the coupled transmission lines.

coupled ones. Recently, the (explicit) M1-D FDTD method has been extended for coupled transmission lines in [15]. Based on even- and odd-mode approach, the coupled line parameters can be incorporated directly into the update equations. However, due to its explicit nature, the method of [15] is still constrained by its stability condition, which is generally more stringent and restrictive than that of the single transmission line.

In this paper, we propose the M1-D alternating direction implicit (ADI)-FDTD method for coupled transmission lines, which features unconditional stability circumventing the CFL stability constraint above. In Section II-A, the differential equations for coupled transmission lines will be formulated in compact matrix form. Proper 4×4 split matrices are to be introduced, which feature unconditional stability for the ADI procedures. These matrices also lead to update equations with left-hand sides (LHS) involving tridiagonal matrices that can be solved efficiently. The update equations will be given in Section II-B. The analytical proof of unconditional stability will also be provided based on von Neumann method in Section III. The M1-D ADI-FDTD method allows quick simulation and visualization of electromagnetic fields everywhere along the coupled transmission lines. To demonstrate the usefulness of M1-D ADI-FDTD method, various coupled line structures for filter and filtering antenna will be analyzed using the method in Section IV.

II. M1-D ADI-FDTD METHOD FOR COUPLED TRANSMISSION LINES

A. Formulation of split matrices

Fig. 1 shows the schematic of coupled transmission lines including the fields and coordinate system. The lines are aligned along the z -axis, E_{x1} , H_{y1} are the electric and magnetic fields along line 1, and E_{x2} , H_{y2} are the electric and magnetic fields along line 2. The governing differential equations for coupled transmission lines can be derived from [9], [16]. We shall formulate them in compact matrix form as

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{W} \mathbf{u} = (\mathbf{A} + \mathbf{B}) \mathbf{u} \quad (1)$$

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where $\mathbf{A}$ and $\mathbf{B}$ are 4×4 split matrices (to be given later),

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & \frac{-1}{\epsilon_s} \frac{\partial}{\partial z} & \frac{-1}{\epsilon_m} \frac{\partial}{\partial z} \\ 0 & 0 & \frac{-1}{\epsilon_m} \frac{\partial}{\partial z} & \frac{-1}{\epsilon_s} \frac{\partial}{\partial z} \\ \frac{-1}{\mu_s} \frac{\partial}{\partial z} & \frac{1}{\mu_m} \frac{\partial}{\partial z} & 0 & 0 \\ \frac{1}{\mu_m} \frac{\partial}{\partial z} & \frac{-1}{\mu_s} \frac{\partial}{\partial z} & 0 & 0 \end{bmatrix}, \quad (2a)$$

$$\mathbf{u} = [E_{x1}, E_{x2}, H_{y1}, H_{y2}]^T, \quad (2b)$$

$$\frac{1}{\epsilon_s} = \frac{v_e Z_{0e} + v_o Z_{0o}}{2}, \quad \frac{1}{\epsilon_m} = \frac{v_e Z_{0e} - v_o Z_{0o}}{2}, \quad (2c)$$

$$\frac{1}{\mu_s} = \frac{1}{2} \left(\frac{v_o}{Z_{0o}} + \frac{v_e}{Z_{0e}} \right), \quad \frac{1}{\mu_m} = \frac{1}{2} \left(\frac{v_o}{Z_{0o}} - \frac{v_e}{Z_{0e}} \right). \quad (2d)$$

In the above, ϵ_s , ϵ_m and μ_s , μ_m are the self and mutual permittivities and permeabilities. They can be expressed in terms of even- and odd-mode characteristic impedances Z_{0e} , Z_{0o} and phase velocities v_e , v_o . In addition, the latter are related to the even- and odd-mode effective permittivities as

$$v_e = c / \sqrt{\epsilon_{eff}^e}, \quad v_o = c / \sqrt{\epsilon_{eff}^o}. \quad (3)$$

For practical microstrip coupled lines whose medium is inhomogeneous, the phase velocities and effective permittivities are different for even and odd modes. Note that both even and odd modes are generally required for analysis and design of coupled transmission lines.

To solve (1), we propose the use of unconditionally stable ADI-FDTD method, which calls for the following two update procedures:

$$\left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{A} \right) \mathbf{u}^{n+\frac{1}{2}} = \left(\mathbf{I} + \frac{\Delta t}{2} \mathbf{B} \right) \mathbf{u}^n \quad (4a)$$

$$\left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B} \right) \mathbf{u}^{n+1} = \left(\mathbf{I} + \frac{\Delta t}{2} \mathbf{A} \right) \mathbf{u}^{n+\frac{1}{2}} \quad (4b)$$

where Δt is the time step and $\mathbf{I}$ is the 4×4 identity matrix. It should be noted here that although the procedures of (4) bear resemblance to those of ADI-FDTD for 3-D, 2-D or even 1-D Maxwell's equations, the respective $\mathbf{u}$ and $\mathbf{I}$ all differ in their sizes and elements, e.g. $\mathbf{u}$ is of 6×1 , 3×1 , 2×1 and 4×1 for 3-D, 2-D, 1-D and (2b) respectively. Moreover, the split matrices $\mathbf{A}$ and $\mathbf{B}$ used in the previous unconditionally stable FDTD methods for 3-D [17]-[19], or 2-D [20]-[23], etc., are all inapplicable for coupled transmission lines.

Here, we introduce the 4×4 split matrices $\mathbf{A}$ and $\mathbf{B}$ as

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & \frac{-1}{\epsilon_s} \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{-1}{\epsilon_m} \frac{\partial}{\partial z} & 0 \\ \frac{-1}{\mu_s} \frac{\partial}{\partial z} & 0 & 0 & 0 \\ \frac{1}{\mu_m} \frac{\partial}{\partial z} & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & \frac{-1}{\epsilon_m} \frac{\partial}{\partial z} \\ 0 & 0 & 0 & \frac{-1}{\epsilon_s} \frac{\partial}{\partial z} \\ 0 & \frac{1}{\mu_m} \frac{\partial}{\partial z} & 0 & 0 \\ 0 & \frac{-1}{\mu_s} \frac{\partial}{\partial z} & 0 & 0 \end{bmatrix}. \quad (5)$$

Using (5), one could find that the LHS of (4) leads to tridiagonal matrices. It can be shown that using the split matrices in (5), the M1-D ADI-FDTD method is unconditionally stable whereby the time step is unrestricted by the CFL condition. The analytical proof of unconditional stability shall be given in the next section.

B. Update equations of M1-D ADI-FDTD

Using (5), the first update procedure of M1-D ADI-FDTD method in (4a) can be expanded in detailed components as

$$E_{x1}|_k^{n+\frac{1}{2}} + \frac{\Delta t}{2\epsilon_s|_k \Delta z} \left(H_{y1}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} - H_{y1}|_{k-\frac{1}{2}}^{n+\frac{1}{2}} \right) = E_{x1}|_k^n - \frac{\Delta t}{2\epsilon_m|_k \Delta z} \left(H_{y2}|_{k+\frac{1}{2}}^n - H_{y2}|_{k-\frac{1}{2}}^n \right) \quad (6a)$$

$$E_{x2}|_k^{n+\frac{1}{2}} + \frac{\Delta t}{2\epsilon_m|_k \Delta z} \left(H_{y1}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} - H_{y1}|_{k-\frac{1}{2}}^{n+\frac{1}{2}} \right) = E_{x2}|_k^n - \frac{\Delta t}{2\epsilon_s|_k \Delta z} \left(H_{y2}|_{k+\frac{1}{2}}^n - H_{y2}|_{k-\frac{1}{2}}^n \right) \quad (6b)$$

$$H_{y1}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} + \frac{\Delta t}{2\mu_s|_{k+\frac{1}{2}} \Delta z} \left(E_{x1}|_{k+1}^{n+\frac{1}{2}} - E_{x1}|_k^{n+\frac{1}{2}} \right) = H_{y1}|_{k+\frac{1}{2}}^n + \frac{\Delta t}{2\mu_m|_{k+\frac{1}{2}} \Delta z} \left(E_{x2}|_{k+1}^n - E_{x2}|_k^n \right) \quad (6c)$$

$$H_{y2}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} - \frac{\Delta t}{2\mu_m|_{k+\frac{1}{2}} \Delta z} \left(E_{x1}|_{k+1}^{n+\frac{1}{2}} - E_{x1}|_k^{n+\frac{1}{2}} \right) = H_{y2}|_{k+\frac{1}{2}}^n - \frac{\Delta t}{2\mu_s|_{k+\frac{1}{2}} \Delta z} \left(E_{x2}|_{k+1}^n - E_{x2}|_k^n \right) \quad (6d)$$

where Δz is the spatial step. The second update procedure in (4b) can also be similarly expanded. Substituting (6c) into (6a) for the first procedure, one obtains the implicit update equation for E_{x1} as

$$\begin{aligned} & \left(1 + \frac{\Delta t^2}{4\epsilon_s|_k \mu_s|_{k+\frac{1}{2}} \Delta z^2} + \frac{\Delta t^2}{4\epsilon_s|_k \mu_s|_{k-\frac{1}{2}} \Delta z^2} \right) E_{x1}|_k^{n+\frac{1}{2}} \\ & - \frac{\Delta t^2}{4\epsilon_s|_k \mu_s|_{k-\frac{1}{2}} \Delta z^2} E_{x1}|_{k-1}^{n+\frac{1}{2}} - \frac{\Delta t^2}{4\epsilon_s|_k \mu_s|_{k+\frac{1}{2}} \Delta z^2} E_{x1}|_{k+1}^{n+\frac{1}{2}} \\ & = E_{x1}|_k^n - \frac{\Delta t}{2\epsilon_s|_k \Delta z} \left(H_{y1}|_{k+\frac{1}{2}}^n - H_{y1}|_{k-\frac{1}{2}}^n \right) \\ & - \frac{\Delta t^2}{4\epsilon_s|_k \mu_s|_{k+\frac{1}{2}} \Delta z^2} \left(E_{x2}|_{k+1}^n - E_{x2}|_k^n \right) \\ & + \frac{\Delta t^2}{4\epsilon_s|_k \mu_s|_{k-\frac{1}{2}} \Delta z^2} \left(E_{x2}|_k^n - E_{x2}|_{k-1}^n \right) \\ & - \frac{\Delta t}{2\epsilon_s|_k \Delta z} \left(H_{y2}|_{k+\frac{1}{2}}^n - H_{y2}|_{k-\frac{1}{2}}^n \right). \end{aligned} \quad (7)$$

It can be seen that the LHS of (7) results in tridiagonal matrix that can be solved efficiently. After obtaining E_{x1} implicitly, the remaining three fields can be solved explicitly. For the second procedure, one can similarly obtain the implicit update equation for E_{x2} , while the remaining three fields are solved explicitly. For clarity, the M1-D ADI-FDTD method can be summarized in two procedures below, which comprise one implicit and three explicit updates in each procedure as

First Procedure

- 1) Update E_{x1} implicitly
- 2) Update H_{y1} explicitly using E_{x1} obtained in step 1)
- 3) Update E_{x2} and H_{y2} explicitly using E_{x1} and H_{y1} obtained in steps 1) and 2)

Second Procedure

- 4) Update E_{x2} implicitly
- 5) Update H_{y2} explicitly using E_{x2} obtained in step 4)

- 6) Update E_{x1} and H_{y1} explicitly using E_{x2} and H_{y2} obtained in steps 4) and 5)

Using the M1-D ADI-FDTD method, not only can the fine meshing for line width and spacing of the coupled lines be omitted, the method is also unconditionally stable, i.e. free from CFL time step restriction.

III. ANALYTICAL PROOF OF UNCONDITIONAL STABILITY

Based on von Neumann method, the Fourier amplification matrix for M1-D ADI-FDTD method is given by

$$\mathbf{M} = (\mathbf{I} - \tilde{\mathbf{B}})^{-1} (\mathbf{I} + \tilde{\mathbf{A}}) (\mathbf{I} - \tilde{\mathbf{A}})^{-1} (\mathbf{I} + \tilde{\mathbf{B}}) \quad (8)$$

where

$$\tilde{\mathbf{A}} = \begin{bmatrix} 0 & 0 & \frac{j\bar{K}_z}{\epsilon_s} & 0 \\ 0 & 0 & \frac{j\bar{K}_z}{\epsilon_m} & 0 \\ \frac{j\bar{K}_z}{\mu_s} & 0 & 0 & 0 \\ -\frac{j\bar{K}_z}{\mu_m} & 0 & 0 & 0 \end{bmatrix}, \quad \tilde{\mathbf{B}} = \begin{bmatrix} 0 & 0 & 0 & \frac{j\bar{K}_z}{\epsilon_s} \\ 0 & 0 & 0 & \frac{j\bar{K}_z}{\epsilon_m} \\ 0 & -\frac{j\bar{K}_z}{\mu_s} & 0 & 0 \\ 0 & \frac{j\bar{K}_z}{\mu_m} & 0 & 0 \end{bmatrix} \quad (9a)$$

$j = \sqrt{-1}$, $\bar{K}_z = \frac{\Delta t}{\Delta z} \sin\left(\frac{k_z \Delta z}{2}\right)$ and k_z is the wave number along the z direction. The analytical expressions for the eigenvalues of Fourier amplification matrix are

$$\lambda_{1,2} = \frac{P + 2Q \pm \sqrt{R + 4PQ}}{S} \quad (10a)$$

$$\lambda_{3,4} = \frac{P - 2Q \pm \sqrt{R - 4PQ}}{S} \quad (10b)$$

where

$$P = 2\bar{K}_z^4 [\epsilon_m^2 \mu_s^2 - \mu_m^2 (\epsilon_m^2 - \epsilon_s^2)] + 4\bar{K}_z^2 \epsilon_m \epsilon_s^2 \mu_m \mu_s^2 + 2\epsilon_m^2 \epsilon_s^2 \mu_m^2 \mu_s^2 \quad (11a)$$

$$Q = \{\bar{K}_z^8 (\epsilon_m^2 \mu_s^2 - \epsilon_s^2 \mu_m^2)^2 + 4\bar{K}_z^4 (\epsilon_m \mu_s - \epsilon_s \mu_m)^2 [\bar{K}_z^2 \epsilon_m \epsilon_s \mu_m \mu_s (\epsilon_m \mu_m + \epsilon_s \mu_s) + \epsilon_m^2 \epsilon_s^2 \mu_m^2 \mu_s^2]\}^{\frac{1}{2}} \quad (11b)$$

$$R = 8\bar{K}_z^8 [\mu_m^4 (\epsilon_s^4 - \epsilon_m^2 \epsilon_s^2) + \epsilon_m^4 \mu_s^2 (\mu_s^2 - \mu_m^2)] - 16\bar{K}_z^6 \epsilon_m \epsilon_s \mu_m \mu_s (\epsilon_m \mu_m + 2\epsilon_s \mu_s) \cdot (\epsilon_m^2 \mu_m^2 - \epsilon_m^2 \mu_s^2 + \epsilon_m \epsilon_s \mu_m \mu_s - \epsilon_s^2 \mu_m^2) \quad (11c)$$

$$+ 8\bar{K}_z^4 \epsilon_m^2 \epsilon_s^2 \mu_m^2 \mu_s^2 (-4\epsilon_m^2 \mu_m^2 + 3\epsilon_m^2 \mu_s^2 - 4\epsilon_s \epsilon_s \mu_m \mu_s + 3\epsilon_s^2 \mu_m^2 + 2\epsilon_s^2 \mu_s^2) - 16\bar{K}_z^2 \epsilon_m^3 \epsilon_s^3 \mu_m^3 \mu_s^3 (\epsilon_m \mu_m - \epsilon_s \mu_s) \quad (11d)$$

$$S = 2\epsilon_m^2 \mu_m^2 (\bar{K}_z^2 + \epsilon_s \mu_s)^2. \quad (11e)$$

Note that all the terms in (11) are real. It can be shown that $R + 4PQ < 0$ and $R - 4PQ < 0$. Subsequently, the magnitude of eigenvalues can be found as

$$|\lambda_1| = |\lambda_2| = |\lambda_3| = |\lambda_4| = \frac{\sqrt{P^2 + 4Q^2 - R}}{S}. \quad (12)$$

Substituting the terms in (11) into (12) and upon some manipulations, we obtain

$$|\lambda_1| = |\lambda_2| = |\lambda_3| = |\lambda_4| = \frac{\sqrt{4\epsilon_m^4 \mu_m^4 (\bar{K}_z^2 + \epsilon_s \mu_s)^4}}{2\epsilon_m^2 \mu_m^2 (\bar{K}_z^2 + \epsilon_s \mu_s)^2} = 1. \quad (13)$$

Fig. 2. Schematic of the coupled line filter in [24]

Fig. 3. Time domain plot of (a) reflected and (b) transmitted E fields for the coupled line filter in [24].

Since all eigenvalues have unity magnitude regardless of the chosen time step Δt , the M1-D ADI-FDTD method is unconditionally stable.

IV. NUMERICAL RESULTS

Using the proposed M1-D ADI-FDTD method, we first compute the time domain electric fields everywhere along the coupled transmission lines in Fig. 1. The even- and odd-mode parameters of the coupled lines are: $Z_{0e} = 173.95$, $Z_{0o} = 70.41$, $\epsilon_{eff}^e = 3.33$, $\epsilon_{eff}^o = 2.92$. Mesh size is set as $\Delta z = 0.3$ mm and the time step size is specified by $CFLN = \Delta t / \Delta t_{CFL}$, where Δt_{CFL} is the CFL time step in [15]. In this case, $CFLN = 10$, so we are using time step size much larger than Δt_{CFL} . A Gaussian pulse is injected into line 1 from the left side and the time snapshot of the electric fields is depicted in Fig. 1. One can see clearly the dynamics of pulse being coupled into line 2 and it travels from left to right along the line. The results are stable and show good agreement with those using the (explicit) M1-D FDTD method in [15].

We next consider a practical coupled line bandpass filter from [24] designed at center frequency $f_0 = 3$ GHz. The schematic of the filter is shown in Fig. 2. It consists of two coupled line sections connected by a transmission line with characteristic impedance Z_c and effective permittivity ϵ_{effc} . The parameters of the filter are: $Z_{0e1} = 173.95$, $Z_{0o1} = 70.41$, $\epsilon_{eff1}^e = 3.33$, $\epsilon_{eff1}^o = 2.92$, $l_1 = 13.84$ mm, $Z_c = 47.33$,

Fig. 4. S parameters of the coupled line filter in [24] computed using various methods.

Fig. 5. Schematic of the filtering antenna in [1].

$\epsilon_{effc} = 3.65$, $l_c = 12.67$ mm. Δz is generally set as 0.3 mm. For the interface between different sections, we have used a smaller Δz , which is set at three times smaller than the general Δz along the main parts of lines. The parameters at the interface are simply set as the arithmetic mean of those neighboring ones. Note that we have not investigated other means of setting the parameters at the interface such as those in [25], [26]. Fig. 3 shows the time domain plot of reflected and transmitted E fields for the coupled line filter. At CFLN = 10 and 20, good agreement is observed between the M1-D ADI-FDTD method and the M1-D FDTD method. Fig. 4 shows the S parameters of the coupled line filter computed using various methods, including those obtained directly from Advanced Design System (ADS) as reference. Generally good agreement is observed among the S parameter plots, which validates our proposed method. Note that ADS does not provide convenient time-domain visualization of field distribution and coupling everywhere like Fig. 3 or Fig. 1. Moreover, using the M1-D ADI-FDTD method, the time step size can be selected much larger than Δt_{CFL} without sacrificing much accuracy.

Finally, we apply our unconditionally stable M1-D ADI-FDTD method to analyze the printed filtering antenna from [1]. The filtering antenna is realized using an inverted-L antenna and coupled line sections at center frequency $f_0 = 2.45$ GHz. The schematic of the filtering antenna is shown in Fig. 5. It consists of three coupled line sections and an

Fig. 6. Normalized radiated and reflected powers of the filtering antenna in [1].

equivalent circuit model of the inverted-L antenna, which is modeled as a network of RLC lumped elements. The parameters for the coupled line sections are: $Z_{0e1} = 83.74$, $Z_{0o1} = 37.57$, $\epsilon_{eff1}^e = 2.74$, $\epsilon_{eff1}^o = 2.23$, $l_1 = 19.42$ mm, $Z_{0e2} = 62.15$, $Z_{0o2} = 41.94$, $\epsilon_{eff2}^e = 2.84$, $\epsilon_{eff2}^o = 2.35$, $l_2 = 19.02$ mm, $Z_{0e3} = 75.67$, $Z_{0o3} = 42.58$, $\epsilon_{eff3}^e = 2.77$, $\epsilon_{eff3}^o = 2.29$, $l_3 = 19.24$ mm. For the equivalent circuit model of inverted-L antenna, the circuit parameters are: $R_A = 28.6 \Omega$, $L_A = 14.2$ nH, $C_A = 0.3$ pF, $C_g = 0.37$ pF. The lumped elements are incorporated into our proposed method based on the approach in [27]. Δz is generally set as 0.4 mm and three times smaller along the interface between line sections and lumped elements. Using our proposed M1-D ADI-FDTD method, the radiated and reflected powers of the filtering antenna are computed. Both powers are normalized against the incident power fed into the filtering antenna. The radiated power is computed as the power dissipated across the radiated resistance R_A . Fig. 6 shows the normalized radiated and reflected powers of the filtering antenna. It can be observed that within the frequency band around f_0 , the power is radiated while it is reflected outside this band. The results for M1-D ADI-FDTD method at CFLN = 10 and 20 agree well with those from M1-D FDTD method and ADS. Hence, the accuracy of M1-D ADI-FDTD method is again well validated even at rather high CFLN. From these numerical examples, it can be seen that the M1-D ADI-FDTD method is useful in simulating EM wave propagation in coupled line structures with high CFLN while maintaining good accuracy.

V. CONCLUSION

This paper has presented the unconditionally stable M1-D ADI-FDTD method for coupled transmission lines. The differential equations for coupled transmission lines have been formulated in compact matrix form. Proper 4×4 split matrices have been introduced, which feature unconditional stability for the ADI procedures. These matrices also lead to update equations with left-hand sides involving tridiagonal matrices that

can be solved efficiently. The update equations of M1-D ADI-FDTD method have been given, along with the analytical proof of unconditional stability based on von Neumann method. The M1-D ADI-FDTD method allows quick simulation and visualization of electromagnetic fields everywhere along the coupled transmission lines. To demonstrate the usefulness of M1-D ADI-FDTD method, various coupled line structures for filter and filtering antenna have been analyzed using the method. Noting that the judicious formulation of split matrices is essential for unconditional stability and efficiency, the proposed method herein may be extended for more general configurations with multiple coupled lines in the future.

REFERENCES

- [1] C. -T. Chuang and S. -J. Chung, "Synthesis and design of a new printed filtering antenna," *IEEE Trans. Antennas Propag.*, vol. 59, no. 3, pp. 1036-1042, Mar. 2011.
- [2] C. -T. Chuang and S. -J. Chung, "A compact printed filtering antenna using a ground-intruded coupled line resonator," *IEEE Trans. Antennas Propag.*, vol. 59, no. 10, pp. 3630-3637, Oct. 2011.
- [3] C. -K. Lin and S. -J. Chung, "A compact filtering microstrip antenna with quasi-elliptic broadside antenna gain response," *IEEE Antennas Wireless Propag. Lett.*, vol. 10, pp. 381-384, 2011.
- [4] T. H. Gan, Z. Yang and E. L. Tan, "A polarization-reconfigurable filtering antenna system," *IEEE Antennas Propag. Mag.*, vol. 55, no. 6, pp. 198-219, Dec. 2013.
- [5] C. -Y. Hsieh, C. -H. Wu and T. -G. Ma, "A compact dual-band filtering patch antenna using step impedance resonators," *IEEE Antennas Wireless Propag. Lett.*, vol. 14, pp. 1056-1059, 2015.
- [6] L. Li and G. Liu, "A differential microstrip antenna with filtering response," *IEEE Antennas Wireless Propag. Lett.*, vol. 15, pp. 1983-1986, 2016.
- [7] D. M. Pozar, *Microwave Engineering*, 4th ed., New York: Wiley, 2011.
- [8] R. Garg, I. Bahl, and M. Bozzi, *Microstrip Lines and Slotlines*, 3rd ed., Norwood, MA, USA: Artech House, 2013.
- [9] R. K. Mongia, I. J. Bahl, P. Bhartia, and J. Hong, *RF and Microwave Coupled-Line Circuits*, 2nd ed., Norwood, MA, USA: Artech House, 2007.
- [10] K. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antennas Propag.*, vol. 14, no. 3, pp. 302-307, May 1966.
- [11] A. Taflov and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*, Boston, M. A.: Artech House, 2005.
- [12] Z. Yang and E. L. Tan, "Microstrip circuit tool kit app with FDTD analysis including lumped elements," *IEEE Microw. Mag.*, vol. 16, no. 1, pp. 74-80, Feb. 2015.
- [13] Z. Yang and E. L. Tan, "Interconnected multi-1-D FADI- and FLOD-FDTD methods for transmission lines with interjunctions," *IEEE Trans. Microw. Theory Tech.*, vol. 65, no. 3, pp. 684-692, Mar. 2017.
- [14] Q. Lei, J. -B. Wang, J. L. Wang and L. -H. Shi, "An efficient unconditionally stable FDTD method for analyzing composite right/left-handed transmission line," *IEEE Antennas Wireless Propag. Lett.*, vol. 16, pp. 2824-2827, 2017.
- [15] Z. Yang and E. L. Tan, "Multiple one-dimensional FDTD method for coupled transmission lines and stability condition," *IEEE Microw. Wireless Compon. Lett.*, vol. 26, no. 11, pp. 864-866, Nov. 2016.
- [16] C. R. Paul, *Analysis of Multiconductor Transmission Lines*, 2nd ed., New York, NY, USA: Wiley, 2008.
- [17] F. Zheng, Z. Chen, and J. Zhang, "Toward the development of a three-dimensional unconditionally stable finite-difference time-domain method," *IEEE Trans. Microw. Theory Tech.*, vol. 48, no. 9, pp. 1550-1558, Sep. 2000.
- [18] T. Namiki, "3-D ADI-FDTD method- unconditionally stable time-domain algorithm for solving full vector Maxwell's equations," *IEEE Trans. Microw. Theory Tech.*, vol. 48, no. 10, pp. 1743-1748, Oct. 2000.
- [19] E. L. Tan, "Fundamental schemes for efficient unconditionally stable implicit finite-difference time-domain methods," *IEEE Trans. Antennas Propag.*, vol. 56, no. 1, pp. 170-177, Jan. 2008.
- [20] T. Namiki, "A new FDTD algorithm based on alternating-direction implicit method," *IEEE Trans. Microw. Theory Tech.*, vol. 47, no. 10, pp. 2003-2007, Oct. 1999.
- [21] F. Zheng, Z. Chen, and J. Zhang, "A finite-difference time-domain method without the Courant stability conditions," *IEEE Microw. Guided Wave Lett.*, vol. 9, no. 11, pp. 441-443, Nov. 1999.
- [22] W. Fu and E. L. Tan, "Development of split-step FDTD method with higher-order spatial accuracy," *Electron. Lett.*, vol. 40, no. 20, pp. 1252-1253, Sep. 2000.
- [23] G. Singh, E. L. Tan and Z. N. Chen, "Implementation of total-field/scattered-field technique in the 2-D LOD-FDTD method," in *2009 Asia Pacific Microw. Conf.*, Singapore, Dec. 2009, pp. 1505-1508.
- [24] S. S. Gao and S. Sun, "Synthesis of wideband parallel-coupled line bandpass filters with non-equiripple responses," *IEEE Microw. Wireless Compon. Lett.*, vol. 24, no. 9, pp. 587-589, Sep. 2014.
- [25] K. -P. Hwang and A. C. Cangellaris, "Effective permittivities for second-order accurate FDTD equations at dielectric interfaces," *IEEE Microw. Wireless Compon. Lett.*, vol. 11, no. 4, pp. 158-160, Apr. 2001.
- [26] W. Fu and E. L. Tan, "Effective permittivity scheme for ADI-FDTD method at the interface of dispersive media" *Appl. Computational Electromagn. Soc. J.*, vol. 23, no. 2, pp. 120-125, Jun. 2008.
- [27] W. Fu and E. L. Tan, "Unconditionally stable ADI-FDTD method including passive lumped elements," *IEEE Trans. Electromag. Compat.*, vol. 48, no. 4, pp. 661-668, Nov. 2006.