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Application of Belevitch Theorem for Pole-Zero Analysis of Microwave Filters with Transmission Lines and Lumped Elements

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Abstract—This paper presents the application of Belevitch theorem for pole-zero analysis of microwave filters synthesized with transmission lines and lumped elements. The scattering (S) matrix determinant (Δ) based on Belevitch theorem, aptly called Belevitch determinant, comprises poles and zeros that are separated in different half-plane regions. Using the Belevitch determinant, the poles and zeros of filter transfer functions can be determined separately with certainty, e.g. by applying the contour integration method based on argument principle. Note that the contour integration can be evaluated numerically without requiring complicated overall analytical expressions. The proposed method is able to solve the poles and zeros for filters synthesized with non-commensurate transmission lines and lumped elements, where the transform method and the eigenvalue approach are inapplicable. Several applications are discussed to demonstrate the use of Belevitch theorem and contour integration method to determine the poles and zeros of various microwave filters on the complex plane.

Index Terms—Argument principle, Belevitch theorem, contour integration, microwave filters, numerical method, poles and zeros, S matrix determinant.

I. INTRODUCTION

MICROWAVE filters have been the subject of much research due to their wide applications in wireless communication. Various filter synthesis methods, design and transformations can be found in numerous classic microwave textbooks [1]–[4]. Over the years, many filter structures have also been analyzed and synthesized involving (lossless) lumped elements and transmission line structures such as stepped lines, stubs and coupled lines [5]–[8]. However, most analyses have not considered in detail the order and locations of poles and zeros of synthesized filters. In particular, most filter analyses of microwave textbooks are subjected only for low pass filter prototype and typically in lumped elements. When replaced by transmission line structures after various transformations, the transfer functions of these filters may change considerably. The numbers of poles and zeros as well as their locations for a synthesized filter may be different from the originally specified or designed ones. For instance, the poles of a synthesized Chebyshev filter may be deviated from the reference Chebyshev ellipse after realization. It is to be shown later that

the number of poles of a synthesized classical coupled-line filter [1] turns out to be more. Hence, the number of poles and zeros should be ascertained after synthesis, while their locations should be verified and checked for further tuning wherever needed, especially for filters with transmission lines transformed from lumped element prototypes. Since poles and zeros control the amplitude response, phase and group delay of a filter directly, they need to be analyzed for better understanding and manipulation of the filter response.

Unfortunately for microwave filters synthesized from transmission line structures, it is often difficult to determine the poles and zeros since the overall analytical expressions may be very complicated due to involvement of (non-polynomial) transcendental functions. The difficulties in solving poles and zeros are further exacerbated when the filters consist of both transmission lines and lumped elements. Their resultant scattering (S) parameters expressions, if one could ever derive, would comprise combinations of various integer power terms and trigonometric power terms. This makes the existing methods such as Richard, Euler or digital transform method [1], [9], [10] and coupling matrix eigenvalue approach [11]–[15] inapplicable for pole-zero analysis of the filters. For blind modeling methods such as vector fitting technique [16], they also involve many uncertainties and inconsistencies of poles and zeros depending on chosen number of poles and fitting bandwidth, etc.

In this paper, we present the application of Belevitch theorem for pole-zero analysis of microwave filters synthesized with transmission lines and lumped elements. In Section II, the challenges in solving poles and zeros for such filters will be exemplified in detail, demonstrating the difficulties and deficiencies of existing methods including transform method, eigenvalue approach and vector fitting technique. In Section III, the Belevitch theorem will be discussed. Using the Belevitch theorem, it will be shown that the scattering (S) matrix determinant (Δ), called Belevitch determinant, comprises poles and zeros that are separated in different half-plane regions. Once the poles and zeros are separated completely, the contour integration method based on argument principle will be applied, which allows them to be determined with certainty. Several applications will be discussed in Section IV to demonstrate the use of Belevitch theorem and contour integration method to determine the poles and zeros of various microwave filters on the complex plane.

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Fig. 1. Schematic of parallel transmission line filter.

II. POLE-ZERO ANALYSIS OF MICROWAVE FILTERS WITH TRANSMISSION LINES AND LUMPED ELEMENTS

For microwave filters involving transmission lines, it is often challenging to solve for the poles and zeros of transfer functions due to involvement of transcendental functions such as the exponential and trigonometric functions. Consider the example from [9] where the filter section consists of two parallel transmission line sections with characteristics impedances and electrical lengths Z_1 , Z_2 and θ_1 , θ_2 , respectively, c.f. Fig. 1(a). The expression of S_{21} for such filter section is given by

$$S_{21}(\omega) = N_{S21}(\omega)/D_{S21}(\omega) \quad (1)$$

where

$$N_{S21}(\omega) = -2jZ_0Z_1Z_2(Z_1\sin(\theta_1\omega/\omega_0) + Z_2\sin(\theta_2\omega/\omega_0)) \quad (2)$$

$$\begin{aligned} D_{S21}(\omega) &= (Z_0^2Z_1^2 + Z_0^2Z_2^2 + Z_1^2Z_2^2)(\sin(\theta_1\omega/\omega_0)\sin(\theta_2\omega/\omega_0)) \\ &+ Z_0^2Z_1Z_2(\cos^2(\theta_1\omega/\omega_0) + \cos^2(\theta_2\omega/\omega_0) + \sin^2(\theta_1\omega/\omega_0) \\ &+ \sin^2(\theta_2\omega/\omega_0) - 2\cos(\theta_1\omega/\omega_0)\cos(\theta_2\omega/\omega_0)) \\ &- 2j\{(Z_0Z_1^2Z_2)\cos(\theta_2\omega/\omega_0)\sin(\theta_1\omega/\omega_0) + \\ &(Z_0Z_1Z_2^2)\cos(\theta_1\omega/\omega_0)\sin(\theta_2\omega/\omega_0)\} \end{aligned} \quad (3)$$

ω_0 is the center angular frequency. To convert (1)-(3) into complex frequency s domain where $s = \sigma + j\omega$, ω is substituted with $-j \cdot s = -j \cdot (\sigma + j\omega)$. It is worth pointing out that the S_{21} denominator of the filter section is already rather complicated even for one section only. As exemplified from (1)-(3), the roots cannot be directly solved for these nonlinear functions. Furthermore, as the trigonometric functions are periodic, there are infinitely many poles on the whole s domain. It is therefore difficult to ascertain and solve for the poles within certain frequency band of interest.

For commensurate lines whose electrical lengths are the same (or multiples) for all sections, one possible way to solve for the poles is by utilizing the Richard transform or Euler's identities [1]. In particular, using

$$\cos(-js\theta_1/\omega_0) = \frac{z + 1/z}{2}; \sin(-js\theta_1/\omega_0) = \frac{z - 1/z}{2j}; \quad (4)$$

$$\cos(-js\theta_2/\omega_0) = \frac{z^a + 1/z^a}{2}; \sin(-js\theta_2/\omega_0) = \frac{z^a - 1/z^a}{2j} \quad (5)$$

where $a = \theta_2/\theta_1$, the S_{21} of (1) is transformed into z domain as

$$S_{21}(z) = \frac{2z^{2a} + 2z^{a+1} - 2z^{a-1} - 2}{4z^{2a+1} - z^{2a-1} - 2z^a - z} \quad (6)$$

If a is an integer, the numerator and denominator of (6) contain only polynomials (with integer powers) for which the roots may be solved directly [9], [10]. Following [9], we set $Z_0 = 50$, $Z_1 = Z_2 = 2Z_0$ and the electrical lengths of the two parallel transmission lines are $\theta_1 = \pi/2$, $\theta_2 = 5\pi/2$ which give $a = 5$. Substituting this a into (6) and upon changing the variable $z = u^{1/2}$, we obtain an equation that is consistent with [9, (3)] by noting $u = e^{j\pi(f/f_d)}$ and the same a_k , b_k in Table I therein.

Thus far, the transform method has been applied for the case of integer a to solve for polynomial roots. However, it is no longer able to simplify root solving for non-commensurate lines or when a is not an integer. To demonstrate this, we let $\theta_2 = 3.6514\pi$ instead of $5\pi/2$ in Fig. 1(a). Now, substituting $a = 7.3028$ into the denominator of (6) gives an equation which consists of fractional powers that can no longer be solved easily as polynomial roots. In fact, when a is not integer, it is also mentioned by [9] that the filter cannot be analyzed within the digital-inspired (z or u) framework (e.g. only approximate when $\theta_1 = 0.497\pi$).

The difficulties in solving poles and zeros of transfer functions are further exacerbated when the filters consist of both transmission lines and lumped elements. To illustrate this, the same filter section in Fig. 1(a) is added with two capacitors of capacitance C_1 and C_2 , as shown in Fig. 1(b). The denominator of S_{21} can be derived as

$$\begin{aligned} D_{S21}(\omega) &= -10^5 C_1 C_2 \omega^2 (p_1^4 - 2p_1^2 p_2^2 - j8p_1^2 p_2 q_2 + 2p_1^2 q_1^2 \\ &+ 2p_1^2 q_2^2 - j8p_1 p_2^2 q_1 + 24p_1 p_2 q_1 q_2 + j8p_1 q_1 q_2^2 + p_2^4 \\ &+ 2p_2^2 q_1^2 + 2p_2^2 q_2^2 + j8p_2 q_1^2 q_2 + q_1^4 - 2q_1^2 q_2^2 + q_2^4) \\ &+ 200\omega(2p_1 p_2 + j p_1 q_2 + j p_2 q_1)(2C_1 p_2 q_1 - j C_2 p_1 p_2 \\ &- j C_1 p_1 p_2 + 2C_2 p_1 q_2 + j C_1 q_1 q_2 + j C_2 q_1 q_2) \\ &- (2p_1 p_2 + j p_1 q_2 + j p_2 q_1)^2 \end{aligned} \quad (7)$$

where

$$p_1 = \cos(\frac{\theta_1}{2}\omega/\omega_0); \quad p_2 = \cos(\frac{\theta_2}{2}\omega/\omega_0) \quad (8)$$

$$q_1 = \sin(\frac{\theta_1}{2}\omega/\omega_0); \quad q_2 = \sin(\frac{\theta_2}{2}\omega/\omega_0) \quad (9)$$

It is evident that due to the combinations of various integer power terms (ω^2 , ω) and trigonometric power terms ($\sin(\frac{\theta}{2}\omega/\omega_0)$, $\cos(\frac{\theta}{2}\omega/\omega_0)$), (7) cannot be transformed into another simpler form to solve for the roots directly. Hence, the transform method is inapplicable for pole-zero analysis of filters with transmission lines and lumped elements.

For certain microwave filter synthesis [11]-[15], its coupling matrix may also be used to find the S parameter poles via eigenvalue approach. Consider for example an n -coupled

lumped element resonator filter, its S_{21} and S_{11} can be derived using the coupling matrix as [3]

$$S_{21} = \frac{2}{\sqrt{q_{e1}q_{en}}} \bar{\bar{A}}_{n1}^{-1} \quad (10a)$$

$$S_{11} = \pm \left(1 - \frac{2}{q_{e1}} \bar{\bar{A}}_{11}^{-1} \right) \quad (10b)$$

where

$$\bar{\bar{A}} = \bar{\bar{q}} + p\bar{\bar{I}} - j\bar{\bar{m}} \quad (11)$$

$$p = \frac{j}{FBW} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \quad (12)$$

q_{e1} and q_{en} are the scaled external quality factors of input and output resonators, FBW is the fractional bandwidth, ω_0 is the center angular frequency, $\bar{\bar{I}}$ is the $n \times n$ identity matrix, $\bar{\bar{q}}$ is an $n \times n$ matrix with all entries zeros except $q_{11} = 1/q_{e1}$ and $q_{nn} = 1/q_{en}$, and $\bar{\bar{m}}$ is the $n \times n$ coupling matrix. If a narrow-band filter can be represented in the coupling matrix form of (11), the poles can be found by first solving p from the eigenvalues of matrix comprising $\bar{\bar{m}}$ in (11). Thereafter, the poles are obtained via (12).

However, it should be noted that a broadband filter can no longer be representable in the coupling matrix form of (11) should it comprise combinations of non-commensurate transmission lines and lumped elements. Indeed, the impedance or admittance matrix or $\bar{\bar{A}}$ in (11) for such filter should have various powers of ω , $\frac{1}{\omega}$ as well as $\sin$ and $\cos$ terms, in much the same manner like (7) above. In other words, the p term of (12) not only involves $\frac{\omega}{\omega_0}$ and $\frac{\omega_0}{\omega}$, while the $\bar{\bar{m}}$ matrix of (11) not only contains constant coupling coefficients, but also various nonlinear functions of ω . This makes it difficult or impossible to extract the poles using the eigenvalue approach directly. Hence, the eigenvalue approach is inapplicable for pole-zero analysis of filters with transmission lines and lumped elements.

The blind system modeling methods such as the vector fitting technique, Loewner matrix or Cauchy method are sometimes used to locate the system poles and zeros through fitting of measurement data points. We shall comment briefly on the vector fitting technique [16] that it is one of the most popular modeling methods. While it is a blind fitting/modeling method, the number of poles to estimate is often uncertain and the poles obtained are very much dependent on the fitting bandwidth. For a filter with no complex zeros, spurious complex zeros may even appear that are not symmetrical about the $j\omega$ axis, thus violating the S parameters unitary conditions [11]. Henceforth, this technique as well as other blind fitting/modeling methods are beyond the scope of this paper.

III. APPLICATION OF BELEVITCH THEOREM FOR POLE-ZERO ANALYSIS

To overcome the difficulties and deficiencies above, we shall present the application of Belevitch theorem for poles and zeros analysis of microwave filters synthesized with transmission lines and lumped elements. The Belevitch theorem states that

the S matrix of a lossless 2-port network can be represented as [17]

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{g} \begin{bmatrix} u & v \\ v_* & -u_* \end{bmatrix} \quad (13)$$

where g is a strictly Hurwitz polynomial and g, u, v are related by

$$uu_* + vv_* = gg_* \quad (14)$$

The notation g_* (with subscript “*”) denotes $g_* = [g(-s^*)]^*$, where the superscript “*” indicates complex conjugation. Note that the effect of g_* is to reflect all complex roots of g symmetrically about the imaginary axis. Therefore, the roots of g and g_* are symmetrical about the imaginary axis. Applying the Belevitch theorem (13)-(14), it can be shown that the S matrix determinant, aptly called Belevitch determinant, takes the form

$$\Delta = S_{11}S_{22} - S_{21}S_{12} = \frac{-g_*}{g}. \quad (15)$$

Since g is strictly Hurwitz, (15) shows that all poles of Δ are located in the left-half plane (LHP) of complex s plane, with the same number of zeros (roots of g_*) located exactly opposite of $j\omega$ axis in the right-half plane (RHP). Note that there will be no zeros in the LHP and no poles in the RHP for Δ . As such, the poles and zeros have been effectively separated into different half-plane regions through (15). This is crucial to allow them to be determined separately with certainty in the subsequent analysis. Furthermore, the Belevitch determinant in (15) is an irreducible expression due to g being strictly Hurwitz. Consequently, the degree of a lossless 2 port is the degree of the polynomial g of the canonic form (13) [17]. By retrieving the poles of Δ , we have essentially retrieved the same poles for S_{21} sharing the common denominator g .

Once the poles and zeros are separated completely, one can proceed to solve for them using several possible ways. To that end, the contour integration method based on argument principle is one possible way that may be applied conveniently. The argument principle is given by [18], [19]

$$Z - P = \frac{1}{2\pi} \Delta_C \arg f(s) = \frac{1}{2\pi j} \oint_C \frac{f'(s)}{f(s)} ds \quad (16)$$

where $\Delta_C \arg f(s)$ is the change in argument of $f(s)$ along closed contour C in counter-clockwise direction, $f(s)$ is a meromorphic function, Z and P are the number of zeros and poles of $f(s)$, including their multiplicities within the closed contour, and $f'(s)$ is the derivative of $f(s)$. The contour C should also not pass through any poles and zeros. The complex function $f(s)$ in (16) is to be replaced by the Belevitch determinant $\Delta(s)$ of a microwave filter on complex s plane. Since Δ has no zeros in the LHP ($Z = 0$), we are able to evaluate (16) to find the number of poles in the LHP. It should be emphasized here that without the use of Belevitch determinant in (15), the argument principle could not be applied directly if $f(s)$ contains both poles and zeros in the LHP. Most argument principle based methods such as [20], [21] are able to solve only for the zeros of $f(s)$ when it is analytic within C ($P = 0$).

The contour integration of $f \equiv \Delta$ can be evaluated numerically via

$$\frac{1}{2\pi j} \oint_C \frac{\Delta'(s)}{\Delta(s)} ds = \frac{1}{\pi j} \sum_s \frac{\Delta(s + \delta s/2) - \Delta(s - \delta s/2)}{\Delta(s + \delta s/2) + \Delta(s - \delta s/2)}. \quad (17)$$

Here, central finite-difference is used for the derivative $\Delta'(s)$, while averaging scheme is used for $\Delta(s)$. This reduces the number of stencils from three to two for every point on the contour. δs is the spatial step size chosen along the path to ensure convergence. It should be emphasized that to evaluate (17), the overall analytical expression of Δ is not required, while only the numerical values of Δ along the contour path are needed. In practice, for microwave filter circuits, the network parameters for individual sections (such as transmission lines, lumped elements, etc.) can be computed and manipulated readily to obtain the Δ values of the overall circuits.

To locate the poles, the contour can be successively divided into smaller sections until they are tightly enclosed. The contour method could even be combined with other efficient iterative root searching algorithms, such as Newton's method, Muller's method, etc., which often require rather good initial guess. Having divided the contour into small sections, the contour centroid may serve as good initial guess to these iterative algorithms. After the poles have been retrieved, we can further determine (if any) the complex zeros of microwave filter. This can be done by repeating (16) for $f \equiv S_{21}$ and dividing the contour into smaller sections successively to find Z with known P .

The poles and zeros may also be solved using the approach in [22] employing the Newton's identities. However, one would need to estimate the number of distinct zeros and poles, which is done through forming successive system matrices and checking if they are singular. This is often difficult in practice to numerically ascertain whether a matrix is singular. Furthermore, large number of zeros and poles often result in high polynomial order and ill-conditioned problem. On the other hand, alternative searching technique such as [23] involves exhaustive triangulation of 2-D domain. Still, it does not guarantee that all the roots can be found and the risk of missing root is higher due to irregular discretization of the domain. Overall, there are still uncertainties and difficulties in solving for poles and zeros using these methods. Unlike the approach above, by using the Belevitch determinant (15), one will have the poles and zeros separated completely, with only poles and no zeros in the LHP. There is no need to estimate the number of poles or zeros, thus obviating the forming of successive system matrices and checking if they are singular. The poles can then be solved as the roots of polynomial. Hence, there is no need to divide the contour excessively until the poles are tightly enclosed. In practice, one would still consider dividing the contour sufficiently to reduce the number of enclosed poles in each contour. This should lower the polynomial order to avoid ill-conditioned problem.

The Belevitch determinant in (15) further allows us to evaluate (16) on the contours in the RHP with $f(s) = \Delta$ (instead of in the LHP with $f(s) = 1/\Delta$). The poles of

Fig. 2. S parameters of coupled-line filter with Chebyshev response ($N = 3$). Inset shows the layout of $N + 1$ sections coupled-line filter along with even and odd characteristic impedances of individual section.

Δ can then be obtained subsequently by reflecting all the zeros of $f(s)$ symmetrically along $j\omega$ axis. This will bypass the computation of $1/\Delta$ inversion for greater simplicity and robustness. Apart from the sole determinant in (15), the Belevitch theorem can also be applied for other S parameter expressions or combinations. For instance, one may consider $f(s) = S_{21}/\Delta$ to solve directly for the transmission zeros (zeros of S_{21}), or consider $f(s) = S_{11}/\Delta$ or $f(s) = S_{22}/\Delta$ to solve directly for the reflection zeros of a microwave filter. All in all, many opportunities arise from the application of Belevitch theorem which could be further explored for pole-zero analysis of microwave filters.

IV. APPLICATIONS FOR MICROWAVE FILTERS

A. Applications I: Classical Filters

We shall demonstrate the applications of Belevitch theorem and contour integration method to solve for poles and zeros of several microwave filters. First, consider the classical coupled-line filter in Fig. 2 [1]. The specifications of the bandpass filter are Chebyshev response with $N = 3$, passband ripple $L_{Ar} = 0.5$ dB, center frequency $f_0 = 2$ GHz and fractional bandwidth $FBW = 0.1$. The filter is realized by four ($N + 1 = 4$) coupled-line sections. The S parameters of the filter are plotted in Fig. 2. From the S_{11} plot, three reflection zeros can be observed, which may lead one to deduce that there are three poles within its passband in line with the third-order specification. Using the contour integration method for the Belevitch determinant with contour paths enclosing the passband around $f_0 = 2$ GHz, the poles are determined as (normalized by 1 GHz)

$$\begin{aligned} s_{p1} &= 2\pi(-0.03124 + j1.89849), \\ s_{p2} &= 2\pi(-0.06259 + j2.00000), \\ s_{p3} &= 2\pi(-0.03124 + j2.10151), \\ s_{p4} &= 2\pi(-1.88540 + j2.00000), \end{aligned}$$

Fig. 3. The (a) real and (b) imaginary parts of S_{21} , absolute errors in (c) real and (d) imaginary parts of S_{21} .

$$s_{p5} = 2\pi(-1.88549 + j2.00000). \quad (18)$$

It can be seen that there are five poles within the contour. For verification, we also solve the poles by utilizing the transform method and find the polynomial roots (with positive imaginary part only) in z domain as

$$\begin{aligned} z_{p1} &= 0.07771 + j0.97266, \\ z_{p2} &= j0.95203, \\ z_{p3} &= -0.07771 + j0.97266, \\ z_{p4} &= j0.22746, \\ z_{p5} &= j0.22744. \end{aligned} \quad (19)$$

The poles in s domain can then be deduced from the roots (19) through the inverse of Richard transform or Euler's identities

$$s = \omega_0/\theta_0 \cdot \ln z. \quad (20)$$

It is found that they agree well with (18) obtained via our Belevitch determinant contour integration method, thus validating our proposed method.

Interestingly from (18), it can be seen that the last two poles s_{p4} and s_{p5} are located further away from the $j\omega$ axis. To investigate further the consequence of retaining all five poles or only first three poles s_{p1} , s_{p2} and s_{p3} , Fig. 3(a) and (b) show the real and imaginary parts of S_{21} constructed using all 5 poles and only first 3 poles, and they are compared to the S_{21} obtained directly from simulation. The real zeros located on the $j\omega$ axis have been included also, considering as well the negative frequencies and periodicity of this coupled-line filter. Fig. 3(c) and (d) show the absolute errors in real and imaginary parts of S_{21} for the results constructed using 5 and 3 poles compared to the simulation. It can be observed that if one excludes the last two poles in (18), the errors are larger in both real and imaginary parts of S_{21} . These errors may at times be tolerable, which explain why most literatures still regard the

($N = 3$) coupled-line filter as “third”-order effectively with three poles only.

Let us now modify slightly the specifications of the coupled-line filter with increased $FBW = 0.2$ and reduced passband ripple $L_{Ar} = 0.01$ dB. f_0 is still maintained at 2 GHz. The S parameters are also shown in Fig. 2. Three reflection zeros are still visible from the S_{11} plot. Meanwhile, the number of poles determined from the determinant contour integration method is still five around the passband. The poles are subsequently solved as

$$\begin{aligned} s_{p1} &= 2\pi(-0.15601 + j1.67367), \\ s_{p2} &= 2\pi(-0.71330 + j2.00000), \\ s_{p3} &= 2\pi(-0.58349 + j2.00000), \\ s_{p4} &= 2\pi(-0.44182 + j2.00000), \\ s_{p5} &= 2\pi(-0.15601 + j2.32633). \end{aligned} \quad (21)$$

For verification, the transform method is again utilized to obtain the roots in z domain as

$$\begin{aligned} z_{p1} &= 0.22427 + j0.85578, \\ z_{p2} &= j0.57108, \\ z_{p3} &= j0.63237, \\ z_{p4} &= j0.70680, \\ z_{p5} &= -0.22427 + j0.85578. \end{aligned} \quad (22)$$

Using (20), the poles (in s domain) are in agreement with (21). It can be seen that all five poles are now near to each other and to $j\omega$ axis. Also, notice that the two extra poles in (18) and (21) do not lie on/near the Chebyshev ellipse on the complex s plane. While the other three poles that are around the Chebyshev ellipse may contribute most to the third order Chebyshev response, the presence of two extra poles would still affect the filter response to certain extent, c.f. Fig. 3. From these two examples, one should appreciate the importance of poles analysis whereby the number and locations of poles of synthesized filter may be different from those originally specified.

B. Applications II: Advanced Filters

Complex zeros may sometimes be utilized to optimize the filter phase response for better group delay equalization. We now apply the Belevitch theorem and contour integration method to analyze a linear phase filter with complex zeros from [3]. The four-pole linear phase filter has a passband from 920 to 975 MHz and can be synthesized by coupling matrix as

$$\overline{\overline{m}} = \begin{bmatrix} 0 & 0.9371 & 0 & 0.1953 \\ 0.9371 & 0 & 0.6196 & 0 \\ 0 & 0.6196 & 0 & 0.9371 \\ 0.1953 & 0 & 0.9371 & 0 \end{bmatrix}. \quad (23)$$

The poles and zeros are solved using contour integration method with Belevitch determinant. The poles and zeros are plotted on the complex s plane in Fig. 4, marked by “x” and “o”, respectively. It can be observed that four poles are visible within the passband. Moreover, a complex zero is also present.

Fig. 4. Poles and zeros plot of linear phase filter.

Fig. 5. Poles and zeros plot of linear phase filter cascaded with parallel transmission lines and lumped elements.

This further shows the effectiveness of our proposed method in retrieving both poles and zeros on the complex plane. For verification, the poles are also calculated using the eigenvalue approach. To that end, p can be solved from the eigenvalues of matrix comprising (23) in (11). Then the poles are obtained via (12) and it is found that they are consistent with those shown in Fig. 4.

Consider next the linear phase filter of (23) being cascaded with the parallel transmission lines and lumped elements in Fig. 1(b). The capacitances are set as $C_1 = 1$ pF and $C_2 = 2$ pF respectively, while $\theta_1 = \pi/2$ and $\theta_2 = 3.6514\pi$. This cascaded filter could no longer be analyzed by the transform method or the eigenvalue approach as it now consists of transmission lines and lumped elements. Using the Belevitch theorem and contour integration method, we are still able to determine the poles and zeros of the cascaded filter as

TABLE I
ROOTS OBTAINED USING MULLER'S METHOD WITH DIFFERENT INITIAL GUESSES. (NORMALIZED TO 1 GHz)

Example	Initial Guesses	Converged Root	Remark
Fig. 4	$s_0 = 2\pi(-0.1 + j1)$, $s_1 = 2\pi(-0.09 + j1)$, $s_2 = 2\pi(-0.1 + j0.99)$	None	Failed to converge
	$s_0 = 2\pi(-0.05 + j0.95)$, $s_1 = 2\pi(-0.04 + j0.95)$, $s_2 = 2\pi(-0.05 + j0.94)$	$2\pi(-0.04262 + j0.94614)$ (Zero)	Within range
Fig. 5	$s_0 = 2\pi(-0.05 + j0.6)$, $s_1 = 2\pi(-0.04 + j0.6)$, $s_2 = 2\pi(-0.05 + j0.61)$	$2\pi(3.77645 + j3.26225)$	Out of range
	$s_0 = 2\pi(-0.03 + j0.9)$, $s_1 = 2\pi(-0.02 + j0.9)$, $s_2 = 2\pi(-0.03 + j0.91)$	$2\pi(3.84705 + j7.25760)$	Out of range
	$s_0 = 2\pi(-0.02 + j0.7)$, $s_1 = 2\pi(-0.01 + j0.7)$, $s_2 = 2\pi(-0.02 + j0.71)$	$2\pi(-0.02664 + j0.71770)$ (Zero)	Within range

shown in Fig. 5. It is observed that there are additional poles and complex zeros introduced due to the cascaded parallel transmission lines and capacitors. This example has shown the capabilities of the proposed method in solving poles and zeros of filters with transmission lines and lumped elements.

The contour integration method is useful to identify the region that is certain to contain one or more zeros/poles. Once identified, one may use various efficient methods such as Muller's method to locate the roots for zeros/poles. Although being more efficient, the roots may at times go out of range or may not even converge as exemplified for some cases in Table I. To demonstrate this, the zeros of filters in Figs. 4 and 5 are searched within the complex s plane using the Muller's method with different initial guesses. From the table, we can see that if the initial guesses are not sufficiently close to the zeros, the roots may go out of range, e.g. converging to the RHP zeros or other harmonic band zeros, or may even fail to converge. For sufficiently close initial guesses, the roots eventually converge within range and they agree well to the zeros in Figs. 4 and 5. Hence, this underlines the importance of identifying the correct zeros/poles region for sufficiently close initial guess, which could be provided for by the contour integration method.

We next proceed to analyze a realized dual band filter that we have designed, fabricated and measured as in [24]. In the design therein, the poles of each passband are properly distributed on the complex s plane to provide different passband responses that meet different specifications of each band. In our case, it is a Butterworth response in the first passband (lower band) and a Chebyshev response in the second passband (higher band). Fig. 6 inset shows the photo of the fabricated filter, realized by non-commensurate transmission lines and stubs. The simulation and measurement of the S parameters have been performed and found to be in good agreement, c.f. [24, Fig. 7]. Actually our main concern here is not only the S parameters but also the poles and zeros, as well as whether they have been conforming to our specifications (e.g., different passband Butterworth/Chebyshev responses). With the presence of non-commensurate transmission lines and stubs, it is difficult to analyze such filter for its poles using existing methods. Using Belevitch theorem and contour

Fig. 6. Poles plot of dual band filter in [24]. Inset shows the photo of the fabricated filter.

integration method, the poles can be solved and plotted in Fig. 6, showing three poles present around each passband for third order responses. To further verify each passband response, the reference Butterworth circle (first passband) and Chebyshev ellipse (second passband) are also drawn on the complex s plane based on specification. It can be seen that the poles of each passband lie quite closely on the reference circle and ellipse, which further ascertain each passband response. To the best of our knowledge, this should be the first time that the poles of such filter synthesized with transmission lines have been analyzed and compared against the reference circle/ellipse on the complex s plane during actual TL synthesis (although many might have done so using only lumped LC elements or commensurate lines via existing transform method or eigenvalue approach, etc.). Such analysis and comparison have been made convenient by using our Belevitch determinant contour integration method to retrieve all poles and zeros on the complex s plane.

We shall analyze another realized dual band filter from [25]. The dual band filter is designed as third-order filter and realized by both non-commensurate transmission lines and lumped elements, c.f., [25, Fig. 9]. For such filter or even more complex designs with multiple branches and cross connections, the corresponding S parameters may be calculated numerically using network analysis techniques involving various network parameters. The poles are subsequently solved and plotted in Fig. 7 within the one-octave-width spectral range of 0.6–

Fig. 7. Poles and zeros plot of dual band filter in [25].

Fig. 8. S parameters of dual band filter in [25].

1.2 GHz. It can be observed that there are three poles around each passband. There are also other poles and zeros present outside these passbands. Utilizing the poles and zeros, the S_{21} is constructed and plotted in Fig. 8. For comparison, the simulated S parameters are also shown, along with the measurement data. It is evident that the S_{21} constructed from poles and zeros agree well with the simulated and measured data, which further verifies our proposed method. For the present filter that consists of transmission lines and lumped elements, the existing methods including transform method and eigenvalue approach face difficulties and deficiencies. On the other hand, our Belevitch determinant contour integration method is still effective to determine all the poles and zeros with certainty within certain frequency band of interest.

V. CONCLUSION

This paper has presented the application of Belevitch theorem for pole-zero analysis of microwave filters synthesized with transmission lines and lumped elements. For such filters, the S parameters expressions comprise combinations of various integer power terms and trigonometric power terms, which render the existing methods such as the transform method and the eigenvalue approach inapplicable. Hence, the Belevitch determinant based on Belevitch theorem has been proposed to separate the poles and zeros into different half-plane regions. Using the Belevitch determinant, the contour integration method based on argument principle has been applied to determine the poles and zeros of filter transfer functions separately and with certainty. Several applications have demonstrated the capabilities of the proposed method to effectively retrieve the poles and zeros of various microwave filters on the complex plane.

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