Essays on innovation, central bank digital currency and asset pricing

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**NANYANG
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**ESSAYS ON INNOVATION,
CENTRAL BANK DIGITAL
CURRENCY AND ASSET PRICING**

AMMU GEORGE

SCHOOL OF SOCIAL SCIENCES

2019

**ESSAYS ON INNOVATION,
CENTRAL BANK DIGITAL
CURRENCY AND ASSET PRICING**

AMMU GEORGE

School of Social Sciences

A thesis submitted to the Nanyang Technological University
in partial fulfilment of the requirement for the degree of

Doctor of Philosophy

2019

Statement of originality

I certify that all work submitted for this thesis is my original work. I declare that no other person's work has been used without due acknowledgment. Except where it is clearly stated that I have used some of this material elsewhere, this work has not been presented by me for assessment in any other institution or University. I certify that the data collected for this project are authentic and the investigations were conducted in accordance with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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I have reviewed the content of this thesis and to the best of my knowledge, it does not contain plagiarised materials. The presentation style is also consistent with what is expected of the degree awarded. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accordance with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

17/ July/19
Date

Joseph Dennis Alba

Authorship Attribution Statement

This thesis contains material from one paper accepted at conferences in which I am listed as an author.

Part of the content of Chapter 3 was presented as Ammu George. “Asset Prices, Growth and Wage Inertia” at *NTU-Nottingham Joint Conference on International Economics and Finance* (Singapore) in March 2018, *25th Annual Meeting of the German Finance Association* (Trier, Germany) in September 2018 and *European Economics and Finance Society 18th Annual Conference* (Genoa, Italy) in June 2019.

06- February-2020

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Date

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Ammu George

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Executive Summary

Globally, economies are preparing for a future where innovations accelerate. Governments are vying to harness new technologies and developing financial innovations to drive progress. Against such a backdrop, this thesis comprises of three independent essays on institutions incorporating innovation and the resulting impact on macroeconomic and financial stability. The first two essays pertains to financial innovation by a central bank who replaces the cash in circulation with a central bank issued digital currency. Several countries like China, France, Sweden etc. have already taken concrete steps or announced their intentions to launch a CBDC in the future (Desouza and Ye, 2019). In this context, the first two essays studies the welfare and macroeconomic implications of the central banks using a financial innovation like CBDC as a secondary monetary policy instrument. The third and final essay deviates from financial innovation to a framework of technological innovation where firms create new patents through Research and Development (R&D). According to World Economic Forum, South Korea and Sweden are the most innovative countries in the world as they filed the highest number of patents in 2017.

The distributed ledger technology has sparked the interests in policy makers to consider a digital replacement of physical cash - the central bank digital currency (CBDC). Theories suggest that CBDC facilitates an interest-bearing design that complements existing monetary policy framework, but in reality cash has never been associated with an adjustable return¹. Chapter 1 tries to

¹Adjustable return or adjustable interest rate refers to letting the interest rate on money

bridge this gap by examining the economic consequences of an interest-bearing design of CBDC, and extend the discussion to an open-economy context with trade and capital flows. Through the lens of a dynamic stochastic general equilibrium (DSGE) model, the study simulate a baseline scenario which resembles a cash economy, and two counter-factual scenarios associated with interest-bearing CBDC — the price rule and the quantity rule regimes. The simulations show that 1) the price rule regime is welfare-improving; 2) the adjustable interest rate on CBDC causes uneven distributional effects between households and financial investors; and 3) macroeconomic stability is enhanced with the adjustable interest rate.

In the context of declining cash in circulation, emerging market economies find CBDC as a potential solution to improve financial inclusion. Economies like Uruguay, Thailand, South Africa etc. are actively involved in pilot projects to implement CBDC. In this context, chapter 2 investigates the macroeconomic consequences of implementing an interest-bearing CBDC in the monetary policy framework of an emerging market economy. The model setup in chapter 2 deviates from chapter 1 in terms of; a) households buy both domestic and imported consumption goods using CBDC ² b) firms use labour alone as the factor of production³ c) central bank use CBDC interest rate (price rule⁴) to target CPI inflation instead of domestic price inflation⁵ d) central bank en-

adjust to equate the demand and supply of money in the market

²Households use CBDC to purchase only domestic consumer goods in chapter 1.

³In chapter 1, firms use imported capital in addition to labour to produce final goods. As chapter 2 framework incorporates imported consumption goods (to see the impact of CBDC on CPI inflation), we do not include imported capital for the ease of identifying the transmission mechanism.

⁴We do not consider quantity rule framework based on the Chapter 1 finding that quantity rule does not improve economy welfare.

⁵Domestic price inflation captures the change in domestic goods price. Consumer price

gages in sterilised foreign exchange (FX) intervention to target exchange rate. The study simulate a baseline cash economy, a cash economy with FX intervention and a CBDC economy with FX intervention. The simulations show that 1) interest-bearing CBDC is welfare improving for an emerging market economy; 2) the adjustable interest rate on CBDC causes uneven distributional effects between households and financial investors; and 3) exchange rate management becomes even more effective when interest-bearing CBDC interacts with sterilised FX intervention.

Chapter 3 deviates from the framework of CBDC to a production based asset pricing model where innovation is driven by firms through R&D. Chapter 3 examines the implications of wage inertia on macroeconomic aggregates and asset prices in a production economy where growth is endogenously determined by innovation as developed by Kung and Schmid (2015). Wage inertia such as wage stickiness with endogenous labour supply (Uhlig, 2007; Donadelli and Grüning, 2016) and search and matching models with Nash and with alternative offer bargaining (AOB) (Christiano et al., 2016) are incorporated in the Kung and Schmid (KS) model. Following a positive productivity shock, the AOB model shows a) labour market and asset pricing moments are closer to the data compared to other types of wage inertia models and the benchmark KS model b) the impulse response of dividends is larger – and closer to the data - under AOB compared to the other models since wages rise the least, as labour, vacancies and unemployment are more responsive following a positive productivity shock.

index (CPI) is an index of both domestic and foreign good price. CPI inflation corresponds to change in CPI.

Chapter 1

Central Bank Digital Currency with Adjustable Interest Rate in Small Open Economies

1.1 Introduction

Digitisation of banking since the last decade has led to reduced usage of notes and coins for the purpose of transactions (see Figure 1.1). Whether central banks should ride the wave by digitising its money, commonly known as the central bank digital currency (CBDC), has become an increasingly popular topic of discussion.

The digitisation of modern money through electronic records has been in existence for some time. Electronic access to deposits or cashless transaction through debit cards, credit cards, internet banking etc has also been prevalent in the economies for some time. CBDC differs from the aforementioned electronic money through distributed ledger technology(DLT) that constitutes

the infrastructure facilitating payments in CBDC. With DLT, payment system would be decentralised with the central bank's ledger distributed among all the holders of CBDC. The existing form of electronic money (debit card, credit card, online banking etc) in comparison has centralised payment systems where payments between agents of the same bank are settled at the bank's ledger, payments between agents of different banks at a common clearing bank's ledger and payments between agents of separate clearing bank require settlement at the central bank. Advocates of DLT propose that a decentralised ledger system would improve payment settlement efficiency (Desouza and Ye, 2019). The marginal cost of operating the payment system in developed economies would be lower. Hence, CBDC could provide more technological efficiency gains. The proposed DLT framework has its origins in the distributed payment system of cryptocurrency (e.g. bitcoin). Cryptocurrency is denominated in its own unit of account, privately created, and is issued on a blockchain. Cryptocurrency is considered to be very risky as its value in fiat currency can fluctuate significantly (Adrian and Mancini-Griffoli, 2019). In contrast, CBDC is deemed to be stable as a store of value as it is fully backed by the government. This makes CBDC as a viable means of payment for economic transactions.

In a working paper released by Bank of England, Barrdear and Kumhof (2016) define CBDC as:

... a universally accessible and interest-bearing central bank liability, implemented via distributed ledgers, that competes with bank deposits as medium of exchange...

Of the features named in this definition, being a universally accessible central bank liability, and a competing medium of exchange with bank deposits, is nothing different from the existing definition of narrow money, or cash, issued

by the central bank. The interest-bearing feature is the key economic innovation in central bank money. Kumhof and Noone (2018) highlight that the CBDC needs to bear an “adjustable interest rate”, among other design principles, so as to serve as a secondary monetary tool. However, the extent to which an adjustable interest on central bank money will improve the policy outcomes remains unclear.

The present study aims to assess the macroeconomic implications of a CBDC with an adjustable interest rate, particularly in the context of an economy open to trade and capital flows. Although the emergence of private digital currency as an alternative unit of account (e.g. bitcoin) is a motivation for our study, our model does not deal with currency competition. We focus only on government issued unit of account (CBDC) and deposits. Privately created digital currency does not exist in our model. To this end, we propose a DSGE framework with CBDC and bank deposits¹, bearing differentiated interests, as competing media of exchange. In this framework, the central bank coordinates among fiscal, monetary and CBDC tools to achieve policy objectives. An exogenous foreign sector that interacts with the model economy’s consumption-goods and bonds markets is introduced, so that uncertainties originated from outside the country are considered in designing monetary policies. This model allows us to easily toggle between a CBDC with a constant return, e.g. 0 per cent, and one with an adjustable interest rate following either a price rule that specifies the response of the nominal CBDC interest rate, or a quantity rule that specifies the response of the quantity of CBDC supply to macroeconomic conditions. Through simulations, we show that welfare of the economy improves when the

¹Deposits include electronic form of money in terms of credit card, debit card, online banking etc.

central bank adjusts the CBDC interest rate using a price rule in comparison to an economy where the central bank keeps the CBDC interest rate fixed.

Our work contributes to the emerging literature of CBDC in three aspects. First, we are the first to formally examine CBDC in an open economy's context. The case of CBDC in an open economy is more complicated as compared to a standard closed economy such as Barrdear and Kumhof (2016). Issues such as granting access to foreign entities (Bank for International Settlements, 2018), and the transmission channel through exchange rate (Meaning et al., 2018), all need to be carefully examined prior to actual implementation of CBDC. The design principles set out in Kumhof and Noone (2018) also disregard the exchange rate regimes. To our best knowledge, most existing discussions of CBDC with reference to open economy are qualitative. Our study therefore contributes to the literature by discussing CBDC in a framework with an exogenous foreign country. The benefits of such a framework includes understanding the impact of CBDC regimes on exchange rate stability and to check whether CBDC could be an effective monetary policy instrument for price stability in the presence of small open economy shocks like foreign interest rate shock and foreign demand shock. However, we do not discuss cross border flows of CBDC in our paper. Central banks are collaborating with each other to experiment cross-border payment based on DLT. Collaborations include Project Stella by the ECB and the Bank of Japan, joint project by the Bank of Canada, the Monetary Authority of Singapore and the Bank of England. However, incorporating cross border CBDC usage in our model would entail more complicated dynamics via the exchange rate channel. Hence, we abstract from cross border CBDC payments.

Second, we assess the welfare implications of CBDC policy regimes. One benefit of the DSGE framework is that it enables assessment of societal welfare

due to alternative, counter-factual policies. Barrdear and Kumhof (2016) simulate the counter-factual scenarios but do not provide a welfare assessment. In this paper, we simulate different combinations of conventional, and futuristic CBDC monetary policies. We provide both the welfare implications and the stabilising effects of these policy options.

Third, we assess in an unified framework the dynamic effects of CBDCs with adjustable interest and the constant or zero-interest CBDCs on other assets such as bank deposits, bank loans and government securities. Interest-bearing CBDCs have advantages in enhancing price stability (Bordo and Levin, 2017), and enabling negative interest rate (Rogoff, 2014). In addition, CBDCs also offer transaction efficiency compared with bank deposits. The attractiveness of CBDCs vis-à-vis bank deposits may have implications on the stability of the financial system and on the overall economy (Bank for International Settlements, 2018).

The main findings of the paper are as follows. First, welfare of the society improves when the central bank follows an adjustable CBDC interest rate system using the price rule. However, we find the welfare of society to decline when the central bank uses the quantity rule instead of the price rule. Second, we find distributional consequences associated with adjustable CBDC interest rates. Although the welfare of households (who form 95% of the population) improves with the price rule, the welfare of the minority population consisting of financial investors declines when CBDC interest rate system is adjustable (both the price rule and the quantity rule). It is also of some interest that the rate of financial investor's welfare decline is relatively less if the central bank follows the price rule. Third, macroeconomic variables like GDP, consumption, exchange rate and inflation are found to be more stable in an environment of

exogenous shocks when the CBDC interest rate is adjustable under the price rule regime. In addition, financial variables like domestic deposits and lending are also found to be more stable under the adjustable CBDC interest rate system under the price rule regime when the economy faces foreign shocks.

The remainder of the paper is organised as follows. Section 1.2 explains the transmission mechanism of CBDC monetary policies. Section 1.3 provides detailed descriptions of the model with CBDC. Section 1.4 discusses the results from our simulation exercises. Section 1.5 concludes.

1.2 Economics of CBDC monetary policies

The concept of an adjustable interest rate on central bank money has not been thoroughly explored, though it is not new. Woodford (2003) describes a policy framework with a set of three monetary instruments in Section 3.2 of Chapter 2 of his seminal book. This set of policy instruments includes the bond interest rate, the supply of monetary assets, and the interest rate of monetary assets. The money demand, after log-linearisation, is a function of the interest differential between bonds and monetary assets as the following:

$$\hat{m}_t = \eta_y \hat{Y}_t - \eta_i (\hat{i}_t - \hat{i}_t^m) + \epsilon_t \quad (1.1)$$

where $\hat{m}_t$ is real money balance, $\hat{Y}_t$ is output, $\hat{i}_t$ and $\hat{i}_t^m$ are interest rates of bonds and monetary assets, respectively, and η_y and η_i are elasticity parameters. It follows from the above equation that, to determine the money circulated in the economy, the central bank needs to specify both the interest rates on bonds and on monetary assets. Alternatively, the central bank may also specify the bond interest rate and the supply of monetary assets, letting the interest rate

on monetary assets adjust to equate the demand and supply of money. In this scenario, the interest rate on monetary assets is equivalent to an adjustable interest rate on central bank money. However, because the central bank money held by the households have long been in physical form, the interest-bearing feature is costly to be implemented. Hence, the interest rate of monetary assets is often taken to be zero and unchanged.

Technology has advanced to the extent that central bank money may also exist in digital form like bank deposits. It is then feasible for a central bank to introduce an interest rate to households' holdings of central-bank issued digital money. Following the simple framework presented in Equation 1.1, and using the conventional Taylor rule as the mandatory monetary policy to specify $\hat{i}_t$, the central bank may adopt a secondary monetary policy which can be either a price-based one that specifies the interest rate on monetary assets, or a quantity-based one that specifies the supply of monetary assets. The last unspecified monetary policy instrument endogenously adjusts to clear the money market.

It is important to note that the money demand is associated differently with the two interest rates. Because the interest differential between bonds and monetary assets measures the opportunity cost in holding the latter, an increased demand for monetary assets is associated with a lower interest on bonds and/or a higher interest on monetary assets. As a result, to use either the price-based or the quantity-based measure as a secondary monetary policy tool to enhance the Taylor rule, the interest on monetary assets or the money supply entails a counter-cyclical component against the bond interest rate.

In Barrdear and Kumhof (2016), a price rule and a quantity rule are introduced. The price rule specifies the response of the interest rate on CBDC to inflation. It has two components. While one of it moves with the bond inter-

est rate, the other component responds counter-cyclically to inflation: In times of high (low) inflation, the interest on CBDC decreases (increases) relative to bond interest rate, reducing (increasing) the attractiveness of holding money, hence withdrawing (injecting) liquidity from (into) the economy. Similarly, in a quantity rule, money supply is increased in times of lower inflation. The interest on monetary assets adjust upwards to clear the money market. We will analyse these two policies in Section 1.3.

In sum, CBDC serves as secondary tools to enhance existing monetary policy framework. It provides the central bank with greater flexibility in responding to macroeconomic conditions. Therefore, analysing the outcomes of these secondary tools delivers important policy implications for central banks in the digital era. In the next section, we describe a framework for analysing the CBDC monetary policies in a small open economy model.

1.3 Model

Our model features a small-open economy with deposits and CBDC being competing media of exchange. The domestic economy is a simplified variant of the Barrdear and Kumhof (2016) model. It retains the consumption loans in Barrdear and Kumhof (2016) as the means of credit creation in the economy. Without loss of generality, and for ease of identifying the transmission mechanism, we abstract from Barrdear and Kumhof (2016) the real-estate and capital-goods sectors. The domestic economy is small relative to the rest of the world in the sense that any disturbance in the domestic economy is negligible to the world, but not vice versa.

Figure 1.2 depicts the flows of funds among the sectors. As in Barrdear and

Kumhof (2016), the model consists of households, financial investors, banks, firms, a central bank (government), and the world economy. Two types of agents live in an extended family, in which a fraction $1 - \omega$ are the financially constrained households, and a fraction ω are the financially unconstrained financial investors. Both agents work in firms which produce consumption goods sold in both the domestic and global markets. They receive wages as reward for their labour.

The banks are relevant for credit creation. Banks are not financial intermediaries who collect deposits from savers and lends the savings to borrowers. Instead, banks finance the creation of money by creating deposits through the act of lending. Bank deposits are created as households borrow for the sole objective of generating sufficient deposits to finance their consumption transactions. This means that households simultaneously borrow from and deposit with banks, which makes them financially constrained. Financial investors are financially unconstrained as they do not borrow from banks.

The assets held by households are CBDC and deposits whereas the asset portfolio of financial investors comprise of bonds and deposits. Financial investors as an asset holding group helps to create an arbitrage condition between deposits and government bonds. Among the three types of assets, deposits and CBDCs provide liquidity and allow agents to use them for daily transactions. Bonds are relatively illiquid and are kept as a store of value.

The government issues CBDCs in exchange for government bonds through open market operations. Government bonds are sold to (bought from) the public in exchange for CBDC when there is a need to reduce (increase) liquidity in the economy.

The interactions between the domestic and world economies are via two

channels. Firstly, firms import intermediate capital goods as a production input, and sell final consumption goods to the world economy as exports. Secondly, financial investors borrow from the world economy by issuing foreign-currency denominated bonds (foreign debt) and pay the prevailing foreign interests.

In the remainder of this section, we describe the details of the model. All the variables given in the section below (in small letters) are in *real terms* unless specified otherwise.

1.3.1 Banks

Banks follow the financing model of money creation where deposits are created as an output when banks make loans to households. Households use the loans to finance their consumption needs. The ‘new money’ through loans are credited to the households’ deposit accounts. Banks are understood to face different exposure to non-credit risks, and are indexed by i .

The per-capita loan stocks held by households from bank i during period t to $t+1$ are given by $\ell_t^c(i)$ with the total loan stocks of bank i denoted by $\ell_t^\ell(i)$ where $\ell_t^\ell(i) = (1 - \omega)\ell_t^c(i)$. The bank’s real gross wholesale lending rates (including premium on cost of regulation) and real retail lending rates (including premium on credit risk) are given by r_t^ℓ and r_t^r , respectively. The bank can incur loan losses which are given by $\mathcal{L}_t^b(i) = (1 - \omega)\mathcal{L}_t^c(i)$.

Both households and financial investors hold deposits with banks. The per capita deposit stocks of households and financial investors in bank i are denoted by $d_t^c(i)$ and $d_t^u(i)$, respectively. Total deposits in the bank i are given by $d_t(i) = (1 - \omega)d_t^c(i) + \omega d_t^u(i)$. The bank’s ex-post real deposit rate is denoted by r_t^d .

The balance sheet of bank i is

$$\ell_t^\ell(i) = d_t(i) + n_t^b(i) \quad (1.2)$$

where $n_t^b(i)$ refers to net worth of the bank. Net worth corresponds to the difference between gross return on loans ($r_{t-1}^\ell \ell_{t-1}^\ell(i)$) and expenses on deposits interest payments ($r_{d,t-1} d_{t-1}(i)$), on loan losses ($\mathcal{L}_{t+1}^b(i)$), on ex-post cost of penalty ($\mathcal{M}_t^b(i)$) and dividends ($\delta^b n_t^b(i)$)². Hence, net worth is expressed as:

$$n_t^b(i) = r_{t-1}^\ell \ell_{t-1}^\ell(i) - r_{d,t-1} d_{t-1}(i) - \mathcal{L}_t^b(i) - \delta^b n_t^b(i) - \mathcal{M}_t^b(i) \quad (1.3)$$

Bank i incurs the ex-post cost of penalty, $\mathcal{M}_t^b(i)$, only if the pre-dividend and pre-penalty net worth ($r_{t-1}^\ell \ell_{t-1}^\ell(i) - r_{d,t-1} d_{t-1}(i) - \mathcal{L}_t^b(i)$) falls below the minimum capital adequacy ratio (MCAR), which is a fraction (Υ) of the gross returns on the bank's loan book weighted for solvency risk. Should bank i fail to meet the MCAR, government regulations mandate that it pays a penalty as portion, χ , of its total loans ($\ell_t^\ell(i)$) at $t + 1$.

The solvency risks of banks are incorporated into the model since the gross loan returns of banks are subject to an idiosyncratic shock $\ln(\omega_{t+1}^b) \sim N(0, (\sigma^b)^2)$. $f_t^b(\omega_{t+1}^b) = f_{t+1}^b$ and $F_t^b(\omega_{t+1}^b) = F_{t+1}^b$ corresponds to the probability density function and cumulative density function, respectively of ω_{t+1}^b . Since there is a continuum of i banks in the model, such a shock makes the banks heterogeneous in their loan book returns. This idiosyncratic risk pertains to a number of individual factors that are not directly related to standard banking operation of

²In line with Barrdear and Kumhof (2016), we also assume that banks follow a fixed dividend policy where households and financial investors receive part of bankers' net worth at an exogenously fixed rate. This is based on the "extended family approach" of Gertler and Karadi (2011).

lending. Examples of such bank factors include different success rates of building non-interest income and reducing non-interest expenses. The aggregate of these individual factors over all banks equal zero.

The cut-off condition in which bank i would pay a penalty for falling below MCAR is given by

$$r_t^\ell \ell_t^\ell(i) \omega_{t+1}^b - r_{d,t} d_t(i) - \mathcal{L}_{t+1}^b(i) < \Upsilon \zeta^c r_t^\ell \ell_t^\ell(i) \omega_{t+1}^b \quad (1.4)$$

where ζ^c is the risk weight associated with consumer loans. The realized cutoff idiosyncratic shock $(\bar{\omega}_t^b)$ below which the bank has to pay penalty is given by

$$\bar{\omega}_t^b = \frac{r_{d,t-1} d_{t-1}(i) + \mathcal{L}_t^b(i)}{(1 - \Upsilon \zeta^c) r_{t-1}^\ell \ell_{t-1}^\ell(i)} \quad (1.5)$$

The (ex-post) penalty if paid by the bank is given by $\mathcal{M}_t^b(i) = \chi \ell_t^\ell(i) F_t^b(\bar{\omega}_{t+1}^b)$, noting that $E_t(\mathcal{L}_{t+1}^b(i)) = 0$.

The objective of the bank is to maximize its pre-dividend net worth given by

$$\max_{\ell_t^\ell(i), d_t(i)} E_t \left\{ r_t^\ell \ell_t^\ell(i) \omega_{t+1}^b - r_{d,t} d_t(i) - \mathcal{L}_{t+1}^b(i) - \chi \ell_t^\ell(i) F_t^b(\bar{\omega}_{t+1}^b) \right\} \quad (1.6)$$

Substituting Equation 1.2 to Equation 1.6, and then taking the first order condition with respect to $\ell_t^\ell(i)$, we get

$$0 = E_t \left\{ r_t^\ell - r_{d,t} - \chi F_t^b(\bar{\omega}_{t+1}^b) - \chi f_t^b(\bar{\omega}_{t+1}^b) \frac{r_{d,t}}{r_t^\ell} \frac{\ell_t^\ell(i)}{n_t^b(i)} \frac{1}{(1 - \Upsilon \zeta^c) \left[\frac{\ell_t^\ell(i)}{n_t^b(i)} \right]^2} \right\} \quad (1.7)$$

1.3.2 Households

The utility function of the household i depends on v , which determines the degree of habit formation of lagged aggregate per capita consumption c_{t-1}^c in preferences. The consumption of household i is given by $c_t^c(i)$ with θ_p the elasticity of substitution among j varieties of Dixit Stiglitz consumption aggregates given by $c_t^c(i) = \left[\int_0^1 c_t^c(i, j)^{\frac{\theta_p-1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p-1}}$. The utility function also comprises of hours worked $h_t^c(i)$ with η as labor supply elasticity. ψ_h refers to the weight of hours worked in the utility function. β_c is the discount factor. The household maximizes its lifetime utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta_c^t \left\{ (1-v) \ln (c_t^c(i) - v c_{t-1}^c) - \psi_h \frac{h_t^c(i)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right\} \quad (1.8)$$

Subject to its budget constraint:

$$\begin{aligned} & c_t^c(i)(1 + \tau_{c,t})(1 + s_t^c(i)) + (d_t^c(i) + m_t^c(i)) (1 + \phi_b(b_t^{rat} - \bar{b}^{rat})) - \Psi_{c,t}(i) \\ & - \ell_t^c(i) \left(1 - \frac{\varphi_c}{2} (\ell_t^c(i) - \ell_{t-1}^c(i))^2 \right) = r_{d,t-1} d_{t-1}^c(i) + r_{m,t-1} m_{t-1}^c(i) \\ & + \Pi_t(i) + w_t^{hh} h_t^c(i)(1 - \tau_{L,t}) + \frac{\iota}{1 - \omega} \Omega_t(i) - \Gamma_t Col_{t-1}(i) \end{aligned} \quad (1.9)$$

From Equation 1.9, we can see that household i pays taxes on consumption, $\tau_{c,t}$, and incurs monetary transaction cost, $s_t^c(i)$, while spending on consumption, $c_t^c(i)$. It holds bank deposits, $d_t^c(i)$, which has a real return, $r_{d,t}$. The household also holds CBDC, $m_t^c(i)$, with real return, $r_{m,t}$. In holding financial assets, the household incurs financial transaction costs, $\phi_b(b_t^{rat} - \bar{b}^{rat})$, where b_t^{rat} is government debt-to-GDP ratio and $\bar{b}^{rat}$ is its steady state value³. These financial transaction costs are treated as exogenous and rebated back to the household as

³Barrdear and Kumhof (2016) note that empirical literature finds the level of government debt to be positively related to the equilibrium real interest rates. In addition, Schmitt-

lump-sum transfer, $\Psi_{c,t}(i)$. The household receives loans from the bank, $\ell_t^c(i)$, subject to quadratic adjustment costs, $(1 - \frac{\varphi_c}{2}(\ell_t^c(i) - \ell_{t-1}^c(i))^2)$, which impose a penalty on rapid changes in the level of loans.

Household income in the budget constraint comes from the following sources: returns from financial assets $(r_{d,t-1}d_{t-1}^c(i) + r_{m,t-1}m_{t-1}^c(i))$; profits from firms which the household owns $(\Pi_t(i))$; after tax wages from hours worked $(1 - \tau_{L,t})w_t^{hh}h_t^c(i)$, where $\tau_{L,t}$ is the labour income tax, w_t^{hh} is real wage and $h_t^c(i)$ is hours of work; and a share (ι) of the real lump-sum income, $\Omega_t(i)$, accruing to the household. $\Gamma_t Col_{t-1}(i)$ in the budget constraint is the fraction (Γ_t) of the collateral value $(Col_{t-1}(i))$ paid by household to the bank to cover the loan interest payments.

Real lump-sum income (Ω_t) consists of dividends from banks $(\delta^b n_t^b)$, the lump-sum component (ς) of the monitoring costs $(\mathcal{M}_t)$ and the aggregate monetary transactions costs $(\mathcal{T}_t)$.

$$\Omega_t = \delta^b n_t^b + (1 - \varsigma)(\mathcal{M}_t + \mathcal{T}_t) \quad (1.10)$$

where $\mathcal{T}_t = (1 - \omega)c_t^c(1 + \tau_{c,t})s_t^c$

Monetary transaction costs of household i , $s_t^c(i)$, follow the specification of Schmitt-Grohé and Uribe (2004) expressed as

$$s_t^c(i) = A_c v_t^c(i) + \frac{B_c}{v_t^c(i)} - 2\sqrt{A_c B_c} \quad (1.11)$$

Grohé and Uribe (2004) among others, assume that transaction cost is a quadratic function of government debt-to-output ratio.

$v_t^c(i)$ is velocity and given by

$$v_t^c(i) = \frac{c_t^c(i)(1 + \tau_{c,t})}{f_t^c(i)} \quad (1.12)$$

$f_t^c(i)$ refers to liquidity generating function of the household and given by

$$f_t^c(i) = (d_t^c(i))^\theta + (T^{fintech} m_t^c(i))^\theta \quad (1.13)$$

where $\frac{1}{1-\theta}$ is the elasticity of substitution between CBDC and deposits. $T^{fintech}$ is a relative productivity coefficient for CBDC which makes monetary transactions more efficient while using CBDC. Hence, the household will hold CBDCs even if the CBDC interest rate is lower than the bank deposit interest rate.

Collateral pledged by the household to secure loans, $Col_t(i)$, in Equation 1.9 includes returns on financial assets such as deposits ($r_{d,t}d_t^c(i)$) and CBDC ($r_{m,t}m_t^c(i)$) in addition to real returns of annualized current labour income ($r_{n,t}4w_t^{hh}h_t^c(i)(1 - \tau_{L,t})$). $r_{n,t}$ is the return on nominal cash flow and equal to $\frac{1}{\pi_t}$. Hence,

$$Col_t(i) = \kappa_t^r \kappa_t^f (r_{d,t}d_t^c(i) + r_{m,t}m_t^c(i)) + \kappa_t^r r_{n,t}4w_t^{hh}h_t^c(i)(1 - \tau_{L,t}) \quad (1.14)$$

where κ_t^r is the proportion of pledged real collateral and $\kappa_t^f \kappa_t^r$ is the proportion of pledged financial collateral.

The collateral value given in Equation 1.14 is subject to an idiosyncratic shock which changes $Col_t(i)$ to $\omega_{t+1}^c Col_t(i)$ in beginning of period $t + 1$, where $\ln(\omega_{t+1}^c) \sim N(0, (\sigma^c)^2)$. The density function and cumulative density function of ω_{t+1}^c are $f_{t+1}^c = f_t^c(\bar{\omega}_{t+1}^c)$ and $F_{t+1}^c = F_t^c(\bar{\omega}_{t+1}^c)$, respectively.

According to a standard debt contract, the household receives the loan

amount $\ell_t^c(i)$ at a gross nominal retail interest of i_t^r . The nominal retail interest rate i_t^r is pre-committed at t and not determined at $t + 1$. This makes expected profits zero for banks but the realized ex post profits are different from zero.

Borrowers who experience ω_{t+1}^c below a cut off level $\bar{\omega}_{t+1}^c$ are unable to pay the committed retail interest rate and enter into bankruptcy. All the assets of such borrowers must be handed over to the bank. However, the bank incurs monitoring costs on their lending activity. Hence, the bank is able to recover only a fraction $(1 - \xi)$ of the collateral value. The bank's ex ante zero profit condition is given by

$$E_t \left\{ \left[(1 - F_t(\bar{\omega}_{t+1}^c)) r_t^r \ell_t^c(i) + (1 - \xi) \int_0^{\bar{\omega}_{t+1}^c} Col_t(i) \omega^c f_t^c(\omega^c) d\omega^c \right] - r_t^\ell \ell_t^c(i) \right\} = 0 \quad (1.15)$$

The terms in square brackets refer to the total payoff from lending: the gross retail interest payments of borrowers on their loans if $\omega_{t+1}^c \geq \bar{\omega}_{t+1}^c$; and the recovery from collateral assets if $\omega_{t+1}^c < \bar{\omega}_{t+1}^c$. $\bar{\omega}_t^c$ is determined by equating the gross interest payments (if borrower continues to operate) to the gross returns on collateral assets (if the borrower becomes bankrupt).

$$\bar{\omega}_t^c = \frac{r_{t-1}^r \ell_{t-1}^c(i)}{Col_{t-1}(i)} \quad (1.16)$$

We can rewrite Equation 1.15 as

$$0 = E_t \{ Col_t(i) [\Gamma_{t+1} - \xi G_{t+1}] - r_t^\ell \ell_t^c(i) \} \quad (1.17)$$

where by definition, the bank's gross share in the value of collateral is:

$$\Gamma_{t+1} = \Gamma_t(\bar{\omega}_{t+1}^c) = \bar{\omega}_{t+1}^c \int_{\bar{\omega}_{t+1}^c}^{\infty} f_t(\omega_{t+1}^c) d\omega_{t+1}^c + \int_0^{\bar{\omega}_{t+1}^c} \omega_{t+1}^c f_t(\omega_{t+1}^c) d\omega_{t+1}^c; \text{ and}$$

the proportion of collateral value that the lender has to spend on monitoring costs is:

$$\xi G_{t+1} = \xi G_t(\bar{\omega}_{t+1}^c) = \xi \int_0^{\bar{\omega}_{t+1}^c} \omega_{t+1}^c f_t^c(\omega_{t+1}^c) d\omega_{t+1}^c.$$

The representative household maximizes Equation 1.8 subject to Equation 1.9 and Equation 1.17. After taking into account symmetry across households, we get the first order conditions as follows

$$c_t^c : \frac{(1-v)}{c_t^c - v c_{t-1}^c} = \lambda_{1,t}(1 + \tau_{c,t})(1 + s_t^c + s_t^{cc'} v_t^c) \quad (1.18)$$

where $s_t^{cc'} = A_c \frac{v_t^c}{c_t^c} - \frac{B_c}{c_t^c v_t^c}$

$$\ell_t^c : \lambda_{1,t} \left(1 - \frac{\varphi_c}{2} (\ell_t^c - \ell_{t-1}^c)^2 - \varphi_c \ell_t^c (\ell_t^c - \ell_{t-1}^c) \right) = \beta_c E_t \{ \lambda_{1,t+1} r_t^\ell \lambda_{2,t+1} \} \quad (1.19)$$

where $\lambda_{2,t+1} = \frac{\Gamma'_{t+1}}{(\Gamma'_{t+1} - \xi G'_{t+1})}$ and Γ'_{t+1} and G'_{t+1} are derivatives of Γ_{t+1} and G_{t+1} with respect to $\bar{\omega}_{t+1}^c$.

$$\begin{aligned} h_t^c : \psi_h(h_t^c)^{\frac{1}{\eta}} &= \lambda_{1,t} w_t^{hh} (1 - \tau_{L,t}) \\ &+ \beta_c \lambda_{1,t+1} r_{n,t} 4w_t^{hh} (1 - \tau_{L,t}) (-\kappa^r \Gamma_{t+1} + \lambda_{2,t+1} \kappa^r (\Gamma_{t+1} - \xi G_{t+1})) \end{aligned} \quad (1.20)$$

$$\begin{aligned} d_t^c : \lambda_{1,t} \left(1 + \phi_b(b_t^{rat} - \bar{b}^{rat}) + c_t^c (1 + \tau_{c,t}) s_t^{cd'} \right) &= \\ \beta_c E_t \{ \lambda_{1,t+1} r_{d,t} [(1 - \kappa^r \kappa^f \Gamma_{t+1}) + \lambda_{2,t+1} \kappa^r \kappa^f (\Gamma_{t+1} - \xi G_{t+1})] \} \end{aligned} \quad (1.21)$$

where $s_t^{cd'} = -A_c \frac{v_t^c}{f_t^c} f_t^{cd'} + \frac{B_c}{v_t^c f_t^c} f_t^{cd'}$, $f_t^{cd'} = \theta(d_t^c)^{\theta-1}$

$$m_t^c : \lambda_{1,t} \left(1 + \phi_b(b_t^{rat} - \overline{b^{rat}}) + c_t^c(1 + \tau_{c,t})s_t^{cm'} \right) = \beta_c E_t \left\{ \lambda_{1,t+1} r_{m,t} \left[(1 - \kappa^r \kappa^f \Gamma_{t+1}) + \lambda_{2,t+1} \kappa^r \kappa^f (\Gamma_{t+1} - \xi G_{t+1}) \right] \right\} \quad (1.22)$$

where $s_t^{cm'} = -A_c \frac{v_t^c}{f_t^c} f_t^{cm'} + \frac{B_c}{v_t^c f_t^c} f_t^{cm'}$, $f_t^{cm'} = \theta(T^{fintech} m_t^c)^{\theta-1}$

The monetary frictions feature prominently in the above optimal conditions. Equation 1.18 shows that household expenditures on consumption goods are higher by a "mark up" of $(1 + s_t^c + s_t^{cc'} v_t^c)$ with monetary frictions than without it. Barrdear and Kumhof liken this markup to a "liquidity" tax with a distortionary tax rate, $\tau_{c,t}^{liq}$, expressed as

$$\tau_{c,t}^{liq} = 1 + s_t^c + s_t^{cc'} v_t^c \quad (1.23)$$

They note the likeness between monetary frictions and distortionary taxes on their impact on real economy as shown in the optimal conditions. Furthermore, Equation 1.18 shows the distortions caused by changes in $\tau_{c,t}^{liq}$ and changes in $\tau_{c,t}$. Similarly, if Equation 1.18 is combined with Equation 1.20, the resulting equation expressing the marginal rate of substitution of consumption and labor would show the distortions caused by changes in $\tau_{c,t}^{liq}$ and changes in $\tau_{L,t}$. Hence, due to the monetary frictions, changes in the quantity of bank deposits and in the quantity of CBDCs or in the quantity of monetary transaction balances get transmitted to the real economy.

Foreign sector

As in Galí and Monacelli (2005), the economy with the global city is small, and foreigners are assumed to live in a large economy. Trade flows depend on exports of domestic goods to foreigners (exp_t) and by imports of intermediate goods from foreigners (k_t^*). Capital flows include borrowing by financial investors from the foreign sector in form of financial investors issuing foreign-currency denominated bonds (b_t^*). Import price $f_t^{k^*}$, export demand exp_t and foreign interest rate r_t^* are exogenously determined in the model.

1.3.3 Firms

The production sector in the economy is owned by the households. The labor to the firm is supplied by both households and financial investors. The representative firm sells the output to households, financial investors and the foreign sector (exports). Since there is a continuum of output variety populating the unit interval, we can index the variety by j . The final good (y_t) is a CES aggregate of these output varieties.

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\theta_p-1}{\theta_p}} \right)^{\frac{\theta_p}{\theta_p-1}} \quad (1.24)$$

The Cobb Douglas production function for the output variety i is given below.

$$y_t(j) = A_t k_t^*(j)^{1-\alpha} h_t(j)^\alpha \quad (1.25)$$

where $h_t(j)$ is the labor input and $k_t^*(j)$ is the imported intermediate good input for production of good j . A_t is the factor productivity of the firm. Firm pays wages w_t^{hh} for the labor input and f_t^k for the imported intermediate good. The

intermediate good input is imported at import price f_t^{k*} which is exogenously determined in our model. f_t^k is the import price denoted in domestic currency using the nominal exchange rate.

$$f_t^k = e_t f_t^{k*} \quad (1.26)$$

The demand function of each output variety is given by

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta_p} y_t \quad (1.27)$$

where θ_p is the elasticity of substitution between the differentiated good variety j .

We follow Calvo-style pricing in our model where there is a probability $1 - \phi$ that the firm resets its price in a given period. Thus with probability ϕ , firms keep their prices unchanged. Let $P_t^*(j)$ denote the price set by a firm j adjusting its price in period t . Using the Calvo price setting model, $P_{t+s}(j) = P_t^*(j)$ with probability ϕ^s for $s = 0, 1, 2, \dots$. We drop the j subscript as all firms optimizing the price in any given period will choose the same price. The objective of the firm while setting the price in period t is to maximize the present value of its profits.

$$Max_{P_t^*} \sum_{s=0}^{\infty} \phi^s E_t \{ M_{t,t+s} [Y_{t+s} (P_t^* - MC_{t+s})] \} \quad (1.28)$$

subject to the below demand constraint

$$Y_{t+s}(j) \leq \left(\frac{P_t^*}{P_{t+s}} \right)^{-\theta_p} Y_{t+s} \quad (1.29)$$

where $M_{t,t+s}$ is the stochastic discount factor, Y_t is the nominal output and MC_t is the nominal marginal cost. Through optimization of the above pricing

problem, we get the first order condition as :

$$\sum_{s=0}^{\infty} \phi^s E_t \left\{ M_{t,t+s} Y_{t+s} \left(P_t^* - \frac{\theta_p}{\theta_p - 1} MC_{t+s} \right) \right\} = 0 \quad (1.30)$$

We can rewrite this equation as

$$\sum_{s=0}^{\infty} \phi^s E_t \left\{ M_{t,t+s} Y_{t+s} \left(\frac{P_t^*}{P_{t-1}} - \frac{\theta_p}{\theta_p - 1} mc_{t+s} \pi_{t-1,t+s} \right) \right\} = 0 \quad (1.31)$$

where $mc_t = \left(\frac{f_t^k}{1-\alpha} \right)^{1-\alpha} \left(\frac{w_t^{hh}}{\alpha} \right)^{\alpha} \frac{1}{A_t}$ is the real marginal cost and $\pi_{t-1,t+s} = \frac{P_{t+s}}{P_{t-1}}$ is the gross inflation between $t-1$ and $t+s$. Using the fact that ϕ is the fraction of firms which keep past period's price unchanged in the current period and $1-\phi$ is the fraction of firms adjusting its price in the current period, the price index in the current period is given by

$$P_t = \left[\phi P_{t-1}^{1-\theta_p} + (1-\phi) P_t^{*1-\theta_p} \right]^{\frac{1}{1-\theta_p}} \implies \pi_t^{1-\theta_p} = \phi + (1-\phi) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\theta_p} \quad (1.32)$$

By loglinearizing Equations 1.31 and 1.32, we obtain the Phillips curve equation given by

$$\log(\pi_t) = \beta_c \lambda_{t+1}^{tot} \log(\pi_{t+1}) + \frac{(1-\phi)(1-\beta_c \phi \lambda_t^{tot})}{\phi} \log(mc_t) \quad (1.33)$$

The consumption adjustment costs enter the discount factor in the above inflation dynamics equation through $\lambda_t^{tot} = \lambda_{1,t}(1 + \tau_{c,t})(1 + s_t^c + s_t^{cc'} v_t^c)$. (see Equation 1.18)

1.3.4 Financial Investors

Like the households, the utility function of the financial investors also depends on v which determines the degree of habit formation of lagged per capita consumption c_{t-1}^u in preferences. Utility also depends on hours worked h_t^u with η as labour supply elasticity. ψ_h refers to the weight of hours worked in the utility function. β_c is the discount factor. The utility function of financial investor differs from the household with respect to the inclusion of monetary transactions balances (f_t^u) with weight ψ_f . The interest semi-elasticity of deposit demand by financial investors is determined by ϑ . Interest semi-elasticity of deposit demand refers to the percentage change in bank deposit demand due to a rise in the opportunity cost of bank deposits (interest rate on domestic bonds) by one percentage point.

The objective of the financial investor is to maximize the utility function

$$Max : E_0 \sum_{t=0}^{\infty} \beta_u^t \left\{ (1-v) \log(c_t^u(i) - v c_{t-1}^u) - \psi_h \frac{h_t^u(i)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \psi_f \frac{(f_t^u(i))^{1-\frac{1}{\vartheta}}}{1-\frac{1}{\vartheta}} \right\} \quad (1.34)$$

where $c_t^u(i) = \left[\int_0^1 c_t^u(i, j)^{\frac{\theta_p-1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p-1}}$ and $f_t^u = d_t^{u\theta}$.

Domestic government bonds b_t^u and bank deposits d_t^u are the financial assets owned by financial investors. Similar to the framework in Schmitt-Grohé and Uribe (2003), our model has foreign debt in the form of foreign-currency denominated bonds, $b_t^{u*}(i)$, issued by the financial investors in international markets. r_t^* refers to the interest rate at which financial investors can borrow in international markets in period t . The consumption expenses of the financial investors amount to $c_t^u(i)(1 + \tau_{c,t})$ and their after tax income equals $w_t^{hh} h_t^u(i)(1 - \tau_{L,t})$. In addition to wage income, they also receive a share $(1 - \iota)$ of the lump-sum

income Ω_t . The budget constraint faced by financial investor i is given below.

$$\begin{aligned}
(b_t^u(i) + d_t^u(i)) (1 + \phi_b(b_t^{rat} - \overline{b^{rat}})) + e_t \zeta_{t-1}^q r_{t-1}^* b_{t-1}^{u*}(i) = \\
q_t b_t^{u*}(i) + \Psi_{u,t}(i) + r_{t-1} b_{t-1}^u(i) + r_{d,t-1} d_{t-1}^u(i) \\
- c_t^u(i)(1 + \tau_{c,t}) + w_t^{hh} h_t^u(i)(1 - \tau_{L,t}) + \frac{1 - \iota}{\omega} \Omega_t \quad (1.35)
\end{aligned}$$

where q_t is the real exchange rate defined as $q_t = f_t^k = e_t f_t^{k*}$ (see Equation 1.26). ζ_t^e is the risk premium following Schmitt-Grohé and Uribe (2004). It is a positive convex function of foreign debt to aggregate output ratio to prevent unlimited borrowing from abroad.

$$\zeta_t^e = \exp \left[\varphi_e \left(\frac{q_t b_t^{u*}}{y_t} - \frac{\overline{q b^{u*}}}{\overline{y}} \right) \right] \quad (1.36)$$

The transaction costs associated with holding domestic assets such as government bonds and deposits, $\phi_b(b_t^{rat} - \overline{b^{rat}})(b_t^u(i) + d_t^u(i))$, is rebated back to financial investors as a lump sum transfer $\Psi_{u,t}(i)$. Financial investors maximize Equation 1.34 subject to Equation 1.35. Since all financial investors face same initial endowment, we can drop i index and we get the below first order conditions.

$$c_t^u : \frac{(1 - v)}{c_t^u - v c_{t-1}^u} = \lambda_{3,t}(1 - \tau_{c,t}) \quad (1.37)$$

$$h_t^u : \psi_h(h_t^u)^{\frac{1}{\eta}} = \lambda_{3,t} w_t^{hh}(1 - \tau_{L,t}) \quad (1.38)$$

$$b_t^u : \beta_u \lambda_{3,t+1} r_t = \lambda_{3,t} (1 + \phi_b(b_t^{rat} - \overline{b^{rat}})) \quad (1.39)$$

$$b_t^{u*} : \beta_u \lambda_{3,t+1} q_{t+1} \zeta_t^e r_t^* = \lambda_{3,t} q_t \quad (1.40)$$

$$d_t^u : \beta_u \lambda_{3,t+1} r_{d,t} = \lambda_{3,t} (1 + \phi_b(b_t^{rat} - \overline{b^{rat}})) - \psi_f \theta (d_t^u)^{\theta-1-\frac{\theta}{\vartheta}} \quad (1.41)$$

1.3.5 Fiscal policy

The government budget constraint is given by

$$b_t + m_t = r_{t-1}b_{t-1} + r_{m,t-1}m_{t-1} + g_t - \tau_{L,t}w_t^{hh}h_t - \tau_{c,t}c_t \quad (1.42)$$

where b_t and m_t are government supply of bonds and CBDCs, respectively, and government spending g_t is a fraction s_g of GDP. Hence, $g_t = s_g gdp_t$. gdp_t ⁴ is defined as

$$gdp_t = c_t + g_t + exp_t - (1 - \omega)k_t^* \quad (1.43)$$

Following the fiscal policy rule of Barrdear and Kumhof (2016), we also protect the government budget from potentially high volatile seigniorage revenue through CBDC creation.

$$gdx_t^{rat} = \overline{gdx^{rat}} - 100d^{gdp} \ln \left(\frac{gdp_t}{\overline{gdp}} \right) \quad (1.44)$$

where gdx_t^{rat} is the adjusted budget deficit ratio and given by

$gdx_t^{rat} = 100 \frac{(b_t - b_{t-1}) + (m_t - m_{t-1})}{gdp_t}$. d^{gdp} is the response coefficient which allows the deficit to fluctuate when there is an output gap, $\ln \left(\frac{gdp_t}{\overline{gdp}} \right)$. Equation 1.44 is assumed to endogenise income tax $\tau_{L,t}$ whereas consumption tax $\tau_{c,t}$ is endogenised by the following equation.

$$\frac{\tau_{c,t} - \overline{\tau_c}}{\overline{\tau_c}} = \frac{\tau_{L,t} - \overline{\tau_L}}{\overline{\tau_L}} \quad (1.45)$$

⁴We use the GDP definition from Kumhof and Laxton (2009) where imported goods are inputs in production.

1.3.6 Monetary policy

The monetary policy follows the conventional (nominal) interest rate rule of Barrdear and Kumhof (2016) which includes interest rate smoothing and deviations of one-period ahead forecast inflation from target inflation, but modified to include deviations of real exchange rate and output from their steady state values.

$$i_t = i_{t-1}^{i_i} \left(\frac{1 + \phi_b(brat_t - \overline{brat})}{\beta_u} \right)^{1-i_i} \left(\frac{\pi_t}{\bar{\pi}} \frac{\pi_{t+1}}{\bar{\pi}} \right)^{(1-i_i)w_\pi} \left(\frac{q_t}{\bar{q}} \right)^{(1-i_i)w_q} \left(\frac{y_t}{\bar{y}} \right)^{(1-i_i)w_y} \quad (1.46)$$

where i_t is nominal interest rate, ϕ_b determines the elasticity of real-interest-rate to government debt-to-GDP ratio ($brat_t$), i_i is the smoothing parameter associated with nominal interest rate and w_π , w_q and w_y are the feedback coefficients on inflation, real exchange rate and output, respectively. $\overline{brat}$, $\bar{\pi}$, $\bar{q}$ and $\bar{y}$ refer to the steady state values of government debt-to-GDP ratio, inflation, real exchange rate and real output, respectively.

CBDC regimes: Given the interest rate rule in Equation 1.46, the CBDC may be used as a secondary monetary policy instrument to maintain price stability. The secondary policy instrument involves using either a CBDC price rule or a CBDC quantity rule as described below.

1. Baseline regime

The baseline CBDC regime assumes the central bank only follows the conventional interest rate rule in Equation 1.46 so the nominal CBDC interest rate is fixed at its steady level, $\overline{i_m}$:

$$i_{m,t} = \overline{i_m} \quad (1.47)$$

2. Price Rule Regime

The price rule regime assumes the central bank uses a secondary monetary policy instrument based on the following rule from Barrdear and Kumhof (2016):

$$i_{m,t} = (i_{m,t-1})^{w_{i_{\pi}^m}} \left(\frac{i_t}{\mathbf{s}_p} \left(\frac{\pi_t}{\bar{\pi}} \frac{\pi_{t+1}}{\bar{\pi}} \right)^{(-i_{\pi}^m)} \right)^{1-w_{i_{\pi}^m}} \quad (1.48)$$

where $w_{i_{\pi}^m}$ is the smoothing parameter of price rule, $\mathbf{s}_p$ is the steady state spread between risk free bond rate and CBDC interest rate and i_{π}^m is the price rule inflation feedback coefficient.

Equation 1.48 indicates that when inflation increases above the target, $(\pi_t \pi_{t+1} > \bar{\pi}^2)$, the CBDC interest rate, $i_{m,t}$, declines relative to the interest rate, i_t , making CBDCs less attractive relative to government bonds. Households demand less CBDCs so the central bank reduces CBDCs held by households and supplies more government bonds to financial investors. As CBDCs are exchanged for government bonds, liquidity is endogenously reduced. This reduction in liquidity would have effects beyond that of the policy interest rate, i_t , specified in Equation 1.46, since households could only replace their reduced CBDC holdings with additional bank deposits. However, bank deposits are more expensive and are imperfect substitutes of CBDCs so financial transaction costs, $\phi_b(b_t^{rat} - \bar{b}^{rat})$, and "liquidity" taxes, $\tau_{c,t}^{liq}$, increase causing real activity to decrease.

3. Quantity Rule Regime

Under the quantity rule regime, the central bank adjusts the CBDC-to-GDP ratio (m_t^{rat}) by the following rule according to Barrdear and Kumhof (2016)

$$m_t^{rat} = \bar{m}^{rat} - m_{\pi} \ln \left(\frac{\pi_t}{\bar{\pi}} \frac{\pi_{t+1}}{\bar{\pi}} \right) \quad (1.49)$$

where $m_t^{rat} = \frac{m}{gdp}$ and m_π is the quantity rule inflation feedback coefficient. The rule states that when inflation is expected to be above target ($\pi_t \pi_{t+1} > \bar{\pi}^2$), the central bank reduces CBDCs from the economy through open market sales of government bonds. This reduces the amount of liquidity in the economy. The impact of reduced liquidity on the real economy is the same as in the price-rule regime. Since the supply of CBDCs decreases relative to demand of CBDCs, and CBDCs are imperfect substitutes to other assets, the remaining CBDCs circulating in the economy become more valuable. Therefore, households would be willing to hold CBDCs with lower returns so the CBDC interest rate, $i_{m,t}$, decreases.

1.3.7 Independence of CBDC monetary policy

Before closing the model, we outline the implication of central bank's independence of CBDC monetary policy through the substitutability between CBDC and deposits. Log linear approximation of Equation 1.21 and Equation 1.22 yields the following relationship between the rates on deposits and CBDC.⁵

$$\hat{i}_{d,t} - \hat{i}_{m,t} = (1 - \theta) (\hat{m}_t^c - \hat{d}_t^c) \quad (1.50)$$

where $i_{m,t}$ and $i_{d,t}$ refer to nominal CBDC rate and nominal deposit rate, respectively. The elasticity of substitution between CBDC and deposits equal $\frac{1}{1-\theta}$ via Equation 1.13.

In the case of CBDC and deposits being perfect substitutes, then $\theta = 1$.

$$\hat{i}_{m,t} = \hat{i}_{d,t} \quad (1.51)$$

⁵We use $\phi_b \rightarrow 0$ to derive the equation.

It is evident from Equation 1.51 that the nominal CBDC interest rate would simply reflect the changes in nominal deposit rate when CBDCs and deposits are perfect substitutes. This would constrain the central bank from using nominal CBDC rate or the CBDC-to-GPP ratio as secondary monetary policy instruments. Hence, CBDCs and deposits must be imperfect substitutes ($\theta < 1$) so that the central bank could use CBDCs as secondary monetary policy instruments under the price rule and quantity rule regimes.

1.3.8 Market Clearing

The equilibrium conditions of our model are detailed in this sub-section. In the domestic consumption goods market, the output of goods produced by the firms for domestic consumption equal the demand from both households and financial investors.

$$c_t = \omega c_t^u + (1 - \omega) c_t^c \quad (1.52)$$

The labour supply from households and financial investors are assumed to be homogeneous. The market clears when labour demand from firms equal labour supply from both households and financial investors.

$$h_t = \omega h_t^u + (1 - \omega) h_t^c \quad (1.53)$$

The market for domestic government bonds and CBDC clear when the demand for bonds from financial investors and the demand for CBDC from households

equal the government's supply of bonds and CBDC respectively.

$$b_t = \omega b_t^u \quad (1.54)$$

$$m_t = (1 - \omega) m_t^c \quad (1.55)$$

In the banking sector, we have equilibrium conditions associated with deposits and loans. In equilibrium, the supply of deposits from banks equal the demand for deposits from both households and financial investors. On the other hand, loan market clears when the banks' loan supply covers the households' demand for credit.

$$d_t = \omega d_t^u + (1 - \omega) d_t^c \quad (1.56)$$

$$\ell_t = (1 - \omega) \ell_t^c \quad (1.57)$$

The goods market is in equilibrium when the total output of firms equal the demand for consumption goods from both households and foreigners, government spending on goods and real resource cost on monitoring and monetary transactions (see Equation 1.10).

$$(1 - \omega) y_t = c_t + g_t + exp_t + \varsigma(\mathcal{M}_t + \mathcal{T}_t) \quad (1.58)$$

Finally, we have the below balance of payments identity where the trade balance measures the change in the net foreign liabilities position of the economy.

$$q_t \omega b_t^{u*} = q_t \zeta_{t-1}^e r_{t-1}^* \omega b_{t-1}^{u*} + q_t (1 - \omega) k_t^* - exp_t \quad (1.59)$$

1.3.9 Shocks

The exogenous variables such as domestic productivity, foreign interest rate and exports, $\mathbf{e}_t = \{A_t, r_t^*, exp_t\}$ are assumed to follow autoregressive process of order 1.

$$\log \mathbf{e}_t = \rho_{\mathbf{e}} \log(\mathbf{e}_t) + (1 - \rho_{\mathbf{e}}) \log(\bar{\mathbf{e}}) + \xi_{\mathbf{e},t}; \xi_{\mathbf{e},t} \sim N(0, \sigma_{\mathbf{e}}^2) \quad (1.60)$$

1.3.10 Welfare

Following Schmitt-Grohé and Uribe (2007), the welfare function is given by

$$\mathcal{W}_{i,t} = u_{i,t} + \beta_i E_t \mathcal{W}_{i,t+1}, i \in c, u \quad (1.61)$$

where $u_{i,t}$ is the utility function and i denotes whether the agent is the household or the financial investor. The weighted average of the welfare of the households and the financial investors is the economy's social welfare and given by.

$$\tilde{\mathcal{W}}_t \equiv \omega \mathcal{W}_{c,t} + (1 - \omega) \mathcal{W}_{u,t} \quad (1.62)$$

1.3.11 Calibration

We solve the the model numerically, and then generate the impulse responses and calculate the welfare function using Dynare. We specify the parameter values based on the literature. The parameter values and the corresponding sources are reported in Table 1.1. The values of the welfare measures in Equation 1.62 are obtained as the unconditional means of $\mathcal{W}_{c,t}$, $\mathcal{W}_{u,t}$ and $\tilde{\mathcal{W}}_t$. In the simulations, the DSGE model is solved at the second order. We assume that

the policy maker's objective is to optimize the policy coefficients to achieve the highest level of social welfare, $\tilde{\mathcal{W}}$.

1.4 Results

1.4.1 Optimal coefficient values from welfare and policy instrument volatility

In the baseline regime, we find the optimum policy coefficient values of the interest rate rule (Equation 1.46) and fiscal policy rule (Equation 1.44) by conducting a five dimensional grid search for the coefficients i_i , w_π , w_q , w_y and d^{gdp} to maximise the social welfare of the economy ($\tilde{\mathcal{W}}_t$). We consider the ranges of 0 to 0.9 for i_i , 0 to 1 for w_q and w_y , 1 to 3 for w_π , and 0 to 2 for d^{gdp} . We find that the values of the optimal exchange rate feedback coefficient and output feedback coefficient are always close to zero. Hence, we simplify further analysis by keeping $w_q = 0$ and $w_y = 0$.⁶ We conduct a grid search over the range of $i_i \in [0, 0.9]$, $d^{gdp} \in [0, 2]$ and $w_\pi \in [1, 3]$ and find social welfare is maximised at $i_i = 0.9$, $d^{gdp} = 2$ and $w_\pi = 1$ for the baseline regime.

Now we discuss the reasoning for the consideration of each range of coefficient values in the optimisation process. In choosing the relevant optimal coefficients (i_i , w_π , d^{gdp} , i_π^m and m_π), Benes and Kumhof (2015) discuss the importance of considering both welfare and volatility of the policy instruments. The policy instruments are i_t in the interest rate rule (Equation 1.46), gdx_t^{rat} in the fiscal rule (Equation 1.44), $i_{m,t}$ in the price rule (Equation 1.48) and m_t^{rat} in the quantity rule (Equation 1.49). Some coefficient values may result in high

⁶Schmitt-Grohé and Uribe (2007) also finds output response to be muted in the optimal monetary policy.

welfare estimates but these values may be accompanied by extremely volatile instruments. Hence, we look at plots of social welfare and the standard deviations (SD) of policy instruments [henceforth welfare-SD curve] in our selection of coefficient values since our objective is to find coefficient values that are not only welfare enhancing but also practically feasible.

In the baseline regime where the CBDC interest rate, $i_{m,t}$, is fixed at $\overline{i_m}$, the central bank follows the conventional interest rate rule expressed by Equation 1.46. To find the optimum, we keep the range of the smoothing coefficient of interest rate rule, i_i , within $[0, 0.9]$. As discussed above, we consider both social welfare and volatility of adjusted budget deficit (standard deviation (SD) of gdx_t^{rat}) while choosing a range of values for d^{gdp} in the fiscal policy rule. Figure 1.3.a shows the welfare-SD of gdx_t^{rat} curve for different values of d^{gdp} indicated by the data labels below the curve. In estimating the welfare-SD curve, w_π and i_i are held at the optimal value. Figure 1.3.a shows the welfare-SD curve to be relative flat for values of d^{gdp} between 0.5 to 2. However, the welfare-SD curve rises steeply for values of d^{gdp} above 2. This implies that at values of d^{gdp} above 2, gdx_t^{rat} becomes extremely volatile. Hence, we limit the range of $d^{gdp} \in [0, 2]$ for welfare optimisation in the baseline regime.

With regard to w_π , we followed Schmitt-Grohé and Uribe (2007) who also consider a money demand function analogous to CBDC in our framework for keeping the values of w_π greater than or equal to 1. We also estimate the range of feasible values of w_π by plotting the curve of social welfare and volatility of nominal policy interest rate (SD of i_t), for different coefficient values of w_π , indicated by data labels from 1 to 2.9 in Figure 1.3.b⁷. The figure shows the welfare-SD curve declines for values of w_π in the range $[1, 1.6]$, so social welfare

⁷ d^{gdp} and i_i are held at the optimal value.

decreases as the standard deviation of i_t increases. However, when w_π is greater than 1.6, the direction of the welfare-SD curve reverses but with a flatter slope so social welfare rises while the standard deviation of i_t drops. Nevertheless, values of w_π greater than 1.6 have lower social welfare than at $w_\pi = 1$ with no significant reduction in standard deviation of i_t .

Next, we optimise the coefficients associated with the CBDC price rule regime (Equation 1.48) where the nominal CBDC interest rate ($i_{m,t}$) acts a secondary monetary policy instrument in addition to the primary policy interest rate rule (Equation 1.46). We keep the coefficients i_i , w_π and d^{gdp} at the optimised values as in the baseline regime and conduct a grid search for i_π^m and $w^{i_\pi^m}$. Since $w^{i_\pi^m}$ is a smoothing coefficient of the price rule, we keep the range of $w^{i_\pi^m}$ in $[0, 0.9]$. With regard to i_π^m , we take the range $[0, 0.8]$ as values of the price rule coefficient considered by Barrdear and Kumhof (2016). We initially conduct a two dimensional grid search over the range $i_\pi^m \in [0, 0.8]$ and $w^{i_\pi^m} \in [0, 0.9]$ and find the optimum coefficients at $i_\pi^m = 0.8$ and $w^{i_\pi^m} = 0.9$. However, we also consider the implications of the volatility of the instrument $i_{m,t}$, for a feasible CBDC price rule so we plot the social welfare-standard deviation (SD) of $i_{m,t}$ for the corresponding values of the coefficient i_π^m in Figure 1.4. The welfare-SD of $i_{m,t}$ curve shows that social welfare increases rapidly as the SD of $i_{m,t}$ declines slowly when i_π^m increases between 0 to 0.2. However, as i_π^m rises for $i_\pi^m > 0.2$, social welfare rises at a decreasing rate relative to the rise in SD of $i_{m,t}$ since the welfare-SD curve becomes flatter. The SD of $i_{m,t}$ is at its minimum at $i_\pi^m = 0.2$. Hence, we assign $i_\pi^m = 0.2$ and keep $w^{i_\pi^m} = 0.9$ as the optimal coefficient values in CBDC price rule.

The final step of welfare optimisation is performed under the CBDC quantity rule regime (Equation 1.49) where the CBDC-to-GDP ratio (m_t^{rat}) acts as the

secondary monetary policy instrument instead of the nominal CBDC interest rate under the price rule regime (Equation 1.48). As in the optimisation in the CBDC price rule regime, we keep the optimal values of coefficients i_i , w_π and d^{gdp} under the baseline regime and conduct a grid search for the optimal values of coefficients m_π . Following Barrdear and Kumhof (2016), we keep the range of m_π in $[0, 8]$. We do a one dimensional grid search over the range $m_\pi \in [0, 8]$ and find the optimum m_π at 8. We also plot the social welfare and standard deviation (SD) of m_t^{rat} corresponding to the values of m_π in the range $[0, 8]$ in Figure 1.5. The figure shows a U-shaped welfare-SD curve such that social welfare initially declines when m_π increases from 0 to 4, reaches its minimum at $m_\pi = 4$, and then starts to rise when m_π is greater than 4. The standard deviation (SD) of m_t^{rat} increases as m_π increases. Figure 1.5 also shows that the slope of the welfare-SD curve becomes steeper as m_π increases for $m_\pi > 4$. This indicates that the rate of increase in volatility (SD) of m_t^{rat} is less than the rate of increase of social welfare when m_π is greater than 4. Hence, we continue to keep the optimum value of m_π at 8.

Table 1.2 shows the welfare associated with the coefficients of the optimal policies. The results show that the households' welfare is considerably less when compared to financial investors under all the three regimes. This could be explained by the fact that households face monetary transactions costs in consumption spending (Equation 1.11) whereas financial investors do not. However, given the monetary transaction costs, households would prefer to use CBDC for consumption spending as it improves efficiency and lowers monetary transaction costs compared to deposits. Table 1.2 also shows that compared to the baseline regime, the welfare of households improves under the price rule regime with the CBDC interest rate ($i_{m,t}$) as the central bank's secondary policy in-

strument. However, the welfare of households is same under both the quantity rule regime and the baseline regime. As we will discuss in the next section, the CBDC interest rate is more volatile under the quantity rule compared to the price rule. In contrast, Table 1.2 shows that financial investors' welfare is worse under the quantity rule and the price rule compared to the baseline regime. This is intuitive since financial investors do not hold CBDCs and could not derive benefit from it. Nevertheless, financial investors are indirectly affected by the changes under the CBDC regimes through changes in other macroeconomic variables. Table 1.2 also shows that overall social welfare is best under the price rule regime. This is due to the large improvement in households' welfare, which more than compensates for the decline in financial investors' welfare. Overall welfare is worst under the quantity rule regime since households' welfare did not change but financial investors' welfare declined relative to the baseline regime. This confirms the argument of Poole (1970) that monetary authorities should operate through nominal interest rates (the CBDC interest rate of the price rule in our model) rather than money stock (the CBDC-to-GDP ratio of the quantity rule in our model) for price stability. Table 1.3 displays the welfare gains for society, households, and financial investors when the economy is hit by one shock at a time. Overall, society's welfare improves only under foreign interest rate shock when CBDC interest rate is adjustable. Households witness a higher welfare from the price rule regime. However, financial investors welfare declines the most in the price rule regime under a foreign interest rate shock. Under domestic productivity shock, there is no change to the welfare of society, households and financial investors when CBDC interest rate is adjustable. Under the foreign demand shock, the welfare of society, households and financial investors decline under a foreign demand shock.

In addition to welfare, the volatility implications measured by the standard deviations of the key variables under the three regimes are shown in Table 1.4. The standard deviations of GDP, consumption, inflation and exchange rate decline when the economy moves from the baseline regime to the adjustable CBDC interest bearing system of price rule and quantity rule. The standard deviations of inflation and GDP are the same under both the price rule and the quantity rule. However, the standard deviations of consumption and exchange rate are smaller in the price rule regime compared to the quantity rule regime.

1.4.2 Impulse responses

In this section, we discuss the impulse responses under the three regimes following a domestic productivity shock, a foreign interest rate shock and a foreign demand shock in the economy.

Productivity shock

Figure 1.6 shows the impulse responses of key macroeconomic variables following a 0.01 basis points productivity shock. The productivity shock increases the supply of domestic (consumption) goods, which in turn initially causes deflation. With forecast inflation lower than its target, the nominal policy interest rate (i_t) declines. The drop in the nominal policy interest rate is more drastic under the interest rate rule in the baseline regime than in the regimes with CBDC as a secondary policy instrument. Under the baseline regime, the drastic drop in the nominal interest rate makes government bonds less attractive to financial investors so the central bank exchanges government bonds for CBDCs. With more CBDCs in the economy, households switch from deposits to CBDCs since

CBDCs are more efficient and incur lower monetary transaction costs than deposits.

With the CBDC as secondary policy instrument, the CBDC interest rate rises (albeit slightly) when forecast inflation is below its target under the price rule regime while the CBDC-GDP ratio (m_t^{rat}) increases when forecast inflation is below its target under the quantity rule. CBDCs become more attractive to households who require higher liquidity as consumption rises while government bonds are less attractive to financial investors as nominal interest rate drops. To meet the changing demands of households and financial investors, the central bank increases CBDCs and reduces government bonds. This endogenously increases the liquidity (refer to subsection 1.3.6.2). In contrast, under the CBDC quantity rule, the central bank increases the supply of CBDC (in exchange of government bonds) as forecast inflation is less than target inflation so the CBDC become less valuable and households are only willing to hold CBDCs at higher CBDC interest rate (refer to subsection 1.3.6.3). These are reflected in the impulse responses of CBDC-to-GDP ratio which are less volatile under the CBDC price rule than the CBDC quantity rule. As discussed above, government bonds decline under all regimes, with the decline significantly worse and the increase in the quantity of CBDC also greater under the quantity rule regime.

Figure 1.6 also shows that as households switch to and hold more CBDC assets, deposits and loan decline in all the three regimes. The decline of deposits and loans is greater under the baseline regime compared with the other regimes since the amount of CBDC increases more under the baseline regime. Nominal deposit and wholesale lending interest rates follow the decline of the nominal CBDC interest rate but the nominal deposit and wholesale rates are

least volatile under the price rule regime, followed by the baseline and then the quantity rule regimes. This is due to the less volatile (and lower decline of) nominal policy interest rate under the price rule compared to either the quantity rule or the baseline regime. While nominal deposit and lending rates drop, there is an even larger deflation so real deposit rate (and correspondingly the real lending rate) increases.

CBDCs are also more efficient and incur lower monetary transaction costs than bank deposits so the rise in CBDCs and the fall in deposits further strengthens the rise in real economic activity. This is reflected in the figure by the sharp rise in consumption. However, as real lending rate rises and bank loans fall, the rise in consumption is not sustained so consumption drops sharply after the initial increase. Since foreign interest rate is exogenous, the decline in the nominal policy interest rate with uncovered interest parity causes the exchange rate to depreciate. The depreciation causes marginal cost of imported intermediate inputs to rise. The marginal cost of imported inputs eventually rises more than the drop in marginal cost due to higher productivity. Hence, inflation moves from a large negative inflation to a small positive inflation. However, we note the uninterrupted adjustment of nominal policy interest rate and the nominal CBDC interest rate to their steady state values. In contrast, the CBDC-to-GDP ratio and the amount of CBDCs decline by more than its steady state values. During the same period, bank deposits and bank loans rise above their steady state values while the real deposit and real lending rates drop lower than their steady state values. Following the rise in bank loans above its steady state value, there is an uptick in consumption.

We note that the exchange rate is most volatile under the benchmark regime compared to the price rule and quantity rule regimes. Furthermore, the depre-

ciation of the exchange rate causes the exchange rate adjusted cost of foreign borrowing to rise together with the risk premium so foreign debt initially falls. However, the exchange rate peaks and starts to appreciate in the fifth period, much more than the decline in consumption. Hence, risk premium drops and foreign debt eventually rise. The volatility of foreign debt, following the volatility of the exchange rate and the risk premium, is largest under the baseline regime.

Foreign interest rate shock

Figure 1.7 shows the impulse responses of the key variables following a positive foreign interest rate shock (0.25 basis points increase in the foreign interest rate). The rise in foreign interest rate depreciates the exchange rate through the uncovered interest parity condition. Firms now face higher marginal cost as the imported intermediate goods for production becomes more expensive. This has a feedback effect on the domestic good prices, causing inflation to rise in the economy. With the rise in marginal cost and domestic prices, demand for consumption goods initially declines.

In light of higher inflation, the monetary policy instruments become active to help the economy move back to price stability. Nominal policy interest rate rises in all the three regimes with baseline regime witnessing the highest rise. On account of higher nominal policy interest rate, financial investors increase their holding of government bonds in exchange of bank deposits. The higher inflation also reduces the nominal CBDC interest rate for the price rule regime (through the downward adjustment in the CBDC interest rate itself) and the quantity rule regime (through downward adjustment in the CBDC-to-GDP ra-

tio). Hence, the CBDC-to-GDP ratio declines, with the decline greater under the quantity rule regime.

With the lower amount of CBDCs in circulation, households switch to bank deposits increasing the amount of deposits available to banks. This in turn drives down the real deposit interest rate as well as the real lending interest rate as the rise in inflation is greater than the rise in nominal deposit and lending interest rates (see baseline and price rule regime). With lower real lending rates, households demand more loans. The rise in the bank loans allows households to increase their consumption after an initial decline. The increase in consumption is sustained over several periods as shown in Figure 1.7.

While the exchange rate depreciation increases the risk premium (see Equation 1.36), the rise in consumption has the opposite effect. There is further downward pressure on the risk premium as foreign borrowing by financial investors (foreign debt) declines as the cost of borrowing rise due to the exchange rate depreciation and rise in foreign interest rate.

Foreign demand shock

The impulse responses of the key variables following a foreign demand shock are shown in Figure 1.8. In our model, exports endogenously increase immediately reflecting the increase in foreign demand. Higher exports cause domestic production and employment to expand which results in increased domestic consumption. Higher exports also increases the current account which initially appreciates the exchange rate. With exchange rate appreciation, the cost of imported intermediate imports decline, which in turn reduces prices and inflation. This causes the first monetary policy instrument of nominal policy interest rate

to initially fall under all the three regimes. As nominal interest rate declines, financial investors' demand for government bonds initially declines.

The amount of decline of nominal policy interest rate is slightly less under the adjustable CBDC interest rate regimes following the foreign demand shock. Under the adjustable CBDC interest rate regimes, the lower than expected inflation causes either the nominal CBDC interest rate to rise through the price rule (with the corresponding rise in CBDC-to-GDP ratio) or the CBDC-to-GDP ratio in the quantity rule to rise (with the rise in the CBDC interest rate). Nominal deposit and lending interest rates follow the movements in the nominal policy and CBDC interest rates. However, the decline in the nominal deposit and lending interest rates are the least under the price rule regime followed by the baseline regime and with the quantity rule regime. As the amount of CBDCs rises, households switch to CBDCs from bank deposits so bank deposits and bank loan drop. Since deflation is greater than the drop in nominal deposit and lending interest rates, the real deposit rate and bank lending rates rise.

In our model, the rise in export demand and consumption and production eventually cause the imported intermediate good inputs (k^*) to rise more than the rise in export demand around 5 periods after the shock. This turns the trade and current account from surplus into deficit which eventually depreciates the exchange rate. The exchange rate depreciation increases marginal cost of the intermediate imports causing the inflation to become positive 5 periods after the shock. Hence, the nominal policy interest rate turns positive following the reversal in expected inflation. Correspondingly, the nominal CBDC interest rate rises as the CBDC-to-GDP ratio drops sharply under the quantity rule regime. It is noteworthy that the adjustment in nominal CBDC interest rate is smoother under the price rule regime compared to the other regimes. With the reversal

in the nominal policy interest rate and the amount of CBDCs in circulation, bank deposits and bank loans turn positive while real deposit and loan rates turn negative. As bank loans of households become positive, there is an uptick in households consumption and it returns to its steady more gradually.

With foreign demand shock, the impulse responses of the risk premium and the foreign debt closely follow with a lag, the impulse response of the exchange rate. We also note that price rule regime is relatively more stable than the quantity rule regime when we consider the response of both nominal CBDC interest rate and CBDC GDP ratio. The same can be said about the response in terms of domestic bonds as well.

1.5 Conclusion

While the idea of CBDCs is not new, breakthroughs in technology in the use of digital currencies have made central banks seriously consider the introduction of CBDCs. Our study contributes to the discussion on the benefits of CBDCs and its policy implications. We find that CBDC interest rate could be an effective secondary monetary policy instrument for maintaining price stability in a small open economy. We also show that society's welfare in an economy with adjustable CBDC interest rate regime is higher than a regime with fixed CBDC interest rate (analogous to cash). However, in a heterogeneous agent framework and with adjustable CBDC interest rate regime, we find that those with access to CBDCs (households) are made better off while those without access to CBDCs (financial investors) are made worse off when there are shocks to the economy. We also find that key macroeconomic variables are less volatile and returns more quickly to the steady state under the adjustable CBDC interest

rate regime. Nevertheless, since CBDCs are imperfect substitutes to bank deposits, more CBDCs in the economy imply less bank deposits and loans. The reduction in bank loans affect households' borrowings which in turn affect the sustainability of economic activity.

Table 1.1: Parameter values

Parameter	Value	Description	Source
A_c	0.2425	Monetary transaction cost function parameter	Barrdear and Kumhof (2016)
α	0.700 0	Share of labour in production function	Author's choice
B_c	0.0161	Monetary transaction cost function parameter	Barrdear and Kumhof (2016)
β_c	0.9550	Discount factor (households)	Author's choice
β_a	0.9760	Discount factor (financial investors)	Author's choice
φ_c	0.00001	Consumption loan adjustment costs	Barrdear and Kumhof (2016)
χ	0.0117	Penalty parameter for breaching MCAR	Barrdear and Kumhof (2016)
δ^b	0.0573	Share of networth paid as dividends	Barrdear and Kumhof (2016)
η	1.0000	Labour supply elasticity	Barrdear and Kumhof (2016)
ι	0.9629	Share of lump sum income recieved by households	Barrdear and Kumhof (2016)
κ^f	1.0000	Financial collateral coefficient	Barrdear and Kumhof (2016)
κ^r	0.8786	Real collateral coefficient	Barrdear and Kumhof (2016)
ω	0.0500	Proportion of financial investors	Barrdear and Kumhof (2016)
ϕ_b	0.00005	Elasticity of real risk free policy rate with respect to government debt GDP ratio	Barrdear and Kumhof (2016)
ψ_f	0.0114	Utility weight of liquidity	Barrdear and Kumhof (2016)
ψ_h	0.5798	Utility weight of hours worked	Barrdear and Kumhof (2016)
s_g	0.1800	Share of GDP in govt spending	Barrdear and Kumhof (2016)
σ_b	0.0300	Bank riskiness parameter	Author's choice
σ_c	0.6327	Collateral asset value riskiness parameter	Barrdear and Kumhof (2016)
θ	0.9500	Liquidity generating function parameter	Barrdear and Kumhof (2016)
θ_π	0.500	Price stickiness parameter	Author's choice
v	0.7000	Consumption habit	Barrdear and Kumhof (2016)
Υ	0.0800	MCAR	Barrdear and Kumhof (2016)
ς	0.2500	Share of monitoring costs and monetary transaction cost in goods market clearing	Barrdear and Kumhof (2016)
ϑ	1.5000	Interest semi-Elasticities of financial investors Deposit Demand	Author's choice
ξ	0.0585	Proportion of collateral value lost through bankrupt borrowers	Barrdear and Kumhof (2016)
ζ^c	0.7500	Risk weight of consumption loan	Barrdear and Kumhof (2016)
T^{fintec}	1.1530	Technical efficiency parameter of the technology used by CBDC	Barrdear and Kumhof (2016)
sp	1.0221	Steady state interest rate spread between government bonds and CBDC	Author's choice
φ_c	0.001	Risk premium elasticity	Author's choice
ρ_{exp}	0.8000	Autoregression coefficient of exports shock	Author's choice
ρ_{r*}	0.8000	Autoregression coefficient of foreign interest rate shock	Author's choice
ρ_a	0.8000	Autoregression coefficient of domestic productivity shock	Author's choice

Table 1.2: Coefficients of optimal policies

i_i	w_π	d^{gdp}	i_{mpi}	$w^{i_{mpi}}$	m_π	Welfare		
						Society	Household	Financial Investor
Baseline	0.9	1	2	-	-	-2.1227	-2.2495	0.2865
Price rule	0.9	1	2	0.2	0.9	-2.1224	-2.2490	0.2832
Quantity rule	0.9	1	2	-	-	-2.1228	-2.2495	0.2851

Table 1.3: Welfare gains and sources of shocks

	Productivity shock			Foreign interest rate shock			Foreign demand shock		
	Society	Household	Fin. inv.	Society	Household	Fin. inv.	Society	Household	Fin. inv.
Baseline	-2.1269	-2.2543	0.2945	-2.1232	-2.2500	0.2866	-2.1264	-2.2538	0.2944
Price rule	-2.1269	-2.2543	0.2945	-2.1227	-2.2493	0.2832	-2.1265	-2.2540	0.2945
Quantity rule	-2.1269	-2.2543	0.2945	-2.1232	-2.2500	0.2854	-2.1265	-2.2539	0.2943

Note: This table reports the optimal coefficients and the associated unconditional mean of the society welfare, household welfare and financial investor welfare obtained from a stochastic simulation of the models using second-order perturbations around the stochastic steady state in Dynare 4.5.3. Model (1) Baseline regime. Model (2) Price rule regime. Model (3) Quantity rule regime.

Table 1.4: Volatility of key variables

	Baseline	Price Rule	Quantity Rule
GDP	0.0089	0.0074	0.0074
Consumption	0.0271	0.0268	0.0269
Inflation	0.0057	0.0047	0.0047
Exchange rate	0.0496	0.0486	0.0489

Note: This table reports the standard deviations of key variables obtained from a stochastic simulation of the models using second-order perturbations around the stochastic steady state in Dynare 4.5.3. Model (1) Baseline regime. Model (2) Price rule regime. Model (3) Quantity rule regime. Variables are expressed in percentage deviations from the steady states.

Figure 1.1: Declining usages of small-denomination cash for transactions (2007-2017)

Note: Small denominations are defined to be denominations smaller than that of the notes available at ATMs. These small denominations are usually used for daily transactions. Declining usages of this group of cash imply increasing adoption of electronic payment facilities. Countries in the advanced economies sample include Australia(AU), Canada(CA), Japan(JP), Sweden(SE), Switzerland(CH), United Kingdom(GB) and United States(US). The emerging economies sample includes Brazil(BR), India(IN), Mexico(MX), Russia(RU), Saudi Arabia(SA), Singapore(SG), South Africa(ZA), South Korea(KR) and Turkey(TR). Change in part (b) reflects from 2012 to 2017 for Euro Area and from 2007 to 2016 for United Kingdom. *Source: Bank for International Settlements*

Figure 1.2: Model outline

Note: Arrows indicate directions of fund flows between sectors.

Figure 1.3: Baseline regime welfare-SD curve

Note: d^{gdp} is the response coefficient of the fiscal policy rule and w_π is the inflation feedback coefficient of the nominal policy rate rule.

Figure 1.4: Price rule regime welfare-SD curve

Note: $i_\pi^m \in [0, 0.8]$ is the inflation feedback coefficient in price rule.

Figure 1.5: Quantity rule regime welfare-SD curve

Note: $m_\pi \in [0, 8]$ is the inflation feedback coefficient in quantity rule.

Figure 1.6: Productivity shock

Impulse response of variables to a 0.01 basis point increase in productivity; Baseline vs. Price rule vs. Quantity rule; x-axis in quarters; y-axis in deviation from steady state

Figure 1.7: Foreign interest rate shock

Impulse response of variables to a 0.25 basis point increase in foreign interest rate; Baseline vs. Price rule vs. Quantity rule; x-axis in quarters; y-axis in deviation from steady state

Figure 1.8: Foreign demand shock

Impulse response of variables to a 2 basis point increase in foreign export demand; Baseline vs. Price rule vs. Quantity rule; x-axis in quarters; y-axis in deviation from steady state

Chapter 2

Central Bank Digital Currency in the monetary policy framework of an emerging economy

2.1 Introduction

According to a survey conducted by Bank for International Settlements (Barontini and Holden, 2019), 70% of the respondent central banks are currently engaging in research related to central bank issued digital currency (CBDC). The survey also finds that the central banks which identify the benefits to outweigh the costs of launching CBDC are predominantly from the emerging market economies (EMEs). In the current climate of declining cash in circulation in spite of increasing the money dispensing outlets (e.g. ATM's), central banks in EMEs find issuing CBDC as a potential solution for promoting financial in-

clusion. Although cashless transactions through mobile payments and other payments means like credit cards, debit cards etc are prevalent in emerging economies, this requires individuals to hold bank accounts. However, banking sector penetration is still considered to be low in emerging low income economies like India. CBDC becomes extremely relevant in this context as it provides a safe payment channel fully backed by the government that does not require individuals to even hold a bank account. Some central banks view this as essential to promote financial inclusion in a digital world in which cash use is progressively diminishing, especially in emerging economies where banking sector penetration is low.

Some of the recent examples of emerging economies researching CBDC includes the Central Bank of Uruguay who conducted a pilot program on e-peso. The pilot program is deemed successful (International Monetary Fund, 2019) as transfers increased after e-peso was issued. The Bank of Thailand and the South African Reserve Bank are conducting experiments using large-scale pilots to understand the benefit of CBDC for interbank payments and efficient settlements (World Economic Forum, 2019).

In the context of the EME central banks preference for issuing CBDC, our study aims to understand the interaction of CBDC with the prevailing monetary policy instruments of EMEs - the policy interest rate and sterilised foreign exchange (FX) market intervention. We utilise a New Keynesian DSGE model where the central bank uses the policy interest rate to respond to inflation and output gap in the economy (Filosa, 2001). Since the growth of emerging market economies are primarily export driven, deviation of exchange rate from the equilibrium are costly (Adler and Tovar, 2011; Ostry et al., 2012). Many

EME central banks use sterilised FX interventions as the main instrument to control exchange rate volatility (Benes et al., 2015).

Exchange rate movements are an important factor shaping the outlook in emerging market economies as a large share of their trade and debt are priced in foreign currency (Ferrari, 2019). Ostry et al. (2012) mentions that pass through from exchange rate to inflation is high in emerging market economies. In the wake of the severe financial crises faced by them in the 1990s, EME country authorities are also skeptic about sharp depreciations that could result in widespread bankruptcies and mass capital outflow. Thus, exchange rate is a policy target for EME central banks in addition to inflation. The textbook version of the inflation targeting(IT) framework, which prescribes pursuing inflation stability with floating exchange rates through adjustments of a short-term interest rate, is too narrow for EME central banks. In comparison, IT cum sterilized FX intervention framework proves to be ideal as the interest rate would be used to meet the inflation target, while sterilized intervention would serve towards exchange rate objective (De Gregorio, 2010). Brazil, Chile, Colombia, Turkey, and other emerging markets with announced inflation targeting regimes have increased both the frequency and the size of their interventions (Cavallino, 2019).

The prerequisite for sterilised FX intervention is the ability of the central bank to purchase or sell FX reserves to influence the exchange value of the domestic currency. Using event study analysis, Fatum and Hutchison (2003) finds sterilised FX intervention to have significant but short term effect on exchange rate of Germany and United States. Other studies like Égert (2007) and Gersl and Holub (2006) find evidence that sterilised foreign exchange affect exchange rate specifically in EMEs. A more recent study by Catalán-Herrera

(2016) finds intervention to have a dampening effect over the daily exchange rate return's volatility of Guatemala. Adler and Tovar (2011) find some evidence that interventions can affect the pace of appreciation, particularly in countries that have a relatively closed capital account according to the Chinn and Ito (2008) index. Hence, the central bank in our model manages the exchange rate through the purchase and sale of FX reserves.

Within the current EME monetary framework of policy interest rate and sterilised FX intervention, the central bank of our model use CBDC interest rate as an additional monetary policy instrument. Kumhof and Noone (2018) emphasise the adjustable interest-bearing nature of CBDC as a core principle for CBDC issuance. Barrdear and Kumhof (2016) provide a framework for using the CBDC interest rate in a "price rule" to maintain price stability. It is our objective to understand the monetary policy implications of CBDC in an emerging small open economy. Hence, the central bank of our model deploys two instruments for price stability - nominal policy interest rate and nominal CBDC interest rate, and a third instrument for exchange rate management - FX reserves. The model enables us to make comparisons between a CBDC with constant return (analogous to cash) and CBDC with adjustable return following a CBDC interest rate rule. A banking sector which follows financing through money creation (Jakab and Kumhof, 2015) makes CBDC compete with deposits as media of exchange to purchase both domestic and foreign consumption goods. The model also includes an exogenous foreign sector that engages in consumption goods trade, foreign bonds market and FX market of the domestic economy. In Barrdear and Kumhof (2016), CBDC is issued against government debt. In comparison, CBDC in our model is issued against both government

debt and FX reserves as central bank holds FX reserves to manage exchange rate through sterilised FX intervention.

Our work contributes to the emerging literature of CBDC in three aspects. First, we are the first to study the implications of CBDC in the monetary policy framework of an *emerging* open economy context. Most of the current literature comprise qualitative studies to identify the CBDC benefits in emerging economies (Barontini and Holden, 2019) or point out the declining cash in circulation trend as a motivation to implement CBDC (Shirai, 2019) in emerging economies. The few quantitative studies on CBDC monetary policy framework consider a closed economy framework (Barrdear and Kumhof, 2016; Davoodalhosseini, 2018). Our contribution to the literature stems from understanding the effectiveness of using CBDC interest rate as a monetary policy instrument in an emerging market small open economy.

Second, we examine whether the interest rate bearing nature of CBDC would complement the exchange rate management using sterilised FX intervention. Our study finds that using a CBDC interest rate rule to maintain stability in prices reduces the volatility of the exchange rate. The closed economy framework in Barrdear and Kumhof (2016) uses CBDC interest rate to stabilise domestic prices, whereas the current study uses CBDC interest rate to target CPI prices. An analysis of the transmission channel of CBDC adjustable interest rate through exchange rate is required before CBDC implementation in emerging market economies (Meaning et al., 2018; Engel, 2018). Our present study contributes to this gap by examining the effect of adjustable CBDC interest on exchange rate dynamics when the economy is subject to exogenous foreign shocks.

Third, we determine the welfare effects of an emerging small open economy

moving from conventional monetary policies (nominal policy interest rate and sterilised FX intervention) to futuristic monetary policy (adjustable CBDC interest rate). The nominal CBDC interest rate is kept fixed in the conventional monetary framework to make CBDC analogous to cash. Using counterfactual simulations, we make welfare comparisons between 1) a cash economy using only nominal policy interest rate as a monetary policy instrument which resembles an IT framework, 2) a cash economy using nominal policy interest rate and sterilised FX intervention which corresponds to an IT cum FX intervention regime of emerging market economies and 3) a CBDC economy using nominal policy interest rate, sterilised FX intervention and adjustable CBDC interest rate as monetary policy instruments. The CBDC economy uses three instruments for two policy targets - inflation and exchange rate. The central bank targets inflation through both policy interest rate and CBDC interest rate, whereas exchange rate is managed using sterilised FX intervention. A stable exchange rate through sterilised FX intervention can directly counteract exchange rate swings that would have undesired effects on the inflation rate. Hence, sterilised FX intervention takes some of the burden off monetary policy conducted through the interest rates.

The main findings of the paper are as follows. First, the welfare of society improves when the central bank uses the adjustable CBDC interest rate. The rise in household welfare (who forms 95% of the population) drives this improvement in social welfare. However, there are distributional consequences associated with using a CBDC interest rate rule. The welfare of one agent type (household) improves at the cost of welfare deterioration of the other agent type (financial investors) when CBDC interest rate is adjustable. Second, exchange rate management becomes even more effective when interest-bearing CBDC in-

teracts with sterilised FX intervention. However, this comes at the cost of a more volatile CPI inflation. Third, volatility of key variables like consumption, GDP and exchange rate decline when the central bank uses CBDC interest rate to combat CPI inflation. Fourth, the financial stability of the economy under an adjustable CBDC interest rate depends on the type of shock. Financial variables like bank deposits, bank loan and foreign debt are more stable in the interest-bearing CBDC regime within the context of a foreign interest rate shock. However, an export demand shock results in more volatility of financial variables when CBDC interest rate is adjustable.

The remainder of the paper is organised as follows. Section 2.2 provides detailed descriptions of the model with CBDC. Section 2.4 discusses the results from our simulation exercises. Section 2.6 concludes.

2.2 Model

The model analyses a small open economy consisting of heterogeneous agents (households and financial investors), banks, firms, a central bank (government) and the world economy. The domestic framework is a simplified version of the Barrdear and Kumhof (2016) model as we abstract from the real estate and capital goods sectors. The economy is made up of 95% households and 5% financial investors. Both agents hold deposits with banks. Banks issue consumer loans to households as in the financing through money creation (FMC) model (Jakab and Kumhof, 2015; Benes and Kumhof, 2015; Barrdear and Kumhof, 2016). In addition to deposits, households hold CBDCs. Bank deposits and CBDCs are competing media of exchange.

Figure 2.1 shows the flow of funds between sectors. Households and financial

investors provide labour to the firms producing final consumption goods and receive wages for their services. Households, financial investors and the rest of the world (through exports) consume the home-produced goods. As households own the firms, they receive profits from firms. The population proportion of households and financial investors are given by ω and $1 - \omega$, respectively.

Banks create deposits through lending to households for their consumption needs. The asset portfolio of households comprises of CBDC and deposits, whereas financial investors portfolio consists of deposits and government bonds. Deposits and CBDC provide liquidity to finance consumption expenses. Bonds are relatively illiquid and kept as a store of value. The central bank manages liquidity in the economy through CBDC issuance against government bonds and stock of FX reserves.

The interactions between the domestic and world economies are via four channels. First, households and financial investors import goods from the world economy for consumption. Second, firms sell final consumption goods to the world economy as exports. Third, financial investors borrow from the world economy by issuing foreign-currency denominated bonds (foreign debt) and pay the current foreign interests. And finally, the central bank purchase/sell foreign exchange reserves for exchange rate management. The central bank receives interest on the stock of foreign exchange reserves from the world economy. In the remainder of this section, we describe the details of the model. All the variables given in the section below (in small letters) are in real terms unless specified otherwise.

2.2.1 Banks

Bank i provides consumer loans to household ($\ell_t^c(i)$) and maintains deposits for the household ($d_t^c(i)$) and the financial investor ($d_t^u(i)$). Net worth ($n_t^b(i)$) form another major source of funds for the bank apart from deposits. The total real loan stock and deposits held by bank i from period t to period $t + 1$ are given by $\ell_t(i)$ and $d_t(i)$ respectively where $\ell_t(i) = (1 - \omega)\ell_t^c(i)$ and $d_t(i) = (1 - \omega)d_t^c(i) + \omega d_t^u(i)$. The balance sheet of bank i is given by

$$\ell_t^\ell(i) = d_t(i) + n_t^b(i) \quad (2.1)$$

The bank's real gross wholesale lending rates (including premium on the cost of regulation) and real retail lending rates (including premium on credit risk) are given by r_t^ℓ and r_t^r , respectively. Real deposit rate incurred by the bank is denoted by r_t^d . The bank can incur losses on their consumer loans, which are given by $\mathcal{L}_t^b(i) = (1 - \omega)\mathcal{L}_t^c(i)$.

Net worth ($n_t^b(i)$) is given by gross return on loans ($r_t^\ell \ell_{t-1}^\ell(i)$) minus sum of the expenses on deposit interest payments ($r_{d,t} d_{t-1}(i)$), loan losses ($\mathcal{L}_{t+1}^b(i)$), ex-post cost of penalty ($\mathcal{M}_t^b$) and dividends ($\delta^b n_t^b$).¹ Hence, net worth is expressed as:

$$n_t^b(i) = r_t^\ell \ell_{t-1}^\ell(i) - r_{d,t} d_{t-1}(i) - \mathcal{L}_t^b(i) - \delta^b n_t^b(i) - \mathcal{M}_t^b \quad (2.2)$$

Banks are indexed by i based on the assumption that they face heterogeneous exposure to credit risk. This credit risk is given by the idiosyncratic shock ω_{t+1}^b which is lognormally distributed with $\ln(\omega_{t+1}^b) \sim N(0, (\sigma^b)^2)$. The probability

¹In line with Barrdear and Kumhof (2016), we also assume that banks follow a fixed dividend policy where households and financial investors receive part of bankers' net worth at an exogenously fixed rate. This is based on the "extended family approach" of Gertler and Karadi (2011).

density function and cumulative density function of ω_{t+1}^b are given by $f_t^b(\omega_{t+1}^b) = f_{t+1}^b$ and $F_t^b(\omega_{t+1}^b) = F_{t+1}^b$, respectively. The idiosyncratic credit risk reflects the individual bank characteristics not related to the normal banking operation of lending, such as differing loan recovery rates, differing success at raising non-interest income and minimising non-interest expenses. The sum of the last two categories is assumed to be zero over all banks.

The regulatory framework requires that bank i should incur the ex-post penalty of $\mathcal{M}_t^b$ if the gross returns from loan minus expenses through interest payments and loans losses is less than a fraction (Υ) of the risk weighted returns on their loan book. Υ refers to the minimum capital adequacy ratio. The penalty ($\mathcal{M}_t^b$) amounts to a portion of total loans given by $\chi_{\ell_t}(i)$. The cut off condition for the penalty is given below

$$r_t^{\ell} \ell_t^{\ell}(i) \omega_{t+1}^b - r_{d,t} d_t(i) - \mathcal{L}_{t+1}^b(i) < \Upsilon \zeta^c r_t^{\ell} \ell_t^{\ell}(i) \omega_{t+1}^b \quad (2.3)$$

where ζ^c is the risk weight associated with consumer loan. The realized cutoff of the idiosyncratic shock ($\bar{\omega}_t^b$) below which the bank has to pay penalty is given by

$$\bar{\omega}_t^b = \frac{r_{d,t-1} d_{t-1}(i) + \mathcal{L}_t^b(i)}{(1 - \Upsilon \zeta^c) r_{t-1}^{\ell} \ell_{t-1}^{\ell}(i)} \quad (2.4)$$

Hence, the (ex-post) penalty, if paid by the bank, is given by $\mathcal{M}_t^b = \chi_{\ell_t}^{\ell}(i) F_t^b(\bar{\omega}_{t+1}^b)$, noting that $E_t(\mathcal{L}_{t+1}^b(i)) = 0$.

The objective of the bank is to maximise its pre-dividend net worth given by

$$\max_{\ell_t^{\ell}(i), d_t(i)} E_t \{ r_t^{\ell} \ell_t^{\ell}(i) \omega_{t+1}^b - r_{d,t} d_t(i) - \mathcal{L}_{t+1}^b(i) - \chi_{\ell_t}^{\ell}(i) F_t^b(\bar{\omega}_{t+1}^b) \} \quad (2.5)$$

Using equation (2.1), we can rewrite the above as

$$\max_{\ell_t^\ell(i)} E_t \{ r_t^\ell \ell_t^\ell(i) - r_{d,t} [\ell_t^\ell(i) - n_t^b(i)] - \chi \ell_t^\ell(i) F_t^b(\bar{\omega}_{t+1}^b) \} \quad (2.6)$$

and the first order condition gives

$$0 = E_t \left\{ r_t^\ell - r_{d,t} - \chi F_t^b(\bar{\omega}_{t+1}^b) - \chi f_t^b(\bar{\omega}_{t+1}^b) \frac{r_{d,t}}{r_t^\ell} \frac{\ell_t^\ell(i)}{n_t^b(i)} \frac{1}{(1 - \Upsilon \zeta^c) \left[\frac{\ell_t^\ell(i)}{n_t^b(i)} \right]^2} \right\} \quad (2.7)$$

Lending Technologies: Households secure loans from the bank by pledging collateral. The value of the collateral (Col_t) held by bank i is subject to an idiosyncratic shock which changes $Col_t(i)$ to $\omega_{t+1}^c Col_t(i)$ in beginning of period $t+1$ where $\ln(\omega_{t+1}^c) \sim N(0, (\sigma^c)^2)$. The density function and cumulative density function of ω_{t+1}^c are $f_{t+1}^c = f_t^c(\bar{\omega}_{t+1}^c)$ and $F_{t+1}^c = F_t^c(\bar{\omega}_{t+1}^c)$, respectively.

The bank provides a standard debt contract to each household. According to the contract, each household receive the loan amount $\ell_t^c(i)$ at a gross nominal retail interest of i_t^r . The nominal retail interest rate i_t^r is pre-committed at t and not determined at $t+1$. This makes expected profits zero for banks but the realised ex post profits are different from zero.

When ω_{t+1}^c falls below a cut off level $\bar{\omega}_{t+1}^c$, households are unable to pay the committed retail interest rate and enter bankruptcy. All the assets of such households must be handed over to the bank. However, on account of monitoring costs on their lending activity, the banks are able to recover only a fraction $(1 - \xi)$ of the collateral value. The bank's ex ante zero profit condition is given

by

$$E_t \left\{ \left[(1 - F_t(\bar{\omega}_{t+1}^c)) r_t^r \ell_t^c(i) + (1 - \xi) \int_0^{\bar{\omega}_{t+1}^c} Col_t(i) \omega^c f_t^c(\omega^c) d\omega^c \right] - r_t^\ell \ell_t^c(i) \right\} = 0 \quad (2.8)$$

The terms in square brackets refer to the total payoff from lending: the gross retail interest payments of borrowers on their loans if $\omega_{t+1}^c \geq \bar{\omega}_{t+1}^c$; and the recovery from collateral assets if $\omega_{t+1}^c < \bar{\omega}_{t+1}^c$. $\bar{\omega}_t^c$ is determined by equating the gross interest payments if the borrower continues to operate with the gross returns on collateral assets under bankruptcy.

$$\bar{\omega}_t^c = \frac{r_{t-1}^r \ell_{t-1}^c(i)}{Col_{t-1}(i)} \quad (2.9)$$

We can rewrite Equation 2.8 as

$$0 = E_t \{ Col_t(i) [\Gamma_{t+1} - \xi G_{t+1}] - r_t^\ell \ell_t^c(i) \} \quad (2.10)$$

where by definition, the bank's gross share in the value of collateral is:

$$\Gamma_{t+1} = \Gamma_t(\bar{\omega}_{t+1}^c) = \bar{\omega}_{t+1}^c \int_{\bar{\omega}_{t+1}^c}^{\infty} f_t(\omega_{t+1}^c) d\omega_{t+1}^c + \int_0^{\bar{\omega}_{t+1}^c} \omega_{t+1}^c f_t^c(\omega_{t+1}^c) d\omega_{t+1}^c; \text{ and}$$

the proportion of collateral value that the lender has to spend on monitoring costs is:

$$\xi G_{t+1} = \xi G_t(\bar{\omega}_{t+1}^c) = \xi \int_0^{\bar{\omega}_{t+1}^c} \omega_{t+1}^c f_t^c(\omega_{t+1}^c) d\omega_{t+1}^c.$$

On account of the linear form of banks' technology, balance sheet items can be easily aggregated over all banks, and we can therefore drop bank-specific index i .

2.2.2 Households

A representative household i 's utility function depends on a habit formation parameter (v) that determines the degree of persistence of lagged aggregate per capita consumption c_{t-1}^c in preferences. Following Galí and Monacelli (2016), $c_t(i)$ is the individual consumption which is a composite consumption index defined by

$$c_t^c(i) \equiv \frac{1}{(1-\alpha)^{1-\alpha} \alpha^\alpha} c_t^{c,h}(i)^{(1-\alpha)} c_t^{c,f}(i)^\alpha \quad (2.11)$$

where α is the openness parameter. $c_t^{c,h}(i)$ is an index of home good consumption given by $c_t^{c,h}(i) \equiv \left[\int_0^1 c_t^{c,h}(i, j)^{\frac{\theta_p-1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p-1}}$ where $j \in [0, 1]$ denotes the continuum of differentiated goods produced by domestic firms and θ_p is the elasticity of substitution between the good varieties. $c_t^{c,f}(i)$ denotes the household i 's consumption of foreign (imported) good.

Household's utility also depends on labour hours, $h_t(i)$, with η as labour supply elasticity. ψ_h refers to the weight of hours worked in the utility function. β_c is the discount factor.

The household maximises its lifetime utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta_c^t \left\{ (1-v) \ln (c_t^c(i) - v c_{t-1}^c) - \psi_h \frac{h_t^c(i)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right\} \quad (2.12)$$

Following Galí and Monacelli (2005), the optimal allocation of nominal consumption spending by households on home and foreign goods are given by $P_t^h C_t^{c,h}(i) = (1-\alpha) P_t C_t^c(i)$ and $P_t^f C_t^{c,f}(i) = \alpha P_t C_t^c(i)$ respectively.² P_t^h denotes the aggregate price of home goods while P_t^f is the aggregate price of foreign goods. P_t refers to the consumer price index which takes the form

² $C_t^{c,h}(i)$ and $C_t^{c,f}(i)$ corresponds to nominal consumption of home and foreign goods respectively.

$P_t = (P_t^h)^{1-\alpha}(P_t^f)^\alpha$. We can rewrite the consumption spending of the household in real terms using the terms of trade equation ($s_t = \frac{P_t^f}{P_t^h}$):

$$\begin{aligned} c_t^{c,h}(i) &= (1 - \alpha)s_t^{(1-\alpha)}c_t^c(i) \\ c_t^{c,f}(i) &= \alpha s_t^{-(1-\alpha)}c_t^c(i) \end{aligned} \tag{2.13}$$

Households pay consumption taxes, $\tau_{c,t}$, and incur monetary transaction cost, $s_t^c(i)$, while spending on their composite consumption, $c_t^c(i)$.

The budget constraint faced by the household i is given in the below equation:

$$\begin{aligned} & c_t^c(i)(1 + \tau_{c,t})(1 + s_t^c(i)) + (d_t^c(i) + m_t^c(i)) (1 + \phi_b(b_t^{rat} - \overline{b^{rat}})) - \Psi_{c,t}(i) \\ & - \ell_t^c(i) \left(1 - \frac{\varphi^c}{2} (\ell_t^c(i) - \ell_{t-1}^c(i))^2 \right) \\ & = r_{d,t-1}d_{t-1}^c(i) + r_{m,t-1}m_{t-1}^c(i) + \Pi_t(i) + w_t^{hh}h_t^c(i)(1 - \tau_{L,t}) + \frac{\iota}{1 - \omega}\Omega_t(i) - \Gamma_t Col_{t-1}(i) \end{aligned} \tag{2.14}$$

In the above equation, the assets held by the household i comprise of real deposits and CBDC given by $d_t^c(i)$ and $m_t^c(i)$, respectively. The gross real interest rate on holding deposits and CBDC from period t to $t + 1$ are denoted by $r_{d,t}$ and $r_{m,t}$, respectively. In holding financial assets, the household incurs financial transaction costs, $\phi_b(b_t^{rat} - \overline{b^{rat}})$, where b_t^{rat} is government debt-to-GDP ratio and $\overline{b^{rat}}$ is its steady state value. In addition to interest income, households also earn after tax real wages from hours worked, $(1 - \tau_{L,t})w_t^{hh}h_t^c(i)$, and profits from firms which the household owns, $\Pi_t(i)$, and a share (ι) of the real lump-sum income, $\Omega_t(i)$, accruing to households.

Lump sum-income (Ω_t) consists of dividends from the bank ($\delta^b n_t^b$) and a

lump-sum component $(1 - \varsigma)$ of the monitoring costs $(\mathcal{M}_t)$ and monetary transactions costs $(\mathcal{T}_t)$ transferred back to the household.

$$\Omega_t = \delta^b n_t^b + (1 - \varsigma)(\mathcal{M}_t + \mathcal{T}_t) \quad (2.15)$$

The budget constraint in Equation 2.14 also consists of loans secured by the household from banks, denoted by $\ell_t^c(i)$. Swift changes in loan levels are penalised by quadratic adjustment costs $\frac{\varphi_c}{2}(\ell_t^c(i) - \ell_{t-1}^c(i))^2$. Households pay a fraction (Γ_t) of the collateral value (Col_{t-1}) to the bank every period to cover the loan interest payments. The collateral pledged by the household in time period t comprises of both real collateral (wages income) and financial collateral (interest income from CBDC and deposits).

$$Col_t(i) = \kappa_t^r \kappa_t^f (r_{d,t} d_t^c(i) + r_{m,t} m_t^c(i)) + \kappa_t^r r_{n,t} 4w_t^{hh} h_t^c(i)(1 - \tau_{L,t}) \quad (2.16)$$

where κ_t^f is the proportion of pledged financial collateral and $\kappa_t^f \kappa_t^r$ is the proportion of pledged real collateral.

Transactions Cost Technologies: Following Schmitt-Grohé and Uribe (2004), monetary transaction costs of households, $s_t^c(i)$, found in the household budget constraint (see Equation 2.14), are specified as

$$s_t^c(i) = A_c v_t^c(i) + \frac{B_c}{v_t^c(i)} - 2\sqrt{A_c B_c} \quad (2.17)$$

where $v_t^c(i)$ is velocity and given by

$$v_t^c(i) = \frac{c_t^c(i)(1 + \tau_{c,t})}{f_t^c(i)} \quad (2.18)$$

$f_t^c(i)$ refers to liquidity generating function of the households and given by

$$f_t^c(i) = (d_t^c)^\theta + (T^{fintech} m_t^c)^\theta \quad (2.19)$$

where $\frac{1}{1-\theta}$ is the elasticity of substitution between CBDC and deposits. $T^{fintech}$ is a relative productivity coefficient for CBDC which makes the monetary transactions more efficient while using CBDC. Hence, households will hold CBDCs even if the CBDC interest rate is lower than the bank deposit interest rate.

The representative household maximises Equation 2.12 subject to Equation 2.14 and Equation 2.10. We get the first order conditions after taking into account the symmetry across households as follows:

$$c_t^c : \frac{(1-v)}{c_t^c - v c_{t-1}^c} = \lambda_{1,t}(1 + \tau_{c,t})(1 + s_t^c + s_t^{cc'} v_t^c) \quad (2.20)$$

where $s_t^{cc'} = A_c \frac{v_t^c}{c_t^c} - \frac{B_c}{c_t^c v_t^c}$

$$\ell_t^c : \lambda_{1,t} \left(1 - \frac{\varphi_c}{2} (\ell_t^c - \ell_{t-1}^c)^2 - \varphi_c \ell_t^c (\ell_t^c - \ell_{t-1}^c) \right) = \beta_c E_t \{ \lambda_{1,t+1} r_t^\ell \lambda_{2,t+1} \} \quad (2.21)$$

where $\lambda_{2,t+1} = \frac{\Gamma'_{t+1}}{(\Gamma'_{t+1} - \xi G'_{t+1})}$ and Γ'_{t+1} and G'_{t+1} are derivatives of Γ_{t+1} and G_{t+1} with respect to $\bar{\omega}_{t+1}^c$.

$$\begin{aligned} h_t^c : \psi_h(h_t^c)^{\frac{1}{\eta}} &= \lambda_{1,t} w_t^{hh} (1 - \tau_{L,t}) \\ &+ \beta_c \lambda_{1,t+1} r_{n,t} 4 w_t^{hh} (1 - \tau_{L,t}) (-\kappa^r \Gamma_{t+1} + \lambda_{2,t+1} \kappa^r (\Gamma_{t+1} - \xi G_{t+1})) \end{aligned} \quad (2.22)$$

$$\begin{aligned} d_t^c : \lambda_{1,t} \left(1 + \phi_b(b_t^{rat} - \bar{b}^{rat}) + c_t^c (1 + \tau_{c,t}) s_t^{cd'} \right) &= \\ \beta_c E_t \{ \lambda_{1,t+1} r_{d,t} [(1 - \kappa^r \kappa^f \Gamma_{t+1}) + \lambda_{2,t+1} \kappa^r \kappa^f (\Gamma_{t+1} - \xi G_{t+1})] \} \end{aligned} \quad (2.23)$$

where $s_t^{cd'} = -A_c \frac{v_t^c}{f_t^c} f_t^{cd'} + \frac{B_c}{v_t^c f_t^c} f_t^{cd'}$, $f_t^{cd'} = \theta(d_t^c)^{\theta-1}$

$$m_t^c : \lambda_{1,t} \left(1 + \phi_b(b_t^{rat} - \overline{b^{rat}}) + c_t^c(1 + \tau_{c,t})s_t^{cm'} \right) = \beta_c E_t \left\{ \lambda_{1,t+1} r_{m,t} \left[(1 - \kappa^r \kappa^f \Gamma_{t+1}) + \lambda_{2,t+1} \kappa^r \kappa^f (\Gamma_{t+1} - \xi G_{t+1}) \right] \right\} \quad (2.24)$$

where $s_t^{cm'} = -A_c \frac{v_t^c}{f_t^c} f_t^{cm'} + \frac{B_c}{v_t^c f_t^c} f_t^{cm'}$, $f_t^{cm'} = \theta(T^{fintech} m_t^c)^{\theta-1}$

2.2.3 Firms

The representative firm manufactures a differentiated good using the production function with linear technology

$$y_t(j) = A_t h_t(j) \quad (2.25)$$

where $h_t(j)$ is the labour input for production of good j and A_t is the factor productivity of the firm. Both households and financial investors supply labour to the firm. The final good produced is y_t , which is a CES aggregate of the differentiated variety of goods populating the unit interval.

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\theta_p-1}{\theta_p}} \right)^{\frac{\theta_p}{\theta_p-1}} \quad (2.26)$$

where θ_p is the elasticity of substitution between differentiated goods indexed by j . The demand function for each differentiated good is given by

$$y_t(j) = \left(\frac{P_t^h(j)}{P_t^h} \right)^{-\theta_p} y_t \quad (2.27)$$

where $P_t^h(j)$ is the price of output variety j and P_t^h denotes the price of CES aggregate of the output variety.

Firms incur costs on wages, w_t^{hh} , paid to the labour input. The objective of the representative firm j is to maximise the profit function given by

$$\Pi_t(j) = y_t(j) - w_t^{hh} h_t(j) \quad (2.28)$$

Using the demand function in Equation 2.27, we can rewrite the profit function as

$$\Pi_t(j) = \left(\frac{P_t^h(j)}{P_t^h} \right)^{-\theta_p} y_t - w_t^{hh} h_t(j) \quad (2.29)$$

We assume Calvo-style pricing in our model where the representative firm j optimises the price with probability $1 - \phi$ in a given period. Hence, firms keep their prices unchanged in a given period with probability ϕ . $P_t^{h*}(j)$ denotes the price set by a firm j who optimises the price in period t . Using the Calvo price setting, $P_{t+s}^h(j) = P_t^{h*}(j)$ with probability ϕ^s for $s = 0, 1, 2, \dots$. As all firms optimising the price in any given period will use the same price, we drop the j subscript. The objective of the firm while setting the price in period t is to maximise the present value of its profits.

$$\text{Max}_{P_t^*} \sum_{s=0}^{\infty} \phi^s E_t \{ M_{t,t+s} [Y_{t+s} (P_t^{h*} - MC_{t+s})] \}$$

subject to the demand constraint

$$Y_{t+s}(j) \leq \left(\frac{P_t^{h*}}{P_{t+s}^h} \right)^{-\theta_p} Y_{t+s}$$

where $M_{t,t+s}$ is the stochastic discount factor, Y_t is the nominal output and MC_t is the nominal marginal cost. Through optimisation of the above pricing

problem, we get the first order condition as :

$$\sum_{s=0}^{\infty} \phi^s E_t \left\{ M_{t,t+s} Y_{t+s} \left(P_t^{h*} - \frac{\theta_p}{\theta_p - 1} M C_{t+s} \right) \right\} = 0$$

We can rewrite this equation as

$$\sum_{s=0}^{\infty} \phi^s E_t \left\{ M_{t,t+s} Y_{t+s} \left(\frac{P_t^{h*}}{P_{t-1}^h} - \frac{\theta_p}{\theta_p - 1} m c_{t+s} \pi_{t-1,t+s}^h \right) \right\} = 0 \quad (2.30)$$

where $m c_t = \left(\frac{w_t^{hh}}{A_t} \right)$ is the real marginal cost and $\pi_{t-1,t+s}^h = \frac{P_{t+s}^h}{P_{t-1}^h}$ is the gross home price inflation between $t - 1$ and $t + s$. A CES function of price index p_t^h in Equation 2.31 is derived based on our assumption of ϕ as the fraction of firms which continue to use the past period's price in the current period and $1 - \phi$ as the fraction of firms optimising the price in the current period.

$$P_t^h = \left[\phi P_{t-1}^{h^{1-\theta_p}} + (1 - \phi) P_t^{h*^{1-\theta_p}} \right]^{\frac{1}{1-\theta_p}} \implies \pi_t^{h^{1-\theta_p}} = \phi + (1 - \phi) \left(\frac{P_t^{h*}}{P_{t-1}^h} \right)^{1-\theta_p} \quad (2.31)$$

By loglinearising equations 2.30 and 2.31, we obtain the Phillips curve equation

$$\log(\pi_t^h) = \beta_c \lambda_{t+1}^{tot} \log(\pi_{t+1}^h) + \frac{(1 - \phi)(1 - \beta_c \phi \lambda_t^{tot})}{\phi} \log(m c_t) \quad (2.32)$$

The consumption adjustment costs enter the discount factor in the above inflation dynamics equation through $\lambda_t^{tot} = \lambda_{1,t}(1 + \tau_{c,t})(1 + s_t^c + s_t^{cc'} v_t^c)$ (see Equation 2.20).

2.2.4 Foreign sector

Our economy is small with domestic policy having no impact on the world. The interaction of the economy with the world occurs through exports, imports, foreign borrowing and purchase and sale of foreign exchange reserves. Firms export home goods to foreigners (y_t^f). Following Galí and Monacelli (2016), the world demand for exports of home good variety i is given by $y_t^f(i) = \left(\frac{P_t^h(i)}{P_t^h}\right)^{-\theta_p} y_t^f$ for $i \in [0, 1]$. The aggregate export index y_t^f is determined by

$$y_t^f = \alpha s_t y_t^* \quad (2.33)$$

where α is the openness parameter, $s_t = \frac{P_t^f}{P_t^h}$ is the terms of trade and y_t^* is the world output per capita.

Households and financial investors import foreign goods for consumption ($c_t^{c,f}, c_t^{u,f}$) at price P_t^f (foreign goods price denoted in home currency).

$$P_t^f = e_t P_t^* \quad (2.34)$$

where e_t is the nominal exchange rate and P_t^* refers to foreign goods price denoted in foreign currency. Foreign price, P_t^f , and home price, P_t^h , are used to derive the CPI price, P_t , as given below:

$$P_t = P_t^{h^{1-\alpha}} P_t^{f\alpha} \quad (2.35)$$

We use Equation 2.35 to express CPI inflation, π_t , with terms of trade and

domestic price inflation.

$$\begin{aligned}
P_t &= (P_t^h)^{1-\alpha} \left(\frac{P_t^f}{P_t^h} \right)^\alpha P_t^{h^\alpha} \\
P_t &= P_t^h s_t^\alpha \\
\frac{P_t}{P_{t-1}} &= \frac{P_t^h}{P_{t-1}^h} \left(\frac{s_t}{s_{t-1}} \right)^\alpha \\
\pi_t &= \pi_t^h \left(\frac{s_t}{s_{t-1}} \right)^\alpha
\end{aligned} \tag{2.36}$$

Apart from exports and imports, domestic economy also interacts with the world through foreign borrowing and FX reserves. Financial investors borrow from abroad through the issuance of foreign currency denominated bonds b_t^* . They are liable to pay interest, r_t^* , on this foreign borrowing. Central bank also interacts with the world economy through the purchase and sale of FX reserves. The stock of real FX reserves held by the central bank, f_t , receives interest payments from abroad (r_t^*) every year. Foreign currency denominated bonds (b_t^*), foreign interest payments (r_t^*) and foreign exchange reserves (f_t) are converted to home currency using real exchange rate, q_t , which is given by $\frac{e_t P_t^f}{P_t}$. Foreign price inflation $\pi_t^* = \frac{P_t^*}{P_{t-1}^*}$, world output y_t^* and foreign interest rate r_t^* are exogenously determined in the model.

2.2.5 Financial Investors

The functional form of the financial investor utility function is similar to the household's utility function except for an additional term on monetary transaction balances. Utility of the financial investor i in Equation 2.37 depends on consumption ($c_t^u(i)$), the degree of habit formation (v) from lagged per capita consumption of financial investors (c_{t-1}^u), hours worked (h_t^u) with weight ψ_h

and monetary transactions balances ($f_t^u = d_t^{u\theta}$) with weight ψ_f . Parameter η corresponds to labour supply elasticity. The interest semi-elasticity of deposit demand coefficient (ϑ) refers to the percentage change in demand for bank deposits resulting from a one percentage point increase in the opportunity cost of bank deposits (interest rate on domestic bonds).

The objective of the financial investor i is to maximise the utility function below

$$Max : E_0 \Sigma_{t=0}^{\infty} \beta_u^t \left\{ (1-v) \log(c_t^u(i) - v c_t^u) - \psi_h \frac{h_t^u(i)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \psi_f \frac{(f_t^u(i))^{1-\frac{1}{\vartheta}}}{1-\frac{1}{\vartheta}} \right\} \quad (2.37)$$

where β_u refers to the discount factor.

As in the case of households, $c_t^u(i)$ is considered as a consumption index which is an aggregate of home goods ($c_t^{u,h}(i)$) and foreign goods ($c_t^{u,f}(i)$) and is given by

$$c_t^u(i) \equiv \frac{1}{(1-\alpha)^{1-\alpha} \alpha^\alpha} c_t^{u,h}(i)^{(1-\alpha)} c_t^{u,f}(i)^\alpha \quad (2.38)$$

where α is the openness parameter.³ $c_t^{u,h}(i)$ is an index of home good consumption given by $c_t^{u,h}(i) \equiv \left[\int_0^1 c_t^{u,h}(i, j)^{\frac{\theta_p-1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p-1}}$ where $j \in [0, 1]$ denotes the continuum of differentiated goods produced by domestic firms and θ_p is the elasticity of substitution between the good varieties. $c_t^{u,f}(i)$ denotes the financial investor i 's consumption of foreign (imported) good.

Following Galí and Monacelli (2005), the optimal allocation of nominal spending by financial investors on home and foreign goods are given by $P_t^h C_t^{u,h}(i) = (1-\alpha)P_t C_t^u(i)$ and $P_t^f C_t^{u,f}(i) = \alpha P_t C_t^u(i)$ respectively. We can rewrite the consumption spending of the financial investor in real terms using the terms of

³We assume that openness parameter α is same for both households and financial investors.

trade equation ($s_t = \frac{P_t^f}{P_t^h}$):

$$\begin{aligned} c_t^{u,h}(i) &= (1 - \alpha)s_t^{(1-\alpha)}c_t^u(i) \\ c_t^{u,f}(i) &= \alpha s_t^{-(1-\alpha)}c_t^u(i) \end{aligned} \quad (2.39)$$

Financial investors incur consumption tax while spending on their composite consumption, $c_t^u(i)$. Contrary to households, financial investors do not face any monetary transaction cost to finance their consumption needs.

The budget constraint faced by financial investor i is given below.

$$\begin{aligned} (b_t^u(i) + d_t^u(i)) (1 + \phi_b(b_t^{rat} - \overline{b^{rat}})) + (1 + \tau_t^{b*})q_t\zeta_{t-1}^e r_{t-1}^* b_{t-1}^{u*}(i) = \\ q_t b_t^{u*}(i) + \Psi_{u,t}(i) + r_{t-1} b_{t-1}^u(i) + r_{d,t-1} d_{t-1}^u(i) \\ - c_t^u(i)(1 + \tau_{c,t}) + w_t^{hh} h_t^u(i)(1 - \tau_{L,t}) + \frac{1 - \iota}{\omega} \Omega_t \end{aligned} \quad (2.40)$$

Financial investor receives income in the form of after tax wage income, $w_t^{hh} h_t^u(i)(1 - \tau_{L,t})$, and a share $(1 - \iota)$ of lump sum income Ω_t . Additionally, financial investors also obtain interest income from their asset holding of deposits, $d_t^u(i)$, and government bonds, $b_t^u(i)$. Like households, financial investors also face asset transaction costs $(1 + \phi_b(b_t^{rat} - \overline{b^{rat}}))$ which are rebated back to them as lump sum transfers $(\Psi_{u,t}(i))$.

Financial investors also issue foreign debt in the form of foreign-currency denominated bonds in international markets (b_t^{u*}). They are liable to pay the international interest rate (r_t^*) on their foreign borrowing. Foreign bonds and foreign interest payments are converted into home currency using the real exchange rate, q_t , in the budget constraint.

The risk premium, ζ_t^e , is increasing in the stock of real FX reserves to GDP ratio ($\frac{q_t f_t}{gdp_t}$) and the foreign bonds to GDP ratio ($\frac{q_t b_t^{u*}}{gdp_t}$) where gdp_t is defined as

the sum of home goods consumption, government spending and total exports (Equation 2.49 below). ζ_t^e increases the spread between domestic nominal policy interest rate and foreign interest rate.

$$\zeta_t^e = \exp \left[\varphi_e^f \left(\frac{q_t f_t}{gdp_t} - \frac{\bar{q}\bar{f}}{\bar{gdp}} \right) + \varphi_e^{b^{u*}} \left(\frac{q_t b^{u*}}{gdp_t} - \frac{\bar{q}\bar{b}^{u*}}{\bar{gdp}} \right) \right] \quad (2.41)$$

Financial investors maximise Equation 2.37 subject to Equation 2.40. Since all financial investors face same initial endowment, we can drop i index and we get the below first order conditions.

$$c_t^u : \frac{(1-v)}{c_t^u - v c_{t-1}^u} = \lambda_{3,t}(1 - \tau_{c,t}) \quad (2.42)$$

$$h_t^u : \psi_h(h_t^u)^{\frac{1}{\eta}} = \lambda_{3,t} w_t^{hh}(1 - \tau_{L,t}) \quad (2.43)$$

$$b_t^u : \beta_u \lambda_{3,t+1} r_t = \lambda_{3,t} (1 + \phi_b(b_t^{rat} - \bar{b}^{rat})) \quad (2.44)$$

$$b_t^{u*} : \beta_u \lambda_{3,t+1} q_{t+1} \zeta_t^e r_t^* = \lambda_{3,t} q_t \quad (2.45)$$

$$d_t^u : \beta_u \lambda_{3,t+1} r_{d,t} = \lambda_{3,t} (1 + \phi_b(b_t^{rat} - \bar{b}^{rat})) - \psi_f \theta (d_t^u)^{\theta-1-\frac{\theta}{\vartheta}} \quad (2.46)$$

2.2.6 Fiscal policy

Following Barrdear and Kumhof (2016), the central bank and the government are treated as the same entity. The revenue of the government is derived from interest payment receipts, r_t^* , on its stock of real foreign reserves (f_t), labour income tax ($\tau_{L,t} w_t^{hh} h_t$) and consumption tax ($\tau_{c,t} c_t$). Government also receives income from the sale of CBDC and bonds. The expenses faced by the government arise from purchase of foreign exchange reserves, f_t , government spending on goods, g_t , and interest payments on CBDC (r_t^m) and bonds (r_t). The gov-

ernment budget constraint is given by

$$b_t + m_t - q_t f_t = r_{t-1} b_{t-1} + r_{m,t-1} m_{t-1} - q_t r_{t-1}^* f_{t-1} + g_t - \tau_{L,t} w_t^{hh} h_t - \tau_{c,t} c_t \quad (2.47)$$

where government spending g_t is a fraction s_g of gdp_t .

$$g_t = s_g gdp_t \quad (2.48)$$

The sum of home goods consumption of households and financial investors ($c_t^h = (1 - \omega)c_t^{c,h} + \omega c_t^{u,h}$), government spending (g_t) and total exports (y_t^f) form the GDP of the economy.

$$gdp_t = c_t^h + g_t + y_t^f \quad (2.49)$$

CBDC issuance could lead to volatile seigniorage revenue. The government budget is protected through the fiscal policy rule given below.

$$gdx_t^{rat} = \overline{gdx^{rat}} - 100 d^{gdp} \ln \left(\frac{gdp_t}{\overline{gdp}} \right) \quad (2.50)$$

where gdx_t^{rat} is the adjusted budget deficit ratio and given by

$gdx_t^{rat} = 100 \frac{(b_t - b_{t-1}) + (m_t - m_{t-1})}{gdp_t}$. d^{gdp} is the response coefficient which allows the deficit to fluctuate when there is a output gap, $\ln \left(\frac{gdp_t}{\overline{gdp}} \right)$. Equation 2.50 is assumed to endogenise income tax $\tau_{L,t}$ whereas consumption tax $\tau_{c,t}$ is endogenised by the below equation.

$$\frac{\tau_{c,t} - \overline{\tau_c}}{\overline{\tau_c}} = \frac{\tau_{L,t} - \overline{\tau_L}}{\overline{\tau_L}} \quad (2.51)$$

2.2.7 Monetary policy

The primary monetary policy instrument of the central bank is the nominal policy interest of bonds, i_t . Following Barrdear and Kumhof (2016), the nominal policy interest rate rule of the central bank is given below

$$i_t = i_{t-1}^{i_i} \left(\frac{1 + \phi_b(brat_t - \overline{brat})}{\beta_u} \right)^{1-i_i} \left(\frac{\pi_t}{\bar{\pi}} \frac{\pi_{t+1}}{\bar{\pi}} \right)^{(1-i_i)w_\pi} \left(\frac{gdp_t}{\overline{gdp}} \right)^{(1-i_i)w_y} \quad (2.52)$$

where ϕ_b determines the elasticity of real-interest-rate to government debt-to-GDP ratio ($brat_t$), i_i is the smoothing coefficient and w_π and w_y are the feedback coefficients on CPI inflation and GDP, respectively. $\overline{brat}$, $\bar{\pi}$ and $\overline{gdp}$ refer to the steady state values of government debt-to-GDP ratio, CPI inflation and GDP, respectively. The real policy interest is defined by $r_t = \frac{i_t}{\pi_{t+1}}$. The central bank may also use CBDC as an additional monetary policy instrument to maintain price stability through the below price rule on nominal CBDC interest rate $i_{m,t}$:

$$i_{m,t} = (i_{m,t-1})^{w_\pi^m} \left(\frac{i_t}{\mathbf{s}_p} \left(\frac{\pi_t}{\bar{\pi}} \frac{\pi_{t+1}}{\bar{\pi}} \right)^{(-i_\pi^m)} \right)^{1-w_\pi^m} \quad (2.53)$$

where w_π^m is the smoothing parameter of price rule, $\mathbf{s}_p$ is the steady state spread between risk free bond rate and CBDC interest rate and i_π^m is the price rule inflation feedback coefficient. The real CBDC interest is defined by $r_{m,t} = \frac{i_{m,t}}{\pi_{t+1}}$.

According to Equation 2.53, the CBDC nominal interest rate ($i_{m,t}$) declines relative to the nominal policy rate (i_t) when CPI inflation rises above the target. If the real CBDC interest rate declines on account of a lower nominal CBDC interest rate, CBDC becomes less attractive than bonds. Liquidity declines endogenously as households reduce their CBDC holdings.

In addition to controlling CPI inflation through nominal policy interest rate and nominal CBDC interest rate, the central bank may target and manage

exchange rate through sterilised FX intervention (Benes et al., 2015; Xie et al., 2017) using the rule⁴:

$$\log(f_t) = \rho_f \log(f_{t-1}) + (1 - \rho_f) \left[\log(\bar{f}) - \frac{w_e}{1 - w_e} \log\left(\frac{q_t}{\bar{q}}\right) - \frac{v_e}{1 - v_e} \log\left(\frac{q_t}{q_{t-1}}\right) \right] \quad (2.54)$$

ρ_f determines the central bank's preference in smoothing the FX reserves. According to the above rule, the central bank aims to use its real FX reserves to keep the real exchange rate at the desired level when $w_e \rightarrow 1$ whereas $w_e \rightarrow 0$ implies that the central bank does not use the FX reserves for exchange rate targeting. v_e is the response coefficient in the FX rule to smooth exchange rates ("leaning against the wind").

2.2.8 Market Clearing

This subsection details the market equilibrium conditions of the different sectors in the model. The aggregate consumption good market clears when the total supply of consumption goods (c_t) is equal to the aggregate demand from both households (c_t^c) and financial investors (c_t^u).

$$c_t = \omega c_t^u + (1 - \omega) c_t^c \quad (2.55)$$

The consumption demand of financial investors, c_t^u , is a composite index of their demand for foreign goods, $c_t^{f,u}$, and home goods, $c_t^{h,u}$ (see Equation 2.38). In a similar manner, the consumption demand of households, c_t^c , is derived from a

⁴All the variables in our model are expressed in real terms. The FX rule slightly deviates from Benes et al. (2015) as we use real exchange rate instead of nominal exchange rate for sterilised FX intervention by the central bank. Other studies like Ostry et al. (2012) incorporate real exchange rate in sterilised FX intervention while studying the monetary policy framework of emerging market economies.

composite index of their demand for foreign goods, $c_t^{f,c}$, and home goods, $c_t^{h,c}$ (see Equation 2.11).

In the home consumption goods market, the sum of the home consumption goods demand from households ($c_t^{h,c}$) and financial investors ($c_t^{h,u}$) equal the home consumption goods supply (c_t^h).

$$c_t^h = \omega c_t^{h,u} + (1 - \omega) c_t^{h,c} \quad (2.56)$$

In the foreign consumption goods market equilibrium, the supply (imports) of foreign consumption goods (c_t^f) equal the demand from households ($c_t^{f,c}$) and financial investors ($c_t^{f,u}$) for foreign consumption goods.

$$c_t^f = \omega c_t^{f,u} + (1 - \omega) c_t^{f,c} \quad (2.57)$$

The goods market is in equilibrium when the total output of firms, $(1 - \omega)y_t$, equal the sum of home consumption good demand, export demand (see Equation 2.33), government spending (see Equation 2.48) and real resource cost on monitoring and monetary transactions (see Equation 2.15)⁵.

$$(1 - \omega)y_t = c_t^h + g_t + y_t^f + \varsigma(\mathcal{M}_t + \mathcal{T}_t) \quad (2.58)$$

Next, we have the balance of payments identity in the below equation which indicates that the trade balance measures the change in the net foreign liabilities position of the economy. The difference between the foreign bond liability (b_t^{u*}) of the financial investors and the foreign reserve assets (f_t) of the central bank

⁵The market clearing condition for GDP slightly differs from the output (y_t) clearing condition. GDP identity in Equation 2.49 does not include the real resource cost component that is present in y_t

reflects the net foreign liability of the economy. c_t^f and y_t^f denote imports and exports, respectively in trade balance and q_t denotes real exchange rate.

$$q_t(\omega b_t^{u*} - f_t) = q_t r_{t-1}^*(\omega b_{t-1}^{u*} - f_{t-1}) + q_t c_t^f - y_t^f \quad (2.59)$$

In the labour market, the labour supply from households and financial investors is assumed to be homogeneous. The market clears when labour demand from firms equal labour supply from both households and financial investors.

$$h_t = \omega h_t^u + (1 - \omega) h_t^c \quad (2.60)$$

The market for domestic government bonds and CBDC clear when the demand for bonds from financial investors and the demand for CBDC from households equal the government's supply of bonds and CBDC, respectively.

$$b_t^g = \omega b_t^u \quad (2.61)$$

$$m_t^g = (1 - \omega) m_t^c \quad (2.62)$$

In the banking sector, we have equilibrium conditions associated with deposits and loans. The supply of deposits from banks equal the demand for deposits from both households and financial investors. On the other hand, loan market clears when the banks' loan supply covers the households' demand for credit.

$$d_t = \omega d_t^u + (1 - \omega) d_t^c \quad (2.63)$$

$$\ell_t = (1 - \omega) \ell_t^c \quad (2.64)$$

2.2.9 Shocks

The exogenous variables like labour productivity, world output and foreign interest rate, $\mathbf{e}_t = \{A_t, y_t^*, r_t^*\}$, are assumed to follow autoregressive process of order 1.

$$\log \mathbf{e}_t = \rho_{\mathbf{e}} \log(\mathbf{e}_t) + (1 - \rho_{\mathbf{e}}) \log(\bar{\mathbf{e}}) + \xi_{\mathbf{e},t}; \xi_{\mathbf{e},t} \sim N(0, \sigma_{\mathbf{e}}^2) \quad (2.65)$$

2.2.10 Welfare

Following Schmitt-Grohé and Uribe (2007), the welfare function is given by:

$$\mathcal{W}_{i,t} = u_{i,t} + \beta_i E_t \mathcal{W}_{i,t+1}, i \in c, u \quad (2.66)$$

where $u_{i,t}$ is the utility function and i denotes whether the agent is the household or the financial investor. The weighted average of the welfare of the households and the financial investors is the economy's social welfare and given by:

$$\tilde{\mathcal{W}}_t \equiv \omega \mathcal{W}_{c,t} + (1 - \omega) \mathcal{W}_{u,t} \quad (2.67)$$

2.3 Calibration

We solve the the model numerically, and then generate the impulse responses and calculate the welfare function using Dynare. The parameter values and the corresponding sources are reported in Table 2.1. The values of the welfare measures in Equation 2.67 are obtained as the unconditional means of $\mathcal{W}_{c,t}$, $\mathcal{W}_{u,t}$ and $\tilde{\mathcal{W}}_t$. In the simulations, the DSGE model is solved at the second order. We

assume that the policy maker's objective is to optimise the policy coefficients to achieve the highest level of social welfare, $\tilde{W}$.

It is our objective to understand the implications of CBDC in a small open economy where the functions of the central bank include price stability, exchange rate targeting and/or controlling the rate of exchange rate movements. To this end, we make comparisons using the following three types of monetary policy settings:

(i) **Baseline regime**, in which the monetary policy of the central bank only follows the conventional interest rate rule in Equation 2.52. The nominal CBDC interest rate and real FX reserves are fixed at their steady state levels. CBDC is analogous to cash in the baseline regime.

$$\begin{aligned} i_{m,t} &= \overline{i_m} \\ f_t &= \overline{f} \end{aligned} \tag{2.68}$$

(iii) **Sterilised FX intervention regime**, where the central bank uses sterilised FX intervention in Equation 2.54 to smooth and target real exchange rate in addition to the conventional interest rate rule in Equation 2.52. Nominal CBDC interest rate is fixed at its steady state level.

$$i_{m,t} = \overline{i_m} \tag{2.69}$$

(iv) **CBDC with sterilised FX intervention regime**, in which the central bank follows nominal interest rate rule in Equation 2.52, CBDC rule in Equation 2.53 and FX rule in 2.54.

The parametrisation of each of the monetary policy regimes are given in Ta-

ble 2.2. The values are selected on the basis of welfare and instrument volatility, which is detailed in section 2.4.1.

2.4 Results

2.4.1 Optimal coefficient values from welfare and policy instrument volatility

We determine the coefficients of the nominal interest rate (Equation 2.52) and fiscal policy rule (Equation 2.50) by conducting a four dimensional grid search for the coefficients i_i , w_π , w_y and d^{gdp} to maximise the social welfare of the economy ($\tilde{W}_t$) in the baseline regime. In the optimisation for the baseline regime, we consider the ranges of 0 to 0.9 for i_i , 1 to 3 for w_π and 0 to 3 for w_y and 0 to 2 for d^{gdp} . We keep the upper limit of the coefficients w_π and w_y at 3. Schmitt-Grohé and Uribe (2007) find policy coefficients greater than 3 to be challenging to convey to policymakers. The upper limit of d^{gdp} is kept at 2 following the welfare results in Chapter 1. The maximum welfare for the baseline regime is achieved at $i_i = 0.9$, $w_\pi = 1$, $w_y = 0.9$ and $d^{gdp} = 2$.

Our objective is to find coefficient values that are not only welfare enhancing but also practically feasible. Hence, we now look at plots of social welfare and the standard deviations (SD) of policy instruments [henceforth welfare-SD curve] in our selection of coefficient values. Figure 2.2a and Figure 2.2b correspond to welfare SD curves of coefficients in the nominal policy interest rate rule. Figure 2.2a plots the welfare SD curve for different values of w_π indicated by the data labels on the curve. We keep the values of w_y and d^{gdp} at their optimal values of 0.9 and 2, respectively; while estimating the welfare-SD

curve in Figure 2.2a. In the figure, we can see that the welfare declines and SD of the nominal interest rate increases when w_π increases. Next, we look at Figure 2.2b which plots the welfare SD curve for different values of w_y . We find that welfare increase and SD of nominal interest rate decline as w_y increases from 0.7 to 0.9. In the next step, we look at the welfare SD curve of adjusted budget deficit (gdx^{rat}) for different values of coefficient d^{gdp} in Figure 2.2c. The rise in social welfare is accompanied by a rise in SD of adjusted budget deficit (gdx^{rat}) when d^{gdp} increases.

We next proceed to optimise the coefficients of sterilised FX intervention regime where the central bank uses the nominal interest rate rule (Equation 2.52) and FX rule (Equation 2.54). We fix i_i , w_π and d^{gdp} at their optimised values in the baseline regime. A three dimensional grid search over the range $\rho_f \in [0, 0.9]$, $w_e \in [0, 0.9]$ and $v_e \in [0, 0.9]$ find the social welfare maximised at $\rho_f = 0$, $w_e = 0.7$ and $v_e = 0.9$. We next examine the instrument volatility of foreign exchange reserves (f_t) through the welfare-SD curves in Figure 2.3. In Figure 2.3a, we can see that an increase in the value of w_e from 0 to 0.6 causes a gradual rise in social welfare and SD of foreign exchange reserves. Although welfare increases when w_e rises from 0.6 to 0.7, significant rise in FX reserves volatility accompanies this welfare increase. We keep the feasible range of w_e in $[0, 0.6]$. Figure 2.3b reveals social welfare and SD of foreign exchange reserves to rise when v_e increases. The rise in SD is steeper when v_e increases from 0.8 to 0.9. Hence, we keep the feasible range of v_e in $[0, 0.8]$. A three dimensional grid search in $\rho_f \in [0, 0.9]$, $v_e \in [0, 0.6]$ and $w_e \in [0, 0.8]$ gives the optimal coefficient values of the sterilised FX intervention regime at $\rho_f = 0$, $w_e = 0.6$ and $v_e = 0.8$.

The final step of welfare optimisation involves finding the optimal coefficients of the CBDC with sterilised FX intervention regime where the central bank

uses three monetary policy instruments; nominal interest rate (Equation 2.52), CBDC rule (Equation 2.53) and FX rule (Equation 2.54). A grid search of the coefficients $\rho_f \in [0, 0.9]$, $w_e \in [0, 0.9]$, $v_e \in [0, 0.9]$, $i_\pi^m \in [0, 0.8]$ and $w^{i_\pi^m} \in [0, 0.9]$ finds the social welfare maximised at $\rho_f = 0$, $w_e = 0.7$, $v_e = 0.9$, $i_\pi^m = 0.8$ and $w^{i_\pi^m} = 0.9$. We next examine the social welfare and SD of policy coefficients in Figure 2.4. The feasible range of w_e is limited to $[0, 0.6]$ as welfare improvement is accompanied by high FX reserves volatility when w_e increases from 0.6 to 0.7. Figure 2.4b shows the SD of foreign exchange reserves to rise steeply when v_e increases from 0.8 to 0.9. Thus, in line with the sterilised FX intervention regime, we keep the feasible range of v_e in 0 to 0.8. In Figure 2.4c, we find that the social welfare increases when i_π^m rises from 0 to 1.5. However, SD of the nominal CBDC rate declines only when i_π^m rises from 0 to 1.2. However, we keep the upper limit of i_π^m at 0.8 by following Barrdear and Kumhof (2016). A grid search of coefficients $\rho_f \in [0, 0.9]$, $w_e \in [0, 0.6]$, $v_e \in [0, 0.8]$, $i_\pi^m \in [0, 0.8]$ and $w^{i_\pi^m} \in [0, 0.9]$ gives the optimal values at $\rho_f = 0$, $w_e = 0.6$, $v_e = 0.8$, $i_\pi^m = 0.8$ and $w^{i_\pi^m} = 0.7$.

Table 2.2 provides the welfare values of all the regimes at their respective optimal coefficient values. The presence of monetary transaction costs in the consumption spending of households causes household welfare to be lower than that of financial investors. Financial investors do not incur monetary transaction costs. Table 2.2 also shows that social welfare improves in the FX intervention regime when compared to the baseline regime. Exchange rate management by the central bank through sterilised FX intervention enables households to enjoy a relatively more stable foreign goods consumption. Households prefer to use CBDC for consumption spending (both home and foreign) as it improves efficiency and lowers monetary transaction costs compared to deposits. Hence,

Table 2.2 reveals CBDC with sterilised FX intervention regime to enjoy the highest social welfare. This social welfare improvement is primarily driven by household welfare as they form 95% of the total labour force. The welfare of financial investors deteriorates the most in CBDC with sterilised FX intervention regime as they do not have access to CBDC.

Table 2.3 gives the volatility implications of the regimes in terms of key macroeconomic variables. When compared to the baseline regime, the volatility of all the key variables except for CPI inflation is lower in the FX intervention regime. Intervention in the FX market reduces the nominal policy interest rate's ability to maintain their price-stability credentials (Miguel A Savastano et al., 1997). Hence, CPI inflation volatility is higher in FX intervention regime. CPI inflation witnesses even higher volatility in CBDC with FX intervention regime. This could imply that FX intervention reduces the ability of both CBDC interest rate and policy interest rate to target CPI inflation. However, all the other key variables like GDP, real exchange rate and consumption are found to be the most stable in CBDC with FX intervention regime.

2.4.2 Impulse responses

In this section, we discuss the impulse responses under the three regimes following a domestic productivity shock, a foreign interest rate shock and a foreign demand shock in the economy.

Domestic productivity shock

Figure 2.5 shows the impulse responses of the key variables following a positive domestic productivity shock (0.01 basis points increase in domestic productivity). Higher productivity causes aggregate consumption to increase following

the shock. Increase in the supply of goods from higher productivity causes deflation in the economy. In spite of inflation lower than the target, the nominal policy interest rate rises on account of the productivity-driven GDP increase (see Equation 2.52). However, a rise in expected inflation causes the real policy interest rate to decline immediately after the shock. The lower real policy interest rate makes government bonds less attractive to financial investors and hence, the central bank exchanges government bonds for CBDC. Govt bond-GDP ratio decline in Figure 2.5.

Since the foreign interest rate is exogenous, the decline in the real policy interest rate causes the real exchange rate to depreciate (see Equation 2.45). The depreciation causes the price of imported consumer goods to rise. Hence, the economy moves from a substantial deflation to small inflation in period 2. Trade balance deteriorates initially on account of the initial productivity shock driven increase in the consumption of imported goods. However, trade balance starts to improve from the third period as exports increase and imports decline following the real exchange rate depreciation.

FX intervention regime and CBDC with FX intervention regime respond to real exchange rate depreciation (see Equation 2.54) by selling FX reserves. As the FX intervention is sterilised, any change in the FX reserves does not affect the nominal CBDC interest rate and nominal policy interest rate. Hence, there is no change in the monetary base on account of changes in FX reserves. Hence, there is not much difference between the FX reserves response in FX intervention regimes (with and without CBDC). The fall in FX reserves causes the risk premium to decline in the the regimes with FX intervention⁶. In contrast, the baseline witness an opposing effect (initial rise) in the risk premium.

⁶Henceforth, *regimes with FX intervention* refer to FX intervention regime and CBDC with FX intervention regime.

This rise in risk premium can be attributed to the real exchange rate depreciation (see Equation 2.41) as FX reserves are fixed at the steady state in the baseline regime. In the regimes with FX intervention, the risk premium decline reduces the size of the foreign interest payment liability on foreign debt. Hence, foreign debt eventually rises more (period 1 to period 5) in regimes utilising FX intervention.

CBDC with FX intervention regime also uses its additional monetary policy instrument of nominal CBDC interest rate to target CPI inflation. By Equation 2.53, nominal CBDC rate increases when CPI inflation falls below its target level. The resulting rise in real CBDC rate makes CBDC more attractive to households. CBDC with FX intervention regime witnesses a more massive increase in CBDC-GDP ratio when compared to other regimes. With bonds less attractive to financial investors (lower real policy rate) and CBDC more appealing to households (higher real CBDC rate), the central bank caters to the agents demand by issuing more CBDC in exchange for government bonds. From the central bank's balance sheet perspective, the rise (decline) of CBDC (government bonds) is relatively higher in the CBDC with FX intervention regime. On the other hand, the response of balance sheet asset through FX reserves (owing to FX intervention) is almost the same in regimes with FX intervention.

As households switch from deposits to CBDC, deposits and loan decline in all the regimes. In the CBDC with FX intervention regime, nominal deposit and lending rate initially rise by following the path of the nominal CBDC rate. The real deposit interest rate as well as the real lending interest rate decline as the rise in expected CPI inflation is higher than the increase in nominal deposit and lending interest rates. With lower real lending rates, households start to demand more loans. Figure 2.5 show bank loan/deposit to rise after the initial

decline. The rise in bank loan allow households to increase their consumption after period 5.

Foreign interest rate shock

Figure 2.6 shows the impulse responses of the key variables following a positive foreign interest rate shock (0.25 basis points increase in the foreign interest rate). The rise in foreign interest rate depreciates the real exchange rate (see Equation 2.45). CBDC with FX intervention regime witnesses a more stable real exchange rate when compared to the other regimes. Trade balance improves as the real exchange rate depreciation causes imports to decline and exports to increase. Imported consumption goods become more expensive, causing CPI inflation to rise. With the rise in CPI prices, the demand for consumption goods initially declines.

According to Equation 2.52, nominal policy interest rate responds to both current period inflation and expected inflation. Following the rise in CPI inflation, the response of nominal policy interest rate is muted due to the steep decline of expected CPI inflation (see period 2). The decline in expected CPI inflation also causes the real policy interest rate to rise in period 1. However, the surge in the real deposit rate is more than the increase in real policy interest rate. Hence, financial investors increase their share of deposits by reducing their portfolio composition of government bonds. Figure 2.6 shows that govt bond-GDP ratio decline after the shock.

Regimes with FX intervention respond to the exchange rate depreciation by selling the stock of the foreign exchange reserves (see Equation 2.54). Hence, these regimes witness a decline in risk premium due to the drop in FX reserves. The reduction in risk premium is relatively lower in the CBDC with FX in-

intervention regime on account of the more modest decline in FX reserves. In contrast, baseline regime witnesses a rise in the risk premium due to real exchange rate depreciation coupled with fixed FX reserves. Foreign debt declines in all the regimes as financial investors fear the increased liability of higher foreign interest payments (due to foreign interest rate shock). The fall in foreign debt is higher in the baseline regime as rise in exogenous foreign interest rate is accompanied by an increase in risk premium.

In the CBDC with FX intervention regime, CBDC nominal interest rate declines slightly as a response to CPI inflation deviation from the steady state. The rise in CPI inflation together with the big dip in expected inflation (see period 2 inflation in Figure 2.6) causes such a fizzled out response from nominal CBDC interest rate (see Equation 2.53). CBDC-GDP ratio drops, with the decline relatively lower in CBDC with FX intervention regime. The sharp rise in nominal CBDC interest rate in period 2 drives CBDC-GDP ratio upwards above the steady state. Across all the regimes, the central bank's balance sheet witnesses a decline in both asset (FX reserves) and liabilities (CBDC and bonds) following the foreign interest rate shock.

With the lower amount of CBDCs in circulation, households switch to bank deposits, increasing the demand for deposits. This, in turn, leads to a hike in the nominal deposit and nominal lending rate. As a consequence, real deposit and real lending rate also rise. The eventual decline in real lending rate leads to more household demand for bank loans around period 5. The rise in bank loans allows households to increase their consumption after an initial decline. The increase in consumption is sustained over several periods, as shown in Figure 2.6.

Export demand shock

Figure 2.7 shows the impulse responses of the key variables following a positive foreign demand shock (2 basis points increase in the export demand). Exports endogenously rise on account of the higher export demand. With an improvement in the current account balance, the real exchange rate appreciates initially causing import prices to decline. Lower import prices coupled with the rise in exports improve the trade balance of the economy. Increase in the aggregate demand for exports leads to a rise in the home prices, causing CPI inflation to rise ⁷.

The rise in CPI inflation causes the first monetary policy instrument of the nominal policy interest rate to initially rise under all the three regimes. As a consequence, the real policy interest rate also increases. Government bonds become more attractive to financial investors, which enables the central bank to issue more government bonds in exchange for CBDC. High real policy interest rate coupled with lower CBDC quantity leads to a decline in aggregate consumption.

FX intervention regime and CBDC with FX intervention regime responds to the exchange rate appreciation by buying FX reserves. The real exchange rate is found to be more stable in the regimes with FX intervention. Increase in the stock of FX reserves causes risk premium to increase, thereby increasing the foreign interest payment liability of financial investors. Hence, foreign debt eventually declines more in the regimes with FX intervention in comparison to the baseline regime.

CBDC with FX intervention regime uses the adjustable CBDC interest rate

⁷The increase in home prices exceeds the decline in import prices following the shock (not shown).

to combat inflation. CBDC nominal interest rate declines as CPI inflation rises above the target. As households switch from CBDC to deposits (CBDC GDP ratio decline in Figure 2.7), aggregate bank deposits initially increase in the economy. Figure 2.7 reveals that real policy interest rate declines from period 2 on wards. This forces financial investors to reduce their bond holding and result in the central bank exchanging CBDC for government bonds. Hence the CBDC GDP ratio rises eventually. As households switch back to the now more available CBDC, there is a sustained uptick in aggregate consumption (from period 2 to around period 12). Higher demand for imported consumption goods due to rising CBDC quantity turns the trade balance from surplus into deficit around period 5.

2.5 Sensitivity analysis

We explore the policy effectiveness in the CBDC with FX intervention regime when the exchange rate pass through to CPI inflation increases. We perform a sensitivity analyses on the economy's responses following a foreign interest rate shock by varying the value of α .

Figure 2.8 shows the impulse responses for α equals 0.2, 0.4 and 0.642 for the CBDC with FX intervention regime with all other calibration parameters remaining the same as in Table 2.1 and policy coefficient values as in Table 2.2. We find that exchange rate and in response, FX reserves are more volatile in the initial periods after the shock for higher values of α . Larger depreciation of exchange rate causes CPI inflation to rise higher when $\alpha = 0.4$ and $\alpha = 0.642$. Next, we see that the nominal policy interest rate (which responds to both output and inflation) declines steeply initially for three periods when

$\alpha = 0.4, 0.642$. Finally, CBDC interest rate which responds counter-cyclically to CPI inflation is more volatile for higher values of α .

2.6 Conclusion

Within the context of EME central banks preference to adopt CBDC, our current study contributes to the policy research regarding the impact of integrating CBDC in the monetary policy framework of EMEs. We find that the CBDC interest rate could complement the exchange rate management mandate of sterilised FX intervention. We also show that in an economy which follows sterilised FX intervention, society's welfare is higher when the central bank uses an adjustable CBDC interest rate than a regime with fixed CBDC interest rate (analogous to cash). However, there are distributional consequences as we find that those with access to CBDCs (households) are made better off while those without access to CBDCs (financial investors) are made worse off when CBDC interest rate is adjustable. We also find that the key macroeconomic variables are more stable when there are shocks to the economy. However, we also find that CPI inflation is more volatile as exchange rate management becomes more effective under an adjustable CBDC interest rate.

Table 2.1: Parameter values

Parameter	Value	Description	Source
A_c	0.2425	Monetary transaction cost function parameter	Barrdear and Kumhof (2016)
B_c	0.0253	Monetary transaction cost function parameter	Barrdear and Kumhof (2016)
β_c	0.9450	Discount factor (households)	Author's choice
β_u	0.9760	Discount factor (financial investors)	Author's choice
φ_c	0.0100	Consumption loan adjustment costs	Benes et al. (2015)
χ	0.0117	Penalty parameter for breaching MCAR	Barrdear and Kumhof (2016)
δ^b	0.0573	Share of networth paid as dividends	Barrdear and Kumhof (2016)
η	1.0000	Labour supply elasticity	Barrdear and Kumhof (2016)
ι	0.9629	Share of lump sum income recieved by households	Barrdear and Kumhof (2016)
κ^f	1.0000	Financial collateral coefficient	Barrdear and Kumhof (2016)
κ^r	0.8786	Real collateral coefficient	Barrdear and Kumhof (2016)
ω	0.0500	Proportion of financial investors	Barrdear and Kumhof (2016)
ϕ_b	0.02	Elasticity of real risk free policy rate with respect to government debt GDP ratio	Author's choice
ψ_f	0.0114	Utility weight of liquidity	Barrdear and Kumhof (2016)
ψ_h	0.5798	Utility weight of hours worked	Barrdear and Kumhof (2016)
s_g	0.1800	Share of GDP in govt spending	Barrdear and Kumhof (2016)
σ_b	0.0300	Bank riskiness parameter	Author's choice
σ_c	0.6327	Collateral asset value riskiness parameter	Barrdear and Kumhof (2016)
θ	0.9500	Liquidity generating function parameter	Barrdear and Kumhof (2016)
ϕ	0.500	Price stickiness parameter	Author's choice
v	0.7000	Consumption habit	Barrdear and Kumhof (2016)
Υ	0.0800	MCAR	Barrdear and Kumhof (2016)
ς	0.2500	Share of monitoring costs and monetary transaction cost in goods market clearing	Barrdear and Kumhof (2016)
ϑ	1.5000	Interest semi-Elasticities of financial investors Deposit Demand	Author's choice
ξ	0.0585	Proportion of collateral value lost through bankrupt borrowers	Barrdear and Kumhof (2016)
ζ^c	0.7500	Risk weight of consumption loan	Barrdear and Kumhof (2016)
T^{fintec}	1.1530	Technical efficiency parameter of the technology used by CBDC	Barrdear and Kumhof (2016)
sp	1.0153	Steady state interest rate spread between government bonds and CBDC	Author's choice
φ_e^f	0.1	Risk premium elasticity parameter	Benes et al. (2015)
$\varphi_e^{\delta^{**}}$	0.00001	Risk premium elasticity parameter	Author's choice
α	0.642	openness parameter	Xie et al. (2017)
ρ_{exp}	0.8000	Autoregression coefficient of exports shock	Author's choice
ρ_{r**}	0.8000	Autoregression coefficient of foreign interest rate shock	Author's choice
ρ_a	0.8000	Autoregression coefficient of domestic productivity shock	Author's choice

Table 2.2: Coefficients of optimal policies

	i_i	w_π	w_y	d^{gdp}	ρ_f	w_f	v_f	i_{mpi}	$w^{i_{mpi}}$		Welfare		
											Society	Household	Financial Investor
Baseline		0.9	1	0.9	2	-	-	-	-	-	-1.7302	-1.8524	0.5901
FX intervention		0.9	1	0.9	2	0	0.6	0.8	-	-	-1.7094	-1.8282	0.5473
CBDC with FX intervention		0.9	1	0.9	2	0	0.6	0.8	0.8	0.7	-1.7066	-1.8240	0.5222

Table 2.3: Volatility of key variables

	Baseline	FX intervention	CBDC with FX intervention
GDP	0.0148	0.0134	0.0114
Consumption	0.0890	0.0862	0.0696
CPI Inflation	0.0080	0.0081	0.0082
Exchange rate	0.0168	0.0154	0.0131

Note: This table reports the standard deviations of key variables obtained from a stochastic simulation of the models using second-order perturbations around the stochastic steady state in Dynare 4.5.3. Variables are expressed in percentage deviations from the steady states.

Figure 2.1: Model outline

Arrows indicate directions of fund flows between sectors

Figure 2.2: Baseline regime welfare-SD curve

w_π and w_y are the inflation and output feedback coefficients of the nominal policy rate rule respectively and d^{gdp} is the response coefficient of the fiscal policy rule.

Figure 2.3: Sterilised FX intervention regime welfare-SD curve

w_e and v_e are the responses of FX reserves to deviations from fixed exchange rate target and exchange rate depreciation respectively.

Figure 2.4: CBDC with sterilised FX intervention regime welfare-SD curve

w_e and v_e are the responses of FX reserves to deviations from fixed exchange rate target and exchange rate depreciation respectively. i_π^m is the inflation feedback coefficient in the CBDC rule.

Figure 2.5: Productivity shock

Impulse response of variables to a 0.01 basis point increase in productivity; Baseline vs. FX intervention vs. CBDC with FX intervention; x-axis in quarters; y-axis in deviation from steady state.

Figure 2.6: Foreign interest rate shock

Impulse response to a 0.25 basis point increase in foreign interest rate; Baseline vs. FX intervention vs. CBDC with FX intervention; x-axis in quarters; y-axis in deviation from steady state.

Figure 2.7: Foreign demand shock

Impulse response to a 2 basis point increase in foreign export demand; Baseline vs. FX intervention vs. CBDC with FX intervention; x-axis in quarters; y-axis in deviation from steady state.

Figure 2.8: Effect of varying α in CBDC with FX intervention regime

Impulse response to a 0.25 basis point increase in foreign interest rate; $\alpha = 0.2$ vs. $\alpha = 0.4$ vs. $\alpha = 0.642$; x-axis in quarters; y-axis in deviation from steady state.

Chapter 3

Asset Prices, Growth and Wage inertia

3.1 Introduction

The paradigm shift from consumption based asset pricing models to the more recent production based asset pricing models pioneered by Cochrane (1991) and Jermann (1998) has mostly ignored the dynamics of the labour market. Recent papers by Tallarini (2000), Kaltenbrunner and Lochstoer (2010) and Croce (2014) have used non trivial production sectors to explain asset pricing moments, but have assumed a frictionless labour market so wage equals marginal productivity of labour. This could have far fetching implications on the asset pricing as dynamics of wage form an integral part of the value of the firm. For example, in standard frictionless labour models, dividends equal profits minus investment, and profits equal output minus wages. Since wages equal the marginal product of labour, it is procyclical as output. This makes prof-

its smoother and dividends acyclical or countercyclical contradicting empirical evidence¹.

The few studies which have integrated the more realistic wage dynamics in the asset pricing models have mostly assumed wage-setting subject to nominal rigidities. Li and Palomino (2014) belong to this category in the labour-asset pricing literature. They show that in the Kaltenbrunner and Lochstoer (2010) model of permanent and transitory technology shocks, the absence or presence of nominal rigidities – nominal rigidities are modeled following the Calvo (1983) staggered price and wage setting – can determine whether the risk premium is negative or positive, respectively. Though the model captures key macroeconomic moments and the Sharpe ratio of stock returns, equity premium falls short of its empirical counterpart. Favilukis and Lin (2016) consider a different variant of wage modeling in terms of infrequent wage resetting where average wage paid by firms is taken to be equal to the weighted average of historical spot wages. On account of this wage modeling, the average wage becomes smoother than the marginal product of labour. They find that the profit and dividend behavior look very much like the data. However, the volatility of equity and the value premium fall short of the data.

While Li and Palomino (2014) and Favilukis and Lin (2016) consider labour supply as exogenous, a recent paper by Donadelli and Grüning (2016) departs from this framework by considering endogenous labour supply in addition to wage rigidities. Donadelli and Grüning (2016) find that including endogenous labour supply leads to higher aggregate risk as labour becomes highly procyclical leading to a rise of about 250 basis points in the risk premia. However, these

¹Please refer to Favilukis and Lin (2016) among others.

models have been criticised for simply assuming wage inertia especially since wage inertia itself is what drives the results.

Instead of simply assuming wage inertia, search and matching labour market models derive wage inertia as an equilibrium outcome. As part of a recent emerging literature on asset pricing, Petrosky-Nadeau et al. (2018) integrate the standard Diamond-Mortensen-Pissarides (DMP) labour search and matching model to the asset pricing framework to avoid the common pitfall of countercyclical dividends in standard asset pricing models². They use the labour search and matching framework to delink wages from the marginal product of labour to explain why wages fall during bad times, but not as much as the fall in output. This causes profits to drop disproportionately and makes the dividends more procyclical leading to higher equity premium. However, the risk free rate is higher than in the data and volatility of interest rate falls short of the data. Kuehn et al. (2017) develop a partial equilibrium labour search model and show that labour search frictions are vital in determining cross-section equity returns. For tractability, they use a partial equilibrium asset pricing model focusing on a labour market model with labour as the only input for production, and its implications on stock returns. They do not model endogenous labour supply decisions from households but instead consider the pricing kernels as exogenous. However, an endogenous pricing kernel would capture preferences concerning uncertainty about long-run growth prospects as mentioned by Croce (2014) and Kung and Schmid (2015). With such preferences, households fear that persistent downturns in economic growth are accompanied by low asset valuations and command high-risk premia in asset markets, explaining the equity premium puzzle. Apart from pricing kernel, partial equilibrium labour

²The standard search and matching model in the labour market was developed by Diamond (1982), Mortensen (1982) and Pissarides (1985)

asset pricing model would ignore the contribution of the returns on physical capital on equity premium. Hence, a general equilibrium production based asset pricing model with both capital and labour can provide a unified framework to explain both asset pricing and macroeconomic dynamics. In this context, Branger et al. (2016) consider the asset pricing implications of DMP search and matching model in an endogenous growth model of Kung and Schmid (2015). However, they assume vacancy posting cost to be fixed and equal to one and their model is able to explain only 60 percent of the equity premium in the data. In addition, DMP search models are subject to Shimer (2005)'s critique that higher labour productivity will have small effects on unemployment, vacancy and job finding rates since the increase in productivity is absorbed by higher wages arising from the increase in workers' threat point in wage bargaining as unemployment duration decreases with lower unemployment and higher vacancy rates.

We examine the implications on asset prices and macroeconomic aggregates of incorporating labour market frictions in a production-based, asset-pricing, general equilibrium framework of Kung and Schmid (2015). Through endogenous creation of new patented technologies through R&D, Kung and Schmid (2015) model small but persistent component in the growth rate of productivity (long run productivity shocks). Hence, they provide production-based micro foundations for long run risks proposed by Bansal and Yaron (2004). While other papers like Gârleanu et al. (2009) and Gârleanu et al. (2012) examine the link between technological growth and asset prices, the arrival of new technologies is assumed to be exogenous. In comparison, Kung and Schmid (2015) examine the asset pricing and growth implications of the endogenous creation of new technologies through R&D. Kung and Schmid (2015) is the most recent

effort at studying asset pricing puzzles within a DSGE framework. Another advantage of Kung and Schmid (2015) model lies in its ability to jointly capturing the dynamics of aggregate quantities and asset markets.

In this paper, we study the labour market implications in the production based asset pricing economy of Kung and Schmid (2015). Our paper differs from other labour asset pricing papers on account of the specification of the search and matching framework of the labour market. We pursue a variant of Christiano et al. (2016) approach to labour market in contrast to den Haan et al. (2000). Instead of simply assuming wage inertia, Christiano et al. (2016) derives wage inertia as an equilibrium outcome. Christiano et al. (2016) performed the novel idea of integrating search and matching model labour market into a New Keynesian economic framework. They capture the key empirical properties of labour market variables like job finding rate, vacancies and unemployment of United States using the alternate offer bargaining (AOB) framework. This is a significant improvement from the other new Keynesian models that build on Diamond (1982), Mortensen (1982) and Pissarides (1985) who have difficulty accounting for the volatility of labor markets (Shimer, 2005).

Our motivation for using Christiano et al. (2016) labour market specification lies with the objective of using different bargaining regimes like alternate offer sharing rule in addition to Nash sharing rule to model rent sharing between firm and its workforce to see the impact on asset prices. Christiano et al. (2016) finds that AOB framework is not subject to Shimer (2005) critique of search and matching models (unlike Nash bargaining) and makes the unemployment volatility of the model more consistent with its empirical counterparts. For comparison, we also consider labour market friction which assumes nominal wage rigidities as in Uhlig (2007) and endogenous labour supply following Donadelli

and Grüning (2016), and the frictionless labour market in Kung and Schmid (2015)³.

We find that following a positive productivity shock, the models with wage inertia show slight improvements to the benchmark KS model compared with US data in terms of moments like proportion of standard deviation of output explained by standard deviation of dividends, correlation of dividends and output, correlation of dividend growth and output growth, equity premium, standard deviation of risk free rate. For the search and matching labour market models, the AOB shows labour market moments closer to the data than the Nash bargaining consistent with the findings of Christiano et al. (2016). However, the AOB labour market framework shows asset pricing moments including the standard deviations of the growth of dividends and the correlation between dividend and output, which are much closer to the data compared with the benchmark KS model and the other types of models of wage inertia. However, while the standard deviation of the price to dividend is underestimated in models with other types of wage inertia, it is overestimated by 78% in the AOB model compared to the data. The impulse responses show that under AOB, wages rise the least, as labour, vacancies and unemployment are more responsive following a positive productivity shock. Hence, the response of dividends is larger – and closer to the data – under AOB than the benchmark KS model or the models with other types of wage inertia.

The higher asset pricing moments in our model arise from endogenous long run risk associated with consumption in the Kung and Schmid (2015) model which is aggravated by the higher unemployment volatility in the AOB framework of Christiano et al. (2016). In the Kung and Schmid (2015) model, the

³Christiano et al. (2016) do not consider the impact of alternate offer bargaining on asset prices.

long run risk is endogenously created by a persistent component of economic growth. With recursive preferences, people fear long-term low growth and economic downturn will lower asset prices, and require a higher risk premium to hold assets even with frictionless labour markets. Incorporating search and matching labour market frictions of the AOB model into the Kung and Schmid (2015) model means economic downturns and low growth are accompanied by high unemployment. The high unemployment rate creates cash flow risk with respect to wages during economic downturn for the household. In order to smooth consumption during periods of high unemployment, household agents would require an even higher equity premium to hold assets compared to the benchmark KS model.

The paper is organised as follows: section 3.1 describes the benchmark KS model without labour market friction and modifications of the KS model with various types of labour market frictions; section 3.3 describes the calibration of the models; Sections section 3.4 discuss the labour market dynamics, aggregate quantity dynamics and asset pricing implications, respectively: and section 3.5 concludes.

3.2 Model

This section embeds the Kung and Schmid (2015) model with wage rigidities, Nash bargaining and alternate offer bargaining frameworks of the search and matching frictions in the labour market.

In the KS model, the labour market is perfectly competitive (i.e. wages equal marginal productivity of labour) resulting in 1) counter-factual highly pro-cyclical swings in wages, and 2) the inability of the model to capture the

dynamics of dividends. Hence, we introduce labour market frictions in the model through wage rigidities (Donadelli and Grüning, 2016) and search and matching framework of labour market (Christiano et al., 2016). The search and matching labour market model includes alternate offer bargaining (henceforth AOB) and the Nash bargaining framework by Christiano et al. (2016). We compare the KS models with labour market frictions to see which of the models could better account for key properties of asset pricing and macroeconomic aggregates, including labour market variables.

3.2.1 Benchmark KS model without labour market frictions

Our benchmark model is the Kung and Schmid (2015) (henceforth KS) model.⁴ Final goods in the economy are produced by the final goods sector using capital, labour and intermediate goods. Final goods sector receive the intermediate good input from a monopolistic competitive intermediate goods sector. There are i varieties of intermediate goods produced by the intermediate goods sector. Newer varieties of intermediate goods are developed by the research and development (R&D) sector, who sells the associated production patents to the intermediate goods sector. R&D sector driven innovation leads to a sustained growth of intermediate goods variety patents. Long run risk endogenously arises due to this sustained growth, which affects the expected consumption growth from Epstein-Zin utility preferences⁵.

⁴Benchmark model is the Kung and Schmid (2015) asset pricing model. We have made no extensions to Kung and Schmid (2015) in the benchmark model.

⁵Subsection 3.2.1 mostly follows the presentation and notations of Kung and Schmid (2015) model in the earlier versions of their paper particularly their section on "benchmark endogenous growth model (ENDO)."

Final goods sector

The representative firm uses capital K_t , labour L_t and a composite of intermediate goods G_t in the below production function to supply final goods for consumption

$$Y_t = (K_t^\alpha (\Omega_t L_t)^{1-\alpha})^{1-\xi} G_t^\xi \quad (3.1)$$

G_t is the intermediate good bundle given by

$$G_t \equiv \left[\int_0^{N_t} X_{i,t}^{\frac{1}{\nu}} di \right]^\nu \quad (3.2)$$

where $X_{i,t}$ is intermediate good $i \in [0, N_t]$. α and ξ denote the share of capital and intermediate goods respectively in the production function. ν refers to the elasticity of substitution among i varieties of intermediate goods.

The objective of the firm is to maximise shareholder's wealth

$$\max_{\{I_t, L_t, K_{t+1}, X_{i,t}\}_{t \geq 0, i \in [0, N_t]}} E_0 \left[\sum_{t=0}^{\infty} M_t D_t \right]$$

where M_t is the stochastic discount factor and D_t is the firm's dividends. Dividend is given by output (Y_t) minus capital investment (I_t), wage expenses ($W_t L_t$) and intermediate goods expenses where $P_{i,t}$ is the price per unit of intermediate good i .

$$D_t = Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t} di$$

The evolution of capital stock is given by

$$K_{t+1} = (1 - \delta)K_t + \Lambda\left(\frac{I_t}{K_t}\right) K_t \quad (3.3)$$

where δ denotes the depreciation rate of capital. The capital adjustment cost function $\Lambda\left(\frac{I_t}{K_t}\right)$ is given by Jermann (1998).

$$\Lambda\left(\frac{I_t}{K_t}\right) \equiv \frac{\alpha_1}{1 - \frac{1}{\zeta}} \left(\frac{I_t}{K_t}\right)^{1 - \frac{1}{\zeta}} + \alpha_2 \quad (3.4)$$

where parameter ζ refers to the elasticity of the investment rate. When $\zeta \rightarrow 0$, capital adjustment cost is infinite. On the other hand, $\zeta \rightarrow \infty$ implies frictionless adjustment. α_1 and α_2 are specified so that there are no adjustment costs in the steady state.⁶

The Lagrangian for maximizing shareholder wealth is

$$\mathcal{L} = E_0 \left[\sum_{t=0}^{\infty} M_t \left\{ Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t} di + q_t \left((1 - \delta)K_t + \Lambda\left(\frac{I_t}{K_t}\right) K_t - K_{t+1} \right) \right\} \right] \quad (3.5)$$

The resulting first order conditions are

$$\begin{aligned} I_t : -1 + q_t \cdot \Lambda'\left(\frac{I_t}{K_t}\right) &= 0 \\ L_t : (1 - \alpha)(1 - \xi) \frac{Y_t}{L_t} - W_t &= 0 \\ X_{i,t} : (K_t^\alpha (\Omega_t L_t)^{1-\alpha})^{1-\xi} \nu \xi \left[\int_0^{N_t} X_{i,t}^{\frac{1}{\nu}} di \right]^{\nu\xi-1} \frac{1}{\nu} X_{i,t}^{\frac{1}{\nu}-1} - P_{i,t} &= 0 \\ K_{t+1} : E_t \left[M_{t+1} \left\{ \alpha(1-\xi) \frac{Y_{t+1}}{K_{t+1}} + q_{t+1} \left((1-\delta) - \Lambda'\left(\frac{I_{t+1}}{K_{t+1}}\right) \cdot \left(\frac{I_{t+1}}{K_{t+1}}\right) + \Lambda\left(\frac{I_{t+1}}{K_{t+1}}\right) \right) \right\} \right] - q_t &= 0 \end{aligned}$$

⁶In the deterministic steady state with no adjustment cost, $\Lambda\left(\frac{\bar{I}}{\bar{K}}\right) = \frac{\bar{I}}{\bar{K}}$. Using the steady state version of Equation 3.3 and Equation 3.4, we can derive $\alpha_1 = (\overline{\Delta N} - 1 + \delta)^{\frac{1}{\zeta}}$ and $\alpha_2 = \frac{1}{\zeta-1}(1 - \delta - \overline{\Delta N})$. $\overline{\Delta N}$ pertains to the steady state growth rate of the economy which will be discussed further in section 3.2.1.

$\Lambda_t \equiv \Lambda \left(\frac{I_t}{K_t} \right)$ and $\Lambda'_t \equiv \Lambda' \left(\frac{I_t}{K_t} \right)$ are defined so as to simplify the notations. This changes the above equations to:

$$q_t = \frac{1}{\Lambda'_t} \quad (3.6)$$

$$W_t = (1 - \alpha)(1 - \xi) \frac{Y_t}{L_t} \quad (3.7)$$

$$1 = E_t \left[M_{t+1} \left\{ \frac{1}{q_t} \left(\alpha(1 - \xi) \frac{Y_{t+1}}{K_{t+1}} + q_{t+1}(1 - \delta) - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}\Lambda_{t+1} \right) \right\} \right] \quad (3.8)$$

$$P_{i,t} = (K_t^\alpha (\Omega_t L_t)^{1-\alpha})^{1-\xi} \nu \xi \left[\int_0^{N_t} X_{i,t}^{\frac{1}{\nu}} di \right]^{\nu\xi-1} \frac{1}{\nu} X_{i,t}^{\frac{1}{\nu}-1} \quad (3.9)$$

Intermediate goods sector

A monopolistic competitive firm i at time period t produces intermediate good $X_{i,t}$ where $i \in [0, N_t]$. According to the equilibrium condition with respect to $X_{i,t}$ (Equation 3.9), the demand for intermediate good i depends on price $P_{i,t}$. The monopolistically competitive i produces at unit cost by considering the demand equation for $X_{i,t}(P_{i,t})$ as given. The objective of intermediate good firm i is to maximise the below profit function.

$$\max_{P_{i,t}} \Pi_{i,t} \equiv P_{i,t} \cdot X_{i,t}(P_{i,t}) - X_{i,t}(P_{i,t})$$

Using the framework in Dixit and Stiglitz (1977), we obtain the following symmetric equilibrium conditions:

$$X_{i,t} = X_t \quad (3.10)$$

$$P_{i,t} = P_t = \nu \quad (3.11)$$

where $\nu > 1$ is the markup. Substituting Equation 3.10 and Equation 3.11 into the definition for G_t (Equation 3.2) and the first order condition with respect to $X_{i,t}$ (Equation 3.9), we get

$$G_t = N_t^\nu X_t \quad (3.12)$$

$$X_t = \left(\frac{\xi}{\nu} (K_t^\alpha (\Omega_t L_t)^{1-\alpha})^{1-\xi} N_t^{\nu\xi-1} \right)^{\frac{1}{1-\xi}} \quad (3.13)$$

Therefore, the profit $\Pi_{i,t}$ and dividend D_t become

$$\Pi_t = (\nu - 1)X_t \quad (3.14)$$

$$D_t = Y_t - I_t - W_t L_t - \nu N_t X_t \quad (3.15)$$

The sum of current year profits and the discounted future profits account for the value ($V_{i,t}$) of owning the patents to produce the intermediate good i .

$$V_{i,t} = \Pi_{i,t} + \phi E_t[M_{t+1} V_{i,t+1}]$$

where ϕ indicates the intermediate goods rate of survival. The symmetric equilibrium conditions are imposed so the i subscript could be dropped and the above is rewritten as

$$V_t = \Pi_t + \phi E_t[M_{t+1} V_{t+1}] \quad (3.16)$$

R&D sector

Innovation in the economy is driven by the R&D sector who develops new intermediate goods variety i and sells the new patent to firms in the intermediate goods sector. Evolution of the number of intermediate goods patents, N_t , is

given by:

$$N_{t+1} = \vartheta_t S_t + \phi N_t \quad (3.17)$$

where S_t refers to R&D expenditures and ϑ_t indicates the productivity of the R&D sector that is taken as given by the R&D firms. The functional form of ϑ_t is given by

$$\vartheta_t = \frac{\chi \cdot N_t}{S_t^{1-\eta} N_t^\eta} \quad (3.18)$$

where $\chi > 0$ refers to the scale parameter and $\eta \in [0, 1]$ denotes the elasticity of new intermediate goods with respect to R&D. The growth rate of patents provide the equilibrium foundation of long run risk in this model.

$$\Delta N_{t+1} = \vartheta_t \frac{S_t}{N_t} + \phi \quad (3.19)$$

Free entry into the R&D sector implies expected sales revenue equals costs:

$$E_t[M_{t+1} V_{t+1}](N_{t+1} - \phi N_t) = S_t$$

which could also be stated in terms of marginal cost equal expected marginal revenue:

$$\frac{1}{\vartheta_t} = E_t[M_{t+1} V_{t+1}]. \quad (3.20)$$

Forcing process

The stochastic process Ω_t provides the ground for exogenous stochastic productivity shocks to the model with dynamics:

$$\begin{aligned}\Omega_t &= e^{a_t} \\ a_t &= \rho a_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2)\end{aligned}$$

Condition for balanced growth

Substituting equations (3.12) and (3.13) into equation (3.1), yields the following expression for output:

$$Y_t = \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{1-\xi}} K_t^\alpha (\Omega_t L_t)^{1-\alpha} N_t^{\frac{\nu\xi-\xi}{1-\xi}} \quad (3.21)$$

The below restriction is in place to ensure balanced growth:

$$\alpha + \frac{\nu\xi - \xi}{1 - \xi} = 1$$

When this restriction is imposed, it implies the standard neoclassical production function with labour augmenting technology as follows:

$$Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha} \quad (3.22)$$

where Z_t is the total factor productivity (TFP):

$$Z_t \equiv \bar{A} \Omega_t N_t \quad (3.23)$$

and $\bar{A} \equiv \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{(1-\xi)(1-\alpha)}} > 0$ is a constant.

Household

The utility function of the household is assumed to follow Epstein-Zin preferences over consumption $\{C_t\}$

$$Util_t = \left\{ (1 - \beta)C_t^{1-\frac{1}{\psi}} + \beta \left(E_t[Util_{t+1}^{1-\gamma}] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad (3.24)$$

where β is the subjective discount rate, γ is the coefficient of risk aversion and ψ is the elasticity of intertemporal substitution. As in the long run risk literature, KS assume $\psi > \frac{1}{\gamma}$ so the agent prefers the early resolution of uncertainty. The preferences imply the stochastic discount factor

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\frac{-1}{\psi}} \left(\frac{Util_{t+1}}{E_t(Util_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi} - \gamma} \quad (3.25)$$

The budget constraint of the household is

$$C_t + V_{m,t}\mathcal{Z}_{t+1} + B_{t+1} = W_t L_t + (V_{m,t} + \mathcal{D}_t)\mathcal{Z}_t + (1 + r_{f,t})B_t \quad (3.26)$$

where $V_{m,t}$ is the stock price, $\mathcal{Z}_t$ is the household's stock holding which pays aggregate dividend $\mathcal{D}_t$, B_t indicates bonds, $r_{f,t}$ is the risk free rate, W_t is the wage rate and L_t is the hours worked.

Following Kung and Schmid (2015), stocks are assumed to be claims to all the production sectors, such as the final good sector, the intermediate goods sector, and the R&D sector. Aggregate dividend is defined as the net payout from the production sector,

$$\mathcal{D}_t = D_t + \int_0^{N_t} \pi_{i,t} di - S_t \quad (3.27)$$

On account of symmetric equilibrium conditions, equation (3.27) could be rewritten as:

$$\mathcal{D}_t = D_t + N_t \pi_t - S_t \quad (3.28)$$

Market clearing

Final output is used for consumption, physical capital investment, purchasing intermediate goods, and R&D expenditure:

$$Y_t = C_t + I_t + N_t X_t + S_t \quad (3.29)$$

In the KS model without labour market friction and disutility for labour, households supply their whole labour so the labour force is normalised to one:

$$L_t = 1 \quad (3.30)$$

Asset prices

The dynamics of risk-free bond rate, the final good sector firm's stock price and the aggregate market's stock price are studied in the model. The risk-free rate is:

$$r_{f,t} = \ln(R_{f,t}), \quad R_{f,t} = \frac{1}{M_{t+1}}$$

Stock price, stock return and the risk premium of final goods sector firm are given by:

$$\begin{aligned}
V_{d,t} &= D_t + M_{t+1}V_{d,t+1}, \\
R_{d,t} &= \frac{V_{d,t}}{V_{d,t-1} - D_{t-1}}, \text{ and} \\
r_{d,t} - r_{f,t} &= (1 + l)(\ln(R_{d,t}) - rf, t),
\end{aligned} \tag{3.31}$$

respectively. The final goods sector excess return is leveraged by imposing $l = \frac{2}{3}$ as in Boldrin et al. (2001). Similarly, for aggregate market we have

$$\begin{aligned}
V_{m,t} &= \mathcal{D}_t + M_{t+1}V_{m,t+1}, \\
R_{m,t} &= \frac{V_{m,t}}{V_{m,t-1} - \mathcal{D}_{t-1}}, \\
r_{m,t} - r_{f,t} &= (1 + l)(\ln(R_{m,t}) - rf, t)
\end{aligned} \tag{3.32}$$

where $V_{m,t}$ is the aggregate stock price, $R_{m,t}$ is the aggregate stock return and $r_{m,t} - r_{f,t}$ is the risk premium of the aggregate market.

3.2.2 KS model with endogenous labour and assumed wage rigidity

Donadelli and Grüning (2016) incorporate wage rigidity into the KS model. They modify the Epstein-Zin preferences (see Equation 3.24) to include leisure, $leis_t$:

$$Util_t = \left\{ (1 - \beta)util_t^{1-\frac{1}{\psi}} + \beta \left(E_t[Util_{t+1}^{1-\gamma}] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} ; \quad util_t = C_t(leis_t)^\tau \quad (3.33)$$

where τ refers to the Frisch elasticity of labour supply, leisure is one minus the hours of work, ($leis_t = 1 - L_t$). The optimal (hours of) labour supply is endogenously determined from the behavior of household from the below equation.

$$W_t^u = \frac{\partial util_t / \partial L_t}{\partial util_t / \partial C_t} = \frac{\tau C_t}{leis_t} \quad (3.34)$$

where W_t^u refers to optimal or frictionless wages. Donadelli and Grüning (2016) follow Uhlig (2007) in assuming that wages are sticky and evolve as

$$W_t = (W_{t-1})^\mu (W_t^u)^{1-\mu} \quad (3.35)$$

where $\mu \in [0, 1]$ is the fraction of sticky wages.

3.2.3 KS model with search and matching labour frictions

Instead of assuming wage stickiness as in the previous section (Equation 3.35), we extend the KS model with the search and matching labour market model specifications as described by Christiano et al. (2016) in this section. We con-

tinue with the Epstein-Zin preferences specified in section 3.2.1 (see Equation 3.24). However, we follow the definition of Christiano et al. (2016) in specifying L_t as the number of workers employed in period t with the law of motion given by

$$L_{t+1} = (\rho_1 + x_{1,t})L_t \quad (3.36)$$

where ρ_1 is the probability that a given match between the firm and the worker continues from one period to the next. Hence, $\rho_1 L_t$ refers to the number of workers that stays with the firm to period $t+1$. Also, $x_{1,t} L_t$ denotes the quantity of newly hired workers employed in period $t+1$. The timing convention of the law of motion of labour in equation (3.36) differs from Christiano et al. (2016), where the workers are engaged in production as soon as they are hired. We assume that when workers and firms meet, bargaining and work do not occur until the next period, which is standard in the labour literature ⁷.

In contrast to equation (3.30), population now includes both the employed and the unemployed in the work force so that

$$L_t + U_t = 1 \quad (3.37)$$

Hence, the number of workers searching for work in period t is the number of unemployed workers in period t , U_t or $1 - L_t$. The probability, f_t , that a period t unemployed worker is employed in period $t+1$ is given by the following equation:

$$f_t = \frac{x_{1,t} L_t}{1 - L_t} \quad (3.38)$$

If a firm decides in period t that it wants to meet a worker, then it must

⁷We use the framework of standard labour literature timing convention of alternate and Nash bargaining framework from the teaching website of Lawrence Christiano: http://faculty.wcas.northwestern.edu/~lchrist/d16/d1614/take_home_final_exam.pdf

post a vacancy in period t at a cost, s_t . The firm then meets a worker in period t with probability Q_t . The firm must also pay a cost κ_t upon meeting a worker. Work begins in $t + 1$ as soon as a match is formed. Let J_t denote the value of a worker to the firm.

$$J_t = \eta_{1,t} - W_t + \rho_1 E_t M_{t+1} J_{t+1} \quad (3.39)$$

where $\eta_{1,t} = (1 - \alpha)(1 - \xi) \frac{Y_t}{L_t}$ is the marginal revenue associated with an additional worker hired by the firm, W_t refers to wage, J_{t+1} is the value of worker to the firm in period $t + 1$ and M_{t+1} is the stochastic discount factor.

In equilibrium, the firm's cost of posting a vacancy, s_t , equals the firm's expected benefit of a vacancy (net of the firm's meeting cost):

$$s_t = Q_t (E_t M_{t+1} J_{t+1} - \kappa_t) \quad (3.40)$$

Letting V_t^{job} denote the value of a job to a worker that pays W_t in period t then

$$V_t^{job} = W_t + E_t M_{t+1} \left[\rho_1 V_{t+1}^{job} + (1 - \rho_1) V_{t+1}^{unem} \right] \quad (3.41)$$

The value of being an unemployed worker, V_t^{unem} is

$$V_t^{unem} = b_t + E_t M_{t+1} \left[f_t V_{t+1}^{job} + (1 - f_t) V_{t+1}^{unem} \right] \quad (3.42)$$

where b_t refers to the unemployment benefits.

The quantity of new hires, $x_{1,t} L_t$, is a function of the number of people searching for jobs in period t , $1 - L_t$ and the number of vacancies posted in t ,

$vac_t L_t$, based on the following matching function:

$$x_{1,t} L_t = \sigma_m (vac_t L_t)^{1-\sigma_1} (1 - L_t)^{\sigma_1}, \sigma_1 \in (0, 1) \quad (3.43)$$

where σ_1 and σ_m refer to the share of job searchers and level parameter respectively in the matching function.

The job finding rate for workers f_t and the vacancy filling rate Q_t are assumed to be related to labour market tightness, Γ_t , as

$$\Gamma_t = \frac{vac_t L_t}{1 - L_t} \quad (3.44)$$

$$f_t = \frac{x_{1,t} L_t}{1 - L_t} = \sigma_m \Gamma_t^{1-\sigma}, \quad Q_t = \frac{x_{1,t} L_t}{vac_t L_t} = \sigma_m \Gamma_t^{-\sigma} \quad (3.45)$$

Since a firm faces vacancy costs and hiring costs, the dividend specification changes from benchmark KS model equation (3.15) to the following

$$D_t = Y_t - I_t - W_t L_t - \left(\frac{s_t}{Q_t} + \kappa_t\right) x_{1,t} L_t - \nu N_t X_t \quad (3.46)$$

The household budget constraint now includes the unemployment benefit and lump sum taxes. Following Christiano et al. (2013), we assume that unemployment benefit b_t is provided by the government to an unemployed worker. The lump sum tax T_t paid by the household finances the unemployment benefit. The budget constraint changes from Equation 3.26 to the below equation.

$$C_t + V_{m,t} \mathcal{Z}_{t+1} + B_{t+1} = W_t L_t + b_t (1 - L_t) + (V_{m,t} + \mathcal{D}_t) \mathcal{Z}_t + (1 + r_{f,t}) B_t - T_t \quad (3.47)$$

Also, the market clearing condition from Equation 3.29 changes to incorpo-

rate hiring costs and vacancy costs.

$$Y_t = C_t + I_t + N_t X_t + S_t + \left(\frac{S_t}{Q_t} + \kappa_t\right) x_{1,t} L_t \quad (3.48)$$

Based on the above framework, we have two variants of wage bargaining: Nash wage bargaining framework and Alternate Offer wage Bargaining framework (henceforth AOB). The subsequent sections provide details of wage determination under both bargaining frameworks.

Wage determination: Nash Bargaining

Under Nash bargaining, the wage rate corresponds to the value of W_t which is determined by the following Nash sharing rule.

$$J_t = \frac{1 - \eta_{bargain}}{\eta_{bargain}} (V_t^{job} - V_t^{unem}) \quad (3.49)$$

where $\eta_{bargain}$ is the share of total surplus received by the worker.

Wage determination: Alternate Offer Bargaining

This section provides the details of alternate offer bargaining arrangements between firms and workers following Christiano et al. (2016). As mentioned above, the timing specification makes the definition of offers and counter offers in our model different from Christiano et al. (2016) to be consistent with the standard labour literature.

As soon as the meeting occurs between a firm and a worker, a wage offer is made by the firm. The worker can accept the offer or reject it. If he accepts it, he can start working right away. If he rejects the offer, he can a) terminate bargaining or b) he can commit to come back next period to make a counteroffer.

If he terminates bargaining, the worker only has the outside option. If he commits to comeback to make a counter offer, negotiations could break down with probability δ_1 in which case the firm and worker are limited to their outside options. The outside option for the worker is unemployment and the workers' value of being unemployed is denoted by V_t^{unem} . The firms' value of the outside option is zero. With probability $1 - \delta_1$, the worker returns the next period and makes a counter offer to the firm. The utility the worker receives by choosing to make a counter offer is called the disagreement payoff.

The firm's wage offer in period t is given by W_t^f . This could either be the firm's opening offer in period t or the counter offer to a worker's asking wage rejected by the firm in the previous period. In the same way, the worker's wage offer to the firm is denoted by W_t^w in case the worker had rejected the firm's wage offer in previous period.

To detail the bargaining problem, the firm would obviously prefer to propose the lowest possible wage. As it is pointless to offer a low wage which would trigger the worker to reject the offer, the firm proposes a wage that just makes the worker indifferent between accepting it and rejecting it in favor of making a counteroffer. The wage offered by the firm satisfies

$$V_t^{job,f} = \max \left\{ V_t^{unem}, \delta_1 V_t^{unem} + (1 - \delta_1) \left[b_t + E_t M_{t+1} V_{t+1}^{job,w} \right] \right\} \quad (3.50)$$

where

$$V_t^{job,f} = W_t^f + E_t M_{t+1} [\rho_1 V_{t+1}^{job} + (1 - \rho_1) V_{t+1}^{unem}]$$

$$V_t^{job,w} = W_t^w + E_t M_{t+1} [\rho_1 V_{t+1}^{job} + (1 - \rho_1) V_{t+1}^{unem}]$$

$V_t^{job,f}$ is the worker's value of being employed at wage W_t^f in period t . In a

similar manner, $V_t^{job,w}$ is the worker's value of being employed at wage W_t^w . He can be employed or unemployed in period $t + 1$ with probability ρ_1 and $1 - \rho_1$, respectively.

Equation 3.50 shows a maximum among two options; the worker's outside option and worker's disagreement payoff. In practice, it is assumed that the latter is greater than the former. Implicit in Equation 3.50 is the assumption that when the worker is indifferent between accepting and rejecting an offer, he accepts it.

On the part of worker, the asking wage, W_t^w , is offered to be as high as possible without triggering a rejection by the firm. That is,

$$J_t^w = \max \left\{ 0, \delta_1 \times 0 + (1 - \delta_1)[- \gamma_{1,t} + E_t M_{t+1} J_{t+1}^f] \right\} \quad (3.51)$$

where

$$J_t^f = \eta_{1,t} - W_t^f + \rho_1 E_t M_{t+1} J_{t+1}$$

$$J_t^w = \eta_{1,t} - W_t^w + \rho_1 E_t M_{t+1} J_{t+1}$$

J_t^f is the firm's value of hiring the worker at wage W_t^f in period t . In a similar manner, J_t^w is the firm's value of hiring the worker at wage W_t^w . With probability ρ_1 , the worker will continue being employed with the firm in period $t + 1$.

In Equation 3.51, δ_1 is the probability that negotiations breakdown and the firm is thrown to its outside option (which is simply zero). With probability $1 - \delta_1$, the firm gets to make a counteroffer at the start of the next period. The cost of making a counter offer is $-\gamma_{1,t}$ and the benefit from making a counteroffer is $E_t M_{t+1} J_{t+1}^f$. $E_t M_{t+1} J_{t+1}^f$ is the value of employing the worker at the wage

W_{t+1}^f in the next period when the firm makes the counter offer. Equation 3.51 also shows a maximum of firm's outside option and its disagreement payoff. It is assumed that the firm's disagreement payoff is higher than its outside option.

With the assumptions of no mistakes in bargaining and of perfect information, the firm's opening offer is accepted

$$W_t = W_t^f, J_t = J_t^f, V_t^{job} = V_t^{job,f} \quad (3.52)$$

$V_t^{job,w}$ and J_t^w can be written as

$$V_t^{job,w} = V_t^{job} + W_t^w - W_t = V_t^{job} + dW_t \quad (3.53)$$

$$J_t^w = J_t - (W_t^w - W_t) = J_t - dW_t \quad (3.54)$$

respectively, where

$$dW_t = W_t^w - W_t$$

Using equations (3.50), (3.52) and (3.53), we can rewrite the indifference condition that restricts firm's offer as

$$V_t^{job} = \delta_1 V_t^{unem} + (1 - \delta_1) \left[b_t + E_t M_{t+1} (V_{t+1}^{job} + dW_{t+1}) \right] \quad (3.55)$$

Substituting equations (3.52) and (3.54) in equation (3.51), we obtain the below functional form for dW_t

$$dW_t = J_t - (1 - \delta_1) [-\gamma_{1,t} + E_t M_{t+1} J_{t+1}] \quad (3.56)$$

Using equation (3.56) in (3.55), we get a dynamic version of the alternate

offer sharing rule in the following equation

$$\begin{aligned}
V_t^{job} &= \delta_1 V_t^{unem} \\
&+ (1 - \delta_1) \left[b_t + E_t M_{t+1} (V_{t+1}^{job} + J_{t+1} + (1 - \delta_1) \gamma_{1,t+1} - (1 - \delta_1) E_t M_{t+2} J_{t+2}) \right]
\end{aligned} \tag{3.57}$$

Balanced growth

The source of growth in the benchmark KS model comes from the growth of intermediate goods $\Delta N_t = \frac{N_t}{N_{t+1}}$. To ensure balanced growth in the non-stochastic steady state, all elements in $[s_t, \kappa_t, \gamma_{1,t}, b_t]$ are considered to grow at the rate ΔN_t in the steady state. Hence, we have the following specification:

$$[s_t, \kappa_t, \gamma_{1,t}, b_t]' = [s, \kappa, \gamma_1, b]' \Delta N_t \tag{3.58}$$

3.3 Calibration

Parameter values used to estimate the models are presented in Panel A, Panel B and Panel C of Table 3.1. Panel A shows the benchmark KS model parameters calibrated following Kung and Schmid (2015). These parameters are common across all labour market specifications except for the R&D productivity parameter, χ , which is slightly adjusted to obtain identical consumption growth rates across models. Panel B shows the parameters pertaining to the wage rigidity framework following Uhlig (2007). The labour elasticity parameter, τ , is derived from the condition that household works 0.7 of its time endowment in the

deterministic steady state. The wage rigidity parameter, μ , is taken from Uhlig (2007).

Panel C of Table 3.1 reports the parameter values of both Nash bargaining and AOB framework used in Christiano et al. (2016). The probability that negotiations break down after an offer is rejected, δ_1 , is set to 0.19% , the match survival rate, ρ_1 , is set to be 0.9 and the share of job searchers in the labour matching function, σ_1 , is set as 0.5. These values correspond to the calibration in Christiano et al. (2016). We set the bargaining weight of worker, $\eta_{bargain}$, to equal 0.2 following Hagedorn and Manovskii (2008). The derivations of remaining model parameters such as hiring costs κ and cost of making a counter offer γ_1 are implied by the steady state properties given by Christiano et al. (2016). The first property is a value of 1 percent for the steady state ratio of hiring costs to gross output. The second property states 0.59 as counteroffer costs as a share of daily revenue. The number of business days in a quarter is assumed to be 60. These steady state properties are given in Panel C.

Based on Christiano et al. (2016), we also give empirically plausible values of 0.055 and 0.7 to unemployment rate (U_t) and vacancy filling rate (Q_t), respectively at steady state. Since we have given exogenous values to unemployment and vacancy filling rate at the steady state, the estimation of vacancy costs (s) and level parameter in matching function (σ_m) becomes endogenous.

Using the parameters and implied steady state values, we solve the model using third order perturbations around the stochastic steady state in Dynare++ 4.5.3. Moments are obtained from 200 simulations of 58 years at quarterly frequency, time-aggregated to annual frequency.

3.4 Results

3.4.1 Labour Market Dynamics

Figure 3.1 shows that impulse responses of labour and of wages to a productivity shock vary across models. In the benchmark KS model, labour is normalised to one since it is always fully utilised by households. Hence, labour does not respond to a shock. In the model with endogenous labour and assumed wage rigidity, labour responds the most to a productivity shock. This is due to households' willingness to supply more labour with the increase in optimal wages and the eventual increase in their actual wages. Hence, households could fully exploit the increase in productivity. In the search and matching models of Nash bargaining and AOB labour market models, the responses of labour to the productivity shock are muted in the AOB model and even more so in the Nash bargaining model. As expected, wage responds the most to a productivity shock in the frictionless benchmark KS model, followed by the responses of wage in the assumed-wage-rigidity model and then in the Nash bargaining model. The wage responds to a productivity shock the least in the AOB model. The different responses of labour and wage in the Nash bargaining and AOB models are explained below using Figure 3.2.

Figure 3.2 shows only responses of unemployment, job vacancies and job finding rates to a productivity shock for Nash bargaining and AOB models—whereby the Nash bargaining model shows more muted responses compared with the AOB model. There are no impulse responses associated with vacancy, unemployment and job finding rates in the benchmark KS model since labour is normalised as one. The same applies to the wage rigidity model as the sum of labour—defined as hours of work—and leisure is equal to one.

The weak effects of a productivity shock on unemployment, vacancies and job finding rate in the Nash bargaining model are closely related to the Shimer (2005) critique of the conventional search and matching models as in Diamond (1982), Mortensen (1982) and Pissarides (1985) (DMP). Nash bargained wage is a weighted average of the worker's job productivity and the value of being unemployed. Value of the outside option (being unemployed) is dependant on the wages offered in other jobs. According to Shimer (2005), a positive productivity shock increases all employers reservation wages. If both the job productivity and the value of being unemployed (derived from all other employers reservation wages) increase by the same amount, there would not be any change in the employer's recruiting effort and unemployment would be stable in a Nash bargaining framework. However, the alternate offer bargained wage no longer depends on the procyclical outside options (value of being unemployed) but instead on the a-cyclical disagreement payoffs. This affects the recruiting effort of employer and unemployment fluctuates drastically in the AOB framework. Hence, Figure 3.2 shows that the AOB model generates a sharp decline in the unemployment rate together with a large increase in job vacancies and the job finding rate. Therefore, the estimated AOB model is not subject to Shimer (2005) critique of search and matching models with low replacement rates.

Table 3.2 reports the aggregate labour market moments from the data and from stochastic simulation of the Nash bargaining and AOB models. We can see from Table 3.2 that AOB in the search and matching framework produces volatility statistics closer to the data than Nash bargaining. The AOB model also accounts for the unconditional correlations between these variables. The

results of Table 3.2 make it evident that AOB does better in accounting for the cyclical properties of key labour market variables than the other models.⁸

3.4.2 Aggregate Quantity Dynamics

Figure 3.3 shows the impulse responses of consumption, investment and output to a positive productivity shock in the benchmark KS, the wage rigidity, Nash bargaining and the AOB models. The figure shows that consumption volatility is highest in the benchmark KS model followed by the wage-rigidity model, then the Nash bargaining model and the least in the AOB model. Households' consumption rises the most in the benchmark KS model as it uses its full endowment of labour and its wages rise the most at close to 0.14% (as shown in Figure 3.1) in response to a positive productivity shock. The response of consumption is second highest in the wage-rigidity-with- endogenous-labour-supply model in response to a productivity shock. This is due to households providing more labour (hours) as wages rise around 0.13%. More labour (hours)–but still less than full endowment of labour–translate to higher earnings so households increase consumption but by less than in the benchmark KS model.

Figure 3.3 also shows the lower responses of consumption to a positive productivity shock in the search and matching models compared to the benchmark KS and wage rigidity models. In the search and matching models, the presence of unemployment benefits acts as an automatic stabiliser which reduces the sensitivity of consumption to the productivity shock. A positive productivity shock results in a rise in unemployment benefits. By the wage sharing rules, the effect of rise in unemployment benefits on wages is less in AOB when com-

⁸Christiano et al. (2013) report higher volatilities for the Nash bargaining and AOB framework but unlike the benchmark KS model, their model has Calvo-type price rigidities.

pared to Nash bargaining framework as alternate offer sharing rule consists of a cost of delay in making a counteroffer. On account of relatively low increase in earnings following a positive productivity shock, agents under alternate offer sharing rule consume less when compared to agents under Nash sharing rule.⁹

The impulse responses of investment and output to a positive productivity shock are also shown in Figure 3.3. The figure shows the responses of both investment and output are the most in the benchmark KS model followed by the AOB model, the Nash bargaining model, and the least in the wage rigidity model.

3.4.3 Asset Pricing Implications

One of the main shortcoming of the standard production based asset pricing models is the inability to capture the cyclical behavior of dividend (Kung and Schmid, 2015; Favilukis and Lin, 2016). In standard models, dividend equals output minus investment minus labour cost. Since wages are perfectly correlated to output and investment is procyclical, dividend is countercyclical in standard models. In contrast, Figure 3.4 shows dividend to be acyclical in the benchmark KS model, slightly procyclical in the wage rigidity model, moderately procyclical in the Nash bargaining model and highly procyclical in the AOB model. The acyclicity in the KS model can be attributed to the different definition of dividend in benchmark KS model compared to the other standard asset pricing models. Dividend in the KS model includes expenses on intermediate goods costs.

The same definition of dividend is used when wage rigidity is assumed in the

⁹Households also earn from dividends but these are small compared to labour income. The responses of dividends to a positive productivity shock in various models are discussed the next section.

KS model. However, dividend becomes slightly procyclical on account of wages being rigid and smoother than the marginal product of labour and output. Under search and matching models, the dividend definition now incorporates both vacancy cost and the cost of meeting a worker by the firm. Both these costs decline during periods of expansion, thereby enabling dividends to be more procyclical. The response of AOB determined wage to a positive productivity shock is more muted than the response witnessed by Nash determined wage. This causes the AOB model dividend to be more procyclical than the Nash model dividend.

Table 3.3 confirms this procyclical behaviour of dividends as the correlation between dividend and output is highest under the AOB model. The moments associated with volatility of dividends also slightly improves under the AOB model. However, the volatility moment of P/D ratio overshoots the data counterpart under the AOB model.

Table 3.4 shows marginal improvement in the the asset pricing moments after labour market model is incorporated in the benchmark KS model. The equity premium moment under AOB model is very close to its data counterpart. This can be explained using the long run risk component of the benchmark model. Kung and Schmid (2015) describe about how agent's fear about periods of economic downturn and the associated low valuation of assets cause the agent to demand a higher equity premium. There are two points to the search and matching model associated with long run risk that makes the equity premium more closer to the data. Firstly, the presence of unemployment in the model increases long run risk. Periods of economic downturn are associated with relatively higher levels of unemployment. Hence, the agent may fear of being unemployed and being limited to unemployment benefits instead of wages. Since

wages are not perfectly correlated with output, wages are relatively higher than unemployment benefits during periods of economic downturn. Such a fear of cashflow risk may force the agent to command an even higher equity premium under the search and matching market model

The second point is with respect to the cyclical behaviour of dividends and the performance of AOB model to explain equity premium. In the benchmark KS model, the agent's cashflow associated with dividend is acyclical. But in the AOB model, highly pro cyclical dividend causes agent's dividend to decline even further when compared to benchmark KS model during periods of economic downturn. With long run risk and the agent's preference for consumption smoothing in the model, higher equity premium under AOB can be associated with the agent's perceived fear of being limited to unemployment benefits and smaller dividend during economic downturn. The volatility of risk premium falls short of their empirical counterparts as given in Table 3.4. As mentioned by Kung and Schmid (2015), this shortcoming could be due to the non incorporation of short run risks, which may not be entirely productivity driven.

With regard to the risk free rate moment, we find that the mean of the risk free rate estimated by the models with labour market dynamics falls short of the benchmark KS model. Intuitively, an explanation for such a slight decrease in risk free rate lies with the agents preference for saving to prepare for persistent low growth episodes. With dividends being more procyclical when compared to the benchmark KS model, agents tend to save more to smooth the consumption during periods of low growth. Table 3.4 also shows that the volatility of the risk free asset improves in the presence of wage inertia with AOB model more closer to the data.

In the context of declining labour share and rising firm concentration, we also conduct sensitivity analysis to find the impact of labour share $(1-\alpha)$ and intermediate good firm monopoly markup (ν) on the performance of equity premium in the AOB model. Table 3.5 shows the equity premium for $1-\alpha$ equals 0.7, 0.65 and 0.6 in the AOB model with all other calibration parameters remaining the same as in Table 3.1. We find that the equity premium rises when labour share declines in the production function. On the other hand, Table 3.6 shows the equity premium for ν equals 1.55, 1.65 and 1.75 in the AOB model with all other calibration parameters remaining the same as in Table 3.1. Table 3.6 shows that increase in firm markup (ν) results in a decline of the equity premium.

3.5 Conclusion

The results from our study depict the importance of incorporating labour market elements in production based asset pricing model. We extend the Kung and Schmid (2015) with a search and matching labour market with Nash bargaining framework and alternate offer bargaining framework following Christiano et al. (2016). We also replicate Donadelli and Grüning (2016)'s extension of the Kung and Schmid (2015) to include wage rigidities as in Uhlig (2007). Among the aforementioned labour market models, rent sharing rule based on alternate bargaining framework in a search and matching model framework captures the key asset pricing moments and aggregate macroeconomic quantities found empirically.

Our contribution to the labour asset pricing literature stems from understanding the labour market implications of long run risk which endogenously

arise in the Kung and Schmid (2015) model. The long run risk channel which makes the agents fear about persistent economic downturn in the future is aggravated further with the unemployment that accompanies such downturns. As alternate offer bargaining framework is better able to replicate the responses of unemployment to productivity shock, this channel makes the cash flow risk (uncertainty over wage flow) of agents higher resulting in them commanding higher risk premia over assets held.

Table 3.1: Parameters and steady state values

Parameter	Description	Value	Source	
Panel A: Common parameters				
β	Subjective discount factor	0.9945	KS (2015)	
ψ	Elasticity of intertemporal substitution	1.8		
γ	Risk aversion	10		
ξ	Patent share	0.5		
ν	Inverse markup	1.65		
α	Capital share	0.35		
ρ	Productivity shock persistence	0.9925		
χ	R&D productivity parameter	0.343		
ϕ	Patent obsolescence rate	0.0375		
η	Elasticity of new patents with respect to R&D	0.83		
δ	Depreciation rate of capital stock	0.02		
ζ	Investment adjustment cost parameter	0.8		
Panel B: Wage rigidity				
χ	R&D productivity parameter	0.462	KS (2015) and authors' calculations	
τ	labour elasticity	0.38	Uhlig (2007)	
μ	Sticky wage parameter	0.2	Uhlig (2007)	
Panel C: Search and matching				
Parameters		Nash rule	AOB	
$100\delta_1$	Prob. of bargaining session break-up	-	0.19	CET (2016)
ρ_1	Job survival probability	0.9	0.9	CET (2016)
σ_1	Share of job searchers	0.5	0.5	CET (2016)
$\eta_{bargain}$	Total surplus share received by workers	0.2	-	Hagedorn and Manovskii (2008)
κ	Hiring cost to meet a worker	0.0431	0.0431	CET (2016) and authors' calculations
γ_1	Counteroffer costs	-	0.0014	CET (2016) and authors' calculations
b	Unemployment benefits	0.0043	0.0031	CET (2016) and authors' calculations
s	Vacancy cost	0.80	0.83	CET (2016) and authors' calculations
σ_m	Level parameter in matching function	1.097	1.097	CET (2016) and authors' calculations

Notes: The table reports the quarterly calibrations of the four models considered in this study. Panel A: KS benchmark model. Panel B: Wage rigidities following Uhlig (2007). Panel C: Search and matching model following Christiano et al. (2016). CET (2016) refers to Christiano et al. (2016)

Table 3.2: Labour market moments

Moments	Data	Benchmark KS	Wage rigidity	Nash Bargaining	AOB
$\sigma(u)$	13.00%	-	-	6.77%	11.34%
$\sigma(vac)$	14.00%	-	-	8.13%	12.8%
$\sigma(\frac{vac}{u})$	26.00%	-	-	14.81%	24.04%
$corr(u, v)$	-0.860	-	-	-0.975	-0.984
$corr(u, v/u)$	-0.960	-	-	-0.999	-0.999
$corr(v, v/u)$	0.98	-	-	0.977	0.986

Note: This table reports the aggregate labour market moments obtained from a stochastic simulations of the relevant models. The model is solved using third-order perturbations around the stochastic steady state in Dynare++ 4.4.3. The moments in the data column are from Christiano et al. (2013) for the period 1951–2008. The benchmark KS model and the wage rigidity model show no moments associated with vacancy and unemployment since the labour force is fixed at unity in KS benchmark model and the sum of labour (defined as hours of work) and leisure is equal to one in the wage rigidity model. Model (1): KS benchmark model. Model (2): Wage rigidities following Uhlig (2007). Model (3): Nash bargaining framework in search and matching labour model following Christiano et al. (2016) Model(4):Alternate offer bargaining framework in search and matching labour model following Christiano et al. (2016). All data are in log levels.

Table 3.3: Dividend and P/D ratio moments

Moments	Data	Benchmark KS	Wage rigidity	Nash Bargaining	Alternate Offer Bargaining
$\sigma[\Delta d]$	2.60%	2.34%	2.93%	1.82%	2.68%
$\sigma[p - d]$	41.54%	15.06%	11.92%	10.73%	74.08%
$\sigma[d]/\sigma[y]$	1.40	0.20	0.35	1.18	1.30
$corr(d, y)$	0.968	0.093	0.842	0.986	0.988
$\sigma[\Delta d]/\sigma[\Delta y]$	2.62	1.02	1.18	0.77	1.13
$corr(\Delta d, \Delta y)$	0.42	0.39	0.60	0.40	0.50

Note: This table reports the dividend and output ratio moments obtained from a stochastic simulation of the four models considered in this study. The model is solved using third-order perturbations around the stochastic steady state in Dynare++ 4.5.3. The moments in the data column except for P/D ratio are calculated from the data obtained from Federal Reserve Bank of St. Louis for the period 1953–2008. The volatility moment of P/D ratio in the data column is taken from Kung and Schmid (2015). Model (1): KS benchmark model. Model (2): Wage rigidities following Uhlig (2007). Model (3): Nash bargaining framework in search and matching labour model following Christiano et al. (2016) Model(4):Alternate offer bargaining framework in search and matching labour model following Christiano et al. (2016). Variables considered are log dividend (d) and log output (y)

Table 3.4: Asset pricing moments

Moments	Data	Benchmark KS	Wage rigidity	Nash Bargaining	Alternate Offer Bargaining
$E(r_m - r_f)$	4.89%	3.07%	3.46%	3.93%	4.41%
$\sigma(r_m - r_f)$	17.92%	3.88%	4.05%	3.67%	3.52%
$E(r_f)$	2.90%	1.09%	1.02%	0.78%	1.05%
$\sigma(r_f)$	3.00%	0.35%	0.38%	0.39%	0.41%

Note: This table reports the asset pricing moments obtained from a stochastic simulation of the four models considered in this study. The model is solved using third-order perturbations around the stochastic steady state in Dynare++ 4.5.3. The moments in the data column are from Papanikolaou (2011) for the period 1951–2008. Model (1): benchmark KS model. Model (2): Wage rigidities following Uhlig (2007). Model (3): Nash bargaining framework in search and matching labour model following Christiano et al. (2016) Model(4):Alternate offer bargaining framework in search and matching labour model following Christiano et al. (2016).

Table 3.5: Effect of varying α on the equity premium in AOB model

	$1 - \alpha = 0.7$	$1 - \alpha = 0.65$	$1 - \alpha = 0.6$
$E(r_m - r_f)$	3.81%	4.41%	7.29%

Note: This table reports the asset pricing moments obtained from a stochastic simulation of the four models considered in this study. The model is solved using third-order perturbations around the stochastic steady state in Dynare++ 4.5.3. The moments are computed using alternate offer bargaining framework in search and matching labour model following Christiano et al. (2016).

Table 3.6: Effect of varying ν on the equity premium in AOB model

	$\nu = 1.55$	$\nu = 1.65$	$\nu = 1.75$
$E(r_m - r_f)$	4.58%	4.41%	4.31%

Note: This table reports the asset pricing moments obtained from a stochastic simulation of the four models considered in this study. The model is solved using third-order perturbations around the stochastic steady state in Dynare++ 4.5.3. The moments are computed using alternate offer bargaining framework in search and matching labour model following Christiano et al. (2016).

Figure 3.1: Response of wages (W_t) and labour (L_t) to productivity shock

Impulse responses to a 1.75 basis point increase in productivity; Benchmark vs. Wage rigidity vs. Nash vs. AOB; x-axis in quarters; y-axis in percentage

Figure 3.2: Response of unemployment (U_t), total vacancies ($Vac_t L_t$) and job finding rate (f_t) to productivity shock

Impulse responses to a 1.75 basis point increase in productivity; Benchmark vs. Wage rigidity vs. Nash vs. AOB; x-axis in quarters; y-axis in percentage

Figure 3.3: Response of consumption (C_t), investment (I_t) and output (Y_t) to productivity shock

Impulse responses to a 1.75 basis point increase in productivity; Benchmark vs. Wage rigidity vs. Nash vs. AOB; x-axis in quarters; y-axis in percentage

Figure 3.4: Response of dividend (D_t) to productivity shock

Impulse responses to a 1.75 basis point increase in productivity; Benchmark vs. Wage rigidity vs. Nash vs. AOB; x-axis in quarters; y-axis in percentage

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