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Nonlinear Energy Harvesting for Millimeter Wave Networks with Large-Scale Antennas

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Abstract—In this paper, an analytical framework is proposed to explore the potential of wireless power transfer (WPT) for a millimeter wave (mmWave) network with large-scale antennas. Prior works mostly focus on WPT systems with a linear energy harvesting (EH) model which cannot properly capture the power dependent EH efficiency. On the other hand, based on a nonlinear EH model, the effect of practical EH circuit specifications on the EH performance can be analyzed. By using stochastic geometry approach, analytical and asymptotic expressions for the energy coverage probability, average harvested energy, and achievable rate are derived under different base station (BS) configuration and deployment scenarios. Numerical results provide interesting design insights that the EH circuit specifications significantly affect the network performance. Increasing the BS density and number of BS antennas can improve the network performance. Due to saturation of EH circuits, the typical receiver can only achieve finite maximum amount of harvested energy and achievable rate even when the BS has a large number of antennas.

Index Terms—mmWave, massive MIMO, nonlinear energy harvesting, stochastic geometry, wireless power transfer.

I. INTRODUCTION

THE growth of mobile and internet of things (IoT) devices is considered to provide a better quality of life and new business opportunities. Some of these devices can be small battery-powered sensors located in various areas, where it is not convenient to replace and recharge their batteries, such as underground and tunnels. Wireless power transfer (WPT) is an enabling technology to make those devices harvest energy from radio frequency (RF) signals. This method is more efficient than the conventional energy harvesting (EH) from solar and wind sources which greatly depend on the outdoor conditions. However, it raises new challenges such as huge connectivity, diverse traffic, as well as sharp signal beams for long-distance WPT, which cannot be accommodated by the current (below 6 GHz) cellular networks. Recently, millimeter-wave (mmWave) communication has attracted a lot of interests due to its large available bandwidth [1]. It is considered as one of the key technologies for the fifth generation (5G) network. The wireless transmission at the mmWave frequencies exhibits specific channel behaviours, consisting of high propagation loss and poor penetration (blocking) when passing through common materials such as concrete, water, and tinted glass. With a smaller wavelength, the mmWave network can employ

large antenna arrays which provide highly directional transmission and reception [1]–[3]. This factor helps to greatly compensate the propagation loss. Moreover, more BSs can be deployed in the mmWave network which makes the cells smaller [3], [4]. Due to cell shrinking, a dense deployment of mmWave base stations (BSs) has numerous advantages for the WPT, such as improving frequency reuse across a geographic region, reducing number of users which compete for given radio resources at each BS, as well as decreasing the path loss over the transmission links significantly. Recent research studies have shown that mmWave communication is feasible for communication ranges of 150 – 200 m in dense urban environments [1], [2]. These above factors ensure the suitability of the mmWave communication for the WPT application in the future 5G networks.

There are two practical receiver designs for the simultaneous wireless information and power transfer (SWIPT) technique, namely time-switching (TS) and power-splitting (PS) designs. Based on the TS design, the receiver is either in the EH mode or the data transmission mode at any point in time. On the other hand, with the PS design, the receiver is able to split the received signal power into two streams, i.e., one stream is devoted to the information decoding circuit and another one is devoted to the EH circuit at the same time. For microwave networks, a review of recent advances in the area of EH wireless communications was summarized in [5], consisting of the information-theoretic performance, transmission scheduling policies, as well as resource allocation. Optimal time allocation of the wireless powered systems was studied in [6], whereas optimal power splitting at the receiver was discussed in [7]. A resource allocation scheme was proposed to jointly optimize transmit power, transfer time and spatial beam for maximizing the secrecy rate [8]. A hybrid network architecture including BSs and power beacons for enabling the WPT was presented in [9]. Optimal spectrum access and energy supply for cognitive radio systems with opportunistic EH was investigated in [10]. For a wireless powered co-operative communication system with a buffer-aided hybrid relay, the long-term throughput of two different block-wise cooperative protocols was studied in [11]. However, the EH issues of mmWave networks are not well understood due to different physical characteristics and design approaches at higher frequencies. Thus, prior works on the WPT in microwave networks cannot directly be applied to mmWave networks. The potential EH of the mmWave network was considered in [12], where it was shown that mmWave EH generally can outperform lower frequency solutions. By using maximum-ratio transmission at the BSs and maximum-ratio combining at the devices, a joint complementary cumulative

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distribution function of SWIPT was studied based on simulation results [13]. In both [12] and [13], the mmWave networks employ the same PS design. Since each data frame is divided into three continuous EH segments, beam-searching and data transmissions, a joint optimal ratio of EH and beamwidth selection scheme was presented to optimize the achievable rate [14]. In [15], based on a 5G network consisting of both microwave and mmWave BSs, the authors studied the number of mmWave BSs that are needed to outperform the microwave BSs from the perspectives of harvested energy and data rate.

Note that the above prior works [5]–[15] focus on the WPT system designs with an ideal hardware model, in which the harvested energy at the output of EH circuit is considered as a linear function of the input power and the energy conversion efficiency is independent of the input power level. However, based on the measurement of realistic EH circuit data, it has been shown that the behaviour of wireless EH circuits is usually expressed as a nonlinear process rather than the linear one [16]. The traditional linear EH model may not accurately characterize the property of the practical EH circuit, which greatly affects the EH performance. Therefore, the nonlinear EH model should be applied to study the WPT issues based on realistic measured data. So far, there are a few works focusing on the microwave WPT systems with the nonlinear EH model. In particular, a resource allocation algorithm for SWIPT systems has been shown in [16]. This model has also been adopted in [17] to investigate rate-energy region of SWIPT for the multiple-input multiple-output (MIMO) broadcasting. The tradeoffs between the energy transfer and information rate for both TS and PS receiver structures have been studied. In [18], optimal mode switching between information decoding and EH over the fading channels has been obtained by solving the problem of maximizing the achievable rate under a harvested energy constraint. In [19], a practical quadratic model for the power conversion efficiency in EH circuitry has been proposed to study resource allocation for a multiuser orthogonal frequency-division multiple-access system. Resource allocation in wireless powered cognitive radio networks has been considered in [20]. To the best of the authors' knowledge, a study on the EH for mmWave massive MIMO networks with the practical nonlinear EH model has not been reported in the literature yet.

In this paper, we study the fundamental performance limits of a wireless powered mmWave cellular network based on the nonlinear EH model under different BS deployment scenarios. The BSs first transfer energy to the user equipments by sending RF signals in the downlink (DL). The users use the harvested energy to perform uplink (UL) wireless information transmission (WIT) to their own serving mmWave BSs. The main contributions of this paper are summarized as follows

- *Energy harvesting:* Based on the stochastic geometry approach, a tractable framework is presented to analyze the energy coverage probability (ECP) and average harvested energy of mmWave networks. Under the assumption of noise limited scenario, a closed-form expression of the ECP is derived. Asymptotic average harvested energy is obtained when the number of BS antennas is very large.
- *Rate performance:* With the harvested energy, the receiver

TABLE I: Summary of notations

Notation	Definition
Φ, λ	PPP modelling BS locations with density λ
$p_L(x) = A$	Probability that the LoS link is present when $x \leq B$
m_L, m_N	Nakagami fading parameters of the small-scale fading terms of LoS and NLoS links
D_L, D_N	Association probabilities of LoS and NLoS links
α_L, α_N	LoS and NLoS path-loss exponents
F_l, F_{nl}	Harvested energies of the typical user in the linear and non-linear EH models
M, w_1, w_2	Specific maximum harvested energy and parameter constants of the EH circuit in the non-linear EH model
a_k, b_k	Total directivity gain a_k between the typical user and BS with a probability b_k
$\tau T, R_u$	Downlink WPT duration, uplink achievable rate
P_e, P_c	DL energy and UL SINR coverage probabilities
G_i, g_i, N_i, P_i	Main-lobe gain, side-lobe gain and number of antennas, transmit power for BSs and users (i.e., $i \in \{B, U\}$)
$ h_i ^2$	Small-scale fading power gains of LoS and NLoS links
S_i, I_i	Desired signal and aggregate interference powers of the typical user in the DL and UL, i.e., $i \in \{d, u\}$

ceiver begins to transmit data information. Analytical and asymptotic expressions for the achievable rate are derived when the network is interference limited, noise limited and/or the number of BS antennas is very large.

- *Key insights:* Numerical results are provided to verify our theoretical analysis. It has been shown that: (i) the line-of-sight (LoS) BSs dominate the harvested energy, while the effect of the small-scale fading on the EH can be negligible due to highly directional transmissions; (ii) the practical circuit specifications greatly affect the EH performance. Increasing the number of BS antennas and BS density can improve the network performance; (iii) different from the linear EH model, the receiver can only achieve a finite maximal amount of harvested energy in nonlinear EH model even when the BS has a very large number of antennas or a high level of transmit power because the EH circuit is saturated.

II. SYSTEM MODEL

We consider an mmWave network, where the locations of BSs are modelled as a homogeneous Poisson point process (PPP) Φ with density λ . The analysis is conducted for a typical user located at the origin [21]. The typical user first harvests energy in the DL and then it uses the harvested energy to transmit data in the UL [22]. In particular, the slot duration T is divided into downlink WPT of duration τT and uplink WIT of duration $(1 - \tau)T$, respectively [22]. To capture the blockage statistics, the probability that the LoS link is present is defined as $p_L(x) = A$, if $x \leq B$, and otherwise $p_L(x) = 0$, where $A \in [0, 1]$ is the average fraction of the LoS area in a circular region of radius B around the investigative point [23]. The pair (A, B) depends on the specific building geography and network deployment scenarios. The path losses of LoS

and non-line-of-sight (NLoS) links are given by [23]–[26]

$$L(x) = \begin{cases} C_L |x|^{-\alpha_L}, & \text{with probability } p_L(x), \\ C_N |x|^{-\alpha_N}, & \text{with probability } 1 - p_L(x), \end{cases} \quad (1)$$

where α_L and α_N are the LoS and NLoS path-loss exponents, respectively. The path loss intercepts of LoS and NLoS links at the reference distance are defined as $C_L = 10^{-\beta_L/10}$ and $C_N = 10^{-\beta_N/10}$, where β_L and β_N are given in [27, Table 1]. The small-scale fading terms of LoS and NLoS links follow independent Nakagami fading with parameters m_L and m_N , respectively. Thus, the small-scale fading power gain of each link is modelled as $|h_j|^2 \sim \Gamma(m_j, \frac{j}{m_j})$ with $j \in \{L, N\}$ [25]. The uniform planar square arrays with a half-wavelength antenna element spacing are used for both BS and user to obtain directional beamforming, and the actual array pattern is defined by a sectored antenna model [25]. Specifically, we define Δ_j as the total directivity gain between the j th BS and the typical user which is defined as a discrete random variable (RV) a_k with a probability b_k ($k \in \{1, 2, 3, 4\}$). We define G_i, g_i, θ_i , and N_i with $i \in \{B, U\}$ as the main-lobe gain, the side-lobe gain, the main-lobe beamwidth, and the number of antenna elements for the BS and users, respectively. The values of a_k are given as $a_k \in \{G_B G_U, G_B g_U, g_B G_U, g_B g_U\}$ with the corresponding probabilities $b_k \in \{q_B q_U, q_B(1 - q_U), (1 - q_B)q_U, (1 - q_B)(1 - q_U)\}$, where $q_B = \frac{\theta_B}{2\pi}$ and $q_U = \frac{\theta_U}{2\pi}$. The main-lobe and side-lobe gains are approximated as [28]

$$G_i = N_i, \quad (2)$$

and

$$g_i = \frac{\sqrt{N_i} - \frac{\sqrt{3}}{2\pi} N_i \sin(\frac{\sqrt{3}}{2\sqrt{N_i}})}{\sqrt{N_i} - \frac{\sqrt{3}}{2\pi} \sin(\frac{\sqrt{3}}{2\sqrt{N_i}})}. \quad (3)$$

These approximations have been commonly used for the conventional and large-scale antenna systems in the literature [28]–[30]. Specifically, theoretical and simulation results of coverage and rate performances for different types of mmWave networks have been obtained by setting $N_B = 16$ in [28], $N_B = 1024$ in [29], as well as a very large value of N_B in [30], respectively. The directivity gain for the desired signal link is defined as $\Delta_0 = G_B G_U$. It is assumed that each user is equipped with an energy harvesting circuit to harvest energy from radio frequency signals. As the energy harvested from the noise is negligible, during the WPT phase, with the linear EH model, the total harvested energy at the typical user is given by [22]

$$F_l = \xi \tau T P_{in}, \quad (4)$$

where $\xi \in (0, 1)$ is the radio frequency to direct current conversion efficiency, $P_{in} = S_d + I_d$, where $S_d = P_B \Delta_0 |h_0|^2 L(x_0)$, $I_d = \sum_{j \in \Phi \setminus \{0\}} P_B \Delta_j |h_j|^2 L(x_j)$, and P_B is the transmit power of the BS. Note that ξ is independent of P_{in} . With the non-linear EH model, the total harvested energy of the typical user is defined as [16]

$$\begin{aligned} F_{nl} &= \tau T \frac{\frac{M}{1+e^{-w_1(P_{in}-w_2)}} - \frac{M}{1+e^{w_1 w_2}}}{1 - \frac{1}{1+e^{w_1 w_2}}} \\ &= \frac{\tau T M (e^{w_1 P_{in}} - 1)}{e^{w_1 P_{in}} + e^{w_1 w_2}}, \end{aligned} \quad (5)$$

where M is the maximum harvested energy when the EH circuit is saturated, w_1 and w_2 are constants which are dependent on the EH circuit specifications such as resistance, capacitance and diode turn-on voltage. The values of M, w_1 , and w_2 can be defined by using a standard curve fitting tool [16]. The notation of parameters is summarized in Table 1.

Remark 1: The harvested energy based on linear EH model increases with P_B . On the other hand, the harvested energy based on non-linear EH model can be increased with P_B when P_B is relatively low, and it converges to a saturated value when P_B is high. Specifically, as P_B is very high, F_{nl} converges to a finite value of $\tau T M$, while F_l becomes a very large value. The non-linear EH model is more general and accurate for realistic EH systems, hence we will study its effect on the mmWave network performance.

After EH, the typical user uses a specific power P_U to transmit its own data. The received signal-to-interference-plus-noise ratio (SINR) in the UL is given by

$$\kappa_u = \frac{S_u}{I_u + \sigma^2}, \quad (6)$$

where $S_u = P_U \Delta_0 |h_0|^2 L(x_0)$ and $I_u = \sum_{j \in \Phi \setminus \{0\}} P_U \Delta_j |h_j|^2 L(x_j)$. The noise power is defined as $\sigma^2 = -174 + 10 \log_{10}(W) + Q$ (in dB), where W is the channel bandwidth and Q is the noise figure [27].

III. ENERGY HARVESTING ANALYSIS

In this section, we analytically evaluate the energy coverage probability and average harvested energy. We then provide network level design insights.

The probability density function (PDF) of the distance between a typical user and its serving BS via LoS link is defined as

$$f_L(x) = \frac{2\pi\lambda A x}{D_L} e^{-\pi\lambda A x^2 - (1-A)\pi\lambda\left(\frac{C_N}{C_L}\right)^{\frac{2}{\alpha_N}} x^{\frac{2\alpha_L}{\alpha_N}}}, \quad (7)$$

while the PDF of the distance between a typical user and its serving BS via NLoS link is defined as [31]

$$f_N(x) = \begin{cases} \frac{2(1-A)\pi\lambda x e^{-(1-A)\pi\lambda x^2}}{D_N} e^{-d_0 x^{\frac{2\alpha_N}{\alpha_L}}}, & x \leq z_0, \\ \frac{2(1-A)\pi\lambda x e^{\pi\lambda A(x^2 - B^2)}}{D_N} e^{-\pi\lambda x^2}, & z_0 \leq x \leq B, \\ \frac{2\pi\lambda x}{D_N} e^{-\pi\lambda x^2}, & x \geq B, \end{cases} \quad (8)$$

where D_L and D_N are the association probabilities of NLoS and LoS links defined as $D_L = 1 - D_N$ and $D_N = e^{-\pi\lambda A B^2} + \int_0^{z_0} 2(1-A)\pi\lambda x e^{-(1-A)\pi\lambda x^2} (e^{-d_4 x^{\frac{2\alpha_N}{\alpha_L}}} - e^{-\pi\lambda A B^2}) dx$ with $z_0 = (\frac{C_L}{C_N})^{\frac{1}{\alpha_L}} B^{\frac{\alpha_N}{\alpha_L}}$ and $d_0 = \pi\lambda A (\frac{C_L}{C_N})^{\frac{2}{\alpha_L}}$. The following theorem characterizes the corresponding harvested energy.

Theorem 1. In an mmWave network with density λ , the energy coverage probability can be tightly lower bounded as

$$P_e = D_L P_e^L + D_N P_e^N, \quad (9)$$

with

$$P_e^L = \int_0^B (1 + \sum_{i=1}^V (-1)^i \binom{V}{i} (1 + \frac{P_B \Delta_0 C_L}{d_1 m_L x^{\alpha_L}})^{-m_L} \\ \times \prod_{k=1}^4 e^{-2\pi\lambda b_k [(1-A)\vartheta(m_N, u_2, y, \alpha_N) + A\vartheta(m_N, u_2, B, \alpha_N)]} \\ \times \prod_{k=1}^4 e^{-2\pi\lambda b_k A[\vartheta(m_L, u_1, x, \alpha_L) - \vartheta(m_L, u_1, B, \alpha_L)]}) f_L(x) dx,$$

and

$$P_e^N = \int_0^\infty (1 + \sum_{i=1}^V (-1)^i \binom{V}{i} (1 + \frac{P_B \Delta_0 C_N}{d_1 m_N x^{\alpha_N}})^{-m_N} \\ \times \prod_{k=1}^4 e^{-2\pi\lambda b_k [(1-A)\vartheta(m_N, u_2, x, \alpha_N) + A\vartheta(m_N, u_2, B, \alpha_N)]} \\ \times \prod_{k=1}^4 e^{-2\pi\lambda b_k A[\vartheta(m_L, u_1, j, \alpha_L) - \vartheta(m_L, u_1, B, \alpha_L)]}) f_N(x) dx,$$

where $\vartheta(a, b, c, d) = \sum_{i=1}^a \binom{a}{i} \frac{b^i c^{2-i} d}{id-2} {}_2F_1(a, i - \frac{2}{d}, i - \frac{2}{d} + 1, -bc^{-d})$, ${}_2F_1(\cdot)$ is the hypergeometric function, $y = (\frac{C_N}{C_L})^{\frac{1}{\alpha_N}} x^{\frac{\alpha_L}{\alpha_N}}$, $j = (\frac{C_L}{C_N})^{\frac{1}{\alpha_L}} x^{\frac{\alpha_N}{\alpha_L}}$, $u_1 = \frac{P_B a_k C_L}{d_1 m_L}$, $u_2 = \frac{P_B a_k C_N}{d_1 m_N}$, $d_1 = \frac{\ln((\gamma_0 e^{w_1 w_2} + 1)/(1 - \gamma_0))}{w_1 \eta}$, $\eta = V(V!)^{-1/V}$, and $\gamma_0 = \frac{\gamma}{\tau T M}$ for given energy threshold γ and parameter V .

Proof: Please refer to Appendix A. ■

Theorem 1 is expressed in a single integral form which can be readily evaluated by using numerical tools over a well-behaving function. Moreover, due to highly directional transmission, high signal attenuation and sensitivity to blockage, mmWave coverage trends are different from the current sub-6GHz cellular networks [25]. Based on results of recent link level experiments, system simulations and analytical studies in [23]–[25], [27], [32], it has been shown that large bandwidth mmWave networks often tend to be noise limited in dense urban scenarios. In that case, the lower bound on the ECP in (9) is obtained by replacing $P_e^L = \int_0^B (1 + \sum_{i=1}^V (-1)^i \binom{V}{i} (1 + \frac{P_B \Delta_0 C_L}{d_1 m_L x^{\alpha_L}})^{-m_L} f_L(x) dx$ and $P_e^N = \int_0^\infty (1 + \sum_{i=1}^V (-1)^i \binom{V}{i} (1 + \frac{P_B \Delta_0 C_N}{d_1 m_N x^{\alpha_N}})^{-m_N} f_N(x) dx$. In order to obtain insightful design guidelines and simplify the analysis, we provide a closed-form expression of the corresponding ECP in the following theorem.

Theorem 2. When the network is noise limited and $p_L(x) = 1$ for a given B , the energy coverage probability is expressed as

$$P_e = 1 - e^{-\pi\lambda\varphi_1^2} + \varphi_2, \quad (10)$$

where $\varphi_1 = \min(B, (\frac{P_B \Delta_0 C_L}{d_1})^{1/\alpha_L})$. If $B < (\frac{P_B \Delta_0 C_N}{d_1})^{1/\alpha_N}$, $\varphi_2 = e^{-\pi\lambda B^2} - e^{-\pi\lambda(\frac{P_B \Delta_0 C_N}{d_1})^{2/\alpha_N}}$ and otherwise $\varphi_2 = 0$.

Proof: As $p_L(x) = A = 1$, the blockage model is a simple LoS ball model, thus $f_L(x) = \frac{2\pi\lambda x}{D_L} e^{-\pi\lambda x^2}$, $x \leq B$ and $f_N(x) = \frac{2\pi\lambda x}{D_N} e^{-\pi\lambda x^2}$, $x \geq B$, where $D_N = e^{-\pi\lambda B^2}$. Based on the channel measurements, it has been shown that the influence of the small-scale fading on the mmWave channels is limited in the highly directional transmission [1]. Hence, by neglecting the small-scale fading effect, from (5), as the

network is noise limited, the conditional ECP of the typical user via LoS link is obtained as

$$P_e^L = \mathbb{P}_L(F_{nl} > \gamma) = \mathbb{P}_L(P_B \Delta_0 C_L x^{-\alpha_L} > d_1) \\ = \frac{1}{D_L} \int_0^{\varphi_1} 2\pi\lambda x e^{-\pi\lambda x^2} dx = \frac{1}{D_L} (1 - e^{-\pi\lambda\varphi_1^2}). \quad (11)$$

Similarly, the conditional ECP via NLoS link is obtained as

$$P_e^N = \mathbb{P}_N(P_B \Delta_0 C_N x^{-\alpha_N} > d_1) \\ = \begin{cases} \frac{1}{D_N} (e^{-\pi\lambda B^2} - e^{-\pi\lambda\varphi_2^2}), & B < (\frac{P_B \Delta_0 C_N}{d_1})^{1/\alpha_N}, \\ 0, & \text{otherwise.} \end{cases} \quad (12)$$

By using the law of total probability, (10) can be defined. ■

We observe that one of the main terms of (10) is $\pi\lambda B^2$ which shows the average number of LoS BSs. According to [15], the association probability that the typical user selects the NLoS BSs for the WPT is much smaller than the LoS BSs. Hence, if only LoS BSs are able to transfer energy, the ECP in (10) is simplified as $P_e = 1 - e^{-\pi\lambda\varphi_1^2}$ [15]. Since λ increases, the number of LoS BSs is increased and the mmWave network can achieve a better WPT performance. This is because the distance between the typical user and its serving BS is decreased. Moreover, from a mathematical point of view, as $\alpha_L \geq 2$, the increasing ECP with λ might be faster than that with the directional mmWave antenna gain $\Delta_0 = N_B N_U$ [15], [25]. It implies that a suitable approach to enhance the WPT performance is to make the cells smaller. These discussions are illustrated by numerical results in Section V.

Furthermore, only the received signal power which is greater than the activation threshold $\gamma_{\min}$ can be harvested. The average harvested energy of the typical user for an energy threshold $\gamma \in [\gamma_{\min}, \infty)$ can be defined as

$$\bar{F}_{nl} = \gamma P_e(\gamma) + \int_{\gamma_{\min}}^\infty P_e(\gamma) d\gamma, \quad (13)$$

where $P_e(\gamma)$ is given in Theorem 1. The expression (13) can be evaluated by numerical methods. In order to get better understanding and further insights, we simplify the average harvested energy in several specific cases as follows.

Theorem 3. When the number of BS antennas is very large, the average harvested energy of the typical user converges to

$$\bar{F}_{nl}^{as} = \tau T M. \quad (14)$$

Proof: The harvested energy is given in (5), where P_{in} has been defined earlier as $P_{in} = P_B N_B N_U |h_0|^2 L(x_0) + \sum_{j \in \Phi \setminus \{0\}} P_B \Delta_j |h_j|^2 L(x_j)$. It is observed that P_{in} is a linear function of N_B . For a given P_B , as N_B is very large, P_{in} becomes a sufficiently high value. According to the measurement of realistic EH circuit data, it has been presented that w_1 is large (i.e., $w_1 > 1$), while w_2 is very small (i.e., $w_2 \ll 1$). Specifically, in [16], [17], and [33], the authors have provided two groups of circuit parameters such as $(w_1, w_2) = \{(6400, 0.003), (1500, 0.0022)\}$. Based on the above discussions, it is shown that $e^{w_1 P_{in}} \gg 1$ and $e^{w_1 P_{in}} \gg e^{w_1 w_2}$. As a results, since N_B is very large, from (5), we have

$$F_{nl} = \frac{\tau T M (e^{w_1 P_{in}} - 1)}{e^{w_1 P_{in}} + e^{w_1 w_2}} \xrightarrow{a.s.} \tau T M, \quad (15)$$

where $\xrightarrow{a.s.}$ denotes the convergence. Therefore, the asymptotic average harvested energy is defined as $\bar{F}_{nl}^{as} = \tau TM (\int_0^B D_L f_L(x) dx + \int_0^\infty D_N f_N(x) dx) = \tau TM$. ■

Theorem 4. When the network is noise limited and $p_L(x) = 1$, the average harvested energy of the typical user is given by

$$\bar{F}_{nl} = \tau TM \left(1 - \int_0^B \frac{(e^{w_1 w_2} + 1) 2\pi \lambda x e^{-\pi \lambda x^2} dx}{e^{w_1 P_B \Delta_0 C_L x^{-\alpha_L}} + e^{w_1 w_2}} - \int_B^\infty \frac{(e^{w_1 w_2} + 1) 2\pi \lambda x e^{-\pi \lambda x^2} dx}{e^{w_1 P_B \Delta_0 C_N x^{-\alpha_N}} + e^{w_1 w_2}} \right). \quad (16)$$

Proof: By neglecting the small-scale fading effect, (16) can be obtained from (5) as the network is noise limited. ■

The expressions (10) and (16) are functions of BS density, the blockage statistics, and the antenna geometry parameters.

Remark 2: By increasing the number of BS antennas, it is beneficial to the EH. When the number of BS antennas is very large, the typical user can achieve a maximum amount of the harvested energy τTM wherever it is located. This amount only depends on M for given values of τ and T , i.e., the design structure.

IV. UPLINK ACHIEVABLE RATE ANALYSIS

After harvesting the energy, it is assumed that the typical user uses all the received power $P_U = \frac{\bar{F}_{nl}}{(1-\tau)T}$ to maximize the UL data transmission [9, Lemma 2], [22]. Similar to [8] and [22], the power control is not considered in the UL because the harvested energy is limited. The achievable rates of the typical user in different network deployments are given below.

Theorem 5. The uplink achievable rate of the typical user in mmWave network is tightly upper bounded as

$$R_u = \frac{W_0}{\ln 2} \int_0^\infty \frac{D_L P_c^L(t) + D_N P_c^N(t)}{1+t} dt, \quad (17)$$

with

$$P_c^L(t) = \int_0^B \sum_{i=1}^{m_L} (-1)^{i+1} \binom{m_L}{i} e^{-c_1 \sigma^2} J_1 J_2 f_L(x) dx,$$

and

$$P_c^N(t) = \int_0^\infty \sum_{i=1}^{m_N} (-1)^{i+1} \binom{m_N}{i} e^{-c_2 \sigma^2} J_3 J_4 f_N(x) dx,$$

where $J_1 = \prod_{k=1}^4 e^{-2\pi \lambda b_k A(\vartheta(m_L, \phi_1, x, \alpha_L) - \vartheta(m_L, \phi_1, B, \alpha_L))}$, $J_2 = \prod_{k=1}^4 e^{-2\pi \lambda b_k [(1-A)\vartheta(m_N, \phi_2, y, \alpha_N) + A\vartheta(m_N, \phi_2, B, \alpha_N)]}$, $J_3 = \prod_{k=1}^4 e^{-2\pi \lambda b_k A(\vartheta(m_L, \phi_3, j, \alpha_L) - \vartheta(m_L, \phi_3, B, \alpha_L))}$, $J_4 = \prod_{k=1}^4 e^{-2\pi \lambda b_k [(1-A)\vartheta(m_N, \phi_4, x, \alpha_N) + A\vartheta(m_N, \phi_4, B, \alpha_N)]}$, $c_1 = \frac{it\eta_L x^{\alpha_L}}{\Delta_0 C_L}$, $c_2 = \frac{it\eta_N x^{\alpha_N}}{\Delta_0 C_N}$, $\phi_1 = \frac{c_1 a_k C_L}{m_L}$, $\phi_2 = \frac{c_1 a_k C_N}{m_N}$, $\phi_3 = \frac{c_2 a_k C_L}{m_L}$, $\phi_4 = \frac{c_2 a_k C_N}{m_N}$, $\eta_L = m_L(m_L!)^{\frac{-1}{m_L}}$, $\eta_N = m_N(m_N!)^{\frac{-1}{m_N}}$, and $W_0 = (1-\tau)W$.

Proof: Please refer to Appendix B. ■

Theorem 6. When the number of BS antennas is very large, the achievable rate of the user converges to

$$R_u^{as} = \frac{W_0}{\ln 2} \int_0^\infty \frac{D_L P_{c,as}^L(t) + D_N P_{c,as}^N(t)}{1+t} dt, \quad (18)$$

with

$$P_{c,as}^L(t) = \int_0^B \sum_{i=1}^{m_L} (-1)^{i+1} \binom{m_L}{i} \tilde{J}_1 \tilde{J}_2 f_L(x) dx,$$

and

$$P_{c,as}^N(t) = \int_0^\infty \sum_{i=1}^{m_N} (-1)^{i+1} \binom{m_N}{i} \tilde{J}_3 \tilde{J}_4 f_N(x) dx,$$

where $\tilde{J}_1 = \prod_{k=1}^2 e^{-2\pi \lambda \tilde{b}_k A(\vartheta(m_L, \tilde{\phi}_1, x, \alpha_L) - \vartheta(m_L, \tilde{\phi}_1, B, \alpha_L))}$, $\tilde{J}_2 = \prod_{k=1}^2 e^{-2\pi \lambda \tilde{b}_k [(1-A)\vartheta(m_N, \tilde{\phi}_2, y, \alpha_N) + A\vartheta(m_N, \tilde{\phi}_2, B, \alpha_N)]}$, $\tilde{J}_3 = \prod_{k=1}^2 e^{-2\pi \lambda \tilde{b}_k A(\vartheta(m_L, \tilde{\phi}_3, z, \alpha_L) - \vartheta(m_L, \tilde{\phi}_3, B, \alpha_L))}$, $\tilde{J}_4 = \prod_{k=1}^2 e^{-2\pi \lambda \tilde{b}_k [(1-A)\vartheta(m_N, \tilde{\phi}_4, x, \alpha_N) + A\vartheta(m_N, \tilde{\phi}_4, B, \alpha_N)]}$, $\tilde{c}_1 = \frac{it\eta_L x^{\alpha_L}}{G_U C_L}$, $\tilde{c}_2 = \frac{it\eta_N x^{\alpha_N}}{G_U C_N}$, $\tilde{\phi}_1 = \frac{\tilde{c}_1 \tilde{a}_k C_L}{m_L}$, $\tilde{\phi}_2 = \frac{\tilde{c}_1 \tilde{a}_k C_N}{m_N}$, $\tilde{\phi}_3 = \frac{\tilde{c}_2 \tilde{a}_k C_L}{m_L}$, and $\tilde{\phi}_4 = \frac{\tilde{c}_2 \tilde{a}_k C_N}{m_N}$.

Proof: As N_B is sufficiently large, the typical user can harvest a maximum amount of energy $\bar{F}_{nl}^{as} = \tau TM$ as shown in Theorem 3. In addition, the SINR (6) converges to

$$\kappa_u \xrightarrow{a.s.} \frac{G_U |h_0|^2 L(x_0)}{\sum_{j \in \Phi \setminus \{0\}} \tilde{\Delta}_j |h_j|^2 L(x_j)}, \quad (19)$$

where $\tilde{\Delta}_j = \tilde{a}_k \in \{G_U, g_U\}$ with corresponding probabilities $\tilde{b}_k \in \{q_B q_U, q_B(1-q_U)\}$. Note that asymptotic SINR in (19) is independent of P_U . The remaining steps are similar to that of the proof for Theorem 5 in Appendix B. ■

Remark 3: We observe that (i) Theorems 3 and 6 present the performance limits in term of the ECP and achievable rate of the typical user as N_B is very large; (ii) if all network users use the same P_U to transmit data, P_U does not affect the achievable rate as the network is interference limited or N_B is very large.

In a dense deployment of BSs, i.e., λ is large, mmWave network is interference limited, (6) is rewritten as $\kappa_u = \frac{S_u}{I_u}$ [25], [26]. Hence, the achievable rate of the typical user can be rewritten as (17) by neglecting $e^{-c_1 \sigma^2}$ and $e^{-c_2 \sigma^2}$. Moreover, as λ is large, it has been shown in [1] and [26] that the performance is mainly limited by the LoS interferers and small-scale fading leads to little change in the received power. To simplify the analysis, we assume that both NLoS BSs and small-scale fading are ignored in this case.

Corollary 1. As the network is interference limited, N_B is very large and $p_L(x) = 1$, the asymptotic rate (18) is lower bounded as

$$R_{u,lb}^{as} = W_0 \log_2 \left(1 + \frac{G_U (\alpha_L - 2)}{c_7} \right), \quad (20)$$

where $c_7 = 2[\gamma(2, \pi \lambda B^2) - B^{2-\alpha_L} (\pi \lambda)^{1-\frac{\alpha_L}{2}} \gamma(\frac{\alpha_L}{2} + 1, \pi \lambda B^2)] \sum_{k=1}^2 \tilde{a}_k \tilde{b}_k$ and $\gamma(a, b)$ is the lower incomplete Gamma function.

Proof: By ignoring all NLoS BSs and small-scale fading, from (19), we use an inequality $\mathbb{E}\{\log_2(1 + \frac{\varpi}{x})\} \geq \log_2(1 + \frac{\varpi}{\mathbb{E}\{x\}})$ with $\varpi > 0$ to obtain (20) directly. ■

Clearly, the expression (20) is much simpler than the expression (18), as it is expressed in terms of the incomplete Gamma function only. It is dependent of the circular region

of radius B , BS density, the LoS path-loss exponent, and the antenna geometry parameters.

Theorem 7. *When the network is noise limited, the exact achievable rate of the typical user is given by*

$$R_u = W_0(D_L R_u^L + D_N R_u^N), \quad (21)$$

with

$$R_u^L = \int_0^B \frac{e^{\frac{m_L x^{\alpha_L}}{C_L c_4}}}{\ln 2} \sum_{k=1}^{m_L} E_{\bar{m}_L} \left(\frac{m_L x^{\alpha_L}}{C_L c_4} \right) f_L(x) dx, \quad (22)$$

and

$$R_u^N = \int_0^\infty \frac{e^{\frac{m_N x^{\alpha_N}}{C_N c_4}}}{\ln 2} \sum_{k=1}^{m_N} E_{\bar{m}_N} \left(\frac{m_N x^{\alpha_N}}{C_N c_4} \right) f_N(x) dx, \quad (23)$$

where $c_4 = P_U \Delta_0 / \sigma^2$, $\bar{m}_L = m_L - k + 1$, $\bar{m}_N = m_N - k + 1$.

Proof: Please refer to Appendix C. ■

To obtain insightful designs, we provide the following corollary.

Corollary 2. *When the network is noise limited and $p_L(x) = 1$, the achievable rate of the user is tightly lower bounded as*

$$R_u^{lb} = W_0(D_L \log_2(1 + C_L c_4 e^{c_5}) + D_N \log_2(1 + C_N c_4 e^{c_6})), \quad (24)$$

where $c_5 = \frac{\alpha_L}{2D_L} (\ln(B^2) e^{-\pi \lambda B^2} + E_1(\pi \lambda B^2) + \varphi_0 + \ln(\pi \lambda)) + \psi(m_L) - \ln(m_L)$, $c_6 = \frac{\alpha_N}{2D_N} (\ln(B^2) e^{-\pi \lambda B^2} + E_1(\pi \lambda B^2)) + \psi(m_N) - \ln(m_N)$, $\varphi_0 \approx 0.5771$ is Euler-Mascheroni constant, $\psi(\cdot)$ is the Psi function and $\bar{F}_{nl}$ is given in (16).

Proof: Please refer to Appendix D. ■

When the effect of small-scale fading is negligible, the lower bound on the achievable rate (24) is defined as

$$R_u^{lb} = W_0(D_L \log_2(1 + C_L c_4 e^{\bar{c}_5}) + D_N \log_2(1 + C_N c_4 e^{\bar{c}_6})), \quad (25)$$

where $\bar{c}_5 = \frac{\alpha_L}{2D_L} (\ln(B^2) e^{-\pi \lambda B^2} + E_1(\pi \lambda B^2) + \varphi_0 + \ln(\pi \lambda))$ and $\bar{c}_6 = \frac{\alpha_N}{2D_N} (\ln(B^2) e^{-\pi \lambda B^2} + E_1(\pi \lambda B^2))$, respectively.

Similar to an observation of the expression (10), it is shown that the expressions (24) and (25) are the functions of $\pi \lambda B^2$ that indicates an average number of LoS BSs in the coverage area πB^2 from the typical user's location. It suggests that the achievable rate is mainly determined by the LoS links. It can be explained that the harvested energy from NLoS links is usually much lower than that from LoS links [15] and the typical user only has a limited transmit power to transmit information from the harvested energy, hence the NLoS uplinks could be blocked.

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we provide simulation results to verify the accuracy of our theoretical analysis and investigate the network performance with different parameter settings. The values of simulation parameters follow the settings of various works [23]–[25], [27]. According to [24], the actual blockage statistics at specific Manhattan areas are given by $(A, B) = \{(0.117, 200), (1, 77.2)\}$. Furthermore, we set $T = 1$, $W = 200$ MHz, $Q = 10$ dB, $\theta_B = \theta_U =$

Fig. 1: Energy coverage probability with different densities λ and circuit parameters when $N_B = 200$, $f_c = 28$ GHz, $P_B = 43$ dBm, $\tau TM = 0.024$, and $(A, B) = (0.0117, 200)$.

Fig. 2: Energy coverage probability with different N_B and carrier frequencies f_c when $\lambda = 5 \times 10^{-5}$, $P_B = 43$ dBm, $(w_1, w_2) = (1500, 0.0022)$, and $(A, B) = (0.0117, 200)$.

10° , $N_U = 30$, $m_L = 3$, $m_N = 2$ $\beta_L = 61.4$, $\beta_N = 72$ dB, $\alpha_L = 2.1$, $\alpha_N = 2.92$ for carrier frequency $f_c = 28$ GHz; as well as $\beta_L = 69.8$, $\beta_N = 82.7$, $\alpha_L = 2.1$ and $\alpha_N = 2.69$ for carrier frequency $f_c = 73$ GHz [27].

Figure 1 demonstrates the energy coverage probability P_e of the mmWave network with different BS densities. We employ two groups of circuit parameters, i.e., $(w_1, w_2) = \{(6400, 0.003), (1500, 0.0022)\}$, which have been provided by [16], [17] and [33], to generate simulation results. The analytical results are defined based on (9) of Theorem 1 by using small $V = 5$ terms in the approximation (i.e., we use an approximation for deriving (9) as shown in Appendix A). We observe that as λ increases from 1×10^{-5} to 5×10^{-4} (m^{-2}), the cell sizes become smaller, then the typical user can harvest more energy due to significantly reduced distances

Fig. 3: Energy coverage probability with different λ as the network is noise limited, $f_c = 28$ GHz and $(A, B) = (1, 77.2)$.

between it and the BSs, hence the P_e increases. Moreover, the circuit parameters greatly influence the EH performance. Specifically, there is a big EH performance gap between the mmWave network with $(w_1, w_2) = (6400, 0.003)$ and that with $(w_1, w_2) = (1500, 0.0022)$. In practice, the values of circuit parameters can be different. Therefore, system engineers should carefully consider the quality and reliability of realistic EH circuit components while designing the EH circuits. In addition, the EH performance increases with N_B because the received power gain at the typical user increases with N_B . This fact can be shown evidently by observing Fig. 2 when the number of BS antennas N_B is increased from 50 to 1000. Another observation is that the energy coverage probability P_e decreases as f_c increases from 28 GHz to 73 GHz due to strong attenuation in higher frequency band. In general, as expected, the analytical results are very close to the corresponding simulation results, which validate our theoretical analysis.

Under the assumption of noise-limited scenarios, the energy coverage probability is shown in Fig. 3 when $N_B = 200$, $f_c = 28$ GHz, $(w_1, w_2) = (1500, 0.0022)$ and $(A, B) = (1, 77.2)$. The curve labelled “Analytical (Theorem 1)” refers to the corresponding ECP results which are obtained by applying Theorem 1, whereas the curve labelled “Analytical (Theorem 2)” refers to the corresponding ECP results which are obtained by applying Theorem 2. It is shown that there are good agreements between those curves, as well as between the analytical and simulation results. These results confirm that the effect of the small-scale fading coefficient on the received power at the typical user can be negligible in the mmWave network with highly directional transmission. In addition, the amount of harvested energy from the directed power transfer link is much higher than that from the ambient RF signals (i.e., the interference) [12], [15].

We study the average harvested energy of the mmWave network with respect to the finite number of BS antennas in Fig. 4. As N_B increases from 100 to 1000, the average harvested energy $\bar{F}_{nl}$ is increased because the more antennas

Fig. 4: Average harvested energy of the typical user with different (w_1, w_2) when $f_c = 28$ GHz and $(A, B) = (1, 77.2)$.

Fig. 5: Asymptotic harvested energy of the typical user with different (w_1, w_2) when $f_c = 28$ GHz and $(A, B) = (1, 77.2)$.

the BS is equipped with, the higher directional antenna gain can be achieved. In that case, the BS can concentrate transmission power effectively with a narrower beam on small areas where the typical user is located. In addition, for the same pair (w_1, w_2) and τT , the larger the value of M , the greater $\bar{F}_{nl}$ can be achieved. This can be explained that a larger value of M provides a higher maximum power which could be harvested at the typical user. For a given M , the harvested energies of the mmWave network with different pairs (w_1, w_2) , which characterize different EH behaviours, are different.

In Fig. 5, we demonstrate the asymptotic behaviour of the average harvested energy when N_B is very large. Herein, the average harvested energy is a function of N_B . Similar to [34] and [35], we use a large scale on the horizontal axis in order to illustrate the convergence to the asymptotic limit defined in Theorem 3. This is because that the convergence is very slow. Specifically, since N_B is very large, the EH circuit is saturated, hence the harvested energy approaches to an asymptotic value

of τTM , such as $\tau TM = 0.024$ and 0.039 as presented in Fig. 5. It is shown that $N_B = 2.25 \times 10^{10}$ antennas are required for the convergence of $\bar{F}_{nl}$ when $\tau TM = 0.024$, whereas $N_B = 2.5 \times 10^{10}$ antennas are required for the convergence of $\bar{F}_{nl}$ when $\tau TM = 0.039$. That validates the theoretical analysis in Theorem 3. Moreover, for a given τTM , each asymptotic value is the same for different pairs (w_1, w_2) . On the other words, the WPT performance loss is negligible due to different EH circuit behaviours (w_1, w_2) when N_B is very large. However, the value of M can greatly affect the WPT as observed from Figs. 4 and 5. Thus, it is important that the engineers need to obtain a higher value of M for obtaining better WPT performance while designing the EH circuits.

After harvesting the energy, the typical user uses the received power to transmit information. The asymptotic behaviour of the achievable user rate is presented in Fig. 6 when N_B is very large. In this example, we set $\tau = 0.4$, $\tau TM = 0.024$, $(w_1, w_2) = (1500, 0.0022)$, $P_B = 30$ dBm. The analytical and asymptotic results are obtained by using Theorem 5 and Theorem 6, respectively. We employ a logarithmic scale on the horizontal axis to present the rate convergence to the asymptotic limit [34]. It is shown that the achievable rate R_u increases with N_B due to increasing the received power and antenna gain. In addition, R_u converges to an asymptotic limit R_u^{as} due to the convergence of SINR given in (19). The rate also converges slowly. It can be explained that the received power of the typical user is low in the nonlinear EH model, as the BS transmit power is low in this case, i.e., $P_B = 30$ dBm, and the large mmWave bandwidth can bring in more noise power. In particular, as N_B is very large, the typical user can achieve a maximum power of $P_U = \frac{\tau M}{1-\tau} = 40$ mW. Hence, with a finite number of BS antennas such as $N_B = 1000$, P_U is lower than 40 mW. Moreover, in order to improve the network performance, more BSs should be deployed. Specifically, as the BS density λ increases from 1×10^{-5} to 5×10^{-5} , the rate performance increases significantly and the rate convergence is faster because of the cell shrinking.

Lastly, we study the achievable rate of the typical user when N_B is a finite number in Fig. 7. Under the assumption of noise-limited scenarios, the analytical results obtained from (21) and (24) are shown in Fig. 7 when $N_B = 200$, $N_U = 40$, $P_B = 43$ dBm, $\tau = 0.4$, $(w_1, w_2) = (1500, 0.0022)$, and $M = 0.4$ W. As f_c increases from 28 GHz to 73 GHz, R_u is significantly decreased. Furthermore, the analytical rates are very close to the corresponding simulation rates. As observed from Figs. 6 and 7, the average harvested energy and achievable rate depend on λ and N_B . The optimum values of λ and N_B can be achieved by considering the environmental and health concerns, quality of service constraints, as well as business view points.

VI. CONCLUSION

The WPT performance at low-power mobile and IoT devices powered by a mmWave large-scale antenna system has been analyzed in this paper. Analytical and asymptotic expressions for the energy coverage probability, the average harvested energy, and the achievable rate have been expressed

Fig. 6: Achievable rates of the typical user with different λ when $(w_1, w_2) = (1500, 0.0022)$, $(A, B) = (1, 77.2)$, $f_c = 28$ GHz, and $P_B = 30$ dBm.

Fig. 7: Achievable rate of the typical user versus λ when $N_B = 200$, $N_U = 50$, $P_B = 43$ dBm, and $(A, B) = (1, 77.2)$.

in terms of the BS density, mmWave channel parameters, the antenna geometry parameters, as well as the EH circuit specifications. Numerical results have verified the accuracy of our theoretical analysis. It has been shown that the harvested energy at the typical user greatly depends on the number of LoS BSs. Due to highly directional transmissions, it has been confirmed that the effect of the small-scale fading on the network performance can be negligible. On the other hand, the network performance is significantly influenced by the practical EH circuit specifications. Since N_B is very large, asymptotic values of harvested energy and achievable rate can be achieved at the typical receiver. Note that the asymptotic average harvested energy is independent of the BS density, whereas the asymptotic rate is dependent on the BS density.

APPENDIX

A. Proof of Theorem 1

From (5), we obtain the conditional ECP of the typical user served by BS via LoS link as

$$P_e^L = \int_0^B \mathbb{P}(F_{nl} > \gamma) f_L(x) dx = \int_0^B \mathbb{P}\left(1 < \frac{S_d + I_d}{\Omega}\right) f_X(x) dx$$

$$\stackrel{(i)}{\approx} \int_0^B \mathbb{P}\left(v < \frac{S_d + I_d}{\Omega}\right) f_X(x) dx, \quad (26)$$

where $\Omega = \ln(\frac{\gamma_0 e^{ab} + 1}{1 - \gamma_0})/a$ and (i) is obtained by replacing 1 with a normalized Gamma random variable (RV) v with low variance, such as $v \sim \Gamma(V, \frac{1}{V})$. This is because the normalized Gamma distribution converges to identity when V approaches to infinity, i.e., $\lim_{V \rightarrow \infty} \frac{V^V x^{V-1} e^{-Vx}}{\Gamma(V)} = \delta(x-1)$, where $\delta(\cdot)$ is the Dirac delta function [36]. Moreover, the cumulative distribution function of v is tightly lower bounded as $\mathbb{P}(v < z) > (1 - e^{-\eta z})^V$, where $\eta = V(V!)^{-1/V}$ [37]. By using Binomial Theorem, we define

$$\mathbb{P}\left(v < \frac{S_d + I_d}{\Omega}\right) > 1 + \sum_{i=1}^V \binom{V}{i} (-1)^i \mathbb{E}\left\{e^{-\frac{\eta i(S_d + I_d)}{\Omega}}\right\} \quad (27)$$

As the main link is LoS, i.e., $|h_0|^2 \sim \Gamma(m_L, \frac{L}{m_L})$, we have

$$\mathbb{E}_L\left\{e^{-\frac{\eta i S_d}{\Omega}}\right\} = \left(1 + \frac{P_{BS} \Delta_0 C_L}{d_1 m_L x^{\alpha_L}}\right)^{-m_L}. \quad (28)$$

Furthermore, due to independent LoS and NLoS interfering links, i.e. $I_d = I_d^L + I_d^N$, we can define

$$\mathbb{E}_{\Phi_L}\left\{e^{-\frac{\eta i I_d^L}{\Omega}}\right\} = \mathbb{E}\left\{e^{-\frac{\eta i \sum_{j \in \Phi \setminus \{0\}} P_B \Delta_j |h_j|^2 C_L x^{-\alpha_L}}{\Omega}}\right\}$$

$$\stackrel{(ii)}{=} e^{-2\pi \lambda \sum_{k=1}^4 b_k \int_x^B (1 - \mathbb{E}_h\{e^{-\eta i P_B \alpha_k h C_L x^{-\alpha_L} / \Omega}\}) A t dt}$$

$$\stackrel{(iii)}{=} \prod_{k=1}^4 e^{-2\pi \lambda b_k \int_x^B (1 - (\frac{-\eta i P_B \alpha_k h C_L x^{-\alpha_L}}{m_L \Omega})^{-m_L}) A t dt}$$

$$\stackrel{(iv)}{=} \prod_{k=1}^4 e^{-2\pi \lambda b_k A (\vartheta(m_L, u_1, x, \alpha_L) - \vartheta(m_L, u_1, B, \alpha_L))} \quad (29)$$

where (ii) is obtained by using the Laplace functional of the PPP Φ_L , (iii) is obtained by using the moment generating function (MGF) of a normalized Gamma RV, and (iv) is done by using the Binomial Theorem and Eq. (3.194.1) of [38]. We use the same steps to define

$$\mathbb{E}_{\Phi_N}\left\{e^{-\frac{\eta i I_d^N}{\Omega}}\right\} = \mathbb{E}\left\{e^{-\frac{\eta i \sum_{j \in \Phi \setminus \{0\}} P_B \Delta_j |h_j|^2 C_L x^{-\alpha_N}}{\Omega}}\right\}$$

$$= \prod_{k=1}^4 e^{-2\pi \lambda b_k [(1-A)\vartheta(m_N, u_2, y, \alpha_N) + A\vartheta(m_N, u_2, B, \alpha_N)]}. \quad (30)$$

By substituting (27)-(30) into (26), P_e^L is defined. Similarly, the conditional ECP of the typical user served by BS via LoS link can be defined as P_e^N in (9). That completes the proof.

B. Proof of Theorem 5

The achievable rate of the user is defined as

$$R_u = \frac{W_0}{\ln 2} \int_0^\infty \frac{P_c(t)}{1+t} dt, \quad (31)$$

where $P_c(t) = D_L P_c^L(t) + D_N P_c^N(t)$ is the coverage probability of (6). The conditional coverage probability of LoS link is obtained as

$$P_c^L(t) = \int_0^B \mathbb{P}[h_0^2 > \frac{x^{\alpha_L} t (I_u + \sigma^2)}{P_U \Delta_0 C_L}] f_L(x) dx$$

$$\stackrel{(i_1)}{<} \int_0^B \sum_{i=1}^{m_L} (-1)^{i+1} \binom{m_L}{i} \mathbb{E}_{\Phi}[e^{-c_1(I_u + \sigma^2)}] f_L(x) dx, \quad (32)$$

where (i₁) is obtained by using similar method for deriving (27) [37]. Moreover, $I_u = I_u^L + I_u^N$, we can obtain

$$\mathbb{E}_{\Phi}[e^{-c_1 I_u^L}] \stackrel{(i_2)}{=} \prod_{k=1}^4 e^{-2\pi \lambda b_k \int_x^B (1 - \frac{1}{(1 + \phi_1 v^{-\frac{1}{\alpha_L}})^{m_L}}) p_L(v) v dv}$$

$$= \prod_{k=1}^4 e^{-2\pi \lambda b_k A (\psi(m_L, \phi_1, x, \alpha_L) - \psi(m_L, \phi_1, B, \alpha_L))} = J_1, \quad (33)$$

where (i₂) results from obtaining the probability generating functional of PPP and Laplace transform of $|h_L|^2 \sim \Gamma(m_L, \frac{1}{m_L})$. Similarly, we get

$$\mathbb{E}_{\Phi}[e^{-c_1 I_u^N}] = \prod_{k=1}^4 \exp(-2\pi \lambda b_k [(1-A)\psi(m_N, u_2, y, \alpha_N) + A\psi(m_N, u_2, B, \alpha_N)]) = J_2. \quad (34)$$

Hence, the upper bound (UB) (32) can be rewritten as (17). A similar approach is used to define the UB on the conditional coverage probability of NLoS link as given in (17). That completes the proof.

C. Proof of Theorem 6

By rewriting (6) as $\kappa_u = \frac{S_u}{\sigma^2}$ and denoting $l = |h_0|^2$, as the UL is a LoS link, the conditional rate is given by

$$R_u^L = \mathbb{E}_x\left\{\int_0^\infty \frac{m_L l^{m_L-1} e^{-m_L l}}{\Gamma(m_L)} \log_2(1 + C_L c_4 l x^{-\alpha_L}) dl\right\}$$

$$\stackrel{(i_1)}{=} \int_0^B \frac{e^{\rho_L}}{\ln 2} \sum_{k=1}^{m_L} (\rho_L)^{m_L-k} \Gamma(k - m_L, \rho_L) f_L(x) dx$$

$$\stackrel{(i_2)}{=} \int_0^B \frac{e^{\rho_L}}{\ln 2} \sum_{k=1}^{m_L} E_{\bar{m}_L}(\rho_L) f_L(x) dx, \quad (35)$$

where $\rho_L = \frac{m_L x^{\alpha_L}}{C_L c_4}$, (i₁) is obtained due to $\int_0^\infty \ln(1 + y) y^{n-1} e^{-\mu t} dt = (n-1)! e^{\mu} \sum_{k=1}^n \frac{\Gamma(k-n, \mu)}{\mu^k}$, (i₂) is obtained due to the relation between the exponential integration $E_n(y)$ and incomplete Gamma function $\Gamma(a, b)$, i.e., $E_n(y) = \int_1^\infty e^{-yt} y^n dy = y^{n-1} \Gamma(1-n, y)$. Similarly, as the UL is a NLoS link, the conditional rate is given by

$$R_u^N = \int_0^\infty \frac{e^{\frac{m_N x^{\alpha_N}}{C_N c_4}}}{\ln 2} \sum_{k=1}^{m_N} E_{\bar{m}_N}\left(\frac{m_N x^{\alpha_N}}{C_N c_4}\right) f_N(x) dx. \quad (36)$$

By using (35) and (36), (21) can be obtained.

D. Proof of Corollary 2

An inequality $\mathbb{E}\{\log_2(1 + \varpi x)\} \geq \log_2(1 + \varpi e^{\mathbb{E}\{\ln(x)\}})$ with $\varpi > 0$ is used to define (24). We denote $l = |h_0|^2$, the uplink conditional rate via LoS link is lower bounded as

$$R_u^L \geq \log_2(1 + C_L c_4 e^{\mathbb{E}_{l,x}\{\ln(l) + \ln(x^{-\alpha_L})\}}). \quad (37)$$

We have $\mathbb{E}_l\{\ln(l)\} = \psi(m_L) - \ln(m_L)$ due to $l \sim \Gamma(m_L, \frac{1}{m_L})$ and $\int_0^\infty \ln(t) t^{n-1} e^{-\mu t} dt = \frac{\Gamma(n)}{\mu^n} (\psi(n) - \ln(n))$, where $\psi(\cdot)$ is the Psi function [38]. Next, we obtain $\mathbb{E}_x\{\ln(x^{-\alpha_L})\}$ as

$$\begin{aligned} \mathbb{E}_x\{\ln(x^{-\alpha_L})\} &= \frac{-\alpha_L}{D_L} \int_0^B \ln(x) 2\pi \lambda x e^{-\pi \lambda x^2} dx \\ &= \frac{\alpha_L}{2D_L} (\ln(B^2) e^{-\pi \lambda B^2} + E_1(\pi \lambda B^2) + \varphi_0 + \ln(\pi \lambda)). \end{aligned} \quad (38)$$

By using the above results, we get $\mathbb{E}_{l,x}\{\ln(l) + \ln(x^{-\alpha_L})\} = c_5$ which is defined in (24). We use similar steps to obtain the conditional rate of the typical user when the UL is a NLoS link. Hence, (24) is defined by using the law of total probability.

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