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THEORETICAL AND EXPERIMENTAL STUDY OF A NOVEL COMPRESSOR

PRADEEP SHAKYA

**SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING**

2019

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Pradeep Shakya

**SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING**

A thesis submitted to the Nanyang Technology University
in partial fulfilment of the requirement for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

31 July 2019

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Pradeep Shakya

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity for it to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

31 JUL 2019

.....
Date

.....
Prof Ooi Kim Tiow

Authorship Attribution Statement

Chapter 4 and section 7.4.1. of Chapter 7 have been submitted to International Journal of Refrigeration. The journal is in press as: Shakya, P. and Ooi, K. T., “Introduction to Coupled Vane compressor: mathematical modelling with validation”, International Journal of Refrigeration, 2020. ISSN 0140-7007, <https://doi.org/10.1016/j.ijrefrig.2020.01.027>.

The contribution of the co-authors are as follows:

- Prof Ooi provided the initial project direction.
- I formulated the mathematical models, developed the simulation codes and investigated the performance of Coupled Vane compressor under the supervision of Prof Ooi.
- For the experimental section, Prof Ooi helped with the funding acquisition for the fabrication of Coupled Vane compressor prototype and its test bed. I performed the experimental testing of the prototype. The test procedure and results obtained were checked by Prof Ooi.
- I prepared the manuscript and the manuscript was thoroughly reviewed and edited by Prof Ooi.

Chapter 5 and section 7.4.2 of Chapter 7 have been submitted to International Journal of Refrigeration. The journal has been as: Shakya, P. and Ooi, K. T., “Vane and rotor dynamics of a coupled vane compressor”, *International Journal of Refrigeration*, 2019. [Under review since 6 Jan 2020]

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- Prof Ooi provided the initial project direction.

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- Prof Ooi assisted with the fundings for the fabrication of compressor prototype and test bed. He also facilitated essential equipment for instrumentation and measurement of compressor prototype.
- I prepared the manuscript and the manuscript was thoroughly reviewed and edited by Prof Ooi.

24 Jan 2020

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Date

.....
Pradeep Shakya

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Abstract

Many different types of the positive displacement rotary compressors are currently used in air-conditioning, refrigeration and heating applications. According to Japanese Air-conditioning and Refrigeration News, the production for positive displacement rotary compressors in 2018/19 alone exceeded 200 million pieces. Obviously, large amount of materials, especially metal such as steel, are being used every year to produce these compressors. Saving these materials will lead towards a more sustainable environment. This thesis investigates the development of a novel compressor, namely, Coupled Vane Compressor, which is significantly material saving, and to our knowledge, it is one of the most compact rotary vane compressors.

Coupled Vane Compressor (CVC) as the name suggests, has two vanes coupled together. Its unique feature is that the coupled vanes cut diametrically through the rotor. Hence, the design of CVC, theoretically, requires the rotor to be as small as the motor shaft for it to work. Due to its compact design, CVC has the potential in saving a significant amount of material during its production and thus leading to smaller carbon footprint over its lifecycle compared to the existing rotary compressors.

The mathematical models of CVC were formulated to study its operational characteristics. The mathematical models developed for the studies include the mathematical representations of the geometry of its working chamber, thermodynamics of the working fluid, main flows through the inlet and outlet ports, secondary flows through internal leakages, kinematics and dynamics of the moving parts and lubrication of the rubbing parts.

A simulation program was developed in Fortran and the program included the mathematical models developed to predict the performance of CVC. REFPROP was used to calculate thermodynamic properties of the working fluid. Moreover, parametric studies of CVC were performed, and the performance of CVC for various vane material, operating pressure ratio and rotor-to-cylinder radii ratio were studied. The results obtained showed that, for steel vanes, CVC can operate at the minimum speed of 1000 r min^{-1} and minimum pressure ratio of 2 without vane chattering. Using aluminium vanes, which is lighter than steel vanes, the frictional losses at the vane tips was reduced and thus the improvement of over 3% in mechanical efficiency was predicted.

For simplicity and to save costs, an open-type test circuit was designed using air as the working fluid to experimentally test the CVC prototype. The CVC prototype was designed to have the maximum suction chamber volume of 44 cm^3 . The measured parameters include the discharge pressure, temperature and the flowrate. At 1500 r min^{-1} , the pressure ratio of 6.1 was measured. The predicted results were then compared with the measured data to validate the mathematical models developed. The comparison showed the maximum discrepancy of 15% between the predicted results and the measured results.

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Nomenclature

A	area	$[m^2]$
b	distance between the rotor centre and the cylinder centre	$[m]$
C	coefficient ; specific heat capacity	$[- ; J\ kg^{-1}\ K^{-1}]$
c	damping coefficient	$[-]$
D	diameter ; hydraulic diameter	$[m]$
E	young's modulus ; energy	$[N\ m^{-2} ; J]$
e	eccentric distance	$[m]$
F	force	$[N]$
f	motor operating frequency	$[Hz]$
g	acceleration due to gravity	$[m\ s^{-2}]$
h	specific enthalpy; heat transfer coefficient ; oil film thickness	$[J\ kg^{-1} ; W\ m^{-2}\ K^{-1} ; m]$
I	moment of inertia	$[m^4]$
k	thermal conductivity	$[W\ m^{-1}\ K^{-1}]$
$k-\epsilon$	$k-\epsilon$ turbulence model	
$k-\omega$	$k-\omega$ turbulence model	
l	length ; current flowing into stator	$[m ; A]$
M	mach number; moment	$[- ; N\cdot m]$
m	mass; number of stator poles in an induction motor	$[kg ; -]$
N	number of items	$[-]$
O	origin	$[-]$
P	power; load per unit length	$[W ; N\ m^{-1}]$
p	pressure	$[Pa]$
Q	heat ; volumetric flowrate of oil	$[J ; m^3\ s^{-1}]$
q	specific heat ; mode participation factor	$[J\ kg^{-1} ; m]$

R	radius; gas constant ; flow resistance ; electrical resistance	[m ; K kg ⁻¹ K ⁻¹ ; Pa·s m ⁻³ ; Ω]
Re	reynolds number	[-]
r	radial coordinate	[m]
s	entropy	[J kg ⁻¹ K ⁻¹]
T	temperature	[K]
t	time; thickness	[s ; m]
u	specific internal energy	[J kg ⁻¹]
V	volume; shear force	[m ³ ; N]
v	velocity; specific volume	[m s ⁻¹ ; m ³ kg ⁻¹]
W	work ; load	[J ; N]
w	width	[m]
x	x-coordinate	[m]
y	y-coordinate	[m]
z	z-coordinate	[m]

Greek symbols

α	contact angle at the leading vane	[rad]
β	contact angle at the trailing vane	[rad]
γ	angle of the resultant force on the vane tip	[rad]
ε	eccentricity ratio	[-]
δ	deflection in y-axis ; clearance	[m ; m]
η	efficiency	[-]
Λ	slenderness ratio	[-]
λ	friction factor	[-]
μ	dynamic viscosity; frictional coefficient	[Pa·s ; -]
ρ	density	[kg m ⁻³]
σ	normal stress	[N m ⁻²]

φ	mode shape ; attitude angle	[- ; rad]
T	torque	[N·m]
τ	shear force	[N]
θ	rotation angle	[rad]
ω	angular speed ; natural frequency	[rad s ⁻¹ ; Hz]
ζ	damping ratio	[-]

Subscripts (only abbreviated subscripts are covered)

b	bearing
b,f	bearing friction
c	cylinder
c,st	at the start of the suction with respect to the cylinder centre
$cen,1$	centrifugal force at the vane 1 (trailing vane)
$cen,2$	centrifugal force at the vane 2 (leading vane)
$cor,1$	coriolis force at the vane 1 (trailing vane)
$cor,2$	coriolis force at the vane 2 (leading vane)
$conv$	convective
$comp$	compression
clr	clearance
cv	control volume
d	discharge
dis	discharge chamber
dis,tip	flow at the vane tip through the discharge port
e	exit
f	flow ; fluid ; friction
f,enf,l	flow at the endface of the leading vane
f,enf,t	flow at the endface of the trailing vane
f,rot	frictional force between the rotor and the vane

f, vn	frictional force between the vanes
enf, vn	vane endface
g	gas
gap	gap
H	at the neck part of the vane
in	inlet
is	isentropic
j	journal
l, vn	leading vane
$leak, enf$	leakage through the endface
$leak, in$	leakage into the chamber
$leak, out$	leakage out of the chamber
$leak, sa$	leakage through the sealing arc
N, rot	normal force on the vane by the rotor
N, vn	normal force the at the point of contact between the vane
n	mode number
oil	oil as the fluid
$orif$	orifice
out	outlet
$p1$	pressure force at the suction chamber on the trailing face of the trailing vane side
$p2$	pressure force at the compression chamber on the leading face of the trailing vane side
$p3$	pressure force at the compression chamber on the trailing face of the leading vane
$p4$	pressure force at the discharge chamber on the leading face of the leading vane
$R, p1$	resultant force at the rotor due to $p1$
$R, p2$	resultant force at the rotor due to $p2$
$R, p4$	resultant force at the rotor due to $p4$

<i>r</i>	rotor
<i>r,enf</i>	endface of the rotor
<i>r,sa</i>	sealing arc
<i>r,st</i>	at the start of the suction with respect to the rotor centre
<i>s</i>	suction
<i>suc</i>	suction chamber

Chapter 1: Introduction

1.1 Background

A Compressor is a mechanical device that increases the pressure of the fluid passing through it by transferring the mechanical energy to the fluid. A conventional compressor is a device which compresses the fluid generally by a set of periodic mechanical action that involves either reciprocating or rotary motion of scientifically designed mechanical parts, whereby the motion alters the energy content of the fluid such that the end state of the fluid has a higher pressure. These are found in gas compression and fluid pumping used in many industries including those in cooling or heating devices [1, 2].

There are different types of compressor and they are generally classified into two main types: the dynamic and the positive displacement [3]. A dynamic compressor converts the kinetic energy of the fluid into the pressure rise of the fluid. Examples of a dynamic compressor include a centrifugal and an axial compressor. In general, dynamic compressors are used in applications where large volumetric capacities are required while the positive displacement compressors are better suited for low volumetric capacities and higher discharge pressures per stage [3].

A positive displacement compressor is a compressor, in which its principle of operation is to induce the working fluid into its working chamber by increasing its volume during the suction process. The volume of the working fluid is then reduced to increase its pressure. At the end of the compression process, since the volume is continued to decrease while the discharge port is open, the fluid will be displaced out of the working chamber. Some of the popular examples of the positive displacement compressor include reciprocating compressor, rolling piston compressor, sliding vane compressor, scroll compressor and screw compressor. Figure 1.1 shows the typical application ranges of the several types of compressor. General advantages of the positive displacement compressors over the dynamic compressors include:

- a higher compression ratio per stage,

- capable of working at lower operating speed, and
- a large variety of positive displacement compressors.

Figure 1.1: Typical application ranges of various compressor types [3]

1.2 Motivation

One of the largest applications of the positive displacement compressors is in the air-conditioning and refrigeration industries. In 2018, the compressor market size for the refrigerators and room air-conditioners stood at 200 million pieces of compressors [4]. The rotary compressors included the rolling piston and sliding vane compressors. Figure 1.2 shows the trend of the global demand for the rotary and scroll compressor from 2011 to 2018. The drop in the production in the year 2015 was caused by weaker global economic activities [5].

Figure 1.2: Trend of compressor market [4]

Global sustainability and the development of greener technologies remain the primary motivating factor in the development of the compressors. Any reduction in the material usage during their production leads to a more sustainable usage of the raw materials. Therefore, reducing the size of a compressor, developing a compressor which requires fewer parts and making a compressor more energy efficient have been important considerations in designing and developing the new compressors.

A simple analysis can be done to understand how the reduction in the metal required for manufacturing the compressors can be beneficial. Generally, existing rotary compressors in refrigeration applications with an input power of around 500-730 W weigh around 10 ± 1 kg [6, 7]. This means the manufacturers worldwide are expending about 2 billion kg of metal for manufacturing 200 million compressors. Assuming 40% reduction in metal required for each compressor means saving 800 million kg of metal every year during their production. Additionally, smaller and simpler compressor parts generally would require less amount of manufacturing time. Thus, it can be inferred that the smaller compressor parts would also save a significant amount of energy during their manufacturing.

1.3 Novelty of the proposed design

General schematics of the rotary compressors such as the rolling piston and sliding vane compressor are shown in Figures 1.3 and 1.4 respectively. In these rotary compressors, the rotor is housed inside the stator. A single vane or multiple vanes divide the space inside the stator into the working chambers. The volume of the working chamber of the compressor together with its operational speed dictate its volumetric capacity.

Figure 1.3: (a) Schematic of a rolling piston compressor [8] and (b) sliding vane compressor [9]

Closer examination of the design of these rotary compressors revealed that the ratio of the diameter of the rotor, D_R , to the internal diameter of the stator, D_S , were generally greater than 0.7. This implied the rotor of these compressors occupied a large volume of space inside the stator. These rotary compressors require relatively larger rotor for them to work properly. In this dissertation, a new compressor named Coupled Vane Compressor (CVC) has been proposed. The unique feature of this compressor includes the two vanes which are coupled together (see figure 3.14 for the schematic of CVC). These coupled vanes can slide diametrically through the rotor. Hence, theoretically, any rotor size which is as big as the motor-shaft will allow this compressor to function. Furthermore, the relatively smaller rotor size of this compressor allows the stator which is also known as the cylinder of the compressor to be designed smaller compared to the existing rotary compressors with matching chamber volume, and therefore, making this compressor potentially the world's most compact rotary compressor. Due to its compact design, it has the potential to save a significant amount of metal, during its manufacturing. It is estimated that CVC has the potential to save approximately 40% of the total volume of metal required for the fabrication compared to that of existing rotary compressors. In this project, detailed design analyses for CVC including mathematical modelling, simulation studies, prototype design and experimentation are conducted.

1.4 Objective and scope of the project

1.4.1 Objective

In view of the ever increasing number of rotary compressor being fabricated annually; 200 million pieces in 2018 as reported by 25th March issue of JARN [10], and each compressor, on averaged, uses at least 15 kg of metal, this adds up to a very significant amount of metal being consumed each year to produce these compressors. The objective of the project is to design and develop a novel and compact rotary compressor which reduces the materials used in its fabrication and hence brings towards a more sustainable cooling and heating system for all. In order to achieve the above, a new novel design of a rotary compressor, namely Coupled Vane Compressor (CVC) has been introduced. CVC, with its novel design, is able to reduce the compressor-end (excluding the electric motor) materials by up to 50%. Hence to achieve the material saving, especially the metal used in compressor fabrication, CVC is introduced and researched into in this thesis.

1.4.2 Scope of the project

To achieve the feasibility of CVC, scopes of this project should cover steps to assess its feasibility, its operation and its performance. Hence the scopes of the research work include formulating the mathematical models for the geometry of working chamber, thermodynamics of working fluid, main and secondary flows in CVC, kinematics and dynamics of CVC and to validate the models using experimental testing of a CVC prototype. Therefore, scopes of the project are:

- i. Review the existing compressors and identify the key features of these designs.
- ii. Analyse the design of CVC and describe its working principle.
- iii. Develop mathematical models which facilitate the theoretical analysis of CVC.
- iv. Develop a simulation program based on the mathematical models and use the simulation program to study performance of the compressor.

- v. Conduct detailed designing and fabrication of the prototype and the fixtures of CVC.
- vi. Conduct instrumentation, assembly, and experimental testing of the prototype of CVC.
- vii. Compare and validate the mathematical models employed for CVC using the results from the measurement.

1.5 Major contributions

- CVC, a novel rotary compressor, is designed which will require significantly less amount of material for its production.
- A zero-dimensional mathematical modelling of CVC is formulated to study the operational characteristics of this compressor. Mathematical models on the geometry of the working volume, thermodynamics of the working fluid, main flows through the suction and discharge port, secondary flows through the internal leakages, valve dynamics and instantaneous in-chamber convective heat transfer between the chamber and wall are presented. The simulation results predicting the performance of compressor are also presented.
- For the analysis of frictional losses due to rubbing of various components of CVC, mathematical models on the kinetics and dynamics models of CVC are formulated. Additionally, using the simulation results, parametric studies including the effect of vane material, discharge to suction pressure and rotor-to-cylinder radii ratio on the vane dynamics and efficiencies of the compressor are presented.
- An oil lubrication model was designed for a CVC prototype. The mathematical models on lubrication flow in various lubrication channels are presented. The results of the simulation studies are presented which ascertain the lubrication of rubbing components for the reliable operation of CVC.
- A CVC prototype was fabricated and experimentally measured. For simplicity and to save cost, air is used the working fluid in an open-type experimental configuration. The predicted results are then compared with measured results for validation.

1.6 Overview of the thesis

This dissertation includes the detailed development of CVC. The dissertation began with Chapter 1 by presenting background, motivation, objective and scope for designing a new compressor. Chapter 2 includes the literature review of the existing positive displacement compressors. The review on the theoretical and experimental investigations on these compressors are also included. Chapter 3 presents the evolution and working principle of CVC.

Chapters 4 and 5 include the formulation of the mathematical models employed to study the performance characteristics of CVC. Chapter 4 formulates the mathematical models to study aspects related to the compressor on thermodynamics, mass flow, valve dynamics, leakage flow and instantaneous in-chamber convection heat transfer. Chapter 5 presents the study of dynamic forces occurring within the compressor by analysing interactions of forces using free body analysis of the vanes, rotor and cylinder. The model on hydrodynamically lubricated journal bearing design is also described in Chapter 5. Chapter 5 also includes the results and discussions on parametric studies of performance of CVC by varying various key parameters identified for CVC.

In Chapter 6, design of the lubrication network for CVC prototype is presented. The mathematical modelling of oil flow along the various flow paths are discussed. The results from the model and parametric studies of the oil flow are presented in this chapter.

The methodologies, results and discussion of experimental analysis are presented in chapter 7. The prediction from mathematical models proposed in chapter 4 and 5 are compared with the experimental measurements for the validation of the mathematical models.

Finally, in chapter 8, the summary of findings of the research project are presented. The chapter concludes with a list of recommendations of future work for further development of CVC.

Chapter 2: Literature Review

This chapter comprises of five main sections. In the first section, the reviews on the existing positive displacement compressors and their characteristics are discussed. In the second section, the reviews on the theoretical models for the evaluation of the characteristics of a positive displacement compressor are presented. The third and the fourth section consists of the discussions on optimization and experimental studies, respectively. The final section summarizes the key research direction for the development of a compressor.

2.1 Positive displacement compressors

In a positive displacement compressor, the fluid is compressed by reducing the physical size of the working chamber volume which results in the pressure rise. In this section, various existing positive displacement compressors and their performance characteristics are discussed.

2.1.1 Reciprocating compressor

In this section, brief discussions on the working mechanism and performance characteristics of the reciprocating compressor are presented.

A. Working mechanism

Figure 2.1: Adapted schematic of a reciprocating compressor [11]

Figure 2.1 is a schematic of a reciprocating compressor used in a vapour compression cycle [11]. It includes a compressor cylinder and a piston connected to a crank mechanism by a connecting rod. As the motor-shaft rotates, the crank

mechanism reciprocates the piston to change the volume of the working chamber. As the piston withdraws from the cylinder bore, the working chamber volume expands, and the working fluid is induced into the chamber from the suction plenum. As the piston advances into the cylinder bore, the volume of the working chamber reduces, and the working fluid is compressed. The compressed fluid is discharged out to the discharge plenum by the opening of a discharge valve.

As the piston reaches its maximum advanced position into the cylinder bore to compress the working fluid, the piston comes to a momentary stop before it reverses its direction of motion to prevent the piston from contacting the end-face of the cylinder bore. Consequently, there exists a clearance volume which includes the volume of compressed fluid trapped within the compression chamber. As the piston recedes from the cylinder bore, this trapped compressed fluid will undergo expansion which causes the loss of a stroke and reduces the amount of working fluid induced into the suction chamber from the suction plenum. Therefore, the existence of a clearance volume reduces the volumetric efficiency of the reciprocating compressor [8]. Generally, longer the piston stroke, smaller is the clearance volume. However, longer strokes have been studied to have higher frictional losses [12].

B. Performance characteristics studies

Since the first mathematical model of a reciprocating compressor was developed by Costagliola [13], many researchers have focused their study on the performance and losses associated with the valves in a reciprocating compressor. Generally, the valve is modelled using Euler-Bernoulli beam theory in a single degree of freedom. With the advances in the simulation capabilities of the computers, studies of 3D models employing Fluid-Structure Interface (FSI) have been developed. In a study by Zhao et al. [14], the mesh mapping method, the dynamic mesh technique and 3D CFD model were employed to simulate the ring-valve motion and to study the fluid flow through the valve. RNG k- ϵ turbulence model and the standard wall conditions were employed to study the fluid flow. The study also compared the pressure predictions from RNG k- ϵ turbulence model and Detached Eddy Simulation (DES) model with a realizable k- ϵ

turbulence model. The DES model was found to predict the pressure pulsations better than the RNG k- ϵ model.

In a reciprocating compressor, two main internal leakage paths were generally studied. These include leakage through the clearance between the piston and cylinder, and leakage through the clearances in the closed valves. Bradshaw et al. [15] studied the effect of clearances on the compressor efficiencies by analysing the exergy destruction rate due to the leakage and frictional losses. Their study found that for the clearance gaps greater than 10 μm , both volumetric and isentropic efficiency decreased due to the increase in leakage losses. For clearance gaps smaller than 10 μm , the frictional losses tended to dominate in reducing the overall isentropic efficiency.

Silva and Deschamps [16] studied leakage through the clearance gaps formed due to the bending of the reed valve on the port due to the gas pressure. The leakage flow was studied by assuming one-dimensional flow, and considering the effects of viscous friction, slip-flow regime, and compressibility. Their study showed that a clearance of 1 μm , reduced the volumetric and isentropic efficiencies by 2.7% and 4.4% respectively. Rezende et al. [17] measured valve leakages indirectly using the constant volume method. In this method, a mass of the fluid in the reservoir of a fixed volume upstream of the leakage was calculated using the ideal gas law. This requires the pressure and temperature in the reservoir upstream of the leakage to be monitored over the time. Their study found that the leakage gaps formed in the range of 0.18 to 3.2 μm and the leakage gaps reduced with the increase in the pressure load.

C. Noise and vibration characteristics

Generally, in the reciprocating compressors, the main sources of vibration are the gas pulsations in the suction and discharge manifolds which excite the acoustic modes of the interior of the compressor and thus radiate as noise. A study by Ma et al. [18] showed that the installation of a surge tank attenuates the pulsation frequency to a lower level. The periodic fluttering of the valve which is often spring-loaded, valve hitting the retainer and the seat are another source of noise [8].

A piston slap is a phenomenon produced by the lateral impact of a piston on the cylinder wall. It manifests in the form of compressor vibration and radiates as noise [19]. Ungar and Ross [20] analysed the noise and vibration due to piston slaps and they suggested that the piston slap induced vibrations may be reduced by reducing the radial clearance between the piston and cylinder bore, using longer connecting rods, and using thicker structures to minimize the noise.

Often in a reciprocating compressor which employs a slider-crank mechanism, a shaking phenomenon is observed. This phenomenon manifests as the noise and vibration through the housing assembly [21].

D. Development of a free piston compressor

Generally, in a free piston compressor, the operation of the piston is controlled by two electromagnets instead of a crank mechanism. Consequently, the motion of the piston is not constrained by the geometry of the crank mechanism. However, this type of compressor requires a compact electromagnetic mechanism large enough to generate forces needed to drive the piston of the compressor. Therefore, the piston is often attached with a spring so that the piston is driven close to its resonance frequency [22].

A linear compressor is a type of free piston compressor which is driven by a linear motor instead of a rotary motor. There are mainly three types of linear motor, namely, moving coil, moving iron and moving magnet [23]. In a linear compressor, all the driving forces act along the line of motion, therefore, there is no sideways thrust on the piston [24]. Consequently, a linear compressor was claimed to be 20-30% more efficient than a crank-driven reciprocating compressor [25]. Nicholas and Reuven's [22] investigation determined about 33% improvement in the COP of a linear compressor compared to a crank-mechanism driven reciprocating compressor of the same volumetric capacity. A moving coil linear compressor with variable input capacity ranging from few milliwatts to 500 W was studied to have motor efficiencies ranging from 74-84% [26]. This compressor was used in a Stirling-type tube cryocooler to provide the temperatures below 12 K [27].

A reciprocating compressor with the crank mechanism is bulky compared to the existing rotary compressors. Generally, it also has high frictional losses, noise

and vibrational characteristics. Although a linear compressor does not require the crank mechanism, it still requires stiff coil springs to function. Thus, a linear compressor can also be considered bulky compared to the existing rotary compressors.

2.1.2 Rolling piston compressor

A. Working mechanism

As shown in Figure 2.2, a rolling piston compressor includes a rotor which is housed inside the cylinder (also referred to as stator). The rotor centre is eccentric relative to the cylinder centre and the rotor rolls against the inner wall of the cylinder. This compressor also includes a vane which is housed inside the slot in the cylinder wall, rests on the rotor, and reciprocates in the vane slot as the rotor rolls around inside the cylinder. The rotor and vane divide the working chamber into a suction and a compression chamber.

Figure 2.2: Schematic of a rolling piston compressor [8]

B. Performance characteristics

Unlike a reciprocating compressor, a rolling piston compressor does not have any clearance volume. The losses in the volumetric capacity of a rolling piston generally occur due to the internal leakages through the clearance gaps between the piston and cylinder bore, endfaces of the vane and rotor, and through the gap between the vane side and vane slot. Yanagisawa and Shimizu [28] presented theoretical and experimental analysis on the leakage through the radial clearance between the rotor and cylinder. The leakage was modelled by assuming the flow through a converging channel first and then as a frictional flow (Fanno flow)

through a constant cross-section duct. Their study found the leakage flowrate of 5-10% of the ideal suction flowrate for clearance gaps of 10-20 μm . A study by Cai et al. [29] showed that radial leakage losses accounted to nearly 61% of the total leakage losses occurring in a rolling piston compressor.

To minimize the radial leakage, Wu et al. [30] demonstrated the use of radial compliance mechanism which allowed the rolling piston to slide radially to reduce the radial clearance gap. Their prediction of the volumetric efficiency showed an improvement of about 1%.

Motion analysis of a rolling piston compressor by Yanagisawa et al. [31] calculated the frictional losses at the vane tip and vane sides. Their results showed that frictional losses occurred at the vane sides due to the high relative velocity between the reciprocating vane and cylinder were higher than at the vane tip.

The measures to improve the friction and wear characteristics in a rolling piston compressor included the use of oils with better lubricity, surface coatings, and nano-additives. The lubrication characteristics of Polyol Ester (POE) oil and Polyalkylene Glycol (PAG) oil in a rolling piston compressor using carbon dioxide as the refrigerant was studied by Jeon et al. [32]. They found that a large amount of CO_2 dissolved in the PAG oil than in the POE oil which led to the PAG oil having better lubricity due to the improvement in its viscosity-temperature characteristics. In a study by Se-Doo et al. [33], TiN coated vanes were reported to have improved wear resistance in the compressor environment where the refrigerant is dissolved in oil. In a study by Zin et al. [34], the addition of graphene-based nanostructures in PAG oil was found to decrease the friction coefficient by about 18% compared to the same while using raw oil. An investigation by Wang et al. [35] found that using the onion like fullerenes (OLFs) and NiFe_2O_4 nanocomposites in a refrigeration oil had an improvement of 1.23% in the COP of the compressor which was investigated under the air conditioning test conditions.

An optimization study of the rolling piston compressor by Ooi [36] used a deterministic model and predicted a 50% reduction in mechanical losses and 14% improvement in the COP. Similarly, Ooi and Lee [37] studied the optimization of

rolling piston compressor geometry using a non-dominated sorting genetic algorithm. Their study found that the optimized design had a reduction in power consumption by 10% while maintaining the same compressor cooling capacity. Gayeski et al. [38] developed a model predictive algorithm to optimize the COP and their predicted COP had the relative discrepancy of around 5% with the measured data.

C. Noise and vibration characteristics

Asami et al. [39] studied the noise characteristics in a rolling piston compressor. They found that the pump-motor assembly and discharge pulsations were the major sources of noise. They suggested lowering the valve lift to reduce noise. However, if the valve lift became too small, the efficiency tended to decrease because of the over-compression of the fluid in the compression chamber.

Vane jumping phenomenon during the start-up of a rolling piston compressor was studied by Bae et al. [40]. Their study showed that when the differential pressure across the vane was low, the vane failed to make contact with the rotor. They also found that using a longer spring and lighter vane reduced the vane jumping phenomenon.

Due to its simple, compact design and easier lubrication of the moving parts, the rolling piston compressor is one of the most widely used compressors in household refrigeration. However, due to the arrangement of the reciprocating vane and the rotor, the rolling piston compressor requires the rotor size to be sufficiently big for it to function properly.

2.1.3 Sliding vane compressor

Figure 2.3: Schematic of a sliding vane compressor [9]

A. Working mechanism

A schematic of a sliding vane compressor is shown in Figure 2.3. It consists of a cylinder, rotor and multiple vanes that can slide through the radial or non-radial vane slots. The rotor is housed inside the cylinder. The rotor centre is eccentrically located relative to the cylinder centre so that when it rotates, the vanes are set into rotational motion with their tips sliding along the internal wall of the cylinder. The centrifugal, gas or spring force applied at the rear end of the vane ensures that the vane tips always slide on the inner wall of the cylinder. As the vanes slide and rotate, the working chamber formed between the two adjacent vanes, rotor and cylinder successively undergo expansion and reduction in the working chamber volume.

Generally, all positive displacement compressors have characteristic intermittent flow. A sliding vane compressor with more than two vanes can achieve near-continuous suction and discharge throughout the working cycle. This allows the sliding vane compressor to be often designed without suction and discharge valves. This removes the valve-fatigue issue which decreases the reliability of the compressor and also minimizes the noise and vibration issues arising from the opening and closing of the vane. However, a compressor without a discharge valve will allow a small volume of compressed fluid from the discharge chamber to leak into the trailing compression chamber [8].

During a start-up under a low back pressure, vane tips detached from contacting the inner wall of the cylinder. This causes a chattering noise and an impact fatigue on the vanes. The gap formed between the vane tip and the inner cylinder wall allows the compressed fluid to leak into the suction chamber [41] which results in the lower volumetric efficiency of the compressor at the start-up.

Generally, in a sliding vane compressor, there exists a small port volume at the discharge because the valve cannot be designed flush with the internal wall of the cylinder. This volume is often known as the throat volume. The compressed discharge fluid trapped in the throat volume undergoes sudden expansion into the trailing compression chamber and sets up large pressure oscillations [42-44].

B. Performance characteristics

The volumetric efficiency of a sliding vane compressor is affected by at least three internal leakage paths, namely, the leakage between the rotor curved surface and internal wall of the cylinder, leakage along the vane tip and sides, and leakage along the rotor end-face. In a study of leakage losses in a sliding vane compressor by Badr et al. [45], the highest leakage losses occurred through the clearance between the vane tip and cylinder. To reduce this tip leakage and improve the volumetric efficiency of the compressor, Shu et al. [46] used a constant back pressure in the vane slot from the compression chamber to ensure that the vane is sufficiently pushed so that it can make continuous contact with the inner wall of the cylinder.

In a comprehensive theoretical and experimental investigation of a sliding vane compressor by Bianchi and Cipollone [47], the flow through the valve was modelled using 1D unsteady flow and quasi-propagatory model (QPM). In this model the unsteady conservation equations are solved by accounting elements defined as capacitive (pressure as a function of inlet and outlet mass flowrates), inertial (mass flow variation as a function of pressure difference), and resistive (frictional and heat losses). Lumped parameter approach using the first law of the thermodynamics and the real gas properties was employed to study the variation of working fluid properties. The vane dynamics included hydrodynamic effects by assuming the presence of a thin film of oil between the vane tip and cylinder wall. Their study concluded that the vane tip frictional losses had the highest contribution in reducing the mechanical efficiency of the compressor. Optimization of the compressor aspect ratio and the reduction of vane mass were considered as effective measures to improve the compressor performance.

In a sliding vane compressor, the frictional wear due to rubbing of the vane sides with the vane slot, vane tip and cylinder wall and rotor endface and the cover. Badr et al. [48] studied the mechanical losses in a sliding vane machine and found that the highest power loss occurred due to the vane tips rubbing against the cylinder wall. To reduce the frictional losses at the vane tips, Bianchi and Cipollone [49] used lighter vanes which had lower centrifugal force pushing the vane towards the cylinder wall. Badr et al. [50] designed a lighter vane with slots

which allow the pressurized working fluid to act on the bases of the vane. The pressure forces from the fluid aided the vane to maintain sealing contact with the cylinder. Other measures to reduce the frictional losses and to reduce the wear of the vane included the coating of Titanium Nitride (TiN) onto the sliding surfaces of the vane [51].

A sliding vane compressor is also widely used in household refrigeration, in applications requiring gas boosting, vapour recovery, oil recovery and so on. The multiple vanes in a sliding vane compressor mean that the total differential pressure acting at the vane sides is much lower compared to the rolling piston compressor. However, as the vanes are contained in vane slots in a rotor, the rotor in a sliding vane compressor should be sufficiently large for its functioning.

2.1.4 Screw compressor

A. Working mechanism

Figure 2.4 shows a screw compressor or also known as twin screw compressor. It comprises of two meshing helicoid lobed rotors on parallel axes. The rotors are contained within a casing. The space contained within the casing and between the rotors where the lobe of a male rotor meshes with the flute of a female rotor forms the working chamber of a screw compressor. Screw compressors are popular in process industries and are applied in compression of gases and vapours over wide ranges of fluid delivery and pressure ratios [52].

A screw compressor can be an oil-free or an oil-injected type. The oil-injected compressor is also known as an oil-flooded compressor because the working chambers are flooded with oil for lubrication and cooling. A general difference between the two types is that an oil-free screw compressor is well suited for the constant volumetric flow at low pressure ratios and low operating speed. On the other hand, an oil-injected compressor is generally applied where the range of volumetric flows at high pressure ratios are required [53]. Laing and Perry [54] noted that the development of an oil-injected screw compressor greatly widened the application of the machine in attaining high single-stage pressure ratios.

Rotor geometry of screw compressor generally contains four to six male-to-female lobe-to-flute combinations. Each lobe-to-flute combination has lobe tip

clearance to avoid physical contact. Fujiwara and Yosada [55] studied the leakage flowrate along the lobe tip clearance and reported that the clearance has a severe influence on the volumetric efficiency of the screw compressor. Tighter clearance, however, increased the shear of the fluid and contributed to the mechanical losses.

Figure 2.4: Schematic of a scroll compressor [56] and its working principle [57]

In a screw compressor, the 'blow holes' are the geometrical features formed at the region of proximity between the rotor tips and the housing at the rotor cusp. Fujiwara et al. [58] studied the effects of the blowholes and found that the circulating oil and gas through the blowholes caused higher pressure in the working chamber. They also found that blowhole leakage had a significant effect on the adiabatic efficiency of the compressor.

B. Performance characteristics

Computational Fluid Dynamics (CFD) improves the precision to evaluate the leakage and dynamic flow losses of a screw compressor. Despite the

complicated rotor geometry of this compressor, Kovacevic [59] employed a 3-D numerical mesh into a CFD code and was able to predict the effects of two-phase flow due to the phase change and the injection of oil. In another study, Kovacevic et al. [60] studied the rotor deformations due to the effects of variation of pressure and temperature within a working chamber. They concluded that the rotor deformation changed the tip clearances, which greatly increased the leakage and deteriorated the overall compressor performance.

Rane et al. [61] studied an oil-flooded screw compressor. They studied the variation of thermodynamic properties including the effects of heat transfer, leakage and the distribution of the oil in the working chamber. The distribution of pressure in the working chamber was found to be uniform, but the gas temperature was non-uniformly distributed due to the non-uniform distribution of oil in the compression chamber. Their study also showed that the oil injection helped lower the gas temperature.

Screw compressors have often been designed with variable pitch rotors for the steeper pressure rise within the compression chamber [62]. A comparison using CFD analysis on screw compressors with uniform pitch rotors and variable pitch rotors revealed that the variable pitch rotors had lower volumetric efficiency due to the higher internal pressure rise [63].

Screw compressors are known to be efficient, emit low noise and have wide range of flow capacities. But, the complicated geometries of rotors imply that screw compressors require the best available production techniques for its manufacturing.

2.1.5 Scroll compressor

A. Working mechanism

Figure 2.5: Various stages in an operational cycle of a scroll Compressor [64]

As shown in Figure 2.5, a scroll compressor consists of two involute spiral scrolls, a stationary and an orbiting scroll. The two scrolls are assembled at 180° phase difference such that space between the two scrolls form crescent-shaped pockets. The suction process occurs through an open crescent-shaped pocket at the periphery between the scrolls. As the orbiting scroll orbits in a circular motion, the working space volume is reduced and fluid in the working space is compressed. As seen in Figure 2.5, the compression moves the gas to the centre of the scrolls and the compressed gas is discharged through the discharge port.

Leon Creux [65] is credited with the invention of the scroll compressor. Due to the lack of precise production tools and techniques at that time, prototyping the compressor was delayed.

B. Performance characteristics

The scroll compressor suffers from component wear at the axial endface of the scroll due to the rubbing of the rotating scroll and the cover. Tojo et al. [64] developed a mechanism which allows the fluid pressure from the compression chamber to leak into a specially designed back pressure chamber. This allowed the pressure force to balance the axial force which eliminated the use of thrust

bearings or mechanical springs. The authors claimed to have optimized the design of the scroll compressor which was 40% smaller, 15% lighter and had 10% higher compressor efficiency than a conventional reciprocating compressor.

In a scroll compressor, the discharge valve can be designed as a gate valve which opens kinematically when the orbiting scroll arrives at the prescribed position. However, a scroll compressor can be subjected to over-pressure or under-pressure expansion at the pressure ratios that do not conform to the opening conditions for which the valve was designed [8]. This leads to the losses in both mechanical and volumetric efficiency of the compressor.

Discharge pulsations in a scroll compressor were studied by Moteji and Nakashima [66]. To dampen the discharge gas pulsation, they modified the design of the fixed scroll to include a chamber such that diameter of this chamber was greater than the diameter of the discharge port. This way the expansion wave from the discharge plenum which propagated to the compression chamber through discharge port was dampened at that chamber.

Etemad and Nieter [67] reported that two kinds of leakage losses were found to be significant in a scroll compressor, namely, the tip and flank leakages. The tip leakage occurred at the clearance gap at the tip of the scrolls. The flank leakage occurred at the gap between the flank or the curved surfaces of the two scrolls. According to Etemad and Nieter, the effect of the tip leakage on the volumetric efficiency of a scroll compressor was found to be the highest. This leakage was reported to be 2-3 times more than the leakage loss due to the flank leakages. Similarly, a study by Chen et al. [68] also showed that the tip leakage caused higher losses in the mass flowrate, power input and the compressor efficiency than the same due to the flank leakage.

Chang et al. [69] studied the reliability of the scroll compressor using Failure Mode and Effect Analysis (FMEA) and the wear was estimated under various test conditions using the prevalent equations between the wear and stress factors. Total time period of 32420 hours at 90% confidence level was predicted.

Scroll compressors are known for their silent and efficient performance. Compared to rotary compressors such as the rolling piston or the sliding vane compressor, the scroll profile of the scroll compressor is complicated. Therefore,

like screw compressors, the scroll compressors also require the production techniques that can be more expensive compared to the rotary compressor such as the rolling piston compressor.

2.1.6 Rotary spool compressor

The rotary spool compressor was invented by Greg Kemp [70]. A typical rotary spool compressor is shown in Figure 2.6.

A. Working mechanism

In its basic form, a rotary spool compressor consists of a cylinder, spool-shaped rotor, eccentric cam and two vanes. The rotor, with its centre at an eccentric distance relative to the cylinder centre, rotates in a fixed axis. The vanes rotate along with the rotor and they are constrained by an eccentric cam such that the vane partitions the working space between the cylinder and the rotor into the working chambers. As shown in Figure 2.6, the clockwise rotation of the rotor induces the fluid into the suction chamber. The trailing vane tip then disconnects the working chamber from the suction port and the resulting working chamber acts as the compression chamber. Further rotation causes the compression of the fluid until the compressed fluid is discharged out through the discharge port.

Figure 2.6: Illustrations of a rotary spool compressor [71]

B. Performance characteristics

The design of a spool compressor is similar to a sliding vane compressor except that the vane in a spool compressor cut diametrically through the rotor and the vane is constrained by an eccentric cam. Kemp et al. [72] used dynamic sealing to minimize leakage between various paths from compression to suction chamber.

In an experimental investigation of one of the embodiments of the rotary spool compressor, they were able to achieve the pressure ratio of 38:1 in 15 seconds with air as the working fluid [73]. The dynamic sealing used in the compressor prototype included a tip seal and the spring combination utilizing the radial force and gas pressure. The measurement results obtained show the volumetric efficiencies varying from 80% to 95% for various clearance gaps. The maximum isentropic efficiency of 65% was also reported.

Kemp and Groll [74] tested the performance of a liquid-flooded rotating spool compressor. Their measured data show 50% overall isentropic efficiency and 98% volumetric efficiency. Mathison et al. [75] developed a model to predict the performance of a spool compressor with multiple vapour injection ports. Their model predicted that adding one vapour injection port increases the cycle COP by approximately 12% and by adding the second port, 4% improvement in the COP was predicted.

A parametric study to improve the design of a rotating spool compressor was conducted by Bradshaw et al. [76] for a variety of geometrical parameters to explore the operational performance of two working fluids (R410A and R134a). Four main geometrical parameters including rotor radius, cylinder radius, rotor-cylinder eccentric distance and the compressor height were first reduced to two dimensionless parameters, namely eccentricity ratio (the ratio of the rotor-to-cylinder radius) and the cylinder slenderness ratio (the ratio of the cylinder height to the diameter). The results obtained through their investigation generally showed higher volumetric and isentropic efficiency for smaller eccentricity ratio. Meanwhile, the optimum values of efficiencies for varying volumetric displacement were found to occur within the slenderness ratio of 1.4 – 1.85.

Due to their high efficiency, rotating spool compressors have the potential for application in small-scale to medium scale refrigeration applications. However, the vane in a spool compressor requires to be constrained by an eccentric cam which is housed inside the rotor. This results in the spool compressor design which requires the rotor to occupy a large volume inside the cylinder.

2.1.7 Swing vane compressors

A. Working mechanism

Figure 2.7: Illustration of a swing vane compressor [77]

A swing vane compressor is shown in Figure 2.7. It was first introduced by Daikin in 1996 [78]. The operation of this compressor is similar to a rolling piston compressor and it can be said that this compressor was developed to solve the problem of high frictional losses occurring at the vane sides of a rolling piston compressor. A swing vane compressor consists of a cylinder, rotor, and vane. One end of the vane is hinged in the cylinder wall by means of a hinge joint and the other end is inserted into the vane slot in the rotor. The rotor is housed inside the cylinder and it is eccentrically arranged relative to the cylinder. The vanes have greater load bearing capacity due to similarity with the simply supported beam on its two ends by the cylinder wall and rotor slot.

B. Performance characteristics

Studies by Hu et al. [77] show that the total frictional loss is only 35.9% of the loss incurred in a conventional sliding vane compressor when the operating speed of compressor was 1000 r min^{-1} . The total frictional losses amounted to 69% of that of a sliding vane compressor when the operating speed increased to 3000 r min^{-1} .

C. Development of a double-swing vane compressor

The double-swing vane compressor, shown in Figure 2.8, was developed by Xu et al. [79] by introducing an additional vane to the compressor assembly. This allowed the inventors to add in another suction port following the second vane. Consequently, the authors claimed that the design allowed the double-swing

vane compressor to increase the volumetric flowrate out of the compressor by about 1.6 as compared to same of a single-vane swing vane compressor.

Figure 2.8: Illustrations of a double-swing vane compressor [79]

Compared to a rolling piston compressor, swing vane does not require a spring for its operation. Therefore, it is simpler in design. But a swing vane compressor still requires the rotor to occupy large volume inside the cylinder to contain the vane in its vane slot.

2.1.8 Revolving vane compressor

The revolving vane compressor was invented in 2006 by Prof Ooi Kim Tiow and Dr. Teh Yong Liang in Nanyang Technological University, Singapore to reduce the excessive frictional wear and tear of the rubbing components in the rolling piston compressor.

Figure 2.9: (a) Sectional top view and (b) side view of a revolving vane compressor [80]

A. Working mechanism

Figure 2.9 (a) and (b) show the schematics of a revolving vane compressor. In its basic form, a revolving vane compressor consists of a cylinder, rotor and vane which connects the rotor with cylinder. The rotor is housed inside the cylinder and it is eccentric relative to the cylinder centre. The vane is attached to the cylinder wall using a hinge joint. During operation, the vane can swivel like a hinge in the cylinder wall and slides through the slot in the rotor during the rotation. As the rotor rotates eccentrically, it revolves the vane which then rotates the cylinder. The motion of these components causes the working chamber volume within the cylinder to vary. This results in suction, compression, and discharge of the working fluid.

The most significant aspect of the revolving vane compressor is that the cylinder is made to rotate along with the rotor. The resulting effect is that the relative sliding velocity between the vane and the cylinder is reduced and therefore, frictional loss between them is reduced. Analysis of frictional losses in a revolving vane, conducted by Teh and Ooi [81], predicted more than 20% reduction in frictional losses compared to a rolling piston compressor. A parametric study on revolving vane compressor also predicted more than 92% of mechanical efficiency for a compressor with larger rotor-to-cylinder radii ratio [80].

A variant of the revolving vane compressor, named as the fixed-vane revolving vane compressor, was developed by Tan and Ooi [82] in 2011. In this compressor, one end of the vane is rigidly fixed onto the rotor or cylinder. The fixed vane revolving vane compressor was predicted to achieve mechanical efficiency of over 95%. Their experimental investigation of a fixed-vane revolving vane prototype using an open-loop test setup and air as the working fluid found the predicted power consumption to be within the reasonable agreement with the measured data. The maximum discrepancy of 10% with the measured data was reported [83].

Since, in a revolving vane compressor, the cylinder rotates along with rotor, additional journal bearings are required to support the cylinder. Tan and Ooi [84] performed a design analysis on dynamically loaded journal bearing for the

revolving vane compressor. Their study concluded that the bearing frictional loss due to the increment in the journal radius is more significant than due to the increment in the length.

B. Performance characteristics

The performance of a rotating discharge valve on the cylinder of a revolving vane compressor was studied by Teh et al. [85] using Euler-Bernoulli beam theory. Their study found that the opening of a rotating discharge valve had a better response than a stationary valve because of the centrifugal force. Therefore, they concluded that a revolving vane compressor can be designed with stiffer valves for better reliability. However, it is noted that the centrifugal force which results in faster opening of the valve may also result in delayed closing of the valve which may result in the leakage of the fluid from the discharge plenum to the trailing compression chamber.

The leakage characteristics of a revolving vane compressor were studied and compared with the same of a rolling piston compressor by Teh and Ooi [86]. They predicted that the leakage loss at the radial clearance can be reduced by more than 40% in the revolving vane compressor as compared to the same of a rolling piston compressor by designing a shorter compressor with rotor-to-cylinder radii ratio of 0.75. However, the shorter compressor was found to have larger frictional losses.

Tan and Ooi [87] presented a theoretical study including an in-chamber convective heat transfer for predicting the performance of a revolving vane compressor. They claimed that the model predicted to within 2% of the experimentally measured pressure variations for various compressor speeds.

A revolving vane compressor can be efficient in its operation. But it still requires a big rotor inside the cylinder to contain the vane.

General summaries of pros and cons of the positive displacement machines studied in section 2.1 are presented in Table 2.1.

Table 2.1 Summary of pros and cons of various positive displacement compressors

Reciprocating compressor (compared to rotary compressors)
--

Pros	Cons
<ul style="list-style-type: none"> • Fewer leakage paths: clearance between piston circumference and cylinder • Fewer rubbing parts: between piston circumference and cylinder and in slider crank mechanism 	<ul style="list-style-type: none"> • Bulky design as it requires slider crank mechanism or other mechanism to generate reciprocating motion of piston • Higher vibrational problems due to poor balancing of reciprocating piston

Rolling piston compressor

• Simpler and compact design with cylinder, eccentric, roller and a vane • Low vibrational problems (compared to reciprocating compressor) because of well-balanced rotary parts	• Multiple leakage paths: through clearances between roller circumference and cylinder, rotor and vane endfaces • Multiple rubbing components: between eccentric and roller, roller and cylinder inner wall, vane tip and roller and vane sides and vane slot

Sliding vane compressor

• Near-continuous flow due to multiple number of vanes • Its ability to have near-continuous flow allows it to be designed without any valve, thereby removing any valve fatigue issue and noise caused due to opening and closing of the valve	• Vane chattering during start-up under low back pressure and lower operating speed • Multiple sources of frictional loss due multiple number of vanes rubbing against the vane slot • Sliding vane compressors require precise fit between vane and vane slot, which increases precision and complexity of manufacturing.

Screw compressor

• Widely known for its capability to attain high pressure ratio and high volumetric flowrate in single stage	• Complicated geometry which requires best production techniques for manufacturing
-

-
- Fewer rubbing parts compared to other rotary compressors
 - Able to achieve near-continuous flow and therefore have low noise and vibration characteristic
-

- **Scroll compressor**

- They are also able to achieve near continuous flow and hence have low noise and vibration characteristics
 - Complicated geometries of the scrolls which means increase in complexity in production and hence results in higher manufacturing costs
-

- **Spool compressor (compared to sliding vane compressor)**

- Constraints in vane motion by eccentric cam meant lower vane tip friction and better vane tip sealing
 - Eccentric cam housed inside the rotor means rotor-to-cylinder diameter ratio of 0.75 or larger is required, which means larger rotor needs to be housed inside the cylinder
-

- **Swing vane compressor (compared to rolling piston compressor)**

- Simpler design as the vane and spring in rolling piston compressor is replaced by a swing vane and a bush
 - Retains the leakage paths of rolling piston compressor
 - Lower frictional loss at the vane sides compared to the rolling piston compressor
 - Constraints in its vane and rotor size means that it also requires rotor-to-cylinder diameter ratio of 0.75 or larger
-

- **Revolving vane compressor**

- Lower frictional loss because of lower relative sliding speed between the vane and cylinder
 - Additional journal bearings are required to support the rotating cylinder which increases the precision and complexity during manufacturing
-

2.2 Review of the simulation studies

A comprehensive theoretical analysis of a positive displacement compressor requires the study of thermodynamics, mass flow, heat transfer, leakage flow, valve dynamics, dynamic forces acting on the compressor parts, oil lubrication flow and so on. In this section, the simulation studies developed by various authors for the compressors will be presented.

2.2.1 Thermodynamics model

The study of thermodynamics of a working fluid evaluates the variation of pressure, temperature and fluid mass in the working chamber of a compressor. The thermodynamic properties of the working fluid are evaluated as the fluid undergoes three processes, viz., suction, compression, and discharge process. Generally, the suction and discharge processes often depend upon the dynamics of the valve. These processes are affected by the leakage through the clearance gaps, and heat transfer between the fluid and surrounding walls.

Costagliola [13] is credited with the development of the first mathematical model for a reciprocating compressor including reed valves. Costagliola employed the polytropic equation and assumed ideal gas behaviour during all the processes. Other assumptions include uniform and instantaneous propagation of properties in a working chamber, isentropic compression process and perfectly sealed condition, that is, the effect of leakage was assumed negligible. Soedel [88] also employed similar model and validated his predictions with the measurement. The assumption in which the thermodynamic properties uniformly propagate within the working chamber is also known as the lumped parameter approach.

Lee et al. [89] concluded that compared to the polytropic model, the use of the first law analysis for the thermodynamics modelling provided a more accurate prediction of the fluid temperature in the working chamber. Squarer and Kothmann [90] assumed ideal gas behaviour but employed the first law of thermodynamics to predict the gas properties of the working fluid. Prakash and Singh [11] developed a model using both real gas properties and the first law of thermodynamics including the effects of leakage and heat transfer. Rottger and Kruse [91], Hiller and Glicksman [92], Ng et al. [93], Lee et al. [89] all carried out simulation studies using the real gas properties and concluded that the use of

real gas properties gave significant improvement in accuracy as compared to the predictions using ideal gas properties.

A modified polytropic process with the assumption of an ideal gas was used to model compression process in a sliding vane compressor by Osama [94]. Their study illustrated the effect of leakage on the decline of mass flowrate, discharge pressure, power input and mechanical efficiency.

Ooi, Wong and Kwek [95] predicted the variation of properties in a rolling piston compressor using the first law analysis, real gas equations, lumped parameter approach, and assuming no leakage. Employing similar model, Ooi [36] and Ooi and Lee [37] optimized the design variables of a rolling piston compressor based on the same model. Li et al. [56] employed similar model to study a water-injected twin-screw compressor. Their predicted result showed the maximum error of 5.4% compared to the measured data. Using same model, Tan and Ooi [83] claimed their predicted mechanical power was within 10% of the experimentally measured data.

Sun et al. [96] similarly predicted the properties of water-cooled scroll compressor using lumped parameter model but included the effects of leakage assuming isentropic nozzle flow. Similarly, to study a sprayed oil injection technique for cooling a sliding vane compressor, Bianchi et al. [97] also employed similar model. The overall agreement with the measured pressure data was said to be satisfactory.

Many authors employ CFD to study the positive displacement compressors. Generally, generating a grid which can describe the changes in working chamber throughout the working cycle is considered challenging. Kovacevic [59] developed an analytical grid generation method by using moving finite volume numerical mesh method for predefined geometrical definitions, initial and boundary conditions and operational parameters. The two-phase flow was simulated using Euler-Lagrangian approach. A standard κ - ϵ model was employed to include the influence of turbulence.

Bianchi et al. [98] employed CFD to study an oil-injected sliding vane compressor. The grid was generated using user-defined nodal displacement. SST k - ω and standard wall functions were employed for turbulence modelling. Constant

pressure at the inlet and outlet was assumed. Their results showed non-uniform distribution of temperature within the working chamber.

Mendoza-Miranda et al. [99] developed compressor model based on dimensionless correlation approach using Buckingham π -theorem for refrigerant fluids such as R1234yf, R1234ze(E) and R450A as an alternative to R134a. The prediction error of the temperature of the working chamber using this method was found to be lower than ± 2 K. Zenhdeoudi et al. [100] used an Artificial Neural Network (ANN) and an Adaptive Neuro Fuzzy Inference System (ANFIS) to predict parameters such as temperature, pressure, suction, discharge and injection mass flowrates in a scroll compressor. Maximum relative deviation of about 2.4% with the measured data was reported.

Available literature indicates that a lumped parameter approach employing real gas properties and using the first law of thermodynamics gives a good prediction of the real compressor behaviour. However, to study the local mechanisms within a compressor in more detail, it would be advisable to employ CFD analysis.

2.2.2 Valve dynamics model

Most of the positive displacement compressors require valves at the suction and discharge ports to prevent the flow reversal. In general, a dynamic equation which describes the valve movement is coupled with a flow equation which relates the valve opening due to the pressure difference across the valve to the mass flowrate. It is usually assumed that the receivers or plena of infinite volume exist at the suction and discharge such that the suction and discharge pressure remain constant.

In 1950, Costagliola [13] developed a mathematical modelling of a reciprocating compressor with spring-loaded valves. The analysis of the valve dynamics was done by assuming a single degree of freedom for a cantilever beam of uniform width. The pressure and valve displacement diagrams were obtained by solving non-linear differential equations using graphical methods. Although this method was deemed too tedious for its application as an industrial design tool, many models are based to some degree on the mathematical model presented by Costagliola.

Wambsgness and Cohen [101] developed a similar model but included the effect of damping by assuming a constant value of damping coefficient. MacLaren and Kerr [102] studied the delay in valve opening due to the oil stiction. Their study showed that the oil stiction had a significant impact at low values of pressure ratio.

While the previous analyses were made by assuming uniform cross-sectional area throughout the valve length, Gatecliff [103] developed a model for forced vibration of a valve with non-uniform cross-sectional area. The predicted results were in good agreement with the experimentally measured data. Ooi et al. [104] studied the over-compression losses as a function of valve displacement. They developed the model for a valve plate with varying width and used polynomial equations as trial functions to approximate the mode shape. They found that stiffer valves caused higher losses due to over-compression. Teh et al. [85] studied the performance of a rotating discharge valve using Euler-Bernoulli beam theory including the effects of centrifugal force. Their study revealed that the rotating discharge valves displayed better response than the stationary valves.

Finite element method allows the researchers to study the complex or irregular structures. Friley and Hamilton [105] and Piechna [106] employed the finite element method to predict the valve displacement. Piechna's study concluded that oblique valve stops should be used to minimize forces and moments that cause rapid oscillations. Fluid-structure interaction simulations [107-109] allow the researchers to incorporate the finite element method and CFD to study the valve dynamics and fluid flow across the valve in detail.

2.2.3 Heat transfer model

The main sources of heat in the compressor include the compression process and heat generated due to friction. The increment in pressure by reducing the volume is accompanied by the increase in temperature of the fluid. The heat is then transferred to the cylinder walls and then to other components of the compressor such as the suction line. Often the fresh working fluid flowing through the suction line is at lower temperature than the heated suction line, as a result, heat is transferred from the suction line to the fluid. The heated fluid induced into the compressor demands higher power for compression than the fluid at a lower

temperature. Therefore, heat transfer within the compressor affects its volumetric and adiabatic efficiency.

The theoretical models on heat transfer in a reciprocating compressor generally assumed lumped formulation and rely on the empirical correlations. Adair et al. [110] studied the heat transfer from the working fluid to the cylinder walls and developed a correlation for the Nusselt number. Their model was based on the correlations developed by Woschni [111] and Annand [112] for an internal combustion engine. The Nusselt number derived is a function of Reynolds number based on the piston mean velocity. The correlation developed by Adair et al. [110] is shown in equation (2.1).

$$Nu = 0.053(Re)^{0.8}(Pr)^{0.6} \quad (2.1)$$

Liu and Zhou [113] measured the temperature distribution on the cylinder walls of a reciprocating compressor for various pressure ratio, compressor speed, and suction temperature. The heat transfer correlation which was similar to equation (2.1) was derived by applying the first law of thermodynamics. Their correlation is shown in equation (2.2).

$$Nu = 0.75(Re)^{0.8}(Pr)^{0.6} \quad (2.2)$$

A study by Tuhovcak et al. [114] showed that the prediction of isentropic efficiency was influenced according to the type of heat transfer model selected. Similarly, Fagotti et al. [115] assessed various heat transfer correlations by comparing the predicted compressor working characteristics, including, valve performances, thermodynamic losses, cooling capacity and power consumption with the experimentally measured data. According to their analysis, Liu and Zhou's [113] swirl velocity evaluation resulted in inconsistent results for some working conditions. Annand's [112] correlation showed the best agreement with the measured data.

Using a standard κ - ϵ turbulence model in a CFD simulation, Aigner and Steinruck [116] developed a heat transfer correlation based on Stanton number for a reciprocating compressor. The Stanton number is defined as the ratio of heat flux to the wall and energy flux in the flow relative to the wall. The Stanton number was modelled as a function of skin friction coefficient. The correlation is shown in equation (2.3).

$$St = \frac{C_f}{2Pr^{2/3}} \quad (2.3)$$

The friction coefficient was derived as the function of Reynolds number. Equation (2.4) represents the correlation for the coefficient of friction used for compression and expansion process. Equation (2.5) is the modified correlation used for suction and discharge process and it was proposed by Bejan [117].

$$C_f = 0.078Re^{-0.25} \quad (2.4)$$

$$C_f = 0.046Re^{-0.2} \quad (2.5)$$

Padhy and Dwivedi [118] used the correlation developed by Adair et al. [110] for a reciprocating compressor to study the variation of temperature in the compression chamber of a rolling piston. Tan and Ooi [87] studied the effect of heat transfer on pressure variations in the compression chamber of a revolving vane compressor using the correlations proposed by Adair et al. [110], Annand [112] and Liu and Zhou [113]. They reported that the chamber pressure predicted using the correlation proposed by Liu and Zhou [113] had the discrepancy within the range of 3.2-5.4% compared to the correlation proposed by other authors. Chen et al. [119] also used the correlation derived for the spiral heat exchanger to study the compression process in a scroll compressor.

Using a lumped capacitance model, Chen et al. [68] studied heat transfer between the gas and various elements of scroll compressor such as steel and aluminium scrolls, suction line, motor parts, compressor shell and oil. Each of these components were identified as 'lumped capacitance' elements and were associated with a 'lumped temperature.' Then, an analogy with an electrical circuit was made, in which, lumped temperature, heat transfer rate, thermal mass and thermal resistance corresponded to voltage, current, capacitance and resistance in an electrical circuit. The results obtained showed that the suction line heating reduced suction gas density and volumetric efficiency of the compressor.

Lumped capacitance method was also employed by Dutra and Deschamps [120] to develop a comprehensive model for predicting the performance of a hermetic reciprocating compressor. Their predicted volumetric efficiency and isentropic efficiency for high pressure ratio condition (evaporating temperature of -35 °C

and condensing temperature of 70 °C) had a maximum deviation of 10.2% and 7.8% compared to the measured data.

Since the 1990s, CFD analyses to study heat transfer in the compressors are gaining popularity. Disconzi et al. [121] employed RNG k - ϵ turbulence model and the eddy viscosity concept for the valve flow to study the heat transfer in a reciprocating compressor during suction and discharge process. They developed four in-cylinder heat transfer correlations for four main processes, namely, suction, compression, discharge, and re-expansion occurring in a reciprocating compressor. The characteristic velocities during the suction and discharge process were defined based on the mass flowrate through the valves. Based on their investigation, the derived correlations for the Nusselt number are shown in Table 2.2.

Table 2.2: The correlations proposed by Disconzi et al. [121]

Process	Correlations
Compression	$Nu = 0.08(Re)^{0.8}(Pr)^{0.6}$
Discharge	$Nu = 0.08(Re)^{0.8}(Pr)^{0.6}$
Expansion	$Nu = 0.12(Re)^{0.8}(Pr)^{0.6}$
Suction	$Nu = 0.08(Re)^{0.9}(Pr)^{0.6}$

One of the major challenges in employing CFD is to generate a grid able to represent the shapes of working chamber throughout the working cycle. Stosic et al. [122, 123] and Kovacevic et al. [59, 124] have presented comprehensive work on the generation of moving and deforming grid, meshing rotor interfaces and computational models for screw compressors. Their study included the heat exchanged between the spherical oil droplets and gas via convection. Assuming the heat transfer coefficient for the Stokes flow, equation (2.6) was used as the empirical correlation for the study.

$$Nu = 2 + 0.6(Re)^{0.6}(Pr)^{0.33} \quad (2.6)$$

Ooi and Zhu [125] studied the convective heat transfer in a scroll compressor using CFD. A standard k - ϵ turbulence model was employed. Their study revealed that although the gas pressure was uniform and consistent with the results predicted using lumped parameter approach, other gas properties, especially temperature, showed non-uniform spatial distribution. Overall, compared to the

lumped approach, their study predicted higher convective heat transfer between the gas and walls.

Lumped parameter approach allows the calculations to start with relatively coarse initial conditions and establish a full stable solution after several cycles in shorter time. However, the accuracy of the results varied from case to case depending on the empirical correlations chosen for the heat transfer coefficient.

2.2.4 Leakage model

Generally, a simple thermodynamic analysis of a compressor assumes a perfectly sealed condition, i.e., the internal leakages within the compressor are ignored. However, all compressors are known to have a varying degree of internal leakages which reduce the volumetric capacity of a compressor. There are 3 methodologies commonly used in modelling leakage of the fluid: leakage path assuming isentropic nozzle, compressible flow model assuming adiabatic and frictional flow, and hybrid models using correlations obtained using regression techniques. Depending on the nature of the leakage paths, various models of the leakage analysis can be utilized to calculate the leakage mass flowrate of the working fluid within the compressor.

Leakage through the converging paths can be modelled by assuming isentropic flow of an ideal compressible gas through a nozzle. This method requires the calculation of an effective flow cross-sectional area at downstream. The model is used with an empirical discharge coefficient to account for static pressure losses in the flow path and the presence of oil in the clearance. Cho et al. [126] suggested this coefficient could be 0.1 in their study on choked flow data. Various authors such as Margolis [127], Puff and Kreuger [128], Youn et al. [129], Lee et al. [130] and Chen et al. [119] have employed the corrected isentropic flow model. The results obtained in these studies show varying degree of agreement with the measured data.

The leakage flow length in a positive displacement compressor are generally long compared to the flow width. Therefore, the fluid friction will have a significant influence on the leakage flow. In a detailed leakage modelling, the leakage path can be assumed to constitute of a converging duct connected to a uniform cross-section duct of a fixed length. While some authors have considered such leakage

flow to include the effects of compressibility, others have simply assumed the flow to be incompressible.

The leakage flow along the straight duct can be considered as Fanno flow, that is, the flow is assumed to be compressible and adiabatic with the frictional flow. This model was employed by authors such as Tojo et al. [131] to study the flank and tip leakages in a scroll compressor. Yanagisawa and Shimizu [28, 132] employed similar model to compute leakage through the radial clearance in a rolling piston compressor. Their model was able to predict to within 15% of the experimentally measured leakage flowrate for the leakage gap between 23 μm and 46 μm and the lower-to-upper pressure ratio of 0.2 to 1. Suefuji et al. [133] reported that using this model, the predicted volumetric efficiency agreed to within 3% error compared to the measured data.

Ishii et al. [134] evaluated the pressure drop to address the leakage over the flow path by assuming incompressible and viscous flow in a pipe. Yuan et al. [135] and Fan and Chen [136] employed incompressible flow with viscous and inertial terms to predict the leakage flowrate.

Although frictional flow model is computationally expensive than isentropic nozzle flow model, frictional flow model has been reported widely to provide better prediction results.

Hybrid models use frictional correction term obtained by correlating the predictions from the isentropic nozzle model with the detailed models such as frictional flow model and CFD. Thus, hybrid models are considered to maintain the simplicity and accuracy while reducing the computational effort required as compared to the frictional flow model. The correction factors are usually obtained through various statistical tools such as regression analysis, machine learning and so on. Bell et al. [137] developed a hybrid model for a scroll compressor using the correction factors obtained through regression and minimization of root-mean-squared error (rmse) of the correlation. Their model still yielded an average absolute error of 11% for radial leakages and 15% error for the flank leakages.

2.2.5 Dynamic model

The dynamic modelling of a compressor includes the study of motion and forces including frictional forces acting on the components of a compressor. Therefore, dynamic modelling is a valuable tool in evaluating the frictional losses occurring in a compressor. Generally, each component of the compressor is assumed as a rigid body and the analysis using free body diagram of the component is employed.

In a rotary compressor, Teichmann [138] studied the frictional losses at the vane tips by assuming that the frictional losses were caused by viscous drag of the thin film of oil. Qvale [139] also analyzed the frictional losses at the vane tips using hydrodynamic lubrication theory by assuming an uninterrupted thin film at the vane tip. Somayajulu [140], Edwards and McDonald [141], Barszcz [142], Teh and Ooi [81], Subianto and Ooi [143, 144] predicted the frictional losses assuming a constant frictional coefficient of 0.133. Peterson and McGahan [145] used the experimentally determined friction coefficients and described the general procedure for evaluating the frictional losses in an oil-flooded sliding vane compressor.

2.2.6 Oil lubrication model

Modelling of oil lubrication system for a compressor is an essential task to determine the required amount of oil necessary to avoid seizure by lubricating rubbing parts such as a journal bearing. The oil lubricates the rubbing components of the compressor, prevents excessive wear, cools the heated parts and acts to seal the clearance gaps which prevents the leakage. Design of oil lubrication system depends upon the orientation of the rotary compressor which could be either horizontal or vertical. A horizontal compressor has its shaft axis parallel to the floor. A vertical compressor has the shaft axis perpendicular to the floor with the top of the shaft coupled to the motor and the bottom immersed in the oil sump. Generally, the horizontal types are preferred in the applications that require low compressor compartment height [146].

A commonly used way to model the oil flow is by applying an analogy in which the differential pressure, volumetric flowrate and flow resistance correspond to the voltage difference, electric current and electric resistance. The oil flow is

generally assumed to be laminar, Newtonian and viscous. The flow model based on an electrical circuit method has been applied by Itoh et al. [147], Kim and Cho [148], Padhy [149], Kim and Lancey [150] and so on. The authors have generally found good agreement with the experimental data.

CFD simulations of oil flow have been used to get a clearer picture of local mechanisms that affect lubrication. Generally, the volume of fluid method for the two phase flow in a rotating frame of reference is considered. Bernardi [151] employed the volume of fluid technique in rotating frame of reference to study oil flow, pumping head and pressure losses. He reported the discrepancy of 17% with the experimentally measured flowrate. Lückmann et al. [152] also employed the volume of fluid model to study the oil lubrication system of a reciprocating compressor. They obtained similar result that showed good agreement with their experimental data. Kerpici et al. [153] predicted the flowrate and the oil climbing time using the volume of fluid model and Eulerian model. Their predicted results had the discrepancy of about 8% with the measured data.

Introduction of nano-sized particles (metal oxides) suspended in refrigerant fluid and oil mixture led to increasing of conduction and convection coefficients allowing more heat transfer out of coolants [154]. Authors also reported improvement in performance using the nanofluids by returning more oil to the compressor. Manca et al [155] reported that the nanoparticles enhanced the heat transfer characteristics of the oil and reduced the bubble size. However, nanofluids stability and production cost hinder the commercialization of the nanofluids.

2.3 Optimization studies

The compressor design process is described as the complex process involving selections of values of numerous design variables. Optimization studies aim to produce a compressor with specified cooling capacities having highest COP and/or the lowest cost [156]. Historically, the designers resorted to multiple iterations of building an experimental prototype, testing it, and modifying the prototype. This method was extremely expensive and time-consuming. Then, compressor modelling and simulation procedures were developed. The simulation studies reduced the compressor testing and development time by

allowing the designers to evaluate the critical design decisions quickly. To further optimize the compressor design, optimization procedures were developed.

Generally, the optimization techniques can be classified into unconstrained and constrained methods. The constrained optimization methods have been further classified into direct and indirect methods. In general, the aim of these methods is to determine the maximum of a nonlinear, multivariable function known as an objective function which is subjected to non-linear inequality constraints. Depending on the nature of an objective function, its partial derivatives may or may not be available. In a direct method, only the values of an objective function are determined. While in an indirect method or also known as the gradient method, the objective function values, and the derivatives are determined. Generally, gradient methods are said to achieve faster convergence than the direct method.

Most available literature studied have usually employed direct search methods as the objective function involved in an optimization study for compressor design are usually reported to be highly non-linear and may not be differentiable. An optimization study of a sliding vane compressor by Lafrance and Hamilton [156] employed a direct search method developed by Fletcher and Power [157]. In the first step, they optimized for maximum COP using five geometrical design variables including the port sizes, radial clearance and the rotor and cylinder diameter by assuming constant swept volume. In the second step, thirteen geometric design variables were considered. They reported increments of 1.8% and 5% in COP for five-variables and thirteen-variables optimization respectively.

A search technique known as the complex method was developed by M. J. Box [158]. MacLaren et al. [159] used this method to optimize the valve design. The criterion of the optimization was to achieve the best volumetric efficiency for the least power input by minimizing the valve losses. Ooi [36] also employed the complex method to optimize the geometrical dimensions of a rolling piston compressor with the minimum mechanical loss. 14% increment in the mechanical efficiency of the compressor was reported. Box's complex method was also used by Stosic et al. [160] to optimize the various geometrical variables and operating conditions for a single stage and double stage screw compressors.

In an optimization study of frictional losses in a scroll compressor, Liu et al. [161] integrated a solver named “MOST” into their simulation program. The solver was developed using constrained and gradient method. The optimization results show that the frictional losses within the scroll compressor can be minimized in the range of 14-18%. Bell et al. [162] studied the optimization of liquid flooded scroll compressor using an optimization algorithm developed by Byrd et al. [163]. The algorithm is based on the constrained and gradient projection method. By searching for an optimum set of built-in volume ratio and scroll base circle radius, they attempted to maximize the isentropic efficiency by minimizing the leakage losses.

Optimization studies result in better compressor design, improved performances, and they also provide new insights into the relationship between the design variables to produce an optimum performance.

2.4 Experimental studies

Experimental investigation of a compressor prototype includes an instrumentation and measurement of the performance parameters. The experimental investigation of a compressor is performed on an experimental test bench. Generally, a test bench can be used to evaluate the performance of a compressor, reliability and life expectancy tests and quality control tests [164]. In this section, general discussions on experimental studies on evaluating the compressor performance will be presented.

Generally, the experimental test bench can be classified broadly into two groups. The first one is the calorimeter setup which is constructed based on International Standard ISO 917 [165]. The second is a specifically designed laboratory setup. The calorimeter setup consists mainly of a single-stage vapour compression unit. International Standard ISO 917 described the procedures for the determination of refrigerating capacity (cooling capacity), power input, isentropic efficiency, COP and oil circulation. The evaluation of these parameters requires the measurement of mass flowrate and volumetric flowrate through the compressor, suction and discharge pressure and temperature, the power input to the compressor and the compressor speed of operation. Additionally, the specific enthalpies and specific volume of the gas are also required to be evaluated.

Figure 2.10: Schematic diagram of a closed-loop experimental setup by Rigola [166]

Rigola et al. [166] designed and built an experimental unit to study a CO₂ trans-critical refrigeration system. The schematic of the experimental setup is shown in Figure 2.10. The experimental unit contained following components: a compressor prototype, condenser and evaporator circuit, and expansion valve. The thermal cooling unit and the heating unit controlled the water temperature in the condenser and the evaporator auxiliary circuits. The experimental setup was able to capture the thermal and fluid dynamic behaviour of each of the components. Overall, the measured data and the predicted results showed good agreement. They reported that the CO₂ trans-critical system generally showed 10% lower in volumetric efficiency and COP compared to that of the conventional sub-critical cycles.

Figure 2.11: Schematic diagram of a closed-loop experimental setup by Wu et al. [167] for testing compressors in air-conditioning systems

Wu et al. [167] studied the startup characteristics of a rolling piston compressor using R290 in an air-conditioning system. The schematic of the experimental setup is shown in Figure 2.11. The setup was installed in a psychrometric chamber consisting of two rooms, indoor and outdoor. The air conditioning system mainly consisted of a compressor, condenser, expansion valve, evaporator and other auxiliary components such as accumulators and fans. Two air handling units controlled the operating condition of the two rooms. The dry air temperature of 27 °C and 35 °C at the indoor and the outdoor respectively was maintained. The measured results showed that for R290, the startup time to reach the steady state was much longer than that of R410a and R22.

Teh and Ooi [168] conducted an experimental investigation to study the functionality of a revolving vane compressor and to validate the theoretical models. The air was selected as the working fluid. The schematic of the experimental setup is shown in Figure 2.12. The experimental setup is of an open type configuration. It consisted of a compressor prototype, air receiver, flow regulating valve, air filter, and variable-area flowmeter. A torque sensor was coupled between the compressor and motor to measure the shaft torque and speed. The predicted results generally showed good agreement for low pressure ratios. Minor discrepancies between the predicted and measured pressure

variations in the suction and the compression chamber were believed to be due to the heat transfer from the compression and discharge chambers to the suction line which was not considered in the mathematical modelling.

Figure 2.12: An open-loop experimental setup by Teh and Ooi [168]

Bianchi and Cipollone [47] employed open type configuration for the experimental investigation of a sliding vane compressor. The compressor was operated at varying pressure ratios and operating speeds. Comparison between the P-V diagrams of the predicted and the experimental data showed good agreement. Further validation was done by comparing the indicated power and the mass flowrate. Slight discrepancies in the mass flowrate at higher pressure ratios were reported due to the sealing effect of the oil.

2.5 Summary

This chapter presented an overview of the positive displacement compressors, the theoretical analyses and the experimental analyses to study the compressors. In general, the review of the positive displacement compressor designs reveals the following features:

- The existing rotary compressor designs such as rolling piston compressor, sliding vane compressor, swing vane compressor, spool compressor and revolving vane compressors are simple in design and have varying degree

of efficiency. But these compressors consist of rotor which occupy significant space inside the respective compressor cylinders. Consequently, these compressors can be considered bulky in design.

- The scroll compressor and the screw compressors are efficient and silent during their operation. However, their design is complicated and require expensive production tools for their manufacturing.
- When it comes to the theoretical analyses, a zero-dimensional mathematical modelling of the working chambers of the rotary compressors assuming lumped parameter approach is still the conventional and easy to implement modelling approach.

The review generally showed that the design and the performance of the current rotary compressors can still be improved and therefore, the development of a more energy efficient compressor either based on an existing design or even a new compressor is a real possibility. To this aim, an innovative new compressor (discussed in Chapter 3) has been designed and studied.

Chapter 3: Design of Coupled Vane Compressor

In this chapter, the evolution of the design of a new rotary compressor called Coupled Vane Compressor (CVC) are presented and discussed. The first section of this chapter presents the limitation in the design of the existing rotary compressors which prevents these compressors to be designed comparatively more compact. The second section introduces a rotary vane compressor named cardioid compressor which is more compact than any other existing rotary vane compressors. This section presents the stepwise evolution in the design of cardioid compressor by analysing its limitations encountered in its design which ultimately led to the invention of CVC. Subsequently, in the third section, the novel CVC is introduced, and its design and working mechanism will be discussed.

3.1 Analysis of existing rotary compressors

In chapter 2, section 2.1, the positive displacement compressors including the reciprocating compressor and the rotary vane compressors such as a rolling piston, a sliding vane, a revolving vane and so on were reviewed. The comparative illustrations of the design of these rotary compressors are shown in figures 3.1 (a) – (d).

A common feature in the figures 3.1 (a) – (d) is that these rotary compressors have a large rotor relative to their cylinder. In all the rotary compressor designs, the ratio of the rotor diameter to the cylinder diameter is generally more than $3/4$. This implies the rotor occupies significantly large space within the cylinder, which otherwise can be applied as the working chamber.

Figure 3.1: Rotary compressors with their large rotor relative to the cylinder: (a) Rolling piston compressor [169]; (b) Sliding vane compressor [9]; (c) Rotary spool compressor [71]; and (d) Revolving vane compressor [80]

Because of their respective design, these compressors require such a large rotor for them to function properly. A theoretical compressor with comparatively smaller rotor size relative to its cylinder can have many advantages. Assuming the fixed displacement volume and the cylinder height for comparison of two compressors with differing rotor diameter, the one with a smaller rotor diameter will also have smaller cylinder diameter. This further implies the external housing which houses the compressor itself will have smaller external diameter. Therefore, the smaller rotor size of a compressor could mean more compact design. Such a compressor will require significantly less amount of material, especially metal, for its production.

3.2 Design analysis of a cardioid compressor

The search for an extremely compact rotary compressor with a comparatively small rotor size relative to its cylinder led to the study of the design and the operational principle of a compressor named cardioid compressor. Charles Bernard Brull [170] is credited with the invention of the cardioid compressor. A 3D view of this compressor is shown in Figure 3.2.

Figure 3.2: A 3D view of a cardioid compressor

3.2.1 Design of the cardioid compressor

The schematic of a cardioid compressor is shown in Figure 3.3. In its basic form, the cardioid compressor includes a cylinder, rotor and vane. The inner wall of the cylinder is cardioid in shape with its vertex at C_c as shown in Figure 3.4. The rotor is mounted eccentrically inside the cylinder at the position where the cardioid wall of the cylinder constitutes a depression in the shape of a circular arc. This arc is the sealing arc between the cylinder and rotor. The radius of this arc is equal to the sum of the radius of the rotor (R_r) and the radial clearance (δ). The rotor has a diametric slot through which the vane can slide in or out. The vane together with the rotor subdivides the space inside the cardioid cylinder to form the working chambers, namely, the suction, compression and discharge chamber.

Figure 3.3: Schematic of a cardioid compressor

Figure 3.4: Chords of a cardioid

In the cardioid compressor, the length of the vane, L_{vn} , is chosen such that as the vane rotates about the rotor centre, C_r , its endpoints always meet the cardioid geometry. This is illustrated in Figure 3.4, in which the length of vanes XX' and YY' are equal. This implies that theoretically the two tips of the vane always form sealing contacts with the cardioid cylinder wall.

It is noted that, for the manufacturing of a cardioid compressor, especially for the inner cardioid shaped cylinder wall, more sophisticated CNC milling machine is required instead of a conventional lathe.

In Figure 3.3, it can be clearly seen that the rotor of the cardioid compressor is extremely small compared to its cylinder. It is believed that the diameter of the rotor of this compressor can be as small as the diameter of the motor-shaft.

Figure 3.5: Comparative illustration of the overall size assuming fixed volume of (a) Cardioid compressor; and (b) Rolling piston compressor

In Figure 3.5, the overall area of the cardioid compressor is compared with a rolling piston compressor of the same volumetric displacement assuming same cylinder height for both the compressors. As seen from Figure 3.5, due to the smaller rotor size, the overall size of the cardioid compressor is extremely small. As a result, the cardioid compressor will require a significantly smaller volume of metal for fabrication.

Some of the arbitrary values selected for the compressors for the comparison of the total volume of the compressor cylinder and the rotor are shown in Table 3.1.

Assuming the fixed volumetric displacement of 18.3 cm^3 and fixed cylinder height of 30 mm, the volume of cylinder and rotor were determined. The total volume required for a cardioid compressor was then compared with the same for a rolling piston compressor. It was found that the cardioid compressor required approximately only 49% of the total volume of metal required for fabrication. This is excluding the volume of metal required for the compressor housing. The size of the compressor housing depends upon the total size of the compressor. This implies that if the volume of metal required to fabricate the compressor housing is incorporated into the calculation, the percentage of the metal volume saved will be even larger.

Table 3.1: Comparison of total volume of metal required for a cardioid compressor and a rolling piston compressor assuming fixed volumetric displacement and fixed cylinder height

	Cardioid compressor	Rolling piston compressor
Rotor-to-cylinder ratio	0.35	0.75
Cylinder height	30 mm	30 mm
Cylinder wall thickness	8 mm	8 mm
Diametric length	30 mm	42.2 mm (*)
Total volume of metal required (**)	52.5 cm^3	103.4 cm^3
Note:		
*: Calculated for the fixed volumetric displacement ($= 18.3 \text{ cm}^3$)		
**: Sum of the volume of the cylinder and the rotor		

3.2.2 Operational principle

Figure 3.6 shows the illustration of the operational cycle in which the working fluid undergoes suction, compression and discharge from the compressor cylinder. The working chambers are the spaces within the cylinder partitioned by the vane and rotor. The rotational motion of the rotor forces the vane to rotate and to translate within the slot which causes the variation of the volume of the chambers.

The position shown in the Figure 3.6 (a) – 1, which is at $\theta_r = 0^\circ$, is the initial angular position of the vane and the rotor. Following the rotation of the rotor by $\theta_r = \theta_{st}$ in anti-clockwise direction, the vane tip extends out of the slot. The suction chamber is formed at the trailing side of the vane facing suction port. Following the anti-clockwise rotation, the suction chamber expands in volume and the

working fluid is induced into the suction chamber through the suction port. The suction process continues until 270° revolutions of the rotor in anti-clockwise direction, after which, the trailing tip of the vane seals off the suction chamber from the suction port. This results in the volume of fluid induced into the compressor to be trapped within the resultant chamber. Further rotation of the rotor causes the physical volume of the compression chamber to decrease which results in the pressure rise of the fluid. After $\theta_r = 360^\circ - \theta_{st}$, the compression chamber is exposed to the discharge port. The pressure rise in the compression chamber eventually forces the discharge valve to open and the compressed fluid is discharged out through the discharge port. At $\theta_r = 540^\circ - \theta_{st}$, the discharge process is completed and the rotor and vane arrive at $\theta_r = 540^\circ$ to start a new working cycle.

Figure 3.6: Working principle of a single vane cardioid compressor illustrating (a) suction, (b) compression, and (c) discharge

3.2.3 Vane design – from a single vane to twin sliding vanes

In this section, some of the limitations observed in the cardioid compressor are discussed and the viable solutions to the limitations are presented.

A. Leakage losses and the frictional wear at the vane tips

Similar to many other rotary compressors, there are three major leakage paths within the cardioid compressor. They are (a) leakage along the vane tips, (b) leakage along the vane endfaces, and (c) leakage along the rotor endface. These leakage paths are clearly shown in Figure 3.7.

Figure 3.7: Cardioid compressor and its probable leakage paths: (a) leakage along the vane tips, (b) leakage along the vane endfaces, and (c) leakage along the rotor endface

In Figure 3.7, the single vane of the cardioid compressor has a fixed vane length ' L_{vn} ' and two vane tips 'A' and 'B'. Generally, the vane length L_{vn} is designed such that the vane tips maintain sealing contacts with the cylinder wall. However, after long hours of operation, the vane tips 'A' and 'B' experience frictional wear and this results in the vane length L_{vn} to shorten. This means, with continued operation, the vane tip wear will only increase, and the leakage along the vane tips will only get worse. The leakage from the discharge chamber to the compression chamber and then ultimately to the suction chamber will gradually increase the amount of energy required to compress the fluid, heat the working chambers, and therefore diminish the capacity of the compressor to induce the

fluid into the compressor. This is the most influencing factor which inhibits the potential introduction of the cardioid compressor to the market.

B. Twin vane system and its working principle

To mitigate the vane tip wear which causes the vane to be shorter, a single vane was replaced with a system of two diametrically sliding vanes as shown in Figure 3.8. The two sliding vanes are labelled leading and trailing vane. The original single vane thus became two vanes sliding upon each other and into and out of the vane slot in the rotor. Although the suction, compression and discharge process occur in the same way as in the single vane cardioid compressor, the vane dynamics is different in the twin sliding cardioid compressor.

Figure 3.8: A 3D illustration of a twin vane cardioid compressor

As shown in the Figure 3.9, the twin sliding vanes rotate along with the rotor, centrifugal force, $F_{cen,L}$ in leading vane and $F_{cen,T}$ in trailing vane will push the vanes away from the rotor centre to the direction of displacement of centre of mass of the vane. Besides the centrifugal force, the vane dynamics is influenced by the chamber pressures: suction chamber pressure P_s , compression chamber pressure P_c and the discharge chamber pressure P_d . During the operation of the twin sliding vane cardioid compressor, the centrifugal force, $F_{cen,L}$ and the discharge chamber pressure P_d at the rear end will tend to push the vane to form the sealing contact with the cylinder wall. The suction chamber pressure, P_s and the compression chamber pressure, P_c at the tip will oppose the vane to contact the cylinder wall. Similarly, in the case of the trailing vane, the discharge chamber pressure P_d and the compression chamber P_c at the vane tip will oppose the

compression chamber pressure P_c at the rear end and the centrifugal force $F_{cen,L}$. In this way, the fluid pressure forces, and the centrifugal forces can be used to ensure that the vane tips remain in contact with the cylinder wall.

Figure 3.9: Illustration of an embodiment of a cardioid compressor with twin diametric sliding vanes

A significant issue with the vane dynamics of the cardioid twin sliding vane is illustrated in Figure 3.10. In the positions shown in Figure 3.10, the centre of mass of the trailing vane drops below the rotor centre, C_r . This means that the centrifugal force, $F_{cen,T}$ will tend to push the vane downwards while being opposed by the chamber pressure, P_c . This means the leakage gap occurs at the trailing vane tip. This issue can be countered by lowering the position of the rotor and designing the rotor with larger diameter such that its rotor centre is always lower than the centre of mass of the vane.

Figure 3.10: Critical vane positions in a cardioid compressor

C. Is the cardioid shape necessary?

A single vane compressor system including a cylinder, rotor and vane such as the one shown in Figure 3.7 requires the inner wall of the stator to be cardioid. This, as discussed in section 3.2.1, is to ensure vane tips to form sealing contact with the cardioid wall. However, now that the single vane has been replaced with twin diametric sliding vanes, this property loses its significance and allows us to simplify the design of the cylinder inner wall to the more familiar circular shape.

Figure 3.11: Schematic of a system of circular compressor with twin diametric sliding vanes

A schematic of this new compressor with twin diametric sliding vanes is shown in Figure 3.11. This compressor consists of a cylinder with a circular inner wall, rotor with a diametric rotor slot and sliding system of two vanes, namely, leading vane and trailing vane. The rotor is offset into the cylinder wall to form a sealing arc to reduce the leakage of the compressed fluid from the discharge chamber to suction chamber. The rotor rotates in an anti-clockwise direction about the rotor centre C_r and the sliding vanes rotate and protrude out of the rotor slot due to centrifugal force and pressure forces.

The twin diametric sliding vanes system retains the advantages offered by the single vane cardioid compressor, that is, theoretically, this compressor allows the rotor diameter to remain as small as the diameter of the motor-shaft. Additionally, for this system, the vane tip wear, which shortens the length of the vane, does

not deteriorate the performance of the compressor. This is because the vanes can still utilize the centrifugal force and the chamber forces to form a sealing contact with the cylinder wall. Furthermore, this system allows the cylinder wall design to be circular which is simpler to manufacture compared to the cardioid geometry.

D. Analysis of dynamics in twin sliding vane design

In this section, the discussion on the forces influencing the contact between the vane tip and cylinder wall is presented. For the rotary compressor design such as the one shown in Figure 3.12, the vane tip of each vane should remain in contact with the cylinder wall to prevent the leakage of compressed fluid.

Figure 3.12: Illustration of various forces acting on a twin sliding vane compressor

As can be seen from Figure 3.12, excluding the frictional forces, the body forces that act on the trailing vane 'T' which influence the contact between the vane tip and the stator wall are of two types: the centrifugal force, $F_{cen,T}$ from the anti-clockwise rotation of the rotor and the vane, and the pressure forces from the working chambers of the compressor namely: suction, compression and discharge. For the instance shown in Figure 3.12, discharge chamber pressure P_d and the compression chamber pressure P_c are pushing the vane tip away from the stator wall. The centrifugal force and the compression chamber pressure P_c

at the vane rear push the vane tip towards the stator wall. A simple analysis using the free body diagram of a trailing vane is shown in Figure 3.13.

In Figure 3.13, the forces pushing the vane tip towards the stator wall are the compression pressure force, $F_{c,T-r}$ and centrifugal force $F_{cen,T}$. Meanwhile, the forces pushing the vane tip away from the cylinder wall are the discharge chamber and compression chamber pressure forces, $F_{d,T-t}$ and $F_{c,T-t}$ respectively. Thus, for the vane tip to continually maintain sealing contact with the cylinder wall, the equation (3.1) must be satisfied.

$$F_{c,T-r} + F_{cen,T} \geq F_{d,T-t} + F_{c,T-t} \quad (3.1)$$

Figure 3.13: Free body diagram of the trailing vane showing body forces acting to push the vane tip against the stator wall (excluding frictional forces)

The compression pressure force $F_{c,T-r}$ acts on normal surface area A_{T-r} which is equal to the cross-sectional area of the vane. The compression chamber force $F_{c,T-t}$ act on normal surface area $A_{c,T-t}$ and the discharge chamber force act on $A_{d,T-t}$. The sum of normal surface areas $A_{c,T-t}$ and $A_{d,T-t}$ is equal to the normal surface area A_{T-r} . Generally, the discharge pressure can be assumed several times larger than the compression chamber pressure. In this case, the sum of compression and discharge chamber pressure forces $F_{c,T-t}$ and $F_{d,T-t}$ will be greater than the compression chamber pressure force $F_{c,T-r}$.

The centrifugal force, $F_{cen,T}$, depends upon the mass of vane, location of its centre of mass and operating speed of the compressor. Then at lower speeds of operation, the centrifugal force may be small enough such that the total force pushing the vane away from the cylinder wall could be higher than the sum of the centrifugal force and the pressure force acting on the rear end of the vane. This implies that the vane tip will fail to remain in contact with the cylinder wall. This causes the compressed fluid to leak from the discharge chamber into the compression chamber. The leakage will only increase following further rotation

which will eventually result in compressor failure. This issue prompted us to reimagine the design of the two sliding vanes. This led to the invention of CVC.

3.3 Novel coupled vane compressor

In this section, the features of the novel coupled vane and the coupled vane compressor are discussed.

3.3.1 Coupled Vane Compressor (CVC)

Figure 3.14: Schematic of CVC

A 3D view of CVC is shown in Figure 3.14. CVC, in its basic form, consists of 3 main parts: a cylinder, rotor and coupled system of two vanes. The rotor and vanes are housed inside the cylinder. The centre of the rotor, C_r and centre of the cylinder, C_c , are both fixed. The rotor rotates anti-clockwise about the rotor centre C_r . The rotor is offset slightly into the wall of the cylinder to create the sealing arc 'G'. The sealing arc is a depression in the cylinder, and it has a shape of a circular arc. The radius of this sealing arc is equal to the rotor radius (R_r) + δ , where δ is the clearance at the sealing arc.

The rotor includes a diametric slot in which the vanes can rotate about the centre of the rotor, C_r . As the vanes rotate, they slide in and out of the slot and the vane tips remain in sealing contact with inner wall of the cylinder because of the centrifugal force and the pressure forces from the fluid under compression. Three different working chambers are formed bounded by the walls of the cylinder, rotor

and the vanes. These chambers, as shown in Figure 3.14, are suction, compression and discharge chamber. During the operation, the volume of these chambers changes such that the fluid is successively induced into the cylinder for compression.

Similar to the cardioid compressor, the rotor diameter of CVC can be significantly small relative to the cylinder diameter for its effective functioning. Theoretically, the rotor diameter needs to be only as big as motor-shaft for the correct functioning of this machine. This also implies that for a fixed displacement volume of the compressor, the diameter of the rotor can be designed smaller relative to the cylinder which also allows the diameter of the cylinder to be proportionally smaller. Therefore, the design CVC is one of the most compact rotary compressor designs.

3.3.2 Coupled vane system

Figure 3.15: (a) 3D view of a vane with female dovetail (keyway) feature; (b) orthographic view of the vane, (c) 3D view of a vane with male dovetail (key) feature, and (d) orthographic view of the vane

For smaller rotor designs of CVC, the vanes are designed with dovetail features. The schematic drawings of the vanes with dovetail feature are shown in figures 3.14 (a)-(d). Figure 3.15 (a) is the female vane and it consists of a keyway on its

forward planar face. Similarly, Figure 3.15 (c) is the male vane and it consists of a key or guide on its forward planar face. The key and the keyway are fashioned after the dovetail joint which allows the vanes to have longitudinal sliding motion but not the transverse movement of the vane. Therefore, the joint acts as the gripping mechanism and such arrangement of the two vanes do not require the rotor to contain them while allowing CVC to operate without letting the vanes to fall off the rotor.

As shown in Figure 3.15 (a) and (c), other prominent features of the vanes include a vane tip, vane neck, and rear end. Unlike the twin sliding vanes shown in Figure 3.12, the coupled vanes consist of the vane neck which acts as an additional surface where the pressure force can act to push the vane against the cylinder wall.

For larger rotor sizes, the vanes in CVC can also be designed without the dovetail feature as long as the rotor slot can adequately contain the vanes. Figure 3.16 is the schematic of the vanes without dovetail.

Figure 3.16: Vanes without the dovetail features

3.3.3 Analysis of the dynamics of the leading vane

In this section, the role of the forces and the normal force area that influence the leading vane tip contact with the cylinder wall are analysed. The effect of the frictional force is ignored.

In CVC, the vane which initiates the suction process is referred to as the trailing vane as the second vane is already leading the fluid compression in the compression chamber. As shown in Figure 3.17 – 3.18, there are 2 major types of forces influencing the contact point between the vane tip and the stator wall. These 2 types of forces are the centrifugal force from the anti-clockwise rotation

of vanes about the rotor centre C_r , and the pressure forces from the working chambers of the coupled vane compressor.

Both the leading vane and trailing vane are designed such that centrifugal force acting on the vane body is at the direction where the vane tips are pressed against the inner wall of the cylinder.

The centrifugal force and pressure forces acting on the trailing vane are shown in Figure 3.17. The force from the discharge chamber pressure, $F_{d,T-r}$, centrifugal force, $F_{cen,T}$, and pressure force at the vane-neck, $F_{c,T-n}$, push the trailing vane towards the cylinder wall. The pressure forces at the tip $F_{s,T-t}$ and $F_{c,T-t}$, tend to push the vane tip away from the cylinder wall.

Figure 3.17: Forces influencing the contact between the trailing vane tip and the cylinder wall

The free body diagram of the trailing vane and the forces acting on it are shown in Figure 3.18. For the trailing vane tip to remain in contact with the cylinder wall during the operation, the equation must hold true.

$$F_{d,T-r} + F_{c,T-n} + F_{cen,T} \geq F_{s,T-t} + F_{c,T-t} \quad (3.2)$$

Figure 3.18: Free body diagram showing the dynamic forces acting on the trailing vane to form a sealing contact with the cylinder wall

During the operation, normal force areas at the tip $A_{s,T-t}$ and $A_{c,T-t}$ will vary. Generally, $A_{c,T-t}$ will be designed smaller compared to the normal force area at the vane neck $A_{c,T-n}$. So, the compression chamber pressure force $F_{c,T-t}$ will generally be smaller than $F_{c,T-n}$. Discharge chamber pressure P_d , can be assumed to be several times larger than the suction chamber pressure P_s . Also, the normal force area at the vane rear $A_{d,T-r}$ is designed larger than the normal force area $A_{s,T-t}$. Consequently, it can be said that, the sum of the force $F_{c,T-n}$, $F_{cen,T}$, and $F_{d,T-r}$ will generally be larger than the sum of forces $F_{s,T-t}$ and $F_{c,T-t}$. This implies that the trailing vane tip will tend to remain in contact with the cylinder wall irrespective of the operating speed.

3.3.4 Analysis of the dynamics of the leading vane

As shown in Figure 3.20, there is a pressure force $F_{d,L-n}$ from the working fluid in discharge chamber acting along the vane-neck. This force along with the centrifugal force, $F_{cen,L}$ and the pressure force at the rear end, $F_{c,L-r}$, act to push the leading vane against the stator wall. The pressure forces, $F_{d,L-t}$ and $F_{c,L-t}$, act to push the vane away from the stator wall and towards the rotor centre, C_r . Excluding frictional forces, forces acting on the trailing vane to push the vane tip against the cylinder wall is shown in the free body diagram in Figure 3.20.

Figure 3.19: Forces influencing the contact between the leading vane tip and the cylinder wall

Figure 3.20: Free body diagram showing the dynamic forces acting on the leading vane to form a sealing contact with the cylinder wall

The force balance required for the leading vane tip to remain in contact with the cylinder wall can be written as equation (3.3).

$$F_{c,L-r} + F_{d,L-n} + F_{Cen} \geq F_{d,L-t} + F_{c,L-t} \quad (3.3)$$

During operation of the compressor, normal force area at the rear end of the vane, $A_{c,L-r}$, and normal force area at the neck, $A_{d,L-n}$, will remain constant. However, normal force areas at the tip of the vane $A_{d,L-t}$ and $A_{c,L-t}$ will vary with rotation. Since both forces $F_{d,L-t}$ and $F_{d,L-n}$ are due to the same discharge chamber pressure P_d , normal force area $A_{d,L-n}$ is required to be designed greater or at least equal to $A_{d,L-t}$. This implies that the force $F_{d,L-n}$ will be greater or at least equal to $F_{d,L-t}$. Similarly, in case of normal force areas $A_{c,L-t}$ and $A_{c,L-r}$, $A_{c,L-r}$ is designed to be greater than $A_{c,L-t}$. Consequently, the sum of the forces $F_{c,L-r}$, $F_{d,L-n}$, and $F_{Cen,L}$ will be greater than $F_{d,L-t}$ and $F_{c,L-t}$. Then, the resultant force on the vane will

always be towards the cylinder wall such that the vane tip will be pressed against the cylinder wall during the operation.

3.3.5 Working Principle

At the start of the working cycle of CVC, the vanes are assumed to align at the vertical axis as shown in Figure 3.21 (1). As the rotor rotates in the anti-clockwise direction, the trailing vane tip housed inside the rotor slot protrudes out. As it does so, backward planar face of the trailing vane, along with the rotor and cylinder, forms the suction chamber. The suction process is illustrated in Figure 3.21 (a) in steps (1) to (4). The suction process continues until 270° of rotation, after which, the tip of the trailing vane seals off the suction port and the suction chamber becomes the compression chamber.

Following the rotation, the physical volume of compression chamber decreases resulting in the pressure rise of the working fluid. The compression process is shown in steps 5-6 in Figure 3.21 (b). In step 6 in Figure 3.21 (b), the trailing vane tip then exposes the discharge port to the compression chamber. During the compression, the pressure in the compression chamber continues to rise until it is greater than the discharge pressure. Once, the differential pressure across the discharge valve becomes sufficiently greater to overcome the stiffness of the valve, the compressed fluid is discharged out of discharge port by the opening of the valve. The discharge process is completed after the tip of the trailing vane seals off the discharge port. In this way, the working cycle of CVC is completed in 540° revolutions. After 540° revolutions, the trailing vane becomes the leading vane and it initiates the new working cycle.

Figure 3.21: Working principle of CVC showing the (a) suction, (b) compression and (c) discharge process

3.4 Summary

In this chapter, a new positive displacement rotary vane compressor, namely, Coupled vane compressor (CVC) has been introduced. Its unique feature is that its rotor can be significantly smaller relative to its cylinder size compared to all other rotary vane compressors available today. Consequently, the new compressor design is probably the most compact rotary vane design available today. Therefore, as compared to the existing rotary vane compressors, the new compressor has an immense potential in saving the significant amount of metal used during the production.

The working chamber volume model, thermodynamics of working fluid, main and secondary mass flows and instantaneous in-chamber heat transfer model have

been introduced in Chapter 4. While in Chapter 5, the kinematics and dynamics model of CVC are discussed in considerable detail.

Chapter 4: Theoretical Model: Volume, Thermodynamics, Mass and Heat Transfer and Valve Dynamics

In chapter 4, a zero-dimensional mathematical modelling of CVC will be formulated to investigate the operational characteristics to predict the performance of this compressor. The mathematical models formulated include the mathematical derivations of the geometry of the working chamber of CVC, thermodynamics of the working fluid, primary flows through the suction and discharge ports, secondary leakage flows through the internal clearances, and the forced convective heat flow occurring in the working chamber.

4.1 Volume model

Figure 4.1 illustrates the parameters used for the formulation of the working chamber volume of the compressor. As described in section 3.3, CVC consists of a cylinder with an internal radius R_c , a rotor with radius R_r and two vanes. In Figure 4.1, the distance between the centre of the rotor, C_r , and the centre of the cylinder C_c is represented by b . $\delta_{r,sa}$ is the depth of the rotor circumference into the cylinder wall to create the sealing arc GG' . The rotor in the compressor shown in Figure 4.1 rotates about C_r in an anti-clockwise direction with a rotational speed ω . The rotational angle θ_r is an angle measured in an anti-clockwise direction with respect to the vertical central axis containing the two centres C_r and C_c .

The rotor circumference and the internal wall of the cylinder intersect at two points G and G' either side of the vertical axis containing the rotor and cylinder centres: C_r and C_c . A circular arc, with its centre at C_r , spanning from G to G' , is cut onto the inner wall of the cylinder. This arc, which is termed as “sealing arc”, forms a small clearance between the inner wall of the cylinder and the rotor circumference from G to G' . The purpose of the sealing arc is to provide a more positive sealing for the working fluid between the low and the high pressure chambers across the sealing arc while allowing the rotor to rotate without physically touching the inner cylinder wall within the sealing arc region.

The angular position of the point G with respect to this vertical axis is half the sealing arc angle and it marks the start of the suction process. Hence, this angle is labelled as $\theta_{r,st}$. The location of the second point G' which is on the opposite side of the vertical axis marks the closing angular position of the discharge process.

Figure 4.1: Top view of CVC showing different parameters used in describing the volume model

The chamber, bounded by the points GPA as shown in Figure 4.2, is exposed to the suction port and since its volume increases as the rotor rotates, it induces the working fluid through the suction port. Therefore, this chamber is termed as the suction chamber. $\theta_{r,st}$ is the angle after which the suction starts, this angle is calculated as shown in equation (4.1).

$$\theta_{r,st} = \tan^{-1} \left(\frac{\sqrt{4b^2R_r^2 - (R_r^2 + b^2 - R_c^2)^2}}{R_r^2 + b^2 - R_c^2} \right) \quad (4.1)$$

Figure 4.2: Illustration of chamber cross-sectional area

The rotation of the rotor about its centre C_r also forces the vanes to slide out of the slot in the rotor. In Figure 4.1, tips of the two vanes, vane 1 and vane 2 contact the internal wall of the cylinder at two points. The corresponding two points, $P(\theta_r)$ and $P(180^\circ + \theta_r)$ are assumed to be at the intersection of the internal circular cylinder wall of radius R_c and the line which passes through the rotor centre C_r and the mid-point along the thickness of the vane tip.

The locus traced out by the point $P(\theta_r)$ is the circular geometry of the inner wall of the cylinder itself. This locus $r(\theta_r)$ can be calculated using equation (4.2).

It is to be noted that, in the sealing arc region, the length of the vane is $R_r + \delta_{r,sa}$, where $\delta_{r,sa}$ is the sealing arc clearance gap, but since $R_r \gg \delta_{r,sa}$ it is assumed that $R_r + \delta_{r,sa} \approx R_r$.

$$r(\theta_r) = \begin{cases} R_r & \text{if } \theta_r \leq \theta_{r,st} \text{ or } 2\pi - \theta_{r,st} < \theta_r < 2\pi + \theta_{r,st} \\ -b \cos \theta_r + \sqrt{-(b \sin \theta_r)^2 + R_c^2} & \text{otherwise} \end{cases} \quad (4.2)$$

For the illustration purpose, the arbitrary values selected for the compressor geometry, where, $R_c = 27.5$ mm, $R_r = 15.5$ mm and $b = 12.5$ mm, variation of $r(\theta_r)$ with respect to rotor angle is shown in Figure 4.3. Assuming origin of the reference frame is at the rotor centre, $r(\theta_r)$ is the distance between the contact point and rotor centre, where, the contact point is at the point of contact between the vane tip and inner cylinder wall (see Figure 4.2).

Figure 4.3: Variation of $r(\theta_r)$ with respect to the rotor centre C_r

The area under the curve traced by point P (the blue cross-sectional area in Figure 4.2) is determined by integrating the square of $r(\theta_r)$. Whereas, the volume, shown in equation (4.3), is the product of the cross-sectional area and length of cylinder (l_c).

$$\begin{aligned}
 V_c(\theta_r) &= \frac{l_c}{2} \int_0^{\theta_r} (r(\theta_r))^2 d\theta_r \\
 &= \frac{l_c}{2} \left[R_c^2 \theta_r + \frac{b^2}{2} \sin(2\theta_r) - b \sin \theta_r \sqrt{R_c^2 - (b \sin \theta_r)^2} \right. \\
 &\quad \left. - R_c^2 \tan^{-1} \left(\frac{b \sin \theta_r}{\sqrt{R_c^2 - (b \sin \theta_r)^2}} \right) \right] \quad (4.3)
 \end{aligned}$$

Equation (4.4) is the volume of the rotor inside the control volume. The rotor volume is the product of the cross-sectional area of the sector of the rotor and the length of the rotor l_r which is also equal to the length of the cylinder l_c in the control volume.

$$V_r = l_c \left(\pi R_r^2 \times \frac{\theta_r}{2\pi} \right) \quad (4.4)$$

The volume of the working chamber in the coupled vane compressor is the product of the length of the cylinder and the working chamber cross-sectional bounded by the inner wall of the cylinder, rotor circumference and the vane(s). For example, in Figure 4.2, the volume of the suction chamber is the product of the length of cylinder and the cross-sectional area GPA.

I. Rotor angle 0° to $\theta_{r,st}$

From Figure 4.4, it is noted that at the rotational angle $\theta_{r,st}$, the cross-sectional area under the curve YP includes the cross-sectional area C_rYP where the part of rotor area overlaps the chamber cross-sectional area. At this area, the working chamber volume is the volume of the clearance in the sealing arc up to that point.

Figure 4.4: Illustration of the rotor and cylinder volumes

The working chamber volume is calculated by evaluating the working chamber cross-sectional area swept by the vane formed by the vane tip and internal cylinder wall after deducting the cylinder and rotor volumes $V_{c,st}$ and $V_{r,st}$. Equations (4.3) and (4.4) are the respective volumes, $V_{c,st}$ and $V_{r,st}$ obtained by substituting $\theta_{r,st}$ for θ_r .

Therefore, for the rotor angle from 0° to $\theta_{r,st}$, the volume of the working chamber and the rate of change of volume with respect to rotor angle θ_r are as shown in equation (4.5) and (4.6).

If $0 \leq \theta_r \leq \theta_{r,st}$

$$V(\theta_r) = V_{clr}(\theta_{r,st}) \quad (4.5)$$

$$\frac{dV(\theta_r)}{d\theta_r} = 0 \quad (4.6)$$

To this end, it is noted that the cylinder and the rotor volume beyond $\theta_{r,st}$ which are obtained using equations (4.3) and (4.4) requires the deduction of cylinder and rotor volumes $V_{c,st}$ and $V_{r,st}$ which are determined using equations (4.3) and (4.4) by substituting $\theta_{r,st}$ for θ_r .

II. Rotor angle $\theta_{r,st}$ to $(180^\circ + \theta_{r,st})$

Figure 4.5 illustrates the suction chamber area formed within the cylinder bounded by the rotor circumference and the trailing face of vane 1. The cross-sectional area of the working chamber is calculated by deducting the respective rotor area and the vane cross-sectional area from the area under the curve GP. This implies the working chamber volume can be obtained using equation (4.7).

$$V(\theta_r) = \frac{l_c}{2} \int_0^{\theta_r} (r(\theta_r))^2 d\theta_r - V_{c,st} - l_r \left(\pi R_r^2 \times \frac{(\theta_r - \theta_{r,st})}{2\pi} \right) - V_{l,vn}(\theta_r) + V_{clr}(\theta_{r,st}) \quad (4.7)$$

Figure 4.5: Illustration of suction volume boundaries

For simplicity, the tip of the vane is assumed to be semi-circular in shape of radius R_{f1} . As the vane protrudes out of the rotor slot, the area occupied by the trailing vane is due to its round filleted tip APB as illustrated in Figure 4.6. This area is obtained by deducting the area of triangle C_vAB from the sector of the round filleted tip C_vPB . The trailing vane volume in the suction control volume within the rotor angle $\theta_{r,st}$ to $180^\circ + \theta_{r,st}$ is shown in equation (A-1.1) in appendix (A-1).

Figure 4.6: Schematic of a vane tip segment in the control volume

The working chamber volume for rotor angles from $\theta_{r,st}$ to $(180^\circ + \theta_{r,st})$ is expressed in the equation (4.9).

$$\begin{aligned}
 V(\theta_r) = & \frac{l_c}{2} \left[R_c^2 \theta_r + \frac{b^2}{2} \sin(2\theta_r) - b \sin \theta_r \sqrt{R_c^2 - (b \sin \theta_r)^2} \right. \\
 & \left. - R_c^2 \tan^{-1} \left(\frac{b \sin \theta_r}{\sqrt{R_c^2 - (b \sin \theta_r)^2}} \right) \right] - V_{c,st} - l_c \left(R_r^2 \times \frac{(\theta_r - \theta_{r,st})}{2} \right) \\
 & - V_{t,vn}(\theta_r) + V_{clr}(\theta_{r,st})
 \end{aligned} \quad (4.8)$$

For the arbitrarily selected values for the compressor geometry, where, $R_c = 27.5$ mm, $R_r = 15.5$ mm, $b = 12.5$ mm, $l_c = 30$ mm, $R_{f1} = 3$ mm, and $t_{vn} = 6$ mm, the variations of the trailing vane volume in the control volume can be shown in Figure 4.7. This volume increases from 0 after the trailing vane protrudes out of the slot at the rotor angle of about 39° . It reaches a maximum at 180° rotor angle when the vane is fully extended and then reduces back to 0 at around 332° rotor angle as the trailing vane tip goes back into the rotor slot.

Figure 4.7: Variation of the trailing vane volume in the control volume

Equation (4.9) is the corresponding derivative of the working chamber volume shown in equation (4.8) with respect to the rotor angle θ_r .

$$\frac{dV(\theta_r)}{d\theta_r} = \frac{l_c}{2} \left[R_c^2 + b^2 \cos(2\theta_r) - 2b \cos \theta_r \sqrt{R_c^2 - (b \sin \theta_r)^2} \right] - l_c \left(\frac{\pi R_r^2}{2\pi} \right) - \frac{dV_{t,vn}(\theta_r)}{d\theta_r} \quad (4.9)$$

The rate of change of the volume of the vane in the control volume is shown in the equation (A-1.2).

The rate of change of $r(\theta_r)$ can be obtained using equation (4.10).

$$\frac{dr(\theta_r)}{d\theta_r} = \begin{cases} 0 & \text{if } \theta_r \leq \theta_{r,st} \text{ or } 2\pi - \theta_{r,st} < \theta_r < 2\pi - \theta_{r,st} \\ b \sin \theta_r + \frac{(-b^2 \sin 2\theta_r)}{2\sqrt{-(b \sin \theta_r)^2 + R_c^2}} & \text{otherwise} \end{cases} \quad (4.10)$$

III. Rotor angle $(180^\circ + \theta_{r,st})$ to $(360^\circ - \theta_{r,st})$

Following the rotation, suction volume transition occurs from $(180^\circ + \theta_{r,st})$. At the end of rotor angle $(180^\circ + \theta_{r,st})$, the vane tip of leading vane (point P'(θ_r) leading P(θ_r) by 180° as shown in Figure 4.8) tends to protrude out of the rotor slot and adds a new constraint to the boundary of the working chamber. The suction chamber becomes compression chamber after the rotor angle 180° as the suction port is sealed off by the leading vane. The same boundary condition continues up to the angle of $(360^\circ - \theta_{r,st})$.

Figure 4.8: Illustration of compression volume boundaries

gives The leading vane position is given by $r(\theta_r + 180^\circ)$ and it can be determined using equation (4.11).

$$r(\theta_r + 180^\circ) = \begin{cases} R_r, & \text{if } \theta_r \geq 180^\circ - \theta_{r,st} \text{ and } \theta_r \leq 180^\circ + \theta_{r,st} \text{ or } \theta_r \geq 540^\circ - \theta_{r,st} \\ -b \cos(\theta_r + 180^\circ) + \sqrt{-(b \sin(\theta_r + 180^\circ))^2 + R_c^2} & , \text{else} \end{cases} \quad (4.11)$$

The derivative of this equation with respect to the rotor angle is shown in equation (4.12).

$$\frac{dr(\theta_r + 180^\circ)}{d\theta_r} = \begin{cases} 0 & \text{if } \theta_r \leq \theta_{r,st} \text{ or } 360^\circ - \theta_{r,st} < \theta_r < 360^\circ - \theta_{r,st} \\ b \sin(\theta_r + 180^\circ) + \frac{(-b^2 \sin 2(\theta_r + 180^\circ))}{2\sqrt{-(b \sin(\theta_r + 180^\circ))^2 + R_c^2}} & , \text{else} \end{cases} \quad (4.12)$$

Using equation ((4.11), the volume of the working chamber for the rotor angle between $(180^\circ + \theta_{r,st})$ to $(360^\circ - \theta_{r,st})$ can be written as shown in equation (4.13), where $V_{l, vn}(\theta_r)$, and $V_{t, vn}(\theta_r)$ are the leading vane and trailing vane volumes in the working chamber.

$$V(\theta_r) = \frac{l_c}{2} \left[\int_{0^\circ}^{\theta_r} (r(\theta_r))^2 d\theta_r - \int_{0^\circ}^{\theta_r - 180^\circ} (r(\theta_r))^2 d\theta_r \right] - l_c \left(\frac{\pi R_r^2}{2} \right) - V_{l,vn}(\theta_r) - V_{t,vn}(\theta_r) + V_{clr}(\theta_{r,st}) \quad (4.13)$$

After integration of the cylinder volume, the working chamber volume with respect to the rotor angle can be written as shown in the equation (4.14).

$$V(\theta_r) = \frac{l_c}{2} \left[R_c^2 \pi - 2b \sin \theta_r \sqrt{R_c^2 - (b \sin \theta_r)^2} - 2R_c^2 \tan^{-1} \left(\frac{b \sin \theta_r}{\sqrt{R_c^2 - (b \sin \theta_r)^2}} \right) \right] - l_c \left\{ \frac{(\pi R_r^2)}{2} \right\} - V_{l,vn}(\theta_r) - V_{t,vn}(\theta_r) + V_{clr}(\theta_{r,st}) \quad (4.14)$$

The derivative of this equation (4.14) can be obtained and written as shown in equation (4.15).

$$\frac{dV(\theta_r)}{d\theta_r} = \frac{l_c}{2} \left[-4b \cos \theta_r \sqrt{R_c^2 - (b \sin \theta_r)^2} \right] - \frac{dV_{l,vn}(\theta_r)}{d\theta_r} - \frac{dV_{t,vn}(\theta_r)}{d\theta_r} \quad (4.15)$$

The trailing vane volume and its rate of change are defined in the same way as shown in appendix in equation (A-1.1) - (A-1.2). The leading vane, however, needs careful examination of the geometry as the leading vane side consists of the vane gap and the dovetail feature.

As the trailing vane tip protrudes out of the rotor slot, only a segment of the volume of the trailing vane tip is exposed. Further rotation will see the vane tip followed by the space or gap between the neck of the trailing vane and the rear of the leading vane exposed. The trailing vane will further extend to expose the keyway or the dovetail sliding joint in the gap between the coupled vanes. This is visualised using the images shown in Figure 4.9.

Figure 4.9: (a) Schematic of a vane; (b-d) Visualisation of the spaces forming within the coupled vanes at different rotor angles; (e-g) The crescent-shaped spaces forming between the vanes are of the same size in both the vanes

Thus, leading vane volume and the rate of change of the leading vane volume are presented in equations (A-1.3) to (A-1.12) in Appendix A-1.

Figure 4.10 shows the volume for the leading vane in the working chamber using the arbitrary values selected where, $R_c = 27.5$ mm, $R_r = 15.5$ mm and $b = 12.5$ mm, $l_c = 30$ mm, $R_{f2} = 3$ mm and $t_{vn} = 6$ mm. The volume is 0 until $(180^\circ + \theta_{r,st})$ where $\theta_{r,st}$ is about 31° . After $(180^\circ + \theta_{r,st})$, the leading vane volume increases as it gradually protrudes out of the slot and becomes maximum at 360° where the vane fully extends onto the cylinder wall. The first discontinuity is observed around the rotor angle 260° rotor angle where the leading vane volume instantaneously introduces a volume of space between the neck of the leading vane and the rear end of the trailing vane. The second discontinuity (at around 336°) is observed when the crescent-shaped space between the dovetail feature of the vanes is instantaneously introduced into the leading vane volume. Similarly, the third discontinuity point (at around 396°) is the case where the crescent-

shaped volume is now disconnected from the leading vane volume. For the fourth discontinuity for the plot at the rotor angle of about 460° , the gap between the two vanes is disconnected from the working chamber space. Finally, as the leading vane fully enters into the rotor slot (at around 514°), there is no leading vane which protrudes out of the rotor hence the volume of the protruded vane is 0.

Figure 4.10: Variation of the leading vane volume in the control volume

IV. Rotor angle $(360^\circ - \theta_{r,st})$ to $(540^\circ - \theta_{r,st})$

After the rotor angle $(360^\circ - \theta_{r,st})$, the trailing vane tip enters the slot of the rotor as shown in Figure 4.11. Hence the new volume boundary is defined by the cylinder wall, rotor wall and the leading vane as shown by the red thick lines in Figure 4.11. The boundary condition remains the same until the leading vane enters the rotor slot at the rotor angle $(540^\circ - \theta_{r,st})$.

Figure 4.11: Illustration of discharge volume boundaries

The equation to describe the volume of working chamber for the rotor angle between $(360^\circ - \theta_{r,st})$ to $(540^\circ - \theta_{r,st})$ can be written as shown in equation (4.28), where $V_{l,vn}(\theta_r)$ is the leading vane volume.

$$V(\theta_r) = \frac{l_c}{2} \left[\int_{\theta_r - 180^\circ}^{360^\circ} (r(\theta_r))^2 d\theta_r \right] - V_{c,st} - l_c \left(R_r^2 \times \frac{(3\pi - \theta_r - \theta_{st})}{2} \right) - V_{l,vn}(\theta_r) + V_{clr}(\theta_{r,st}) \quad (4.16)$$

After integration of the cylinder volume, the control volume with respect to the rotor angle can be written as shown in the equation (4.17).

$$V(\theta_r) = \frac{l_c}{2} \left[R_c^2 (3\pi - \theta_r) + \frac{b^2}{2} \sin(2\theta_r) + b \sin \theta_r \sqrt{R_c^2 - (b \sin \theta_r)^2} + R_c^2 \tan^{-1} \left(\frac{b \sin(\theta_r - \pi)}{\sqrt{R_c^2 - (b \sin \theta_r)^2}} \right) \right] - V_{c,st} - l_c \left(R_r^2 \times \frac{(3\pi - \theta_r - \theta_{st})}{2} \right) - V_{l,vn}(\theta_r) + V_{clr}(\theta_{r,st}) \quad (4.17)$$

The derivative of this equation (4.17) can be obtained and written as shown in equation (4.18).

$$\frac{dV(\theta_r)}{d\theta_r} = \frac{l_c}{2} \left[-R_c^2 - b^2 \cos(2\theta_r) + 2b \cos \theta_r \sqrt{R_c^2 - (b \sin \theta_r)^2} \right] + l_c \left(\frac{R_r^2}{2} \right) - \frac{dV_{l,vn}(\theta_r)}{d\theta_r} \quad (4.18)$$

V. Rotor angle $(540^\circ - \theta_{r,st})$ to 540°

Working volume from $(540^\circ - \theta_{r,st})$ to 540° is $V_{clr}(\theta_{r,st})$ as the trailing vane tip is inside the rotor slot and as a result, no working volume is formed between the vane tip, rotor wall and the cylinder wall. This is stated in equation (4.19). The corresponding derivative of volume with respect to rotor angle, $\theta_{r,st}$ is 0 as shown in equation (4.20).

$$V(\theta_r) = V_{clr}(\theta_{r,st}), \text{ if } (540^\circ - \theta_{r,st}) \leq \theta_r \leq 540^\circ \quad (4.19)$$

$$\frac{dV(\theta_r)}{d\theta_r} = 0, \text{ if } (540^\circ - \theta_{r,st}) \leq \theta_r \leq 540^\circ \quad (4.20)$$

For the the compressor dimensions of $R_c = 27.5$ mm, $R_r = 15.5$ mm, $l_c = 30$ mm, $b = 12.5$ mm, $R_{f1} = R_{f2} = 3$ mm and $t_{vn} = 6$ mm, the variation of the working chamber volume from 0° to 540° is shown in Figure 4.12. The working chamber

volume increases from the from the rotor angle of about 42.5° . The maximum working chamber volume is achieved at an angle of 270° . In this case, this value is about 43.5 cm^3 . Then after, the volume decreases for compression of the working fluid. The compression ends at around 497.5° .

Figure 4.12: Variation of the working chamber volume for CVC

The corresponding rate of change of the working chamber volume with the rotor angle is shown in Figure 4.13. From 0° to 42.5° rotor angle, the rate of increase of the cylinder volume with respect to the rotor angle is smaller compared to the sum of the rate of increase of the rotor volume and rate of increase of trailing vane volume in the control volume. Hence, the total rate of change of working chamber volume is small and close to 0. But further along the rotation, the rate of increase of the cylinder volume starts exceeding the sum of the rate of the rotor volume and the vane volume. At 180° rotor angle, the rate of increase of the working chamber volume reaches the maximum. The rate of change of volume is positive until 270° rotor angle. This also implies that the working chamber volume is maximum at this rotor angle and the volume starts to decrease after this rotor angle. Therefore, ideally, the suction port should be designed such the suction process is continuous until 270° rotor angle. As the working chamber volume decreases for the compression, the rate of decrement of the working chamber volume goes to minimum at 360° . Finally, as the leading vane enters into the rotor slot, the rate of change of the volume becomes 0.

Figure 4.13: Variation of the rate of change of working chamber volume with the rotor angle

VI. Vane gap volume

In the illustration shown in Figure 4.14, it can be seen that as the leading vane protrudes into the rotor slot, there exists a pocket or a gap of space within the rotor between the rear part of the leading vane and the neck of trailing vane. Using equations (4.21) and (4.22), the derivation for the vane gap volume and the rate of change of this volume are presented as shown in equations (4.23) and (4.24).

Figure 4.14: Illustration of the formation and the evolution of the gap volume (a): the formation of the gap volume, (b): the maximum gap volume, (c) the gap volume before it coalesces with the working chamber

If $(r(\theta_r + 180^\circ) - R_r) < l_{tip}$

$$l_{gap} = r(\theta_r + 180^\circ) + r(\theta_r) - l_{tip} - l_{vn}$$

$$V_{cres}(\theta_r) = \begin{cases} \frac{\pi}{4}(R_{flt}^2 - (l_{gap} - R_{flt})^2)g_h & \text{if } R_{flt} > l_{gap} - R_{flt} \\ (R_{flt}^2 + (l_{gap} - R_{flt})R_{flt})g_h & \text{if } R_{flt} < l_{gap} - R_{flt} \end{cases} \quad (4.21)$$

$$\frac{dl_{gap}(\theta_r)}{d\theta_r} = \frac{dr(\theta_r)}{d\theta_r} + \frac{dr(\theta_r + 180^\circ)}{d\theta_r}$$

$$\frac{dV_{cres}(\theta_r)}{d\theta_r} = \begin{cases} \frac{\pi}{2} \left(-(l_{gap} - R_{flt}) \frac{dl_{gap}(\theta_r)}{d\theta_r} \right) g_h & \text{if } R_{flt} > l_{gap} - R_{flt} \\ \left(\frac{dl_{gap}(\theta_r)}{d\theta_r} R_{flt} \right) g_h & \text{if } R_{flt} < l_{gap} - R_{flt} \end{cases} \quad (4.22)$$

$$V_{gap}(\theta_r) = l_{gap}(t_{vn} - g_h)l_c + V_{cres}(\theta_r) + V_{clr}(\theta_{r,st}) \quad (4.23)$$

$$\frac{dV_{gap}(\theta_r)}{d\theta_r} = \frac{dl_{gap}(\theta_r)}{d\theta_r}(t_{vn} - g_h)l_c + \frac{dV_{cres}(\theta_r)}{d\theta_r} \quad (4.24)$$

Figure 4.15 and Figure 4.16 show the variation of vane gap volume and the rate of change of vane gap volume for compressor dimension, where, $R_c = 27.5$ mm, $R_r = 15.5$ mm, $l_c = 30$ mm, $b = 12.5$ mm, $l_{vn} = 33$ mm, $l_{tip,vn} = 8.8$ mm and $t_{vn} = 6$ mm. At around 95° rotor angle, the leading vane neck enters the rotor slot, resulting in the formation of gap volume of 0.058 cm^3 . We encounter discontinuities at 150° and 210° because of the change in $r(\theta_r + 180^\circ)$ shown and described in equation (4.11). Subsequently, at around 265° , the gap volume communicates with the control volume and the working fluid in the gap mixes with the working fluid in the control volume. Similarly, at around 459° rotor angle, the gap volume forms again because the leading vane neck enters the rotor slot. The working fluid in this gap volume will communicate with the suction chamber at the rotor angle of 265° of the next cycle.

Assuming this gap volume is sealed off from the working chamber, the rate of thermodynamic changes within this gap volume are different than that of the main working chamber within the cylinder because the rate of change of the gap volume and the rate of change of the main working chamber volume are different.

Figure 4.15: Variation of vane gap volume

Figure 4.16: Variation of the rate of change of vane gap volume

4.2 Thermodynamics model

The thermodynamic model for CVC involves the prediction of thermodynamic properties of the working fluid in the working chamber which assumes a control volume including the main (or primary) flows through the ports and the secondary flows through the leakage clearances. A complete working cycle for the compressor includes three processes: a suction, a compression and a discharge. In a suction process, the main flow is the flow into the control volume through the suction port. In a compression process, the working chamber behaves like a closed system. While, in a discharge process, main flow is the flow out of the control volume through the discharge port. Throughout the working cycle, the thermodynamic state of the fluid is also influenced by the secondary flows involving leakage of the working fluid through the clearances between the moving parts of the compressor. Therefore, assuming the steady state condition at the inlet and the outlet of the compressor, the evolution of the thermodynamic state

of the working fluid in the working chamber from the suction till the end of the discharge can be predicted by analysing the energy, mass flow across the boundary and the change in the volume of the working chamber.

Figure 4.17: Cross-section of CVC showing different control volumes

It is assumed that the properties of the working fluid are uniform throughout the control volume. Any changes brought about by the three processes, namely, a suction, a compression and a discharge are evenly and instantaneously propagated throughout the control volume. Suction, compression and discharge chambers and various paths of mass flow to the chambers are shown in Figure 4.17.

It is also assumed that the main flow-processes are steady, that is, initial quantities assumed for the system are the same at the start of every cycle. Consequently, the energy balance of the control volume can be expressed as shown in equation (4.25).

$$\begin{aligned} \sum m_{in} \left(u + pv + \frac{V^2}{2} + gz \right)_{in} - \sum m_{out} \left(u + pv + \frac{V^2}{2} + gz \right)_{out} + Q - W \\ = \Delta \left[m \left(u + \frac{V^2}{2} + gz \right) \right]_{system} \end{aligned} \quad (4.25)$$

It is considered that the operating speed of the compressor is constant for any working cycle. For initial consideration for a perfectly sealed compressor, that is, a compressor without any secondary leakage flows, it has the same mass flowrate at the inlet and at the outlet. Further, with respect to any relative change in the flow area at the inlet and the outlet, the resulting change in the velocity of the fluid and hence its kinetic energy will be negligible when compared to the changes in its internal energy. Additionally, the gravitational potential of the fluid throughout the working cycle can be assumed to remain constant. Equation (4.25) can be written as an ordinary differential equation with respect to temporal dimension as shown in equation (4.26).

$$\sum \dot{m}_{in}(u + pv)_{in} - \sum \dot{m}_{out}(u + pv)_{out} + \dot{Q} - \dot{W} = [\dot{m}(u) + m(\dot{u})]_{system} \quad (4.26)$$

The specific enthalpy is defined in equation (4.27). The rate of change of the specific internal energy is derived in equation (4.28).

$$h = u + pv \quad (4.27)$$

$$\dot{u} = \dot{h} - \dot{p}v - p\dot{v} \quad (4.28)$$

The subscript 'system' in equation (4.26) represents the control volume 'cv' in the energy balance equation at each time step. Equation (4.29) can be obtained by using equations (4.26), (4.27), and (4.28).

$$\begin{aligned} \dot{m}_{in}h_{in} + \dot{m}_{leak,in}h_{leak,in} - \dot{m}_{out}h_{out} - \dot{m}_{leak,out}h_{leak,out} + \dot{Q} - \dot{W} \\ = \left[\dot{m}_{cv}(h - pv)_{cv} + m_{cv}(\dot{h} - \dot{p}v - p\dot{v})_{cv} \right] \end{aligned} \quad (4.29)$$

Using the continuity equation, the mass flow rate of the control volume can be represented as shown in equation (4.30).

$$\dot{m}_{cv} = \dot{m}_{in} + \dot{m}_{leak,in} - \dot{m}_{out} - \dot{m}_{leak,out} \quad (4.30)$$

The variation of compression work, the rate of change of specific volume and the density of fluid within the control volume with time are defined using equations (4.31), (4.32) and (4.33):

$$\dot{W} = p\dot{V}_{cv} \quad (4.31)$$

$$\dot{v} = \frac{1}{m_{cv}} \frac{dV}{dt} - \frac{V_{cv}}{m_{cv}^2} \frac{dm}{dt} \quad (4.32)$$

$$\dot{\rho} = \frac{1}{V_{cv}} \frac{dm}{dt} - \frac{m_{cv}}{V_{cv}^2} \frac{dV}{dt} \quad (4.33)$$

The enthalpy and the pressure of the fluid can be said to be functions of both the temperature and the density. Therefore, the rate of change of enthalpy and the pressure in the control volume are expressed as shown in equation (4.34), (4.35), (4.36) and (4.37).

$$\frac{dh}{dt} = \left(\frac{\partial h}{\partial T} \right)_{\rho} \frac{dT}{dt} + \left(\frac{\partial h}{\partial \rho} \right)_T \frac{d\rho}{dt} \quad (4.34)$$

$$\frac{dh}{dt} = \left(\frac{\partial h}{\partial T} \right)_{\rho} \frac{dT}{dt} + \left(\frac{\partial h}{\partial \rho} \right)_T \left(\frac{1}{V_{cv}} \frac{dm}{dt} - \frac{m_{cv}}{V_{cv}^2} \frac{dV}{dt} \right) \quad (4.35)$$

$$\frac{dp}{dt} = \left(\frac{\partial p}{\partial T} \right)_{\rho} \frac{dT}{dt} + \left(\frac{\partial p}{\partial \rho} \right)_T \frac{d\rho}{dt} \quad (4.36)$$

$$\frac{dp}{dt} = \left(\frac{\partial p}{\partial T} \right)_{\rho} \frac{dT}{dt} + \left(\frac{\partial p}{\partial \rho} \right)_T \left(\frac{1}{V_{cv}} \frac{dm}{dt} - \frac{m_{cv}}{V_{cv}^2} \frac{dV}{dt} \right) \quad (4.37)$$

Substituting equations (4.30), (4.31), (4.32), (4.33), (4.34), (4.35), (4.36) and (4.37) into (4.29), we get an expression deriving the rate of change of temperature as shown in equation (4.38).

$$\begin{aligned} \dot{T}_{cv} &= \frac{\left[\dot{Q} + \rho_{cv} \dot{V}_{cv} \left\{ \rho_{cv} \left(\frac{\partial h}{\partial \rho} \right)_T - \left(\frac{\partial p}{\partial \rho} \right)_T \right\} + \dot{m}_{in} \left\{ h_{in} - h_{cv} - \rho_{cv} \left(\frac{\partial h}{\partial \rho} \right)_T + \left(\frac{\partial p}{\partial \rho} \right)_T \right\} \right. \\ &\quad + \dot{m}_{leak,in} \left\{ h_{leak,in} - h_{cv} - \rho_{cv} \left(\frac{\partial h}{\partial \rho} \right)_T + \left(\frac{\partial p}{\partial \rho} \right)_T \right\} \\ &\quad - \dot{m}_{out} \left\{ h_{out} - h_{cv} - \rho_{cv} \left(\frac{\partial h}{\partial \rho} \right)_T + \left(\frac{\partial p}{\partial \rho} \right)_T \right\} \\ &\quad \left. - \dot{m}_{leak,out} \left\{ h_{leak,out} - h_{cv} - \rho_{cv} \left(\frac{\partial h}{\partial \rho} \right)_T + \left(\frac{\partial p}{\partial \rho} \right)_T \right\} \right]}{m_{cv} \left(\frac{\partial h}{\partial T} \right)_{\rho} - V_{cv} \left(\frac{\partial p}{\partial T} \right)_{\rho}} \end{aligned} \quad (4.38)$$

Equation (4.38) represents a first order differential equation and it needs to be integrated numerically to calculate the instantaneous temperature of the working fluid in the control volume. Pressure, mass and density of the fluid are similarly obtained by numerically integrating equations (4.30), (4.33) and (4.37). Partial

differential functions are obtained from the real gas properties. Using equation (4.32), equation (4.38) can be further simplified into (4.39) by writing the rate of change of volume, $\dot{V}_{cv}$, in terms of the rate of change of specific volume, $\dot{v}$.

$$\dot{T}_{cv} = \frac{\left[\dot{Q} + \dot{v} \left\{ V_{cv} \left(\frac{\partial p}{\partial \rho} \right)_T - m_{cv} \left(\frac{\partial h}{\partial \rho} \right)_T \right\} + \dot{m}_{in} \{ h_{in} - h_{cv} \} \right] + \sum \dot{m}_{leak,in} \{ h_{leak,in} - h_{cv} \}}{m_{cv} \left(\frac{\partial h}{\partial T} \right)_\rho - V_{cv} \left(\frac{\partial p}{\partial T} \right)_\rho} \quad (4.39)$$

4.3 Suction and discharge flow model

The suction and discharge mass flowrates are modelled assuming steady, isentropic flow through the orifice [171], as illustrated in Figure 4.18. At the inlet, the flow at pressure p_1 , temperature T_1 , density ρ_1 and enthalpy h_1 . The flow downstream across the orifice is at pressure p_2 , temperature T_2 , and the enthalpy $h_{2,is}$ assuming isentropic conditions.

Figure 4.18: Illustration of a flow through an orifice

Assuming, steady, compressible, adiabatic, reversible and hence isentropic, the energy balance equation (4.25) can be reduced to the equation (4.40).

$$h_1 + \frac{V_1^2}{2} = h_{2,is} + \frac{V_2^2}{2} \quad (4.40)$$

If the upstream control volume is assumed to be at stagnation condition, then equation (4.41) is the downstream flow velocity.

$$V_2 = \sqrt{2(h_1 - h_{2,is})} \quad (4.41)$$

Further, if it is assumed that the flow area is equal to the orifice cross-sectional area, then mass flowrate determined this way as shown in equation (4.42) is considered as the ideal mass flowrate through the orifice.

$$\dot{m}_{ideal} = \rho_1 A_{orif} V_2 \quad (4.42)$$

During the flow through an orifice, the viscous flow means the presence of the viscous dissipation and the other flow losses, therefore, the flow is non-isentropic. This also means that the real flowrate will always be less than the ideal flowrate obtained using equation (4.42). Dissipation is significant, especially where the boundary change is sudden such as in the illustration of the flow regime shown in Figure 4.18. To account for the flow loss, a coefficient is introduced into the equation (4.42) which is called the coefficient of discharge, C_d . For sharp-edged orifice, such as the one shown in Figure 4.18, a coefficient of discharge of 0.61 [172] to 0.63 [173] has been reported. Hence in the orifice flow, phenomena known as *vena contracta* can be said to be present which implies the constriction of the flow area. The real mass flow rate is then determined using equation (4.43).

$$\dot{m}_{real} = \rho_1 C_d A_{flow} \bar{V} \quad (4.43)$$

Generally, the discharge port in the compressor is often designed with a valve which means the flow area is further affected by the opening and the closing of the valve. However, the suction port in the coupled vane compressor is designed without the valve and suction flowrate can be modelled as shown in equation (4.44).

$$\dot{m}_{in} = \rho_1 C_d A_{orif} \sqrt{2(h_1 - h_{2,is})} \quad (4.44)$$

Discharge flowrate, $\dot{m}_{out}$, is determined using equation (4.43) in which the flow area A_{flow} depends upon the displacement of reed valve from the valve seat due to differential pressure across the valve. The flow area for the discharge port is discussed in section 4.4 using equation (4.68).

4.3.1 Flow area model

Figure 4.19: (a) Sectional view of CVC and the angles that define the starting and ending angular position with respect to the rotor centre; (b) and (c) Illustration of the suction port and the flow area

As it can be seen from Figure 4.19, when the vane tip is passing along the suction orifice area, the suction flow area is partitioned to two suction chamber, namely, the leading suction chamber and the trailing suction chamber. The real flow area for the working fluid is the area of the segment of the total orifice area. Area of the segment can be visualised from the Figure 4.19 (C). $\Delta\theta_{suc}$ is the sector angle. The angular position of the start point of the suction port with respect to the rotor centre C_r is labelled $\theta_{suc,st}$ and angular position of the ending point of the suction port is labelled $\theta_{suc,end}$. At $\theta_{suc,end}$ and beyond, the area of the flow is equal to the circle which is the full cross-sectional area of the suction port. The sector angle $\Delta\theta_{suc}$, when the rotor angle is at $\theta_{suc,st}$, is equal to 0 and when the rotor angle is at $\theta_{suc,end}$, it is equal to 360° . In between these bounding angles, the sector angle $\Delta\theta_{suc}$ varies linearly. Then, the segment area is the sector area minus the area of the isosceles triangle of which the two equal sides are the radius of the port.

This segment area which represents the orifice area towards the trailing suction chamber can be modelled as shown in the given equation (4.45)

If $\theta_{suc,end} \geq \theta_r > \theta_{suc,st}$,

$$A_{flow}(\theta_r) = R_{suc,orif}^2 \left[\frac{\Delta\theta_{suc}}{2} - \frac{\sin\Delta\theta_{suc}}{2} \right]$$

$$\text{where, } \Delta\theta_{suc} = 2\pi \left(\frac{\theta_r - \theta_{suc,st}}{\theta_{suc,end} - \theta_{suc,st}} \right) \quad (4.45)$$

Similarly, for the segment area which represents the orifice area towards the leading suction chamber can be modelled as shown in the equation (4.46).

If $180^\circ + \theta_{suc,st} \leq \theta_r \leq 180^\circ + \theta_{suc,end}$,

$$A_{flow}(\theta_r) = R_{suc,orif}^2 \left[\frac{\Delta\theta_{suc}}{2} - \frac{\sin\Delta\theta_{suc}}{2} \right]$$

$$\text{where, } \Delta\theta_{suc} = 2\pi \left(\frac{\theta_{suc,end} + \pi - \theta_r}{\theta_{suc,end} - \theta_{suc,st}} \right) \quad (4.46)$$

If $\theta_{suc,end} < \theta_r < 180^\circ + \theta_{suc,st}$, then the area of flow is simply the area of the circle as shown in equation (4.47).

$$A_{flow}(\theta_r) = \pi R_{suc,orif}^2 \quad (4.47)$$

Using the equation (4.45), (4.46) and (4.47), the variation of the suction flow area can be plotted as shown in Figure 4.20.

Figure 4.20: Variation of flow area with rotor angle

4.4 Valve dynamics

A thin reed of non-uniform cross-sectional area as shown in Figure 4.21 is designed to be used as a valve at discharge port. This reed consists of a circular free end and a fixed end. At any given instance, the differential pressure Δp acts as the external loading on the circular free end. In CVC, the reed valve opens when the pressure in the discharge chamber becomes greater than the pressure in the discharge plenum and the force acting across the valve is able to overcome the stiffness of the reed. The circular end of the reed has the radius ' R_{val} ' and tends to deflect along the y-axis. The main body of the reed has a uniform rectangular cross-section of width w_{val} between the circular free end and the fixed end. The entire reed has uniform thickness t_{val} throughout its length l_{val} .

To prevent the reed from deflecting beyond its allowable deflection during the operation of the compressor, a valve stopper is installed on top of the reed. This improves the fatigue performance of the reed from unnecessary large deflection which otherwise may lead to premature valve failure.

Figure 4.21: (a) and (b) A thin reed with a non-uniform cross-sectional area

To study the dynamics of the reed valve opening, the reed is modelled assuming the thin beam vibration model. The valve response is characterized as the vibration of the cantilever beam of varying width. An infinitesimally small element with mass ' dm ' in the reed valve is under shear force ' V ' and experiences bending moment ' M ' due to the external loading $P(x,t)$. The valve experiences damping force $c \frac{\partial y}{\partial t}$ due to the air-cushioning effect, where, c is the damping

coefficient. The free body diagram of the infinitesimally small element in the reed valve is shown in Figure 4.22.

Figure 4.22: Free body diagram of an infinitesimally small element of the reed
The force balance across the y-direction or the thickness of the valve is shown in equation (4.48).

$$-V + V + dV + P(x, t)dx - c \frac{\partial y}{\partial t} dx = dm \frac{\partial^2 y}{\partial t^2} \quad (4.48)$$

Similarly, the momentum equilibrium equation for the element is shown in equation (4.49).

$$Vdx - M + M + dM - P(x, t)dx \cdot \frac{dx}{2} + c \frac{\partial y}{\partial t} \cdot \frac{dx}{2} = 0 \quad (4.49)$$

Ignoring second or higher order terms of dx in equation (4.49), equation (4.50) can be obtained. Based on the flexural theory, the relationship between the shear force, bending moment and the beam deflection can be expressed in equation (4.51).

$$V = -\frac{\partial M}{\partial x} \quad (4.50)$$

$$M = EI(x) \frac{\partial^2 y}{\partial x^2} \quad (4.51)$$

Similarly, the mass of the infinitesimally small element 'dm' can be written in terms of density ρ_{val} , its cross-sectional area $A(x)$ and the length as shown in equation (4.52).

$$dm = \rho_{val} A(x) dx \quad (4.52)$$

Cross-sectional area $A(x)$ and the moment of inertia of the valve reed is determined using equations (4.53) and (4.54).

$$A(x) = \begin{cases} 2t_{val}\sqrt{R_{val}^2 - (R_{val} - x)^2} & 0 < x < 2R_{val} \\ w_{val}t_{val} & 2R_{val} < x < l_{val} \end{cases} \quad (4.53)$$

$$I(x) = \begin{cases} \frac{t_{val}^3}{6}\sqrt{R_{val}^2 - (R_{val} - x)^2} & 0 < x < 2R_{val} \\ \frac{w_{val}t_{val}^3}{12} & 2R_{val} < x < l_{val} \end{cases} \quad (4.54)$$

Combination of equations (4.48), (4.49), (4.50), (4.51) and (4.52) gives the equation for the beam deflection in y-direction due to the external loading $P(x,t)$ across its thickness ' t_{val} ' including the fourth order differential term in spatial dimension, ' x ', and second order in time, ' t '. This equation is written as shown in equation (4.55).

$$\frac{\partial^2}{\partial x^2} \left(EI(x) \frac{\partial^2 y}{\partial x^2} \right) + \rho_{val} A(x) \frac{\partial^2 y}{\partial t^2} + c \frac{\partial y}{\partial t} = P(x, t) \quad (4.55)$$

Applying separation of variables method and using the principle of superposition for linear vibration, the solution for the valve deflection can be assumed to be of a form as shown in equation (4.56):

$$y = \sum_{n=1}^{\infty} \varphi_n(x) q_n(t) \quad (4.56)$$

$\varphi_n(x)$ represents the valve shape function which is determined from the free vibration analysis. $q_n(t)$ represents the mode participation factor. ' n ' represents the mode shape number. For the case of flow consideration, the first two mode shapes are sufficient [88]. For each mode shape, equation (4.55) can be reduced to equation (4.57) using equation (4.56).

$$\begin{aligned} & \frac{1}{\rho_{val} A(x) \varphi_n(x)} \frac{\partial^2}{\partial x^2} \left(EI(x) \frac{\partial^2 \varphi_n(x)}{\partial x^2} \right) \\ &= \frac{P(x, t)}{\rho_{val} A(x) \varphi_n(x) q_n(t)} - \frac{1}{q_n(t)} \frac{\partial^2 q_n(t)}{\partial t^2} - \frac{c}{\rho_{val} A(x) q_n(t)} \frac{\partial q_n(t)}{\partial t} \end{aligned} \quad (4.57)$$

Figure 4.23 shows the first mode shape for the free vibration response of the reed valve shown in Figure 4.21 using finite element method on commercially available Dassault Systèmes Solidworks 2018 (Student version). The valve dimension was arbitrarily selected, where, $R_{val} = 4$ mm, $l_{val} = 30$ mm, $w_{val} = 3$ mm and $t_{val} = 0.3$ mm. The material selected for the valve plate was alloy steel whose yield strength is 620.4 MPa, tensile strength is 723.8 MPa, density is 7700 kg m⁻³,

Poisson's ratio is 0.28 and elastic modulus is 210 GPa. The resultant amplitude for each node was normalized and the natural frequency for the first mode shape was obtained to be 356.25 Hz.

Figure 4.23: First modal valve deflection of reed valve using free vibration response

Figure 4.24 shows the second modal reed valve deflection. The second mode for the reed valve is observed at the frequency of 2540.7 Hz.

Figure 4.24: Second modal valve deflection of reed valve using free vibration response

$\varphi_n(x)$ is the eigenfunction of free, damped vibration of the n th mode of the beam, i.e. when the external loading on the right side of equation (4.55) is 0. These eigenfunction are orthogonal with a typical eigenfunction, say, $\varphi_m(x)$. The orthogonality is expressed as equation (4.58):

$$\int_0^l \varphi_n(x) \varphi_m(x) dx = \begin{cases} 0, & n \neq m \\ 1, & n = m \end{cases} \quad (4.58)$$

$\varphi_n(x)$ can be determined assuming an admissible trial function which can be approximated to a standard polynomial function as shown in equation (4.59) [104].

$$\varphi(x) = \left\{ \left(\frac{x}{l_{val}} \right)^4 - 4 \left(\frac{x}{l_{val}} \right) + 3 \right\} \quad (4.59)$$

Following considerations are necessary for the selection of approximate trial functions for them to be accurate:

1. Geometric boundary conditions must be satisfied.
2. It is better to satisfy the generalized force boundary conditions.
3. The trial function should follow the expected mode shape.

The natural and the geometrical boundary conditions for the cantilever beam are:

$$\text{At } x = 0, \frac{\partial^2 \varphi(x)}{\partial x^2} = 0, \frac{\partial^3 \varphi(x)}{\partial x^3} = 0$$

$$\text{At } x = l, \varphi(l) = 0, \frac{\partial \varphi(x)}{\partial x} = 0$$

Using the principle of separation of variables method, both sides of equation (4.57) must be equal to a real constant ω_n^2 . The left side and the right side of the equation (4.57) can be written as two ordinary differential equations (4.60) and (4.61).

$$\frac{1}{\rho_{val} A(x) \varphi_n(x)} \frac{d^2}{dx^2} \left(EI(x) \frac{d^2 \varphi_n(x)}{dx^2} \right) = \omega_n^2 \quad (4.60)$$

$$\frac{P(x, t)}{\rho_{val} A(x) \varphi_n(x) q_n(t)} - \frac{1}{q_n(t)} \frac{d^2 q_n(t)}{dt^2} - \frac{c}{\rho_{val} A(x) q_n(t)} \frac{dq_n(t)}{dt} = \omega_n^2 \quad (4.61)$$

ω_n represents the frequency of the n^{th} mode of vibration of the valve. As the natural modes are orthogonal and satisfy orthonormal conditions, ω_n is obtained using the Rayleigh quotient. The equation (4.62) gives upper bound to the first mode fundamental frequency.

$$\omega_n^2 = \frac{X}{Y} \quad (4.62)$$

X represents the effective stiffness of the valve reed and Y represents the effective mass of the valve reed. Following the geometry of the reed valve, as shown in Figure 4.21, X and Y in the numerator and denominator of equation (4.62) are obtained using equations (4.63) and (4.64) respectively.

$$X = E \int_0^{2R_{val}} \varphi_n(x) \frac{d^2}{dx^2} \left(I(x) \frac{d^2 \varphi_n(x)}{dx^2} \right) dx + E \int_{2R_{val}}^{l_{val}} \varphi_n(x) \frac{d^2}{dx^2} \left(I(x) \frac{d^2 \varphi_n(x)}{dx^2} \right) dx \quad (4.63)$$

$$Y = \rho_{val} \int_0^{2R_{val}} A(x) (\varphi_n(x))^2 dx + \rho_{val} \int_{2R_{val}}^{l_{val}} A(x) (\varphi_n(x))^2 dx \quad (4.64)$$

The natural frequencies for the discharge port of diameter 12 mm and for some arbitrary valve geometries are listed in Table 4.1.

Table 4.1: Natural frequencies for some typical valve geometries

Valve thickness (mm)	Constant width (mm) ' w_{val} ' = 6mm	Vane tip radius (mm) ' R_{val} ' = 12.2 mm	Natural frequency (Hz) ' ω_n '	
	Length of the valve (mm)		Mode shape 1	Mode shape 2
0.25		16.88	521.54	2837.13
0.35		18.88	683.41	3373.47
0.45		20.88	711.67	3566.92

The equation (4.61) can be written as shown in equation (4.66).

$$P(x, t) - \rho_{val} A(x) \frac{d^2 q_n(t)}{dt^2} - c \varphi_n(x) \frac{dq_n(t)}{dt} = \omega_n^2 \rho_{val} A(x) q_n(t) \quad (4.65)$$

Multiplying both sides of equation (4.65) by $\varphi_n(x)$, integrating over the length of the valve and applying the orthogonality of $\varphi_n(x)$ yields equation (4.66).

$$\begin{aligned} \int_0^{l_{val}} P(x, t) \varphi_n(x) dx - \frac{d^2 q_n(t)}{dt^2} \int_0^{l_{val}} \rho_{val} A(x) \varphi_n(x) dx + c \int_0^{l_{val}} \{\varphi_n(x)\}^2 dx \frac{dq_n(t)}{dt} \\ = \omega_n^2 q_n(t) \int_0^{l_{val}} \rho_{val} A(x) \varphi_n(x) dx \end{aligned} \quad (4.66)$$

The net force acting on the valve plate at any given instant during the operation of the compressor is equal to the product of the effective flow area times the differential pressure across the valve. Then total differential force per unit length, $P(x,t)$, can be written as shown in the equation (4.67):

$$\int_0^{l_{val}} P(x,t) \varphi_n(x) dx = \Delta p(t) \int_0^{l_{val}} \varphi_n(x) A_{force} dx \quad (4.67)$$

Here, $\Delta p(t)$ is the differential pressure across the reed valve. A_{force} , is the area acted on the valve affected by the fluid forcing out of discharge fluid. The force area is assumed to be equal to the cross-sectional area of the port. In order to impose restriction to the flow, the effective flow area, A_{flow} , is the curved surface area of the flow or the cross-sectional area of the port, whichever is the minimum [104]. If the port is circular in shape, the curved surface area of the flow is the cylindrical surface area where the cylinder height is the valve deflection height (see equation (4.70)). Thus, the flow area and the force area are shown in equation (4.68) and (4.69).

$$A_{flow}(y) = \delta_{val} \times (\pi d_{dis}), \text{ if } \delta_{val} \times (\pi d_{dis}) < \frac{\pi}{4} d_{dis}^2$$

$$A_{flow}(y) = \left(\frac{\pi}{4} d_{dis}^2\right), \text{ if } \delta_{val} \times (\pi d_{dis}) > \frac{\pi}{4} d_{dis}^2 \quad (4.68)$$

$$A_{force} = \frac{\pi}{4} d_{dis}^2 \quad (4.69)$$

$$\delta_{val} = y(x, t), \text{ for } x = r_{val} \quad (4.70)$$

The damping coefficient, c , in equation (4.66) can be written in terms of damping ratio, ζ , the natural frequency ω_n and the effective mass of the valve as shown in equation (4.71). Combining equations (4.66), (4.67) and (4.71), we obtain the equation (4.72) to calculate the mode participation factor.

$$c = 2\zeta\omega_n \int_0^l \rho_{val} A(x) \varphi_n(x) dx \quad (4.71)$$

$$\frac{d^2 q_n(t)}{dt^2} + 2\zeta\omega_n \int_0^l \{\varphi_n(x)\}^2 dx \frac{dq_n(t)}{dt} + \omega_n^2 q_n(t) = \frac{\Delta p(t) \int_0^l \varphi_n(x) A_{force}(y) dx}{\int_0^l \rho_{val} A(x) \varphi_n(x) dx} \quad (4.72)$$

Second order differential equation (4.72) is coupled with the thermodynamic equation (4.38) because of the differential pressure term $\Delta p(t)$ which dictates the

force to open or close the valve. Hence, equations (4.38) and (4.72) need to be numerically solved simultaneously, using a numerical integration technique such as Runge-Kutta. Runge-Kutta numerical integration requires equation (4.72) to be reduced into a single order differential equation. This is achieved by assuming two variables which are defined as $\alpha_1 = \frac{dq_n(t)}{dt}$ and $\alpha_2 = q_n(t)$. Consequently, we get two simultaneous equations (4.73) and (4.74) from equation (4.72).

$$\frac{d\alpha_1}{dt} + 2\zeta\omega_n\alpha_1 \int_0^l \{\varphi_n(x)\}^2 dx + \omega_n^2\alpha_2 = \frac{\Delta p(t) \int_0^l \varphi_n(x) A_{force}(y) dx}{\int_0^l \rho_{val} A(x) \varphi_n(x) dx} \quad (4.73)$$

$$\alpha_1 = \frac{d\alpha_2}{dt} \quad (4.74)$$

4.5 Leakage flow model

Three secondary flow paths, which are the internal leakage paths, are identified in CVC: leakage through the sealing arc, leakage through the clearance gap between the vane-endface and the cover and the leakage through the discharge port at the vane tip. These leakage paths are illustrated in figures 4.26, 4.29 and 4.32.

4.5.1 Leakage through the sealing arc

Figure 4.25: Geometrical model for radial leakage path through sealing arc

As shown in Figure 4.25, the leakage through the sealing arc is along the circumference in the clearance gap $\delta_{r,sa}$ between the rotor surface and the sealing arc. Assuming adiabatic flow and steady conditions, we can model the flow to be compressible frictional flow (Fanno flow) to evaluate the leakage flowrate through the clearance gap in the sealing arc.

According to Yanagisawa and Shimizu [28], the geometrical path of the leakage can be assumed to consist of a converging section and then a constant cross-sectional area channel. This schematic is illustrated in Figure 4.26.

Figure 4.26: Sealing arc leakage flow model

To calculate the leakage flowrate, the flow is first assumed to choke at the constant cross-sectional area channel exit. The length of flow, l_f , is determined based on the rotor radius R_r , clearance gap $\delta_{r,sa}$ and the angle spanning the flow path which is the sealing arc angle $2\theta_{r,st}$ shown in Figure 4.25. This flow length is derived as shown in equation (4.75).

$$l_f = (2R_r + \delta_{r,sa})\theta_{r,st} \quad (4.75)$$

The cross-sectional area of flow is given by equation (4.76).

$$A_f = l_c \delta_{r,sa} \quad (4.76)$$

$\delta_{r,sa}$ is the clearance gap between two surfaces and l_c is the axial length of the flow path which is equal to the length of the cylinder. The hydraulic diameter is defined by equation (4.77).

$$D_H = \frac{4A_f}{P_f} \quad (4.77)$$

Using equation (4.76), equation (4.77) can be written as equation (4.78).

$$D_H = \frac{2A_f l_c}{l_c^2 + A_f} \quad (4.78)$$

The local Reynolds number of the leakage flow is then obtained by using equation (4.79).

$$Re = \frac{\dot{m}_{leak} D_H}{A_f \mu} \quad (4.79)$$

Here, $\dot{m}_{leak}$, the leakage flow rate, is determined iteratively using a procedure proposed by Yanagisawa and Shimizu [28].

Since the flow is assumed to be in the sonic conditions at the exit ('e' in Figure 4.26) Mach number at the throat (M_t), which is at the cross-section joining the nozzle and the constant cross-sectional area channel, must satisfy Fanno flow conditions given by equations (4.80).

$$\bar{\lambda} \frac{l_f}{2\delta_{r,sa}} = \frac{1 - M_t^2}{\kappa M_t^2} + \frac{\kappa + 1}{2\kappa} \ln \left(\frac{M_t^2 (\kappa + 1)}{2 + (\kappa - 1)M_t^2} \right) \quad (4.80)$$

In equation (4.80), $\bar{\lambda}$, is the average friction factor of the two-dimensional channel and it is related to Reynolds number by the equation (4.81) [36].

$$\bar{\lambda} = \begin{cases} \frac{96}{Re} & \text{if } Re \leq 3560 \\ \frac{0.3614}{Re^{0.25}} & \text{if } Re > 3560 \end{cases} \quad (4.81)$$

For the flow from the discharge chamber to the flow at the exit of the constant cross-sectional area channel, equations (4.82), (4.83) and (4.84) can be applied to evaluate the pressure ratios.

$$\frac{p_c}{p_t} = \left(1 + \frac{\kappa - 1}{2} M_t^2 \right)^{\frac{\kappa}{\kappa - 1}} \quad (4.82)$$

$$\frac{p_t}{p_e} = \frac{1}{M_t} \left(\frac{\kappa + 1}{2 + (\kappa - 1)M_t^2} \right)^{0.5} \quad (4.83)$$

$$\frac{p_c}{p_e} = \frac{p_c}{p_t} \cdot \frac{p_t}{p_e} \quad (4.84)$$

If the ratio given by equation (4.84) is less than the ratio of the chamber pressures, $\frac{p_c}{p_s}$ then the flow is indeed choked. The leakage flow conditions at the exit can be determined as shown in the equations (4.85) and (4.86):

$$p_e = \frac{p_c}{\frac{p_c}{p_t} \cdot \frac{p_t}{p_e}} \quad (4.85)$$

$$T_e = T_c \left(\frac{1}{1 + \left(\frac{\kappa - 1}{2} \right) M_e^2} \right)^{-1} \quad (4.86)$$

If the flow is choked, then M_e is equals to 1 and the flow velocity, V_e at the exit can be obtained using equation (4.87). Assuming ideal gas properties, the leakage mass flowrate can be determined using equation (4.88).

$$V_e = M_e \sqrt{\kappa R_g T_e} \quad (4.87)$$

$$\dot{m}_{leak,sa} = \frac{p_e}{R_g T_e} A_f V_e \quad (4.88)$$

R_g represents the gas constant for each ideal gas or mixture of an ideal gas, $R_g = U/M_{gas}$, where, U denotes Universal gas constant and M_{gas} is the molecular weight of the ideal gas or mixture.

If the flow does not choke, then the critical duct length should be determined by guessing the value for throat Mach number M_t^* . Equation (4.89) is used to calculate the critical duct length for the guessed mach number at the throat.

$$\bar{\lambda} \frac{l_f^*}{2\delta_{r,sa}} = \frac{1 - M_t^{*2}}{\kappa M_t^{*2}} + \frac{\kappa + 1}{2\kappa} \ln \left(\frac{M_t^{*2}(\kappa + 1)}{2 + (\kappa - 1)M_t^{*2}} \right) \quad (4.89)$$

Using the critical duct length derived in equation (4.89), the Mach number at the exit, M_e , is obtained by solving the equation (4.90).

$$\bar{\lambda} \frac{l_f^* - l_f}{2\delta_{r,sa}} = \frac{1 - M_e^2}{\kappa M_e^2} + \frac{\kappa + 1}{2\kappa} \ln \left(\frac{M_e^2(\kappa + 1)}{2 + (\kappa - 1)M_e^2} \right) \quad (4.90)$$

The pressure ratios are obtained using equation (4.91), (4.92), (4.93) and (4.94).

$$\frac{p_t}{p_*} = \frac{1}{M_t^*} \left(\frac{\kappa + 1}{2 + (\kappa - 1)M_t^{*2}} \right)^{0.5} \quad (4.91)$$

$$\frac{p_c}{p_t} = \left(1 + \frac{\kappa - 1}{2} M_t^{*2} \right)^{\frac{\kappa}{\kappa - 1}} \quad (4.92)$$

$$\frac{p_e}{p_*} = \frac{1}{M_e} \left(\frac{\kappa + 1}{2 + (\kappa - 1)M_e^2} \right)^{0.5} \quad (4.93)$$

$$\frac{p_c}{p_e} = \frac{p_c}{p_t} \cdot \frac{p_t}{p_*} \cdot \frac{1}{\frac{p_e}{p_*}} \quad (4.94)$$

If the pressure ratio in equation (4.94) is equal to the ratio of chamber pressures, $\frac{p_c}{p_s}$, that is, $\frac{p_c}{p_e} = \frac{p_c}{p_s}$, then the values for Mach numbers at inlet and exit (M_t^* and M_e) have been correctly chosen and we can use equations (4.86), (4.87) and (4.88) to calculate the temperature of the leaked mass and the leakage flow rate. Otherwise, the iteration must continue from (4.89) till (4.94) until the correct values for M_t^* and M_e have been found.

For the dimensions of CVC, where, $R_c = 27.5$ mm, $R_r = 15.5$ mm, $b = 13$ mm, $l_c = 30$ mm, radial clearance, $\delta_{r,sa}$, of $10 \mu\text{m}$, operating speed of 3000 r/min and R1234yf as the working fluid, Figure 4.27 (a) is the leakage rate obtained for the discharge and the suction pressures shown in Figure 4.27 (b). The leakage rate is calculated for half the revolutions only ($0^\circ - 180^\circ$) because the discharge pressure and suction pressures in the working chambers are periodic or repeat themselves after 180° of revolutions.

(a) Variation of leakage flowrate

(b) Variation of discharge and suction pressure

Figure 4.27: Variation of leakage flowrate at the sealing arc

4.5.2 Leakage through the clearance gap at the vane endface

As shown in Figure 4.28 (a) – (c), there exists the clearance between the vane endface and the cylinder endface. The axial length of this clearance gap is extremely small compared to the width of the vane. Therefore, the leakage along this clearance gap can be modelled as the compressible flow with friction or the fanno flow through the constant area duct.

Figure 4.28: (a) Illustration of leakage through vane endface; (b) Illustration of the leakage flow length; (c) Illustration of the width of the flow

The cross-sectional area of flow for the leakage through the vane endface at the trailing vane and the leading vane are determined using equation (4.95) and (4.96).

$$A_{f, enf, t} = (r(\theta_r) - R_r) \delta_{enf, vn} \quad (4.95)$$

$$A_{f, enf, l} = (r(180^\circ + \theta_r) - R_r) \delta_{enf, vn} \quad (4.96)$$

The length of flow for the leakage through the vane endface is simply the width of the vane and the flow through the convergent section is neglected. The schematic of the flow path is shown in Figure 4.29.

Figure 4.29: Schematic of constant area fanno flow

(4.103) are applied to evaluate the leakage flowrate assuming choked condition. Pressure ratio between the throat and the exit of the channel given by equation (4.98) is compared with the pressure ratio across the two chambers. If the throat to exit pressure ratio is lower or equal to the pressure ratio across the two chambers, the assumed choked flow condition is valid. Assuming ideal gas properties, the leakage flowrate can be determined using equation (4.101), where, $A_{f,enf}$ is the flow area which is obtained using equations (4.95) and (4.96).

$$\bar{\lambda} \frac{l_f}{2\delta_{enf,vn}} = \frac{1 - M_t^2}{\kappa M_t^2} + \frac{\kappa + 1}{2\kappa} \ln \left(\frac{M_t^2(\kappa + 1)}{2 + (\kappa - 1)M_t^2} \right) \quad (4.97)$$

$$\frac{p_t}{p_e} = \frac{1}{M_t} \left(\frac{\kappa + 1}{2 + (\kappa - 1)M_t^2} \right)^{0.5} \quad (4.98)$$

$$T_e = T_d \left(\frac{1}{1 + \left(\frac{\kappa - 1}{2} \right) M_e^2} \right)^{-1} \quad (4.99)$$

$$V_e = M_e \sqrt{\kappa R_g T_e} \quad (4.100)$$

$$\dot{m}_{leak,enf} = \frac{p_e}{R_g T_e} A_{f,enf} V_e \quad (4.101)$$

$$\bar{\lambda} = \begin{cases} \frac{96}{Re} & \text{if } Re \leq 3560 \\ \frac{0.3614}{Re^{0.25}} & \text{if } Re > 3560 \end{cases} \quad (4.102)$$

$$Re = \frac{\dot{m}_{leak,enf} D_H}{A_f \mu} \quad (4.103)$$

However, if the pressure ratio is higher than the pressure ratio across the chambers, then similar to the process described in section 4.5.1, the Mach number at the exit will have to be searched iteratively using equations (4.104) to (4.108) until the pressure ratio is equal to the pressure ratio across the chambers.

$$\bar{\lambda} \frac{l_f^*}{2\delta_{rad}} = \frac{1 - M_t^{*2}}{\kappa M_t^{*2}} + \frac{\kappa + 1}{2\kappa} \ln \left(\frac{M_t^{*2}(\kappa + 1)}{2 + (\kappa - 1)M_t^{*2}} \right) \quad (4.104)$$

$$\bar{\lambda} \frac{l_f^* - l_f}{2\delta_{rad}} = \frac{1 - M_e^2}{\kappa M_e^2} + \frac{\kappa + 1}{2\kappa} \ln \left(\frac{M_e^2(\kappa + 1)}{2 + (\kappa - 1)M_e^2} \right) \quad (4.105)$$

$$\frac{p_t}{p_*} = \frac{1}{M_t^*} \left(\frac{\kappa + 1}{2 + (\kappa - 1)M_t^{*2}} \right)^{0.5} \quad (4.106)$$

$$\frac{p_e}{p_*} = \frac{1}{M_e} \left(\frac{\kappa + 1}{2 + (\kappa - 1)M_e^2} \right)^{0.5} \quad (4.107)$$

$$\frac{p_c}{p_e} = \frac{p_t}{p_*} \cdot \frac{1}{\frac{p_e}{p_*}} \quad (4.108)$$

For an arbitrary size of compressor, where, $R_c = 27.5$ mm, $R_r = 15.5$ mm, $b = 13$ mm, endface clearance, δ_{enf} , of $10 \mu\text{m}$, operating speed of 3000 r min^{-1} and R1234yf as the working fluid, Figure 4.30 (c) shows the leakage rates for the chamber pressures shown in Figure 4.30 (d). The leakage rate is calculated for half the revolutions only ($0^\circ - 180^\circ$) because the chamber pressures in the working chambers repeat after 180° of revolutions. Figure 4.30 (a) is shown here to illustrate the two endface leakages.

Since, constant clearance gap is assumed and the vane thickness is constant, the varying parameter is the width of flow. This variations of width of flow at the vane endface 1 and 2 (see Figure 4.30 (a) for location of vane endface 1 and 2) are presented in Figure 4.30 (b). In case of vane endface 1, as the vane is inside the vane slot in rotor for rotor angles $0 - 30^\circ$, the flow width is 0. As the vane protrudes out of the vane slot, the flow width increases and reaches the maximum of 25 mm at 180° rotor angle. For vane 2, since the vane is extended furthest at 0° rotor angle, the flow width is maximum of 25 mm at this rotor angle.

(a) Illustration of endface leakage locations

(b) Variation of flow widths at vane endfaces

(c) Variation of endface leakage flowrate

(d) Variation pressure in working chambers

Figure 4.30: Variation of endface leakage flowrate

4.5.3 Leakage at the vane tip through the discharge port

As the vane tip passes through the discharge port towards the end of the discharge phase, the discharge chamber is found to connect to the trailing compression chamber. This causes the leakage of the gas from the discharge

chamber to the compression chamber until the pressure in both the chambers equalize. The illustration of this leakage is shown in Figure 4.31.

Figure 4.31: Leakage of fluid through the discharge tip

I. An analytical model for the flowrate

This leakage is modelled using compressible flow through the orifice by assuming isentropic and constant conditions of flow across the *vena contracta*, ignoring the effects of friction, gravity and heat transfer. The leakage flowrate is obtained using equations (4.109) and (4.110).

$$\dot{m}_{dis,tip} = \rho_1 C_d A_{eff,tip} \bar{V} \quad (4.109)$$

$$\bar{V} = \sqrt{2(h_1 - h_{2,is})} \quad (4.110)$$

II. Leakage flow area

The flow area for the leakage is the effective area of flow including the non-isentropic effects. The flow from the discharge port first is visualised to hit the curved tip of the vane first and the flow properties are instantaneously propagated to the rest of the chamber. The minimum between the orifice opening area and the curved surface area bounded by the vane tip is taken as the effective flow area. The illustration of the orifice opening area and the curved surface area of the flow is shown in Figure 4.32 (a) and (b).

(a)

(b)

Figure 4.32: Flow areas for the discharge tip leakage (a): Orifice opening area; (b): Curved flow area

The effective area of the flow will depend upon the angular position of the vane, discharge port diameter, cylinder wall diameter, curvature and the thickness of the vane tip. As the vane passes through the discharge port, an evolution of the curved flow area for an arbitrary compressor dimension of $R_c = 27.5$ mm, $R_r = 15.5$ mm, $b = 12.5$ mm and discharge port diameter of 6 mm is figuratively illustrated in Figure 4.33 (2° position to 12° position). It takes roughly about 13° of rotor angle for the vane to sweep the leading edge of the port and the trailing edge of the port. This implies that the 0° position is where vane tip coincides with the discharge port leading edge shown in Figure 4.31.

Figure 4.33: An evolution of the curved surface area evaluated using Solidworks 2018-2019 (Student version)

The area of the curved flow area for the typical 6 mm diameter circular port is plotted with respect to the angular position in Figure 4.34. Correspondingly, the relation between the flow area and the angular position can be expressed using equation (4.111).

Figure 4.34: Variation of the curved flow area

$$A_{curved,tip} = 7 \times 10^{-8}x^6 - 7 \times 10^{-6}x^5 + 0.0003x^4 - 0.005x^3 + 0.0441x^2 + 0.0428x - 10^{-4} \quad (4.111)$$

III. Determination of leakage flow coefficient

The discharge tip leakage was simulated using CFD. The results obtained were then used to determine the flow coefficient for the tip leakage through discharge port. The assumptions made for the CFD simulation are as follows:

- Constant pressure, temperature and density across the discharge chamber and the compression chamber
- Adiabatic flow
- Stationary walls
- Air as an ideal gas working fluid

Assuming transient conditions and compressible flow, the shear stress transport model (SST) $k-\omega$ turbulence model was used. The time step taken to simulate the

transient condition was 5.57×10^{-7} s. ANSYS Workbench 19.1 software with its meshing and Fluent sub-components was used for simulation.

An arbitrary compressor dimension of cylinder diameter $R_c = 65$ mm, rotor diameter $R_r = 40.5$ mm, vane thickness $t_{vn} = 10$ mm and three discharge ports each with diameter = 8mm was chosen for the simulation. The mesh information and the boundary conditions used are shown in Figure 4.35.

Figure 4.35: Mesh information and boundary conditions for tip leakage simulation
The streamline of the velocity of the flow was visualised for a typical operating pressure ratio of 5 is shown in Figure 4.36.

Figure 4.36: Visualisation of velocity streamlines for the tip leakage

After the comparison of the flowrates predicted by the analytical model vs the CFD simulation was done, it was found that the discharge coefficient of 0.61 was the best fit for the analytical model with the maximum deviation of about 15% for the pressure ratio of 7. Additionally, the curved fit equation (4.111) was used as an effective flow area.

Pressure ratio

○	2.0
□	3.0
◇	4.0
△	5.0
×	6.0
+	7.0

Figure 4.37: Comparison of predicted flowrate using analytical model vs CFD simulation for various pressure ratios and discharge coefficients (C_d)

4.6 Instantaneous in-chamber convective heat transfer model

Assuming the quasi-steady process, the heat transfer rate is proportional to the difference in temperature between the working fluid in the chamber and the bounding walls of the chamber. The instantaneous convective heat transfer coefficient is then represented by equation (4.112).

$$h_{conv} = \frac{\dot{q}}{A(T_{cv} - T_{wall})} \quad (4.112)$$

The heat transfer coefficient, h_{conv} , is determined using equation (4.113).

$$h_{conv} = Nu \frac{k}{D_H} \quad (4.113)$$

' k_f ' is the thermal conductivity of the fluid and ' D_H ' is the hydraulic diameter of the working chamber. Nusselt number, Nu , is obtained based on the empirical correlation proposed by Annand [174]. This correlation is shown in the equation (4.114).

$$Nu = aRe^bPr^c \quad (4.114)$$

Where, Re is Reynolds number, Pr is the Prandtl number, coefficients a and b have values depending upon the vessel containing the fluid. The values of a and b obtained by Annand for the two-stroke engine were 0.76 and 0.64 ± 0.1 ; for four stroke-engine: 0.26 and 0.75 ± 0.15 . The correlation proposed by Adair et al. [175] included $a = 0.053$, $b = 0.8$ and $c = 0.6$. Studies by Tan and Ooi [87] suggested the use of 0.7 for both a and b . Due to the lack of the experimental

data on the heat transfer coefficients for CVC, and in view of the similar flow field of the working fluid in the working chamber, the heat transfer coefficients suggested by Tan and Ooi [87] has been used.

Reynolds number, Re is determined using the equation (4.115).

$$Re = \frac{\rho U D_H}{\mu} \quad (4.115)$$

Figure 4.38: A compression chamber in CVC

As shown in Figure 4.38, the flow of fluid inside the working chamber of CVC is mainly due to the rotating rotor and the vane motion, the average fluid velocity, U , is defined by equation (4.116). Hydraulic diameter, D_h , of the working chamber of the coupled vane compressor is defined as shown in equation (4.117). ' A_{ht} ' is the surface area of the working chamber and ' P_w ' is the wetted perimeter of the working chamber.

$$U = \frac{\omega_r(r(\theta) - R_r)}{2} \quad (4.116)$$

$$D_h = \frac{4A_{ht}}{P_w} \quad (4.117)$$

4.6.1 Heat transfer surface area

The surface area of the control volume where the heat transfer between the control volume and the boundary takes place is presented in equation (4.118). The cross-section of the control volume is shown in Figure 4.17. The control volume is bounded by the curved surfaces of the cylinder wall and rotor, cover on the top and bottom and the vane bounding the control volume.

$$A_{ht}(\theta_r) = \frac{2V(\theta_r)}{l_c} + R_c l_c \Delta\theta_c + R_r l_c \Delta\theta_r + l_{vn,exposed} l_c \quad (4.118)$$

4.7 Simulation results and discussion

The simulation of thermodynamic model including the effects of leakage was performed. The simulation procedure is comprehensively described in Appendix A-2. The working fluid selected was R1234yf, the operating condition was between the evaporating and the condensing temperature of 7.2 °C and 54.4 °C respectively. The inlet temperature assumed was 33 °C. For an arbitrary dimensions of compressor, where, $R_c = 27.5$ mm, $R_r = 15.5$ mm, $b = 13$ mm, $R_{f1} = R_{f2} = 3$ mm, $t_{vn} = 6$ mm and the discharge port diameter of 11 mm, the variation of temperature and pressure for one working cycle (which includes the working fluid through suction, compression and discharge phase) are shown in Figure 4.39 (a) and (b) respectively.

The area under the curve in the Pressure-Volume (P-V) diagram shown in Figure 4.39 (c) is the total indicated power. The total indicated power can be derived using equation (4.40).

$$P_{ind} = \int_0^{540^\circ} p_{cv} \frac{dV(\theta_r)}{d\theta_r} d\theta_r \quad (4.119)$$

The suction loss is the loss associated with the expansion of the fluid below the suction pressure of the compressor. The discharge loss is the loss due to the over-compression of the fluid in the discharge chamber relative to the discharge pressure. The suction loss and the discharge loss can be obtained using equations (4.120) and (4.121).

$$P_{suc} = \int_0^{270^\circ} |p_{suc} - p_{cv}| \frac{dV(\theta_r)}{d\theta_r} d\theta_r \quad (4.120)$$

$$P_{dis} = \int_{\theta_{dis,st}}^{\theta_{dis,end}} |p_{cv} - p_{dis}| \frac{dV(\theta_r)}{d\theta_r} d\theta_r \quad (4.121)$$

Any expansion or the compression of fluid trapped in the vane gap is also considered as the loss of energy. This expansion and the compression loss in the vane gap are derived using equation (4.122).

$$P_{gap} = \int_0^{540^\circ} p_{gap} \frac{dV_{gap}(\theta_r)}{d\theta_r} d\theta_r \quad (4.122)$$

The total indicated power for the P-V diagram shown in Figure 4.39 (c) was found to be 1116 W. The losses were mainly due to the discharge loss, expansion and compression loss in the vane gap and the suction loss. These losses were evaluated to be 27.7 W, 14.3 W and 47.7 W respectively.

(a) Variation of pressure

(b) Variation of temperature

(c) Pressure-volume diagram

(d) Variation of power required for the compressor to complete one revolution

Figure 4.39: Variation of the properties from the thermodynamic model

4.8 Summary

In this chapter, the mathematical modelling of the geometry, thermodynamics, mass flow, valve dynamics, leakage and the instantaneous convective heat transfer using a zero-dimensional approach was presented. Summary of this chapter is as follows:

- The mathematical models for the volume of the working chamber for the suction, compression, and discharge phase was developed.
- The volume model also includes the pocket or gap between the vanes when this pocket is inside the rotor slot. This pocket is treated as a separate working chamber as the fluid leaked into this chamber undergoes compression and expansion. Therefore, the work done to compress or to expand the fluid in this chamber represents the additional energy required to operate the compressor.
- The zero-dimensional thermodynamic model for the working cycle of the compressor using the first law and real gas properties was developed.
- The suction and discharge flow models were developed by incorporating the varying flow areas.
- The valve dynamics model for the valve of varying cross-sectional width was developed.
- Three main leakage model was discussed and developed. The leakage through the sealing arc and the leakage through the vane endface clearances were modelled assuming compressible flow with friction (Fanno flow). The third leakage model, which is the vane tip leakage through the discharge port was modelled assuming the isentropic flow through the orifice. CFD simulation was used to validate the analytical model for the flow through the orifice and a coefficient of discharge was evaluated by comparing the flowrates obtained using the analytical model and the CFD simulation. The comparison study showed that the discharge coefficient of 0.61 was the best fit for the analytical model. The same coefficient can also be used in the suction and discharge flow models.
- A simple instantaneous in-chamber convective heat transfer model was also developed to evaluate the heat flow in the working chambers.

Chapter 5: Theoretical model: Kinematics and Dynamics model

In this chapter, the kinetics and dynamics models of CVC are presented. The dynamics model can be used to determine the minimum operating condition in CVC where the vane tips must be in contact with the inner cylinder wall at all time in order to prevent vane chattering and leakage through vane tip-cylinder contact. This will also be used to evaluate the frictional losses in various rubbing components of the compressor.

5.1 Kinematics model

The operational parameters and the geometrical dimensions of CVC have been presented in section 4.1. Figure 5.1 shows a schematic of CVC. During the operation of CVC, the cylinder is stationary while the rotor rotates in its central axis and the vanes rotate along with the rotor while diametrically extending in and out of the slot in the rotor. The two vanes have been named the trailing and the leading vane. While the trailing face of the trailing vane initiates the suction process, the leading face of the leading vane undergoes the compression and/or discharge process. Through an appropriate design consideration, the two vanes are assisted by the combination of the centrifugal force and the gas pressure forces from the chambers to extend radially and form the sealing contact with the inner wall of the cylinder.

Since the rotor is assumed to rotate clockwise at a constant angular speed ω_r about the centre C_r , the angular speed ω_r can be written as in equation (5.1), where θ_r is the rotational angle of the rotor.

$$\omega_r = \frac{d\theta_r}{dt} \quad (5.1)$$

Figure 5.1: Illustration of the components of CVC

The notation $r(\theta_r)$ denotes the radial distance between the vane tip-cylinder contact and C_r . The $r(\theta_r)$ and the rate of change of $r(\theta_r)$ with respect to the rotor angle θ_r are given by equations (5.2) and (5.3).

$$r(\theta_r) = \begin{cases} R_r & (\theta_r \leq \theta_{r,st} \text{ or } 360^\circ - \theta_{r,st} < \theta_r < 360^\circ + \theta_{r,st}) \\ -b \cos \theta_r + \sqrt{-(b \sin \theta_r)^2 + R_c^2} & \end{cases} \quad (5.2)$$

$$\frac{dr(\theta_r)}{d\theta_r} = \begin{cases} 0 & (\theta_r \leq \theta_{r,st} \text{ or } 360^\circ - \theta_{r,st} < \theta_r < 360^\circ + \theta_{r,st}) \\ b \sin \theta_r + \frac{(-b^2 \sin 2\theta_r)}{2\sqrt{-(b \sin \theta_r)^2 + R_c^2}} & \end{cases} \quad (5.3)$$

For the trailing vane, the corresponding vane-tip-cylinder radial distance will be $r(180^\circ + \theta_r)$. The $r(180^\circ + \theta_r)$ and its angular velocity are shown in equations (5.4) and (5.5).

$$r(180^\circ + \theta_r) = \begin{cases} R_r & (180^\circ - \theta_{r,st} \leq \theta_r \leq 180^\circ + \theta_{r,st}) \\ -b \cos(180^\circ + \theta_r) + \sqrt{-(b \sin(180^\circ + \theta_r))^2 + R_c^2} & \end{cases} \quad (5.4)$$

$$\frac{dr(180^\circ + \theta_r)}{d\theta_r} = \begin{cases} 0 & (180^\circ - \theta_{r,st} \leq \theta_r \leq 180^\circ + \theta_{r,st}) \\ b \sin(180^\circ + \theta_r) + \frac{(-b^2 \sin 2(180^\circ + \theta_r))}{2\sqrt{-(b \sin(180^\circ + \theta_r))^2 + R_c^2}} & \end{cases} \quad (5.5)$$

For the compressor dimensions where, $b = 13$ mm, $R_c = 27.5$ mm and $R_r = 15.5$ mm, the variation of $r(\theta_r)$ and $r(180^\circ + \theta_r)$ are shown in the Figure 5.2. The result is shown for 180° only as the compressor is symmetrical about its mid-plane containing the two centres C_r and C_c (Figure 5.1). This implies that after 180°

rotor angle, leading vane assumes the position of trailing vane at the 0° and vice-versa.

The derivatives shown in equations (5.3) and (5.5) are with respect to the rotor angle θ_r . Their time derivatives, by employing chain rule can be rewritten as equations (5.6) and (5.7).

$$\frac{dr(\theta_r)}{dt} = \frac{dr(\theta_r)}{d\theta_r} \frac{d\theta_r}{dt} \quad (5.6)$$

$$\frac{dr(\theta_r - \pi)}{dt} = \frac{dr(\theta_r - \pi)}{d\theta_r} \frac{d\theta_r}{dt} \quad (5.7)$$

Figure 5.3 shows the radial velocity of the leading and the trailing vane. Until about 30° rotor angle which is the half of the sealing arc angle, the trailing vane remains inside the rotor and the vane tip is in contact with the sealing arc, hence there is no radial motion. The same applies to leading vane after 150° rotor angle.

Figure 5.2: Radial lengths over 180° rotor angle

Figure 5.3: Radial speeds over 180° rotor angle

The acceleration of the leading and the trailing vanes in the radial direction can be shown as equations (5.8) and (5.9).

$$\begin{aligned} & \frac{d^2 r(\theta_r)}{d\theta_r^2} \\ &= \begin{cases} 0 & (\theta_r \leq \theta_{r,st} \text{ or } 360^\circ - \theta_{r,st} < \theta_r < 360^\circ + \theta_{r,st}) \\ b \cos \theta_r + \frac{(-4b^2 \cos 2\theta_r (R_c^2 - (b \sin \theta_r)^2)) - (b^2 \sin 2\theta_r)^2}{4(\sqrt{-(b \sin \theta_r)^2 + R_c^2})^3} & (\text{else}) \end{cases} \quad (5.8) \end{aligned}$$

$$\frac{d^2r(180^\circ + \theta_r)}{d\theta_r^2} = \begin{cases} 0 & (180^\circ - \theta_{r,st} \leq \theta_r \leq 180^\circ + \theta_{r,st}) \\ b \cos(180^\circ + \theta_r) + \\ (-4b^2 \cos 2(180^\circ + \theta_r)(R_c^2 - (b \sin(180^\circ + \theta_r))^2)) \\ - (b^2 \sin 2(180^\circ + \theta_r))^2 \\ \frac{}{4(\sqrt{-(b \sin(180^\circ + \theta_r))^2 + R_c^2})^3} \end{cases} \quad (5.9)$$

5.1.1 Contact point and angle on vane tip

As shown in Figure 5.4, the point of contact P' between the vane tip and the internal wall of the cylinder differs from the assumed point of contact P which is the intersection of the line represented by $r(\theta_r)$ and the internal wall of the cylinder. Since the vane tip is curved and the line normal to the tangent at the point of contact P' between the vane tip and the cylinder circle passes through the geometrical centre of both the vane tip C_{vn} and the cylinder C_c . The normal line P'C_{vn} forms an angle α with PC_{vn}. This angle is referred to as the contact angle for the trailing vane. Similarly, for the leading vane, the normal line at the leading vane forms an angle β with the radial line represented by the radial function $r(180^\circ + \theta_r)$.

Figure 5.4: Broken out view of the vane tip contact point at the cylinder inner wall. The tangent line passing through the contact point P' at various rotor positions are shown as red lines in Figure 5.5. Assuming the positive x-y axis as shown in Figure 5.5, we observe that the slopes of tangents at the contact points are negative and increases for the range of rotor angles of $\theta_r = 0$ to θ_1 and 180° to θ_2 ; meanwhile for the rotor angle ranges θ_1 to 180° and θ_2 to 360° , the slopes of the

tangents are positive and decreasing; as shown in Figure 5.5. The two specific angles θ_1 and θ_2 can be determined using equations (5.10) and (5.11).

$$\theta_1 = \pi - \sin^{-1}\left(\frac{R_c}{\sqrt{R_c^2 + b^2}}\right) \quad (5.10)$$

$$\theta_2 = \pi + \sin^{-1}\left(\frac{R_c}{\sqrt{R_c^2 + b^2}}\right) \quad (5.11)$$

Figure 5.5: Contact point limiting condition

In a coordinate system as shown in Figure 5.5, the origin is at C_R , $\theta_r = 0$ as positive x-axis and $\theta_r = 180^\circ$ as negative x-axis and $\theta_r = 90^\circ$ as positive y-axis, the equation of a circle which is the inner wall of the cylinder can be defined by equation (5.12).

$$(x + b)^2 + y^2 = (R_c)^2 \quad (5.12)$$

Taking the derivative of equation (5.12) with respect to x and determining the slope of the tangent which is $\frac{dy}{dx}$ as shown in equation (5.13).

$$\frac{dy}{dx} = -\frac{x + b}{2y} \quad (5.13)$$

Substituting $x = r\cos(\theta_r)$ and $y = r\sin(\theta_r)$ in equation (5.13), we obtain the slope of the tangent line at the point of contact. This slope can then be used to obtain

the slope angle of the tangent line θ_{tan} with respect to the positive x-axis (illustrated in Figure 5.5) as shown in equation (5.14).

$$\theta_{tan} = \begin{cases} \frac{\pi}{2} & \text{for } \theta_r = 0, \pi \\ 0 & \text{for } \theta_r = \theta_1, \theta_2 \\ \pi - \tan^{-1} \left(\left| \frac{rcos(\theta_r) + b}{2rsin(\theta_r)} \right| \right) & \text{for } \theta_r = (0, \theta_1) \text{ and } (\pi, \theta_2) \\ \tan^{-1} \left(\left| \frac{rcos(\theta_r) + b}{2rsin(\theta_r)} \right| \right) & \text{for } \theta_r = (\theta_1, \pi) \text{ and } (\theta_2, 2\pi) \end{cases} \quad (5.14)$$

Tangent line XY, positive x-axis containing XC_R and the line C_RP represented by $r(\theta_r)$ forms a triangle XPC_R as shown in Figure 5.6. Since the normal line is orthogonal to the tangent line, the external angle QPY to the triangle XPC_R is 90° . The external angle QPY can be obtained using the law for an external angle of a triangle as the sum of two opposite internal angles within that triangle.

Figure 5.6: Illustration of rotor angle, contact angle and the tangent line angle

Thus, the contact angle α is represented by equation (5.15).

$$\alpha = \begin{cases} 0 & \text{for } \theta_r = 0 \text{ and } 180^\circ \\ \theta_1 - \frac{\pi}{2} & \text{for } \theta_r = \theta_1 \\ \frac{3\pi}{2} - \theta_2 & \text{for } \theta_r = \theta_2 \\ \frac{\pi}{2} + \theta_r - \theta_{tan} & \text{for } \theta_r = (0, \theta_1) \\ -\frac{\pi}{2} + \theta_r - \theta_{tan} & \text{for } \theta_r = (\theta_1, 180^\circ) \\ \frac{\pi}{2} - \theta_r + \theta_{tan} & \text{for } \theta_r = (180^\circ, \theta_2) \\ -\frac{\pi}{2} - \theta_r + \theta_{tan} & \text{for } \theta_r = (\theta_2, 360^\circ) \end{cases} \quad (5.15)$$

For the compressor dimensions, where $b = 13$ mm, $R_c = 27.5$ mm and $R_r = 15.5$ mm and $R_{f1} = R_{f2} = 3$ mm, the contact angles for the trailing vane (α) and the leading vane (β) can be plotted as shown in Figure 5.7.

Figure 5.7: Contact angles with respect to rotor angle

5.1.2 Centre of mass of the vane

Figure 5.8 (a) and (b) show the design and subcomponents of the vanes and (c) and (d) show the cross-sectional area of the vane across the X-Y plane. The body forces including the gravitational force and the rotational forces such as the centrifugal force and the coriolis force can be assumed to act at the point of the centre of mass (P_{CM}). The vane is symmetrical along the mid-plane of its z-axis.

Figure 5.8: (a) and (b) Vane design with dovetail feature, (c) and (d) Illustration of x-y plane and location of centre of mass for the dovetail vane with keyway

(female) on the left and the vane with key (male) on the right

The vane section projected in the x-y plane in Figure 5.8 can be broken down to simpler geometries such as rectangles and the semi-circles. And each of these geometries will have their local centres of mass. The cumulative centre of mass in the x-y plane can be written as shown in equation (5.16), (5.17) and (5.18), where, i represents the individual sub-component and n represents the total number of sub-components

$$x_{CM} = \frac{\sum_{i=1}^n x_i V_i}{\sum_{i=1}^n V_i} \quad (5.16)$$

$$y_{CM} = \frac{\sum_{i=1}^n y_i V_i}{\sum_{i=1}^n V_i} \quad (5.17)$$

$$z_{CM} = \frac{l_z}{2} \quad (5.18)$$

5.2 Dynamics model for the vane

The forces acting on a vane can be analysed using a free body diagram. The various forces acting on the free-body of the trailing vane and the leading vane at any arbitrary rotor angular position θ_r are shown in Figure 5.9 and Figure 5.10.

Generally, in CVC, there are three working chambers, namely suction, compression and the discharge chamber. The gas pressure in these working chambers are labelled, p_1 for suction chamber pressure, p_2 for compression chamber pressure acting on the trailing vane side, p_3 which is equal to p_2 but acting on the leading vane side and p_4 for the discharge chamber pressure. The corresponding pressure forces are labelled F_{p1} , F_{p2} , F_{p3} and F_{p4} and they are assumed to act on the mid-point of the side of the vane exposed to the pressure. There are also the pressure forces acting on the curved surface of the vane tip. These pressure forces at the vane tip are labelled $F_{tp, p1}$, $F_{tp, p2}$ for the trailing vane tip and $F_{tp, p3}$ and $F_{tp, p4}$ for the leading vane tip. Finally, there are pressure forces acting on the neck of the vane (Figure 5.8 (a) and (b)). These pressure forces are labelled $F_{H, p2}$ and $F_{H, p4}$.

From Figure 5.9 and Figure 5.10, it is noted that, for CVC to compress the gas without leaking the gas through the vane tip, the vane tip needs to be in sealing contact with the inner cylinder wall. Failure to do so at the leading vane tip would

mean the gas in the discharge chamber will leak to the trailing compression chamber. To ensure the vane tip remains in sealing contact throughout the operation of the compressor, firstly, the vane tip pressure forces $F_{tp, p1}$, $F_{tp, p2}$, $F_{tp, p3}$ and $F_{tp, p4}$ need to be smaller than the vane neck pressure forces $F_{H, p2}$ and $F_{H, p4}$. At the trailing vane tip, trailing side pressure p_1 is always less than or equal to leading side pressure p_2 . This implies that the curved tip area facing the leading side should be smaller whenever possible than the curved tip area facing the trailing side. Similarly, at the leading vane tip, trailing side pressure p_3 is always less than leading side pressure p_4 . Again, this implies that to minimize the vane tip pressure forces, leading tip curved surface area should be significantly smaller than the trailing side curved surface area. Finally, the vane neck pressure forces $F_{H, p2}$ and $F_{H, p4}$ should be allowed to be as large as appropriate to ensure that the vane tip remains in contact with the cylinder wall.

The rotation of the trailing vane and the leading vane about the rotor centre C_r causes two additional rotational forces to exist for each vane. The first one is the centrifugal force (F_{cen}). And the second one is the coriolis force (F_{cor}) which is due to both rotation and translation of the vanes with respect to C_r . The trailing vane is denoted with subscript '1' and the leading vane is '2'. It is also noted that the F_{cen} assists the vane to radially extend out of the rotor slot and towards the inner cylinder wall. Therefore, it is important that vane centre of mass needs to be designed such that the direction of the centrifugal force is always towards the direction which forces the vane tip to form the sealing contact with the inner cylinder wall.

To this end, the forces acting on the vane discussed so far in section 5.2 are termed as 'known' forces. The pressure forces are computed using the chamber pressures from the thermodynamics model and the respective force acting areas. The rotational forces are determined using the vane mass, its centre of mass and the operating speed. The 'unknown' forces are the contact forces between the vane and other components. They are evaluated by assuming static and dynamic equilibrium at any moment of time.

The rotor provides the rotating force, $F_{N,rot}$, which is the main driving force for the compressor. The vane rotation is assumed to be proportional to the frictional forces, $F_{f,rot}$ and $F_{f,vn}$.

Figure 5.9: Free body diagram illustrating the forces acting on the trailing vane

Figure 5.10: Free body diagram of the leading vane

The reaction forces on the vane tip from the inner cylinder wall which are labelled $F_{tp,1}$ and $F_{tp,2}$. Each of these reaction forces is resolved into two normal reaction force ($F_{N,tp,1}$ and $F_{N,tp,2}$) and the tangential reaction force ($F_{T,tp,1}$ and $F_{T,tp,2}$) at

the point of contact. Additionally, $\mathbf{F}_{tp,1}$ and $\mathbf{F}_{tp,2}$ can be resolved into two components in the axis parallel to the $\mathbf{r}(\theta_r)$ and the axis perpendicular to the $\mathbf{r}(\theta_r)$. These two forces are labelled $\mathbf{F}_{//,1}$ and $\mathbf{F}_{\perp,1}$ for the trailing vane and $\mathbf{F}_{//,2}$ and $\mathbf{F}_{\perp,2}$ for the leading vane respectively. It can be inferred that to ascertain that the vane tip is indeed in sliding contact with the cylinder wall, the forces $\mathbf{F}_{//,1}$ and $\mathbf{F}_{//,2}$ must be positive throughout the operation cycle of the compressor.

5.2.1 Calculation of the pressure and the body forces

The magnitude of pressure force along the length of vane exposed can be obtained in the equations (5.19), (5.20), (5.21) and (5.22).

$$F_{p1} = p_1(\theta_r) \times \left[\left(r(\theta_r) - R_r - \frac{w_{vn}}{2} \right) \times l_c \right] \quad (5.19)$$

$$F_{p2} = p_2(\theta_r) \times \left[\left(r(\theta_r) - R_r - \frac{w_{vn}}{2} \right) \times l_c \right] \quad (5.20)$$

$$F_{p3} = p_3(\theta_r) \times \left[\left(r(\theta_r + \pi) - R_r - \frac{w_{vn}}{2} \right) \times l_c \right] \quad (5.21)$$

$$F_{p4} = p_4(\theta_r) \times \left[\left(r(\theta_r + \pi) - R_r - \frac{w_{vn}}{2} \right) \times l_c \right] \quad (5.22)$$

The magnitude of the pressure forces at the vane tip are shown in equations (5.23), (5.24), (5.25) and (5.26). The projected vane tip area is the product of the length of the chord and the axial length of the vane. The chord length is determined using the cosine rule (see Figure 5.6).

$$F_{tp,p1} = p_1(\theta_r) \times \left[\sqrt{\frac{w_{vn}^2}{2} \left(1 - \cos \left(\frac{\pi}{2} + \alpha \right) \right)} \times l_c \times \sin \left(\frac{\pi}{2} + \alpha \right) \right] \quad (5.23)$$

$$F_{tp,p2} = p_2(\theta_r) \times \left[\sqrt{\frac{w_{vn}^2}{2} \left(1 - \cos \left(\frac{\pi}{2} - \alpha \right) \right)} \times l_c \times \sin \left(\frac{\pi}{2} - \alpha \right) \right] \quad (5.24)$$

$$F_{tp,p3} = p_3(\theta_r) \times \left[\sqrt{\frac{w_{vn}^2}{2} \left(1 - \cos \left(\frac{\pi}{2} - \beta \right) \right)} \times l_c \times \sin \left(\frac{\pi}{2} - \beta \right) \right] \quad (5.25)$$

$$F_{tp,p4} = p_4(\theta_r) \times \left[\sqrt{\frac{w_{vn}^2}{2} \left(1 - \cos \left(\frac{\pi}{2} + \beta \right) \right)} \times l_c \times \sin \left(\frac{\pi}{2} + \beta \right) \right] \quad (5.26)$$

At the vane-neck facing the compression chamber, the magnitude of the pressure force is shown in equations (5.27) and (5.28).

$$F_{H,p2} = p_{h,p2}(\theta_r) \times \left[\frac{w_{vn}}{2} \times l_c \right] \quad (5.27)$$

$$F_{H,p4} = p_{h,p4}(\theta_r) \times \left[\frac{w_{vn}}{2} \times l_c \right] \quad (5.28)$$

The variation of the respective pressure forces are illustrated in Figure 5.11 (c) – (f) for the variation of the working chamber pressure shown in Figure 5.11 (b). The working chamber pressure is evaluated using the thermodynamic model for CVC operating R1234yf as the working fluid at 3000 r min⁻¹. Some arbitrary dimension used for the evaluation of the pressure forces are: $R_c = 27.5$ mm, $R_r = 15.5$ mm, $w_{vn} = 6$ mm and $l_c = 30$ mm.

(a) Working chamber pressures

(b) Variation of working chamber pressures

(c) Variation of pressure forces

(d) Variation of pressure forces at the trailing vane tip

(e) Variation of pressure forces at the leading vane tip

(f) Variation of the pressure forces at the hook space of the trailing and the leading vane

Figure 5.11: Variation of pressure forces at different cross-sections

Assuming the vane tip slides against the cylinder inner wall, the sliding accelerations of the trailing and leading vane, $a_{vn,1}$ and $a_{vn,2}$, are given by the second derivative of the radial function shown in equations (5.8) and (5.9) respectively. The radial body force on the vanes due to the centrifugal and sliding acceleration of the vane is obtained using equations (5.29) and (5.30).

$$F_{r,1} = m_{vn,1} \times [\omega^2 r(\theta_r) - a_{vn,1}] \quad (5.29)$$

$$F_{r,2} = m_{vn,2} \times [\omega^2 r(\theta_r + \pi) - a_{vn,2}] \quad (5.30)$$

The vane sliding velocities of the leading and the trailing vane, $v_{vn,1}$ and $v_{vn,2}$ are given in equations (5.6) and (5.7). The magnitude of the coriolis body force acting on the vane is shown in equations (5.31) and (5.32).

$$F_{cor,1} = m_{vn,1} \times [v_{vn,1}\omega] \quad (5.31)$$

$$F_{cor,2} = m_{vn,2} \times [v_{vn,2}\omega] \quad (5.32)$$

The radial displacement of the centre of mass of vane with respect to the rotor centre is shown in equations (5.33) and (5.34).

$$r_{CM,1} = x_{CM,1} - (l_{vn} - r(\theta_r)) \quad (5.33)$$

$$r_{CM,2} = x_{CM,2} - (l_{vn} - r(\theta_r - \pi)) \quad (5.34)$$

When $R_c = 27.5$ mm, $R_r = 15.5$ mm, $l_{vn} = 33$ mm, $l_{vn,tp} = 8.2$ mm, $w_{vn} = 6$ mm, $l_c = 30$ mm, $\rho_{vn} = 7800$ kg m⁻³ and operating speed at 3000 r min⁻¹, the vane mass was 29.7 g. The variation of the centrifugal and the coriolis forces are shown in Figure 5.12 (a) and (b).

(a) Variation of the centrifugal force

(b) Variation of the coriolis force

Figure 5.12: Variation of the centrifugal and the coriolis force on the vane

5.2.2 Calculation of the reaction and the frictional forces

The unknown forces include the normal forces $F_{N,rot}$ and $F_{N,vn}$, the frictional forces applied by the rotor and vane $F_{f,rot}$ and $F_{f,vn}$, the reaction forces at the vane tips parallel to the radial line, $F_{//,1}$ and $F_{//,2}$, the reaction forces at the tip perpendicular to the radial line, $F_{\perp,1}$ and $F_{\perp,2}$. Assuming all components of the compressors are made from steel with uniform properties and density of 7800 kg/m³, a mean value of the friction coefficient $\mu_f = 0.15$ was assumed [48, 176-

180]. The trailing vane while rotating along the rotor protrudes out of the rotor for rotor angles 0° to 180° and then back into the rotor for rotor angles 180° to 360° , therefore, the frictional force $\mathbf{F}_{f, rot}$ and $\mathbf{F}_{f, vn}$ follow the direction opposite to the relative vane motion. The reaction force on the vane tip, $\mathbf{F}_{\parallel,1}$ tends to push the trailing vane opposite to its radial motion. The force normal to the reaction force on the leading vane tip $\mathbf{F}_{\perp,1}$ tends to push the vane towards the direction of rotation for rotor angles 0° to 180° , but the same force acts to push against the direction of rotation for rotor angles 180° to 360° .

Using the dynamic force equilibrium on the free body diagram of the trailing vane (as shown in Figure 5.9) along the radial and normal to $\mathbf{r}(\theta_r)$, equations (5.35) and (5.36), can be derived to calculate the unknown forces. Equation (5.35) represents the force balance assuming the forces pointing towards the rotor centre to be positive. Equation (5.36) represents the force balance orthogonal to the radial forces assuming the forces pointing vertically up to be positive.

$$\begin{aligned} & (F_{\parallel,1} + F_{f,vn} + F_{f,rot}) \\ & = F_{r,1} + F_{H,p2} + F_{H,p4} - F_{tp,p1} \sin\left(\frac{\pi - \alpha}{2}\right) - F_{tp,p2} \sin\left(\frac{\pi + \alpha}{2}\right) \end{aligned} \quad (5.35)$$

$$\begin{aligned} & (F_{\perp,1} + F_{N,vn} - F_{N,rot}) \\ & = F_{p1} - F_{p2} + F_{tp,p1} \cos\left(\frac{\pi - \alpha}{2}\right) - F_{tp,p2} \cos\left(\frac{\pi + \alpha}{2}\right) + F_{cor,1} \end{aligned} \quad (5.36)$$

A moment equation (5.37) can be derived at the centre of mass of the trailing vane assuming the anticlockwise rotation of the free body as a positive and clockwise rotation as negative.

$$\begin{aligned} & (-F_{\perp,1} \times \{r(\theta_r) - r_{CM,1}\} + F_{N,vn} \times \{l_{vn} - r(\theta_r) + r_{CM,1}\} \\ & \quad + F_{N,rot} \times \{R_r - r_{CM,1}\}) \\ & = (F_{p2} - F_{p1}) \times \left[R_r + \left\{ \frac{r(\theta_r) - R_r}{2} \right\} - r_{CM,1} \right] \\ & \quad + \left\{ F_{tp,p2} \cos\left(\frac{\pi + \alpha}{2}\right) - F_{tp,p1} \cos\left(\frac{\pi - \alpha}{2}\right) \right\} \\ & \quad \times (r(\theta_r) - R_r - r_{CM,1}) \end{aligned} \quad (5.37)$$

Similarly, the dynamic force equilibrium on the free body diagram (Figure 5.10) of the leading vane, equations (5.38) and (5.39) are derived.

$$\begin{aligned}
& (F_{\parallel,2} - F_{f,vn} - F_{f,rot}) \\
& = F_{r,2} + F_{H,p2} + F_{H,p4} - F_{tp,p3} \sin\left(\frac{\pi + \beta}{2}\right) - F_{tp,p4} \sin\left(\frac{\pi - \beta}{2}\right) \quad (5.38)
\end{aligned}$$

$$\begin{aligned}
& (F_{\perp,2} + F_{N,vn} - F_{N,rot}) \\
& = F_{p3} - F_{p4} + F_{tp,p3} \cos\left(\frac{\pi + \beta}{2}\right) - F_{tp,p4} \cos\left(\frac{\pi - \beta}{2}\right) - F_{cor,2} \quad (5.39)
\end{aligned}$$

Another moment equation (5.40) can be derived at the centre of mass of the leading vane assuming the anticlockwise rotation of the free body as positive and clockwise rotation as negative.

$$\begin{aligned}
& (-F_{\perp,1} \times \{r(\theta_r + \pi) - r_{CM,2}\} + F_{N,vn} \times \{l_{vn} - r(\theta_r + \pi) + r_{CM,2}\} \\
& + F_{N,rot} \times \{R_r - r_{CM,2}\}) \\
& = (F_{p4} - F_{p3}) \times \left[R_r + \left\{ \frac{r(\theta_r + \pi) - R_r}{2} \right\} - r_{CM,2} \right] \\
& + \left\{ F_{tp,p4} \cos\left(\frac{\pi - \beta}{2}\right) - F_{tp,p3} \cos\left(\frac{\pi + \beta}{2}\right) \right\} \\
& \times (r(\theta_r + \pi) - R_r - r_{CM,2}) \quad (5.40)
\end{aligned}$$

By solving 6 simultaneous equations (5.35), (5.36), (5.37), (5.38), (5.39) and (5.40), we can calculate the unknown reaction forces acting the coupled vanes: the radial ($F_{\parallel,1}$ and $F_{\parallel,2}$) and the normal reaction forces on vane tips ($F_{L,1}$ and $F_{L,2}$) and the vane rotating force $F_{N,rot}$ and $F_{N,vn}$. The corresponding magnitudes of frictional forces, $F_{f,rot}$ and $F_{f,vn}$ can then be determined using equations (5.41) and (5.42).

$$F_{f,rot} = \mu_f F_{N,rot} \quad (5.41)$$

$$F_{f,vn} = \mu_f F_{N,vn} \quad (5.42)$$

At the tip of trailing vane, the normal force $F_{N,tp,1}$, forms an angle α with the radial reaction force $F_{\parallel,1}$. Correspondingly, at the vane tip of the leading vane, the normal force $F_{N,tp,1}$, forms an angle β with radial reaction force $F_{\parallel,2}$. The resultant force $F_{tp,1}$ forms an angle γ_1 with the radial reaction force. This is illustrated in Figure 5.13.

Figure 5.13: Illustration of resultant tip force and its components

The resultant force at the tip of trailing vane $\mathbf{F}_{tp, 1}$ and the angle γ_1 can be obtained as shown in equations (5.43) and (5.44).

$$F_{tp,1} = \sqrt{F_{\parallel,1}^2 + F_{\perp,1}^2} \quad (5.43)$$

$$\gamma_1 = \tan^{-1} \left(\frac{F_{\perp,1}}{F_{\parallel,1}} \right) \quad (5.44)$$

The resultant force at the tip of leading vane $\mathbf{F}_{tp, 1}$ and the angle γ_2 can be obtained using equations (5.45) and (5.46).

$$F_{tp,2} = \sqrt{F_{\parallel,2}^2 + F_{\perp,2}^2} \quad (5.45)$$

$$\gamma_2 = \tan^{-1} \left(\frac{F_{\perp,2}}{F_{\parallel,2}} \right) \quad (5.46)$$

The resultant force normal to the contact point, $\mathbf{F}_{N, tp, 1}$ and $\mathbf{F}_{N, tp, 2}$ are determined using equations (5.47) and (5.48).

$$F_{N,tp,1} = F_{tp,1} \cos(\gamma_1 - \alpha) \quad (5.47)$$

$$F_{N,tp,2} = F_{tp,2} \cos(\gamma_2 - \beta) \quad (5.48)$$

Equations (5.49) and (5.50) are the resultant force tangential to the contact point, $\mathbf{F}_{T, tp, 1}$ and $\mathbf{F}_{T, tp, 2}$.

$$F_{T,tp,1} = F_{tp,1} \sin(\gamma_1 - \alpha) \quad (5.49)$$

$$F_{T,tp,2} = F_{tp,2} \sin(\gamma_2 - \beta) \quad (5.50)$$

The magnitude of the frictional forces at the tip of the leading vane and the trailing vane can be obtained using equations (5.51) and (5.52).

$$F_{f,tp,1} = \mu_f F_{tp,1} \cos(\gamma_1 - \alpha) \quad (5.51)$$

$$F_{f,tp,2} = \mu_f F_{tp,2} \cos(\gamma_2 - \beta) \quad (5.52)$$

For $R_c = 27.5$ mm, $R_r = 15.5$ mm, $w_{vn} = 6$ mm, $h_{vn} = 30$ mm, operating speed at 3000 r min^{-1} and the varying gas pressures in the working chambers as shown in Figure 5.11 (b), the variation of the dynamic forces acting on the vanes can be plotted as shown in Figure 5.14 (a) – (e). In figures (a) – (e), a discontinuity or the sudden change of the curve is common at 85° . This is because of the variation of vane pocket pressures $p_{h,p2}$ and $p_{h,p4}$ in equations (5.27) and (5.28). The variation of $p_{h,p2}$ and $p_{h,p4}$ is also demonstrated in Figure 5.11 (f). In this Figure 5.11 (f), $p_{h,p2}$ has a discontinuity at 85° which is because from 0° to 85° , this gas pressure is undergoing compression faster than compression chamber pressure p_2 . After 85° , this $p_{h,p2}$ becomes equal to the compression chamber pressure p_2 . Therefore, this sudden change in $p_{h,p2}$ has a knock-on effect on the dynamic forces as well.

(a) Variation of $F_{//,1}$ and $F_{//,2}$ forces

(b) Variation of $F_{\perp,1}$ and $F_{\perp,2}$ forces

(c) Variation of $F_{N, tp, 1}$ and $F_{N, tp, 2}$ forces

(d) Variation of $F_{T, tp, 1}$ and $F_{T, tp, 2}$ forces

(e) Variation of the frictional force at the tip

Figure 5.14: Variation of dynamic forces for half revolutions (180°)

5.3 Dynamics model for the rotor

Figure 5.15: Chamber pressure forces acting on the rotor

Figure 5.15 demonstrates various forces acting on the rotor of CVC. Assuming two vane tips are always in contact with the cylinder wall, dynamic force acting on the rotor is determined from the resultant of gas pressure forces $F_{R,p1}$, $F_{R,p2}$ and $F_{R,p4}$. Besides gas pressure at the rotor circumference, the rotor also experiences shearing of fluid at the sealing arc and endfaces of the rotor.

5.3.1 Gas pressure forces

For the evaluation of the gas pressure forces, firstly, it is observed that the resultant gas pressure forces $F_{R,p1}$, $F_{R,p2}$ and $F_{R,p4}$ act on the lines AB, AC and BC respectively. Secondly, it is assumed that uniform pressure p_1 acts at the line AB and p_4 acts at the line BC.

These pressure forces for the rotor angle range of 0° to 180° can then be determined using equations (5.53), (5.54) and (5.55).

$$F_{R,p1} = p_1(\theta_r) \times [l_{p1} \times l_c] \quad (5.53)$$

$$F_{R,p2} = p_2(\theta_r) \times [l_{p2} \times l_c] \quad (5.54)$$

$$F_{R,p4} = p_4(\theta_r) \times [l_{p4} \times l_c] \quad (5.55)$$

The respective lengths l_{p1} , l_{p2} and l_{p4} can be determined as shown in equation (5.56), (5.57) and (5.58) using the cosine laws for triangles ABC_r and BCC_r (see Figure 5.15).

$$l_{p1} = \sqrt{\{r(\theta_r)\}^2 + R_r^2 - 2r(\theta_r)R_r \cos \theta_r} \quad (5.56)$$

$$l_{p2} = r(\theta_r) + r(180^\circ + \theta_r) \quad (5.57)$$

$$l_{p4} = \sqrt{\{r(180^\circ + \theta_r)\}^2 + R_r^2 - 2r(180^\circ + \theta_r)R_r \cos(180^\circ - \theta_r)} \quad (5.58)$$

The resultant of the gas pressure forces $F_{R,p1}$, $F_{R,p2}$ and $F_{R,p4}$ can be determined with respect to the **x-y** coordinate shown in Figure 5.15. For some arbitrary parameters selected for CVC, where, $R_c = 27.5$ mm, $R_r = 15.5$ mm, R1234fy as the working fluid and the operating speed at 3000 r min^{-1} , the resultant rotor load is shown in Figure 5.16.

Figure 5.16: Variation of the resultant of the gas pressure force on the rotor

5.3.2 Endface friction

Figure 5.17: Illustration of the rotor endfaces

The lubrication fluid can be assumed to fill the clearance gap between the rotor endface and the cover of CVC shown in Figure 5.17. Assuming incompressible flow with a viscosity of the lubricating fluid (μ_{oil}) at an assumed temperature, and a constant clearance gap $\delta_{r,enf}$ at the endface, the flow can be assumed to be under shear stress due to the rotating rotor and the stationary cover. It is also assumed that the effect of the differential pressure across the rotor gap is negligible compared to the shear stress. Thus, the fluid shear stress assuming the Couette flow model can be derived using equation (5.59).

$$\vec{\tau}_{enf} = \frac{\mu_{oil}}{\delta_{r,enf}} \Delta \vec{v}_{enf} \quad (5.59)$$

$\Delta \vec{v}$ is the relative velocity across the control volume which is determined as shown in the equation (5.60).

$$\Delta \vec{v}_{enf} = \vec{\omega}_r \times \vec{r} \quad (5.60)$$

The resultant frictional force and the torque can be derived using equations (5.61) and (5.62).

$$\vec{F}_{r,enf} = \frac{\mu_{oil}}{\delta_{r,enf}} \int_A \Delta \vec{v} dA \quad (5.61)$$

$$\vec{\mathcal{T}}_{r,enf} = \frac{\mu_{oil}}{\delta_{r,enf}} \int_A (\Delta \vec{v} \times \vec{r}) dA \quad (5.62)$$

The directions of the frictional force and the torque required to overcome fluid friction are opposite to the rotor motion. Using equation (5.60) and $dA = r \cdot dr \cdot d\theta$, equations (5.61) and (5.62) can be solved to obtain equations (5.63) and (5.64).

$$F_{r,enf} = \frac{\mu_{oil}}{\delta_{r,enf}} \int_0^{2\pi} \int_{R_{sh}}^{R_r} (\omega_r r) r dr d\theta \quad (5.63)$$

$$T_{r,enf} = \frac{\mu_{oil}}{\delta_{r,enf}} \int_0^{2\pi} \int_{R_{sh}}^{R_r} (\omega_r r^2) r dr d\theta \quad (5.64)$$

Equations (5.63) and (5.64) can then be integrated to obtain equations (5.65) and (5.66).

$$F_{r,enf} = \frac{\mu_{oil}}{\delta_{r,enf}} \frac{2\pi\omega_r(R_r^3 - R_{sh}^3)}{3} \quad (5.65)$$

$$T_{r,enf} = \frac{\mu_{oil}}{\delta_{r,enf}} \frac{\pi\omega_r(R_r^4 - R_{sh}^4)}{2} \quad (5.66)$$

From equations (5.65) and (5.66), it is observed that the frictional force in the rotor endface for CVC depends on the rotor size, lubricant viscosity and the clearance gap. Even though the frictional force is larger for smaller clearance gaps, smaller clearance gaps are preferred to reduce working fluid leakage and the axial vibration of the shaft.

5.3.3 Friction in the sealing arc

(a) Illustration of the sealing arc

(b) Assumption for the flow control volume at the sealing arc

Figure 5.18: Illustration of the sealing arc clearance

It is assumed that the sealing arc in CVC (see Figure 5.18 (a) and (b)) is filled with the lubricating oil. The fluid friction model in the sealing arc clearance is derived in a similar way to the endface friction model described in section 5.3.2. Assuming incompressible flow, constant viscosity of the lubricating fluid and constant area of flow, the shear stress using the Couette flow can be derived as shown in equation (5.67).

$$\vec{\tau}_{sa} = \mu_{oil} \frac{du}{dy} \quad (5.67)$$

As the pressure decreases in the direction of the flow, the pressure gradient dp/dx is negative and the pressure drop can be assumed to be equal to the chamber pressure across the two ends (equation (5.68)). For the Couette flow, the relative strain rate of the fluid du/dy between the rotating rotor and the stationary cylinder is given by equation (5.69).

$$\frac{p_4 - p_1}{L_{r,sa}} = -\frac{dp}{dx} \quad (5.68)$$

$$\frac{du}{dy} = \frac{1}{\mu_{oil}} \frac{dp}{dx} y + \left[\frac{\omega_r R_r}{\delta_{r,sa}} - \frac{\delta_{r,sa}}{2\mu_{oil}} \frac{dp}{dx} \right] \quad (5.69)$$

The resultant frictional force and the torque is derived using equation (5.70) and (5.71).

$$\vec{F}_{r,sa} = \mu_{oil} \int_{-L_{r,sa}/2}^{L_{r,sa}/2} \int_0^{\delta_{r,sa}} \frac{du}{dy} dy dx \quad (5.70)$$

$$\vec{T}_{r,sa} = \mu_{oil} \int_{-L_{r,sa}/2}^{L_{r,sa}/2} \int_0^{\delta_{r,sa}} y \frac{du}{dy} dy dx \quad (5.71)$$

Equations (5.70) and (5.71) can be solved to obtain the equations (5.72) and (5.73).

$$\vec{F}_{r,sa} = L_{r,sa} \left[\frac{\delta_{r,sa}^2}{2} \frac{dp}{dx} + \mu_{oil} \delta_{r,sa} \left(\frac{\omega_r R_r}{\delta_{r,sa}} - \frac{\delta_{r,sa}}{2\mu_{oil}} \frac{dp}{dx} \right) \right] \quad (5.72)$$

$$\vec{T}_{r,sa} = L_{r,sa} \left[\frac{\delta_{r,sa}^3}{3} \frac{dp}{dx} + \frac{\mu_{oil} \delta_{r,sa}}{2} \left(\omega_r R_r - \frac{\delta_{r,sa}^2}{2\mu_{oil}} \frac{dp}{dx} \right) \right] \quad (5.73)$$

5.4 Journal bearing design

Figure 5.19: Illustration of a hydrodynamically lubricated journal bearing

A hydrodynamically lubricated journal bearing such as the one illustrated in the Figure 5.19 are extensively used in rotary devices because of their slow wear, minimum energy consumption and good damping characteristics.

In Figure 5.19, the shaft of radius R_j is supported by the bearing with internal radius R_b and length l_b . We can assume the steady state condition where the shaft rotates at a constant speed in an axis and the lubricating oil is dragged into the clearance gap to form a thin film of lubricating oil which hydrodynamically supports the shaft. The journal centre C_j and the bearing centre C_b are eccentric and the eccentric distance e_b exists between the centres of the journal and the bearing. Consequently, along with the axis containing the two centres C_b and C_j , there exists a minimum thickness of oil film ' h_{min} '.

Assuming incompressible lubricating oil, the oil is dragged into the converging section of the clearance gap, an oil film pressure develops (mainly) in the converging region of the journal-bearing. For a good design, adequate minimum oil film thickness and large film pressure are critical in ensuring continuous functioning of the journal bearing system. This helps to avoid the seizing of the journal bearing due to potential friction welding. It is important to ensure that the fresh lubricating oil flows into the clearance gap to minimize the frictional heating of the lubricating oil which degrades the viscosity and thus the load carrying capacity of the oil.

The oil film thickness, h_b , within the clearance gap between the journal and bearing can be geometrically derived as shown in equation (5.74).

$$h_b = \delta_b(1 + \varepsilon \cos \theta) \quad (5.74)$$

In the equation (5.74), the radial clearance, δ_b , is defined as the difference between the bearing radius and the journal radius, that is, $\delta_b = R_b - R_j$ and the eccentricity is defined as the ratio of the eccentric distance e_b to the radial clearance, that is $\varepsilon = e_b / \delta_b$. The Reynolds equation governing the thin film fluid flow is shown in equation (5.75).

$$\begin{aligned} \frac{\partial}{\partial \theta} \left\{ (1 + \varepsilon \cos \theta)^3 \frac{\partial p}{\partial \theta} \right\} + R_j^2 \frac{\partial}{\partial z} \left\{ (1 + \varepsilon \cos \theta)^3 \frac{\partial p}{\partial z} \right\} \\ = 12\mu_{oil} \left(\frac{R_j}{\delta_b} \right)^2 \left\{ \dot{\varepsilon} \cos \theta + \varepsilon \left(\dot{\phi} - \frac{\omega}{2} \right) \sin \theta \right\} \end{aligned} \quad (5.75)$$

The equation (5.75) is derived for steady state conditions assuming the viscosity of the fluid μ_{oil} does not change, inertial and body forces are negligible and the pressure variation across the thin film in the axial direction is negligible. The film thickness, h_b , is also assumed to be much smaller compared to the diameter and the length of the bearing.

According to Hirani et al [181], the equation (5.75) has two closed form solutions assuming two limiting cases: The first one is assuming infinitely short bearing approximation where the slenderness ratio defined by $\Lambda = l_b / 2R_b < 0.25$ and Ocvirk's solution, P_o , as shown in equation (5.76) can be used. The second one is assuming infinitely long bearing where the slenderness ratio defined by $\Lambda > 2$ and Sommerfield's solution P_s as shown in equation (5.77) can be used.

$$P_o = \frac{3\mu_{oil}U_j l_{br}^2}{R_j \delta_b} \left[\frac{1}{4} - \left(\frac{z}{l_{br}} \right)^2 \right] \frac{\varepsilon \sin \theta}{(1 + \varepsilon \cos \theta)^3} \quad (5.76)$$

$$P_s = \frac{6\mu_{oil}U_j R_j}{\delta_b^2 (2 + \varepsilon^2)} \left[\frac{\varepsilon \sin \theta (2 + \varepsilon \cos \theta)}{(1 + \varepsilon \cos \theta)^2} \right] \quad (5.77)$$

Most bearings used in practical applications have slenderness ratios defined between $0.25 < \Lambda < 2$. Reason and Narang [182] proposed the approximation based on the harmonic mean of the pressure distribution shown in equations (5.76) and (5.77). To improve the accuracy of the solution, Equation (5.78) was

developed by Hirani et al. [181] who used the pressure correction factors to the harmonic mean pressure distribution.

$$\frac{1}{p(\theta)} = \frac{1 + \varepsilon \Lambda^{1.2} (e^{\varepsilon^5} - 1)}{P_o} + \frac{e^{(1-\varepsilon)^3}}{P_s} \quad (5.78)$$

The film pressure obtained by solving equation (5.78) is then integrated to obtain the total load acting on the bearing surface. Integration of bearing pressure over the region of the thin film, the components of the total load carrying capacity of the bearing, W , are written as shown in equation (5.79) and (5.80).

$$W \cos \varphi = - \int_0^\pi \int_{-l_{br}/2}^{l_{br}/2} R_j p \cos \theta dz d\theta \quad (5.79)$$

$$W \sin \varphi = \int_0^\pi \int_{-l_{br}/2}^{l_{br}/2} R_j p \sin \theta dz d\theta \quad (5.80)$$

The attitude angle, φ is the angle between the load W acting on the bearing and the axis containing the shaft and the bearing centres. This angle is obtained using equation (5.81).

$$\varphi = \tan^{-1} \left(\frac{W \sin \varphi}{W \cos \varphi} \right) \quad (5.81)$$

The shear stress of the oil film is mainly due to shear of the oil film because the journal is rotating in high speed and due to the variation of hydrodynamic film pressure. As shown in the equation (5.82), the fluid friction is the integration of the shear stress over the journal surface.

$$F_{b,f} = \int_A \tau dA = \int_A \left(\frac{\mu_{oil} U_j}{h_b} + \frac{h_b}{2R_j} \frac{\partial p}{\partial \theta} \right) dA \quad (5.82)$$

Three different theoretical assumptions for the solution for equation (5.82) were proposed by Hirani et al. [181]. First is assuming the angular extent of the oil film spanning 2π . The second one assumes the oil film shearing occurs only in the convergent region, that is, the angular extent was limited to π . It was reported that the 2π -extent model overestimated the frictional force and the π -extent model underestimated the frictional force. This is because, in the divergent region, due to cavitation, the oil film is incompletely filled and thus the bearing is partially covered with lubricant over the bearing length. Hence, the third model using π -

extent for the convergent region and the effective length of the fluid film for the divergent region was proposed. The effective length of the fluid film in the divergent region is the sum of the length of the oil stream flows along the axial direction. By solving equation (5.82) using the effective length approximation model, equation (5.83) for the total frictional force is obtained.

$$F_{b,f} = \frac{\pi\mu_{oil}U_j l_b R_j}{\delta_b \sqrt{1-\varepsilon^2}} \left(\frac{2+\varepsilon}{1+\varepsilon} \right) + \frac{\delta_b \varepsilon}{2R_j} W \sin \varphi \quad (5.83)$$

The torque due to the lubricant fluid friction in a bearing is given by equation (5.84).

$$T_{b,f} = F_{b,f} R_j \quad (5.84)$$

The frictional force relates to the energy loss and the fraction of this energy lost due to friction results in the generation of heat which increases the temperature of the lubricant oil and decreases the viscosity of the oil. Hirani et al. [181] used isothermal theory and the energy balance equation where the rate of heat carried by the lubricant flow should be equal to the effective power lost due to heat generated. Then if ρ_{oil} is the density of the lubricant at the inlet, C_{oil} is the specific heat capacity of the lubricant, $Q_{oil, leak}$ is the lubricant flow leaked away from the bearing and σ is the fraction of the total heat carried away by the oil then the effective final temperature of the lubricant, $T_{oil,f}$, after one revolution is shown in the equation (5.85). Observations from the experimental studies by Cole [183] show that the value of σ can be chosen equal to the eccentricity ratio.

$$T_{oil,f} = T_{oil,in} + \frac{\sigma F_{b,f} U_j}{\rho_{oil} C_{oil} Q_{oil,leak}} \quad (5.85)$$

Figure 5.20: Illustration of two bearings to support the rotor

For CVC, it is assumed that two bearings of the same length and same internal diameter. The bearing length assumed is $l_b = 30$ mm, the internal diameter $r_b = 14$

mm and the radial clearance $\delta_b = 10 \mu\text{m}$. For the compressor dimensions, $R_c = 27.5 \text{ mm}$, $R_r = 15.5 \text{ mm}$, $w_{vn} = 6 \text{ mm}$, $h_{vn} = 30 \text{ mm}$, operating speed at 3000 r min^{-1} and the varying gas pressures in the working chambers as shown in Figure 5.11 (b), the variation of the load on the bearing and other parameters are shown in Figure 5.21 (a) – (e).

(a) Variation of the bearing load

(b) Variation of the eccentricity ratio

(c) Variation of the minimum oil film thickness

(d) Variation of the frictional force

(e) Variation of the oil film temperature in the bearing clearance

Figure 5.21: (a) – (e) Variation of the bearing parameters in CVC

5.5 Power loss due to friction

To this end, the frictional forces acting on the components of CVC have been derived. In this section, the power loss due to friction for CVC will be derived. The power loss due to friction at the vane tips can be derived as shown in equations (5.86) and (5.87).

$$P_{tp,1}(\theta_r) = \left| (F_{f,tp,1} \cos \alpha) \times \{\omega_r \cdot r(\theta_r)\} + (F_{f,tp,1} \sin \alpha) \times \left\{ \omega_r \cdot \frac{dr(\theta_r)}{d\theta_r} \right\} \right| \quad (5.86)$$

$$P_{tp,2}(\theta_r) = \left| (F_{f,tp,2} \cos \beta) \times \{\omega_r \cdot r(\theta_r + \pi)\} + (F_{f,tp,2} \sin \beta) \times \left\{ \omega_r \cdot \frac{dr(\theta_r + \pi)}{d\theta_r} \right\} \right| \quad (5.87)$$

The power loss due to the friction between the rotor and the vane and between the two vanes can be derived as shown in equations (5.88) and (5.89).

$$P_{r,vn}(\theta_r) = \left| F_{f,rot} \times \left\{ \omega_r \cdot \frac{dr(\theta_r)}{d\theta_r} \right\} \right| + \left| F_{f,rot} \times \left\{ \omega_r \cdot \frac{dr(\theta_r + \pi)}{d\theta_r} \right\} \right| \quad (5.88)$$

$$P_{vn}(\theta_r) = \left(\left| F_{f,vn} \times \omega_r \cdot \left\{ \frac{dr(\theta_r)}{d\theta_r} - \frac{dr(\theta_r + \pi)}{d\theta_r} \right\} \right| \right) \quad (5.89)$$

The power loss due to the endface friction and the friction in the sealing arc can be derived as shown in equations (5.90) and (5.91).

$$P_{r,enf} = \omega_r \cdot \mathcal{T}_{r,enf} \quad (5.90)$$

$$P_{r,sa} = \omega_r \cdot \mathcal{T}_{r,sa} \quad (5.91)$$

The power loss at the journal bearing can be derived as shown in equation (5.92).

$$P_{b,f}(\theta_r) = \omega_r \cdot \mathcal{T}_{b,f} \quad (5.92)$$

Then, the total power loss in CVC is the sum of all the power losses obtained in equations (5.86) to (5.92). The total power loss is obtained in equation (5.93).

$$P_{f,total}(\theta_r) = P_{tp,1}(\theta_r) + P_{tp,2}(\theta_r) + P_{r,vn}(\theta_r) + P_{vn}(\theta_r) + P_{r,enf}(\theta_r) + P_{r,sa}(\theta_r) + P_{b,f}(\theta_r) \quad (5.93)$$

The corresponding energy loss due to friction is determined using equation (5.94).

$$E_f = \frac{1}{\omega_r} \int_{0^\circ}^{540^\circ} P_{f,total}(\theta_r) d\theta_r \quad (5.94)$$

For a pressure-volume graph shown in Figure 5.22 (a), the area under the curve is the total indicated work which is the total energy required to induce, compress and discharge the gas using a positive displacement compressor such as CVC.

Finally, the mechanical efficiency of the compressor can be derived using the energy lost due to friction and the total indicated work obtained from the pressure-volume curve. The equation derived for the mechanical equation is shown in (5.95).

$$\eta_{mech} = \frac{E_{comp}}{E_{comp} + E_f} \times 100\% \quad (5.95)$$

For some arbitrary dimension of CVC, $R_c = 27.5$ mm, $R_r = 15.5$ mm, $w_{vn} = 6$ mm, $h_{vn} = 30$ mm, operating speed at 3000 r min^{-1} and R1234yf as the working fluid, the total indicated work was evaluated to be 22.5 J, the energy loss due to the frictional losses for one operating cycle was 6.3 J. The discharge and the suction losses were evaluated to be 0.6 J and 1 J respectively. The mechanical efficiency determined was 77%. The P-V diagram and the variation of the mechanical power loss are shown in Figure 5.22 (a) and (b) respectively.

Figure 5.22: Indicator diagram and the power loss variation in CVC

5.6 Parametric studies for the vane dynamics

One of the key requirements for CVC to operate as a compressor is that its vane tips must form sealing contacts with the inner cylinder wall. Various fluid pressure forces acting on the vanes are schematically illustrated in Figure 5.23 (a) and (b). It can be inferred from Figure 5.23 (a) and (b) that, if the vane tips fail to make sealing contact with the inner cylinder wall during the operation, there will be the leakage of fluid along the vane tips which may result in significant communication

of neighbouring chambers and resulting in serious internal leakages and the loss of compression effects.. This condition in CVC arises when the radially inward fluid pressure forces at the vane tip become larger than the sum of the radially outward forces.

In this section, various parameters that influence the centrifugal forces and the fluid pressure forces acting on the vane body are investigated.

Figure 5.23: Illustration of the forces acting on the vane during the operation

5.6.1 Effect of the vane material used

The centrifugal force acting on the vane depends upon three main parameters, namely, the mass of the vane, the relative distance between the centre of mass of the vane and the rotor centre and the operating speed of the compressor. In section 5.6.1, the effect of the vane material selected on the dynamics of the vane is studied.

Table 5.1 includes the vane materials and the respective material densities selected for the simulation.

Table 5.1: Vane materials and their densities

Material	Vane density, ρ_{vn} (kg m^{-3})
Aluminium 7178 alloy	2830

17-7 PH Stainless steel	7850
Haynes® 188 alloy (Cobalt-Nickel-chromium-tungsten alloy)	8980

Some of the arbitrary values selected for the compressor geometry and the operating conditions for the simulation are shown in Table 5.2. R1234yf was selected as the working fluid. The suction and the discharge pressure of 377 kPa and 1470 kPa respectively was used based on the saturation vapour pressures at the evaporating and the condensing temperature of 7.2°C and 54.4°C respectively specified by ASHRAE/T condition for the compressor testing.

Table 5.2: Parameters selected for the simulation studies

R_c	27.5 mm
R_r	15.5 mm
l_c	30 mm
t_{vn}	6 mm
R_{f1}	1 mm
R_{f2}	5 mm
μ_f	0.1
Operating speed	3000 r min ⁻¹

Figure 5.24 (a) and (b) show the results obtained for the variation of the net radial force acting on the trailing vane and the leading vane tip respectively for the various vane materials selected in Table 5.1. The results shown in Figure 5.24 (a) and (b) are for 180° rotor angle only because the forces acting on the vane was periodic every 180° rotation angles. From Figure 5.24 (a) and (b), the predicted results showed that, for the vane materials selected, both the vane tips were pressed against the cylinder inner wall resulting in the positive magnitude of the net radial force acting on the vane tip.

In case of the lighter vanes, as the centrifugal forces acting on the vanes were lower, the total frictional losses at the vane tips were found to be lower. Consequently, as shown in Table 5.3, the mechanical efficiency obtained for CVC with lighter vanes were found to be higher.

In Figure 5.24 (c) and (d), the minimum operating speed required to ensure that the vane tip radially extends out towards the cylinder wall was investigated. It was

found that for Aluminium 7178 vane, the minimum operating speed was 800 r min⁻¹. Similarly, for stainless steel vane and the Haynes® 188 alloy vane, the minimum operating speed required was equal to 700 r min⁻¹.

Figure 5.24: (a) and (b) Variation of the net radial forces at the vane tips for various vane material selected

Table 5.3: Predicted frictional losses and the mechanical efficiencies for various vane densities

ρ_{vn} (kg m ⁻³)	Operating speed = 3000 r min ⁻¹		
	Total frictional loss at the vane tips (W)	Total frictional loss (W)	Mechanical efficiency (%)
2830	45.7	70.4	83.4
7850	61.7	86.5	80.7
8980	65.5	90.1	80.1

5.6.2 Effect of the discharge to suction pressure ratio

The effect of various discharge to suction pressure ratio for a fixed vane density of 2830 kg m^{-3} and for a fixed operating speed of 3000 r min^{-1} was also studied. The results obtained shown in Figure 5.25 indicate that, for the pressure ratio of 2 or less, the leading vane tip tended to retract away from the cylinder wall and back into the rotor slot for the rotor angles of 142° to 167° . Upon closer inspection of the fluid pressure forces acting on the leading vane, it was found that the sudden drop of the pressure occurred at the leading vane rear end at the rotor angle of 120° . This sudden drop in the pressure occurred because the fluid undergoing compression in the vane gap was suddenly expanded to the chamber undergoing suction process.

(a) Variation of the net radial force at the trailing vane tip

(b) Variation of the net radial force at the leading vane tip

Figure 5.25: (a) and (b) Variation of the net radial force at the vane tips for various operating pressure ratios

The effect of various operating speed assuming fixed vane densities of 2830 kg m^{-3} and 7850 kg m^{-3} and for a fixed pressure ratio of 2 was studied. The results obtained are shown in Figure 5.26 (a) and (b). It was found that, for the Aluminium 7178 vane and at the pressure ratio of 2 or lower, the leading vane tip always retracted back into the rotor slot at the rotor angle of 146° regardless of the operating speed selected for the study. Although the centrifugal force increased with the increase in the operating speed, drop in the fluid pressure at the rear end still caused the leading vane tip to retract back into the rotor slot.

As shown in Figure 5.26 (c) and (d), the stainless steel vane, which is heavier than the Aluminium 7178 vane, was found to be able to operate at the pressure ratio of 2 and the operating speed as low as 1000 r min⁻¹.

(a) Variation of the net radial force at the trailing vane tip for $\rho_{vn} = 2830 \text{ kg m}^{-3}$

(b) Variation of the net radial force at the leading vane tip for $\rho_{vn} = 2830 \text{ kg m}^{-3}$

(c) Variation of the net radial force at the trailing vane tip for $\rho_{vn} = 7850 \text{ kg m}^{-3}$

(d) Variation of the net radial force at the leading vane tip for $\rho_{vn} = 7850 \text{ kg m}^{-3}$

Figure 5.26: (a) and (b) Variation of the net radial force at the vane tips for various operating speeds at the pressure ratio of 2

5.6.3 Effect of rotor-to-cylinder ratio on the efficiencies

Using equation (4.14), the volumetric displacement of CVC is given by equation (5.96).

$$V_{max} = \frac{l_c}{2} \left[R_c^2 \pi + 2b\sqrt{R_c^2 - b^2} - 2R_c^2 \tan^{-1} \left(\frac{b}{\sqrt{R_c^2 - b^2}} \right) - (\pi R_r^2) \right] - V_{l,vn}(270^\circ) - V_{t,vn}(270^\circ) \quad (5.96)$$

The basic geometrical configuration of CVC is mainly dictated by rotor radius, cylinder radius and axial length of the compressor. The rotor radius is divided by the cylinder radius to form a non-dimension parameter termed as rotor-to-cylinder ratio R_r/R_c . The simulation procedure is presented in Appendix A-2. Some major operating parameters and the main dimensions of CVC selected are shown in Table 5.4.

Table 5.4: Operating condition and the main dimensions

Operating condition	
Volumetric displacement	44 cm ³
Operating speed	3000 r min ⁻¹
Working fluid	R1234yf
Evaporating temperature	7.2 °C
Condensing temperature	54.4 °C
Lubricant dynamic viscosity	14.8 mPa·s
Main dimensions	
Cylinder radius	27.5 mm
Rotor radius	15.5 mm
Cylinder length	30 mm
Distance between rotor to cylinder centre	13 mm
Compressor height	30 mm
Clearance gaps	
Sealing arc clearance	10 µm
Vane endface clearance	10 µm

The effects of varying R_r/R_c and the axial length l_c of CVC on its performance are investigated by fixing the volumetric displacement of CVC and other parameters presented in Table 5.4. Figure 5.27 shows the variation of l_c with respect to the R_r/R_c .

Figure 5.27: Variation of the compressor axial length for varying R_r/R_c

Figure 5.28: Variation of the mechanical and the volumetric efficiency of CVC for varying rotor-to-cylinder ratio

Figure 5.28 shows the increment in volumetric efficiency of CVC when R_r/R_c is decreased from 0.8 to 0.5. This is due to the increase in l_c when R_r/R_c is increased. This results in larger leakage path through the sealing arc.

The mechanical efficiencies predicted for various operating speed also predict an increasing trend when R_r/R_c is decreased. This is because, for longer l_c , the vane height is longer and therefore, the vanes are heavier. This results in larger centrifugal force acting on the vanes. The forces acting on the vane neck (see

equations (5.27) and (5.28)) also increase with l_c . Consequently, the vane tip rubbing against the inner cylinder wall is higher for longer l_c .

Additionally, various sources of power losses in CVC were investigated, the obtained results are presented in Figure 5.29. The variation in power losses due to rubbing at the vane tip and cylinder wall were found to be more significant compared to the variation in power losses due to rubbing at the vane and rotor and between vanes. It was found that, when R_r/R_c , was varied from 0.5 to 0.8, at 0.8 R_r/R_c , the power losses at the trailing and leading vane tip increased more than 330% and 180% respectively compared to the same at 0.5 R_r/R_c (see Figure 5.29 (a)). This is again due to the increment in l_c with respect to R_r/R_c , which resulted in larger pressure force area pressing the vane tip against the cylinder wall (see equation (5.27) and (5.28)).

Figure 5.29: Variation of power losses due to rubbing of various components with respect to R_r/R_c

Meanwhile, increasing R_r/R_c decreases the power losses between the vane and rotor and between the vanes (see Figure 5.29 (b)). This is because of decrease in the pressure forces acting on vane sides which are the consequences of an increase in the rotor radius R_r and decrease in $r(\theta)$ (see equations (5.19) to (5.22)).

5.7 Summary

In this chapter, the mathematical models to predict the dynamic forces within the compressor are formulated. The dynamic model is then used to analyse the frictional losses occurring within the compressor. The summary of this chapter can be presented using following points:

- The mathematical models for the kinematics of the rotor and the vanes were developed.
- The dynamic model for the vane and rotor were presented. The frictional forces at the vane tips, between the rotor and vane and between the two sliding vanes were formulated. Assuming the presence of a film of lubricating oil, the fluid frictional losses at the sealing arc and the endfaces were formulated.
- The mathematical model for the design of the journal bearing design was presented.
- The mathematical model for the instantaneous power loss due to friction was formulated. Energy lost due to friction was derived using the integration of the instantaneous power loss over an operating cycle of the compressor. Finally, the mechanical efficiency of the compressor was also derived as the ratio of the indicated work to the sum of indicated work and the energy lost due to friction.
- For some arbitrary dimension of CVC, the results from the simulation using the mathematical models formulated for the variation of the kinematics, dynamics and power losses in the compressor were also presented.
- The effect of vane material and operating pressure ratio on the vane dynamics were studied using the parametric studies. The parametric study showed that, using lighter vanes generally reduces the frictional losses. However, at pressure ratio of 2 or lower, the lighter vanes may result in failure because the vane tips retract and fail to contact the cylinder wall.
- It was also found that the vanes which are made of stainless steel ($\rho_{vn} = 7850 \text{ kg m}^{-3}$) or heavier material can operate in CVC even at pressure ratio as low as 2 at the operating speed of 1000 r min^{-1} .
- The numerical investigation of the compressor performance by varying R_r/R_c showed the volumetric efficiency and the mechanical efficiency of

CVC generally increased by lowering R_r/R_c . The maximum efficiencies were calculated at the $R_r/R_c = 0.5$. Higher efficiencies predicted at smaller R_r/R_c ratio implies that CVC can be designed such that its rotor occupies significantly smaller space inside the cylinder, which allows the CVC to be designed as an extremely compact compressor compared to the existing rotary compressors. In chapter 7, the design of CVC prototype is presented which uses R_r/R_c ratio of 0.57.

- Variation of R_r/R_c had significant effect on power losses due the rubbing of vane tips against cylinder wall. The study showed that the power losses at the trailing and leading vane tip increased by over 330% and 180% respectively as R_r/R_c was increased from 0.5 to 0.8.

Chapter 6: Design of Lubrication Model

In this chapter, the mathematical modelling of lubrication system of a CVC prototype (shown in Figure 6.1 (a) and (b)) are presented. The mechanical design of components of the CVC prototype are discussed in Appendix A-4. The CVC prototype includes a compressor housing which houses a compressor cylinder including a reed, valve stopper, rotor-shaft and oil sump. Two shaft bushings were installed to support the shaft. The compressor shaft is oriented vertically for the experimental investigation.

(a) A CVC prototype

Overall prototype dimension:

Height	155 mm
Diameter	150 mm

(b) A sectional view of the CVC prototype

Figure 6.1: Illustration of assembled CVC prototype

6.1 Oil lubrication model

In this section, the studies on the lubrication model for CVC are presented. Figure 6.2 shows the oil sump, lubrication pathways and oil sealing mechanisms such as O-rings and an oil seal designed for CVC prototype. The orientation of the shaft is vertical and the oil sump, which is housed inside the compressor housing, is designed at the lowest position for the collection and circulation of oil.

In this section, the working mechanism, the mathematical modelling of the lubrication flow and the simulation results for the oil flowrate within CVC prototype are presented. The mathematical modelling of the lubrication flow is performed by assuming the oil flow to be analogous to the current flow in an electrical circuit.

Figure 6.2: Oil lubrication model for CVC prototype

6.1.1 Working mechanism of the lubrication model

As seen in Figure 6.3, the CVC shaft is symmetrical about the mid-plane which is why there are 2 identical sets of radial feed holes and the vertical holes either side of the line of symmetry (at 180° apart). The advantages of having 2 sets of oil feeding holes are that the shaft becomes well balanced while the lubrication flow into the CVC is increased.

The discharge pressure acting on the oil sump is responsible for pumping and circulating the oil through the compressor. The oil lubrication flow into the CVC prototype can be described as follows:

- 1) The oil is pumped into the 1st radial feed hole (see Figure 6.3). Since the shaft is rotating in an anti-clockwise direction, the centrifugal force acts against the oil flow.
- 2) The oil is then pumped into the 1st vertical hole. Steps 1) and 2) repeat for the 2nd vertical hole after 180° rotation of the shaft.
- 3) The oil is then pumped radially out to the lower bearing through 2nd radial feed holes. Since the oil sump is at the discharge pressure, the oil is then pumped into the suction chamber.
- 4) The oil in the 1st and the 2nd vertical hole continue to flow to the 3rd radial holes. From the 3rd radial feed holes, the oil is pumped into the upper bearing. The oil in the upper bearing eventually flows into the suction chamber in CVC.
- 5) The remaining oil in the 1st and the 2nd vertical hole continue to flow up through the 3rd vertical hole. The oil is then pumped out to the upper endface of the shaft through the pair of 4th radial hole. The oil at the upper endface is first radially thrown out because of the centrifugal force and then seeps into the upper bearing.
- 6) The oil collected in the suction chamber through the lower and the upper bearing are swept towards the discharge chamber by the vanes and then expelled out to the oil sump through the discharge port.

Figure 6.3: Lubrication pathways for the CVC prototype

6.1.2 Mathematical modelling of the oil lubrication network

The objective of the mathematical modelling of the lubrication network is to ensure that the adequate amount of oil is circulating within the CVC prototype. Lack of adequate lubrication may also cause the compressor heating due to the friction between the rubbing parts. This will also prevent excessive wear and tear of the rubbing parts.

Assuming the oil to be incompressible, the oil flow in an oil lubrication network in a compressor can be analogous to an electric current flowing a circuit. This implies the pressure drop acting along the flowpath to be analogous to the potential difference (or voltage) across the resistor. The flow resistance, similar to electrical resistance in a resistor, depends upon the dimension of the flowpath. Consequently, using the pressure drop and the flow resistance, the oil flowrates through the various flowpaths can be determined.

A. Flow through a straight hole

Flow through a straight circular hole of uniform cross-section can be modelled as Hagen-Poiseuille flow in a pipe [172]. The flow upwards the vertical pipe is driven by the pressure difference across the pipe and the flow must overcome the flow resistance and the gravitational effect. The flow resistance across the straight pipe is given by following equation (6.1).

$$R_n = \frac{\Delta p_n}{Q_n} = \frac{128\mu L_n}{\pi d_n^4} \quad (6.1)$$

B. Flow through radial hole

The radial flow in a shaft is accelerated by the centrifugal force. The differential pressure across the two ends of the radial hole is given by equation (6.2) [150].

$$\Delta p_{cf,n} = \frac{\rho \omega^2}{2} (R_{o,n}^2 - R_{i,n}^2) \quad (6.2)$$

The flow resistance in a radial hole is also given by equation (6.1).

C. Flow through journal bearing clearance gap

As the shaft rotates eccentrically with respect to the bearing centre, the converging and diverging sections are formed (see Figure 5.19) and the shaft drags the oil into the converging section. Assuming the oil to be incompressible,

the flow is modelled similar to Couette flow [150]. The total oil flowing into the bearing clearance can be written as shown in equation (6.3).

$$Q_{br,n} = \int_0^\pi -\frac{h^3}{12\mu} \frac{\partial p}{\partial z} \frac{D_j}{2} d\theta \quad (6.3)$$

where, h is the oil film thickness.

Equation (6.4) is obtained by integrating equation (6.3) by assuming constant pressure gradient along the axial direction of the bearing. The oil film thickness varies according to the equation: $h = \delta_{br,n}(1 + \varepsilon \cos \theta)$.

$$Q_{br,n} = \frac{\Delta p_n}{L_{br,n}} \frac{\pi \delta_{br,n}^3 D_{j,n}}{12\mu} \left(1 + \frac{3}{2} \varepsilon^2\right) \quad (6.4)$$

where, ε is the eccentricity ratio. ε is from the Journal bearing model presented in section 5.4.

Using equation (6.4), the flow resistance can be determined as shown in equation (6.5).

$$R_n = \frac{\Delta p_n}{Q_{br,n}} = \frac{12\mu L_{br,n}}{\pi \delta_{br,n}^3 D_{j,n} \left(1 + \frac{3}{2} \varepsilon^2\right)} \quad (6.5)$$

where, $\delta_{br,n}$ is the radial clearance in the bearing.

D. Flow through shaft end-face gap

Flow through the gap between the shaft end-face is due to the centrifugal force pushing the fluid radially outwards. The flow rate can be determined using equation (6.6) and the corresponding flow resistance due to the radial pressure difference is obtained using equation (6.6) [150].

$$R_{p,n} = \frac{\Delta p_n}{Q_{p,n}} = \frac{6\mu \ln\left(\frac{r_2}{r_1}\right)}{\pi \delta_{rot}^3} \quad (6.6)$$

where, r_1 and r_2 represent the initial and the final flow radii.

Although the oil flow in the left side of feed holes (Figure 6.3) lags by 180° rotor angle to the right side of the feed holes, identical flows in the left side of feed holes to the right side of the feed holes can be assumed for the same differential pressure. The oil flow network in CVC prototype is presented in Figure 6.4.

Figure 6.4: Oil flow network using electrical circuit analogy for CVC prototype

In Figure 6.4, Q_1 to Q_6 are the seven unknown flowrates. Q_1 is the flow through the bearing clearance, Q_2 is the flow to through the 1st radial feed hole. Since the flowrate is a conserved property, Q_2 is also the flow through 1st vertical hole (between the 1st radial feed hole and 2nd radial feed hole), Q_3 is the 2nd radial feed hole, Q_4 is the flow through the 1st vertical hole (between the 2nd radial feed hole and the 3rd radial feed hole), Q_5 is the flow through the 3rd radial feed hole, Q_6 is the flow through the 2nd vertical hole (between the 3rd radial feed hole and the 4th radial feed hole). Since Q_6 is the remaining flow in the 2nd vertical hole, from the flow conservation, Q_6 is also the flow through the 4th radial feed hole and the upper shaft endface.

Using Kirchoff's current law at 2 nodes N_1 and N_2 (see Figure 6.4), equations (6.7) and (6.8) can be derived.

$$Q_3 + Q_5 + Q_6 = Q_2 \quad (6.7)$$

$$Q_2 = Q_3 + Q_4 \quad (6.8)$$

Equations (6.9) to (6.12) are derived using Kirchoff's voltage law which is applied to the 4 loops within the circuit in Figure 6.4.

$$P_{dis} - P_{suc} + \rho g(h_1 - h_2) = Q_1 R_1 \quad (6.9)$$

$$\Delta p_{cf,1} - \Delta p_{cf,2} + \rho g(h_3 - h_1) = Q_1 R_1 - Q_3 R_4 - Q_2(R_2 + R_3) \quad (6.10)$$

$$\Delta p_{cf,2} - \Delta p_{cf,3} + \rho g(h_4 - h_5) = Q_3 R_4 - Q_5(R_6 + R_7) - Q_6 R_7 - Q_4 R_5 \quad (6.11)$$

$$\begin{aligned} P_{dis} - P_{suc} + \rho g(h_1 - h_3 - h_4 - h_6 + h_7) - \Delta p_{cf,1} + \Delta p_{cf,4} \\ = Q_2 R_1 + Q_4 R_5 + Q_6(R_8 + R_9 + R_{10}) \end{aligned} \quad (6.12)$$

The 6 unknown flowrates, Q_1 to Q_6 can be obtained by solving 6 equations (6.7) to (6.12).

6.1.3 Simulation results

Dimensions of the oil flow pathways are presented in Table 6.1.

Table 6.1: Dimension of the oil flow pathways

Flow resistance	Path-type	Description	Dimension (mm)
R ₁	Lower bearing	Diameter	30 mm
		Length	35 mm
		Radial	
		clearance	45 μm
R ₂	1 st radial feed hole	Diameter	4 mm
		Length	9 mm
R ₃	1 st vertical hole	Diameter	5 mm
		Length	31 mm
R ₄	2 nd radial feed hole and the flow into the bearing clearance	Diameter	3 mm
		Length	9 mm
R ₅	1 st vertical hole	Diameter	5 mm
		Length	21 mm
R ₆	3 rd radial feed hole	Diameter	5 mm
		Length	1.5 mm
R ₇	Upper bearing	Diameter	28 mm
		Length	30 mm
		Radial	
		clearance	48 μm

R ₈	2 nd vertical hole	Diameter	8 mm
		Length	2 mm
R ₉	4 th radial feed hole	Diameter	3 mm
		Length	6.8 mm
R ₁₀	Upper shaft endface	Axial clearance	0.1 mm
		Outer radius	14 mm
		Inner radius	7 mm

The simulation study is performed using Shell Refrigeration Oil S4 FR-F 68 which has density of 991 kg m⁻³. The physical characteristics of the oil is presented in Appendix A-3. The oil sump temperature assumed is 80°C. At 80°C, the kinematic viscosity of oil was 15 mm² s⁻¹. Constant density and viscosity were assumed.

Table 6.2: Operating condition and the main dimensions of CVC prototype

Operating condition	
Volumetric displacement	44 cm ³
Operating speed	3000 r min ⁻¹
Working fluid	Air
Suction pressure	1 bar
Discharge pressure	10 bar
Lubricant dynamic viscosity	14.8 mPa s
Main dimensions	
Cylinder radius	27.5 mm
Rotor radius	15.5 mm
Cylinder length	30 mm
Distance between rotor to cylinder centre	13 mm

Using the dimensions of oil pathways presented in Table 6.1, the operating condition and the main dimensions of CVC presented in Table 6.2, the variation of flow resistances are shown in Figure 6.5 (a) – (d). The variation of the flow resistances at the bearing clearances are affected by the variation of eccentricity ratio (see Figure 6.5 (c)).

The variations of the oil flowrates are shown in Figure 6.6.

(a) Flow resistance at the oil feed holes

(b) Flow resistance at the lower bearing (R_1) and the upper bearing (R_7) clearance

(c) Variation of eccentricity ratio at the lower bearing and the upper bearing

(d) Flow resistance at R_4

Figure 6.5 Variation of flow resistances at various flow paths

(a) Prediction of the oil flowrate using lubrication model

(b) Prediction of the minimum oil flowrate using the Journal bearing model (see section 5.4)

Figure 6.6 Variation of the oil flowrates predicted using the lubrication model and the comparison with the minimum oil flowrate required using the journal bearing model

Figure 6.6 (b) is the prediction of the minimum oil flowrate required in the journal bearing using the journal bearing model presented in section 5.4. From the lubrication model (Figure 6.4), the oil flowrate in the lower bearing is the sum of Q_1 and Q_2 . Likewise, the net oil flowrate in the upper bearing is the sum of Q_5 and Q_6 . In both the bearings, the predicted flowrate using lubrication model is greater than the minimum flowrate required for journal bearing model.

The lubrication flow predicted in the lower bearing is much higher than the lubrication flow predicted in the upper bearing. This is because the lubrication flow path at the lower bearing is shorter from the oil sump (at the discharge pressure) to the suction chamber. Whereas, the oil flow path through the oil feeding holes in the shaft is much longer before it can flow through the upper bearing clearance.

The effect of the discharge pressure and the operating speed on oil flowrates are studied with the objective to determine the critical discharge pressure and the operating speed required for the operation of the CVC prototype. The results are shown in Appendix A-4.

6.2 Summary

Oil lubrication model developed for a CVC prototype was presented in this chapter. This chapter may be summarised using following points:

- A 44 cm³ CVC prototype was designed.
- For fabrication, 17 – 4 PH stainless steel was selected for the cylinder and shaft.
- A simple plain sliding vane without the dovetail feature was selected for the CVC prototype.
- From the stress-strain analysis of the vanes, the maximum discharge pressure recommended during operation is 10 bars. This will serve as the operating limit for the testing of CVC prototype.
- An oil lubrication model was designed for the CVC prototype. The simulation of the oil lubrication model indicate that the oil flow is more than sufficient for reliable operation of the compressor prototype.

Chapter 7: Experimental Study and Validation

A CVC prototype was designed with the concepts discussed in chapter 3. A CVC prototype was fabricated and evaluated through the experimental investigations. The measured results were then used to validate the accuracy of the mathematical models developed in Chapters 4 to 6.

7.1 Physical dimension of the prototype

The CVC prototype was fabricated by Woo and Woo Precision Industries Pte Ltd. The components of CVC prototype were manufactured from 17-4 PH (UNS S17400) stainless steel. After receiving the prototype, the compressor dimensions were measured to determine the exact sizes of the clearance gaps. The major compressor dimensions were measured using micrometer screw gauge and digital Vernier calipers with accuracies of ± 0.001 mm and 0.01 mm respectively. Coordinate Measurement Machine (CMM), with the accuracy of ± 0.1 μm , was used to measure bearing diameter and length. Multiple measurements were made, their arithmetic mean was computed and recorded as the measured value. Key dimensions measured are shown in the following Table 7.1.

Table 7.1: Measured prototype dimensions

Cylinder	
Inner cylinder wall diameter	54.990 mm
Cylinder height	29.998 mm
*Sealing arc radius	15.610 mm
Rotor	
Rotor diameter	30.981 mm
Lower journal diameter	29.989 mm
Upper journal diameter	27.970 mm
Slot width	5.940 mm
Slot height	30.005 mm
Housing	
Lower bearing diameter	30.011 mm

Bearing length	39.958 mm
Cover	
Upper Bearing diameter	28.011 mm
Bearing length	15.021 mm
Aluminium Bronze Vane	
*Vane head thickness	5.947 mm
Vane body thickness	2.961 mm
*Vane height	29.933 mm
Vane length	33.847 mm
Reed	
Thickness	0.21 mm
Effective length	20.50 mm
Head diameter	6.01 mm
Body width	3.02 mm

Note: The asterisk (*) mark in Table 7.1 indicates the component dimensions were reworked for improved clearance in consideration with the fits for the assembly.

Key leakage path clearances have been measured and shown in following Table 7.2.

Table 7.2: Leakage path clearance measured

Description	Clearance (mm)
Lower radial clearance	0.022
Upper radial clearance	0.041
Sealing arc clearance	0.119
Vane endface clearance	0.065

The sealing arc clearance and the vane endface clearances are the two significant leakage paths identified in the prototype. The measured dimensions shown in Table 7.2 are the values of the clearances measured for the prototype before the experimental measurement of the prototype was completed.

The surface roughness of the prototype was measured using Talyscan 150 Surface profiler which has the vertical resolution of $\pm 0.06 \mu\text{m}$. The measured surface roughness parameters are tabulated below in Table 7.3.

Table 7.3: Measured surface roughness values

Description	Average roughness, R_a (μm)	RMS roughness, R_q (μm)	Average peak-peak roughness, R_z , (μm)
Lower shaft	0.248	0.304	1.615
Upper shaft	0.221	0.274	1.480
Rotor	0.224	0.276	1.440
Top sealing shaft	0.203	0.257	1.380

7.2 Experimental setup

Figure 7.1 is the schematic of the experimental setup designed to measure the performance of the CVC prototype in an open-loop air cycle. The discharge tank pressure and flowrate and the total power input to the compressor are to be measured.

The compressor is powered by ABB 2.2 kW two-pole induction motor and a frequency controller is used to regulate the operating speed of the compressor. The power input into the compressor is measured using Fluke MDA-510 scope meter. The atmospheric air is drawn directly into the compressor through the suction port and the discharged into a Swagelok 304L-HDF4-300-PD receiver tank. The discharge pressure is measured by a WIKA A-10 series pressure transducer which can measure gas pressure up to 40 bar. The volumetric flowrate was measured using Aalborg 044-40-GL 150 mm flowtube and $\frac{1}{4}$ " diameter Tantalum float. The flowmeter has the measurement range between 2015 – 69940 ml min⁻¹. The discharge flow temperature was measured by a Type-K thermocouple. The compressor housing temperature was monitored using a Type-J thermocouple.

The measurement uncertainties of the instruments are listed in Table 7.4.

Table 7.4: Measurement uncertainties

Pressure transducer	± 0.2 bar
Flowmeter	1398 ml min ⁻¹
Type-K thermocouple	1.0 K

Figure 7.1: Schematic of the experimental setup

Figure 7.2: Actual experimental setup

The pressure transducer was connected to a NI 9421 system for data acquisition. The type-K thermocouple was connected to PICO TC-08 data logger. The calibration data and the product data of the instruments used can be found in Appendix A-5.

7.3 Experimental procedure

The experimental procedures for the measurement of CVC prototype performance are as follows:

- a. Compressor prototype and the measuring instruments were adequately cleaned before assembly.
- b. After assembly, pipes and fittings were checked for secure connection to prevent external leakage.
- c. The power supply was turned on to the frequency controller, transducers and DAQ systems. The motor frequency controller was initially adjusted to 12 Hz, which is equal to the synchronous speed of 720 r min^{-1} .
- d. With all the valves closed, the pressure was allowed to build for approximately 2 min.
- e. With the ball valve fully open, the needle valve was adjusted such that the flowmeter reading was at 5 mm.
- f. The motor frequency controller was adjusted to higher synchronous speed. The flowrate and the discharge pressure were allowed to stabilize for 3-5 mins.
- g. The needle valve is adjusted such that the flowmeter reading rose by 5 mm more. The flow was then allowed to stabilize, and the discharge tank pressure and temperature readings were recorded. The voltage and the current into the motor were recorder using the scope meter. 4 sets of reading at the fixed synchronous speed were recorded.
- h. The discharge tank pressure was constantly monitored so that the discharge pressure does not fall below 2.5 bar (abs). This is to ensure the lubrication system for the prototype works smoothly.
- i. The prototype housing temperature was also monitored, and the experiment was allowed to continue until the housing temperature

reached 60°C. A standing fan was also used to externally cool the compressor.

- j. After 4 sets of data were recorded at the fixed synchronous speed, the steps (f) – (i) were repeated.

7.4 Validation of the theoretical models

7.4.1 Validation of thermodynamics and leakage model

The measured results were compared with the predicted results to validate the thermodynamics and the leakage model derived in Chapter 4. The theoretical model requires the discharge pressure and the operating speed as an input and the predicted flowrate is compared with the measured result. The density of the compressed air in the discharge tank was determined using ideal gas laws. Assuming negligible pressure drop across the flowmeter, pressure and the temperature at the inlet of the flowmeter were used for the density measurement. (See Appendix A-5, section D for the validation of this assumption).

To account for the effect of sealing of the internal clearances because of oil, leakage coefficients were used. The orifice flow coefficients and the internal leakage coefficients used are presented in Table 7.5.

Table 7.5: Flow coefficients used in the theoretical model

Flow coefficients	Value
Orifice flow coefficients [172]	0.61
Sealing arc leakage coefficient	0.5
Vane endface leakage coefficient	0.5

The measured discharge pressure and the flowrates are shown in Figure 7.3 (a). The comparison between the measured and the predicted flowrates for various operating conditions is shown in Figure 7.3. The predicted flowrate and the measured flowrate were generally found to be in good agreement with maximum discrepancy of $\pm 15\%$.

(a) Measured results

(b) Comparison of the measured and predicted flowrate

Figure 7.3: (a) Measured discharge pressure and flowrate (b) Comparison of the measured and predicted flowrate

Volumetric efficiency, in equation (7.1), is defined as the ratio of the measured flowrate to theoretical mass flow rate assuming no internal leakage.

$$\eta_{vol} = \frac{\rho_{measured} \dot{V}_{measured}}{\rho_{suc} V_{c,max} f_{measured}} \quad (7.1)$$

Using the measured flowrate and the operating speed, volumetric efficiencies of CVC at various operating conditions are shown in Figure 7.4. Among the measured data, the lowest volumetric efficiency was 38% for 4.7 bar (abs) and 1200 r min⁻¹. The highest volumetric efficiency was 79% at 1500 r min⁻¹ and 4.6 bar (abs). The uncertainties determined for volumetric efficiency is presented in Table 7.8. It was found that potential uncertainty in the measurement of volumetric efficiency could be approximately 6%.

Figure 7.4: Volumetric efficiencies computed from measurements

7.4.2 Validation of dynamics model

In addition to the frictional losses discussed in chapter 5, the frictional loss at the shaft-seal interface should also be considered. Based on the method proposed by Muller and Nau [184], the power loss due to the shaft-seal friction is obtained using equation (7.2).

$$P_{f,s} = 2\pi R_s^2 \bar{\varphi} \omega_s \quad (7.2)$$

The predicted total power input to the compressor is determined using equation (7.3).

$$P_{in} = P_{indicated} + P_{f,total} + P_{f,s} \quad (7.3)$$

where, P_{in} is the total power input, $P_{indicated}$ is the indicated power and $P_{f,total}$ is the power loss due to friction (see equation (5.93)).

The total mechanical power input to the compressor is calculated using the method proposed by Bhimbhra [185]. In equation (7.4), m is the number of poles in the induction motor, I_1 is the stator current, R_f is resistance offered to the stator by the rotating air-gap field, P_g is the total air-gap power, s is motor slip and P_m is the total mechanical power developed by the motor. (See equation (7.7) for the derivation of R_f)

$$P_g = mI_1^2 R_f \quad (7.4)$$

$$P_m = (1 - s)P_g \quad (7.5)$$

From the motor catalogue (see Appendix A-5), $m = 2$ and $s = 0.041$. The stator current I_1 is measured using the current probe and a scope meter.

Figure 7.5 is the comparison between the predicted and the measured power input to CVC. The frictional coefficient assumed for the prediction was 0.2. The predicted result had the maximum discrepancy of $\pm 15\%$ with the measured result. The uncertainties associated with the measurement of power input is presented in Table 7.7. It was found that approximately 14% of uncertainty on the measurement of power input.

Figure 7.5: Comparison of the measured and predicted power input

7.4.3 Uncertainty Analysis

The uncertainties of various measuring devices used in the testing of CVC are shown in Table 7.6: Uncertainties of Measuring Devices

Table 7.6: Uncertainties of Measuring Devices

Operating frequency, Δf	± 2.050 Hz
Futek scope meter, ΔI_1	± 0.215 A
Aalborg rotameter, $\Delta \dot{V}$	± 1.359 L min ⁻¹

Using equation (7.4) and (7.5), the uncertainty of the power input is calculated using approximation shown in equation (7.6).

$$\Delta P_m \approx \sqrt{\left(\frac{\partial P_m}{\partial I_1} \Delta I_1\right)^2 + \left(\frac{\partial P_m}{\partial R_f} \Delta R_f\right)^2}$$

$$\Delta P_m \approx (1-s)mI_1 \sqrt{(2\Delta I_1 R_f)^2 + (I_1 \Delta R_f)^2} \quad (7.6)$$

R_f is the resistance offered to the stator by the rotating air-gap field which varies with the operating motor frequency. Equation (7.7) is used to determine R_f .

$$Z_f = R_f + jX_f = \frac{\left(\frac{r_2}{s} + jx_2 \left(\frac{f_m}{f_R}\right)\right) jX_m \left(\frac{f_m}{f_R}\right)}{\frac{r_2}{s} + j\left(x_2 \left(\frac{\omega_m}{\omega_R}\right) + X_m \left(\frac{f_m}{f_R}\right)\right)} \quad (7.7)$$

where, f_m is the motor operating frequency, f_R is the rated motor frequency (= 50 Hz), r_2 is the rotor ohmic loss, x_2 is the leakage impedance at the rotor, X_m is the magnetizing reactance. Equation (7.7) is simplified to obtain equation (7.8).

$$R_f = \frac{\frac{r_2}{s} X_m^2 \left(\frac{f_m^2}{f_R^2}\right)}{\left(\frac{r_2}{s}\right)^2 + \left\{(x_2 + X_m) \left(\frac{f_m}{f_R}\right)\right\}^2} \quad (7.8)$$

The uncertainties determined for input power are presented in Table 7.7.

Table 7.7: Uncertainties of power input

Operating Motor frequency (ω_m)	Discharge pressure (Abs. bar)	Predicted power (W)	Measured power (W)	Discrepancy	Uncertainties (%)
20	3.5	254.82	309.41	17.64	14.48645
20	3.9	273.85	310.69	11.86	13.39405
20	4.0	281.43	315.20	10.71	14.48701
20	4.3	316.38	320.66	1.33	12.97744
21	4.9	345.90	359.73	3.84	10.34507
21	5.4	371.83	361.58	2.84	12.11126
21	5.8	381.85	371.38	2.82	11.86849
25	5.2	603.42	599.16	0.71	11.02658
25	6.1	673.02	606.85	12.33	11.40457

The volumetric efficiency determined using (7.9), is defined as the ratio of measured flowrate to ideal flowrate into the compressor. The uncertainty of the volumetric efficiency is determined using equation (7.10):

$$\eta_{vol} = \frac{\rho_{measured} \dot{V}_{measured}}{\rho_{suc} V_{c,max} f_{measured}} \quad (7.9)$$

$$\begin{aligned} \Delta\eta_{vol} &\approx \sqrt{\left(\frac{\partial\eta_{vol}}{\partial\dot{V}} \Delta\dot{V}\right)^2 + \left(\frac{\partial\eta_{vol}}{\partial\omega} \Delta\omega\right)^2} \\ &= \sqrt{\left(\frac{\rho_{measured}}{\rho_{suc} V_{c,max} f_{measured}} \Delta\dot{V}\right)^2 + \left(\frac{\rho_{measured} \dot{V}_{measured}}{\rho_{suc} V_{c,max} f_{measured}^2} \Delta f\right)^2} \end{aligned} \quad (7.10)$$

The uncertainties determined for volumetric efficiencies are presented in Table 7.8.

Table 7.8: Uncertainties of volumetric efficiencies

Operating Motor frequency (ω_m)	Discharge pressure (Abs. bar)	Predicted Ideal mass flowrate (g/s)	Measured mass flowrate (g/s)	Volumetric efficiency (%)	Uncertainties (%)
20	4.7	2.12	1.01	47.67	6.25
20	4.0	2.12	0.96	45.47	6.18
20	3.9	2.12	1.16	54.91	6.14
20	3.5	2.12	1.19	56.19	6.08
21	5.6	2.23	0.85	38.06	5.77
21	5.4	2.23	0.89	39.96	5.75
21	4.9	2.23	0.92	41.47	5.70
21	4.6	2.23	0.98	44.02	5.67
25	4.6	2.65	2.12	79.80	4.01
25	5.2	2.65	1.93	72.81	4.04
25	6.1	2.65	1.66	62.44	4.11

7.4.4 Simulation results

In this section, comparison of predicted results for two cases of operating conditions shown in Table 7.9, is presented. The clearances and flow coefficients used for the simulation were shown in Table 7.2 and Table 7.5. The two cases presented are for the highest and the lowest volumetric efficiency measured during the experimental testing of CVC. The obtained results are shown in Figure 7.6 (a) – (f).

Variations in instantaneous pressure for the two cases are shown in Figure 7.6 (a). In Figure 7.6 (a), the raise in chamber pressure at about 40° rotor angle is due to the small suction chamber volume and immediate leakage to this chamber through the sealing arc (shown in Figure 7.6 (d)). Figure 7.6 (b) shows p-V diagrams of the two cases. The indicated power obtained for the operating condition 1 and 2 were 290.5 W and 161.1 W respectively. Leakage through the vane endfaces 1 and 2 are presented in Figure 7.6 (e) and (f).

Table 7.9 Operating conditions for simulation studies for 2 cases: for the lowest volumetric efficiency and the highest volumetric efficiency measured

	Operating Condition 1	Operating Condition 2
Working fluid	Air	Air
Discharge pressure (bar, abs)	4.6	5.6
Inlet temperature (°C)	33	31
Operating speed (r min ⁻¹)	1500	1200
Measured volumetric efficiency (%)	79.8	38.1

(a) Variation of pressure

(b) p-V diagrams

(c) Valve displacement

(d) Sealing arc leakage

(e) Vane endface 1 leakage

(f) Vane endface 2 leakage

Figure 7.6: Simulation results for the operating conditions with the lowest and the highest volumetric efficiency

7.5 General observations after the experiment

CVC prototype was carefully disassembled for the post-experiment study. Figure 7.7 (a) – (f) show the polished surfaces of the components of CVC prototype due to rubbing.

The wear marks on the leading face of the vane and the polished inner wall of the cylinder, shown in Figure 7.7 (b) and (d), indicate that the vane extends radially during the operation of CVC.

(a) Rotor and shaft surfaces

(b) Leading face of the vane

(c) Trailing face of the vane

(d) Inner wall of the cylinder

(e) Sealing arc

(f) Lower cover

(g) Upper cover

Figure 7.7: Post experiment observation of the CVC components

7.6 Summary

The performance of a CVC prototype was measured experimentally. Based on the experimental results and observations, the summary can be listed as follows:

- The prediction from the mathematical model shows the maximum discrepancy of $\pm 15\%$ with the measurement.

- Based on the measured results, at lower operating speeds, low volumetric efficiency was observed. This is because the internal leakage is severe at lower operating speeds.
- Since the experimental setup was designed without the closed-loop oil circulation, the oil loss was significant.

For improvement in the performance of CVC in future development, following points are recommended:

- During the mechanical design phase, tighter tolerance control, including the assembly fits, should be practiced.
- The leading and the trailing face of the vanes require adequate lubrication to minimize the frictional wear due to the rubbing between the vane and the vane slot. The vanes maybe designed with lubrication pathways.
- For the further validation of the thermodynamics model, the instantaneous pressure at the suction and the compression chamber may be measured.

Chapter 8: Conclusions and Future Work

An extremely compact, novel rotary compressor, known as Coupled Vane Compressor (CVC), was studied. In this chapter, the key findings and the summary on theoretical and experimental investigations made during the project are presented. Finally, a list of potential works to be conducted in the future are also recommended.

8.1 Design motivation and objective

The compressor design of CVC is most probably the most compact rotary vane design available today. Its unique feature is that its rotor can be significantly smaller relative to its cylinder size compared to all other rotary vane compressors available today. As compared to the existing rotary vane compressors, the new compressor has an immense potential in saving the significant amount of material, especially, the metal used during the production. From the preliminary analysis, the metal saved could be up to 40%. The application of CVC will be in gas compression, fluid pumping and heating applications.

8.2 Compressor design

The design of CVC was derived from the cardioid compressor which had cardioid shaped inner cylinder wall and the rotor with diametric slot through which a singular vane maintains the slide-able contact with the rotor slot and the inner cylinder wall. Among the rotary positive displacement compressors, the cardioid compressor had the smallest rotor-to-cylinder ratio of less than 0.45. The major drawback identified with the cardioid compressor was that the rubbing of vane tips at the inner cylinder wall led to gradual shortening of the vane length which eventually led to an extremely inefficient performance of the compressor. The proposed solution in CVC was to replace the single vane with a couple of extendable vanes. Following are the key points for the consideration of the vane design:

- The vane tip, vane neck and the vane rear are carefully designed such that the fluid pressure and the centrifugal force acting on the vane result in

the vane tip being pressed against the inner wall of the cylinder during the operation.

- The centrifugal force acting on the vane must always press the vane tip against the inner wall of the cylinder.
- For extremely small rotor sizes of CVC, the vanes are designed with dovetail feature to allow the rotor to contain the vanes within the rotor slot.

8.3 Mathematical modelling

The mathematical models incorporating the working chamber geometries, thermodynamics, main flows through the suction and discharge port, secondary flows through the clearance gaps, in-chamber convective heat transfer, kinematics and dynamics of vane, rotor, journal bearing, and oil lubrication model have been formulated. Following predictions were made:

- CFD was used to model the internal leakage through the discharge port at the vane tip. The analytical model for the leakage was also derived assuming the isentropic flow through the orifice. It was found that the predicted flowrate from the analytical model using discharge coefficient of 0.61 had the maximum discrepancy of $\pm 15\%$ with the same from CFD model.
- The parametric study of the effect of vane material and operating pressure ratio on the vane dynamics showed that using lighter vanes (Aluminium vanes) generally reduced the frictional losses. However, at pressure ratio of 2 or lower, the lighter vanes may fail to remain in contact with the inner wall of the cylinder.
- It was also found that heavier vanes ($\rho_{vn} = 7850 \text{ kg m}^{-3}$ or greater) can operate in CVC even at pressure ratio as low as 2 at the operating speed of 1000 r min^{-1} . At higher operating speeds, the centrifugal force was sufficient to press the vane tip against the cylinder wall.
- The effect of rotor-to-cylinder (R_r/R_c) ratio was studied. It was found that the mechanical efficiency and volumetric efficiency of CVC were both higher for smaller rotor-to-cylinder ratio. For the compressor dimensions selected with R_r/R_c of 0.5, the mechanical efficiency of 82.7% and the volumetric efficiency of 98% was predicted.

- The key effect of variation of R_r/R_c was studied on the power losses at the trailing and leading vane tip, which increased by over 330% and 180% respectively as R_r/R_c was increased from 0.5 to 0.8
- Comparison of the oil flowrates predicted using the journal bearing model and the lubrication model, indicate that the oil flow is more than sufficient for reliable operation of the compressor prototype. The minimum flowrate predicted by the lubrication model at 1.8 bar (abs) discharge pressure and 900 r min⁻¹ was 0.8 cm³/s while the maximum flowrate required predicted by the journal bearing model was 0.5 cm³/s.

8.4 Key findings and observations

Key findings and observations obtained during the measurement of the performances of CVC are presented as follows:

- CVC prototype was experimentally tested in an open-loop experimental setup using air as the working fluid. The measured parameters included discharge pressure, flowrate and the power input to the compressor. Aluminium Bronze was selected as the material for the fabrication of the vane while the 17-4 PH stainless steel was used for the fabrication of the cylinder and the rotor. Because of differences in the material properties such as the melting point, the CVC prototype was able to operate without any seizure during the experimental testing.
- The compressor was tested for the operating speed of 1200 - 1500 r min⁻¹ and the maximum discharge pressure obtained was 6.1 bar (abs).
- The flowrate was determined assuming the ideal gas laws, pressure and temperature measured at the inlet of the flowrate. The leakage coefficient of 0.5 was used for the prediction. The predicted flowrates had the maximum discrepancy of $\pm 15\%$ with the measured data.
- Due to the severity of the leakage, at the operating speeds lower than 1200 r min⁻¹ and discharge pressures greater than 4.7 bar (abs), the volumetric efficiency was found to be lower than 40%. The volumetric efficiency increased at higher operating speed. The maximum volumetric efficiency measured was 79% at 1500 r min⁻¹ and 4.6 bar (abs).

- Using the friction coefficient of 0.2, the predicted power input had the maximum discrepancy of $\pm 15\%$ with the measured data.

8.5 Future work

The list of recommendations for the further development of CVC are presented as follows:

A. Study of heat transfer between the components of CVC

Lumped capacitance model [68] and [120] can be used to study the heat transfer between the working fluid and the components of CVC prototype such as the rotor, cylinder, vanes and the compressor housing. Each of these components are the elements with 'lumped capacitance' with a 'lumped temperature'. The overall heat transfer model can be obtained using an analogy to the electrical circuit, in which, lumped temperature, the rate of heat transfer and the thermal mass are analogous to the voltage, current and the resistance of an electric load in a circuit. The addition of the heat transfer model of CVC is expected to improve the accuracy of the prediction.

B. Vane dynamics and vane design

Due to the rapid wear and tear of the vane sides, the vane design of CVC can be further improved by redesigning the vanes with lubrication pathways to allow for the oil to shear instead of metal-to-metal rubbing of the vane and the rotor slot. The trailing face of the vanes can be designed with the shallow and wide tapered cuts (see Figure 8.1) which allow the squeezing of oil into the rubbing interfaces. Various authors such as Teichmann [138] and Qvale [139] have studied the frictional losses due to the shearing of oil film present between the rubbing components using the hydrodynamic lubrication theory.

Figure 8.1: Redesigned vane with the tapered cuts on the trailing face of the vane

C. Multi-variable multi-objective optimization study of CVC

The volumetric displacement of CVC is shown again in equation (8.1). The key geometrical parameters of CVC include the rotor and the cylinder radii, axial length, and the distance between the rotor and the cylinder centre. Besides these geometrical parameters, the performance of CVC is influenced by suction and discharge port diameters. A multi-variable and multi-objective optimization study use these key parameters to maximize the performance of CVC such as the mechanical efficiency, volumetric efficiency and COP.

$$V_{max} = \frac{l_c}{2} \left[R_c^2 \pi + 2b \sqrt{R_c^2 - b^2} - 2R_c^2 \tan^{-1} \left(\frac{b}{\sqrt{R_c^2 - b^2}} \right) - (\pi R_r^2) \right] - V_{l,vn}(270^\circ) - V_{t,vn}(270^\circ) \quad (8.1)$$

D. Experimental study

The experimental analysis of CVC may be improved further by incorporating following points:

- A new prototype should be designed with tighter tolerance control for improved mechanical and volumetric efficiencies.
- The vane and shaft of the new CVC prototype should be surface hardened for reducing the wear and tear.
- A new improved closed-loop lubrication design can be designed for recirculating the oil and minimizing the oil loss.
- The compression chamber and the suction chamber pressure can be measured for the further validation of the thermodynamics model.
- The temperature of working chamber of CVC can be measured for the development and the validation of comprehensive heat transfer model.
- The performance of a newly designed CVC can be measured using a closed-loop refrigeration cycle. The schematic of an experimental setup is shown in Figure 8.2. From the measured data, the cooling capacity and the COP of CVC can be determined.

Figure 8.2: Schematic of a closed-loop refrigeration cycle to test the performance of a CVC prototype [165]

E. Study of pulsating flow at the suction and discharge

In section 4.3, flows through the suction and discharge port have been modelled assuming steady and isentropic condition. This assumption limits the ability to predict the dynamic characteristics of CVC which are due to the pulsating flow at the suction and discharge ports [186]. For example, the dynamics of the valve discharge port is affected by the pressure pulsations [187]. Therefore, for improving the accuracy of prediction, dynamic characteristics of CVC and the pulsating flow should be studied.

8.6 Concluding Remarks

CVC has shown a great potential as a rotary compressor which is extremely compact and material saving in design. It is hoped that the studies presented in this thesis lays a good foundation for the further development of CVC. The dissertation ends here, but the author will continue to strive for innovation on greener technologies such as CVC.

Author's Publications

Journals

- Shakya, P. and Ooi, K. T., "Introduction to Coupled Vane compressor: mathematical modelling with validation", *International Journal of Refrigeration*, 2020. ISSN 0140-7007, <https://doi.org/10.1016/j.ijrefrig.2020.01.027>.
- Shakya, P. and Ooi, K. T., "Vane and rotor dynamics of a coupled vane compressor", *International Journal of Refrigeration*, 2019. [Submitted: 13 Dec 2019]

Conferences

- Ooi, K. T. and Shakya, P. (2018). A New Compact Rotary Compressor: Coupled Vane compressor, *International Compressor Engineering Conference*, Purdue University. Paper 2613.
- Ooi, K. T. and Shakya, P. (2019). Simulation studies of a coupled vane compressor, *IOP Conference Series: Materials Science and Engineering*, 604(1): 012069.

Patent

- Ooi, K. T., Shakya, P., Sin, K. and Ang, C. L. (2018). WO/2018/217173 (PCT/SG2018/050260). Retrieved from: https://patentscope.wipo.int/search/en/detail.jsf?docId=WO2018217173&_cid=P20-K0UK34-28805-1

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Appendix A-1: Vane Volume Calculations

A. Trailing vane volume

If $(r(\theta_r) - R_r) < R_{f1}$

$$V_{t,vn}(\theta_r) = \left(\frac{\pi}{4} R_{f1}^2 \tan^{-1} \left(\frac{\sqrt{R_{f1}^2 - (R_{f1} - r(\theta_r) + R_r)^2}}{(R_{f1} - r(\theta_r) + R_r)} \right) - \frac{(R_{f1} - r(\theta_r) + R_r) \sqrt{R_{f1}^2 - (R_{f1} - r(\theta_r) + R_r)^2}}{2} \right) l_c$$

$$\text{Else, } V_{t,vn}(\theta_r) = \left(\frac{\pi}{4} R_{f1}^2 + (r(\theta_r) - R_r - R_{f1}) \frac{t_{vn}}{2} \right) l_c \quad (\text{A-1.1})$$

If $(r(\theta_r) - R_r) < R_{f1}$

$$\frac{dV_{t,vn}(\theta_r)}{d\theta_r} = l_c \sqrt{R_{f1}^2 - (R_{f1} - r(\theta_r) + R_r)^2} \left(\frac{dr(\theta_r)}{d\theta_r} \right)$$

Else,

$$\frac{dV_{t,vn}(\theta_r)}{d\theta_r} = \left(\frac{dr(\theta_r)}{d\theta_r} \frac{t_{vn}}{2} \right) l_c \quad (\text{A-1.2})$$

B. Leading vane volume

If $(r(\theta_r + 180^\circ) - R_r) < R_{f2}$

$$V_{l,vn}(\theta_r) = \left(\pi \times R_{f2}^2 \tan^{-1} \left(\frac{\sqrt{R_{f2}^2 - (R_{f2} - r(\theta_r + 180^\circ) + R_r)^2}}{(R_{f2} - r(\theta_r + 180^\circ) + R_r)} \right) \times \frac{1}{2\pi} - \frac{(R_{f2} - r(\theta_r + 180^\circ) + R_r) \sqrt{R_{f2}^2 - (R_{f2} - r(\theta_r + 180^\circ) + R_r)^2}}{2} \right) l_c \quad (\text{A-1.3})$$

$$\frac{dV_{l,vn}(\theta_r)}{d\theta_r} = l_c \sqrt{R_{f2}^2 - (R_{f2} - r(\theta_r + 180^\circ) + R_r)^2} \left(\frac{dr(\theta_r + 180^\circ)}{d\theta_r} \right) \quad (\text{A-1.4})$$

Else if, $R_{f2} < (r(\theta_r + 180^\circ) - R_r) < l_{tip}$

$$V_{l,vn}(\theta_r) = \left(\frac{\pi}{4} R_{f2}^2 + (r(\theta_r + 180^\circ) - R_r - R_{f2}) \frac{t_{vn}}{2} \right) l_c \quad (\text{A-1.5})$$

$$\frac{dV_{l,vn}(\theta_r)}{d\theta_r} = \left(\frac{dr(\theta_r + 180^\circ)}{d\theta_r} \frac{t_{vn}}{2} \right) l_c \quad (\text{A-1.6})$$

Else,

$$l_{gap} = r(\theta_r + 180^\circ) + r(\theta_r) - l_{tip} - l_{vn}$$

$$V_{cres}(\theta_r) = \begin{cases} \frac{\pi}{4} (R_{flt}^2 - (l_{gap} - R_{flt})^2) g_h & \text{if } R_{flt} > l_{gap} - R_{flt} \\ (R_{flt}^2 + (l_{gap} - R_{flt}) R_{flt}) g_h & \text{if } R_{flt} < l_{gap} - R_{flt} \end{cases} \quad (\text{A-1.7})$$

$$\frac{dl_{gap}(\theta_r)}{d\theta_r} = \frac{dr(\theta_r)}{d\theta_r} + \frac{dr(\theta_r + 180^\circ)}{d\theta_r}$$

$$\frac{dV_{cres}(\theta_r)}{d\theta_r} = \begin{cases} \frac{\pi}{2} \left(-(l_{gap} - R_{flt}) \frac{dl_{gap}(\theta_r)}{d\theta_r} \right) g_h & \text{if } R_{flt} > l_{gap} - R_{flt} \\ \left(\frac{dl_{gap}(\theta_r)}{d\theta_r} R_{flt} \right) g_h & \text{if } R_{flt} < l_{gap} - R_{flt} \end{cases} \quad (\text{A-1.8})$$

If $l_{gap} < g_{len}$

$$V_{l,vn}(\theta_r) = \left(\frac{\pi}{4} R_{f2}^2 + (r(\theta_r + 180^\circ) - R_r - R_{f2}) \frac{t_{vn}}{2} \right) l_c - l_{gap}(a - g_h) l_c \quad (\text{A-1.9})$$

$$\frac{dV_{l,vn}(\theta_r)}{d\theta_r} = \left(\frac{dr(\theta_r + 180^\circ)}{d\theta_r} \frac{t_{vn}}{2} \right) l_c - \frac{dl_{gap}(\theta_r)}{d\theta_r} (a - g_h) l_c \quad (\text{A-1.10})$$

Else,

$$V_{l,vn}(\theta_r) = \left(\frac{\pi}{4} R_{f2}^2 + (r(\theta_r + 180^\circ) - R_r - R_{f2}) \frac{t_{vn}}{2} \right) l_c - l_{gap}(a - g_h) l_c - V_{cres}(\theta_r) \quad (\text{A-1.11})$$

$$\frac{dV_{l,vn}(\theta_r)}{d\theta_r} = \left(\frac{dr(\theta_r + 180^\circ)}{d\theta_r} \frac{t_{vn}}{2} \right) l_c - \frac{dl_{gap}(\theta_r)}{d\theta_r} (a - g_h) l_c - \frac{dV_{cres}(\theta_r)}{d\theta_r} \quad (\text{A-1.12})$$

Appendix A-2: Simulation Procedure

FORTTRAN (Formula Translation) was selected as the programming language to develop the codes to simulate the numerical analysis for the mathematical models developed for the compressor. FORTTRAN is especially suited for intensive numerical and scientific computational applications. Development of FORTTRAN has seen successive version with added support for structured programming, array programming, modular programming, generic and high-performance optimizations. To evaluate the thermodynamic states of the working fluid during various process REFPROP derived from Reference Fluid Thermodynamic and Transport Properties routines [188] are incorporated into the code. The REFPROP program uses the latest high accuracy equations (such as ones discussed above) based on Helmholtz energy for thermodynamic properties with typical uncertainties of 0.1% in densities, vapour pressures and speeds of sound, 0.5% in heat capacities, and 0.1% in pressure in the critical region.

The flow of simulation is depicted in Figure A-2.1. The simulation code consists of five main blocks and each block contain numerous other modules as shown below.

1. Initial condition, operating conditions and evaluation of other physical constants of the compressor components such as vane weight, valve natural frequencies and so on
2. Kinematics and geometric models
3. Thermodynamic model (including valve dynamics, Mass flow and heat transfer)
4. Internal leakage flow models
5. Dynamic model

The geometric model calculates the working volume and rate of change of working volume of the coupled vane compressor using the initial and constant conditions. For the first cycle, the thermodynamic model evaluates the

thermodynamic properties of the working fluid such as the pressure, temperature, enthalpy, entropy, partial differential functions and the density assuming the adiabatic and perfectly sealed condition. Using the values obtained for temperature, the heat transfer rate is determined for the next iteration. Then using the value obtained for the pressure and enthalpy and employing the mass flow model, the flow rate across the inlet port is calculated. In case of discharge flow, the differential pressure across the valve is determined which leads to the evaluation of the extent of valve opening and then the mass flow model is employed to evaluate the discharge. These processes are repeated until 1 operational cycle including the suction, compression and discharge phase are completed. The end of discharge marks the end of the first cycle.

The second cycle begins by establishing the pressure and the temperature of the control volume at each positions of the rotor angle. Then, the leakage flow models are employed to evaluate the leakage flowrates across the clearance gaps. The leakage flowrates are fed into the thermodynamic model which evaluates the latest thermodynamic state of the working fluid by including the effects of the leakage and the heat transfer. The resulting data from the thermodynamic model is then fed into the heat transfer, mass flow, valve dynamics and the leakage flow model. At the end of the second cycle, the convergence criteria are checked. For this simulation test, the convergence is defined to be achieved if the instantaneous pressure at each rotor angle is equal to or within 1% deviation of the same from the previous cycle for each rotor angle.

The evaluations of the thermodynamic properties, mass flow and the valve dynamics require the application of the numerical integration. In this case, Runge-Kutta 4th order method is employed to solve the first order ordinary differential equations. Therefore, the local truncation error is on the order of $O(h^5)$ and the total accumulated error is on the order of $O(h^4)$, where, h is the angle step.

Similarly, the fanno flow leakage models require a numerical scheme which searches for the correct exit Mach number (M_e) by guessing a throat Mach number (M_t). For this purpose, the golden-section root search method was

employed. The convergence was said to be achieved if the pressure ratio for the guessed M_t was within 5% of the discharge to the suction chamber pressure ratio. To solve the dynamic model which includes six simultaneous equations, inverse matrix method was used.

Figure A-2.1: Flowchart depicting the algorithm of the coupled vane compressor simulation code

A. Operating conditions

The operating condition of the compressor in a vapour compression cycle includes the selection of the refrigerant fluid, the evaporating and the condensing temperature, the compressor inlet temperature after the superheat, the liquid temperature after the subcool and the operating speed of the compressor shaft. The operating temperatures are selected as per the rated conditions set by American society of heating, refrigerating and air-conditioning engineers (ASHRAE) [189]. Table A-2.1 summarizes these operating conditions selected.

Table A-2.1: Operating condition selected for refrigerants other than air

Operating speed (ω_r)	3000 r min ⁻¹
Evaporating temperature (T_{evap})	7.2 °C
Condensing temperature (T_{cond})	54.4 °C
Liquid temperature (T_{liq})	46.1 °C
Compressor inlet temperature (T_{in})	35 °C

B. Initial conditions

The operating cycle of the coupled vane compressor is shown in Figure A-2.2 through steps (1) to (8). The starting orientation of the vanes, assumed to be at $\theta_r = 0^\circ$, is shown in step (1) of Figure A-2.2. For the orientation of the compressor shown in Figure A-2.2, assuming the rotation of the rotor in anti-clockwise direction, a control volume evolves into the working chambers, namely, suction, compression and discharge chamber for which the thermodynamic properties are evaluated using the simulation model. This implies one operational cycle for the coupled vane compressor, in which the fluid undergoes suction, compression and discharge phase respectively, is equal to 540° of rotor angle.

For $\theta_r = 0^\circ$ to $\theta_{r,st}$, the control volume is not yet exposed to the suction port and the physical size of the control volume is constant. This control volume is illustrated in Figure A-2.3 (a). At the start of the cycle, the initial condition for the thermodynamic properties of this control volume is assumed to be at the same state as the state of the working fluid arriving at the suction port from the suction plenum, that is, the gas pressure and the temperature at this control volume at $\theta_r = 0^\circ$ is at the suction pressure and temperature. However, it is noted that, at the end of the discharge phase, that is at the rotor angle $\theta_r = 540^\circ$, the working fluid

in the control volume (illustrated in Figure A-2.3 (b)) will approximately be close to the discharge pressure and temperature.

Figure A-2.2: An operational cycle of CVC

Figure A-2.3: (a) Illustration of the assumed initial volume, (b) Illustration of the control volume at the end of the cycle

The other constants including the flow coefficients, physical and mechanical properties used in the simulation are shown in Table A-2.2.

Table A-2.2: Flow coefficients, physical and mechanical properties used in the simulation

Flow coefficients:	
Suction port coefficient of discharge ($C_{d,suc}$) [172]	0.61
Discharge port coefficient of discharge ($C_{d,dis}$) [172]	0.61
Physical and mechanical properties of the material used for reed:	
Density of the material used for reed(ρ_{val})	7800 kg m ⁻³
Young's modulus (E_{val})	210 GPa
Damping ratio (ζ_{val})	0.2
Vane Dynamics	
Friction coefficient (μ_{fric})	0.15

C. Step size test

A step size test was conducted to select an optimum angle step for which the parametric studies will be carried out. For a compressor operating on R134a at 2000 r min⁻¹, other operating conditions defined in section 6.1.1 and with arbitrary dimensions, cylinder radius (R_c) = 32.5 mm, rotor radius (R_r) = 20.25 mm, axial cylinder length (l_c) = 45 mm, including 3 discharge ports of discharge port diameter (d_{dis}) = 7 mm and a suction port of equivalent diameter (d_{suc}) = 22.14 mm, the maximum over-compression pressure, discharge loss, suction loss and the total indicated power was obtained and the percentage deviation with respect to the corresponding values obtained for the step size of 0.005° are as shown in Table A-2.3 and

Table A-2.4. The result obtained showed that for step sizes greater than 0.0125°, the absolute deviation of the suction loss was more than 0.1%. Therefore, considering the computation time required for the calculation of once cycle (shown in Table A-2.4, the step size of 0.01° was selected.

Table A-2.3: Step size test using various losses

Step size (°)	Maximum Over compression pressure (kPa)	Deviation (%)	Discharge loss (W)	Deviation (%)	Suction loss (W)	Deviation (%)
0.005	1678.630	0	63.623	0	7.389	0
0.0075	1678.630	0	63.621	-0.003	7.387	-0.03
0.01	1678.630	0	63.622	-0.002	7.384	-0.06
0.0125	1678.755	0.007	63.627	0.006	7.383	-0.08
0.015	1678.437	-0.01	63.612	-0.02	7.381	-0.11
0.0175	1678.768	0.008	63.628	0.008	7.379	-0.14
0.02	1678.768	0.008	63.614	-0.010	7.377	-0.17

Table A-2.4: Step size test using total indicated power

Step size (°)	Indicated power (W)	Deviation (%)	Approx. computation time required (per cycle)
0.005	1510.223	0	3' 55"
0.0075	1510.175	-0.003	2' 45"
0.01	1510.118	-0.007	2' 4"
0.0125	1510.076	-0.009	1' 39"
0.015	1510.046	-0.012	1' 22"
0.0175	1509.979	-0.016	1' 10"
0.02	1509.912	-0.021	1' 1"

Appendix A-3: Material Properties

A. 17-4PH (UNS S17400) stainless steel

Table A-3.1: Material properties of 17-4PH stainless steel

	AISI 1020 Carbon Steel, Cold rolled	AISI 4140	P20 tool steel	SST304	SST316	17-4PH
Tensile Strength (MPa)	394	655	965-1030	515	515	1020
0.2% Offset Yield Strength (MPa)	294	415	827-862	241	205	1110.74
Elastic Modulus (GPa)	200	205	205	193	193	196
Rockwell Hardness HB	64	92	30	92	95	36
Machinability	High	Low	High	Low	Low	High
Mean coefficient of Thermal Expansion ($\mu\text{m}/\text{m}/^{\circ}\text{C}$) (0-100 $^{\circ}\text{C}$)	11.7	12.2	12.8	17.2	15.9	11.3
Thermal conductivity at 100 $^{\circ}\text{C}$ (W/m.K)	51.9	42.6	29-41	16.2	16.2	18.3
Corrosion resistance	Poor	Good	Good	Good	Good	Good

B. Aluminium bronze

Table A-3.2: Material properties of Aluminium bronze

Specification:	C95810
Typical Chemical composition:	Cu: min 79.0% Al: 8.5-9.5% Sn: max 0.1% Zn: max 0.5% Ni: 4.0-5.0% Pb: max 0.05% Fe: 3.5-4.5% Mn: 0.8-1.5%
Typical Mechanical Properties (C95810):	
Tensile Strength:	610 N/mm ²
Elongation:	12%
Typical Hardness:	160 HB
0.2% Yield Strength:	245 N/mm ²

C. Shell Refrigerant Oil S4 FR-F 68

Technical Data Sheet

Previous Name: Shell Clavus Oil R

Shell Refrigeration Oil S4 FR-F 68 : *Reliable Performance For use with HFC Refrigerants*

Advanced Synthetic Refrigerator Compressor Lubricant

Shell Refrigeration Oil S4 FR-F is a synthetic refrigeration lubricant with a polyol ester base fluid. It has been developed for use with R134a and other HFC refrigerants.

DESIGNED TO MEET CHALLENGES

Performance, Features & Benefits

- Extended maintenance intervals**
 Shell Refrigeration Oil S4 FR-F has excellent high temperature and oxidation stability providing long service life even where high compressor discharge temperatures are found.
- Wear protection**
 Shell Refrigeration Oil S4 FR-F has been designed to minimise the wear in bearings and pistons – the most critical parts of a reciprocating compressor.

- Refrigerator Compressors**
 Shell Refrigeration Oil S4 FR-F is recommended for use in open, semi-open and hermetic compressors in domestic, commercial and industrial refrigeration systems. It can be used in both rotary and reciprocating compressor types.
- Refrigerant Compatibility**
 Shell Refrigeration Oil S4 FR-F is recommended for use with R134a and other types of HFC refrigerant.
- Seal Compatibility**
 Shell Refrigeration Oil S4 FR-F is compatible with all sealing materials commonly used with HFC refrigerant systems.

Main Applications

Specifications, Approvals & Recommendations

- Shell Refrigeration Oil S4 FR-F meets the requirements of DIN 51503 KD.

For a full listing of equipment approvals and recommendations, please consult your local Shell Technical Helpdesk.

Typical Physical Characteristics

Properties	Method	Refrigeration Oil S4 FR-F 68
ISO Viscosity Grade	ISO 3448	68
Refrigerator Oil Group	DIN 51503	KD
Kinematic Viscosity @40°C mm ² /s	ISO 3104	66
Kinematic Viscosity @100°C mm ² /s	ISO 3104	8.8
Density @15°C kg/m ³	ISO 12185	991
Flash Point (COC) °C	ISO 2592	>230
Pour Point °C	ISO 3016	-42
Neutralisation Number mg KOH/g	ASTM D664 (TAN)	<0.06
Characteristics when used with R134a - Flocc-point °C	DIN 51351	<-30
Characteristics when used with R134a - Refrigerant Stability @250°C	DIN 51393	>96
Miscibility		Miscible over the whole range of typical refrigeration temperatures

These characteristics are typical of current production. Whilst future production will conform to Shell's specification, variations in these characteristics may occur.

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Shell Refrigeration Oil S4 FR-F 68, v 1.1
09.06.2014.14.03

Figure A-3.1: Physical properties of Shell Refrigerant Oil S4 FR-F 68

Figure A-3.2: Variation of viscosity of Shell Refrigerant Oil S4 FR-F 68

Appendix A-4: Design of CVC Prototype

A. Operating condition of CVC prototype

The CVC prototype will be experimentally measured in an open-loop experimental setup with air as the working fluid. In an open-loop experimental setup, the ambient air is induced into the compressor through the suction port and discharged into a discharge tank for the measurement. With consideration to the available space, cost constraints and the instruments available for experimental investigation, the parameters imposed for the design of the CVC prototype is presented in Table A-4.1.

Table A-4.1: Parameters selected for the design of CVC prototype

Working fluid	Air
Maximum differential pressure	15 bar
Maximum operating speed	3000 r min ⁻¹

B. Compressor cylinder

17 – 4PH (UNS S17400) stainless steel was selected for the fabrication of the compressor cylinder. The material properties for 17 – 4PH stainless steel are presented in appendix A-3.

Figure A-4.1 shows the compressor cylinder and its key components.

Figure A-4.1: Design of a compressor cylinder

Assuming 15 bar differential pressure across the cylinder wall, the minimum cylinder wall thickness can be approximated using hoop stress, $\sigma_{h,cyl}$ for the pressurised vessel (see equation (A-4.1)).

$$\sigma_{hoop,cyl} = \frac{\Delta p \times R}{t_w} \quad (A-4.1)$$

where, Δp is the differential pressure, R is the external diameter and t_w is the wall thickness. The yield strength of 17-4 PH stainless steel is 1110 MPa (see Appendix A-3). Assuming the safety factor of 10, we obtain the minimum allowable thickness of the cylinder wall as shown in equation (A-4.2).

$$t_{w,min} = N \times \frac{\Delta p \times R_c}{\sigma_{h,cyl}} = 10 \times \frac{(15 \times 10^5) \times 0.030}{1130 \times 10^6} = 398 \mu m \quad (A-4.2)$$

Figure A-4.2 (a) and (b) are the stress and the strain distribution on the cylinder wall respectively when the 15 bar differential pressure was applied. The maximum von-Mises stress obtained was 60 MPa which is much lower than the yield strength of 1110 MPa.

Figure A-4.2: Stress-strain simulation study of cylinder wall in Solidworks 2018

C. Rotor-shaft

The rotor-shaft designed for CVC prototype is shown in Figure A-4.3 (a) and (b). The shaft includes a vane slot and multiple oil feeding holes. These internal flow paths are critical to the oil lubrication of the compressor prototype which will be presented in section 6.6. 17-4 PH stainless steel is selected for the fabrication of

the shaft. The total rotor load F_{rot} acting on the shaft and the resulting reaction forces on the bearing are illustrated in Figure A-4.4. Assuming 15 bar as maximum differential pressure acting across the vane, the maximum rotor load, $F_{rot, max}$ was 1125 N. The reaction forces on the bearing are obtained in equation (A-4.3) and (A-4.4).

$$F_{br,1,max} = F_{rot,max} \times \frac{BC}{AC} = 1125 \times \frac{35}{70} = 562.5N \quad (A-4.3)$$

$$F_{br,2,max} = F_{rot,max} \times \frac{AB}{AC} = 1125 \times \frac{35}{70} = 562.5N \quad (A-4.4)$$

(a) CVC rotor-shaft

(b) A sectional view of the rotor-shaft

Figure A-4.3: Schematics of a CVC rotor-shaft

Figure A-4.4: Various forces acting on the shaft

The maximum torque applied on the shaft was 45.56 N m. Figure A-4.5 shows simulation study performed on Solidworks 2018 to study the distribution of stress and strain in the shaft. The maximum von-Mises stress was 473 MPa. The minimum factor of safety was 2.35.

Figure A-4.5: Stress-strain simulation study of the shaft in Solidworks 2018

Based on the bearing load calculations, the maximum shear forces and the bending moment of the shaft can be obtained using Figure A-4.6 (a) and (b).

Figure A-4.6: Shear force and bending moment diagrams for the shaft

From the Figure A-4.6, the maximum shear force is 562.5N and maximum bending moment is 19.7 N m.

Using Figure A-4.6, various mechanical stress criteria can be used to evaluate the shaft design. Due to the presence of the lubrication pathways, the mechanical stresses are evaluated at various cross-sections shown in Figure A-4.7.

Figure A-4.7: Illustration of various cross-sections of the shaft

Bending stress:

The bending stress determined for section B-B shown in Figure A-4.7 is presented in equation (A-4.5) for the chosen outer diameter of 31 mm, oil feeding holes of 5mm each and slot width of 6mm.

$$\sigma_b = \frac{M \times \frac{d_o}{2}}{\frac{\pi d_o^4}{64} - \frac{d_o w_{slot}^3}{12} - \frac{2\pi d_{oil}^4}{64}} = 6.9 \text{ MPa} \quad (\text{A-4.5})$$

where,

M is Bending moment, d_o is outer diameter and w_{slot} is the width of the rectangular slot and d_{oil} is the diameter of oil feeding holes.

Stress due to torsion:

The stress due to torsion for section B-B shown in Figure A-4.7 is determined using equation (A-4.6).

$$\tau_{xy} = \frac{T \times \frac{d_o}{2}}{\frac{\pi d_o^4}{32} - \frac{d_o w_{slot}}{12} (w_{slot}^2 + d_o^2) - \frac{2\pi d_{oil}^4}{32}} = 12.1 \text{ MPa} \quad (\text{A-4.6})$$

Maximum shear stress theory:

The maximum allowable shear stress is calculated as shown in equation (A-4.7) using the bending stress and the torsional stress derived in equations (A-4.5) and (A-4.6).

$$\tau_{shear,max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 12.6 \text{ MPa} \quad (\text{A-4.7})$$

Maximum normal stress theory:

The maximum allowable normal stress is determined using equation (A-4.8).

$$\tau_{normal,max} = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 16 \text{ MPa} \quad (\text{A-4.8})$$

Von Mises/Distortion-Energy theory:

The Von Mises stress is obtained using equation (A-4.9):

$$\tau_{von,max} = \sqrt{\sigma_b^2 + 3\tau_{xy}^2} = 22 \text{ MPa} \quad (\text{A-4.9})$$

ASME design code (ductile material):

The allowable stress is determined in equation (A-4.10) according to the ASME design code using bending and torsion factors for gradually applied load are $k_m = 1.5$ and $k_t = 1.0$ for rotating shaft.

$$\tau_{allowable} = \sqrt{(k_m \sigma_b)^2 + (k_t \tau_{xy})^2} = 15.1 \text{ MPa} \quad (\text{A-4.10})$$

From equations (A-4.7) to (A-4.10) we can see that allowable stress is larger than the maximum stress for different criteria studied. Hence, the selected shaft size is unlikely to fail during operation.

For section A-A shown in Figure A-4.7, where, d_o is the outer diameter, d_i is the inner diameter, d_{oil} is the oil feed hole diameter and l_{oil} the length of the oil feed hole across the cross-section. The stress due to torsion and the von Mises stress is as shown below in equation (A-4.11) and (A-4.12):

$$\tau_{xy} = \frac{T \times \frac{d_o}{2}}{\frac{\pi d_o^4}{32} - \frac{d_{oil} l_{oil}}{12} (l_{oil}^2 + d_{oil}^2) - \frac{\pi d_i^4}{32}} = 98.5 \text{ MPa} \quad (\text{A-4.11})$$

$$\tau_{von,max} = \sqrt{3\tau_{xy}^2} = 170.6 \text{ MPa} \quad (\text{A-4.12})$$

From the calculations in equations (A-4.11) and (A-4.12), we again see that the maximum stresses are lower than the yield strength of the material (1110 MPa).

D. Vane

Since the rotor design presented in section 7.3 is sufficiently big to contain the vanes without the dovetail feature, the vane design shown in Figure A-4.8 (a) is chosen for the CVC prototype.

Figure A-4.8 (a) shows the main body parts of the vane designed for CVC prototype. Since the oil lubrication system will be used to lubricate the rubbing parts of the CVC prototype, 3 mm wide and 0.5 mm deep oil grooves were added at the leading face of the vane tip to allow the oil to flow into the vane gap.

Figure A-4.8: Vane design

Figure A-4.8 (b) shows the orientation of vanes where the maximum differential pressure is assumed. For this orientation of the vanes, the stress analysis of the vanes can be performed by assuming the vanes to be similar to a cantilever beam.

Figure A-4.9 (a) and (b) are the stress analysis performed on the vane in Solidworks 2018 assuming the vane shown in Figure A-4.8 (b) as the cantilever beam. At 15 bar differential pressure, it was found that the von-Mises stress exceeded the allowable stress at the interface between the leading face of the vane 2 and the rear end of vane 1 (see Figure A-4.9 (a)). Maximum differential

pressure was reduced to 10 bar and the stress analysis was performed again. Figure A-4.9 (c) is the variation of the factor of safety on the vane body assuming 10 bar differential pressure.

(a) Stress distribution at 15 bar differential pressure

(b) Stress distribution at 10 bar differential pressure

(b) Factor of safety at 10 bar differential pressure

Figure A-4.9: Stress analysis of the vane

E. Fasteners

Minimum number of fasteners required at the CVC cover to hold 15 bar differential pressure is determined using equation (A-4.13).

$$N_{fastener,min} = \frac{FOS \times F_{actual,max}}{A_{tensile} \times \sigma_{allowable}} \quad (A-4.13)$$

where, Factor of safety (FOS) assumed = 2.5, Load acting on the circular cover of 150 mm diameter ($F_{actual,max}$) assuming 15 bar differential pressure = 24.9 kN, $A_{tensile}$ = 14.2 mm² and $\sigma_{allowable}$ is the allowable stress of the bolt. For M5 x 0.8 (ISO 898/I-1988) the property class, allowable stress and the minimum number of fasteners required are presented in Table A-4.2. Based on the calculation presented in Table A-4.2, 12 x M5 x 0.8 bolts with 8.8 property class were selected to be used on the cover of the CVC prototype (see Figure A-4.10).

Figure A-4.10: Fasteners used in CVC prototype (Top view of prototype)

Table A-4.2: Minimum number of fasteners

Property class	Allowable stress (MPa)	Minimum number of fasteners
6.8	440	11.9
8.8	579.6	9.1
9.8	650	8.1

Appendix A-5: Parametric Study of Oil Lubrication

Model Designed for CVC Prototype

A. Effect of the discharge pressure and operating speed

The effect of the discharge pressure and the operating speed on oil flowrates are studied with the objective to determine the critical discharge pressure and the operating speed required for the operation of the CVC prototype. The critical flowrates in the lubrication model are the oil flowrates at the bearing clearances. Hence, in this study, the oil flowrate predicted using lubrication model is compared with the minimum flowrate required by the journal bearing in various operating condition. The results obtained are shown through figures A-4.1 to A-4.3

900 r min⁻¹

(a)

(b)

(c)

(d)

Figure A-5.1: (a) and (c): Variation of the oil flowrate predicted at the lower bearing and the upper bearing at 900 r min⁻¹; (b) Prediction of the minimum oil flowrate required using journal bearing model at the lower and upper bearing

respectively

1800 r min⁻¹

(a)

(b)

(c)

(d)

Figure A-5.2: (a) and (c) Variation of the oil flowrate predicted at the lower bearing and the upper bearing at 1800 r min⁻¹; (b) Prediction of the minimum oil flowrate required using journal bearing model at the lower and upper bearing respectively

3000 r min⁻¹

(a)

(b)

(c)

(d)

Figure A-5.3: (a) and (c): Variation of the oil flowrate predicted at the lower bearing and the upper bearing at 3000 r min⁻¹; (b) Prediction of the minimum oil flowrate required using journal bearing model at the lower and upper bearing respectively

Appendix A-6: Specifications for Measurement Instruments and Induction Motor

A. ABB Induction Motor – Datasheet

ABB Motors and Generators	Diagram for equivalent-star connected motor
	Project	Location	
Department/Author	Customer name	Customer ref.	Item name 1.00001
Our ref.	Rev/Changed by Date of issue A 4/4/2018	Saving ident untitled.xls	Pages
Product	TEFC, 3-phase, squirrel cage induction motor		
Type/Frame	M2AA 90LB 2		
Product code	3GAA 091 213-ASE	Calc. ref.	GEN_3GAA091213-E_D
Rated output P_N	2.2 kW		
Duty	S1(IEC) 100%		
Actual Motor:			
Voltage (V) 400 D	Current I_N (A) 4.4	Power factor at P_N 0.85	
Frequency (Hz) 50	Speed (r/min) 2875	Efficiency (%) at P_N 84.6	
Equivalent motor Volt/phase 231 V	R1s [Ohms] 2.7	X1s [Ohms] 3.1	
	Xmagnetizing [Ohms] 144	Rfriction+iron [Ohms] 1559	
	X2 r nom [Ohms] 3.9	R2 r nom [Ohms] 2.4	
	X2 start [Ohms] 3.1	R2 start [Ohms] 2.3	
	X2 max [Ohms] 3.2	R2 max [Ohms] 2.3

Data based on situation 5/17/2016

All data subject to tolerances in accordance with IEC

Figure A-6.1: ABB Induction Motor – Datasheet II

B. Aalborg 044-40-GL 150 mm flowtube

044-40

FLOW DATA

DOCUMENT No. 587

Tube Number: 044-40(GL, SA, ST, CA, TA)
Units : std ml/min

Metering Temperature: 70 deg.F
Metering Pressure : 14.70 psia

STD. Conditions: 1 atmos @ 70 deg.F
Tolerance : $\pm 2\%$ F.S.

SCALE READINGS AT CENTER OF FLOAT

Float Material:	Glass (GL)	Sapphire (SA)	Stainless Steel (ST)	Carboloy (CA)	Tantalum (TA)
Float Density :	2.53 g/ml	3.98 g/ml	8.04 g/ml	14.98 g/ml	16.58 g/ml

A I R	Scale Readings		Scale Readings		Scale Readings		Scale Readings		Scale Readings	
	Flow [ml/min]	Scale Readings	Flow [ml/min]	Scale Readings	Flow [ml/min]	Scale Readings	Flow [ml/min]	Scale Readings	Flow [ml/min]	Scale Readings
150 ---	23742	150 ---	30711	150 ---	45227	150 ---	66346	150 ---	69940	
140 ---	22126	140 ---	28568	140 ---	41968	140 ---	61524	140 ---	64831	
130 ---	20585	130 ---	26223	130 ---	38315	130 ---	55632	130 ---	58644	
120 ---	18651	120 ---	23998	120 ---	35172	120 ---	50253	120 ---	52900	
110 ---	16985	110 ---	21745	110 ---	31655	110 ---	44997	110 ---	47467	
100 ---	15076	100 ---	19322	100 ---	28245	100 ---	39909	100 ---	42076	
90 ---	13542	90 ---	17294	90 ---	25161	90 ---	35286	90 ---	37215	
80 ---	11961	80 ---	15262	80 ---	22199	80 ---	30710	80 ---	32410	
70 ---	10252	70 ---	13051	70 ---	19057	70 ---	26288	70 ---	27748	
60 ---	8637	60 ---	11007	60 ---	16007	60 ---	21976	60 ---	23200	
50 ---	6785	50 ---	8769	50 ---	12988	50 ---	17771	50 ---	18783	
40 ---	5102	40 ---	6693	40 ---	10040	40 ---	14063	40 ---	14916	
30 ---	3701	30 ---	4826	30 ---	7256	30 ---	10218	30 ---	10813	
20 ---	2248	20 ---	2998	20 ---	4668	20 ---	6181	20 ---	6532	
10 ---	791	10 ---	1208	10 ---	2182	10 ---	1842	10 ---	2015	

W A T E R	Scale Readings		Scale Readings		Scale Readings		Scale Readings		Scale Readings	
	Flow [ml/min]	Scale Readings	Flow [ml/min]	Scale Readings	Flow [ml/min]	Scale Readings	Flow [ml/min]	Scale Readings	Flow [ml/min]	Scale Readings
150 ---	541	150 ---	806	150 ---	1288	150 ---	1881	150 ---	2001	
140 ---	508	140 ---	759	140 ---	1215	140 ---	1778	140 ---	1885	
130 ---	470	130 ---	704	130 ---	1140	130 ---	1652	130 ---	1775	
120 ---	431	120 ---	650	120 ---	1053	120 ---	1528	120 ---	1640	
110 ---	395	110 ---	593	110 ---	965	110 ---	1392	110 ---	1500	
100 ---	355	100 ---	530	100 ---	868	100 ---	1252	100 ---	1368	
90 ---	313	90 ---	470	90 ---	771	90 ---	1117	90 ---	1222	
80 ---	274	80 ---	411	80 ---	678	80 ---	981	80 ---	1071	
70 ---	231	70 ---	355	70 ---	578	70 ---	848	70 ---	920	
60 ---	193	60 ---	296	60 ---	488	60 ---	714	60 ---	782	
50 ---	156	50 ---	240	50 ---	397	50 ---	586	50 ---	640	
40 ---	120	40 ---	185	40 ---	315	40 ---	457	40 ---	500	
30 ---	84	30 ---	130	30 ---	222	30 ---	356	30 ---	365	
20 ---	45	20 ---	80	20 ---	141	20 ---	212	20 ---	239	
10 ---	15	10 ---	30	10 ---	57	10 ---	96	10 ---	106	

AALBORG INSTRUMENTS & CONTROLS, INC

20 Corporate Drive, Orangeburg NY 10962 USA
Phone: +1 (845) 770-3000 or Toll-Free 1-800-866-3837 * FAX: +1 (845) 770-3010

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Figure A-6.2: Correlated flow data of Aalborg 044-40-GL 150 mm flowtube

B. Effect of the discharge pressure and operating speed

The effect of the discharge pressure and the operating speed on oil flowrates are studied with the objective to determine the critical discharge pressure and the operating speed required for the operation of the CVC prototype. The critical flowrates in the lubrication model are the oil flowrates at the bearing clearances. Hence, in this study, the oil flowrate predicted using lubrication model is compared with the minimum flowrate required by the journal bearing in various operating condition. The results obtained are shown through figures A-4.1 to A-4.3

900 r min⁻¹

(e)

(f)

(g)

(h)

Figure A-6.3: (a) and (c): Variation of the oil flowrate predicted at the lower bearing and the upper bearing at 900 r min⁻¹; (b) Prediction of the minimum oil flowrate required using journal bearing model at the lower and upper bearing respectively

1800 r min⁻¹

(e)

(f)

(g)

(h)

Figure A-6.4: (a) and (c) Variation of the oil flowrate predicted at the lower bearing and the upper bearing at 1800 r min⁻¹; (b) Prediction of the minimum oil flowrate required using journal bearing model at the lower and upper bearing respectively

3000 r min⁻¹

(e)

(f)

(g)

(h)

Figure A-6.5: (a) and (c): Variation of the oil flowrate predicted at the lower bearing and the upper bearing at 3000 r min⁻¹; (b) Prediction of the minimum oil flowrate required using journal bearing model at the lower and upper bearing respectively

C. WIKA Pressor transducer

WIKA S-10 Pressure Transducer (PT) was calibrated using Oil-type Deadweight tester. WIKA S-10 PT selected for the measurement had the measurement range from 1 bar (abs) to 40 bar (abs). The sensing part of the pressure transducer was connected to tester and the signal transmission cables were connected to the DAQ system to measure the signal voltage. The range of measurement was selected from 1 bar (g) to 2 bar (g). The voltage reading for each pressure applied were taken using the same DAQ system used to take measurement for the experimental testing. Figure A-6.6 is the calibration data of WIKA S-10 PT.

Figure A-6.6: Calibration data of WIKA S-10

D. Pressure drop measured across the flowmeter

Figure A-6.7: Experimental setup for pressure drop measurement

Figure A-6.8: Pressure drop measured across the flowmeter at various operating conditions

E. Shaft seal

Figure A-6.9: Friction at various lip seals as a function of pressure