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Discrete Time Optimal Control and Its Applications in Power Systems

HEHONG ZHANG
Interdisciplinary Graduate School
Institute of Catastrophe Risk Management

Discrete Time Optimal Control and Its Applications in Power Systems

HEHONG ZHANG

Interdisciplinary Graduate School
Institute of Catastrophe Risk Management

A thesis submitted to the Nanyang Technological University in partial fulfillment
of the requirement for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Date

hehong zhang

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HEHONG ZHANG

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Authorship Attribution Statement

This thesis contains material from 7 papers published in the following peer-reviewed journal(s) where I was the first and/or corresponding author.

Chapter 2 is published as

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The contributions of the co-authors are as follows:

- A/Prof Xiao provided the initial project direction.
- A/Prof Xiao, Dr. Zhai and I discussed the methodologies.
- I prepared the manuscript drafts. The manuscript was revised by A/Prof Xiao, Dr. Zhai, Prof. Pan and Prof. Huang.

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- I prepared the manuscript drafts. The manuscript was revised by A/Prof Xiao, Dr. Zhai and Prof. Pan.
- Dr. She conducted the simulation tests and collected data.

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- A/Prof Xiao provided the initial project direction.
- A/Prof Xiao, A/Prof Xie and I discussed the methodologies.
- I prepared the manuscript drafts. The manuscript was revised by A/Prof Xiao, Dr. Zhai and Prof. Pan.
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- Mr. Yang provided PMU data and did some pre-processing.

20/09/2019

Date

hehong zhang

HEHONG ZHANG

Abstract

Almost all human systems and activities strongly depend on reliable electricity. However, with the expansion of interconnected power systems and the adoption of new cyber-physical technologies in smart grid, the challenges including occasional wide-area black-outs and stability issues have emerged. State estimation (SE) that serves to estimate the real-time operating system states is among the most fundamental and critical functions in enhancing the power system resilience, since it provides system operators with better situational awareness for decision-making. Motivated by the urgent need of accurate and efficient SE algorithms to help operators make appropriate decisions, my research has two focuses:

- i Design practical, accurate and efficient state estimators.
- ii Manage state estimations in power systems integrating PMUs measurements.

The first part of the thesis focuses on construction and theoretical analysis of SE algorithms via discrete-time optimal control (DTOC). A closed-form mathematical derivation for the DTOC algorithm was first proposed by Han, which has been widely used to design estimators, observers and noise-tolerant differentiators. I first investigate the convergence of this DTOC algorithm to make its application technically sound. To cream off the best and filter out the impurities, a novel time optimal control algorithm with a simple structure is constructed with the method of the isochronic regions. Specifically, a linearised criterion is adopted for determining the control signal sequence in this approach. The convergence of this new time optimal control algorithm is proved by demonstrating the convergence path of the state point sequence driven by the corresponding control signal sequence. Furthermore, a novel tracking differentiator (TD) based on the proposed DTOC algorithm is presented. Simulation and experiment results show that it performs well in signal-tracking, differentiation acquisition and helps reduce the computational resources needed.

Then, I conduct state estimations in power systems. Specifically, the field PMUs data are filtered using the proposed TD based on the DTOC algorithm. To test its effectiveness, I compare the system emergency control schemes' solutions and the corresponding control results during cascading failures based on the filtered data and the noise-free PMUs data, respectively. Not requiring complex power system modelling and historical data, the proposed TD is suitable for real-time synchrophasor estimation application especially when the states are corrupted by noises.

The aforementioned SE algorithms can effectively reduce the computational resources needed and meet the requirements of state estimations in power systems. The proposed approaches may provide a new methodology to realize more accurate and efficient SE in power systems. The obtained results can provide system operators with better situational awareness to take prompt emergency control actions against failures, and in turn, improve the power systems' resilience.

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Chapter 1

Introduction

In this chapter, the background regarding cascading failures in power systems and resilience of power systems is presented first. Then, we specifically discuss the main measures that we can take to enhance the power system resilience. As state estimation lays the basis for realizing most of the mentioned measures. Exploring better state estimation algorithms becomes the main focus of our research. Finally, the main work and thesis framework are presented.

1.1 Background

1.1.1 Cascading Outages

Power system is a complex system [1]. Consisting of generators, transformers, transmission network, distribution network and loads makes this interconnected system complex. In addition, a power system encompasses a vast array of other electrical components such as FACTS devices, phasor measurement units (PMUs), power electronic devices and so forth (see Fig. 1.1).

Almost all human systems and activities strongly depend on reliable electricity [2]. However, with the expansion of interconnected power systems and the adoption of new cyber-physical technologies in smart grid, the challenges including occasional wide-area blackouts and stability issues have emerged [3]. Large blackouts, although infrequent,

Figure 1.1: Basic structure of the electric system.

may result in huge loss [4] [5]. For instance, on August 14, 2003, US and Canada experienced a wide-area blackout that affected around 50 million people and caused a loss of about 10 billion U.S. dollars [6]. On September 8, 2011, the cascading blackout occurred in the Pacific Southwest, resulting in approximately 2.7 million people with no access to power [7].

Cascading outage is not uncommon in power systems, where a single failure leads to a sudden spike across all nodes of the system. This is considered as one of critical factors to result in power blackout [8] [9]. Generally, the process of a blackout covers two stages including the pre-cascading stage and the cascading one. In the first stage, system operators may have opportunities and time to take appropriate control and emergency control actions to stop system from evolving into the second phase. While in the second stage, electrical components may trip in high speed that it becomes hard, if not impossible, for the system operators to react promptly [10] [11]. According to the practical data, Fig. 1.2 demonstrates the blackout phases of Italy blackout in September, 2003.

It is therefore of a high importance for the system operators to react in the pre-cascading phase, before the failures become uncontrollable. This requires the development of adequate situation awareness based on state estimation results during the

Figure 1.2: Propagation phases of a cascading blackout (Italy blackout September, 2003).

operating process [12]. Advanced monitoring and estimation of system states and the corresponding decision-support measures and tools must be handy to aid operators to respond effectively facing an incident [13] [14].

1.1.2 Resilience of Power Systems

Secure and reliable power plays an essential part in social well-being. The cascading blackouts categorized as low-probability high-impact events are not uncommon. Resilience to these extreme events is of high importance [15] [16].

Resilience is a tool to measure a system's capacity of absorbing disturbances and retaining its essential functions [17] [18]. From the perspective of energy, absorbing disturbances represents the system's capacity of storing and transforming energy. While

Figure 1.3: Resilience as response behaviour of a self-organising systems to endogenous or exogenous shocks.

retaining its functions refers to how does system release disturbances' energy, recover and restore from failures to retain its structure and functions (see Fig. 1.3).

Power system resilience covers the capacity of absorbing disturbance and recovery from high-impact, low-frequency events including cascading failures [19] [20]. Such events usually lead to huge loss not only in power systems, but also in transport and telecommunication systems [21]. To enhance power system resilience, the following measures can be taken into consideration (see Fig. 1.4):

- Hardening and resilience investment: using advanced technologies, such as advanced power components, that help systems limit damage
- Fault identification and appropriate control actions: the use of appropriate control actions from operators during pre-cascading phase based on state estimation results

Figure 1.4: Resilience consists of hardening, fault identification, corrective actions, and restoration in power systems.

- Corrective actions and emergency response: the use of fast automatic actions based on state estimation results to prevent systems from evolving into an uncontrollable high-speed cascading phase
- Restoration and damage assessment: the use of techniques and tools to restore electricity service so as to help consumers access to normal power sources for their normal work

The above four aspects are potential ways to improve the power system's resilience [22] [23] [24]. Therein, SE accounts for the fundamental function in power systems [25] [26]. It serves to estimate the real-time the operating states and helps operators carry out the appropriate emergency control actions [27].

1.2 Research contents

We aim to present practical, accurate and efficient state estimators based on discrete-time optimal control (DTOC) and its derivative algorithm, that is, tracking differentiator (TD) to manage state estimations in power systems. Specifically, we aim to propose new state estimation algorithms with strict convergence proof. To carry out state estimations, we conduct state estimation based on TD using real-time data from PMUs.

A. State estimation algorithms

- A closed-form mathematical derivation for the DTOC algorithm was first proposed by Han. It has been widely used to design controllers, observers and noise-tolerant differentiators, and it can be extended to general second-order plants easily. However, the convergence of this DTOC algorithm is still an open question, which makes its application technically unsound. We first test the convergence of this DTOC algorithm via some easily checkable procedures.
- A novel time optimal control algorithm with simple structure is constructed. A linearised criterion is adopted for determining the control signal sequence in this approach. By demonstrating the state point sequence's motion trajectories, the algorithm's convergence is presented.
- A novel tracking differentiator via the above DTOC algorithm is presented. The analysis of the proposed tracking differentiator reveals its filtering mechanism. Simulation results show that it performs well in signal-tracking, differentiation acquisition and can reduce the computational resources needed.

B. State estimations

- To perform state estimation, the field phasor measurement units (PMUs) data are processed using the proposed TD. Not requiring complex power system modeling

Figure 1.5: Main work and thesis framework.

and historical data, the proposed TD is suitable for real-time synchrophasor estimation application especially when the states are corrupted by noises.

- To test its effectiveness, we compare the system emergency control schemes' solutions and the corresponding control results during cascading failures based on the filtered data and the noise-free PMUs data, respectively.

1.3 Layout of the Chapters

Figure 1.5 demonstrates the main research work and thesis framework. Specifically, there exist two main parts including state estimation algorithms and the corresponding estimation results based on the proposed algorithms in power systems. The first part introduces TD based state estimator. The second part of the thesis presents the SE results via the proposed state estimation algorithms.

Chapters' layout is shown as follows:

- **Chapter 1** introduces background information on power system's cascading failures and resilience.
- **Chapter 2** shows research motivation in this thesis of doing state estimations for improving power systems' resilience by presenting how different state estimation results may affect system emergency control schemes and the corresponding control results during cascading failures.
- **Chapter 3** tests the convergence of the DTOC algorithm proposed by Han via some easily checkable procedures.
- **Chapter 4** investigates an improved discrete-time optimal control algorithm with a simple structure that can construct the state estimator.
- **Chapter 5** comes up with a novel DTOC-based tracking differentiator that characterizes excellent signal-tracking filtering, differentiation acquisition.
- **Chapter 6** demonstrates the experimental results in the state estimation based on the proposed TD.
- **Chapter 7** gives a summary of thesis work and some future work for state estimations in power systems.

Chapter 2

Effects of State Estimation Results on Emergency Control Schemes for Power Systems against Cascading Failures

2.1 Introduction

The real-time and reliable state estimation (SE) plays an essential role in monitoring the operation of power systems [28]. The advancements of phasor measure units (PMUs) technologies and their applications in power systems now make near real-time monitoring of power systems become accessible [29]. Moreover, not like traditional SCADA measurements, PMUs provide high-speed coherent and time-stamped data, which offers the chance to observe the system transient process even including cascading failures [30].

The presence of noises in PMU data, however, is unavoidable due to measurement and sensor inaccuracies, electromagnetic interference, and ambient temperature effects and so forth [31] [32]. This may lead to variations of SE and the power systems' overall behaviours. Recently, the importance of noises on the analysis of devices and processes of power grids has received considerable attention. Hwang et al. [33] analysed frequency errors and angle variations when the noises were taken into consideration by the method of discrete Fourier transform and found that noises result in main frequency estimate errors.

Sangswang et al. [34] investigated the characteristics of random noises from experiments; and a switching pulse-width modulated dc-dc converter performance under the existence of the random noises was studied. Yang and Cao [35] studied the power hybrid harvester based harvested under the influence of Gaussian coloured noises and found that noise intensities will bring the stochastic bifurcation. The chaos situation of power grids under noises with certain a boundary was investigated in [36]. All these work suggest that the noises can make a big difference to analysis of power systems both at component level and system level.

In this chapter, we investigate effects of different SEs due to noises in PMU data on intelligent system emergency control schemes for power systems against cascading failures. As there is no IEEE standard regarding the noises in PMU data, different researchers apply different types of noises with various degrees of noise power [37] [38] [39]. In [40], with some convincing evidences, authors claimed that 45dB's SNR may be selected as a practical estimate of noise power. Thus, in this chapter, unless otherwise specified, we shall mainly adopt this SNR in our extensive numerical simulations.

For the remaining content, in Section 2.2, we show the protection framework that is adopted to prevent power systems from cascading failures, followed by the emergency control scheme formulated as an optimization problem. Section 2.3 presents comparisons between system emergency control solutions and the corresponding system control results according to the SE results subject to different levels of noises in PMU data. Finally this chapter is concluded in Section 2.4.

2.2 Portection Architecture

Near real-time operating states of power systems can be monitored as PMUs are installed and applied at most key nodes [41]. The control centre offers the functions of estimating the branches' parameters, identifying the disturbances and taking the proper

Figure 2.1: Protection framework of power grids against cascading outages with different SE results.

protection actions against propagation of failures [42]. The protection framework based on emergency control schemes (see Figure 2.1) is composed of three main blocks including state estimation where branches' parameters and contingencies are identified, prediction of cascading failures by cascading loop and protection schemes. The emergency control scheme in our research mainly cover the adjustments of power on the corresponding buses for preventing grids from cascading blackouts.

As mentioned before, different SE results subject to different levels of noises in PMU data will be investigated. The key task is to test how different SE results will affect the emergency control scheme' solutions and the corresponding control results during cascading failures.

2.2.1 Outage Model

Cascading failure is not uncommon in power systems, where the initial disturbances trigger a sequence of one or more dependent component failures [43]. Reasonably, we assume such disturbances (natural or anthropic factors) give rise to changes in admittances of certain transmission branches. All these disturbances can be identified through SE algorithms based on PMU data. However, different SE results may be generated based on PMU data of different qualities.

We assume there is a power grid with n transmission lines and n_b buses. The branch admittance affected by disturbances δ is

$$Y_p^1 = Y_p^0 + \delta$$

where Y_p^0 is the initial branch admittance with n -dimension. Here, we adopt the DC model as our study is only on high-voltage transmission networks [44], that is

$$P = A^T \text{diag}(Y_P^k) A \theta^k \quad (2.1)$$

where P with n_b -dimensional vector stands for flowing power on each bus and $A_{n \times n_b}$ denotes the branch-bus incidence matrix. During each cascading step, such as the k -th, θ^k refers to the voltage angle. The solution to (2.1) is

$$\theta^k = (A^T \text{diag}(Y_P^k) A)^{-1*} P. \quad (2.2)$$

To perform the operation of inverse of a square matrix, the operator $-1*$ is applied in (2.2). Now, the flowing power between Bus i to Bus j is

$$P_{ij}^k = e_i^T A^T \text{diag}(Y_P^k) A e_j (e_i - e_j)^T (A^T \text{diag}(Y_P^k) A)^{-1*} P \quad (2.3)$$

where e_i refers to the unit vector. We have the change of branch admittance as follows

$$Y_p^{k+1} = \begin{cases} Y_p^k + \delta, & k = 0 \\ G(P_{ij}^k) \star Y_p^k, & k \geq 1. \end{cases} \quad (2.4)$$

where the Hadamard product ($\star$) is applied and $G(P_{ij}^k)$ is

$$G(P_{ij}^k) = (g(P_{i1j1}^k, c_{i1j1}), \dots, g(P_{injn}^k, c_{injn}))^T$$

Therein $g(\cdot)$ is the approximation function

$$g(P_{ij}^k, c_{ij}) = \begin{cases} \frac{1 - \sin\sigma[(P_{ij}^k)^2 - c_{ij}^2]}{2}, & \text{otherwise} \\ 0, & |P_{ij}^k| \geq \sqrt{c_{ij}^2 + \frac{\pi}{2\sigma}} \\ 1, & |P_{ij}^k| \leq \sqrt{c_{ij}^2 - \frac{\pi}{2\sigma}} \end{cases} \quad (2.5)$$

where c_{ij} denotes the flowing power threshold for the corresponding line. The relay protection's mechanism may be approximately represented by (2.5), i.e., if the real load of a branch is less than or equal to the threshold value, its circuit breaker is in the status of *on*; otherwise, it is in the status of *off*. Derivative calculations are facilitated by introducing a smooth function $g(\cdot)$ that resembles the step function, allowing differentiation of the function at switching points.

2.2.2 Emergency control Scheme

For preventing cascading outages, a wide variety of studies have been carried out to apply various heuristics to recommend actions that may mitigate stress, typically with no consideration of noises or uncertainties in states. To evaluate the effects of noises in PMU data on system emergency control schemes against cascading outages, without loss of generality, we adopt the general method of Nonrecurring Emergency Control Scheme (NECS) that we proposed in [45] to calculate system emergency control solutions for preventing cascading blackouts. Note that the control actions will only be applied once at a given step (such as m -th cascading step) in this scheme. For integrity of discussions,

we present the main results of NECS here. Readers may refer to ref. [45] for more details.

$$\min \quad R(P, W) \quad (2.6)$$

$$s.t. \quad Y_p^m = G(P_{ij}^{m-1}) \star G(P_{ij}^{m-2}) \star \cdots \star G(P_{ij}^1) \star Y_p^1$$

$$P_{ij}^m = e_i^T A^T \text{diag}(Y_P^k) A e_j (e_i - e_j)^T (A^T \text{diag}(Y_P^k) A)^{-1} P \quad (2.7)$$

$$(P_{ij}^m)^2 \leq c_{ij}^2$$

$$\underline{P}_i \leq P_i \leq \bar{P}_i, \quad i \in I_{n_b}.$$

The cost function $R(P, W)$ is

$$R(P, W) = \|W \star (P - P^0)\|^2 \quad (2.8)$$

where P^0 is the initial power, and $P = (P_1, P_2, \dots, P_{n_b})^T$ denotes the injected power vector when system adopts the adjustment scheme. The boundary constraints of power on Bus i are denoted as $\underline{P}_i$ and $\bar{P}_i$, respectively. The bus significance is reflected by the weight vector $W = (W_1, W_2, \dots, W_{n_b})^T$. The Saddle Point Dynamics, which has been highly successful in designing distributed network control protocols, is applied to solve the problem. We summarize the NECS in Table 2.1.

2.3 Comparisons of Simulation results

In this section, based on the proposed emergency control scheme against cascading failures, we compare the system control schemes' solutions and the corresponding control results on the IEEE 57 bus system based on different SEs subject to different levels of noises. Specifically, the accurate SE represents the case where all the states including parameters of branches are estimated using noise-free PMU data, while the inaccurate SE is generated where all states are estimated from PMU data subject to certain noises. As discussed earlier, existing studies show that an SNR value of 45 dB could be a reasonable approximation for a wide range of PMU data covering from high voltage grids to low

Table 2.1: Nonrecurring Emergency Control Scheme [45]

1:	Monitor the real-time states via PMUs with different data quality
2:	Estimate $Y_p(t)$
3:	if ($Y_p(t) \neq Y_p^0$)
4:	Find the disturbance $\delta = Y_p(t) - Y_p^0$
5:	Initiate cascading failures with (2.3) and (2.4)
6:	Record m -th cascading step's states
7:	Solve Optimization problem
8:	if ($k = m$)
9:	Carry out emergency control schemes
10:	end if
11:	Determine the flowing power
12:	else
13:	Turn to Step 1
14:	end if

voltage girds. In our simulations, unless otherwise specified, we will add an SNR value of 45 dB to PMU data in all cascading steps. Specifically, a certain disturbance is added on the selected branch in the initial cascading step $k = 0$. The flowing power on each branch will be determined according to (2.3) at every cascading step. Once there exists overloading compared with the threshold for each branch, the corresponding branches will be tripped. The emergency control scheme calculated based on accurate and inaccurate SE respectively will be activated if the cascading failures reach at the certain step ($k = m$), to prevent system from cascading blackouts.

2.3.1 Parameter Setting

To investigate the main purpose of the effects of noises on system emergency control schemes' solutions during cascading failures. We set the parameters for solving the optimization problem as follows.

The weight vector is $W = (1, 1, \dots, 1)^T$ that stands for every bus has the same significance level. For simplicity, we adopt the per unit values with the 100MV power

as base. The negative value of total load on each single bus is selected as the minimum injected power. Note that for the load buses, $\bar{P}_i = 0$. Every branch has $1pu$ as the threshold of flowing power. The $\sigma = 10^3$ is set in (2.5). The flowing power threshold for each line is set as $1pu$ with $\sigma = 10^3$ in the approximation function.

2.3.2 Simulation Results

In our graphic representation of power systems, different marks are used to represent different components in power grids. The generator bus is denoted as red rectangle. Green rectangle refers to the load bus. The cyan links stand for branches. Each branch is possible to be selected to initiate the cascade failures. The initial real power (generation) of IEEE 57-bus test system is $12.8393 pu$.

Here, Branch 7 connecting between Bus 6 and Bus 8 (both are generator buses) is selected as the initial disturbance, the cascading outage will end after 8 steps without any control measures.

With the proposed control scheme being implemented, we demonstrate the distributions of changes of power on each bus, according to the calculated system emergency control schemes' solutions, at the 4-*th* cascading step based on accurate or inaccurate SE (with an SNR value of 45dB) in Fig. 2.2 and Fig. 2.3, respectively. The corresponding system control results for these two scenarios are then presented in Figs. 2.4 and 2.5, respectively.

Note that for both scenarios, from the final system topology of IEEE 57 bus system (see Fig. 2.4 and Fig. 2.5), most lines that have flowing power are connected. This indicates that the proposed emergency control actions are effective. Specifically, with accurate SE results, there exist 53 connected branches and the total flowing power with the value of $6.6542 pu$. The objective function R is 0.0968. For inaccurate SE results, the total power flow left is $2.314 pu$ with 44 connected branches.

Figure 2.2: Adjustments of power for each bus under accurate state estimation when branch 7 fails.

Figure 2.3: Adjustments of power for each bus under inaccurate state estimation when branch 7 fails.

With accurate or inaccurate SE results, the distributions of power's changes for each bus is significantly different (see Fig. 2.2 and Fig. 2.3), which in turn may affect adopting correct control measures in practice. The above comparisons show that noises in PMU data may lead to SE variations and further affect the system emergency control schemes' solutions and the corresponding control results.

Now we show the system emergency control schemes' solutions (see Figs. 2.6 and 2.7) and the corresponding control results (see Figs. 2.8 and 2.9) for the above mentioned two scenarios when a branch connects between load buses instead of generator buses fails.

Figure 2.4: Final system topology under accurate state estimation when branch 7 fails.

Figure 2.5: Final system topology under inaccurate state estimation when branch 7 fails.

We choose the Branch 13 connecting between load Bus 13 and load Bus 14 to apply the initial disturbance.

From the simulations results, we can see that the system emergency control schemes' solutions and the corresponding control results under different SE results differ significantly when the initial identified disturbance is added to the branch connected between load buses. With accurate SE results, the system remains 69 connected branches and a total power flow of 8.3456 pu , while there exist 64 connected branches and 7.8324 pu power flow remaining in the inaccurate SE results scenario.

To test the vulnerability of different types of branches against noises, we randomly

Figure 2.6: Adjustments of power for each bus under accurate state estimation when branch 13 fails.

Figure 2.7: Adjustments of power for each bus under inaccurate state estimation when branch 13 fails.

select 5 branches that connect between two load buses and 5 branches that connect at least one generator bus, respectively. The final number of connected branches and a total power flow left under accurate/inaccurate SE results are presented in Table 2.2.

In Table 2.2, the former 5 branches are connected with at least 1 generator bus while the latter 5 are connected between load buses. For the former cases, the average difference of the number of connected branch under accurate and inaccurate SEs is 10.8 while for the latter cases, the average is at a smaller value of 3.8. As to the average difference of power flow left, it is 2.982 per unit for the former cases and only 0.50768 per unit

Figure 2.8: Final system topology under accurate state estimation when branch 13 fails.

Figure 2.9: Final system topology under inaccurate state estimation when branch 13 fails.

for latter ones. The simulation results on different types of buses indicate that different cases with outage of different branches may have different vulnerability levels against noises. Specifically, cases with outages of generator branches may be more vulnerable against noises. This may result from the fact that a relatively large real power and/or reactive power is typically attached to all generator bus directly, which makes these buses strongly depend upon accurate SE to realize appropriate dispatches.

Now we will test the variations of emergency control schemes' solutions and the corresponding control results when the SNR degrades. In practice, the SNR of PMU data may

Table 2.2: The final number of connected branches and total power flow left under accurate/inaccurate SEs.

Branch ID	Bus information		No. connected branches		Power flow left (p.u.)	
	From Bus	To Bus	Accurate	Inaccurate	Accurate	Inaccurate
1	1	2	61	49	7.2843	3.284
2	2	3	67	58	7.8458	4.312
5	4	6	72	60	8.5432	6.583
21	5	6	69	58	7.9326	5.7436
27	12	17	69	59	7.8587	4.632
14	13	15	68	65	8.0324	7.5204
19	4	18	70	66	8.6438	8.1576
33	22	23	72	68	9.8364	9.3379
45	32	33	73	70	9.454	8.9372
64	50	51	67	62	8.533	8.0081

Figure 2.10: Adjustments of power for each bus under inaccurate state estimation (with an SNR value of 30dB) when branch 7 fails.

degrade because of the operation conditions of PMUs and/or measurement inaccuracies. To compare the results between the cases with a 45dB SNR and a 30dB SNR in PMU data respectively, the Branch 7 is again selected to be applied the initial disturbance here. The results when a 30dB SNR is applied for SE are presented in Figs. 2.10 and 2.11, respectively.

As shown in Fig. 2.10, with a 30dB SNR, more than twice of buses are involved in adjustments of injected power as compared with the results in Fig. 2.3 when there is with an SNR of 45dB in PMU data. Also the magnitude of adjustment is bigger when SNR degrades to 30dB. With 38 branches remaining connected and a total 1.9842 *pu* power flow left, a degraded SNR in PMU data results in more unconnected branches and less power flow left. This indicates that the effects of noises for SE play an important role in

Chapter 3

On Convergence Performance of Discrete-Time Optimal Control Based Tracking Differentiator

In this chapter, the convergence performance of a rapid-convergent discrete-time optimal control (DTOC) algorithm with chattering alleviation that was first proposed by Han will be investigated. This DTOC algorithm is the theoretical basis of construction of state estimators in this thesis. In practice, the discrete-time optimal control (DTOC) algorithm is implemented in the form of state feedback for a double-integral system, which is widely used to design controllers, observers and noise-tolerant differentiators. The convergence performance of this DTOC algorithm, however, is still open, which makes its applications not technically sound. This work first presents the convergence analysis of this DTOC algorithm by demonstrating state convergence trajectories. In particular, we demonstrate the state convergence trajectories driven by the corresponding DTOC algorithm within different the isochronic regions (IRs). The comparison simulation results show the advantage of chattering alleviation of the TD in signal-tracking filtering and differentiation acquisition.

3.1 Introduction

The servo control system problems in 1950s have brought the time optimal control (TOC) [46]. The bang-bang control [47], its solution, led to the development of optimal control theory [48] [49]. For that, bang-bang control of the double-integral system is

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u, \end{cases} \quad (3.1)$$

where $|u| \leq r$ gives the limitation of system acceleration. To drag any system state into the steady state within the minimum time, the following control inputs are taken [50] [51]

$$u = -r \text{sign}(s) \quad (3.2)$$

where s is the switching curve function, defined as $\Gamma(x_1, x_2) = x_1 + \frac{x_2|x_2|}{2r}$.

The applications of this TOC solution are limited because the control signals switch frequently [52] [53]. In [54] [55] [56], the non-closed-form of the shortest time feedback control algorithms for linear systems are proposed. In [57] [58] [59], the time-optimal model predictive control (MPC) schemes are presented. One of the pioneer researchers of TOC in the early 1960s was Jinqing Han. He came up with a discrete-time closed-form solution for TOC problem [60]. This algorithm is constructed by the method of the isochronic regions (IRs) for the double-integral system [61] [62], and it has advantageous smoothness [63]. Meanwhile, Han built a tracking differentiator (TD) via this discrete-time optimal control (DTOC) algorithm. The TD baed on the proposed DTOC algorithm allows one to avoid a setpoint jump in the emerging active disturbance-rejection controller (ADRC) [64] [65].

Many engineering applications have used this DTOC algorithm and/or its derivatives. As verified in various numerical and practical results [66] [67] [68], this algorithm is convergent and fast. However, a rigorous convergence proof remains open. Therefore, the objective of our investigation is to test its convergence performance under some

easily checkable procedures. In particular, the state convergence trajectories driven by the corresponding DTOC algorithm in different IRs are presented.

The layout of this chapter is given as follows: the construction of the DTOC algorithm is presented by adopting boundary curves in Section 3.2. The analysis on the convergence of the DTOC algorithm is shown in Section 3.3. Section 3.4 carries out the comparison simulation results to show that this DTOC algorithm based tracking differentiator outperforms the existing one. Section 3.5 concludes the work in this chapter.

3.2 The DTOC algorithm

In this section, for integrity of the work, the construction of the DTOC algorithm, that is, *ghan* algorithm is presented by adopting three boundary curves (Γ_A , Γ_B and Γ_C).

We consider the following discrete-time double-integral system

$$x(k+1) = Ax(k) + Bu(k) \quad (3.3)$$

where $|u| \leq r$ gives the limitation of system acceleration, $A = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix}$, and $B = \begin{bmatrix} 0 & h \end{bmatrix}^T$. We aim to drag any system state to the steady state within the shortest time in discrete-time domain. The following definition of this problem is given.

Definition 3.1: Under given system (3.3) and its initial condition, determine the control inputs, from $u(0)$ to $u(k)$, so as to drive $x(k)$ back to the steady state (the origin) in a finite and minimum steps.

Definition 3.2: $G(k)$ refers to the states' set. These states ($x(0) \in G(k)$), exist a possible control strategy, from $u(0)$ to $u(k)$, that can make the solution of (3.3) satisfy $x(k) = 0$.

We determine the DTOC algorithm as follows. First, for any initial state sequence, at least one admissible control signal sequence exists to make the solution of (3.3) meet

Figure 3.1: $G(1)$ and $G(2)$ on the phase plane

$x(k) = 0$. Under initial condition $x(0)$, the solution is

$$x(k) = A^k x(0) + \sum_{i=1}^k A^{k-i} B u(i-1) \quad (3.4)$$

Let $x(k) = 0$, there exist

$$x(0) = \sum_{i=1}^k \begin{bmatrix} ih^2 \\ -h \end{bmatrix} u(i-1) \quad (3.5)$$

Second,

$$G(k) = \left\{ \sum_{i=1}^k \begin{bmatrix} ih^2 \\ -h \end{bmatrix} u(i-1) \right\} \quad (3.6)$$

Consider

$$G(1) = \left\{ \begin{bmatrix} h^2 \\ -h \end{bmatrix} u(0) \right\}$$

$$G(2) = \left\{ \begin{bmatrix} h^2 \\ -h \end{bmatrix} u(0) + \begin{bmatrix} 2h^2 \\ -h \end{bmatrix} u(1) \right\}$$

When the value of r or $-r$ is applied to the control input $u(i)$, the $G(k)$, $k = 1, 2$ can be shown in Fig. 3.1.

$G(1)$ is a line connected by the following two points

$$\{a_{-1} = \begin{bmatrix} -h^2 r \\ hr \end{bmatrix}, a_1 = \begin{bmatrix} h^2 r \\ -hr \end{bmatrix}\}.$$

Determined by the following four points, $G(2)$ is a parallelogram.

$$\{a_2 = \begin{bmatrix} 3h^2r \\ -2hr \end{bmatrix}, a_{-2} = \begin{bmatrix} -3h^2r \\ 2hr \end{bmatrix}\}$$

and

$$\{b_2 = \begin{bmatrix} -h^2r \\ 0 \end{bmatrix}, b_{-2} = \begin{bmatrix} h^2r \\ 0 \end{bmatrix}\}$$

We assume

$$y = x_1 + hx_2 \tag{3.7}$$

$G(2)$ is surrounded by two pairs of parallel lines described as $y = x_1 + hx_2 = \pm h^2r$ and $y + hx_2 = \pm h^2r$. The states of $G(2)$ are

$$\Omega_0 = \{(x_1, x_2) : |y| \leq h^2r \cap |y + hx_2| \leq h^2r\}$$

Let

$$a_k = \left\{ \sum_{i=1}^k \begin{bmatrix} ih^2 \\ -h \end{bmatrix} u(i-1), u(i) = r \right\}$$

$$a_{-k} = \left\{ \sum_{i=1}^k \begin{bmatrix} ih^2 \\ -h \end{bmatrix} u(i-1), u(i) = -r \right\}$$

$$b_k = \left\{ \sum_{i=1}^k \begin{bmatrix} ih^2 \\ -h \end{bmatrix} u(i-1), u(0) = r, u(i) = -r \text{ for } i > 0 \right\}$$

$$b_{-k} = \left\{ \sum_{i=1}^k \begin{bmatrix} ih^2 \\ -h \end{bmatrix} u(i-1), u(0) = -r, u(i) = r \text{ for } i > 0 \right\}$$

According to (3.5) and the control sequence taken above, we obtain a curve that depicts the minimum time state trajectory, denoted as Γ_A , as follows:

$$\Gamma_A : x_1 + \frac{x_2|x_2|}{2r} + \frac{1}{2}hx_2 = 0 \tag{3.8}$$

where $\Gamma_A = \Gamma_A^+ \cup \Gamma_A^-$ (see Figure 4.2). This state trajectory curve overlaps the above broken line at point $\{a_k, a_{-k}\}$.

Connecting $\{b_{-2}, \dots, b_{-(k-1)}, b_{-k}, \dots\}$ and $\{\dots, b_k, b_{k-1}, \dots, b_2\}$ forms a broken line. We can determine the expression of its corresponding curve, denoted by Γ_B , as follows:

$$\Gamma_B : x_1 - s \frac{x_2^2}{2r} + \frac{5}{2} h x_2 - s h^2 r = 0 \quad (3.9)$$

where $s = \text{sign}(x_1 + h x_2)$ and $\Gamma_B = \Gamma_B^+ \cup \Gamma_B^-$ (see Fig. 3.2). This curve overlaps the corresponding broken line at point $\{b_k, b_{-k}, k \geq 2\}$.

Note that segments $[b_{-k} a_k]$ ($[b_k a_{-k}]$) are all parallel to each other and their midpoints, c_k (c_{-k}). The broken line is denoted as Γ^0 . We can also obtain the expression of its corresponding curve, denoted by Γ_C (see Fig. 3.2), as follows:

$$\Gamma_C : x_1 + \frac{x_2 |x_2|}{2r} + \frac{3}{2} h x_2 = 0 \quad (3.10)$$

Connecting the points $\{\dots, a_k, a_{k-1}, \dots, a_2, b_2, \dots, b_{k-1}, b_k\}$ forms a boundary, Γ^+ , and points $\{\dots, b_{-k}, b_{-(k-1)}, \dots, b_{-2}, a_{-2}, \dots, a_{-(k-1)}, a_{-k}, \dots\}$ as Γ^- . Combining the functions of Γ_A and Γ_B with the boundary transformation in (3.7) produces the following:

$$T(x_1, x_2, r, h) = x_2 + \frac{1}{2} (\sqrt{h^2 r^2 + 8r|y|} - hr) \text{sign}(y), |y| \geq h^2 r \quad (3.11)$$

From this, $T(x_1, x_2, r, h) = -hr$ corresponds to the parabolas connecting a_k and b_k , respectively, and this curve, denoted as $\tilde{\Gamma}^+$, overlaps Γ^+ at points $a_k, b_k, k \geq 2$. Similarly, $T(x_1, x_2, r, h) = hr$ corresponds to the parabolas connecting a_{-k} and b_{-k} , respectively. This curve, $\tilde{\Gamma}^-$, overlaps Γ^- at points $a_{-k}, b_{-k}, k \geq 2$. In addition, $T(x_1, x_2, r, h) = 0$ represents $\tilde{\Gamma}^0$, which overlaps Γ^0 at points $c_{-k}, c_k, k \geq 2$. Moreover, $\tilde{\Gamma}^+$ and $\tilde{\Gamma}^-$ partition the phase plane in the manner of

$$\begin{cases} T(x_1, x_2, r, h) \leq -hr, x = [x_1, x_2]^T \text{ is below } \tilde{\Gamma}^+ \\ T(x_1, x_2, r, h) \geq hr, x = [x_1, x_2]^T \text{ is above } \tilde{\Gamma}^- \\ T(x_1, x_2, r, h) \leq hr, x = [x_1, x_2]^T \text{ is between } \tilde{\Gamma}^+ \text{ and } \tilde{\Gamma}^- \end{cases} \quad (3.12)$$

Figure 3.2: Illustration of Γ_A , Γ_B , and Γ_C , where $\Gamma_A = \Gamma_A^+ \cup \Gamma_A^-$, and $\Gamma_B = \Gamma_B^+ \cup \Gamma_B^-$

Figure 3.3: Illustration of $\tilde{\Gamma}^+$, $\tilde{\Gamma}^-$, and $\tilde{\Gamma}^0$

Up to now, the state points can be divided into four parts: $G(1)$, $G(2)$, the initial condition ($|y| \geq h^2r$) inside the area bounded by $\tilde{\Gamma}^+$ and $\tilde{\Gamma}^-$ and the remaining portion (see Fig. 3.3).

As described by Z. Gao [62], the complete DTOC algorithm is

$$u = -rsat(T(x_1, x_2, r, h), hr) \quad (3.13)$$

where

$$T(x_1, x_2, r, h) = \begin{cases} x_2 + \frac{(\sqrt{h^2r^2 + 8r|y|} - hr)}{2} \text{sign}(y), & |y| > h^2r \\ x_2 + y/h, & |y| \leq h^2r \end{cases} \quad (3.14)$$

The above law can be shown in (3.15).

$$\left\{ \begin{array}{l} u = fhan(x_1, x_2, r, h) \\ d = rh; \ d_0 = hd \\ y = x_1 + hx_2 \\ a_0 = \sqrt{d^2 + 8r|y|} \\ T = \begin{cases} x_2 + \frac{(a_0-d)}{2}\text{sign}(y), & |y| > h^2r \\ x_2 + y/h, & |y| \leq h^2r \end{cases} \\ fhan = - \begin{cases} r\text{sign}(T), & |T| > d \\ r\frac{T}{d}, & |T| \leq d \end{cases} \end{array} \right. \quad (3.15)$$

3.3 Convergence analysis

The main purpose of this section is to rigorously analyse the convergence of the DTOC algorithm in (3.13) for system (3.3) by demonstrating the state convergence trajectories. For $k \geq 2$ in that quadrant, curve $\tilde{\Gamma}^+$ overlaps Γ_B^+ . Similarly, for $k \geq 2$, curve $\tilde{\Gamma}^-$ overlaps Γ_A^+ . In the latter proof, we use both curves without instructions. The main result is stated as Theorem 3.1 as follows.

Theorem 3.1 *For system (3.3) and the determined control inputs, $u(0), u(1), \dots, u(k)$ ($|u(k)| \leq r$) in (3.13), any initial state point $M(x_1(k), x_2(k))$ can converge into the origin in a minimum and finite number of steps.*

Before presenting the convergence analysis on this DTOC algorithm, we need to figure out whether isochronic region for any initial condition is unique or not. In other words, we have to show the monotonic characteristic of IR boundaries to clarify that any initial condition belongs to only one IR. In order to show this, equation in (3.11) is introduced, which partitions the whole phase plane in some manners (see Fig. 3.3).

Let

$$T(x_1, x_2, r, h) = x_2 + \frac{1}{2}(\sqrt{w} - hr)\text{sign}(y)$$

where $\sqrt{w} = \sqrt{h^2 r^2 + 8r|y|}$. We have

$$\begin{cases} \frac{\partial T}{\partial x_1} = \frac{2r}{\sqrt{w}} > 0 \\ \frac{\partial T}{\partial x_2} = \frac{2hr}{\sqrt{w}} > 0 \end{cases} \quad (3.16)$$

which indicates that the boundaries of IR are monotonic. Thus, according to (3.12), different values for h and r lead to different boundaries, i.e.,

$$\begin{cases} T(x_1, x_2, r, h) = c_1 \\ T(x_1, x_2, r, h) = c_2 \end{cases}$$

where $c_1 \neq c_2$. Therefore, a curve family $T(x_1, x_2, r, h) = c$ must partition the entire phase plane into different IRs (Fig. 3.3). In doing so, we confirm that any initial condition can only belong to one IR.

Proof: Then convergence analysis of the *fhan* algorithm can be carried out in the following several steps.

Step 1: Any initial state point $M(x_1(k), x_2(k))$ located below $\tilde{\Gamma}^+$ or above $\tilde{\Gamma}^-$ (see (3.12)) can converge into the region between $\tilde{\Gamma}^+$ and $\tilde{\Gamma}^-$.

Under this condition, the control law takes on an extreme value according to the *fhan* algorithm. That is,

$$u(k) = -r \text{sign}(s(k)) \quad (3.17)$$

where $s(k) = x_1 + \frac{x_2|x_2|}{2r} + \frac{1}{2}hx_2$ is curve Γ_A . We assume that the initial state point $M(x_1(0), x_2(0))$ (denoted as $M(x_1, x_2)$ for simplicity) located above $\tilde{\Gamma}^-$ on the right side of the phase plane, where $s(k) > 0$. The following Lyapunov function is the constructed:

$$\begin{aligned} \Delta s(k) &= s(k+1) - s(k) \\ &= hx_2(k) - \frac{1}{2}h^2r - h(x_2(k) - \frac{1}{2}hr)\text{sign}(x_2(k)) \\ &= h(x_2(k) - |x_2(k)|) - \frac{1}{2}h^2r(1 - \text{sign}(x_2(k))) \leq 0 \end{aligned} \quad (3.18)$$

Two cases are possible for the value of $\Delta s(k)$.

(1) If $x_2(k) > 0$, then $\Delta s(k) = 0$.

Figure 3.4: Illustration of state trajectory when initial state is located above Γ_B^+ or below Γ_A^+ .

According to (3.13), the initial state $x_2(k)$ would keep decreasing until it arrives at Γ_A . There exists $x_2(k+1) = x_2(k) - hr$, or $x_2(k) = x_2(0) - khr$. This produces a positive constant $k_0 = \frac{x_2(0)}{hr}$ that makes $x_2(k) < 0$ when $k > k_0$. Therefore, any initial state located above Γ_A on the upper-phase plane can be driven to the lower-phase plane where $x_2(k) < 0$.

(2) If $x_2(k) < 0$, then $\triangle s(k) = -2h|x_2(k)| - h^2r < -h^2r < 0$.

When $s(k) > 0$, $s(k+1) - s(k) < -h^2r$ or $s(k) < s(0) - kh^2r$. Clearly there is a positive constant $k_1 = \frac{s(0)}{h^2r}$ that guarantees $s(k) < 0$ when $k > k_1$.

A similar conclusion can be obtained when the initial state $M(x_1(k), x_2(k))$ is located below $\tilde{\Gamma}^+$. This demonstrates that any initial state $M(x_1, x_2)$ located below $\tilde{\Gamma}^+$ or above $\tilde{\Gamma}^-$ can converge into the region between $\tilde{\Gamma}^+$ and $\tilde{\Gamma}^-$ (see Fig. 3.4).

Step 2: Any initial state point $M(x_1(k), x_2(k))$ located inside the IR between $\tilde{\Gamma}^+$ and $\tilde{\Gamma}^-$ and beyond $G(2)$ cannot extend beyond that region but instead converges into $G(2)$ along Γ_A^+ (i.e., $\tilde{\Gamma}^-$).

When any initial state point $M(x_1(k), x_2(k))$ satisfies the above condition, according

to (3.11), then the following applies:

$$x_2(k) + \frac{1}{2}(\sqrt{w(k)} - hr) = \theta(k)hr \quad (3.19)$$

where $\sqrt{w(k)} = \sqrt{h^2r^2 + 8r|y(k)|}$ and $\theta(k) \in [-1, 1]$ is a parameter used to determine a different curve. For example, when $\theta(k) = -1$, it corresponds to curve Γ_A^+ (i.e., $\tilde{\Gamma}^-$); when $\theta(k) = 0$, it corresponds to curve Γ_C^+ (i.e., $\tilde{\Gamma}^0$). Note that $\text{sign}(y(k)) = 1$ in (11) because the initial state point in **Step 2** satisfies $y(k) = x_1(k) + hx_2(k) < 0$.

According to the *fhan* algorithm, the control law takes on $u = -\frac{T(x_1, x_2, r, h)}{h} = -r\theta$. Inserting it into (4.3) and combining with (3.19), we achieve $x_2(k+1) = \frac{1}{2}(hr - \sqrt{w(k)}) < 0$. Furthermore, without a loss of generality, we assume that $M(x_1(k+1), x_2(k+1))$ is located on curve $\tilde{\Gamma}(k+1)$, which then leads to

$$\begin{aligned} \theta(k+1) &= \frac{1}{hr}[x_2(k+1) + \frac{1}{2}(\sqrt{w(k+1)} - hr)] \\ &= \frac{1}{hr}[\frac{1}{2}(hr - \sqrt{w(k)}) + \frac{1}{2}(\sqrt{w(k+1)} - hr)] \\ &= \frac{\sqrt{w(k+1)} - \sqrt{w(k)}}{2hr} < 0 \end{aligned} \quad (3.20)$$

Based on the analysis above, any initial state point $M(x_1(k), x_2(k))$ located inside the IR between $\tilde{\Gamma}^+$ and $\tilde{\Gamma}^-$ and beyond $G(2)$ cannot step outside of this region but would simultaneously enter the region below curve Γ_C^+ , i.e., $\tilde{\Gamma}^0$ (see Fig. 3.5).

When states locate in the region beneath curve Γ_C^+ , the position of $M(x_1(k+1), x_2(k+1))$ must be determined on the phase plane. In doing so, this introduces an auxiliary curve, $y = x_2(k+1)$, that intersects with curve Γ_A^+ at point $A(x_1(A), x_2(k+1))$ (Fig. 3.5). According to (3.8), the following is produced:

Figure 3.5: Illustration of state trajectory when initial state is located inside IR between Γ_B^+ and Γ_A^+ and beyond $G(2)$.

$$\begin{aligned}
 x_1(A) &= \frac{x_2(k+1)^2}{2r} - \frac{1}{2}hx_2(k+1) \\
 &= \frac{x_2(k+1)}{2r}[(x_2(k+1) - hr)] \\
 &= -\frac{1}{2r} \frac{(hr - \sqrt{w(k)})}{2} \frac{(hr + \sqrt{w(k)})}{2} \\
 &= \frac{1}{8r}[w(k) - h^2r^2] \\
 &= \frac{8r|y(k)|}{8r} \\
 &= x_1(k+1)
 \end{aligned} \tag{3.21}$$

The above formula demonstrates that state point $M(x_1(k+1), x_2(k+1))$ coincides with point A , representing the state point that will arrive at curve Γ_A^+ . Meanwhile, because $\nabla x_2(k) = hu(k)$ and $u = -\frac{T(x_1, x_2, r, h)}{h}$, $|x_2(k)|$ will decrease monotonically, meaning that initial state point $M(x_1(k), x_2(k))$ converges to $G(2)$ along Γ_A^+ (Fig. 3.5).

Step 3: Any initial state point $M(x_1(0), x_2(0))$ located inside $G(2)$ can converge into the origin with at most two steps.

When the initial state point $M(x_1(0), x_2(0))$ is inside $G(2)$, then

$$u(0) = -\frac{x_1(0) + 2hx_2(0)}{h^2} = \frac{x_2(0) + y(0)/h}{h} \tag{3.22}$$

Furthermore, we have

$$x(1) = \begin{bmatrix} x_1(0) + hx_2(0) \\ x_2(0) + hu(0) \end{bmatrix} = \begin{bmatrix} y(0) \\ -y(0)/h \end{bmatrix} \in G(1) \quad (3.23)$$

Therefore, any initial condition $M(x_1(0), x_2(0)) \in G(2)$ will be driven into $G(1)$ and then back to the origin. ■

3.4 Simulations

Numerical simulations are presented to compare the *Fhan*-based TD with the differentiator based on the sliding-mode technique. According to *Fhan* algorithm, a discrete-time solution for (3.3) could be obtained as [55]

$$u = fhan(x_1 - v, x_2, r, h) \quad (3.24)$$

where v is the input signal.

DI. The robust exact differentiator utilizes the sliding-mode technique [73].

$$\begin{cases} \dot{x}_1 = x_2 - \alpha|x_1 - v|^{0.5}\text{sign}(x_1 - v), \\ \dot{x}_2 = -\beta\text{sign}(x_1 - v) \end{cases}$$

where $\alpha, \beta > 0$ are tuning parameters.

DII. The *Fhan* based TD [60] [61].

$$\begin{cases} u(k) = Fhan(x_1(k) - v(k), x_2(k), r_0, c_0h), \\ x_1(k+1) = x_1(k) + hx_2(k), \\ x_2(k+1) = x_2(k) + hu(k), |u(k)| \leq r \end{cases}$$

The same initial value, that is, $x_1(0) = 0$, $x_2(0) = 2$ and input signal sequence $v(t) = 1 + \sin(3t) + \gamma(t)$ are adopted for all simulations. Therein, $\gamma(t)$ is a white noise with an intensity of 0.001. For differentiator **DI**, the parameters are $\alpha = 1.5$, $\beta = 8$, and $h = 0.0001$. For the differentiator **DII**, the sampling step is $h = 0.005$; the quickness factor, $r_0 = 200$; the damping factor, $c_1 = 2$; and the filtering factor, $c_0 = 5$. The results

Figure 3.6: Comparison of signal-tracking between two differentiators

Figure 3.7: Comparison of differentiation acquisition between two differentiators

of filtering and differentiation tracking under the above two algorithms are demonstrated in the following figures.

According to the simulations above, we determine that, the **DII** has better filtering and differentiation tracking than the differentiator produced via the sliding-mode technique **DI** in the aspects of dynamic process and steady exactness.

3.5 Conclusions

The convergence analysis of *fhan* algorithm has been first investigated in this work by adopting Lyapunov functions. The proof guarantees that any given initial state can converge to the steady state within a finite number of steps, which not only makes its applications technically sound, but also provides the theoretical foundation of ADRC. Comparison simulation results showed that this DTOC algorithm based TD has better performance in signal-tracking filtering and differentiation acquisition than the other existing ones, in particular, in dynamic process and chattering alleviation. The readers can refer to [70] [71] [72] for more details of the applications.

Chapter 4

On Convergence of an Enhanced Discrete-Time Optimal Control Algorithm

In this chapter, the convergence of a new and simple DTOC algorithm that will be used to estimate states of power systems in later chapters is presented. First, a new time optimal control algorithm with simple structure is constructed. The control signal sequence in this approach is determined by the linearized criterion according to the position of the initial state point on the phase plane. Second, the proposed DTOC algorithm's convergence is proved by demonstrating the convergence path of the state point sequence driven by the corresponding control signal sequence. Finally, some numerical simulations are presented to show the effectiveness of this new DTOC algorithm.

4.1 Introduction

Here, to make the chapter to be more readable, we briefly present the bang-bang control once again as follows:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u, \quad |u| \leq r \end{cases} \quad (4.1)$$

where r is a constant constraint of the control inputs. In order to drive any initial condition to the origin in the minimum time, the following control inputs are taken

$$u = -r\text{sign}(s) \quad (4.2)$$

and the switching curve function is

$$\Gamma(x_1, x_2) = x_1 + \frac{x_2|x_2|}{2r}. \quad (4.3)$$

In [61] [62], a closed-form solution of DTOC is attained. However, the structure of the algorithm (*Fhan*) is complex with non-linear calculations including square roots calculation, which makes this algorithm difficult to be applied in engineering applications.

Facing with the problems above, we present a new DTOC algorithm with simple structure and prove its convergence. The features of this new DTOC algorithm are given as follows.

I: The mathematical derivation of a new closed-form discrete optimal control algorithm for the discrete form of system (4.1) is presented. This control algorithm avoids the overshoot and the high frequency chattering of the control signals that exist in digitized bang-bang control. Compared with the algorithm proposed in [61] [62], the discrete optimal control algorithm is determined by the linearized criterion according to the position of the initial state point on the phase plane, which equips this control algorithm with a simpler structure, making it much easier to be applied in the practical engineering.

II: The convergence of the discrete-time optimal control algorithm is proved by showing the convergence path of the state point sequence driven by the derived control algorithm.

The layout of this chapter is as follows: the discrete-time optimal control (DTOC) algorithm is constructed in Section 4.2. In Section 4.3, the convergence of DTOC algorithm is proved. Comparison experiments are conducted to illustrate the effectiveness of the proposed DTOC algorithm in Section 4.4, followed by concluding remarks in Section 4.5.

4.2 Discrete-Time Optimal Control algorithm

We consider the following discrete-time double-integral system

$$x(k+1) = Ax(k) + Bu(k) \quad (4.4)$$

where $|u(k)| \leq r$ is the constraint of control inputs, $A = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix}$, and $B = \begin{bmatrix} 0 \\ h \end{bmatrix}$. We aim to drive the initial condition to the origin within the shortest time in discrete-time domain. The following definition of this problem is given.

DTOC problem: Under given system (3.3) and its initial condition, determine the control inputs, from $u(0)$ to $u(k)$, such that $x(k)$ is driven back to the origin in a finite and minimum steps.

For driving the initial state back to the origin in the continuous system in (4.1), the control signal switches between its two extreme values around the switching curve $\Gamma(x_1, x_2)$ in (4.3). That is, when the initial state is located over the switching curve, the control signal takes on the extreme values, i.e., $u = -r$; otherwise the control signal takes on $u = +r$. The control signal switches the sign after reaching the switching curve. For a continuous system, the control signal can switch instantaneously. For a discrete time system, however, the process of sign switching of the control signal will take place within the sampling period h . During the process, corresponding state sequences would locate in a certain region (denoted as Ω) near the switching curve. The control signals for the state sequences in the region Ω are determined by the linearized criterion. The control signal varies from a certain positive (negative) value to a negative (positive) value when the control signal u passes from one side of the region Ω to the other. All initial state sequences located outside the region Ω when the control signal takes on extreme value, i.e., $u = +r$ or $u = -r$, would locate at certain curves, referred as boundary curves Γ_A and Γ_B . The region Ω is surrounded by these boundary curves.

The basic idea in deriving the DTOC algorithm is to find the control signal sequence for any initial state point $x(0) \in \Omega$ or $x(0) \notin \Omega$. The whole task is divided into two parts:

- i: Determine the boundary curves of the region Ω based on state back-stepping approach.
- ii: For $x(0) \in \Omega$ or $x(0) \notin \Omega$, find the corresponding control signal sequence as a function of $x(0)$.

4.2.1 Determination of Boundary Curves

The solution of (4.4) with the initial condition $x(0)$ is

$$x(k+1) = A^{k+1}x(0) + \sum_{i=0}^k A^{k-i}Bu(i) \quad (4.5)$$

where $x(0) = [x_1(0), x_2(0)]^T$ and $i = 0, 1, 2, \dots, k$. It manifests that $x(k+1) = 0$. Therefore, the initial condition satisfies

$$x(0) = \sum_{i=0}^k \begin{bmatrix} (i+1)h^2 \\ -h \end{bmatrix} u(i). \quad (4.6)$$

Based on the state back-stepping approach above, we determine the two boundary curves denoted as Γ_A and Γ_B in the followings.

We first determine the boundary curve Γ_A . Suppose that $\{a_{+k}\}$, $\{a_{-k}\}$ are the sets of any $x(0)$ that can be driven back to the origin with the control signal sequence $u(i) = +r$ and $u(i) = -r$, $i = 0, 1, 2, \dots, k$, respectively. Denote that all initial states in the set $\{a_{+k}\}$ consist of Γ_A^+ and all initial states in the set $\{a_{-k}\}$ consist of Γ_A^- .

For the set $\{a_{+k}\}$, the following result can be obtained when the control signal sequence takes on $u(i) = +r$ according to (4.6)

$$x(0) = r \sum_{i=0}^k \begin{bmatrix} (i+1)h^2 \\ -h \end{bmatrix}. \quad (4.7)$$

Then there exist $x_1(0) = rh^2(\frac{k^2}{2} + \frac{3k}{2} + 1)$ and $x_2(0) = -rh(k+1) < 0$. Simplifying $x(0)$ into x and eliminating the variable k , the boundary curve Γ_A^+ is $x_1 = \frac{x_2^2}{2r} - \frac{1}{2}hx_2$, where

$x_2 < 0$. Similarly, we can get the boundary curve Γ_A^- : $x_1 = -\frac{x_2^2}{2r} - \frac{1}{2}hx_2$, where $x_2 > 0$.

Therefore, the whole boundary curve Γ_A is

$$\Gamma_A : x_1 + \frac{x_2|x_2|}{2r} + \frac{1}{2}hx_2 = 0. \quad (4.8)$$

We then determine the boundary curve Γ_B . Suppose that $\{b_{+k}\}, \{b_{-k}\} (k > 2)$ are the sets of any initial state $x(0)$ that can be driven back to the origin when the control signal takes on $u(0) = -r$ or $u(0) = +r$ in the first step, and from the second step on, the control sequence takes on $u(i) = +r$ or $u(i) = -r, i = 1, 2, \dots, k$. Similarly, the boundary curve Γ_B consists of Γ_B^+ and Γ_B^- .

For the set $\{b_{+k}\}$, according to the rule of choosing the control signal sequence above, we can obtain $x_1 = rh^2(\frac{k^2}{2} + \frac{3k}{2} - 1)$ and $x_2 = -rh(k-1) < 0$. After eliminating the variable k , the boundary curve Γ_B^+ is $x_1 = \frac{x_2^2}{2r} - \frac{5}{2}hx_2 + h^2r$ and there exists $x_1 + hx_2 = \frac{1}{2}rh^2k(k+1) > 0$. Similarly, we can obtain the boundary curve Γ_B^- : $x_1 = -\frac{x_2^2}{2r} - \frac{5}{2}hx_2 - h^2r, x_1 + hx_2 < 0$. Therefore, the whole boundary curve Γ_B is

$$\Gamma_B : x_1 - \tilde{s}\frac{x_2^2}{2r} + \frac{5}{2}hx_2 - \tilde{s}h^2r = 0 \quad (4.9)$$

where $\tilde{s} = \text{sign}(x_1 + hx_2)$. The above two boundary curves of the region Ω are determined with the state back-stepping method, and they are shown on the phase plane in Fig. 4.1.

Remark: The whole phase plane is divided into two regions, the region Ω surrounded by the two boundary curves Γ_A and Γ_B , and the rest part $R^2 - \Omega$.

4.2.2 Construction of the Discrete TOC Law

In this subsection, the DTOC law is obtained constructively based on the boundary curves and regions proposed above. As shown in Fig. 4.1, we assume that for any initial state $M(x_1, x_2)$ in the fourth quadrant ($x_1 > 0, x_2 < 0$), there is an auxiliary line $x_2 =$

Figure 4.1: Illustration of the region Ω , the two boundary curves Γ_A , Γ_B , the switching curve Γ and two intersection points A , B that intersected by an auxiliary line and Γ_A , Γ_B .

$x_2(M)$ that intersects with the boundary curves at points A and B (in the direction of x_1). Their x -axis value x_A and x_B are

$$\begin{cases} x_A = \frac{x_2^2}{2r} + \frac{1}{2}h|x_2| \\ x_B = \frac{x_2^2}{2r} + \frac{5}{2}h|x_2| + h^2r \end{cases} \quad (4.10)$$

For any initial state $M(x_1, x_2)$ satisfying $x_1 < x_A$ or $x_1 > x_B$, the control signal is taken on $u = +r$ or $u = -r$. For any initial state $M(x_1, x_2)$ satisfying $x_1 \in [x_A, x_B]$, the control signal can be taken on as follows:

$$u = -r\epsilon \text{sign}(x_2) \quad (4.11)$$

where $\epsilon = \frac{x_B + x_A - 2|x_1|}{x_B - x_A}$.

When the initial state $M(x_1, x_2)$ is in the second quadrant, the control signal sequence can be constructed similarly.

However, when the initial state $M(x_1, x_2)$ (located outside the region Ω) is in the first or third quadrant, there are two different cases for choosing the control signal. When

$M(x_1, x_2)$ cannot be driven back to the origin within two steps, that is, the initial state does not satisfy the condition $x_1^2 + x_2^2 = 0$, let $u = -r \text{sign}(x_1 + hx_2)$. When $M(x_1, x_2)$ can be driven back to the origin within two steps, the initial state $x(0)$ and the corresponding control signal sequence satisfies (4.6), i.e.,

$$\begin{cases} x_1(1) = x_1(0) + hx_2(0) \\ x_2(1) = x_2(0) + hu(0) \\ x_1(2) = x_1(1) + hx_2(1) \\ x_2(2) = x_2(1) + hu(1). \end{cases}$$

Furthermore, when $M(x_1, x_2)$ can be driven back to the origin within two steps, the corresponding control signals can be derived as follows:

$$\begin{cases} u(0) = -\frac{x_1(0) + 2hx_2(0)}{h^2} \\ u(1) = \frac{x_1(0) + hx_2(0)}{h^2}. \end{cases} \quad (4.12)$$

The condition $u(1) \leq r$ is a necessary condition for driving the initial state back to the origin within two steps. If it is not satisfied, the initial state cannot be driven back to the origin within two steps. If it is satisfied, the control signal can take on $u(0)$ and $u(1)$ in (4.12) to drive the initial state back to the origin.

The region in which any $x(0)$ can be driven back to the origin within two steps, denotes as Ω_2 , is surrounded by two pairs of parallel lines $x_1 + hx_2 = \pm h^2r$ and $x_1 + 2hx_2 = \pm h^2r$. As can be shown in Fig. 4.2, Ω_2 is shaped by $(-h^2r, 0)$, $(-3h^2r, 2hr)$, $(h^2r, 0)$ and $(3h^2r, -2hr)$.

The complete DTOC algorithm is described as follows:

Step 1: Setting $z_1 = x_1 + \lambda hx_2$, $z_2 = z_1 + hx_2$, where $\lambda \in (0, 1]$ is a tuning parameter to determine the different characteristic points x_A, x_B . If $|z_1| > h^2r$ or $|z_2| > h^2r$, then $M(x_1, x_2)$ cannot be driven back to the origin within two steps, i.e., $M(x_1, x_2) \notin \Omega_2$, and go to next step; otherwise, go to Step 5;

Step 2: If the initial state $M(x_1, x_2)$ satisfies $x_1x_2 \geq 0$ and $M(x_1, x_2) \notin \Omega_2 \cup \Omega$, then the control signal takes on $u = -r \text{sign}(x_1 + hx_2)$;

Figure 4.2: Illustration of the region Ω_2 .

Step 3: Determine the boundary of the region Ω , i.e., $x_A = \frac{x_2^2}{2r} + \frac{1}{2}h|x_2|$ and $x_B = \frac{x_2^2}{2r} + \frac{5}{2}h|x_2| + h^2r$;

Step 4: If $|x_1| \geq x_B$, then the control signal takes on $u = -r\text{sign}(x_1)$; if $|x_1| \leq x_A$, then the control signal takes on $u = r\text{sign}(x_1)$; otherwise, the control signal takes on $u = -r\epsilon\text{sign}(x_2)$ where $\epsilon = \frac{x_B + x_A - 2|x_1|}{x_B - x_A}$;

Step 5: If the initial state $M(x_1, x_2) \in \Omega_2$, then the control signal takes on $u = -\frac{x_1 + 2hx_2}{h^2}$;

Step 6: The algorithm ends.

From the deduction above, the discrete time optimal control algorithm (**DTOC algorithm**) as a function of x_1 , x_2 , r and h , denoted as $u(k) = Ftd(x_1(k), x_2(k), r, h)$, is obtained.

Remark: The DTOC algorithm presented above demonstrates that the control signal u in discrete time does not always take on the extreme value, which is different from its continuous time counterpart.

Remark: The essence of DTOC algorithm proposed in [61] is a non-linear boundary transformation, which includes complex non-linear calculations. The control signal of the

proposed DTOC algorithm is determined by piecewise linear function according to the relative positions of the initial state and the corresponding x-axis value of the intersection points A and B , which allows the DTOC algorithm to have a simple structure.

Remark: For the **Step 1** of the algorithm, the different value of λ can lead to different points x_A, x_B and different control signal according to (4.11) when the initial state $M(x_1, x_2)$ is located inside the region Ω . For different applications, one needs to choose a corresponding λ to make systems perform better.

4.3 Convergence proof of time optimal control algorithm

The main result in this section is stated as the following Theorem.

Theorem 4.1: Given the system (4.4) and any initial state $x(0)$, the state $x(k)$ can converge to the origin in the minimum and finite number of steps with the discrete time optimal control signal sequence $u(0), u(1), \dots, u(k)$ determined by $u(k) = Ftd(x_1(k), x_2(k), r, h)$, subject to the constraint of $|u(k)| \leq r$. That is, $x_1(k) \rightarrow 0$ and $x_2(k) \rightarrow 0$ with $u(k^*)$, where $|u(k)| \leq r$ and $k^* = \min[k | x(k+1) = 0]$.

Proof: Here we will split the proof into two steps according to the position of the initial state on the phase plane.

Step A: Any initial state $x(0)$ located outside the region Ω can converge to the region Ω when the control signal sequence takes on the extreme value.

Consider the following control system when the initial state is located outside the region Ω :

$$\begin{cases} x_1(k+1) = x_1(k) + hx_2(k) \\ x_2(k+1) = x_2(k) + hu(k) \\ u(k) = -r \text{sign}(s(k)) \end{cases} \quad (4.13)$$

where $s(k)$ is the boundary curve $\Gamma_A : x_1 + \frac{x_2|x_2|}{2r} + \frac{1}{2}hx_2 = 0$.

When the initial state $M(x_1, x_2)$ is located above the Γ_A , there exists $s(k) > 0$. The following Lyapunov function is constructed

$$\begin{aligned}\Delta s(k) &= s(k+1) - s(k) \\ &= hx_2(k) - \frac{1}{2}h^2r - h(x_2(k) - \frac{1}{2}hr)\text{sign}(x_2(k)) \\ &= h(x_2(k) - |x_2(k)|) - \frac{1}{2}h^2r(1 - \text{sign}(x_2(k))) \leq 0\end{aligned}\tag{4.14}$$

There are two possible cases for the value of $\Delta s(k)$.

(1) If $x_2(k) > 0$, then $\Delta s(k) = 0$.

According to (4.13), the initial state $x_2(k)$ will keep decreasing until it arrives at Γ_A . There exists $x_2(k+1) = x_2(k) - hr$, that is, $x_2(k) = x_2(0) - khr$. Hence there exists a positive constant $k_0 = \frac{x_2(0)}{hr}$ that can make $x_2(k) < 0$ when $k > k_0$. Therefore, any initial state located above Γ_A on upper phase plane can be driven to lower phase plane, i.e., $x_2(k) < 0$.

(2) If $x_2(k) < 0$, then $\Delta s(k) = -2h|x_2(k)| - h^2r < -h^2r < 0$.

When $s(k) > 0$, there exists $s(k+1) - s(k) < -h^2r$, that is, $s(k) < s(0) - kh^2r$ and clearly there is a positive constant $k_1 = \frac{s(0)}{h^2r}$ that can guarantee $s(k) < 0$ when $k > k_1$.

The above statement manifests that the initial state $x(0)$ located outside the region Ω can approach the boundary curve Γ_A and converge to the region Ω when the control signal sequence takes on the extreme value described in (4.13). Similar conclusion can be obtained when the initial state $M(x_1, x_2)$ is located below Γ_A .

Step B: The state sequence located inside the region Ω can be driven into the region Ω_2 with the control signal in (4.11) in a limited number of steps.

In this step, we only need to prove the convergence of the state sequence on the fourth quadrant because the region Ω is symmetric in the second and fourth quadrant. For simplicity, we may still adopt parametric expressions by introducing the parameters α and β ($\alpha > 0, \beta > 0$), where any state point on the phase plane can be expressed as

$x_1 = \alpha h^2 r, x_2 = -\beta h r$. Further, for convenience, we may as well adopt dimensionless expression by introducing another parameter θ to ignore the value of h and r .

Based on the above, the boundary curves Γ_A and Γ_B on the fourth quadrant in parametric form are

$$\begin{cases} \Gamma_A^+ : \alpha = \frac{\beta}{2}(\beta + 1) \\ \Gamma_B^+ : \alpha = \frac{\beta}{2}(\beta + 1) + (2\beta + 1). \end{cases} \quad (4.15)$$

Any initial state point $M(x_1, x_2)$ located inside the region Ω in dimensionless form can be

$$\begin{cases} \alpha = \frac{\beta}{2}(\beta + 1) + (2\beta + 1)\theta \\ \beta = \beta \end{cases} \quad (4.16)$$

where $0 \leq \theta \leq 1$. Its corresponding control signal sequence according to **Step 4** in Section 4.2 is

$$\begin{aligned} u &= -r \frac{x_B + x_A - 2|x_1|}{x_B - x_A} \text{sign}(x_2) \\ &= r(1 - 2\theta) \end{aligned} \quad (4.17)$$

where $x_A = \frac{\beta(\beta+1)}{2}h^2r$ and $x_B = [\frac{\beta(\beta+5)}{2} + 1]h^2r$.

The state variables at steps k and $(k + 1)$ in dimensionless form can be

$$\begin{cases} x_1(k) = \alpha(k)h^2r \\ x_2(k) = -\beta(k)hr \\ x_1(k + 1) = \alpha(k + 1)h^2r \\ x_2(k + 1) = -\beta(k + 1)hr \end{cases}$$

Because of (4.4), there exist

$$\begin{cases} \alpha(k + 1) = \alpha(k) - \beta(k) \\ \beta(k + 1) = \beta(k) - 1 + 2\theta(k) \end{cases}$$

According to (4.16), we can derive $\alpha(k + 1) = \frac{\beta(k+1)[\beta(k+1)+1]}{2} + [2\beta(k + 1) + 1]\theta(k + 1)$.

Therefore, the expressions of $\alpha(k + 1)$, $\beta(k + 1)$ and $\theta(k + 1)$ can be derived as follows:

$$\begin{cases} \alpha(k + 1) = \frac{\beta(k+1)[\beta(k+1)+1]}{2} + [2\beta(k + 1) + 1]\theta(k + 1) \\ \beta(k + 1) = \beta(k) + 2\theta(k) - 1 \\ \theta(k + 1) = \frac{2\theta(k)[1-\theta(k)]}{2\beta(k)-1+4\theta(k)}. \end{cases} \quad (4.18)$$

According to (4.17), there exists a curve (denoted as Γ_C) between the boundary curves Γ_A and Γ_B . It can be determined by choosing $\theta = 0.5$ as follows.

Suppose that $\{c_{+k}\}$ and $\{c_{-k}\}$ ($k > 2$) are the sets of any initial state $x(0)$ that can be driven back to the origin when the control signal takes on $u(0) = 0$ in the first step. Then the control sequence takes on $u(i) = +r$ or $u(i) = -r, i = 1, 2, \dots, k$. Similarly, the boundary curve Γ_C consists of Γ_C^+ and Γ_C^- .

For set $\{c_{+k}\}$, there exists $x_1 = \frac{1}{2}rh^2(k^2 + 3k + 2)$ and $x_2 = -rhk < 0$ according to the rule for choosing the control signal sequence, as shown above. By eliminating the variable k , we can obtain the control characteristic curve Γ_C^+ : $x_1 = \frac{x_2^2}{2r} - \frac{3}{2}hx_2 + \frac{1}{2}h^2r$. Similarly, we can obtain the control characteristic curve Γ_C^- : $x_1 = -\frac{x_2^2}{2r} - \frac{3}{2}hx_2 + \frac{1}{2}h^2r$. Therefore, the entire control characteristic curve Γ_C is $x_1 + \frac{x_2|x_2|}{2r} + \frac{3}{2}hx_2 - \frac{1}{2}h^2r = 0$.

We assume that the curve Γ_C and the region Ω_2 intersect at point C , which can be determined by their simultaneous equations:

$$\begin{cases} x_1 - \frac{x_2^2}{2r} + \frac{3}{2}hx_2 - \frac{1}{2}h^2r = 0 \\ x_1 + \frac{x_2|x_2|}{2r} + \frac{3}{2}hx_2 - \frac{1}{2}h^2r = 0 \end{cases} \quad (4.19)$$

Solving the above equation, we can get the intersection point $(\frac{\sqrt{5}+1}{2}h^2r, -\frac{\sqrt{5}-1}{2}hr)$.

The region Ω is divided into two regions (denoted as $\Omega_{\theta>0.5}$ and $\Omega_{\theta\leq 0.5}$) by the curve Γ_C . The definitions of these two regions are given as follows and shown in Fig. 4.3.

Definition 4.1: $\Omega_{\theta>0.5} = \{M(\alpha, \beta) | M \in \Omega, M \notin \Omega_2, \theta > 0.5\}$.

Definition 4.2: $\Omega_{\theta\leq 0.5} = \{M(\alpha, \beta) | M \in \Omega, M \notin \Omega_2, \theta \leq 0.5\}$.

From the definitions above, the region $\Omega - \Omega_2$ is divided into two different regions. What we should prove is that any initial state $M(\alpha, \beta) \in \Omega - \Omega_2$ can be driven into the region Ω_2 with the control signal sequence described in (4.17). The whole proof can be split into two steps.

Step 1: When the initial state satisfies $M(x_1, x_2) \in \Omega_{\theta>0.5}$ as shown in Fig. 4.4, there exists $\beta(0) \geq 0, \theta > 0.5$.

Figure 4.3: Illustration of the region Ω_2 , $\Omega_{\theta>0.5}$ and $\Omega_{\theta\leq 0.5}$.

Figure 4.4: Convergence path for any initial state M located in the region $\Omega_{\theta>0.5}$.

Suppose that $\theta(0) = 0.5 + \delta$ where $0 < \delta < 0.5$. According to (4.18), we can derive

$$\begin{aligned} 0 \leq \theta(1) &= \frac{2\theta(0)[1 - \theta(0)]}{2\beta(0) + 4\theta(0) - 1} \\ &= \frac{\frac{1}{2}}{2\beta(0) + 4\delta + 1} \leq \frac{1}{2} \cdot \frac{1}{2\beta(0) + 1} \leq \frac{1}{2}. \end{aligned} \quad (4.20)$$

However, this contradicts with the condition that the initial state satisfies $\theta > 0.5$. Therefore we can derive the conclusion that any initial state can be driven into the

Figure 4.5: Convergence path for any initial state M located in the region $\Omega_{\theta \leq 0.5}$.

region Ω_2 or the region $\Omega_{\theta \leq 0.5}$ with the control signal in one step, and the control signal is $u(1) = r(1 - 2\theta(1)) \geq 0.5r$ according to (4.16).

Step 2: When the initial state satisfies $M(x_1, x_2) \in \Omega_{\theta \leq 0.5}$ shown in Fig.4.5, there exists $x_2(k) = -\beta(k)hr < -\frac{\sqrt{5}-1}{2}hr$, i.e., $\beta(0) > \frac{\sqrt{5}-1}{2}$.

If the initial state cannot be driven out of the region $\Omega_{\theta \leq 0.5}$ with the corresponding control signal in one step, according to (5.18), there exists

$$\begin{aligned} \theta(0) &= \frac{2\theta(0)[1 - \theta(0)]}{2\beta(0) + 4\theta(0) - 1} \\ &= \frac{2\theta(0)[1 - \theta(0)]}{2\beta(1) + 1} < \frac{1}{2} \cdot \frac{1}{2\beta(1) + 1} < \frac{1}{2\sqrt{5}}. \end{aligned} \quad (4.21)$$

From (4.16) and (4.20), $u(1) = r(1 - 2\theta(1)) > (1 - \frac{1}{\sqrt{5}})r$ can be obtained.

According to **Step 1** and **Step 2** above, where the initial state is in the region $\Omega_{\theta > 0.5}$ or $\Omega_{\theta \leq 0.5}$, the control signal satisfies $u > \frac{1}{2}r$. From (4.5), we can obtain that

$$\begin{aligned} x_2(k+1) &= x_2(0) + h \sum_{i=0}^k u(i) \\ &> x_2(0) + (1 - \frac{1}{\sqrt{5}})(k-2)hr - \frac{1}{2}hr \end{aligned} \quad (4.22)$$

Further, we have

$$\beta(k+1) < \beta(0) - (1 - \frac{1}{\sqrt{5}})(k-2) - \frac{1}{2}. \quad (4.23)$$

From (4.23), we know $\beta(k+1)$ is a monotonic function. However, for any initial state located in the region $\Omega_{\theta \leq 0.5}$, the condition $\beta > 0$ holds. Therefore, the state must be driven out of the region $\Omega_{\theta \leq 0.5}$ with the corresponding control signal in a limited number of steps.

Suppose that the state $M(\alpha(k), \beta(k))$ located in the region $\Omega_{\theta \leq 0.5}$ becomes $M(\alpha(k+1), \beta(k+1))$ located outside the region $\Omega_{\theta \leq 0.5}$ by adding the control signal in one step, we need to prove that the state $M(\alpha(k+1), \beta(k+1))$ is in the region Ω .

From (4.18), we can obtain

$$\begin{cases} \alpha(k+1) = \frac{\beta^2(k)}{2} - \frac{\beta(k)}{2} + 2\theta(k)\beta(k) + \theta(k) \\ \beta(k+1) = \beta(k) + 2\theta(k) - 1. \end{cases} \quad (4.24)$$

Further, since $\beta(k) > \frac{\sqrt{5}-1}{2}$ and $0 \leq \theta \leq 1$, the following equations can be obtained

$$\begin{cases} -2\theta^2(k) - \beta(k) + \frac{1}{2} < 0 \\ -2\theta^2(k) + 2\theta(k) \geq 0. \end{cases} \quad (4.25)$$

According to (4.25), the state $M(\alpha(k+1), \beta(k+1))$ satisfies the condition that $\beta(k+1)$ is in the region surrounded by Γ_A^+ and Γ_C ; meanwhile, the state is not in the region $\Omega_{\theta \leq 0.5}$. Thus it can be concluded that the state must locate in the region Ω_2 .

However, we prove that the state sequence located inside of the region Ω can be driven into the region Ω_2 with the corresponding control signal sequence in a minimum and finite number of steps. Once the initial state enters the region Ω_2 , it can be driven back to the origin according to (4.12). Combining **Step A** and **Step B**, we prove the convergence of the time optimal control law described in **Theorem 4.1**.

4.4 Numerical Simulations

We may as well denote the DTOC law derived in this chapter as $u(k) = Ftd(x_1(k), x_2(k), r, h)$, where r is the quickness factor and h is the sampling period. In this section, numerical

Figure 4.6: Illustration of state trajectory under two different algorithms.

simulations will be conducted to compare the performance of the proposed DTOC law with the algorithm (denoted by $Fhan$) proposed by Han.

Firstly, we illustrate the state trajectory under two different algorithms, $Fhan$ and Ftd . From Fig. 4.6, we can see that the state trajectory for Ftd is in accordance with the optimal trajectory while the state trajectory for $Fhan$ lags the optimal trajectory.

Based on the discrete time optimal feedback control, i.e., $u(k) = Ftd(x_1(k), x_2(k), r, h)$, we can easily construct the following time optimal system-based tracking differentiator by replacing x_1 in (4.4) with $x_1 - v$.

$$\begin{cases} u(k) = Ftd(x_1(k) - v(k), x_2(k), r, c_0 h) \\ x_1(k+1) = x_1(k) + hx_2(k) \\ x_2(k+1) = x_2(k) + hu(k), k = 0, 1, 2, \dots \end{cases} \quad (4.26)$$

where r is the quickness factor, c_0 is the filtering factor, and h is the sampling period. We then demonstrate the errors of tracking and differentiation of signals between the algorithm $Fhan$ and Ftd by using the input signal sequence $V(t) = \sin(2\pi t)$. In the simulations, the parameters, such as the sampling step $h = 0.01$, the quickness factor

Figure 4.7: Comparison of tracking errors between *Fhan* and *Ftd* algorithms.

$r_0 = 100$, and the filtering factor $c_0 = 1.5$ are chosen by means of a trial-and-error approach, so that the algorithm *Fhan* can track input signal quickly and derive excellent derivative signal as much as possible. We use the same parameters for the proposed algorithm *Ftd*.

The simulation results are plotted in Figs. 4.7 and 4.8. The algorithm *Ftd* quickly tracks an input signal without overshooting or chattering, while also obtaining an excellent differentiation of the input signal. This algorithm is more accurate in tracking and differentiation compared with the other algorithm.

4.5 Conclusions

In this chapter, the convergence of an improved discrete-time optimal control (DTOC) algorithm has been proved. This algorithm was determined by a linearized criterion for a discrete-time double-integral system, which equips this DTOC algorithm with a simple structure. This closed-form non-linear state feedback control algorithm avoids the

Figure 4.8: Differentiation of signal errors compared between $Fhan$ and Ftd algorithms.

problem associated with the time optimal control of instant switching between extreme values in the control signals. The control algorithm proposed in this paper is ready for implementation in some digital control systems including state estimations in power systems, much more practical than the bang-bang control algorithm. The numerical simulation results showed that the proposed DTOC algorithm Ftd is effective and noise-tolerant.

Chapter 5

A Simple Tracking Differentiator via Discret-Time Optimal Control

Based on the proposed DTOC algorithm in Chapter 4, a simple tracking differentiator is proposed in this chapter. Note that the control characteristic curve (Γ_C) is adopted to enhance the control precision of the DTOC algorithm (Ftd) in Chapter 4 where the new algorithm is denoted as *Fast*. The analysis of the proposed tracking differentiator reveals its filtering mechanism. Simulation results show that it performs well in filtering and differentiation, and can reduce the computational resources needed. Experiments conducted on the maglev train demonstrate that the proposed the tracking differentiator group, with moving-average algorithm, can filter noises, amend distortion signals effectively and compensate for phase delays when the train is passing over track joints.

5.1 Introduction

The real-time and noise-tolerant differential signals is essential in control related fields [73] [74]. For example, the typical PID controller is still the most practical control strategy in control engineering practice [75] [76]. The PID control, however, is usually degraded to PI control since differential signals are hard to obtain [77]. To deal with this, researchers have proposed many different approaches for differentiator design, including those based

on a high-gain observer [78], a linear time-derivative tracker [79], a super-twisting second-order sliding mode algorithm [80], robust exact differentiation [73] [81], and a finite time convergent differentiator [82] etc.

Firstly proposed by Han, a bang-bang control-based tracking differentiator (TD) allows one to avoid a setpoint jump in the emerging active disturbance rejection controller [61]. This TD features excellent dynamic process and smoothness regarding the differential signals [64]. The following presents a brief outline for the construction of this TD.

The double-integral system is defined as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u, \quad |u| \leq r \end{cases} \quad (5.1)$$

where r is a constant constraint for inputs. According to the bang-bang control theory, we have

$$u = -r \operatorname{sign}(x_1 - v + \frac{x_2|x_2|}{2r}) \quad (5.2)$$

where v is the desired value for x_1 . The switching curve function is $\Gamma(x_1, x_2) = x_1 + \frac{x_2|x_2|}{2r}$.

To access to the differential signals, there exist

$$\begin{cases} \dot{v}_1 = v_2, \\ \dot{v}_2 = -r \operatorname{sign}(v_1 - v + \frac{v_2|v_2|}{2r}) \end{cases} \quad (5.3)$$

where v_1 is the tracking signals and v_2 gives its differential signals.

Since most control algorithms now are in discrete-time domain, a discrete-time algorithm corresponds to the bang-bang control is needed. The chattering problem will exist when one digitizes the bang-bang control directly. For the discrete-time optimal control algorithm (*Fhan*), the evaluation of the initial state's position on the phase plane involves non-linear boundary transformation. This makes the structure of a TD to be complex with non-linear calculations, including square-root calculations. Unlike the control algorithm *Fhan*, the DTOC algorithm proposed in Chapter 4 is based on a linearized

criterion that depends upon the position of the initial state point on the phase plane. In doing so, the new control algorithm has a simpler structure that is much easier to be applied in practical engineering scenarios. Experiments are carried out on position signal processing for position sensing in maglev train [83]. In practice, the signals from the position sensor may be aberrant due to the existence of track joints, which leads to low efficiency of the traction system or even safety misadventures. The proposed TD can be used to construct a TD group with a moving-average algorithm that filters the noises, compensates for phase delays, and amends distortion signals when the train is passing over track joints.

This chapter is organized as follows: the structure of the TD and its filtering characteristic are discussed in Section 5.2. In Section 5.3, numerical simulations are demonstrated to compare the signal tracking, differentiation acquisition and the computational resources needed in field-programmable gate array (FPGA) application between *Fhan* algorithm and the proposed one, followed by experimental results on position signal processing for a maglev train. Finally Section 5.4 concludes the paper.

5.2 Structure Analysis and Filtering Characteristic of *Fast*

Note that we introduce the control characteristic curve (Γ_C) proposed in Chapter 4 into constructing the new DTOC algorithm in this chapter so as to enhance the precision of control. As shown in Fig. 5.1, we denote that, for any initial state $M(x_1, x_2)$ in the fourth quadrant ($x_1 > 0, x_2 < 0$), an auxiliary line $x_2 = x_2(M)$ intersects with the curves at points A , C and B (in the direction of x_1). The coordinates of x_A , x_B , and x_C along the x-axis are

$$\begin{cases} x_A = \frac{x_2^2}{2r} + \frac{1}{2}h|x_2| \\ x_B = \frac{x_2^2}{2r} + \frac{5}{2}h|x_2| + h^2r \\ x_C = \frac{x_2^2}{2r} + \frac{3}{2}h|x_2|. \end{cases} \quad (5.4)$$

Figure 5.1: Illustration of two boundary curves (Γ_A and Γ_B), control characteristic curve Γ_C , region Ω (surrounded by two boundary curves), and three intersection points A , B , and C .

For any initial state $M(x_1, x_2)$ satisfying $x_1 < x_A$ or $x_1 > x_B$, the control signal is taken as $u = +r$ or $u = -r$. For any initial state $M(x_1, x_2)$ satisfying $x_1 \in [x_A, x_C]$, the control signal can be determined as follows:

$$u = -r\alpha \text{sign}(x_2) \quad (5.5)$$

where $\alpha = \frac{x_C - x_1}{x_C - x_A}$. For any initial state $M(x_1, x_2)$ satisfying $x_1 \in [x_C, x_B]$, the control signal is calculated as:

$$u = r\beta \text{sign}(x_2) \quad (5.6)$$

where $\beta = \frac{x_1 - x_C}{x_B - x_C}$.

According to Chapter 4, any initial state $M(x_1, x_2)$ on the $x_1 - x_2$ plane can be dragged back to the origin in the shortest time according to the control signal sequence above. The complete DTOC algorithm is described as follows:

Step 1: Set $z_1 = x_1 + \lambda h x_2$, $z_2 = z_1 + h x_2$, where $\lambda \in (0, 1]$ is a tuning parameter to determine the different coordinates of x_A , x_B , and x_C . Here, we choose $\lambda = 1$. If

$|z_1| > h^2 r$ or $|z_2| > h^2 r$, then $M(x_1, x_2)$ cannot be dragged back to the origin within two steps, i.e., $M(x_1, x_2) \notin \Omega_r$, go to the next step; otherwise, go to **Step 5**;

Step 2: If the initial state $M(x_1, x_2)$ satisfies $x_1 x_2 \geq 0$ and $M(x_1, x_2) \notin \Omega_2 \cup \Omega$, then the control signal takes on $u = -r \text{sign}(x_1 + h x_2)$;

Step 3: Determine the boundary of the region Ω , i.e., x_A , x_C and x_B according to (5.4).

Step 4: If $|x_1| \geq x_B$, then the control signal takes on $u = -r \text{sign}(x_1)$; if $|x_1| \leq x_A$, then the control signal takes on $u = r \text{sign}(x_2)$;

Step 5: If $|x_1| \leq x_C$, then the control signal takes on $u = -r \alpha \text{sign}(x_2)$; otherwise $u = r \beta \text{sign}(x_2)$, where $\alpha = \frac{x_C - |x_1|}{x_C - x_A}$ and $\beta = \frac{|x_1| - x_C}{x_B - x_C}$;

Step 6: If the initial state $M(x_1, x_2) \in \Omega_2$, then the control signal takes on $u = -\frac{z_2}{h^2}$;

Step 7: The algorithm ends.

From the deduction above, the mathematical derivation of a closed-form the DTOC algorithm as a function of x_1, x_2, r , and h , denoted by $u(k) = \text{Fast}(x_1(k), x_2(k), r, h, x_A, x_B, x_C)$, is obtained.

Based on the discrete-time optimal feedback control, i.e., $u(k) = \text{Fast}(x_1(k), x_2(k), r, h)$, we can easily construct the following time optimal system-based tracking differentiator

$$\begin{cases} u(k) = \text{Fast}(x_1(k) - v(k), x_2(k), r, c_0 h) \\ x_1(k+1) = x_1(k) + h x_2(k) \\ x_2(k+1) = x_2(k) + h u(k), k = 0, 1, 2, \dots \end{cases} \quad (5.7)$$

where r is the quickness factor, c_0 is the filtering related factor, and h is the sampling time.

The approximate linear format of the proposed TD in (5.7) can be shown as follows if one properly selects parameter r :

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \begin{pmatrix} 1 - \frac{0.5}{c_0^2} & (1 - \frac{0.75}{c_0})h \\ -\frac{1}{c_0^2 h} & 1 - \frac{1.5}{c_0} \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \frac{1}{c_0^2 h} \begin{pmatrix} 1 \\ 0.5h \end{pmatrix} v(k). \quad (5.8)$$

For convenience, the above equation can be expressed as

$$x(k+1) = Gx(k) + \Gamma v(k), k = 0, 1, 2, \dots \quad (5.9)$$

where G, Γ are the corresponding matrices in (5.9). Consider a input signal $v(t) = \sum_{i=1}^N A_i e^{j(w_i t + \phi_{i0})} + \xi(t)$, where $A_i, w_i \in \mathbf{R}^+$, $\phi_{i0} \in \mathbf{R}$, we have

$$x(k) = G^k p_0 + \sum_{i=1}^N (e^{jw_i h} I_2 - G)^{-1} \Gamma A_i e^{j(w_i k h + \phi_{i0})} + \eta(k) \quad (5.10)$$

where $\eta(k+1) = G\eta(k) + \Gamma\xi(k)$, and p_0 is determined by initial condition $x(0)$ and $\eta(0)$.

By adopting $\Phi(z) = C(zI_2 - G)^{-1}\Gamma$, $x_1(k)$ is rewritten as

$$x_1(k) = CG^k p_0 + \sum_{i=1}^N \Phi(e^{jw_i h}) A_i e^{j(w_i k h + \phi_{i0})} + C\eta(k). \quad (5.11)$$

For comparisons of filtering characteristics between *Fast* and *Fhan* algorithms, we only consider the width-steady random process. If $\xi(t)$ is a white noise sequence, there exists $R_\xi = Q\delta(\tau)$, where $\delta(\tau)$ is the Kronecker delta function and Q is the constant matrix [84] [85]. The $R_\eta(k)$ converges to the constant matrix, i.e.,

$$R_\eta = GR_\eta G^T + \Gamma Q \Gamma^T. \quad (5.12)$$

For *Fhan* algorithm, the matrix G and Γ are

$$G_1 = \begin{pmatrix} 1 & h \\ \frac{1}{c_0 h} & 1 - \frac{2}{c_0} \end{pmatrix}, \Gamma_1 = \begin{pmatrix} 0 \\ \frac{1}{c_0^2 h} \end{pmatrix}$$

respectively. Assume that the white-noise power spectrum density is $Q = 1$. Fig. 5.2 shows the relationship between the output variance sequence R and the parameter c_0 .

As shown in Fig. 5.2, choosing the proper filtering factor c_0 allows that the proposed TD based on the control law *Fast* to effectively filter random noise. Compared with the control law *Fhan*, the *Fast* algorithm performs better in signal-filtering.

Figure 5.2: Output variance sequence R vs filtering factor c_0 .

5.3 Simulations and Experiments

By numerical simulations, we compare the state trajectory, signal-tracking, and differentiation of input signals between the algorithm *Fhan* and the algorithm *Fast*. The comparisons of the computational complexities of the proposed *Fhh* algorithm and normal *Fhan* algorithm are demonstrated by showing computational resources needed and the time consumptions in real-time executions using field-programmable gate array (FPGA) and MCU (STM32F405), respectively. Experiments have also been conducted on signal processing for the position sensing in a maglev train. Our objective is to show that using the proposed tracking differentiator to construct a tracking differentiator group with a moving-average algorithm can filter noise effectively, compensate for phase delays, and amend distortion signals at track joints, and it performs better than the algorithm *Fhan*.

5.3.0.1 Numerical Simulation

The switching curve function of the algorithm *Fhan* is $\Gamma_A : x_1 + \frac{x_2|x_2|}{2r} + \frac{1}{2}hx_2 = 0$, while the switching curve function of TOC is $\Gamma_0 : x_1 + \frac{x_2|x_2|}{2r} = 0$. Figure 5.3 illustrates

Figure 5.3: Illustration of state trajectory under two different algorithms.

the state trajectory under two different algorithms, *Fhan* and *Fast*. We can see that the state trajectory for *Fast* is in accordance with the optimal trajectory while the state trajectory for *Fast* lags the optimal trajectory.

We then demonstrate the errors of tracking and differentiation of signals between the algorithms *Fhan* and *Fast*. The same initial value ($x_{10} = 0$, $x_{20} = 2$) and the input signal sequence $v(t) = \sin(2\pi t) + \gamma(t)$ are adopted, where $\gamma(t)$ is the evenly distributed white noise with an intensity of 0.01. The sampling step is $h = 0.001s$, the quickness factor is $r_0 = 100$, and the filtering factor is $c_0 = 1.5$.

From simulations in Figs. 5.4 and 5.5, the algorithm *Fast* quickly tracks an input signal without overshooting or chattering, while obtaining an excellent differentiation of the input signal. This algorithm is more accurate in tracking and differentiation compared with the other algorithm.

To compare the computational complexities of the *Fhh* based TD and normal *Fhan*

Figure 5.4: Comparison of tracking errors between *Fhan* and *Fast* algorithms.

Figure 5.5: Differentiation of signal errors compared between *Fhan* and *Fast* algorithms.

based TD, we may count what and how many operations have been taken during each sampling interval to measure the computational resources needed. Also, we can carry out a certain number of tests (like 1000 times) to observe the average time consumptions for real-time executions. Specially, we make comparisons between the performances of the two algorithms in (i) computational resources needed while using field-programmable gate array (FPGA); and (ii) the average time consumptions in real-time executions using MCU (STM32F405), respectively.

For the comparisons on computational resources needed, we introduce the FPGA with item No. Stratix II EP2S30484C5 under *QuartusII*5.1 programming context. A great amount of multipliers and logic-element resources are consumed in FPGA implementations, especially for square root calculations. We adopted the method of successive approximations to calculate square root using the VHSIC Hardware Description Language (VHDL) language in FPGA implementations. The comparisons of the FPGA resources consumed by a single run of the two different algorithms are presented in Table 1. For simplification, we regard multiplication and division as the same type of operations.

Table 5.1: Comparisons of the computational resources needed in FPGA of *Fhan* and *Fast*

Algorithm	Multiplications	Square-root	Logic-Element	Clock cycles
<i>Fhan</i> (32 bit)	14	1	290	74
<i>Fast</i> (32 bit)	6	0	6	5

From Table 1, we see that the proposed *Fast* algorithm can significantly reduce the computational resources needed. In our extensive testing, for different initial states on the phase plane, the number of multiplication operations carried out by the *Fast* algorithm has never been larger than 6. Starting from some initial states, the *Fhh* algorithm requests only 4 multiplication operations.

To compare the average time consumptions of the two different algorithms in processing the same input signals in real-time execution, we adopt the STM32F405 with an

upgraded CPU clock speed of 200MHz. We calculate the average execution time for a single run of the algorithms (with 1000 times). Table 2 gives the results.

Table 5.2: Comparisons of the average execution time of $Fhan$ and Fhh

Algorithm	Average execution time
$Fhan$	$5.25us$
$Fast$	$2.45us$

From the above comparisons, we see that the $Fast$ algorithm reduces both the computational resources needed and the time consumptions for real-time executions.

5.3.1 Experiment Validation

A linear motor is adopted to realize the traction aim in maglev train [86] [87]. As shown in Fig. 5.6, the speed and position detection system is a core part of the synchronous traction system, which is applied to receive and process the sensors' information. It sends the processed data of the speed and position of the train to the traction system in terms of the agreement to realize traction function. Therein, the position sensor can detect inductance changes of long stators' alveolar structures (see Fig. 5.6) to acquire the position information about the train.

In practice, the detection coils of the position sensor face long stators' alveolar structures. When the position sensor moves along the long stators, it can distinguish the teeth and the slots by detecting changes of inductance, and meanwhile, the position of the train can be detected by counting the number of passed alveolar structures.

To prevent the system against the thermal expansion and contraction, the long stator track features numerous stators with many joints (80mm of the actual displacement) along the track (see Fig. 5.7). These joints distort the outputs of the magnetic pole phase from the position sensors. The expected output of the position sensor when maglev train

Figure 5.6: Synchronous traction system.

Figure 5.7: Track joint in traction system.

Figure 5.8: The expected output of the position sensor when the maglev train passes over track joint.

Figure 5.9: Illustration of TD group

is passing over track joint is shown in Fig. 5.8. However, because of the track joints, the position signal will be more or less aberrant as the train passes.

To amend the distortion of position signals at track joints, the TD based on algorithm *Fast* is introduced. We use the proposed TD to construct the TD group (see Fig. 5.9), employing a moving-average algorithm to compensate for any phase delay. The moving-average algorithm is determined as

$$\hat{V}(t) = \sum_{k=1}^n C_n^k (-1)^{(k+1)} V_k(t) \quad (5.13)$$

We choose $n = 3$, leading to $\hat{V}(t) = 3V_1(t) - 3V_2(t) + V_3(t)$.

The experimental data are acquired from the speed and position detection system when the train is passing over a track joint. In the experiments, the parameters are set by the trial and error method, adopting values as follows: for the algorithm *Fast*, the quickness factor is $r_0 = 100$, and the filtering factor is $c_0 = 40$, while for the algorithm *Fhan*, the quickness factor is $r_0 = 300$, and the filtering factor is $c_0 = 45$. For each algorithm, sampling time step h corresponds to 0.005 second. Figure. 5.10 shows that, when the proposed TD group is used based on the algorithm *Fast*, the aberrant magnetic pole phase from the position sensor becomes smoother and the aberrant signal is greatly improved. The phase delay is small enough to meet the need of traction system. Furthermore, compared with the TD based on the algorithm *Fhan*, we can find that the

Figure 5.10: Results of magnetic pole phase filtering and phase compensation with the algorithm *Fast* and *Fhan*.

TD based on *Fast* is superior to the *Fhan* in filtering and phase compensation. Such findings verify the effectiveness of the TD based on the *Fast* algorithm.

5.4 Conclusion

A nonlinear TD based on the proposed discrete-time optimal control algorithm was presented in this chapter. The boundary curves and the control characteristic curve were obtained using the state back-stepping method. Using a linearized criterion in control algorithm, we obtained a new TD with a simple structure. The filtering mechanism was identified through structure analysis. Numerical simulations showed that the TD based on the algorithm *Fast* is more effective and has better performances filtering and obtaining differentiation than algorithm *Fhan*. Furthermore, the computational resources needed in FPGA application are greatly reduced using the proposed *Fast* algorithm. Experiments in a maglev train verified the effectiveness of the TD based on

the algorithm *Fast* in signal processing.

Chapter 6

State Estimation based on Tracking differentiator and Its Effects on System Emergency Control Scheme

In this chapter, a brief discussion on current state estimation (SE) algorithms in power systems is made, in particular, for static state estimation algorithms. Then we carry out the state estimation regarding the measurements from phasor measurement units (PMUs). Specifically, the proposed differentiator is applied to reliable estimation filtering of the system states. The experiments and simulations demonstrate the excellent estimation filtering results using the proposed differentiator. Finally, we show how the proposed state estimation performs in helping generate the system emergency control schemes' solutions and the corresponding control results during cascading failures as those have been demonstrated in Chapter 2.

6.1 Introduction

Initially proposed by F.C. Schweppe in 1970, power system state estimation (SE) has played a critical role in control and protection of the systems [88] [89]. State estimation, known as filtering as well, it is a way that using the redundancy of real-time measurements to improve data accuracy. It aims to exclude the random noise-caused error message,

monitor and estimate the running states of a system [90] [91].

At the beginning, remote terminal units (RTUs) collect the information on current/voltage magnitudes and flowing power. Such information can only connect power and current magnitude to the voltage-related states with non-linear function, which leads to the SE algorithm at this stage as nonlinear and iterative weighted least-squares (WLS) problem [92] [93].

RTUs measurements are usually not time-stamped. In the processing, some uncertain errors are introduced, resulting in estimated states being polluted. With the advancements of PMUs technologies and their deployments in power systems, the time-stamped and accurate phasors can be recorded at a high sampling rate [94]. This can equip systems with the ability of measuring the running states directly. Thanks to PMUs, the equations linking measurements and state variables, called measurement model, become linear [95]. Because of such characteristics of PMUs data, there are two alternative SE algorithms we can use, namely the Kalman filter (KF) and the least absolute value (LAV) [96], respectively. Note that LAV and WLS are static estimators since they only work based on the measurements. While the KF belongs to the dynamic estimators, because the process model that describes the time-evolution of running states in power systems is needed as well [97] [98].

6.2 State Estimation based on TD

A real-time, accurate state estimation (SE) is crucial if we are to enhance the power systems' resilience. Driven by this concern, an increasing number of PMUs are being deployed in power systems. Collecting near real-time and accurate phasors makes PMUs capable of directly measuring the power-system state. Here, we used real-time PMU data to compare between SE filtering by our proposed discrete TD and Kalman filter (KF).

Figure 6.1: Output of current magnitude SE filtering using the real-time PMU data.

The raw data, including current and voltage phasors, were recorded during normal operations at a 20-KV power plant in Jiangsu Province, China. The sampling frequency was 4960 Hz. For this processing, we set parameters for TD as follows: sampling period, $h = 0.001s$; quickness factor, $r_0 = 800$; and filtering factor, $c_0 = 8$. Figure 6.1 shows the raw PMU data and the corresponding processing effect. From Fig.6.1, compared with the Kalman filter algorithm, the proposed TD obtains more effective SE filtering outputs than KF.

Also, to compare the time consumptions of the two different algorithms in processing the PMUs data with the same data window width in real-time execution, we again adopt the STM32F405 with an upgraded CPU clock speed of 200MHz. The results are presented in Table 6.1. From the above comparisons, we see that the *Fast* algorithm reduces the

Table 6.1: Comparisons of the execution time of *Fast* and *KF* with same data window width

Algorithm	Average execution time
<i>Fast</i>	15.3s
<i>KF</i>	20.5s

time consumptions for real-time executions. Together with filtering results, we can see that the *Fast* based state estimation algorithm is more accurate and efficient.

6.3 Effects of estimated states via TD on system emergency control scheme

In this section, to test TD based SE results' effectiveness, we compare the system emergency control schemes' solutions and the corresponding control results during cascading failures based on the filtered data and the noise-free PMUs data (benchmark data) on the IEEE 57-bus test system, respectively. The system details are presented in Chapter 2. We adopt the same emergency control scheme shown in Fig. 2.1.

The Branch 7 connecting between Bus 6 and Bus 8 (both are generator buses) is again selected to be applied the initial disturbance. The generating PMU data with a 45dB SNR is fed into the proposed TD based state estimation algorithm. Then we use the outputs of SE algorithm as inputs to participate in estimating and identifying branches' parameters and disturbances. The system emergency control schemes' solutions and the corresponding control results at the 4-th cascading step are shown in Fig. 6.2 and Fig. 6.3, respectively. When the SNR degrades to 30dB, the relevant results are demonstrated in Fig. 6.4 and Fig. 6.5, respectively.

According to the simulations, we can see that the proposed emergency control scheme is effective after the proposed TD based SE algorithm is incorporated. More importantly, compared with results in Figs. 2.2 and 2.4, for the PMU data with an SNR value of 45dB and the proposed TD based SE algorithm, the system emergency control schemes' solutions and the corresponding control results are quite similar to those based on noise-free PMU data, except that there exist three fewer buses being involved in adjustments of injected power, as shown in Fig. 6.2. Also, with 50 connected branches, three more branches are tripped and the remaining power flow changes from 6.6542 pu to 6.3274 pu.

Figure 6.2: Adjustments of power on each bus with the proposed SE when exists an SNR value of 45dB.

Figure 6.3: Final system topology with the proposed SE when exists an SNR value of 45dB.

When an SNR degrades to 30dB, from simulation results in Fig. 6.4 , we can see that there are 48 connected branches and total remaining flowing power is 6.2964 *pu*. This indicates that the proposed SE algorithm plays its role even when an SNR of PMU data degrades.

Figure 6.4: Final system topology with the proposed SE when exists an SNR value of 30dB.

6.4 Conclusion

The proposed tracking differentiator (TD) was applied to carry out the SE in power systems. From results, we found that the TD-based SE algorithm has advantages of better accuracy and efficiency over Kalman filter. Further, the effectiveness of the proposed SE was verified in calculating system emergency control schemes' solutions and the corresponding control results during cascading failures. This implies that the proposed SE algorithm may provide power systems with a new potential method to improve the situation awareness for operators and hence helps the power system resilience.

Chapter 7

Summary and Future Work

We conclude the thesis and make a discussion on the future work on state estimation via discrete-time optimal control (DTOC) and its derivative in power systems.

7.1 Summary of Contributions

We investigated the state estimation algorithms and their implementations. In particular, we first studied the new close-form solution of DTOC and its derivative, that is, tracking differentiator. We then focused on the implementations of proposed algorithms in state estimation. We give the summary of contributions below.

- We investigated the effects of noise in PMU data on intelligent system emergency control scheme of power grids against cascading failures in Chapter 2. Specifically, the intelligent system emergency control schemes' solutions and the corresponding system control results under different SE results that are affected by the noise in PMU data were studied on IEEE 57-bus test system. It was found that a relatively factual noise in PMU data with a 45dB signal-to-ratio (SNR) may make differences to the emergency control schemes' selections and results. These results demonstrate that the importance and necessity of having good SE in calculating system emergency control schemes' solutions during cascading failures. This chapter showed

the research motivation of doing state estimation for enhancing the resilience of power systems.

- The DTOC algorithm's convergence was investigated with some easy checkable procedures. Specifically, the state convergence trajectories driven by the corresponding DTOC algorithm (*Fhan*) in different Isochronic Region was demonstrated. Numerical simulations showed that this DTOC algorithm based tracking differentiator outperforms the sliding mode method based differentiator in the aspects of dynamic process and stability exactness.
- In Chapter 4, the convergence of the improved DTOC algorithm was proved. This new DTOC algorithm (*Ftd*) was determined by a linearized criterion, which equips it with a simple structure. This closed-form non-linear state feedback control algorithm avoids the problem associated with the bang-bang control algorithm. The proposed DTOC algorithm is ready for implementation in some digital control systems. Numerical simulations showed that *Ftd* algorithm based tracking differentiator outperforms *Fhan* algorithm in signal-tracking filtering and differentiation acquisition.
- In Chapter 5, a novel second-order TD based on discrete-time optimal control is proposed. By adopting a linearized criterion in designing the control algorithm, we obtained a new TD with a simple structure. The filtering mechanism was identified through structure analysis. Numerical simulations showed that *Fast* algorithm based TD is effective in filtering and obtaining differentiation. Furthermore, the computational resources needed in FPGA application are greatly reduced using the proposed *Fast* algorithm. Experiments on the speed and position detection system in a maglev train verified the effectiveness of the TD based on the algorithm *Fast* in signal-processing.

- In Chapter 6, we applied the proposed tracking differentiator to carry out the SE in power systems. From the results, we found that the TD-based SE algorithm has advantages of better accuracy and efficiency over Kalman filter. Further, the effectiveness of the proposed SE was verified in calculation of emergency control schemes' solutions and the corresponding control results during cascading failures. This implies that the proposed SE algorithm may provide power systems with a new potential method to improve the situation awareness for operators and hence helps the power system resilience.

7.2 Future Work

We come up with some potential future work as presented below.

7.2.1 Extended DTOC algorithm

The disturbances are not investigated in (3.3) for constructing the corresponding DTOC algorithm. Thus, we need to consider the following double-integral plant with perturbations

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u + w(x, t), \quad |u| \leq r \end{cases}$$

where $w(x, t)$ are perturbations. Perturbations, including uncertainties and external disturbances, can be functions of time t .

In the future work, we will investigate the time-optimal solution for the above system and its discrete-time form, where we call it as extended DTOC algorithm. Specifically, we first need to find its corresponding boundary curves and the characteristic control curve to define the regions that initial state on the phase plane belongs to. Then, we should determine the control signal sequences inside and/or outside the different regions to drag the state points back to the origin.

7.2.2 Tracking Differentiator

In practice, the sensors' information may feature quick changes within a certain frequency band and/or high-frequency noises. Our extensive testing showed that under such cases, the proposed tracking differentiator with fixed setting parameters may degrade in the aspects of filtering and differentiation. To improve the performance of the tracking differentiator for the above mentioned input signals, we will develop an adaptive tracking differentiator in our future work. This adaptive tracking differentiator shall be able to automatically adjust the differentiator's parameters based on errors between the input signals and the feedback of filtering outputs.

As mentioned in Chapter 5, compared with the existing differentiators, the tracking differentiator has many advantages. For example, it requests a weaker constraint on input signals. However, a rigorous proof for bang-bang algorithm based TD is still open, other than the *Fhan* algorithm based TD, though filtering properties and stability of some special kinds of TD have been discussed. We shall investigate these open questions in the near future.

Other future investigations on TD will include an analysis of the accuracy of tracking and differentiation for the proposed TDs, and the selection of the optimal TD's parameters using a heuristic algorithm.

7.2.3 Effects of noises in PMU data on system emergency control

From simulation results in Chapter 2, we can see that it may be a non-trivial issue to carefully control noises in PMU data. Also, the proposed SE algorithm may help us re-formulate the optimization problem for designing protection scheme. Future work includes the following aspects:

- (i) Design proper signal processing methods to achieve a higher SNR in PMU data collection and processing when such data are to be used in designing intelligent system emergency control scheme.
- (ii) Carefully avoid filtering out the real risk signals, especially the early-stage risk signals, when trying to increase SNR.
- (iii) Investigate the mechanism that makes different failures be of different vulnerability levels against noises.
- (iv) Devise emergency control schemes that have stronger tolerance against noises and errors in PMU data or other sensing data.

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