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Multiple 1-D Fundamental ADI-FDTD Method for Coupled Transmission Lines on Mobile Devices

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Abstract—This paper presents the multiple one-dimensional (M1-D) fundamental alternating direction implicit (FADI) finite-difference time-domain (FDTD) method for coupled transmission lines on mobile devices. The method is aptly called the M1-D FADI coupled line (CL)-FDTD method. It is based on the fundamental implicit scheme, which features matrix operator-free right-hand sides (RHS). The formulations of the M1-D FADI CL-FDTD method and update equations are provided. Various sets of split matrices including those with self symmetry, mutual symmetry and self-mutual separation are proposed and discussed. Their stability analyses are performed using the Fourier amplification matrix formulated in fundamental form, which is simpler with only one inverse term each without RHS operator. It is found that the sets of split matrices with self symmetry are unconditionally stable, while those with mutual symmetry and self-mutual separation are not. Using two auxiliary quadratic polynomials along with single necessary and sufficient condition, the analytical proof of unconditional stability is made more convenient and concise. The efficiency of both stable sets of split matrices with self symmetry is discussed based on the number of RHS terms and floating-point operations (flops) count. Numerical results are presented to validate the accuracy of the proposed method at time step larger than the stability limit. Several EM simulations of coupled line structures are demonstrated on mobile devices. The CPU time incurred on various platforms is provided for M1-D FADI CL-FDTD method using both sets of split matrices with self symmetry. Using mobile devices, the EM simulations can be performed efficiently and ubiquitously anytime, anywhere.

Index Terms—Multiple 1-D method, fundamental ADI, FDTD, coupled transmission lines, EM simulations on mobile devices

I. INTRODUCTION

Coupled transmission lines form an integral part of many electromagnetic (EM) engineering applications involving directional couplers, filters and other microwave circuits [1]–[3]. To simulate EM fields along coupled transmission lines, the 3-D finite-difference time-domain (FDTD) method [4] can be used due to its flexibility. However, the conventional FDTD method is explicit in nature and hence has its time step restricted by the Courant-Friedrichs-Lewy (CFL) stability constraint dictated by the finest mesh in the spatial domain. Furthermore, the EM problems involving coupled transmission lines are often of multiscale nature since the conductor thickness, line width and spacing are generally very small compared to the length of the coupled line structures. This poses a big challenge for the explicit 3-D FDTD method due to the required fine mesh that greatly inhibits its time step size. While

one could resort to unconditionally stable FDTD methods such as the alternating direction implicit (ADI) [5], [6] and the locally one-dimensional (LOD) [7], [8] FDTD methods, these full wave 3-D implicit methods are still computationally intensive and call for huge computing resources.

To reduce the required computing resources, the multiple one-dimensional (M1-D) explicit FDTD method has been proposed in [9] for coupled transmission lines. The coupled line parameters are incorporated directly into the M1-D method based on even- and odd-mode approach, which requires only discretization along the propagation direction z . This obviates the fine mesh needed for the conductor thickness, line width and spacing, but the method is still explicit and subjected to stability constraint. Subsequent efforts to overcome such stability constraint lead to the unconditionally stable M1-D ADI-FDTD method for coupled transmission lines [10]. These M1-D methods for coupled transmission lines are henceforth aptly referred to as the M1-D explicit and ADI coupled line (CL)-FDTD methods respectively.

Meanwhile, the increasing computing power of mobile devices has enabled EM simulations on mobile platforms besides traditional desktop computers or workstations. Mobile devices could serve to complement the workstations by providing quick initial design and analysis using more computationally efficient methods to engineers, students and researchers alike. Due to their wide availability and affordances, they allow seamless and ubiquitous EM simulations anytime, anywhere. With unconditionally stable methods, mobile users can even accelerate EM simulations for real-time demonstration or field visualization. However, the M1-D ADI CL-FDTD method is still rather complicated for implementation on mobile devices. This is because not only the left-hand sides (LHS) involve tridiagonal matrices to be solved, the right-hand sides (RHS) of the update procedures also involve matrix operators, which would incur considerable memory indexing, access and arithmetic operations. It would be of great interest to improve the simplicity and efficiency of the M1-D ADI CL-FDTD method in [10]. To that end, we have attempted alternative set of split matrices based on self-mutual separation in [11]. However, the numerical stability analysis indicates that it is not unconditionally stable. Therefore, our efforts continue to seek other stable split matrices that are potentially simpler and more efficient, especially for implementation on mobile devices. Note that the previous stability analysis in [11] is only performed numerically to help detect quickly any potential instability. It is not sufficient and requires more rigorous analysis to prove the unconditional stability of split matrices.

In this paper, we present the M1-D fundamental ADI

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(FADI)-FDTD method for coupled transmission lines on mobile devices. The method is aptly called the M1-D FADI CL-FDTD method. It is based on the fundamental implicit scheme, which features matrix operator-free RHS. In Section II, the formulations of the M1-D FADI CL-FDTD method and update equations will be provided. Various sets of split matrices including those with self symmetry, mutual symmetry and self-mutual separation will be proposed and discussed. In Section III, their stability analyses will be performed both numerically and analytically using the Fourier amplification matrix formulated in fundamental form. To prove analytically the unconditional stability, instead of solving the eigenvalues explicitly from the characteristic polynomial, we shall examine their magnitudes more simply via two auxiliary quadratic polynomials along with single necessary and sufficient condition. In Section IV, the efficiency will be discussed based on the number of RHS terms and floating-point operations (flops) count. The accuracy of the proposed method will be validated through numerical results. Several EM simulations of coupled line structures will be demonstrated on mobile devices. The CPU time incurred on various platforms will be provided for M1-D FADI CL-FDTD method and compared with M1-D explicit CL-FDTD method.

II. M1-D FUNDAMENTAL ADI CL-FDTD METHOD

The EM fields in coupled transmission lines are governed by the differential equations [12] in compact form as [10]

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{W} \mathbf{u} \quad (1a)$$

$$\mathbf{u} = [E_{x1}, E_{x2}, H_{y1}, H_{y2}]^T \quad (1b)$$

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & \frac{-1}{\epsilon_s} \frac{\partial}{\partial z} & \frac{-1}{\epsilon_m} \frac{\partial}{\partial z} \\ 0 & 0 & \frac{-1}{\epsilon_m} \frac{\partial}{\partial z} & \frac{-1}{\epsilon_s} \frac{\partial}{\partial z} \\ \frac{-1}{\mu_s} \frac{\partial}{\partial z} & \frac{1}{\mu_m} \frac{\partial}{\partial z} & 0 & 0 \\ \frac{1}{\mu_m} \frac{\partial}{\partial z} & \frac{-1}{\mu_s} \frac{\partial}{\partial z} & 0 & 0 \end{bmatrix}. \quad (1c)$$

E_{x1} , H_{y1} are the electric and magnetic fields along line 1, while E_{x2} , H_{y2} are the electric and magnetic fields along line 2. ϵ_s , ϵ_m and μ_s , μ_m are the self and mutual permittivities and permeabilities, which can be expressed in terms of even- and odd-mode characteristic impedances (Z_{0e} , Z_{0o}), phase velocities (v_e , v_o) and effective permittivities (ϵ_{eff}^e , ϵ_{eff}^o) as [3]

$$\frac{1}{\epsilon_s} = \frac{v_e Z_{0e} + v_o Z_{0o}}{2}, \quad \frac{1}{\epsilon_m} = \frac{v_e Z_{0e} - v_o Z_{0o}}{2}, \quad (2a)$$

$$\frac{1}{\mu_s} = \frac{1}{2} \left(\frac{v_o}{Z_{0o}} + \frac{v_e}{Z_{0e}} \right), \quad \frac{1}{\mu_m} = \frac{1}{2} \left(\frac{v_o}{Z_{0o}} - \frac{v_e}{Z_{0e}} \right), \quad (2b)$$

$$v_e = c / \sqrt{\epsilon_{eff}^e}, \quad v_o = c / \sqrt{\epsilon_{eff}^o}. \quad (2c)$$

The M1-D ADI CL-FDTD method [10] solves (1) in two update procedures as

$$\left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{A} \right) \mathbf{u}^{n+\frac{1}{2}} = \left(\mathbf{I} + \frac{\Delta t}{2} \mathbf{B} \right) \mathbf{u}^n \quad (3a)$$

$$\left(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B} \right) \mathbf{u}^{n+1} = \left(\mathbf{I} + \frac{\Delta t}{2} \mathbf{A} \right) \mathbf{u}^{n+\frac{1}{2}} \quad (3b)$$

where $\mathbf{A}$ and $\mathbf{B}$ are the 4×4 split matrices obtained by proper splitting of matrix $\mathbf{W}$ as $\mathbf{W} = \mathbf{A} + \mathbf{B}$. Various

split matrices shall be discussed later. We first see that the RHS of (3) still consist of matrix operators. This would incur considerable memory indexing and arithmetic operations which inhibit the efficiency. To enhance the efficiency of the M1-D ADI CL-FDTD method, we shall propose the M1-D fundamental ADI (FADI) CL-FDTD method. It is based on the fundamental implicit scheme [13], which features matrix operator-free right-hand sides (RHS). While the principle of fundamental scheme is similar, it should be pointed out that the split matrices $\mathbf{A}$ and $\mathbf{B}$ here are different and not reducible from those of 3-D FADI or fundamental LOD FDTD methods [13], [14]. Introducing auxiliary vector $\mathbf{v}$ and upon some manipulations, the update procedures for M1-D FADI CL-FDTD method can be obtained as

$$\mathbf{v}^n = \mathbf{u}^n - \mathbf{v}^{n-\frac{1}{2}}; \quad \left(\frac{1}{2} \mathbf{I} - \frac{\Delta t}{4} \mathbf{A} \right) \mathbf{u}^{n+\frac{1}{2}} = \mathbf{v}^n \quad (4a)$$

$$\mathbf{v}^{n+\frac{1}{2}} = \mathbf{u}^{n+\frac{1}{2}} - \mathbf{v}^n; \quad \left(\frac{1}{2} \mathbf{I} - \frac{\Delta t}{4} \mathbf{B} \right) \mathbf{u}^{n+1} = \mathbf{v}^{n+\frac{1}{2}}. \quad (4b)$$

One can see that compared to (3), the RHS of (4) are now matrix operator-free. Equations (4a) form the first procedure, while (4b) form the second one. Next, we shall discuss various split matrices. Note that while the RHS of (4) are matrix operator-free, the split matrices should be chosen such that the LHS of (4) result in tridiagonal matrices that can be solved efficiently. Not only that, they must ensure stability which will also be analyzed subsequently.

A. 1st Set of Split Matrices with Self Symmetry

We begin with the set of split matrices proposed in [10], given below as

$$\mathbf{A}_{s1} = \begin{bmatrix} 0 & 0 & \boxed{\frac{-1}{\epsilon_s} \frac{\partial}{\partial z}} & 0 \\ 0 & 0 & \boxed{\frac{-1}{\epsilon_m} \frac{\partial}{\partial z}} & 0 \\ \boxed{\frac{-1}{\mu_s} \frac{\partial}{\partial z}} & 0 & 0 & 0 \\ \boxed{\frac{1}{\mu_m} \frac{\partial}{\partial z}} & 0 & 0 & 0 \end{bmatrix}, \quad (5a)$$

$$\mathbf{B}_{s1} = \begin{bmatrix} 0 & 0 & 0 & \boxed{\frac{-1}{\epsilon_m} \frac{\partial}{\partial z}} \\ 0 & 0 & 0 & \boxed{\frac{-1}{\epsilon_s} \frac{\partial}{\partial z}} \\ 0 & \boxed{\frac{1}{\mu_m} \frac{\partial}{\partial z}} & 0 & 0 \\ 0 & \boxed{\frac{-1}{\mu_s} \frac{\partial}{\partial z}} & 0 & 0 \end{bmatrix}. \quad (5b)$$

Notice that the self permittivity and permeability terms (marked with boxes) are symmetrically located in the split matrices. Hence, this set of matrices in (5) shall be regarded as the 1st set of split matrices with self symmetry.

Defining $\mathbf{v} = [e_{x1}, e_{x2}, h_{y1}, h_{y2}]^T$, the update equations are obtained by substituting (5) into (4). The update equations for the first procedure read

$$e_{x\xi}^n = E_{x\xi}^n - e_{x\xi}^{n-\frac{1}{2}}; \quad h_{y\xi}^n = H_{y\xi}^n - h_{y\xi}^{n-\frac{1}{2}}, \quad \xi = 1, 2 \quad (6a)$$

$$\frac{1}{2} E_{x1}^{n+\frac{1}{2}} + \frac{\Delta t}{4\epsilon_s} \frac{\partial H_{y1}^{n+\frac{1}{2}}}{\partial z} = e_{x1}^n \quad (6b)$$

$$\frac{1}{2} E_{x2}^{n+\frac{1}{2}} + \frac{\Delta t}{4\epsilon_m} \frac{\partial H_{y1}^{n+\frac{1}{2}}}{\partial z} = e_{x2}^n \quad (6c)$$

$$\frac{1}{2}H_{y1}^{n+\frac{1}{2}} + \frac{\Delta t}{4\mu_s} \frac{\partial E_{x1}^{n+\frac{1}{2}}}{\partial z} = h_{y1}^n \quad (6d)$$

$$\frac{1}{2}H_{y2}^{n+\frac{1}{2}} - \frac{\Delta t}{4\mu_m} \frac{\partial E_{x1}^{n+\frac{1}{2}}}{\partial z} = h_{y2}^n. \quad (6e)$$

The update equations for the second procedure can be obtained by interchanging the subscripts of fields in lines 1 and 2, e.g. $E_{x1} \leftrightarrow E_{x2}$ and $e_{x1} \leftrightarrow e_{x2}$. Based on the fundamental scheme, one can see that the RHS of (6) are indeed free from any spatial operator of the split matrices.

To solve for E_{x1} in the first procedure, (6d) is substituted into (6b) to yield

$$\frac{1}{2}E_{x1}^{n+\frac{1}{2}} - \frac{\Delta t}{4\epsilon_s} \frac{\partial}{\partial z} \left(\frac{\Delta t}{2\mu_s} \frac{\partial E_{x1}^{n+\frac{1}{2}}}{\partial z} \right) = e_{x1}^n - \frac{\Delta t}{2\epsilon_s} \frac{\partial h_{y1}^n}{\partial z}. \quad (7)$$

If the spatial derivatives are discretized using central differencing operated on standard staggered cells, the LHS of (7) yields tridiagonal matrices which can be solved efficiently. Besides that, the LHS has only self permittivity and permeability terms (due to the symmetrically located self terms in the split matrices). Compared to the M1-D ADI CL-FDTD method [10], the RHS of (7) has fewer terms and only first-order spatial derivative. Further simplicity and efficiency can be achieved by combining (6d) and (6e) with $h_{y\xi}$ in (6a) of the second procedure as

$$h_{y1}^{n+\frac{1}{2}} = h_{y1}^n - \frac{\Delta t}{2\mu_s} \frac{\partial E_{x1}^{n+\frac{1}{2}}}{\partial z}; \quad h_{y2}^{n+\frac{1}{2}} = h_{y2}^n + \frac{\Delta t}{2\mu_m} \frac{\partial E_{x1}^{n+\frac{1}{2}}}{\partial z}. \quad (8)$$

B. 2nd Set of Split Matrices with Self Symmetry

We present another set of split matrices as

$$\mathbf{A}_{s2} = \begin{bmatrix} 0 & 0 & \boxed{\frac{-1}{\epsilon_s} \frac{\partial}{\partial z}} & \boxed{\frac{-1}{\epsilon_m} \frac{\partial}{\partial z}} \\ 0 & 0 & 0 & 0 \\ \boxed{\frac{-1}{\mu_s} \frac{\partial}{\partial z}} & \frac{1}{\mu_m} \frac{\partial}{\partial z} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (9a)$$

$$\mathbf{B}_{s2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{\epsilon_m} \frac{\partial}{\partial z} & \boxed{\frac{-1}{\epsilon_s} \frac{\partial}{\partial z}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{\mu_m} \frac{\partial}{\partial z} & \boxed{\frac{-1}{\mu_s} \frac{\partial}{\partial z}} & 0 & 0 \end{bmatrix}. \quad (9b)$$

The self permittivity and permeability terms (marked with boxes) are again symmetrically located in the split matrices. This set shall be regarded as the 2nd set of split matrices with self symmetry.

Substituting (9) into (4), the update equations for the first procedure read

$$e_{x\xi}^n = E_{x\xi}^n - e_{x\xi}^{n-\frac{1}{2}}; \quad h_{y\xi}^n = H_{y\xi}^n - h_{y\xi}^{n-\frac{1}{2}}, \quad \xi = 1, 2 \quad (10a)$$

$$\frac{1}{2}E_{x1}^{n+\frac{1}{2}} + \frac{\Delta t}{4\epsilon_s} \frac{\partial H_{y1}^{n+\frac{1}{2}}}{\partial z} + \frac{\Delta t}{4\epsilon_m} \frac{\partial H_{y2}^{n+\frac{1}{2}}}{\partial z} = e_{x1}^n \quad (10b)$$

$$\frac{1}{2}E_{x2}^{n+\frac{1}{2}} = e_{x2}^n \quad (10c)$$

$$\frac{1}{2}H_{y1}^{n+\frac{1}{2}} + \frac{\Delta t}{4\mu_s} \frac{\partial E_{x1}^{n+\frac{1}{2}}}{\partial z} - \frac{\Delta t}{4\mu_m} \frac{\partial E_{x2}^{n+\frac{1}{2}}}{\partial z} = h_{y1}^n \quad (10d)$$

$$\frac{1}{2}H_{y2}^{n+\frac{1}{2}} = h_{y2}^n. \quad (10e)$$

The update equations for the second procedure can again be obtained by interchanging the subscripts of fields in lines 1 and 2. The RHS of (10) are also free from any spatial operator of the split matrices based on the fundamental scheme.

By substituting (10d) into (10b), E_{x1} can be solved to yield

$$\begin{aligned} \frac{1}{2}E_{x1}^{n+\frac{1}{2}} - \frac{\Delta t}{4\epsilon_s} \frac{\partial}{\partial z} \left(\frac{\Delta t}{2\mu_s} \frac{\partial E_{x1}^{n+\frac{1}{2}}}{\partial z} \right) \\ = e_{x1}^n - \frac{\Delta t}{2\epsilon_m} \frac{\partial h_{y2}^n}{\partial z} - \frac{\Delta t}{4\epsilon_s} \frac{\partial}{\partial z} \left(\frac{\Delta t}{2\mu_m} \frac{\partial e_{x2}^n}{\partial z} \right) - \frac{\Delta t}{2\epsilon_s} \frac{\partial h_{y1}^n}{\partial z}. \end{aligned} \quad (11)$$

Using central differencing, the LHS of (11) also yields tridiagonal matrices and has only self permittivity and permeability terms. On the other hand, the RHS of (11) still has second-order spatial derivative and the same amount of terms as the corresponding equation in M1-D ADI CL-FDTD method (without fundamental scheme). However, we can combine (10c)-(10e) with (10a) at the second procedure as

$$e_{x2}^{n+\frac{1}{2}} = e_{x2}^n \quad (12a)$$

$$h_{y1}^{n+\frac{1}{2}} = h_{y1}^n - \frac{\Delta t}{2\mu_s} \frac{\partial E_{x1}^{n+\frac{1}{2}}}{\partial z} + \frac{\Delta t}{2\mu_m} \frac{\partial E_{x2}^{n+\frac{1}{2}}}{\partial z} \quad (12b)$$

$$h_{y2}^{n+\frac{1}{2}} = h_{y2}^n. \quad (12c)$$

It should be noted that we could omit (12a) and (12c). Hence, only one explicit update equation (12b) is needed in each procedure.

C. 1st Set of Split Matrices with Mutual Symmetry

Instead of focusing on the self terms like above, we could look at the mutual terms and propose the set of split matrices as follows:

$$\mathbf{A}_{m1} = \begin{bmatrix} 0 & 0 & 0 & \boxed{\frac{-1}{\epsilon_m} \frac{\partial}{\partial z}} \\ 0 & 0 & 0 & \boxed{\frac{-1}{\epsilon_s} \frac{\partial}{\partial z}} \\ \boxed{\frac{-1}{\mu_s} \frac{\partial}{\partial z}} & 0 & 0 & 0 \\ \boxed{\frac{1}{\mu_m} \frac{\partial}{\partial z}} & 0 & 0 & 0 \end{bmatrix}, \quad (13a)$$

$$\mathbf{B}_{m1} = \begin{bmatrix} 0 & 0 & \frac{-1}{\epsilon_s} \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \boxed{\frac{-1}{\epsilon_m} \frac{\partial}{\partial z}} & 0 \\ 0 & \boxed{\frac{1}{\mu_m} \frac{\partial}{\partial z}} & 0 & 0 \\ 0 & \boxed{\frac{-1}{\mu_s} \frac{\partial}{\partial z}} & 0 & 0 \end{bmatrix} \quad (13b)$$

Unlike the previous sets of split matrices, the mutual permittivity and permeability terms (marked with boxes) are now symmetrically located in the split matrices. Hence, this set of split matrices is regarded as the 1st set of split matrices with mutual symmetry. The update equations can be similarly written down by substituting (13) into (4). Without going into details, it can be shown that the LHS of the update equation yields tridiagonal matrices that can be solved efficiently.

D. 2nd Set of Split Matrices with Mutual Symmetry

We consider the next set of split matrices shown below.

$$\mathbf{A}_{m2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{\epsilon_m} \frac{\partial}{\partial z} & \frac{-1}{\epsilon_s} \frac{\partial}{\partial z} \\ \frac{-1}{\mu_s} \frac{\partial}{\partial z} & \frac{1}{\mu_m} \frac{\partial}{\partial z} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (14a)$$

$$\mathbf{B}_{m2} = \begin{bmatrix} 0 & 0 & \frac{-1}{\epsilon_s} \frac{\partial}{\partial z} & \frac{-1}{\epsilon_m} \frac{\partial}{\partial z} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{\mu_m} \frac{\partial}{\partial z} & \frac{-1}{\mu_s} \frac{\partial}{\partial z} & 0 & 0 \end{bmatrix} \quad (14b)$$

This set of split matrices is regarded as the 2nd set of split matrices with mutual symmetry. Using this set of split matrices, it can also be shown that the LHS of the update equation yields tridiagonal matrices.

E. Set of Split Matrices with Self-Mutual Separation

For completeness, we include the set of split matrices with self-mutual separation in [11, (5a)-(5b)]. This set of split matrices will reduce naturally to the equations for uncoupled transmission lines when all the mutual terms are omitted, i.e. $\mathbf{B} = 0$. The LHS of the update equation again yields tridiagonal matrices.

III. STABILITY ANALYSIS

We shall analyze the stability of the M1-D FADI CL-FDTD method using the von Neumann method. The Fourier amplification matrix $\tilde{\mathbf{M}}$ can be derived as

$$\tilde{\mathbf{u}}^{n+1} = \left(\frac{1}{2} \mathbf{I} - \frac{\Delta t}{4} \tilde{\mathbf{B}} \right)^{-1} \left(\frac{1}{2} \mathbf{I} + \frac{\Delta t}{4} \tilde{\mathbf{A}} \right) \left(\frac{1}{2} \mathbf{I} - \frac{\Delta t}{4} \tilde{\mathbf{A}} \right)^{-1} \cdot \left(\frac{1}{2} \mathbf{I} + \frac{\Delta t}{4} \tilde{\mathbf{B}} \right) \tilde{\mathbf{u}}^n = \tilde{\mathbf{M}} \tilde{\mathbf{u}}^n \quad (15)$$

where $\tilde{\mathbf{u}}$, $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ are the field vector and split matrices in the Fourier domain. Using the relation $\tilde{\mathbf{v}} = \left(\frac{1}{2} \mathbf{I} + \frac{\Delta t}{4} \tilde{\mathbf{B}} \right) \tilde{\mathbf{u}}$, (15) can be transformed into

$$\tilde{\mathbf{v}}^{n+1} = \left(\frac{1}{2} \mathbf{I} + \frac{\Delta t}{4} \tilde{\mathbf{B}} \right) \left(\frac{1}{2} \mathbf{I} - \frac{\Delta t}{4} \tilde{\mathbf{B}} \right)^{-1} \left(\frac{1}{2} \mathbf{I} + \frac{\Delta t}{4} \tilde{\mathbf{A}} \right) \cdot \left(\frac{1}{2} \mathbf{I} - \frac{\Delta t}{4} \tilde{\mathbf{A}} \right)^{-1} \tilde{\mathbf{v}}^n = \tilde{\mathbf{M}}' \tilde{\mathbf{v}}^n. \quad (16)$$

Note that $\tilde{\mathbf{M}}'$ is the similarity transformation of $\tilde{\mathbf{M}}$ which preserves all the eigenvalues. Upon further manipulation, we obtain

$$\tilde{\mathbf{M}}' = \left[\left(\frac{1}{2} \mathbf{I} - \frac{\Delta t}{4} \tilde{\mathbf{B}} \right)^{-1} - \mathbf{I} \right] \left[\left(\frac{1}{2} \mathbf{I} - \frac{\Delta t}{4} \tilde{\mathbf{A}} \right)^{-1} - \mathbf{I} \right]. \quad (17)$$

The Fourier amplification matrix (17) is represented in fundamental form [15], which is simpler with only one inverse term each without RHS operator.

We first examine numerically the eigenvalues of the Fourier amplification matrix for M1-D FADI CL-FDTD method using

Fig. 1. Eigenvalues of the Fourier amplification matrix around semi unit circle for both sets of split matrices with self symmetry.

Fig. 2. Eigenvalues of the Fourier amplification matrix around semi unit circle for both sets of split matrices with mutual symmetry.

various sets of split matrices. The parameters of the coupled transmission lines are: $Z_{0e} = 59.59$, $Z_{0o} = 36.99$, $\epsilon_{eff}^e = 3.15$, $\epsilon_{eff}^o = 2.59$. Cell size is set as $\Delta z = 1$ mm. The time step is specified in terms of $CFLN = \Delta t / \Delta t_{CFL}$, where Δt_{CFL} is the CFL time step [9]. The eigenvalues are computed for some random Fourier wavenumbers along z direction. Fig. 1 plots the eigenvalues of the Fourier amplification matrix around semi unit circle for both sets of split matrices with self symmetry. It is found that their eigenvalues are the same and all have unity magnitude. Moreover, even at high $CFLN = 20$, all the eigenvalues are still located on the semi unit circle, indicating stability. Fig. 2 next plots the eigenvalues of the Fourier amplification matrix around semi unit circle for both sets of split matrices with mutual symmetry. The eigenvalues for these sets of split matrices are also the same. However, there are clearly some eigenvalues outside the semi unit circle especially for $CFLN = 2$. Therefore, both sets of split matrices with mutual symmetry are not unconditionally stable. Fig. 3 plots the eigenvalues of the Fourier amplification matrix around semi unit circle for the set of split matrices with self-mutual separation. It can be seen that some eigenvalues for $CFLN = 2$ also lie outside the circle, making this set of split matrices not unconditionally stable. This is consistent with the findings of [11] using M1-D ADI CL-FDTD method (without fundamental scheme).

We now prove analytically the unconditional stability for the sets of split matrices with self symmetry. It can be shown that their Fourier amplification matrices share the same characteristic polynomial as

$$\lambda^4 + c_1 \lambda^3 + c_2 \lambda^2 + c_1 \lambda + 1 = 0 \quad (18)$$

with real coefficients

$$c_1 = \frac{4 \left[\bar{K}_z^4 (\epsilon_m^2 \mu_m^2 - \epsilon_m^2 \mu_s^2 - \epsilon_s^2 \mu_m^2) - 2 \bar{K}_z^2 \epsilon_m \epsilon_s^2 \mu_m \mu_s^2 - \epsilon_m^2 \epsilon_s^2 \mu_m^2 \mu_s^2 \right]}{\epsilon_m^2 \mu_m^2 (\bar{K}_z^2 + \epsilon_s \mu_s)^2} \quad (19a)$$

Fig. 3. Eigenvalues of the Fourier amplification matrix around semi unit circle for set of split matrices with self-mutual separation.

$$c_2 = \frac{2\bar{K}_z^4(3\epsilon_m^2\mu_m^2 - 4\epsilon_m^2\mu_s^2 - 4\epsilon_s^2\mu_m^2 + 8\epsilon_s^2\mu_s^2) + 4\bar{K}_z^2(4\epsilon_m\epsilon_s^2\mu_m\mu_s^2 - \epsilon_m^2\epsilon_s\mu_m^2\mu_s) + 6\epsilon_m^2\epsilon_s^2\mu_m^2\mu_s^2}{\epsilon_m^2\mu_m^2(\bar{K}_z^2 + \epsilon_s\mu_s)^2}. \quad (19b)$$

$\bar{K}_z = \frac{\Delta t}{\Delta z} \sin\left(\frac{k_z \Delta z}{2}\right)$ and k_z is the Fourier wavenumber along the z direction. Instead of solving (18) for all four eigenvalues explicitly, we shall examine their magnitudes more simply via the following two auxiliary quadratic polynomials:

$$\lambda^2 - 2u\lambda + 1 = 0 \quad (20a)$$

$$u^2 + \frac{c_1}{2}u + \frac{c_2 - 2}{4} = 0. \quad (20b)$$

By substituting u from (20a) into (20b), the equivalence of (20) and (18) can be shown readily.

We first state that all roots of (20b) are real since from (19),

$$\begin{aligned} \text{discriminant} &\equiv c_1^2/4 - (c_2 - 2) \\ &= \frac{16\bar{K}_z^8(\epsilon_m^2\mu_s^2 - \epsilon_s^2\mu_m^2)^2 + 64\bar{K}_z^4(\epsilon_m\mu_s - \epsilon_s\mu_m)^2}{\epsilon_m^4\mu_m^4(\bar{K}_z^2 + \epsilon_m\mu_m)^4} > 0. \end{aligned} \quad (21)$$

Furthermore, it is of interest to determine whether $|u| \leq 1$ is satisfied. To that end, we shall consider two cases of $\bar{K}_z = 0$ and $\bar{K}_z \neq 0$. For $\bar{K}_z = 0$, the roots of (20b) are solved easily as $u = 1$ using $c_1 = -4$ and $c_2 = 6$. For $\bar{K}_z \neq 0$, we refer to (32) in Appendix and test

$$1 - \left| \frac{c_2 - 2}{4} \right| > \left| \frac{c_1}{2} - \frac{c_1^*}{2} \left(\frac{c_2 - 2}{4} \right) \right|? \quad (22)$$

If true, the condition (22) is the necessary and sufficient one for $|u| < 1$. It is derived based on the Schur-Cohn theorem [16], [17], whereby two conditions have been combined into a single one conveniently, cf. Appendix. Applying (19) into (22), we obtain the following relations:

If $\frac{c_1}{2} > \frac{c_1}{2} \left(\frac{c_2 - 2}{4} \right)$,

$$\begin{aligned} 1 - \left| \frac{c_2 - 2}{4} \right| &- \left| \frac{c_1}{2} - \frac{c_1}{2} \left(\frac{c_2 - 2}{4} \right) \right| \\ &= \frac{8\bar{K}_z^2\epsilon_s^2\mu_s^2(\bar{K}_z^2 + \epsilon_m\mu_m)^2 \left\{ \bar{K}_z^2[\mu_s^2(\epsilon_m^2 - \epsilon_s^2) + \epsilon_s^2(\mu_m^2 - \mu_s^2)] + 2\epsilon_m\epsilon_s\mu_m\mu_s(\epsilon_m\mu_m - \epsilon_s\mu_s) \right\}}{\epsilon_m^4\mu_m^4(\bar{K}_z^2 + \epsilon_m\mu_m)^4}; \end{aligned} \quad (23)$$

If $\frac{c_1}{2} < \frac{c_1}{2} \left(\frac{c_2 - 2}{4} \right)$,

$$\begin{aligned} 1 - \left| \frac{c_2 - 2}{4} \right| &- \left| \frac{c_1}{2} - \frac{c_1}{2} \left(\frac{c_2 - 2}{4} \right) \right| \\ &= \frac{8\bar{K}_z^6(\epsilon_m^2 - \epsilon_s^2)(\mu_m^2 - \mu_s^2) \left\{ \bar{K}_z^2[\mu_s^2(\epsilon_m^2 - \epsilon_s^2) + \epsilon_s^2(\mu_m^2 - \mu_s^2)] + 2\epsilon_m\epsilon_s\mu_m\mu_s(\epsilon_m\mu_m - \epsilon_s\mu_s) \right\}}{\epsilon_m^4\mu_m^4(\bar{K}_z^2 + \epsilon_m\mu_m)^4} \end{aligned} \quad (24)$$

Since $\epsilon_m > \epsilon_s > 0$ and $\mu_m > \mu_s > 0$ as are evident from (2),

$$1 - \left| \frac{c_2 - 2}{4} \right| - \left| \frac{c_1}{2} - \frac{c_1}{2} \left(\frac{c_2 - 2}{4} \right) \right| > 0 \quad (25)$$

for both (23) and (24). Hence, the necessary and sufficient condition (22) is indeed true and all roots of (20b) are inside the unit circle for $\bar{K}_z \neq 0$, i.e. $|u| < 1$. Overall, one can deduce that

$$u^2 \leq 1, \quad \Im(u) = 0. \quad (26)$$

We are now ready to analyze the magnitudes of eigenvalues, which can be obtained from the roots of (20a) as

$$\lambda = u \pm \sqrt{u^2 - 1} = u \pm j\sqrt{1 - u^2}. \quad (27)$$

Having ascertained (26), it can be readily shown that

$$|\lambda| = \sqrt{u^2 + (1 - u^2)} = 1. \quad (28)$$

Note that the analytical eigenvalues λ (or u) need not be solved explicitly, and the stability can be deduced solely based on the coefficients of characteristic polynomial. Using two auxiliary quadratic polynomials (20) along with single necessary and sufficient condition (22), the analytical proof of unconditional stability is made more convenient and concise. Since all eigenvalues have magnitudes of one for any time step Δt , the sets of split matrices with self symmetry for M1-D FADI CL-FDTD method are unconditionally stable.

IV. NUMERICAL RESULTS AND SIMULATIONS ON MOBILE DEVICES

Since the sets of split matrices with mutual symmetry and self-mutual separation are not unconditionally stable, they will be omitted in the subsequent discussions. For both stable sets of split matrices with self symmetry, we compare the number of RHS terms and flops count between M1-D FADI and ADI CL-FDTD methods in Table I. The number of terms includes those of field variables and coefficients, while the flops count includes those of multiplication/division (M/D) and addition/subtraction (A/S) operations. From Table I, we see that for the 1st set of split matrices with self symmetry, most savings in the number of RHS terms and flops count of FADI over ADI come from the implicit E update equations. This is exemplified in (7), which is much simpler than that of M1-D ADI method, cf. [10, (7)]. On the other hand, for the 2nd set of split matrices with self symmetry, most savings in the number of RHS terms and flops count of FADI over ADI come from the explicit equations. As seen in (12), only one explicit update equation (12b) is required in each procedure. Note that

TABLE I
COMPARISONS OF RHS TERMS AND FLOPS COUNT PER TIME STEP
BETWEEN M1-D FADI AND ADI CL-FDTD METHODS

Scheme	Update Equations	No. of Terms Field	No. of Terms Coeff.	No. of Flops M/D	No. of Flops A/S
M1-D FADI 1 st self sym.	Implicit E	6	2	2	4
	Explicit E	14	2	2	8
	Auxiliary e				
	Explicit H	16	4	4	10
	Auxiliary h				
	Total	36	8	8	22
M1-D FADI 2 nd self sym.	Implicit E	18	8	8	16
	Explicit E	4	0	0	2
	Auxiliary e				
	Explicit H	10	4	0	6
	Auxiliary h				
	Total	32	12	8	24
M1-D ADI 1 st or 2 nd self sym.	Implicit E	18	8	8	16
	Explicit E	10	4	4	8
	Explicit H	20	8	8	16
	Total	48	20	20	40

Fig. 4. Time-domain E fields at (a) port 2 and (b) port 3 of directional coupler computed using various methods. Inset shows the schematic.

the total number of RHS field terms for the 2nd set of split matrices with self symmetry are also fewer than those of the 1st set. This saving may be more significant due to the larger memory required for the field variables. Overall, both sets of split matrices with self symmetry for M1-D FADI CL-FDTD method have fewer number of RHS terms and flops count than the M1-D ADI CL-FDTD method. While maintaining unconditional stability, the M1-D FADI CL-FDTD method achieves greater simplicity and higher efficiency.

To validate the accuracy of M1-D FADI CL-FDTD method, we consider a directional coupler whose schematic is shown in inset of Fig. 4. The parameters of the directional coupler are: $Z_{0e} = Z_0 \sqrt{(1+C)/(1-C)}$, $Z_{0o} = Z_0 \sqrt{(1-C)/(1+C)}$, $\epsilon_{eff}^e = \epsilon_{eff}^h = 2.948$, where C is the coupling coefficient chosen to be $1/\sqrt{2}$ (i.e. 3 dB) at design frequency 1 GHz, and the reference impedance Z_0 is 50 Ω . The even- and odd-mode parameters (Z_{0e} , Z_{0o} , ϵ_{eff}^e , ϵ_{eff}^h) can be converted to and from the physical dimensions such as line width and spacing

Fig. 5. Time-domain (a) transmitted and (b) reflected E fields for coupled line filter computed using various methods. Inset shows the schematic.

Fig. 6. S parameters of coupled line filter computed using various methods.

using some commercial software tools, e.g. Advanced Design System (ADS). The length of directional coupler is quarter wavelength and cell size is $\Delta z = 0.175$ mm. A sinusoidal source at 5 GHz is excited from port 1. Fig. 4(a) shows the time-domain E fields at port 2 computed using various methods, including M1-D FADI, ADI and explicit CL-FDTD methods. We can see that the results for M1-D FADI and ADI CL-FDTD methods are consistent with each other for both sets of split matrices with self symmetry. At CFLN = 10, they are also in good agreement with the M1-D explicit CL-FDTD method. Fig. 4(b) now shows the time-domain E fields at port 3. Again, the fields computed by M1-D FADI, ADI and explicit CL-FDTD methods are all in good agreement with each other. Furthermore, it is observed that the field amplitudes at ports 2 and 3 are equal and there is a 90° phase difference between them. Hence, the accuracy of the proposed M1-D FADI CL-FDTD method is well validated.

We next show the usefulness of the M1-D FADI CL-FDTD method to efficiently simulate and visualize the time-domain fields of coupled line structures. We consider a coupled line bandpass filter from [1]. The schematic of the coupled line

Fig. 7. Snapshot of E fields for directional coupler computed using (a) M1-D FADI CL-FDTD and (b) M1-D explicit CL-FDTD methods on iPhone.

filter is shown in inset of Fig. 5. The filter consists of four coupled line sections and the (inhomogeneous) parameters are given as: $Z_{0e1} = 70.61$, $Z_{0o1} = 39.24$, $\epsilon_{eff1}^e = 3.18$, $\epsilon_{eff1}^o = 2.52$, $Z_{0e2} = 56.64$, $Z_{0o2} = 44.77$, $\epsilon_{eff2}^e = 3.14$, $\epsilon_{eff2}^o = 2.69$, $l_1 = 21.88$ mm, $l_2 = l_1$. Nonuniform cell sizes are adopted: the cell size Δz is generally set as 0.55 mm, while along the interface between the coupled line sections, Δz is set at three times smaller. A modulated Gaussian pulse is injected from the left side of the structure and the relevant E fields are recorded. Fig. 5 shows the time-domain (a) transmitted and (b) reflected E fields for coupled line filter computed using various methods. The fields computed by M1-D FADI CL-FDTD method with both sets of split matrices with self symmetry are compared with the M1-D explicit CL-FDTD method. Good agreement is observed at CFLN = 10 and 20, thus validating the proposed method. Fig. 6 next shows the S parameters of coupled line filter computed using various methods. For comparison, the results obtained using the ADS are included. Again, good agreement is observed among the methods. It should be noted that unlike the M1-D FADI and explicit CL-FDTD methods, the ADS does not directly provide time-domain fields along the coupled transmission lines. Moreover, the proposed M1-D FADI CL-FDTD method can be executed efficiently at time step larger than the stability limit with good accuracy.

Having validated the accuracy of the M1-D FADI CL-FDTD method, we proceed to demonstrate several EM simulations of coupled transmission lines on mobile devices. Since the M1-D ADI CL-FDTD method is more complicated with RHS involving matrix operators, it shall not be implemented on mobile devices herein. Fig. 7 shows the snapshot of E fields for directional coupler on iPhone. The fields are computed using M1-D FADI CL-FDTD method at CFLN = 4 in Fig. 7(a), and M1-D explicit CL-FDTD method in Fig. 7(b). The fields computed using both methods are consistent with each other. The equal amplitude and 90° phase difference between ports 2 and 3 can be visualized on mobile devices. We can also observe that there is no wave in port 4, which is the isolated port. Due to its unconditional stability, the CFLN of M1-D FADI CL-FDTD method can be adjusted to be larger than one using the slider. Hence, the simulation

TABLE II
CPU TIME INCURRED IN SECONDS FOR 20000 ITERATIONS OF M1-D FADI AND EXPLICIT CL-FDTD METHODS ON VARIOUS PLATFORMS

Platform	M1-D FADI 1 st self sym.	M1-D FADI 2 nd self sym.	M1-D explicit
MacBook Pro Simulator	2.0	2.0	0.7
iPad 3	39.6	36.3	11.7
iPad Pro	4.4	3.8	1.3
iPhone 7 Plus	4.3	3.6	1.3

can be ‘fast-forwarded’ and accelerated with higher efficiency. Using mobile devices, the EM simulations can be performed efficiently and ubiquitously anytime, anywhere.

To analyze the efficiency, we record the CPU time incurred on various platforms for M1-D FADI CL-FDTD method using both sets of split matrices with self symmetry. The CPU time incurred in seconds for 20000 iterations is provided and compared with M1-D explicit CL-FDTD method in Table II. The platforms tested include MacBook Pro (Retina) with XCode simulator, iPad 3, iPad Pro and iPhone 7 Plus. On MacBook Pro, the incurred CPU time for M1-D FADI CL-FDTD method is the same using both sets of split matrices with self symmetry. Interestingly, the incurred CPU time using the 2nd set of split matrices is shorter than the 1st set on mobile platforms iPad 3, iPad Pro and iPhone 7 Plus. This may be attributed to fewer RHS field variable terms for the 2nd set of split matrices. Among all, the CPU time incurred on MacBook Pro is the shortest, while it is the longest on the older iPad 3. To the best of our knowledge, there is not much EM simulation on mobile devices to date and no CPU time comparison reported so far, which may be due to less efficiency and insufficient memory in previous mobile devices. With the advances in more contemporary mobile platforms such as iPad Pro and iPhone 7 Plus, the CPUs become more powerful thus allowing more pervasive EM simulations. Note that with unconditional stability, the M1-D FADI CL-FDTD method can be executed at time step larger than the stability limit, thus requiring fewer iterations than the explicit method. For instance, at CFLN = 10, the simulation by M1-D FADI CL-FDTD method using the 2nd set of split matrices can be completed at 3.6 times faster than the M1-D explicit CL-FDTD method on iPhone 7 Plus. Thus, high efficiency can be achieved on recent mobile devices, thanks to not only the improved hardware but also the unconditionally stable numerical method.

We also perform the EM simulation of coupled line filter on mobile devices. Fig. 8 shows the snapshot of E fields for coupled line filter computed using (a) M1-D FADI CL-FDTD and (b) M1-D explicit CL-FDTD methods on iPhone. For M1-D FADI CL-FDTD method, CFLN is set as 10 and the fields are consistent with those of the explicit method. The CFLN of the M1-D FADI CL-FDTD method can again be adjusted to be larger than one using the slider. The EM dynamics of the coupled line filter such as coupling and resonating mechanisms on each coupled line section can be visualized clearly on mobile devices. Therefore, one is able to simulate and visualize the EM fields along the coupled line filter ubiquitously on mobile devices.

Fig. 8. Snapshot of E fields for coupled line filter computed using (a) M1-D FADI CL-FDTD and (b) M1-D explicit CL-FDTD methods on iPhone.

V. CONCLUSION

This paper has presented the M1-D FADI CL-FDTD method for coupled transmission lines on mobile devices. The formulations of the M1-D FADI CL-FDTD method and update equations have been provided. Various sets of split matrices including those with self symmetry, mutual symmetry and self-mutual separation have been proposed and discussed. Their stability analysis have been performed using the Fourier amplification matrix formulated in fundamental form, which is simpler with only one inverse term each without RHS operator. It has been found that the sets of split matrices with self symmetry are unconditionally stable, while those with mutual symmetry and self-mutual separation are not. Using two auxiliary quadratic polynomials along with single necessary and sufficient condition, the analytical proof of unconditional stability has been made more convenient and concise. The efficiency has been discussed based on the number of RHS terms and flops count. Numerical results have been presented to validate the accuracy of the proposed method at time step larger than the stability limit. Several EM simulations of coupled line structures have been demonstrated on mobile devices. Due to its unconditional stability and efficiency, the M1-D FADI CL-FDTD method can be generalized to multiple coupled lines including transmission lines with stubs as well.

APPENDIX

For a quadratic polynomial with complex coefficients:

$$au^2 + bu + c = 0, \quad (29)$$

an Hermitian matrix can be constructed according to the Schur-Cohn theorem [16], [17] as

$$\mathbf{H} = \begin{bmatrix} |a|^2 - |c|^2 & ab^* - c^*b \\ a^*b - cb^* & |a|^2 - |c|^2 \end{bmatrix} \quad (30)$$

(* denotes complex conjugate). Let H_1 and H_2 be the two leading principal minors of $\mathbf{H}$ and assume that they are nonzero. The theorem states that there are v roots outside and $(2-v)$ roots inside the unit circle, where v corresponds to the number of sign variation in the sequence 1, H_1 , H_2 . For all

roots to be inside the unit circle, i.e. $v = 0$, there will be no sign variation in the sequence 1, H_1 , H_2 when

$$H_1 = |a|^2 - |c|^2 > 0 \quad (31a)$$

$$H_2 = (|a|^2 - |c|^2)^2 - |a^*b - cb^*|^2 > 0. \quad (31b)$$

Combining both conditions in (31), we arrive at the single necessary and sufficient condition for $|u| < 1$:

$$|a|^2 - |c|^2 > |a^*b - cb^*|. \quad (32)$$

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