Robust optimal operation of future power systems under uncertainties

Lahanda Purage Mohasha Isuru Sampath

2020

Lahanda, P. M. I. S. (2020). Robust optimal operation of future power systems under uncertainties. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/137301>

<https://doi.org/10.32657/10356/137301>

This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License (CC BY-NC 4.0).

Downloaded on 26 Apr 2023 15:20:08 SGT

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

SINGAPORE

**ROBUST OPTIMAL OPERATION OF
FUTURE POWER SYSTEMS UNDER
UNCERTAINTIES**

LAHANDA PURAGE MOHASHA ISURU SAMPATH

Interdisciplinary Graduate School
Energy Research Institute @ NTU

2020

ROBUST OPTIMAL OPERATION OF FUTURE POWER SYSTEMS UNDER UNCERTAINTIES

LAHANDA PURAGE MOHASHA ISURU SAMPATH

INTERDISCIPLINARY GRADUATE SCHOOL

A thesis submitted to **Nanyang Technological University**
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

17-DEC-2019

.....

Date

.....

Lahanda Purage
Mohasha Isuru Sampath

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

17-DEC-2019

.....

Date

.....

Assoc. Prof. H. B. Gooi

Authorship Attribution Statement

This thesis contains material from one journal paper published and two papers under review in the following peer-reviewed journals where I was the first and corresponding author.

Chapter 4 is published as: **L. P. M. I. Sampath**, B. V. Patil, H. B. Gooi, J. M. Maciejowski, and K. V. Ling, “A trust-region based sequential linear programming approach for AC optimal power flow problems,” *Electric Power Systems Research*, 165 (2018), 134-143.

The contributions of the co-authors are as follows:

- Dr. B. V. Patil and I jointly developed the concept of trust-region based sequential linear program and implemented the algorithm in MATLAB.
- I obtained the simulation results, and prepared the manuscript and the responses to reviewers’ comments.
- Dr. B. V. Patil assisted me to get the initial results. He proofread the manuscript and suggested improvements to the content and organization of the manuscript. He further assisted me to address the reviewers’ comments.
- Assoc. Prof. H. B. Gooi and Prof. J. M. Maciejowski supervised the research work, proofread the manuscript at various stages, suggested improvements to the content and organization of the manuscript, and guided me to address reviewers’ comments.
- Assoc. Prof. K. V. Ling provided technical inputs during the discussions.

Chapter 5 and 6 is under revision as: **L. P. M. I. Sampath**, M. Hotz, H. B. Gooi, and W. Utschick, “Network-constrained thermal unit commitment for hybrid AC/DC transmission grids under wind power uncertainty,” *Applied Energy*, vol. 258, pp. 114031, 2020.

The contributions of the co-authors are as follows:

- I developed the concept of day-ahead energy and reserve scheduling framework for hybrid AC/DC transmission grids, implemented the program in MATLAB, produced the results, and prepared the manuscript and the responses to reviewers’ comments.
- Mr. M. Hotz suggested working on combining the unit commitment and optimal power flow for hybrid AC/DC transmission grids, guided to avoid numerical issues in the program, proofread the manuscript, assisted me in improving the content and organization of the manuscript, and helped me during the revision of the manuscript.
- Assoc. Prof. H. B. Gooi supervised the research work, proofread the manuscript at various stages, suggested improvements to the content and organization of the manuscript, and guided me to address reviewers’ comments.
- Prof. W. Utschick suggested the concept of the hybrid AC/DC transmission grid architecture (jointly with Mr. M. Hotz), supervised the research work and suggested improvements to the content and organization of the manuscript.

Chapter 7 is under revision as: **L. P. M. I. Sampath**, Ashok Krishnan, Y. S. Foo Eddy, and H. B. Gooi, “Cooperative bidding-based robust optimal energy management of multi-microgrids,” *IEEE Transactions on Industrial Informatics*, pp. 1-1, 2019.

The contributions of the co-authors are as follows:

- I have developed the concepts of nonanticipative reserve scheduling and co-operative trading in multi-microgrid systems, implemented the program in MATLAB, produced the results, and prepared the manuscript and the responses to reviewers' comments.
- Mr. Ashok Krishnan assisted me in improving the content and organization of the manuscript, and assisted me in the process of addressing the reviewers' comments.
- Dr. Y. S. Foo Eddy proofread the manuscript at various stages, suggested improvements to the content and organization of the manuscript, and guided me to address reviewers' comments.
- Assoc. Prof. H. B. Gooi supervised the research work, proofread the manuscript at various stages, suggested improvements to the content and organization of the manuscript, and guided me to address reviewer comments.

17-DEC-2019

.....

Date

.....

Lahanda Purage
Mohasha Isuru Sampath

To My Family.

Acknowledgment

First and foremost, I would like to express my sincere gratitude to my supervisor, Associate Professor Gooi Hoay Beng, for providing me the opportunity to pursue a Ph.D. under his supervision. He always encouraged me to work hard and shape my goals for better career prospects. His constant encouragement, technical inputs, and guidance during my research study always built my enthusiasm and helped me to explore new ideas.

I would also like to sincerely thank my Thesis Advisory Committee members Professor Ong Yew Soon and Professor Wolfgang Utschick (Technische Universität München, Germany) for their guidance and encouragement during my Ph.D. journey. I would like to further thank Professor Wolfgang Utschick for suggesting the concept of hybrid AC/DC transmission grids and hosting me in Technische Universität München, Germany during a visit in May 2016. I would also like to sincerely thank Professor Jan Maciejowski of Cambridge University for all his technical inputs during various discussions. The interactions with these professors certainly enriched my Ph.D. experience.

I am thankful to Mr. Matthias Hotz for various suggestions regarding convex relaxations, software implementation issues, and paper submissions. I would also like to thank Mr. Ashok Krishnan, Dr. Bhagyesh Patil, and Dr. Foo Yi Shyh Eddy for various technical collaborations and providing me an encouraging research environment a researcher could ask for.

I would like to thank Ms. Huang Minying, Ms. Lilly Lim Seok Kim and all the IGS staff for their timely support with administrative requirements. I would like to thank Madam Chia-Nge Tak Heng and Ms. Lin Zhiren, in the Clean Energy Research Lab. I am also thankful for all my colleges in the Clean Energy Research Lab, and the graduate students in NTU from Sri Lanka who helped me to spend a wonderful time in Singapore during my Ph.D. studies.

I am grateful to my grandmother, my parents, my wife, and my brothers for their love and support which helped me a lot to overcome many difficult situations during my Ph.D. journey.

Finally, this research would not have been possible without the financial support from the International Center of Energy Research (ICER), established by Nanyang Technological University, Singapore and Technische Universität München, Germany.

Abstract

As a result of the government incentives based on the policies on the decarbonization of the energy sector, renewable energy sources (RESs) are increasingly integrated into the power systems. This will introduce significant real-time variability in power injections due to the stochastic nature of these sources and the demand. As such, optimal reserve allocation is essential for economical and reliable operation of future power systems. Further, as many of these RESs are remote generation facilities, transmission congestion can be an upcoming issue faced by the system operators. Therefore, multi-terminal DC systems may be an essential part of future power systems due to their inherent capability to integrate RESs, to support voltage management, and to reduce transmission congestion via flexible power flow control. Furthermore, the concept of the deregulated markets promotes the proliferation of microgrids (MGs) in future power systems. The MGs comprise various types of flexible sources, which make them more promising for locally managing uncertainties. As such, the aforementioned challenges and the structural changes necessitate the development of efficient and scalable unit commitment (UC), economic dispatch, and optimal power flow (OPF) tools incorporating accurate optimization models for the analysis, planning, and operation of future power systems.

This thesis is broadly divided into three parts. The first part of the thesis proposes a new trust-region based sequential linear programming algorithm to solve the AC-OPF problem. Therein, the OPF problem is linearized using the Taylor series and a trust-region constraint is used to control the validity of the linear model. A feasibility restoration phase is introduced to locate a feasible point. The algorithm converges satisfying the first-order optimality conditions for the AC-OPF problem. The case studies illustrate the efficiency and computational performance of the proposed approach based on the standard IEEE and large-scale Polish systems.

In the second part of the thesis, a two-stage robust optimization (RO) framework is proposed for the joint scheduling of power dispatch and reserves for hybrid AC/DC transmission grids (HTGs). The optimal day-ahead schedule, including the commitment, power dispatch, and reserve allocations is computed subject to the feasibility of nonlinear AC and DC network constraints (NCs) while respecting the nonanticipativity under a specified set of scenarios. The proposed decomposition approach renders network security evaluations into standard OPF problems. As such, the scalability issues observed with existing methods are resolved.

Further, a second-order cone relaxation is applied to nonlinear AC and DC NCs to improve the computational tractability and the globally optimal solvability. In simulations, the proposed RO framework is illustrated using a recently proposed HTG architecture with a specific topology. The results demonstrate the effective utilization of the grid capacity to accommodate more demand and RESs. This HTG architecture induces exactness of the convex relaxation, supporting the validity of the optimal day-ahead schedule. The efficiency and computational performance of the proposed approach are compared to the existing literature and the robustness is verified using out-of-sample analyses.

Recently, the concept of multi-microgrid (MMG) systems has come into prominence due to the economic benefits accrued through the sharing of resources between the constituent MGs. The final part of the thesis proposes an RO framework to determine the day-ahead schedule of an MMG system. Unlike the existing works in the literature, the proposed RO framework preserves the nonanticipativity in reserve scheduling. The proposed RO framework also includes a cooperative bidding-based trading scheme to facilitate the sharing of power and reserves between the constituent MGs of the MMG system while respecting privacy concerns. The results highlight the economic benefits obtained through the sharing of resources between the constituent MGs in an MMG system. Furthermore, the proposed nonanticipative RO framework outperformed the existing RO frameworks in the literature in terms of robustness for MMG systems.

Contents

Acknowledgment	i
Abstract	iii
List of Figures	ix
List of Tables	xiii
Abbreviations and Nomenclature	xvii
1 Introduction	1
1.1 Background and Motivation	1
1.1.1 Challenges in Optimal Operation of the Future Power Systems	1
1.1.2 Motivation	4
1.2 Contributions	5
1.3 Thesis Organization	6
2 Literature Review	9
2.1 Optimal Power Flow	9
2.1.1 Linear Approximations of Optimal Power Flow	10
2.1.2 Convex Relaxations of Optimal Power Flow	11
2.1.3 Optimal Power Flow of Hybrid AC/DC Transmission Grids	12
2.2 Network-Constrained Unit Commitment	13
2.3 Optimal Scheduling under Uncertainties	15
2.3.1 Stochastic and Robust Optimization	15
2.3.2 Multi-Stage Optimization Frameworks	16
2.3.2.1 Column and Constraints Generation Method	16
2.3.2.2 Benders Decomposition	17
2.3.3 Nonanticipativity in Day-Ahead Scheduling	18
2.4 Multi-Microgrid Systems	19
3 Optimal Power Flow Formulation	23
3.1 System Model	23
3.1.1 Architecture	23
3.1.2 AC Branch Model	24

3.1.3	AC Bus Model	25
3.1.4	AC Network Constraints	26
3.1.4.1	Bus Voltage Magnitude	26
3.1.4.2	AC Branch Ampacity	26
3.1.4.3	AC Branch Voltage Magnitude Drop	27
3.1.4.4	AC Branch Voltage Angle Difference	27
3.1.5	DC Branch Model	29
3.1.6	DC Bus Model	29
3.1.7	DC Network Constraints	30
3.1.7.1	Bus Voltage Magnitude	30
3.1.7.2	DC Branch Power Flow	30
3.1.7.3	DC Branch Voltage Magnitude Drop	31
3.1.8	Converter Model	31
3.1.9	Generator Model	34
3.2	Problem Formulation	36
3.2.1	Power Balance	36
3.2.1.1	Power Balance of the AC Network	37
3.2.1.2	Power Balance of the DC Network	37
3.2.1.3	Total Power Loss	38
3.2.2	Optimal Power Flow Problem	38
3.3	Summary	39
4	A TR-SLP Approach for AC-OPF	41
4.1	Introduction	41
4.2	Mathematical Formulation	42
4.2.1	AC-OPF Problem Formulation	42
4.2.2	Linear Programming Formulation	44
4.3	TR based SLP Algorithm	46
4.3.1	Trust-Region Linear Program	46
4.3.2	Feasibility Restoration	47
4.3.3	Step Acceptance/Rejection Criterion	48
4.3.4	Discussion on Convergence	50
4.4	Numerical Experiments and Discussion	51
4.4.1	Test Systems for Benchmarking TR-SLP	52
4.4.2	Performance Comparison of TR-SLP with Existing Approaches	53
4.4.2.1	Case Study 1: Computation Time Comparison	54
4.4.2.2	Case Study 2: Relative Optimality Comparison	56
4.4.2.3	Performance Comparison with MATPOWER Solver Package	60
4.5	Summary	61
5	Network-Constrained Robust UC	63
5.1	Introduction	63
5.2	Wind Power Uncertainty	64

5.2.1	Uncertainty Set	64
5.2.2	Scenario Selection	65
5.3	Robust Day-Ahead Power Dispatch and Reserve Management with NCs	67
5.3.1	Modeling the Generator Ramp Constraints while Preserving Nonanticipativity in Power Dispatch	68
5.3.2	Robust Unit Commitment Formulation	70
5.3.3	Network Constraint Formulation	72
5.4	Two-stage Iterative Algorithm based on Benders Decomposition	73
5.4.1	Master Problem Formulation	74
5.4.2	Subproblem Formulation	74
5.4.3	Benders Cut Formulation	76
5.4.4	Algorithm for Robust Day-Ahead Optimal Power Dispatch and Reserve Management	78
5.5	Summary	78
6	Convex Relaxations for Network Constraints of HTGs	81
6.1	Introduction	81
6.2	Convex Relaxations for Network Constraints	82
6.2.1	Second-Order Cone Relaxation	82
6.2.2	Exactness and the Hybrid AC/DC Grid Architecture	85
6.2.3	Interpretation and Handling of Inexactness	87
6.2.4	Day-Ahead Robust Scheduling Algorithm Considering SOCR for NCs	88
6.3	Simulation Results	88
6.3.1	Test System	88
6.3.2	Numerical Simulations	90
6.3.3	Case Study 1: Computational Performance	91
6.3.4	Case Study 2: Economic Efficiency and Grid Loadability	96
6.3.5	Case Study 3: Generation Utilization	98
6.3.6	Comparison with Existing Methods	100
6.3.6.1	Efficiency and Computational Performance	103
6.3.6.2	Validation of the Generation Schedule Considering Different RO Models	103
6.3.6.3	Validation of the Reserve Margins for Different Scenario Selection Schemes	105
6.4	Summary	106
7	Robust Optimal Energy Management of MMGs	109
7.1	Introduction	109
7.2	Uncertainty Model for Robust Optimization	110
7.2.1	Uncertainty Characterization	110
7.2.2	Scenario Selection based on the TOAT	111
7.3	Robust Day-Ahead Optimal Energy Management of Multi-Microgrids	113

7.3.1	Multi-Microgrid System Structure	113
7.3.2	Nonanticipativity in Reserve Scheduling	114
7.3.3	Microgrid Component Modeling	114
7.3.3.1	Energy Storage System	115
7.3.3.2	Diesel Generators	117
7.3.3.3	Dispatchable Loads	117
7.3.3.4	Interactions with the PSO	118
7.3.4	Energy Management Model for a Single Microgrid	119
7.4	Optimal Energy and Reserve Sharing via Cooperative Bidding	121
7.4.1	Distributed Algorithm for Market Clearing	121
7.4.2	Convergence Characteristics	123
7.5	Case Studies	124
7.5.1	Simulation Settings	124
7.5.2	Convergence Analysis	126
7.5.3	Robust Optimal Operation of the MMG System Considering Optimal PFs	127
7.5.3.1	Role of the PSO and the MGOs in Real-Time Op- eration	128
7.5.3.2	Validation of the RO-PF Solution	130
7.5.4	Comparison Between the Cooperative and Isolated Trading Schemes	132
7.6	Summary	135
8	Conclusions and Recommendation for Future Works	137
8.1	Conclusions	137
8.2	Recommendations for Future Works	140
A	Taguchi's Orthogonal Array Testing	143
B	Latin Hypercube Sampling Method	145
C	List of Publications	147
	Bibliography	149

List of Figures

3.1	Model for AC branch $l \in \mathcal{L}_{ac}$ which connects bus $\hat{\varepsilon}(l)$ and bus $\check{\varepsilon}(l)$ [1].	25
3.2	Model for AC bus $n \in \mathcal{N}_{ac}$ [1].	25
3.3	Converter which couples AC bus $\tilde{o}(z)$ and DC bus $\bar{o}(z)$, where bus ① is the point-of-common coupling with the AC subgrid; bus ② is the transformer/filter bus; bus ③ is the AC side of the VSC; and bus ④ is the DC side of the VSC and also the point-of-common coupling with the DC subgrid.	32
3.4	Approximated P/Q-capability region of converter $z \in \mathcal{Z}$. (a) More conservative box representation and (b) more accurate polyhedral representation using the intersection of 8 half-planes (linear segments which enclose the capability domain). Curve ① is the maximum apparent power flow limit based on the current handling capability of the VSC.	34
3.5	Approximated polyhedral P/Q-capability region of generator $g \in \mathcal{G}$. Curve ① is the apparent power generation capability; Lines ② and ③ are the power factor limits.	35
4.1	The computational scalability comparison among all solvers/algorithms for various IEEE (118, 300) and Polish (2383, 2746, 3012) systems. TR-SLP is simulated under the flat start strategy.	56
4.2	Relative comparison in iterations for the convergence of TR-SLP under flat start and decoupled warm start strategies for various IEEE (118, 300) and Polish (2383, 2746, 3012) systems.	60
5.1	Power dispatch trajectories (dark dashed lines) of generator $g \in \mathcal{G}$ under each anticipated scenario vector.	68
5.2	Reserve allocations of generator $g \in \mathcal{G}$ for two consecutive time periods.	69
6.1	Convergence trajectories of the objective value and constraint violations for (a) the ACG and (b) the HTG.	92
6.2	Convergence trajectories of the power loss over the optimization horizon under the base scenario.	93
6.3	Convergence trajectories of the aggregated power dispatch and the total number of online units for (a) the ACG and (b) the HTG during hour 4 (the graph on the left side) and hour 18 (the graph on the right side).	93

6.4	Computational time of the MP and SPs over the iterations for (a) the ACG and (b) the HTG.	94
6.5	Voltage reconstruction error for 1560 SPs of (a) AC and (b) DC subgrids of the HTG. Black and gray lines show the maximum and average reconstruction errors for each SP respectively.	95
6.6	Day-ahead hourly operational schedule for generator 1 in the HTG. Curve 1 is the base scenario power dispatch. Curve 2 and 3 are the up and down reserve margins with respect to the power dispatch for the robust operation. Curve 4 and 5 are the maximum and minimum generation capabilities respectively.	95
6.7	Convergence trajectories of the aggregated power dispatch and total number of online units under 10% higher demand for (a) the ACG and (b) the HTG during hour 4 (the graph on the left side) and hour 18 (the graph on the right side).	97
6.8	Computation time of the MP and SPs over the iterations for (a) the ACG and (b) the HTG.	98
6.9	Voltage reconstruction error for 1560 SPs of (a) AC and (b) DC subgrids of the HTG.	99
6.10	Curve 1 is the total power dispatch as a fraction of online generation capability for the base-case. Curve 2 and 3 are the up and down reserve margins as a fraction of online generation capability respectively. Curve 4 is the up reserve utilization as a fraction of the up reserve capability of online units. Curve 5 is the total wind power generation as a fraction of demand.	100
6.11	Aggregated generation profiles of the HTG at 32% increased demand and 45% increased wind power. Curve 1 is the total power dispatch for the base-case. Curve 2 and 3 are the up and down reserve margins respectively. Curve 4 is the total generation capacity of the online units. Curve 5 is the total generation capacity of the system.	101
6.12	Computation time of the four schemes over the iterations based on the (a) time per MP evaluation and (b) maximum time per SP evaluation.	104
6.13	Histogram of the normalized cost for the Scenario-wise RO Approach (Scheme 1) and the proposed Nonanticipative RO Approach (Scheme 2) with the LHS method considering feasible wind power scenarios (out of 1,000 scenarios). Bin width of the histogram is 5.5×10^{-4}	105
6.14	Histogram of the normalized cost for the proposed Nonanticipative RO Approach with the LHS method (Scheme 2), the TOAT (Scheme 3), and the Monte Carlo method (Scheme 4) considering feasible wind power scenarios (out of 1,000 scenarios). Bin width of the histogram is 5.5×10^{-4}	107
7.1	The typical structure of an MMG system.	113
7.2	Forecasts for (a) load demand and (b) aggregated RES generation.	126

7.3	Convergence characteristics of Algorithm 7.1 for (a) DO and (b) RO-PF.	127
7.4	Number of iterations required for Algorithm 7.1 to converge for different values of α	127
7.5	(a)-(d) Base scenario power dispatch profiles (black curves), and up and down reserve margins (red and blue curves) obtained using RO-PF approaches and power dispatch profile (magenta curve) obtained using DO approach of DG 1 in 4 MGs. (e)-(h) PFs of DGs against uncertainties of each MG.	129
7.6	(a)-(d) Power and reserve exchanges between the MGOs and the PSO, where $\Delta P_{O,0}^{m,t} = P_{O,0}^{B,m,t} - P_{O,0}^{S,m,t}$, and the upper and lower operational bounds respectively. (e)-(h) PFs of power exchange against uncertainties of each MG.	129
7.7	Profiles of ESS 1 in MG 2 and 3 for the base scenario power dispatch (black curve), up and down reserve margins (red and blue curves) obtained using the RO-PF approach.	130
7.8	Flexible demand dispatch profiles under the base scenario (black curve) and the worst scenario (blue curve) in MG 2 and 3 obtained using the RO-PF approach.	130
7.9	Base scenario power dispatch profile and up and down reserve margins of the DGs under the isolated trading scheme in the 4 MGs. . .	133
7.10	Market equilibrium price at the base scenario of the robust operation.	133
7.11	Interaction in between PSO and the utility grid.	133
7.12	Expected TOC in cooperative and isolated trading for the 4 MGs along with the percentage cost reduction.	134

List of Tables

4.1	Details of the Test Cases [2]	52
4.2	Computation Time Comparison among All Solvers/Algorithms. TR-SLP is Executed under the Flat Start Strategy.	55
4.3	Relative Efficiency in the Optimality of TR-SLP Solution in Comparison to KNITRO-MS. TR-SLP is Executed under the Flat Start Strategy.	57
4.4	Relative Efficiency in the Optimality of TR-SLP Solution in Comparison to KNITRO-MS. TR-SLP is Executed under the Decoupled Warm Start Strategy.	58
4.5	Performance Comparison between the Best Known Solution of TR-SLP with MATPOWER Interior Point Solver [2]	61
5.1	Orthogonal Array L_42^3	66
6.1	Comparison of Economic Efficiency w.r.t. Grid Loadability	96
6.2	Generator Parameters	102
7.1	Selected Scenarios based on the OA L_82^7	112
7.2	Linear Cost Parameters of DGs	125
7.3	Cost Associated with DO, RO, and RO-PF Models	128
7.4	Optimality and Robustness Evaluation of the RO Solutions under Different Degrees of Uncertainties	132
A.1	Scenarios Specified by TOAT based on L_42^3	144

List of Algorithms

- 4.1 Trust-Region Sequential Linear Program 50
- 5.1 Day-ahead optimal power dispatch and reserve management 79
- 6.1 Day-ahead robust optimal scheduling with an SOCR of NCs. 89
- 7.1 Cooperative Optimal Power Sharing Coordination 121

Abbreviations and Nomenclature

A. Abbreviations

ACG	AC grid.
BC	Benders cut.
BD	Benders decomposition.
CCG	Column and constraints generation.
COP	Centralized optimization problem.
DG	Diesel generator.
DO	Deterministic optimization.
ESS	Energy storage system.
HTG	Hybrid AC/DC transmission grid.
KNITRO-MS	KNITRO multi-start.
LHS	Latin Hypercube Sampling.
LP	Linear programming.
MEP	Market equilibrium price.
MG	Microgrid.
MGO	Microgrid operator.
MMG	Multi-microgrid.
MP	Master problem.
NC	Network constraint.
NLP	Nonlinear programming.
OA	Orthogonal array.
OPF	Optimal power flow.
PCC	Point-of-common-coupling.
PF	Participation factor.
PSD	Positive semidefinite.
PSO	Power-sharing operator.
RES	Renewable energy source.
RO	Robust optimization.

SDR	Semidefinite relaxation.
SLP	Successive/Sequential linear programming.
SO	Stochastic optimization.
SoC	State-of-charge.
SOCR	Second-order cone relaxation.
SP	Subproblem.
TOAT	Taguchi's orthogonal array testing.
TOC	Total operating cost.
TR	Trust-region.
TR-SLP	Trust-region based sequential linear program.
UC	Unit commitment.
VSC	Voltage source converter.

B. Indexes and Sets

$\hat{\mathcal{L}}_{ac}(n)/\check{\mathcal{L}}_{ac}(n)$	Set of AC lines outgoing/incoming at bus n .
$\hat{\mathcal{L}}_{dc}(n)/\check{\mathcal{L}}_{dc}(n)$	Set of DC lines outgoing/incoming at bus n .
$\hat{\varepsilon}/\check{\varepsilon}$	Set which maps an AC line to its source/destination bus.
$\hat{\vartheta}/\check{\vartheta}$	Set which maps a DC line to its source/destination bus.
$\mathbb{R}$	Set of real numbers.
$\mathbb{S}$	Set of Hermitian matrices.
Ω^t	Uncertainty set for time period t .
$\tilde{o}/\bar{o}$	Set which maps a VSC to its AC/DC bus.
ϱ/ς	Set which maps a generator/RES to its injection bus.
$k/\mathcal{K}$	Index/Set of iterations.
$l/\mathcal{L}_{ac}/\mathcal{L}_{dc}$	Index/Set of AC/DC transmission lines.
$m/\mathcal{M}$	Index/Set of MGs.
$n/\mathcal{N}_{ac}/\mathcal{N}_{dc}$	Index/Set of AC/DC buses.
$\mathcal{H}_G/\mathcal{H}_C$	Polyhedral capability set for generators/VSCs.
ω/Υ	Index/Set of scenarios.
$e/\mathcal{E}$	Index/Set of ESSs.
$f/\mathcal{F}$	Index/Set of RESs.
$g/\mathcal{G}$	Index/Set of generators or DGs.
$t/\mathcal{T}$	Index/Set of time instances (hours).

C. Operators

$(\cdot)^*$	Complex conjugate of complex quantity $(\cdot)$.
$(\cdot)^H$	Hermitian of the complex matrix $(\cdot)$.

$(\cdot)^T$	Transpose of the matrix $(\cdot)$.
$\text{tr}(\cdot)$	Trace operator of square matrix $(\cdot)$.
D. Parameters	
$(\cdot)^{\min}/(\cdot)^{\max}$	Minimum/Maximum limit of the variable $(\cdot)$.
$N_{\text{ac}}/N_{\text{dc}}$	Number of AC/DC buses.
$L_{\text{ac}}/L_{\text{dc}}$	Number of AC/DC transmission lines.
$\tilde{y}_l/\hat{y}_l/\check{y}_l$	Series admittance/Source bus shunt admittance/Destination bus shunt admittance for AC line l .
$\dot{y}_n$	Shunt admittance at AC bus n .
$\hat{\rho}_l/\check{\rho}_l$	Source side/Destination side complex voltage ratio for AC line l .
$\hat{Y}_{\text{ac}}/\check{Y}_{\text{ac}}$	Source/Destination branch admittance matrix of AC subgrid.
e_n	n^{th} standard basis vector of appropriate dimension.
$Y_{\text{ac}}/Y_{\text{dc}}$	AC/DC bus admittance matrix.
$\hat{Y}_{\text{dc}}/\check{Y}_{\text{dc}}$	Source/Destination branch admittance matrix of DC subgrid.
$\tilde{M}_n/\bar{M}_n$	System matrix of voltage magnitude constraint of AC/DC bus n .
$\hat{I}_{\text{ac},l}/\check{I}_{\text{ac},l}$	System matrix of source/destination current flow capacity constraint of AC line l .
$\nu_{\text{ac},l}/\nu_{\text{dc},l}$	Per unit voltage drop across AC/DC line l .
$\hat{\mathcal{V}}_{\text{ac},l}/\check{\mathcal{V}}_{\text{ac},l}$	System matrix of source/destination voltage drop constraint of AC line l .
$\tilde{\delta}_l$	Voltage phase angle difference across AC line l .
$A_l/\hat{A}_l/\check{A}_l$	System matrices of angle difference constraints of AC line l .
$\bar{y}_l$	Series conductance for DC line l .
$\hat{P}_{\text{dc},l}/\check{P}_{\text{dc},l}$	System matrix of source/destination power flow capacity constraint of DC line l .
$\hat{\mathcal{V}}_{\text{dc},l}/\check{\mathcal{V}}_{\text{dc},l}$	System matrix of source/destination voltage drop constraint of DC line l .
$\hat{\eta}_z/\check{\eta}_z$	Forward (rectifier mode)/Backward (inverter mode) conversion loss factors for VSC z .
m_z	Modulation index for VSC z .
$c_Q/c_L/c_F/c_S/c_D$	Quadratic/Linear/Fixed/Startup/Shutdown cost coefficients of generators.

$\tilde{S}_n/\tilde{P}_n/\tilde{Q}_n$	System matrix of apparent/active/reactive power balance equation at AC bus n .
Γ^G	Matrix which maps P_G and Q_G to the AC bus vector.
Γ^F	Matrix which maps P_F and Q_F to the AC bus vector.
$\hat{\Gamma}^C/\check{\Gamma}^C/\tilde{\Gamma}^C$	Matrix which maps $\hat{p}_C/\check{p}_C/q_C$ to the AC bus vector.
$\bar{\Gamma}^C$	Matrix which maps $\hat{p}_C$ and $\check{p}_C$ to the DC bus vector.
$\bar{P}_n$	System matrix of power balance equation at DC bus n .
$\epsilon_d/\epsilon/\epsilon_\pi$	Tolerances used in TR-SLP.
K	Maximum number of iterations.
$P_{F,f}^{t,\zeta+}/P_{F,f}^{t,\zeta-}$	Deviations in the upper/lower confidence bounds of the power output of RES f in HTG during time period t .
$\hat{R}_{G,g}^\delta/\check{R}_{G,g}^\delta$	Reserve capability of generator g during hour t in HTG.
$\hat{R}_{G,g}^\Delta/\check{R}_{G,g}^\Delta$	Maximum operating ramp up/down rate of generator g during hour t in HTG.
$\hat{T}_G/\check{T}_G$	Minimum uptime/downtime for generator g in HTG.
Λ^t	Parameter to adjust down reserve requirement during hour t in HTG.
β	Penalty parameter in ψ .
$\tilde{\kappa}_l/\bar{\kappa}_l$	Reconstruction error relative to AC/DC line l .
$\tilde{\kappa}_{\max}/\tilde{\kappa}_{\text{avg}}$	Maximum/Mean of $\tilde{\kappa}_l$.
$\bar{\kappa}_{\max}/\bar{\kappa}_{\text{avg}}$	Maximum/Mean of $\bar{\kappa}_l$.
$\mathcal{W}$	Number of scenarios specified by TOAT.
F	Number of random factors considered in TOAT.
B	Number of levels (the two confidence bounds) considered for each random factor in TOAT.
$P_{\text{ND},\omega}^{m,t}$	Nominal demand of MG m during hour t under scenario ω .
$P_{\text{ND}}^{m,t,\zeta+}/P_{\text{ND}}^{m,t,\zeta-}$	Deviation in the upper/lower confidence bounds of the nominal demand of MG m during hour t
$P_{F,f,\omega}^{m,t}$	Power generation of RES f of MG m during hour t under scenario ω .
$\lambda_{N,\omega}^{\text{B},t}/\lambda_{N,\omega}^{\text{S},t}$	Price at which electricity is imported/exported from/to the utility grid during hour t under scenario ω .
$a_{\text{E},e}^m$	Coefficient of the quadratic degradation cost function of ESS e of MG m .

B_e^m	Energy capacity of ESS e of MG m .
$E_{I,e}^m$	Initial SoC of ESS e of MG m .
η_e^C / η_e^D	Charging/Discharging efficiencies of ESS e .
$\zeta_\omega^{m,t}$	Uncertain power injections of MG m during hour t under scenario ω .
$a_{G,g}^m / b_{G,g}^m$	Quadratic/Linear coefficients of the fuel cost function of DG g of MG m .
$\hat{R}_{G,g}^{\delta,m} / \check{R}_{G,g}^{\delta,m}$	Up/Down reserve capability limits of DG g of MG m .
$\hat{R}_{G,g}^{\Delta,m} / \check{R}_{G,g}^{\Delta,m}$	Up/Down operating ramp rate limits of DG g of MG m .
a_D^m	Coefficient of the demand redispatch/curtailment cost function of MG m .
E_{SH}^m	Maximum permissible total load shedding of MG m .
$V_O / V_m^B / V_m^S$	Voltage magnitudes of PCC/MG m while buying/selling.
Z_R^m	Resistance of the line which connects MG m to the PCC.
μ	Probability of the base scenario.
α	step size, $\alpha \in \mathbb{R}_{>0}$.
τ	Service fee per unit of electricity.
η_T	Efficiency of the transformer.

E. Power System Terminology

Base scenario	Scenario comprising the forecast values for all the uncertain variables.
Baseline case	OPF test case without line flow limits.
Committed schedule	Operating schedule of the generators which comprises the commitment and power dispatch over the time horizon.
Flat-start	Initiating the algorithm using the mid-values of all the variables, i.e., voltage angles, voltage magnitudes, active power dispatch, and reactive power dispatch in their domain.
Hourly reserve allocations	Amount of active power reserves allocated for each generator during each hour.
Multi-start	Initiating the algorithm with several starting points.
Thermally constrained case	OPF test case with line flow limits.
Warm-start	Initiating the algorithm using a known suboptimal OPF solution.

Worst scenario

Scenario which potentially causes the highest impact on the operational security of the power system.

F. Variables

$\tilde{v}/\bar{v}$

AC/DC bus voltage vector.

i_{ac}/i_{dc}

AC/DC bus injection current vector.

$\hat{i}_{ac}/\check{i}_{ac}$

Source/Destination current vector for AC lines.

$\hat{i}_{dc}/\check{i}_{dc}$

Source/Destination current vector for DC lines.

$\hat{p}_{dc,l}/\check{p}_{dc,l}$

Source/Destination power flow for DC line l .

$S_{C,z}^{ac}/P_{C,z}^{dc}$

Apparent power extraction/real power injection for VSC z .

$\Theta^{t,\omega}$

Tuple pertaining to the OPF variables under scenario ω during hour t .

V/δ

Vector of bus voltage magnitude/phase angle.

$P_{X,n}/Q_{X,n}$

Sum of active/reactive power extractions from bus n .

J_C^{k-1}

Jacobian of C_G at iteration $k-1$.

$J_{P,n}^{k-1}/J_{Q,n}^{k-1}$

Jacobian of $P_{X,n}/Q_{X,n}$ at iteration $k-1$.

$J_{I,l}^{k-1}/J_{\check{I},l}^{k-1}$

Jacobian of $\hat{i}_{ac,l}/\check{i}_{ac,l}$ at iteration $k-1$.

$f_o/f_e/f_i$

Objective function/Equality constraints/Inequality constraints.

d^k

Step to define a new solution at iteration k .

$\nabla f_o(\Theta^{k-1})$

First-order partial derivatives of $f_o(\Theta)$ with respect to Θ^{k-1} .

$\nabla f_e(\Theta^{k-1})$

First-order partial derivatives of $f_e(\Theta)$ with respect to Θ^{k-1} .

$\nabla f_i(\Theta^{k-1})$

First-order partial derivatives of $f_i(\Theta)$ with respect to Θ^{k-1} .

Δ^k

TR radius at iteration k .

π_e^k/π_i^k

Lagrange multipliers of the f_e/f_i at iteration k .

γ_i

Slack variable vector used to relax $f_i(\Theta)$.

$\hat{\gamma}_e, \check{\gamma}_e$

Slack variable vectors used to relax $f_e(\Theta)$.

$\Delta\phi_{act}^k/\Delta\phi_{pre}^k$

Actual/Predicted reduction in $f_o(\Theta^k)$.

$\phi(\Theta^k)$

Merit function defined in TR-SLP at iteration k .

β_e^k/β_i^k

Penalty factors used for $f_e(\Theta^k)/f_i(\Theta^k)$ in $\phi(\Theta^k)$.

Ξ^k

Actual-to-predicted cost ratio at iteration k .

$P_{F,f}^{t,\omega}/Q_{F,f}^{t,\omega}$

Active/Reactive power generation of RES f under scenario ω during hour t in HTG.

$u_{G,g}^t$

Commitment of generator g during time period t .

$\hat{u}_{G,g}^t / \check{u}_{G,g}^t$	Start up/Shut down state of generator g during time period t .
$P_{G,g}^t, P_{G,g}^{t,0}$	Base scenario power dispatch of generator g during hour t in HTG.
$\hat{r}_{G,g}^t / \check{r}_{G,g}^t$	Up/Down reserve allocations of generator g during hour t in HTG.
Φ_g^t	Day-ahead decision variables of UC pertaining to generator g during hour t in HTG.
Φ_g^T	Tuple of day-ahead decision variables of UC in HTG.
$P_{G,g}^{t,\omega} / Q_{G,g}^{t,\omega}$	Active/Reactive power dispatch of generator g under scenario ω during hour t in HTG.
$P_{D,n}^{t,\omega} / Q_{D,n}^{t,\omega}$	Active/Reactive power demand at bus n under scenario ω during hour t in HTG.
$\hat{p}_{C,z}^{t,\omega} / \check{p}_{C,z}^{t,\omega}$	Nonnegative active power injected/extracted of VSC z to/from the DC bus $\bar{o}(z)$ under scenario ω during hour t in HTG.
$q_{C,z}^{t,\omega}$	Reactive power extracted of VSC z from the AC bus $\bar{o}(z)$ under scenario ω during hour t in HTG.
$\tilde{v}^{t,\omega} / \bar{v}^{t,\omega}$	AC/DC bus voltage vector under scenario ω during hour t in HTG.
ψ	Piece-wise linear function with three segments.
$\varphi^{t,\omega}$	Optimal objective value of SP under scenario ω during hour t .
$\pi_{P,n}^{t,\omega} / \pi_{Q,n}^{t,\omega}$	Dual variables of the active/reactive power balance constraint at AC bus n in HTG.
$\hat{\pi}_{R,g}^{t,\omega} / \check{\pi}_{R,g}^{t,\omega}$	Dual variables of the up/down reserve capability bound of generator g in HTG.
$\hat{\pi}_{Q,g}^{t,\omega} / \check{\pi}_{Q,g}^{t,\omega}$	Dual variables of the reactive power capability upper/lower bound of generator g in HTG.
$P_{\text{loss}}^{t,\omega}$	Total power loss under scenario ω during hour t in HTG.
$\tilde{W}^{t,\omega} / \bar{W}^{t,\omega}$	PSD matrix variable pertaining to $\tilde{v}^{t,\omega} / \bar{v}^{t,\omega}$.
$\tilde{w}^{t,\omega} / \bar{w}^{t,\omega}$	Vectorized variable pertaining to $\tilde{W}^{t,\omega} / \bar{W}^{t,\omega}$.
$\tilde{\xi} / \bar{\xi}$	Vectorization pertaining to $\tilde{W}^{t,\omega} / \bar{W}^{t,\omega}$.
$\tilde{S}_l / \bar{S}_l$	Second-order cone pertaining to AC/DC line l .
$P_{E,e,\omega}^{C,m,t} / P_{E,e,\omega}^{D,m,t}$	Charging/discharging power of ESS e of MG m during hour t under scenario ω .

$E_{e,0}^{m,t}$	SoC of ESS e of MG m during hour t under the base scenario.
$\bar{P}_{E,e}^{C,m,t} / \underline{P}_{E,e}^{C,m,t}$	Auxiliary bounding variables of charging power of ESS e of MG m during hour t .
$\bar{P}_{E,e}^{D,m,t} / \underline{P}_{E,e}^{D,m,t}$	Auxiliary bounding variables of discharging power of ESS e of MG m during hour t .
$\bar{E}_e^{m,t} / \underline{E}_e^{m,t}$	Auxiliary bounding variables of SoC of ESS e of MG m during hour t .
$P_{G,g,\omega}^{m,t}$	Power dsipatch of DG g of MG m during hour t under scenario ω .
$\bar{P}_{G,g}^{m,t} / \underline{P}_{G,g}^{m,t}$	Auxiliary bounding variables of power dispatch of DG g of MG m during hour t .
$P_{RD,\omega}^{m,t} / P_{CD,\omega}^{m,t}$	Demand redispatch/curtailment of MG m during hour t under scenario ω .
$\underline{P}_{RD}^{m,t} / \bar{P}_{CD}^{m,t}$	Auxiliary bounding variables of demand redispatch/curtailment of MG m during hour t .
$P_{O,\omega}^{B,m,t} / P_{O,\omega}^{S,m,t}$	Power import/export by MG m at the MG bus from/to the PSO during hour t under scenario ω .
$\ddot{P}_{O,\omega}^{B,m,t} / \ddot{P}_{O,\omega}^{S,m,t}$	Power import/export by MG m at the PCC from/to the PSO.
$\beta_{O,\omega}^{B,t} / \beta_{O,\omega}^{S,t}$	Buying/Selling prices specified by the PSO during hour t under scenario ω .
$\Psi_{\omega}^{m,t}$	Decision variable vector pertaining to the local optimal dispatch problem of MG m during hour t under scenario ω .
$C_{G,g,\omega}^{m,t}$	Cost of DG g in MG m during hour t under scenario ω .
$C_{E,e,\omega}^{m,t}$	Cost of ESS e in MG m during hour t under scenario ω .
$C_{D,\omega}^{m,t}$	Cost of demand redispatch and curtailmet of MG m during hour t under scenario ω .
$C_{O,\omega}^{m,t}$	Cost of imports and exports from and to the PSO of MG m during hour t under scenario ω .
$C_{\omega}^{m,t}$	TOC of MG m during hour t under scenario ω .
C_E^m / C_R^m	Expected total energy/reserve dispatch cost of MG m .
C_{exp}^m	Expected TOC of MG m .
$\sigma_{G,g}^{m,t}$	PFs of DG g in MG m during hour t .
$\sigma_{E,e}^{m,t}$	PFs of ESS e in MG m during hour t .

$\sigma_D^{m,t}$	PFs of demand redispatch and curtailment of MG m during hour t .
$\sigma_O^{m,t}$	PFs of power imports from and power exports to the PSO of MG m during hour t .
$\lambda_{O,\omega}^t/\lambda_{eq,\omega}^t$	Updated price by the PSO/MEP during hour t under scenario ω .
k	Current iteration.
$I_{O,\omega}^t$	Aggregated income of the PSO during hour t under scenario ω .

Chapter 1

Introduction

1.1 Background and Motivation

1.1.1 Challenges in Optimal Operation of the Future Power Systems

Power systems are large-scale entities that transmit power from sources to the loads in real time, which are geographically dispersed in general. The optimal scheduling of generators is an essential task of the system operators to operate the system at the minimum cost. As such, there are multi-layer energy management systems deployed for resource utilization based on different time scales [3]. For instance, UC is day-ahead scheduling of resources while respecting the capability restrictions, so as to meet the next day's power supply requirements considering daily forecasts. Conversely, OPF is used to dispatch the committed generators for the next hour based on the hourly forecasts while respecting the generation capabilities and network constraints [1, 4]. Traditionally, these two problems were solved separately. Due to the increasing demand and the limited generation resources, the generator commitment and power dispatch must be essentially determined considering the transmission capacity limitations to alleviate transmission congestion issues involved with existing grids. Hence, the day-ahead scheduling decisions must be

taken combining the UC and hourly OPF problems for the optimal resource utilization of the system. Typically, the former involves mixed-integer variables, and the latter involves nonlinear constraints. As the practical power systems are generally larger in scale, the combined problem has computational challenges due to the problem structure and the larger dimensionality [5–7].

With the new policies on the decarbonization of the energy sector to act on climate change, many countries are increasingly replacing the conventional fossil fuel-based generation units with RESs [8]. Higher penetration of RESs in the generation mix creates more challenges to the system operators due to their stochastic (and hence nondispatchable) nature. Accordingly, it is necessary to allocate sufficient reserves when scheduling generators to mitigate the power mismatches in the system due to the possible real-time variations. As such, new modeling methodologies are necessary to reformulate the combined UC and hourly OPF problem to find sufficiently robust scheduling solutions while scalable solution approaches are required to address the resulting computational challenges.

The transmission system is concerned with the bulk transfer of electrical power from generation facilities to load centers. With the increasing demand, transmission congestion can cause the dispatch of more expensive generators while cheaper generators are not fully utilized, due to the capacity limitation of some lines. Further, the integration of renewable-based remote generation facilities can also be restricted due to transmission capacity limitations. The conventional approach to mitigate transmission congestion is the construction of new lines. This is often complicated and protracted by the construction of new corridors due to issues with right-of-way and public acceptance. As discussed in [9–11], existing AC transmission lines can be converted to DC operation without corridor adjustments. The active power transmission capacity of the converted DC line can be two or more times the apparent power transmission capacity of the respective AC line, depending on the configuration. On this basis, an HTG architecture comprising an AC subgrid with multiple multi-terminal DC subgrids was recently proposed in [1],

which serves as an upgrade strategy for traditional AC transmission grids to enhance its economic utilization and loadability [1, 6, 7, 12]. However, integrating AC and DC systems further demands new modeling methodologies and optimization approaches which are capable of accurately representing their electrical behavior while maintaining the computational tractability.

Many of the RESs are remotely located and connected to the distribution system. Furthermore, MGs are gaining popularity in future power systems due to their ability to integrate more RESs [13–15]. MGs bring the sources and loads closer. This reduces the power transfer losses. In addition to RESs, a typical MG may include numerous controllable sources and (price-responsive) loads which provide additional flexibility to counteract against power injection uncertainties for reliable operation. In addition to the increased penetration of RESs, the advancements in communication technologies, and the liberalization of energy markets have supported the proliferation of MMG systems [16–18] in the distribution grid. In general, the constituent MGs in an MMG system belongs to separate entities. The development of distributed optimization frameworks that enable the efficient sharing of resources while preserving the privacy concerns between the individual MGs within MMGs has been an active research area in recent years.

In addition to the power sharing, the reserve sharing among the constituent MGs will further improve the economic efficiency and makes them less dependent on the utility grid. Most of the existing robust optimization schemes are centralized which do not satisfy the privacy requirements of the constituent MGs within MMGs. In this context, distributed robust optimization approaches and both power and reserve trading schemes are essential which allows both power and reserve sharing agreements among the constituent MGs within MMGs.

1.1.2 Motivation

The above discussions indicate the necessity for new operational strategies, market mechanisms, mathematical models, and optimization methods to efficiently operate future power systems. The optimization models must accurately represent the electrical behaviour of the future power system and its components such as generators, energy storage systems (ESSs), voltage source converters (VSCs), flexible loads, multi-terminal DC systems, MMGs, etc.

Developing new optimization frameworks for optimal resource utilization of emerging power systems which incorporate the above-mentioned components is one of the key contributions of this thesis to support reliable day-ahead market clearing. As such, robust optimization schemes are proposed to determine the optimal generation commitment, power dispatch and reserve allocation schedules while respecting the generation and reserve capabilities, transmission restrictions, and voltage security of the system. The robustness of the optimal solutions is also validated against variable stochastic power injections.

As the future power systems are larger in scale, new solution approaches, such as decomposition techniques which reduce the dimensionality and enable provisions for parallel computations, convex relaxations/approximations, etc., are required to ensure the scalability and computational tractability of the proposed optimization models.

Considering the operation of MMGs, the robust optimization approaches are proposed which support distributed computations. Further, market clearing mechanisms are proposed to enhance the sharing of resources for both power and reserve requirements which ensure economic benefits for the constituent MGs. As such, this thesis primarily proposes such optimization frameworks and solution approaches to enable the efficient operation of future power systems.

1.2 Contributions

Based on the earlier discussions and compared to the existing works in the literature, the notable contributions of this thesis are briefly described below:

1. In Chapter 4, a synergistic solution approach for the OPF problem is developed based on a trust-region (TR) method and SLP, which is referred to as the trust-region based sequential linear program (TR-SLP). The proposed approach adopts the polar form of complex quantities to capture the quasi-linear relationship between the active power and bus voltage angles, as well as the reactive power and voltage magnitudes of the original OPF problem. This supports faster convergence. Additionally, a TR radius constraint is included to improve the validity of the linear approximations in subsequent SLP steps. Furthermore, a feasibility restoration phase is included to avoid infeasibilities of intermediate linearizations.
2. In Chapter 5, a novel two-stage scenario-based RO framework is proposed for day-ahead robust UC considering nonlinear NCs under wind power uncertainty. The proposed RO framework determines generator-wise hourly reserve margins while preserving nonanticipativity under the selected set of scenarios. The proposed RO framework enables a complete decomposition of the day-ahead energy and reserve scheduling problem into one MP and multiple SPs (one per scenario per time instance/hour) without resorting to modeling simplifications. Accordingly, parallel computing platforms can be utilized to enhance the computational performance which is (partially) restricted in the existing RO frameworks. Therewith, the proposed RO framework is scalable, facilitating its application to large-scale power systems. The SPs are also formulated in a practical, scalable, and unified manner, which enables the potential utilization of established OPF packages.

In Chapter 6, the performance of the proposed RO framework in Chapter 5 is extended with a convex relaxation to both AC and DC NCs in the SPs.

In addition, the performance of the RO framework is extensively studied considering a large-scale HTG based on the Polish 2383-bus system. Therein, the efficiency, computational performance, and the robustness of the proposed RO framework are compared with the state-of-the-art approaches via out-of-sample analyses.

3. In Chapter 7, a convex and scalable robust day-ahead optimal scheduling framework is proposed for MMG systems. The proposed RO framework determines optimal participation factors for the components in constituent MGs to mitigate the power mismatch caused by the uncertainties while preserving the nonanticipativity in reserve-scheduling. Further, the proposed RO framework incorporates a cooperative bidding-based trading scheme to facilitate the optimal sharing of energy and reserves between the constituent MGs in an MMG system while respecting privacy requirements. The proposed RO framework also accounts for the power losses in the lines connecting the point-of-common-coupling and the individual MGs and the power losses in the distribution transformer connecting the MMG system and the utility grid.

1.3 Thesis Organization

This thesis is organized as described below.

Chapter 1 overviews the challenges faced in the optimal operation of future power systems and outlines the motivation and the contributions of this thesis.

Chapter 2 broadly discusses the optimization problems related to the operation of power systems, for instance, OPF, UC, etc. The discussion mainly highlights the structural changes and state-of-the-art solution approaches which are proposed to address the future challenges in power systems and their limitations.

Derivations of the OPF problem are presented in Chapter 3 for an HTG from the first principles based on the bus-injection model for power flow. The formulations

include the modeling of power flow through the AC and DC subgrids and the component-wise models for generators and VSCs. The NCs are modeled considering the operational requirements to be satisfied while transferring power through the AC and DC lines in the HTGs.

The trust-region based sequential linear program for OPF is presented in Chapter 4. The detailed formulations of the polar form of the OPF, linear approximation based on the Taylor series, and the TR constraints are presented. Further, the algorithm of the proposed iterative framework and its convergence criteria are also presented. The results section presents the application of the proposed TR-SLP method for large-scale power systems, and its computational performance is compared with existing sequential linear programming approaches of OPF.

Chapter 5 develops a novel two-stage scenario-based RO framework for day-ahead robust UC of HTGs considering nonlinear NCs under wind power uncertainties. The uncertainties are characterized by the polyhedral sets based on the confidence bounds. The modeling of nonanticipative reserve allocation is discussed within the Benders decomposition (BD) framework. Further, the scalability improvements of the proposed RO framework via the decomposition and the SP formulation are discussed.

Chapter 6 extends the RO framework proposed in Chapter 5 with a convex relaxation to both the AC and DC NCs of the SPs. Moreover, the efficiency, computational performance, and the robustness of the proposed RO framework are compared with the state-of-the-art RO frameworks based on a large-scale HTG.

The RO framework is extended to an MMG system in Chapter 7. Therein, the detailed MG component models are presented respecting the technical constraints. A cooperative trading scheme is also developed to facilitate efficient energy and reserve sharing among the constituent MGs within the MMG system via an independent entity called the power-sharing operator. The case studies illustrate the efficiency and robustness of the proposed RO framework compared to a state-of-the-art RO framework in the context of MMG systems. Further, the economic

benefits of the proposed cooperative trading scheme are compared with those of the isolated trading in which the individual MGs are only trading with the utility grid.

Chapter 8 concludes the thesis and provides some recommendations for future research.

Chapter 2

Literature Review

The following sections review some existing literature which is relevant to the scope of the various topics covered in this thesis.

2.1 Optimal Power Flow

OPF is an operational optimization problem executed by system operators to satisfy the power demand by utilizing the most economical generation resources without violating generator capabilities and NCs [19, 20]. The objective of OPF is to minimize the generation cost, which is typically a convex quadratic function. NCs include the active and reactive power balance of each bus considering the power flow equations which respect Kirchhoff's current and voltage laws, bus voltage security limits, and the capacity, voltage drop, and angle difference restrictions of transmission lines. OPF is a quadratically-constrained quadratic optimization program which is a nonconvex problem due to *bilinear* representation of voltage variables in power flow equations. Thus, for a given OPF problem there is a potential that it may have multiple equilibrium points (solutions). Hence, searching for a global solution is in principle NP-hard (cf. [20–22]). Thus, local constrained-optimization is traditionally used to obtain a feasible solution for OPF.

In the context of OPF, the solution approaches, such as linear programming (LP) methods [23–25], quadratic programming methods [26], Lagrangian relaxation [27], and interior-point methods [28] have been extensively investigated in the literature. It is worth noting that among all these approaches, interior-point methods have emerged as a promising direct solution approach for OPF problems [2]. The development, implementation, and testing of interior-point methods, and their application for power systems are discussed in [29]. Among many variants of interior-point methods, the primal-dual logarithmic barrier interior-point method and its predictor-corrector extension are known to be efficient for OPF solution algorithms due to their superior computational efficiency [2, 30]. A detailed survey, including the other solution approaches for OPF problems can be found in [31] and references therein.

2.1.1 Linear Approximations of Optimal Power Flow

Hourly market-clearing strategies of power systems are mainly based on nodal prices, which are the dual variables of power balance constraints of the OPF problem [7]. This highlights the importance of the convexity and scalability features for any algorithm to be useful in OPF calculations [19, 23]. In addition, real-world OPF problems involve large numbers of decision variables [4, 7]. This makes them challenging for a solution technique, both in terms of memory and computation time requirements. Consequently, there is a great need for computationally efficient approaches which can handle the nonconvex AC NCs.

To improve the scalability and global solvability, the feasibility region of the OPF problem is convexified using linear approximations for power flow equations [3], for instance, the decoupled OPF problem. However, the feasibility of the approximated solution must be verified separately using an appropriate nonlinear power flow solver [23]. In case if it reports infeasibility, suitable alterations such as transformer tap adjustment, static var compensator adjustment, etc., are made to avoid

constraint violations. This is a useful practice for feasible operation, which is followed by the industry so far [32]. However, resource utilization might still be suboptimal. Moreover, a 5% increase in efficiency of OPF solution would save approximately six billion dollars per year in the United States alone as per United States Federal Energy Regulatory Commission [33]. Thus, there is a significant need for potential improvements in solving the OPF problem.

According to the aforementioned discussion of OPF approaches, LP methods can be an attractive candidate for OPF formulations due to their inherent scalable nature. Recent works [23, 34] have proposed successive/sequential linear programming (SLP)¹ approaches to demonstrate this fact. Specifically, [23] has shown the scalability of LP tools against the well-known interior-point solver IPOPT [35] as well as the nonlinear optimization solver KNITRO [36]. However, the OPF formulation in [23] is based on the Cartesian form of the power flow model, which disregards quasi-linear relationships of active power and bus voltage angles, and the reactive power and voltage magnitudes [32]. In addition, that formulation results in nonconvex voltage limit constraints which demand additional slack variables for linearization.

2.1.2 Convex Relaxations of Optimal Power Flow

Recently, the OPF formulations based on the convex relaxations of nonlinear power flow constraints are gaining more attention due to their computational tractability and lesser optimality gap in comparison to the decoupled OPF approximation. One of the widely used techniques in the last decade is semidefinite relaxation (SDR), which determines a lower bound for the optimal cost of the OPF problems. In addition, SDR provides the global optimal solution of the OPF problem for radial networks under mild operating conditions [1, 21, 37–41]. However, in the case of meshed networks, SDR possesses a relaxation gap and necessitates the use of structural modifications, such as adding phase shifters to recover bus voltage angles

¹The words ‘successive’ or ‘sequential’ are used interchangeably in the context of LP approximation schemes.

[21, 42]. However, semidefinite programs do not scale well for large-scale problems. This limits the application of SDR to OPF problems of large-scale power systems [7, 43].

To circumvent the scalability issue associated with SDR, recently a second-order cone relaxation (SOCR) has been introduced. The SOCR explores the inherent sparsity of OPF problems which enhances the computational performance, enabling the solvability for large-scale power systems [44, 45]. SOCR formulations have been presented based on the bus-injection model [7] and the branch-flow model [44, 45]. Recently, the optimization models in [46, 47] introduced additional linear cuts in a branch-flow model based OPF framework to guarantee the exactness of SOCR for active distribution networks. Similarly, an improved quadratic convex relaxation is proposed in [43] as an extension of SDR, in which voltage magnitudes are coupled with voltage angles using additional polyhedral constraints. This improves the relaxation gap in comparison to SOCR without sacrificing computational performance to a certain extent. However, a significant relaxation gap persists in many power system cases [43]. Solution approaches, such as convex envelopes [48, 49], and decomposition methods [50], which utilize the convex relaxations with the global optimization philosophy, have also been reported in the literature.

2.1.3 Optimal Power Flow of Hybrid AC/DC Transmission Grids

The transmission capacity limitations cause congestion in power systems leading to trading restrictions in deregulated power market context [7]. According to the physical laws, congestion of a single line in a cycle of lines in the power network can stagnate the power flow over the cycle [51]. To reduce the impact of congestion, optimal transmission switching was considered in the literature [51, 52], where transmission line switching is incorporated into the traditional OPF problem to open undesirable lines to increase the utilization of cheaper generation resources.

However, this is a combinatorial problem and is challenging to realize with AC-OPF constraints [52]. Further, system operators may be reluctant to follow this approach as it causes wearing of breakers and introduces additional transients to the system.

Further, a higher penetration of RESs demands more transmission capacity and flexibility to handle the underlying uncertainties. Considering the advantages in long-distance transmission, flexible power flow control, and reactive power compensation, VSC coupled high-voltage DC systems are a potentially applicable key technology to resolve the aforementioned issues [9–11]. With the recent technological developments, several point-to-point DC systems and multi-terminal DC systems are already installed and planned [7]. With the continuation of this trend, future power systems would be large-scale HTGs with significant penetration of RESs. However, operational planning of these HTGs via OPF requires component modeling and optimization techniques to address these structural changes [53] and uncertainties.

Regarding the OPF of HTGs, various models and optimization techniques are available in the literature [54–57]. Therein, [54] developed an iterative optimization scheme for HTGs based on a linear approximation of NCs. An SOCR approach is proposed in [55] for NCs in HTGs. The references, [56, 57] proposed the SDR, in which the former shows the conditions for the zero-relaxation gap, while the latter suggests an iterative scheme to reach zero-relaxation gap solution. Moreover, [53] developed an open-source OPF framework featuring a rigorous mathematical model and convex relaxation for HTGs with provision to interface with different solvers.

2.2 Network-Constrained Unit Commitment

UC is a day-ahead optimal resource management problem to satisfy the forecasted demand subject to operational restrictions and the availability of generators [6, 58]. Several UC formulations were proposed in the literature. They include operating

cost minimization while maximizing voltage stability in [59], consumer payment minimization in [60], the development of a distributed framework for UC avoiding grid-related uncertainties in [61], demand response incorporation in [62], and the impact of modeling inflexibilities in [58]. However, none of the above works consider a network model which restricts the applicability of those mathematical models in practical power systems.

Notably, in case of large-scale power systems, network-constrained UC is essential for a reliable power market clearing as the network congestions affect the power dispatch of generation units [63–65]. In [63], the authors discuss the challenges involved in solving large-scale UC problems. The impact of the computational performance of the day-ahead UC algorithms on the market-clearing is discussed in [64], while [65] discusses a transmission constraint filtering technique to improve computational performance. Many notable studies including [63–65] utilize decoupled NCs based on a simplified system model, which constitutes a tractable approximation of the network laws. However, the model mismatch requires more conservative system constraints in a practical implementation that leads to a sub-optimal utilization [5]. For instance, power losses can be significant, and the active and reactive power may be strongly coupled, restricting the reactive power transmission, which may lead to voltage security issues at some buses.

Consequently, the day-ahead generation schedule must be validated over the considered time horizon against the *nonlinear* NCs, which include nodal power balance, voltage bounds, power flow limits, and so forth [6]. The combined problem of UC with accurate but nonlinear NCs is a mixed-integer nonlinear program which is typically extensive and even intractable to solve directly using existing solvers for large-scale transmission grids. Thus, the utilization of decomposition methods such as multi-stage optimization algorithms is necessary [5]. Many notable studies utilize BD, in which the first stage comprises UC, referred to as the master problem (MP), and the second stage comprises subproblems (SPs) to verify the feasibility of the day-ahead generation schedule for nonlinear NCs under possible real-time variations [66–69].

2.3 Optimal Scheduling under Uncertainties

2.3.1 Stochastic and Robust Optimization

As a result of higher penetration of RESs, different classes of network-constrained UC problems are modeled in the literature to derive the optimal operating point with sufficient reserves ensuring the immunization against the possible real-time fluctuations in power injections. Two widely used approaches to co-optimize power dispatch and reserve allocations include stochastic optimization (SO) [68, 70, 71] and RO [67, 72–75]. In general, these methods adopt a two-stage iterative structure in which the day-ahead generation schedule is computed in the first stage at the minimum operating cost, while its feasibility on uncertain realizations is verified in the second stage [68].

In SO, a finite number of scenarios is used to represent plausible uncertain power generation realizations. A UC solution is generated to minimize the generation cost while allocating a sufficient amount of reserves to take recourse actions necessary for robust operation in all representative scenarios [72]. However, the scenarios generated in SO are dependent on probability distributions of uncertain power outputs which are not readily available [68, 70]. Also, the number of scenarios is a trade-off between the accuracy of the representation and the solution efficiency [68, 71].

In contrast, RO methods typically require the interval of uncertain variables and require less or even no information of the probability distributions of the uncertain variables [73, 76]. Accordingly, RO methods consider minimizing the operating cost under the worst scenario, ensuring the feasibility of the computed schedule for all possible uncertain realizations over the intervals of the uncertain variables [67, 73]. The amount of reliability/security level required to be satisfied by the solution can be adapted using constraints for budgeting of uncertainties, as the uncertainties are considered as variables of the optimization problem within their confidence bounds.

2.3.2 Multi-Stage Optimization Frameworks

There are two widely used multi-stage optimization frameworks for day-ahead generation scheduling of power systems.

2.3.2.1 Column and Constraints Generation Method

Recently, a column and constraints generation (CCG) method was developed for the day-ahead robust UC in [74, 75]. Therein, a two-stage approach is adopted in [67, 73–75, 77], which computes the day-ahead generation schedule in the first stage for fixed uncertain variables. In the second stage, dual-domain information of the recourse problem is utilized to obtain the least cost recourse actions under the maximum impact of uncertainties for the computed day-ahead generation schedule. Therefore, the second stage aims at determining a scenario which has the potential to violate the NCs under the current day-ahead generation schedule. Therein, the worst scenario is determined subject to the budgeting of uncertainties. These scenarios are iteratively integrated into the first stage to recompute a day-ahead generation schedule satisfying the NCs under those scenarios found in the second stage. Accordingly, the first stage in the CCG method includes *all* the integer and continuous decision variables (and the recourse actions) and the respective constraints associated with every scenario [75].

Consequently, the CCG method forms a mixed-integer nonlinear program in the first stage when nonlinear NCs are considered. Such a problem may be intractable using standard solvers. Further, the first stage problem in the CCG method *expands* in every iteration in both the number of variables and constraints, which can be extensive or potentially intractable for large-scale power systems [72]. Moreover, to the best of the author’s knowledge, the CCG method has not been reported in the literature for robust network-constrained UC considering large-scale power systems.

In contrast to computing the worst scenarios within the optimization problem, numerous scenario-based RO frameworks and scenario selection methods have been proposed in [72, 78, 79] and some of them have been applied in the OPF literature, for instance, [72, 80, 81]. Therein, [78] and [72] propose two alternative tractable solution approaches to capture a reduced set of extreme points while considering the correlation between uncertain variables. Latin Hypercube Sampling (LHS) method [70, 82] is also a widely used approach for scenario reduction. In comparison, the Taguchi's Orthogonal Array Testing (TOAT) has been successfully applied for studying the impact of worst uncertainty in robust OPF related works [80, 81].

2.3.2.2 Benders Decomposition

BD is a widely used iterative approach for such scenario-based multistage optimization problems [67, 68, 83]. Therein, the dual information of the violated NCs under the considered set of scenarios is used to rectify the day-ahead generation schedule in the first stage of subsequent iterations. In comparison to the CCG method, the BD remains with the same set of variables in the first stage and integrate a smaller number of additional constraints in every iteration [83]. Further, the aforementioned scenario selection methods can be combined with BD to compute robust scheduling solutions effectively.

A two-stage optimization framework is proposed in [66, 69, 84] which combines UC with nonlinear NCs. In [66], the authors consider an MP combined with decoupled NCs and SPs are formulated to determine the violations of nonlinear NCs. This framework is extended in [69] considering the operation with DC links and in [84] with a SOCR of NCs in the MP instead of the decoupled NCs. However, these optimization frameworks neglect the presence of uncertainties. A SO framework based on BD is presented in [71] that considers uncertainties of the day-ahead generation scheduling problem. However, the intertemporal constraints associated with the scenarios are neglected. An adaptive RO scheme is proposed in [67] for UC with nonlinear NCs. Therein, the worst scenarios are obtained based on the

decoupled NCs and multi-period OPF problems are considered as SPs to incorporate the intertemporal constraints associated with the scenarios. That work is extended in [77] considering an alternative linear formulation of NCs incorporating the decoupled NC formulation with voltage and reactive power variables. However, the applicability to large-scale networks is limited due to the multi-period OPFs and the inaccurate uncertainty representation.

To avoid the multi-period OPFs, [68] proposes an SO-based UC with a heuristic approach to sequentially evaluate the nonlinear NC satisfaction of each scenario as SPs. However, the optimality and the convergence of the day-ahead generation schedule cannot be guaranteed [68, Sec. IV-D]. Thus, there are limitations in all these RO frameworks that restrict the utilization of parallel computing platforms. Consequently, the scalability and hence, the application of those RO frameworks to large-scale power systems is limited. Furthermore, in all the above-mentioned approaches, the reserves are arbitrarily allocated from the committed generators. Such a day-ahead scheduling solution does not provide a proper guideline for system operators on which generators should be used for reserve support in real-time operation.

2.3.3 Nonanticipativity in Day-Ahead Scheduling

In the following day's operation, the system operators follow the generation schedule for generator commitment, and the base scenario power dispatch is used as hourly reference set-points for the generators. Accordingly, in the case of scenario-based RO schemes, the *nonanticipativity* of the day-ahead generation schedule is vital to ensure the reserve deliverability in real time while respecting generator ramp constraints. Preserving nonanticipativity implies compliance with the generator ramp constraints under all possible transitions that might occur in between two adjacent time periods under all the scenarios in the pool [72, 73].

The RO frameworks in [67, 68] violate the aforementioned nonanticipativity requirements. Two recent works develop formulations of a robust nonanticipative

UC with decoupled NCs using the CCG method [73] and a single-stage optimization approach [72]. However, the RO frameworks in [72, 73] consider multi-period OPF formulations involving (mixed-integer) commitment decisions together with NCs. The resulting problem is extensive and intractable with standard solvers, which cannot be directly extended to robust UC with nonlinear NCs for large-scale systems.

2.4 Multi-Microgrid Systems

MMG systems may include a coordinating entity to enable power sharing between the constituent MGs [16, 17, 85]. This concept was first proposed in [85] based on a multi-agent system. Power-sharing frameworks in MMG systems promote microgrid operators (MGOs) of the constituent MGs to optimize their local resources and cooperatively contribute to the MMG energy and reserve markets via the coordinating entity. Furthermore, this framework is also privacy-preserving as it only requires the individual MGOs to communicate their supply/demand bidding information [16–18, 85–95].

Cooperative control frameworks based on the multi-agent systems and game theoretic approaches were proposed in [88] to coordinate the operations of multiple MGs, while self-adaptively forming an MG coalition. Reference [89] adopted an event-based scheme to reduce the communication between agent MGs for achieving power balance in an MMG system. An auction architecture was also modeled in [90] and the bidding behavior of the MG agents was formulated as a Markov process for maximizing the profits of the individual MGs. Reference [91] proposed a Lagrangian relaxation-based bidding framework for an MMG system having multi-energy MGs. Reference [92] focused on a multi-level multi-agent systems architecture for an MMG system including a community-based ESS. Furthermore, multi-level market structures were reported in [93–95], wherein the upper levels considered the power balance within each MG and the day-ahead market-clearing

of the MMG system while the lower levels were designed for optimal hourly and real-time rescheduling of resources to mitigate the impact of forecast uncertainties.

However, none of the aforementioned works related to MMG systems include a proper day-ahead reserve allocation mechanism which accounts for the impact of uncertain factors within their optimization models. In this context, the day-ahead optimization models proposed in [87–91, 93, 94] may not be sufficiently robust against any potential real-time power mismatches. There are two major sources of uncertainties that affect the economical and reliable operation of MMG systems. Firstly, the RES generation and the fixed load demand are stochastic in nature. Consequently, an arbitrary allocation of reserves may result in a suboptimal or unreliable operation of the MMG system as the reserves might not be sufficient to mitigate the uncertainties in forecasts of the load demand and the RES generation. Secondly, in energy markets such as the National Electricity Market of Singapore, the prices at which electricity is purchased from and sold to the utility grid are allowed to change up to 65 minutes before the actual trade [16, 18].

In general, RO approaches minimize the total cost while providing a feasible solution under the worst impact of the uncertainties present in the optimization model [16, 18, 78, 79, 86]. In the context of MG energy management systems, an RO framework with a polyhedral uncertainty set was used in [78] to incorporate a vertex enumeration algorithm for exploring the worst scenario. Conversely, [79], and [80] adopted scenario-based RO frameworks in which the TOAT was used to select a reduced set of vertex permutations from the uncertainty polyhedron.

RO based approaches for the day-ahead power dispatch and the reserve allocation in MMG systems were recently proposed in [16, 18]. To facilitate the clearance of the MMG energy and reserve markets, an alternating direction method of multipliers based approach was used in [18] while a game-theoretic approach was investigated in [16]. Reference [18] only considered the sharing of power between the individual MGs while the reserves were managed locally within each individual MG, thereby diluting the benefits offered by the MMG framework and resulting in a possibly

suboptimal dispatch solution. Furthermore, in both [16] and [18], the realized worst scenario assumed that the lowest RES generation and the highest load demand (and vice-versa) coincides for all the constituent MGs during each hour of the entire optimization horizon. However, such a scenario is highly unlikely in practice.

Chapter 3

Optimal Power Flow Formulation

This chapter discusses the Cartesian formulation of the classical OPF problem for an HTG from the first principles. Section 3.1 describes the system architecture and the network constraint models for the OPF problem. Section 3.2 formulates the OPF problem followed by the summary of the chapter in Section 3.3.

3.1 System Model

Consider an HTG consisting of multi-terminal AC and DC grids. The coupling in between the AC and DC grids are achieved through VSCs at some buses. In the following, the components in the HTG and their electrical behavior is explained aiming to formulate the OPF problem.

3.1.1 Architecture

The HTG architecture is described by the sets $\mathcal{G}$, $\mathcal{F}$, $\mathcal{Z}$, $\mathcal{N}_{ac}$, $\mathcal{N}_{dc}$, $\mathcal{L}_{ac}$, and $\mathcal{L}_{dc}$. Further, let $\hat{\varepsilon} : \mathcal{L}_{ac} \rightarrow \mathcal{N}_{ac}$, $\check{\varepsilon} : \mathcal{L}_{ac} \rightarrow \mathcal{N}_{ac}$, $\hat{\vartheta} : \mathcal{L}_{dc} \rightarrow \mathcal{N}_{dc}$, $\check{\vartheta} : \mathcal{L}_{dc} \rightarrow \mathcal{N}_{dc}$, $\tilde{o} : \mathcal{Z} \rightarrow \mathcal{N}_{ac}$, $\bar{o} : \mathcal{Z} \rightarrow \mathcal{N}_{dc}$, $\varrho : \mathcal{G} \rightarrow \mathcal{N}_{ac}$, and $\varsigma : \mathcal{F} \rightarrow \mathcal{N}_{ac}$. Here, the directionality of AC lines, DC lines and VSCs is not related to the direction of power flow and can be chosen

arbitrarily. In addition, some of the sets are defined as follows to support the mathematical exposition later on.

Definition 3.1.1. $\hat{\mathcal{L}}_{\text{ac}}(n) \subseteq \mathcal{L}_{\text{ac}}$ and $\check{\mathcal{L}}_{\text{ac}}(n) \subseteq \mathcal{L}_{\text{ac}}$ are defined as

$$\hat{\mathcal{L}}_{\text{ac}}(n) = \{l \in \mathcal{L}_{\text{ac}} : \hat{\varepsilon}_{\text{ac}}(l) = n\} \quad (3.1a)$$

$$\check{\mathcal{L}}_{\text{ac}}(n) = \{l \in \mathcal{L}_{\text{ac}} : \check{\varepsilon}_{\text{ac}}(l) = n\} \quad (3.1b)$$

Definition 3.1.2. $\hat{\mathcal{L}}_{\text{dc}}(n) \subseteq \mathcal{L}_{\text{dc}}$ and $\check{\mathcal{L}}_{\text{dc}}(n) \subseteq \mathcal{L}_{\text{dc}}$ are defined as

$$\hat{\mathcal{L}}_{\text{dc}}(n) = \{l \in \mathcal{L}_{\text{dc}} : \hat{\varepsilon}_{\text{dc}}(l) = n\} \quad (3.2a)$$

$$\check{\mathcal{L}}_{\text{dc}}(n) = \{l \in \mathcal{L}_{\text{dc}} : \check{\varepsilon}_{\text{dc}}(l) = n\} \quad (3.2b)$$

Definition 3.1.3. The hybrid architecture does not comprise any self-loops.

$$\nexists l \in \mathcal{L}_{\text{ac}} : \hat{\varepsilon}(l) = \check{\varepsilon}(l) \quad (3.3a)$$

$$\nexists l \in \mathcal{L}_{\text{dc}} : \hat{\vartheta}(l) = \check{\vartheta}(l) \quad (3.3b)$$

Further, the parallel transmission lines are assumed to be combined together with their equivalent line admittances and the transmission capacity of the new line is the aggregated capacities of the original lines.

3.1.2 AC Branch Model

In this section, AC lines, transformers, and phase shifters are referred as AC branches. They are commonly represented by the well-known Π -model in Fig. 3.1. It comprises $\tilde{y}_l, \hat{y}_l, \check{y}_l \in \mathbb{C}$ for AC line $l \in \mathcal{L}_{\text{ac}}$. $\hat{\rho}_l, \check{\rho}_l \in \mathbb{C} \setminus \{0\}$, where $|\hat{\rho}_l|$ and $|\check{\rho}_l|$ are the tap ratios and $\arg(\hat{\rho}_l)$ and $\arg(\check{\rho}_l)$ are the phase shift of the respective transformer at the source and destination buses of AC line $l \in \mathcal{L}_{\text{ac}}$.

$$\begin{bmatrix} y_l^{(1)} & y_l^{(2)} \\ y_l^{(3)} & y_l^{(4)} \end{bmatrix} = \begin{bmatrix} |\hat{\rho}_l|^2 (\hat{y}_l + \tilde{y}_l) & -\hat{\rho}_l^* \check{\rho}_l \tilde{y}_l \\ -\hat{\rho}_l \check{\rho}_l^* \tilde{y}_l & |\check{\rho}_l|^2 (\check{y}_l + \tilde{y}_l) \end{bmatrix}; \forall l \in \mathcal{L}_{\text{ac}} \quad (3.4)$$

FIGURE 3.1: Model for AC branch $l \in \mathcal{L}_{ac}$ which connects bus $\hat{\epsilon}(l)$ and bus $\check{\epsilon}(l)$ [1].

Therewith, $\hat{Y}_{ac} \in \mathbb{C}^{L_{ac} \times N_{ac}}$ and $\check{Y}_{ac} \in \mathbb{C}^{L_{ac} \times N_{ac}}$ are given by

$$\hat{Y}_{ac} = \sum_{l \in \mathcal{L}_{ac}} e_l \left[y_l^{(1)} e_{\hat{\epsilon}(l)}^T + y_l^{(2)} e_{\check{\epsilon}(l)}^T \right] \quad (3.5a)$$

$$\check{Y}_{ac} = \sum_{l \in \mathcal{L}_{ac}} e_l \left[y_l^{(4)} e_{\hat{\epsilon}(l)}^T + y_l^{(3)} e_{\check{\epsilon}(l)}^T \right] \quad (3.5b)$$

Then, $\hat{i}_{ac} \in \mathbb{C}^{L_{ac}}$, and $\check{i}_{ac} \in \mathbb{C}^{L_{ac}}$ are given by

$$\hat{i}_{ac} = \hat{Y}_{ac} \tilde{v}, \quad \check{i}_{ac} = \check{Y}_{ac} \tilde{v} \quad (3.6)$$

3.1.3 AC Bus Model

AC buses are modeled as illustrated in Fig. 3.2. AC bus $n \in \mathcal{N}_{ac}$ comprises power injections, a shunt admittance $y_n \in \mathbb{C}$ for reactive power compensation, and connections to the outgoing AC lines $l \in \hat{\mathcal{L}}_{ac}$ and to the incoming AC lines $l \in \check{\mathcal{L}}_{ac}$.

FIGURE 3.2: Model for AC bus $n \in \mathcal{N}_{ac}$ [1].

Then, $Y_{ac} \in \mathbb{C}^{N_{ac} \times N_{ac}}$ and $i_{ac} \in \mathbb{C}^{N_{ac}}$ are given by

$$Y_{ac} = \sum_{n \in \mathcal{N}_{ac}} e_n \left[\left(\dot{y}_n + \sum_{l \in \hat{\mathcal{L}}_{ac}(n)} y_l^{(1)} + \sum_{l \in \check{\mathcal{L}}_{ac}(n)} y_l^{(4)} \right) e_n^T + \sum_{l \in \hat{\mathcal{L}}_{ac}(n)} y_l^{(2)} e_{\hat{\varepsilon}(l)}^T + \sum_{l \in \check{\mathcal{L}}_{ac}(n)} y_l^{(3)} e_{\check{\varepsilon}(l)}^T \right] \quad (3.7a)$$

$$i_{ac} = Y_{ac} \tilde{v} \quad (3.7b)$$

3.1.4 AC Network Constraints

The power flow through the AC subgrid of the HTG must respect certain requirements which are discussed in the following to establish a reliable power supply.

3.1.4.1 Bus Voltage Magnitude

AC transmission grids are designed with a certain voltage security range for every bus in the grid to maintain a reliable power supply. Therein, the voltage magnitude at AC bus $n \in \mathcal{N}_{ac}$ must be within the closed interval $[\tilde{v}_n^{\min}, \tilde{v}_n^{\max}] \subset \mathbb{R}_+$ with $\tilde{v}_n^{\min} < \tilde{v}_n^{\max}$. This is represented by

$$\left(\tilde{v}_n^{\min} \right)^2 \leq \tilde{v}^H \tilde{M}_n \tilde{v} \leq \left(\tilde{v}_n^{\max} \right)^2; \quad \forall n \in \mathcal{N}_{ac} \quad (3.8)$$

with $\tilde{M}_n = e_n e_n^T \in \mathbb{R}^{N_{ac} \times N_{ac}}$.

3.1.4.2 AC Branch Ampacity

The current flow through AC branch $l \in \mathcal{L}_{ac}$ must be less than its thermal limit $i_{ac,l}^{\max} \in \mathbb{R}_+ \setminus \{0\}$. This physical limit can be enforced by

$$\left| \hat{i}_{ac,l} \right|^2 = \left(\hat{i}_{ac,l} \right)^* \hat{i}_{ac,l} = \left(e_l^T \hat{Y}_{ac} \tilde{v} \right)^* \left(e_l^T \hat{Y}_{ac} \tilde{v} \right) = \tilde{v}^H \hat{J}_{ac,l} \tilde{v} \leq \left(i_{ac,l}^{\max} \right)^2; \quad \forall l \in \mathcal{L}_{ac} \quad (3.9)$$

with $\hat{I}_{ac,l} = \hat{Y}_{ac}^H e_l e_l^T \hat{Y}_{ac} \in \mathbb{S}^{N_{ac}}$ on the source AC branch current. Similarly, the destination AC branch current must respect

$$\tilde{v}^H \check{I}_{ac,l} \tilde{v} \leq \left(i_{ac,l}^{\max}\right)^2; \forall l \in \mathcal{L}_{ac} \quad (3.10)$$

with $\check{I}_{ac,l} = \check{Y}_{ac}^H e_l e_l^T \check{Y}_{ac} \in \mathbb{S}^{N_{ac}}$.

3.1.4.3 AC Branch Voltage Magnitude Drop

The voltage drop along the transmission lines must be within the maximum and minimum allowable limits for stable operation. For AC branch $l \in \mathcal{L}_{ac}$, $\nu_{ac,l} \in \mathbb{R}$ is given by

$$\nu_{ac,l} = \frac{|\tilde{v}_{\hat{\varepsilon}(l)}|}{|\tilde{v}_{\check{\varepsilon}(l)}|} - 1 \quad (3.11)$$

which must be maintained within the closed interval $[\nu_{ac,l}^{\min}, \nu_{ac,l}^{\max}] \subset [-1, 1]$ with $\nu_{ac,l}^{\min} < \nu_{ac,l}^{\max}$. This is equivalently represented by

$$\tilde{v}^H \hat{\mathcal{V}}_{ac,l} \tilde{v} \leq 0, \quad \tilde{v}^H \check{\mathcal{V}}_{ac,l} \tilde{v} \leq 0 \quad (3.12)$$

where $\hat{\mathcal{V}}_{ac,l}, \check{\mathcal{V}}_{ac,l} \in \mathbb{S}^{N_{ac}}$ are

$$\hat{\mathcal{V}}_{ac,l} = \tilde{M}_{\check{\varepsilon}(l)} - \left(1 + \nu_{ac,l}^{\max}\right)^2 \tilde{M}_{\hat{\varepsilon}(l)} \quad (3.13a)$$

$$\check{\mathcal{V}}_{ac,l} = \left(1 + \nu_{ac,l}^{\min}\right)^2 \tilde{M}_{\hat{\varepsilon}(l)} - \tilde{M}_{\check{\varepsilon}(l)} \quad (3.13b)$$

3.1.4.4 AC Branch Voltage Angle Difference

The voltage angle difference along the transmission lines must be within the maximum and minimum allowable limits for stable operation. For AC branch $l \in \mathcal{L}_{ac}$,

$\tilde{\delta}_l \in \mathbb{R}$ is given by

$$\tilde{\delta}_l = \arg\left(\tilde{v}_{\hat{\varepsilon}(l)}^* \tilde{v}_{\check{\varepsilon}(l)}\right) \quad (3.14)$$

which must be maintained within the closed interval $[\tilde{\delta}_l^{\min}, \tilde{\delta}_l^{\max}] \subset [-\pi/2, \pi/2]$ with $\tilde{\delta}_l^{\min} < \tilde{\delta}_l^{\max}$. This is equivalently represented by

$$\operatorname{Re}\left(\tilde{v}_{\hat{\varepsilon}(l)}^* \tilde{v}_{\check{\varepsilon}(l)}\right) \geq 0 \quad (3.15a)$$

$$\tilde{\delta}_l^{\min} \leq \frac{\operatorname{Im}\left(\tilde{v}_{\hat{\varepsilon}(l)}^* \tilde{v}_{\check{\varepsilon}(l)}\right)}{\operatorname{Re}\left(\tilde{v}_{\hat{\varepsilon}(l)}^* \tilde{v}_{\check{\varepsilon}(l)}\right)} \leq \tilde{\delta}_l^{\max} \quad (3.15b)$$

Further, the substitutions given by

$$\operatorname{Re}\left(\tilde{v}_{\hat{\varepsilon}(l)}^* \tilde{v}_{\check{\varepsilon}(l)}\right) = \frac{\tilde{v}_{\hat{\varepsilon}(l)}^* \tilde{v}_{\check{\varepsilon}(l)} + \tilde{v}_{\check{\varepsilon}(l)}^* \tilde{v}_{\hat{\varepsilon}(l)}}{2} = \frac{\tilde{v}^H [e_{\hat{\varepsilon}(l)} e_{\check{\varepsilon}(l)}^T + e_{\check{\varepsilon}(l)} e_{\hat{\varepsilon}(l)}^T] \tilde{v}}{2} \quad (3.16a)$$

$$\operatorname{Im}\left(\tilde{v}_{\hat{\varepsilon}(l)}^* \tilde{v}_{\check{\varepsilon}(l)}\right) = \frac{\tilde{v}_{\hat{\varepsilon}(l)}^* \tilde{v}_{\check{\varepsilon}(l)} - \tilde{v}_{\check{\varepsilon}(l)}^* \tilde{v}_{\hat{\varepsilon}(l)}}{2\mathbf{i}} = \frac{\tilde{v}^H [e_{\hat{\varepsilon}(l)} e_{\check{\varepsilon}(l)}^T - e_{\check{\varepsilon}(l)} e_{\hat{\varepsilon}(l)}^T] \tilde{v}}{2\mathbf{i}} \quad (3.16b)$$

can be used to convert (3.15) to a quadratic form as

$$\tilde{v}^H A_l \tilde{v} \leq 0, \quad \tilde{v}^H \hat{A}_l \tilde{v} \leq 0, \quad \tilde{v}^H \check{A}_l \tilde{v} \leq 0 \quad (3.17)$$

where

$$A_l = -e_{\hat{\varepsilon}(l)} e_{\check{\varepsilon}(l)}^T - e_{\check{\varepsilon}(l)} e_{\hat{\varepsilon}(l)}^T \quad (3.18a)$$

$$\hat{A}_l = -[\tan(\tilde{\delta}_l^{\max}) + \mathbf{i}] e_{\hat{\varepsilon}(l)} e_{\check{\varepsilon}(l)}^T - [\tan(\tilde{\delta}_l^{\max}) - \mathbf{i}] e_{\check{\varepsilon}(l)} e_{\hat{\varepsilon}(l)}^T \quad (3.18b)$$

$$\check{A}_l = [\tan(\tilde{\delta}_l^{\min}) + \mathbf{i}] e_{\hat{\varepsilon}(l)} e_{\check{\varepsilon}(l)}^T + [\tan(\tilde{\delta}_l^{\min}) - \mathbf{i}] e_{\check{\varepsilon}(l)} e_{\hat{\varepsilon}(l)}^T \quad (3.18c)$$

However, the following conditions are necessary for the feasibility of (3.17), i.e.,

$$-\pi/2 < \tilde{\delta}_l^{\min} < -\arg(\hat{\rho}_l^* \check{\rho}_l) \quad (3.19a)$$

$$-\arg(\hat{\rho}_l^* \check{\rho}_l) < \tilde{\delta}_l^{\max} < \pi/2 \quad (3.19b)$$

which are normally satisfied in practice as the magnitude of the phase-shift $|\arg(\hat{\rho}_l^* \check{\rho}_l)|$ is smaller than the usual stability-related angle limits $|\tilde{\delta}_l^{\min}|$ and $|\tilde{\delta}_l^{\max}|$.

3.1.5 DC Branch Model

The DC subgrid can be modeled as an AC grid operating at *unity* power factor having purely resistive transmission lines and cables under steady-state operation [56]. Therein, DC lines and cables are referred as DC branches, which only comprise $\bar{y}_l \in \mathbb{R}$ for DC branch $l \in \mathcal{L}_{\text{dc}}$. Therewith, $\hat{Y}_{\text{dc}} \in \mathbb{R}^{L_{\text{dc}} \times N_{\text{dc}}}$ and $\check{Y}_{\text{dc}} \in \mathbb{R}^{L_{\text{dc}} \times N_{\text{dc}}}$ of the DC subgrid are given by

$$\hat{Y}_{\text{dc}} = \sum_{l \in \mathcal{L}_{\text{dc}}} e_l \left[\bar{y}_l e_{\hat{\vartheta}(l)}^{\text{T}} - \bar{y}_l e_{\check{\vartheta}(l)}^{\text{T}} \right] \quad (3.20a)$$

$$\check{Y}_{\text{dc}} = \sum_{l \in \mathcal{L}_{\text{dc}}} e_l \left[\bar{y}_l e_{\check{\vartheta}(l)}^{\text{T}} - \bar{y}_l e_{\hat{\vartheta}(l)}^{\text{T}} \right] \quad (3.20b)$$

Then, $\hat{i}_{\text{dc}} \in \mathbb{R}^{L_{\text{dc}}}$ and $\check{i}_{\text{dc}} \in \mathbb{R}^{L_{\text{dc}}}$ are given by

$$\hat{i}_{\text{dc}} = \hat{Y}_{\text{dc}} \bar{v}, \quad \check{i}_{\text{dc}} = \check{Y}_{\text{dc}} \bar{v} \quad (3.21)$$

3.1.6 DC Bus Model

DC bus $n \in \mathcal{N}_{\text{dc}}$ comprises power injections, and connections to the outgoing DC lines $l \in \hat{\mathcal{L}}_{\text{dc}}$ and to the incoming DC lines $l \in \check{\mathcal{L}}_{\text{dc}}$. Then, $Y_{\text{dc}} \in \mathbb{R}^{N_{\text{dc}} \times N_{\text{dc}}}$ and $i_{\text{dc}} \in \mathbb{R}^{N_{\text{dc}}}$ are given by

$$Y_{\text{dc}} = \sum_{n \in \mathcal{N}_{\text{dc}}} e_n \left[\left(\sum_{l \in \hat{\mathcal{L}}_{\text{dc}}(n)} \bar{y}_l + \sum_{l \in \check{\mathcal{L}}_{\text{dc}}(n)} \bar{y}_l \right) e_n^{\text{T}} - \sum_{l \in \hat{\mathcal{L}}_{\text{dc}}(n)} \bar{y}_l e_{\check{\vartheta}(l)}^{\text{T}} - \sum_{l \in \check{\mathcal{L}}_{\text{dc}}(n)} \bar{y}_l e_{\hat{\vartheta}(l)}^{\text{T}} \right] \quad (3.22a)$$

$$i_{\text{dc}} = Y_{\text{dc}} \bar{v}. \quad (3.22b)$$

3.1.7 DC Network Constraints

The power flow through the AC subgrid of the HTG must respect certain requirements which are discussed in the following to establish a reliable power supply.

3.1.7.1 Bus Voltage Magnitude

DC grids are designed with a certain voltage security range for every bus in the grid to maintain a reliable power supply. Therein, the voltage magnitude at DC bus $n \in \mathcal{N}_{\text{dc}}$ must be within the closed interval $[\bar{v}_n^{\min}, \bar{v}_n^{\max}] \subset \mathbb{R}_+$ with $\bar{v}_n^{\min} < \bar{v}_n^{\max}$. This is represented by

$$\left(\bar{v}_n^{\min}\right)^2 \leq \bar{v}^T \bar{M}_n \bar{v} \leq \left(\bar{v}_n^{\max}\right)^2; \forall n \in \mathcal{N}_{\text{ac}} \quad (3.23)$$

with $\bar{M}_n = e_n e_n^T \in \mathbb{R}^{N_{\text{dc}} \times N_{\text{dc}}}$.

3.1.7.2 DC Branch Power Flow

The power flow through DC branch $l \in \mathcal{L}_{\text{dc}}$ must be less than its thermal limit $p_{\text{dc},l}^{\max} \in \mathbb{R}_+ \setminus \{0\}$. This physical limit can be enforced by

$$\begin{aligned} \hat{p}_{\text{dc},l} &= \frac{1}{2} \left(\bar{v}_{\hat{\vartheta}(l)} \hat{i}_{\text{dc},l} + \hat{i}_{\text{dc},l} \bar{v}_{\hat{\vartheta}(l)} \right) = \frac{1}{2} \left(\bar{v}^T e_{\hat{\vartheta}(l)} e_l^T \hat{Y}_{\text{dc}} \bar{v} + \bar{v}^T \hat{Y}_{\text{dc}}^T e_l e_{\hat{\vartheta}(l)}^T \bar{v} \right) \\ &= \bar{v}^T \hat{P}_{\text{dc},l} \bar{v} \leq \left(p_{\text{dc},l}^{\max}\right)^2; \forall l \in \mathcal{L}_{\text{dc}} \end{aligned} \quad (3.24)$$

with $\hat{P}_{\text{dc},l} = \frac{1}{2} \left(e_{\hat{\vartheta}(l)} e_l^T \hat{Y}_{\text{dc}} + \hat{Y}_{\text{dc}}^T e_l e_{\hat{\vartheta}(l)}^T \right) \in \mathbb{R}^{N_{\text{dc}} \times N_{\text{dc}}}$ on the source bus DC branch power flow, where $\hat{P}_{\text{dc},l} = \hat{P}_{\text{dc},l}^T$. Similarly, the destination bus DC branch power flow must respect

$$\bar{v}^T \check{P}_{\text{dc},l} \bar{v} \leq \left(p_{\text{dc},l}^{\max}\right)^2; \forall l \in \mathcal{L}_{\text{dc}} \quad (3.25)$$

with $\check{P}_{\text{dc},l} = \frac{1}{2} \left(e_{\check{\vartheta}(l)} e_l^T \check{Y}_{\text{dc}} + \check{Y}_{\text{dc}}^T e_l e_{\check{\vartheta}(l)}^T \right) \in \mathbb{R}^{N_{\text{dc}} \times N_{\text{dc}}}$, where $\check{P}_{\text{dc},l} = \check{P}_{\text{dc},l}^T$.

3.1.7.3 DC Branch Voltage Magnitude Drop

The voltage drop along the DC branches must not exceed certain limits for stable operation. For AC branch $l \in \mathcal{L}_{\text{dc}}$, $\nu_{\text{dc},l} \in \mathbb{R}$ is given by

$$\nu_{\text{dc},l} = \frac{|\bar{v}_{\hat{\vartheta}(l)}|}{|\bar{v}_{\check{\vartheta}(l)}|} - 1 \quad (3.26)$$

which must be maintained within the closed interval $[\nu_{\text{dc},l}^{\min}, \nu_{\text{dc},l}^{\max}] \subset [-1, 1]$ with $\nu_{\text{dc},l}^{\min} < \nu_{\text{dc},l}^{\max}$. This is equivalently represented by

$$\bar{v}^T \hat{\mathcal{V}}_{\text{dc},l} \bar{v} \leq 0, \quad \bar{v}^T \check{\mathcal{V}}_{\text{dc},l} \bar{v} \leq 0 \quad (3.27)$$

where $\hat{\mathcal{V}}_{\text{dc},l}, \check{\mathcal{V}}_{\text{dc},l} \in \mathbb{R}^{N_{\text{dc}} \times N_{\text{dc}}}$ are

$$\hat{\mathcal{V}}_{\text{dc},l} = \bar{M}_{\check{\vartheta}(l)} - \left(1 + \nu_{\text{dc},l}^{\max}\right)^2 \bar{M}_{\hat{\vartheta}(l)} \quad (3.28a)$$

$$\check{\mathcal{V}}_{\text{dc},l} = \left(1 + \nu_{\text{dc},l}^{\min}\right)^2 \bar{M}_{\hat{\vartheta}(l)} - \bar{M}_{\check{\vartheta}(l)} \quad (3.28b)$$

3.1.8 Converter Model

The AC and DC subgrids are coupled at some buses through bidirectional VSCs. The converter model used in this work is illustrated in Fig. 3.3. The resistances of the phase reactor and the filter are negligible. The transformer, phase reactor, and filter at the AC side of the VSC can be modeled using two AC branches and a capacitive shunt, cf. [53, 96, 97]. Therein, the additional buses of each VSC, i.e., the transformer/filter bus and AC side converter bus are integrated into the set of AC buses $\mathcal{N}_{\text{ac}}$ and standard active and reactive power flow equations are enforced. In case if the filter or transformer is omitted from the design, the remaining component can be lumped together with the phase reactor either to eliminate the transformer/filter bus from the formulation or to combine that with the point-of-common coupling with the AC subgrid respectively.

FIGURE 3.3: Converter which couples AC bus $\tilde{o}(z)$ and DC bus $\bar{o}(z)$, where bus ① is the point-of-common coupling with the AC subgrid; bus ② is the transformer/filter bus; bus ③ is the AC side of the VSC; and bus ④ is the DC side of the VSC and also the point-of-common coupling with the DC subgrid.

There are two modes of operation for VSCs, in which power flows from the AC side to DC side and vice versa in the rectifier mode and in the inverter mode respectively. In the mathematical formulations, the rectifier mode of operation is taken as the (forward) positive direction of power flow for each VSC as shown in Fig. 3.3. VSCs typically comprise switching and conduction losses. The total loss per VSC station has been reduced to less than 1% over the years of technological developments [11, 98]. An analysis on the accuracy of different loss models in the literature for VSCs in power flow studies is presented in [97]. Based on the difference of the “Complete” model [97, Eq. (1)] and the “Avg” model [97, Eq. (2)], it was emphasized that individual loss parameterization of the rectifier and inverter modes is essential. Further, the “Avg” model and the “Prop” model [97, Eq. (3)] were shown similar performance suggesting that individual proportional loss models for two modes potentially offer sufficient accuracy. Thus, the proportional loss model is used in this thesis to formulate the losses in VSC operation. To this end, $S_{C,z}^{ac} \in \mathbb{C}$ and $P_{C,z}^{dc} \in \mathbb{R}$ for VSC $z \in \mathcal{Z}$ are formulated in (3.29) as below.

$$P_{C,z}^{dc} = \hat{p}_{C,z} - \check{p}_{C,z}; z \in \mathcal{Z} \quad (3.29a)$$

$$S_{C,z}^{ac} = \left[\frac{1}{1 - \hat{\eta}_z} \right] \hat{p}_{C,z} - [1 - \check{\eta}_z] \check{p}_{C,z} + \mathbf{i}q_{C,z}; z \in \mathcal{Z} \quad (3.29b)$$

where $\hat{p}_{C,z} \in \mathbb{R}_+$ (and $\check{p}_{C,z} \in \mathbb{R}_+$) are the nonnegative active power injected (and extracted), respectively to (and from) the DC bus $\bar{o}(z)$, while $q_{C,z} \in \mathbb{R}$ is the reactive power extracted at the AC bus $\tilde{o}(z)$ of VSC $z \in \mathcal{Z}$. Therein, the active power is

converted subject to $\hat{\eta}_z, \check{\eta}_z \in (0, 1)$.

In addition to the security limits of the converter AC and DC bus voltages explained by (3.8) and (3.23) respectively, the DC bus voltage exerts a restriction on the maximum AC bus voltage depending on the modulation mode, which is given by

$$\left| \tilde{v}_{\tilde{o}(z)} \right| - m_z \bar{v}_{\tilde{o}(z)} \leq 0 \quad (3.30a)$$

$$\tilde{v}^H \tilde{M}_{\tilde{o}(z)} \tilde{v} - m_z \bar{v}^T \bar{M}_{\tilde{o}(z)} \bar{v} \leq 0 \quad (3.30b)$$

where m_z is the modulation factor. A VSC can independently control its active and the reactive power flow, and hence the AC and DC bus voltages. The P/Q-capability of a VSC is restricted by the following aspects [98]:

1. The maximum allowable current flow through the converter depends on the design limits of semiconductor devices. This can be alternatively represented by enforcing a limitation on apparent power flow in the converter P/Q-plane as illustrated in curve ① of Fig. 3.4. Therefore, the active power conversion capability $p_{C,z}^{\max}$ is decided based on this limit.
2. The maximum reactive power absorption $q_C^{\max}$ and injection $q_C^{\min}$ capabilities are typically decided based on the voltage limits of the converter AC bus and the effect of the transformer and filter.

A detailed discussion on P/Q-capability of VSCs is presented in [99]. Considering aforementioned constraints, a P/Q-capability region can be represented more conservatively using a box domain [1, 6] as in Fig. 3.4(a) or more accurately using a polyhedral set [53] as shown in Fig. 3.4(b). The accuracy and the complexity of the polyhedral P/Q-capability region increases with the number of half-planes considered. The box domain representation of the P/Q-capability region for all the converters is given by

$$(\hat{p}_C, \check{p}_C) \in [\mathbf{0}, p_C^{\max}] \subset \mathbb{R}_+^{|Z|} \quad (3.31a)$$

$$q_C \in [q_C^{\min}, q_C^{\max}] \subset \mathbb{R}^{|Z|} \quad (3.31b)$$

FIGURE 3.4: Approximated P/Q-capability region of converter $z \in \mathcal{Z}$. (a) More conservative box representation and (b) more accurate polyhedral representation using the intersection of 8 half-planes (linear segments which enclose the capability domain). Curve ① is the maximum apparent power flow limit based on the current handling capability of the VSC.

Alternatively, the polyhedral P/Q-capability region can be represented by the set $\mathcal{H}_{C,z}$ which comprises the intersection of 8 half-planes enclosing the area shown in Fig. 3.4(b). Using compact notations, the polyhedral P/Q-capability region is expressed by

$$\mathcal{H}_C = \mathcal{H}_{C,1} \times \mathcal{H}_{C,2} \times \cdots \times \mathcal{H}_{C,|\mathcal{Z}|} \quad (3.32a)$$

$$\mathcal{H}_C = \left\{ (\hat{p}_C, \check{p}_C, q_C) \in \mathbb{R}_+^Z \times \mathbb{R}_+^Z \times \mathbb{R}^Z : \hat{H}_C^P \hat{p}_C + \check{H}_C^P \check{p}_C + H_C^Q q_C \leq h_C \right\} \quad (3.32b)$$

with $\hat{H}_C^P, \check{H}_C^P, H_C^Q \in \mathbb{R}^{8|\mathcal{Z}| \times |\mathcal{Z}|}$ and $h_C \in \mathbb{R}^{8|\mathcal{Z}|}$.

3.1.9 Generator Model

Power injections at AC buses may comprise generation sources (both dispatchable sources and RESs), (critical and flexible) loads, and VSCs. The P/Q-capability region of a conventional generator is illustrated in Fig. 3.5. The conventional generators comprise nonnegative P-capability, i.e., $0 < P_{G,g}^{\text{min}} < P_{G,g}^{\text{max}}$ as shown in Fig. 3.5. In most of the OPF literature, the P/Q-capability of generators are

characterized by the box injection domain given by

$$P_G \in [P_G^{\min}, P_G^{\max}] \subset \mathbb{R}_+^{|\mathcal{G}|} \quad (3.33a)$$

$$Q_G \in [Q_G^{\min}, Q_G^{\max}] \subset \mathbb{R}^{|\mathcal{G}|} \quad (3.33b)$$

In general, $Q_{G,g}^{\max}$ and $Q_{G,g}^{\min}$ depends on the excitation limitations and power-angle stability requirements of generator $g \in \mathcal{G}$ respectively.

FIGURE 3.5: Approximated polyhedral P/Q-capability region of generator $g \in \mathcal{G}$. Curve ① is the apparent power generation capability; Lines ② and ③ are the power factor limits.

Practically, a generator comprise a certain apparent power generation capability as shown in curve ① of Fig. 3.5 and some of the generators further maintains power factor within a closed interval as shown in lines ② and ③ of Fig. 3.5. These constraints can be more accurately represented using a polyhedral set. The shaded area in Fig. 3.5 illustrates the approximated P/Q-capability domain $\mathcal{H}_{G,g}$ for generator $g \in \mathcal{G}$ using the intersection of 8 half-planes (linear segments which enclose the capability domain). Using compact notations, the polyhedral P/Q-capability domain is expressed by

$$\mathcal{H}_G = \mathcal{H}_{G,1} \times \mathcal{H}_{G,2} \times \cdots \times \mathcal{H}_{G,|\mathcal{G}|} \quad (3.34a)$$

$$\mathcal{H}_G = \left\{ (P_G, Q_G) \in \mathbb{R}_+^{|\mathcal{G}|} \times \mathbb{R}^{|\mathcal{G}|} : H_G^P P_G + H_G^Q Q_G \leq h_G \right\} \quad (3.34b)$$

with $H_G^P, H_G^Q \in \mathbb{R}^{8|\mathcal{G}| \times |\mathcal{G}|}$ and $h_G \in \mathbb{R}^{8|\mathcal{G}|}$.

The power generation of units typically involves a certain cost associated with every operating point on their P/Q-capability regions. Let $C_G : \mathcal{H}_G \rightarrow \mathbb{R}$ be a convex cost function associated with total generation which is typically defined by a quadratic cost function as in (3.35).

$$C_G(P_G) = P_G^T \text{diag}(c_Q) P_G + c_L^T P_G + c_F^T \mathbf{1} \quad (3.35)$$

The flexible loads typically constitute a polyhedral set (usually a box set) around their nominal power demand on the third quadrant (negative active and reactive power generation) of the P/Q-plane depending on the flexibility offered by each load. Normally, the flexible loads must be incentivized considering inconvenience of the consumers due to the demand shift. Moreover, the critical (i.e., non-flexible) loads constitute a (fixed) point on the third quadrant of the P/Q-plane, corresponding to their active and reactive power demands.

3.2 Problem Formulation

The objective function of the OPF problem is generally formulated as the total cost minimization. The constraints are formulated to satisfy the power balance at each AC and DC buses, the generation capability, conversion capability of VSCs, and AC and DC network constraints.

3.2.1 Power Balance

Kirchhoff's current law emphasizes the nodal current balance of electrical networks. In OPF studies, this is respected via enforcing the *power balance* equations which balance the power injections and extractions at every bus of the system.

3.2.1.1 Power Balance of the AC Network

Bus injections of the AC subgrid comprise conventional generators, stochastic sources, loads, VSCs, incoming and outgoing AC branches. Considering the active and reactive power flow from and to the bus, the two equations (3.36a) and (3.36b) are formulated respectively for AC bus $n \in \mathcal{N}_{\text{ac}}$ of the system as follows:

$$\left(\Gamma_n^{\text{G}}\right)^{\text{T}} P_{\text{G}} - P_{\text{D},n} + \left(\Gamma_n^{\text{F}}\right)^{\text{T}} P_{\text{F}} = \tilde{v}^{\text{H}} \tilde{P}_n \tilde{v} - \left(\tilde{\Gamma}_n^{\text{C}}\right)^{\text{T}} \tilde{p}_{\text{C}} + \left(\hat{\Gamma}_n^{\text{C}}\right)^{\text{T}} \hat{p}_{\text{C}}; \quad \forall n \in \mathcal{N}_{\text{ac}} \quad (3.36\text{a})$$

$$\left(\Gamma_n^{\text{G}}\right)^{\text{T}} Q_{\text{G}} - Q_{\text{D},n} + \left(\Gamma_n^{\text{F}}\right)^{\text{T}} Q_{\text{F}} = \tilde{v}^{\text{H}} \tilde{Q}_n \tilde{v} + \left(\tilde{\Gamma}_n^{\text{C}}\right)^{\text{T}} q_{\text{C}}; \quad \forall n \in \mathcal{N}_{\text{ac}} \quad (3.36\text{b})$$

The matrices $\tilde{P}_n$ and $\tilde{Q}_n \in \mathbb{S}^{N_{\text{ac}}}$ are

$$\tilde{P}_n = \frac{1}{2} \left(\tilde{S}_n + \tilde{S}_n^{\text{H}} \right) \quad (3.37\text{a})$$

$$\tilde{Q}_n = \frac{1}{2\text{j}} \left(\tilde{S}_n - \tilde{S}_n^{\text{H}} \right) \quad (3.37\text{b})$$

and $\tilde{S}_n = Y_{\text{ac}}^{\text{H}} e_n e_n^{\text{T}} \in \mathbb{C}^{N_{\text{ac}} \times N_{\text{ac}}}$. The vectors Γ_n^{G} , Γ_n^{F} , $\hat{\Gamma}_n^{\text{C}}$, $\check{\Gamma}_n^{\text{C}}$, and $\tilde{\Gamma}_n^{\text{C}}$ are the n^{th} column of the matrices Γ^{G} , Γ^{F} , $\hat{\Gamma}^{\text{C}}$, $\check{\Gamma}^{\text{C}}$, and $\tilde{\Gamma}^{\text{C}}$ respectively, which are defined below.

$$\Gamma^{\text{G}} = \sum_{g \in \mathcal{G}} e_g e_{\varrho(g)}^{\text{T}} \in \{0, 1\}^{|\mathcal{G}| \times N_{\text{ac}}} \quad (3.38\text{a})$$

$$\Gamma^{\text{F}} = \sum_{f \in \mathcal{F}} e_f e_{\varsigma(f)}^{\text{T}} \in \{0, 1\}^{|\mathcal{F}| \times N_{\text{ac}}} \quad (3.38\text{b})$$

$$\hat{\Gamma}^{\text{C}} = \sum_{z \in \mathcal{Z}} \frac{1}{1 - \hat{\eta}_z} e_z e_{\hat{o}(z)}^{\text{T}} \in \{0, 1\}^{|\mathcal{Z}| \times N_{\text{ac}}} \quad (3.38\text{c})$$

$$\check{\Gamma}^{\text{C}} = \sum_{z \in \mathcal{Z}} (1 - \check{\eta}_z) e_z e_{\check{o}(z)}^{\text{T}} \in \{0, 1\}^{|\mathcal{Z}| \times N_{\text{ac}}} \quad (3.38\text{d})$$

$$\tilde{\Gamma}^{\text{C}} = \sum_{z \in \mathcal{Z}} e_z e_{\tilde{o}(z)}^{\text{T}} \in \{0, 1\}^{|\mathcal{Z}| \times N_{\text{ac}}} \quad (3.38\text{e})$$

3.2.1.2 Power Balance of the DC Network

The DC subgrid constitutes only an active power flow. In the study of this thesis, the bus injections of the DC subgrid only comprise VSCs. However, stochastic

sources and DC loads can also be considered similar to (3.36a) where necessary. To this end, the active power balance at DC bus $n \in \mathcal{N}_{\text{dc}}$ is given by

$$\left(\bar{\Gamma}_n^{\text{C}}\right)^{\text{T}} \hat{p}_{\text{C}} - \left(\bar{\Gamma}_n^{\text{C}}\right)^{\text{T}} \check{p}_{\text{C}} = \bar{v}^{\text{T}} \bar{P}_n \bar{v}; \quad \forall n \in \mathcal{N}_{\text{dc}} \quad (3.39)$$

where $\bar{P}_n = Y_{\text{dc}}^{\text{T}} e_n e_n^{\text{T}} \in \mathbb{R}^{N_{\text{dc}} \times N_{\text{dc}}}$. The vector $\bar{\Gamma}_n^{\text{C}}$ is the n^{th} column of the matrix $\bar{\Gamma}^{\text{C}}$ which is defined below.

$$\bar{\Gamma}^{\text{C}} = \sum_{z \in \mathcal{Z}} e_z e_{\bar{o}(z)}^{\text{T}} \in \{0, 1\}^{|\mathcal{Z}| \times N_{\text{dc}}} \quad (3.40)$$

3.2.1.3 Total Power Loss

The total power loss of the HTG comprises the losses in AC branches, DC branches, and VSCs. Therein, the three terms of (3.41a) compute the aggregated power losses in aforementioned components. Alternatively, the total power loss can be computed by the difference in aggregated generation and demand as in (3.41b), considering the power balance of the system.

$$P_{\text{loss}} = \frac{1}{2} \tilde{v}^{\text{H}} \left(Y_{\text{ac}} + Y_{\text{ac}}^{\text{H}} \right) \tilde{v} + \bar{v}^{\text{T}} Y_{\text{dc}} \bar{v} + \sum_{z \in \mathcal{Z}} \left[\frac{\hat{\eta}_z}{1 - \hat{\eta}_z} \hat{p}_{\text{C},z} + \check{\eta}_z \check{p}_{\text{C},z} \right] \quad (3.41a)$$

$$P_{\text{loss}} = \mathbf{1}^{\text{T}} P_{\text{G}} + \mathbf{1}^{\text{T}} P_{\text{F}} - \sum_{n \in \mathcal{N}_{\text{ac}}} P_{\text{D},n} \quad (3.41b)$$

3.2.2 Optimal Power Flow Problem

Let $\Theta = (P_{\text{G}}, Q_{\text{G}}, \hat{p}_{\text{C}}, \check{p}_{\text{C}}, q_{\text{C}}, \tilde{v}, \bar{v})$ be a tuple pertaining to all the decision variables of the OPF problem. To this end, the OPF problem for an HTG can be summarized by

$$\min_{\Theta} C_{\text{G}}(P_{\text{G}}) \quad (3.42a)$$

$$\begin{aligned} \text{s. t. } & \left(\Gamma_n^{\text{G}}\right)^{\text{T}} P_{\text{G}} - P_{\text{D},n} + \left(\Gamma_n^{\text{F}}\right)^{\text{T}} P_{\text{F}} = \tilde{v}^{\text{H}} \tilde{P}_n \tilde{v} \\ & - \left(\check{\Gamma}_n^{\text{C}}\right)^{\text{T}} \check{p}_{\text{C}} + \left(\hat{\Gamma}_n^{\text{C}}\right)^{\text{T}} \hat{p}_{\text{C}}; \quad \forall n \in \mathcal{N}_{\text{ac}} \end{aligned} \quad (3.42b)$$

$$\left(\Gamma_n^G\right)^T Q_G - Q_{D,n} + \left(\Gamma_n^F\right)^T Q_F = \tilde{v}^H \tilde{Q}_n \tilde{v} + \left(\tilde{\Gamma}_n^C\right)^T q_C; \quad \forall n \in \mathcal{N}_{ac} \quad (3.42c)$$

$$\left(\bar{\Gamma}_n^C\right)^T \hat{p}_C - \left(\bar{\Gamma}_n^C\right)^T \check{p}_C = \bar{v}^T \bar{P}_n \bar{v}; \quad \forall n \in \mathcal{N}_{dc} \quad (3.42d)$$

$$(\tilde{v}_n^{\min})^2 \leq \tilde{v}^H \tilde{M}_n \tilde{v} \leq (\tilde{v}_n^{\max})^2; \quad \forall n \in \mathcal{N}_{ac} \quad (3.42e)$$

$$(\bar{v}_n^{\min})^2 \leq \bar{v}^T \bar{M}_n \bar{v} \leq (\bar{v}_n^{\max})^2; \quad \forall n \in \mathcal{N}_{dc} \quad (3.42f)$$

$$\tilde{v}^H \hat{I}_{ac,l} \tilde{v} \leq \left(i_{ac,l}^{\max}\right)^2; \quad \forall l \in \mathcal{L}_{ac} \quad (3.42g)$$

$$\tilde{v}^H \check{I}_{ac,l} \tilde{v} \leq \left(i_{ac,l}^{\max}\right)^2; \quad \forall l \in \mathcal{L}_{ac} \quad (3.42h)$$

$$\bar{v}^T \hat{P}_{dc,l} \bar{v} \leq \left(p_{dc,l}^{\max}\right)^2; \quad \forall l \in \mathcal{L}_{dc} \quad (3.42i)$$

$$\bar{v}^T \check{P}_{dc,l} \bar{v} \leq \left(p_{dc,l}^{\max}\right)^2; \quad \forall l \in \mathcal{L}_{dc} \quad (3.42j)$$

$$\tilde{v}^H \hat{\mathcal{V}}_{ac,l} \tilde{v} \leq 0, \quad \tilde{v}^H \check{\mathcal{V}}_{ac,l} \tilde{v} \leq 0; \quad \forall l \in \mathcal{L}_{ac} \quad (3.42k)$$

$$\bar{v}^T \hat{\mathcal{V}}_{dc,l} \bar{v} \leq 0, \quad \bar{v}^T \check{\mathcal{V}}_{dc,l} \bar{v} \leq 0; \quad \forall l \in \mathcal{L}_{dc} \quad (3.42l)$$

$$\tilde{v}^H \hat{A}_l \tilde{v} \leq 0, \quad \tilde{v}^H \check{A}_l \tilde{v} \leq 0, \quad \tilde{v}^H \check{A}_l \tilde{v} \leq 0; \quad \forall l \in \mathcal{L}_{ac} \quad (3.42m)$$

$$\tilde{v}^H \tilde{M}_{\tilde{o}(z)} \tilde{v} - m_z \bar{v}^T \bar{M}_{\bar{o}(z)} \bar{v} \leq 0; \quad \forall z \in \mathcal{Z} \quad (3.42n)$$

$$(P_G, Q_G) \in \mathcal{H}_G, \quad (\hat{p}_C, \check{p}_C, q_C) \in \mathcal{H}_C \quad (3.42o)$$

where the total cost is *minimized* in (3.42a) while respecting the NCs (3.42b)-(3.42m) and the capability constraints of generators and VSCs in (3.42n)-(3.42o). The dimensionality of the problem $|\Theta|$ is $2|\mathcal{G}| + 3|\mathcal{Z}| + N_{ac} + N_{dc}$. The number of NCs is $3N_{ac} + 2N_{dc} + 7L_{ac} + 4L_{dc}$.

3.3 Summary

This chapter discusses the quadratic complex formulation of the classical OPF problem for an HTG from first principles. The well-known Π -model is applied for modeling the transmission lines, and the power flow relations are developed based on the bus-injection model. Detailed models were developed for the generators, VSCs, and AC and DC transmission systems which are typically coupled through VSCs. The NCs represent the operational requirements to be satisfied while transferring power through the transmission lines in HTGs. The OPF model developed

in this chapter is embedded in the formulations discussed in the following chapters of this thesis.

Chapter 4

A Trust-Region based Sequential Linear Programming Approach for AC-OPF Problems

4.1 Introduction

LP approximations are preferred for large-scale optimization problems due to their inherent computational performance and scalability. This chapter presents the details of the TR-SLP which is proposed to solve the AC-OPF problem. It should be noted that an SLP approach starting at an arbitrary point far from a solution to the original OPF may not converge to a feasible solution. In such circumstances, TR-based methods have proven to be a viable alternative; see for instance TR-based state estimation [100], TR-based interior-point [101], [102], and TR-based quadratic program [103]. In TR methods, an approximation problem is solved within a small radius, which is called the *TR*. This enables a closer approximation for the original OPF to be obtained at each solution step within the given TR.

The proposed TR-SLP exploits the benefits of both the TR method and SLPs. In brief, the polar formulation of the OPF problem is presented in Section 4.2, which

utilizes the quasi-linear relationship between active power and bus voltage angles, and the reactive power and voltage magnitudes for faster convergence. Further, the first-order Taylor series is used to construct a local linear model for the original OPF problem. A TR constraint is designed to ensure the validity of the constructed linear model, i.e., to ensure the original nonlinear constraints are satisfied. In Section 4.3, the algorithm of the iterative procedure is presented. Section 4.4 presents the numerical results on various IEEE and Polish systems, wherein the proposed TR-SLP terminates in a finite number of iterations, returning an OPF solution satisfying the convergence criteria given in Section 4.3.4. Finally, the chapter is concluded in Section 4.5.

4.2 Mathematical Formulation

In this section, the AC-OPF problem is formulated in terms of the polar form of the complex bus voltage. Then, an LP approximation of the AC-OPF problem is derived using first-order Taylor series. This linear approximation is later embedded in an iterative procedure to form the TR-SLP algorithm (see Section 4.3).

4.2.1 AC-OPF Problem Formulation

To formulate the OPF problem using real-valued variables, the polar form of the complex bus voltage $\tilde{v} \in \mathbb{C}^{N_{ac}}$ is used. The n^{th} element of $\tilde{v}$ is denoted by $\tilde{v}_n = V_n e^{i\delta_n}$ which is the voltage phasor at bus $n \in \mathcal{N}_{ac}$. In the following, the system matrices defined in the OPF formulation in Chapter 3 are modified to model the polar formulation of the OPF problem, which is given in (4.1).

$$\min_{\delta, V, P_G, Q_G} C_G(P_G) \quad (4.1a)$$

$$\text{s.t. } (\Gamma_n^G)^T P_G - P_{D,n} = P_{X,n}; \quad \forall n \in \mathcal{N}_{ac} \quad (4.1b)$$

$$\begin{aligned} P_{X,n} = & (e_n^T \tilde{P}_n e_n) V_n^2 - V_n \sum_{m \in \mathcal{B}_{ac}(n)} V_m \left(e_m^T \tilde{P}_n e_n + e_n^T \tilde{P}_n e_m \right) \cos(\delta_n - \delta_m) \\ & + V_n \sum_{m \in \mathcal{B}_{ac}(n)} V_m \frac{1}{i} \left(e_m^T \tilde{P}_n e_n - e_n^T \tilde{P}_n e_m \right) \sin(\delta_n - \delta_m); \quad \forall n \in \mathcal{N}_{ac} \end{aligned} \quad (4.1c)$$

$$(\Gamma_n^G)^T Q_G - Q_{D,n} = Q_{X,n}; \quad \forall n \in \mathcal{N}_{ac} \quad (4.1d)$$

$$\begin{aligned} Q_{X,n} = & (e_n^T \tilde{Q}_n e_n) V_n^2 - V_n \sum_{m \in \mathcal{B}_{ac}(n)} V_m \left(e_m^T \tilde{Q}_n e_n + e_n^T \tilde{Q}_n e_m \right) \cos(\delta_n - \delta_m) \\ & - V_n \sum_{m \in \mathcal{B}_{ac}(n)} V_m \frac{1}{i} \left(e_m^T \tilde{Q}_n e_n - e_n^T \tilde{Q}_n e_m \right) \sin(\delta_n - \delta_m); \quad \forall n \in \mathcal{N}_{ac} \end{aligned} \quad (4.1e)$$

$$\begin{aligned} |\hat{i}_{ac,l}|^2 = & \hat{I}_{ac,l}^{(1)} V_{\hat{\varepsilon}(l)}^2 + \hat{I}_{ac,l}^{(2)} V_{\check{\varepsilon}(l)}^2 - 2V_{\hat{\varepsilon}(l)} V_{\check{\varepsilon}(l)} \hat{I}_{ac,l}^{(3)} \cos(\delta_{\hat{\varepsilon}(l)} - \delta_{\check{\varepsilon}(l)}) \\ & + 2V_{\hat{\varepsilon}(l)} V_{\check{\varepsilon}(l)} \hat{I}_{ac,l}^{(4)} \sin(\delta_{\hat{\varepsilon}(l)} - \delta_{\check{\varepsilon}(l)}); \quad \forall l \in \mathcal{L}_{ac} \end{aligned} \quad (4.1f)$$

$$\begin{aligned} |\check{i}_{ac,l}|^2 = & \check{I}_{ac,l}^{(1)} V_{\hat{\varepsilon}(l)}^2 + \check{I}_{ac,l}^{(2)} V_{\check{\varepsilon}(l)}^2 - 2V_{\check{\varepsilon}(l)} V_{\hat{\varepsilon}(l)} \check{I}_{ac,l}^{(3)} \cos(\delta_{\hat{\varepsilon}(l)} - \delta_{\check{\varepsilon}(l)}) \\ & + 2V_{\check{\varepsilon}(l)} V_{\hat{\varepsilon}(l)} \check{I}_{ac,l}^{(4)} \sin(\delta_{\hat{\varepsilon}(l)} - \delta_{\check{\varepsilon}(l)}); \quad \forall l \in \mathcal{L}_{ac} \end{aligned} \quad (4.1g)$$

$$|\hat{i}_{ac,l}|^2 \leq (i_{ac,l}^{\max})^2; \quad \forall l \in \mathcal{L}_{ac} \quad (4.1h)$$

$$|\check{i}_{ac,l}|^2 \leq (i_{ac,l}^{\max})^2; \quad \forall l \in \mathcal{L}_{ac} \quad (4.1i)$$

$$V_n \in [\tilde{v}_n^{\min}, \tilde{v}_n^{\max}]; \quad \forall n \in \mathcal{N}_{ac} \quad (4.1j)$$

$$P_{G,g} \in [P_{G,g}^{\min}, P_{G,g}^{\max}]; \quad \forall g \in \mathcal{G} \quad (4.1k)$$

$$Q_{G,g} \in [Q_{G,g}^{\min}, Q_{G,g}^{\max}]; \quad \forall g \in \mathcal{G} \quad (4.1l)$$

where $\mathcal{B}_{ac}(n) = \check{\varepsilon}(\hat{\mathcal{L}}_{ac}(n)) \cup \hat{\varepsilon}(\check{\mathcal{L}}_{ac}(n)) \subset \mathcal{N}_{ac}$. The objective function for minimizing the generation cost is given in (4.1a). Considering the physical laws of power flow, (4.1b)-(4.1e) represent the active power balance, active power extractions, reactive power balance, and reactive power extractions at bus $n \in \mathcal{N}_{ac}$ respectively. Equations (4.1f) and (4.1g) model the apparent current flow from source bus $\hat{\varepsilon}(l)$

to destination bus $\check{\varepsilon}(l)$ through transmission line $l \in \mathcal{L}_{ac}$ respectively, where

$$\hat{I}_{ac,l}^{(1)} = e_{\hat{\varepsilon}(l)}^T \hat{I}_{ac,l} e_{\hat{\varepsilon}(l)}, \quad (4.2a)$$

$$\hat{I}_{ac,l}^{(2)} = e_{\check{\varepsilon}(l)}^T \hat{I}_{ac,l} e_{\check{\varepsilon}(l)}, \quad (4.2b)$$

$$\hat{I}_{ac,l}^{(3)} = \frac{e_{\hat{\varepsilon}(l)}^T \hat{I}_{ac,l} e_{\check{\varepsilon}(l)} + e_{\check{\varepsilon}(l)}^T \hat{I}_{ac,l} e_{\hat{\varepsilon}(l)}}{2}, \quad (4.2c)$$

$$\hat{I}_{ac,l}^{(4)} = \frac{e_{\hat{\varepsilon}(l)}^T \hat{I}_{ac,l} e_{\check{\varepsilon}(l)} - e_{\check{\varepsilon}(l)}^T \hat{I}_{ac,l} e_{\hat{\varepsilon}(l)}}{2i} \quad (4.2d)$$

Following the analogy, $\check{I}_{ac,l}^{(1)}$, $\check{I}_{ac,l}^{(2)}$, $\check{I}_{ac,l}^{(3)}$, and $\check{I}_{ac,l}^{(4)}$ can be derived from $\check{I}_{ac,l}$ which is defined in Section 3.1.4.2 as well. Constraints (4.1h) and (4.1i) restrict the maximum current flow through each transmission line.

Constraint (4.1j) bounds the engineering limits of the voltage at each bus; and (4.1k) and (4.1l) bound the active and reactive power generation capabilities of each generator respectively. The optimization problem consists of $2(N_{ac} + |\mathcal{G}|)$ number of variables to be solved subject to the variable bounds and $2(N_{ac} + L_{ac})$ number of constraints.

4.2.2 Linear Programming Formulation

The nonlinearity in the aforementioned OPF problem comes from equations (4.1a), (4.1c), and (4.1e)-(4.1g). In the proposed iterative procedure (i.e., TR-SLP), the nonlinear terms in these equations are linearized by applying first-order Taylor series approximations evaluated at the solution of the previous iteration. Assume the decision variable vector pertaining to the nonlinear programming (NLP) problem (4.1) as

$$\Theta := [\delta^T, V^T, P_G^T, Q_G^T]^T \in \mathbb{R}^{2(N_{ac} + |\mathcal{G}|)}.$$

Then, the partial derivatives of (4.1a), (4.1c), and (4.1e)-(4.1g) are used to compute the Jacobian matrices as follows.

$$J_C^{k-1} = \left[\mathbf{0}_{2N_{ac}}^T, \frac{\partial C_G}{\partial P_{G,1}}, \dots, \frac{\partial C_G}{\partial P_{G,|\mathcal{G}|}}, \mathbf{0}_{|\mathcal{G}|}^T \right] \Big|_{\Theta^{k-1}} \quad (4.3a)$$

$$J_{P,n}^{k-1} = \left[\frac{\partial P_{X,n}}{\partial \delta_1}, \dots, \frac{\partial P_{X,n}}{\partial \delta_{N_{ac}}}, \frac{\partial P_{X,n}}{\partial V_1}, \dots, \frac{\partial P_{X,n}}{\partial V_{N_{ac}}}, -(\Gamma_n^G)^T, \mathbf{0}_{|\mathcal{G}|}^T \right] \Big|_{\Theta^{k-1}}; \forall n \in \mathcal{N}_{ac} \quad (4.3b)$$

$$J_{Q,n}^{k-1} = \left[\frac{\partial Q_{X,n}}{\partial \delta_1}, \dots, \frac{\partial Q_{X,n}}{\partial \delta_{N_{ac}}}, \frac{\partial Q_{X,n}}{\partial V_1}, \dots, \frac{\partial Q_{X,n}}{\partial V_{N_{ac}}}, \mathbf{0}_{|\mathcal{G}|}^T, -(\Gamma_n^G)^T \right] \Big|_{\Theta^{k-1}}; \forall n \in \mathcal{N}_{ac} \quad (4.3c)$$

$$J_{\hat{i},l}^{k-1} = \left[\frac{\partial |\hat{i}_{ac,l}|^2}{\partial \delta_1}, \dots, \frac{\partial |\hat{i}_{ac,l}|^2}{\partial \delta_{N_{ac}}}, \frac{\partial |\hat{i}_{ac,l}|^2}{\partial V_1}, \dots, \frac{\partial |\hat{i}_{ac,l}|^2}{\partial V_{N_{ac}}}, \mathbf{0}_{2|\mathcal{G}|}^T \right] \Big|_{\Theta^{k-1}}; \forall l \in \mathcal{L}_{ac} \quad (4.3d)$$

$$J_{\check{i},l}^{k-1} = \left[\frac{\partial |\check{i}_{ac,l}|^2}{\partial \delta_1}, \dots, \frac{\partial |\check{i}_{ac,l}|^2}{\partial \delta_{N_{ac}}}, \frac{\partial |\check{i}_{ac,l}|^2}{\partial V_1}, \dots, \frac{\partial |\check{i}_{ac,l}|^2}{\partial V_{N_{ac}}}, \mathbf{0}_{2|\mathcal{G}|}^T \right] \Big|_{\Theta^{k-1}}; \forall l \in \mathcal{L}_{ac} \quad (4.3e)$$

Equations (4.3a), (4.3b), (4.3c), (4.3d), and (4.3e) represent the Jacobian matrices of (4.1a), (4.1c), (4.1e), (4.1f), and (4.1g) respectively, which are originally nonlinear. At the k^{th} iteration of TR-SLP, those Jacobian matrices in (4.3) are updated based on the solution of the previous $(k-1)^{\text{th}}$ iteration. Finally, the LP approximation of the OPF problem (4.1) which is solved at the k^{th} iteration can be deduced as follows:

$$\text{LP}(\Theta^{k-1}) \left\{ \begin{array}{l} \min_{\Theta} J_C^{k-1}(\Theta - \Theta^{k-1}) + C_G|_{\Theta^{k-1}} \\ \text{s.t. } J_{P,n}^{k-1}(\Theta - \Theta^{k-1}) + P_{X,n}|_{\Theta^{k-1}} - (\Gamma_n^G)^T P_G^{k-1} = -P_{D,n}, \forall n \in \mathcal{N}_{ac} \\ J_{Q,n}^{k-1}(\Theta - \Theta^{k-1}) + Q_{X,n}|_{\Theta^{k-1}} - (\Gamma_n^G)^T Q_G^{k-1} = -Q_{D,n}, \forall n \in \mathcal{N}_{ac} \\ J_{\hat{i},l}^{k-1}(\Theta - \Theta^{k-1}) + |\hat{i}_{ac,l}|^2|_{\Theta^{k-1}} \leq (i_{ac,l}^{\max})^2; \forall l \in \mathcal{L}_{ac} \\ J_{\check{i},l}^{k-1}(\Theta - \Theta^{k-1}) + |\check{i}_{ac,l}|^2|_{\Theta^{k-1}} \leq (i_{ac,l}^{\max})^2; \forall l \in \mathcal{L}_{ac} \\ (4.1j) - (4.1l) \end{array} \right. \quad (4.4)$$

It should be noted that (4.4) is tightly-coupled to the original OPF problem (4.1) at the evaluated point Θ^{k-1} .

4.3 Trust-Region based Sequential Linear Programming Algorithm

This section first introduces components such as TR-LP formulation, feasibility restoration phase, and step acceptance/rejection criterion. Then, the pseudo-code of the main algorithm TR-SLP comprising all these components is presented. For ease of explanation, the AC-OPF problem (4.1) is represented using a generic NLP form as follows:

$$\text{NLP} \left\{ \begin{array}{l} \min_{\Theta} f_o(\Theta) \\ \text{s.t. } f_e(\Theta) = 0 \\ f_i(\Theta) \leq 0 \\ \Theta^{\min} \leq \Theta \leq \Theta^{\max} \end{array} \right. \quad (4.5)$$

where $f_o : \mathbb{R}^{2(N_{ac}+|\mathcal{G}|)} \rightarrow \mathbb{R}$ represents (4.1a); $f_e : \mathbb{R}^{2(N_{ac}+|\mathcal{G}|)} \rightarrow \mathbb{R}^{2N_{ac}}$ include (4.1b)-(4.1e); $f_i : \mathbb{R}^{2(N_{ac}+|\mathcal{G}|)} \rightarrow \mathbb{R}^{2L_{ac}}$ include (4.1f)-(4.1i); and $\Theta^{\min}$ and $\Theta^{\max}$ in (4.5) represent the variable bounds (4.1j)-(4.1l).

4.3.1 Trust-Region Linear Program

At the k^{th} iteration, the LP(Θ^{k-1}) approximates the original OPF problem (4.1) at Θ^{k-1} . However, it may be a very poor representation of (4.1) if $\|\Theta^k - \Theta^{k-1}\|$ is not sufficiently small. To circumvent this issue, the variation of Θ is bounded within a small closed region called TR using Δ^k . In particular, the TR constraint is added to the LP approximation (4.4), and the following LP problem is formed.

$$\text{TR-LP}(\Theta^{k-1}, \Delta^k) \left\{ \begin{array}{l} \min_d f_o(\Theta^{k-1}) + [\nabla f_o(\Theta^{k-1})]^T d \\ \text{s.t. } f_e(\Theta^{k-1}) + [\nabla f_e(\Theta^{k-1})]^T d = 0 : \pi_e^k \\ f_i(\Theta^{k-1}) + [\nabla f_i(\Theta^{k-1})]^T d \leq 0 : \pi_i^k \\ \max(\Theta^{\min} - \Theta^{k-1}, -\Delta^k) \leq d \\ d \leq \min(\Theta^{\max} - \Theta^{k-1}, \Delta^k) \end{array} \right. \quad (4.6)$$

where $d := \Theta - \Theta^{k-1}$; $\Delta^k > 0 \in \mathbb{R}^{2(N+G)}$; and $\pi^k = \begin{bmatrix} (\pi_e^k)^T & (\pi_i^k)^T \end{bmatrix}^T$. Then, d^k (i.e., d at iteration k) is used to define the new solution approximation, i.e., $\Theta^k = \Theta^{k-1} + d^k$ (see Section 4.3.3). The Karush-Kuhn-Tucker conditions for (4.6) are:

$$f_e(\Theta^{k-1}) + \left[\nabla f_e(\Theta^{k-1}) \right]^T d^k = 0 \quad (4.7a)$$

$$f_i(\Theta^{k-1}) + \left[\nabla f_i(\Theta^{k-1}) \right]^T d^k \leq 0 \quad (4.7b)$$

$$\nabla f_o(\Theta^{k-1}) + \nabla f_i(\Theta^{k-1})\pi_i^k + \nabla f_e(\Theta^{k-1})\pi_e^k = 0 \quad (4.7c)$$

$$\left(f_i(\Theta^{k-1}) + \left[\nabla f_i(\Theta^{k-1}) \right]^T d^k \right) \pi_i^k = 0 \quad (4.7d)$$

$$\pi_i^k \geq 0 \quad (4.7e)$$

Equations (4.7) will be satisfied at every successful TR-LP computation.

It should be noted that a smaller TR radius may cause constraint infeasibilities or may reduce the speed of convergence. Similarly, a larger TR radius will weaken the validity of linear models that represent nonlinear constraints in (4.1). Therefore Δ^k is modified at each iteration of the algorithm, depending on the improvement in optimality of the solution.

4.3.2 Feasibility Restoration

In practice, TR-LP(Θ^{k-1}, Δ^k) can be infeasible due to two reasons: 1) The constraint gradients $\left[\nabla f_e(\Theta^{k-1}) \right]^T$ can become degenerated at the point Θ^{k-1} , leading to infeasible linearized constraints. Then the system $\left[\nabla f_e(\Theta^{k-1}) \right]^T d = -f_e(\Theta^{k-1})$ simply has no solution. 2) If the TR is too small, the TR-LP (4.6) may be infeasible. In such circumstances, the linear constraints, $\left[\nabla f_e(\Theta^{k-1}) \right]^T d = -f_e(\Theta^{k-1})$, cannot be satisfied within the TR radius Δ^k of Θ^{k-1} .

Feasibility restoration (NLP-FR) searches for a feasible point by solving the following problem, so that the next TR-LP subproblem would be feasible.

$$\text{NLP-FR} \left\{ \begin{array}{l} \min_{\Theta, \gamma_i, \hat{\gamma}_e, \check{\gamma}_e} \gamma_i + \hat{\gamma}_e + \check{\gamma}_e \\ \text{s. t. } f_i(\Theta) - \gamma_i \leq 0 \\ f_e(\Theta) - \hat{\gamma}_e + \check{\gamma}_e = 0 \\ \Theta^{\min} \leq \Theta \leq \Theta^{\max} \\ \gamma_i, \hat{\gamma}_e, \check{\gamma}_e \geq 0 \end{array} \right. \quad (4.8)$$

where $\gamma_i \in \mathbb{R}_+^{2L_{ac}}$, and $\hat{\gamma}_e, \check{\gamma}_e \in \mathbb{R}_+^{2N_{ac}}$.

If the NLP-FR cannot find a solution, then the OPF problem (4.1) is declared as *infeasible*. Otherwise, Θ^k is a feasible point, which is used to compute the step size $d^k := \Theta^k - \Theta^{k-1}$.

4.3.3 Step Acceptance/Rejection Criterion

The following metrics are defined to accept or reject the current step size d^k and to update the TR radius Δ^k for the next TR-SLP iteration.

Let d^k be a solution of TR-LP(Θ^{k-1}, Δ^k). Then the predicted reduction in the objective is defined as

$$\Delta\phi_{\text{pre}}^k = \left[\nabla f_o(\Theta^{k-1}) \right]^T d^k. \quad (4.9)$$

To account for any constraint violations, as well as the actual value of the objective of the NLP (4.5), the following merit function is defined.

$$\phi(\Theta^k) = f_o(\Theta^k) + (\beta_e^k)^T |f_e(\Theta^k)| + (\beta_i^k)^T \max\{f_i(\Theta^k), 0\}, \quad (4.10)$$

where $\beta_e^k \in \mathbb{R}_+^{2N_{ac}} \setminus \{0\}$ and $\beta_i^k \in \mathbb{R}_+^{2L_{ac}} \setminus \{0\}$. These are derived in each iteration k based on (4.11a) and (4.11b) using dual variables π_e^k and π_i^k as follows.

$$\beta_e^k = \max \{ \beta_e^{k-1}, \pi_e^k \} \quad (4.11a)$$

$$\beta_i^k = \max \{ \beta_i^{k-1}, \pi_i^k \} \quad (4.11b)$$

$$\beta_{e,n}^0 = \frac{\| \nabla f_o(\Theta^0) \|_2}{\| \nabla f_{e,n}(\Theta^0) \|_2}, \quad \forall n \in \{1, \dots, 2N_{ac}\} \quad (4.11c)$$

$$\beta_{i,l}^0 = \frac{\| \nabla f_o(\Theta^0) \|_2}{\| \nabla f_{i,l}(\Theta^0) \|_2}, \quad \forall l \in \{1, \dots, 2L_{ac}\} \quad (4.11d)$$

Further, (4.11c) and (4.11d) are used to calculate the penalty factors for the first iteration. The actual reduction in the objective is

$$\Delta \phi_{act}^k = \phi(\Theta^k) - \phi(\Theta^{k-1}). \quad (4.12)$$

The ratio Ξ^k is then defined as

$$\Xi^k = \frac{\Delta \phi_{act}^k}{\Delta \phi_{pre}^k} \quad (4.13)$$

Then, the TR radius is determined by

$$\Delta^{k+1} = \begin{cases} \alpha_1 \Delta^k & \text{if } \Xi^k \leq 0 \\ \alpha_2 \Delta^k & \text{if } 0 < \Xi^k \leq 0.25 \\ \Delta^k & \text{if } 0.25 < \Xi^k \leq 0.75 \\ \min(2\Delta^k, \Delta^{\max}) & \text{if } 0.75 < \Xi^k \end{cases} \quad (4.14)$$

where $\alpha_1 \in (0, 1)$, $\alpha_2 \in (0, 1)$ and $\Delta^{\max}$ are constants. This is a heuristic approach, and values for these parameters should be determined on a case-by-case basis for the better performance of TR-SLP in the context of the OPF problem (4.1).

Remark 1: If $\Xi^k < 0$, then the iteration is considered as a failure. In that case, the new point Θ^k is rejected, and the TR radius Δ^k for the next iteration is reduced to α_1 times its present value, and the TR-LP(Θ^{k-1}, Δ^k) is solved again. If $\Xi^k \geq 0$,

then the new point $\Theta^k = \Theta^{k-1} + d^k$ is accepted and the algorithm proceeds to the next step with the updated TR radius Δ^{k+1} .

The TR-SLP algorithm is summarized below via the pseudo-code in Algorithm 4.1, utilizing the aforementioned TR-LP, NLP-FR, and step acceptance/rejection (i.e., StepQuality) components. It may be noted that the output (f_o^k, Θ^k) of the TR-SLP algorithm which satisfies the *convergence* conditions given in Section 4.3.4, is a local optimal solution of the OPF problem (4.1).

Algorithm 4.1: Trust-Region Sequential Linear Program

Input : $f_o, f_e, f_i, \Theta^0, \alpha_1, \alpha_2, \Delta^1, \Delta^{\max}, K$

Output: f_o^k, Θ^k

```

1  while convergence not satisfied do
    // solve trust-region linear program
2   $(f_o^k, d^k, \pi^k) \leftarrow \text{solve TR-LP}(\Theta^{k-1}, \Delta^k)$ 
3  if TR-LP $(\Theta^{k-1}, \Delta^k)$  is not feasible then
    // feasibility restoration phase
4  |  $d^k \leftarrow \text{solve NLP-FR}$ 
5  end
    // step quality determination phase
6   $(\Delta^{k+1}, \Xi^k) \leftarrow \text{StepQuality}(f_o, \nabla f_o, \Theta^{k-1}, d^k, \alpha_1, \alpha_2, \Delta^{\max})$ 
7  if  $\Xi^k < 0$  then
    // reject step
8  |  $\Theta^k \leftarrow \Theta^{k-1}$ 
9  else
    // accept step
10 |  $\Theta^k \leftarrow \Theta^{k-1} + d^k$ 
11 |  $k \leftarrow k + 1$ 
12 end
13 end

```

4.3.4 Discussion on Convergence

The Karush-Kuhn-Tucker conditions adopted from [104] are listed below for TR-SLP described in Algorithm 1. The TR-SLP terminates when the following conditions are satisfied.

- (a) $\|d^k\|_\infty \leq \epsilon_d$
- (b) $\|f_e(\Theta^k)\|_\infty \leq \epsilon, \max\{f_i(\Theta^k)\} \leq \epsilon$
- (c) $\max\{\|\nabla f_o(\Theta^k) + \nabla f_i(\Theta^k)\pi_i^k + \nabla f_e(\Theta^k)\pi_e^k\|_\infty, \|f_i(\Theta^k)\pi_i^k\|_\infty\} < \epsilon_\pi (1 + \|\pi^k\|_2)$
- (d) $\pi_i^k \geq 0$

where ϵ_d , ϵ , and ϵ_π are tolerances chosen for the step change, constraint satisfaction, and KKT condition satisfaction respectively. Condition (a) implies that the step size has reached the user-specified accuracy ϵ_d . Condition (b) implies that within the TR radius, the original nonlinear constraints f_e and f_i in the NLP (4.5) are satisfied to a user-specified accuracy of ϵ . Conditions (c) and (d) provide a measure of the closeness of the computed solution to a point satisfying the first-order optimality conditions for the NLP problem (4.5).

To establish the convergence of TR-SLP (Algorithm 4.1), consider the following. TR-SLP solves a TR-based linear approximation, i.e., TR-LP (4.6), of the original NLP (4.5) at each iteration k . If TR-LP (4.6) is feasible, it will compute d^k in Step 2 of Algorithm 4.1. Subsequently, based on the previous iterate Θ^{k-1} and d^k , the actual-to-predicted cost ratio Ξ^k and the new TR radius Δ^{k+1} are determined in Step 6 of Algorithm 4.1. As the TR-SLP progresses, Δ^{k+1} shrinks based on Ξ^k , ensuring the tightness of the linear approximation TR-LP (4.6). This leads the successive iterations of Θ^k to converge to a local solution of NLP (4.5), satisfying the aforementioned conditions (a)-(d).

4.4 Numerical Experiments and Discussion

In this section, numerical results are reported with the proposed TR-SLP algorithm for the OPF problem (4.1).

4.4.1 Test Systems for Benchmarking TR-SLP

The TR-SLP is analyzed on a benchmark test suite consisting of the IEEE 14-bus, 30-bus, 57-bus, 118-bus and 300-bus systems, and Polish 2383wp, 2746wop and 3012wp systems (the number refers to the number of buses in the respective test case) available in [2]. The details such as the number of buses, lines, generators, and loads of the aforementioned test cases are reported in Table 4.1.

TABLE 4.1: Details of the Test Cases [2]

Test Case	# Buses	# Lines	# Generators	# Loads
IEEE 14	14	20	5	11
IEEE 30	30	41	6	20
IEEE 57	57	80	7	42
IEEE 118	118	186	54	99
IEEE 300	300	411	69	196
Polish 2383wp	2383	2896	327	1822
Polish 2746wop	2746	3514	514	1996
Polish 3012wp	3012	3572	520	2269

These standard systems serve as good platforms to demonstrate the the functionality of new algorithms for solving OPF problems. Some of these test systems are modified versions of actual systems. For instance, the IEEE 14-bus, 30-bus, 57-bus, and 118-bus systems represent portions of the American Electric Power System (in the Midwestern US) as of February 1962, December 1961, early 1960's, and December 1962 respectively [2]. Further, Polish 2383wp represents the Polish 400, 220 and 110 kV networks during winter 1999-2000 peak conditions. It is part of the 7500+ bus European UCTE system. To decrease the number of buses, the tie lines to foreign networks were replaced by artificial load or generator buses (180-186). Multiple generators at a bus have been aggregated. Generators that are not centrally dispatchable in the Polish energy market are given a cost of zero (cf., MATPOWER manual [2]). Similarly, Polish 2746wop, and 3012wp systems represent the Polish 400, 220 and 110 kV networks during winter 2003-04 off-peak and winter 2007-08 evening peak conditions respectively. Therefore, these systems

represent real-world data relevant to OPF-based studies. As such, the use of the aforementioned standard test systems for performance benchmarking adequately demonstrates the potential real-world applicability of the TR-SLP algorithm proposed in this chapter.

4.4.2 Performance Comparison of TR-SLP with Existing Approaches

For performance comparison, the author mainly refers to [23], which is a recently published computational study on the AC-OPF problem for the aforementioned systems. It reports the OPF (4.1) results using general-purpose optimization solvers IPOPT [35] and KNITRO [36], and an SLP approach based on a penalty reformulation. In [23], two types of OPF problems are solved for each system, i.e., without line flow limits (baseline case), and with line flow limits (thermally constrained case). The line flow limits data used in the simulations are obtained from [23, Table II] for a fair comparison. Further, [23, Tables IV, V] reports the results obtained using the SLP, IPOPT, and KNITRO for the aforementioned IEEE and Polish systems. Those results have been obtained with constraint satisfaction up to 0.001 tolerance. In the following, those results are used in this study to compare the performance of the proposed TR-SLP in terms of optimality and computation time.

The convergence and optimality of the solution of SLP algorithms depend on the selected initial point (cf. [23]). In this study, two different initialization strategies, i.e., flat start, and decoupled warm start, are considered for TR-SLP to demonstrate the effect of the starting point on its performance. In the flat start, it is assumed unity voltage phasors and half-max outputs for all generation. The decoupled warm start is constructed with the solution obtained from the decoupled OPF problem combined with unit voltage magnitudes and half-max reactive power generation. Three case studies are conducted to showcase the performance of the proposed TR-SLP. Firstly, the computational time of the TR-SLP with the

aforementioned test cases is compared against that of KNITRO, IPOPT, and SLP. Secondly, the optimality of the solution obtained using the TR-SLP is compared to that of the same test cases against the KNITRO solver executed in a multi-start mode (henceforth referred to as KNITRO-MS). It should be noted that KNITRO executed in a multi-start mode results in improved local optimal solutions [36].

The TR-SLP algorithm is implemented in MATLAB and the optimization problems are formulated based on the MATPOWER library [2]. The LP subproblems are solved using CPLEX 12.6 [105] and the feasibility restoration subproblems using the MATLAB *fmincon* solver with the *interior-point method*. All experiments are carried out on a desktop PC with an Intel®Core i7-5500U four-core CPU processor running at 2.40GHz with 8GB RAM. The results reported in the following are based on the parameters chosen as follows: $\alpha_1 = 0.1$, $\alpha_2 = 0.25$, $\Delta^{(0)} = 0.4$, $\Delta^{\max} = 2$, $K = 30$, $\epsilon_d = 0.1$, and $\epsilon = \epsilon_\pi = 0.01$. Note that in all the numerical studies, the original nonlinear constraints (4.1b)–(4.1g) are satisfied up to the same accuracy employed by [23], i.e., 0.001 or lesser.

4.4.2.1 Case Study 1: Computation Time Comparison

In this study, TR-SLP is executed with the flat start strategy for OPF problem (4.1) and is compared in Table 4.2 with different solution approaches for several test cases. It should be noted that in [23], four different initialization strategies, i.e., flat start, decoupled warm start, AC warm start, and uniform cold start are used and the best solver time recorded for each test case out of all the starting points is reported (see Table 4.2; KNITRO, IPOPT, and SLP).

It can be observed that for all the IEEE systems, the CPU times of KNITRO and IPOPT are almost the same. Comparatively, in the Polish systems, KNITRO is noted to be slower. SLP is found to be the slowest for all the IEEE systems. However, its performance is observed to be improved for the Polish systems with respect to KNITRO and IPOPT. It is observed that TR-SLP is the fastest among all solution approaches in many test cases (except IEEE 300 and Polish 2383 thermally

TABLE 4.2: Computation Time Comparison among All Solvers/Algorithms. TR-SLP is Executed under the Flat Start Strategy.

Test Case	Computation Time (s)							
	Baseline Case				Thermally Constrained Case			
	KNITRO	IPOPT	SLP	TR-SLP	KNITRO	IPOPT	SLP	TR-SLP
IEEE 14	0.12	0.16	0.24	0.06	0.12	0.19	0.19	0.1
IEEE 30	0.19	0.2	0.84	0.07	0.19	0.19	0.58	0.08
IEEE 57	0.32	0.38	0.76	0.14	0.32	0.35	0.79	0.17
IEEE 118	0.84	1.18	2.53	0.94	1.23	1.18	2.86	1.01
IEEE 300	2.27	2.63	4.88	2.17	1.96	2.22	5.23	3.61
Polish 2383	38.65	88.47	37.05	15.94	43.63	26.32	36.42	28.09
Polish 2746	42.43	20.67	51.71	8.97	82.02	39.66	43.78	12.85
Polish 3012	96.51	33.15	49.73	16.40	99.53	35.56	46.34	20.07

constrained cases, where KNITRO and IPOPT are slightly faster). The TR-SLP reports comparatively the best CPU time for the largest Polish 3012 system, approximately 6, 2, and 3 times faster than KNITRO, IPOPT, and SLP, respectively, for the baseline case and approximately 5, 1.5, and 2 times faster than KNITRO, IPOPT, and SLP, respectively, for the thermally constrained case.

FIGURE 4.1: The computational scalability comparison among all solvers/algorithms for various IEEE (118, 300) and Polish (2383, 2746, 3012) systems. TR-SLP is simulated under the flat start strategy.

Figure 4.1 reports the computation time growth among all solution approaches for both the baseline and thermally constrained cases. It can be seen that both SLP and TR-SLP result in an almost linear increase in time against the test case size. This demonstrates the better scalability of these LP-based approaches to large-size optimization problems.

4.4.2.2 Case Study 2: Relative Optimality Comparison

In this study, TR-SLP is executed with the flat start strategy as well as with the decoupled warm start strategy for the OPF problem (4.1). As pointed out in [23, Section V], multi-start mode increases the probability of finding better local

TABLE 4.4: Relative Efficiency in the Optimality of TR-SLP Solution in Comparison to KNITRO-MS. TR-SLP is Executed under the Decoupled Warm Start Strategy.

Test Case	Baseline Case		Efficiency	Thermally Constrained Case		Efficiency
	Optimality (\$/h)			Optimality (\$/h)		
	KNITRO-MS	TR-SLP		KNITRO-MS	TR-SLP	
IEEE 14	8,091	8,088	0.03%	9,294	9,270	0.26%
IEEE 30	575	575	0%	582	583	−0.51%
IEEE 57	41,817	41,769	0.15%	41,978	41,961	0.04%
IEEE 118	129,903	129,895	0%	135,189	135,017	0.13%
IEEE 300	720,149	720,414	−0.03%	726,794	726,192	0.08%
Polish 2383	1,858,447	1,857,929	0.03%	1,863,627	1,862,377	0.07%
Polish 2746	1,185,115	1,208,277	−1.91%	1,185,507	1,208,281	−1.80%
Polish 3012	2,581,020	2,581,810	−0.03%	2,597,386	2,583,008	0.6%

solutions for KNITRO-MS. Hence, it is assumed that KNITRO-MS solutions as one of the best-known set of solutions for benchmarking. Therefore, the optimality of the solution obtained by the TR-SLP approach is compared to the KNITRO-MS solution. For Table 4.3 and Table 4.4, the following performance metric is defined:

$$\text{Efficiency} = \frac{\text{KNITRO-MS} - \text{TR-SLP}}{\text{KNITRO-MS}} \times 100\% \quad (4.15)$$

where “Efficiency” indicates the relative improvement (if Efficiency is positive) or deterioration (if Efficiency is negative) in optimality of the solution obtained by TR-SLP with respect to the best known KNITRO-MS solution.

Table 4.3 reports the optimal solution obtained using KNITRO-MS and TR-SLP (with the flat start strategy) for the baseline and thermally constrained cases. In both cases, it can be observed that for half of the IEEE and Polish systems, TR-SLP results in slightly improved optimality compared to the KNITRO-MS (ranging 0.03% to 0.6%). The performance of TR-SLP for the Polish 2746 system is observed to be slightly suboptimal for both the baseline and thermally constrained cases. However, KNITRO-MS is computationally slower than TR-SLP due to the multi-start feature and hence, represents a trade-off for using multi-start in practical OPF applications.

Table 4.4 reports the optimal solution obtained using KNITRO-MS and TR-SLP with the decoupled warm start strategy for the baseline and thermally constrained cases. The convergence and optimality of the solution of TR-SLP depend upon the initial point. As such, it can be observed that TR-SLP with the decoupled warm start strategy locates slightly different optimal values compared to those of the flat start strategy. Hence, to compare the relative effectiveness between the flat start and decoupled warm start strategies, the number of iterations taken under each strategy to converge to the final solution are compared. Figure 4.2 depicts the TR-SLP iterations of the IEEE (118, 300) and Polish (2383, 2746, 3012) systems for both the baseline and thermally constrained cases. It is observed that the decoupled warm start strategy speeds up the convergence compared to the flat start strategy,

except for the Polish 2383 system. However, both strategies converge within the maximum number of iterations set for TR-SLP, i.e., $K = 30$.

FIGURE 4.2: Relative comparison in iterations for the convergence of TR-SLP under flat start and decoupled warm start strategies for various IEEE (118, 300) and Polish (2383, 2746, 3012) systems.

4.4.2.3 Performance Comparison with MATPOWER Solver Package

MATPOWER [2] is a well-known open-access power flow and OPF solver package which interfaces many nonlinear solvers. The default solver in MATPOWER for solving the OPF problem is an interior-point method based solver (referred to as *mips*). Table 4.5 shows the performance comparison between the TR-SLP approach and the MATPOWER solver while solving OPF problems of varying sizes. It is observed that MATPOWER is faster than TR-SLP for all the test cases. The author notes that the computation times significantly depend on the implementation of the algorithm. However, the optimality of the solution obtained with the TR-SLP approach is better than that obtained with MATPOWER for large-scale Polish test systems. The slower computation times of the TR-SLP approach and other sequential optimization approaches can possibly be attributed to their iterative nature. Nevertheless, when compared with nonlinear solvers such as MATPOWER's interior-point solver, the applicability of sequential convex optimization approaches

can be extended to find feasible paths from an initial set-point to a better optimal set-point, cf. [106]. Consequently, such methods have the potential to compute OPF solutions which are robust against plausible uncertain power injections by RESs.

TABLE 4.5: Performance Comparison between the Best Known Solution of TR-SLP with MATPOWER Interior Point Solver [2]

Test Case	MATPOWER [2]		TR-SLP
	Time (s)	Optimality (\$/h)	Optimality (\$/h)
IEEE 14	0.04	9,271	9,270
IEEE 30	0.06	582	583
IEEE 57	0.07	41,907	41,961
IEEE 118	0.31	134,992	135,017
IEEE 300	0.51	726,170	726,192
Polish 2383	9.03	1,868,512	1,862,370
Polish 2746	8.77	1,208,280	1,208,278
Polish 3012	13.68	2,591,707	2,583,008

4.5 Summary

This chapter discussed the polar formulation of the classical AC-OPF. The SLP-based approach is proposed to solve it. The linear models formulated in the SLP approach are valid only at the vicinity of the linearization point. Therefore, a trust-region was introduced that bounds variations in the decision variables to tighten the SLP approximation. This ensured the convergence of TR-SLP for the OPF problem. In addition, the feasibility restoration phase was introduced to find a feasible point when the SLP approximation is infeasible. The results illustrated that the proposed TR-SLP outperforms KNITRO, IPOPT, and a recently reported SLP method based on penalty reformulations [23] in terms of computation time. Based on IEEE and Polish test systems, the optimality of TR-SLP solution is compared

with KNITRO-MS, wherein KNITRO-MS has the potential to find better local optima due to the utilization of multiple starting points. It was observed that TR-SLP converges to better solutions than KNITRO-MS for some of the benchmarking test systems. The results can be further improved with an investigation on selecting better starting points for the proposed TR-SLP.

Chapter 5

Network-Constrained Robust Unit Commitment: Formulation and Decomposition

5.1 Introduction

This chapter develops a day-ahead robust generation scheduling framework for power systems aiming to address the limitations of the existing approaches. In particular, the author’s work in [6] is extended to nonanticipative day-ahead power dispatch and reserve scheduling under wind power uncertainty. The proposed RO framework incorporates accurate modeling of HTGs while offering a scalable procedure to support the application to large-scale power systems. In brief, the main contributions of this chapter are summarized below.

In comparison to the state-of-the-art RO frameworks in the literature, for instance [67, 68], this chapter develops a novel two-stage scenario-based nonanticipative RO framework for the day-ahead robust UC of HTGs considering nonlinear AC and DC NCs under wind power uncertainties. The Benders cuts (BCs) are customized to efficiently rectify the generator-wise hourly reserve allocations

considering the actual impact on nonlinear NCs. In addition, the proposed RO framework enables a complete decomposition of the day-ahead power dispatch and reserve scheduling problem supporting the utilization of parallel computing platforms which improves its scalability to large-scale power systems. Furthermore, SPs are formulated supporting the utilization of standard OPF solver packages.

To this end, this chapter is organized as follows. Uncertainties of wind power injections are characterized in Section 5.2. The nonanticipative day-ahead optimal power dispatch and reserve management problem is formulated in Section 5.3. The two-stage iterative algorithm based on BD is presented in Section 5.4. The chapter is summarized in Section 5.5.

5.2 Wind Power Uncertainty

5.2.1 Uncertainty Set

In this chapter, the uncertain wind power injections are assumed to be varying within a polyhedron. Such polyhedral uncertainty sets are widely used in the literature [67, 72, 73]. This polyhedron is defined by the deterministic confidence bounds for each uncertain variable over the time horizon which can be computed based on historical data using statistical inference techniques [76].

In general, the day-ahead optimal planning involves decisions over a predefined time horizon $\mathcal{T} = \{1, \dots, T\}$. Let Ω be the uncertainty set having the following form (Cartesian product):

$$\Omega = \Omega^1 \times \Omega^2 \times \dots \times \Omega^T \tag{5.1}$$

Then, Ω^t is an $|\mathcal{F}|$ -dimensional polyhedral uncertainty set for time period $t \in \mathcal{T}$.

$$\Omega^t = \left\{ \left(P_{F,1}^{t,\omega}, \dots, P_{F,|\mathcal{F}|}^{t,\omega} \right)^T \middle| P_{F,f}^{t,\omega} = P_{F,f}^t - P_{F,f}^{t,\zeta^-} + \theta_f^t \left(P_{F,f}^{t,\zeta^+} + P_{F,f}^{t,\zeta^-} \right); \right. \\ \left. \sum_{f \in \mathcal{F}} \theta_f^t = \mathcal{I}^t, \theta_f^t \in [0, 1]; f \in \mathcal{F} \right\} \quad (5.2)$$

where Ω^t is a non-empty, convex and compact set associated with a budget constraint as in (5.2), in which $\mathcal{I}^t$ is a nonnegative real number or an integer.

5.2.2 Scenario Selection

The feasibility of the day-ahead schedule against the worst uncertain scenario can be achieved by maximizing the uncertainty induced by the variables $P_{F,f}^{t,\omega}$ over the polyhedral uncertainty set Ω^t , while minimizing the recourse actions of the generators to avoid network constraint violations [107]. Considering that the uncertainties are pertained to bus injections, the worst scenario is an extreme point of the uncertainty polyhedron which is a permutation of the confidence bounds of the uncertainty set [108, Sec. 2.4]. In the case of scenario-based RO methods, the robustness and conservatism of the day-ahead optimal schedule depend on the selected set of scenarios. However, exploring all the extreme points is computationally expensive or even intractable as their number rises exponentially with the dimension of the uncertainty set [78, 79, 107]. For instance if $\mathcal{I}^t$ equals $|\mathcal{F}|$ for all $t \in \mathcal{T}$ then the number extreme points of the polyhedral uncertainty set Ω is $2^{|\mathcal{F}|T}$.

The TOAT utilizes a unique set of *orthogonal arrays* (OAs) [109, 110] which select a fraction of the full set of extreme points of the uncertainty polyhedron [79–81, 111]. Further details on modeling uncertain scenarios via TOAT is presented in Appendix A. Interesting readers may refer to [111] for an extensive review on the application of the TOAT in power system uncertainty characterization. However, the RO framework developed in this work is not limited to the TOAT. Any other scenario-based characterization of uncertain realizations such as in [72, 78] can also be utilized within the proposed RO framework. To illustrate how the scenarios are

selected in TOAT, consider a three-wind-farm case. Theoretically, the polyhedral set Ω^t during time period t constitutes 8 extreme points. Table 5.1 depicts the OA L_42^3 specifying only four scenarios that involve three two-level random factors. Therein, random factors are the wind farms in the system and levels denote the boundary values of uncertain variables, where -1 denotes the lower level $P_{F,f}^t - P_{F,f}^{t,\zeta^-}$ and $+1$ denotes the upper level $P_{F,f}^t + P_{F,f}^{t,\zeta^+}$ of the confidence interval of wind power generation at wind farm $f \in \mathcal{F}$ as defined in (5.2).

TABLE 5.1: Orthogonal Array L_42^3

Scenario ω	WF 1	WF 2	WF 3
1	-1	-1	-1
2	-1	$+1$	$+1$
3	$+1$	-1	$+1$
4	$+1$	$+1$	-1

The example in [72, Sec. II] discusses the importance of selecting scenarios which represent the whole space of the uncertainty polyhedron. The TOAT combines the least possible scenarios which are *uniformly distributed* over the uncertainty space and, thus, has the potential to represent plausible uncertain scenarios [81, 111]. Each row in Table 5.1 describes an extreme point of the uncertainty polyhedron for each time interval. In the representation of Table 5.1, four extreme points are neglected, assuming that their effect is reflected in the combination of selected extreme points/scenarios [111]. In general, wind power outputs are assumed to have an asymmetrical probability distribution. In this study, it is assumed that the probability of the confidence lower bounds (i.e., wind power shortfall) is higher compared to the confidence upper bounds [111]. Further, that requires an increase of the power dispatch or sometimes commitment of new generation resources to fulfill the supply shortfall, which results in increased cost operation. Due to this, the selected scenarios in Table 5.1 are biased towards the lower bounds of the respective confidence intervals, aiming to capture the worst possible uncertain scenarios.

Let $\Upsilon = \{0, 1, \dots, \mathcal{W}\}$.

$$P_{F,f}^{t,0} = P_{F,f}^t \quad (5.3a)$$

$$P_{F,f}^{t,\omega} = \left\{ P_{F,f}^t - P_{F,f}^{t,\zeta^-}, P_{F,f}^t + P_{F,f}^{t,\zeta^+} \right\}; \forall \omega \in \Upsilon \setminus \{0\} \quad (5.3b)$$

where $\omega = 0$ represents the base scenario at which wind power outputs are assumed to take the forecasted values. All other uncertain scenarios constitute extreme points of the uncertainty polyhedron related to wind power variability.

5.3 Robust Day-Ahead Power Dispatch and Reserve Management with Network Constraints

The day-ahead power dispatch and reserve management problem involves the preparation of the system to operate with a minimum generation cost schedule while assuring the operational feasibility for worst possible real-time operating conditions. However, scheduling the generators while minimizing the cost associated with the worst scenario is conservative in practice. The proposed RO framework minimize the operating cost associated with the base scenario (which usually has the highest probability of occurrence) while allocating sufficient reserves to ensure the feasibility under the worst possible uncertainties. Thus, the resulting scheduling decisions are less conservative. The proposed RO framework computes the commitment schedule, base scenario power dispatch, and hourly reserve allocations for each generator in the system. These values provide a reference point and hourly operational margins for each generator, which can be used by the system operator to mitigate real-time variations (forecast errors). The exact power dispatch during each hour can be computed by evaluating an OPF problem under short-term forecasts considering the specified reserve bounds (by the RO framework) for each generator. In this section, the mathematical formulation of the proposed scenario-based RO framework is presented.

5.3.1 Modeling the Generator Ramp Constraints while Preserving Nonanticipativity in Power Dispatch

Numerous RO frameworks such as in [67, 68, 107], schedule the generation resources while enforcing ramp constraints of generators separately for each anticipated wind power realization over the time horizon. In practice, the system operators will strictly follow the generator commitment schedule, the base scenario power dispatch (as hourly reference points), and reserve allocations in the next day's operation. Consequently, the nonoccurrence of an uncertainty realization as anticipated might potentially lead to a reserve undeliverability due to the ramping infeasibilities for some generators, as the generation schedules initially depended on a specific sequence of wind power realizations with time. This condition is illustrated in Fig. 5.1. For instance, the power dispatch levels at point ② and point ③ in Fig. 5.1 are associated with two plausible uncertain realizations during time period t . As per the anticipated uncertain scenario vector at $\omega = \mathcal{W}$ in Fig. 5.1, ramp constraints are only imposed for the transition of power dispatch levels from point ① to point ②. Scheduling decisions based on such anticipation of complete information on uncertain scenarios does not necessarily ensure the feasibility of the transition from the power dispatch level at point ① to that at point ③. Therefore, the violation of *nonanticipativity* of day-ahead scheduling decisions might potentially lead to reserve undeliverability in next day's operation.

FIGURE 5.1: Power dispatch trajectories (dark dashed lines) of generator $g \in \mathcal{G}$ under each anticipated scenario vector.

In the proposed RO framework, hourly up and down reserve allocations of each generator are considered as scheduling variables bounded by the individual generator reserve capability limits. Accordingly, the proposed RO framework allocates sufficient reserves, with respect to the the hourly power requirements under all the uncertain scenarios. In addition, these hourly reserve allocations (requirements) are used to ensure compliance with the ramp constraints of all generators to guarantee a valid transition between all plausible power requirements associated with wind power realizations of two adjacent time periods. For instance from point ① to point ③ in Fig. 5.1. This allocation of reserves is nonanticipative as it does not depend on a specific sequence wind power realization over time, and the reserve deliverability in the real-time operation is ensured to be accomplished within the allocated reserve margins for each generator.

This is illustrated in Fig. 5.2, which depicts the allocated hourly reserve margins for generator g during two consecutive time periods.

FIGURE 5.2: Reserve allocations of generator $g \in \mathcal{G}$ for two consecutive time periods.

As illustrated in Fig. 5.2, the power dispatch and reserve allocation of generator g are constrained by (5.4a) and (5.4b), respecting the generation capability bounds $P_{G,g}^{\max}$ and $P_{G,g}^{\min}$ for the committed generators, i.e., for all generators $g \in \mathcal{G}$ with the commitment $u_{G,g}^t$ equal to 1. Otherwise, both power dispatch and reserve allocation are zero. Similarly, (5.4c) and (5.4d) limit the up and down reserve

capability respectively.

$$P_{G,g}^t + \hat{r}_{G,g}^t \leq u_{G,g}^t P_{G,g}^{\max}; \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \quad (5.4a)$$

$$P_{G,g}^t - \check{r}_{G,g}^t \geq u_{G,g}^t P_{G,g}^{\min}; \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \quad (5.4b)$$

$$0 \leq \hat{r}_{G,g}^t \leq u_{G,g}^t \hat{R}_{G,g}^\delta; \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \quad (5.4c)$$

$$0 \leq \check{r}_{G,g}^t \leq u_{G,g}^t \check{R}_{G,g}^\delta; \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \quad (5.4d)$$

The intertemporal startup, shutdown, and operating ramp constraints for generators are given by

$$\begin{aligned} P_{G,g}^t + \hat{r}_{G,g}^t - P_{G,g}^{t-1} + \check{r}_{G,g}^{t-1} &\leq u_{G,g}^{t-1} \hat{R}_{G,g}^\Delta \\ &\quad + (1 - u_{G,g}^{t-1}) P_{G,g}^{\min}; \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \setminus \{1\} \end{aligned} \quad (5.5a)$$

$$\begin{aligned} P_{G,g}^{t-1} + \hat{r}_{G,g}^{t-1} - P_{G,g}^t + \check{r}_{G,g}^t &\leq u_{G,g}^t \check{R}_{G,g}^\Delta \\ &\quad + (1 - u_{G,g}^t) P_{G,g}^{\min}; \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \setminus \{1\} \end{aligned} \quad (5.5b)$$

The startup and shutdown ramp rates are set to be $P_{G,g}^{\min}$ which means the generator $g \in \mathcal{G}$ must first reach its minimum power level after startup and before shutdown.

5.3.2 Robust Unit Commitment Formulation

The day-ahead optimal schedule includes the generator commitment plan, power dispatch values, and the reserve allocation for all the generators in the system given by

$$\Phi_g^t = \left\{ \hat{u}_{G,g}^t, \check{u}_{G,g}^t, u_{G,g}^t, P_{G,g}^t, \hat{r}_{G,g}^t, \check{r}_{G,g}^t \right\}; \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \quad (5.6)$$

The proposed robust UC problem (5.7) derives the day-ahead optimal schedule.

$$\min_{\Phi_{\mathcal{G}}^{\mathcal{T}}} \sum_{t \in \mathcal{T}} \left[(P_{\mathcal{G}}^t)^T \text{diag}(c_{\mathcal{Q}}) P_{\mathcal{G}}^t + c_{\mathcal{L}}^T P_{\mathcal{G}}^t + c_{\mathcal{F}}^T u_{\mathcal{G}}^t + c_{\mathcal{S}}^T \hat{u}_{\mathcal{G}}^t + c_{\mathcal{D}}^T \check{u}_{\mathcal{G}}^t \right] \quad (5.7a)$$

$$\text{s.t. } u_{\mathcal{G}}^t, \hat{u}_{\mathcal{G}}^t, \check{u}_{\mathcal{G}}^t \in \{0, 1\}^{|\mathcal{G}|}; \forall t \in \mathcal{T} \quad (5.7b)$$

$$\hat{u}_{\mathcal{G}}^t - \check{u}_{\mathcal{G}}^t = u_{\mathcal{G}}^t - u_{\mathcal{G}}^{t-1}; \forall t \in \mathcal{T} \setminus \{1\} \quad (5.7c)$$

$$\sum_{\tau=t}^{t+\hat{T}_{\mathcal{G},g}-1} u_{\mathcal{G},g}^{\tau} \geq \hat{T}_{\mathcal{G},g} \hat{u}_{\mathcal{G},g}^t; \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \quad (5.7d)$$

$$\sum_{\tau=t}^{t+\check{T}_{\mathcal{G},g}-1} [1 - u_{\mathcal{G},g}^{\tau}] \geq \check{T}_{\mathcal{G},g} \check{u}_{\mathcal{G},g}^t; \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \quad (5.7e)$$

$$\sum_{g \in \mathcal{G}} P_{\mathcal{G},g}^t \geq \sum_{n \in \mathcal{N}_{\text{ac}}} P_{\mathcal{D},n}^t - \sum_{f \in \mathcal{F}} P_{\mathcal{F},f}^t; \forall t \in \mathcal{T} \quad (5.7f)$$

$$\sum_{g \in \mathcal{G}} \hat{r}_{\mathcal{G},g}^t \geq \sum_{f \in \mathcal{F}} \left[P_{\mathcal{F},f}^t - \min_{\omega \in \Upsilon \setminus \{0\}} P_{\mathcal{F},f}^{t,\omega} \right]; \forall t \in \mathcal{T} \quad (5.7g)$$

$$\frac{1}{1 + \Lambda^t} \sum_{g \in \mathcal{G}} \check{r}_{\mathcal{G},g}^t \geq \sum_{f \in \mathcal{F}} \left[\max_{\omega \in \Upsilon \setminus \{0\}} P_{\mathcal{F},f}^{t,\omega} - P_{\mathcal{F},f}^t \right]; \forall t \in \mathcal{T} \quad (5.7h)$$

$$(5.4), (5.5) \quad (5.7i)$$

In (5.7), $\Phi_{\mathcal{G}}^{\mathcal{T}}$ denotes $|\mathcal{G}|T$ -tuple $\Phi_{\mathcal{G}}^{\mathcal{T}} = (\Phi_g^t)_{g \in \mathcal{G}}^{t \in \mathcal{T}}$. The objective function of the UC in (5.7a) is the minimization of the total generation cost over the specified time horizon $\mathcal{T}$, wherein $c_{\mathcal{Q}} \in \mathbb{R}_+^{|\mathcal{G}|}$, $c_{\mathcal{L}} \in \mathbb{R}_+^{|\mathcal{G}|}$, $c_{\mathcal{F}} \in \mathbb{R}_+^{|\mathcal{G}|}$, $c_{\mathcal{S}} \in \mathbb{R}_+^{|\mathcal{G}|}$, and $c_{\mathcal{D}} \in \mathbb{R}_+^{|\mathcal{G}|}$. Equation (5.7c) explains the relation of the binary variables $\hat{u}_{\mathcal{G}}^t$, $\check{u}_{\mathcal{G}}^t$ and $u_{\mathcal{G}}^t$. The minimum uptime $\hat{T}_{\mathcal{G}} \in \mathbb{R}_+^{|\mathcal{G}|}$ and downtime $\check{T}_{\mathcal{G}} \in \mathbb{R}_+^{|\mathcal{G}|}$ requirements of all the generation units are enforced with (5.7d) and (5.7e) respectively.

The minimum generation requirement for the base scenario operation is enforced by (5.7f) over the time horizon. The minimum required up and down reserve allocations are enforced by (5.7g) and (5.7h), considering the possible fluctuations in wind power generation. In fact, the constraints (5.7f)-(5.7h) are redundant as the power dispatch requirement that satisfies the losses and the reserve requirements are iteratively rectified through the feedback from the second stage SPs, which will be discussed later in Section 5.4.3. However, these are valid inequalities that support the decision making in the MP and, therewith the convergence of the

algorithm which will be presented later on. Finally, Λ^t is a constant parameter with the default value of 0 for all $t \in \mathcal{T}$. Its relevance to the solution process is explained in Section 6.2.3.

5.3.3 Network Constraint Formulation

The study considers a hybrid grid which includes both AC and DC transmission lines. Therein, AC and DC buses are coupled through bidirectional VSCs. The converter model used in this work is illustrated in Fig. 3.3. Therein, the active power is converted subject to $\hat{\eta}_z, \check{\eta}_z \in (0, 1)$, respectively. The system variables pertaining to the NCs are given below

$$\Theta^{t,\omega} = (P_G^{t,\omega}, Q_G^{t,\omega}, \hat{p}_C^{t,\omega}, \check{p}_C^{t,\omega}, q_C^{t,\omega}, \tilde{v}^{t,\omega}, \bar{v}^{t,\omega}); \forall t \in \mathcal{T}, \forall \omega \in \Upsilon \quad (5.8)$$

To this end, the nonlinear NCs of the HTG under scenario ω during time period t can be formulated as below. The feasibility of the day-ahead schedule $\Phi_{\mathcal{G}}^{\mathcal{T}}$ computed in (5.7) is verified against these nonlinear NCs under the plausible scenarios $\Upsilon = \{0, 1, \dots, \mathcal{W}\}$ (including the base scenario) over the time horizon $\mathcal{T}$.

$$-\check{r}_{G,g}^{t,\omega} \leq P_{G,g}^{t,\omega} - P_{G,g}^t \leq \hat{r}_{G,g}^{t,\omega}; \forall g \in \mathcal{G} \quad (5.9a)$$

$$u_{G,g}^t Q_{G,g}^{\min} \leq Q_{G,g}^{t,\omega} \leq u_{G,g}^t Q_{G,g}^{\max}; \forall g \in \mathcal{G} \quad (5.9b)$$

$$(3.42b) - (3.42n) \quad (5.9c)$$

In (5.9), all the NCs are consolidated in (5.9c). Their formulations from the first principles are presented in (3.42) in Chapter 3 of the thesis.

The proposed RO framework schedules hourly reserves for each generator independent of the scenarios. Accordingly, up and down reserve allocations, i.e., $\hat{r}_{G,g}^{t,\omega}$ and $\check{r}_{G,g}^{t,\omega}$, under scenario ω during time period t are assigned as below.

$$\left\{ \hat{r}_{G,g}^{t,\omega}, \check{r}_{G,g}^{t,\omega} \right\} = \begin{cases} \left\{ \hat{r}_{G,g}^t, \check{r}_{G,g}^t \right\}; & \omega \in \Upsilon \setminus \{0\} \\ \{0, 0\}; & \omega = 0 \end{cases} \quad (5.10)$$

The system variables under the base scenario and the corresponding NCs are also implemented separately (similar to other scenarios). This reduces the problem complexity and the computational burden of (5.7). Therein, no reserves are allocated in (5.10) under the base scenario (i.e., at $\omega = 0$) as the power dispatch $P_{G,g}^{t,0} (= P_{G,g}^t)$ must satisfy the corresponding NCs under the base scenario. In case of all other scenarios $\omega \in \Upsilon \setminus \{0\}$ (except the base scenario), the generators are allowed to change their power dispatch values in response to wind power variability, within the respective ranges of allocated reserves, i.e., $[P_{G,g}^t - \check{r}_{G,g}^t, P_{G,g}^t + \hat{r}_{G,g}^t]$ for generator g during time period t . Further, the reactive power dispatch of generator $g \in \mathcal{G}$ is constrained by the commitment $u_{G,g}^t$ and the reactive power capability bounds $Q_{G,g}^{\max}$ and $Q_{G,g}^{\min}$ as in (5.9b). Finally, $P_{\text{loss}}^{t,\omega}$ of the HTG can be computed as in (3.41), which includes the line losses in AC and DC subgrids, and the losses of VSCs.

5.4 Two-stage Iterative Algorithm based on Benders Decomposition

It follows from Section 5.3 that the network-constrained robust day-ahead power dispatch and reserve management problem is a combination of (5.7) and the NCs (5.9), $\forall t \in \mathcal{T}, \forall \omega \in \Upsilon$. The dimensionality of the combined problem is equal to $|\Theta^{t,\omega}|T(\mathcal{W} + 1)$ (where $|\Theta^{t,\omega}| = 2|\mathcal{G}| + 3|\mathcal{Z}| + N_{\text{ac}} + N_{\text{dc}}$) which is extensive for a direct solution approach. To this end, a two-stage iterative approach is proposed in this section based on BD. The latter is a widely used and effective framework for decomposing large-scale mixed-integer optimization problems with guaranteed ϵ -convergence in finite number of iterations, provided that given conditions, e.g. [112, Theorem 2.5], hold. Therein, the BD comprises an MP and several convex SPs which sequentially iterate in the solution process. BCs are constructed at the end of each iteration based on the results of the SPs. The MP is reformulated integrating these BCs, which aims in driving the solution towards convergence.

Therewith, the MP and SPs in the proposed RO framework are formulated as follows:

5.4.1 Master Problem Formulation

In the first stage, the day-ahead power dispatch and reserve management problem (5.7) is considered as the MP, which computes the day-ahead optimal schedule $\Phi_{\mathcal{G}}^{\mathcal{T}}$. Here, the MP formulation accounts for all the inter-temporal constraints and the corresponding variables, which include the generator commitment decisions, the base scenario power dispatch, and the reserve allocations. In the proposed MP formulation, hourly reserves of each generator are computed considering all plausible uncertain scenarios, while ensuring the ramping feasibility in between adjacent time intervals. Such a reserve allocation enables a full decomposition of NCs (5.9) per time period $t \in \mathcal{T}$ and scenario $\omega \in \Upsilon$. Thus, SPs can be executed in parallel as no information is required to pass in between them.

5.4.2 Subproblem Formulation

The operational feasibility of the day-ahead schedule $\Phi_{\mathcal{G}}^{\mathcal{T}}$ computed by the MP is verified considering the nonlinear NCs (5.9), under all scenarios $\omega \in \Upsilon$ over the optimization horizon $\mathcal{T}$ in the second stage. The formulation in (5.9) shows that the structure of the nonlinear NCs is consistent for all the scenarios $\omega \in \Upsilon$ over the time horizon $\mathcal{T}$. Thus, the feasibility verification is established via decomposition into individual SPs, each corresponding to nonlinear NCs (5.9) under a scenario $\omega \in \Upsilon$ during a time period $t \in \mathcal{T}$. Hereafter, the day-ahead scheduling decisions $\{u_{G,g}^t, P_{G,g}^t, \hat{r}_{G,g}^t, \check{r}_{G,g}^t\}$, $\forall t \in \mathcal{T}$ of (5.7) which interact with the nonlinear NCs (5.9) are referred to as *complicating variables*. In every iteration, these complicating variables are fixed in the coupling constraints (5.9a) and (5.9b) at the values computed by the MP. Therefore, a particular day-ahead schedule $\Phi_{\mathcal{G}}^{\mathcal{T}}$ might

cause infeasibilities in some of the SPs due to insufficient generator commitment, active power dispatch, or reserves.

To formulate the SP, let the convex piece-wise linear function $\psi : \mathbb{R}^3 \rightarrow \mathbb{R}$ comprise three linear segments with positive, zero and negative gradients, i.e.,

$$\psi(x, \bar{x}, \underline{x}) = \begin{cases} \beta(x - \bar{x}) & ; \bar{x} < x \\ 0 & ; \underline{x} \leq x \leq \bar{x} \\ -\beta(x - \underline{x}); & x < \underline{x} \end{cases} \quad (5.11)$$

where β is the penalty associated with bound violations. Therewith, the proposed SP under scenario $\omega \in \Upsilon$ during time period $t \in \mathcal{T}$ reads

$$\begin{aligned} \varphi^{t,\omega} = \min_{\Theta_{\mathcal{R}}^{t,\omega}} \sum_{g \in \mathcal{G}} \psi & \left(P_{G,g}^{t,\omega}, P_{G,g}^t + \hat{r}_{G,g}^{t,\omega}, P_{G,g}^t - \check{r}_{G,g}^{t,\omega} \right) \\ & + \sum_{g \in \mathcal{G}} \psi \left(Q_{G,g}^{t,\omega}, u_{G,g}^t Q_{G,g}^{\max}, u_{G,g}^t Q_{G,g}^{\min} \right) \end{aligned} \quad (5.12a)$$

$$\text{s.t. (3.42b)-(3.42n)} \quad (5.12b)$$

$$P_G^{t,\omega}, Q_G^{t,\omega} \in \mathbb{R}^{|\mathcal{G}|} \quad (5.12c)$$

The SP in (5.12) minimizes the aggregated amount of constraint violations $\varphi^{t,\omega}$ caused by the day-ahead schedule under scenario ω during time period t . The bounds on active and reactive power dispatch $P_{G,g}^{t,\omega}$ and $Q_{G,g}^{t,\omega}$ in (5.9a) and (5.9b) are relaxed using soft limits in the (convex) piece-wise linear objective function (5.12a), which assigns a per unit penalty cost β for the violations of their actual bounds. Here, the penalty cost must be higher than the maximum marginal cost of all the generators, i.e., $\beta > \max[2 \text{diag}(c_Q) P_G^{\max} + c_L]$, in order to discourage bound violations. In contrast to the related works [67, 68], this formulation of SPs requires neither constraint manipulation nor additional variables. The SPs are compact and scalable, hence potentially applicable for large-scale power systems. Further, the structure of the SP formulation is consistent with the standard OPF problem which supports the utilization of existing well established OPF packages such as

hynet [53], PowerModels [113], MATPOWER [2] which are flexible with user defined convex objective functions on active and reactive power dispatch variables. The dual information is also available to be extracted from the output.

5.4.3 Benders Cut Formulation

In BD, the dual domain of SPs is utilized to deduce the sensitivities of complicating variables on constraint violations. Therewith, BCs are constructed in every iteration and integrated into the MP for subsequent iterations. Here, the role of BCs is to iteratively rectify $\Phi_{\mathcal{G}}^T$ aiming to avoid violations in generator capability bounds. In the proposed SP formulation, the coupling constraints (5.9a) and (5.9b) are included in the objective function (5.12a). Hence, the following procedure is pursued to extract the corresponding dual variables of the coupling constraints which provide the sensitivities of reserve allocations and generator commitment.

Let $\pi_{P,n}^{t,\omega}$ be the dual variable of the active power balance constraint (3.42b) at AC bus $n \in \mathcal{N}_{ac}$ and $\hat{\pi}_{R,g}^{t,\omega}$ and $\check{\pi}_{R,g}^{t,\omega}$ be the dual variables of the up and down reserve capability bounds (5.9a) of generator $g \in \mathcal{G}$ under scenario ω during time period t respectively.

$$\begin{aligned} &\text{If } P_{G,g}^{t,\omega} > P_{G,g}^t + \hat{r}_{G,g}^t, \text{ then } \hat{\pi}_{R,g}^{t,\omega} = \pi_{P,\varrho(g)}^{t,\omega}, \check{\pi}_{R,g}^{t,\omega} = 0, \\ &\text{else if } P_{G,g}^{t,\omega} < P_{G,g}^t - \check{r}_{G,g}^t, \text{ then } \hat{\pi}_{R,g}^{t,\omega} = 0, \check{\pi}_{R,g}^{t,\omega} = -\pi_{P,\varrho(g)}^{t,\omega}, \\ &\text{otherwise } \hat{\pi}_{R,g}^{t,\omega} = \check{\pi}_{R,g}^{t,\omega} = 0. \end{aligned}$$

Similarly, let $\pi_{Q,n}^{t,\omega}$ be the dual variable of the reactive power balance constraint (3.42c) at AC bus $n \in \mathcal{N}_{ac}$ and $\hat{\pi}_{Q,g}^{t,\omega}$ and $\check{\pi}_{Q,g}^{t,\omega}$ be the dual variables of reactive power capability upper and lower bounds in (5.9b) of generator $g \in \mathcal{G}$ under scenario ω during time period t respectively.

$$\text{If } Q_{G,g}^{t,\omega} > u_{G,g}^t Q_{G,g}^{\max}, \text{ then } \hat{\pi}_{Q,g}^{t,\omega} = \pi_{Q,\varrho(g)}^{t,\omega}, \check{\pi}_{Q,g}^{t,\omega} = 0$$

else if $Q_{G,g}^{t,\omega} < u_{G,g}^t Q_{G,g}^{\min}$, then $\hat{\pi}_{Q,g}^{t,\omega} = 0$, $\check{\pi}_{Q,g}^{t,\omega} = -\pi_{Q,g}^{t,\omega}$
 otherwise $\hat{\pi}_{Q,g}^{t,\omega} = \check{\pi}_{Q,g}^{t,\omega} = 0$.

Then, the dual variables $\pi_{P,n}^t$, $\hat{\pi}_{R,g}^t$, $\check{\pi}_{R,g}^t$, $\hat{\pi}_{Q,g}^t$, and $\check{\pi}_{Q,g}^t$ and the penalization of constraint violations $\varphi^{t,\omega}$ for all scenarios $\omega \in \Upsilon$ during time period t are averaged as in (5.13a) to reduce the number of constraints imposed on the MP at every iteration [68]. Therein, a higher weight is assigned on the constraint violations of the base scenario (i.e., at $\omega = 0$) as the convergence of P_G^t supports the convergence of all other scenarios considered during time period t .

$$\varphi^t = \varphi^{t,0} + \frac{1}{\mathcal{W}} \sum_{\omega \in \Upsilon \setminus \{0\}} \varphi^{t,\omega}; \forall t \in \mathcal{T} \quad (5.13a)$$

$$\hat{\pi}_{Q,g}^t = \hat{\pi}_{Q,g}^{t,0} + \frac{1}{\mathcal{W}} \sum_{\omega \in \Upsilon \setminus \{0\}} \hat{\pi}_{Q,g}^{t,\omega}; \forall t \in \mathcal{T} \quad (5.13b)$$

$$\check{\pi}_{Q,g}^t = \check{\pi}_{Q,g}^{t,0} + \frac{1}{\mathcal{W}} \sum_{\omega \in \Upsilon \setminus \{0\}} \check{\pi}_{Q,g}^{t,\omega}; \forall t \in \mathcal{T} \quad (5.13c)$$

$$\pi_{P,n}^t = \pi_{P,n}^{t,0} + \frac{1}{\mathcal{W}} \sum_{\omega \in \Upsilon \setminus \{0\}} \pi_{P,n}^{t,\omega}; \forall t \in \mathcal{T} \quad (5.13d)$$

$$\hat{\pi}_{R,g}^t = \frac{1}{\mathcal{W}} \sum_{\omega \in \Upsilon \setminus \{0\}} \hat{\pi}_{R,g}^{t,\omega}; \forall t \in \mathcal{T} \quad (5.13e)$$

$$\check{\pi}_{R,g}^t = \frac{1}{\mathcal{W}} \sum_{\omega \in \Upsilon \setminus \{0\}} \check{\pi}_{R,g}^{t,\omega}; \forall t \in \mathcal{T} \quad (5.13f)$$

These values can be used as follows to construct the BCs during time period t for iteration $k+1$ of the algorithm.

$$\begin{aligned} & \varphi^{t,(j)} + \sum_{g \in \mathcal{G}} \left[\hat{\pi}_{Q,g}^{t,(j)} Q_{G,g}^{\max} - \check{\pi}_{Q,g}^{t,(j)} Q_{G,g}^{\min} \right] \left(u_{G,g}^t - u_{G,g}^{t,(j)} \right) \\ & + \sum_{g \in \mathcal{G}} \hat{\pi}_{R,g}^{t,(j)} \left(\hat{r}_{G,g}^t - \hat{r}_{G,g}^{t,(j)} \right) + \sum_{g \in \mathcal{G}} \check{\pi}_{R,g}^{t,(j)} \left(\check{r}_{G,g}^t - \check{r}_{G,g}^{t,(j)} \right) \\ & + \sum_{g \in \mathcal{G}} \pi_{P,g}^{t,(j)} \left(P_{G,g}^t - P_{G,g}^{t,(j)} \right) \leq 0; \forall t \in \mathcal{T}, \forall j \in \mathcal{K} = \{1, 2, \dots, k\} \end{aligned} \quad (5.14)$$

Therein, the complicating variables of the previous iterations and their corresponding sensitivities are termed with the superscript (j) , where $j \in \mathcal{K}$. These BCs are successively integrated into the MP, wherein (5.14) combines all the BCs which

would be integrated to the MP in iteration $k + 1$. Moreover, the sensitivity information provided by the BCs for each time period is used to rectify the commitment as well as the generation and reserve scheduling to eliminate violations in NCs.

To this end, the MP for iteration $k + 1$ can be reformulated as follows, including the BCs from previous iterations.

$$\min_{\Phi_G^T} \sum_{t \in \mathcal{T}} \left[(P_G^t)^T \text{diag}(c_Q) P_G^t + c_L^T P_G^t + c_F^T u_G^t + c_S^T \hat{u}_G^t + c_D^T \check{u}_G^t \right] \quad (5.15a)$$

$$\text{s.t. } (5.7b) - (5.7e), (5.4), (5.5) \quad (5.15b)$$

$$(5.7f) - (5.7h) \quad (5.15c)$$

$$(5.14) \quad (5.15d)$$

5.4.4 Algorithm for Robust Day-Ahead Optimal Power Dispatch and Reserve Management

The proposed algorithm developed for the robust day-ahead optimal power dispatch and reserve management can be summarized as shown in Algorithm 5.1. The MP (5.15) is solved in Step 3 for iteration $k + 1$. Steps 6 and 7 explain the execution of SPs and power loss computation for each time period t under uncertainty ω . In Step 12, the BCs are computed for time periods during which NCs are violated. The algorithm terminates if no NCs are violated by the day-ahead generation schedule Φ_G^T . Moreover, if infeasibility occurs in Steps 3 or 6, this indicates the infeasibility of the original day-ahead power dispatch and reserve management problem.

5.5 Summary

This chapter discussed the electrical system model and mathematical formulation of the robust unit commitment problem with nonlinear network constraints for

Algorithm 5.1: Day-ahead optimal power dispatch and reserve management

Initialize: $k = 0, \epsilon = 0.005, \Lambda^t = 0; \forall t \in \mathcal{T}$ and
 $P_{\text{loss}}^{t,\omega} = 0, \varphi^{t,\omega} = \infty; \forall \omega \in \Upsilon, \forall t \in \mathcal{T}.$

```

1 while  $\max_{t \in \mathcal{T}} \max_{\omega \in \Upsilon} \varphi^{t,\omega} \geq \beta \epsilon$  do
2    $k \leftarrow k + 1.$ 
3   Execute the MP (5.15) and extract optimal  $\Phi_{\mathcal{G}}^{\mathcal{T}}.$ 
4   for  $t \in \mathcal{T}$  do
5     for  $\omega \in \Upsilon$  do
6       Execute the SP (6.8) to compute  $\varphi^{t,\omega}.$ 
7       Calculate  $P_{\text{loss}}^{t,\omega}$  using (3.41).
8     end
9   end
10  for  $t \in \mathcal{T}$  do
11    if  $\max_{\omega \in \Upsilon} \varphi^{t,\omega} \geq \beta \epsilon$  then
12      Compute the corresponding BC as in (5.14).
13    end
14  end
15  Reformulate the MP (5.15) including the latest BCs.
16 end

```

hybrid AC/DC transmission grids under wind power uncertainties. Therein, confidence bounds are used to characterize the wind power uncertainty and orthogonal arrays are used to select representative scenarios. A two-stage iterative algorithm was proposed based on the Benders decomposition framework in which the day-ahead optimal schedule is computed in the first stage while the second stage verifies its viability against the nonlinear network constraints under the selected scenarios. Moreover, nonanticipativity of the day-ahead schedule is preserved in the proposed robust optimization framework, which ensures reserve deliverability under all plausible scenarios. In addition, the subproblems in the second stage of the proposed RO framework are formulated uniquely and effectively to utilize the standard solver packages. The performance evaluation of the proposed RO framework is presented in Chapter 6 of the thesis.

Chapter 6

Convex Relaxations for Network Constraints of Hybrid AC/DC Transmission Grids

6.1 Introduction

Chapter 5 developed an RO framework based on BD for nonanticipative reserve scheduling of HTGs. BD requires convex SPs, as the strong duality of SPs guarantees the validity of the BCs and thereby the convergence of the RO algorithm [112]. This chapter extends the RO framework proposed in Chapter 5 with convex relaxations for AC and DC NCs. Further, the performance of the proposed RO framework is extensively studied considering a recently proposed topology-preserving HTG architecture [1] based on the Polish 2383-bus system. Further, the efficiency, computational performance, and robustness of the solution of the proposed RO framework are compared with that of a state-of-the-art RO framework.

The rest of this chapter is organized as follows. Section 6.2 discusses the convex relaxation of nonlinear AC and DC NCs. Simulations results are shown in Section 6.3, followed by the summary in Section 6.4.

6.2 Convex Relaxations for Network Constraints

The SP in (5.12) is nonconvex. As discussed in Section 5.4.3, BCs are feedback of sensitivities of the complicating variables on constraint violations. Here, the sensitivities are extracted based on the dual variables of the SPs. Accordingly, if the SPs possess a *nonzero* duality gap, the coupling between sensitivities and dual variables, and hence the coupling between the MP and SPs are invalidated. This potentially leads to a slower convergence or even divergence of the solution process [112]. Moreover, the conditions in Theorem 2.5 of [112] states that there must exist a feasible day-ahead schedule $\Phi_{\mathcal{G}}^{\mathcal{T}}$ for the MP of the generalized BD framework, while the SPs must be convex and feasible for each solution of the MP in the solution process, for a finite ϵ -convergence of Algorithm 5.1. In the following, we develop the *convex relaxation* for both AC and DC NCs. The convex relaxation will establish a *zero duality gap* in primal-dual optimal solutions for SPs supporting convergence via valid BCs. Moreover, the convex relaxation also improves the computational properties of SPs in terms of scalability and global optimal solvability for large-scale power systems [1, 7, 39–41].

6.2.1 Second-Order Cone Relaxation

The SOCR has been widely applied on numerous AC-OPF problems, cf. [7, 39] and the references therein. In this section, the application of SOCR on nonlinear NCs of both AC and DC subgrids is discussed in order to guarantee the finite ϵ -convergence of RO Algorithm 5.1. In general, power systems are sparsely meshed. This renders the constraint matrices of problem (5.9) sparse. This sparsity pattern of the constraint matrices in (3.42) such as $\tilde{P}_n, \tilde{Q}_n$, etc., are defined by the graph of the AC subgrid and the DC subgrid, i.e., the element (n_1, n_2) in those matrices will only be nonzero if bus n_1 and bus n_2 of the system are connected by a line [7, 39].

To this end, let $X = [x_{i,j}] \in \mathbb{S}^N$. As AC line l connects bus $\hat{\varepsilon}(l)$ to bus $\check{\varepsilon}(l)$, let $\ddot{x}_l^{\Re} = \frac{\sqrt{2}}{2} (e_{\hat{\varepsilon}(l)}^T X e_{\check{\varepsilon}(l)} + e_{\check{\varepsilon}(l)}^T X e_{\hat{\varepsilon}(l)})$ and $\ddot{x}_l^{\Im} = \frac{\sqrt{2}}{2i} (e_{\hat{\varepsilon}(l)}^T X e_{\check{\varepsilon}(l)} - e_{\check{\varepsilon}(l)}^T X e_{\hat{\varepsilon}(l)})$. Therewith, the vectorization $\tilde{\xi} : \mathbb{S}^{N_{\text{ac}}} \rightarrow \mathbb{R}^{N_{\text{ac}}+2L_{\text{ac}}}$ of this partial Hermitian matrix X is defined as

$$\tilde{\xi}(X) := \left[e_1^T X e_1, \dots, e_{N_{\text{ac}}}^T X e_{N_{\text{ac}}}, \ddot{x}_1^{\Re}, \dots, \ddot{x}_{L_{\text{ac}}}^{\Re}, \ddot{x}_1^{\Im}, \dots, \ddot{x}_{L_{\text{ac}}}^{\Im} \right]^T \in \mathbb{R}^{N_{\text{ac}}+2L_{\text{ac}}}. \quad (6.1)$$

Let $\tilde{W}^{t,\omega}$ substitute $\tilde{v}^{t,\omega}(\tilde{v}^{t,\omega})^H$ and let $\tilde{w}^{t,\omega} = \tilde{\xi}(\tilde{W}^{t,\omega}) = [\tilde{w}_n^{t,\omega}] \in \mathbb{R}^{N_{\text{ac}}+2L_{\text{ac}}}$. Therein, $\tilde{w}^{t,\omega}$ comprises all the diagonal elements and all the *active* off-diagonal elements (i.e., off-diagonal elements which will be multiplied with nonzero entries in the constraint matrices) of $\tilde{W}^{t,\omega}$. Considering the vectorization (6.1) and the *trace* operator $\text{tr}(\cdot)$ of square matrices, the constraint matrices related to the AC subgrid in (5.9), for instance $(\tilde{v}^{t,\omega})^H \tilde{P}_n \tilde{v}^{t,\omega}$ can be linearized as follows.

$$(\tilde{v}^{t,\omega})^H \tilde{P}_n \tilde{v}^{t,\omega} = \text{tr} \left((\tilde{v}^{t,\omega})^H \tilde{P}_n \tilde{v}^{t,\omega} \right) = \text{tr} \left(\tilde{P}_n \tilde{v}^{t,\omega} (\tilde{v}^{t,\omega})^H \right) = \tilde{\xi}(\tilde{P}_n)^T \tilde{w}^{t,\omega} \quad (6.2)$$

Similarly, constraint matrices related to the DC subgrid in (5.9) can be linearized using the vectorization in (6.3). Let $X = [x_{i,j}] \in \mathbb{R}^{N_{\text{dc}} \times N_{\text{dc}}}$. As DC line l connects bus $\hat{\vartheta}(l)$ to bus $\check{\vartheta}(l)$ and let $\ddot{x}_l = \frac{\sqrt{2}}{2} (e_{\hat{\vartheta}(l)}^T X e_{\check{\vartheta}(l)} + e_{\check{\vartheta}(l)}^T X e_{\hat{\vartheta}(l)})$. Therewith, the vectorization $\bar{\xi} : \mathbb{R}^{N_{\text{dc}} \times N_{\text{dc}}} \rightarrow \mathbb{R}^{N_{\text{dc}}+L_{\text{dc}}}$ of this partial square matrix Y is defined as

$$\bar{\xi}(X) := \left[e_1^T X e_1, \dots, e_{N_{\text{dc}}}^T X e_{N_{\text{dc}}}, \ddot{x}_1, \dots, \ddot{x}_{L_{\text{dc}}} \right]^T \in \mathbb{R}_+^{N_{\text{dc}}+L_{\text{dc}}}. \quad (6.3)$$

Let $\bar{W}^{t,\omega}$ substitute $\bar{v}^{t,\omega}(\bar{v}^{t,\omega})^T$ and let $\bar{w}^{t,\omega} = \bar{\xi}(\bar{W}^{t,\omega}) = [\bar{w}_n^{t,\omega}] \in \mathbb{R}^{N_{\text{dc}}+L_{\text{dc}}}$. Thus, the quadratic terms of (5.9) in $\tilde{v}^{t,\omega}$ and $\bar{v}^{t,\omega}$ can be linearized in terms of $\tilde{w}^{t,\omega}$ and $\bar{w}^{t,\omega}$ respectively. However, the *exactness* (or the *equivalence*) of this convex relaxation depends on the accuracy of the optimal $\tilde{v}^{t,\omega\star}$ and $\bar{v}^{t,\omega\star}$ recovered from the optimal $\tilde{w}^{t,\omega\star}$ and $\bar{w}^{t,\omega\star}$ respectively. That demands both $\tilde{W}^{t,\omega\star}$ and $\bar{W}^{t,\omega\star}$ must permit positive semidefinite (PSD) rank 1 completion [39, 40]. The necessary condition for $\tilde{W}^{t,\omega\star}$ and $\bar{W}^{t,\omega\star}$ to be PSD is that all the active 2×2 principal submatrices of $\tilde{W}^{t,\omega\star}$ and $\bar{W}^{t,\omega\star}$ are PSD [7].

To this end, the active 2×2 principal submatrices of $\tilde{W}^{t,\omega}$ can be formulated as second-order cones which are given by

$$\tilde{S}_l(\tilde{w}^{t,\omega}) = \left\| \begin{array}{c} \frac{1}{2}(\tilde{w}_{\hat{\varepsilon}(l)}^{t,\omega} - \tilde{w}_{\check{\varepsilon}(l)}^{t,\omega}) \\ \tilde{w}_{N_{ac}+l}^{t,\omega} \\ \tilde{w}_{N_{ac}+L_{ac}+l}^{t,\omega} \end{array} \right\|_2 - \frac{1}{2}(\tilde{w}_{\hat{\varepsilon}(l)}^{t,\omega} + \tilde{w}_{\check{\varepsilon}(l)}^{t,\omega}); \forall l \in \mathcal{L}_{ac} \quad (6.4)$$

Similarly, the active 2×2 principal submatrices of $\bar{W}^{t,\omega}$ can be formulated as second-order cones which are given by

$$\bar{S}_l(\bar{w}^{t,\omega}) = \left\| \begin{array}{c} \frac{1}{2}(\bar{w}_{\hat{\vartheta}(l)}^{t,\omega} - \bar{w}_{\check{\vartheta}(l)}^{t,\omega}) \\ \bar{w}_{N_{dc}+l}^{t,\omega} \end{array} \right\|_2 - \frac{1}{2}(\bar{w}_{\hat{\vartheta}(l)}^{t,\omega} + \bar{w}_{\check{\vartheta}(l)}^{t,\omega}); \forall l \in \mathcal{L}_{dc} \quad (6.5)$$

Therewith, the second-order cone (SOC) relaxation of NCs (3.42b)-(3.42n), which are used in (5.9c), can be formulated as in (6.6).

$$\begin{aligned} (\Gamma_n^G)^T P_G^{t,\omega} - P_{D,n}^{t,\omega} + (\Gamma_n^F)^T P_F^{t,\omega} &= \tilde{\xi}(\tilde{P}_n)^T \tilde{w}^{t,\omega} \\ &\quad - (\check{\Gamma}_n^C)^T \check{p}_C^{t,\omega} + (\hat{\Gamma}_n^C)^T \hat{p}_C^{t,\omega}; \forall n \in \mathcal{N}_{ac} \end{aligned} \quad (6.6a)$$

$$\begin{aligned} (\Gamma_n^G)^T Q_G^{t,\omega} - Q_{D,n}^{t,\omega} + (\Gamma_n^F)^T Q_F^{t,\omega} &= \tilde{\xi}(\tilde{Q}_n)^T \tilde{w}^{t,\omega} \\ &\quad + (\check{\Gamma}_n^C)^T \check{q}_C^{t,\omega}; \forall n \in \mathcal{N}_{ac} \end{aligned} \quad (6.6b)$$

$$(\bar{\Gamma}_n^C)^T \hat{p}_C^{t,\omega} - (\bar{\Gamma}_n^C)^T \check{p}_C^{t,\omega} = \bar{\xi}(\bar{P}_n)^T \bar{w}^{t,\omega}; \forall n \in \mathcal{N}_{dc} \quad (6.6c)$$

$$(\tilde{v}_n^{\min})^2 \leq \tilde{\xi}(\tilde{M}_n)^T \tilde{w}^{t,\omega} \leq (\tilde{v}_n^{\max})^2; \forall n \in \mathcal{N}_{ac} \quad (6.6d)$$

$$(\bar{v}_n^{\min})^2 \leq \bar{\xi}(\bar{M}_n)^T \bar{w}^{t,\omega} \leq (\bar{v}_n^{\max})^2; \forall n \in \mathcal{N}_{dc} \quad (6.6e)$$

$$\tilde{\xi}(\hat{I}_{ac,l})^T \tilde{w}^{t,\omega} \leq (i_{ac,l}^{\max})^2; \forall l \in \mathcal{L}_{ac} \quad (6.6f)$$

$$\tilde{\xi}(\check{I}_{ac,l})^T \tilde{w}^{t,\omega} \leq (i_{ac,l}^{\max})^2; \forall l \in \mathcal{L}_{ac} \quad (6.6g)$$

$$\bar{\xi}(\hat{P}_{dc,l})^T \bar{w}^{t,\omega} \leq (p_{dc,l}^{\max})^2; \forall l \in \mathcal{L}_{dc} \quad (6.6h)$$

$$\bar{\xi}(\check{P}_{dc,l})^T \bar{w}^{t,\omega} \leq (p_{dc,l}^{\max})^2; \forall l \in \mathcal{L}_{dc} \quad (6.6i)$$

$$\tilde{\xi}(\hat{\mathcal{V}}_{ac,l})^T \tilde{w}^{t,\omega} \leq 0, \tilde{\xi}(\check{\mathcal{V}}_{ac,l})^T \tilde{w}^{t,\omega} \leq 0; \forall l \in \mathcal{L}_{ac} \quad (6.6j)$$

$$\bar{\xi}(\hat{\mathcal{V}}_{dc,l})^T \bar{w}^{t,\omega} \leq 0, \bar{\xi}(\check{\mathcal{V}}_{dc,l})^T \bar{w}^{t,\omega} \leq 0; \forall l \in \mathcal{L}_{dc} \quad (6.6k)$$

$$\tilde{\xi}(\hat{A}_l)^T \tilde{w}^{t,\omega} \leq 0, \tilde{\xi}(\hat{A}_l)^T \tilde{w}^{t,\omega} \leq 0, \tilde{\xi}(\check{A}_l)^T \tilde{w}^{t,\omega} \leq 0; \forall l \in \mathcal{L}_{ac} \quad (6.6l)$$

$$\tilde{\xi}(\tilde{M}_{\tilde{o}(z)})^T \tilde{w}^{t,\omega} - m_z \bar{\xi}(\bar{M}_{\bar{o}(z)})^T \bar{w}^{t,\omega} \leq 0; \forall z \in \mathcal{Z} \quad (6.6m)$$

$$\tilde{S}_l(\tilde{w}^{t,\omega}) \leq 0; \forall l \in \mathcal{L}_{ac} \quad (6.6n)$$

$$\bar{S}_l(\bar{w}^{t,\omega}) \leq 0; \forall l \in \mathcal{L}_{dc} \quad (6.6o)$$

where (6.6a)-(6.6m) are the linearized formulations of (3.42b)-(3.42n). The PSD constraints on the 2×2 principal submatrices of $\tilde{W}^{t,\omega}$ and $\bar{W}^{t,\omega}$ are enforced via the second-order cone constraints in (6.6n) and (6.6o) respectively. Accordingly, the equivalent system variables pertaining to the SOCR of the NCs are given by

$$\Theta_{\mathcal{R}}^{t,\omega} = (P_G^{t,\omega}, Q_G^{t,\omega}, \hat{p}_C^{t,\omega}, \check{p}_C^{t,\omega}, q_C^{t,\omega}, \tilde{w}^{t,\omega}, \bar{w}^{t,\omega}); \forall t \in \mathcal{T}, \forall \omega \in \Upsilon \quad (6.7)$$

To this end, the SOCR of the feasibility verification SP under scenario $\omega \in \Upsilon$ during time period $t \in \mathcal{T}$ can be formulated as in (6.8) following the structure in (5.12).

$$\begin{aligned} \varphi^{t,\omega} = \min_{\Theta_{\mathcal{R}}^{t,\omega}} \sum_{g \in \mathcal{G}} \psi & \left(P_{G,g}^{t,\omega}, P_{G,g}^t + \hat{r}_{G,g}^{t,\omega}, P_{G,g}^t - \check{r}_{G,g}^{t,\omega} \right) \\ & + \sum_{g \in \mathcal{G}} \psi \left(Q_{G,g}^{t,\omega}, u_{G,g}^t Q_{G,g}^{\max}, u_{G,g}^t Q_{G,g}^{\min} \right) \end{aligned} \quad (6.8a)$$

$$\text{s.t. (6.6a)-(6.6o)} \quad (6.8b)$$

$$P_G^{t,\omega}, Q_G^{t,\omega} \in \mathbb{R}^{|\mathcal{G}|} \quad (6.8c)$$

The resulting SOCR (6.8) of the feasibility verification SP (5.12) can be efficiently solved using standard convex optimization solvers like MOSEK [114].

6.2.2 Exactness and the Hybrid AC/DC Grid Architecture

The SOCR (6.8) of (5.12) is *exact* if and only if the partial matrices $\tilde{W}^{t,\omega^\star}$ and $\bar{W}^{t,\omega^\star}$ associated with the solutions $\tilde{v}^{t,\omega^\star}$ and $\bar{v}^{t,\omega^\star}$ of (6.8) permit PSD rank-1 completions respectively. For conventional transmission grids, this is typically not preserved [7]. The hybrid architecture proposed in [1], which comprises a radial AC subgrid with multi-terminal DC subgrids, supports the exactness of the SOCR under normal operating conditions unless the dual variables of power

injections combine to a point in a union of linear subspaces as proven in [7, Sec. VII]. Therefore, the hybrid architecture ensures the global optimal solvability of SPs (6.8) and the validity of the day-ahead optimal schedule $\Phi_{\mathcal{G}}^{\mathcal{T}}$.

It shall be noted that the optimal AC bus voltage vectors $\tilde{v}^{t,\omega^\star}$ associated with the optimal solutions $\tilde{w}^{t,\omega^\star}$ can be recovered using the method given in [115, Sec. III-B-3]. Therein, the voltage magnitudes are computed by the (positive) square root of the diagonal elements of $\tilde{W}^{t,\omega^\star}$, i.e., the first N_{ac} elements of $\tilde{w}^{t,\omega^\star}$. Voltage angle differences across AC lines are computed using the angles of (complex) off-diagonal elements of $\tilde{W}^{t,\omega^\star}$, wherein, the off-diagonal term relative to AC line l is $\tilde{w}_{N_{\text{ac}}+l}^{t,\omega^\star} + \mathbf{i}\tilde{w}_{N_{\text{ac}}+L_{\text{ac}}+l}^{t,\omega^\star}$. Then, by assuming the voltage angle of *slack* bus is known (usually 0), the voltage angles corresponding to $\tilde{v}^{t,\omega^\star}$ are computed by traversing the graph of AC subgrid [115, Sec. III-B-3]. The voltage vector of the DC subgrid $\bar{v}^{t,\omega^\star}$ is computed by the (positive) square root of the first N_{dc} elements of $\bar{w}^{t,\omega^\star}$.

Therefore, if the SOCR is inexact, the magnitudes of the off-diagonal elements are inconsistent. Thus, the accuracy of the exactness can be verified a posteriori via the reconstruction error which is given in (6.9a) relative to AC line l and in (6.9b) relative to DC line l respectively, cf. [7, Sec. VI-C].

$$\tilde{\kappa}_l(\tilde{v}^{t,\omega^\star}) = \left| \frac{\tilde{v}_{\hat{\varepsilon}(l)}^{t,\omega^\star} (\tilde{v}_{\check{\varepsilon}(l)}^{t,\omega^\star})^* - (\tilde{w}_{N_{\text{ac}}+l}^{t,\omega^\star} + \mathbf{i}\tilde{w}_{N_{\text{ac}}+L_{\text{ac}}+l}^{t,\omega^\star})}{\tilde{v}_{\hat{\varepsilon}(l)}^{t,\omega^\star} (\tilde{v}_{\check{\varepsilon}(l)}^{t,\omega^\star})^*} \right|; \forall l \in \mathcal{L}_{\text{ac}} \quad (6.9a)$$

$$\bar{\kappa}_l(\bar{v}^{t,\omega^\star}) = \left| \frac{\bar{v}_{\hat{\vartheta}(l)}^{t,\omega^\star} \cdot \bar{v}_{\check{\vartheta}(l)}^{t,\omega^\star} - \bar{w}_{N_{\text{dc}}+l}^{t,\omega^\star}}{\bar{v}_{\hat{\vartheta}(l)}^{t,\omega^\star} \cdot \bar{v}_{\check{\vartheta}(l)}^{t,\omega^\star}} \right|; \forall l \in \mathcal{L}_{\text{dc}} \quad (6.9b)$$

Therewith, the maximum and mean reconstruction errors of AC and DC bus voltages can be measured as follows. These reconstruction errors quantify the relaxation gap of the SOCR (6.8) in comparison to the original SP (5.12).

$$\tilde{\kappa}_{\max}(\tilde{v}^{t,\omega^\star}) = \max_{l \in \mathcal{L}_{\text{ac}}} \tilde{\kappa}_l(\tilde{v}^{t,\omega^\star}) \quad (6.10a)$$

$$\tilde{\kappa}_{\text{avg}}(\tilde{v}^{t,\omega^\star}) = \frac{1}{L_{\text{ac}}} \sum_{l \in \mathcal{L}_{\text{ac}}} \tilde{\kappa}_l(\tilde{v}^{t,\omega^\star}) \quad (6.10b)$$

$$\bar{\kappa}_{\max}(\bar{v}^{t,\omega^\star}) = \max_{l \in \mathcal{L}_{\text{dc}}} \bar{\kappa}_l(\bar{v}^{t,\omega^\star}) \quad (6.10c)$$

$$\bar{\kappa}_{\text{avg}}(\bar{v}^{t,\omega^\star}) = \frac{1}{L_{\text{dc}}} \sum_{l \in \mathcal{L}_{\text{dc}}} \bar{\kappa}_l(\bar{v}^{t,\omega^\star}) \quad (6.10d)$$

6.2.3 Interpretation and Handling of Inexactness

As discussed in Section 6.2.2, the exactness of the SOCR (6.8) is guaranteed for the hybrid AC/DC transmission grid (HTG) described in [7], if the dual variables of power injections do not form a pathological profile, which is very unlikely if the objective function (6.8a) has a positive gradient of the optimizer [7]. However, the objective function of the SP (6.8a) comprises three linear segments in which the second and third segments have zero and negative gradients respectively. If the reserve allocation is sufficient or in case of insufficient down reserves, the solution of the SP can be inexact. In the following, a convex auxiliary problem (AP) is defined as in (6.11) to verify the feasibility of the day-ahead schedule under scenario ω during time period t .

$$\min_{\Theta_{\mathcal{R}}^{t,\omega}} \left\{ \mathbf{1}^T P_G^{t,\omega} \mid (5.9a), (5.9b), (6.6) \right\} \quad (6.11)$$

The objective function in (6.11) minimizes the power loss subject to the coupling constraints (5.9a), (5.9b), and the SOCR of NCs (6.6) under scenario ω during time period t . The feasibility of (6.11) confirms the validity of the day-ahead generation schedule $\Phi_{\mathcal{G}}^{\mathcal{T}}$ computed by (5.7) for scenario ω during time period t , i.e., $\varphi^{t,\omega} = 0$ and no BC is required. Here, the computational complexity of both SPs and APs are similar.

6.2.4 Day-Ahead Robust Scheduling Algorithm Considering Second-Order Cone Relaxation for Network Constraints

Algorithm 6.1 is a modified version of Algorithm 5.1 which utilizes the SOCR of the nonlinear NCs. Therein, the AP is included to verify the exactness of the SP solution, and hence the day-ahead generation schedule $\Phi_{\mathcal{G}}^{\mathcal{T}}$ computed at iteration $k+1$ under the corresponding scenario $\omega \in \Upsilon$ during time period $t \in \mathcal{T}$. If the AP (6.11) is infeasible, Λ^t is adjusted for the particular time period $t \in \mathcal{T}$ as in Step 16 and the MP is re-executed to allocate more down reserves for the operation of that time period.

6.3 Simulation Results

6.3.1 Test System

The proposed day-ahead power dispatch and reserve optimization method is illustrated using the 2383-bus test case “case2383wp.m” which represents the Polish transmission grid during winter peak conditions [2]. The test case is adapted to support the case studies, as the demand increase of the original test case is limited to 5% due to the capacity restrictions of only two (out of 2896) lines, i.e., line 2239 and 2862. To this end, their individual line ratings are increased by 35% to enable simulations under higher demand. Further, parallel lines in the original test case are combined into a single line using equivalent parameters. To support the exactness of the SOCR [7, 39], all lossless lines in the original test case are imposed with a negligible series resistance of 10^{-5} p.u.

In the following, the upgrade strategy to an HTG in [7] is applied. In general, DC lines are suitable for long distance transmission [11]. Accordingly, a minimum spanning tree based on the series resistance is identified as the AC subgrid and the

Algorithm 6.1: Day-ahead robust optimal scheduling with an SOCR of NCs.

```

Initialize:  $k = 0, \epsilon = 0.005, \Lambda^t = 0; \forall t \in \mathcal{T}$  and
                $P_{\text{loss}}^{t,\omega} = 0, \varphi^{t,\omega} = \infty; \forall \omega \in \Upsilon, \forall t \in \mathcal{T}$ .
1  while  $\max_{t \in \mathcal{T}} \max_{\omega \in \Upsilon} \varphi^{t,\omega} \geq \beta \epsilon$  do
2       $k \leftarrow k + 1$ .
3      Execute the MP (5.15) and extract optimal  $\Phi_{\mathcal{G}}^{\mathcal{T}}$ .
4      for  $t \in \mathcal{T}$  do
5          for  $\omega \in \Upsilon$  do
6              Execute the SP (6.8) to compute  $\varphi^{t,\omega}$ .
7              Compute the voltage vectors  $\tilde{v}^{t,\omega*}$  and  $\bar{v}^{t,\omega*}$ .
8              if  $\tilde{\kappa}_{\max}(\tilde{v}^{t,\omega*}) > \epsilon_e$  and  $\bar{\kappa}_{\max}(\bar{v}^{t,\omega*}) > \epsilon_e$  then
9                  Execute the AP (6.11).
10                 if feasible then
11                      $\varphi^{t,\omega} = 0$ .
12                 else
13                     Adjust  $\Lambda^t \leftarrow \Lambda^t + 0.1$ . Return to Step 3.
14                 end
15             end
16             Calculate  $P_{\text{loss}}^{t,\omega}$  using (3.41).
17         end
18     end
19     if  $\max_{\omega \in \Upsilon} \varphi^{t,\omega} \geq \beta \epsilon$  then
20         Compute the BCs as in (5.14).
21     end
22 end
23 Reformulate the MP (5.15) including the latest BCs.
    
```

lines outside this subgraph are upgraded for DC operation. Therewith, 17.46% of the original transmission lines in the test case are converted to DC lines which form the multi-terminal DC subgrid for the study. The AC subgrid and DC subgrid are coupled at several buses through bidirectional VSCs as shown in Fig. 3.3. The active power capacity and the line resistance of the DC lines are set to the original line rating and the series resistance in the test case. The active power capability $p_C^{\max}$ of the VSCs are selected as the summation of the capacity of DC lines connected to it. The reactive power capability $q_C^{\max}$ of the VSCs are selected conservatively as 25% of the respective $p_C^{\max}$, cf. e.g. [11]. The conversion loss factors $\hat{\eta}_z$ and $\check{\eta}_z$ for both directions are selected as 1% which is reasonable as

in [11]. The resulting test system is referred to as the HTG which is compared with the original test case, referred to as the AC grid (ACG) in the case studies.

In addition to the generator data of the test case, the following parameters related to the UC formulation (5.7) are assumed: $P_{G,g}^{\min} = \max\{P_{G,g}^{\min}, 10 \text{ MW}\}$, $c_{F,g} = \$20/\text{h}$, $c_{S,g} = \$100$, $c_{D,g} = \$10$, $\hat{R}_{G,g}^{\Delta} = \check{R}_{G,g}^{\Delta} = 0.5(P_{G,g}^{\max} - P_{G,g}^{\min})$, $\hat{T}_{G,g} = 4 \text{ h}$, $\check{T}_{G,g} = 2 \text{ h}$ and $\hat{R}_{G,g}^{\delta} = \check{R}_{G,g}^{\delta} = 0.25(P_{G,g}^{\max} - P_{G,g}^{\min})$, for all the generators in the system. Further, for generators in which $c_{L,g}$ is 0 is adjusted to $\$30/\text{MWh}$. The day load profile which is used to scale the individual active power demand at each bus over the time horizon, is taken from [116, Table IV]. Similarly, the reactive power demand is also scaled assuming a constant power factor over the entire time horizon for all loads. Further, wind profiles of 60 wind farms from [117] are integrated in to the AC subgrid. The peak value of all wind profiles is adjusted to 100MW and the rest of the values are scaled proportionally¹. The variability of each wind farm in a particular scenario corresponds to the respective column in the selected OA. To this end, the first 60 columns of the OA $L_{64}2^{63}$ are adopted to generate 64 scenarios (except the base scenario) for each time period $t \in \mathcal{T}$ [111]. Altogether 1560 ($= 24 \times 65$) SPs are solved in each iteration. Each scenario represents information on either of the confidence bounds (as defined in (5.3b)) of the 60 uncertain variables of the respective time period t . Therein, $P_{F,f}^{t,\zeta-} = 0.4P_{F,f}^t$ and $P_{F,f}^{t,\zeta+} = 0.1P_{F,f}^t$, for all the wind farms in all case studies.

6.3.2 Numerical Simulations

The MP in (5.15) is a mixed-integer linear program which is solved using CPLEX. For the HTG, the SP in (5.12) is convexified using the SOCR discussed in Section 6.2. The convex SPs (6.8) and APs (6.11) are solved using the primal-dual interior-point solver MOSEK. For the ACG, SOCR (6.8) of the nonlinear NCs is inexact. Consequently, the NCs and hence (5.12) are approximated for the ACG using the well-known decoupled network representation cf. e.g. [73]. Therein, the

¹The wind power generation under the base scenario and the respective AC buses can be found in <http://dx.doi.org/10.13140/RG.2.2.23955.32809>

active power losses and reactive power flows are neglected while assuming a uniform voltage profile [73]. The resulting linear program is solved using CPLEX. Subsequently, AC power flow simulations are performed on the day-ahead optimal schedule for the ACG via MATPOWER [2] in order to verify the viability of the base scenario power dispatch at every hour. The algorithms were programmed using MATLAB and all simulations were performed on a notebook PC with an Intel®Core i7-7500U two-core CPU processor running at 2.70GHz with 16GB of RAM. The base value used for apparent power in all computations is 100MVA.

6.3.3 Case Study 1: Computational Performance

In this case study the optimal cost is computed and the validity of the results are examined for both the ACG and HTG. Fig. 6.1 depicts the convergence trends at the nominal demand. Therein, Fig. 6.1(a) illustrates the increment of the objective value of the MP (5.15) and the reduction in constraint violations for the ACG over the iterations of Algorithm 5.1. Similarly, Fig. 6.1(b) illustrates the same for the HTG over the iterations of Algorithm 6.1. The algorithms successfully drive the day-ahead schedule towards negligible violations in coupling constraints which are the real power capability bounds for the ACG, and both real and reactive power capability bounds for the HTG respectively. In comparison, the congestion issues in the ACG demands more BCs to confine the solution space of the MP. Thus, it takes more iterations to find the optimal solution for the ACG. Conversely, the DC lines in the HTG offer more flexibility in power flow routing which expands the feasibility space of the combined UC with nonlinear NCs, thereby improving the accessibility of economic generation facilities. Therefore, the number of iterations taken for the HTG is less, as its inherent flexibility due to controllable DC lines induces less constraint violations for a particular day-ahead schedule.

Fig. 6.2 depicts the convergence trends of the hourly power loss under the base scenario for the HTG. Therein, comparatively higher ripples can be seen at the peak period (i.e., during hours 18-19), while power losses stabilize after a few

FIGURE 6.1: Convergence trajectories of the objective value and constraint violations for (a) the ACG and (b) the HTG.

iterations. To further illustrate the convergence of the day-ahead schedule during peak and off-peak periods, Fig. 6.3 shows the evolution of the total power dispatch and the total number of online units during hours 4 and 18 for the ACG and the HTG. It is observed that the variations in the aggregated power dispatch of the ACG is almost negligible due to the negligence of power losses in the decoupled NCs formulation. However, significant variability can be seen in the commitment decisions. Conversely, for the HTG, the commitment schedule converges faster, while slight changes can be seen in the aggregated power dispatch due to variations in the power losses over the iterations.

Fig. 6.4 illustrates the computation time for solving the MP and SPs over the iterations for the ACG and the HTG. Therein, one MP and 1560 SPs are solved in each iteration. The proposed nonanticipative RO framework supports parallel execution of SPs within the hourly reserve margins prespecified by the MP solution. Therefore, the total computational time depends on the available hardware setup. Due to this, Fig. 6.4 shows the maximum and average computation times for SPs in every iteration. It is observed that the MP computations are slightly more expensive in the ACG, especially before iteration 20 due to the constraint

FIGURE 6.2: Convergence trajectories of the power loss over the optimization horizon under the base scenario.

FIGURE 6.3: Convergence trajectories of the aggregated power dispatch and the total number of online units for (a) the ACG and (b) the HTG during hour 4 (the graph on the left side) and hour 18 (the graph on the right side).

violations present in SPs. In comparison, fewer constraint violations are induced by the HTG for a particular day-ahead schedule which imposes less burden on the subsequent iteration of the MP. As a result, the MP computations are comparatively faster. However, the SP computations are significantly faster for the ACG, due to the simplified network model with reduced dimensionality. Conversely, the HTG considers an accurate network model while applying SOCR for tractability, which results in reasonable computation time for SPs.

Further, it should be noted that hourly AC power flow simulations performed on the

FIGURE 6.4: Computational time of the MP and SPs over the iterations for (a) the ACG and (b) the HTG.

day-ahead schedule of the ACG reports significant number of violations in voltage limits, transmission line flow limits, and reactive power capabilities of generators during 22 hours (out of the 24 hours), thereby its applicability is invalidated. Moreover, the validity of the optimal day-ahead schedule computed for the HTG requires the exactness of the SOCR for all the SP computations. Fig. 6.5 shows the corresponding reconstruction errors $\tilde{\kappa}_{\max}(\tilde{v}^{t,\omega\star})$ and $\bar{\kappa}_{\max}(\bar{v}^{t,\omega\star})$ of the voltage vectors of the AC and DC subgrids respectively. In both graphs, black lines show the maximum error and gray lines show the average error for all the SP calculations. It can be observed that the errors are negligible for both the AC and DC subgrids, thereby validating the practicality of the optimal day-ahead schedule computed using the SOCR for nonlinear NCs.

In the optimal day-ahead schedules computed for the ACG and HTG, 267-301 and 270-303 units are committed with 245 and 239 units consistently operating at their maximum capability respectively. Fig. 6.6 illustrates the hourly operational schedule for generator 1 in the HTG. It can be observed that the generator is committed from hour 7 to 23. The startup and shutdown ramp rates are respected between hours 6 to 7 and hours 23 to 24 respectively. Generator 1 is capable of

FIGURE 6.5: Voltage reconstruction error for 1560 SPs of (a) AC and (b) DC subgrids of the HTG. Black and gray lines show the maximum and average reconstruction errors for each SP respectively.

deploying reserves in between hours 8 to 22 within the up and down reserve margins (curve 2 and 3 of Fig. 6.6) allocated by the RO algorithm. Those reserve allocations complied with their ramping capability, as discussed in Section 5.3.1. Usually, the online units possess sufficient down reserve capacities to absorb generation variability. However, the up reserve allocations typically increase the base scenario operating cost as they often cause an inferior utilization of economic units for the base scenario power dispatch.

FIGURE 6.6: Day-ahead hourly operational schedule for generator 1 in the HTG. Curve 1 is the base scenario power dispatch. Curve 2 and 3 are the up and down reserve margins with respect to the power dispatch for the robust operation. Curve 4 and 5 are the maximum and minimum generation capabilities respectively.

6.3.4 Case Study 2: Economic Efficiency and Grid Loadability

This case study examines the impact of high demand on the day-ahead robust utilization of resources. Table 6.1 reports the aggregated UCs over the day, i.e., $\sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} u_{G,g}^t$ and the total generation cost at different demands (via scale factors in column 1) for both ACG and HTG. Furthermore, the cost reduction in the operation of the HTG is compared with the operational cost of the ACG. The total cost for the ACG is calculated, including the generation cost of the slack power, which is required to satisfy the power losses. The power losses are computed by the hourly AC power flow simulations performed on the base scenario power dispatch values via MATPOWER [2]. It can be observed in Table 6.1 that the HTG offers better utilization of generation resources, thereby enabling consistent economic benefits at different demands in comparison to the ACG. Although the total generation capacity is sufficient, the ACG *cannot* be loaded more than 10% above the nominal demand due to transmission congestion issues.

In contrast, the flexibility provided by the DC lines in power flow routing results in improved loadability for the HTG up to 25% above the nominal demand. The total cost of operation is also increasing at almost a linear rate for the HTG with the load increment. The results illustrate that the HTG architecture can be an effective capacity expansion procedure which supports a significant amount of increase in demand.

TABLE 6.1: Comparison of Economic Efficiency w.r.t. Grid Loadability

Scale Factor	# Online Units		Total Cost (M\$/day)		Cost Reduction
	ACG	HTG	ACG	HTG	
1.00	6,925	6,986	25.01	24.05	3.80%
1.05	6,998	7,063	28.36	26.96	4.91%
1.10	7,023	7,106	32.85	29.99	8.72%
1.15	—	7,204	—	33.12	—
1.20	—	7,319	—	36.37	—
1.25	—	7,418	—	39.76	—

Further, it should be noted that the generation schedules of the ACG, which are computed based on a decoupled representation of the network, *cannot* be implemented due to a significant number of violations in voltage limits, transmission line flow limits, and reactive power capabilities of generators according to AC power flow simulations performed via MATPOWER [2]. Consequently, the operation cost of the ACG is potentially even higher than the values listed in Table 6.1. Moreover, these results suggest that the related day-ahead RO methods [67, 68, 73] which partially/completely rely on the decoupled network model might not be applicable in practice, at least in the case of large-scale networks.

The convergence of the aggregated generation and the unit commitment at 110% demand is shown in Fig. 6.7 during hours 4, and 18 for both ACG and HTG. It is observed that the number of iterations for the ACG at 110% demand has been reduced compared to that at nominal demand. Conversely, the number of iterations for the HTG at 110% demand has been increased compared to that at nominal demand in Fig. 6.3.

FIGURE 6.7: Convergence trajectories of the aggregated power dispatch and total number of online units under 10% higher demand for (a) the ACG and (b) the HTG during hour 4 (the graph on the left side) and hour 18 (the graph on the right side).

It is further observed in Fig. 6.8 that the computation time of SPs at 110% demand for both the ACG and HTG are comparable to that at nominal demand in Fig. 6.4 respectively. Thus, the overall computation time mostly depends on the complexity of the MPs for both grids. It is observed that the MP computations for

the ACG are more demanding at 110% in comparison to those at nominal demand in Fig. 6.4(a). Further, computation time increases over the iterations. Conversely, the MP computations for the HTG are of the same order compared to those at nominal demand in Fig. 6.4(b).

FIGURE 6.8: Computation time of the MP and SPs over the iterations for (a) the ACG and (b) the HTG.

Moreover, Fig. 6.9 shows the AC and DC voltage reconstruction errors $\tilde{\kappa}_{\max}(\tilde{v}^{t,\omega*})$ and $\bar{\kappa}_{\max}(\bar{v}^{t,\omega*})$ in AC subgrid and DC subgrid of HTG respectively for all the SPs at 10% and 25% above the nominal demand. The corresponding voltage reconstruction error estimates at other demand levels are not shown here for brevity. However, it can be seen that the voltage reconstruction errors of these two representative demand levels are almost of the same order of the magnitude and they are negligible ($< 10^{-5}$), thereby validating the practicality of the day-ahead generation schedule computed using the SOCR for nonlinear NCs.

6.3.5 Case Study 3: Generation Utilization

In this case study, the wind penetration (and its variability) is increased by 45% to investigate the hosting capacity of wind power based generation. The simulation of the ACG reported *infeasibility*, indicating that the ACG *cannot* accommodate the increased wind-based power injections with significant variability. The results

FIGURE 6.9: Voltage reconstruction error for 1560 SPs of (a) AC and (b) DC subgrids of the HTG.

demonstrate the inflexibility of the ACG to deploy reserves owing to the transmission congestion, even though the available generation capacity is sufficient.

For the HTG, curves 1-3 in Fig. 6.10 illustrate the hourly aggregated base scenario power dispatch, and up and down reserve allocations respectively, under 45% higher wind generation as a fraction of the generation capability of online units, i.e., $\sum_{g \in \mathcal{G}} u_{G,g}^t P_{G,g}^{\max}$ at nominal demand. In addition, curve 4 in Fig. 6.10 shows the hourly utilization of the reserves as a fraction of the up reserve capability of online units, i.e., $\sum_{g \in \mathcal{G}} u_{G,g}^t \hat{R}_{G,g}^{\delta}$. Finally, curve 5 in Fig. 6.10 shows the fraction of the hourly demand shared by the wind power generation under the base scenario. Therein, it is observed that significant penetration of wind power, which is above 46% of the demand during hours 4-6. However, wind power penetration during hours 9-20 (which include the peak period) is less than 30% which is due to less availability of wind and the higher demand during daytime. Moreover, the up reserve allocation is also significantly high (more than 80% of the online up reserve capacity) during the first six hours and the last two hours of the day to cater the higher wind power variability. During these hours, most of the units are running near the minimum capability supporting up reserve requirements. It is further observed that the maximum up reserve capacity of online units is reached during hour 5 (see curve 4 in Fig. 6.10) to act upon the highest wind power variability. Thus, wind power

hosting capacity of the HTG is restricted only by the up reserve capability of the generators.

FIGURE 6.10: Curve 1 is the total power dispatch as a fraction of online generation capability for the base-case. Curve 2 and 3 are the up and down reserve margins as a fraction of online generation capability respectively. Curve 4 is the up reserve utilization as a fraction of the up reserve capability of online units. Curve 5 is the total wind power generation as a fraction of demand.

In addition, the generation utilization is examined against grid loadability under 45% increased wind power generation. Fig. 6.11 illustrates the aggregated generation profile of the system at 132% of the nominal demand. It is observed that during hour 18 (peak hour), the total generation capacity is almost exploited (up to 99.64%) for the base scenario power dispatch and up reserve requirements. Therein, 90.88% of the generation capacity is being used for the base scenario power dispatch and 8.76% of the generation capacity is allocated for up-reserve requirements. Therefore, it can be assumed that the flexibility offered by the HTG supports the effective utilization of *all* the generation capacity in the system and thereby resolves transmission congestion issues in the ACG.

6.3.6 Comparison with Existing Methods

In this section, the efficiency, computational performance, and robustness of the proposed nonanticipative RO framework is compared to those of scenario-wise reserve scheduling approaches in [67, 68, 77]. To the best of authors' knowledge, these are the most recent RO frameworks in the context of robust UC with nonlinear NCs. As multi-period OPF with nonlinear NCs is extensive, we adopt the

FIGURE 6.11: Aggregated generation profiles of the HTG at 32% increased demand and 45% increased wind power. Curve 1 is the total power dispatch for the base-case. Curve 2 and 3 are the up and down reserve margins respectively. Curve 4 is the total generation capacity of the online units. Curve 5 is the total generation capacity of the system.

sequential evaluation procedure in [68, Sec. III-E]. It should be noted that the reserve allocations are not considered as scheduling variables in the MP of the RO approach in [68]. Accordingly, the ramp constraints (5.4)-(5.5) are modified as in [68, Eq. (2d),(2e),(7e),(7f)] and the BCs (5.14) are modified as in [68, Eq. (7b)]. Further, ramp constraints associated with each scenario [68, Eq. (3h),(3i)] are sequentially updated following the procedure in [68, Sec. III-E]. In the following, the resulting problem is referred to as the “Scenario-wise RO Approach” (Scheme 1).

To verify the compatibility of the proposed RO framework with other scenario selection tools, the authors generated scenarios using the LHS method and a Monte Carlo based scenario reduction technique. The LHS method is widely used in scenario-based reserve scheduling frameworks [70, 82]. For a fair comparison with TOAT (64 samples), in terms of efficiency, computation time, and robustness, 100 samples were generated based on the LHS method following the procedure in [82, Sec. II]. It should be noted that one of the main differences of TOAT and the LHS method is that the scenarios provided by TOAT consists of the vertex information of the polyhedral set during each hour, while the LHS method specifies the complete information of scenario vectors over the entire optimization horizon. Further details on modeling uncertain scenarios via the LHS method is presented in Appendix B. Further, 1,000 samples were generated based on the Monte Carlo method and 100 samples were selected following the procedure given in Algorithm 2 of [118,

Appendix].

In the following, the performance of the proposed RO framework for the day-ahead power dispatch and reserve scheduling is evaluated under the following three schemes:

1. *Scenario-wise RO Approach* with the LHS method for scenario selection.
2. *Nonanticipative RO Approach* with the LHS method for scenario selection.
3. *Nonanticipative RO Approach* with TOAT for scenario selection.
4. *Nonanticipative RO Approach* with the Monte Carlo method based scenario reduction technique for scenario selection.

Here, Schemes 1 and 2 compare the effectiveness of the proposed RO framework (Nonanticipative RO Approach) against the recently proposed RO framework (Scenario-wise RO Approach) in [68]. Schemes 2, 3 and 4 compare the effectiveness of the LHS method [82, Sec. II], TOAT, and the Monte Carlo method (with scenario reduction technique proposed in [118]) for the proposed Nonanticipative RO Approach. It should be noted that the Scenario-wise RO Approach based on [68] requires complete information of scenario vectors over the entire optimization horizon. Thus, it cannot be implemented with TOAT.

Further, to illustrate that the proposed RO framework supports different generator parameters, we use the set of parameters in Table 6.2 in all three schemes.

TABLE 6.2: Generator Parameters

Generator g	$c_{F,g}$ (\$/h)	$c_{S,g}$ (\$)	$c_{D,g}$ (\$)	$\hat{T}_{G,g}$ (h)	$\check{T}_{G,g}$ (h)	$\hat{R}_{G,g}^\delta, \check{R}_{G,g}^\delta$ (p.u.) ²
1-100	30	110	20	3	1	0.4
101-200	20	100	15	4	2	0.4
201-323	10	90	10	5	3	0.4

²Proportional to $P_{G,g}^{\max} - P_{G,g}^{\min}$

6.3.6.1 Efficiency and Computational Performance

In the simulations, the total operating costs were 27.79 M\$/day, 27.59 M\$/day, 27.72 M\$/day, and 27.59 M\$/day for Scheme 1-4 respectively. Accordingly, the total operating cost associated with Scheme 1 is 0.72% higher compared to that of Schemes 2 and 4 and it is 0.26% higher compared to that of Scheme 3, respectively. As mentioned in [68, Sec. IV-D], the suboptimality associated with the solution approach in Scheme 1 might be due to its heuristic nature. The cost differences in between Schemes 2-4 are due to the different scenarios considered in the LHS method [82], TOAT [111] and Monte Carlo based scenario selection [118].

Fig. 6.12 shows the evolution of the computation time of the MP and SPs under the three schemes. It can be observed that the proposed Nonanticipative RO Approach (in Schemes 2-4) converges in a few iterations and that the computation time of the MPs (which is the dominant component of the overall computation time) at each iteration is significantly faster than the Scenario-wise RO Approach (Scheme 1). Therefore, the optimization of reserve allocations in the MP based on the feedback of the SPs (a salient feature of the proposed RO framework) in Schemes 2-4 can support both faster convergence and faster MP evaluations with tighter BCs. Further, it is observed that the computational performances of Scheme 2-4 are comparable and, thus, the performance of the proposed nonanticipative RO framework is essentially independent of the considered scenario selection tool.

6.3.6.2 Validation of the Generation Schedule Considering Different RO Models

To validate the robustness of the day-ahead scheduling solutions of the three schemes, a set of 1,000 wind power scenarios are generated by sampling randomly and independently from a uniform probability distribution within the original polyhedral confidence bounds. Each scenario comprises wind power realizations for all the integrated wind farms over the entire optimization horizon.

FIGURE 6.12: Computation time of the four schemes over the iterations based on the (a) time per MP evaluation and (b) maximum time per SP evaluation.

In this section, the robustness of the proposed nonanticipative RO framework is compared to the recently proposed scenario-wise RO framework under the same scenario selection tool (LHS method), i.e., the validity of the scheduling solution of Scheme 2 is compared to that of Scheme 1. The hourly redispatch decisions for each scenario are determined by executing T ($= 24$) single-period OPF problems over the time horizon, i.e., 24,000 OPF computations. For a fair comparison of the two schemes, the generation of each unit is bounded by the original reserve capability limits around the base scenario power dispatch, as the RO framework in [68] does not compute reserve margins for the generators. Further, the power dispatch is subjected to the ramp rates considering the base scenario power dispatch during the previous hour. If at least one of the OPF problems report infeasibility or inexactness, the day-ahead scheduling solution is considered infeasible for the corresponding scenario.

In the OPF computations, it is observed that 993 scenarios are feasible (which is a 99.3% success) for the generation schedule computed based on Scheme 2, while only 741 scenarios are feasible (which is a 74.1% success) for that based on Scheme 1. Therefore, the cost increase in the scheduling solution of Scheme 1, i.e., the recently

proposed scenario-wise RO framework, is *not* justified. Even with a lower total operating cost, the scheduling solution of Scheme 2, which is based on the proposed nonanticipative RO framework, outperforms Scheme 1 in terms of robustness, as it ensures a proper reserve scheduling under all considered scenarios. These results illustrate the importance of preserving nonanticipativity of the scheduling solutions in the day-ahead optimization of generation resources. In Fig. 6.13, the two histograms show the number of feasible scenarios with respect to the normalized total operating cost of *wait-and-see* decisions for both the schemes. The expected total operating cost (considering feasible scenarios only) of wait-and-see decisions of Scheme 1 and 2 are 6.44% (29.58 M\$/day) and 6.67% (29.43 M\$/day) higher than the cost of the day-ahead scheduling solutions 27.79 M\$/day and 27.59 M\$/day found in Section 6.3.6.1 respectively.

FIGURE 6.13: Histogram of the normalized cost for the Scenario-wise RO Approach (Scheme 1) and the proposed Nonanticipative RO Approach (Scheme 2) with the LHS method considering feasible wind power scenarios (out of 1,000 scenarios). Bin width of the histogram is 5.5×10^{-4} .

6.3.6.3 Validation of the Reserve Margins for Different Scenario Selection Schemes

In this section, the reserve margins computed by Scheme 2-4 for different scenario selection tools, i.e., the LHS method, the TOAT and the Monte Carlo method (with the scenario reduction technique proposed in [118]), are validated using the

set of scenarios of Section 6.3.6.2. The RO framework proposed in this paper determines the expected hourly operating range of each generator in the day-ahead schedule. Accordingly, these reserve bounds are enforced on the generators in OPF computations of this validation study, in addition to the constraints mentioned in Section 6.3.6.2.

In the OPF computations, it is observed that all scenarios are feasible (which is a 100% success) for the generation schedule computed based on Scheme 3 while only 916 and 980 scenarios are feasible (which are 91.6% and 98.0% success) for the generation schedule based on Schemes 2 and 4. Therefore, the cost increase in the scheduling solution of Scheme 3 compared to Schemes 2 and 4 is the compensation to obtain robust reserve margins for the generators. Thus, TOAT is more successful by providing vertex information of the polyhedral uncertainty set, which is more likely capturing the worst scenarios. In Fig. 6.14, the two histograms show the number of feasible scenarios with respect to the normalized total operating cost of wait-and-see decisions for both the schemes. The expected total operating cost (considering feasible scenarios only) of wait-and-see decisions of Schemes 2-4 are 6.91% (29.50 M\$/day), 6.70% (29.58 M\$/day) and 6.87% (29.49 M\$/day) higher than the cost of the respective day-ahead scheduling solutions 27.59 M\$/day, 27.72 M\$/day and 27.59 M\$/day found in Section 6.3.6.1 respectively.

6.4 Summary

In this chapter, a convex relaxation was applied to the subproblems of the day-ahead robust network-constrained unit commitment problem formulated in Chapter 5, which enabled their globally optimal solution and supported convergence. Further, numerical simulations were performed for the Polish 2383-bus system during winter peak conditions. Therein, the merits of a hybrid AC/DC transmission grid architecture were illustrated in comparison to the reference grid in a day-ahead

FIGURE 6.14: Histogram of the normalized cost for the proposed Nonanticipative RO Approach with the LHS method (Scheme 2), the TOAT (Scheme 3), and the Monte Carlo method (Scheme 4) considering feasible wind power scenarios (out of 1,000 scenarios). Bin width of the histogram is 5.5×10^{-4} .

market context. The hybrid AC/DC transmission grid architecture ensures the exactness of the convex relaxation of both AC and DC network constraints and the transmission congestion issues are alleviated by the flexibility offered by the DC systems. The results demonstrate consistently lower generation costs over different load levels.

Furthermore, the enhanced loadability and reserve provision enable the utilization of all generation in the system to accommodate an increase in demand and variable wind power generation. Consequently, the hybrid AC/DC transmission grid architecture may be utilized as a topology-preserving capacity expansion strategy for existing congested grids to support the efficient utilization of the available generation facilities. A validation study shows that the proposed robust optimization framework outperforms a state-of-the-art robust optimization framework in terms of optimality, computation time, and robustness. Furthermore, the beneficial computational performance of the proposed robust optimization framework was shown to be almost unaffected by the choice of the scenario selection tool.

Chapter 7

Cooperative Bidding-based Robust Optimal Energy Management of Multi-Microgrids

7.1 Introduction

As discussed in Chapter 1, there is a higher potential for MGs to locally manage the uncertainties caused by the demand and the RESs. As such, the concept of MMG systems provides better financial benefits for nearby MGs via power-sharing. This chapter develops a convex and scalable RO framework which is interfaced with a cooperative trading scheme for power dispatch and reserve scheduling of an MMG system. The MGs participating in MMG systems typically belong to separate entities. Hence, distributed computations are essential to promote the proliferation of MMG systems. In the proposed RO framework, the optimization of local resource utilization of each constituent MG is performed by the individual MGOs. An independent coordinating entity is introduced to manage power and reserve sharing among the constituent MGs as and when it is economical.

The RO framework developed in this chapter determines optimal participation of MG components and imports/exports from/to constituent MGs to mitigate the real-time power mismatches while preserving nonanticipativity. The power and reserve sharing is facilitated via cooperative bidding, depending on the import/export preferences of the constituent MGs. In contrast to the existing approaches, the proposed RO framework accounts for the power losses in the lines connecting the point-of-common-coupling (PCC) and the individual MGs, and the power losses in the distribution transformer connecting the MMG system and the utility grid.

To this end, the uncertainty representation is presented in Section 7.2. The MMG architecture and the optimization models for constituent MGs are presented in Section 7.3. The cooperative trading scheme and the market-clearing mechanism of the MMG system are presented in Section 7.4. Finally, the results are presented in Section 7.5. The results demonstrate the performance of the proposed RO framework and the benefits of energy and reserve sharing between individual MGs when compared with the isolated operation of individual MGs. The proposed nonanticipative RO framework is also compared to recently proposed RO frameworks in [16, 18], and its robustness is verified via an out-of-sample analysis. The chapter is summarized in Section 7.6.

7.2 Uncertainty Model for Robust Optimization

7.2.1 Uncertainty Characterization

A detailed discussion on polyhedral constraints is given in Chapter 5 of this thesis. In the RO literature, the uncertain variables are generally assumed to lie within a polyhedral set with known confidence bounds. To obtain feasible results under the worst scenario, the minimum cost resource management problem needs to be *maximized* over the polyhedral uncertainty set. The resulting *max-min* problem is nonconvex and hence, NP-hard to solve in polynomial time. The specific structure

of this resource management problem enables the worst scenario to be a permutation of the extreme points of the uncertainty polytope [108, Sec. 2.4]. As such, the global solution can be obtained if the resource optimization problem is evaluated at all the vertexes of the uncertainty polytope. However, the computational complexity involved in exploring all the vertex (full factorial) combinations increases exponentially with the dimensions of the uncertainty set [78].

Thus, [78] and [72] proposed two alternative tractable solution approaches to capture a reduced set of vertex permutations while considering the correlation between the uncertain variables. In the context of MMG systems, the uncertain variables belong to different entities. Hence, the correlations between uncertain variables might not be readily available due to privacy concerns. Conversely, the TOAT generates a fraction of the full factorial combinations of vertexes based on a special set of OAs without requiring any information about the confidence bounds [111]. The readers can refer to the detailed discussion on the TOAT which is included in Section 5.2.2 of Chapter 5.

7.2.2 Scenario Selection based on the TOAT

The general representation of an OA is $L_{\mathcal{W}}B^F$. In this chapter, an MMG with four MGs is considered. Table 7.1 depicts the first five columns of the OA L_82^7 specifying eight scenarios involving five two-level factors (five cooperative entities) [111]. Regarding the columns titled MGs 1-4 in Table 7.1, the +1 and -1 levels (symbolically) represent the corresponding vertexes of the demand and RES uncertainty polyhedrons which require up reserve and down reserve allocations during hour t respectively. Each row is associated with a scenario.

The expected levels of the nominal demand can be defined as shown in (7.1), wherein the base scenario is represented by $\omega = 0$.

$$\left[P_{\text{ND}}^{m,t}\right]_+ := P_{\text{ND},0}^{m,t} + P_{\text{ND}}^{m,t,\zeta^+}, \quad \left[P_{\text{ND}}^{m,t}\right]_- := P_{\text{ND},0}^{m,t} - P_{\text{ND}}^{m,t,\zeta^-} \quad (7.1)$$

TABLE 7.1: Selected Scenarios based on the OA L_82^7

Scenario ω	Levels of each Random Factor				
	Price	MG 1	MG 2	MG 3	MG 4
1	+1	+1	+1	+1	+1
2	+1	+1	+1	-1	-1
3	+1	-1	-1	+1	+1
4	+1	-1	-1	-1	-1
5	-1	+1	-1	+1	-1
6	-1	+1	-1	-1	+1
7	-1	-1	+1	+1	-1
8	-1	-1	+1	-1	+1

Let $\Upsilon = \{0, 1, \dots, \mathcal{W}\}$. Then, the nominal demand $P_{\text{ND},\omega}^{m,t}$ of MG m under scenario $\omega \in \Upsilon \setminus \{0\}$ during hour $t \in \mathcal{T}$ is a vertex of the uncertain polyhedron as defined below:

$$P_{\text{ND},\omega}^{m,t} \in \left\{ [P_{\text{ND}}^{m,t}]_+, [P_{\text{ND}}^{m,t}]_- \right\} \quad (7.2)$$

Furthermore, the expected levels of the RES generation (wind and solar PV power) can be defined as follows:

$$[P_{\text{F},f}^{m,t}]_+ := P_{\text{F},f,0}^{m,t} - P_{\text{F},f}^{m,t,\zeta-}, \quad [P_{\text{F},f}^{m,t}]_- := P_{\text{F},f,0}^{m,t} + P_{\text{F},f}^{m,t,\zeta+} \quad (7.3)$$

Regarding the column titled Price in Table 7.1, the +1 and -1 levels represent the upper and lower bounds respectively of the price at which electricity is purchased from the utility grid during time t .

$$[\lambda_{\text{N}}^{\text{B},t}]_+ := \lambda_{\text{N},0}^{\text{B},t} + \lambda_{\text{N}}^{\text{B},t,\zeta+}, \quad [\lambda_{\text{N}}^{\text{B},t}]_- := \lambda_{\text{N},0}^{\text{B},t} - \lambda_{\text{N}}^{\text{B},t,\zeta-} \quad (7.4)$$

$[\lambda_{\text{N}}^{\text{S},t}]_+$ and $[\lambda_{\text{N}}^{\text{S},t}]_-$ can also be defined in a similar fashion.

7.3 Robust Day-Ahead Optimal Energy Management of Multi-Microgrids

7.3.1 Multi-Microgrid System Structure

An MMG system is a cluster of MGs in which each MG is connected to a common busbar called the PCC via a circuit breaker as shown in Fig. 7.1. Each MG in an MMG system can operate in either the islanded mode or the grid-connected mode. The PCC is connected to the utility grid through a transformer.

FIGURE 7.1: The typical structure of an MMG system.

Each MG in the MMG system considered in this chapter comprises DGs, dispatchable loads, fixed loads, ESSs, and RESs. Each MGO optimally utilizes the locally available resources to meet the local load demand while interfacing with the power-sharing operator (PSO) for power trading (buying/selling). The PSO collects all the supply/demand bids of the constituent MGs and decides the market equilibrium price (MEP). Therein, the PSO imports/exports the power shortfall/surplus from/to the utility grid at predetermined buying/selling prices. Accordingly, the PSO facilitates the market clearing of the MMG system based on the supply and demand bids received from the constituent MGs. In real-time operation, the PSO monitors the power flows of the individual MGs using the metering infrastructure located at the PCC as shown in Fig. 7.1.

7.3.2 Nonanticipativity in Reserve Scheduling

The state-of-the-art RO frameworks developed in the context of MMG systems, cf. [16, 18], enforce the intertemporal ramp constraints of the DGs and the state-of-charge (SoC) constraints of the ESSs only for the transition between power dispatch values corresponding to each scenario vector (a sequence of realizations over the time) separately. This condition is illustrated in Fig. 5.1 of Chapter 5 for power dispatch of generators. Further, the discussion in Section 5.3.1 of Chapter 5 explains the impact of such an anticipation for the reserve deliverability. Consequently, the nonoccurrence of an uncertainty realization as anticipated might potentially lead to an infeasible power dispatch (reserve undeliverability) for some DGs and/or ESSs due to ramping infeasibility of the DGs and/or SoC limit violations in real-time operation. Hence, the violation of *nonanticipativity* of the day-ahead scheduling might cause unwanted load shedding or RES spillage.

In the proposed RO framework, auxiliary bounding variables are used for the power dispatch values of the DGs and ESSs, the SoC values of the ESSs, and the demand curtailment and redispatch values (details are presented in Section 7.3.3). Therein, the reserves are dispatched considering the requirements in each scenario and intertemporal constraints are enforced via the auxiliary bounding variables ensuring a valid transition between *any* two plausible power dispatch values. Such an allocation of reserves is nonanticipative as it does not depend on a specific sequence uncertain realization over time. Hence, the robustness of the day-ahead optimal schedule against all the plausible uncertain scenarios is implicitly ensured by the proposed RO framework.

7.3.3 Microgrid Component Modeling

This section discusses the operational constraints of the MG components. In addition to the dispatch variables, auxiliary bounding variables are introduced to preserve the nonanticipativity of reserve scheduling which is a salient feature of

the proposed RO framework in comparison to that in [16, 18]. The superscripts ‘max’ and ‘min’ represent the upper and lower capability limits respectively of the corresponding variable.

7.3.3.1 Energy Storage System

Constraints related to the robust operation of ESSs are presented in the following.

For all $e \in \mathcal{E}^m$:

$$C_{E,e,\omega}^{m,t} = a_{E,e}^m (P_{E,e,\omega}^{C,m,t})^2 + a_{E,e}^m (P_{E,e,\omega}^{D,m,t})^2 \quad (7.5a)$$

$$0 \leq P_{E,e,\omega}^{C,m,t} \leq (P_{E,e}^{C,m})^{\max}; \forall \omega \in \Upsilon \quad (7.5b)$$

$$0 \leq P_{E,e,\omega}^{D,m,t} \leq (P_{E,e}^{D,m})^{\max}; \forall \omega \in \Upsilon \quad (7.5c)$$

$$E_{e,0}^{m,t} = E_{e,0}^{m,t-1} + \frac{\Delta t}{B_e^m} \left(\eta_e^C P_{E,e,0}^{C,m,t} - \frac{1}{\eta_e^D} P_{E,e,0}^{D,m,t} \right) \quad (7.5d)$$

$$E_{e,0}^{m,1} = \frac{\Delta t}{B_e^m} \left(\eta_e^C P_{E,e,0}^{C,m,1} - \frac{1}{\eta_e^D} P_{E,e,0}^{D,m,1} \right) + E_{I,e}^m \quad (7.5e)$$

$$(E_e^m)^{\min} \leq E_{e,0}^{m,t} \leq (E_e^m)^{\max} \quad (7.5f)$$

$$E_{e,0}^{m,T} = E_{I,e}^m \quad (7.5g)$$

$$P_{E,e,\omega}^{D,m,t} - P_{E,e,\omega}^{C,m,t} = P_{E,e,0}^{D,m,t} - P_{E,e,0}^{C,m,t} + \sigma_{E,e}^{m,t} (\zeta_\omega^{m,t} - \zeta_0^{m,t}); \forall \omega \in \Upsilon \setminus \{0\} \quad (7.5h)$$

$$\bar{P}_{E,e}^{C,m,t} \geq P_{E,e,\omega}^{C,m,t}, \quad \underline{P}_{E,e}^{C,m,t} \leq P_{E,e,\omega}^{C,m,t},$$

$$\bar{P}_{E,e}^{D,m,t} \geq P_{E,e,\omega}^{D,m,t}, \quad \underline{P}_{E,e}^{D,m,t} \leq P_{E,e,\omega}^{D,m,t}; \forall \omega \in \Upsilon \quad (7.5i)$$

$$\bar{E}_e^{m,t} = \bar{E}_e^{m,t-1} + \frac{\Delta t}{B_e^m} \left(\eta_e^C \bar{P}_{E,e}^{C,m,t} - \frac{1}{\eta_e^D} \bar{P}_{E,e}^{D,m,t} \right) \quad (7.5j)$$

$$\bar{E}_e^{m,1} = \frac{\Delta t}{B_e^m} \left(\eta_e^C \bar{P}_{E,e}^{C,m,1} - \frac{1}{\eta_e^D} \bar{P}_{E,e}^{D,m,1} \right) + E_{I,e}^m \quad (7.5k)$$

$$\underline{E}_e^{m,t} = \underline{E}_e^{m,t-1} + \frac{\Delta t}{B_e^m} \left(\eta_e^C \underline{P}_{E,e}^{C,m,t} - \frac{1}{\eta_e^D} \underline{P}_{E,e}^{D,m,t} \right) \quad (7.5l)$$

$$\underline{E}_e^{m,1} = \frac{\Delta t}{B_e^m} \left(\eta_e^C \underline{P}_{E,e}^{C,m,1} - \frac{1}{\eta_e^D} \underline{P}_{E,e}^{D,m,1} \right) + E_{I,e}^m \quad (7.5m)$$

$$(E_e^m)^{\min} \leq \{ \bar{E}_e^{m,t}, \underline{E}_e^{m,t} \} \leq (E_e^m)^{\max} \quad (7.5n)$$

$$(1 - \Delta_E) E_{I,e}^m \leq \{ \bar{E}_e^{m,T}, \underline{E}_e^{m,T} \} \leq (1 + \Delta_E) E_{I,e}^m \quad (7.5o)$$

The ESSs have the following parameters for all $m \in \mathcal{M}$, $e \in \mathcal{E}^m$: $a_{E,e}^m = \$5/\text{MW}^2\text{h}$, $B_e^m = 1 \text{ MWh}$, $\eta_e^C = \eta_e^D = 0.97$, $(P_{E,e}^{C,m})^{\max} = (P_{E,e}^{D,m})^{\max} = 0.2 \text{ MW}$, $(E_e^m)^{\max} = 0.9$, $(E_e^m)^{\min} = 0.1$, $E_{R,e}^m = 0.5$, and $\delta = 0.2$.

Equation (7.5a) is the quadratic degradation cost function for ESS e . The charging and discharging power limits of ESS e are shown in (7.5b) and (7.5c) respectively. The time varying SoC of ESS e for the base-case scenario is computed in (7.5d)-(7.5e) and is bounded by (7.5f). Under the base scenario, (7.5g) ensures that the SoC of ESS e at the end of the optimization horizon is the same as $E_{I,e}^m$. The duration of the optimization interval denoted by Δt is 1 h. Constraint (7.5h) enforces the participation factor (PF) of the charging/discharging powers of ESS e to mitigate any power mismatch due to the uncertainties.

In (7.5i), the auxiliary variables $\bar{P}_{E,e}^{C,m,t}$, $\bar{P}_{E,e}^{D,m,t}$ and $\underline{P}_{E,e}^{C,m,t}$, $\underline{P}_{E,e}^{D,m,t}$ in the proposed RO framework represent the highest and lowest dispatch values of the variables $P_{E,e,\omega}^{C,m,t}$ and $P_{E,e,\omega}^{D,m,t}$ respectively under all $\omega \in \Upsilon$ during hour t . For instance, $\bar{P}_{E,e}^{C,m,t} \geq \max_{\omega \in \Upsilon} P_{E,e,\omega}^{C,m,t}$, $\underline{P}_{E,e}^{C,m,t} \leq \min_{\omega \in \Upsilon} P_{E,e,\omega}^{C,m,t}$ and so on. Equations (7.5j)-(7.5m) account for the effects of charging and discharging events on the SoC of ESS e under all the uncertain scenarios $\omega \in \Upsilon$. The maximum expected SoC during hour t (denoted by $\bar{E}_e^{m,t}$) is computed in (7.5j)-(7.5k) on the basis of the maximum SoC value during hour $t-1$ and the maximum (or minimum) possible energy that could be stored (or extracted) during hour t under all the plausible uncertain scenarios. The minimum expected SoC (denoted by $\underline{E}_e^{m,t}$) is also computed similarly in (7.5l)-(7.5m). Thus, $\bar{E}_e^{m,t}$ and $\underline{E}_e^{m,t}$ implicitly represent the upper and lower margins of $E_e^{m,t}$ respectively under all $\omega \in \Upsilon$ during hour t . These two variables are bounded within the physical SoC limits of ESS e by (7.5n). Thus, the nonanticipativity of the charging power and discharging power dispatch values apart from the SoC of ESS e is preserved under all the plausible uncertain scenarios. The maximum and minimum expected SoC values at the end of the optimization horizon are constrained around $E_{I,e}^m$ within a predefined radius Δ_E by (7.5o) to permit sufficient flexibility for the ESS operation during the following day.

7.3.3.2 Diesel Generators

Constraints related to the robust operation of ESSs are presented in the following.

For all $g \in \mathcal{G}^m$:

$$C_{G,g,\omega}^{m,t} = a_{G,g}^m (P_{G,g,\omega}^{m,t})^2 + b_{G,g}^m P_{G,g,\omega}^{m,t} \quad (7.6a)$$

$$(P_{G,g}^m)^{\min} \leq P_{G,g,\omega}^{m,t} \leq (P_{G,g}^m)^{\max}; \forall \omega \in \Upsilon \quad (7.6b)$$

$$-\check{R}_{G,g}^{\delta,m} \leq P_{G,g,\omega}^{m,t} - P_{G,g,0}^{m,t} \leq \hat{R}_{G,g}^{\delta,m}; \forall \omega \in \Upsilon \setminus \{0\} \quad (7.6c)$$

$$P_{G,g,\omega}^{m,t} = P_{G,g,0}^{m,t} + \sigma_{G,g}^{m,t} (\zeta_{\omega}^{m,t} - \zeta_0^{m,t}); \forall \omega \in \Upsilon \setminus \{0\} \quad (7.6d)$$

$$\bar{P}_{G,g}^{m,t} \geq P_{G,g,\omega}^{m,t}, \underline{P}_{G,g}^{m,t} \leq P_{G,g,\omega}^{m,t}; \forall \omega \in \Upsilon \quad (7.6e)$$

$$\bar{P}_{G,g}^{m,t} - \underline{P}_{G,g}^{m,t-1} \leq \hat{R}_{G,g}^{\Delta,m} \quad (7.6f)$$

$$\bar{P}_{G,g}^{m,t-1} - \underline{P}_{G,g}^{m,t} \leq \check{R}_{G,g}^{\Delta,m} \quad (7.6g)$$

Equation (7.6a) is the fuel cost function. (7.6b) bounds the generation of DG g under scenario ω . Constraint (7.6c) ensures that the power dispatch under scenario ω respects the reserve capability bounds. Constraint (7.6d) enforces the PF of the power dispatch of DG g to mitigate any power mismatch due to the uncertainties. Equations (7.6f) and (7.6g) ensure that the up and down operating ramp rate constraints are respected between any two successive time periods under all plausible scenarios ω while considering the worst possible ramping rate of DG g . This preserves the nonanticipativity of the day-ahead power dispatch and reserve allocation of DG g .

7.3.3.3 Dispatchable Loads

The MGs support the demand response scheme formulated below.

$$C_{D,\omega}^{m,t} = a_D^m (P_{RD,\omega}^{m,t})^2 + a_D^m (P_{CD,\omega}^{m,t})^2 \quad (7.7a)$$

$$0 \leq P_{RD,\omega}^{m,t} \leq (P_{RD}^m)^{\max}; \forall \omega \in \Upsilon \quad (7.7b)$$

$$0 \leq P_{CD,\omega}^{m,t} \leq (P_{CD}^m)^{\max}; \forall \omega \in \Upsilon \quad (7.7c)$$

$$\sum_{t \in \mathcal{T}} [P_{\text{RD},0}^{m,t} - P_{\text{CD},0}^{m,t}] = 0 \quad (7.7d)$$

$$P_{\text{CD},\omega}^{m,t} - P_{\text{RD},\omega}^{m,t} = P_{\text{CD},0}^{m,t} - P_{\text{RD},0}^{m,t} + \sigma_{\text{D}}^{m,t} (\zeta_{\omega}^{m,t} - \zeta_0^{m,t}); \forall \omega \in \Upsilon \setminus \{0\} \quad (7.7e)$$

$$\underline{P}_{\text{RD}}^{m,t} \leq P_{\text{RD},\omega}^{m,t}, \bar{P}_{\text{CD}}^{m,t} \geq P_{\text{CD},\omega}^{m,t}; \forall \omega \in \Upsilon \quad (7.7f)$$

$$0 \leq \underline{P}_{\text{RD}}^{m,t} \leq (P_{\text{RD}}^{m,t})^{\max}, 0 \leq \bar{P}_{\text{CD}}^{m,t} \leq (P_{\text{CD}}^{m,t})^{\max} \quad (7.7g)$$

$$\Delta t \sum_{t \in \mathcal{T}} [\bar{P}_{\text{CD}}^{m,t} - \underline{P}_{\text{RD}}^{m,t}] \leq E_{\text{SH}}^m \quad (7.7h)$$

The cost function in (7.7a) reflects the sensitivity of the inconvenience felt by the consumers due to the load shifting. The demand redispatch and curtailment are constrained by (7.7b) and (7.7c) respectively. No load shedding is permitted under the base scenario as described in (7.7d). Constraint (7.7e) enforces the PFs of the demand redispatch/curtailment to mitigate any power mismatch due to the uncertainties. The additional variables $\underline{P}_{\text{RD}}^{m,t}$ and $\bar{P}_{\text{CD}}^{m,t}$ are defined for the redispatched and the curtailed load demands respectively as shown in (7.7f) and (7.7g). Using these variables, (7.7h) restricts the worst possible total load shedding below E_{SH}^m under all the plausible uncertain scenarios, thereby satisfying the nonanticipativity in demand shifting.

7.3.3.4 Interactions with the PSO

The PSO supervises the power and price bids submitted by each MGO for buying/selling electricity.

$$C_{\text{O},\omega}^{m,t} = \beta_{\text{O},\omega}^{\text{B},t} \left[P_{\text{O},\omega}^{\text{B},m,t} + Z_{\text{R}}^m (P_{\text{O},\omega}^{\text{B},m,t})^2 / (V_m^{\text{B}})^2 \right] - \beta_{\text{O},\omega}^{\text{S},t} \left[P_{\text{O},\omega}^{\text{S},m,t} - Z_{\text{R}}^m (P_{\text{O},\omega}^{\text{S},m,t})^2 / (V_m^{\text{S}})^2 \right] \quad (7.8a)$$

$$P_{\text{O},\omega}^{\text{B},m,t} - P_{\text{O},\omega}^{\text{S},m,t} = P_{\text{O},0}^{\text{B},m,t} - P_{\text{O},0}^{\text{S},m,t} + \sigma_{\text{O}}^{m,t} (\zeta_{\omega}^{m,t} - \zeta_0^{m,t}); \forall \omega \in \Upsilon \setminus \{0\} \quad (7.8b)$$

$$0 \leq P_{\text{O},\omega}^{\text{B},m,t}, P_{\text{O},\omega}^{\text{S},m,t} \leq (P_{\text{O}}^m)^{\max}; \forall \omega \in \Upsilon \quad (7.8c)$$

In general, the R/X ratios of distribution lines are larger than those of transmission lines. Hence, the source side voltage magnitude must be higher than the destination side voltage magnitude [119]. It is assumed that if MG $m \in \mathcal{M}$ is a seller/buyer MG, the corresponding bus voltage must be higher/lower than V_O and V_O is fixed as it is strongly coupled with the utility grid. Thus, two values are assumed for the voltage magnitudes in the respective MG buses to compute losses which may be incurred while transferring active power from and to the PSO with $V_m^S > V_O$ and $V_m^B < V_O$ for all $m \in \mathcal{M}$. These two values can be approximately estimated a priori using historical voltage data measured at the respective MG bus while operating as a seller or buyer MG respectively. Accordingly, power losses are assumed to be $Z_R^m (P_{O,\omega}^{S,m,t})^2 / (V_m^S)^2$ while selling and $Z_R^m (P_{O,\omega}^{B,m,t})^2 / (V_m^B)^2$ while buying. The cost of buying power from the PSO and the income gained by selling power to the PSO are calculated while approximately accounting for *power transfer losses* as in (7.8a). As explained later in Section 7.4.1, $\beta_{O,\omega}^{B,t} > \beta_{O,\omega}^{S,t}$. This convexifies the power trading cost function in (7.8a) and avoids the simultaneous buying and selling of electricity. Constraint (7.8b) enforces the PF of the power import/export to mitigate any power mismatch due to the uncertainties. Constraint (7.8c) limits the power imported from and exported to the PSO respectively.

7.3.4 Energy Management Model for a Single Microgrid

The total operating cost (TOC) function under scenario ω during hour t is formulated as follows.

$$C_\omega^{m,t}(\Psi_\omega^{m,t}) = \sum_{g \in \mathcal{G}^m} C_{G,g,\omega}^{m,t} + \sum_{e \in \mathcal{E}^m} C_{E,e,\omega}^{m,t} + C_{D,\omega}^{m,t} + C_{O,\omega}^{m,t} \quad (7.9)$$

To formulate the expected TOC function over the entire optimization horizon, it is assumed that the probability of the base scenario is known. The sampling average approximation method [120] is applied to compute an *average* reserve cost in (7.10b) by assuming the same probability of occurrence for all the other plausible uncertain scenarios. Accordingly, C_E^m and C_R^m for the robust operation of MG m

can be formulated as shown below in (7.10).

$$C_E^m = \sum_{t \in \mathcal{T}} C_0^{m,t}(\Psi_0^{m,t}) \quad (7.10a)$$

$$C_R^m = \sum_{t \in \mathcal{T}} \left[\frac{1}{\mathcal{W}} \sum_{\omega \in \Upsilon \setminus \{0\}} C_\omega^{m,t}(\Psi_\omega^{m,t}) - C_0^{m,t}(\Psi_0^{m,t}) \right] \quad (7.10b)$$

To this end, the robust day-ahead optimal dispatch problem of each MG $m \in \mathcal{M}$ can be formulated as shown below in (7.11) to obtain the optimal *expected* TOC while respecting all the technical constraints under all the plausible uncertain scenarios.

$$\min_{\Psi_\omega^{m,t}} C_{\text{exp}}^m = C_E^m + (1 - \mu)C_R^m \quad (7.11a)$$

$$\begin{aligned} \text{s.t. } \mathbf{1}^T P_{G,0}^{m,t} + \mathbf{1}^T [P_{E,0}^{D,m,t} - P_{E,0}^{C,m,t}] + P_{O,0}^{B,m,t} - P_{O,0}^{S,m,t} \\ = P_{RD,0}^{m,t} - P_{CD,0}^{m,t} + \zeta_0^{m,t}; \forall t \in \mathcal{T} \end{aligned} \quad (7.11b)$$

$$\zeta_\omega^{m,t} = P_{ND,\omega}^{m,t} - \mathbf{1}^T P_{F,\omega}^{m,t}; \forall t \in \mathcal{T}, \forall \omega \in \Upsilon \quad (7.11c)$$

$$\mathbf{1}^T \sigma_G^{m,t} + \mathbf{1}^T \sigma_E^{m,t} + \sigma_D^{m,t} + \sigma_O^{m,t} = 1; \forall t \in \mathcal{T} \quad (7.11d)$$

$$(7.5b)-(7.5o), (7.6b)-(7.6g), (7.7b)-(7.7h), (7.8b)-(7.8c) \quad (7.11e)$$

where $\mathbf{1}$ is a vector of ones with appropriate dimension; (7.11a) is the expected TOC function; μ can be adjusted on the basis of historical data or as per the preference of each MGO. However, it has no impact on the worst scenario *feasibility* of the solution; (7.11b) is the power balance constraint; (7.11c) calculates the uncertain power injections under each scenario; (7.11d) enforces the sum of the PFs to be 1 at every hour so that the uncertain power mismatches are automatically compensated through optimal contributions from the MG components; and (7.11e) consolidates the technical constraints explained earlier. The optimization problem in (7.11) is a quadratic programming problem which can be solved efficiently using convex optimization solvers. Essentially, in (7.11a), the behaviour of μ is similar to a weight factor representing a trade-off between the base scenario (higher μ) and the worst scenario (lower μ) cost functions.

7.4 Optimal Energy and Reserve Sharing via Cooperative Bidding

7.4.1 Distributed Algorithm for Market Clearing

The centralized optimization problem (COP) for the MMG system aggregates the sub-problems in (7.11) for all the MGs constituting the MMG system. The COP turns out to be a convex quadratic programming problem having a *unique* optimum owing to the convex characteristics of (7.11). However, solving the COP requires the full system information of all the MGs to be known. In practice, individual MGs in the MMG system may be owned by separate entities. As such, these entities may be reluctant to share information such as generation costs and setpoints owing to privacy concerns. Furthermore, simultaneously accessing large amounts of data may lead to data traffic congestion issues [18].

Algorithm 7.1: Cooperative Optimal Power Sharing Coordination

Initialize: $k \leftarrow 0$, $\epsilon \leftarrow 10^{-3}$, $\lambda_{\text{eq},\omega}^t = \lambda_{\text{O},\omega}^t(0)$ and $\Delta P_{\text{N},\omega}^t(0) \leftarrow 0$.

```

1 while  $\max_{t \in \mathcal{T}} \max_{\omega \in \mathcal{Y}} |\Delta P_{\text{O},\omega}^t(k) - \Delta P_{\text{N},\omega}^t(k)| \geq \epsilon$  do
2    $k \leftarrow k + 1$ 
3   Update  $\beta_{\text{O},\omega}^{\text{B},t}$  and  $\beta_{\text{O},\omega}^{\text{S},t}$  using (7.14).
   // Waiting for the responses from MGOs
4   Receive  $\ddot{P}_{\text{O},\omega}^{\text{B},m,t}(k)/\ddot{P}_{\text{O},\omega}^{\text{S},m,t}(k)$  from each MGO.
5   Compute  $\lambda_{\omega}^t(k+1)$  using (7.13).
   // Deciding the MEP and the trading with the utility grid
6   if  $\lambda_{\text{O},\omega}^t(k+1) \geq \lambda_{\text{N},\omega}^{\text{B},t}/\eta_{\text{T}}$  then
7      $\lambda_{\text{eq},\omega}^t = \lambda_{\text{N},\omega}^{\text{B},t}/\eta_{\text{T}}$ ,  $P_{\text{N},\omega}^{\text{B},t} = \Delta P_{\text{O},\omega}^t(k)$  ; // Import
8   else if  $\lambda_{\text{O},\omega}^t(k+1) \leq \eta_{\text{T}}\lambda_{\text{N},\omega}^{\text{S},t}$  then
9      $\lambda_{\text{eq},\omega}^t = \eta_{\text{T}}\lambda_{\text{N},\omega}^{\text{S},t}$ ,  $P_{\text{N},\omega}^{\text{S},t} = -\Delta P_{\text{O},\omega}^t(k)$  ; // Export
10  else
11     $\lambda_{\text{eq},\omega}^t = \lambda_{\text{O},\omega}^t(k+1)$ ,  $P_{\text{N},\omega}^{\text{B},t} = P_{\text{N},\omega}^{\text{S},t} = 0$ 
12  end
13   $\Delta P_{\text{N},\omega}^t(k) = P_{\text{N},\omega}^{\text{B},t} - P_{\text{N},\omega}^{\text{S},t}$ 
14 end
```

Algorithm 7.1 summarizes the cooperative bidding scheme proposed in this chapter which mitigates the aforementioned issues. Therein, the COP is decomposed into $|\mathcal{M}|$ local sub-problems of the form in (7.11). Algorithm 7.1 is executed by the PSO. Each MGO separately solves (7.11) and communicates $\ddot{P}_{O,\omega}^{B,m,t} / \ddot{P}_{O,\omega}^{S,m,t}$ (calculated as shown below) to the PSO under all the scenarios $\omega \in \Upsilon$ over the optimization horizon $t \in \mathcal{T}$, as described in Step 4 of Algorithm 7.1.

$$\ddot{P}_{O,\omega}^{B,m,t} = P_{O,\omega}^{B,m,t} + Z_R^m \left(P_{O,\omega}^{B,m,t} \right)^2 / (V_m^B)^2 \quad (7.12a)$$

$$\ddot{P}_{O,\omega}^{S,m,t} = P_{O,\omega}^{S,m,t} - Z_R^m \left(P_{O,\omega}^{S,m,t} \right)^2 / (V_m^S)^2 \quad (7.12b)$$

Subsequently, the PSO estimates the difference between the energy demand and supply bids to recalculate the MEP $\lambda_{eq,\omega}^t$ using (7.13) as described in Steps 5-12 of Algorithm 7.1.

$$\Delta P_{O,\omega}^t(k) = \sum_{m \in \mathcal{M}} \left[\ddot{P}_{O,\omega}^{B,m,t}(k) - \ddot{P}_{O,\omega}^{S,m,t}(k) \right] \quad (7.13a)$$

$$\lambda_{O,\omega}^t(k+1) = \lambda_{O,\omega}^t(k) + \alpha \Delta P_{O,\omega}^t(k) \quad (7.13b)$$

The power trading prices $\beta_{O,\omega}^{B,t}$ and $\beta_{O,\omega}^{S,t}$ for the MGOs are computed as shown in (7.14) using $\lambda_{eq,\omega}^t$.

$$\beta_{O,\omega}^{B,t} = \lambda_{eq,\omega}^t + \tau, \quad \beta_{O,\omega}^{S,t} = \lambda_{eq,\omega}^t - \tau; \quad \forall t \in \mathcal{T}, \quad \forall \omega \in \Upsilon \quad (7.14)$$

The $I_{O,\omega}^t$ in (7.15) should cover any possible expenses incurred for maintaining the system which facilitates the power sharing.

$$I_{O,\omega}^t = \tau \sum_{m \in \mathcal{M}} \left[\ddot{P}_{O,\omega}^{B,m,t} + \ddot{P}_{O,\omega}^{S,m,t} \right]; \quad \forall t \in \mathcal{T}, \quad \forall \omega \in \Upsilon \quad (7.15)$$

In this model, each MGO m solves (7.11) to minimize its TOC for $\beta_{O,\omega}^{B,t}$ and $\beta_{O,\omega}^{S,t}$ provided by the PSO. Consequently, (7.13) arrives at a higher MEP for the next iteration if the net demand is higher to encourage more suppliers or fewer buyers. Conversely, (7.13) arrives at a lower MEP for the next iteration if the net supply is

higher to encourage fewer suppliers or more buyers. Accordingly, (7.13) describes a typical demand and supply market scenario. This can be interpreted as a *market clearing* mechanism since the trading prices are modified with the aim of balancing the net power demand and supply.

The MMG system considered in this chapter is assumed to be operated in the grid-connected mode. The PSO manages the electric power transactions between the MMG system and utility grid based on the per unit spot prices $\lambda_{N,\omega}^{B,t}$ and $\lambda_{N,\omega}^{S,t}$. In practice, $\lambda_{N,\omega}^{B,t} > \lambda_{N,\omega}^{S,t}$ to avoid energy arbitrage [18]. Further, the losses incurred while importing/exporting power from/to the utility grid are mainly associated with the efficiency of the transformer η_T . Accordingly, the buying and selling prices are corrected so that the corresponding buyers and sellers are charged for the losses incurred while importing and exporting power respectively. If the net supply bid is less than the net demand bid at $\lambda_{eq,\omega}^t \geq \lambda_{N,\omega}^{B,t}/\eta_T$, the resultant shortfall is met through power imports from the utility grid as described in Steps 6-7 of Algorithm 7.1. Then, $P_{N,\omega}^{B,t}$ is the power received (at the MMG system side of the transformer) from the utility grid and the cost of the losses is $\frac{1-\eta_T}{\eta_T} P_{N,\omega}^{B,t} \lambda_{N,\omega}^{B,t}$. Similarly, if the net supply bid exceeds the net demand bid at $\lambda_{eq,\omega}^t \leq \eta_T \lambda_{N,\omega}^{S,t}$, the resultant surplus power is exported to the utility grid as described in Steps 8-9 of Algorithm 7.1. Then, $P_{N,\omega}^{S,t}$ is the power injected (at the MMG system side of the transformer) to the utility grid and the cost of the losses is $(1 - \eta_T) P_{N,\omega}^{S,t} \lambda_{N,\omega}^{S,t}$. Thus, $\lambda_{eq,\omega}^t$ will lie in between $\lambda_{N,\omega}^{B,t}/\eta_T$ and $\eta_T \lambda_{N,\omega}^{S,t}$ if the demand and supply bids can be internally matched as shown in Step 11 of Algorithm 7.1.

7.4.2 Convergence Characteristics

The dual problem of the COP can be defined as follows:

$$\max_{\lambda_{eq,\omega}^t} \sum_{m \in \mathcal{M}} \min_{\Psi_{\omega}^{m,t}} C_{\text{exp}}^m \left(\Psi_{\omega}^{m,t}, \lambda_{eq,\omega}^t \right) \quad (7.16a)$$

$$\text{s.t. (7.11b), (7.11e); } \forall m \in \mathcal{M} \quad (7.16b)$$

$$\sum_{m \in \mathcal{M}} [\ddot{P}_{O,\omega}^{B,m,t} - \ddot{P}_{O,\omega}^{S,m,t}] = P_{N,\omega}^{B,t} - P_{N,\omega}^{S,t}, \forall t \in \mathcal{T}, \forall \omega \in \Upsilon \quad (7.16c)$$

According to (7.16), the local cost minimization subproblem described in (7.11) for each MG m is the contribution of the corresponding MG to the Lagrangian function of (7.16) at a given $\lambda_{\text{eq},\omega}^t$. Thus, $\boldsymbol{\lambda}_{\text{eq}} = [\lambda_{\text{eq},0}^1, \lambda_{\text{eq},1}^1, \dots, \lambda_{\text{eq},\mathcal{W}}^T]^T$ comprises the Lagrangian multipliers $\lambda_{\text{eq},\omega}^t$ under scenario ω during hour t relative to the coupling constraint (7.16c), which is the power balance constraint for the MMG system. Algorithm 7.1 is based on the *iterative sub-gradient method* in [121, Ch. 8]. Moreover, the coupling constraint (7.16c) is interpreted in (7.13) and Steps 5-12 of Algorithm 7.1 as sub-gradients of (7.16) at $\lambda_{\text{eq},\omega}^t$. As such, MGs optimize their local resources at a given $\boldsymbol{\lambda}_{\text{eq}}$ as in (7.11) in every iteration and $\boldsymbol{\lambda}_{\text{eq}}$ is iteratively updated in a direction which satisfies (7.16c). The *zero duality gap* in convex optimization problems ensures the uniqueness and the global optimality of the solution obtained for (7.16). Hence, the convergence of the sequence $\{\boldsymbol{\lambda}_{\text{eq}}(k)\}$ is guaranteed within a finite number of iterations for iterative sub-gradient methods depending on the selected α in (7.13).

7.5 Case Studies

7.5.1 Simulation Settings

Numerical simulations are performed on an exemplar MMG system comprising four MGs. In simulations, each MG contains two DGs, two ESSs, dispatchable loads, fixed loads, and RESs. The linear cost coefficients of all the DGs in the MMG system are listed in Table 7.2. Further, for all $m \in \mathcal{M}, g \in \mathcal{G}^m$: $a_{G,g}^m = \$5/\text{MW}^2\text{h}$, $(P_{G,g}^m)^{\max} = 0.5 \text{ MW}$, $(P_{G,g}^m)^{\min} = 0 \text{ MW}$, $\hat{R}_{G,g}^{\Delta,m} = \check{R}_{G,g}^{\Delta,m} = 0.25 \text{ MW/h}$ and $\hat{R}_{G,g}^{\delta,m} = \check{R}_{G,g}^{\delta,m} = 0.2 \text{ MW}$. The ESSs have the following parameters for all $m \in \mathcal{M}, e \in \mathcal{E}^m$: $a_{E,e}^m = \$5/\text{MW}^2\text{h}$, $B_e^m = 1 \text{ MWh}$, $\eta_e^C = \eta_e^D = 0.97$, $(P_{E,e}^{C,m})^{\max} = (P_{E,e}^{D,m})^{\max} = 0.2 \text{ MW}$, $(E_e^m)^{\max} = 0.9$, $(E_e^m)^{\min} = 0.1$, $E_{R,e}^m = 0.5$, and $\delta = 0.2$. The dispatchable

loads have the following parameters for all $m \in \mathcal{M}$: $a_{\text{FD}}^m = \$15/\text{MW}^2\text{h}$, $(P_{\text{RD}}^{m,t})^{\max} = (P_{\text{CD}}^{m,t})^{\max} = 0.4\text{MW}$, and $E_{\text{SH}}^m = 0.75\text{MWh}$. Further, $Z_{\text{R}}^m = 0.005\text{p.u.}$ under 1 MVA base, $(P_{\text{O}}^m)^{\max} = 1.5\text{MW}$, $V_m^{\text{S}} = 1.02\text{p.u.}$, and $V_m^{\text{B}} = 0.98\text{p.u.}$ for all $m \in \mathcal{M}$.

TABLE 7.2: Linear Cost Parameters of DGs

Parameter	MG 1	MG 2	MG 3	MG 4
$b_{\text{G},g}^m$ (\$/MWh)	85	75	80	95

The forecasts for the load demand and RES generation are shown in Fig. 7.2. The electricity buying price forecasts, i.e., $\lambda_{\text{N},0}^{\text{B},t}$ for all $t \in \mathcal{T}$, are the hourly average values extracted from [122]. $\lambda_{\text{N},0}^{\text{S},t} = \lambda_{\text{N},0}^{\text{B},t} - \20 for all $t \in \mathcal{T}$. Symmetrical forecast errors of $[10\%, -10\%]$ and $[5\%, -5\%]$ are considered for the nominal load demand of each MG and the utility grid spot prices respectively. The RES generation forecast errors are considered to be in the range of $[10\%, -40\%]$. Moreover, τ is taken as $\$3/\text{MWh}$ for cooperative trading and $\$1/\text{MWh}$ for isolated trading respectively. Algorithm 7.1 and the MG energy management problem described in (7.11) were programmed in MATLAB and (7.11) was solved using CPLEX. All the simulations were performed on a notebook PC with an Intel®Core i7-7500U two-core CPU processor running at 2.70GHz with 16GB of RAM.

The performance of the proposed RO framework for MMG systems is evaluated under the following three schemes.

1. Deterministic optimization (DO) in which forecasts for the RES generation, load demand, and energy market prices are assumed to be 100% accurate.
2. RO which allocates reserves to mitigate the variations in the load demand and RES generation.
3. RO-PF which allocates reserves while considering optimal PFs for the MG components to mitigate the variations in the load demand and RES generation.

Here, Cases 1-3 are listed in the order of increasing computational complexity in terms of both the number of variables and the constraints. Further, the economical benefits of the cooperative trading scheme is compared with the isolated trading.

FIGURE 7.2: Forecasts for (a) load demand and (b) aggregated RES generation.

7.5.2 Convergence Analysis

Fig. 7.3 illustrates the evolution of the power mismatch and the objective function value for the MMG system with each iteration for both DO and RO-PF at $\alpha = 4$, $\lambda_{O,\omega}^t(0) = 0.5(\lambda_{N,\omega}^{B,t} + \lambda_{N,\omega}^{S,t})$, and $\mu = 0.5$. From Fig. 7.3, it is observed that the change in the objective function value is marginal after the first few iterations. The convergence of $\Delta P = \max_{t \in \mathcal{T}} \max_{\omega \in \Upsilon} |\Delta P_{O,\omega}^t(k) - \Delta P_{N,\omega}^t(k)|$ is almost linear in the log-scale, i.e., an *exponential* convergence in linear-scale. Further, the convergence of the DO scheme is faster as it is less complex (less number of variables and constraints) when compared with the RO-PF scheme. The computation times of the DO and RO-PF schemes are 0.62s (for 16 iterations) and 41.23s (for 120 iterations) respectively, which are reasonably fast for day-ahead computations. It should be noted that the convergence of Algorithm 7.1 is influenced by the step size α . To investigate the sensitivity of α in the speed of convergence, Fig. 7.4 shows the number of iterations required by Algorithm 7.1 to converge to the optimal solution for different values of α . It is observed in Fig. 7.4 that Algorithm 7.1 converges slowly when α is smaller. However, the number of iterations, and the computation time reduce exponentially when α increases. Higher values of α induce toggling in the power mismatch calculated as ΔP , after a few iterations. It is observed that the

cooperative solution *always* converges to the same solution. Consequently, the optimal solution attained using the cooperative trading scheme is independent of α .

FIGURE 7.3: Convergence characteristics of Algorithm 7.1 for (a) DO and (b) RO-PF.

FIGURE 7.4: Number of iterations required for Algorithm 7.1 to converge for different values of α .

7.5.3 Robust Optimal Operation of the MMG System Considering Optimal PFs

Fig. 7.5 shows the dispatches of the DGs in the MMG system at $\mu = 0.5$ under the aforementioned simulation scenarios. It is observed that the dispatches obtained under the base scenario using the DO and the RO-PF schemes significantly differ. The RO-PF framework computes hourly optimal reserve margins while ensuring the ramping feasibility in delivering reserves. It should be noted that the optimal

reserve margins are *not uniform*, i.e., optimal reserve allocation is non-trivial, and hence, a suitable RO framework is required for optimal reserve scheduling. Fig. 7.6 illustrates the trading of energy and reserves between the individual MGOs and the PSO, wherein upper operating bound $\Delta \bar{P}_O^{m,t} = \max_{\omega \in \Upsilon} \{P_{O,\omega}^{B,m,t} - P_{O,\omega}^{S,m,t}\}$, and lower operating bound $\Delta \underline{P}_O^{m,t} = \min_{\omega \in \Upsilon} \{P_{O,\omega}^{B,m,t} - P_{O,\omega}^{S,m,t}\}$. By and large, it is observed that MG 4 behaves as a buyer, while MGs 1-3 behave as both buyers and sellers. Table 7.3 shows the energy and reserve costs of each MG and the revenue of the PSO while using the DO, RO, and RO-PF schemes. It is observed that the expected TOCs of the MGs increase to ensure the robustness of operation and it is further increased for the automated real-time operation with predetermined participation of MG components. The cost of MG 4 is the highest since it mostly depends on power imports as seen in Fig. 7.6(d).

TABLE 7.3: Cost Associated with DO, RO, and RO-PF Models

Market Players	DO (\$/day)	RO (\$/day)		RO-PF (\$/day)	
		$\mu = 0.5$	$\mu = 0.1$	$\mu = 0.5$	$\mu = 0.1$
MG 1	673	1,083	1,405	1,117	1,454
MG 2	541	967	1,304	995	1,354
MG 3	881	1,270	1,552	1,311	1,614
MG 4	2,131	2,546	2,874	2,593	2,942
PSO	-163	-195	-227	-192	-222

7.5.3.1 Role of the PSO and the MGOs in Real-Time Operation

The base scenario power exchange $P_{O,0}^{m,t}$ and the upper and lower bounds are plotted in Figs. 7.6(a)-(d). This bidding information is available with the PSO and the PSO will use this information as a reference to govern the real-time power exchanges of the constituent MGs. In real-time operation, the PSO monitors the power flows of each MG at the PCC, i.e., $\ddot{P}_O^{B,m,t} / \ddot{P}_O^{S,m,t}$. If the real-time power flows of any MG violate the agreed operating interval (i.e., in between red and blue curves for each MG), the PSO imposes a penalty on the respective MG. Accordingly, the PSO maintains the power balance of the MMG system in real-time operation. Moreover,

FIGURE 7.5: (a)-(d) Base scenario power dispatch profiles (black curves), and up and down reserve margins (red and blue curves) obtained using RO-PF approaches and power dispatch profile (magenta curve) obtained using DO approach of DG 1 in 4 MGs. (e)-(h) PFs of DGs against uncertainties of each MG.

FIGURE 7.6: (a)-(d) Power and reserve exchanges between the MGOs and the PSO, where $\Delta P_{O,0}^{m,t} = P_{O,0}^{B,m,t} - P_{O,0}^{S,m,t}$, and the upper and lower operational bounds respectively. (e)-(h) PFs of power exchange against uncertainties of each MG.

Figs. 7.6(e)-(h) depict the optimal PFs (an internal information of MGOs) of power exchanges with the PSO under real-time power mismatch (due to forecast errors). Accordingly, each MGO uses PFs of its own components and the power imports/-exports to mitigate the real-time power mismatches. It can be seen at some hours $\sigma_O^{m,t} = 1$, i.e., uncertainties are completely handled by the power imports/exports. Further, Fig. 7.7 shows the SoC and charging/discharging trajectories of ESS 1 in MG 2 and 3. Fig. 7.8 shows the demand redispatch and curtailment trajectories of MG 2 and 3. Therein, the area in between the two curves (black and blue) of

FIGURE 7.7: Profiles of ESS 1 in MG 2 and 3 for the base scenario power dispatch (black curve), up and down reserve margins (red and blue curves) obtained using the RO-PF approach.

FIGURE 7.8: Flexible demand dispatch profiles under the base scenario (black curve) and the worst scenario (blue curve) in MG 2 and 3 obtained using the RO-PF approach.

Fig. 7.8 represent the possible amount of load shedding under the worst scenario. Furthermore, it can be observed that the contributions of both ESS and flexible demand are mostly in energy management under the base scenario. Their roles in reserve support are less compared to DGs and power imports/exports.

7.5.3.2 Validation of the RO-PF Solution

In this section, the optimality and robustness of the proposed nonanticipative RO-PF framework are compared to those of the state-of-the-art RO-PF frameworks

in [16, 18] in the context of MMG systems. It should be noted that the RO-PF frameworks in [16, Eq. (5)-(11)] and [18, Eq. 9(d)-9(h)] enforce ramp constraints for DGs and ESSs for each scenario separately and auxiliary variables are not considered. In the following, the resulting RO-PF framework is referred to as the scenario-wise RO-PF framework, which is used to benchmark the performance of the proposed nonanticipative RO-PF framework.

Simulations are conducted under four degrees of uncertainties, wherein the sizes of uncertain bounds are varied as,

1. 50% for both demand and RESs,
2. 50% for demand and 100% for RESs,
3. 100% for demand and 50% for RESs, and
4. 100% (original) for both demand and RESs.

An out-of-sample analysis is used to validate the robustness of the optimal scheduling decisions including the PFs computed by the RO-PF frameworks. A set of scenarios comprising 50,000 uncertain realizations are randomly and independently generated following a uniform probability distribution within the hourly confidence intervals of the demand and RESs considered (i.e., the four degrees mentioned above) in the optimization problem. Hourly redispatch decisions of each MG component are computed for each scenario considering the optimal base scenario power dispatch values and PFs.

Table 7.4 reports the expected TOCs and the percentage of the scenarios (out of 200,000 samples for four MGs) which cause constraint violations. It is observed that the cost increase when using the nonanticipative RO-PF solution is marginal ($< 0.2\%$) and is *fully robust* against the pool of scenarios under all degrees of uncertainties, in comparison to the scenario-wise RO-PF solution.

TABLE 7.4: Optimality and Robustness Evaluation of the RO Solutions under Different Degrees of Uncertainties

Case	Scenario-wise RO-PF [16, 18]		Nonanticipative RO-PF		
	TOC (\$/day)	Infeasible Scenarios	TOC (\$/day)	Infeasible Scenarios	Optimality Gap
1	5,039	29.05%	5,041	0%	0.04%
2	5,950	24.98%	5,954	0%	0.08%
3	5,084	34.22%	5,088	0%	0.08%
4	6,007	33.36%	6,016	0%	0.16%

7.5.4 Comparison Between the Cooperative and Isolated Trading Schemes

In this case study, the performances of the cooperative and isolated trading schemes are compared for the RO-PF approach based operation of the MMG system. In the isolated trading scheme, it is assumed that the individual MGOs trade with the utility grid alone at the predetermined $\lambda_{N,\omega}^{S,t}$ and $\lambda_{N,\omega}^{B,t}$. Thus, the energy management problem in (7.11) is individually executed by each MGO at $\beta_{O,\omega}^{S,t} = \lambda_{N,\omega}^{S,t} - \tau$ and $\beta_{O,\omega}^{B,t} = \lambda_{N,\omega}^{B,t} + \tau$. The dispatch profiles of all the DGs in the MMG system obtained for the isolated trading scheme at $\mu = 0.5$ are shown in Fig. 7.9. Compared with the operation under cooperative trading scheme (dispatch profiles shown in Fig. 7.5), the utilization of the cheaper DGs in MGs 2 and 3 under the isolated trading scheme is lower. From Fig. 7.9, it is observed that the most expensive DG in MG 4 is dispatched at full capacity during hours 19-23 under the isolated trading scheme while the relatively cheaper DGs in MGs 2 and 3 are not fully utilized. The comparison illustrates that the bidding framework embedded in the cooperative trading scheme allows MGs 2 and 3 to enhance their revenues while offering MG 4 an opportunity to reduce its expenditure.

This can be further observed in Figs. 7.10 and 7.11 which illustrate the MEPs for the cooperative trading, and the aggregated power exchanges of the MGs with the utility grid under both trading schemes respectively. During some of the hours, for instance in between hours 3-9, etc., the MMG system is self-sustained with

FIGURE 7.9: Base scenario power dispatch profile and up and down reserve margins of the DGs under the isolated trading scheme in the 4 MGs.

cooperative trading under the base scenario and the MEP settles in between $\lambda_{N,0}^{B,t}$ and $\lambda_{N,0}^{S,t}$. This accrues economic benefits to both the buyers (as $\lambda_{eq,0}^t < \lambda_{N,0}^{B,t}$) and sellers (as $\lambda_{eq,0}^t > \lambda_{N,0}^{S,t}$) when compared with the isolated trading scheme.

FIGURE 7.10: Market equilibrium price at the base scenario of the robust operation.

FIGURE 7.11: Interaction in between PSO and the utility grid.

Finally, Fig. 7.12 depicts the variations in the TOC incurred with μ while using the RO-PF approach under the cooperative and isolated trading schemes. A lower μ results in a higher reserve cost which leads to a higher total cost for every MG. Also, it is seen that the cooperative trading scheme enables the sharing of economic resources between the constituent MGs in the MMG system, thereby leading to a lower cost for all the MGs when compared with the isolated trading scheme. The percentage reduction in the expected TOC is around 3%-6% for every MG. In practice, such consistent financial benefits will encourage participants to engage in the cooperative power trading process.

FIGURE 7.12: Expected TOC in cooperative and isolated trading for the 4 MGs along with the percentage cost reduction.

7.6 Summary

An RO framework for the joint optimal scheduling of energy and reserves in an MMG system was presented in this chapter. Participation factors of MG components are optimized to mitigate the power mismatches caused by the uncertainties in demand and RES generation while preserving nonanticipativity of reserve-scheduling. This ensures the component-wise reserve deliverability and the reliability of the load scheduling under all plausible uncertain scenarios. The RO framework included a cooperative trading scheme based on the iterative sub-gradient method to enable the energy and reserve trading among the constituent MGs within the MMG system. The TOAT was used to identify a reduced set of vertex permutations from the polyhedral uncertainty set, which comprises the confidence bounds of the RES generation, demand, and energy market prices.

The proposed RO framework was illustrated via an MMG system comprising four MGs. The results demonstrated the cost differences of the deterministic optimization, robust optimization, and robust optimization with optimal participation factors. The significant differences in the dispatch values and cost for the robust and deterministic optimization reflected the necessity of proper planning of resources to achieve robustness. The convergence characteristics of the trading scheme demonstrated reasonable speed in the context of day-ahead optimization. A validation study based on an out-of-sample analysis showcased that the proposed RO framework outperforms state-of-the-art RO frameworks in terms of robustness with an almost negligible cost increase. Subsequently, the cost-efficiency of the cooperative trading scheme was demonstrated compared to the isolated trading scheme. Therein, the cooperative trading scheme resulted in economic benefits to all the participating MGs owing to the shared utilization of economic resources.

Chapter 8

Conclusions and Recommendation for Future Works

8.1 Conclusions

This thesis aimed to enhance the efficiency, computational performance, and robustness of the operation in future power systems. Novel modeling and solution approaches were developed to accurately address the structural changes and uncertainties, while ensuring the scalability and solution tractability. The solutions of the proposed optimization frameworks in this thesis illustrated improved efficiency, computation time, and robustness compared to the state-of-the-art approaches based on practical-sized systems.

The OPF problem was formulated in Chapter 3 from the first principles for an HTG having multi-terminal DC subgrids coupled with the AC subgrid via VSCs. In Chapter 4 a scalable OPF framework was developed which follows an SLP approach. The polar formulation of the AC-OPF is used to retain the quasi-linearity between active and reactive power dispatches to voltage angles and magnitudes, respectively, which supports faster convergence. A TR was introduced that bounds variations in the decision variables to tighten the linear model, which ensured the

convergence of the TR-SLP to a local optimum of the OPF problem. A feasibility restoration phase was introduced to locate a feasible solution when the SLP approximation is infeasible. The results illustrated the capability of the proposed TR-SLP to find better local optima for the IEEE and Polish systems compared to the best-known set of OPF solutions obtained using KNITRO-MS. Further, TR-SLP approach outperformed KNITRO, IPOPT, and a recently reported SLP method based on penalty reformulations in terms of computation time.

Chapter 5 developed a novel day-ahead power dispatch and reserve scheduling framework for HTGs. The optimization framework combines the unit commitment problem with nonlinear NCs under wind power uncertainties. Polyhedral confidence bounds are used to characterize wind power uncertainties. TOAT is used to select representative scenarios based on the vertex permutations of the polyhedral uncertainty set. Under the selected scenarios, generator-wise hourly reserve allocations are determined while preserving nonanticipativity. That ensures reserve deliverability under all the selected scenarios. A two-stage iterative algorithm was proposed based on the Benders decomposition framework with a complete decomposition of the SPs in the second stage to an OPF problem. In addition, the SPs are formulated uniquely and effectively to utilize the standard OPF solver packages. These salient features of the proposed RO framework enhanced its scalability compared to the state-of-the-art approaches.

In Chapter 6, the proposed RO framework in Chapter 5 was extended with a convex relaxation of AC and DC NCs, which enabled the globally optimal solvability of SPs while ensuring the convergence of RO framework. The results illustrated the merits of a recently proposed HTG architecture to alleviate the transmission congestion due to additional flexibility offered by the multi-terminal DC systems. As such, loadability and RES hosting capacity was enhanced while offering consistently lower generation costs over different load levels. Consequently, the HTG architecture has the potential to support the efficient utilization of the available generation facilities of the existing grids via suitable upgrade measures. Further, the performance of the proposed RO framework was validated using out-of-sample analyses.

In the simulations, the proposed RO framework outperformed a state-of-the-art approach in terms of optimality, computation time, and robustness. Thus, the results showcase the merits of the scalability and the nonanticipativity properties of the proposed RO framework. Furthermore, the improvements in computational performance of the proposed RO framework was shown to be almost unaffected by choice of the scenario selection tool.

An RO framework was developed in Chapter 7 for the joint optimal scheduling of power dispatch and reserves in an MMG system under the uncertainties caused by RES generation, demand, and energy market prices. Reserve scheduling was implemented via optimal proportional participation of MG components to mitigate the real-time power mismatches while preserving nonanticipativity. The RO framework included a cooperative trading scheme based on the iterative sub-gradient method to enable the trading of power and reserves among the constituent MGs within the MMG system. The proposed RO framework was illustrated via an MMG system comprising four MGs. The significant differences in the dispatch values and cost for the robust and deterministic optimization reflected the necessity of proper planning of resources to achieve robustness.

A validation study based on an out-of-sample analysis showcased that the proposed RO framework outperforms a state-of-the-art RO framework in terms of robustness with an almost negligible cost increase. Consequently, the results highlight the importance of nonanticipativity for the component-wise reserve deliverability, and the reliability of the load scheduling in real-time operation. Subsequently, the cost-efficiency of the cooperative trading scheme was demonstrated compared to the isolated trading scheme. Therein, the cooperative trading scheme resulted in economic benefits to all the participating MGs owing to the shared utilization of economic resources.

8.2 Recommendations for Future Works

The TR-SLP approach presented in Chapter 4 in this thesis is very efficient when it is initiated with a feasible point. In general, daily, weekly, and monthly variations of the load profiles are available with the system operators, and they usually follow similar trends. As such, there is a potential to obtain (predict) feasible operating points to initiate the algorithm in practice. Moreover, OPF is traditionally an hourly task. However, with the advancements in communication technologies, hardware setups, and solution approaches, OPF can be executed more frequently (say in 5 minutes intervals). That will improve the economy of operation as well. Further, with lesser interval size, lesser variations in demand and RESs can be expected, leading to more accurate forecasts. Moreover, lesser expected variations in demand and RESs might support for faster convergence of the algorithm with suitable starting points.

Convex relaxations are capable of providing a lower bound for the optimal objective value of OPF problems with lesser optimality gaps in many of the cases. As such, it may be promising to initiate TR-SLP with solutions given by convex relaxations to find feasible and near-optimal solutions. Another possible direction for TR-SLP approach is to develop a robust AC-OPF framework satisfying nonlinear NCs by utilizing the convexity to extract valid dual information. Further, the application to HTGs and distribution grids can also be studied.

The RO framework presented in Chapters 5 and 6 considered interval forecasts to realize a robust optimal operation of the HTGs. The present framework provides hourly generator-wise reserve margins within that the feasibility of the real-time operation is ensured. As a next step, the framework can be extended to utilize the information from both interval and point forecasts. Accordingly, affine decision rules can be determined for the proportional participation of each committed generator to counteract on real-time power mismatches. As such, the robustness of the operation can still be ensured with the less conservative operation.

In addition, the proposed day-ahead power dispatch and reserve scheduling framework in Chapter 5 only consider electrical power supply and demand. In many countries, electricity demand and heat demand are supplied using combined heat (thermal energy) and power (electrical energy) units. As such, the heat dispatch and power dispatch are interrelated and the maximum power capability at a time is restricted by the heat dispatch and vice versa. Therefore, mathematical models proposed in Chapter 5 can be extended considering the day-ahead energy and reserve scheduling of multi-energy systems. Therein, scalable and accurate models are required for boilers, thermal energy storages, etc. Further, thermal reserve scheduling for mitigating the uncertainties involved with heat demand will also be a vital aspect of the optimization problem.

Furthermore, the power dispatch and reserve sharing framework proposed for MMG systems in Chapter 7 is based on localized MGs. In the future, the RO framework can be extended to distant power-sharing of MGs located at different nodes of the distribution grid. As such, benefits can be accrued via power-sharing over the distribution network, in addition to the sharing among localized MGs. This requires convex and scalable NCs models of the distribution network to respect voltage management and power flow congestion limits while power transfer. Therein, convex relaxations and linear approximations can be utilized. The distribution grids are typically radial, and power flows (unidirectionally) from the root-node (PCC with the transmission grid) to the other nodes. The proposed power-sharing would result in reverse power flows which might need reactive power support at some of the nodes as well.

Appendix A

Taguchi's Orthogonal Array Testing

TOAT [111] generates a special set of *orthogonal arrays* (OAs) which represents a fraction of the full factorial combinations of experiments. The general representation of an OA is $L_{\mathcal{W}}B^F$, where $\mathcal{W}$ is the number of scenarios; F is the number of random factors (uncertain variables) and B is the number of levels considered for each random factor [109, 110]. In this thesis, random factors are considered with only two levels, i.e., $B = 2$, defined by the polyhedral confidence bounds. Some examples of 2-level OAs in OA libraries [109, 110] are L_42^3 , L_82^7 , $L_{16}2^{15}$, etc. For instance, the OA L_42^3 specifying four experiments/scenarios that involve (at most) three two-level random factors is depicted below.

$$L_42^3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \quad (\text{A.1})$$

where random factors correspond to the columns and the scenarios correspond to the rows. There are two main properties of OAs. 1) Each column contains equal

number of all levels. For instance, there are two 1s and two 2s in each column of L_42^3 given in (A.1). Thus, equal chances are given for each level of the random factors to influence the output. 2) In any two columns, all combination of levels are appeared equal number of times. Hence, the scenarios given by TOAT are uniformly distributed over the polyhedral uncertainty space.

In this thesis, appropriate OAs are extracted from OA libraries [109, 110]. In case there is no OA in OA libraries which has the number of columns equal to the random factors of the problem, the next biggest OA with more number of columns must be chosen. As per the properties of OAs mentioned above, an OA can be formed for TOAT by ignoring the redundant columns of the chosen OA [111].

Let ζ_f be a random factor with two levels/confidence bounds $\zeta_f(1)$ and $\zeta_f(2)$ with $f \in \{1, 2, 3\}$. Then, the corresponding 4 scenarios specified by the TOAT based on L_42^3 in (A.1) can be represented as in Table A.1.

TABLE A.1: Scenarios Specified by TOAT based on L_42^3

Scenario	Random factor		
ω	1	2	3
1	$\zeta_1(1)$	$\zeta_2(1)$	$\zeta_3(1)$
2	$\zeta_1(1)$	$\zeta_2(2)$	$\zeta_3(2)$
3	$\zeta_1(2)$	$\zeta_2(1)$	$\zeta_3(2)$
4	$\zeta_1(2)$	$\zeta_2(2)$	$\zeta_3(1)$

Appendix B

Latin Hypercube Sampling Method

In the following, the steps of the LHS method (based on [82]) which was used to generate the wind power output samples for the validation study in Section 6.3.6 are described.

1. Specify sample size, $\mathcal{S}$.
2. Divide the range of variability (defined by the polyhedral bounds) of wind power injections during each time period into subintervals with equal probability, $1/\mathcal{S}$.
3. Randomly sample and record a value from each subinterval.
4. Perform a random permutation to generate sample scenarios with wind power outputs from each wind plant over the time horizon.

Appendix C

List of Publications

The author's contributions related to this thesis are included in the following publications.

Journal Publications

1. **L. P. M. I. Sampath**, B. V. Patil, H. B. Gooi, J. M. Maciejowski, and K. V. Ling, "A trust-region based sequential linear programming approach for AC optimal power flow problems," *Electric Power Systems Research*, 165 (2018), 134-143.
2. **L. P. M. I. Sampath**, M. Hotz, H. B. Gooi, and W. Utschick, "Network-constrained thermal unit commitment for hybrid AC/DC transmission grids under wind power uncertainty," *Applied Energy*, vol. 258, pp. 114031, 2020.
3. **L. P. M. I. Sampath**, Ashok Krishnan, Y. S. Foo Eddy, and H. B. Gooi, "Cooperative bidding-based robust optimal energy management of multi-microgrids," *IEEE Transactions on Industrial Informatics*, pp. 1-1, 2019.
4. Ashok Krishnan, **L. P. M. I. Sampath**, Y. S. Foo Eddy and H. B. Gooi "Optimal scheduling of a microgrid including pump scheduling and network constraints," *Complexity*, 2018.

5. B. V. Patil, **L. P. M. I. Sampath**, Ashok Krishnan, K. V. Ling, Jan Maciejowski and H. B. Gooi, "Experiments with hybrid Bernstein global optimization algorithm for the OPF problem in power systems," *Engineering Optimization* (2018), 1-16.

Conference Publications

1. **L. P. M. I. Sampath**, Ashok Krishnan, Kalpesh Chaudhari, H. B. Gooi and Abhisek Ukil, "A control architecture for optimal power sharing between interconnected microgrids," *2017 IEEE Power and Energy Society General Meeting*, Chicago, IL, 2017, pp. 1-5.
2. **L. P. M. I. Sampath**, M. Hotz, H. B. Gooi, and W. Utschick, Unit commitment with AC power flow constraints for a hybrid transmission grid," in *2018 Power Systems Computation Conference (PSCC)*, pp. 1-7, Jun. 2018.
3. B. V. Patil, **L. P. M. I. Sampath**, Ashok Krishnan, K. V. Ling, and H. B. Gooi "Examining the Bernstein global optimization approach to optimal power flow problem," *AIP Conference Proceedings*. Eds. Yaroslav D. Sergeyev, et al. Vol. 1776. No. 1. AIP Publishing, 2016.
4. B. V. Patil, **L. P. M. I. Sampath**, B. K. Ranjan, Ashok Krishnan, and Y. S. F. Eddy, "Bernstein global optimization approach for distributed optimal power flow problem incorporating emission costs," *2017 International Conference on Technological Advancements in Power and Energy (TAP Energy)*, Kollam, 2017, pp. 1-6.
5. Ashok Krishnan, **L. P. M. I. Sampath**, Foo Y. S. Eddy, B. V. Patil and H. B. Gooi, "Multi-energy scheduling using a hybrid systems approach," *IFAC-PapersOnLine*, vol. 51, no. 16, pp. 229-234, 2018. 6th IFAC Conference on Analysis and Design of Hybrid Systems ADHS 2018.

References

- [1] M. Hotz and W. Utschick, “A hybrid transmission grid architecture enabling efficient optimal power flow,” *IEEE Transactions on Power Systems*, vol. 31, pp. 4504–4516, Nov. 2016. [ix](#), [1](#), [2](#), [3](#), [11](#), [25](#), [33](#), [81](#), [82](#), [85](#)
- [2] R. D. Zimmerman, C. E. Murillo-Sanchez, and R. J. Thomas, “MATPOWER: Steady-state operations, planning, and analysis tools for power systems research and education,” *IEEE Transactions on Power Systems*, vol. 26, pp. 12–19, Feb. 2011. [xiii](#), [10](#), [52](#), [54](#), [60](#), [61](#), [76](#), [88](#), [91](#), [96](#), [97](#)
- [3] A. J. Wood, B. F. Wollenberg, and G. B. Sheblé, *Power generation, operation, and control*. John Wiley & Sons, 2013. [1](#), [10](#)
- [4] L. Sampath, B. V. Patil, H. Gooi, J. Maciejowski, and K. Ling, “A trust-region based sequential linear programming approach for AC optimal power flow problems,” *Electric Power Systems Research*, vol. 165, pp. 134 – 143, 2018. [1](#), [10](#)
- [5] A. Castillo, C. Laird, C. A. Silva-Monroy, J. P. Watson, and R. P. O’Neill, “The unit commitment problem with AC optimal power flow constraints,” *IEEE Transactions on Power Systems*, vol. 31, pp. 4853–4866, Nov. 2016. [2](#), [14](#)
- [6] L. P. M. I. Sampath, M. Hotz, H. B. Gooi, and W. Utschick, “Unit commitment with AC power flow constraints for a hybrid transmission grid,” in *2018 Power Systems Computation Conference (PSCC)*, pp. 1–7, Jun. 2018. [3](#), [13](#), [14](#), [33](#), [63](#)

-
- [7] M. Hotz and W. Utschick, “The hybrid transmission grid architecture: Benefits in nodal pricing,” *IEEE Transactions on Power Systems*, vol. 33, pp. 1431–1442, Mar. 2018. [2](#), [3](#), [10](#), [12](#), [13](#), [82](#), [83](#), [85](#), [86](#), [87](#), [88](#)
- [8] G. Arcia-Garibaldi, P. Cruz-Romero, and A. Gómez-Expósito, “Future power transmission: Visions, technologies and challenges,” *Renewable and Sustainable Energy Reviews*, vol. 94, pp. 285 – 301, 2018. [2](#)
- [9] B. Sander, J. Lundquist, I. Gutman, C. Neumann, B. Rusek, and K. Weck, “Conversion of AC multi-circuit lines to AC-DC hybrid lines with respect to the environmental impact,” *Cigre Session, Paris*, 2014. [2](#), [13](#)
- [10] J. Lundkvist, I. Gutman, and L. Weimers, “Feasibility study for converting 380 kV AC lines to hybrid AC/DC lines,” in *Proc. EPRI’s High-Voltage Direct Current & Flexible AC Transmission Systems Conference*, p. 11, 2009.
- [11] ABB Grid Systems, “HVDC Light – It’s time to connect,” tech. rep., ABB AB, Ludvika, Sweden, Dec. 2012. [2](#), [13](#), [32](#), [88](#), [89](#), [90](#)
- [12] M. Hotz, I. Boiarchuk, D. Hewes, R. Witzmann, and W. Utschick, “Reducing the need for new lines in Germany’s energy transition: The hybrid transmission grid architecture,” in *International ETG Congress 2017*, pp. 1–6, Nov. 2017. [3](#)
- [13] A. Hamidi, D. Nazarpour, and S. Golshannavaz, “Multiobjective scheduling of microgrids to harvest higher photovoltaic energy,” *IEEE Transactions on Industrial Informatics*, vol. 14, pp. 47–57, Jan. 2018. [3](#)
- [14] L. Ma, N. Liu, J. Zhang, W. Tushar, and C. Yuen, “Energy management for joint operation of CHP and PV prosumers inside a grid-connected microgrid: A game theoretic approach,” *IEEE Transactions on Industrial Informatics*, vol. 12, pp. 1930–1942, Oct. 2016.
- [15] H. Farzin, M. Fotuhi-Firuzabad, and M. Moeini-Aghaie, “Stochastic energy management of microgrids during unscheduled islanding period,” *IEEE Transactions on Industrial Informatics*, vol. 13, pp. 1079–1087, Jun. 2017. [3](#)

- [16] Y. Li, T. Zhao, P. Wang, H. B. Gooi, L. Wu, Y. Liu, and J. Ye, “Optimal operation of multimicrogrids via cooperative energy and reserve scheduling,” *IEEE Transactions on Industrial Informatics*, vol. 14, pp. 3459–3468, Aug. 2018. [3](#), [19](#), [20](#), [21](#), [110](#), [114](#), [115](#), [131](#), [132](#)
- [17] L. P. M. I. Sampath, A. Krishnan, K. Chaudhari, H. B. Gooi, and A. Ukil, “A control architecture for optimal power sharing among interconnected microgrids,” in *Proc. IEEE PES General Meeting*, Jul. 2017. [19](#)
- [18] Y. Liu, Y. Li, H. B. Gooi, Y. Jian, H. Xin, X. Jiang, and J. Pan, “Distributed robust energy management of a multimicrogrid system in the real-time energy market,” *IEEE Transactions on Sustainable Energy*, vol. 10, pp. 396–406, Jan. 2019. [3](#), [19](#), [20](#), [21](#), [110](#), [114](#), [115](#), [121](#), [123](#), [131](#), [132](#)
- [19] F. Capitanescu, J. M. Ramos, P. Panciatici, D. Kirschen, A. M. Marcolini, L. Platbrood, and L. Wehenkel, “State-of-the-art, challenges, and future trends in security constrained optimal power flow,” *Electric Power Systems Research*, vol. 81, no. 8, pp. 1731 – 1741, 2011. [9](#), [10](#)
- [20] F. Capitanescu, “Critical review of recent advances and further developments needed in ac optimal power flow,” *Electric Power Systems Research*, vol. 136, pp. 57 – 68, 2016. [9](#)
- [21] J. Lavaei and S. H. Low, “Zero duality gap in optimal power flow problem,” *IEEE Transactions on Power Systems*, vol. 27, pp. 92–107, Feb. 2012. [11](#), [12](#)
- [22] W. A. Bukhsh, A. Grothey, . I. M. McKinnon, and P. A. Trodde, “Local solutions of the optimal power flow problem,” *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4780–4788, 2013. [9](#)
- [23] A. Castillo, P. Lipka, J. Watson, S. S. Oren, and R. P. O’Neill, “A successive linear programming approach to solving the IV-ACOPF,” *IEEE Transactions on Power Systems*, vol. 31, pp. 2752–2763, Jul. 2016. [10](#), [11](#), [53](#), [54](#), [56](#), [61](#)

-
- [24] S. Mhanna, G. Verbič, and A. C. Chapman, “Tight LP approximations for the optimal power flow problem,” in *2016 Power Systems Computation Conference (PSCC)*, pp. 1–7, Jun. 2016.
- [25] J. Hörsch, H. Ronellenfitsch, D. Witthaut, and T. Brown, “Linear optimal power flow using cycle flows,” *Electric Power Systems Research*, vol. 158, pp. 126–135, 2018. [10](#)
- [26] A. Garces, “A quadratic approximation for the optimal power flow in power distribution systems,” *Electric Power Systems Research*, vol. 130, pp. 222–229, 2016. [10](#)
- [27] D. Phan and J. Kalagnanam, “Some efficient optimization methods for solving the security-constrained optimal power flow problem,” *IEEE Transactions on Power Systems*, vol. 29, pp. 863–872, Mar. 2014. [10](#)
- [28] F. Capitanescu and L. Wehenkel, “Experiments with the interior-point method for solving large scale optimal power flow problems,” *Electric Power Systems Research*, vol. 95, pp. 276–283, 2013. [10](#)
- [29] V. H. Quintana, G. L. Torres, and J. Medina-Palomo, “Interior-point methods and their applications to power systems: a classification of publications and software codes,” *IEEE Transactions on Power Systems*, vol. 15, pp. 170–176, Feb. 2000. [10](#)
- [30] R. A. Jabr, “A primal-dual interior-point method to solve the optimal power flow dispatching problem,” *Optimization and Engineering*, vol. 4, no. 4, pp. 309–336, 2003. [10](#)
- [31] S. Frank, I. Steponavice, and S. Rebennack, “Optimal power flow: a bibliographic survey I,” *Energy Systems*, vol. 3, no. 3, pp. 221–258, 2012. [10](#)
- [32] B. Stott and O. Alsac, “Optimal power flow—Basic requirements for real-life problems and their solutions,” in *SEPOPE XII Symposium, Rio de Janeiro, Brazil*, 2012. [11](#)

- [33] A. Castillo and R. P. O'Neill, "Survey of approaches to solving the ACOPF," *Federal Energy Regulatory Commission, Tech. Rep*, 2013. [11](#)
- [34] Z. Yang, H. Zhong, Q. Xia, A. Bose, and C. Kang, "Optimal power flow based on successive linear approximation of power flow equations," *IET Generation, Transmission Distribution*, vol. 10, no. 14, pp. 3654–3662, 2016. [11](#)
- [35] A. Wächter and L. T. Biegler, "On the implementation of a primal-dual interior point filter line search algorithm for large-scale nonlinear programming," *Mathematical Programming*, vol. 106, no. 1, pp. 25–57, 2006. [11](#), [53](#)
- [36] R. H. Byrd, J. Nocedal, and R. A. Waltz, *KNITRO: An integrated package for nonlinear optimization*. Springer, 2006. [11](#), [53](#), [54](#)
- [37] X. Bai, H. Wei, K. Fujisawa, and Y. Wang, "Semidefinite programming for optimal power flow problems," *International Journal of Electrical Power and Energy Systems*, vol. 30, no. 6-7, pp. 383–392, 2008. [11](#)
- [38] H. Hijazi, C. Coffrin, and P. V. Hentenryck, "Polynomial SDP cuts for optimal power flow," in *2016 Power Systems Computation Conference (PSCC)*, pp. 1–7, Jun. 2016.
- [39] S. H. Low, "Convex relaxation of optimal power flow—Part I: Formulations and equivalence," *IEEE Transactions on Control of Network Systems*, vol. 1, pp. 15–27, Mar. 2014. [82](#), [83](#), [88](#)
- [40] S. H. Low, "Convex relaxation of optimal power flow—Part II: Exactness," *IEEE Transactions on Control of Network Systems*, vol. 1, pp. 177–189, Jun. 2014. [83](#)
- [41] R. Madani, S. Sojoudi, and J. Lavaei, "Convex relaxation for optimal power flow problem: Mesh networks," *IEEE Transactions on Power Systems*, vol. 30, pp. 199–211, Jan. 2015. [11](#), [82](#)

-
- [42] B. Kocuk, S. S. Dey, and X. A. Sun, “Inexactness of SDP relaxation and valid inequalities for optimal power flow,” *IEEE Transactions on Power Systems*, vol. 31, pp. 642–651, Jan. 2016. [12](#)
- [43] C. Coffrin, H. L. Hijazi, and P. Van Hentenryck, “The qc relaxation: A theoretical and computational study on optimal power flow,” *IEEE Transactions on Power Systems*, vol. 31, pp. 3008–3018, Jul. 2016. [12](#)
- [44] M. Farivar and S. H. Low, “Branch flow model: Relaxations and convexification—Part I,” *IEEE Transactions on Power Systems*, vol. 28, pp. 2554–2564, Aug. 2013. [12](#)
- [45] M. Farivar and S. H. Low, “Branch flow model: Relaxations and convexification—Part II,” *IEEE Transactions on Power Systems*, vol. 28, pp. 2565–2572, Aug. 2013. [12](#)
- [46] H. Gao, J. Liu, L. Wang, and Y. Liu, “Cutting planes based relaxed optimal power flow in active distribution systems,” *Electric Power Systems Research*, vol. 143, pp. 272–280, 2017. [12](#)
- [47] W. Wei, J. Wang, N. Li, and S. Mei, “Optimal power flow of radial networks and its variations: A sequential convex optimization approach,” *IEEE Transactions on Smart Grid*, vol. 8, pp. 2974–2987, Nov. 2017. [12](#)
- [48] M. Lu, H. Nagarajan, R. Bent, S. D. Eksioglu, and S. J. Mason, “Tight piecewise convex relaxations for global optimization of optimal power flow,” in *2018 Power Systems Computation Conference (PSCC)*, pp. 1–7, Jun. 2018. [12](#)
- [49] Z. Qin, Y. Hou, and Yiping Chen, “Convex envelopes of optimal power flow with branch flow model in rectangular form,” in *2015 IEEE Power Energy Society General Meeting*, pp. 1–5, Jul. 2015. [12](#)
- [50] D. Li and X. Li, “Decomposition-based global optimization for optimal design of power distribution systems,” in *2016 IEEE 55th Conference on Decision and Control (CDC)*, pp. 3265–3270, Dec. 2016. [12](#)

- [51] P. A. Ruiz, J. M. Foster, A. Rudkevich, and M. C. Caramanis, “Tractable transmission topology control using sensitivity analysis,” *IEEE Transactions on Power Systems*, vol. 27, pp. 1550–1559, Aug. 2012. [12](#)
- [52] B. Kocuk, S. S. Dey, and X. Sun, “New formulation and strong MISOCP relaxations for AC optimal transmission switching problem,” *IEEE Transactions on Power Systems*, vol. PP, no. 99, pp. 1–1, 2017. [12](#), [13](#)
- [53] M. Hotz and W. Utschick, “*hynet*: An optimal power flow framework for hybrid AC/DC power systems,” *arXiv:1811.10496*, Nov. 2018. [13](#), [31](#), [33](#), [76](#)
- [54] Z. Yang, H. Zhong, A. Bose, Q. Xia, and C. Kang, “Optimal power flow in AC-DC grids with discrete control devices,” *IEEE Transactions on Power Systems*, vol. 33, pp. 1461–1472, Mar. 2018. [13](#)
- [55] M. Baradar, M. R. Hesamzadeh, and M. Ghandhari, “Second-order cone programming for optimal power flow in VSC-type AC-DC grids,” *IEEE Transactions on Power Systems*, vol. 28, pp. 4282–4291, Nov. 2013. [13](#)
- [56] S. Bahrami, F. Therrien, V. W. S. Wong, and J. Jatskevich, “Semidefinite relaxation of optimal power flow for AC-DC grids,” *IEEE Transactions on Power Systems*, vol. 32, pp. 289–304, Jan. 2017. [13](#), [29](#)
- [57] A. Venzke and S. Chatzivasileiadis, “Convex relaxations of probabilistic AC optimal power flow for interconnected AC and HVDC grids,” *IEEE Transactions on Power Systems*, vol. 34, pp. 2706–2718, Jul. 2019. [13](#)
- [58] G. Morales-España, L. Ramírez-Elizondo, and B. F. Hobbs, “Hidden power system inflexibilities imposed by traditional unit commitment formulations,” *Applied Energy*, vol. 191, pp. 223 – 238, 2017. [13](#), [14](#)
- [59] M. Furukakoi, O. B. Adewuyi, H. Matayoshi, A. M. Howlader, and T. Senjyu, “Multi objective unit commitment with voltage stability and PV uncertainty,” *Applied Energy*, vol. 228, pp. 618 – 623, 2018. [14](#)

-
- [60] R. Fernández-Blanco, J. M. Arroyo, and N. Alguacil, “Consumer payment minimization under uniform pricing: A mixed-integer linear programming approach,” *Applied Energy*, vol. 114, pp. 676 – 686, 2014. [14](#)
- [61] H. Quan, D. Srinivasan, A. M. Khambadkone, and A. Khosravi, “A computational framework for uncertainty integration in stochastic unit commitment with intermittent renewable energy sources,” *Applied Energy*, vol. 152, pp. 71 – 82, 2015. [14](#)
- [62] R. Azizipanah-Abarghooee, F. Golestaneh, H. B. Gooi, J. Lin, F. Bavafa, and V. Terzija, “Corrective economic dispatch and operational cycles for probabilistic unit commitment with demand response and high wind power,” *Applied Energy*, vol. 182, pp. 634 – 651, 2016. [14](#)
- [63] Y. Fu, Z. Li, and L. Wu, “Modeling and solution of the large-scale security-constrained unit commitment,” *IEEE Transactions on Power Systems*, vol. 28, pp. 3524–3533, Nov. 2013. [14](#)
- [64] Y. Chen, A. Casto, F. Wang, Q. Wang, X. Wang, and J. Wan, “Improving large scale day-ahead security constrained unit commitment performance,” *IEEE Transactions on Power Systems*, vol. 31, pp. 4732–4743, Nov. 2016. [14](#)
- [65] Á. S. Xavier, F. Qiu, F. Wang, and P. R. Thimmapuram, “Transmission constraint filtering in large-scale security-constrained unit commitment,” *IEEE Transactions on Power Systems*, vol. 34, pp. 2457–2460, May 2019. [14](#)
- [66] Y. Fu, M. Shahidehpour, and Z. Li, “Security-constrained unit commitment with AC constraints,” *IEEE Transactions on Power Systems*, vol. 20, pp. 1538–1550, Aug. 2005. [14](#), [17](#)
- [67] N. Amjady, S. Dehghan, A. Attarha, and A. J. Conejo, “Adaptive robust network-constrained AC unit commitment,” *IEEE Transactions on Power Systems*, vol. 32, pp. 672–683, Jan. 2017. [15](#), [16](#), [17](#), [18](#), [63](#), [64](#), [68](#), [75](#), [97](#), [100](#)

- [68] A. Nasri, S. J. Kazempour, A. J. Conejo, and M. Ghandhari, “Network-constrained AC unit commitment under uncertainty: A Benders’ decomposition approach,” *IEEE Transactions on Power Systems*, vol. 31, pp. 412–422, Jan. 2016. [15](#), [17](#), [18](#), [63](#), [68](#), [75](#), [77](#), [97](#), [100](#), [101](#), [102](#), [103](#), [104](#)
- [69] A. Lotfjou, M. Shahidehpour, Y. Fu, and Z. Li, “Security-constrained unit commitment with AC/DC transmission systems,” *IEEE Transactions on Power Systems*, vol. 25, pp. 531–542, Feb. 2010. [14](#), [17](#)
- [70] M. Kabir, Y. Mishra, and R. Bansal, “Probabilistic load flow for distribution systems with uncertain PV generation,” *Applied Energy*, vol. 163, pp. 343 – 351, 2016. [15](#), [17](#), [101](#)
- [71] J. Aghaei, A. Nikoobakht, P. Siano, M. Nayeripour, A. Heidari, and M. Mardaneh, “Exploring the reliability effects on the short term AC security-constrained unit commitment: A stochastic evaluation,” *Energy*, vol. 114, pp. 1016 – 1032, 2016. [15](#), [17](#)
- [72] Q. Zhai, X. Li, X. Lei, and X. Guan, “Transmission constrained UC with wind power: An all-scenario-feasible MILP formulation with strong nonanticipativity,” *IEEE Transactions on Power Systems*, vol. 32, pp. 1805–1817, May 2017. [15](#), [16](#), [17](#), [18](#), [19](#), [64](#), [65](#), [66](#), [111](#)
- [73] N. G. Cobos, J. M. Arroyo, N. Alguacil, and A. Street, “Network-constrained unit commitment under significant wind penetration: A multistage robust approach with non-fixed recourse,” *Applied Energy*, vol. 232, pp. 489 – 503, 2018. [15](#), [16](#), [18](#), [19](#), [64](#), [90](#), [91](#), [97](#)
- [74] D. Bertsimas, E. Litvinov, X. A. Sun, J. Zhao, and T. Zheng, “Adaptive robust optimization for the security constrained unit commitment problem,” *IEEE Transactions on Power Systems*, vol. 28, pp. 52–63, Feb. 2013. [16](#)
- [75] B. Zeng and L. Zhao, “Solving two-stage robust optimization problems using a column-and-constraint generation method,” *Operations Research Letters*, vol. 41, no. 5, pp. 457 – 461, 2013. [15](#), [16](#)

-
- [76] P. Pinson and G. Kariniotakis, “Conditional prediction intervals of wind power generation,” *IEEE Transactions on Power Systems*, vol. 25, pp. 1845–1856, Nov. 2010. [15](#), [64](#)
- [77] A. Attarha, N. Amjady, and A. J. Conejo, “Adaptive robust AC optimal power flow considering load and wind power uncertainties,” *International Journal of Electrical Power & Energy Systems*, vol. 96, pp. 132 – 142, 2018. [16](#), [18](#), [100](#)
- [78] Y. Zhang, N. Gatsis, and G. B. Giannakis, “Robust energy management for microgrids with high-penetration renewables,” *IEEE Transactions on Sustainable Energy*, vol. 4, pp. 944–953, Oct. 2013. [17](#), [20](#), [65](#), [111](#)
- [79] Y. Xiang, J. Liu, and Y. Liu, “Robust energy management of microgrid with uncertain renewable generation and load,” *IEEE Transactions on Smart Grid*, vol. 7, pp. 1034–1043, Mar. 2016. [17](#), [20](#), [65](#)
- [80] W. Zhang, Y. Xu, Z. Dong, and K. P. Wong, “Robust security constrained-optimal power flow using multiple microgrids for corrective control of power systems under uncertainty,” *IEEE Transactions on Industrial Informatics*, vol. 13, pp. 1704–1713, Aug. 2017. [17](#), [20](#)
- [81] H. Yu and W. D. Rosehart, “An optimal power flow algorithm to achieve robust operation considering load and renewable generation uncertainties,” *IEEE Transactions on Power Systems*, vol. 27, pp. 1808–1817, Nov. 2012. [17](#), [65](#), [66](#)
- [82] P. Jirutitijaroen and C. Singh, “Comparison of simulation methods for power system reliability indexes and their distributions,” *IEEE Transactions on Power Systems*, vol. 23, pp. 486–493, May 2008. [17](#), [101](#), [102](#), [103](#), [145](#)
- [83] W. Gan, X. Ai, J. Fang, M. Yan, W. Yao, W. Zuo, and J. Wen, “Security constrained co-planning of transmission expansion and energy storage,” *Applied Energy*, vol. 239, pp. 383 – 394, 2019. [17](#)

- [84] Y. Bai, H. Zhong, Q. Xia, C. Kang, and L. Xie, “A decomposition method for network-constrained unit commitment with AC power flow constraints,” *Energy*, vol. 88, pp. 595 – 603, 2015. [17](#)
- [85] E. J. Ng and R. A. El-Shatshat, “Multi-microgrid control systems (MMCS),” in *IEEE PES General Meeting*, pp. 1–6, Jul. 2010. [19](#)
- [86] Y. Wang, S. Mao, and R. M. Nelms, “On hierarchical power scheduling for the macrogrid and cooperative microgrids,” *IEEE Transactions on Industrial Informatics*, vol. 11, pp. 1574–1584, Dec. 2015. [20](#)
- [87] D. Gregoratti and J. Matamoros, “Distributed energy trading: The multiple-microgrid case,” *IEEE Transactions on Industrial Electronics*, vol. 62, pp. 2551–2559, Apr. 2015. [20](#)
- [88] F. Luo, Y. Chen, Z. Xu, G. Liang, Y. Zheng, and J. Qiu, “Multiagent-based cooperative control framework for microgrids’ energy imbalance,” *IEEE Transactions on Industrial Informatics*, vol. 13, pp. 1046–1056, Jun. 2017. [19](#)
- [89] T. Zhao, Z. Li, and Z. Ding, “Consensus-based distributed optimal energy management with less communication in a microgrid,” *IEEE Transactions on Industrial Informatics*, pp. 1–1, 2018. [19](#)
- [90] M. H. Cintuglu and O. A. Mohammed, “Behavior modeling and auction architecture of networked microgrids for frequency support,” *IEEE Transactions on Industrial Informatics*, vol. 13, pp. 1772–1782, Aug. 2017. [19](#)
- [91] D. Xu, B. Zhou, K. W. Chan, C. Li, Q. Wu, B. Chen, and S. Xia, “Distributed multi-energy coordination of multi-microgrids with biogas-solar-wind renewables,” *IEEE Transactions on Industrial Informatics*, pp. 1–1, 2018. [19](#), [20](#)
- [92] V. Bui, A. Hussain, and H. Kim, “A multiagent-based hierarchical energy management strategy for multi-microgrids considering adjustable power and demand response,” *IEEE Transactions on Smart Grid*, vol. 9, pp. 1323–1333, Mar. 2018. [19](#)

-
- [93] M. M. Esfahani, A. Hariri, and O. A. Mohammed, “A multiagent-based game-theoretic and optimization approach for market operation of multimicrogrid systems,” *IEEE Transactions on Industrial Informatics*, vol. 15, pp. 280–292, Jan. 2019. [19](#), [20](#)
- [94] C. Dou, D. Yue, X. Li, and Y. Xue, “MAS-based management and control strategies for integrated hybrid energy system,” *IEEE Transactions on Industrial Informatics*, vol. 12, pp. 1332–1349, Aug. 2016. [20](#)
- [95] X. Yang, H. He, Y. Zhang, Y. Chen, and G. Weng, “Interactive energy management for enhancing power balances in multi-microgrids,” *IEEE Transactions on Smart Grid*, pp. 1–1, 2019. [19](#)
- [96] CIGRE Working Group B4.57, “Guide for the development of models for HVDC converters in a HVDC grid,” tech. rep., Dec. 2014. [31](#)
- [97] Q. Zhao, J. García-González, O. Gomis-Bellmunt, E. Prieto-Araujo, and F. Echavarren, “Impact of converter losses on the optimal power flow solution of hybrid networks based on VSC-MTDC,” *Electric Power Systems Research*, vol. 151, pp. 395 – 403, 2017. [31](#), [32](#)
- [98] K. Friedrich, “Modern HVDC PLUS application of VSC in modular multi-level converter topology,” in *2010 IEEE International Symposium on Industrial Electronics*, pp. 3807–3810, Jul. 2010. [32](#), [33](#)
- [99] J. Beerten, S. Cole, and R. Belmans, “Generalized steady-state VSC MTDC model for sequential AC/DC power flow algorithms,” *IEEE Transactions on Power Systems*, vol. 27, pp. 821–829, May 2012. [33](#)
- [100] S. Pajic and K. A. Clements, “Power system state estimation via globally convergent methods,” *IEEE Transactions on Power Systems*, vol. 20, pp. 1683–1689, Nov. 2005. [41](#)
- [101] W. Min and L. Shengsong, “A trust region interior point algorithm for optimal power flow problems,” *International Journal of Electrical Power and Energy Systems*, vol. 27, no. 4, pp. 293–300, 2005. [41](#)

REFERENCES

- [102] A. A. Sousa, G. L. Torres, and C. A. Canizares, “Robust optimal power flow solution using trust region and interior-point methods,” *IEEE Transactions on Power Systems*, vol. 26, pp. 487–499, May 2011. [41](#)
- [103] W. Sheng, K. Liu, and S. Cheng, “Optimal power flow algorithm and analysis in distribution system considering distributed generation,” *IET Generation, Transmission and Distribution*, vol. 8, no. 2, pp. 261–272, 2014. [41](#)
- [104] R. H. Byrd, Nicholas I.M. Gould, J. Nocedal, and R. A. Waltz, “An algorithm for nonlinear optimization using linear programming and equality constrained subproblems,” *Mathematical Programming*, vol. 100, no. 1, pp. 27–48, 2004. [50](#)
- [105] The IBM Corp., CPLEX Version 12.6.3, New York, US, 2016. [54](#)
- [106] D. Lee, K. Turitsyn, D. K. Molzahn, and L. A. Roald, “Feasible path identification in optimal power flow with sequential convex restriction,” *arXiv:1906.09483*, Nov. 2019. [61](#)
- [107] H. Ye and Z. Li, “Robust security-constrained unit commitment and dispatch with recourse cost requirement,” *IEEE Transactions on Power Systems*, vol. 31, pp. 3527–3536, Sep. 2016. [65](#), [68](#)
- [108] D. P. Bertsekas, *Convex optimization theory*. Athena Scientific Belmont, 2009. [65](#), [111](#)
- [109] “Orthogonal arrays (Taguchi designs).” <https://www.york.ac.uk/depts/maths/tables/orthogonal.htm>. Accessed: 2019-10-25. [65](#), [143](#), [144](#)
- [110] M. S. Phadke, *Quality Engineering Using Robust Design*. Upper Saddle River, NJ, USA: Prentice Hall PTR, 1st ed., 1995. [65](#), [143](#), [144](#)
- [111] Y. Hong, F. Lin, and T. Yu, “Taguchi method-based probabilistic load flow studies considering uncertain renewables and loads,” *IET Renewable Power Generation*, vol. 10, no. 2, pp. 221–227, 2016. [65](#), [66](#), [90](#), [103](#), [111](#), [143](#), [144](#)

-
- [112] A. M. Geoffrion, “Generalized Benders decomposition,” *Journal of optimization theory and applications*, vol. 10, no. 4, pp. 237–260, 1972. [73](#), [81](#), [82](#)
- [113] C. Coffrin, R. Bent, K. Sundar, Y. Ng, and M. Lubin, “PowerModels.jl: An open-source framework for exploring power flow formulations,” in *2018 Power Systems Computation Conference (PSCC)*, pp. 1–8, Jun. 2018. [76](#)
- [114] MOSEK ApS, “The MOSEK optimization toolbox for MATLAB manual. version 7.1 (revision 53),” 2016. [85](#)
- [115] B. Subhonmesh, S. H. Low, and K. M. Chandy, “Equivalence of branch flow and bus injection models,” in *Proc. 50th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pp. 1893–1899, Oct. 2012. [86](#)
- [116] Reliability Test System Task Force, “The IEEE reliability test system-1996. A report prepared by the reliability test system task force of the application of probability methods subcommittee,” *IEEE Transactions on Power Systems*, vol. 14, pp. 1010–1020, Aug. 1999. [90](#)
- [117] I. Staffell and S. Pfenninger, “Using bias-corrected reanalysis to simulate current and future wind power output,” *Energy*, vol. 114, pp. 1224 – 1239, 2016. [90](#)
- [118] J. M. Morales, S. Pineda, A. J. Conejo, and M. Carrion, “Scenario reduction for futures market trading in electricity markets,” *IEEE Transactions on Power Systems*, vol. 24, pp. 878–888, May 2009. [101](#), [102](#), [103](#), [105](#)
- [119] B. M. Weedy, B. J. Cory, N. Jenkins, J. B. Ekanayake, and G. Strbac, *Electric power systems*. John Wiley & Sons, 2012. [119](#)
- [120] N. Liu, X. Yu, C. Wang, C. Li, L. Ma, and J. Lei, “Energy-sharing model with price-based demand response for microgrids of peer-to-peer prosumers,” *IEEE Transactions on Power Systems*, vol. 32, pp. 3569–3583, Sep. 2017. [119](#)

REFERENCES

- [121] D. P. Bertsekas, *Convex Analysis and Optimization*. Athena Scientific, 2003.
124
- [122] Energy Market Company, *Uniform Singapore Energy Price (USEP) Information*. <https://www.emcsg.com/marketdata/priceinformation>, 16 Jul. 2018.
125