Game theoretic sponsored content management in mobile data market

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2020

https://hdl.handle.net/10356/137399

https://doi.org/10.32657/10356/137399

Nanyang Technological University

Game Theoretic Sponsored Content Management in Mobile Data Market

A thesis submitted to
School of Computer Science and Engineering
Nanyang Technological University

by

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in partial fulfilment of the requirement for the degree of
Doctor of Philosophy

March 13, 2020

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

Xiong Zehui

October 30, 2019
Date

Xiong Zehui

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

October 30, 2019
Date

Dusit Niyato

Authorship Attribution Statement

This thesis contains material from 2 paper published in the following peer-reviewed journal and from 3 papers accepted at conferences in which I am listed as an author.

Chapter 3 is published as
"Z. Xiong, S. Feng, D. Niyato, P. Wang, Y. Zhang, and B. Lin, "A Stackelberg game approach for sponsored content management in mobile data market with network effects", *IEEE Internet of Things Journal*, DOI: 10.1109/JIOT.2020.2975804, to appear, 2020,"
and
"Z. Xiong, S. Feng, D. Niyato, P. Wang, and Y. Zhang, "Economic analysis of network effects on sponsored content: A hierarchical game theoretic approach", in *IEEE Global Communications Conference (GLOBECOM)*, Singapore, December 2017,"
and
"Z. Xiong, S. Feng, D. Niyato, P. Wang, and Y. Zhang, "Competition and cooperation analysis for data sponsored market: A network effects model", in *IEEE Wireless Communications and Networking Conference (WCNC)*, Barcelona, Spain, April 2018."

The contributions of the co-authors are as follows:

- Prof. Niyato provided the initial research direction.
- I proposed the system model, formulated the optimization problem, performed the game theory analysis, designed the game theory algorithm, and implement all the simulation codes. I derived and explored the engineering insights from the numerical results.

- I finished the manuscript draft. Mr. Feng contributed in the mathematical proof parts. Prof. Niyato and Prof. Wang edited the manuscript. Dr. Zhang, and Dr. Lin contributed to the revision.

Chapter 4 is published as
"Z. Xiong, J. Zhao, D. Niyato, P. Wang, and Y. Zhang, "Design of contract-based sponsorship scheme in Stackelberg game for sponsored content market", to be presented in *IEEE Global Communications Conference (GLOBECOM)*, Waikoloa, HI, USA, December 2019."

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- Prof. Niyato provided the initial research direction.
- I proposed the system model, developed the contract problem, performed the contract analysis, designed the algorithm, and implement all the simulation codes.
- I derived and explored the engineering insights from the numerical results.
- I finished the manuscript draft. Prof. Zhao contributed in the mathematical proof parts. Prof. Niyato and Prof. Wang edited the manuscript. Dr. Zhang contributed to the revision.

Chapter 5 is published as
"Z. Xiong, S. Feng, D. Niyato, P. Wang, A. Leshem, and Z. Han, "Joint sponsored and edge caching content service market: A game-theoretic approach", *IEEE Transactions on Wireless Communications*, vol. 18, no. 2, pp. 1166-1181, 2019,"
and
"Z. Xiong, S. Feng, D. Niyato, P. Wang, A. Leshem, and Y. Zhang, "Game theoretic analysis for joint sponsored and edge caching content service market", in *IEEE Global*

Communications Conference (GLOBECOM), Abu Dhabi, UAE, December 2018."

The contributions of the co-authors are as follows:

- Prof. Niyato and Prof. Wang provided the initial project direction.
- I proposed the system model as well as the problem formulation, studied and addressed the problem using game theory.
- I designed the algorithm, implemented all the relevant simulation coding to conducted the performance evaluation. I obtained and analyzed the engineering insights from the numerical results.
- I prepared and finished the manuscript drafts. Mr. Feng contributed in the mathematical proof parts. Prof. Niyato, Prof. Wang, and Prof. Han edited the manuscript. All authors including Prof. Amir and Dr. Zhang contributed to the revision.

Part of Chapter 6 is published as
"Z. Xiong, Y. Zhang, D. Niyato, R. Deng, P. Wang, and L-C. Wang, "Deep reinforcement learning for mobile 5G and beyond: Fundamentals, applications and challenges", *IEEE Vehicular Technology Magazine*, vol. 14, no. 2, pp. 44-52, 2019."

The contributions of the co-authors are as follows:

- Prof. Niyato and Prof. Wang provided the initial project direction.
- I performed the survey of literature review, and proposed the system model. I designed the framework of deep reinforcement learning for resource management in 5G with the assistance of Dr. Zhang.
- I co-implemented the relevant simulation coding with Dr. Zhang to conducted the performance evaluation. I analyzed the engineering insights from the numerical results.

- I wrote the drafts of the manuscript. The manuscript was modified together with Dr. Zhang and Prof. Niyato. All authors contributed to the revision.

Xiong Zehui

October 30, 2019
Date

Xiong Zehui

Acknowledgments

First of all, I would like to express my most sincere gratitude towards my supervisor, Prof. Dusit Niyato for his excellent guidance and consistent support during the past years of my doctoral study at Nanyang Technological University (NTU). I am truly inspired by his immense knowledge and research enthusiasm. He always shows me the cutting-edge research topics and novel ideas, along with his unwavering encouragements. It has been a wonderful time working with him, for his patience, guidance, excellent personality as well as passion for the research.

In addition, I would greatly thank my co-supervisor, Professor Wang Ping for her helpful and patient guidance. Her advice on both research and future career are invaluable to me. I would like to thank my thesis thesis advisory committee including Prof. Ma Maode and Prof. Wen Yonggang, for offering valuable insights on my future research directions and possible methods. I would also like to thank my all my collaborators: Prof. Han Zhu, Prof. Dong In Kim, Prof. Amir Leshem, Prof. Luo Jun, Prof. Wang (Shuxun) Shaun, Prof. Deng Ruilong, Prof. Zhao Jun, Prof. Wang Li-Chun, Prof. Mohsen Guizani, Prof. Zhang Junshan, Prof. Liang Ying-Chang, Prof. Shen (Sherman) Xuemin, and Prof. H. Vincent Poor. I really appreciate all of their efforts on the research work and I enjoy working with them together. Especially, I appreciate Prof. H. Vincent Poor a lot for his kind host at Princeton University, where I had a very fulfilling research visit.

I am also greatly indebted to my great group colleagues, Dr. Zhang Yang, Dr. Kang Jiawen, Dr. Wang Wenbo, Dr. Dinh Thai Hoang, Dr. Mohammad Abu Alsheikh, Feng Shaohan, Jiao Yutao, Nguyen Cong Luong, Suttinee Sawadsitang, Kongrath Suankaew-manee, and Ong Shen Hoong Kevin, for their friendship and insightful discussions. I also want to thank many other friends at NTU for their help and support, where the thesis has been finished and most time of my Ph.D research studies has been spent. Thanks to them, I live a wonderful life here.

Last but no least, I sincerely thank my parents, from the bottom of my heart. With their unconditional love, encouragement, and constant company, my Ph.D life has become much more delightful. Their love will endless motivate me to overcome any difficulties in the rest of my life.

Contents

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Abstract

A sponsored content policy enables a content provider to pay a mobile network operator, and thereby mobile users can access contents from the content provider through network services from the network operator with lower charge. As a result, the content provider and users are both actively engaged into the sponsored content ecosystem. As such, a key challenge is how to provide proper sponsorship in the situation of complex tripartite interactions among the three entities, namely, the network operator, the content provider, and users. In this thesis, we mainly study the game-theoretic interactions among the three entities under the sponsored content policy.

In the first part of the thesis, we model the interactions as a hierarchical Stackelberg game, where the network operator and the content provider act as the leaders determining the pricing and sponsoring strategies, respectively, and the users act as the followers deciding on their content demand. The model incorporates the network effects in a social domain and congestion in a network domain which enables us to obtain more insights from the sponsored content policy. We adopt the matrix formulation to characterize the game equilibrium solution and leverage the theory of matrix to assist our analysis. Through backward induction, the existence and uniqueness of Stackelberg equilibrium are validated analytically.

In the second part of the thesis, we propose to tackle the challenge of the information asymmetry between the content provider and users using a Stackelberg game-based framework. In the framework, the network operator acts as the leader, and the content provider as well as users act as the followers. We model the interaction between the content provider and the users as a contract game in the presence of asymmetric information. In the contract game, the content provider designs a contract that contains its sponsorship strategies toward all types of users. We then derive the necessary and

sufficient conditions of feasible contracts and obtain an optimal contract to maximize the profit of the content provider. Taking into account the optimal contract of contract game, we also investigate the optimal pricing of the network operator through backward induction.

Meanwhile, with the forthcoming of 5G, edge caching becomes a promising technology for traffic offloading and improve service quality of the contents. The key idea is that an edge caching content provider caches content on edge networks. The cached content is then delivered to users locally, reducing latency substantially. In the third part of the thesis, we propose the joint sponsored and edge caching content service market model. We investigate an interplay between the normal sponsored content provider and the edge caching content provider under the non-cooperative game framework. Furthermore, the interactions among the network operator, content providers and users are modeled as a hierarchical three-stage Stackelberg game. In the game model, we explore the sub-game perfect equilibrium in each stage systematically and analytically. The existence of the proposed Stackelberg equilibrium is validated by capitalizing on the bilevel optimization programming. Based on the analysis of the game properties, we propose a sub-gradient based iterative algorithm, which guarantees to converge to the Stackelberg equilibrium.

In summary, this thesis addresses a few urgent challenging problems with the tools of game theory in the real implementation of sponsored content. We also conduct extensive numerical simulations to illustrate some important properties of the equilibrium, and confirm the effectiveness of the proposed game models and sponsorship schemes in sponsored content. Finally, we outline several promising research directions for the future work.

Chapter 1

Introduction

In this chapter, we present the research scope of our work, i.e. ,the overview of sponsored content in mobile data market. The research challenges and motivations therein are given. The key research contributions and the thesis organization are then summarized and provided.

1.1 Research Scope

Currently, smart phones facilitate people to interact and share information online with their friends, which leads to a huge information flow in the forms of cellular data traffic. Due to the booming increase in the consumption and usage of cellular services, the data cost becomes one of the critical concerns for mobile users. Thus, the mobile users are implicitly or explicitly forced to limit their data consumption on content access from content providers, e.g., Youtube and Facebook. However, the content providers are economically dependent on the content volume consumed by mobile users, mainly via displayed advertisements. For this reason, the content providers have an incentive to help partially sponsor the mobile users' data usage. In response to this, the concept of *sponsored content* has been introduced by the mobile network operator as a new business model. *Sponsored Content* in mobile data market has been a promising business approach to accommodate the data market growth and ensure the network operator profit. This

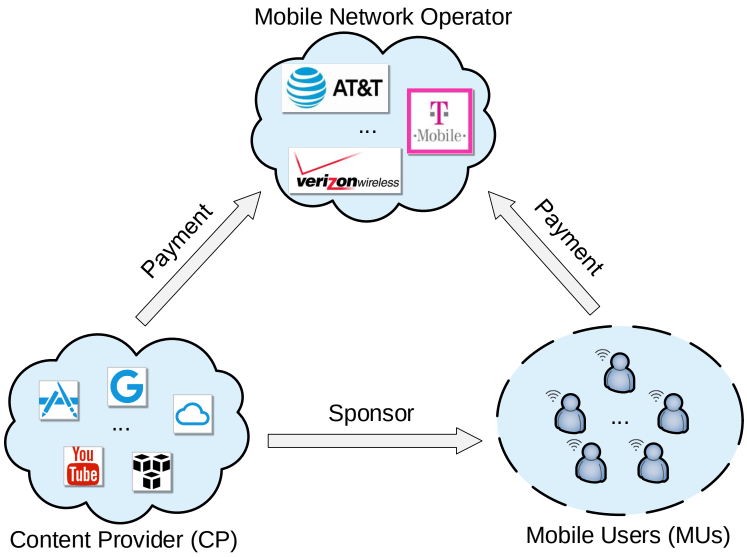


Figure 1.1: A sketch of sponsored content.

scheme exploits a properly designed incentive mechanism to motivate mobile users to access and consume more content actively, yielding a higher revenue to the mobile service businesses.

One representative example is that AT&T launched a data sponsored policy in 2014 [1], where the content providers can absorb their users' cellular data cost, and thereby the users access the content provider's contents through the network operator with lower charge. Another real-world example is that Google joined with Indian operator Airtel to offer free access to certain Google-based services such as Google's email and Google+ via Google search without ringing up data charges [2]. Subsequently, a huge number of third-party companies now are constructing and providing interface platforms between content providers and mobile network operators, such as Aquto and Datami [3].

Figure 1.1 depicts the interactions among different entities in the sponsored content ecosystem. Clearly, the concept of sponsored content provides a fundamentally novel paradigm with a virtuous cycle. First, the mobile users, i.e., the subscribers of network content services, are encouraged to have a deeper engagement with the content providers.

Second, the content provider gains more advertisement earnings which compensate for or even surpass the sponsorship cost. Third, the active engagement of mobile users, in turn, contributes to the revenue gain of the mobile network operator. Intuitively, such a market scheme creates a novel paradigm with respect to who should pay for the network bandwidth, which potentially leads to a triple-win outcome for mobile users, content providers and mobile network operator. Since it was introduced by AT&T in 2014, the sponsored content scheme has been adopted rapidly with more and more companies identifying its potential business value.

However, this concept creates complex interactions among the entities in sponsored content ecosystem: mobile network operator, content providers and mobile users [4]. As such, a key challenge is how to provide proper sponsorship in the situation of complex tripartite interactions among the three entities, especially when they seek for optimal market strategies non-cooperatively. We discern the fact that all the entities in sponsored content can be fully self-interested or selfish, i.e., they make rational decisions with the only consideration of their individual objectives. Furthermore, considering the requirements for distributed operation of the autonomous individuals and the algorithms that can efficiently represent strategic interactions (e.g., competitive or collaborative scenarios), we are motivated to employ the tool of game theory. *Game theory* is a powerful tool to study complex interdependent interactions among selfish players and predict their strategy choices, which has been widely applied in wireless communications and mobile computing systems [5, 6]. In this thesis, we aim to address the sponsorship problem in a unified game theory-based framework.

Naturally, the coexistence of the mobile network operator, the content provider, and mobile users leads to a hierarchical market structure in mobile data market. As such, we resort to the hierarchical Stackelberg game to capture the hierarchical interactions among the main entities in sponsored content. The reason is that the Stackelberg game is especially suitable to be applied in the networks including multiple entities within a hierarchical market structure. A Stackelberg game models the sequential decision making among players in a hierarchical way, where a leader chooses its strategy before followers [7, 8]. The Stackelberg game has been applied in various wireless networks and applications, such as cognitive radio networks [9], fog computing [10], and data center networks [11]. For example, the authors in [9] studied a spectrum sharing problem in cognitive radio networks, in which the Stackelberg game is adopted for deciding on the best strategies towards potential spectrum recall. In [10], the authors formulated a hierarchical Stackelberg game to solve the resource management in fog computing, where the game-theoretic modeling of the market and pricing strategies are proposed. In [11], the authors studied the resource allocation issue for the data center networks by modeling the interactions among the data center operators and service subscribers as a Stackelberg game.

Nevertheless, there exists a few urgent challenges in sponsored content which have not been well addressed in the literature, as shown in the next section.

1.2 Research Challenges and Motivations

In this section, we list a series of urgent research challenges in sponsored content, which exactly motivate us to conduct the research studies in this thesis.

1.2.1 Impacts of Network Effects on Sponsored Content

Inherently, the contents demanded by mobile users belong to the information goods, and (social) network effects are a predominant characteristic of information economies [12, 13]. Intuitively, information services can encourage mobile users to be deeply engaged by promoting their interactions with each other through these services (e.g., online games and social networking communities). For example, when one user watches and posts a video on the social network, the likelihood that his/her friends will do the same is very high. That is, the participation of one user in such content service can potentially promote his/her social friends to join and enjoy the content service. In turn, the user may feel more motivated and assured to consume the content service provided that his/her social friends are also in the same content service. Accordingly, the network effect in the social domain potentially increases the benefit (in terms of utility) of the user since more of them can share information and motivate each other. Formally, network effect refers to such positive effect that one user of social service has on the value of that service to other users [14, 15, 16]. Moreover, such network effect can capture word-of-mouth communication among users: users typically form their opinions about the quality of service based on the information that they obtain from their peers. In traditional product or service market, the seller may offer the good at a discounted price to certain buyers and subsequently exploit the network effect of their usage on the rest of the buyers. However, such a pricing strategy is not applicable for the advertisement-supported content service market, since most online content providers such as Youtube do not charge users for additional fees. Specifically, the users access the content from the content provider but pay the mobile network operator (e.g., AT&T) for the data cost.

Nevertheless, with the sponsored content policy, the content provider is allowed to pay the mobile network operator on behalf of the mobile users to lower the data cost of the mobile users. As such, the content provider is able to design its sponsoring strategy to influence the pricing strategy of the mobile network operator. For example, the content provider may offer more sponsorship to certain users and subsequently exploit the network effect of their content consumption on the rest of the users. With network effects, the sponsoring strategy will significantly promote users to engage in consuming the social content [12, 13]. Therefore, by utilizing the network effects, the sponsored content policy can create greater economic benefit by intuition. Thus, it is more appropriate to consider the network effects in sponsored content where the social contents are popular and prevalent. However, the network effects complicate the interactions among the three entities including mobile network operator, content provider and mobile users, posting a remarkable challenge to the operation of the sponsored content applications. Nevertheless, to the best of our knowledge, none of the existing works on sponsored content takes the social network effects into consideration. Hence, this motivates us to address this challenge in Chapter 3.

1.2.2 Impacts of Information Asymmetry on Sponsored Content

To our best knowledge, the existing work in the literature merely studied the transparent sponsored content market, in which the information is publicly known to all the participant entities. Specifically, they assume that the mobile users will truthfully report their personal information (type) to the content provider and/or mobile network operator. This situation happens when there exists a supervising entity in the market that is capable of monitoring and sharing all behaviors to ensure that the mobile users always truthfully report the correct information and to penalize if any of mobile users deviate. However, without such a supervising entity which is often the case in practice, the mobile user does not reveal private information to the content provider or mobile network operator, which is called "information asymmetry".

A selfish mobile user inherently has an incentive to provide falsified information and pretend to be the mobile user with other types to gain more benefit. This information asymmetry, not being taken into consideration in the previous works, makes the existing incentive schemes less applicable to real-world sponsored content applications. Furthermore, the tools such as typical game theory that model the incentive mechanisms with symmetric information are not suitable to adopt here. In this regard, the contract theory that was specifically designed to handle such a situation can provide the effective way to address the above issue under the circumstance of asymmetric information [17]. Moreover, the general idea of contract theory has been employed in practice for data plan design, such as [18]. With the successful implementations of contract theory in wireless networks [19], we envision that contract theory can also be leveraged for developing effective incentive mechanisms for sponsored content applications. We will address the challenge of information asymmetry in Chapter 4.

1.2.3 Coexistence of Sponsored Content and Edge Caching

With the forthcoming of the 5th generation (5G) cellular networks, edge caching is emerging as a promising technology to deliver content services with lower cost and higher quality. The key idea that the edge caching content provider caches the content on edge devices in advance. As such, when users request the content, it can be delivered directly. Furthermore, with edge caching, mobile users can obtain requested content without incurring the cellular data cost, e.g., through a WiFi connection, and thus edge caching can be viewed as a new sponsorship scheme for mobile users. In this sense, the edge caching has the potential to alleviate backbone network burden [20]. In the future 5G cellular network, to meet explosive content traffic demands and support sustainable development, edge caching is one of the most effective solutions. One the one hand, more and more cache-enabled small base stations will be deployed as the infrastructure of edge networks. On the other hand, the storage capacity has been growing rapidly and becomes much cheaper [21]. As a result, equipping caches at small base stations offers a promising way to exploit the potential of edge networks in addition to densifying the existing cellular networks. Therefore, both sponsored and edge caching content are accessible for mobile users simultaneously. The coexistence of sponsored content and edge caching content inevitably introduces new challenges: the interplay between the normal sponsored content provider and the edge caching content provider will further complicate the tripartite interactions among the previous three entities in sponsored content. Motivated by this, we will address the above challenging problem in Chapter 5.

1.3 Summary of Contributions and Thesis Organization

In Chapter 1, we briefly present the research scope of our work. Specifically, we introduce the overview of sponsored content as well as its real-world applications, and we present its three potential challenges and our research motivations.

In Chapter 2, we provide the literature review of existing studies that are related to our research. We discuss and highlight the novel contribution of this thesis and its significance compared with other existing work in the literature.

We investigates the role of underlying network effects on sponsored content using a hierarchical game in Chapter 3. Specifically, we employ the hierarchical Stackelberg game to model their interactions to jointly maximize the utilities of users, the profit of the content provider, and the revenue of the network operator. The game model incorporates the network effects in the social domain utilizing the structural properties of the underlying wireless network. The model also incorporates the congestion in the network domain to realistically capture the scarcity of radio resource in wireless network environment. Through backward induction, we derive the unique Nash equilibrium point among the users. We investigate the mutual interplay between the network operator and the content provider by characterizing the scenarios where the network operator and the content provider compete sequentially, the network operator and the content provider compete simultaneously, and the network operator and the content provider cooperate for a common goal, respectively. The existence and uniqueness of the Stackelberg equilibrium are validated analytically in the three scenarios.

We develop the contract design in hierarchical game to resolve the information asymmetry from sponsored content in Chapter 4. Specifically, we propose a novel framework that integrates contract theory and Stackelberg game to analyze the hierarchical interactions among the mobile network operator, the sponsored content provider and mobile users with information asymmetry. Essentially, we employ the contract game to tackle the asymmetric information for which the contract theory has been inherently developed. Through backward induction, we analyze the sub-game perfect equilibrium in each stage systematically. We also derive the necessary and sufficient conditions of the feasible contract, and we derive the optimal contract that maximizes the profit of sponsored content provider. Then, we investigate the optimal pricing of the mobile network operator. We analytically validate that the optimal pricing strategy of the network operator is unique, and hence validate the uniqueness of the Stackelberg equilibrium.

We propose a joint sponsored and edge caching content market, and analyze its system performance in Chapter 5. We develop a novel hierarchical three-stage Stackelberg game to jointly maximize the payoff of the mobile network operator, the profit of each content providers, and the individual utilities of mobile users. By using backward induction methods, we analyze the sub-game perfect equilibrium in each stage. Essentially, we prove that the optimal strategy of the mobile user in Stage III is unique, and accordingly demonstrate the uniqueness of the Nash equilibrium between the content providers in Stage II. In addition, the existence of the Stackelberg equilibrium is validated by capitalizing on the bilevel optimization technique. We finally propose a sub-gradient based iterative algorithm that guarantees the convergence to the Stackelberg equilibrium.

Finally, Chapter 6 concludes this thesis and provide a number of promising open directions for the future work. Chapter 7 presents the appendix of the mathematical proof.

Chapter 2

Literature Review

In this chapter, we provide an overview of state-of-the-art studies on sponsored content, and review the related work regarding three challenges of the real implementation of sponsored content: network effects, information asymmetry, and coexistence with edge caching. We also discuss and highlight the novel contribution of each chapter and their significance compared with other existing work in the literature.

2.1 Sponsored Content Management

Recently, the mobile network operators have been experiencing several emerging and innovative pricing schemes, e.g., secondary market scheme [22] and time-dependent pricing [23]. In addition to these, the sponsored data policy is also introduced to into the actual data market. In 2014, AT&T launched a data sponsored policy [1], where the content providers can absorb their users' cellular data cost, and thereby the users access the content provider's contents through the network operator with lower charge. Moreover, Google joined with Indian operator Airtel to offer free access to certain Google-based services such as Google's email and Google+ via Google search without ringing up data charges [2]. Subsequently, a huge number of third-party companies now are constructing and providing interface platforms between content providers and mobile network operators, such as Aquto and Datami [3].

With remarkable interests from industry, the sponsored content concept has attracted many researchers to investigate and innovate better management schemes. For example, the authors in [24] studied the sponsorship competition among multiple content providers in Internet content market and demonstrated that the competitions improve the welfare of the market. The authors in [25] studied the interaction between the content provider and the network operator using a Stackelberg game, in which the network operator determines a pricing schedule and the content provider responds by deciding on how much content to sponsor. In [26], the authors presented a pricing scheme that optimizes the network operator's profit, where content providers are incentivized to reveal their valuation and underlying traffic in a truthful manner. In [27], the authors focused on one-to-one interaction between the network operator and the content provider, and investigated the market dynamics using the bargaining game framework. In [28], the authors explored the interplay between a single network operator and multiple content providers and presented a pricing mechanism for sponsored content that is truthful in content provider's valuation as well as its underlying traffic. The authors in [29] considered the service-selection process among the users as an evolutionary population game and demonstrated how sponsoring helps improve the network operator's revenue and the user's experience. However, the authors simplified the game model ignoring the interplay between the network operator and content providers.

The above studies focused on either the interplay between the content provider and the network operator, or the interplay between the content provider and users. In response to this, the authors in [30, 31, 32] investigated the interactions among the three main entities are. The interactions among the network operator, content provider and users were modeled as a Stackelberg game in [30, 31], where their payoffs are jointly analyzed and optimized. The authors in [30] mainly compare the impacts of sponsored content policy on different scales of content providers from short-run (market shares are fixed) and long-run (market shares are dynamic) perspectives, and the authors in [31] showed that the sponsorship benefits more to users than to content provider. The authors in [32] studied the similar problem proposed in [30], where the non-sponsored and sponsored content provider coexist in the market, where the competition between the two providers is explored under different network capacity.

2.2 Existing Studies on Network Effects in Communication and Wireless Networks

However, most of the existing studies neglect the strategic interactions among mobile users, i.e., all users are homogeneous whose utilities are decoupled. On one hand, it has been found that social network effects among users exist in many of the real-world data communication applications, e.g., crowdsensing [33], mobile data trading [34, 35], traffic offloading [36] and security protection [37]. The social aspect of mobile networking is an emerging paradigm for network design and optimization [34]. In [38], the authors found that the information obtained from social tie connections will influence in decision making. Inspired by [38], the social group utility maximization framework was proposed in [39, 40], which captures the impact of users' diverse social ties that are subject to diverse social relationships. The authors in [41] showed the evidence of network effect in communication service using the real data analytic, and quantified such effect using a simple metric. The finding of [41] provides the motivation for planning pricing mechanisms with network effect in the real markets. Recently, the network effect has been considered jointly with service pricing in data communication networks. For example, in the pioneering work [34], the authors investigated the static pricing schemes of the wireless service provider in the presence of network effects. In [42], the authors discussed the dynamic pricing strategy of mobile social data with network effects. Similarly, the authors in [43] analyzed the impacts of network effects on cloud computing resource allocation.

The authors in [44] exploited the network effects to design the incentive mechanism for engaging more participants in the mobile crowdsensing platform.

In addition, the Quality of Experience (QoE) of the users can also be negatively affected by other users because of radio resource occupation (congestion effects). Since users typically share limited resources in data communication networks, congestion effect from physical domain on users' behavior is common [45]. For example, if an Internet service provider becomes oversubscribed, its subscribers suffer from the congestion due to limited bandwidth resources. Thus, insightful discoveries behind the strategic interaction among users deserve indepth studies in this emerging platform. Along with this direction, we have conducted economic analysis on sponsored content with network effects and congestion effects in Chapter 3. Therein, we studied the interactions among the network operator, content provider and users using Stackelberg game with the modeling of network effects and congestion effects on the demand side. Furthermore, we provide the detailed and rigorous mathematical analysis on resource management in sponsored content with network effects. In addition, we justify that the cooperation between the network operator and the content provider is the best choice for themselves through extensive numerical results.

2.3 Existing Studies on Information Asymmetry in Communication and Wireless Networks

Moreover, most of the previous work on sponsored content have a restrictive assumption that the heterogeneous private information of mobile users is publicly accessible for the content provider or the mobile network operator, which is the information symmetric environment. However, the information asymmetry scenario is more applicable for the real-world sponsored content applications, i.e., the user side has certain private information which can not be observed by other sides. Without knowing mobile users' private information, it is difficult for the content provider and network operator to design the efficient sponsoring and pricing strategies, respectively. As such, the information asymmetry make the operation of such emerging applications more challenging. In this case, contract theory is treated as an promising tool to solve this kind of problem [17]. The basic idea of contract theory is that the contract designer offers the right contract to the agents, such that all the agents are induced to truthfully reveal their private information and the designer obtains the maximum profit in the same time.

Information asymmetry is widespread in communication and wireless networks, and hence contract theory [46, 47, 48, 49, 50, 51, 18, 52] has been applied into many areas such as spectrum allocation in cognitive radio networks, traffic offloading in cellular networks, and energy trading in smart grid. In the pioneering work [46], the authors modeled the spectrum trading process as a contract game to achieve dynamic spectrum sharing. Therein, the primary user offers a contract that includes a number of "quality-price" contract items for the spectrum it sells, and the secondary users select the appropriate item for purchasing. The feasibility of contract is then analyzed and the optimal contract that maximizes the profit of the primary user is derived. The authors in [47] studied a similar spectrum trading problem to that in [46], and discussed the strongly incomplete information scenario in which the primary user does not even know the total number of secondary users. In [48], the authors proposed a contract-theoretic model for promoting users to participate in D2D communication, in which the user type is not available at the base station. The type of users under the discrete type case and continuum type case are both considered. Under the contract theoretic framework proposed in [46], the authors in [49] investigated the traffic offloading problem in cellular networks and the authors in [50] studied the energy trading problem in smart grids. To address the incentive mechanism in crowdsensing, the authors in [51] proposed an incentive contract which rewards users by evaluating their performances from multiple dimensions. The optimal multi-dimensional contract that solves the principal's profit maximization problem is analyzed, and the optimal solutions are obtained. Likewise, the authors in [18] proposed a multi-dimensional contract-theoretic approach, and studied the optimal design of data caps as well subscription fees systematically.

Nevertheless, the impact of private information of mobile users on sponsored content still remains an open issue, which becomes the exact motivation for our research in Chapter 4.

2.4 Existing Studies on Edge Caching in 5G Mobile Networks

Recent studies have shown that edge caching emerges as a promising paradigm to alleviate the increasing data/content traffic burden of traditional cellular networks [53, 20, 21, 54, 55, 56]. The key idea is that an edge caching content provider caches content on edge networks. The cached content is then delivered to mobile users locally, reducing latency substantially. The authors in [53] provided a review on the techniques related to caching in existing wireless networks. The authors introduced the methods to predict the content popularity distributions and users' preferences in [20], and further investigated the impact of content delivery and placement on wireless caching. In [21], the authors demonstrated that local caching at base stations can improve the energy efficiency of the cellular networks. In [57], the authors proposed an optimal caching scheme where the cached content in the cache server/node is dynamically determined by the price and the demand of users. The interaction between the cache server and users are modeled as a Stackelberg game.

The authors in [54] studied the competition among multiple edge caching content providers and formulated a resource allocation problem. Each content provider decides its optimal reserved cache memory in the edge network for enhancing the QoS of users.

The authors in [55] explored optimal pricing for the caching service, in which content is dynamically stored in the edge memory. The network operator offers the price for storing the content on the shared cache node, which engenders the competition for cache memory sharing among edge caching content providers. In [58], the authors considered a simple model, where the single content provider offers both sponsored and edge caching content service to users and the requests for the content service from users are assumed to be random. A two-stage decision problem is then formulated to model the interaction between the content provider and users. However, the interactions between the normal sponsored content provider and the edge caching provider is neglected. Considering the similar problem to that in [58], the authors in [56] proposed an online sponsoring optimization problem and solved the problem using the Lyapunov optimization technique.

To the best of our knowledge, none of the existing work has explored the great benefit of sponsored content concept and edge caching. They can clearly complement each other, from the users' perspective, which encourages the users to consume more services while improving service quality, and hence this is the objective of Chapter 5.

Chapter 3

A Hierarchical Game for Sponsored Content with Network Effects

In this chapter[1], we study the optimal strategies of the mobile network operator, content provider, and mobile users in the sponsored content platform, where the impacts of the network effects are formally considered for the first time. The main contributions of this chapter are summarized as follows:

- We formulate the pricing, sponsoring and content demand problem to analyze the interactions among the mobile network operator, content provider and mobile users under the sponsored content policy. In particular, we adopt the hierarchical Stackelberg game to model their interactions to jointly maximize the utilities of users, the profit of the content provider and the revenue of the mobile network operator. Therein, the mobile network operator and the content provider act as the leaders determining the pricing and sponsoring strategies, respectively, and the users act as the followers deciding on their content demand.

- The game model exploits the network effects in the social domain utilizing the structural properties of the underlying wireless network, which improve the content demand of users to a large extent. Furthermore, the model incorporates the

[1] The work in this chapter has been published in [59, 60, 61].

congestion in the network domain to realistically capture the scarcity of radio resource in wireless network environment.

- Through backward induction, we derive the unique Nash equilibrium point among the users. We also investigate the mutual interplay between the mobile network operator and the content provider by characterizing the scenarios where the mobile network operator and the content provider compete sequentially, the mobile network operator and the content provider compete simultaneously, and the mobile network operator and the content provider cooperate for a common goal, respectively. The existence and uniqueness of the Stackelberg equilibrium are validated analytically in the three scenarios.

- Our evaluation results reveal the fact that the network effects significantly improve the payoff of three entities, i.e., the utilities of users, the profit of the content provider and the revenue of the mobile network operator. Additionally, the results show that the cooperation of the mobile network operator and the content provider is the best choice for themselves.

For our considered hierarchical game problem, there are two major difficulties which have not been well addressed in the literature. Firstly, there is no closed-form solution for content demand sub-game at the users, which impedes us from solving the hierarchical three-stage game by directly employing the conventional backward induction method. Secondly, the underlying externalities including (social) network effects and congestion effects at the users further complicate the efficient tracking of the equilibrium. Consequently, we resort to the matrix formulation to characterize the game equilibrium solution and leverage the theory of matrix to assist our analysis.

The rest of the chapter is structured as follows. We present the system model and formulate the hierarchical Stackelberg game in Section 3.1. In Section 3.2, we characterize different scenarios for the mutual interplay between the content provider and the mobile network operator. Then, we examine the optimal content demand of users, the optimal sponsoring of the content provider as well as the optimal pricing of the mobile network operator using backward induction methods. The performance evaluation is given in Section 3.4. Section 3.5 summarizes the chapter.

3.1 System Model and Game Formulation

In this section, we first present the system model of sponsored content platform under our consideration. We next propose the hierarchical Stackelberg formulation for modeling the interactions among the mobile network operator, the content provider and mobile users.

3.1.1 System Model

We consider the sponsored content platform as a market consisting of three entities: Mobile Network Operator (MNO), Content Provider (CP) and Mobile Users (MUs). We model their interactions as a hierarchical Stackelberg game [62, 63]. The MNO interacts with mobile users directly and hence it is likely to be able to set the price before CP [30, 31]. The monopolist MNO determines how to price its basic data services. The strategy of the CP is to decide how much data to sponsor, and obtain the advertisement revenue in return. The MUs make the decision on the demand for contents that they need based on the pricing strategies of the MNO and sponsoring strategies of the CP. We further assume that the complete information of the underlying network social structure is available for all the players in the market, i.e., the utility functions, strategies and "types" of market players are common knowledge.

3.1.1.1 Mobile Users' Content Demand

Consider a group of N MUs, the set of which is denoted as $\mathcal{N} \triangleq \{1, \dots, N\}$. Each MU $i \in \mathcal{N}$ decides on the demand for contents, denoted by x_i, where $x_i \geq 0$. Then, let

$\mathbf{x} \triangleq (x_1, \ldots, x_N)$ and $\mathbf{x}_{-i}$ denote the content demand of all the MUs and all other MUs except MU i, respectively. The utility of MU i is given by:

$$u_i(x_i, \mathbf{x}_{-i}, \theta_i, p^{\mathrm{u}}) = f_i(x_i) + \Phi_n(x_i, \mathbf{x}_{-i}) - p(x_i) + \theta(x_i) - \mathscr{C}(\mathbf{x}). \tag{3.1}$$

The first term $f_i(x_i)$ represents the internal effects that MU i gains from consuming and enjoying the contents, for which we use the linear-quadratic function to capture the decreasing marginal returns for tractability reason (matrix formulation). Following [14, 34], it is defined as $f_i(x_i) = a_i x_i - b_i {x_i}^2$, where $a_i, b_i > 0$ are the personal type coefficients that capture the MUs heterogeneity. As in [14], the quadratic form of the internal utility not only allows for tractable analysis, but also serves as a good second-order approximation for a broad class of concave utility functions. It is worth noting that the authors in [34, 36, 64] have conducted empirical studies with real dataset to evaluate the network performance including the linear-quadratic formulation of $f_i(x_i)$. For future work, we will examine the formulation of other utility functions such as the logarithmic function, however, we expect that the major insights should remain the same. In particular, a_i models the maximum internal demand willingness rate, and b_i models such willingness elasticity factor. The maximum internal demand rate a_i can also indicate the variation of relative cellular signal power/strength or signal-to-noise ratio (SNR) of MUs because of different signal attenuation for different locations of MUs. Note that the SNR of MU i (the value of a_i) will also affect its enjoyment of consuming the content. For example, when the MU i watches the video in Youtube, the low SNR (small a_i) will negatively affect its viewing experience.

$\Phi_n(x_i, \mathbf{x}_{-i})$ represents the external benefits because of the network effects. For example, in social networks, the users affect each other by social behaviors via their relationships, especially in social-enabled service market [65, 66]. The sponsored content platform exhibits significant network effects [12, 13]. Thus, the users' behaviors in terms of content demand are directly dependent on others' demand. Here we introduce the adjacency matrix $\mathcal{G} = [g_{ij}], i, j \in \mathcal{N}$. The element $g_{ij} \geq 0$ is the influence strength of MU i's social tie with MU j that quantifies the marginal social effect of MU j on MU i. We consider the bidirectional social relations, i.e., g_{ij} or g_{ji} denotes the social tie between the MUs i and j. This is also supported by the idea of social reciprocity [67, 68, 34]: a MU's social behavior to another is likely to imitate the latter's behavior to the former. As a result, two MU social ties to each other tend to be the same. Note that $g_{ii} = 0$ means the MU cannot influence itself. Nevertheless, the same model can be applied to bidirectional social relations straightforwardly. Specifically, we use $x_i \sum_{j\in\mathcal{N}} g_{ij} x_j$ to represent the second term $\Phi_n(x_i, \mathbf{x}_{-i})$ in (3.1), similar to [14]. As in [14] (equation (1) in Section 3), the term $x_i \sum_{j\in\mathcal{N}} g_{ij} x_j$ indicates the fact an MU derives more utility from the content as its demand increases, and the marginal gain of utility increases as its socially-connected MUs increase their demand. Generally, the neighbours of MU i in the network social structure have stronger social relations with MU i. Moreover, such product form $x_i \sum g_{ij} x_j$ has been adopted widely for social network effects modeling in the literature [14, 34, 35, 36]. The existing works such as [34, 36] have conducted empirical studies with real dataset to evaluate the network performance.

The third term indicates the cost, and the price per unit of content charged to the MU is expressed by p^{u}. Then MU pays the MNO with the amount $p^{\mathrm{u}} x_i$ with demand based pricing, as a function of x_i. The usage-based pricing is widely used in practice by network operators to control the demand [69], and we consider that the price p^{u} is the same for all MUs to ensure fairness. The fourth term $\theta(x_i)$ denotes the benefits from the sponsorship. Similar to that in [31], we apply $p^{\mathrm{u}} \theta_i x_i$ as the sponsorship fee from the CP, where the sponsorship factor θ_i is decided by the CP. Since the CP has a limited network capacity, we further have the last term to indicate the congestions. Following [34, 36, 35], we apply the quadratic sum form $c(\sum_{j\in\mathcal{N}} x_j)^2$ to indicate the congestion experience of MUs, where $c > 0$ is the congestion coefficient. The quadratic sum form reflects that the congestion of one MU is affected by the demand of all the MUs. Also, the marginal cost of congestion increases as the total demand increases. It is worth noting that the authors in [34, 36, 64] have conducted empirical studies with real dataset to evaluate the network performance including the formulation of $c(\sum_{j\in\mathcal{N}} x_j)^2$. Considering the congestion experience of MUs (e.g., video content delays) is practical because of the limited capacity in physical communication networks, which discourages them to demand more content. Note that the congestion effects can happen because of the bottleneck of the CP or the MNO. Nevertheless, the modeling of the congestion effects can straightforwardly absorb the impacts of the bottleneck from both the CP and the MNO. The reason is that the presence of congestion only affects the user side instead of other sides, no matter the limited-resource situation exists in the CP side or the MNO side. Therefore, the utility of MU i is expressed as follows:

$$u_i(x_i, \mathbf{x}_{-i}, \theta_i, p^{\mathrm{u}}) = a_i x_i - b_i {x_i}^2 + x_i \sum_{j\in\mathcal{N}} g_{ij} x_j - p^{\mathrm{u}}(1-\theta_i)x_i - c\left(\sum_{j\in\mathcal{N}} x_j\right)^2. \tag{3.2}$$

Note that our model can straightforwardly include the heterogeneous contents consideration. Specifically, we can consider a content pool managed by the CP, in which there exist a number of heterogeneous contents. The users can freely select different contents to view and consume based on their preferences. In this case, the strategy of each MU $i \in \mathcal{N}$, x_i, is its total content demand from heterogeneous contents instead of the single content demand from homogeneous contents. For example, in each Youtube channel, there has a number of heterogeneous video contents with similar themes such as a movie or cartoon. In this regard, although the contents are heterogeneous, the themes are similar and the network effects function in the same way.

3.1.1.2 Content Provider's Sponsorship

The CP provides discriminatory sponsorship for different MUs. The sponsorship factor θ_i ($\theta_i \in [0,1]$) for each MU i is decided by the CP, which denotes the percentage of content demand of mobile users that are fully subsidized. Note that the sponsored content platform allows the CP to provide different sponsorships to MUs with different activity level. Nevertheless, uniform sponsorship where $\theta = \theta_i, \forall i$ is just a special case of the analysis. The CP's payoff function includes an advertisement utility, and a component depending on its sponsorship. The cost of the CP associated to the sponsoring, is denoted as $p^{\mathrm{u}} \sum_{i \in \mathcal{N}} x_i \theta_i$. Note that the CP is also charged by the MNO for entering the sponsored content market, which is considered to be fixed and thus ignored for ease of presentation. Thus, the payoff, i.e., profit of the CP, is formulated as:

$$\mathscr{P} = \gamma \sum_{i \in \mathcal{N}} \left(s x_i - t {x_i}^2 \right) - p^{\mathrm{u}} \sum_{i \in \mathcal{N}} x_i \theta_i, \tag{3.3}$$

where γ is the adjustable parameter denoting the equivalent monetary worth of MUs' content demand, and $s, t > 0$ are coefficients characterizing the concavity extent of the function. Similar to $f_i(x)$ in (3.1), we also use the linear-quadratic function with the decreasing marginal return property to transform the MUs' content demand to the monetary revenue for the CP. In fact, we could also adopt a linear dependency between demand and ads revenue [32]. However, most of the related works have proved that a function with diminishing return would model the ads revenue more closely [30, 31, 70, 71].

3.1.1.3 Mobile Network Operator's Pricing

The decision variable of the MNO is the price p^{u}. In the model, we consider the situation where the MNO charges the MUs with the same price. Generally, the data traffic service fee is the same for all MUs in actual data market. The revenue of the MNO is composed of the payment from MUs and the sponsorship fee from the CP. Then, the revenue of the MNO is obtained as:

$$\Pi = p^{\mathrm{u}} \sum_{i \in \mathcal{N}} x_i. \tag{3.4}$$

3.1.2 Hierarchical Stackelberg Game Formulation

As illustrated in Fig. 3.1, we model the interaction among the MNO, CP and MUs as the Stackelberg game. In Stage I, the MNO, i.e., the 1st-tier leader, determines the price p^{u} to maximize its revenue. The optimization problem of the MNO is defined as follows:

$$\textbf{Problem 3.1}: \; p^{\mathrm{u}*} = \arg\max_{p^{\mathrm{u}}>0} \Pi.$$

In Stage II, the CP, the 2nd-tier leader, determines the sponsorship factor $\boldsymbol{\theta} = \{\theta_1, \theta_2, \ldots, \theta_N\}$ to maximize its profit. It is obtained by solving the optimization problem:

$$\textbf{Problem 3.2}: \; \boldsymbol{\theta}^* = \arg\max_{\theta_i \in [0,1]} \mathscr{P}.$$

In Stage III, the MUs decide on content demand to maximize their individual utility, acting as followers of the game. Mathematically, the problem can be formulated as:

$$\textbf{Problem 3.3}: \; {x_i}^* = \arg\max_{x_i \geq 0} u_i(x_i, \mathbf{x}_{-i}, p^{\mathrm{u}}, \theta_i).$$

The **Problems 3.1-3.3** together form the hierarchical three-stage Stackelberg game. The objective of the game is to find the Stackelberg equilibrium. The Stackelberg equilibrium is a point where the payoff of the leader is maximized given that the followers adopt their best responses, i.e., the Nash equilibrium [5]. In the following, we address the follower game and leader game to investigate the Stackelberg game.

3.2 Follower Game: Content Demand Equilibrium Analysis

From this section, we use backward induction to investigate the Stackelberg game. We first analyze the content demand equilibrium in follower game in the following.

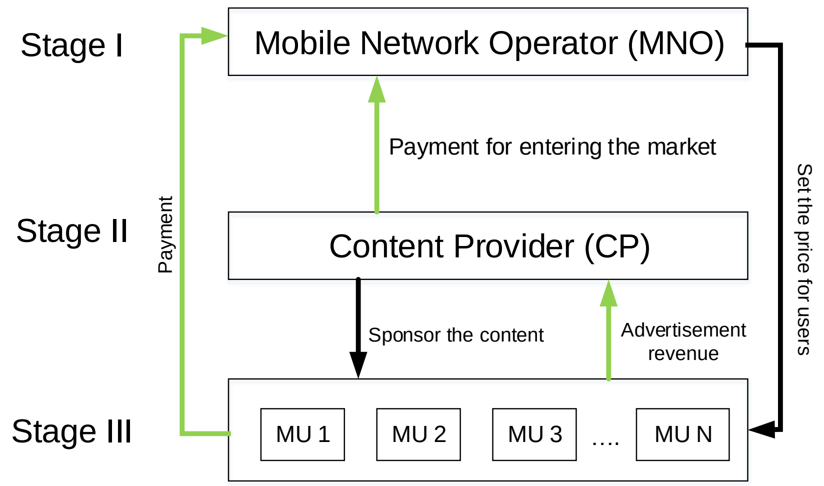


Figure 3.1: Three-stage Stackelberg game model of the interactions among the MNO, CP, and MUs in the sponsored content market.

3.2.1 Best Response Function of Followers (MUs)

In the non-cooperative sub-game $\mathcal{G}^u$, the best response function of MU i can be obtained by solving **Problem 3.3**. According to the first order condition, by setting the derivative $\frac{\partial u_i(x_i,\mathbf{x}_{-i},\theta_i,p^{\mathrm{u}})}{\partial x_i} = 0$, we obtain the following proposition.

Proposition 3.1 *Given price p^{u} and the sponsorship factor $\boldsymbol{\theta}$, and the content demand profile without MU i, $\mathbf{x}_{-i}$, the best response function $\mathscr{F}$ of MU i is expressed as follows:*

$$\mathscr{F}(\mathbf{x}_{-i},\theta_i,p^{\mathrm{u}}) = \left(\frac{a_i - p^{\mathrm{u}}(1-\theta_i)}{2b_i+2c} + \sum_{j\in\mathcal{N}\setminus i} x_j \frac{g_{ij}-2c}{2b_i+2c}\right)^+, \tag{3.5}$$

$\forall i$, where $(\cdot)^+ \triangleq \max\{0,\cdot\}$.

From Proposition 3.1, the best response of content demand of each MU i consists of two parts. The left part, i.e., $\frac{a_i-p^{\mathrm{u}}(1-\theta_i)}{2b_i+2c}$ is its internal demand which is independent from other MUs' strategies. The right part, i.e., $\sum\limits_{j\in\mathcal{N}\setminus i} x_j \frac{g_{ij}-2c}{2b_i+2c}$ is its external content demand depending on other MUs' strategies. Similar to [34] and [36], we make a general and realistic assumption to ensure that each MU has no incentive to unboundedly increase its content demand as shown as follows:

Assumption 3.1 $\sum\limits_{j\in\mathcal{N}\setminus i}\frac{g_{ij}-2c}{2b_i+2c}<1,\forall i.$

For ease of tractable analysis, we set the assumption in order to guarantee the existence of an equilibrium of sub-game $\mathcal{G}^u$. It is observed that $\sum_{j\in\mathcal{N}\setminus i}x_j\frac{g_{ij}-2c}{2b_i+2c}\leq\left|\sum\limits_{j\in\mathcal{N}\setminus i}x_j\frac{g_{ij}-2c}{2b_i+2c}\right|\leq\left|\sum_{j\in\mathcal{N}}x_j\frac{g_{ij}-2c}{2b_i+2c}\right|\leq\sum\limits_{j\in\mathcal{N}}x_j\frac{|g_{ij}-2c|}{2b_i+2c}=\sum\limits_{j\in\mathcal{N}}x_j\left|\frac{g_{ij}-2c}{2b_i+2c}\right|<\max_{j\in\mathcal{N}}\{x_j\}$. Accordingly, Assumption 3.1 indicates that any MU's external content demand is no more than the maximum content demand among all the other MUs. For the non-cooperative game, the Nash equilibrium is defined as the point at which no player can improve the utility by changing its strategy unilaterally. Next, we discuss the existence and uniqueness of the Nash equilibrium of sub-game $\mathcal{G}^u$ under this sufficient condition. We will relax the assumptions made in the thesis to make the modeling more practical and general in the future works.

3.2.2 Existence of the Equilibrium of Sub-game $\mathcal{G}^u$

Theorem 3.1 *Under Assumption 3.1, sub-game* $\mathcal{G}^u=\{\mathcal{N},\{u_i\}_{i\in\mathcal{N}},[0,+\infty)^N\}$ *admits the Nash equilibrium.*

Proof. We denote $\max_{j\in\mathcal{N}}\{x_j^*\}$ as x_i^*, i.e., $x_i^*\geq x_j^*,\forall i\neq j$, and we have $x_i^*=\left(\frac{a_i-p^{\mathrm{u}}(1-\theta_i)}{2b_i+2c}+\sum\limits_{j\in\mathcal{N}\setminus i}x_j^*\frac{g_{ij}-2c}{2b_i+2c}\right)^+\leq\frac{|a_i-p^{\mathrm{u}}(1-\theta_i)|}{2b_i+2c}+\sum\limits_{j\in\mathcal{N}\setminus i}x_i^*\frac{g_{ij}-2c}{2b_i+2c}\leq\frac{|a_i-p^{\mathrm{u}}(1-\theta_i)|}{2b_i+2c}+x_i^*\frac{\sum\limits_{j\in\mathcal{N}\setminus i}|g_{ij}-2c|}{2b_i+2c}$. With simple transformations, we have $x_i^*\leq\frac{|a_i-p^{\mathrm{u}}(1-\theta_i)|}{2b_i+2c-\sum\limits_{j\in\mathcal{N}\setminus i}|g_{ij}-2c|}$. Meanwhile, x_i^* is the maximum in $\mathbf{x}^*$, and we can conclude that there exists $\widehat{x}$ guaranteeing the boundedness of strategy space in $[0,\widehat{x}]$. As $u_i(x_i,\mathbf{x}_{-i},p^{\mathrm{u}},\boldsymbol{\theta})$ is continuous and concave in each x_i, the sub-game $\mathcal{G}^u$ admits the Nash equilibrium [72]. The proof is now completed. □

3.2.3 Uniqueness of the Equilibrium of Sub-game $\mathcal{G}^u$

Theorem 3.2 *Under Assumption 3.1, the Jacobian matrix of point-to-set mapping with respect to the utility profile, i.e.,* $\nabla\mathbf{F}(\mathbf{u}(\mathbf{x}))$, *is strictly diagonally dominant, and thus the Nash equilibrium of sub-game* $\mathcal{G}^u=\{\mathcal{N},\{u_i\}_{i\in\mathcal{N}},[0,+\infty)^N\}$ *is unique.*

Proof. From $\mathbf{u}(\mathbf{x}) \triangleq \{u_1(\mathbf{x}), \ldots, u_N(\mathbf{x})\}$, we have point-to-set mapping $\mathbf{F} = \mathbf{F}(\mathbf{u}(\mathbf{x})) = [\nabla_{x_i} u_i(\mathbf{x})]_{i=1}^N$ [72]. $\nabla \mathbf{F} = \nabla \mathbf{F}(\mathbf{u}(\mathbf{x})) = \begin{bmatrix} \nabla^2_{1,1} u_1(x) & \nabla^2_{1,2} u_1(x) & \cdots & \nabla^2_{1,N} u_1(x) \\ \nabla^2_{2,1} u_2(x) & \nabla^2_{2,2} u_2(x) & \cdots & \nabla^2_{2,N} u_2(x) \\ \vdots & \vdots & \ddots & \vdots \\ \nabla^2_{N,1} u_N(x) & \nabla^2_{N,2} u_N(x) & \cdots & \nabla^2_{N,N} u_N(x) \end{bmatrix} = \begin{bmatrix} -2b_1 - 2c & g_{12} - 2c & \cdots & g_{1N} - 2c \\ g_{21} - 2c & -2b_2 - 2c & \cdots & g_{2N} - 2c \\ \vdots & \vdots & \ddots & \vdots \\ g_{N1} - 2c & g_{N2} - 2c & \cdots & -2b_N - 2c \end{bmatrix}$. Based on Assumption 3.1, we have $[-\nabla \mathbf{F}]_{ii} > \sum_{j \neq i} |[-\nabla \mathbf{F}]_{ij}|, \forall i$. Thus, $-\nabla \mathbf{F}$ is strictly diagonally dominant, and positive definite accordingly. Then, $\nabla \mathbf{F} + \nabla \mathbf{F}^\top$ is negative definite. Therefore, $\mathbf{u}(\mathbf{x})$ is diagonally strictly concave [72]. The proof is now completed. □

We denote $\nabla \mathbf{F}$ as $\mathbf{G} - 2\mathbf{\Lambda}$, where $\mathbf{\Lambda} = \begin{bmatrix} b_1 + c & 0 & \cdots & 0 \\ 0 & b_2 + c & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_N + c \end{bmatrix}$ and $\mathbf{G} = \begin{bmatrix} 0 & g_{12} - 2c & \cdots & g_{1N} - 2c \\ g_{21} - 2c & 0 & \cdots & g_{2N} - 2c \\ \vdots & \vdots & \ddots & \vdots \\ g_{N1} - 2c & g_{N2} - 2c & \cdots & 0 \end{bmatrix}$ for ease of presentation. Moreover, $\mathbf{1} = [1, \ldots, 1]^\top$, $\mathbf{a} = [a_1, a_2, \ldots, a_N]^\top$, $\mathbf{b} = \boldsymbol{diag}\{[2b_1, 2b_2, \ldots, 2b_N]^\top\}$, $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_N]^\top$, $\mathbf{s} = [s, \ldots, s]$, $\mathbf{c} = [2c, \ldots, 2c]$, and $\mathbf{I}$ is the $n \times n$ identity matrix. Based on Proposition 3.1, we can conclude that there exists a set $\mathcal{P}$ and $x_i > 0$ only for $i \in \mathcal{P}$. If $i \notin \mathcal{P}$, $x_i = 0$. Specifically, the MUs in set $\mathcal{P}$ are the MUs in $\mathcal{N}$ which have the positive content demand. We denote $\mathbf{x}_\mathcal{P}$ as the vector of all x_i such that $i \in \mathcal{P}$, and define $\mathbf{G}_\mathcal{P}$, $\mathbf{\Lambda}_\mathcal{P}$, $\mathbf{c}_\mathcal{P}$ and $\mathbf{b}_\mathcal{P}$ similarly. We then have the following proposition.

Proposition 3.2 *Denoting $\mathbf{x}^* = \mathbf{x}_\mathcal{N}$ as the unique equilibrium of the sub-game $\mathcal{G}^u$ and $\mathbf{x}_\mathcal{P}$ as the vectors of all x_i such that $x_i > 0$, then the matrix form of the equilibrium of the sub-game $\mathcal{G}^u$ is as follows:*

$$\mathbf{x}_\mathcal{P} = (\mathbf{G}_\mathcal{P} - \mathbf{b}_\mathcal{P} - \mathbf{1}_\mathcal{P}\mathbf{c}_\mathcal{P})^{-1}[p^{\mathrm{u}}(\mathbf{1}_\mathcal{P} - \boldsymbol{\theta}_\mathcal{P}) - \mathbf{a}_\mathcal{P}],$$
$$\mathbf{x}_{\mathcal{N}\setminus\mathcal{P}} = \mathbf{0}.$$

Proof. With $(\mathbf{b}_{\mathcal{P}})^\top = \mathbf{b}_{\mathcal{P}}$, $(\mathbf{1}_{\mathcal{P}}\mathbf{c}_{\mathcal{P}})^\top = \mathbf{1}_{\mathcal{P}}\mathbf{c}_{\mathcal{P}}$ and $(\mathbf{G}_{\mathcal{P}})^\top = \mathbf{G}_{\mathcal{P}}$, the matrix form of the derivative of $u_i(x_i, \mathbf{x}_{-i}, p^{\mathrm{u}}, \theta_i)$ in (3.2) can be written as:

$$\mathbf{a}_{\mathcal{P}} - \mathbf{b}_{\mathcal{P}}\mathbf{x}_{\mathcal{P}} + \mathbf{G}_{\mathcal{P}}\mathbf{x}_{\mathcal{P}} - p^{\mathrm{u}}(\mathbf{1}_{\mathcal{P}} - \boldsymbol{\theta}_{\mathcal{P}}) - \mathbf{1}_{\mathcal{P}}\mathbf{c}_{\mathcal{P}}\mathbf{x}_{\mathcal{P}} = \mathbf{0}. \tag{3.6}$$

Similar to [34], we can easily prove that $(\mathbf{G}_{\mathcal{P}} - \mathbf{b}_{\mathcal{P}} - \mathbf{1}_{\mathcal{P}}\mathbf{c}_{\mathcal{P}})$ is invertible under Assumption 3.1. Then, we obtain $\mathbf{x}_{\mathcal{P}} = (\mathbf{G}_{\mathcal{P}} - \mathbf{b}_{\mathcal{P}} - \mathbf{1}_{\mathcal{P}}\mathbf{c}_{\mathcal{P}})^{-1}\left[p^{\mathrm{u}}\left(\mathbf{1}_{\mathcal{P}} - \boldsymbol{\theta}_{\mathcal{P}}\right) - \mathbf{a}_{\mathcal{P}}\right]$, and we have $\mathbf{G}_{\mathcal{P}} - \mathbf{b}_{\mathcal{P}} - \mathbf{1}_{\mathcal{P}}\mathbf{c}_{\mathcal{P}} = \mathbf{G}_{\mathcal{P}} - 2\boldsymbol{\Lambda}_{\mathcal{P}}$. Meanwhile, we also have $\mathbf{x}_{\mathcal{P}} = (\mathbf{G}_{\mathcal{P}} - 2\boldsymbol{\Lambda}_{\mathcal{P}})^{-1}\left[p^{\mathrm{u}}\left(\mathbf{1}_{\mathcal{P}} - \boldsymbol{\theta}_{\mathcal{P}}\right) - \mathbf{a}_{\mathcal{P}}\right]$. The proof is now completed. □

Then, we adopt the best response dynamics algorithm to obtain the Nash equilibrium with respect to MUs' content demand, similar to that in [59]. The algorithm iteratively updates MUs' strategies based on their best response functions in (3.5) and explicitly converges to the Nash equilibrium of MUs' sub-game $\mathcal{G}^u$. In the subsequent discussions, we consider the practical situation where all MUs have the positive content demand at the Stackelberg equilibrium, i.e., a special case of (3.5). In the typical market, the monopolist seller wants to charge individuals low enough (lower than a threshold) so that all consumers would like to purchase a positive amount of goods [14]. Specifically, if MUs are charged appropriately, none of MUs chooses zero content demand. We introduce the following condition similar to that in [14, 36], as follows: all MUs have the positive content demand at the Stackelberg equilibrium, i.e., $x_i > 0, \forall i$. To ease the description, we can rewrite (3.5) in a matrix form. Specifically, the matrix form of the best response of all the MUs with respect to the content demand is

$$\mathbf{x}^*(\boldsymbol{\theta}, p^{\mathrm{u}}) = \mathbf{K}\left[p^{\mathrm{u}}(\mathbf{1} - \boldsymbol{\theta}) - \mathbf{a}\right], \tag{3.7}$$

where $\mathbf{K} = (\mathbf{G} - 2\boldsymbol{\Lambda})^{-1}$ is a negative definite matrix according to Lemma 3.1 as shown as follows.

Lemma 3.1 $\mathbf{G} - 2\boldsymbol{\Lambda}$ *is a negative definite matrix, which is invertible.*

Proof. If Assumption 3.1 holds, we have $(2\mathbf{\Lambda} - \mathbf{G})_{ii} = 2b_i + 2c > \sum_{j \neq i}(g_{ij} - 2c) > \sum_{j \in \mathcal{N}}(g_{ij} - 2c) = -\sum_{j \in \mathcal{N}}(2\mathbf{\Lambda} - \mathbf{G})_{ij} = -\sum_{j \in \mathcal{N}}\left|(2\mathbf{\Lambda} - \mathbf{G})_{ij}\right|$. Accordingly, $\mathbf{B} - \mathbf{G}$ is strictly diagonally dominant and all the diagonal elements. Based on Gershgorin circle theorem [73], every eigenvalue λ of $\mathbf{G} - 2\mathbf{\Lambda}$ satisfies $|(2\mathbf{\Lambda} - \mathbf{G})_{ii} - \lambda| < \sum_{j \in \mathcal{N}}\left|(2\mathbf{\Lambda} - \mathbf{G})_{ij}\right|$. Therefore, we know $\lambda > 0$, and thus $2\mathbf{\Lambda} - \mathbf{G}$ is a negative definite matrix, and accordingly $\mathbf{G} - 2\mathbf{\Lambda}$ is positive definite matrix, from which its invertibility follows. The proof is now completed. □

3.3 Leader Game: Optimal Sponsoring and Pricing Strategies Analysis

In this section, we address the mutual interaction between the CP and the MNO by characterizing the scenarios where the MNO and the CP compete sequentially, the MNO and the CP compete simultaneously, and the MNO and the CP cooperate for a common goal, respectively.

3.3.1 Sequential Competition Between CP and MNO

We first investigate the sequential competition scenario, where the MNO first optimizes its pricing strategy for revenue maximization, and then the CP optimizes its sponsoring strategy for profit maximization sequentially, as illustrated in Fig. 3.1. Under this setting, the **Problems 3.1** and **3.2** together form a non-cooperative sequential game for modelling the interplay between the CP and the MNO. In the following, we solve the **Problems 3.2** and **3.1** in each stage sequentially using backward induction.

3.3.1.1 Stage II: Optimal Sponsoring (2nd-tier leader)

The CP achieves the profit maximization by solving **Problem 3.2**. Taking into account the optimal content demand given in (3.7), we further observe that profit function of the CP, $\mathscr{P}$ is concave over θ_i. Thus, **Problem 3.2** is a convex optimization problem. Again, the optimal solution must satisfy the first order condition. Specifically, the optimal solution for **Problem 3.2** can be obtained in the following proposition.

Proposition 3.3 *Given price p^{u} charged by the MNO to MUs, the matrix form of the optimal sponsorship factor $\boldsymbol{\theta}^*$ is*

$$\boldsymbol{\theta}^* = \left(-2t\gamma p^{\mathrm{u}}\mathbf{K} + 2p^{\mathrm{u}}\mathbf{I}\right)^{-1}\left[\gamma\mathbf{s} + (\mathbf{I} - 2\gamma t\mathbf{K})(p^{\mathrm{u}}\mathbf{1} - \mathbf{a})\right], \tag{3.8}$$

where $\mathbf{K} = (\mathbf{G} - 2\boldsymbol{\Lambda})^{-1}$.

Proof. Please refer to the Appendix for details. □

Note that the Proposition 3.3 is based on Lemma 3.2 as follows.

Lemma 3.2 $\mathbf{I} - \gamma t\mathbf{K}$ *is invertible.*

Proof. We first decompose $\mathbf{I} - \gamma t\mathbf{K}$ as follows:

$$\begin{aligned} \mathbf{I} - \gamma t\mathbf{K} &= \mathbf{I} - \gamma t\left(\mathbf{G} - \mathbf{b} - \mathbf{1c}\right)^{-1} \\ &= \mathbf{I} - \gamma t\left(\mathbf{G} - 2\boldsymbol{\Lambda}\right)^{-1} \\ &= \left(\mathbf{G} - \mathbf{b} - \mathbf{1c} - \gamma t\mathbf{I}\right)\left(\mathbf{G} - \mathbf{b} - \mathbf{1c}\right)^{-1}. \end{aligned} \tag{3.9}$$

We know that $\left(\mathbf{G} - \mathbf{b} - \mathbf{1c}\right)^{-1}$ is invertible as proved in Lemma 3.1. As for the term $\mathbf{G}-\mathbf{b}-\mathbf{1c}-\gamma t\mathbf{I}$, $\mathbf{b}$ and $\gamma t\mathbf{I}$ are diagonal matrix and can be combined together. Then recall from Assumption 3.1, the condition for the reversibility of this term is $\sum_{j\in\mathcal{N}} \frac{g_{ij}-2c}{2b_i+\gamma t+2c} < 1, \forall i \in \mathcal{N}$. Under Assumption 3.1, we have $\sum_{j\in\mathcal{N}} \frac{g_{ij}-2c}{2b_i+2c} < 1, \forall i \in \mathcal{N}$, and $\sum_{j\in\mathcal{N}} \frac{g_{ij}-2c}{2b_i+\gamma t+2c} < \sum_{j\in\mathcal{N}} \frac{g_{ij}-2c}{2b_i+2c}$. Therefore, the condition for the reversibility of the term $\mathbf{G} - \mathbf{b} - \mathbf{1c} - \gamma t\mathbf{I}$ is achieved under Assumption 3.1. Thus, the matrix $\mathbf{I} - \gamma t\mathbf{K}$ is invertible. The proof is now completed. □

3.3.1.2 Stage I: Optimal Pricing (1st-tier leader)

The MNO obtains its revenue from charging MUs, and the price charged is the same for all the MUs with uniform pricing [74]. Thus, the MNO determines the optimal price p^{u} by solving **Problem 3.1**. Now we obtain the optimal content demand $\mathbf{x}^*(p^{\mathrm{u}}, \boldsymbol{\theta})$ in (3.7) from Stage III and the optimal sponsorship factor $\boldsymbol{\theta}^*(p^{\mathrm{u}})$ in (3.8) from Stage II. We next substitute $\boldsymbol{\theta}^*(p^{\mathrm{u}})$ into (3.7), and obtain the following expression with some simple transformations:

$$\mathbf{x}^*(p^{\mathrm{u}}) = \mathbf{K}(-2t\gamma\mathbf{K} + 2\mathbf{I})^{-1}(p^{\mathrm{u}}\mathbf{1} - \gamma\mathbf{s} - \mathbf{a}). \tag{3.10}$$

Then we can substitute $\mathbf{x}^*(p^{\mathrm{u}})$ from (3.10) into (3.4) to solve **Problem 3.1**. The matrix form of (3.4) is expressed by $p^{\mathrm{u}}\mathbf{x}\mathbf{1}^\top$ or $p^{\mathrm{u}}\mathbf{x}^\top\mathbf{1}$. Clearly, the utility function of the MNO in (3.4) is concave with respect to the price p^{u}, i.e., $\frac{\partial^2\Pi}{\partial(p^{\mathrm{u}})^2} < 0$. The detailed proof is omitted here due to space limit. Therefore, we conclude with the following theorem.

Theorem 3.3 *With the best response of the CP and MUs, the optimal price p^{u} charged by the monopolistic MNO is uniquely determined.*

In our hierarchical three-stage Stackelberg game, we have proved that each stage has its optimal unique solutions. Thus, we know that each stage has a sub-game perfect equilibrium, the Stackelberg equilibrium of the proposed three-stage game model exists. If we know that each sub-game perfect equilibrium in each stage is unique, we can conclude that the Stackelberg equilibrium is also unique. Therefore, the existence and uniqueness of the Stackelberg equilibrium can be guaranteed. Thus, we can apply the best-response dynamics algorithm to achieve the unique Stackelberg equilibrium [5].

3.3.2 Simultaneous Competition Between CP and MNO

It is worth noting that the model for the interaction among the MNO, the CP and MUs can be reduced to a two-stage game, if we consider that the MNO and the CP move at

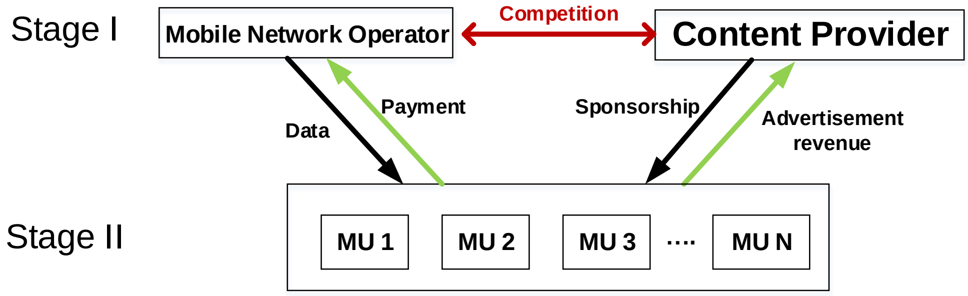


Figure 3.2: Two-stage Stackelberg game model of the interactions among the MNO, CP, and MUs in the sponsored content market.

the same time, as illustrated in Fig. 3.2. In this scenario, the content demand sub-game of MUs is referred to as the lower Stage II of the hierarchical Stackelberg game. In the upper Stage I, two entities, i.e., the MNO and the CP both act as the leaders of the two-stage game. In particular, the MNO and the CP compete with each other simultaneously and selfishly, and thus their interaction can be modeled as a non-cooperative static game. This situation can happen when the MNO and CP have similar market influence in which they cannot decide on their strategies before each other.

Based on the Nash equilibrium of the content demand $\mathbf{x}^*(p^{\mathrm{u}}, \boldsymbol{\theta})$ in (3.7) from Stage II, the MNO and the CP optimize their pricing and sponsoring strategies in Stage I, respectively. Specifically, the MNO determines the price to maximize its revenue defined in (3.4) by solving **Problem 3.1**, and the CP determines the sponsorship factor to maximize its profit defined in (3.3) by solving **Problem 3.2** simultaneously. Thus, the **Problems 3.1** and **3.2** together form a non-cooperative static game. Then, we investigate the Nash equilibrium of the non-cooperative game and conlcude with the following theorem.

Theorem 3.4 *The existence and uniqueness of the Nash equilibrium of the non-cooperative game between the CP and the MNO can be guaranteed if the following condition holds:*

the total payment from MU i to MNO is larger than a threshold, i.e.,

$$p^{\mathrm{u}}(1-\theta_i) > \max\left\{\gamma s, \frac{a_i}{3}, \left[(\sqrt{-2\gamma t\mathbf{K}}+\mathbf{I})^{-1}\sqrt{\frac{-\gamma t\mathbf{K}}{2}}\mathbf{a}\right]_i\right\}.$$

Proof. Please refer to the Appendix for details. □

Then, we use the best-response dynamics for calculating the Nash equilibrium of the two-player non-cooperative game in this stage. So far, we have proved that each Nash equilibrium of sub-game in this Stackelberg game is unique. Therefore, it can be concluded that the Stackelberg equilibrium is also unique. Similar to that in Section 3.3.1, we can apply the best-response dynamics algorithm to achieve the unique Stackelberg equilibrium [5].

3.3.3 Cooperation Between CP and MNO

In the non-cooperative game discussed in Section 3.3.1 and Section 3.3.2, the interaction among selfish players may lead to an inefficient Nash equilibrium. In order to address the well-known inefficiency of Nash equilibrium of the non-cooperative game, we consider another practical cooperative setting between the CP and the MNO. In this scenario, the interaction between the MNO and the CP is modeled as an optimization problem. Thus, the objectives of the CP and the MNO are to maximize their aggregate payoff. Note that the focus of our work is on non-cooperative game among main entities in which we consider rational or selfish behavior models to better capture the reality of their interactions, and the cooperation scenario is presented as a comparison. Nevertheless, the future work on the coalition game between the CP and the MNO is worth studying in the future works.

In particular, we consider the MNO and the CP as a single entity, referred to as a coalition. This situation can happen, for example, when the MNO and CP are close business partners. Then, in Stage I, the CP-MNO coalition determines the sponsoring and the pricing strategies jointly, with the purpose of maximizing their aggregate payoff, i.e., $\mathscr{R} = \mathscr{P} + \Pi$. Therefore, the **Problems 3.1** and **3.2** need to be modified, specifically, the new problem, i.e., the CP-MNO coalition's payoff maximization problem is formulated as follows:

$$\begin{aligned} \underset{\theta_i, p^{\mathrm{u}}}{\text{maximize}} \quad & \mathscr{R} = \gamma \sum_{i \in \mathcal{N}} (s x_i - t {x_i}^2) + p^{\mathrm{u}} \sum_{i \in \mathcal{N}} x_i (1 - \theta_i) \\ \text{subject to} \quad & \mathbf{x} = \mathbf{K} \left[p^{\mathrm{u}} (\mathbf{1} - \boldsymbol{\theta}) - \mathbf{a} \right]. \end{aligned} \tag{3.11}$$

We can rewrite the objective function of (3.11) in a matrix form, and eliminate $\mathbf{x}$ from the objective function with KKT condition. Then, we apply the second-order partial derivative to check its Hessian matrix. Thus, we have the following theorem provided with the same condition as shown in Theorem 3.4.

Theorem 3.5 *The objective function in (3.11) is strictly concave with respect to its decision variables* $\boldsymbol{\theta}$ *and* p^{u}*, and thus there exists a unique globally optimal solution for* $\{\boldsymbol{\theta}^*, p^{\mathrm{u}*}\}$.

Proof. Please refer to the Appendix for details. □

However, it is impossible to derive the closed form solution for $\boldsymbol{\theta}^*$ and $p^{\mathrm{u}*}$, due to their complicated expression. In our performance evaluation, we can apply the low-complexity iterative algorithms based on the gradient assisted binary search algorithm to find the optimal sponsorship factor $\boldsymbol{\theta}^*$ and optimal price $p^{\mathrm{u}*}$, which are the optimal solutions of problem in (3.11).

3.4 Performance Evaluation

In this section, we perform the simulations to evaluate the performance of the strategy adaptation of the MNO, CP, and MUs in sponsored content market under the sequential competition, simultaneous competition and cooperation scenarios. In particular, we simulate and construct the social graph G using the Erdős-Rényi (ER) graph [75] to capture

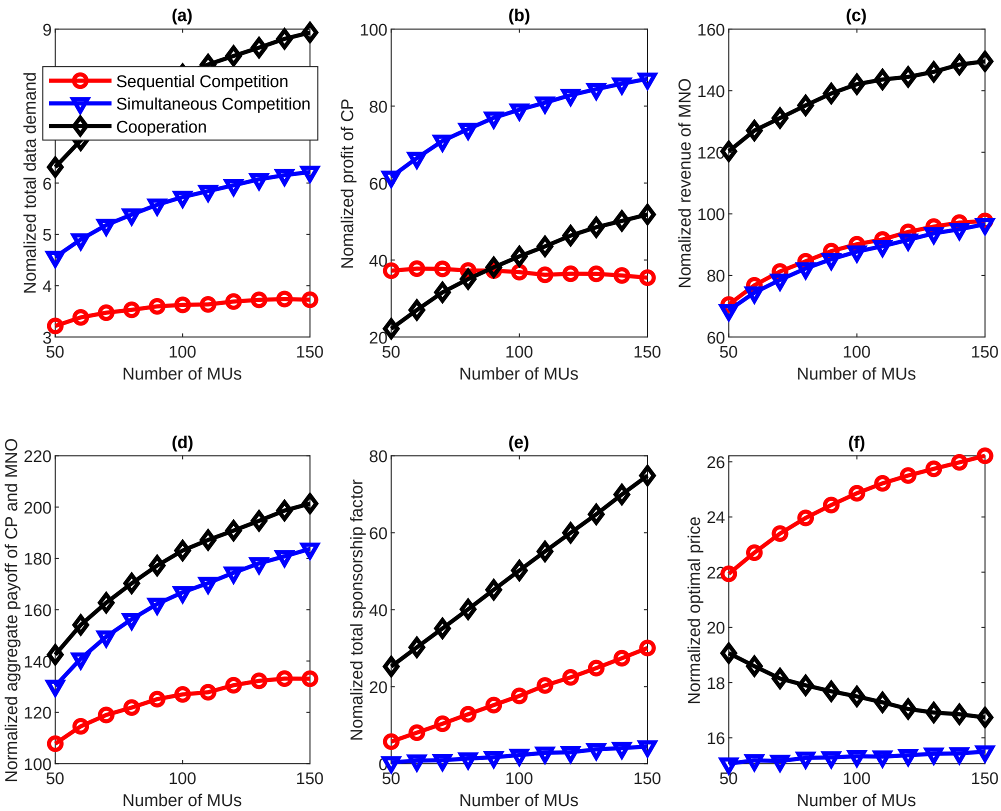


Figure 3.3: The impact of the total number of MUs on three entities of sponsored content market.

the properties from social domain. In the ER graph, the social tie between any two MUs exists with the same probability P_e. We consider a group of N MUs in the ER graph based social network and assume that the parameters a_i and b_i of MUs follow the normal distribution $\mathcal{N}(\mu_a, 1)$ and $\mathcal{N}(\mu_b, 1)$, respectively. Likewise, the social tie g_{ij} between any two MUs i and j follows a normal distribution $\mathcal{N}(\mu_g, 1)$. The default parameters are set as follows: $c = 3$, $\gamma = 3$, $s = t = 5$, $\mu_a = \mu_b = 30$, $\mu_g = 4$, $N = 100$ and $P_e = 0.9$. The presented experimental results are averaged over 500 runs.

3.4.1 The Impacts of the Number of MUs

We first investigate the impact from varying the number of MUs on the three entities of the sponsored content market, i.e., MUs, CP and MNO, as shown in Fig. 3.3. As expected, the total content demand of MUs, the profit of the CP and the revenue of the MNO increase as the number of MUs increases, in the simultaneous competition and cooperation scenarios. The reason is that adding more MUs would enhance each MU's interactions with others, and potentially stimulate more content demand of new coming MUs. However, due to the congestion effects, the marginal increase of the content demand decreases as the number of MUs increases. Meanwhile, the CP provides larger sponsorship for the coming MUs in three scenarios. Furthermore, we observe that in the sequential competition scenario, the profit of the CP decreases as the number of MUs increases. The reason is that the MNO moves before the CP in the sequential competition scenario. The MNO can predict that when the number of MUs increases, a higher price forces the CP to offer larger sponsorship to attract MUs. Consequently, the MNO extracts more surplus from the increasing sponsorship from the CP, and thus the profit of the CP decreases. We also observe that the optimal price increases with the increase of number of MUs in the competition scenarios. The reason is that as the number of MUs increases, more MUs have higher intrinsic demand. Consequently, increasing the price does not result in significant decrease in total demand. However, in the cooperation scenario, we observe that the optimal price decreases with the increase of number of MUs. The reason is that the CP-MNO coalition aims to maximize their aggregate payoff in the cooperation scenario. When the MNO reduces the price, the selfish CP wants to offer smaller sponsorship for saving cost in the competition scenarios. In the cooperation scenario, the MNO and CP are not selfish individuals, and therefore the MNO-CP coalition allows them to reduce the price and offer larger sponsorship at

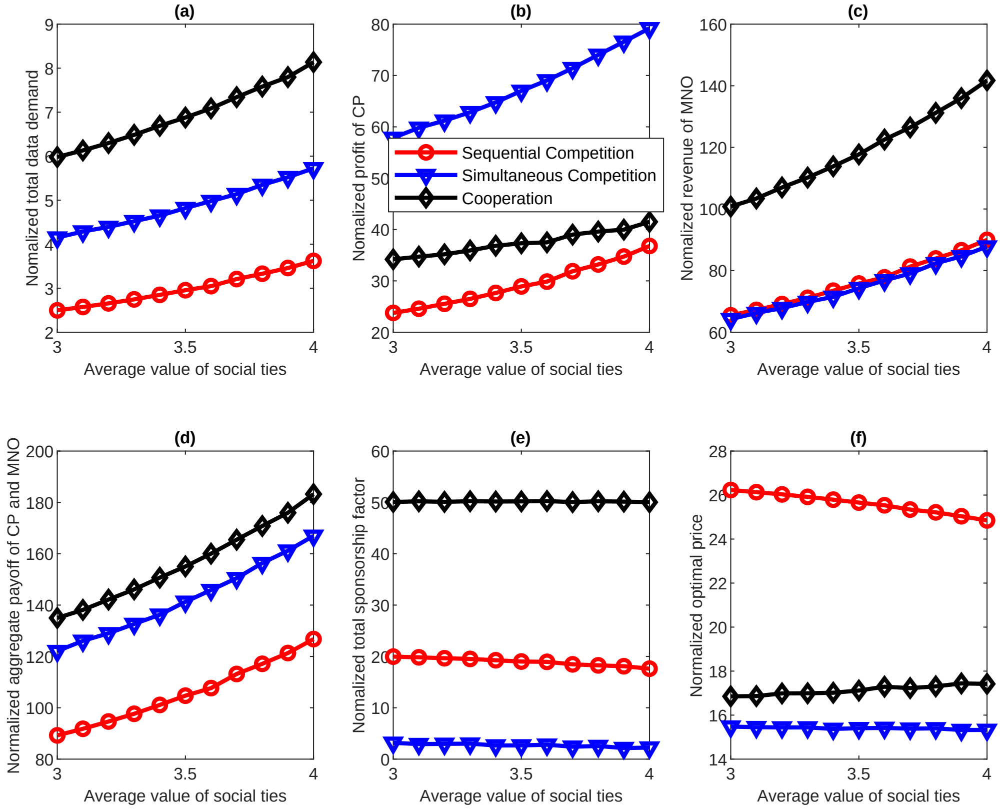


Figure 3.4: The impact of social network effects on three entities of sponsored content market.

the same time, which extracts the surplus from the MUs to the full extent and achieves higher aggregate payoff.

3.4.2 The Impacts of the Social Network Effects

We then investigate the impact of network effects on these three entities, and the results are shown in Fig. 3.4. In all of the three scenarios, the total content demand of MUs, the profit of the CP and the revenue of the MNO increase significantly with stronger network effects. This is from the fact that when the network effects become stronger, the demand of each MU is promoted due to stronger positive interdependency of each other. When the total content demand of MUs is high enough, consequently the CP is able to offer smaller sponsorship to save money. In the competition scenarios, we observe that the optimal price decreases as the network effects become stronger. The reason is that when the network effects are strong, the MUs are positively motivated to have higher content demand. The MNO has the incentive to reduce the price for stimulating more content demand by taking advantage of underlying network effect. However, in the cooperation scenario, we observe that the optimal price increases as the network effects become stronger. This is from the fact that the sponsorship from the CP in the cooperation scenario is higher than that in the competition scenarios. Recall that both the CP and the MNO aim to maximize their aggregate payoff in the cooperation scenario. As such, increasing the price will not significantly reduce the content demand of MUs. Therefore, the MNO has the incentive to increase the price slightly to extract more surplus since the CP will not selfishly offer smaller sponsorship for saving cost. Consequently, the CP-MNO coalition achieves higher aggregate payoff as the network effects become stronger.

3.4.3 The Impacts of the Congestion Effects

Next, we evaluate the impact of congestion effects on three entities, as illustrated in Fig. 3.5. We first observe that the total content demand of MUs, the profit of the CP and the revenue of the MNO decrease as the congestion factor increases in the three scenarios. The reason is that the congestion has a negative impact on the content demand of MUs. As such, when the congestion factor increases, the decreasing content demand of MUs leads to the decrease of the profit of the CP and the revenue of the MNO. In the competition scenarios, we observe that when the congestion factor increases, the optimal offered sponsorship increases and the optimal price decreases. This is due to the fact that the CP needs to offer larger sponsorship with the increase of congestion factor to retain the original MUs at least, which may incur more cost. Meanwhile, the MNO has no

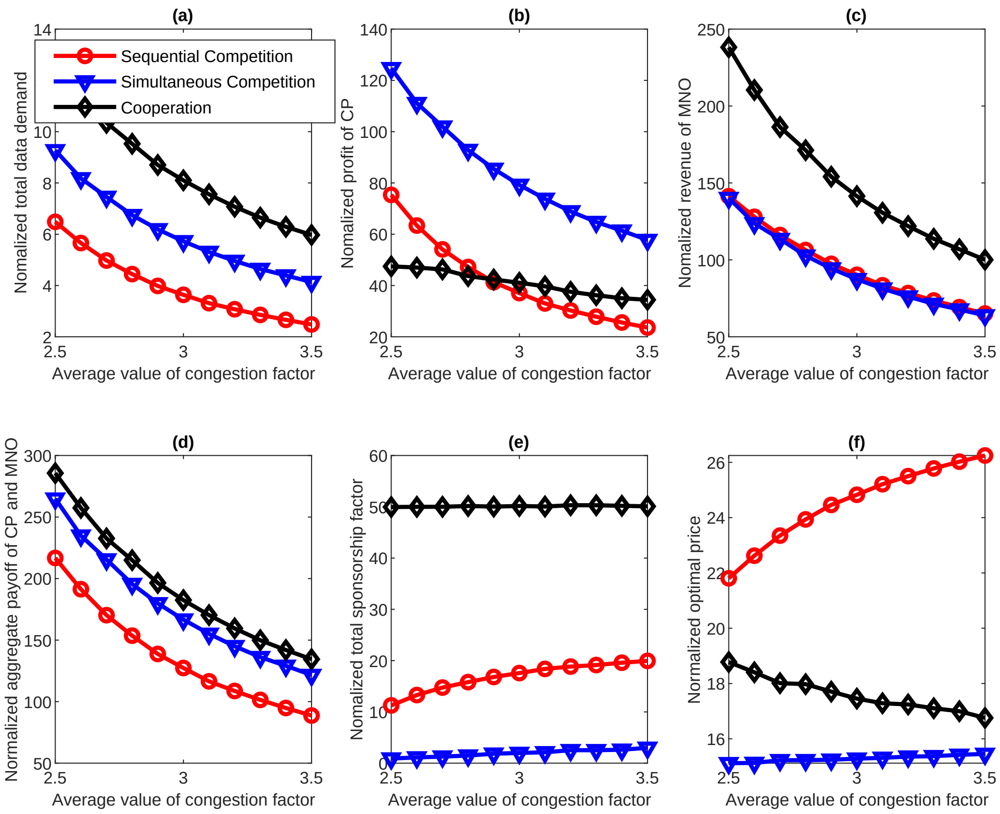


Figure 3.5: The impact of congestion effects on three entities of sponsored content market.

incentive to set lower price to encourage content demand of MUs because the selfish CP will offer smaller sponsorship for saving cost due to the competition. On the contrary, in the cooperation scenario, we observe that the optimal price decreases with the increase of congestion factor. The reason is that the CP-MNO coalition will not optimize their individual payoff selfishly due to the common goal. Therefore, in order to compensate the increasing congestion effects, the CP-MNO coalition offers a larger sponsorship and also reduces the price. This slightly compensates the decreasing content demand of MUs and further extract the surplus from them.

3.4.4 The Impacts of the Advertisements Revenue of CP

Furthermore, we evaluate the impacts of the advertisements (ads) revenue of the CP on three entities, as shown in Fig. 3.6. In the competition scenarios, the optimal price and the optimal sponsorship increase as the ads revenue level of CP increases. The reason is that the CP has the incentive to offer larger sponsorship to attract content demand of MUs when the ads revenue level is improved. This enables the CP to gain higher ads revenue. However, due to the competition, the MNO wants to increase the price since it knows that the increasing sponsorship from the CP slightly compensates the increasing price, which will not significantly reduce the content demand of MUs. In the sequential competition scenario, we also observe that the profit of the CP decreases as the ads revenue level increases. The reason is that the MNO predicts that the higher price leads to larger sponsorship from the CP, and the CP can merely choose the optimal sponsorship given the price determined by the MNO. Recall that the CP cannot unilaterally change the sponsorship to reduce the revenue of the MNO in the sequential competition scenario. When the ads revenue level of the CP increases, the motivation of the CP for offering larger sponsorship is more. By utilizing this motivation, the MNO wants to considerably increase the price and further extract more surplus from the CP, and therefore the profit of the CP decreases. In the cooperation scenario, we observe that the optimal price decreases with the increase of ads revenue level of the CP. This is because when the CP-MNO coalition offers substantial sponsorship, reducing the price can significantly attract the content demand of MUs. Although the revenue of the MNO slightly decreases, the profit of the CP substantially increases due to the increasing content demand. As the ads revenue level increases, the increasing profit is greater, which compensates the decreasing revenue of the MNO. Consequently, the aggregate payoff of the CP and the MNO increases.

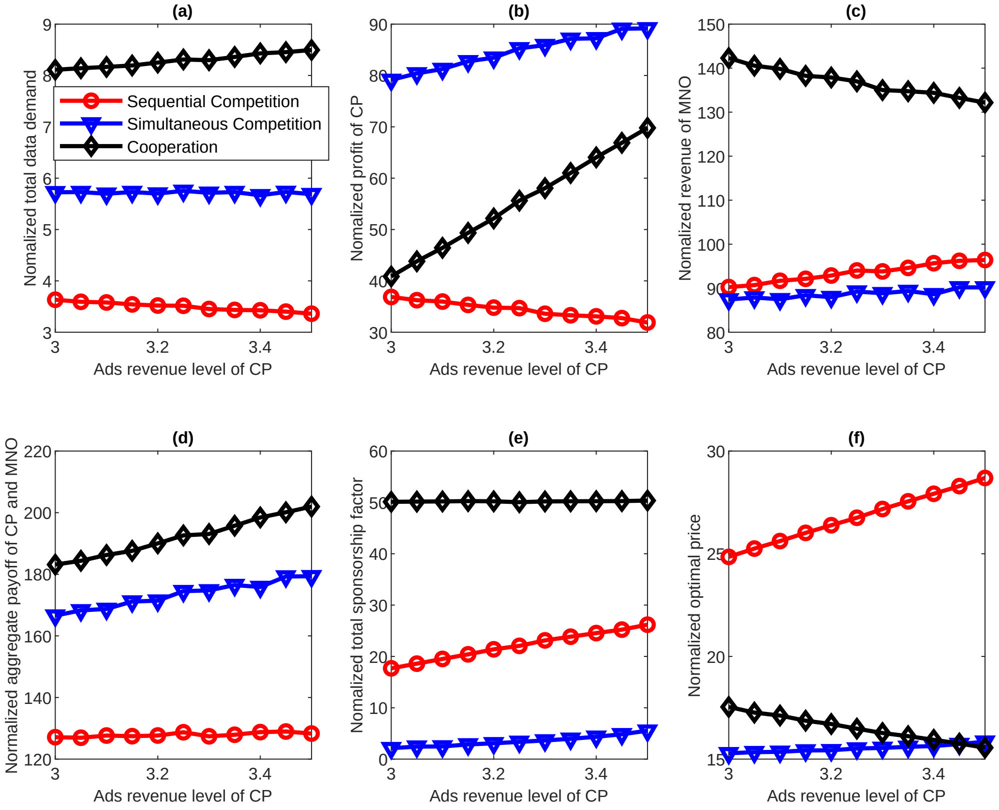


Figure 3.6: The impact of ads revenue level of CP on three entities of sponsored content market.

3.4.5 The Comparison of Three Mutual Interplays Between CP and MNO

At last, we compare three mutual interplays between the CP and the MNO in Figs. 3.3-3.6. We first compare the competition scenarios, i.e., the sequential competition and simultaneous competition scenario. As expected, in the sequential competition scenario, the profit of the CP is lower and the revenue of the MNO is higher than those in the simultaneous competition scenario. Recall that the MNO moves before the CP in the sequential competition scenario. The MNO is able to predict that the CP will offer larger sponsorship to retain the existing MUs if the MNO increases the price. However, the CP can merely choose the sponsorship that maximizes its profit given the price, but cannot increase its profit unilaterally by changing the sponsorship. Thus, in the sequential competition scenario, given the higher price determined by the MNO, the CP offers larger sponsorship. Taking advantage of this, the MNO can increase the price since it can make a market decision before the CP in the sequential competition scenario. Although the content demand of MUs decreases, the revenue of the MNO still increases compared with the simultaneous competition scenario. The reason is that the MNO extracts more surplus from the increasing sponsorship from the CP, and consequently the profit of the CP decreases. Instead, in the simultaneous competition, the MNO and the CP have the same priority. In this case, the CP can change its sponsoring strategy to reduce the profit of the MNO, which in turn changes the pricing strategy of the MNO. The same applies with the MNO. Therefore, the CP and the MNO play against each other for their individual benefit.

Then, we compare the cooperation scenario and the competition scenarios. Recall that in the cooperation scenario, the CP-MNO coalition aims to maximize their aggregate payoff. In other words, the coalition is able to adopt the strategies that fully extract the surplus from MUs. In this scenario, the coalition usually sets a lower price, and provides more sponsorship to better encourage all the MUs. Therefore, the cooperation between the CP and the MNO helps to achieve higher aggregate payoff, as indicated in Figs. 3.3-3.6. However, in the competition scenarios, when the MNO sets a lower price, the selfish CP wants to offer smaller sponsorship to maximize its profit because the sponsorship is not necessary when the price is low enough. As a result, the aggregate payoff of the CP and the MNO decreases, compared with the cooperation scenario. We also observe that the content demand of MUs in the cooperation scenario is higher than those in the competition scenarios. The reason is that the CP and the MNO can cooperate for attracting content demand of MUs and thus fully extract the surplus from MUs. All the results in Figs. 3.3-3.6 clearly show that the cooperation between the CP and the MNO is the best choice for the MNO and the CP in the sponsored content market.

3.5 Summary

In this chapter, we have proposed a hierarchical Stackelberg game to model the interactions among the Mobile Network Operator (MNO), the Content Provider (CP) and Mobile Users (MUs) under sponsored content policy. Furthermore, the network effects in the social domain and congestions in the network domain have been jointly investigated for modeling the interactions among MUs. In the game model, we have examined the interplay between the CP and the MNO by characterizing the scenarios where the MNO and the CP compete sequentially, the MNO and the CP compete simultaneously, and the MNO and the CP cooperate for a common goal. Under the three scenarios, we have derived the unique Nash equilibrium point among the MUs and analytically validated the existence and uniqueness of Stackelberg equilibrium through backward induction. With extensive performance evaluation, it has been revealed that the network effects significantly improve the utilities of MUs, the profit of the CP and the revenue of the MNO. In addition, it has been verified that the cooperation is the best choice for the MNO and the CP.

Chapter 4

Contract Design in Hierarchical Game for Sponsored Content Service Market

The goal of this chapter[1] is to study the interactions among the three entities in the sponsored content market with the presence of private information (type) of mobile users. We introduce a framework combining contract theory and Stackelberg game for the hierarchical sponsored content market. Under the hierarchical market structure with asymmetric information, the mobile network operator, as the leader, naturally decides on the unit data price in Stage I. Then, the sponsored content provider and users, as the followers, determine their strategies in Stage II. We model the situation in Stage II as a contract game. In the game, the sponsored content provider designs a contract that contains its sponsorship strategies toward all types of users. Given the best chosen contract, the mobile network operator adjusts the price to maximize its utility in terms of the revenue minus cost. The major contributions of this chapter are summarized as follows:

- We propose a novel framework that integrates contract theory and Stackelberg game to analyze the hierarchical interactions among the mobile network operator, the sponsored content provider and mobile users with information asymmetry.

[1]The work in this chapter has been published in [76].

- We formulate a two-stage hierarchical Stackelberg game to jointly maximize the utility of the mobile network operator and the profit of the sponsored content provider. The network operator determines the optimal pricing in Stage I, and the contract game is employed to model the interaction between the content provider and mobile users in Stage II. Here, we employ the contract game to handle the asymmetric information for which the contract theory has been inherently developed.

- Through backward induction, we analyze the sub-game perfect equilibrium in each stage analytically. We also derive the necessary and sufficient conditions of the feasible contract, and we obtain the optimal contract that maximizes the profit of sponsored content provider.

- Given the optimal contract from the contract game, we investigate the optimal pricing of the mobile network operator. We prove that the optimal pricing strategy of the network operator is unique, and hence validate the uniqueness of the Stackelberg equilibrium.

- To demonstrate the effectiveness of the proposed framework, we compare the performance of our proposed scheme with that of the existing benchmark scheme through extensive numerical evaluation, e.g., discriminatory sponsorship scheme and uniform scheme.

The rest of the chapter is organized as follows. Section 4.1 presents the system description and formulation of the Stackelberg game to characterize the hierarchical sponsored content market model. We investigate the interactions among the followers as the contract game in Section 4.2, in which the solution of an optimal contract is proposed. The optimal pricing strategy for the network operator is derived in Section 4.3. Some benchmark schemes are elaborated in Section 4.4. Finally, we provide the performance evaluation in Section 4.5 and summarize the chapter in Section 4.6.

4.1 System Description

We consider a sponsored content delivery network consisting of a Mobile Network Operator (MNO), a sponsored Content Provider (CP) and a set of mobile users (MUs) denoted as $\mathcal{N}$. Each MU $i \in \mathcal{N}$ can access and consume the content, e.g., video content, from the CP, and the content can be downloaded directly through the network infrastructure supported by the MNO. In this chapter, we consider a general sponsored content market, where the video content is only given as one example. The video content-specific properties of sponsored content is worth incorporating in the future studies. The MU pays the data price to the MNO for accessing the network infrastructure. Nonetheless, the data usage of each MU, i.e., for downloading contents from the CP, can also be subsidized by the CP under the sponsored content scheme. Note that the CP earns the advertisement revenue, e.g., from the third-party, and the MU obtains the additional sponsorship while enjoying the downloaded content.

4.1.1 Market-Oriented Sponsored Content Model

Let x denote the content (volume) demand from the MU, and $\sigma_i f(x)$ denote the utility obtained from accessing and consuming the content. The content valuation coefficient $\sigma_i > 0$ represents a particular valuation of the content to MU i. The valuation can reflect the willingness-to-participate preference of this MU. For example, if the content valuation coefficient is higher, the MU is more willing to participate in the sponsored content market, accessing and enjoying the content. Moreover, the valuation as the private information is only known to the MU itself, which is treated as its type.

To characterize the heterogeneity of MUs, the MUs are divided into N categories according to their content valuation coefficients. According to the statistical information, the CP can classify MUs into different clusters (types) by using some well-known data mining methods, e.g., k-means. Intuitively, the CP can partition the statistical data into more clusters (MU types) to increase the accuracy at the expense of additional complexity. However, the CP can only design the limited number of contracts in practice due to the market constraints, e.g., the government regulation. In the future work, we will explore the choices of the CP on how many types of MUs to partition, thereby studying the trade-off between the profit and the contract complexity. The MU whose content valuation coefficient corresponds to the i^{th} coefficient level σ_i, $i \in \mathcal{N} = \{1, 2, \ldots, N\}$ is the type-i MU. Without loss of generality, we assume $\sigma_1 > \sigma_2 > \cdots > \sigma_N$ and the higher σ_i indicates higher willingness for consuming the content. It is worth noting that the impacts of Quality of Service (QoS) satisfaction level can still be absorbed into the definition of σ. The lower σ indicates the lower satisfaction level of QoS, which explains the lower willingness to participate from the MU perspective. Practically, the QoS level might be dynamically changing over time depending on traffic demand. However, the QoS level in peak period keeps relatively stable [32]. In this work, following [32], we focus on the interactions between the CP and MUs in peak period and treat QoS level as relatively unchanged. In the future works, we will investigate the impacts of the dynamic QoS on the sponsored content service market.

Similar to that in [31, 30], we first define the following function:

$$f(x) = \frac{1}{1-\alpha} x^{1-\alpha}, \tag{4.1}$$

where $0 < \alpha < 1$ is the given coefficient. In particular, the function $f(\cdot)$ is non-decreasing and concave with decreasing marginal satisfaction, i.e., $\frac{df(x)}{dx} \geq 0$ and $\frac{d^2 f(x)}{dx^2} < 0$, indicating the decreasing marginal preference of MUs to content. Due to its properties and mathematical simplicity, the function defined in (5.1) has been widely used in sponsored content market, such as [31, 30, 32]. It is noteworthy that any increasing concave function can be adopted as the evaluation function, and the change of these functions does not affect our analytical solutions. In traditional mobile content access, the MNO charges each MU a certain data price p for the volume of content downloaded [77]. Thus, the general form of the utility function of the MU i with content demand x is formulated as follows:

$$v_i(x) = \sigma_i f(x) - px. \tag{4.2}$$

According to the sponsored content scheme, the payment for data usage from MUs to the MNO can be partly subsidized by the CP. Let $0 \leq \theta < 1$ denote the sponsorship factor of content subsidized by the CP, which means θx units of the content is sponsored. Thus, the MU pays for the rest $(1-\theta)x$ units of content to the network operator, with the cost $(1-\theta)px$ incurred to the MU [59]. Note that in some cases the sponsorship factor can be negative, which means that the CP would generate additional charges [78]. For example, Youtube provides certain paid video content to charge their users [79]. In this work, we focus on the positive sponsorship case, i.e., $\theta > 0$.

Generally, the utility of an MU is also affected by the amount of advertisement imposed by the CP per volume of content, i.e., l_a. In the following, we assume that l_a is constant for all content. However, we will numerically investigate the impacts of l_a through numerical results. We assume a normalized $l_a \in [0, 1]$ since the CP has the volume of advertisement strictly less than that of the provided content. For example, Pandora Internet Radio plays advertisement regularly between songs. For ease of derivation, we introduce an auxiliary variable τ defined as follows:

$$\tau = \frac{1}{1+l_a}, \tau \in \left[\frac{1}{2}, 1\right]. \tag{4.3}$$

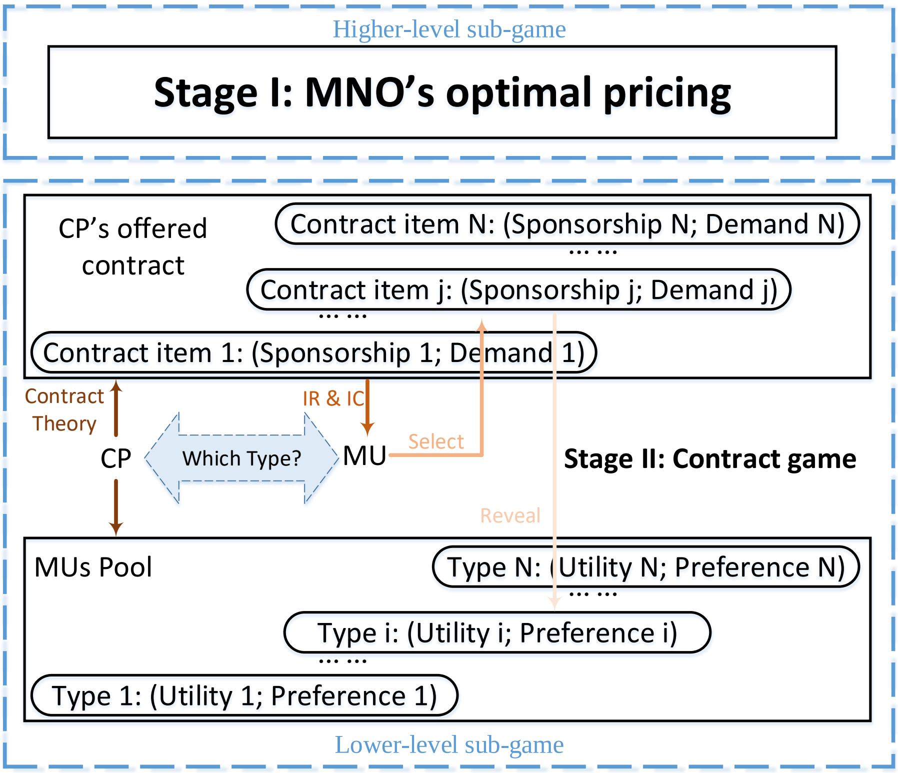


Figure 4.1: A two-stage Stackelberg game framework with information asymmetry for market-oriented sponsored content model.

Therefore, we use $\tau\sigma_i f(x)$ to denote a utility gain of the MU from enjoying the content. Thus, the utility of MU $i \in \mathcal{N}$ which has the content demand x with sponsorship factor θ is expressed as follows:

$$u_i(x) = \tau\sigma_i f(x) - (1-\theta)px. \tag{4.4}$$

4.1.2 Stackelberg Game Formulation

In the following, we model the interaction among the MNO, CP, and MUs as a Stackelberg game with information asymmetry, as illustrated in Fig. 4.1. In Stage I, the MNO, acting as the leader, determines the unit data price p. In Stage II, the CP and MUs, acting as the followers, decide on the sponsorship and the data usage, respectively. The decision making process of followers, in the presence of asymmetric information is modeled as a contract game.

4.1.2.1 CP and MUs in Stage II (Contract Game)

As aforementioned, the MU type is private information which is not known to the CP. Nevertheless, the CP can have the knowledge on the probability distribution of MU types based on statistical information [48]. As there are N types of MUs, the probability of the MU belonging to type-i is denoted as λ_i with $\sum_{i=1}^{N} \lambda_i = 1$.

It is reasonable for the CP to adopt different sponsorship strategies toward different types of MUs, since MUs of different types, i.e., with different content valuation coefficients, have different willingness-to-participate preferences on the sponsored content. However, the private information of MUs posts a challenge for the CP to recognize exactly the type of each MU since some MUs may cheat by untruthfully claiming themselves to have a type different from what they actually do. For example, an MU may claim higher content valuation coefficients, i.e., it has high interest in content, and expect to receive a higher sponsorship. This inevitably causes more sponsorship cost for the CP. Moreover, the untruthful behaviors increase the uncertainty on interactions, thereby complicating the sponsoring and pricing strategies of the CP and the MNO, respectively. To deal with this situation effectively, the contract theory has been developed to provide incentive mechanisms under information asymmetry. In what follows, the interactions between the CP and MUs are modeled as a contract game. In the contract game, the MUs want to consume a certain amount of video content from the CP, and thus each MU needs to decide on how much content it should consume from the CP and how much sponsorship that it can obtain. The decision has to ensure that the MU's demand is fulfilled and the utility is maximized. As the CP designs a contract which contains its sponsorship strategies toward all types of MUs, we define the sponsorship strategies in the form of (x_i, θ_i),

$i \in \mathcal{N}$, where x_i is the volume of content demand of the type-i MU and θ_i is the corresponding sponsorship factor. Let $\boldsymbol{x} = \{x_1, x_2, \ldots, x_N\}$ and $\boldsymbol{\theta} = \{\theta_1, \theta_2, \ldots, \theta_N\}$ denote the content demand vector and the sponsorship factor vector, respectively. Thus, the set containing sponsorship strategies towards all N types of MUs constructs a contract $\phi = \{(x_i, \theta_i), i \in \mathcal{N}\}$, where (x_i, θ_i) is called a contract item.

The utility of the type-i MU selecting the contract item designed for itself can be expressed as follows:

$$u_i(x_i, \theta_i) = \tau\sigma_i f(x_i) - (1 - \theta_i)x_i p, \tag{4.5}$$

where $f(\cdot)$ is defined in (5.1).

By properly designing the contract, the CP can induce each MU to disclose its type by its selection of the contract item and thus resolve the information asymmetry. Based on its designed contract, the CP's obtained profit from all N types of MUs, i.e., the gained advertisement revenue minus the cost of offered sponsorship, is as follows:

$$U_{CP}(\boldsymbol{\theta}, \boldsymbol{x}) = \sum\nolimits_{i=1}^{N} \lambda_i \left[\sigma_c h(x_i) - \theta_i p x_i\right], \tag{4.6}$$

where σ_c represents the advertisement revenue coefficient and $\sigma_c h(x)$ represents the advertisement revenue from the content volume access of MUs. Note that different types of CPs may have different values of σ_c. We numerically explore the impacts of advertisement revenue coefficient σ_c on the system performance in Section 7. In this work, we consider the single CP in order to derive more insights on the interaction between the CP and MUs under information asymmetry. We will reserve the study of the sponsoring competition among CPs for the future work. $h(x)$ is defined using the following function which is similar to that in [31], i.e.,

$$h(x) = \frac{1}{1-\gamma} x^{1-\gamma}, \tag{4.7}$$

where $0 < \gamma < 1$ is the coefficient. In fact, most of the related works have proved that a function with diminishing return would model the ads revenue more closely [30, 31, 71, 70].

4.1.2.2 MNO in Stage I (Optimal Pricing)

We use the MNO's revenue minus cost to represent its utility. The utility is from three sources: 1) the discounted price charged to MUs for the data usage, 2) the sponsorship offered by the CP, and 3) the variable maintaining cost, modeled as cx, where c is the per unit cost. The total maintaining cost includes the variable cost and the fixed cost. In this chapter, we only consider the variable cost, e.g., energy consumption, since the fixed cost is only a constant which does not affect the analytical solutions. Thus, the MNO determines the optimal price p to solve the following payoff, i.e., utility, maximization problem:

$$\begin{aligned} U_{MNO}(p) &= \sum\nolimits_{i=1}^{N} \lambda_i \left[(1-\theta_i)x_i p + \theta_i x_i p - c x_i\right] \\ &= \sum\nolimits_{i=1}^{N} \lambda_i \left(p x_i - c x_i\right). \end{aligned} \tag{4.8}$$

In what follows, we investigate the follower game and the leader game of the Stackelberg game model using backward induction.

4.2 Follower Game: Contract Game between the CP and MUs

In this section, we employ the contract theory to model the interaction between the CP and MUs and resolve the conflicting objectives between them. In particular, given the unit data price determined by the MNO, the CP proposes a set of contracts for all types of MUs. The contract includes a series of "the volume of content demand" and corresponding "sponsorship factor" pairs as contract items. The MUs are free to select the contract item. The goal of the CP is to maximize U_{CP} by offering the optimal contracts $\phi = \{(x_i^*, \theta_i^*), \forall i \in \mathcal{N} = \{1, \ldots, N\}\}$. In the following, we investigate the conditions and feasibility of the contract.

4.2.1 Conditions for Contract Feasibility

A feasible contract is capable of attracting the MUs to access and consume the content provided by the CP and ensuring that each MU only selects the contract item designed for its type, the following individual rationality (IR) and incentive compatibility (IC) constraints need to be guaranteed.

Definition 4.1 (Individual Rationality (IR)) *The MU only chooses to take on its contract if its utility is nonnegative, i.e.,* $u_i(x_i, \theta_i) \geq 0$*, specifically,*

$$\tau \sigma_i f(x_i) - (1 - \theta_i) x_i p \geq 0, \forall i \in \{1, \ldots, N\}, \tag{4.9}$$

Definition 4.2 (Incentive Compatible (IC)) *The MU must choose the contract item designed specifically for its own type instead of any other contract item, i.e.,* $u_i(x_i, \theta_i) \geq u_i(x_j, \theta_j)$, $\forall i, j \in \{1, \ldots, N\}, i \neq j$*, specifically,*

$$\tau \sigma_i f(x_i) - (1 - \theta_i) x_i p \geq \tau \sigma_i f(x_j) - (1 - \theta_j) x_j p, \forall i, j \in \{1, \ldots, N\}, i \neq j. \tag{4.10}$$

The IR constraint is to ensure that the MU has necessary incentives to accept and sign the contract. The IC constraint is to ensure that the MU can only maximize its utility by selecting the contract that is designed for its type. Thus, the type of each MU is revealed to the CP, which is called "self-reveal". If the contract satisfies the IC and IR constraints, the contract is then a feasible contract. From the contract theory, the CP aims at maximizing the profit subject to the IR and IC constraints given in (4.9) and (4.10). Therefore, the optimal contract is the solution to the following optimization problem:

$$
\begin{aligned}
\underset{(\boldsymbol{\theta},\boldsymbol{x})}{\text{maximize}} \quad & U_{CP}(\boldsymbol{\theta},\boldsymbol{x}) = \sum_{i=1}^{N} \lambda_i \left[\sigma_c h(x_i) - \theta_i p x_i\right], \\
\text{subject to} \quad & u_i(x_i,\theta_i) \geq 0,\ \forall i \in \{1,\dots,N\}, \\
& u_i(x_i,\theta_i) \geq u_i(x_j,\theta_j),\ \forall i,j \in \{1,\dots,N\}, i \neq j, \\
& x_i \geq 0, 1 > \theta_i \geq 0, \sigma_i \geq 0,\ \forall i \in \{1,\dots,N\}, \\
& \sigma_1 > \sigma_2 > \ldots > \sigma_N.
\end{aligned}
\tag{4.11}
$$

Since there are N IR constraints and $N(N-1)$ IC constraints which are non-convex, the problem given in (4.11) is not straightforward to solve. As such, we simplify these constraints following a standard method in the contract theory. In particular, we first simplify the IR constraints by Lemma 4.1.

Lemma 4.1 *If $\sigma_1 > \sigma_2 > \cdots > \sigma_N$, then*

- *under the IC constraints, we conclude that $u_i(x_i,\theta_i) \geq u_N(x_N,\theta_N),\ \forall i \in \{1,\dots,N\}$ $\Longleftrightarrow u_N(x_N,\theta_N) \geq 0$, and*
- *in the optimal contract, under the IC constraints, the IR constraint for the lowest type σ_N is binding, i.e., $u_N(x_N,\theta_N) = \tau\sigma_N f(x_N) - (1-\theta_N)x_N p = 0$.*

Proof. Please refer to Appendix for details. □

From Lemma 4.1, we conclude that if the lowest MU type among all IR constraints is binding, then the other types will automatically hold under the IC constraints. Next, we prove that the IC constraints can be simplified by the following lemma. We begin by analyzing the necessary conditions for IC constraints.

Lemma 4.2 *Under the IC constraints, if $\sigma_i > \sigma_j$, then $x_i > x_j$, $\forall i,j \in \{1,\dots,N\}$, $i \neq j$.*

Proof. Please refer to Appendix for details. □

Lemma 4.2 indicates that the MUs of higher type have the higher preference towards accessing and consuming the sponsored content. From Lemma 4.2, we define the following definition on monotonicity.

Definition 4.3 (Monotonicity) *If the contract satisfies IC constraints, then the monotonicity constraint holds, i.e.,* $x_i \geq x_j$ *if* $\sigma_i \geq \sigma_j$, $\forall i, j \in \{1, \ldots, N\}$.

Lemma 4.3 *Under the IC constraints, if* $\sigma_i > \sigma_j$, *then* $(1-\theta_i)x_i p > (1-\theta_j)x_j p$, $\forall i, j \in \{1, \ldots, N\}$.

Proof. Please refer to Appendix for details. □

Lemma 4.3 implies that the discounted content access cost after having the sponsorship, i.e., $(1-\theta_i)x_i p$, should be higher for higher type-i MUs. Otherwise, all types of MUs would like to choose the higher volume of content demand with the lower cost. Furthermore, we can deduce that the higher type MUs have greater utility than those of the MUs with lower types. From the IC constraints and Lemmas 4.1-4.3, we have the following conclusion. If the MU with a higher type selects the contract designed for a lower type MU, even though the discounted content access cost is small, the less gain from consuming the content will deteriorate such MU's utility. Conversely, if the MU with a lower type selects the contract designed for a higher type MU, the gain in terms of enjoying the content cannot compensate for its corresponding cost. Consequently, the cost exceeds the gain. The MU can only receive the maximum utility if it selects the contract item designed for itself. Additionally, the corresponding sufficient conditions for IC constraints are given in the following.

Definition 4.4 (Local Upward Incentive Constraint (LUIC)) *LUIC*$(\sigma_i, \sigma_{i+1}) :$ $u_i(x_i, \theta_i) \geq u_i(x_{i+1}, \theta_{i+1})$, $\forall i \in \{1, \ldots, N-1\}$. *Specifically,*

$$\tau \sigma_i f(x_i) - (1-\theta_i)x_i p \geq \tau \sigma_i f(x_{i+1}) - (1-\theta_{i+1})x_{i+1} p. \tag{4.12}$$

Definition 4.5 (Local Downward Incentive Constraint (LDIC)) *LDIC*(σ_i, σ_{i-1}) : $u_i(x_i, \theta_i) \geq u_i(x_{i-1}, \theta_{i-1})$, $\forall i \in \{2, \dots, N\}$. *Specifically,*

$$\tau\sigma_i f(x_i) - (1-\theta_i)x_i p \geq \tau\sigma_i f(x_{i-1}) - (1-\theta_{i-1})x_{i-1}p. \tag{4.13}$$

Lemma 4.4 *With the monotonicity constraints, the IC constraints can be reduced to LDIC and LUIC contraints, i.e.,*

$$\{LUIC(\sigma_i, \sigma_{i+1}) : \forall i \in \{1, \dots, N-1\}\} \cap$$
$$\{LDIC(\sigma_i, \sigma_{i-1}) : \forall i \in \{2, \dots, N\}\} \Longrightarrow \text{IC constraints}. \tag{4.14}$$

Proof. Please refer to Appendix for details. □

Lemma 4.4 is presented to show that LDIC and LUIC are the sufficient conditions for the IC constraints. Thus, we can replace IC constraints by the LDIC and LUIC constraints. Moreover, we use Lemma 4.5 further to reduce the LDIC and LUIC constraints.

Lemma 4.5 *In the optimal contract, the IC constraints can be replaced by*

$$u_i(x_i, \theta_i) = u_i(x_{i+1}, \theta_{i+1}), \ \forall i \in \{1, \dots, N-1\}, \tag{4.15}$$

i.e.,

$$\tau\sigma_i f(x_i) - (1-\theta_i)x_i p = \tau\sigma_i f(x_{i+1}) - (1-\theta_{i+1})x_{i+1}p,$$
$$\forall i \in \{1, \dots, N-1\}. \tag{4.16}$$

Proof. Please refer to Appendix for details. □

Lemma 4.5 indicates that the IC constraints are ensured in the optimal contract provided that the condition $u_i(x_i, \theta_i) = u_i(x_{i+1}, \theta_{i+1})$, $\forall i \in \{1, \dots, N-1\}$ holds. The condition in Lemma 4.5 means that, for any type-i MU ($i = 1, \dots, N-1$), the utility from selecting the contract item intended for itself and the utility from selecting the contract item intended for type-$(i+1)$ MU have the same value. The Lemma 4.5 can be leveraged to replace $N(N-1)$ IC constraints with only $(N-1)$ constraints. By using the simplified constraints given by Lemmas 4.1 and 4.5, the optimization problem given in (4.11) can be transformed to

$$\begin{aligned}
\underset{(\boldsymbol{\theta},\boldsymbol{x})}{\text{maximize}} \quad & U_{CP}(\boldsymbol{\theta},\boldsymbol{x}) = \sum_{i=1}^{N} \lambda_i \left[\sigma_c h(x_i) - \theta_i p x_i\right], \\
\text{subject to} \quad & u_N(x_N, \theta_N) = 0, \\
& u_i(x_i,\theta_i) = u_i(x_{i+1},\theta_{i+1}),\ \forall i \in \{1,\dots,N-1\}, \\
& x_1 > x_2 > \cdots > x_N, \\
& x_i \geq 0, 1 > \theta_i \geq 0, \sigma_i \geq 0,\ \forall i \in \{1,\dots,N\}, \\
& \sigma_1 > \sigma_2 > \cdots > \sigma_N.
\end{aligned} \tag{4.17}$$

4.2.2 Optimal Contract Solution

We now solve the optimization problem defined in (4.17) to obtain the optimal contract. We first apply a standard method in the contract theory to solve the relaxed problem without the monotonicity constraint and then verify that the solution guarantees the monotonicity constraint. Specifically, by iterating the first and second constraints in (4.17), we have

$$(1-\theta_i)x_i p = \tau\sigma_N f(x_N) + \sum_{i}^{N} w_i,\ \forall i \in \{1,\dots,N-1\}, \tag{4.18}$$

where

$$w_i = \begin{cases} \tau\sigma_i f(x_i) - \tau\sigma_i f(x_{i+1}), & \forall i \in \{1,\dots,N-1\}, \\ 0, & \text{for } i = N. \end{cases}$$

Thus, we have

$$\theta_i x_i p = x_i p - \tau\sigma_N f(x_N) - \sum_{i}^{N} w_i,\ \forall i \in \{1,\dots,N-1\}. \tag{4.19}$$

Algorithm 4.1 The algorithm for solving the infeasible sub-sequences

Initialization: Let $x_i^* = \arg\max_{x_i} S_i$, $i \in \{1, \ldots, N\}$;
while The set $\{x_i^*\}$ does not satisfy the monotonicity constraint, **do**
  In the set $\{x_i^*\}$, search for the infeasible sub-sequence $\{x_i^*, x_{i+i}^*, \ldots, x_j^*\}$, where $x_i^* \geq x_{i+1}^* \geq \cdots \geq x_j^*$, $i, j \in \{1, \ldots, N\}$, and $i < j$;
  Set $x_k^* = \arg\max_x \sum_{r=i}^{j} S_r(x)$, $k \in \{i, i+1, \ldots, j\}$;
end while
Return The feasible set $\{x_i^*\}$, $i \in \{1, \ldots, N\}$;

Substituting (4.19) into the objective function of the CP, i.e., $U_{CP}(\boldsymbol{\theta}, \boldsymbol{x})$, we can remove the dependency of $U_{CP}(\boldsymbol{\theta}, \boldsymbol{x})$ on $\boldsymbol{\theta}$ and rewrite $U_{CP}(\boldsymbol{\theta}^*, \boldsymbol{x})$ as $U_{CP}(\boldsymbol{x})$, i.e.,

$$\begin{aligned} U_{CP}(\boldsymbol{\theta}^*, \boldsymbol{x}) &= \sum_{i=1}^{N} \lambda_i \left[\sigma_c h(x_i) - \theta_i p x_i\right] \\ &= \sum_{i=1}^{N} \lambda_i \left[\sigma_c h(x_i) - x_i p + \tau\sigma_N f(x_N) + \sum_{i}^{N} w_i\right] \\ &= \sum_{i=1}^{N} \lambda_i \left[\sigma_c h(x_i) - x_i p + \tau\sigma_N f(x_N) + \sum_{i}^{N-1} [\tau\sigma_i f(x_i) - \tau\sigma_i f(x_{i+1})]\right] \\ &= \sum_{i=1}^{N} \left\{\lambda_i \left[\sigma_c h(x_i) - x_i p\right] + R_i f(x_i)\right\}, \end{aligned} \tag{4.20}$$

where

$$R_i := \begin{cases} \tau\sigma_i \sum_{j=1}^{i} \lambda_j - \tau\sigma_{i-1} \sum_{j=1}^{i-1} \lambda_j, & \forall i \in \{2, \ldots, N\}, \\ \tau\sigma_1\lambda_1, & \text{for } i = 1. \end{cases}$$

Accordingly, the optimization problem in (4.17) becomes

$$\begin{aligned} \underset{\boldsymbol{x}}{\text{maximize}} \quad & U_{CP}(\boldsymbol{x}) = \sum_{i=1}^{N} \left\{\lambda_i \left[\sigma_c h(x_i) - x_i p\right] + R_i f(x_i)\right\}, \\ \text{subject to} \quad & x_i \geq 0, 1 > \theta_i \geq 0, \sigma_i \geq 0, \ \forall i \in \{1, \ldots, N\}. \end{aligned} \tag{4.21}$$

The objective function given in (4.21) can be presented as $U_{CP}(\boldsymbol{x}) = \sum_{i=1}^{N} S_i$, where

$$S_i = \lambda_i \left[\sigma_c h(x_i) - x_i p\right] + R_i f(x_i). \tag{4.22}$$

Since we know that S_i is independent of S_j, $i, j \in \{1, \ldots, N\}$, $i \neq j$, and thus S_i is only associated with x_i. Therefore, the optimal volume of content demand $\boldsymbol{x}^*$ that maximizes

$U_{CP}(\boldsymbol{x})$ can be derived separately from $x_i^* = \arg\max_{x_i} S_i$. After taking the second-order partial derivative of the objective function given in (4.21), i.e., S_i, with respect to x_i, we can easily know $\frac{\partial^2 S_i}{\partial {x_i}^2} < 0$. Thus, S_i is a concave function in terms of x_i, $\forall i \in \{1, \ldots, N\}$. Taking the first-order partial derivative of S_i with respect to x_i, we obtain

$$\begin{aligned} \frac{\partial S_i}{\partial x_i} &= \lambda_i \left[\sigma_c h'(x_i) - p\right] + R_i f'(x_i) \\ &= \lambda_i \left(\sigma_c {x_i}^{-\gamma} - p\right) + R_i {x_i}^{-\alpha}. \end{aligned} \tag{4.23}$$

According to Fermat's Theorem [80], we derive x_i^* in the optimal contract from solving $\left.\frac{\partial S_i}{\partial x_i}\right|_{x_i = x_i^*} = 0$. This gives x_i^* as the solution to $\lambda_i \sigma_c {x_i}^{-\gamma} + R_i {x_i}^{-\alpha} = \lambda_i \cdot p$. Note that we cannot obtain the closed-form expression of x_i^*, since there exists no general solution expression for variable x_i from the equation $\lambda_i \sigma_c {x_i}^{-\gamma} + R_i {x_i}^{-\alpha} = \lambda_i \cdot p$. Therefore, it's practically impossible to obtain the closed-form expression of x_i^*. Nevertheless, we can numerically obtain the value of x_i^*. After x_i^* is obtained, we solve for θ_i^* via (4.19).

So far, we have attained the optimal contract that maximizes the profit of the CP and satisfies the constraints of IR and IC, which is summarized in the following theorem.

Theorem 4.1 (Optimal Contract in the Lower-Layer Game) *In the optimal contract, for each* $i \in \{1, 2, \ldots, N\}$, x_i^* *can be obtained from*

$$\lambda_i \sigma_c {x_i}^{-\gamma} + R_i {x_i}^{-\alpha} = \lambda_i p, \tag{4.24}$$

where R_i *is defined by*

$$R_i := \begin{cases} \tau\sigma_i \sum_{j=1}^{i} \lambda_j - \tau\sigma_{i-1} \sum_{j=1}^{i-1} \lambda_j, & \forall i \in \{2, \ldots, N\}, \\ \tau\sigma_1\lambda_1, & \textit{for } i = 1. \end{cases}$$

After x_i^* *is obtained via (4.24),* θ_i^* *is derived from* $(1 - \theta_i^*) x_i^* p = \tau\sigma_N f(x_N^*) + \sum_i^N w_i$, *for each* $i \in \{1, 2, \ldots, N\}$, *where* w_i^* *is defined by*

$$w_i = \begin{cases} \tau\sigma_i f(x_i^*) - \tau\sigma_i f(x_{i+1}^*), & \forall i \in \{1, \ldots, N-1\}, \\ 0, & \textit{for } i = N. \end{cases}$$

Collectively, the proposed solution approach achieves all the optimal contract items (x_i^*, θ_i^*) *that form the optimal contract,* $i \in \{1, \ldots, N\}$.

Remark on Theorem 4.1: The solution to the optimal contract allows the CP to design a contract containing its sponsoring strategies toward all types of MUs in the sponsored content market. Through the optimal contract, the CP not only attracts MUs to consume its content but also maximizes its own profit, thereby tackling the information asymmetry between the CP and MUs.

It is worth noting that when we obtain the solution x_i^*, $i \in \{1, \ldots, N\}$ for the relaxation problem, we need to check whether these solutions meet the monotonicity constraint. Since each x_i^* is obtained separately from the corresponding S_i, there may exist some sub-sequences that do not follow the decreasing order. Nevertheless, since $\{S_i$, $i \in \{1, \ldots, N\}\}$ are concave functions, we can solve the problem of such infeasible sub-sequences by using an iterative substitution algorithm, i.e., "Bunching and Ironing" algorithm [17, 46]. The algorithm is presented in Algorithm 4.1. Moreover, the monotonicity constraint is satisfied automatically when the type is uniformly distributed [48, 50, 81].

4.2.3 Practical Implementation

To apply the proposed contract-theoretic approach in practice, the following implementation steps should be followed. First, the CP needs to collect the information required for the calculation of the optimal contract. Practically, the coefficients such as τ and α can be predicted by observing from the historical data or learning from the long-term performance utility records, feedbacks, and satisfaction levels of MUs. Similarly, the CP can estimate the statistical information about the number of MUs and the MU types distribution by learning from user historical behavior or making a user survey. With these system parameters, the CP can calculate the optimal contract given the pricing from the MNO. In the future studies, we will explore how to obtain the values of parameters, e.g., by using machine learning methods.

When there are MUs requesting sponsored contents, e.g., via websites or apps, the CP will broadcast the optimal contracts as the notifications to MUs before the content delivery. The MUs will choose one of the contract item or not by evaluating the contracts, indicating whether they are willing to participate in the sponsored content market. After receiving the feedback from MUs, the CP will sign the contract with the MU that accepts it. Finally, the CP will delivery the demanded sponsored content with the corresponding sponsorship to these MUs according to the contract item they select.

The above implementation framework has been widely adopted in most contract theory-based works, such as Section IV-C in in [48].

4.3 Leader Game: Optimal Pricing of the MNO

Recall from Section 4.2, the optimal contract is derived given the price which is determined by the MNO. In other words, the optimal values of $\boldsymbol{\theta}^*$ and $\boldsymbol{x}^*$ that we obtain are the function of unit data price p. In the optimal contract, for each $i \in \{1, 2, \ldots, N\}$, x_i is the solution to $\lambda_i \sigma_c {x_i}^{-\gamma} + R_i {x_i}^{-\alpha} = \lambda_i p$. We notice that there is no closed-form solution in the contract game, which impedes us from substituting the lower layer solution to the higher layer problem. Consequently, we resort to the properties of composite functions to characterize the Stackelberg equilibrium by analyzing the lower layer equilibrium solution as follows.

We first have the revenue function of the MNO recall from Section 4.1, which is given as

$$U_{MNO} = \sum_{i=1}^{N} [\lambda_i (p - c) x_i]. \tag{4.25}$$

After taking the first-order and the second-order derivatives of (4.25) with respect to p, we have

$$\frac{\partial U_{MNO}}{\partial p} = \sum_{i=1}^{N} \lambda_i \left[x_i + (p - c) \frac{\partial x_i}{\partial p} \right], \tag{4.26}$$

and

$$\frac{\partial^2 U_{MNO}}{\partial p^2} = \sum_{i=1}^{N} \lambda_i \left[(p-c)\frac{\partial^2 x_i}{\partial p^2} + 2\frac{\partial x_i}{\partial p} \right]. \tag{4.27}$$

Taking the partial derivative on both sides of (4.24) with respect to p, we have

$$\frac{\partial x_i}{\partial p} = -\frac{\lambda_i}{\lambda_i \sigma_c \gamma x_i{}^{-\gamma-1} + \alpha R_i x_i{}^{-\alpha-1}}, \tag{4.28}$$

which further implies

$$\begin{aligned}\frac{\partial^2 x_i}{\partial p^2} = & -\frac{\lambda_i}{(\lambda_i \sigma_c \gamma x_i{}^{-\gamma-1} + \alpha R_i x_i{}^{-\alpha-1})^2} \\ & \times \left[\lambda_i \sigma_c \gamma(\gamma+1) x_i{}^{-\gamma-2} + \alpha(\alpha+1) R_i x_i{}^{-\alpha-2} \right] \frac{\partial x_i}{\partial p}. \end{aligned} \tag{4.29}$$

Substituting (4.28) and (4.29) into (4.26) and (4.27), we derive (4.30) and (4.31) as follows:

$$\frac{\partial U_{MNO}}{\partial p} = \sum_{i=1}^{N} \left[\lambda_i x_i - \frac{\lambda_i{}^2 (p-c)}{\lambda_i \sigma_c \gamma x_i{}^{-\gamma-1} + \alpha R_i x_i{}^{-\alpha-1}} \right]. \tag{4.30}$$

$$\begin{aligned}\frac{\partial^2 U_{MNO}}{\partial p^2} = \sum_{i=1}^{N} \Bigg\{ \lambda_i \Bigg\{ & -(p-c) \cdot \frac{\lambda_i}{(\lambda_i \sigma_c \gamma x_i{}^{-\gamma-1} + \alpha R_i x_i{}^{-\alpha-1})^2} \left[\lambda_i \sigma_c \gamma (\gamma+1) x_i{}^{-\gamma-2} \right. \\ & \left. + \alpha(\alpha+1) R_i x_i{}^{-\alpha-2} \right] + 2 \Bigg\} \cdot \left(-\frac{\lambda_i}{\lambda_i \sigma_c \gamma x_i{}^{-\gamma-1} + \alpha R_i x_i{}^{-\alpha-1}} \right) \Bigg\}. \end{aligned} \tag{4.31}$$

In general, it is hard to identify the properties of the objective function of the MNO, and further explore the optimal pricing. To show some interesting insights with tractability, we consider the case where $\alpha = \gamma$ similar to that in [30, 31]. Specifically, the MU's and CP's utility components have the same shape. According to [31], for the CPs that benefit directly from the usage of their content access, the benefit component of such usage ($h(x_i)$ in (5.9)) is usually modeled with the same isoelastic utilities that are modeled for user benefit component ($f(x_i)$ in (4.5)). Furthermore, we numerically demonstrate that our results qualitatively hold for the general case where $\alpha \neq \gamma$.

By analyzing the first order and the second-order derivatives of (4.25), we can achieve the following theorem.

Theorem 4.2 (Optimal Pricing in the Higher-Layer Game) *The optimal pricing for the MNO is uniquely determined, and hence the uniqueness of the Stackelberg equilibrium is thus validated [74]. Specifically, the optimal pricing strategy* p^* *is given by* $p^* = \frac{c}{1-\gamma}$.

Remark on Theorem 4.2: In the sponsored content market, all the rational entities including the MNO, the CP and MUs will keep adjusting the strategies towards the direction of payoff maximization as they are inherently rational or selfish. This may lead to the unstable market with the constantly changing marketing strategies and outcomes. Nevertheless, if the uniqueness of the Stackelberg equilibrium is ensured, the interactions among the MNO, the CP, and MUs under information asymmetry will always reach the unique market equilibrium or agreement on content consumption, content sponsoring and data pricing. The marketing strategies in the unique market equilibrium are able to achieve the joint payoff maximization outcome for the three entities: the MNO, the CP, and MUs. Therefore, the uniqueness of the Stackelberg equilibrium provides the guiding and practical significance on marketing activities.

Proof. When $\alpha = \gamma$, it first follows from (4.24) that $x_i = p^{-\frac{1}{\gamma}} T_i^{-\frac{1}{\gamma}}$ for $T_i := \frac{\lambda_i}{\lambda_i \sigma_c + R_i}$. Applying this to (4.30) and (4.31), we obtain

$$\begin{aligned}
\frac{\partial U_{MNO}}{\partial p} &= \sum_{i=1}^{N} \left[\lambda_i \cdot p^{-\frac{1}{\gamma}} T_i^{-\frac{1}{\gamma}} - \frac{{\lambda_i}^2 (p-c)}{(\lambda_i \sigma_c + R_i)\gamma p^{1+\frac{1}{\gamma}} T_i^{1+\frac{1}{\gamma}}} \right] \\
&= \sum_{i=1}^{N} \left[\lambda_i \cdot p^{-\frac{1}{\gamma}} T_i^{-\frac{1}{\gamma}} - \frac{\lambda_i (p-c)}{\gamma p^{1+\frac{1}{\gamma}} T_i^{\frac{1}{\gamma}}} \right] \\
&= \frac{c}{\gamma} \left[\sum_{i=1}^{N} \left(\lambda_i T_i^{-\frac{1}{\gamma}} \right) \right] p^{-\frac{1}{\gamma}-1} - \frac{(1-\gamma)}{\gamma} \left[\sum_{i=1}^{N} \left(\lambda_i T_i^{-\frac{1}{\gamma}} \right) \right] p^{-\frac{1}{\gamma}}, \qquad (4.32)
\end{aligned}$$

and

$$
\begin{aligned}
\frac{\partial^2 U_{MNO}}{\partial p^2} &= \sum_{i=1}^{N}\left\{\lambda_i\left\{-(p-c)\cdot\frac{\lambda_i}{[(\lambda_i\sigma_c+R_i)\gamma p^{1+\frac{1}{\gamma}}T_i^{1+\frac{1}{\gamma}}]^2}\right.\right.\\
&\quad\left.\left.\times\left[(\lambda_i\sigma_c+R_i)\gamma(\gamma+1)p^{1+\frac{2}{\gamma}}T_i^{1+\frac{2}{\gamma}}\right]+2\right\}\left(-\frac{\lambda_i}{(\lambda_i\sigma_c+R_i)\gamma p^{1+\frac{1}{\gamma}}T_i^{1+\frac{1}{\gamma}}}\right)\right\}\\
&= \sum_{i=1}^{N}\left\{\lambda_i\left\{-(p-c)\cdot\frac{T_i}{[\gamma p^{1+\frac{1}{\gamma}}T_i^{1+\frac{1}{\gamma}}]^2}\right.\right.\\
&\quad\left.\left.\times\left[\gamma(\gamma+1)p^{1+\frac{2}{\gamma}}T_i^{1+\frac{2}{\gamma}}\right]+2\right\}\left(-\frac{T_i}{\gamma p^{1+\frac{1}{\gamma}}T_i^{1+\frac{1}{\gamma}}}\right)\right\}\\
&= \frac{(1-\gamma)}{\gamma^2}\left[\sum_{i=1}^{N}\left(\lambda_i T_i^{-\frac{1}{\gamma}}\right)\right]p^{-\frac{1}{\gamma}-1}-\frac{(1+\gamma)c}{\gamma^2}\left[\sum_{i=1}^{N}\left(\lambda_i T_i^{-\frac{1}{\gamma}}\right)\right]p^{-\frac{1}{\gamma}-2}. \qquad (4.33)
\end{aligned}
$$

After analyzing the properties of (4.32) and (4.33), we have the following conclusion. For $p \leq \frac{c}{1-\gamma}$, we have $\frac{\partial U_{MNO}}{\partial p} \geq 0$. For $p \geq \frac{c}{1-\gamma}$, we have $\frac{\partial U_{MNO}}{\partial p} \leq 0$; For $p \leq \frac{(1+\gamma)c}{1-\gamma}$, we have $\frac{\partial^2 U_{MNO}}{\partial p^2} \leq 0$. For $p \geq \frac{(1+\gamma)c}{1-\gamma}$, we have $\frac{\partial^2 U_{MNO}}{\partial p^2} \geq 0$.

Summarizing the above, we can conclude that U_{MNO} is concavely increasing for $p \leq \frac{c}{1-\gamma}$, concavely decreasing for $\frac{c}{1-\gamma} \leq p \leq \frac{(1+\gamma)c}{1-\gamma}$, and convexly decreasing for $p \geq \frac{(1+\gamma)c}{1-\gamma}$. Thus, we ascertain that the optimal value of unit data price p^* is unique, which validates the uniqueness of the Stackelberg equilibrium. Moreover, the unique value of price p^* exists on the concave parts $p \leq \frac{(1+\gamma)c}{1-\gamma}$. Accordingly, we can obtain p^* from solving $\left.\frac{\partial U_{MNO}}{\partial p}\right|_{p=p^*} = 0$. Specifically, the optimal pricing strategy p^*, and the corresponding volume of content demand from type-i MU x_i^* are given by

$$
\begin{cases}
p^* = \frac{c}{1-\gamma},\\
x_i = \left[\frac{c\lambda_i}{(1-\gamma)(\lambda_i\sigma_c+R_i)}\right]^{-\frac{1}{\gamma}}, \ \forall i \in \{1,2,\ldots,N\}.
\end{cases} \qquad (4.34)
$$

The proof is then completed. □

4.4 Discussions on Benchmark Schemes

Until now, we have obtained the Stackelberg equilibrium under contract-based sponsorship scheme with information asymmetry. To evaluate the benefits of the proposed scheme, we discuss some benchmark schemes in this section, i.e., the discriminatory sponsorship scheme and the uniform sponsorship scheme.

4.4.1 Discriminatory Sponsorship Scheme

The discriminatory sponsorship scheme is obtained from the optimal contract under no information asymmetry, i.e., the CP is aware of the types of MUs, which is the optimal outcome that serves as the upper bound. In such a case, the CP is able to treat each MU separately, and thus the optimization problem has only IR constraints but no IC constraints. Specifically, the optimization problem becomes

$$\begin{aligned} \underset{(\boldsymbol{x},\boldsymbol{\theta})}{\text{maximize}} \quad & U_{CP}(\boldsymbol{x},\boldsymbol{\theta}) = \sum_{i=1}^{N} \lambda_i \left[\sigma_c h(x_i) - \theta_i p x_i\right], \\ \text{subject to} \quad & \text{(IR) } \tau\sigma_i f(x_i) - (1-\theta_i)x_i p \geq 0. \end{aligned} \tag{4.35}$$

The IR constraint for type-i user implies $\theta_i \geq 1 - \frac{\tau\sigma_i f(x_i)}{x_i p}$. Since the IR constraints are binding, θ_i denoting the optimal θ_i^* is given by

$$\theta_i^* = 1 - \frac{\tau\sigma_i f(x_i)}{x_i p}, \forall i \in \{1,\ldots,N\}. \tag{4.36}$$

When θ_i takes the optimal value θ_i^*, the utility of CP can be computed as follows:

$$\begin{aligned} U_{CP}(\boldsymbol{x}) &= \sum_{i=1}^{N} \lambda_i \left[\sigma_c h(x_i) - \theta_i p x_i\right] \\ &= \sum_{i=1}^{N} \lambda_i \left[\sigma_c h(x_i) + \tau\sigma_i f(x_i) - p x_i\right]. \end{aligned} \tag{4.37}$$

Taking the partial derivative of U_{CP} with respect to x_i, $\forall i \in \{1,\ldots,N\}$, we obtain

$$\begin{aligned} \frac{\partial U_{CP}}{\partial x_i} &= \lambda_i \left[\sigma_c h'(x_i) + \tau\sigma_i f'(x_i) - p\right] \\ &= \lambda_i \left[\sigma_c {x_i}^{-r} + \tau\sigma_i {x_i}^{-\alpha} - p\right]. \end{aligned} \tag{4.38}$$

Setting $\frac{\partial U_{CP}(\boldsymbol{x})}{\partial x_i} = 0$, we denote the resulting x_i by x_i^*, and we know that x_i^* satisfies

$$\sigma_c x_i^{*-\gamma} + \tau\sigma_i x_i^{*-\alpha} = p, \forall i \in \{1,\ldots,N\}. \tag{4.39}$$

Then, $U_{CP}(\boldsymbol{x})$ increases as x_i increases for $x_i < x_i^*$, and decreases as x_i increases for $x_i > x_i^*$. Hence, in the optimal contract, x_i equals x_i^*. After x_i is obtained, θ_i is derived via (4.36). With the optimal solutions to the lower-level problem, we can further analyze the higher-level problem using backward induction. Here, the analytical proof for the optimal pricing is structurally the same as the proof given in Section 4.3 and is omitted here because of the space limit. The detailed proof is give in Appendix.

4.4.2 Uniform Sponsorship Scheme

The uniform sponsorship scheme or the linear sponsorship strategy is also under the information asymmetry, in which the CP has no acknowledgement of the MU type [48, 82]. In this scheme, the CP can only specify a uniform sponsorship factor θ without discrimination. The MUs will demand the volume of content that they want to maximize their individual utilities. Given the same sponsorship factor, the MUs which have the higher content demand receive the greater sponsorship fees because of the linear term "$\theta p x_i$". The utility of each type-i MU becomes

$$u_i = \tau\sigma_i f(x_i) - (1-\theta)x_i p, \forall i \in \{1, \ldots, N\}. \tag{4.40}$$

Taking the first-order partial derivative of u_i with respect to x_i, we obtain

$$\frac{\partial u_i}{\partial x_i} = \tau\sigma_i f'(x_i) - (1-\theta)p = \tau\sigma_i {x_i}^{-\alpha} - (1-\theta)p. \tag{4.41}$$

Setting $\frac{\partial u_i}{\partial x_i} = 0$, we denote the resulting x_i by $x_i^*(\theta)$ and we know

$$x_i^*(\theta) = \left[\frac{\tau\sigma_i}{(1-\theta)p}\right]^{\frac{1}{\alpha}} = \left(\frac{\tau\sigma_i}{p}\right)^{\frac{1}{\alpha}} (1-\theta)^{-\frac{1}{\alpha}}. \tag{4.42}$$

Since u_i increases as x_i increases for $x_i < x_i^*$, and decreases as x_i increases for $x_i > x_i^*$, the optimal x_i equals x_i^*. Then it holds that

$$\frac{\partial x_i^*(\theta)}{\partial \theta} = \frac{1}{\alpha}\left(\frac{\tau\sigma_i}{p}\right)^{\frac{1}{\alpha}} (1-\theta)^{-\frac{1}{\alpha}-1}. \tag{4.43}$$

Taking the first-order partial derivative of U_{CP} with respect to θ, we obtain

$$\begin{aligned}\frac{\partial U_{CP}}{\partial \theta} &= \sum_{i=1}^{N} \lambda_i \left[\sigma_c x_i^*(\theta)^{-\gamma} \frac{\partial x_i^*(\theta)}{\partial \theta} - \theta p \frac{\partial x_i^*(\theta)}{\partial \theta} - p x_i^*(\theta)\right] \\ &= \sum_{i=1}^{N} \lambda_i \left[\sigma_c \times \left[\frac{\tau\sigma_i}{(1-\theta)p}\right]^{-\frac{\gamma}{\alpha}} \times \frac{1}{\alpha}\left(\frac{\tau\sigma_i}{p}\right)^{\frac{1}{\alpha}}\right. \\ &\qquad \times (1-\theta)^{-\frac{1}{\alpha}-1} - \theta p \times \frac{1}{\alpha}\left(\frac{\tau\sigma_i}{p}\right)^{\frac{1}{\alpha}} \\ &\qquad \left. \times (1-\theta)^{-\frac{1}{\alpha}-1} - p\left(\frac{\tau\sigma_i}{p}\right)^{\frac{1}{\alpha}} (1-\theta)^{-\frac{1}{\alpha}}\right]. \end{aligned} \tag{4.44}$$

For tractability, we consider the case of $\alpha = \gamma$ similar to that in Section 4.3. Then it follows from (4.44) that

$$\begin{aligned}\frac{\partial U_{CP}}{\partial \theta} &= \sum_{i=1}^{N} \lambda_i \left[\sigma_c \times \left[\frac{\tau\sigma_i}{(1-\theta)p}\right]^{-1} \times \frac{1}{\gamma}\left(\frac{\tau\sigma_i}{p}\right)^{\frac{1}{\gamma}} (1-\theta)^{-\frac{1}{\gamma}-1}\right. \\ &\qquad \left. - \theta p \times \frac{1}{\gamma}\left(\frac{\tau\sigma_i}{p}\right)^{\frac{1}{\gamma}} (1-\theta)^{-\frac{1}{\gamma}-1} - p\left(\frac{\tau\sigma_i}{p}\right)^{\frac{1}{\gamma}} (1-\theta)^{-\frac{1}{\gamma}}\right] \\ &= \sum_{i=1}^{N} \lambda_i (1-\theta)^{-\frac{1}{\gamma}} \left(\frac{\tau\sigma_i}{p}\right)^{\frac{1}{\gamma}} \left[\sigma_c \frac{p}{\gamma\tau\sigma_i} - \theta p \times \frac{1}{\gamma}(1-\theta)^{-1} - p\right]. \end{aligned} \tag{4.45}$$

Since we can easily check that $\frac{\partial^2 U_{CP}}{\partial \theta^2} < 0$ when $0 \leq \theta < 1$, the optimal θ^* is derived from solving $\frac{\partial U_{CP}}{\partial \theta}\Big|_{\theta=\theta^*} = 0$, which is $\theta^* = \frac{\sum_{i=1}^{N} \lambda_i \sigma_i^{\frac{1}{\gamma}}\left(\frac{\sigma_c}{\tau\sigma_i} - \gamma\right)}{\sum_{i=1}^{N} \lambda_i \sigma_i^{\frac{1}{\gamma}}\left(1 + \frac{\sigma_c}{\tau\sigma_i} - \gamma\right)}$. Given the optimal uniform sponsorship factor θ^*, we can obtain the optimal solution x^* from (4.42). By substituting x^* into U_{MNO}, we can investigate the optimal pricing of the MNO using backward induction. Again, the analytical proof for the optimal pricing is structurally the same as the proof given in Section 4.3 and is omitted here because of the space limit. The detailed proof is give in Appendix.

4.5 Performance Evaluation

In this section, we evaluate the performance of the proposed market-oriented sponsored content model through extensive simulations. For the simulation setup, we consider 10

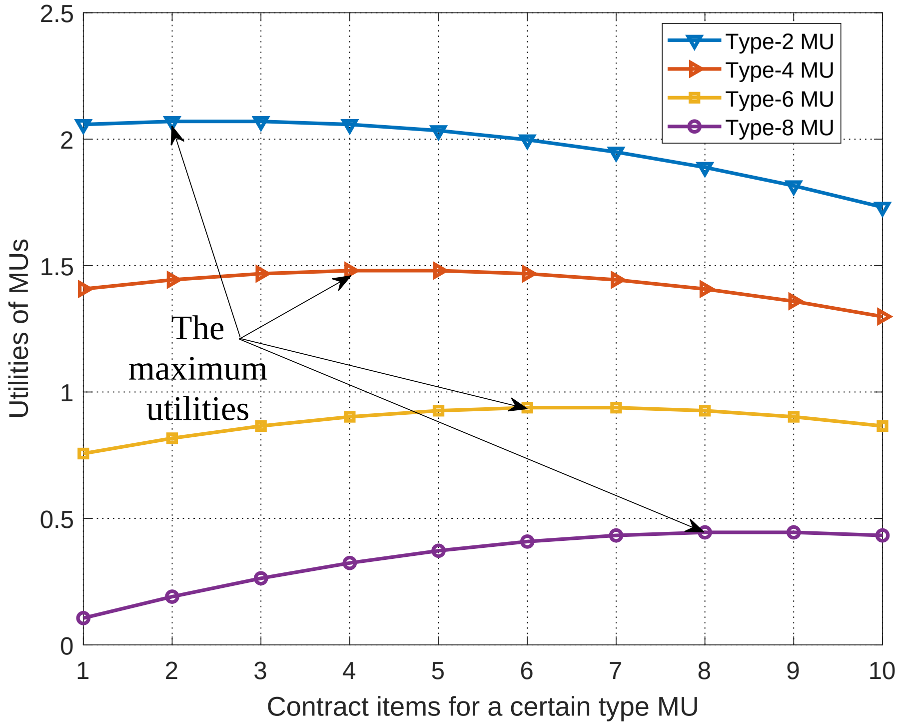
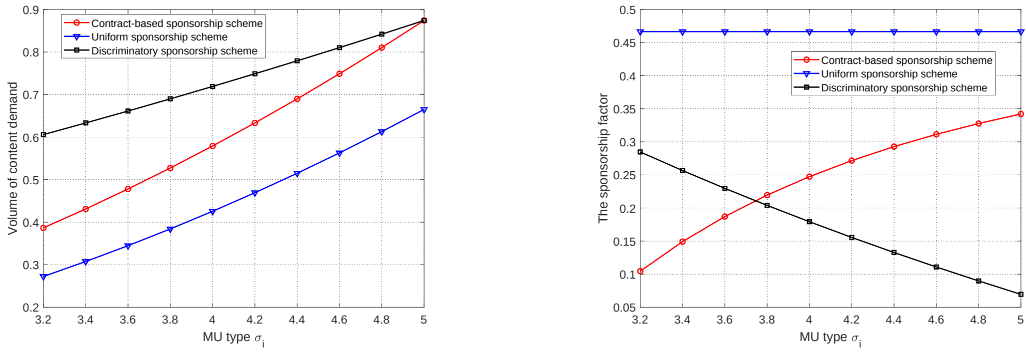


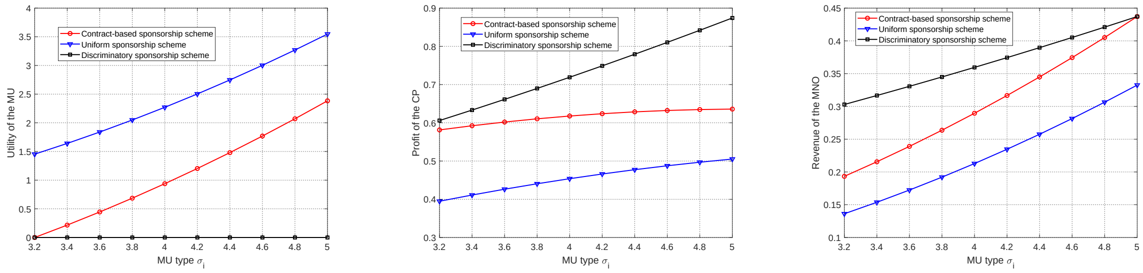
Figure 4.2: Utilities of type-2, type-4, type-6 and type-8 MUs when selecting different optimal contract items offered by the CP.

different types of MUs in the network, i.e., $N = 10$. The MU type σ_i is chosen from the set $\{5, 4.8, 4.6, \ldots, 3.4, 3.2\}$, according to a certain probability distribution so that the monotonicity constraint in Definition 4.3 is satisfied, i.e., the lower the type, the higher the value of σ_i. Specifically, the distribution of different types of MUs is assumed to be the uniform distribution following [48, 83, 84], i.e., $\lambda_i = 1/N$. Unless otherwise stated, the parameters are set as follows: $\alpha = 0.5$, $\gamma = 0.5$, $c = 5$, and $\sigma_c = 5$. From (4.34), we know that the optimal price is $p^* = \frac{c}{1-\gamma} = 10$ ($10/GB approximates current MNO data prices, e.g, AT&T's data plans vary from $7.50/GB to $25/GB [85]). We assume that the CP carries an additional 15% of ads per content volume ($l_a = 0.15$), e.g., a 30-second advertisement for a 200-second video, and thus $\tau = \frac{1}{1+l_a} \approx 0.87$. Note that most of the above parameters are adopted from [31].



(a) The optimal volume of content demand vs. Type of MUs (b) The optimal sponsorship factor vs. Type of MUs

Figure 4.3: The optimal contract $(\boldsymbol{\theta}^*, \boldsymbol{x}^*)$ for different types of MUs under different schemes.



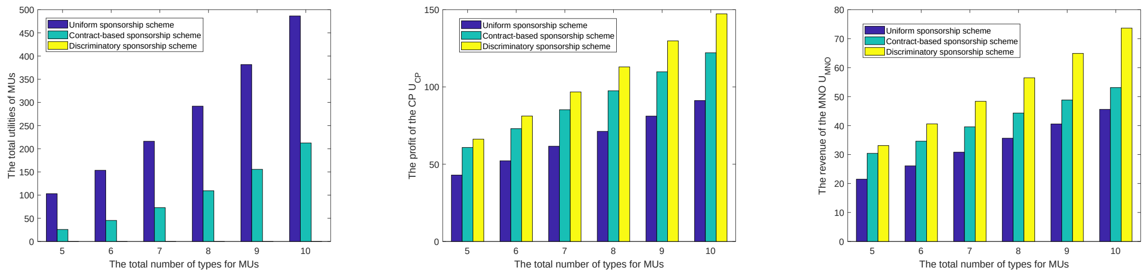
(a) Utility of the MU (b) Profit of the CP (c) Revenue of the MNO

Figure 4.4: System performance for different types of MUs under different schemes.

We first verify the feasibility (i.e., IR and IC) of the proposed contract for the contract game under information asymmetry. We plot the utilities of type-2, type-4, type-6 and type-8 MUs when selecting different contract items in Fig 4.2. From Fig. 4.2, we observe that the utility of the MU achieves the maximum value when it selects the contract item which is exactly designed for its corresponding type, which indicates that the IC constraints are satisfied. Moreover, we observe that each of the MU obtains the non-negative utility, which guarantees the IR constraints. We can conclude that in the proposed contract-based sponsorship scheme, each MU reveals its type to the CP after selecting the contract item, and this effectively addresses the information asymmetry. We also observe that the MUs receive very similar utilities from selecting different contract items. The reason of leading to this result is due to the restriction of IC constraints. Recall that we further relax the IC constraints through Lemma 4.5. The condition in Lemma 4.5 might become the strict restrictions for the optimal contract design, leading to the results that the MUs derive similar utilities from selecting different contracts items.

Moreover, we compare the performance of the proposed scheme with the other benchmark schemes given in Section 4.4, as illustrated in Figs. 4.3 and 4.4. In Fig. 4.3(a), we observe that the optimal volume of content demand increases with the MU type, which is consistent with the monotonicity in Definition 4.3. Moreover, we observe that the sponsorship factor increases with the MU type in Fig. 4.3(b), which confirms Lemma 4.3. In the discriminatory sponsorship scheme, the CP can treat each MU separately without information asymmetry, and offer the lower sponsorship factor to the MU with the higher type. The reason is that the CP expects that the MU with a higher type has greater volume of content demand to maximize its utility. As a result, the utilities of MUs all remain zero in the discriminatory sponsorship scheme which yields the highest profit for the CP, as shown in Figs. 4.4(a) and (b).

In the uniform sponsorship scheme, the CP applies only the uniform sponsorship factor and does not place any restrictions on MUs' content demand selections. Consequently, less information is retrieved, which leads to the more profit loss of the CP. Meanwhile, the MUs have more freedom to optimize their individual utilities and thus can achieve more surplus. Furthermore, we find that the proposed contract-based sponsorship scheme yields a higher profit for the CP than that from the uniform sponsorship scheme. Note that even though the proposed contract-based sponsorship scheme can induce the MUs to reveal their types, the exact value of the MU type is still unavailable to the CP. Therefore, the CP can only achieve a near-optimal profit under information asymmetry, which is always upper bounded by the discriminatory sponsorship scheme. This also explains



(a) Total utilities of MUs (b) Profit of the CP (c) Revenue of the MNO

Figure 4.5: System performance when the number of MU types varies under different schemes.

the revenue-minus-cost performance of the MNO under different schemes in Fig. 4.4(c). Furthermore, we observe that the CP and the MNO always receive the higher payoffs when interacting with the MU with a higher type, as illustrated in Fig. 4.4. This result is also in accordance with the fact that the MUs with the higher type always have a greater volume of content demand given the same sponsorship fees and data price.

We also study the impacts of the number of MU types on system performance, as illustrated in Fig. 4.5. For each type of the MUs, we consider that there are 5 MUs. The increase of the number of MU types will increases the number of MUs. Therefore, the profits of the CP and the revenue of the MNO both increase when the number of MU types increases. Similar to the main conclusions from Fig. 4.4, the CP and the MNO have the highest profit and revenue, respectively, under discriminatory sponsorship scheme. Also, under these schemes, the total utilities of MUs remain zero. The contract-based sponsorship scheme yields the second highest payoff for the CP and the MNO. The uniform sponsorship scheme yields the highest total utilities of MUs, the lowest profit and revenue of the CP and the MNO, respectively.

We next examine the performance when the advertisement revenue coefficient varies in Figs. 4.6 and 4.7. From Fig. 4.6, we observe that the total volume of content demand, the total sponsorship fees, and the payoffs of the CP as well as the MNO all increase as

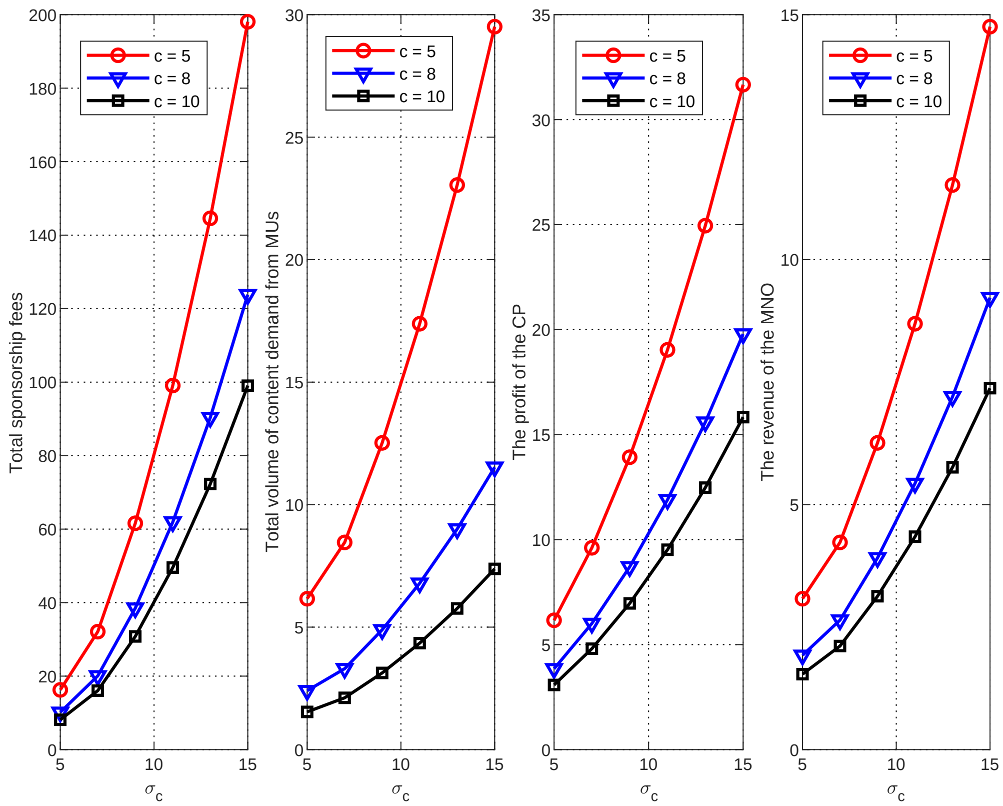


Figure 4.6: System performance when the advertisement revenue coefficient varies under different value of variable maintaining cost.

the advertisement revenue coefficient σ_c increases. The reason is that the advertisement revenue increases with the increase of σ_c, given the same volume of content demand from MUs. In this regard, the CP has an incentive to offer more sponsorship to motivate MUs to demand a greater volume of content. In return, the greater content volume consumption leads to the increase of the profit and the revenue of the CP and the MNO, respectively. In Fig. 4.6, we also observe that the lower variable maintaining cost allows the MNO to reduce the data price, which in turn promotes the CP to offer greater sponsorship for attracting more content consumption from MUs. Consequently, the payoff of the CP and the MNO both increases. Comparing different values of the normalized

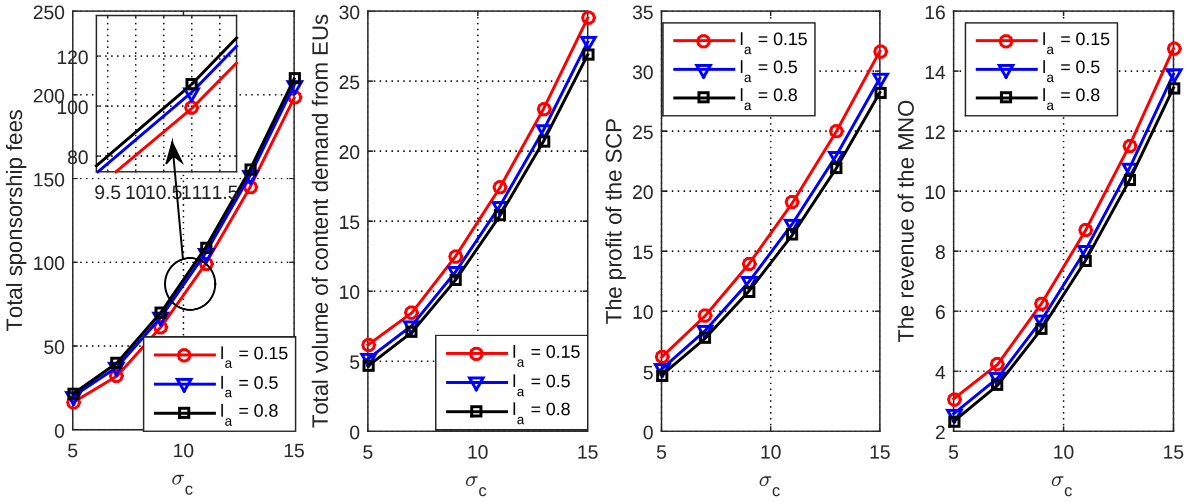


Figure 4.7: System performance when the advertisement revenue coefficient varies under different values of advertisement ratio.

advertisement volume l_a in Fig. 4.7, we observe that the total volume of content demand increases and the total sponsorship decreases when l_a increases. The reason is that the utility of the MU from enjoying the content decreases when l_a increases, and the volume of content demand decreases accordingly. In order to avoid the substantial loss of content consumption, the CP offers greater sponsorship for compensating for the utility loss of the MU. However, the volume of content demand still slightly decreases which consequently leads to the decrease of the profit and the revenue of the CP and the MNO, respectively. In the future work, we will further study the optimization of sponsoring schemes together with regulating the amount of content advertisement.

Lastly, we numerically investigate optimal pricing of the MNO for the general case where $\alpha \neq \gamma$ in Fig. 4.8. Recall from Section 4.3, we analytically prove that when p increases, the revenue of the MNO U_{MNO} first concavely increases, then concavely decreases, and finally convexly decreases. This validates the optimal pricing of the MNO and the uniqueness of the Stackelberg equilibrium in case of $\alpha = \gamma$. Nevertheless, Fig. 4.8 presents the numerical evidence that our conclusions in Section 4.3 still qualitatively holds even if $\alpha \neq \gamma$. Specifically, we find that the revenue functions of the MNO all increase

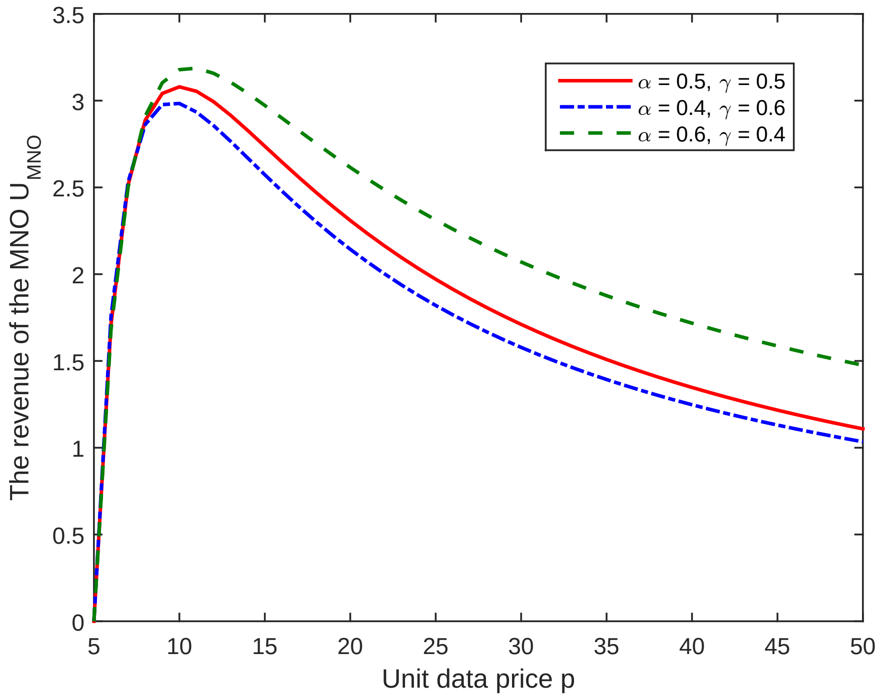


Figure 4.8: The revenue of the MNO when the unit data price p varies.

first and decrease then under different pairs of α and γ, and the only difference is the concavity or convexity degree of the revenue curve. This demonstrates that the optimal pricing of the MNO is uniquely determined even in the general case where $\alpha \neq \gamma$.

4.6 Summary

In this chapter, we have proposed a novel joint optimization approach combining contract theory and Stackelberg game to characterize the market-oriented model for the sponsored content market with information asymmetry, and capture the interactions among the main entities therein. Specifically, we have developed a hierarchical Stackelberg game, where the mobile network operator determines the optimal pricing acting as the leader, the sponsored content provider and mobile users make decisions acting as the followers. The decision making process of followers has been modeled as the contract game. We have derived the necessary and sufficient conditions of the feasible contract, and obtained the optimal contract solutions. Through the backward induction, we have proved that the optimal pricing of the network operator is unique, and accordingly validated the uniqueness of the Stackelberg equilibrium. The performance evaluation has demonstrated the effectiveness of our scheme compared with other benchmark schemes.

Chapter 5

Joint Sponsored and Edge Caching Content Service Market using Game Theory

In this chapter[1], we introduce the integration of the edge caching and sponsored content schemes. In particular, we investigate the effects of their interaction and their coexistence on the mobile user behavior, the network operator in the data/content traffic market. The main contributions of this chapter are summarized as follows:

- We formulate a joint sponsored and edge caching content service market model to analyze the interactions among the mobile network operator, the sponsored content provider as well as the edge caching content provider, and mobile users.
- We formulate a novel hierarchical three-stage Stackelberg game to model their interactions to jointly maximize the payoff of the mobile network operator, the profit of each content providers, and the individual utilities of mobile users.
- Through backward induction, we analyze the sub-game perfect equilibrium in each stage analytically. In particular, we prove that an optimal strategy of the mobile user in Stage III is unique, and accordingly demonstrate the uniqueness of the Nash equilibrium among the content providers in Stage II.

[1] The work in this chapter has been published in [86, 87]

- Furthermore, the existence of the Stackelberg equilibrium is validated by capitalizing on the bilevel optimization technique. Based on our theoretical discoveries regarding the properties of the equilibrium in the Stackelberg game, we propose a sub-gradient based iterative algorithm that guarantees the convergence to the Stackelberg equilibrium.

- We conduct extensive numerical simulations to evaluate the performance of all the players in the proposed Stackelberg game. The results draw some useful engineering insights, e.g., the monopoly mobile network operator intends to set the maximum possible value as the optimal price for payoff maximization.

The rest of the chapter is organized as follows. Section 5.1 presents the system description and Section 5.2 formulates a hierarchical three-stage Stackelberg game to model the interactions among the players in the joint sponsored and edge caching content service market. The equilibrium analysis through backward induction for optimal strategies of players are shown in Section 5.3. Section 5.4 provides the numerical results for performance evaluation. Section 5.5 concludes the chapter with summary.

5.1 System Model

As illustrated in Fig. 5.1(a), we consider a mobile video content delivery network as a market consisting of four entities: Mobile Network Operator (MNO), Sponsored Content Provider (SCP), Edge Caching Content Provider (ECCP), and a pool of mobile users (MUs). As there are two content providers, we call them the SCP and the ECCP in order to distinguish them. Due to the fact that the same video is often available from many content providers, which applies particularly to popular videos [88, 89]. Therefore, there is a competition among different content providers as they may provide the same video contents to the same pool of MUs. In order to obtain some insights, we consider the scenario where the SCP and the ECCP both have the same popular video contents. In particular, the SCP and the ECCP compete for the video users, e.g., Netflix [32] and Hulu [90]. In other words, the video contents requested by MUs are available from both the SCP and the ECCP. The MUs can choose to access and consume the video contents from the SCP and ECCP. If the MU accesses the content from the SCP, the content is downloaded directly through the network infrastructure of MNO. The SCP can sponsor partly or fully the data transfer from the MNO to the MU. On the contrary, the MU can access the content stored in an edge caching device from the ECCP. The MU can download the content from the device locally without involving the MNO. In practice, the MU requests for the content by using an application agent, where the content source, i.e., from the SCP and ECCP, can be chosen proactively by the MU. Again, since the MU can choose the services from two sources, the SCP and ECCP compete with each other to attract demand from the users.

Let y denote the content (volume) demanded by MUs, and $\sigma_e f(y)$ denote the utility obtained from accessing and consuming the content, where the factor $\sigma_e > 0$ represents the utility coefficient of MUs, e.g., a particular valuation between MUs and content. Similar to that in [31, 30], we adopt the following function:

$$f(y) = \frac{1}{1-\alpha} y^{1-\alpha}, \tag{5.1}$$

where $0 < \alpha < 1$ is a given coefficient. In particular, $f(\cdot)$ is a non-decreasing and concave function with decreasing marginal satisfaction. This reflects the decreasing marginal preference of MUs. In traditional wireless content access, the MNO charges each MU p for per unit volume of content downloaded. Thus, the general utility of the MU which has the content demand y is given by

$$v(y) = \sigma_e f(y) - py. \tag{5.2}$$

5.1.1 Content Access from Sponsored Content Provider

Under the sponsored content scheme launched by the MNO, the payment from MUs to the MNO can be partly sponsored by the SCP. Let $\theta \in [0,1]$ represent the sponsorship factor of content decided by the SCP. Again, the sponsorship fee is paid by the SCP to the MNO. Thus, the MU pays for $(1-\theta)y$ units of content to the MNO, and thus the cost incurred to the MU is $(1-\theta)py$ [59]. Generally, the MUs are also affected by another variable l_a which is the amount of advertisement imposed by the SCP and ECCP per volume of content. We assume that l_a is constant for all contents. For example, Pandora Internet Radio plays advertisement at regular intervals between songs. We assume the normalized $l_a \in [0,1]$ since both the ECCP and SCP cannot have more advertisement than its content. For the ease of derivation later, we introduce an auxiliary variable τ defined as:

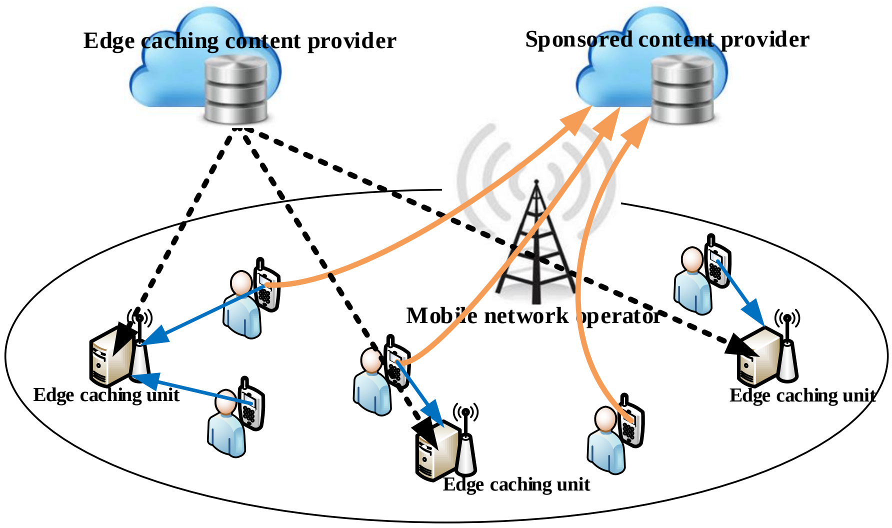
$$\tau = \frac{1}{1+l_a}, \tau \in \left[\frac{1}{2}, 1\right]. \tag{5.3}$$

Thus, the utility of the MU which has the content demand y from the SCP is expressed by

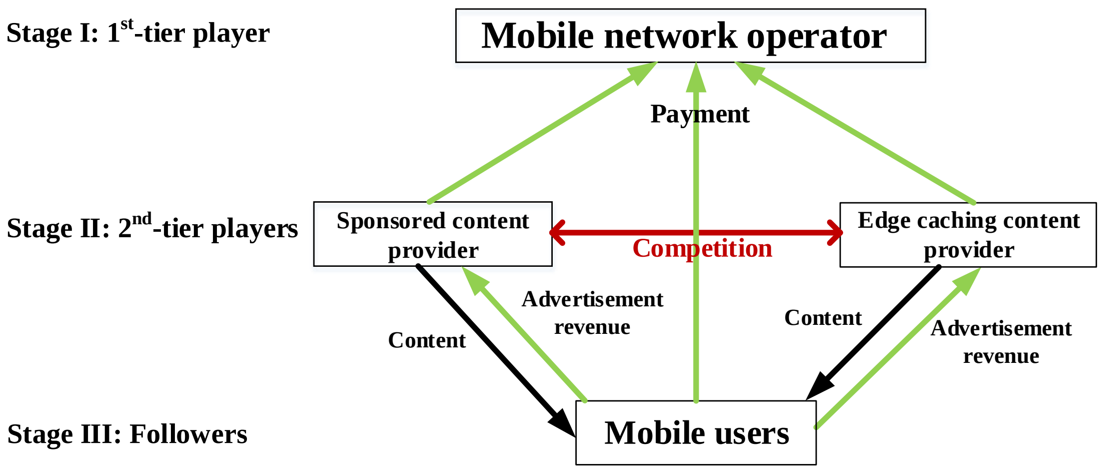
$$u_s(y) = \tau\sigma_e f(y) - (1-\theta)py. \tag{5.4}$$

5.1.2 Content Access from Edge Caching Content Provider

Considering edge caching, the ECCP is able to cache the video contents in the edge caching devices, and thus the MUs can access the cached content from the ECCP through a local network connection. It is worth noting that the ECCP cannot cache all the content it has in the edge devices due to the limited size, and thus the ECCP will refresh the video contents in the edge devices after a relatively long time. We denote t ($t \in [0,1]$) as the caching effort of the ECCP, which indicates the sponsorship from the ECCP to



(a)



(b)

Figure 5.1: (a) System model for joint sponsored and edge caching content service market; (b) A hierarchical three-stage Stackelberg game to model the interactions among the mobile network operator, the sponsored content provider as well as the edge caching content provider, and mobile users.

MUs. Again, the MU accessing the cached content from the ECCP, the advertisement from the ECCP is imposed to the MU which lowers its utility. Nevertheless, accessing the cached content, the MU does not need to pay the MNO [56]. Accordingly, the utility of the MU which has demand z for the cached content from the ECCP is expressed as

$$u_e(z) = \tau\sigma_e f(z)g(t) - cz, \tag{5.5}$$

where c is the network handover cost [91].

The caching effort t indicates the amount of resources, e.g., storage, transmit power, bandwidth, and computing, allocated for delivering the cached content to the MUs. $g(t)$ is defined as content delivery quality. In particular, $g(t)$ is a monotonically increasing function reflecting the positive influence of caching effort t on MUs' experience. That is, the greater t, the better quality $g(t)$, but the increase in the quality becomes lower when t is larger. Similar to (5.1), we adopt the a common function to capture the Quality of Service (QoS) experienced by MUs [92], i.e.,

$$g(t) = \frac{1}{1-\beta}t^{1-\beta}, \tag{5.6}$$

where $0 < \beta < 1$ is a coefficient.

Note that there may exist the radio transmission with multiple access. The impacts of such wireless characteristic on the proposed scheme can be incorporated straightforwardly in the model. Specifically, on view of such wireless characteristic, the service quality experienced by MUs is also negatively affected by other MUs while using a local network connection because of the multiple access technique. Similar to our work in Chapter 3, the congestion component in the utility function of MU which accesses the edge caching content can capture the impacts of congestion, i.e., the negative impacts from other MUs. In addition, different MUs may have different congestion sensitivity factors due to the inherent heterogeneity. In this regard, the caching effort t cannot benefit different

MUs in the same way. For example, the same caching effort may benefit the congestion-tolerant MUs (with lower congestion sensitivity factor) more. We will further explore this direction in the future work. In the next section, based on the system description presented, we provide the detailed analysis for the joint sponsored and edge caching content service model.

5.2 Game Formulation for Joint Sponsored and Edge Caching Content Service Model

In this section, we formulate a hierarchical three-stage Stackelberg game to model the interactions among the MNO, SCP, ECCP, and MUs as illustrated in Fig. 5.1(b). We analyze the sub-game problems from each stage in Sections 5.2.1, 5.2.2 and 5.2.3.

5.2.1 Mobile Users in Stage III (Followers)

Given the popular video contents under our consideration, the content demand requested by the MU can be sponsorable and cachable at the same time. Thus, in addition to accessing the sponsored content from the SCP, the MU has an alternative choice, i.e., to access the cached content from an edge caching device deployed by the ECCP. Each MU determines a fraction of the content demand to access from the SCP denoted by $x \in [0, 1]$. Thus, the fraction of cached content demand from the ECCP through a local network connection is $1 - x$. Note that we normalize the content demand as 1 to facilitate the tractability of the three-stage Stackelberg game analysis and obtain some insights into the problem. In particular, the analytical results will not structurally change even if the content demand is not normalized.

Each myopic MU needs to consider how to balance its content demand from the SCP and ECCP to maximize its utility. In particular, the utility of the MU from taking the action x is expressed as follows:

$$\begin{aligned} u(x;\theta,t,p) &= u_s(x) + u_e(1-x) \\ &= \tau\sigma_e f(x) - (1-\theta)xp + \tau\sigma_e f(1-x)g(t) - (1-x)c, \end{aligned} \tag{5.7}$$

where $f(x)$ and $g(t)$ are given in (5.1) and (5.6), respectively.

Given the volume unit price set by the MNO, the sponsorship factor θ from the SCP and the caching effort t from the ECCP, the MU chooses x to maximize the utility, and thus each MU sub-game problem can be written as follows:

Problem 5.1. (The MU sub-game):

$$\begin{aligned} \underset{x}{\text{maximize}} \quad & u(x;\theta,t,p) \\ \text{subject to} \quad & x \in [0,1]. \end{aligned} \tag{5.8}$$

5.2.2 SCP and ECCP in Stage II (2nd-Tier Players)

Being aware of the pricing strategy of the MNO, both the SCP and ECCP determine their individual strategy competitively and simultaneously.

5.2.2.1 Sponsored Content Provider

The goal of the SCP is to maximize its profit, i.e., advertisement revenue minus the sponsorship fee provided for the MUs. The profit is expressed as follows:

$$\Pi_s(\theta;p) = \sigma_c h(x) - \theta p x. \tag{5.9}$$

We denote $\sigma_c h(x)$ as the advertisement revenue [31, 30], where σ_c is the advertisement revenue coefficient and

$$h(x) = \frac{1}{1-\gamma} x^{1-\gamma}, \tag{5.10}$$

where $0 < \gamma < 1$ which is a coefficient. In fact, we could also adopt a linear dependency between demand and ad revenue. However, most of the related works have proved that a function with diminishing return would model the ad revenue more closely [30, 59, 31, 70].

The SCP is willing to provide the sponsorship to lower the cost for MUs, which in turn attracts more MUs to access and consume the content. However, the more sponsorship may incur excessive cost. Thus, the SCP sub-game is defined as follows:

Problem 5.2-A. (The SCP sub-game):

$$\begin{aligned} \underset{\theta}{\text{maximize}} \quad & \Pi_s(\theta; p) \\ \text{subject to} \quad & \theta \in [0, 1]. \end{aligned} \tag{5.11}$$

5.2.2.2 Edge Caching Content Provider

Recall that the cost of the ECCP for caching the content with caching effort t is Ct, as discussed in Section 5.1. Likewise, the ECCP aims at maximizing its profit, i.e., the advertisement revenue from content traffic minus the cost for content caching, which is expressed as follows:

$$\Pi_e(t; p) = \sigma_c h(1 - x) - Ct, \tag{5.12}$$

where $h(\cdot)$ is given in (5.10). We assume that the baseline content caching cost is C, and thus the cost of the ECCP for caching the content with caching effort t is Ct [54, 93, 56]. The ECCP has an incentive to increase the caching effort which provides better-quality service to the MUs and hence attracts more MUs to access more content from the ECCP. However, increasing the caching effort increases the cost incurred to the ECCP. Note that in order to provide cache services, the ECCP also needs to pay a side payment to the MNO for using the edge cache devices or small base stations deployed by the MNO [94]. In the chapter, we consider that the ECCP "rents" the cache service in which the payment is fixed for a certain time period. Thus, it does not affect the game strategies of ECCP or MNO. Similar to [89, 56], we assume such payment to be a constant cost for the ECCP and thus is omitted for ease of presentation and analysis. Accordingly, the profit maximization of the ECCP is expressed as follows:

Problem 5.2-B. (The ECCP sub-game):

$$\begin{aligned} \underset{t}{\text{maximize}} \quad & \Pi_e(t;p) \\ \text{subject to} \quad & t \in [0,1]. \end{aligned} \tag{5.13}$$

Generally, the ECCP has the potential of offloading as well as relieving the backbone network burden, and reducing content delivery cost especially when the number of mobile users is increasing considerably.

5.2.3 Mobile Network Operator in Stage I (1st-Tier Player)

The MNO sets the volume unit price p for data traffic. In addition to obtaining its revenue from charging the SCP and MUs, the MNO has the content delivery cost. Accordingly, the objective of the MNO is to maximize its payoff, which is expressed as follows:

$$\mathscr{P}(p) = px - wx^2, \tag{5.14}$$

where wx^2 denotes the corresponding cost, and w represents the unit cost of content delivery. The quadratic sum form reflects the marginal cost increases as the total demand increases, e.g., due to congestion effects, which is a widely-accepted assumption [95]. We have the strategy space of the MNO be $\{\mathbf{P} : 0 \le p \le \overline{p}\}$, where $\overline{p}$ is the maximum price. Therefore, the payoff maximization problem of the MNO is formulated as follows:

Problem 5.3. (The MNO sub-game):

$$\begin{aligned} \underset{p}{\text{maximize}} \quad & \mathscr{P}(p) \\ \text{subject to} \quad & p \in [0,\overline{p}]. \end{aligned} \tag{5.15}$$

Note that we assume a monopolistic MNO in this work, while many MNOs, e.g., in the US, are oligopolists. However, due to low churn rates, they are often effective monopolies [31].

The **Problems 5.1**, **5.2-A**, **5.2-B** and **5.3** altogether form a hierarchical three-stage Stackelberg game with complete information. The objective of the game is to find the

Stackelberg equilibrium. The Stackelberg equilibrium is the point where the payoff of the leader is maximized provided that the followers adopt their best responses, i.e., the Nash eqilibrium [5]. In certain scenarios, the information and strategy of the player cannot be predicted accurately by others, which lead to the incomplete information environments. As such, we can employ the Bayesian game to describe the strategic behaviors among players. Comparing with the complete information game, Bayesian game introduces two new concepts: the types of players, and the probability distribution of the types. The probability distribution is the belief about unknown information of the players, referred to as the type of the player, e.g., the utility coefficient of MUs. Such distribution information can be obtained through, e.g., historical information or long-term learning. In this regard, each MU knows it own expected payoff instead of specific payoff. Thus, each player seeks for a strategy profile that maximizes its expected payoff given the belief on the strategies of other players and type distribution. Thereafter, the game equilibrium analysis for the problem can be similarly applied by following [96].

To investigate the Stackelberg equilibrium of the proposed game, we seek for each sub-game ($\mathcal{G}^u$, $\mathcal{G}^c$, and $\mathcal{G}^w$) a perfect equilibrium using backward induction to determine the strategies of all the players. In Stage III, the MUs are not coupled with each other, and thus we can analyze an optimal fraction of content demand of each MU in the sub-game independently. Since there is only one type of players in this sub-game $\mathcal{G}^u$, the best response of the MU can be obtained by directly solving **Problem 5.1**. In Stage II, the sub-game $\mathcal{G}^c$ among the 2nd-tier players, i.e., the SCP and ECCP is a non-cooperative game. For a non-cooperative game, the Nash equilibrium is defined as the point at which no player can improve its payoff by changing its strategy unilaterally. The best response of the 2nd-tier players can be obtained by solving **Problems 2-A** and **5.2-B**. Note that **Problem 5.1** must be solved first since the 2nd-tier players derive their individual best responses based on those of 1st-tier players, i.e., MUs. Similarly, **Problems 5.2-A** and

5.2-B need to be solved before solving **Problem 5.3**, since the best response of the 1st-tier player, the MNO, is dependent on those of the 2nd-tier players, i.e., the ECCP and SCP. In the next section, we solve the **Problems 5.1**, **5.2-A**, **5.2-B** and **5.3** in each stage sequentially.

5.3 Game Equilibrium Analysis

In this section, we consider the sub-game perfect equilibrium in each stage sequentially by employing backward induction. Specifically, we analyze the content demand of MUs in Stage III, the sponsoring strategy of the SCP as well as the caching strategy of the ECCP in Stage II, and the pricing strategy of the MNO in Stage I.

5.3.1 Stage III: MU's Content Demand

Given price p decided by the MNO, sponsorship factor θ and caching effort t determined by the SCP and ECCP, respectively, the MUs decides on their optimal content demand for utility maximization individually in the sub-game $\mathcal{G}^u$. Since the utilities of different MUs are not coupled with each other, the sub-game of the MUs can be modeled as an optimization problem. Thus, we analyze the sub-game $\mathcal{G}^u$ by solving **Problem 5.1**. By substituting $f(\cdot)$ in (5.1) and $g(\cdot)$ in (5.6) into (5.7), we obtain the following expression for the utility of the MU which has the decision variable x,

$$u(x;\theta,t,p) = \frac{\tau\sigma_e}{1-\alpha}x^{1-\alpha} + \frac{\tau\sigma_e t^{1-\beta}}{(1-\alpha)(1-\beta)}(1-x)^{1-\alpha} - (1-x)c - (1-\theta)xp. \quad (5.16)$$

Then, we take the first order and second order derivatives of (5.16) with respect to x to prove its concavity, which can be written as follows:

$$\frac{\partial u}{\partial x} = \tau\sigma_e x^{-\alpha} - \frac{\tau\sigma_e t^{1-\beta}}{1-\beta}(1-x)^{-\alpha} + c - (1-\theta)p, \quad (5.17)$$

$$\frac{\partial^2 u}{\partial x^2} = -\alpha\tau\sigma_e x^{-\alpha-1} - \frac{\alpha\tau\sigma_e t^{1-\beta}}{1-\beta}(1-x)^{-\alpha-1} < 0. \quad (5.18)$$

The second order derivative of $u(\cdot)$ is always negative, and thus u is strictly concave with respect to x. We also know that the strategy space of x is a convex and compact subset of the Euclidean space. Accordingly, we can conclude with the following proposition immediately [5].

Proposition 5.1 *The sub-game perfect equilibrium in the sub-game $\mathcal{G}^u$ is unique.*

Furthermore, based on the first order derivative condition, we have

$$\frac{\partial u}{\partial x} = \tau\sigma_e x^{-\alpha} - \frac{\tau\sigma_e t^{1-\beta}}{1-\beta}(1-x)^{-\alpha} + c - (1-\theta)p = 0, \tag{5.19}$$

and we can show that

$$x^{-\alpha} - \frac{t^{1-\beta}}{1-\beta}(1-x)^{-\alpha} = \frac{1}{\tau\sigma_e}\left[(1-\theta)\,p - c\right]. \tag{5.20}$$

We have the following conclusions immediately. If the best response of the MU is larger than 1, then $x^* = 1$, and if the best response of the MU is smaller than 0, then $x^* = 0$. If the best response of the MU with respect to its content demand x^* is within the strategy space $[0,1]$, the best response of the MU, x^* satisfies the condition in (5.20), i.e.,

$$x^{*-\alpha} - \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha} = \frac{1}{\tau\sigma_e}\left[(1-\theta)\,p - c\right]. \tag{5.21}$$

5.3.2 Stage II: SCP's Sponsoring Strategy and ECCP's Caching Strategy

Based on the sub-game equilibrium in $\mathcal{G}^u$ obtained from the Stage III, the 2nd-tier players, i.e., the SCP and ECCP optimize their sponsoring and caching strategies for profit maximization competitively, respectively. The optimal strategies of both the SCP and ECCP are obtained by solving the **Problems 5.2-A** and **5.2-B**.

We first analyze the optimal sponsoring strategy of the SCP. From (5.9), we have the profit of the SCP, which is reformulated as follows:

$$\Pi_s(\theta; p) = \sigma_c \frac{1}{1-\gamma} x^{*1-\gamma} - \theta p x^*, \tag{5.22}$$

where x^* is the best response of the MU given the strategies of other players in the game. The first and second derivatives of profit $\Pi_s(\theta;p)$ with respect to the sponsorship factor θ are given as $\frac{\partial \Pi_s(\theta;p)}{\partial \theta} = \sigma_c x^{*-\gamma}\frac{\partial x^*}{\partial \theta} - px^* - \theta p \frac{\partial x^*}{\partial \theta}$ and $\frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} = -\gamma\sigma_c x^{*-\gamma-1}\left(\frac{\partial x^*}{\partial \theta}\right)^2 + \sigma_c x^{*-\gamma}\frac{\partial^2 x^*}{\partial \theta^2} - 2p\frac{\partial x^*}{\partial \theta} - \theta p \frac{\partial^2 x^*}{\partial \theta^2}$. From the condition in (5.20), we can obtain $\frac{\partial x^*}{\partial \theta}$ and $\frac{\partial^2 x^*}{\partial \theta^2}$, the steps of which are shown as follows. The first order partial derivatives of (5.20) with respect to θ is expressed as $\left[x^{*-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}\right]\frac{\partial x^*}{\partial \theta} = \frac{p}{\alpha\tau\sigma_e}$. Consequently, we can obtain

$$\frac{\partial x^*}{\partial \theta} = \frac{p}{\alpha\tau\sigma_e\left[x^{*-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}\right]} > 0. \tag{5.23}$$

The second order partial derivatives of (5.20) with respect to θ is expressed as follows:

$$\left[-x^{*-\alpha-2} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-2}\right](\alpha+1)\left(\frac{\partial x^*}{\partial \theta}\right)^2 + \left[x^{*-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}\right]\frac{\partial^2 x^*}{\partial \theta^2} = 0. \tag{5.24}$$

By substituting (5.23) into (5.24), we have

$$\frac{\partial^2 x^*}{\partial \theta^2} = -\left[-x^{*-\alpha-2} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-2}\right]\frac{(\alpha+1)p^2}{\alpha^2\tau^2{\sigma_e}^2\left[x^{*-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}\right]^3}. \tag{5.25}$$

Likewise, we next analyze the optimal caching strategy of the ECCP. From (5.12), we have the profit of the ECCP, which is expressed as follows:

$$\Pi_e(t;p) = \sigma_c\frac{1}{1-\gamma}(1-x^*)^{1-\gamma} - Ct. \tag{5.26}$$

The first and second derivatives of profit $\Pi_e(t;p)$ with respect to caching effort t are given as follows:

$$\frac{\partial \Pi_e(t;p)}{\partial t} = -\sigma_c(1-x^*)^{-\gamma}\frac{\partial x^*}{\partial t} - C, \tag{5.27}$$

and

$$\frac{\partial^2 \Pi_e(t;p)}{\partial t^2} = -\gamma\sigma_c(1-x^*)^{-\gamma-1}\left(\frac{\partial x^*}{\partial t}\right)^2 - \sigma_c(1-x^*)^{-\gamma}\frac{\partial^2 x^*}{\partial t^2}. \tag{5.28}$$

From the condition in (5.20), we derive $\frac{\partial x^*}{\partial t}$ and $\frac{\partial^2 x^*}{\partial t^2}$ with the following steps. The first order partial derivatives of (5.20) with respect to t is expressed as follows:

$$\left[x^{*-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}\right]\frac{\partial x^*}{\partial t} = -\frac{t^{-\beta}}{\alpha}(1-x^*)^{-\alpha}. \tag{5.29}$$

Accordingly, we have

$$\frac{\partial x^*}{\partial t} = \frac{-t^{-\beta}(1-x^*)^{-\alpha}}{\alpha\left[x^{*-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}\right]} < 0. \tag{5.30}$$

Similarly, we obtain the second order partial derivatives of (5.20) with respect to t as follows:

$$\left[x^{*-\alpha-2} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-2}\right](\alpha+1)\left(\frac{\partial x^*}{\partial t}\right)^2 + \left[x^{*-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}\right]\frac{\partial^2 x^*}{\partial t^2}$$
$$+ 2t^{-\beta}(1-x^*)^{-\alpha-1}\frac{\partial x^*}{\partial t} = \frac{\beta}{\alpha}t^{-\beta-1}(1-x^*)^{-\alpha}. \tag{5.31}$$

By substituting (5.30) into (5.31), we have

$$\frac{\partial^2 x^*}{\partial t^2} = \frac{t^{-\beta}(1-x^*)^{-\alpha}}{\alpha\left[x^{*-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}\right]}\left\{\frac{2t^{-\beta}(1-x^*)^{-\alpha-1}}{x^{*-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}} + \frac{\beta}{t}\right.$$
$$\left. + \frac{-(\alpha+1)t^{-\beta}(1-x^*)^{-\alpha}\left[-x^{*-\alpha-2} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-2}\right]}{\alpha\left[x^{*-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}\right]^2}\right\}. \tag{5.32}$$

By analyzing the profits of the SCP and ECCP given in (5.22) and (5.26), respectively, we have the following proposition.

Proposition 5.2 *The existence of the Nash equilibrium in the non-cooperative sub-game $\mathcal{G}^c$ is guaranteed if the following conditions*

$$\sigma_c x^{*-\gamma} - \theta p > 0, \tag{5.33}$$

$$-x^{*-\alpha-2} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-2} > 0, \tag{5.34}$$

and

$$\gamma + \alpha - 1 > 0 \tag{5.35}$$

hold.

Proof. Please refer to the Appendix for details. □

Finally, we prove the uniqueness of the Nash equilibrium in the non-cooperative sub-game between the SCP and ECCP, as shown in the following proposition.

Proposition 5.3 *The Nash equilibrium in the non-cooperative sub-game $\mathcal{G}^c$ is unique provided*

$$2\alpha - 1 > 0. \tag{5.36}$$

Proof. Please refer to the Appendix for details. □

5.3.3 Stage I: MNO's Pricing Strategy

In Stage I, the monopolist MNO determines the optimal price p^* by solving **Problem 5.3** with the optimal sponsorship factor θ^* as well as the optimal caching effort t^* obtained from Stage II, and the optimal content demand x^* obtained from Stage I. Thus, **Problem 5.3** can be reformulated as follows:

$$\begin{aligned}
\text{maximize} \quad & \mathscr{P}(p) = px^* - w{x^*}^2 && (5.37)\\
\text{subject to} \quad & 0 \le p \le \overline{p}, && (5.38)\\
& \theta^* = \arg\max_{\theta} \Pi_s, && (5.39)\\
& t^* = \arg\max_{t} \Pi_e, && (5.40)\\
& \sigma_c {x^*}^{-\gamma} - \theta^* p > 0, && (5.41)\\
& -{x^*}^{-\alpha-2} + \frac{{t^*}^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-2} > 0, && (5.42)\\
& {x^*}^{-\alpha} - \frac{{t^*}^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha} = \frac{1}{\tau\sigma_e}\left[(1-\theta^*)\,p - c\right]. && (5.43)
\end{aligned}$$

The constraint in (5.38) is used for the strategy space of the MNO, the constraints in (5.39) and (5.40) indicate that θ^* and t^* denote the best responses of the SCP and ECCP, respectively, provided that the price p is given. The constraint in (5.43) is derived from the best response of the MU when θ, t and p are given, which represents the implicit function of $x^*(\theta, t, p)$. The constraints in (5.41) and (5.42) are given in Proposition 5.2, which ensure the existence and the uniqueness of the non-cooperative sub-game $\mathcal{G}_c$.

Let $\mathcal{X}$ denote $x^*(\theta, t, p) : \mathcal{D}_\theta \otimes \mathcal{D}_t \otimes \mathcal{D}_p \mapsto \mathcal{D}_x$, where $\otimes$ denotes the Cartesian product, $\mathcal{D}_\theta$, $\mathcal{D}_t$ and $\mathcal{D}_p$ represent the domains of θ, t and p, respectively. Note that these domains are all close sets. Furthermore, we define $\Pi(\theta, t, p)$ and $g(\theta, t, p)$ as follows:

$$\begin{aligned}\Pi(\theta,t,p) = \begin{bmatrix} \Pi_s(\theta,p) \\ \Pi_e(t,p) \end{bmatrix} &= \begin{bmatrix} \sigma_c h(x^*(\theta,t,p)) - \theta p x^*(\theta,t,p) \\ \sigma_c h(1 - x^*(\theta,t,p)) - Ct \end{bmatrix} \\ &= \begin{bmatrix} \sigma_c h(\mathcal{X}) - \theta p \mathcal{X} \\ \sigma_c h(1-\mathcal{X}) - Ct \end{bmatrix},\end{aligned} \tag{5.44}$$

$$\begin{aligned} g(\theta,t,p) &= \begin{bmatrix} -\sigma_c (x^*(\theta,t,p))^{-\alpha} + \theta p + \upsilon \\ (x^*(\theta,t,p))^{-\alpha-2} - \frac{t^{1-\beta}}{1-\beta}(1-(x^*(\theta,t,p)))^{-\alpha-2} + \upsilon \end{bmatrix} \\ &= \begin{bmatrix} -\sigma_c \mathcal{X}^{-\alpha} + \theta p + \upsilon \\ \mathcal{X}^{-\alpha-2} - \frac{t^{1-\beta}}{1-\beta}(1-\mathcal{X})^{-\alpha-2} + \upsilon \end{bmatrix},\end{aligned} \tag{5.45}$$

where υ which is a small number. In addition, we define $\boldsymbol{\rho} = \begin{bmatrix} \theta \\ t \end{bmatrix} : \mathcal{D}_\theta \otimes \mathcal{D}_t \mapsto \mathcal{D}_{\boldsymbol{\rho}}$. As a result, **Problem 5.3** can be redefined as the bilevel programming problem, which is shown as follows:

$$\begin{aligned} \underset{0<p<\overline{p}}{\text{maximize}} \quad & \mathscr{P}(p) = p\mathcal{X} - w\mathcal{X}^2 \\ \text{subject to} \quad & \boldsymbol{\rho}^* = \arg\max_{\boldsymbol{\rho}} \Pi(\boldsymbol{\rho}, p), \\ & g(\boldsymbol{\rho}, p) \le 0, \\ & \boldsymbol{\rho} \in \mathcal{D}_{\boldsymbol{\rho}}. \end{aligned} \tag{5.46}$$

Recall from Section 5.3.2, we prove that for any given p, there exists a unique pair of θ^* and t^* as the optimal solution of **Problem 5.2**, i.e., the Nash equilibrium in non-cooperative sub-game $\mathcal{G}_c$. Accordingly, the optimal solution to the lower-level programming problem $\boldsymbol{\rho}^* = \begin{bmatrix} \theta^* \\ t^* \end{bmatrix}$ is shown to exist and be unique, for any given p. Therefore, the strong sufficient optimality condition of second order (SSOSC) is satisfied for the bilevel programming problem in (5.46) because of the existence and the uniqueness of

$\boldsymbol{\rho}^*$ (Theorem 3.9 in [97]). This indicates that the optimal solution to the lower-level programming problem of the bilevel programming problem is strongly stable. Thus, our bilevel programming problem can be reduced to a single-level problem, which is expressed as follows:

$$\begin{aligned} &\underset{0<p<\overline{p}}{\text{maximize}} \quad \mathscr{P}(p) = p\mathcal{X} - w\mathcal{X}^2, \\ &\text{subject to} \quad \boldsymbol{\rho} = U(p). \end{aligned} \tag{5.47}$$

$\boldsymbol{\rho} = U(p)$ in the constraint can be obtained using the KKT condition to the lower-level programming problem of the bilevel programming problem as follows:

$$\begin{cases} \nabla_{\boldsymbol{\rho}}\Pi(\boldsymbol{\rho}, p) - \lambda^\top \nabla_{\boldsymbol{\rho}} g(\boldsymbol{\rho}, p) = 0, \\ \lambda \perp g(\boldsymbol{\rho}, p), \lambda \geq 0, g(\boldsymbol{\rho}, p) \leq 0, \end{cases} \tag{5.48}$$

where λ denotes the Lagrangian multiplier vector. Moreover, the feasible domain of the single-level programming is defined as follows:

$$\Omega(p, \boldsymbol{\rho}) = \left\{ (p, \boldsymbol{\rho}) \middle| \, \boldsymbol{\rho} = U(p), p \in \mathcal{D}_p \right\}, \tag{5.49}$$

which is a non-empty and closed set according to the Weierstrass Theorem [98]. Since we know that the optimal solution to the lower-level programming problem of the bilevel programming problem is unique, Constant Rank Constraint Qualification (CRCQ) and Mangasarian-Fromovitz Constraint Qualification (MFCQ) are satisfied by all the feasible points in $\Omega(p, \boldsymbol{\rho})$ (Theorem 3.9 in [97]). Following [99], we can easily check the full rank of the gradient of the constraint for satisfying CRCQ, and the existence of the first derivative of the constraint for satisfying MFCQ. Based on Theorem 4.10 in [100], $\boldsymbol{\rho} = U(p)$ is a piecewise continuously differentiable function and $(p, \boldsymbol{\rho}) = (p, U(p))$ is therefore continuous on p. Furthermore, with the closed sets $\Omega(p, \boldsymbol{\rho})$ and $\mathcal{D}_p$, according to the well-known Closed Graph Theorem [101], we can conclude that $\boldsymbol{\rho} = U(p)$ being continuous implies that the mapping $\mathcal{D}_p \mapsto \Omega(p, \boldsymbol{\rho})$ is closed. Therefore, $\Omega(p, \boldsymbol{\rho})$ is non-empty and closed, and thus the bilevel programming problem admits a globally optimal solution, namely the Stackelberg equilibrium. Accordingly, we conclude with the following proposition.

Algorithm 5.1 Sub-gradient based iterative algorithm finding the Stackelberg equilibrium for joint sponsored and edge caching content service market model

Initialization:
Select initial input $p \in [0, \overline{p}]$, $\theta \in [0,1]$ and $t \in [0,1]$, $k \leftarrow 1$, step size δ;
repeat
  Each MU decide on its content demanded from the SCP $x^{[k+1]}$, using a gradient assisted searching algorithm, e.g.,

$$x^{[k+1]} \leftarrow x^{[k]} + \mu \nabla u(x^{[k]}), \tag{5.50}$$

  where $\mu \nabla u(x^{[k]})$ is the gradient with $\frac{\partial u(x^{[k]})}{\partial x^{[k]}}$ and μ is the step size;
  The SCP and ECCP update their sponsorship factor $\theta^{[k+1]}$ and caching effort and $t^{[k+1]}$ using the similar gradient assisted searching algorithm, respectively. The updated sponsorship factor and caching effort are broadcast to all MUs and the MNO;
  The MNO tries to increase or decrease its price p with a small step size δ, and calculate its corresponding payoff;
    if $\mathscr{P}(p^{[k]}) < \mathscr{P}\left(p^{[k]} + \delta\right)$ **then**
      $p^{\text{new}} \leftarrow \min\left\{\overline{p}, p^{[k]} + \delta\right\}$; % Increase the price
    else
      if $\mathscr{P}(p^{[k]}) < \mathscr{P}\left(p^{[k]} - \delta\right)$ **then**
      $p^{\text{new}} \leftarrow \max\left\{0, p^{[k]} - \delta\right\}$; % Decrease the price
      else
      $p^{\text{new}} \leftarrow p^{[k]}$; % Keep the price unchanged
      end if
    end if
  $p^{[k+1]} \leftarrow p^{\text{new}}$;
  The updated price information is broadcast to all MUs and both the SCP and ECCP;
  $k \leftarrow k+1$;
until $\left\|p^{[k]} - p^{[k-1]}\right\|_1 < \delta$

Proposition 5.4 *There exists at least one Stackelberg equilibrium in the proposed hierarchical three-stage Stackelberg game.*

Up to now, each stage of the proposed hierarchical three-stage Stackelberg game has been investigated. Similar to that in [102, 11], we then present the sub-gradient based algorithm[2] to obtain the Stackelberg equilibrium of the proposed game in Algorithm 5.1. Specifically, the proposed iterative algorithm to obtain the equilibrium can be implemented as follows:

(1) Initially, the price p, sponsorship factor θ, and caching effort t are randomly offered, and the information is broadcast.

(2) The MUs receive the information concerning the price, sponsorship factor and caching effort. Next, they choose the amount of content to consume from SCP and ECCP using a gradient assisted searching algorithm and feed back the content demand information to the MNO and the SCP and ECCP.

(3) Then, the SCP and ECCP receive the information and determine the sponsorship factor and caching effort using a gradient assisted searching algorithm, respectively.

(4) Lastly, the MNO adjusts its price in each iteration. If increasing or decreasing the price improves the payoff of the MNO, in the next round, the MNO increases or decreases its price with δ, and steps (2), (3), (4) are repeated until the price p converges.

When Algorithm 5.1 converges, the MNO cannot increase or decrease the price unilaterally for improving its payoff. The convergence property of the sub-gradient based iterative algorithm has been proved (Lemma 3 in [102]). Specifically, when the initial

[2]Note that the best response dynamics algorithm cannot be applied here since it cannot guarantee the convergence. It can be applied in the situation where the uniqueness of the Stackelberg equilibrium has been proved.

price value and the step size δ are fixed, the results in the subsequent iterations are fixed. For example, we consider that at the kth iteration, the price of the MNO is given with a fixed value. Then, at the $(k+1)$th iteration, the step size is fixed and the search direction of the algorithm from the current iteration to the next iteration is unique because of the property of sub-gradient strategy [11, 102]. Thus, the price of the MNO at the $(k+1)$th iteration is also fixed. Based on the above mentioned fact, therefore, the game can converge to a unique outcome, when the initial price and the step size δ are fixed.

5.3.4 Extension Analysis on Multi-MNO Game

In this part, we briefly discuss a general scenario that incorporates multiple MNOs in the game. In this regard, the individual MNO competes with each other to reach an equilibrium, while constrained by the lower level equilibrium among the lower-layer players, i.e., the SCP, the ECCP and MUs. Modeling this competition leads to the Equilibrium Problems with Equilibrium Constraints (EPEC) formulation [103]. Note that our previous analysis on the market with a single MNO can be similarly extended to a multiple MNO scenario with no logical difficulty, but at the price of cumbersome derivation and equilibrium computation. We also adopt the backward induction and first consider the content demand problem of MUs in Stage III with fixed upper strategies, i.e., sponsorship, caching effort and price. Let $\boldsymbol{x} = \{x_1, \ldots, x_m, \ldots, x_M\}$ denote the strategy of the MU, where $x_m \in [0, 1]$ represents the fraction of the content demand to access through a cellular link of the MNO m. Let $\boldsymbol{\theta} = \{\theta_1, \ldots, \theta_m, \ldots, \theta_M\}$ denote the strategy of the SCP, where θ_m represents the sponsorship offered to the MU using the cellular link provided by the MNO m. Likewise, the strategy of the ECCP is similarly defined as $\boldsymbol{t} = \{t_1, \ldots, t_m, \ldots, t_M\}$. p_m is the price set by the MNO m, $m = 1, 2, \ldots, M$.

Then the analysis on the lower equilibrium in the previous section can be readily presented here. The utility of the MU is modified as follows:

$$u(\boldsymbol{x}) = \sum_{m=1}^{M} (\tau\sigma_e f(x_m) - (1-\theta_m)x_m p_m + \tau\sigma_e f(1-x_m)g(t_m) - (1-x_m)c). \quad (5.51)$$

The profit of the SCP and the ECCP are respectively reformulated as (5.52) and (5.53):

$$\Pi_s(\boldsymbol{\theta}) = \sum_{m=1}^{M} (\sigma_c h(x_m) - \theta_m p_m x_m), \quad (5.52)$$

$$\Pi_e(\boldsymbol{t}) = \sum_{m=1}^{M} (\sigma_c h(1-x_m) - Ct_m). \quad (5.53)$$

Following the similar analysis, we can also prove that

$$\frac{\partial^2 u(\boldsymbol{x})}{\partial {x_m}^2} = -[\alpha\tau\sigma_e {x_m}^{-\alpha-1} - \frac{\alpha\tau\sigma_e {t_m}^{1-\beta}}{1-\beta}(1-x_m)^{-\alpha-1}] < 0, \quad (5.54)$$

and we also know that the strategy space of x_m is a convex and compact subset of the Euclidean space. Thus, the sub-game perfect equilibrium of MUs can be validated to be unique. With the optimal content demand solution obtained from Stage III, we can check properties of the second order derivative of the objective function of the SCP and the ECCP, i.e., $\frac{\partial^2 \Pi_s(\boldsymbol{\theta})}{\partial {\theta_m}^2}$ and $\frac{\partial^2 \Pi_e(\boldsymbol{t})}{\partial {t_m}^2}$. The existence and uniqueness of the non-cooperative sub-game between the SCP and the ECCP can be proved in a way similar to that presented in Section 5.3.2 and the details are omitted. Clearly, we can see that the properties of the lower equilibrium remain the same with the case with one MNO.

After solving the problems in the lower layers, we can consider the upper layer multi-objective optimization problems, which are expressed as follows:

$$\underset{0 \le p_m \le \overline{p}}{\text{maximize}} \quad \mathscr{P}(p_m) = p_m x_m - w_m {x_m}^2, m = 1, 2, \ldots, M. \quad (5.55)$$

Note that such problems are also constrained by the lower level equilibrium among the lower-layer players. The MNOs, as the leaders, are able to predict the lower-layer equilibrium to assist the decision-making at the upper layer. Such EPEC formulation can be solved by following the EPEC decomposition method proposed in [104]. To find the equilibrium solutions from the competition among MNOs, we resort to the sub-gradient based algorithm (Algorithm 1 in [11]), and its convergence is proved in [11]. The intuition is that when MNOs adopt the sub-gradient algorithm, each MNO initially assumes that there is no competition with other MNOs and hence sets its price at a certain value to receive high utility. After several iterations, when the MNO discovers that there exist other MNOs trying to attract MUs with appealing prices, the MNO predicts the responses of the other MNOs and tries to adjust its price competitively. Within the finite number of iterations, all MNOs are able to determine the best pricing decisions to achieve the highest utilities. Consequently, the EPEC problem is solved in an iterative manner.

5.4 Performance Evaluation

In this section, we conduct simulations to evaluate performance of the players in the proposed joint sponsored and edge caching content service market through the Stackelberg game. Unless otherwise stated, we set $\alpha = 0.8$, $\beta = 0.5$, $\gamma = 0.8$, $l_a = 1$, $\sigma_e = 40$, $\sigma_c = 120$, $c = 80$, $C = 120$, $w = 1$, and $\overline{p} = 100$.

Figure 5.2(a) illustrates the Nash equilibrium (NE) of the non-cooperative game between the 2nd-tier players, i.e., the SCP and ECCP. The NE is the point at which the best responses of the SCP and ECCP intersect. Under different prices, different NE points are observed. Additionally, when the price increases, the optimal sponsorship fee increases and the optimal caching effort decreases. This is because when the price is high, the MUs are reluctant to choose the cellular link for content access. In this regard, the SCP needs to offer higher sponsorship fee to compensate the cost for MUs, i.e., the price paid to the MNO, in order to attract content demand of the MUs. Conversely, the ECCP has an incentive to decrease its caching effort to save the cost. Furthermore, Fig. 5.2(b) shows the number of iterations needed for convergence versus different step

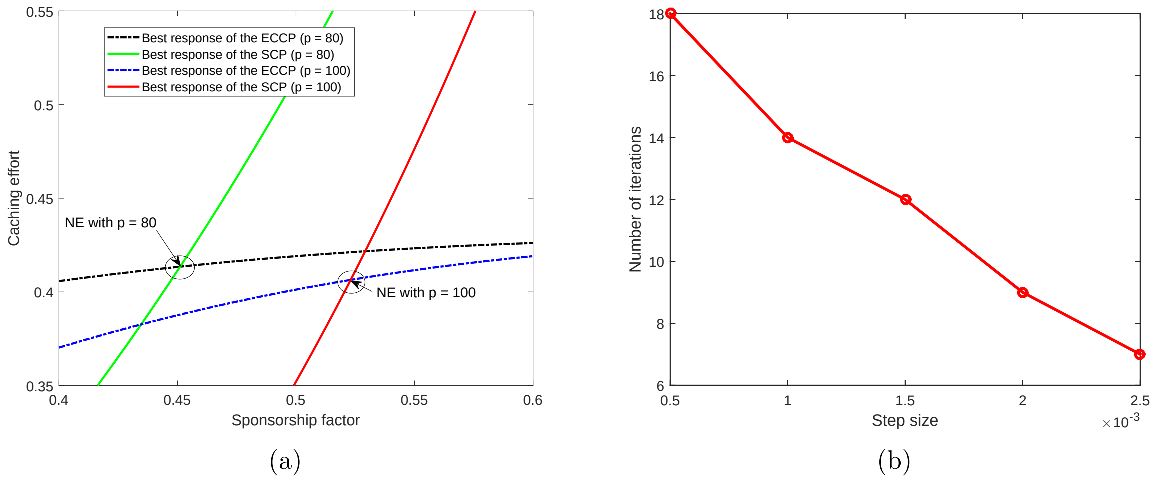


Figure 5.2: (a) Nash equilibrium for caching effort t and sponsorship factor θ under different price levels; (b) Number of iterations versus the step size.

sizes. As expected, the convergence rate of the proposed algorithm depends on the step size. When the step size is small, the delay is large because of the slow convergence time in the sub-gradient algorithm, but the achieved results are more accurate. The accuracy means that the gap between the achieved results and the optimal values is small because of the small step size. Conversely, when the step size is big, the delay is small but the achieved results are not accurate. We next investigate the impact of the price constraint on the MNO, as illustrated in Fig. 5.3. It is worth noting that the optimal price offered by the MNO is the same as the maximum price constraint. The intuition is that the lower price can attract the MU to consume more sponsored content from the SCP, which may incur higher delivery cost maintained by the MNO. Thus, the MNO is reluctant to lower the offered price. In addition, we observe that as the price constraint, i.e., the optimal price increases, the payoff of the MNO increases and the sponsored content demand of the MU decreases. This is because the MNO is able to set a higher price and extract more surplus from the MU, and achieves higher payoff consequently. Moreover, as expected, the payoff of the MNO decreases with the increase of w, i.e., the content delivery cost

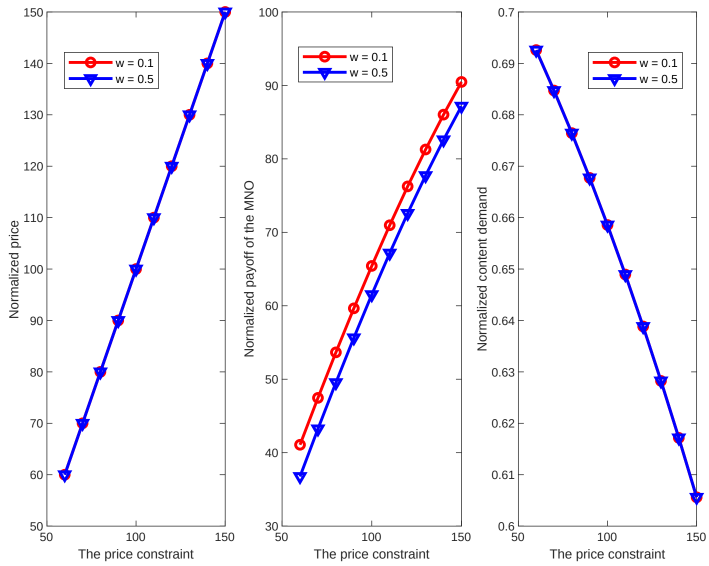


Figure 5.3: The impact of the price constraint $\overline{p}$ on the MNO from joint sponsored and edge caching content service market. (lower delivery cost factor: $w = 1$, higher delivery cost factor: $w = 2$)

factor.

Then, we study the impact of the utility coefficient of MUs on the strategies and payoff of players in the game model, as illustrated in Fig. 5.4. We observe that the sponsored content demand of the MU decreases with the increase of σ_e. This is because when σ_e increases, the sponsorship fee from the SCP becomes relative lower compared with the improved utility of the MU from consuming the content. Recall that the caching effort positively affects the utility of the MU from consuming the content, therefore, the sponsored content demand of the MU decreases. As a result, the SCP wants to offer more sponsorship fee to attract the MU to consume the sponsored content, and the ECCP has an incentive to lower its caching effort to reduce the caching cost. Once the sponsorship fee from the SCP is large enough, the SCP is not willing to offer more sponsorship fee to save its cost. Accordingly, the sponsorship fee from the SCP increases first and then decreases. Therefore, the SCP's profit decreases and the ECCP's profit increases,

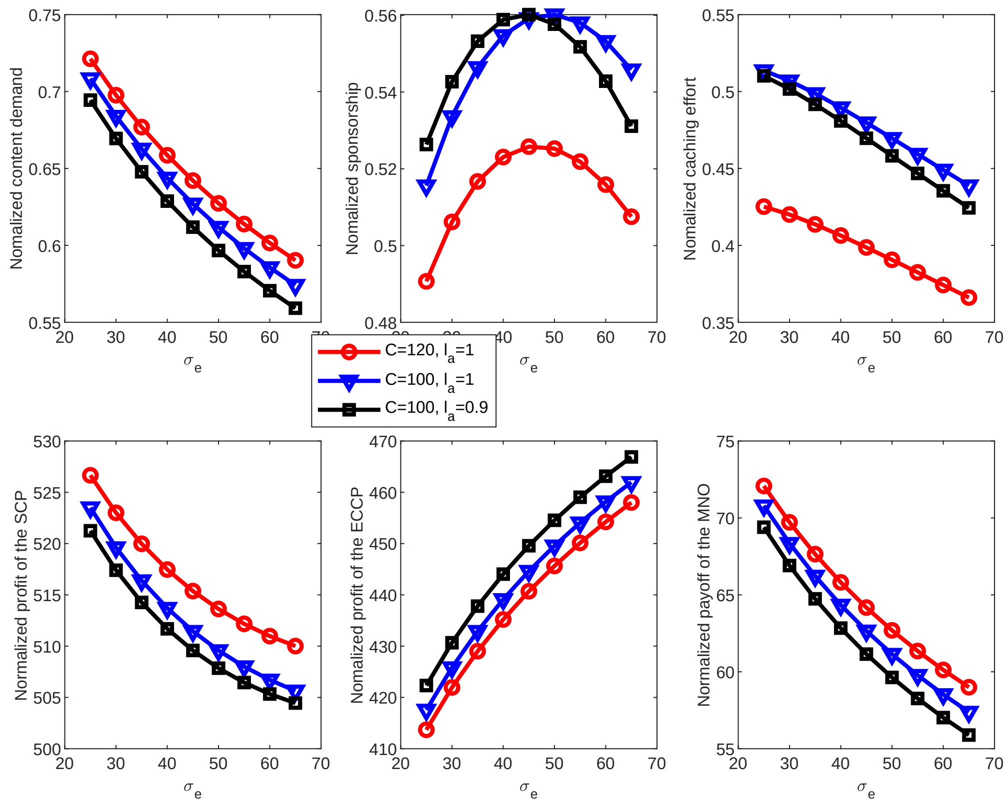


Figure 5.4: The impact of the utility level of MUs on the strategies and payoff of players (MUs, SCP, ECCP, and MNO) from joint sponsored and edge caching content service market.

which is consistent with the results in Fig. 5.4. Since the sponsored content demand of the MU decreases, and thus the payment from the MU to the MNO is reduced, which leads to the decrease of the MNO's payoff. As expected, we observe that the sponsored content demand of the MU increases and the caching effort of the ECCP decreases as the content caching cost C increases. This is due to the fact that, with the increase of C, the cost of the ECCP for increasing caching effort becomes greater. In this case, the ECCP is willing to reduce its caching effort for saving cost and thus compensate for its decreasing profit. Meanwhile, the SCP wants to offer higher sponsorship fee to the MU for encouraging the MU's higher sponsored content demand. Consequently, both the

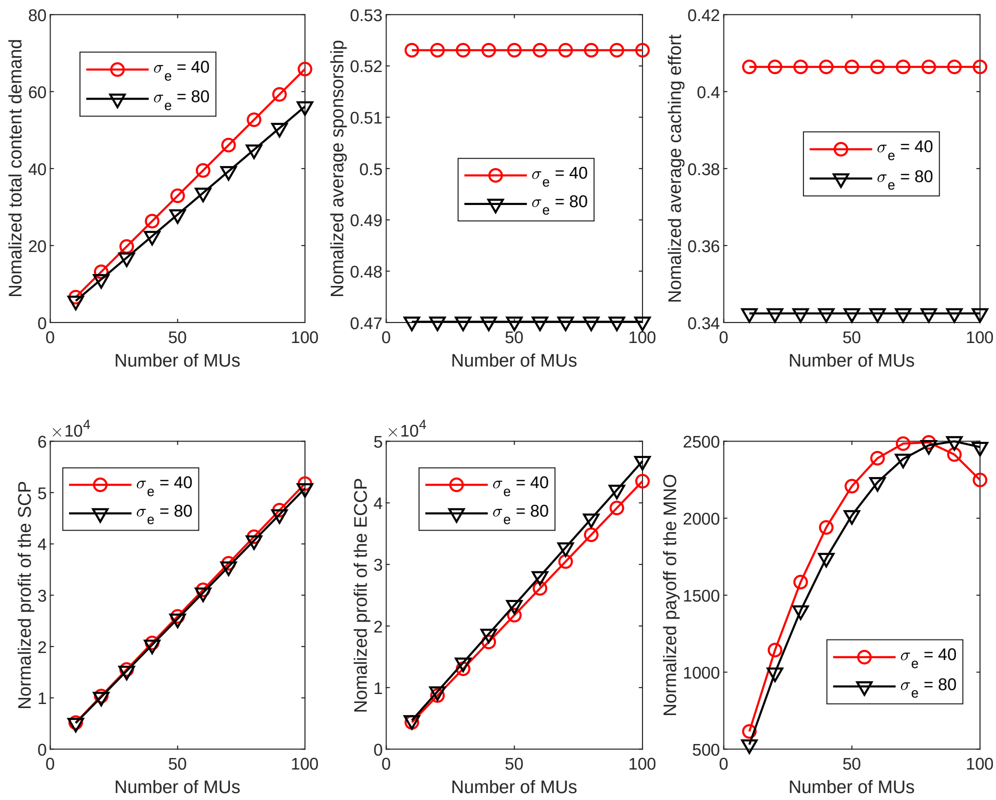
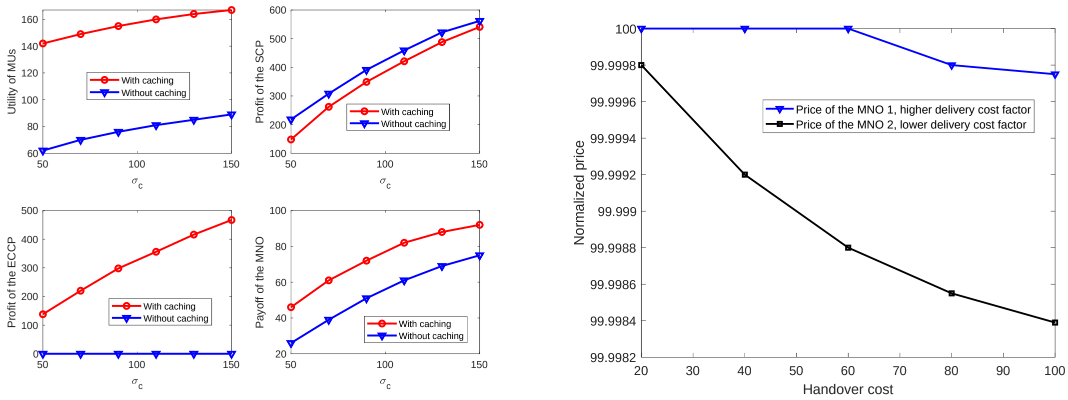


Figure 5.5: The impact of the number of MUs on the strategies and payoff of players (MUs, SCP, ECCP, and MNO) from joint sponsored and edge caching content service market.

SCP's profit and the MNO's payoff increase. Moreover, comparing curves with different value of the advertisement length l_a in the second row of Fig. 5.4, we observe that the increase of l_a leads to the increase of the SCP's profit and the MNO's payoff. This is because when l_a increases, the utility of the MU from consuming the content decreases, and the offered sponsorship fee becomes significant for the MU. This encourages higher sponsored content demand of the MU, which benefits the SCP and the MNO accordingly. Therefore, the profit of the ECCP decreases because of its decreasing content traffic.

Furthermore, in Fig. 5.5, we evaluate the impact of the number of MUs on the strategies and payoff of players in the game model. We consider a set of MUs, in which the utility coefficients of MUs are assumed to follow the normal distribution $\mathcal{N}(\sigma_e, 1)$. Since the MUs are not coupled with each other in the model, the SCP and the ECCP can determine their strategies on each MU individually without involvements from other MUs. In this regard, the strategies of both providers only depend on the utility coefficient of the MU, i.e., σ_e. As expected, when σ_e increases, the optimal sponsorship fee decreases and the optimal caching effort decreases. This is consistent with what we have discussed for Fig. 5.4. We also find that the average sponsorship and average caching effort remain unchanged, when the number of MUs increases. Note that the terms "average sponsorship" or "average caching effort" are the mean value of the offered "sponsorship fees" or devoted "caching resources", respectively, for each MU. Thus, the total investments of the SCP and the ECCP, i.e., the total sponsorship fees and total caching resources, respectively, are increasing, which scales with the number of MUs. Moreover, the increase of the number of MUs leads to the profit improvement of both providers since they can sell more services to the incoming customers. We also find that the marginal increase of the payoff of the MNO decreases when the number of MUs increases. Furthermore, when the number of MUs reaches a certain level, the payoff of the MNO even decreases. When the number of MUs increases, the backhaul traffic increases that causes congestion, which in turn leads to the rapid increase of the maintenance cost because of the limited capacity and bandwidth in the wireless environments.

We also study the performance comparison between the joint sponsored and edge caching scenario and the scenario without caching, as shown in Fig. 5.6(a). We find that the utilities of MUs are higher under the joint sponsored and edge caching scenario. The intuition is that, with caching, the ECCP enters the market for selling the content, and extracts the surplus from MUs. Meanwhile, the MUs have the positive payoff from purchasing and enjoying the content with more efficient content delivery. With the competition of the ECCP, the profit of the SCP decreases consequently. The payoff



(a) (b)

Figure 5.6: (a) The performance comparison between the joint sponsored and edge caching scenario and the scenario without caching; (b) The normalized price versus the handover cost in the duopolistic MNO scenario.

of the MNO is also higher under the joint sponsored and edge caching scenario. The reason is that the ECCP helps to offload some traffic load from the cellular link. As aforementioned, when the data traffic (number of MUs) reaches a certain level, the payoff of the MNO may decrease. Thus, the edge caching in the model relieves the congestion of the backhaul network maintained by the MNO, especially when the data traffic is high. Under the joint sponsored and edge caching scenario, the profit of the ECCP is higher than the profit loss of the SCP when the ECCP exists in the market. Therefore, the social welfare of all the entities under the joint sponsored and edge caching scenario is also higher than that under the scenario without caching.

Lastly, we investigate the price competitions in the multiple MNO scenario. Figure 5.6(b) illustrates the normalized price versus the handover cost. For ease of presentation, we consider the duopoly MNO scenario that consists of two MNOs: MNO 1 with a higher delivery cost factor, MNO 2 with a lower delivery cost factor. We find that the MNO 2 is willing to reduce the price when the handover cost increases. The reason is that the MUs are reluctant to access edge caching content as the handover cost is high. In this regard, the MNO 2 has an incentive to reduce its price to attract MUs competing with MNO 1. Thus, when the handover cost increases, the price set by the MNO 2 decreases. However, when the handover cost is relatively small, the MNO 1 is not willing to reduce its price. The reason is that the lower price may attract more MUs and meanwhile cause higher data traffic, therefore, the delivery cost is even higher than the gained revenue. When the handover cost decreases to a certain level, the MNO 1 reduces its price slightly because of the competition since the price of the MNO 2 is relatively low. Note that the prices set by two MNOs approach to their maximum values, i.e., the price constraint. The reason is that the lower price may lead to higher data traffic, which may even decrease the payoff of the MNO. Thus, the price is not reduced significantly.

In summary, we draw the following engineering insights of the proposed scheme:

- In the proposed scheme, the MUs are encouraged to access and consume more services while improving service quality. Furthermore, in the presence of competition between the SCP and the ECCP, they are tempted to put more investment, i.e., sponsorship fee or caching resources. This further benefits MUs at the expense of the content providers, and thus the utility of user is improved compared with that under the scenario without caching. Consequently, the profit of the SCP decreases because of the competition. Nevertheless, we find that the total profit of the SCP and the ECCP in the proposed model is higher than the profit of the SCP in the model without caching. This provides a motivation for cooperation between the SCP and the ECCP. Indeed, this may even encourage them to collaborate to extract more surplus from MUs. The monopoly MNO is willing to set the price as high as possible to reduce the congestion cost. In the scenario with multiple MNOs, the prices set by the MNOs also approach their maximum values. However, the MNO with a lower delivery cost factor inclines to set a lower price than those of the MNOs.

- With the demand of cellular data/content traffic increasing sharply, the resource scarcity in wireless networks becomes more serious which deteriorates the service quality of users. In the presence of limited bandwidth and capacity of the MNO, we find that when the number of users is increasing which is often the case in practice, the backhaul traffic significantly increases that causes severe congestion and in turn leads to rapid increase of the maintenance cost, i.e., the content delivery cost. Nevertheless, the edge caching can alleviate such backbone network burden, and thus improve the payoff of the MNO in a large extent. This is also confirmed in numerical results. Therefore, the proposed scheme is shown to be of high importance to well accommodate the growing content traffic demand of users. On the one hand, the proposed scheme relieves the congestion of the backhaul network. On the other hand, the proposed scheme also improves the service quality. This benefit is especially valuable in consideration of radio resource scarcity in wireless networks. Finally, the social welfare of the proposed model is higher than that of the model without caching.

5.5 Summary

In this work, we have formulated a joint sponsored and edge caching content service market model, where the interactions among the mobile network operator, the sponsored content provider as well as the edge caching content provider have been modeled as a hierarchical three-stage Stackelberg game. Then, we have analytically analyzed the sub-game perfect equilibrium in each stage using backward induction. Furthermore, we have validated the existence of the Stackelberg equilibrium by capitalizing on the bilevel optimization technique. Additionally, we have proposed a sub-gradient based iterative algorithm, which is ensured to converge to the Stackelberg equilibrium. At last, we have presented the performance evaluation.

Chapter 6

Conclusions and Future Work

The main theme of this thesis is to study the sponsored content management using a unified game theory-based framework in response to some urgent challenges therein, i.e., network effects in Chapter 3, information asymmetry in Chapter 4, and coexistence with edge caching in Chapter 5. In this chapter[1], we first summarize the main contributions of this thesis. Then, we provide several promising open directions for the future work.

6.1 Conclusions

In sponsored content, a network operator negotiates with a content provider in which the latter can pay the former to lower the cost of the mobile users to access certain content. The payment can be treated as the sponsorship from the content provider to users.The key challenge is how to provide proper sponsorship given the content demand from users and the service fee charged by the network operator. Therefore, in this thesis, we aim to address the sponsorship problems in a unified game theory-based framework in response to some major challenges in real implementation of the sponsored content. In Chapter 1, we have presented the overview of sponsored content as well as its real-world applications, and pointed out several challenges and the main research contributions of this thesis. In Chapter 2, we have provided the literature review of existing studies that are related to

[1]Part of the work in this chapter has been published in [105, 87].

our research, where we have also highlighted the novel contribution of this thesis and its significance compared with other existing work in the literature.

In Chapter 3, we have proposed a hierarchical Stackelberg game to model the interactions among the mobile network operator, the content provider and mobile users under sponsored content policy. Furthermore, the network effects in the social domain and congestions in the network domain have been jointly investigated for modeling the interactions among users. In the game model, we have examined the interplay between the content provider and the network operator by characterizing the scenarios where the network operator and the content provider compete sequentially, the network operator and the content provider compete simultaneously, and the network operator and the content provider cooperate for a common goal. Under the three scenarios, we have derived the unique Nash equilibrium point among the users and analytically validated the existence and uniqueness of Stackelberg equilibrium through backward induction. With extensive performance evaluation, it has been revealed that the network effects significantly improve the utilities of users, the profit of the content provider and the revenue of the network operator. In addition, it has been verified that the cooperation is the best choice for the network operator and the content provider.

In Chapter 4, we have proposed a novel joint optimization approach combining contract theory and Stackelberg game to characterize the market-oriented model for the sponsored content market with information asymmetry, and capture the interactions among the main entities therein. Specifically, we have developed a hierarchical Stackelberg game, where the mobile network operator determines the optimal pricing acting as the leader, the content provider and mobile users make decisions acting as the followers. The decision making process of followers has been modeled as the contract game. We have derived the necessary and sufficient conditions of the feasible contract, and obtained the optimal contract solutions. Through the backward induction, we have proved that the optimal pricing of the network operator is unique, and accordingly validated the uniqueness of the Stackelberg equilibrium. The performance evaluation has demonstrated the effectiveness of our scheme compared with other benchmark schemes.

In Chapter 5, we have formulated a joint sponsored and edge caching content service market model, where the interactions among the mobile network operator, the normal sponsored content provider as well as the edge caching content provider have been modeled as a hierarchical three-stage Stackelberg game. Then, we have analytically analyzed the sub-game perfect equilibrium in each stage using backward induction. Furthermore, we have validated the existence of the Stackelberg equilibrium by capitalizing on the bilevel optimization technique. Additionally, we have proposed a sub-gradient based iterative algorithm, which is ensured to converge to the Stackelberg equilibrium. At last, we have presented the performance evaluation.

To sum up, in this thesis, we addressed a series of urgent challenges in sponsored content with the tools of game theory. The hope of our work is that the novel insights from an economic point of view will be helpful to the researchers regarding the real implementation of sponsored content, and enable the engineers to apply the economics strategies from game theory in the future mobile data market.

6.2 Future Work

There are a number of potential open directions for the future work as follows.

6.2.1 Scalability of the Algorithm Implementation

In some real-world cases of sponsored content system, there may exist a large number of content providers and a huge number of mobile users. In this regard, it is challenging for the conventional game theory methods or algorithms to tackle the hierarchical multi-leader multi-follower game problem because of the high dimensionality of each player's strategy space. Furthermore, the centralized optimization solution that simultaneously maximizes all the payoff of players is practically impossible. To address this problem, we may resort to the ADMM algorithm to reach the hierarchical social optimum point in a distributed manner. The ADMM algorithm can efficiently cope with the networks with a large number of players including leaders and followers, which is confirmed in [106, 107, 108]: the algorithm converges linearly and is independent of the network size.

6.2.2 Bounded Rationality

It is worth noting that the decision making of players in sponsored content model proposed in this thesis is considered to be rational and selfish, such as content provider, and mobile users. Nonetheless, the model can be extended to capture human behaviors, i.e., internal bounded rationality. In this regard, the prediction of user strategies can be intractable that may change the actual decision-making process. Therefore, similar to [109], we can seek for more general theoretical model called prospect theory from behavioral economics [110] to analyze the human behaviors with more psychologically accurate description.

6.2.3 Bayesian Game

In this thesis, we study the sponsored content management in a game of complete information, where the strategies of players are perfectly observed by each other. In certain scenarios, the strategies of players cannot be observed accurately by others, which we call the incomplete information environment. In this regard, we can employ the Bayesian game to describe the game behaviors among players [7]. Comparing with the typical complete information game, the definition of Bayesian game introduces two more components: the types of players, and the probability distribution of the types. The probability distribution is the belief about unknown information of the players, referred to as the type of the player, e.g., the utility coefficient of MUs. Such distribution information can be obtained through, e.g., historical information or long-term learning. In this regard, each MU knows it own expected payoff instead of specific payoff. Thus, each player seeks for a strategy profile that maximizes its expected payoff given the belief on the strategies of other players and type distribution. Thereafter, the game equilibrium analysis for the sponsorship problem can be similarly applied by following this thesis.

6.2.4 Multi-dimensional Contract

In Chapter 4, we addressed the information asymmetry of sponsored content. We consider that the mobile users have only one-dimensional private information in order to characterize the uncertain nature of heterogeneous user preferences. However, the users may have multi-dimensional private information in some realistic scenarios [111]. In this regard, the problem involving users' multi-dimensional private information leads to a multi-dimensional contract design problem. The traditional contract theory approach cannot be applied straightforwardly since the single-crossing condition does not always hold when there are more than one user type. As such, we can resort to the multi-dimensional contract [102, 51, 18] to address the information asymmetry, in which users are involved with multi-dimensional private information.

6.2.5 Game Theory and Deep Reinforcement Learning

In game theory, we consider that the payoff formulation of each player is fixed, where the resource and service requirement is given in advance. However, with the dynamics and uncertainty inherently existing in the wireless network environments, conventional approaches of service and resource management that require complete and perfect knowledge of the systems become inefficient or even inapplicable. In such time-varying and unpredictable network environments, Reinforcement Learning (RL) has shown to be a viable tool to tackle the real-time dynamic decision-making problems [112]. On the one hand, instead of myopically optimizing the current benefits, RL naturally incorporates farsighted system evolution when making decisions, which is essential for time-variant 5G networks. On the other hand, RL can update decision policies to reach optimal system performance through the reward feedback of the previous decisions, i.e., reinforcement, even without up-to-date information [112]. Therefore, RL-based approaches can be an promising option for solving resource and service management problems in mobile networks. Nevertheless, conventional RL algorithms, such as Q learning, suffer from slow convergence speed, especially if the state space and action space of the problem are large. Furthermore, the algorithms have to store full tables of an immediate value, e.g., Q-value, for each state-action pair. The tables can be too large to be maintained in mobile devices. In this regard, the RL often leads to poor performance. As a branch of artificial intelligence, Deep Reinforcement Learning (DRL), which is the combination of RL and deep learning, has been promisingly proposed to overcome the limitations of RL. DRL has shown vast successes in many mobile networks and applications such as [105].

6.2.6 Emerging Networking Applications Management

The unified game theory-based framework proposed in this thesis can also be straightforwardly applied in other innovative data pricing schemes in mobile data market. For example, mobile data rewards is a novel business model leading a new economic trend in wireless networks, in which the operators stimulate mobile users to watch ads with data rewards and ask for corresponding payments from advertisers. Different from the sponsored content where the users can only consume the sponsored data for viewing the content provider's content, the users under the data rewarding scheme can consume the rewarded data in any mobile content or service access [113]. Another major difference is that the sponsored content providers benefit from the data usage of users for the contents.

Instead, the advertiser under the data rewarding scheme aims to deliver the ads to users instead of benefiting from the data consumption of users. Both of the promising schemes exploit a well-designed motivation pattern to incentivize users to access more content with more data consumption, thereby benefiting the practice of the telecommunication industry. However, the inherent incentive mechanism design for *mobile data rewards* remains largely unexplored yet in the literature, which poses a great challenge for an efficient adoption and operation of data rewarding ecosystems. The interactions among the mobile network operator, the advertiser, and mobile users can be modeled and studied in the same manner as the proposed game theory framework. Furthermore, the framework of game theory can also be similarly leveraged to address the resource management in other emerging networking applications, such as 5G [105], smart IoT [114, 115] and blockchain networks [116, 117].

Chapter 7

Appendix

7.1 Appendix of Chapter 3

7.1.1 Proof of Proposition 3.3

Proof. Based on (3.3), the best response of the CP is obtained by setting the derivative $\frac{\partial \mathscr{P}(\theta)}{\partial \theta_j} = 0$, and thus we have

$$\frac{\partial \mathscr{P}(\theta)}{\partial \theta_j} = \gamma \sum_{i \in \mathcal{N}} (s - 2tx_i^*) \frac{\partial x_i^*}{\partial \theta_j} - p^{\mathrm{u}} \left(\sum_{i \in \mathcal{N}} \theta_i \frac{\partial x_i^*}{\partial \theta_j} + \sum_{i \in \mathcal{N}} x_i^* \right) = 0, \tag{7.1}$$

$$\Rightarrow \gamma(\mathbf{s} - 2t\mathbf{x}^*)^\top \frac{\partial \mathbf{x}^*}{\partial \boldsymbol{\theta}} - p^{\mathrm{u}} \left(\boldsymbol{\theta}^\top \frac{\partial \mathbf{x}^*}{\partial \boldsymbol{\theta}} + \mathbf{x}^* \right) = 0. \tag{7.2}$$

According to (3.7), we can easily obtain

$$\frac{\partial \mathbf{x}^*}{\partial \boldsymbol{\theta}} = -\mathbf{K} p^{\mathrm{u}}, \tag{7.3}$$

and we substitute (7.3) into (7.2). Then, we have

$$\left[-2t\gamma p^{\mathrm{u}} \mathbf{K}\mathbf{K}^\top - p^{\mathrm{u}} \left(\mathbf{K} + \mathbf{K}^\top\right)\right] \left[p^{\mathrm{u}}(\mathbf{1} - \boldsymbol{\theta}) - \mathbf{a}\right] = \gamma p^{\mathrm{u}} \mathbf{K}^\top \mathbf{s} - p^{\mathrm{u}} \mathbf{K}^\top \left(p^{\mathrm{u}} \mathbf{1} - \mathbf{a}\right). \tag{7.4}$$

Since we know that $\mathbf{K} = (\mathbf{G} - 2\boldsymbol{\Lambda})^{-1}$ is symmetric, with simple transformations, we obtain

$$\boldsymbol{\theta}^* = \left[-2t\gamma p^{\mathrm{u}} \mathbf{K}^2 + 2p^{\mathrm{u}} \mathbf{K}\right]^{-1} \left\{ \gamma \mathbf{K}^\top (\mathbf{K} p^{\mathrm{u}})^\top \left[\mathbf{s} - 2t\mathbf{K}\left(p^{\mathrm{u}} \mathbf{1} - \mathbf{a}\right)\right] + \left[\mathbf{K}\left(p^{\mathrm{u}} \mathbf{1} - \mathbf{a}\right)\right] \right\}. \tag{7.5}$$

Lastly through simplifying the right hand side of the equation in (7.5), the expression in (3.8) can be obtained. The proof is now completed. □

7.1.2 Proof of Theorem 3.4

Proof. First, using the first-order condition, we derive the best response function of the CP, given the strategy of the MNO. The steps of obtaining the best response function of the CP are as follows:

$$\frac{\partial \mathscr{P}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = -\gamma s p^{\mathrm{u}} \mathbf{K}\mathbf{1} - 2t\gamma p^{\mathrm{u}} \mathbf{K}^2[\mathbf{a} - p^{\mathrm{u}}(\mathbf{1} - \boldsymbol{\theta})] + p^{\mathrm{u}}\mathbf{K}\mathbf{a} - (p^{\mathrm{u}})^2\mathbf{K}(\mathbf{1} - \boldsymbol{\theta}) + (p^{\mathrm{u}})^2\mathbf{K}\boldsymbol{\theta} = 0, \quad (7.6)$$

$$\Rightarrow -\gamma s p^{\mathrm{u}} \mathbf{K}\mathbf{1} - 2t\gamma p^{\mathrm{u}} \mathbf{K}^2(\mathbf{a} - p^{\mathrm{u}}\mathbf{1}) - 2t\gamma (p^{\mathrm{u}})^2 \mathbf{K}^2 \boldsymbol{\theta} + p^{\mathrm{u}}\mathbf{K}\mathbf{a} - (p^{\mathrm{u}})^2\mathbf{K}\mathbf{1} + 2(p^{\mathrm{u}})^2 K\boldsymbol{\theta} = 0, \quad (7.7)$$

$$\Rightarrow (-2t\gamma p^{\mathrm{u}}\mathbf{K} + 2p^{\mathrm{u}}\mathbf{I})\boldsymbol{\theta} = \gamma s\mathbf{1} + 2t\gamma \mathbf{K}(\mathbf{a} - p^{\mathrm{u}}\mathbf{1}) - \mathbf{a} + p^{\mathrm{u}}\mathbf{1}, \quad (7.8)$$

$$\Rightarrow \boldsymbol{\theta}^*(p^{\mathrm{u}}, \mathbf{x}^*) = \frac{1}{2p^{\mathrm{u}}}(-t\gamma\mathbf{K} + \mathbf{I})^{-1}\big[\gamma s\mathbf{1} + (-2t\gamma\mathbf{K} + \mathbf{I})\,(p^{\mathrm{u}}\mathbf{1} - \mathbf{a})\,\big]. \quad (7.9)$$

Similarly, given the strategy of the CP, we obtain the best response of the MNO as shown in (7.13). From (3.7), we can easily derive that

$$\frac{\partial \mathbf{x}}{\partial \boldsymbol{\theta}} = -p^{\mathrm{u}}\mathbf{K} \quad (7.10)$$

and

$$\frac{\partial \mathbf{x}}{\partial p^{\mathrm{u}}} = \mathbf{K}(\mathbf{1} - \boldsymbol{\theta}). \quad (7.11)$$

The steps of obtaining the best response function of the MNO are described as follows:

$$\begin{aligned} \frac{\partial \Pi(p^{\mathrm{u}})}{\partial p^{\mathrm{u}}} &= -\mathbf{1}^{\top}\mathbf{K}\left[\mathbf{a} - p^{\mathrm{u}}\left(\mathbf{1} - \boldsymbol{\theta}\right)\right] + p^{\mathrm{u}}\mathbf{1}^{\top}\mathbf{K}\left(\mathbf{1} - \boldsymbol{\theta}\right) \\ &= -\mathbf{1}^{\top}\mathbf{K}\mathbf{a} + 2p^{\mathrm{u}}\mathbf{1}^{\top}\mathbf{K}\left(\mathbf{1} - \boldsymbol{\theta}\right) = 0, \end{aligned} \quad (7.12)$$

$$\Rightarrow p^{\mathrm{u}*}(\boldsymbol{\theta}, \mathbf{x}^*) = \big[2\mathbf{1}^{\top}\mathbf{K}(\mathbf{1} - \boldsymbol{\theta})\big]^{-1}\mathbf{1}^{\top}\mathbf{K}\mathbf{a}. \quad (7.13)$$

Existence: The CP's strategy space is defined to be within $[0, 1]$, which is a nonempty, convex, and compact subset of the Euclidean space. Then, we take the second partial derivative of the CP's objective function, $\mathscr{P}(\boldsymbol{\theta})$, with respect to its own decision variable $\boldsymbol{\theta}$. Recall that $\mathbf{K}$ is a negative definite matrix, we have:

$$\frac{\partial^2 \mathscr{P}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^2} = -2t\gamma(p^{\mathrm{u}})^2\mathbf{K}^2 + 2(p^{\mathrm{u}})^2\mathbf{K} < 0. \tag{7.14}$$

Therefore, the objective function of the CP, $\mathscr{P}(\boldsymbol{\theta})$, is continuous and strictly concave with respect to $\boldsymbol{\theta}$. Similarly, the objective function of the MNO, $\Pi(p^{\mathrm{u}})$, is strictly concave with respect to its decision variable p^{u}, since we conclude that the second-order derivative of Π with respect to p^{u} is less than 0, i.e.,

$$\frac{\partial^2 \Pi(p^{\mathrm{u}})}{\partial (p^{\mathrm{u}})^2} = 2\mathbf{1}^\top \mathbf{K}(\mathbf{1} - \boldsymbol{\theta}) < 0. \tag{7.15}$$

Since we know p^{u} is smaller than a threshold, and p^{u} is larger than 0. Accordingly, the price p^{u} is a nonempty convex and compact subset of the Euclidean space. Thus, the Nash equilibrium exists in this non-cooperative sub-game between the CP and the MNO [5].

Uniqueness: From the Jacobian matrix of point-to-set mapping with respect to the utility profile of the CP and the MNO, we have $\nabla \mathbf{F} = \begin{bmatrix} \frac{\partial^2 \mathscr{P}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^2} & \frac{\partial^2 \mathscr{P}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial p^{\mathrm{u}}} \\ \left(\frac{\partial^2 \Pi(p^{\mathrm{u}})}{\partial \boldsymbol{\theta} \partial p^{\mathrm{u}}}\right)^\top & \frac{\partial^2 \Pi(p^{\mathrm{u}})}{\partial (p^{\mathrm{u}})^2} \end{bmatrix}$. With simple calculations, $\nabla \mathbf{F} + \nabla \mathbf{F}^\top$ is shown as follows:

$$\begin{bmatrix} -4t\gamma(p^{\mathrm{u}})^2\mathbf{K}^2 + 4(p^{\mathrm{u}})^2\mathbf{K} & -\gamma s\mathbf{K}\mathbf{1} - 2\gamma t\mathbf{K}^2\left[\mathbf{a} - 2p^{\mathrm{u}}\left(\mathbf{1} - \boldsymbol{\theta}\right)\right] + \mathbf{K}\left[\mathbf{a} - 4p^{\mathrm{u}}(\mathbf{1} - \boldsymbol{\theta})\right] \\ \left(-\gamma s\mathbf{K}\mathbf{1} - 2\gamma t\mathbf{K}^2\left[\mathbf{a} - 2p^{\mathrm{u}}(\mathbf{1} - \boldsymbol{\theta})\right] + \mathbf{K}\left[\mathbf{a} - 4p^{\mathrm{u}}(\mathbf{1} - \boldsymbol{\theta})\right]\right)^\top & 4\mathbf{1}^\top\mathbf{K}(\mathbf{1} - \boldsymbol{\theta}) \end{bmatrix}. \tag{7.16}$$

We decompose the matrix in (7.16) into the form of $-\mathrm{A} - \mathrm{B} - \mathrm{C}$, the expression of which is shown as follows:

$$\begin{aligned} & - \underbrace{\begin{bmatrix} 4t\gamma(p^{\mathrm{u}})^2\mathbf{K}^2 & 2\gamma t\mathbf{K}^2[\mathbf{a} - 2p^{\mathrm{u}}(\mathbf{1} - \boldsymbol{\theta})] \\ 2\gamma t[\mathbf{a} - 2p^{\mathrm{u}}(\mathbf{1} - \boldsymbol{\theta})]^\top\mathbf{K}^2 & -2\mathbf{1}^\top\mathbf{K}(\mathbf{1} - \boldsymbol{\theta}) \end{bmatrix}}_{\mathrm{A}} \\ & - \underbrace{\begin{bmatrix} -3(p^{\mathrm{u}})^2\mathbf{K} & \gamma s\mathbf{K}\mathbf{1} \\ \gamma s\mathbf{1}^\top\mathbf{K} & -\mathbf{1}^\top\mathbf{K}(\mathbf{1} - \boldsymbol{\theta}) \end{bmatrix}}_{\mathrm{B}} \\ & - \underbrace{\begin{bmatrix} -(p^{\mathrm{u}})^2\mathbf{K} & -\mathbf{K}[\mathbf{a} - 4p^{\mathrm{u}}(\mathbf{1} - \boldsymbol{\theta})] \\ -[\mathbf{a} - 4p^{\mathrm{u}}(\mathbf{1} - \boldsymbol{\theta})]^\top\mathbf{K} & -\mathbf{1}^\top\mathbf{K}(\mathbf{1} - \boldsymbol{\theta}) \end{bmatrix}}_{\mathrm{C}}. \end{aligned} \tag{7.17}$$

If we can conclude that $\nabla\mathbf{F}+\nabla\mathbf{F}^\top$ is negative definite, then $\nabla\mathbf{F}$ is diagonally strictly concave, and accordingly the Nash equilibrium of this non-cooperative sub-game is unique [74]. To prove $-\mathrm{A}-\mathrm{B}-\mathrm{C}$ is negative definite, we can prove that matrices A, B and C are all positive definite. Based on the matrix congruence theorem, if $\mathrm{Q}' = \mathrm{P}^\top\mathrm{QP}$ and P is invertible, then Q and Q′ have the same numbers of positive, negative, and zero eigenvalues [118]. We use the matrix $\mathrm{P}_1 = \begin{bmatrix} \mathbf{I} & \left(-4\gamma t(p^{\mathrm{u}})^2\mathbf{K}^2\right)^{-1} 2\gamma t\mathbf{K}^2\left[\mathbf{a}-2p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)\right] \\ 0 & 1 \end{bmatrix}$ and we obtain the congruence of matrix A, A′ as follows:

$$\mathrm{A}' = \mathrm{P}_1^\top\mathrm{AP}_1 = \begin{bmatrix} 4t\gamma(p^{\mathrm{u}})^2\mathbf{K}^2 & \mathbf{0} \\ \mathbf{0} & -2\mathbf{1}^\top\mathbf{K}(\mathbf{1}-\boldsymbol{\theta}) - \gamma t\mathbf{K}^2[\mathbf{a}-2p^{\mathrm{u}}(\mathbf{1}-\boldsymbol{\theta})]^\top\frac{\mathbf{K}^2}{(p^{\mathrm{u}})^2}[\mathbf{a}-2p^{\mathrm{u}}(\mathbf{1}-\boldsymbol{\theta})] \end{bmatrix}. \tag{7.18}$$

Now we only need to prove that A′ is positive definite, which implies

$$-2\mathbf{1}^\top\mathbf{K}(\mathbf{1}-\boldsymbol{\theta}) - \gamma t\mathbf{K}^2[\mathbf{a}-2p^{\mathrm{u}}(\mathbf{1}-\boldsymbol{\theta})]^\top\frac{\mathbf{K}^2}{(p^{\mathrm{u}})^2}[\mathbf{a}-2p^{\mathrm{u}}(\mathbf{1}-\boldsymbol{\theta})] > 0. \tag{7.19}$$

With simple transformations, we have

$$\begin{aligned} &-2\boldsymbol{\theta}^\top\mathbf{K}(\mathbf{1}-\boldsymbol{\theta}) - 2(\mathbf{1}-\boldsymbol{\theta})^\top\mathbf{K}(\mathbf{1}-\boldsymbol{\theta}) \\ &\qquad -[\frac{\sqrt{t\gamma}}{p^{\mathrm{u}}}\mathbf{a} - 2\sqrt{t\gamma}(\mathbf{1}-\boldsymbol{\theta})]^\top\mathbf{K}^2[\frac{\sqrt{t\gamma}}{p^{\mathrm{u}}}\mathbf{a} - 2\sqrt{t\gamma}(\mathbf{1}-\boldsymbol{\theta})] > 0. \end{aligned} \tag{7.20}$$

Accordingly, it can be concluded that

$$\mathbf{1}-\boldsymbol{\theta} > \sqrt{-\mathbf{K}}[\frac{\sqrt{t\gamma}}{2p^{\mathrm{u}}}\mathbf{a} - 2\sqrt{t\gamma}(\mathbf{1}-\boldsymbol{\theta})]. \tag{7.21}$$

At last, we derive

$$p^{\mathrm{u}}(\mathbf{1}-\boldsymbol{\theta}) > \left[\left(\sqrt{-2\gamma t\mathbf{K}}+\mathbf{I}\right)^{-1}\sqrt{\frac{-\gamma t\mathbf{K}}{2}}\mathbf{a}\right], \tag{7.22}$$

which is ensured if the condition $p^{\mathrm{u}}(1-\theta_i) > \max\left\{\gamma s, \frac{a_i}{3}, \left[(\sqrt{-2\gamma t\mathbf{K}}+\mathbf{I})^{-1}\sqrt{\frac{-\gamma t\mathbf{K}}{2}}\mathbf{a}\right]_i\right\}$ holds. With the same steps, we use the matrix $\mathrm{P}_2 = \begin{bmatrix} \mathbf{I} & \left(3(p^{\mathrm{u}})^2\mathbf{K}\right)^{-1}\gamma s\mathbf{K}\mathbf{1} \\ 0 & 1 \end{bmatrix}$ to obtain B′, which is shown as follows:

$$\mathrm{B}' = \mathrm{P}_2^\top\mathrm{BP}_2 = \begin{bmatrix} -3(p^{\mathrm{u}})^2\mathbf{K} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1}^\top\mathbf{K}(\mathbf{1}-\boldsymbol{\theta}) + \gamma^2 s^2\mathbf{1}^\top\frac{\mathbf{K}}{3(p^{\mathrm{u}})^2}\mathbf{1} \end{bmatrix}. \tag{7.23}$$

Then, we need to prove that $-\mathbf{1}^\top\mathbf{K}(\mathbf{1}-\boldsymbol{\theta})+\gamma^2 s^2\mathbf{1}^\top\frac{\mathbf{K}}{3(p^{\mathrm{u}})^2}\mathbf{1}>0$, which implies

$$-\boldsymbol{\theta}^\top\mathbf{K}(\mathbf{1}-\boldsymbol{\theta})-(\mathbf{1}-\boldsymbol{\theta})^\top\mathbf{K}(\mathbf{1}-\boldsymbol{\theta})-\left(\frac{\gamma s}{\sqrt{3}p^{\mathrm{u}}}\mathbf{1}\right)^\top\mathbf{K}^2\frac{\gamma s}{\sqrt{3}p^{\mathrm{u}}}\mathbf{1}>0. \tag{7.24}$$

Accordingly, we have

$$p^{\mathrm{u}}(\mathbf{1}-\boldsymbol{\theta})>\frac{\gamma s}{\sqrt{3}}\mathbf{1}, \tag{7.25}$$

which is ensured if the condition $p^{\mathrm{u}}(1-\theta_i)>\max\left\{\gamma s,\frac{a_i}{3},\left[(\sqrt{-2\gamma t\mathbf{K}}+\mathbf{I})^{-1}\sqrt{\frac{-\gamma t\mathbf{K}}{2}}\mathbf{a}\right]_i\right\}$ holds.

Likewise, using the matrix $\mathrm{P}_3=\begin{bmatrix}\mathbf{I} & \left((p^{\mathrm{u}})^2\mathbf{K}\right)^{-1}[\mathbf{a}-4p^{\mathrm{u}}(\mathbf{1}-\boldsymbol{\theta})]\\ 0 & 1\end{bmatrix}$, we have C' which is shown as follows:

$$\mathrm{C}'=\mathrm{P}_3^\top\mathrm{C}\mathrm{P}_3=\begin{bmatrix}-(p^{\mathrm{u}})^2\mathbf{K} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}^\top\mathbf{K}(\mathbf{1}-\boldsymbol{\theta})+[\mathbf{a}-4p^{\mathrm{u}}(\mathbf{1}-\boldsymbol{\theta})]^\top\frac{\mathbf{K}}{(p^{\mathrm{u}})^2}[\mathbf{a}-4p^{\mathrm{u}}(\mathbf{1}-\boldsymbol{\theta})]\end{bmatrix}. \tag{7.26}$$

To prove C' is positive definite, we need to prove

$$-\mathbf{1}^\top\mathbf{K}(\mathbf{1}-\boldsymbol{\theta})+[\mathbf{a}-4p^{\mathrm{u}}(\mathbf{1}-\boldsymbol{\theta})]^\top\frac{\mathbf{K}}{(p^{\mathrm{u}})^2}[\mathbf{a}-4p^{\mathrm{u}}(\mathbf{1}-\boldsymbol{\theta})]>0, \tag{7.27}$$

which implies that

$$\boldsymbol{\theta}^\top\mathbf{K}(\mathbf{1}-\boldsymbol{\theta})+(\mathbf{1}-\boldsymbol{\theta})^\top\mathbf{K}(\mathbf{1}-\boldsymbol{\theta})-\left[\frac{\mathbf{a}}{p^{\mathrm{u}}}-4(\mathbf{1}-\boldsymbol{\theta}\right]^\top\mathbf{K}\left[\frac{\mathbf{a}}{p^{\mathrm{u}}}-4(\mathbf{1}-\boldsymbol{\theta})\right]>0. \tag{7.28}$$

In the final step, it can be concluded that

$$\mathbf{1}-\boldsymbol{\theta}>\frac{\mathbf{a}}{p^{\mathrm{u}}}-4(\mathbf{1}-\boldsymbol{\theta}). \tag{7.29}$$

Accordingly, we have $p^{\mathrm{u}}(\mathbf{1}-\boldsymbol{\theta})>\frac{\mathbf{a}}{5}$, which is ensured when the condition $p^{\mathrm{u}}(1-\theta_i)>\max\left\{\gamma s,\frac{a_i}{3},\left[(\sqrt{-2\gamma t\mathbf{K}}+\mathbf{I})^{-1}\sqrt{\frac{-\gamma t\mathbf{K}}{2}}\mathbf{a}\right]_i\right\}$ holds. Accordingly, we have proved that the positive definiteness of A, B and C are guaranteed. Thus, the proof is now completed. □

7.1.3 Proof of Theorem 3.5

Proof. The Hessian matrix of $\mathscr{R}(\boldsymbol{\theta}, p^{\mathrm{u}})$ can be obtained from $\begin{bmatrix} \frac{\partial^2 \mathscr{R}}{\partial \boldsymbol{\theta}^2} & \frac{\partial^2 \mathscr{R}}{\partial \boldsymbol{\theta} \partial p^{\mathrm{u}}} \\ \left(\frac{\partial^2 \mathscr{R}}{\partial \boldsymbol{\theta} \partial p^{\mathrm{u}}}\right)^\top & \frac{\partial^2 \mathscr{R}}{\partial (p^{\mathrm{u}})^2} \end{bmatrix}$, which is shown as follows:

$$\begin{bmatrix} -2t\gamma(p^{\mathrm{u}})^2\mathbf{K}^2 - 2(p^{\mathrm{u}})^2\mathbf{K}^2 & -\gamma s\mathbf{K}\mathbf{1} - 2\gamma t\mathbf{K}^2\left[\mathbf{a} - 2p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)\right] + \mathbf{K}\left[\mathbf{a} - 4p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)\right] \\ \left(-\gamma s\mathbf{K}\mathbf{1} - 2\gamma t\mathbf{K}^2\left[\mathbf{a} - 2p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)\right] + \mathbf{K}\left[\mathbf{a} - 4p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)\right]\right)^\top & -2t\gamma(\mathbf{1}-\boldsymbol{\theta})^\top\mathbf{K}^2\left(\mathbf{1}-\boldsymbol{\theta}\right) + 2(\mathbf{1}-\boldsymbol{\theta})^\top\mathbf{K}\left(\mathbf{1}-\boldsymbol{\theta}\right) \end{bmatrix}. \tag{7.30}$$

Similar to that in the proof of Theorem 3.4, we decompose the Hessian matrix into three matrices, $-\mathrm{D}-\mathrm{E}-\mathrm{F}$, the expression of which is shown as follows:

$$\begin{aligned} &-\underbrace{\begin{bmatrix} 2t\gamma(p^{\mathrm{u}})^2\mathbf{K}^2 & 2\gamma t\mathbf{K}^2\left[\mathbf{a}-2p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)\right] \\ 2\gamma t[\mathbf{a}-2p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)]^\top\mathbf{K}^2 & 2\gamma t(\mathbf{1}-\boldsymbol{\theta})^\top\mathbf{K}^2\left(\mathbf{1}-\boldsymbol{\theta}\right) \end{bmatrix}}_{\mathrm{D}} \\ &-\underbrace{\begin{bmatrix} -(p^{\mathrm{u}})^2\mathbf{K} & -\mathbf{K}\left[\mathbf{a}-4p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)\right] \\ -\left[\mathbf{a}-4p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)\right]^\top\mathbf{K} & -(\mathbf{1}-\boldsymbol{\theta})^\top\mathbf{K}\left(\mathbf{1}-\boldsymbol{\theta}\right) \end{bmatrix}}_{\mathrm{E}} \\ &-\underbrace{\begin{bmatrix} -(p^{\mathrm{u}})^2\mathbf{K} & \gamma s\mathbf{K}\mathbf{1} \\ \gamma s\mathbf{1}^\top\mathbf{K} & -(\mathbf{1}-\boldsymbol{\theta})^\top\mathbf{K}\left(\mathbf{1}-\boldsymbol{\theta}\right) \end{bmatrix}}_{\mathrm{F}}. \end{aligned} \tag{7.31}$$

To prove that (7.30) is negative definite, one sufficient condition is that we can prove D, E and F are all positive definite. Here, we further employ the matrix congruence theorem [118] so as to prove the positivity of its congruence matrix.

Specifically, we use $P_4 = \begin{bmatrix} \mathbf{I} & -\left(2t\gamma(p^{\mathrm{u}})^2\mathbf{K}^2\right)^{-1}2t\gamma\mathbf{K}^2\left[\mathbf{a}-2p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)\right] \\ 0 & 1 \end{bmatrix}$ to obtain D′, which is shown as follows:

$$\mathrm{D}' = \mathrm{P_4}^\top\mathrm{D}\mathrm{P_4} = \begin{bmatrix} 2t\gamma(p^u)^2\mathbf{K}^2 & \mathbf{0} \\ \mathbf{0} & 2t\gamma(\mathbf{1}-\boldsymbol{\theta})^\top\mathbf{K}^2(\mathbf{1}-\boldsymbol{\theta}) - 2\gamma t[\mathbf{a}-2p^u\left(\mathbf{1}-\boldsymbol{\theta}\right)]^\top\frac{\mathbf{K}^2}{(p^u)^2}\left[\mathbf{a}-2p^u\left(\mathbf{1}-\boldsymbol{\theta}\right)\right] \end{bmatrix}. \tag{7.32}$$

To prove that D′ is positive definite, we need to prove

$$-2\gamma t\left[\frac{\mathbf{a}}{p^{\mathrm{u}}} - 2\left(\mathbf{1}-\boldsymbol{\theta}\right)\right]^\top\mathbf{K}^2\left[\frac{\mathbf{a}}{p^{\mathrm{u}}} - 2\left(\mathbf{1}-\boldsymbol{\theta}\right)\right] + 2t\gamma(\mathbf{1}-\boldsymbol{\theta})^\top\mathbf{K}^2(\mathbf{1}-\boldsymbol{\theta}) > 0. \tag{7.33}$$

With simple transformations, we have

$$\left(\mathbf{1}-\boldsymbol{\theta}\right) > \frac{\mathbf{a}}{p^{\mathrm{u}}} - 2\left(\mathbf{1}-\boldsymbol{\theta}\right), \tag{7.34}$$

which implies $p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right) > \frac{\mathbf{a}}{3}$. This is guaranteed when the condition $p^{\mathrm{u}}(1-\theta_i) > \max\left\{\gamma s, \frac{a_i}{3}, \left[(\sqrt{-2\gamma t\mathbf{K}}+\mathbf{I})^{-1}\sqrt{\frac{-\gamma t\mathbf{K}}{2}}\mathbf{a}\right]_i\right\}$ holds.

With the same steps, we use $P_5 = \begin{bmatrix} \mathbf{I} & -\left((p^{\mathrm{u}})^2\mathbf{K}\right)^{-1}\mathbf{K}\left[\mathbf{a}-4p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)\right] \\ 0 & 1 \end{bmatrix}$ and $P_6 = \begin{bmatrix} \mathbf{I} & \left((p^{\mathrm{u}})^2\mathbf{K}\right)^{-1}\gamma s\mathbf{K}\mathbf{1} \\ 0 & 1 \end{bmatrix}$ to obtain E′ and F′, which are shown in (7.35) and (7.36), respectively.

$$\mathrm{E}' = {\mathrm{P}_5}^{\top}\mathrm{E}\mathrm{P}_5 = \begin{bmatrix} -(p^u)^2\mathbf{K} & \mathbf{0} \\ \mathbf{0} & -(\mathbf{1}-\boldsymbol{\theta})^{\top}\mathbf{K}\left(\mathbf{1}-\boldsymbol{\theta}\right)+[\mathbf{a}-4p^u\left(\mathbf{1}-\boldsymbol{\theta}\right)]^{\top}\frac{\mathbf{K}}{(p^u)^2}\left[\mathbf{a}-4p^u\left(\mathbf{1}-\boldsymbol{\theta}\right)\right] \end{bmatrix}, \tag{7.35}$$

$$\mathrm{F}' = {\mathrm{P}_6}^{\top}\mathrm{F}\mathrm{P}_6 = \begin{bmatrix} -(p^u)^2\mathbf{K} & \mathbf{0} \\ \mathbf{0} & -(\mathbf{1}-\boldsymbol{\theta})^{\top}\mathbf{K}\left(\mathbf{1}-\boldsymbol{\theta}\right)+\gamma^2 s^2\mathbf{1}^{\top}\frac{\mathbf{K}}{(p^u)^2}\mathbf{1} \end{bmatrix}. \tag{7.36}$$

Likewise, in order to prove that E′ and F′ are positive definite, we have to ensure that

$$-\left(\mathbf{1}-\boldsymbol{\theta}\right)^{\top}\mathbf{K}\left(\mathbf{1}-\boldsymbol{\theta}\right)+\gamma^2 s^2\mathbf{1}^{\top}\frac{\mathbf{K}}{\left(p^{\mathrm{u}}\right)^2}\mathbf{1} > 0 \tag{7.37}$$

and

$$\left[\mathbf{a}-4p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)\right]^{\top}\frac{\mathbf{K}}{\left(p^{\mathrm{u}}\right)^2}\left[\mathbf{a}-4p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right)\right]-\left(\mathbf{1}-\boldsymbol{\theta}\right)^{\top}\mathbf{K}\left(\mathbf{1}-\boldsymbol{\theta}\right) > \mathbf{0}, \tag{7.38}$$

which correspond to $p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right) > \gamma s\mathbf{1}$ and $p^{\mathrm{u}}\left(\mathbf{1}-\boldsymbol{\theta}\right) > \frac{\mathbf{a}}{5}$, respectively. Therefore, it can be concluded that the condition where the Hessian matrix of $\mathscr{R}$ is negative definite holds. The proof is now completed. □

7.2 Appendix of Chapter 4

7.2.1 Proof of Lemma 4.1

Proof. From the IC constraint $u_i(x_i,\theta_i) \geq u_i(x_N,\theta_N)$, it holds that

$$u_i(x_i,\theta_i) \geq \tau\sigma_i f(x_N)-(1-\theta_N)x_N p. \tag{7.39}$$

Using $\sigma_i \geq \sigma_N$ in (7.39), we obtain

$$u_i(x_i,\theta_i) \geq \tau\sigma_i f(x_N)-(1-\theta_N)x_N p \geq \tau\sigma_N f(x_N)-(1-\theta_N)x_N p = u_N(x_N,\theta_N). \tag{7.40}$$

The proof is then completed. □

7.2.2 Proof of Lemma 4.2

Proof. From the IC constraint $u_i(x_i, \theta_i) \geq u_i(x_j, \theta_j)$, it holds that

$$\tau\sigma_i f(x_i) - (1-\theta_i)x_i p \geq \tau\sigma_i f(x_j) - (1-\theta_j)x_j p. \tag{7.41}$$

From the IC constraint $u_j(x_j, \theta_j) \geq u_j(x_i, \theta_i)$, it holds that

$$\tau\sigma_j f(x_j) - (1-\theta_j)x_j p \geq \tau\sigma_j f(x_i) - (1-\theta_i)x_i p. \tag{7.42}$$

Adding (7.41) and (7.42), we obtain

$$[\tau\sigma_i f(x_i) - (1-\theta_i)x_i p] + [\tau\sigma_j f(x_j) - (1-\theta_j)x_j p] \geq \\ [\tau\sigma_i f(x_j) - (1-\theta_j)x_j p] + [\tau\sigma_j f(x_i) - (1-\theta_i)x_i p], \tag{7.43}$$

which implies

$$\tau(\sigma_i - \sigma_j)[f(x_i) - f(x_j)] \geq 0. \tag{7.44}$$

Since $f(x)$ is an increasing function of x, (7.44) induces that if $\sigma_i > \sigma_j$, then $x_i > x_j$. The proof is then completed. □

7.2.3 Proof of Lemma 4.3

Proof. From (7.42), it follows that

$$\tau\sigma_j[f(x_j) - f(x_i)] \geq (1-\theta_j)x_j p - (1-\theta_i)x_i p. \tag{7.45}$$

If $\sigma_i > \sigma_j$, we use Lemma 4.2 to obtain $x_i > x_j$, which implies $f(x_i) > f(x_j)$ since $f(x)$ is an increasing function of x. Applying $f(x_i) > f(x_j)$ to (7.45), we have $(1-\theta_i)x_i p > (1-\theta_j)x_j p$. The proof is then completed. □

7.2.4 Proof of Lemma 4.4

Proof. We establish

$$\begin{aligned} & u_i(x_i, \theta_i) \geq u_i(x_{i+1}, \theta_{i+1}),\ \forall i = \{1, \ldots, N-1\}, \\ & \implies u_i(x_i, \theta_i) \geq u_i(x_j, \theta_j),\ \forall i, j = \{1, \ldots, N-1\} \text{ and } j > i, \end{aligned} \tag{7.46}$$

and

$$\begin{aligned} & u_i(x_i, \theta_i) \geq u_i(x_{i-1}, \theta_{i-1})\ \forall i = \{2, \ldots, N\}, \\ & \implies u_i(x_i, \theta_i) \geq u_i(x_j, \theta_j),\ \forall i, j = \{2, \ldots, N\}, \text{ and } j < i. \end{aligned} \tag{7.47}$$

We begin with proving (7.46).

Given $\sigma_i > \sigma_{i+1}$ and $x_{i+1} > x_{i+2}$ from Lemma 4.2, as $f(x)$ is an increasing function of x, we have

$$\tau(\sigma_i - \sigma_{i+1})[f(x_{i+1}) - f(x_{i+2})] \geq 0,$$

or specifically,

$$\tau\sigma_i f(x_{i+1}) - \tau\sigma_{i+1} f(x_{i+1}) \geq \tau\sigma_i f(x_{i+2}) - \tau\sigma_{i+1} f(x_{i+2}). \tag{7.48}$$

From the Local Upward Incentive Constraint $u_{i+1}(x_{i+1}, \theta_{i+1}) \geq u_{i+1}(x_{i+2}, \theta_{i+2})$, it holds that

$$\tau\sigma_{i+1} f(x_{i+1}) - (1 - \theta_{i+1})x_{i+1}p \geq \tau\sigma_{i+1} f(x_{i+2}) - (1 - \theta_{i+2})x_{i+2}p. \tag{7.49}$$

The combination of (7.48) and (7.49) gives

$$\tau\sigma_i f(x_{i+1}) - (1 - \theta_{i+1})x_{i+1}p \geq \tau\sigma_i f(x_{i+2}) - (1 - \theta_{i+2})x_{i+2}p,$$

i.e.,

$$u_i(x_{i+1}, \theta_{i+1}) \geq u_i(x_{i+2}, \theta_{i+2}). \tag{7.50}$$

Combining (7.50) and the Local Upward Incentive Constraint $u_i(x_i, \theta_i) \geq u_i(x_{i+1}, \theta_{i+1})$, we obtain $u_i(x_i, \theta_i) \geq u_i(x_{i+2}, \theta_{i+2})$. Thus, we can further prove (7.46).

We can prove (7.47) in a way similar to that of (7.46). The proof is then completed. □

7.2.5 Proof of Lemma 4.5

Proof. Based on Lemma 4.4, it suffices to prove the following:

$$\text{“}\{u_i(x_i, \theta_i) = u_i(x_{i+1}, \theta_{i+1}) : i \in \{1, \ldots, N-1\}\} \\ \Longrightarrow \{\text{LDIC}(\sigma_i, \sigma_{i-1}) : i \in \{2, \ldots, N\}\},\text{”} \tag{7.51}$$

and

“The optimal solution under the constraint LUIC(σ_i, σ_{i+1})
(i.e., $u_i(x_i, \theta_i) \geq u_i(x_{i+1}, \theta_{i+1})$)
is binding; in other words, it satisfies:
$u_i(x_i, \theta_i) = u_i(x_{i+1}, \theta_{i+1})$.” (7.52)

We begin with proving (7.51). First, we write $\{u_i(x_i, \theta_i) = u_i(x_{i+1}, \theta_{i+1}) : i \in \{1, \ldots, N-1\}\}$ as

$$\tau\sigma_i f(x_i) - (1-\theta_i)x_i p = \tau\sigma_i f(x_{i+1}) - (1-\theta_{i+1})x_{i+1}p. \tag{7.53}$$

Second, we write $\{\text{LDIC}(\sigma_{i+1}, \sigma_i) : i \in \{1, \ldots, N-1\}\}$ as $\{\text{LDIC}(\sigma_{i+1}, \sigma_i) : i \in \{1, \ldots, N-1\}\}$, or specifically

$$\{\tau\sigma_{i+1} f(x_{i+1}) - (1-\theta_{i+1})x_{i+1}p \geq \tau\sigma_{i+1} f(x_i) - (1-\theta_i)x_i p : 1 \leq i \leq N-1\}. \tag{7.54}$$

From (7.53), $\sigma_i > \sigma_{i+1}$, and $f(x_i) > f(x_{i+1})$, we obtain

$$(1-\theta_i)x_i p - (1-\theta_{i+1})x_{i+1}p = \tau\sigma_i[f(x_i) - f(x_{i+1})] \\ \geq \tau\sigma_{i+1}[f(x_i) - f(x_{i+1})]. \tag{7.55}$$

Rearranging the terms in (7.55), we obtain the desired $\tau\sigma_{i+1} f(x_{i+1}) - (1-\theta_{i+1})x_{i+1}p \geq \tau\sigma_{i+1} f(x_i) - (1-\theta_i)x_i p$ in (7.54). Hence, (7.51) is proved.

We now prove (7.52). The constraint LUIC(σ_i, σ_{i+1}) (i.e., $u_i(x_i, \theta_i) \geq u_i(x_{i+1}, \theta_{i+1})$) means

$$\tau\sigma_i f(x_i) - (1-\theta_i)x_i p \geq \tau\sigma_i f(x_{i+1}) - (1-\theta_{i+1})x_{i+1}p, \tag{7.56}$$

which can be written as

$$\theta_i \geq 1 - \frac{\tau\sigma_i f(x_{i+1}) - (1-\theta_{i+1})x_{i+1}p - \tau\sigma_i f(x_i)}{x_i p}. \tag{7.57}$$

From Lemma 4.1, in the optimal solution, the CP sets $\theta_N = 1 - \frac{\tau\sigma_N f(x_N)}{px_N}$. Given θ_N, the CP lowers the sponsorship factor θ_{N-1} to take the equal sign of (7.57) for $i = N-1$. Given θ_{N-1}, the CP lowers the sponsorship factor θ_{N-2} to take the equal sign of (7.57) for $i = N-2$. This process continues iteratively. Hence, in the optimal solution, the CP lowers the sponsorship factor θ_i to take the equal sign of (7.57) for $i \in \{1, 2, \ldots, N-1\}$. Hence, we also have the equal sign of (7.56) for $i \in \{1, 2, \ldots, N-1\}$. This establishes (7.52). In addition, we know that the constraint $\theta_i < 1$ will automatically hold from (7.57).

Finally, we use (7.51), (7.52) and Lemma 4.4 to obtain Lemma 4.5. The proof is then completed. □

7.2.6 Optimal Pricing under Discriminatory Sponsorship Scheme

As we have

$$U_{MNO} = \sum_{i=1}^{N} [\lambda_i \, (p-c) \, x_i], \tag{7.58}$$

we can derive

$$\frac{\partial U_{MNO}}{\partial p} = \sum_{i=1}^{N} \lambda_i \left[x_i + (p-c)\frac{\partial x_i}{\partial p} \right], \tag{7.59}$$

and

$$\frac{\partial^2 U_{MNO}}{\partial p^2} = \sum_{i=1}^{N} \lambda_i \left[(p-c)\frac{\partial^2 x_i}{\partial p^2} + 2\frac{\partial x_i}{\partial p} \right]. \tag{7.60}$$

Taking the partial derivative on both sides of (4.39) with respect to p, we establish

$$\frac{\partial x_i}{\partial p} = -\frac{1}{\sigma_c \gamma {x_i}^{-\gamma-1} + \alpha\tau\sigma_i {x_i}^{-\alpha-1}}, \tag{7.61}$$

which further implies

$$\frac{\partial^2 x_i}{\partial p^2} = -\frac{1}{(\sigma_c \gamma {x_i}^{-\gamma-1} + \alpha\tau\sigma_i {x_i}^{-\alpha-1})^2} \times \left[\sigma_c\gamma(\gamma+1){x_i}^{-\gamma-2} + \alpha(\alpha+1)\tau\sigma_i {x_i}^{-\alpha-2}\right]\frac{\partial x_i}{\partial p}. \tag{7.62}$$

Substituting (7.61) and (7.62) into (7.59) and (7.60), we derive

$$\frac{\partial U_{MNO}}{\partial p} = \sum_{i=1}^{N}\left[x_i - \frac{p-c}{\sigma_c \gamma {x_i}^{-\gamma-1} + \alpha\tau\sigma_i {x_i}^{-\alpha-1}}\right], \tag{7.63}$$

and

$$\frac{\partial^2 U_{MNO}}{\partial p^2} = \sum_{i=1}^{N}\Bigg\{\lambda_i\Bigg\{-(p-c)\frac{1}{(\sigma_c \gamma {x_i}^{-\gamma-1} + \alpha\tau\sigma_i {x_i}^{-\alpha-1})^2} \times \left[\sigma_c\gamma(\gamma+1){x_i}^{-\gamma-2} + \alpha(\alpha+1)\tau\sigma_i {x_i}^{-\alpha-2}\right] + 2\Bigg\}\left(-\frac{1}{\sigma_c \gamma {x_i}^{-\gamma-1} + \alpha\tau\sigma_i {x_i}^{-\alpha-1}}\right)\Bigg\}. \tag{7.64}$$

For tractable analysis, we consider the special case of $\alpha = \gamma$. Then it follows from (4.39) that $x_i = p^{-\frac{1}{\gamma}} {T_i}^{-\frac{1}{\gamma}}$, where $T_i = \frac{1}{\sigma_c + \tau\sigma_i}$. Applying this to (7.63) and (7.64), we obtain

$$\begin{aligned}\frac{\partial U_{MNO}}{\partial p} &= \sum_{i=1}^{N}\lambda_i\left[p^{-\frac{1}{\gamma}}{T_i}^{-\frac{1}{\gamma}} - \frac{p-c}{(\sigma_c+\tau\sigma_i)\gamma p^{1+\frac{1}{\gamma}}{T_i}^{1+\frac{1}{\gamma}}}\right] \\ &= \sum_{i=1}^{N}\lambda_i\left[p^{-\frac{1}{\gamma}}{T_i}^{-\frac{1}{\gamma}} - \frac{p-c}{\gamma p^{1+\frac{1}{\gamma}}{T_i}^{\frac{1}{\gamma}}}\right] \\ &= \frac{c}{\gamma}\left[\sum_{i=1}^{N}\left(\lambda_i {T_i}^{-\frac{1}{\gamma}}\right)\right]p^{-\frac{1}{\gamma}-1} - \frac{(1-\gamma)}{\gamma}\left[\sum_{i=1}^{N}\left(\lambda_i {T_i}^{-\frac{1}{\gamma}}\right)\right]p^{-\frac{1}{\gamma}},\end{aligned} \tag{7.65}$$

and

$$
\begin{aligned}
&\frac{\partial^2 U_{MNO}}{\partial p^2} \\
&= \sum_{i=1}^{N} \Bigg\{ \lambda_i \Big\{ -(p-c) \frac{1}{(\sigma_c \gamma x_i{}^{-\gamma-1} + \gamma \tau \sigma_i x_i{}^{-\gamma-1})^2} \\
&\qquad \times \left[\sigma_c \gamma(\gamma+1) x_i{}^{-\gamma-2} + \gamma(\gamma+1)\tau\sigma_i x_i{}^{-\gamma-2}\right] + 2 \Big\} \left(-\frac{1}{\sigma_c \gamma x_i{}^{-\gamma-1} + \gamma \tau \sigma_i x_i{}^{-\gamma-1}} \right) \Bigg\} \\
&= \sum_{i=1}^{N} \Bigg\{ \lambda_i \Big\{ (p-c) \frac{-T_i}{[\gamma p^{1+\frac{1}{\gamma}} T_i{}^{1+\frac{1}{\gamma}}]^2} \left[\gamma(\gamma+1) p^{1+\frac{2}{\gamma}} T_i{}^{1+\frac{2}{\gamma}} \right] + 2 \Big\} \left(-\frac{T_i}{\gamma p^{1+\frac{1}{\gamma}} T_i{}^{1+\frac{1}{\gamma}}} \right) \Bigg\} \\
&= \frac{(1-\gamma)}{\gamma^2} \left[\sum_{i=1}^{N} \left(\lambda_i T_i{}^{-\frac{1}{\gamma}} \right) \right] p^{-\frac{1}{\gamma}-1} - \frac{(1+\gamma)c}{\gamma^2} \left[\sum_{i=1}^{N} \left(\lambda_i T_i{}^{-\frac{1}{\gamma}} \right) \right] p^{-\frac{1}{\gamma}-2}. \qquad (7.66)
\end{aligned}
$$

For $p < \frac{c}{1-\gamma}$, we have $\frac{\partial U_{MNO}}{\partial p} > 0$. For $p > \frac{c}{1-\gamma}$, we have $\frac{\partial U_{MNO}}{\partial p} < 0$. For $p < \frac{(1+\gamma)c}{1-\gamma}$, we have $\frac{\partial U_{MNO}}{\partial^2 p} < 0$. For $p > \frac{(1+\gamma)c}{1-\gamma}$, we have $\frac{\partial U_{MNO}}{\partial^2 p} > 0$.

Summarizing the above, U_{MNO} is concavely increasing for $p < \frac{c}{1-\gamma}$, concavely decreasing for $\frac{c}{1-\gamma} < p < \frac{(1+\gamma)c}{1-\gamma}$, and convexly decreasing for $p > \frac{(1+\gamma)c}{1-\gamma}$.

Hence, in the special case of $\alpha = \gamma$, the optimal values of p^* is given by

$$
p^* = \frac{c}{1-\gamma}. \qquad (7.67)
$$

7.2.7 Optimal Pricing under Uniform Sponsorship Scheme

Similarly, we first have

$$
\begin{aligned}
U_{MNO} &= \sum_{i=1}^{N} [\lambda_i \,(p-c)\, x_i] \\
&= \sum_{i=1}^{N} [\lambda_i \,(p-c) \left(\frac{\tau \sigma_i}{p} \right)^{\frac{1}{\gamma}} (1-\theta)^{-\frac{1}{\gamma}}]. \qquad (7.68)
\end{aligned}
$$

Taking the partial derivative of U_{MNO} with respect to p, we can deduce that

$$
\frac{\partial U_{MNO}}{\partial p} \propto (1 - \frac{1}{\gamma}) p^{-\frac{1}{\gamma}} + c \frac{1}{\gamma} p^{-\frac{1}{\gamma}-1}.
$$

Hence, the optimal value of p^* is given by

$$p^* = \frac{c}{1-\gamma}.$$

Remark: After comparing the optimal prices under different sponsorship schemes, we find an interesting result: the optimal prices in response to different sponsorship schemes keep the same form, i.e., $p^* = \frac{c}{1-\gamma}$. In other words, the optimal price does not depend on which sponsorship scheme is adopted. This result may be due to the fact that the first-mover advantage in the game allows the MNO (i.e., leader) to take the initiative position, and hence the payoff structure for the leader's strategies in our model might be pre-determined by the system parameters. Therefore, the change of the sponsorship scheme will only affect the lower-layer interactions between the CP and MUs instead of the optimal pricing structure from the higher layer.

7.3 Appendix of Chapter 5

7.3.1 Proof of Proposition 5.2

Proof. From (5.25), we can easily know that $\frac{\partial^2 x^*}{\partial \theta^2} < 0$ under the condition in (5.34). Furthermore, we have $\frac{\partial x^*}{\partial \theta} > 0$ and

$$\frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta^2} = -\gamma \sigma_c x^{*-\gamma-1} \left(\frac{\partial x^*}{\partial \theta}\right)^2 - (\sigma_c x^{*-\gamma} - \theta p) \frac{\partial^2 x^*}{\partial \theta^2} - 2p \frac{\partial x^*}{\partial \theta}. \tag{7.69}$$

Consequently, we can conclude that $\frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} < 0$ under the condition in (5.33). Then, we analyze the properties of $\frac{\partial^2 \Pi_e(t;p)}{\partial t^2}$. From (5.28), in order to prove that $\frac{\partial^2 \Pi_e(t;p)}{\partial t^2}$ is negative, we need to prove that

$$\gamma (1-x^*)^{-1} \left(\frac{\partial x^*}{\partial t}\right)^2 + \frac{\partial^2 x^*}{\partial t^2} > 0. \tag{7.70}$$

By substituting (5.30) and (5.32) into (7.70), and with simple steps, we have

$$\underbrace{(\gamma + 2\alpha)(1-x^*)^{-1} x^{*-\alpha-1}}_{>0} + \underbrace{(\alpha+1) x^{*-\alpha-2}}_{>0} + (\gamma + \alpha - 1) \underbrace{\frac{t^{1-\beta}}{1-\beta} (1-x^*)^{-\alpha-2}}_{>0} > 0. \tag{7.71}$$

Accordingly, under the condition in (5.35), the inequality given in (7.71) is satisfied. Therefore, the negativity of $\frac{\partial^2 \Pi_e(t;p)}{\partial t^2}$ is proved.

The strategy space of the SCP is defined to be within $[0, 1]$, which is a non-empty, convex, and compact subset of the Euclidean space. As aforementioned, we prove that the second partial derivative of the SCP's objective function with respect to its decision variable is negative, i.e., $\frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} < 0$. Therefore, the profit function of the SCP, $\Pi_s(\theta; p)$, is continuous and strictly concave with respect to θ. Likewise, the profit function of the ECCP, $\Pi_e(t; p)$, is strictly concave with respect to its decision variable t. This is because we prove that the second order partial derivative of $\Pi_e(t; p)$ with respect to t is nagative, i.e., $\frac{\partial^2 \Pi_e(t;p)}{\partial t^2} < 0$. Thus, the strict concavity of the objective function of the ECCP $\Pi_e(t; p)$ is ensured. Moreover, the strategy space of the ECCP, $[0, 1]$, is also a non-empty convex and compact subset of the Euclidean space. Therefore, the Nash equilibrium exists in this non-cooperative sub-game $\mathcal{G}^c$ between the SCP and ECCP [5]. The proof is now completed. □

7.3.2 Proof of Proposition 5.3

Proof. The Jacobian matrix of point-to-set mapping with respect to the profit profile of the SCP and ECCP, i.e., $\nabla \mathbf{F}$ is defined as follows:

$$\nabla \mathbf{F} = \nabla \mathbf{F}(\Pi_s(\theta; p), \Pi_e(t; p)) = \begin{bmatrix} \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} \\ \left(\frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t}\right)^\top & \frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix}. \tag{7.72}$$

Then, we have

$$\nabla \mathbf{F} + \nabla \mathbf{F}^\top = \begin{bmatrix} 2\frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} + \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} \\ \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} + \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} & 2\frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix}. \tag{7.73}$$

In order to guarantee the uniqueness of the Nash equilibrium in the non-cooperative sub-game $\mathcal{G}^c$, we need to prove that the matrix in (7.73) is negative definite [72]. Recall from Proposition 5.2 that $\frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} < 0$ and $\frac{\partial^2\Pi_e(t;p)}{\partial t^2} < 0$. Then, we only need to ensure the negativity of the determinant for the matrix given in (7.73). We have

$$
\begin{aligned}
& \det\begin{bmatrix} 2\frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t}+\frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} \\ \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t}+\frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} & 2\frac{\partial^2\Pi_e(t;p)}{\partial t^2} \end{bmatrix} \\
= \; & \det\begin{bmatrix} \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & \frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} \\ \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t} & \frac{\partial^2\Pi_e(t;p)}{\partial t^2} \end{bmatrix} + \det\begin{bmatrix} \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t} \\ \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t} & \frac{\partial^2\Pi_e(t;p)}{\partial t^2} \end{bmatrix} \\
& + \det\begin{bmatrix} \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t} \\ \frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} & \frac{\partial^2\Pi_e(t;p)}{\partial t^2} \end{bmatrix} + \det\begin{bmatrix} \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & \frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} \\ \frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} & \frac{\partial^2\Pi_e(t;p)}{\partial t^2} \end{bmatrix}, \qquad (7.74)
\end{aligned}
$$

and we also have

$$
\begin{aligned}
& \det\begin{bmatrix} \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t} \\ \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t} & \frac{\partial^2\Pi_e(t;p)}{\partial t^2} \end{bmatrix} + \det\begin{bmatrix} \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & \frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} \\ \frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} & \frac{\partial^2\Pi_e(t;p)}{\partial t^2} \end{bmatrix} \\
= \; & 2\frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2}\frac{\partial^2\Pi_e(t;p)}{\partial t^2} - \left(\frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t}\right)^2 - \left(\frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t}\right)^2 \\
< \; & 2\frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2}\frac{\partial^2\Pi_e(t;p)}{\partial t^2} - 2\frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t}\frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} \\
= \; & \det\begin{bmatrix} \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & \frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} \\ \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t} & \frac{\partial^2\Pi_e(t;p)}{\partial t^2} \end{bmatrix} + \det\begin{bmatrix} \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t} \\ \frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} & \frac{\partial^2\Pi_e(t;p)}{\partial t^2} \end{bmatrix}. \qquad (7.75)
\end{aligned}
$$

Thus, in order to guarantee the negativity of the determinant for the matrix in (7.73), we only need to ensure the negativity of $\begin{bmatrix} \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & \frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t} \\ \frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} & \frac{\partial^2\Pi_e(t;p)}{\partial t^2} \end{bmatrix}$.

Accordingly, we prove the positivity of $\begin{bmatrix} -\frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & -\frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t} \\ -\frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} & -\frac{\partial^2\Pi_e(t;p)}{\partial t^2} \end{bmatrix}$ with the following steps:

$$
\begin{aligned}
&\det\begin{bmatrix} -\frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & -\frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t} \\ -\frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} & -\frac{\partial^2\Pi_e(t;p)}{\partial t^2}\end{bmatrix} \\
= \quad & \gamma^2{\sigma_c}^2(1-x^*)^{-\gamma-1}\left(\frac{\partial x^*}{\partial\theta}\right)^2\left(\frac{\partial x^*}{\partial t}\right)^2 + \gamma{\sigma_c}^2 x^{*-\gamma-1}(1-x^*)^{-\gamma}\left(\frac{\partial x^*}{\partial\theta}\right)^2\frac{\partial^2 x^*}{\partial t^2} \\
- \quad & \gamma\sigma_c(\sigma_c x^{*-\gamma}-\theta p)(1-x^*)^{-\gamma}\frac{\partial^2 x^*}{\partial\theta^2}\left(\frac{\partial x^*}{\partial t}\right)^2 - \sigma_c(\sigma_c x^{*-\gamma}-\theta p)(1-x^*)^{-\gamma}\frac{\partial^2 x^*}{\partial\theta^2}\frac{\partial^2 x^*}{\partial t^2} \\
+ \quad & 2p\gamma\sigma_c(1-x^*)^{-\gamma-1}\left(\frac{\partial x^*}{\partial t}\right)^2\frac{\partial x^*}{\partial\theta} + 2p\sigma_c(1-x^*)^{-\gamma}\frac{\partial x^*}{\partial\theta}\frac{\partial^2 x^*}{\partial t^2} - \gamma^2{\sigma_c}^2 x^{*-\gamma-1}(1-x^*)^{-\gamma-1} \\
\times \quad & \left(\frac{\partial x^*}{\partial\theta}\right)^2\left(\frac{\partial x^*}{\partial t}\right)^2 - \gamma{\sigma_c}^2 x^{*-\gamma-1}(1-x^*)^{-\gamma}\frac{\partial^2 x^*}{\partial\theta\partial t}\frac{\partial x^*}{\partial\theta}\frac{\partial x^*}{\partial t} + \gamma\sigma_c(\sigma_c x^{*-\gamma}-\theta p)(1-x^*)^{-\gamma} \\
\times \quad & \frac{\partial^2 x^*}{\partial\theta\partial t}\frac{\partial x^*}{\partial\theta}\frac{\partial x^*}{\partial t} + \sigma_c(\sigma_c x^{*-\gamma}-\theta p)(1-x^*)^{-\gamma}\left(\frac{\partial^2 x^*}{\partial\theta\partial t}\right)^2 - p\gamma\sigma_c(1-x^*)^{-\gamma-1}\frac{\partial x^*}{\partial\theta}\left(\frac{\partial x^*}{\partial t}\right)^2 \\
- \quad & p\sigma_c(1-x^*)^{-\gamma}\frac{\partial x^*}{\partial t}\frac{\partial^2 x^*}{\partial\theta\partial t}.
\end{aligned} \tag{7.76}
$$

With simple transformations, we have (7.77) in the final step as follows:

$$
\begin{aligned}
&\det\begin{bmatrix} -\frac{\partial^2\Pi_s(\theta;p)}{\partial\theta^2} & -\frac{\partial^2\Pi_s(\theta;p)}{\partial\theta\partial t} \\ -\frac{\partial^2\Pi_e(t;p)}{\partial\theta\partial t} & -\frac{\partial^2\Pi_e(t;p)}{\partial t^2}\end{bmatrix} = \gamma{\sigma_c}^2 x^{*-\gamma-1}(1-x^*)^{-\gamma-1}\underbrace{\frac{\partial x^*}{\partial\theta}}_{>0}\left(\frac{\partial x^*}{\partial\theta}\frac{\partial^2 x^*}{\partial t^2} - \frac{\partial x^*}{\partial t}\frac{\partial^2 x^*}{\partial\theta\partial t}\right) \\
- \quad & \gamma\sigma_c(\underbrace{\sigma_c x^{*-\gamma}-\theta p}_{>0})(1-x^*)^{-\gamma-1}\underbrace{\frac{\partial x^*}{\partial t}}_{<0}\left(\frac{\partial^2 x^*}{\partial\theta^2}\frac{\partial x^*}{\partial t} - \frac{\partial^2 x^*}{\partial\theta\partial t}\frac{\partial x^*}{\partial\theta}\right) - \sigma_c\underbrace{(\sigma_c x^{*-\gamma}-\theta p)}_{>0} \\
\times \quad & (1-x^*)^{-\gamma}\left[\frac{\partial^2 x^*}{\partial\theta^2}\frac{\partial^2 x^*}{\partial t^2} - \left(\frac{\partial^2 x^*}{\partial\theta\partial t}\right)^2\right] + p\sigma_c(1-x^*)^{-\gamma}\left(2\frac{\partial x^*}{\partial\theta}\frac{\partial^2 x^*}{\partial t^2} - \frac{\partial x^*}{\partial t}\frac{\partial^2 x^*}{\partial\theta\partial t}\right) \\
+ \quad & \underbrace{p\gamma\sigma_c(1-x^*)^{-\gamma-1}\frac{\partial x^*}{\partial\theta}\left(\frac{\partial x^*}{\partial t}\right)^2}_{>0}.
\end{aligned} \tag{7.77}
$$

Consequently, to prove the positivity of above equality, one necessary condition is that the following constraints are satisfied:

$$
\begin{aligned}
&\left(\frac{\partial x^*}{\partial\theta}\frac{\partial^2 x^*}{\partial t^2} - \frac{\partial x^*}{\partial t}\frac{\partial^2 x^*}{\partial\theta\partial t}\right) > 0, \left(\frac{\partial^2 x^*}{\partial\theta^2}\frac{\partial x^*}{\partial t} - \frac{\partial^2 x^*}{\partial\theta\partial t}\frac{\partial x^*}{\partial\theta}\right) > 0, \\
&\left[\frac{\partial^2 x^*}{\partial\theta^2}\frac{\partial^2 x^*}{\partial t^2} - \left(\frac{\partial^2 x^*}{\partial\theta\partial t}\right)^2\right] < 0, \left(2\frac{\partial x^*}{\partial\theta}\frac{\partial^2 x^*}{\partial t^2} - \frac{\partial x^*}{\partial t}\frac{\partial^2 x^*}{\partial\theta\partial t}\right) > 0.
\end{aligned} \tag{7.78}
$$

Since we obtain the expression of $\frac{\partial x^*}{\partial \theta}$, $\frac{\partial^2 x^*}{\partial \theta^2}$, $\frac{\partial x^*}{\partial t}$ and $\frac{\partial^2 x^*}{\partial t^2}$ aforementioned, in the following, we derive the expression for $\frac{\partial^2 x^*}{\partial \theta \partial t}$. From (5.20), we have

$$\left[-x^{*-\alpha-2} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-2}\right](\alpha+1)\frac{\partial x^*}{\partial \theta}\frac{\partial x^*}{\partial t} + \left[x^{*-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}\right]\frac{\partial^2 x^*}{\partial \theta \partial t} + t^{-\beta}(1-x^*)^{-\alpha-1}\frac{\partial x^*}{\partial \theta} = 0. \quad (7.79)$$

With simple manipulations, we obtain the final expression as follows:

$$\frac{\partial^2 x^*}{\partial \theta \partial t} = \frac{pt^{-\beta}(1-x^*)^{-\alpha}}{\alpha\tau\sigma_e\left[x^{*-\alpha-2} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}\right]^2}\left\{\frac{(\alpha+1)\left[-x^{*-\alpha-2} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-2}\right]}{\alpha\left[x^{*-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1}\right]} - (1-x^*)^{-1}\right\}. \quad (7.80)$$

Then, to check whether the constraints in (7.78) are satisfied, we substitute the specific expressions of $\frac{\partial x^*}{\partial \theta}$, $\frac{\partial^2 x^*}{\partial \theta^2}$, $\frac{\partial x^*}{\partial t}$, $\frac{\partial^2 x^*}{\partial t^2}$ and $\frac{\partial^2 x^*}{\partial \theta \partial t}$ into (7.78). Accordingly, for the first constraint in (7.78), we have $\left(\frac{\partial x^*}{\partial \theta}\frac{\partial^2 x^*}{\partial t^2} - \frac{\partial x^*}{\partial t}\frac{\partial^2 x^*}{\partial \theta \partial t}\right) =$

$$\frac{pt^{-\beta}(1-x)^{-\alpha}}{\alpha^2\tau\sigma_e\left[x^{-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x)^{-\alpha-1}\right]^2}\left\{\frac{t^{-\beta}(1-x)^{-\alpha-1}}{\left[x^{-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x)^{-\alpha-1}\right]} + \frac{\beta}{t}\right\}, \quad (7.81)$$

from which the first constraint in (7.78) is satisfied. Furthermore, the second constraint in (7.78) is also satisfied without any conditions, since we have

$$\left(\frac{\partial^2 x^*}{\partial \theta^2}\frac{\partial x^*}{\partial t} - \frac{\partial^2 x^*}{\partial \theta \partial t}\frac{\partial x^*}{\partial \theta}\right) = \frac{p^2 t^{-\beta}(1-x)^{-\alpha-1}}{\alpha^2\tau^2{\sigma_e}^2\left[x^{-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x)^{-\alpha-1}\right]^3}, \quad (7.82)$$

which is positive without any conditions. Also, we have $\left[\frac{\partial^2 x^*}{\partial \theta^2}\frac{\partial^2 x^*}{\partial t^2} - \left(\frac{\partial^2 x^*}{\partial \theta \partial t}\right)^2\right] =$

$$\frac{p^2 t^{-2\beta}(1-x)^{-2\alpha}}{\alpha^2\tau^2{\sigma_e}^2\left[x^{-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x)^{-\alpha-1}\right]^4}\left\{\frac{-(\alpha+1)\beta\left[-x^{-\alpha-2} + \frac{t^{1-\beta}}{1-\beta}(1-x)^{-\alpha-2}\right]}{\alpha t} - t^{-\beta}(1-x)^{-\alpha-2}\right\}. \quad (7.83)$$

Thus, under the condition in (5.34), the negativity of $\left[\frac{\partial^2 x^*}{\partial \theta^2}\frac{\partial^2 x^*}{\partial t^2} - \left(\frac{\partial^2 x^*}{\partial \theta \partial t}\right)^2\right]$ is ensured. Lastly, with some manipulations, we have $\left(2\frac{\partial x^*}{\partial \theta}\frac{\partial^2 x^*}{\partial t^2} - \frac{\partial x^*}{\partial t}\frac{\partial^2 x^*}{\partial \theta \partial t}\right) =$

$$\left\{\frac{(\alpha+1)t^{-\beta}(1-x)^{-\alpha-2} + 3\alpha t^{-\beta}(1-x)^{-\alpha-1} + (2\alpha-1)t^{-\beta}(1-x)^{-2\alpha-2}\frac{t^{1-\beta}}{1-\beta}}{\alpha\left[x^{-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x)^{-\alpha-1}\right]^2} + \frac{\beta}{t}\right\}$$
$$\times \frac{pt^{-\beta}(1-x)^{-\alpha}}{\alpha^2\tau\sigma_e\left[x^{-\alpha-1} + \frac{t^{1-\beta}}{1-\beta}(1-x)^{-\alpha-1}\right]^2}. \quad (7.84)$$

If the condition in (5.36), i.e., $2\alpha - 1 > 0$ holds, the last constraint in (7.78) is satisfied. Therefore, the Jacobian matrix of point-to-set mapping with respect to the profit profile of both the SCP and ECCP is negative definite. Consequently, $\nabla \mathbf{F}$ is diagonally strictly concave, from which the uniqueness of the Nash equilibrium in the non-cooperative subgame $\mathcal{G}^c$ is guaranteed [72]. Thus, the proof is now completed. □

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47) **Zehui Xiong**, Dusit Niyato, and Ping Wang, "Network Effects", in Encyclopedia of Wireless Networks, Edited by X.(S.) Shen, X. Lin, K. Zhang, *Springer*, 2018.