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2018

Feng, S., Xiong, Z., Niyato D., Wang, P., & Wang, S. S. (2018). Joint pricing and security investment for cloud-insurance : a security interdependency perspective. Proceedings of 2018 IEEE Wireless Communications and Networking Conference (WCNC).
doi:10.1109/WCNC.2018.8377068

<https://hdl.handle.net/10356/137438>

<https://doi.org/10.1109/WCNC.2018.8377068>

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Joint Pricing and Security Investment for Cloud-Insurance: A Security Interdependency Perspective

Shaohan Feng*, Zehui Xiong*, Dusit Niyato*, Ping Wang*, and Shaun Shuxun Wang†

*School of Computer Engineering, Nanyang Technological University (NTU), Singapore

†Nanyang Business School, Nanyang Technological University (NTU), Singapore

Abstract—Cyber insurance has been introduced as the mean to transfer cyber risks to an insurance company, namely, insurer. The users are thus covered by the insurance to alleviate the damage from cyber threats. In this paper, we investigate the joint pricing and security investment in a cloud-insurance market. The market is composed of users, cloud providers, and cloud-insurers. The users subscribe to use the cloud service (platform) from the cloud providers. To protect from the damage, the users can buy a cloud-insurance product from the cloud-insurers which will pay a claim to the users if an attack happens to the cloud service. The users are interdependent in which they can take advantage of the positive security effects generated by other users' investments in security. We assume that the cloud provider and cloud-insurer are the business partners. Therefore, the cloud-insurers can invest in the cloud platform to improve the security level, i.e., quality, of the cloud service and hence reduce the probability of paying claim. Our proposed model consists of two stages, i.e., the Stackelberg game. In the first stage, cloud-insurers set the price charging to the users and decide on the investment for improving the cloud security quality. In the second stage, the users decide on the amount of these cloud-insurances to purchase based on the observed prices and qualities. The existence and uniqueness for the equilibrium of the Stackelberg game are proved analytically. The performance evaluation shows some interesting results. For example, when the users have strong interdependency, the price of the cloud-insurance becomes lower. This is from the fact that the users can be influenced more easily by their peers, when one cloud-insurer decreases the price, it can attract more users easily.

Index Terms—Interdependency, security, security investment, cloud-insurance, quality, Stackelberg game

I. INTRODUCTION

Although there are significant improvements on cyber security techniques, e.g., software and hardware, over the last decade, achieving a perfect cyber-security solution is still impossible [1], [2]. In this regard, cyber-insurance has been identified as a potential tool to effectively manage cyber risks recently [3]. Cloud-insurance, which is one of the cyber-insurances, is a risk management technique via transferring cyber risks faced by users to an insurance company with a fee in return, i.e., premium. Proponents of cloud-insurance believe that the cloud-insurance has the potential to be a market solution that can align with economic incentives of cloud-insurers, users (individuals/organizations), and cloud providers. For example, the cloud-insurers can earn profit from appropriately pricing premiums and invest in the cloud platform to improve the cloud security quality. Likewise, the network users will

seek to hedge potential losses by buying cloud-insurance from the cloud-insurers. As such, the cloud-insurers can earn and optimize their profits by setting price/premium offered to the users. Moreover, the cloud-insurers can adjust their pricing strategies by capitalizing the fact that the users' decisions are not independent in the real market scenario.

In this paper, we focus on a joint competitive pricing and security investment problem, in which firms compete for selling substitute cloud-insurances. The substitute product can satisfy the need of a user that another product or service fulfills. For example, AIG, Chubb, and Beazley offer substitute cloud-insurance, e.g., AIG CyberEdge, Chubb Cyber Enterprise Risk Management package, and Beazley Breach Response (BBR), respectively. The user subscribing the cloud service can buy the cloud-insurance in which a claim will be paid by the cloud-insurer to the user to cover the damage from a cyber attack, e.g., data loss. As such, the cloud-insurer which is a partner of a cloud provider has an incentive to invest in the cloud platform to improve the security quality and hence reduce the probability of paying the claim. For example, Zhong'an online insurance company is a partner of Alibaba Cloud. The company sells cloud-insurance product to cloud users [4]. The company also invests in the Alibaba Cloud. The Alibaba Cloud can use that money to improve the service reliability and security by providing more data redundancy, e.g., multiple backup sites. As such, if the cloud service is breached, the users will be less affected and the insurers pay less claim. Note that we focus on the interaction between the Cloud insurers and users. The analysis on the cloud provider is out of scope of our work, which can be addressed in our future works.

We propose a two-stage game model, i.e., the Stackelberg game. In the first stage, cloud-insurers choose the strategy which consists of the price that they charge users for their cloud-insurance product, i.e., the premium, and the investment to strengthen the cloud security quality. In the second stage, the users decide on the fraction of their money to spend on these cloud-insurance products. Specifically, the contributions of the paper are summarized as follows. Firstly, we formulate a joint pricing and security investment problem as a game model to analyze an interaction between cloud-insurers and users. The model is able to also investigate the interdependency among the users. Secondly, in the proposed model, we incorporate the relationship between the security quality of the cloud service

and insurers' investment.

The rest of the paper is organized as follows. Section II describes the system model and Stackelberg game formulation. Section III shows the proof of the existence and uniqueness for the Stackelberg equilibrium under some reasonable assumptions. Section IV shows the numerical performance evaluation, and Section V concludes the paper.

II. SYSTEM DESCRIPTION

We consider the joint pricing and security investment problem in the competitive cloud-insurance market. In the market, there are cloud-insurers competing for selling the substitutable cloud-insurance product by setting appropriate prices and investing to improve the cloud security qualities. The users make their decisions of buying cloud-insurance product considering the prices and qualities as well as the interaction with other users.

A. System Model

Figure 1: The cloud-insurance market.

For ease of derivation and presentation, we consider a duopoly market with two cloud-insurers A and B competing for selling cloud-insurance product to a set of network users, the set of which is denoted by $\mathcal{N}$. We assume that every user $i \in \mathcal{N}$, i.e., the buyer, determines a fraction of the product demand to be purchased from cloud-insurer A , denoted by $x_i \in [0, 1]$. Then, the fraction of product demand of user i to cloud-insurer B is thus $1 - x_i$. As such, the strategy of user i is x_i and $1 - x_i$. Here, fraction x_i can denote either the probability that user i buys from cloud-insurer A or the fraction of a user group i that buys from A .

The security qualities of the cloud services from the investment of the cloud-insurers are denoted by q^A and q^B . The qualities have an impact on the users' purchasing demand. Further, q^A and q^B are functions of the investments which are denoted by z^A and z^B , respectively. The investment is from the cloud-insurer on the security technology of cloud service, which makes the cloud service more reliable and secure. For example, the investment can be used to improve the security levels of the cloud services by inventing more sophisticated intrusion detection algorithms to safeguard the data centers as well as stronger authentication algorithms to prevent unauthorized access to the cloud platform. Alternatively, the investment can be used to improve data redundancy as aforementioned for the Alibaba Cloud.

We adopt the proportional hazard model from [5] to quantify the effect of information security spending in addressing the vulnerability of an organization's data information system. Let $v(z^A)$ denote the probability of that the system covered by the cloud-insurer A 's insurance is breached when its corresponding investment is z^A , i.e.,

$$q^A = 1 - v(z^A) = 1 - \left\{ 1 - [1 - v(1)](z^A)^{-\alpha} \right\} = [1 - v(1)](z^A)^{-\alpha}, \quad (1)$$

where $v(1)$ represents the probability of being breached when the investment $z^A = 1$, and $\alpha > 0$ is the parameter of the function. Then, we obtain an inverse function of (1) according to its bijection as follows:

$$z^A(q^A) = \left\{ \frac{\ln[1 - v(1)]}{\ln(q^A)} \right\}^{\frac{1}{\alpha}}. \quad (2)$$

The marginal cost of cloud-insurers A is:

$$C^A(z^A) = z^A + nv(z^A)\lambda \quad (3)$$

$$= z^A + n \left\{ 1 - [1 - v(1)](z^A)^{-\alpha} \right\} \lambda, \quad (4)$$

$$= z^A(q^A) + n(1 - q^A)\lambda = C_1^A(q^A) + C_2^A(q^A), \quad (5)$$

where n and λ represent the numbers of threats and the monetary value of the data asset in one single area of vulnerability, respectively. Note that the extension of multiple areas of vulnerability given in [5] can be incorporated without any difficulty. Here, the term $nv(z^A)\lambda$ accounts for the claim paid to the users if the breach happens. We separate the cost function into two parts, i.e., $C_1^A(\cdot)$ and $C_2^A(\cdot)$, the physical meanings of which are explained later. The cost function for cloud-insurer B is similar to that for cloud-insurer A : $C^B(z^B) = z^B + nv(z^B)\lambda = z^B + n \left\{ 1 - [1 - v(1)](z^B)^{-\alpha} \right\} \lambda = z^B(q^B) + n(1 - q^B)\lambda = C_1^B(q^B) + C_2^B(q^B)$.

Let $\mathbf{x} \triangleq (x_1, \dots, x_{|\mathcal{N}|})$ denote the product demand from all the users to cloud-insurer A , and $\mathbf{x}_{-i}$ denote the same demand except that from user i . The utility of user i from applying strategy x_i is expressed as follows [6]:

$$u_i(x_i, \mathbf{x}_{-i}, p_i^A, p_i^B, q^A, q^B) = q^A a_i x_i - b_i x_i^2 + q^B a_i (1 - x_i) - b_i (1 - x_i)^2 + \sum_{j \in \mathcal{N}} g_{ij} x_i x_j + \sum_{j \in \mathcal{N}} g_{ij} (1 - x_i)(1 - x_j) - p_i^A x_i - p_i^B (1 - x_i) - \frac{d^A}{2} \left(\sum_{j \in \mathcal{N}} x_j \right)^2 - \frac{d^B}{2} \left(\sum_{j \in \mathcal{N}} (1 - x_j) \right)^2. \quad (6)$$

The first four terms $q^A a_i x_i - b_i x_i^2 + q^B a_i (1 - x_i) - b_i (1 - x_i)^2$, $a_i, b_i \geq 0$ represent the internal effects that user i obtained from buying the cloud-insurance product. We use the linear-quadratic function to capture the decreasing marginal returns. Specifically, coefficient b_i is the intrinsic demand elasticity factor, and coefficient a_i is the maximum intrinsic demand rate [7]. The maximum intrinsic demand rate is strongly dependent on the quality of products. As for the fifth and sixth terms, $\sum_{j \in \mathcal{N}} g_{ij} x_i x_j$ and $\sum_{j \in \mathcal{N}} g_{ij} (1 - x_i)(1 - x_j)$, the relationship among users is represented by an interdependency matrix $\mathbf{G}$. The ij -th element of $\mathbf{G}$ denoted by g_{ij} represents the interdependency degree that user i has on user j [6]. We assume that $g_{ij} \geq 0, \forall i, j \in \mathcal{N}$ and

$g_{ii} = 0, \forall i \in \mathcal{N}$. The seventh and eighth terms, $p_i^A x_i$ and $p_i^B (1 - x_i)$, are the payments of user i for consuming x_i and $(1 - x_i)$ units of insurance products from the cloud-insurers A and B , respectively. The last two terms, $\frac{d^A}{2} \left(\sum_{j \in \mathcal{N}} x_j \right)^2$ and $\frac{d^B}{2} \left(\sum_{j \in \mathcal{N}} (1 - x_j) \right)^2$, in quadratic sum form capture the congestion of one user which is affected by all the users, and the marginal cost of congestion increases as the total demand increases [7]. The marginal cost also captures the degree that the cloud-insurer is sensitive to the demand increment.

The goals of the cloud-insurers A and B are to maximize their individual profits which are expressed as follows:

$$\begin{aligned} \Pi^A &= \sum_{i \in \mathcal{N}} \left(p_i^A - C_2^A(q^A) \right) x_i - C_1^A(q^A) \\ &= \sum_{i \in \mathcal{N}} \left(p_i^A - n \quad 1 - [1 - v(1)]^{(z^A)^{-\alpha}} \quad \lambda \right) x_i - z^A(q^A) \\ &= \sum_{i \in \mathcal{N}} \left(p_i^A - n \quad 1 - q^A \quad \lambda \right) x_i - z^A(q^A), \end{aligned} \quad (7)$$

$$\begin{aligned} \Pi^B &= \sum_{i \in \mathcal{N}} \left(p_i^B - C_2^B(q^B) \right) (1 - x_i) - C_1^B(q^B) \\ &= \sum_{i \in \mathcal{N}} \left(p_i^B - n \quad 1 - [1 - v(1)]^{(z^B)^{-\alpha}} \quad \lambda \right) (1 - x_i) - z^B(q^B) \\ &= \sum_{i \in \mathcal{N}} \left(p_i^B - n \quad 1 - q^B \quad \lambda \right) (1 - x_i) - z^B(q^B), \end{aligned} \quad (8)$$

where p_i^A and p_i^B are the prices charged to user i by cloud-insurers A and B , respectively. Note that the investments, z^A and z^B (i.e., $C_1^A(q^A)$ and $C_1^B(q^B)$), are used to improve the security quality of the cloud service, which are independent of users' demands. However, the costs $C_2^A(q^A)$ and $C_2^B(q^B)$ are the claims amount paid to the users, and hence they are dependent on the users' demands.

B. Stackelberg Game Formulation

Considering the utility and profit functions given in (6)-(8), it is natural to model the market described in Section II-A into two stages. In the first stage, two cloud-insurers A and B decide on the prices of cloud-insurance products for every user and their investment for improving the security quality of the cloud service. In the second stage, each user determines the fraction of the cloud-insurance demand to be purchased from cloud-insurers A and B based on the prices and the qualities. Hence, the cloud-insurance market is modeled as the two-leader-multi-follower Stackelberg game. In particular, cloud-insurers A and B form a noncooperative two-player leader subgame for optimizing their prices and security qualities. Then, a number of users form another subgame for determining the fraction of cloud-insurance demand to be purchased from each cloud-insurer. The Stackelberg game can be defined by the following two subgames:

1) User-level noncooperative game: Given fixed security qualities q^A and q^B , as well as the price vectors $\mathbf{p}^A = [p_1^A, \dots, p_{|\mathcal{N}|}^A]^\top$ and $\mathbf{p}^B = [p_1^B, \dots, p_{|\mathcal{N}|}^B]^\top$, the user-level (follower) noncooperative game is defined as a four-tuple $\mathcal{G}_u = \{\mathcal{N}, \mathbf{x}, \mathcal{X}, \mathbf{u}\}$, where

- $\mathcal{N}$ is the set of active users.
- $\mathcal{X} = \left\{ x_1, \dots, x_{|\mathcal{N}|} \mid x_i \in [0, 1], \forall i \in \mathcal{N} \right\} \subset \mathbb{R}^{|\mathcal{N}|}$ is the domain of definition for $\mathbf{x}$ and an M-polyhedron.
- $\mathbf{x} = [x_1, \dots, x_{|\mathcal{N}|}]^\top$ is the vector of demands, where x_i is the demand of the user i to the product of cloud-insurer A , and $\mathbf{x}$ is defined in $\mathcal{X}$.
- $\mathbf{u} = [u_1, \dots, u_{|\mathcal{N}|}]^\top$ is the vector of users' utilities in the demand $\mathbf{x}$, where $u_i, \forall i \in \mathcal{N}$ is defined in (6).

2) Insurer-level noncooperative game: Given the users' demand $\mathbf{x}$, the cloud-insurers A and B form a noncooperative game, which is defined as a three-tuple: $\mathcal{G}_I = \left\{ [\mathbf{p}^A, q^A]^\top, \mathcal{D}^A, [\mathbf{p}^B, q^B]^\top, \mathcal{D}^B, \Pi \right\}$, where

- $\mathcal{D}^A = \left\{ [\mathbf{p}^A, q^A]^\top \mid p^u \geq p_i^A \geq 0, \forall i \in \mathcal{N}, q^A \in [0, 1] \right\}$ is the domain of definition for the price and quality of cloud-insurer A , where p^u is the upper bound of price p_i^A . The upper bound can be imposed, for example, by a regulator to avoid too high price.
- $[\mathbf{p}^A, q^A]^\top = [p_1^A, \dots, p_{|\mathcal{N}|}^A, q^A]^\top$ is the vector of the price and quality for cloud-insurer A defined in $\mathcal{D}^A$.
- $\mathcal{D}^B = \left\{ [\mathbf{p}^B, q^B]^\top \mid p^u \geq p_i^B \geq 0, \forall i \in \mathcal{N}, q^B \in [0, 1] \right\}$ is the domain of definition for the price and quality of cloud-insurer B .
- $[\mathbf{p}^B, q^B]^\top = [p_1^B, \dots, p_{|\mathcal{N}|}^B, q^B]^\top$ is the vector of the price and quality for cloud-insurer B defined in $\mathcal{D}^B$.
- $\Pi = [\Pi^A, \Pi^B]$ is the vector of profits for cloud-insurers A and B .

III. STACKELBERG EQUILIBRIUM SOLUTION

A. Solutions to the User-Level Game

In order to find the Nash equilibrium (NE) of the user-level game, we first need to find the best responses of the users in (6). To find the best response of each user, we take the derivative of the utility function in (6) with respect to the fraction of demand and set it to zero. Thus, we have $\frac{\partial u_i}{\partial x_i} = q^A a_i - 2b_i x_i - q^B a_i + 2b_i (1 - x_i) + \sum_{j \in \mathcal{N}} g_{ij} x_j + g_{ii} x_i - \sum_{j \in \mathcal{N}} g_{ij} (1 - x_j) - g_{ii} (1 - x_i) - d^A \sum_{j \in \mathcal{N}} x_j + d^B \sum_{j \in \mathcal{N}} (1 - x_j) - p_i^A + p_i^B = (q^A - q^B) a_i - 4b_i x_i + 2b_i + 2 \sum_{j \in \mathcal{N}} g_{ij} x_j - \sum_{j \in \mathcal{N}} g_{ij} - d^A + d^B) \sum_{j \in \mathcal{N}} x_j + |\mathcal{N}| d^B - p_i^A + p_i^B$.

Let $\mathbf{a} = [a_1, a_2, \dots, a_{|\mathcal{N}|}]^\top$, $\mathbf{p}^A = [p_1^A, p_2^A, \dots, p_{|\mathcal{N}|}^A]^\top$, $\mathbf{p}^B = [p_1^B, p_2^B, \dots, p_{|\mathcal{N}|}^B]^\top$, $\mathbf{1} = \text{ones}(|\mathcal{N}|, 1)$, $\mathbf{B} = \text{diag}([2b_1, 2b_2, \dots, 2b_{|\mathcal{N}|}])$ and $\mathbf{I} = \text{ones}(|\mathcal{N}|, |\mathcal{N}|)$. The first derivative of $u_i, i \in \mathcal{N}$ can be rewritten in a matrix form: $\frac{\partial \mathbf{u}}{\partial \mathbf{x}} = (q^A - q^B) \mathbf{a} - 2\mathbf{B}\mathbf{x} + \mathbf{B}\mathbf{1} + 2\mathbf{G}\mathbf{x} - \mathbf{G}\mathbf{1} - (d^A + d^B) \mathbf{I}\mathbf{x} + |\mathcal{N}| d^B \mathbf{1} - \mathbf{p}^A + \mathbf{p}^B$. Let $\frac{\partial u_i}{\partial x_i} = 0, \forall i \in \mathcal{N}$, $\mathbf{x}^*$ is shown in (9).

Assumption 1. $4b_i > \sum_{j \in \mathcal{N}} [2g_{ij} - (d^A + d^B)], \forall i \in \mathcal{N}$.

Lemma 1. The matrix $[2\mathbf{B} - 2\mathbf{G} + (d^A + d^B) \mathbf{I}]$ is invertible and positive definite based on Assumption 1.

Proof: Since $\frac{2g_{ij} - (d^A + d^B)}{4b_i} \geq 0$ and $b_i > 0$, we have $2g_{ij} - (d^A + d^B) \geq 0$ and $4b_i > \sum_{j \in \mathcal{N}} [2g_{ij} - (d^A + d^B)] =$

$$(q^A - q^B) \mathbf{a} + (\mathbf{B} - \mathbf{G}) \mathbf{1} + |\mathcal{N}| d^B \mathbf{1} - \mathbf{p}^A + \mathbf{p}^B = [2\mathbf{B} - 2\mathbf{G} + (d^A + d^B) \mathbf{I}] \mathbf{x}^* \quad (9)$$

$$\Pi^A = [\mathbf{p}^A - n(1 - q^A) \lambda \mathbf{1}]^\top \mathbf{x} - z^A = [\mathbf{p}^A - n(1 - q^A) \lambda \mathbf{1}]^\top \mathbf{K} [(q^A - q^B) \mathbf{a} + (\mathbf{B} - \mathbf{G}) \mathbf{1} + |\mathcal{N}| d^B \mathbf{1} - \mathbf{p}^A + \mathbf{p}^B] - z^A \quad (10)$$

$$\Pi^B = [\mathbf{p}^B - n(1 - q^B) \lambda \mathbf{1}]^\top (\mathbf{1} - \mathbf{x}) - z^B = [\mathbf{p}^B - n(1 - q^B) \lambda \mathbf{1}]^\top \{ \mathbf{1} - \mathbf{K} [(q^A - q^B) \mathbf{a} + (\mathbf{B} - \mathbf{G}) \mathbf{1} + |\mathcal{N}| d^B \mathbf{1} - \mathbf{p}^A + \mathbf{p}^B] \} - z^A \quad (11)$$

$\sum_{j \in \mathcal{N}} |2g_{ij} - (d^A + d^B)|$. Hence, the following inequality $\{[2\mathbf{B} - 2\mathbf{G} + (d^A + d^B) \mathbf{I}]\}_{ii} = 4b_i + (d^A + d^B) > 4b_i > \sum_{j \in \mathcal{N}} |2g_{ij} - (d^A + d^B)| > \sum_{j \neq i} |2g_{ij} - (d^A + d^B)| = \sum_{j \neq i} \{[2\mathbf{B} - 2\mathbf{G} + (d^A + d^B) \mathbf{I}]\}_{ij}$ is certainly satisfied, where $\{\cdot\}_i$ and $\{\cdot\}_{ij}$ represent the i th row and ij th element of matrix, respectively. Therefore, the matrix $[2\mathbf{B} - 2\mathbf{G} + (d^A + d^B) \mathbf{I}]$ is strictly diagonal dominant and its every eigenvalue λ satisfy $|\{[2\mathbf{B} - 2\mathbf{G} + (d^A + d^B) \mathbf{I}]\}_{ii} - \lambda| \leq \sum_{j \neq i} |\{[2\mathbf{B} - 2\mathbf{G} + (d^A + d^B) \mathbf{I}]\}_{ij}|$ according to Gershgorin circle theorem [8]. Since $[2\mathbf{B} - 2\mathbf{G} + (d^A + d^B) \mathbf{I}]$ is strictly diagonal dominant and $\{[2\mathbf{B} - 2\mathbf{G} + (d^A + d^B) \mathbf{I}]\}_{ii} = 4b_i + (d^A + d^B) > 0$, hence $\lambda > 0$. We can conclude that $[2\mathbf{B} - 2\mathbf{G} + (d^A + d^B) \mathbf{I}]$ is positive definite because all its eigenvalues are larger than 0 and hence is invertible. ■

$\mathbf{K} = [2\mathbf{B} - 2\mathbf{G} + (d^A + d^B) \mathbf{I}]^{-1} \succ 0$ since its inverse matrix is positive definite. We multiply both sides of (9) with $\mathbf{K}$, the optimal solution to the user-level game $\mathcal{G}_u$ is

$$\mathbf{x}^* = \mathbf{K} [(q^A - q^B) \mathbf{a} + (\mathbf{B} - \mathbf{G}) \mathbf{1} + |\mathcal{N}| d^B \mathbf{1} - \mathbf{p}^A + \mathbf{p}^B]. \quad (12)$$

Proposition 1. *An NE exists in game $\mathcal{G}_u = \{\mathbf{u}, \mathbf{x}, \mathcal{X}, \mathbf{u}\}$, if the following conditions hold:*

- 1) $\mathcal{X}$ is a nonempty convex compact subset of $\mathbb{R}^{|\mathcal{N}|}$.
- 2) $\mathbf{u}$ is continuous in $\mathbf{x}$, and u_i is concave in x_i .

THEOREM 1. *Under Assumption 1, the user-level noncooperative game $\mathcal{G}_u = \{\mathcal{N}, \mathbf{x}, \mathcal{X}, \mathbf{u}\}$ admits at least one NE.*

Proof: From (6), u_i is continuous and differentiable on x_i . Now we derive the second derivative of $u_i(\cdot)$ with respect to x_i to prove its concavity as follows: $\frac{\partial^2 u_i}{\partial x_i^2} = -4b_i + 2g_{ii} - (d^A + d^B) = -4b_i - (d^A + d^B) \leq 0$, which is obviously always less than 0, and hence u_i is concave in x_i . The strategy space $\mathcal{X}$ is clearly a nonempty convex and compact subset of Euclidean space $\mathbb{R}^{|\mathcal{N}|}$. The NE exists in the user-level noncooperative game $\mathcal{G}_u$ based on Proposition 1. ■

THEOREM 2. *Under Assumption 1, the Jacobi matrix of $\mathbf{u}(\mathbf{x})$ is diagonally strictly concave, and thus the NE of the user-level game $\mathcal{G}_u = \{\mathcal{N}, \mathbf{x}, \mathcal{X}, \mathbf{u}\}$ is unique.*

Proof: The Jacobi matrix of $\mathbf{u}(\mathbf{x})$ is given in (14), where $\nabla_{i,j} u_i(\mathbf{x})$ is the partial derivative of $u_i(\mathbf{x})$ with respect to x_i firstly and x_j secondly. (13) is the specific formulation of (14) after substituting $\nabla_{i,j} u_i(\mathbf{x})$. According to Assumption 1, the specific Jacobi matrix (13) satisfies the dominance solvability condition because $-\nabla_{i,j} u_i(\mathbf{x})]_{\forall i,j \in \mathcal{N}} - [\nabla_{i,j} u_i(\mathbf{x})]_{\forall i,j \in \mathcal{N}}$ is a strictly diagonally dominant matrix. Accordingly, the Jacobi matrix $[\nabla_{i,j} u_i(\mathbf{x})]_{\forall i,j \in \mathcal{N}} + [\nabla_{i,j} u_i(\mathbf{x})]_{\forall i,j \in \mathcal{N}}^\top$ is diagonally strictly concave [9]. Hence, the

NE of the user-level game $\mathcal{G}_u = \{\mathcal{N}, \mathbf{x}, \mathcal{X}, \mathbf{u}\}$ is unique, and the proof is now completed. ■

B. Solution to the Insurer-Level Noncooperative Game

Given the fraction of demand derived by the users, the profit of each cloud-insurer is affected not only by its own price and security quality, but also by the prices and qualities offered by the other cloud-insurer. This interaction between cloud-insurers can be modeled as a competitive game $\mathcal{G}_I$. The players are two cloud-insurers A and B . Their strategies are price and security quality, which is a function of investment. Payoff is the profit. Here, the Nash equilibrium (NE) of the game is a set of strategies that no cyber-insurer can increase its profit by choosing a different strategy given that the other player's strategies do not change [10]. We firstly prove the existence of the NE in $\mathcal{G}_I$ and the uniqueness follows.

To obtain the Nash equilibrium, we can plug-in the fractions of users' demand from (12) into the profit functions for cloud-insurers. We then rewrite (7) and (8) as (10) and (11), respectively. z^A and z^B are the inverse functions of (1) with q^A and q^B as q^A and q^B are the independent variables of z^A and z^B , respectively. Then we calculate the second order partial derivative of (10) with respect to $\mathbf{p}^A$ and q^A , and also (11) with respect to $\mathbf{p}^B$ and q^B as shown in (15). We consider the case that there is a lower bound on the security quality. Hence, we make the following assumption to ensure that the security quality meets the lower bound condition.

Assumption 2. $z^{A''} = \frac{\partial^2 z^A}{\partial q^{A2}} > (\mathbf{a} + n\lambda \mathbf{1})^\top \mathbf{K} (\mathbf{a} + n\lambda \mathbf{1})$ and $z^{B''} = \frac{\partial^2 z^B}{\partial q^{B2}} > (\mathbf{a} + n\lambda \mathbf{1})^\top \mathbf{K} (\mathbf{a} + n\lambda \mathbf{1})$.

Assumption 2 indicates that the minimum value of z^A must be larger than a certain threshold. This means to ensure low probability of paying claim, the insurers' investment should be larger than a certain threshold. Moreover, when z^A is larger than the threshold, z^A is convex and ensures the concavity for $\mathcal{G}_I$. The same rationale applies to z^B .

THEOREM 3. *Under Assumption 2, at least one NE exists in the insurer-level noncooperative game $\mathcal{G}_I$.*

Proof: We use congruent matrix to prove the concavity for the Hessian matrix of Π^A with respect to $[\mathbf{p}^A, q^A]^\top$ under Assumption 2. The same property of Π^B can be similarly proven. Due to the space limit, the proof is omitted. ■

THEOREM 4. *Under Assumption 2, the NE in the insurer-level noncooperative game $\mathcal{G}_I$ is unique.*

Proof: We prove that the Jacobi matrix is negative definite by using its congruent matrix. Due to the space limit, the proof

$$[\nabla_{i,j} u_i(\mathbf{x})]_{\forall i,j \in \mathcal{N}} + [\nabla_{i,j} u_i(\mathbf{x})]_{\forall i,j \in \mathcal{N}}^\top = \begin{bmatrix} -4b_1 - (d^A + d^B) & 2g_{12} - (d^A + d^B) & \cdots & 2g_{1|\mathcal{N}|} - (d^A + d^B) \\ 2g_{21} - (d^A + d^B) & -4b_2 - (d^A + d^B) & \cdots & 2g_{2|\mathcal{N}|} - (d^A + d^B) \\ \vdots & \vdots & \ddots & \vdots \\ 2g_{|\mathcal{N}|1} - (d^A + d^B) & 2g_{|\mathcal{N}|2} - (d^A + d^B) & \cdots & -4b_{|\mathcal{N}|} - (d^A + d^B) \end{bmatrix} \quad (13)$$

$$= \begin{bmatrix} \nabla_{1,1} u_1(\mathbf{x}) & \nabla_{1,2} u_1(\mathbf{x}) & \cdots & \nabla_{1,|\mathcal{N}|} u_1(\mathbf{x}) \\ \nabla_{2,1} u_2(\mathbf{x}) & \nabla_{2,2} u_2(\mathbf{x}) & \cdots & \nabla_{2,|\mathcal{N}|} u_2(\mathbf{x}) \\ \vdots & \vdots & \ddots & \vdots \\ \nabla_{|\mathcal{N}|,1} u_{|\mathcal{N}|}(\mathbf{x}) & \nabla_{|\mathcal{N}|,2} u_{|\mathcal{N}|}(\mathbf{x}) & \cdots & \nabla_{|\mathcal{N}|,|\mathcal{N}|} u_{|\mathcal{N}|}(\mathbf{x}) \end{bmatrix} + \begin{bmatrix} \nabla_{1,1} u_1(\mathbf{x}) & \nabla_{1,2} u_1(\mathbf{x}) & \cdots & \nabla_{1,|\mathcal{N}|} u_1(\mathbf{x}) \\ \nabla_{2,1} u_2(\mathbf{x}) & \nabla_{2,2} u_2(\mathbf{x}) & \cdots & \nabla_{2,|\mathcal{N}|} u_2(\mathbf{x}) \\ \vdots & \vdots & \ddots & \vdots \\ \nabla_{|\mathcal{N}|,1} u_{|\mathcal{N}|}(\mathbf{x}) & \nabla_{|\mathcal{N}|,2} u_{|\mathcal{N}|}(\mathbf{x}) & \cdots & \nabla_{|\mathcal{N}|,|\mathcal{N}|} u_{|\mathcal{N}|}(\mathbf{x}) \end{bmatrix}^\top, \quad (14)$$

$$\frac{\partial^2 \Pi^A}{\partial \mathbf{p}^A \partial \mathbf{p}^A} = \frac{\partial^2 \Pi^B}{\partial \mathbf{p}^B \partial \mathbf{p}^B} = -2\mathbf{K}, \frac{\partial^2 \Pi^B}{\partial \mathbf{p}^A \partial \mathbf{p}^B} = \frac{\partial^2 \Pi^B}{\partial \mathbf{p}^B \partial \mathbf{p}^A} = \mathbf{K}, \frac{\partial^2 \Pi^A}{\partial \mathbf{p}^A \partial q^A} = \mathbf{K}(\mathbf{a} - n\lambda \mathbf{1}), \frac{\partial^2 \Pi^A}{\partial q^A \partial \mathbf{p}^A} = 2n\lambda \mathbf{1}^\top \mathbf{K} \mathbf{a} - z^A'', \frac{\partial^2 \Pi^A}{\partial \mathbf{p}^A \partial q^B} = -\mathbf{K} \mathbf{a}, \frac{\partial^2 \Pi^A}{\partial q^A \partial \mathbf{p}^B} = n\lambda \mathbf{K} \mathbf{1},$$

$$\frac{\partial^2 \Pi^A}{\partial q^A \partial q^B} = -n\lambda \mathbf{1}^\top \mathbf{K} \mathbf{a}, \frac{\partial^2 \Pi^B}{\partial \mathbf{p}^B \partial q^B} = \mathbf{K}(\mathbf{a} - n\lambda \mathbf{1}), \frac{\partial^2 \Pi^B}{\partial q^B \partial \mathbf{p}^B} = 2n\lambda \mathbf{1}^\top \mathbf{K} \mathbf{a} - z^B'', \frac{\partial^2 \Pi^B}{\partial q^A \partial \mathbf{p}^B} = -\mathbf{K} \mathbf{a}, \frac{\partial^2 \Pi^B}{\partial \mathbf{p}^A \partial q^B} = n\lambda \mathbf{K} \mathbf{1}, \frac{\partial^2 \Pi^B}{\partial q^A \partial q^B} = -n\lambda \mathbf{1}^\top \mathbf{K} \mathbf{a}. \quad (15)$$

is omitted.

Figure 2: The relationship between the quality and the investment.

IV. PERFORMANCE EVALUATION

In this section, we conduct the simulations to evaluate the performance of the game in each stage. We consider a group of $|\mathcal{N}|$ users in the cloud-insurance market and set the intrinsic parameters for users as follows: $a_i \sim N(\mu_a, \sigma)$ and $b_i \sim N(\mu_b, \sigma)$, where $\mu_a = 2$, $\mu_b = 4.5$, and $\sigma = 1/3$. $N(\mu, \sigma)$ is a random number generator following the normal distribution with mean μ and standard deviation σ . Due to the diversity of cloud-insurers, e.g., reputation, we set the intrinsic parameters for cloud-insurers as $d^A \sim N(\mu_d, \sigma)$ and $d^B = d^A + d$, where $d \sim N(\sigma, \sigma/3)$. Here, the larger the values of d^A and d^B are, the less resources the cloud-insurers have. The corresponding g_{ij} , $\forall i, j \in \mathcal{N}$ are generated by the random number generator following the uniform distribution according to Assumption 1. The other coefficients are $v(1) = 0.5$, $n = 1$, $\lambda = 1$, $p^u = 10$, and $\alpha = 0.5$. The relationship between the quality and the investment according to (1) is shown in Fig. 2 [5].

Firstly, we evaluate the users' total utilities, the insurers' profits, the average prices and the quality of the cloud-insurance products when the number of users is varied with different degrees of interdependency among users. Figure 3(a) illustrates the total utilities when the number of users increases under different degree of interdependency. From Fig. 3(a), the users' total utility increases as the number of users increases. Further, the total utility in the cloud-insurance market with

Figure 3: Total utility and profit with increasing users and different interdependency.

Figure 4: Price and quality with increasing users and different interdependency.

strong interdependency is observed to be higher than that in the market with weak interdependency. This is due to the fact that the users tend to buy the same cloud-insurance product with the other users. When one user buys a cloud-insurance product, the other associated users will be likely to follow, and hence the security levels of these users improve. Likewise, as show in Fig. 3(b), the profits of the cloud-insurers in the market with strong interdependency are unexpected lower than that with weak interdependency. This is mainly because when the users have strong interdependency, the cloud-insurers need to stay competitive by improving the security quality. However,

Figure 5: Comparison of total utility and profit with increasing number of users between different strategies under strong interdependency.

Figure 6: Comparison of price and quality with increasing number of users between different strategies under strong interdependency.

doing so requires much more investment as the rate of the quality increase becomes smaller when the quality is close to 1 as shown in Fig. 2. Thus, the cloud-insurers may not want to invest too much to improve the near perfect security quality which is shown in Fig. 4(b). Meanwhile, from Fig. 4(b), we observe that the security quality is not affected by the interdependency. As a result, the cloud-insurers have to adjust their prices appropriately.

We next consider two cases of strategy optimization of the cloud-insurers. In the first case, we fix the prices of both cloud-insurers and optimize only their investment strategies. This is referred to “Only Optimizing Security Investment Strategy” in Figs. 5 and 6. In the second case, we optimize both the prices and the investment strategies of the cloud-insurers. This is referred to “Joint Pricing and Security Investment Strategy”. Note that from Figs. 4(a) and (b), only the price strategy is affected by the interdependency of the users. Specifically, the joint pricing and security investment strategy is the same as optimizing only the price strategy.

Figure 5 presents the comparisons of users’ total utilities and insurers’ profits while Fig. 6 shows the comparisons of average price and security quality under two above cases in a strong interdependency market. As shown in Fig. 5, with the investment strategy is optimized while the price is fixed, the difference of the users’ total utility under two cases is small. However, both the cloud-insurers are operating at a

loss, i.e., their profits are negative. This is because when the price is fixed, the cloud-insurers are likely to decrease the investment to reduce their cost. However, the security quality becomes deteriorated which results in more claims and decreasing profit. Additionally, the other results are similar to those in Figs. 3 and 4.

V. CONCLUSION

In this paper, we have formulated a joint pricing and security investment problem to analyze the interaction between the cloud-insurers and users in the cloud-insurance market. In particular, we have presented the hierarchical two-stage Stackelberg game to model their interaction in terms of the utility maximization of users and the profit maximization of cloud-insurers. Moreover, we have incorporated the interdependency among the users in our model. In addition, we have incorporated the relationship between the security quality of the cloud service and insurer’s investment to reconsider the strategies for the cloud-insurers. Further, we have presented the optimal strategies of users, and the optimal pricing as well as security investment strategies of cloud-insurers using backward induction. The existence and uniqueness for the equilibrium of the proposed Stackelberg game have been validated analytically. At last, we have conducted the simulation to evaluate the performance. For the future work, we will further explore the role of reinsurance on this cloud-insurance market.

VI. ACKNOWLEDGEMENT

This work was supported in part by MOE Tier 1 under Grant 2017-T1-002-007, MoE Tier 1 (RG33/16), MOE Tier 2 under Grant MOE2014-T2-2-015 ARC4/15 and NRF2015-NRF-ISF001-2277, and the Cyber Risk Management (CyRiM) project under the Monetary Authority of Singapore.

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