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Limited Feedback Assisted Dynamic Decode-and-Forward Relaying for Multiple Access Relay Channel

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Abstract Feedback about the channel state information (CSI) enables the transmitter nodes to exploit channel conditions to yield large improvements in almost any performance metric. However, in practice, channel adaptive techniques based on full CSI have been deemed impractical due to the finite capacity of the feedback links. This work considers a multiple-access relay channel (MARC) where two source nodes communicate with one destination node assisted by one half-duplex dynamic decode-and-forward (DDF) relay. Using the diversity-multiplexing tradeoff (DMT) as a figure-of-merit, we propose a practical limited-feedback (LF) mechanism for MARC and show that a small number of information bits about the channel conditions leads to near optimal performance. With no CSI, this system incurs diversity-gain loss at higher multiplexing gains. However, by using an LF scheme where the destination sends an ACK or NACK to the relay, we show that the optimal diversity gain can be achieved across all the multiplexing gains for the considered DDF based MARC system.

Keywords Protocols · Relays · Multiplexing · Diversity · Diversity-multiplexing tradeoff (DMT) · Limited feedback · Dynamic decode-and-forward (DDF) · Multiple-access relay channel (MARC)

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1 Introduction

Relay-assisted communication has received significant interest as a means of providing spatial diversity when time, frequency, and antenna diversity are unavailable due to delay, bandwidth, and terminal size constraints, respectively. These techniques provide diversity by enabling users to utilize one another's resources and has been extensively studied from outage probability analysis or diversity-multiplexing tradeoff (DMT) perspective [6], [2], [18]. In [13], closed-form expressions for the bit error rates are derived for amplify-and-forward (AF), and decode-and-forward (DF) relay assisted successive interference cancelers (SIC) for the Rayleigh fading channel. The achievable DMT and diversity gain region (DGR) of a Z-channel with a full-duplex (FD) DF relay have been studied in [10] and [11], respectively. In [12], the achievable DGR for half-duplex (HD) dynamic decode-and forward (DDF) relay-assisted Z-channel is derived. In [14], authors characterized the DGR of the Nakagami-m faded Z-channel.

Practical communication systems usually involve more than one user. One of the most common models is the multiple-access channel (MAC). The capacity region of MAC is well known and the DMT is also developed in [16] for MAC. In the case of multiple-access relay channel (MARC), one relay supports multiple sources improving the overall performance of the system compared to that of MAC. We consider MARC and focus on the DDF protocol [2]. For the DDF protocol, the relay does not decode until it is possible to decode source information message successfully. The relay then re-encodes the message and transmit it in the remaining coding interval. Moreover, since a single relay is shared by multiple users in the MARC, the extra cost of adding the relay per user may thus be more acceptable.

1.1 Motivation

In [3], authors provided the diversity-multiplexing tradeoff (DMT) analysis of traditional DDF for MARC. It is shown that the DDF achieves optimal diversity gain in the lower multiplexing gain regime for two-user MARC. However, there is a performance loss at higher multiplexing gains. On the other hand, multiple-access amplify-and-forward (MAF) [4] and compress-and-forward (CF) [18] schemes achieve optimal diversity at higher multiplexing gains but offer inferior performance in lower multiplexing gain regime. The authors in [1] studied an opportunistic DDF for MARC where the relay selection is based on source-to-destination channel gain and have proved that the scheme achieves optimal DMT only at lower multiplexing gains. In [5], DMT of DDF protocol for generalized channels has been derived. It is shown that DDF fails to achieve optimal performance in higher multiplexing gain regime. An opportunistic selection based DDF scheme for multiple relay-based MARC has been reported in [8]. It is shown to offer better outage performance compared to static DF protocols. In [9], DMT has been characterized for RF

energy harvesting based multiple relay channel using a variant of DDF protocol. This renewed attention on DDF motivated us to revisit the scheme under the purview of limited feedback.

It has been well recognized that complete knowledge of channel state information (CSI) at the transmitting nodes aids in achieving the optimal performance. However, this solution incurs enormous feedback overhead. To reduce this cost, schemes which require extremely low-rate feedback channel have been published in the literature [7, 15, 17]. This work proposes a DDF protocol based on a limited feedback (LF-DDF) to achieve optimal diversity gain across all multiplexing gains.

1.2 Contributions

- Closed-form expression for the DMT of MARC with limited feedback based DDF protocol has been derived
- Shown that the proposed system achieves maximum possible diversity gain across all multiplexing gain regions

1.3 Organization of the paper

The rest of the paper is organized as follows: Section 2 presents the system model and also describes the proposed ACK/NACK based LF protocol. In Section 3, we describe the tools used for the performance analysis. In Section 4, we obtain the closed-form solution for the achievable DMT. The numerical results and concluding remarks are given in Section 5 and Section 6, respectively.

2 System Model

This work considers a wireless network consisting of two sources (S_1, S_2), a destination (D), and a half-duplex relay (T) as shown in Fig. 1 and the fading coefficients g_i and h_i , $i \in \{1, 2\}$ are assumed to be quasi-static, i.e., the path gains are constant over a codeword of length, l . It is also assumed that the destination can transmit feedback information to the relay through a separate noiseless channel. As shown in the inset of Fig. 1, the relay operates in time-switching mode: information decoding and retransmission. Relay decodes the data from the sources during l_d symbol transmissions, and after successful decoding, the information is forwarded to the destination for the rest of the codeword duration (l_t).

In the proposed DDF protocol, S_i , $i \in \{1, 2\}$ transmit data at a rate of $R/2$ bits per channel use (BPCU) during every codeword. We denote the signals transmitted by sources S_i , $i \in \{1, 2\}$ and the relay T as $x_{i,k}$, $k \in [1, l]$, and

Fig. 1 MARC System Model with two source nodes

$\tilde{x}_k$, $k \in [l_d, l]$, respectively. The received signals at the relay ($y_{r,k}$) and the destination ($y_{d,k}$) can be written as:

$$y_{r,k} = \sum_{i=1}^2 h_i x_{i,k} + n_k, \quad 1 \leq k \leq l_d \quad (1)$$

$$y_{d,k} = \begin{cases} \sum_{i=1}^2 g_i x_{i,k} + v_k, & 1 \leq k \leq l_d \\ \sum_{i=1}^2 g_i x_{i,k} + g_r \tilde{x}_k + v_k, & l_d \leq k \leq l \end{cases} \quad (2)$$

where n_k and v_k are the noise samples observed at the relay and the destination, respectively, for the symbol k . The next subsection describes the employed limited feedback protocol and also presents the associated operating modes.

2.1 Limited Feedback: Protocol Description

The feedback signals (ACK/NACK) are generated as shown below:

1. Source nodes S_1 and S_2 transmit their respective pilot packets.

2. Destination estimates the channel gains g_1 and g_2 based on the pilots. If the channel gain of the respective link is sufficient to decode the individual target rate of $R/2$, the destination D sends an *ACK*; otherwise, it sends a *NACK*.

Fig. 2 Limited Feedback: Operating Modes

Based on the feedback signals, the proposed system operates in any of the four modes as shown in Fig. 2. In this figure, C_1 and C_2 represent the instantaneous channel capacities of $S_1 - D$ and $S_2 - D$ links, respectively and are given as

$$C_i = \log(1 + |g_i|^2 \gamma), \quad i \in \{1, 2\}, \quad (3)$$

where γ is the operating SNR ($\triangleq P_S/\sigma_D^2$), P_S is the transmit power, and σ_D^2 is the variance of the noise observed at the destination D . The operating modes are further enumerated in Table 1 and the flags S_{1f} and S_{2f} represent the

Table 1 Operating modes

Mode	S_{1f}	S_{2f}	Remarks
M_1	ACK	ACK	Both the direct links are good
M_2	NACK	ACK	$S_2 - D$ link is good
M_3	ACK	NACK	$S_1 - D$ link is good
M_4	NACK	NACK	Both the direct links are in error

feedback information related to S_1 and S_2 , respectively. The operating modes of the considered limited feedback system are further detailed below:

- Mode M_1 : In this mode, the individual rate ($R/2$) requirement is satisfied for both the users using the direct links alone, i.e.,

$$\log(1 + |g_1|^2\gamma) \geq \frac{R}{2}, \quad \log(1 + |g_2|^2\gamma) \geq \frac{R}{2} \quad (4)$$

In this mode, the relay T stays idle.

- Mode M_2 : In this mode, D can decode only S_2 's data but not the S_1 's data using the direct links, i.e.,

$$\log(1 + |g_1|^2\gamma) < \frac{R}{2}, \quad \log(1 + |g_2|^2\gamma) \geq \frac{R}{2}, \quad (5)$$

thus the relay T forwards the data of S_1 alone.

- Mode M_3 : This mode is equivalent to mode M_2 with the roles of S_1 and S_2 are reversed, and it also exhibits the same DMT due to symmetry. Hence, for brevity, the analysis of this mode has been omitted in this work.
- Mode M_4 : In this mode, destination D cannot decode data from both S_1 and S_2 using the direct links alone, i.e.,

$$\log(1 + |g_1|^2\gamma) < \frac{R}{2}, \quad \log(1 + |g_2|^2\gamma) < \frac{R}{2}, \quad (6)$$

and the relay T forwards the data of both S_1 and S_2 .

3 The Diversity-Multiplexing Tradeoff

The DMT essentially captures the high-SNR tradeoff of data rate and the reliability. Following [2], the asymptotic definitions of multiplexing gain (r_i) and diversity gain (d_i) of user i , $i \in \{1, 2\}$ are given as follows:

$$d_i \triangleq -\lim_{\gamma \rightarrow \infty} \frac{\log(P_E^i(\gamma))}{\log(\gamma)} \quad \text{and} \quad r_i \triangleq \lim_{\gamma \rightarrow \infty} \frac{R_i(\gamma)}{\log(\gamma)}, \quad (7)$$

where $P_E^i(\gamma)$ is the error probability, and R_i is the target data rate, respectively, for user i . In this work, we assume the symmetric scenario, i.e., the multiplexing gains r_1 and r_2 are equal to $r/2$. Thus, the overall multiplexing gain is r and the overall diversity gain $d \triangleq d_1 = d_2$.

3.1 DMT of Reference Schemes:

For comparison, the DMTs of the genie-aided (where both the source messages are provided to relay T a priori, is also the upper-bound (d_{UB}) and the DDF (without feedback, is also the lower-bound (d_{LB})) protocols for the symmetric two-user MARC are provided below [3]

$$d_{UB}(r) = \begin{cases} 2 - r, & 0 \leq r \leq \frac{1}{2} \\ 3(1 - r), & \frac{1}{2} \leq r \leq 1 \end{cases} \quad (8)$$

$$d_{LB}(r) = \begin{cases} 2 - r, & 0 \leq r \leq \frac{1}{2} \\ 3(1 - r), & \frac{1}{2} \leq r \leq \frac{2}{3} \\ 2\left(\frac{1-r}{r}\right), & \frac{2}{3} \leq r \leq 1 \end{cases} \quad (9)$$

4 The DMT of Limited Feedback based DDF Protocol

At asymptotically high SNR, the $P_E(\gamma)$ is upper bounded by the outage probability, P_O [4]. The total outage probability for the considered system, P_O , can be expressed as the sum of four joint probabilities related to the each of the operating modes as given below

$$\begin{aligned} P_O &= P(O|M_1)P(M_1) + P(O|M_2)P(M_2) + P(O|M_3)P(M_3) + P(O|M_4)P(M_4) \\ &= 2 P(O, M_2) + P(O, M_4) \quad (\because P(O|M_1) = 0), \end{aligned} \quad (10)$$

where the factor 2 is due to the symmetry between mode M_2 and mode M_3 , i.e., $P(O, M_2) = P(O, M_3)$. Also, note that in mode M_1 , there is no outage. Hence, this mode is not considered for further analysis.

In the following subsection, we define the possible outage error events for each of the operating modes.

4.1 Outage Error Events

As in [3], for each of the modes, we partition the joint outage error event $\mathcal{E}$ into two types of error events $\mathcal{E}_1$ and $\mathcal{E}_2$, i.e.,

$$\mathcal{E} = \mathcal{E}_1 \cup \mathcal{E}_2, \quad (11)$$

where $\mathcal{E}_1$ denotes the outage event related to individual target rate ($R/2$) of S_i , $i \in \{1, 2\}$ and $\mathcal{E}_2$ denotes the outage event related to the target sum-rate (R). Hence by applying the union bound, the achievable diversity gain $d_{O, M_j}(r)$, $j \in \{2, 4\}$ can be written as

$$d_{O, M_j}(r) = \min\{d_{\mathcal{E}_1}, d_{\mathcal{E}_2}\} \text{ where } P_{\mathcal{E}_1} \doteq \rho^{-d_{\mathcal{E}_1}}, P_{\mathcal{E}_2} \doteq \rho^{-d_{\mathcal{E}_2}}, \quad (12)$$

where $P_{\mathcal{E}_{1(2)}}$ is the error probability corresponding to outage event $\mathcal{E}_{1(2)}$ and $d_{\mathcal{E}_{1(2)}}$ is the diversity gain considering $P_{\mathcal{E}_{1(2)}}$ alone. Finally, the overall DMT for the LF-DDF can be written as

$$d_{LF-DDF}(r) = \min_{j \in \{2,4\}} \{d_{O,M_j}(r)\} \quad (13)$$

In the subsequent subsections, we derive the achievable DMT for each of these modes.

4.2 Mode M_2 : Achievable DMT (d_{O,M_2})

The proposed LF based system operates in mode M_2 if the constraints given in (5) are satisfied, and their corresponding high SNR equivalents are given as:

$$\begin{aligned} \log(1 + |g_1|^2 \gamma) &< \frac{R}{2} \Leftrightarrow v_1 > \left(1 - \frac{r}{2}\right) \\ \log(1 + |g_2|^2 \gamma) &\geq \frac{R}{2} \Leftrightarrow v_2 \leq \left(1 - \frac{r}{2}\right), \end{aligned} \quad (14)$$

where v_j is the exponential order of $1/|g_j|^2$ [2].

Also, in this mode, the relay T forwards only the S_1 's data to the destination. Thus, considering the joint detection [12] of the desired signal (corresponding to source S_1) and the interfering signal (corresponding to source S_2), the decoding interval l_d in mode M_2 is given by

$$l_d = \min \left\{ l, \max \left\{ \left\lceil \frac{lR}{2 \log(1 + |h_1|^2 \gamma_T)} \right\rceil, \left\lceil \frac{lR}{\log(1 + (|h_1|^2 + |h_2|^2) \gamma_T)} \right\rceil \right\} \right\}, \quad (15)$$

where $\lceil \cdot \rceil$ is the ceiling function, and $\gamma_T (= P_S/\sigma_T^2)$ denotes the signal-to-noise ratio (SNR) at the relay T , and σ_T^2 is the variance of the noise observed at the relay T . Note that the second and third terms in (15) are related to individual target rate of $R/2$ from the source S_1 and the target sum-rate R , respectively.

The high signal-to-noise ratio equivalent of (15) is given by

$$f = \min \left\{ 1, \max \left\{ \frac{r}{2(1 - u_1)^+}, \frac{r}{(1 - \min\{u_1, u_2\})^+} \right\} \right\}, \quad (16)$$

where $f(= l_d/l)$ is the fraction of symbol intervals during which the relay decodes the source messages and u_j is the exponential order of $1/|h_j|^2$.

4.2.1 Mode 2: Error event related to individual rate, $\mathcal{E}_1$

The probability of error corresponding to individual rate $P_{\mathcal{E}_1}$ is upper-bounded by [2]

$$P_{\mathcal{E}_1|g_1, g_2, g_r, h_1, h_2} \leq \left[1 + |g_1|^2 \frac{P_S}{2\sigma_D^2} \right]^{-l_d} \cdot \left[1 + |g_1|^2 \frac{P_S}{2\sigma_D^2} + |g_r|^2 \frac{P_S}{2\sigma_D^2} \right]^{-(l-l_d)}, \quad (17)$$

and the diversity gain corresponding to error event $\mathcal{E}_1$, $d_{\mathcal{E}_1}$ is given by

$$d_{\mathcal{E}_1} \triangleq \inf_{O_{\mathcal{E}_1}^+} \{v_1 + v_2 + v_r + u_1 + u_2\}, \quad (18)$$

where $O_{\mathcal{E}_1}^+$ is the nonnegative subset that satisfy the following constraints

$$f(1 - v_1)^+ + (1 - f)(1 - \min\{v_1, v_r\})^+ \leq \frac{r}{2}, \quad (19)$$

$$v_1 \geq \left(1 - \frac{r}{2}\right), \quad v_2 \leq \left(1 - \frac{r}{2}\right) \quad (20)$$

The diversity gain $d_{\mathcal{E}_1}$ in (18) can be expressed as

$$d_{\mathcal{E}_1}(r) = \inf_f (\lambda_{11}(r, f) + \lambda_{12}(r, f)) \quad (21)$$

where $\lambda_{11}(r, f)$ and $\lambda_{12}(r, f)$ are given by

$$\lambda_{11}(r, f) = \inf_{u_1, u_2 \in O_{\mathcal{E}_1}^+} (u_1 + u_2) \quad (22)$$

$$\lambda_{12}(r, f) = \inf_{v_1, v_2, v_r \in O_{\mathcal{E}_1}^+} (v_1 + v_2 + v_r) \quad (23)$$

By using (16), the solution to the optimization problem in (22) can be evaluated as

$$\lambda_{11}(r, f) \triangleq \begin{cases} 2\left(1 - \frac{r}{f}\right), & r \leq f < \frac{3r}{2} \\ 1 - \frac{r}{2f}, & \frac{3r}{2} \leq f \leq 1 \end{cases} \quad (24)$$

Also, by considering the non-trivial case of $v_1 \geq v_r$, $\{v_1, v_r\} \leq 1$, the linear optimization problem in (23) can be formally defined as

$$\begin{aligned} \lambda_{12}(r, f) &= \inf(v_1 + v_2 + v_r) \\ \text{s.t. } &fv_1 + (1 - f)v_r \geq \left(1 - \frac{r}{2}\right) \\ &\left(1 - \frac{r}{2}\right) \leq v_1 \leq 1, \quad 0 \leq v_2 \leq \left(1 - \frac{r}{2}\right) \\ &0 \leq v_r \leq 1, \quad v_1 \geq v_r, \end{aligned} \quad (25)$$

and by noticing that the optimal value for v_2 as zero based on the stand alone constraint ($0 \leq v_2 \leq 1 - r/2$), (25) can be rewritten as

$$\begin{aligned} \lambda_{12}(r, f) &= \inf(v_1 + v_r) \\ \text{s.t. } &fv_1 + (1 - f)v_r \geq \left(1 - \frac{r}{2}\right) \\ &\left(1 - \frac{r}{2}\right) \leq v_1 \leq 1, \quad 0 \leq v_r \leq 1, \quad v_1 \geq v_r, \end{aligned} \quad (26)$$

and the solution to the above optimization problem is given in the following lemma.

Lemma 1 : The solution of the optimization problem defined in (26) is given by

$$\lambda_{12}(r, f) = \begin{cases} 2 - r, & 0 \leq f < \frac{1}{2} \\ 2 - \frac{r}{2(1-f)}, & \frac{1}{2} \leq f \leq (1 - \frac{r}{2}) \\ \frac{2-r}{2f}, & (1 - \frac{r}{2}) \leq f \leq 1 \end{cases} \quad (27)$$

Proof: Please refer to the Appendix A. ■

Using (21), (24) and (27), the diversity gain considering the error event $P_{\mathcal{E}_1}$ can be written as

$$d_{\mathcal{E}_1}(r) = \begin{cases} 2 - r & 0 \leq r \leq \frac{1}{2} \\ \frac{4-5r}{2(1-r)} & \frac{1}{2} \leq r \leq \frac{2}{3} \\ \frac{2-r}{2r}, & \frac{2}{3} \leq r \leq 1 \end{cases} \quad (28)$$

4.2.2 Mode 2: Error event related to sum-rate, $\mathcal{E}_2$

Similar to the earlier case, the probability of error corresponding to sum-rate $P_{\mathcal{E}_2}$ is upper-bounded by

$$P_{\mathcal{E}_2|g_1, g_2, g_r, h_1, h_2} \leq \left[1 + (|g_1|^2 + |g_2|^2) \frac{P_S}{2\sigma_D^2} \right]^{-l_d} \cdot \left[1 + (|g_1|^2 + |g_2|^2 + |g_r|^2) \frac{P_S}{2\sigma_D^2} \right]^{-(l-l_d)}, \quad (29)$$

and the corresponding diversity gain ($d_{\mathcal{E}_2}$) is given by

$$d_{\mathcal{E}_2} \triangleq \inf_{O_{\mathcal{E}_2}^+} \{v_1 + v_2 + v_r + u_1 + u_2\}, \quad (30)$$

where $O_{\mathcal{E}_2}^+$ is the nonnegative subset that satisfy the following constraints

$$f(1 - \min\{v_1, v_2\})^+ + (1 - f)(1 - \min\{v_1, v_2, v_r\})^+ \leq r, \quad (31)$$

$$v_1 \geq \left(1 - \frac{r}{2}\right), \quad v_2 \leq \left(1 - \frac{r}{2}\right) \quad (32)$$

The diversity gain $d_{\mathcal{E}_2}$ in (30) can be expressed as

$$d_{\mathcal{E}_2}(r) = \inf_f (\lambda_{21}(r, f) + \lambda_{22}(r, f)) \quad (33)$$

where $\lambda_{21}(r, f)$ and $\lambda_{22}(r, f)$ are given by

$$\lambda_{21}(r, f) = \inf_{u_1, u_2 \in O_{\mathcal{E}_2}^+} (u_1 + u_2) \quad (34)$$

$$\lambda_{22}(r, f) = \inf_{v_1, v_2, v_r \in O_{\mathcal{E}_2}^+} (v_1 + v_2 + v_r) \quad (35)$$

Note that the $\lambda_{21}(r, f)$ depends only on the constraints defined in (16). Thus, $\lambda_{21}(r, f)$ is equal to $\lambda_{11}(r, f)$ and is given by (24). The function $\lambda_{22}(r, f)$ can be found by solving the following optimization problem:

$$\begin{aligned} \lambda_{22}(r, f) &= \inf \left(\left(1 - \frac{r}{2}\right) + v_2 + v_r \right) \\ \text{s.t. } &fv_2 + (1-f)v_r \geq (1-r) \\ &0 \leq v_2 \leq \left(1 - \frac{r}{2}\right) \\ &0 \leq v_r \leq 1, \quad v_2 \geq v_r, \end{aligned} \quad (36)$$

The corresponding solution is given in the following lemma.

Lemma 2 : *The solution of the optimization problem defined in (36) is given by*

$$\lambda_{22}(r, f) = \begin{cases} \left(1 - \frac{r}{2}\right) + 2(1-r), & 0 \leq f < \frac{1}{2} \\ \left(1 - \frac{r}{2}\right) + 2 - \frac{r}{2} \left(\frac{3-2f}{1-f}\right), & \frac{1}{2} \leq f \leq \frac{1-r}{1-\frac{r}{2}}, \\ \left(1 - \frac{r}{2}\right) + \frac{1-r}{f}, & \frac{1-r}{1-\frac{r}{2}} \leq f \leq 1 \end{cases} \quad (37)$$

and

$$\lambda_{22}(r, f) = \begin{cases} \left(1 - \frac{r}{2}\right) + 2(1-r), & 0 \leq f < \frac{1}{2} \\ \left(1 - \frac{r}{2}\right) + \frac{1-r}{f}, & \frac{1}{2} \leq f \leq 1 \end{cases} \quad (38)$$

Proof: Please refer to the Appendix B. ■

Using (24), (37), and (38), the diversity gain considering the error event $P_{\mathcal{E}_2}$ can be written as

$$d_{\mathcal{E}_2}(r) = \begin{cases} 3 - \frac{5r}{2}, & 0 \leq r < \frac{1}{2} \\ 5 - \frac{3r}{2} - \left(\frac{4-3r}{2(1-r)}\right), & \frac{1}{2} \leq r \leq 2 - \sqrt{2} \\ \left(1 - \frac{r}{2}\right) + \frac{1-r}{r}, & 2 - \sqrt{2} \leq r \leq 1 \end{cases} \quad (39)$$

4.2.3 Mode 2: Diversity gain (d_{O, M_2})

By noticing that the DMT given by (28) and (39) is better than the genie-aided tradeoff given in (8) for $r > 1/2$, the maximum achievable diversity gain based on mode M_2 characterization is given by:

$$d_{O, M_2}(r) = \begin{cases} 2 - r, & 0 \leq r \leq \frac{1}{2} \\ 3(1-r), & \frac{1}{2} \leq r \leq 1 \end{cases} \quad (40)$$

The next subsection derives the DMT for mode M_4 .

4.3 Mode M_4 : Achievable DMT

In this operating mode, none of the direct links can support the required rate. Hence, the relay T forwards the data from both the sources. The decoding interval l_d for this mode is given by

$$l_d = \min \left\{ l, \max \left\{ \left\lceil \frac{lR}{2 \log(1 + \min\{|h_1|^2, |h_2|^2\} \gamma_T)} \right\rceil, \left\lceil \frac{lR}{\log(1 + (|h_1|^2 + |h_2|^2) \gamma_T)} \right\rceil \right\} \right\}, \quad (41)$$

and the high signal-to-noise ratio equivalent of (41) is given by

$$f = \min \left\{ 1, \max \left\{ \frac{r}{2(1 - \max\{u_1, u_2\})^+}, \frac{r}{(1 - \min\{u_1, u_2\})^+} \right\} \right\} \quad (42)$$

The proposed system operates in mode M_4 if the constraints in (6) are satisfied. The corresponding high SNR equivalents are given as:

$$\begin{aligned} \log(1 + |g_1|^2 \gamma) &< \frac{R}{2} \Leftrightarrow v_1 > \left(1 - \frac{r}{2}\right) \\ \log(1 + |g_2|^2 \gamma) &< \frac{R}{2} \Leftrightarrow v_2 > \left(1 - \frac{r}{2}\right), \end{aligned} \quad (43)$$

by incorporating the above constraints and also by considering the non-trivial case of $v_1 \geq v_r$, $\{v_1, v_r\} \leq 1$, the linear optimization problem for $\lambda_{41}(r, f)$ ($= \inf(v_1 + v_2 + v_r)$) related to the error event $\mathcal{E}_1$ in mode M_4 can be formally defined as

$$\begin{aligned} \lambda_{41}(r, f) &= \inf(v_1 + v_2 + v_r) \\ \text{s.t } f v_1 + (1 - f) v_r &\geq \left(1 - \frac{r}{2}\right) \\ \left(1 - \frac{r}{2}\right) &< v_1, v_2 \leq 1 \\ 0 &\leq v_r \leq 1, \quad v_1 \geq v_r, \end{aligned} \quad (44)$$

Hence, $\lambda_{41}(r, f)$ is always greater than $(2-r)$ which in turn greater than the diversity gain given in (40). Now, by following the same argument, $\lambda_{42}(r, f)$ ($= \inf(v_1 + v_2 + v_r)$) related to the error event $\mathcal{E}_2$ in mode M_4 is also greater than $2-r$. In other words, the maximum achievable diversity gain of LF-DDF system hinges on the DMT corresponding to mode M_2 . Thus, by using (40), the DMT of LF-DDF protocol is given as:

$$d_{LF-DDF}(r) = \begin{cases} 2 - r, & 0 \leq r \leq \frac{1}{2} \\ 3(1 - r) & \frac{1}{2} \leq r \leq 1 \end{cases} \quad (45)$$

Fig. 3 DMT of different relaying schemes

5 DISCUSSIONS

LF based DDF achieves the optimal tradeoff across all multiplexing gains and hence dominates the DMT of traditional DDF protocol characterized by [3, Equation 21].

The DMT comparison for the MARC is shown in Fig. 3. As evident from the Fig. 3, DDF and MAF achieves DMT upper bound for $r \leq 2/3$ and $r \geq 2/3$, respectively. However, LF-DDF scheme is optimal across all the multiplexing gains. This result can be intuitively interpreted as follows. As observed in the earlier section, the channel conditions in which relay receives an ACK for one source and a NACK for another source determines the overall DMT of the system. In this case, the relay in traditional DDF protocol continues to forward the data of both the users. Whereas in LF-DDF, relay aids the source whose direct link is weak. The relaxation of relay forwarding constraint helps in achieving the optimality (note the absence of constraints related to v_2 in Fig. 4(a), Fig. 4(b), and Fig. 4(c) in Appendix A).

In [3], authors proved that the DDF with at least two ARQ rounds (ARQ-DDF) achieves the optimality across all multiplexing gains. However, the constraint on ARQ rounds limits the usefulness of protocol in real-time latency constrained applications. Similarly, Abouelseoud et al. [1] proposed an opportunistic CF scheme which achieves genie-aided diversity. However, this CF scheme requires the complex processing at the relay apart from the selec-

tion overhead involved in opportunistic communication. We can see that the proposed LF-DDF incur only a minute overhead compared to classical DDF protocol while achieving full diversity across all multiplexing gains for MARC.

6 Conclusion

In this work, the diversity-multiplexing tradeoff of limited feedback based DDF for MARC has been derived, and its performance has been compared with that of traditional DDF and MAF relaying schemes. Our results conclude that the incorporation of limited feedback drastically improves the performance of DDF relaying for MARC. Finally, it is shown that the proposed scheme achieves maximum achievable diversity gain across all multiplexing gains.

A Proof of Lemma 1

The constraint region corresponding to linear programming problem defined in (26) can be characterized into three parts based on the value of f . These possibilities are graphically depicted in Fig. 4(a), Fig. 4(b) and Fig. 4(c).

Fig. 4 Outage regions for individual rate related error event ($\mathcal{E}_1$) in Mode M_2

1. $0 \leq f \leq \frac{1}{2}$: Using Fig. 4(a), we can conclude that the optimal values $v_1^* = v_r^* = 1 - \frac{r}{2}$, i.e.,

$$\inf(v_1 + v_2 + v_r) = 2 \left(1 - \frac{r}{2}\right) = 2 - r \quad (46)$$

2. $\frac{1}{2} \leq f \leq (1 - \frac{r}{2})$: As shown in Fig. 4(b), solving (26) for v_r^* at $v_1^* = 1$ gives

$$v_r = 1 - \frac{r}{2(1-f)},$$

thus

$$\inf(v_1 + v_2 + v_r) = 2 - \frac{r}{2(1-f)} \quad (47)$$

3. $(1 - \frac{r}{2}) \leq f \leq 1$: As can be seen from Fig. 4(c), in this case $v_1^* = \frac{2-r}{2f}$ and $v_r^* = 0$, hence

$$\inf(v_1 + v_2 + v_r) = \frac{2-r}{2f} \quad (48)$$

Finally, by using (46) (47) and (48), we get (27) and thus completes the proof. $\blacksquare$

B Proof of Lemma 2

We follow essentially similar steps as in the case of individual rate related error event. For $r \leq 2/3$, the constraint region corresponding to the linear programming problem defined in (36) can be characterized into three distinct portions based on the value of f . These possibilities are graphically depicted in Fig. 5(a), Fig. 5(b) and Fig. 5(c). However for $r > 2/3$, the region depicted in Fig. 5(b) is subsumed in to the region defined by Fig. 5(a), i.e., $\frac{1-r}{1-\frac{r}{2}} \leq \frac{1}{2}$.

Fig. 5 Outage regions for sum-rate related error event ($\mathcal{E}_2$) in Mode M_2

1. $0 \leq f \leq \frac{1}{2}$: Using Fig. 5(a), we can conclude that the optimal values $v_1^* = v_r^* = 1 - \frac{r}{2}$, i.e.,

$$\inf(v_1 + v_2 + v_r) = 2(1-r) = \left(1 - \frac{r}{2}\right) + 2(1-r) \quad (49)$$

2. $\frac{1}{2} \leq f \leq \frac{1-r}{1-\frac{r}{2}}$: As shown in Fig. 5(b), solving (36) for v_r^* at $v_1^* = 1 - \frac{r}{2}$ gives

$$v_r = 1 - \frac{r}{2} \left(\frac{2-f}{1-f} \right),$$

thus

$$\inf(v_1 + v_2 + v_r) = \left(1 - \frac{r}{2}\right) + 2 - \frac{r}{2} \left(\frac{3-2f}{1-f} \right) \quad (50)$$

3. $\frac{1-r}{1-\frac{r}{2}} \leq f \leq 1$: As can be seen from Fig. 5(c), in this case $v_1^* = \frac{1-r}{f}$ and $v_r^* = 0$, hence

$$\inf(v_1 + v_2 + v_r) = \left(1 - \frac{r}{2}\right) + \frac{1-r}{f} \quad (51)$$

Finally, by using (49) (50) and (51), we get (37) and (38) and thus completes the proof. ■

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