Investigation of accurate fringe analysis techniques for fringe projection profilometry

Agarwal Nimisha

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**NANYANG
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SINGAPORE

**INVESTIGATION OF ACCURATE FRINGE
ANALYSIS TECHNIQUES FOR FRINGE
PROJECTION PROFILOMETRY**

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SCHOOL OF COMPUTER SCIENCE AND ENGINEERING

2019

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FOR FRINGE PROJECTION PROFILOMETRY

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ACCURATE FRINGE ANALYSIS
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PROJECTION PROFILOMETRY**

**AGARWAL NIMISHA
2019**

School of Computer Science and Engineering

“A thesis submitted to Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy”

2019

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Prof. Qian Kemao

Authorship Attribution Statement

This thesis contains material from 5 papers published in the following peer-reviewed journals / from papers accepted at conferences in which I am listed as an author.

Chapter 3 is published in [1] as [N Agarwal, C Wang and Q Kemao, “Windowed Fourier ridges for demodulation of carrier fringe patterns with nonlinearity: a theoretical analysis,” Applied Optics, Vol. 57 \(21\), 6198-6206 \(2018\).](#)

The explicit contributions are listed below as:

- Prof Qian Kemao provided the initial project direction, guided the work closely, supervised in performing simulation-based validation and further supervised in editing the manuscript drafts.
- I performed the theoretical study to eliminate nonlinear gamma errors using Windowed Fourier Ridges (WFR). I also performed an experimental validation to justify the theoretical analysis for a real scenario and wrote the paper.
- Dr. Chenxing Wang guided the work and revised the paper.

Chapter 4 contains two parts: (a) WFR as a carrier squeezing technique which has been published in [2], and (b) Background removal which is presented in [3] as [N Agarwal, C Wang, H Wang and Q Kemao, “Windowed Fourier Filtering based background intensity removal for fringe enhancement,” IcOpen \(2018\).](#)

The contributions of both these publications are as follows:

- For (a), Prof Qian Kemao provided the initial inspiration and for (b), Dr Chenxing Wang provided the initial inspiration to work on background removal.
- Since gamma nonlinearity has already been analyzed in the previous work, here, I under the supervision of Prof. Qian Kemao analyzed a carrier squeezing technique (CSI) for nonlinear gamma removal to use WFR for the same.
- I also, analyzed the intensity of background removal in existing fringe patterns. Later, I designed some single carrier fringe patterns with discontinuous background using simulation. Under the supervision of Prof Qian Kemao, I designed a novel Windowed Fourier Filtering (WFF) based background removal technique in MATLAB, and validated the effectiveness of the proposed algorithm using both simulations and experiments. I also wrote the paper.
- Dr. Chenxing Wang helped to replicate the state-of-the-art EMD techniques for background removal and guided the work closely.

- Dr. Haixia Wang guided the work and revised the paper.
- Prof. Qian Kemao guided the work closely and revised the paper.

Chapter 5 is published in [4] as R. Shien, A. Nimisha, W. Qianghua and K. Qian, “Comparative study of sampling Moiré and windowed Fourier transform techniques for demodulation of a single fringe pattern,” *Applied Optics*, Vol. 57 (36), 10402-10411 (2018).

The contributions of this publication are as follows:

- Dr Shien Ri provided the initial inspiration and background knowledge to perform a comparative study between two state-of-the-art fringe analysis techniques namely, Sampling Moiré (SM) and Windowed Fourier Transform (WFT).
- All the guidance and the direction to perform this study was provided by Prof. Qian Kemao.
- Dr. Shien Ri and Qinghua Wang supervised me throughout this work. Under their guidance and under the supervision of Prof Qian Kemao, I worked on analyzing both the algorithms namely, Sampling Moire (SM) and Windowed Fourier Ridge (WFR) and performed a comparative study to review the efficiency and effectiveness in tackling existing issues.
- I along with Dr. Shien Ri replicated Sampling Moire (SM) algorithm and we validated the simulation and experimental results for both WFR and SM algorithms.
- I wrote the paper which was further revised together with Dr. Shien Ri, Dr. Qinghua Wang and Prof. Qian Kemao.

Chapter 6 is published in [5] as A. Nimisha and K. Qian, “Windowed Fourier ridges as a spatial carrier phase-shifting algorithm,” *Optical Engineering*, Vol. 56 (8), 080501 (2017). <https://doi.org/10.1117/1.OE.56.8.080501>

The contributions of this publication are as follows:

- The project direction and inspiration came from Prof. Qian Kemao who further supervised in editing the manuscript drafts.
- In order to set up a link between two major fringe analysis methods, Prof Qian Kemao analyzed the two algorithms: Windowed Fourier Ridges (WFR) and Spatial Carrier Phase Shifting (SCPS) both mathematically. He also performed the simulation-based analysis to justify the link between the two algorithms.

- I validated the mathematical analysis and simulation based performance. I also performed experimental results to justify the effectiveness of the proposed algorithm.
- Manuscript drafts were then prepared and further revised along with Prof. Qian Kemao.

16/04/2020

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Summary

Optical 3D measurement techniques have been studied and developed for a long time in literature now due to its non-contact, high speed, high accuracy, and sensitivity, for 3D measurements. Fringe pattern analysis (FPA) is an important aspect of all optical measurement techniques. Fringe patterns are usually generated by a well-known phenomenon of optical interference. The surface of an object is then, reconstructed by analyzing the phase distribution of these fringe patterns- a process called phase extraction. To analyze the phase distributions in both spatial and spectral domains, pixel-wise phase shifting and global Fourier transform have been established as the two pillar techniques for fringe analysis. To use both the properties simultaneously, Windowed Fourier Transform (WFT) has emerged as an in-between technique to analyze the fringe patterns block by block.

In the past decade, FPA has made great progress for 3D measurements. Several methods have been developed for 3D surface measurement based on varied principles. These include time of flight, laser triangulation, stereo vision, shape from focus and defocus, structured light projections and digital fringe projection (DFP). Among these, structured light fringe projections and digital fringe projections (DFP) are commonly used in fringe projection profilometry (FPP) systems due to their simple setup and the ability to achieve high-speed and high-accuracy. However, two key issues still prevail which need further improvement. First, with the rapid demand for 3D reconstruction of object surfaces which can be readily applied and used in various applications such as virtual reality, animations, a precision inspection of optical components and many more, obtaining a high-speed and highly accurate surface measurement has become a necessity. Second, with several

fringe analysis methods being developed over the past decades, it has become increasingly difficult to understand, analyze and introspect some of the best-suited applications and conditions where different fringe analysis methods can prove useful and thus, requires a link to be set up between different algorithms for further investigation. Both these issues revolve around a central idea: robustness and high-accuracy 3D measurement. Since the spectral information of a patch of fringe pattern is much simpler to analyze and thus, more effective, instead of resolving some of the major error sources in existing FPP systems in spatial and spectral domain separately, in this thesis, author proposes to use the “local” in-between Windowed Fourier Transform (WFT) technique for the same with an agenda to be able to investigate the link between some of the best suited techniques for various applications under certain measurement conditions.

First, some state-of-the-art fringe analysis techniques are reviewed. Also, some major issues in fringe analysis for FPP systems are inspected such as nonlinear gamma, uneven background, edge discontinuity, and high dynamic range, and the existing state-of-the-art techniques for resolving these issues including both the phase-shifting and transform-based methods in the past decades are reviewed. It is found that the accuracy of measurement can be improved by either using a large number of fringe patterns or by iterating the algorithm many times thus, leaving scope for further improvement.

Second, to resolve these error issues in fringe analysis one by one, we first try to tackle nonlinear gamma errors caused by the nonlinear nature of the devices-projector and camera. A theoretical analysis for resolving the nonlinear gamma errors using Windowed Fourier Ridges (WFR) for carrier phase demodulation is then proposed. It first, analyzes the local frequency error and phase extraction error for the carrier fringe pattern in order to depict the amount of gamma non-linearity and then, resolves the error with the help of a performance prediction and error control using WFR. It was observed that despite being challenging it is possible to reduce the peak phase error to be as low as 0.05 rad in the presence of the error source.

Third, after tackling the issue of nonlinear gamma in single carrier fringe pattern, we then first try to analyze nonlinear gamma in phase-shifting technique and thus, resolve the error by using WFR for phase-shifted fringe patterns which are squeezed to a single carrier fringe pattern. After that, uneven and discontinuous background is chosen as the second error source which needs to be resolved. To tackle this issue in a single carrier fringe pattern, a Windowed Fourier filtering (WFF) based background removal technique is proposed. It uses a mirroring approach to tackle the error around the corner pixels. An ideal window of $(50, 50)$ is used for both the algorithms for effectively resolving the error sources.

Fourth, after discussing the two major issues in fringe analysis, a comparative study is then performed between two state-of-the-art fringe analysis techniques - WFR and Sampling Moiré (SM), to analyze the effectiveness of the two algorithms for a single carrier fringe pattern for the very first time. While SM converts a single carrier fringe pattern into multiple phase-shifted Moiré patterns to demodulate phase, WFR obtains the phase information in the window-based Fourier domain. Despite appearing two completely different approaches for fringe analysis and despite having separate applications, both simulations and experimental results confirm that both the algorithms appear as two sides of the same coin and thus, can be used interchangeably in various applications. In the presence of noise, both algorithms give outstanding performance.

Finally, a link between WFR and spatial carrier phase-shifting (SCPS) algorithm is proposed for the very first time. Since phase-shifting and Fourier transform are the two pillar techniques for fringe analysis, and as Windowed Fourier transform has already been established as the extension of the Fourier transform, it is crucial to understand the link between the two, in order to select a proper technique for effective fringe analysis. It is observed that whenever SCPS is used to analyze a fringe pattern, WFR can always be attempted for better results.

Key Words: Optical measurement, digital fringe projection (DFP), carrier fringe pattern, Nonlinear gamma, Windowed Fourier ridges (WFR), Windowed Fourier

filtering (WFF), Sampling Moiré (SM), Spatial carrier phase shifting (SCPS), background removal, carrier phase demodulation

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Chapter 1

Introduction

1.1 Background

With the desire to move closer to the real world and bridge the gap between the two-dimensional (2D) and three-dimensional (3D) analysis, there is a rapid requirement for high-speed and highly accurate 3D measurements. Accurate 3D measurement is nowadays in demand for a number of application fields such as mechanical engineering, metallurgy, material science, aerospace industry, non-destructive testing, bio-engineering (to measure the refractive index of cells and the size of drug particles), military (such as terrain evaluation by synthetic aperture radar), computer graphics and animations, virtual and augmented reality etc. Optical interferometric techniques for 3D surface measurements are useful because they provide non-contact and full-field measurements, which are useful and attractive in both research and engineering fields.

Optical interferometry and optical 3D measurement techniques family includes Moiré interferometry [6], photo-elasticity[7], speckle interferometry [8–10], holographic interferometry [8,11] and classical interferometry [12,13]. These techniques have been rigorously developed and widely used in the past decades due to their non-contact, full-field and high accuracy nature which eliminated the issue of point-by-point probe-based measurements. The difficulty in processing fringe patterns turned the research interest towards fringe projection and analysis using optical interferometry.

1.2 Fringe Pattern Analysis

Optical interferometry and optical 3D measurement techniques generally output either a single fringe pattern or multiple fringe patterns which needs to be analyzed to extract the phase distribution to measure the 3D surface of an object. Thus, fringe pattern analysis is a crucial step in all optical measurements and has been studied over decades [14–26].

1.2.1 Fringe pattern generation model

Fringe patterns are usually generated by optical interference of waves[12, 13, 27]. In optical measurements, fringe patterns can be mathematically modeled as:

$$f(x, y; t) = a(x, y; t) + b(x, y; t)\cos[\varphi(x, y; t)] + n(x, y; t) \quad (1.1)$$

where (x, y) are the pixel coordinates and t is the temporal coordinate, $a(x, y; t)$, $b(x, y; t)$ and $\varphi(x, y; t)$ are the background intensity, amplitude of the fringe pattern and desired phase information respectively, and $n(x, y; t)$ is the noise.

Throughout this dissertation we will use a spatial fringe pattern, thus, we ignore the temporal coordinate t and the fringe pattern can thus, be expressed as:

$$f(x, y) = a(x, y) + b(x, y)\cos[\varphi(x, y)] + n(x, y) \quad (1.2)$$

The purpose of fringe analysis is to retrieve the phase information $\varphi(x, y)$. Since the number of unknowns in Eq. 1.2 such as $a(x, y)$, $b(x, y)$ and $n(x, y)$ are more than the unknown $\varphi(x, y)$, thus, it becomes difficult to directly retrieve phase.

Fringe projection profilometry (FPP) uses structured light fringe projection to project the fringe patterns on the object surface and thus, measure the object surface through the distortion of fringe patterns [28]. The FPP system comprises of a camera and a projector to project computer-generated fringe patterns on the object surface. Fringe patterns are mainly categorized into four types.

1.2.1.1 Exponential phase fields

An exponential fringe pattern comprises of only the exponential term and can be mathematically modeled as:

$$f(x, y) = b(x, y) \exp[j\varphi(x, y)] \quad (1.3)$$

Phase information can be easily extracted from these signals as:

$$\hat{\varphi}(x, y) = \text{atan2} \left[\frac{\text{Im}[f(x, y)]}{\text{Re}[f(x, y)]} \right] \quad (1.4)$$

where $\text{Re}[\cdot]$ and $\text{Im}[\cdot]$ are the real and imaginary parts of the fringe pattern respectively. The retrieved phase map is wrapped due to the characteristic of arc-tangent function between $[-\pi, \pi]$. For simplicity, we replace atan2 by $\arctan$ or $\tan^{-1}$ function and is used interchangeably in this dissertation.

Exponential phase fields (EPFs) [10] are rather easy to process which can be directly obtained by using multiple fringe patterns which are similar to each other and differ by only a shift. These multiple fringe patterns are collectively used in phase shifting techniques and can be modeled as:

$$f_i(x, y) = a(x, y) + b(x, y) \cos[\varphi(x, y) + \delta_i] \quad i = 1, 2, \dots, N \quad (1.5)$$

where δ_i is the amount of phase shift and N is the total number of fringe patterns. There are various phase shifting techniques depending on the number of fringe patterns used and the type of phase shift introduced. N-step phase shifting techniques [29–38] require a known phase-shift. A typical 4-step phase shifting algorithm uses 4 fringe patterns with a phase shift $\delta_i = (i - 1)\frac{\pi}{2}$. The EPFs for this can thus, be obtained by a combination of $\frac{1}{2}[f_1(x, y) - f_3(x, y) + jf_2(x, y) - jf_4(x, y)]$ resulting in Eq. 1.3. EPFs can also be used for processing a single fringe pattern for simplicity.

1.2.1.2 Carrier Fringe Patterns

When analyzing the fringe patterns in frequency domain, two peaks (fundamental frequency components) can be observed without overlapping if the fringe patterns

have high-frequency. The phase then can be easily retrieved by using any of the fundamental frequency components. However, if the fringe pattern is not dense enough i.e. the frequency of fringe pattern is low, overlapping might occur making the phase retrieval process difficult as the two fundamental components cannot be separated. Thus, a carrier frequency is usually added in such fringe patterns to increase the frequency and be able to obtain two separate peaks and such patterns are named as Carrier fringe patterns [14].

Carrier fringe patterns can be expressed as:

$$\begin{aligned} f(x, y) &= a(x, y) + b(x, y)\cos[\omega_{cx}x + \omega_{cy}y + \varphi(x, y)], \\ &= a(x, y) + c(x, y)\exp[j(\omega_{cx}x + \omega_{cy}y)] + c^*(x, y)\exp[-j(\omega_{cx}x + \omega_{cy}y)], \end{aligned} \quad (1.6)$$

where ω_{cx} and ω_{cy} are the carrier frequencies along x and y directions respectively, superscript $*$ is the conjugate of a complex number and is used uniformly throughout this dissertation and $c(x, y) = \frac{b(x, y)}{2}\exp[j\varphi(x, y)]$ which in a way is itself an EPF.

To retrieve the phase from carrier fringe patterns, Fourier transform [28, 39–42], windowed Fourier transform [43–48] and Wavelet transform [49–52] are often used. For this, fringe patterns need to be analyzed in the frequency domain to be able to use any one of the fundamental peaks. We will use carrier fringe patterns widely in this dissertation due to its simplicity.

1.2.1.3 Wrapped Phase maps

A wrapped phase map can be estimated once the EPF is obtained as:

$$f(x, y) = \varphi_w(x, y) \quad (1.7)$$

indicating that the wrapped phase map is the angle of an EPF which needs to be further processed and unwrapped to obtain a continuous phase map for 3D surface reconstruction. This wrapped phase map can be easily converted to an EPF by keeping $b(x, y) = 1$.

1.2.1.4 Single Closed fringe patterns

Fringe patterns without a carrier are termed as closed fringe patterns which can be represented as:

$$f(x, y) = a(x, y) + b(x, y)\cos[\varphi(x, y)] + n(x, y) \quad (1.8)$$

Retrieving phase information from closed fringe patterns [53–55] is comparatively difficult than the above mentioned three fringe patterns due to the lack of a temporal or spatial carrier frequency.

Figure 1.1: An Example of a simulated fringe pattern: (a) simulated fringe pattern, (b) wrapped phase map and (c) continuous unwrapped phase. Image courtesy [26].

Figure 1.1(a) shows a simulated fringe pattern with background intensity $a(x, y)$ and fringe amplitude $b(x, y)$. Figure 1.1(b) is the wrapped phase map which is the extracted phase information from the fringe analysis. Due to the special characteristic of the cosine function, the phase obtained is usually in an interval of $(-\pi, \pi]$ and thus is wrapped. In order to reconstruct a smooth phase distribution as shown in figure 1.1(c), a phase unwrapping procedure is incorporated.

1.2.2 Phase-shifting Technique

The Phase-shifting [29–38] and Fourier transform techniques [28, 39–42] discussed in the following sections are the two main pillars of fringe analysis techniques due to their simplicity and high-accuracy. Phase-shifting techniques use multiple fringe

Figure 1.2: Four step phase shifting fringe patterns of an angel sculpture. (a) Fringe image I1 ($\delta = 0$) (b) Fringe image I2 ($\delta = 90^\circ$) (c) Fringe image I3 ($\delta = 180^\circ$) (d) Fringe image I4 ($\delta = 270^\circ$).

patterns for analysis which are created with a phase shift as in Eq. 1.5. Phase-shifts in optical interferometry are usually introduced by moving a mirror attached to a Piezoelectric transducer (PZT). However, in Digital fringe projection (DFP), a known phase shift is generally introduced by computer. For example, a four-step phase shifting algorithm [38] as shown in figure 1.2, with a known phase shift $\delta_i = (i - 1)\frac{\pi}{2}$ can be performed as:

$$\begin{aligned}
 f_0(x, y) &= a(x, y) + b(x, y)\cos[\varphi(x, y)], \\
 f_1(x, y) &= a(x, y) + b(x, y)\cos\left[\varphi(x, y) + \frac{\pi}{2}\right], \\
 f_2(x, y) &= a(x, y) + b(x, y)\cos[\varphi(x, y) + \pi], \\
 f_3(x, y) &= a(x, y) + b(x, y)\cos\left[\varphi(x, y) + \frac{3\pi}{2}\right].
 \end{aligned} \tag{1.9}$$

The phase can thus be retrieved, assuming the background and amplitude are constant, as:

$$\varphi(x, y) = \text{angle} \left[\frac{f_3(x, y) - f_1(x, y)}{f_0(x, y) - f_2(x, y)} \right] \quad (1.10)$$

Temporal phase-shifting (TPS) [56], spatial carrier phase shifting (SCPS) [37] and spatial phase shifting (SPS) [29–36, 38] techniques are majorly used. TPS records the fringe patterns at different time instances. Phase shifts thus are temporal carriers. In order to use phase shifts as spatial carriers, a method called spatial carrier phase shifting (SCPS) is used [37]. In spatial phase-shifting techniques, phase-shifted fringe patterns are simultaneously acquired at the same time instance for each (x, y) at different locations. For effective phase retrieval, researchers usually consider a number of fringe patterns to be $N \geq 3$.

1.2.3 Transform-based techniques

Transform-based techniques are another successful fringe analysis technique that analyzes a carrier fringe pattern into the frequency domain. A number of transform-based techniques [28, 39–42, 57–59] have been reported in the literature. These techniques require either a temporal or a spatial carrier to separate the components in the frequency domain. Transform-based techniques are further categorized as: Fourier transform techniques, Windowed Fourier transform techniques and Wavelet transform techniques.

1.2.3.1 Fourier Transform techniques

Fourier transform method was first proposed by Takeda in 1982 [39] for a 1D fringe pattern. Later, a number of Fourier transform methods have been developed [40, 41, 60–68] for 2D and 3D fringe pattern analysis. Fourier transform is the most widely used technique due to its simple implementation and dense analysis. A 2D forward and inverse Fourier transform pair is defined as:

$$Ff(\xi_x, \xi_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \exp(-j\xi_x x - j\xi_y y) dx dy \quad (1.11)$$

$$f(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Ff(\xi_x, \xi_y) \exp(j\xi_x x + j\xi_y y) d\xi_x d\xi_y \quad (1.12)$$

where $Ff(\xi_x, \xi_y)$ is the Fourier transform of $f(x, y)$ and $\xi = (\xi_x, \xi_y)^T$ is the frequency coordinate in Fourier domain. For a carrier fringe pattern $f(x, y)$ defined in Eq. 1.6, Fourier transform gives the spectrum of $f(x, y)$ as:

$$Ff(\xi_x, \xi_y) = Fa(\xi_x, \xi_y) + Fc(\xi_x - \omega_{cx}x, \xi_y - \omega_{cy}y) + Fc^*(\xi_x - \omega_{cx}x, \xi_y - \omega_{cy}y) \quad (1.13)$$

where F is the Fourier transform. Due to the presence of carrier, the three components $Fa(\xi_x, \xi_y)$, $Fc(\xi_x - \omega_{cx}x, \xi_y - \omega_{cy}y)$ and $Fc^*(\xi_x - \omega_{cx}x, \xi_y - \omega_{cy}y)$ can be easily separated in the frequency domain. The phase can then be retrieved from the fundamental component $c(x, y)$ by applying a band-pass filter centered at $(\omega_{cx}x, \omega_{cy}y)$ such that only $Fc(\xi_x - \omega_{cx}x, \xi_y - \omega_{cy}y)$ remains after filtering. Therefore, the fundamental component $c(x, y)$ is obtained after inverse Fourier transform, and the phase can be extracted as in the case of EPF as:

$$\varphi(x, y) = \text{angle}[c(x, y)], \quad (1.14)$$

1.2.3.2 Windowed Fourier Transform techniques

In optical measurements, the point undergoing measurement shows a significantly low relation with the point far off in comparison to a point close-by. This local similarity is useful in suppressing noise and estimating local parameters. Thus, analyzing a fringe pattern window-by-window or block-by-block is useful and this approach is called Windowed Fringe pattern analysis.

Windowed Fourier transform (WFT) is an extension of Fourier transform [26, 45–47], which analyzes the fringe pattern window-by-window in the frequency domain. Since, Fourier transform is a global operator, it is affected by signals in various positions. Thus, in order to make use of the local properties in a fringe pattern, we need both the spatial and frequency information which is where Windowed Fourier transform (WFT) or short-term Fourier transform (STFT) comes into play. A 2D WFT and inverse WFT pair can be expressed as:

$$Sf(u, v; \xi_x, \xi_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)g(x - u, y - v)\exp(-j\xi_x x - j\xi_y y)dx dy, \quad (1.15)$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Sf(u, v; \xi_x, \xi_y)g(x - u, y - v)\exp(j\xi_x x + j\xi_y y)d\xi_x d\xi_y \quad (1.16)$$

where $Sf(u, v; \xi_x, \xi_y)$ denotes the WFT spectrum and $g(x, y)$ is the Gaussian window function defined as:

$$g(x, y) = \exp\left(\frac{-x^2}{2\sigma^2} - \frac{y^2}{2\sigma^2}\right) \quad (1.17)$$

where σ controls the extension of $g(x, y)$.

The two main approaches for Windowed Fourier transform are: Windowed Fourier Ridges (WFR) and Windowed Fourier filtering (WFF). WFR technique is based on the selection of the best possible frequency match in the fringe pattern while WFF is based on filtering. These two approaches are discussed in detail in the next chapter.

1.2.3.3 Wavelet Transform techniques

Wavelet transform (WT) is another time-frequency analysis method for fringe demodulation which was first studied for interferometric measurements in [18]. An adaptive window is used in WT which changes corresponding to a frequency where some techniques use the phase gradient directly from the WT to obtain the phase distribution of the fringe patterns without any unwrapping [50, 51, 69]. Several wavelet transform methods for fringe analysis have been proposed in literature over time [51, 52, 70, 71]

A continuous 1D Wavelet transform (WT) is defined as:

$$W(a, b) = \int_{-\infty}^{\infty} f(x) M_{a,b}^*(x) dx \quad (1.18)$$

where $a > 0$ is the parameter related to frequency, b is the shift parameter at the position (x) and $M_{a,b}^*(x) = \frac{1}{a} M\left[\frac{x-b}{a}\right]$ is obtained by shifting and scaling the Morlet wavelet function $M(x)$. Morlet wavelet comprises of both a sine wave and a cosine wave which can further be modulated by a Gaussian function as:

$$M(x) = \frac{1}{\sqrt[4]{\pi}} \sqrt{\frac{2\pi}{\gamma}} \exp\left[-\frac{(2\pi/\gamma)^2 x^2}{2} + j2\pi x\right] \quad (1.19)$$

where $\gamma = \pi\sqrt{2/\ln 2}$. 1D phase $\varphi(b)$ can thus, be retrieved as:

$$\varphi(b) = \arctan \left[\frac{\text{Im}Wf(a_{r,b}, b)}{\text{Re}Wf(a_{r,b}, b)} \right] \quad (1.20)$$

where $a_{r,b}$ is the scale parameter a relating to the ridge point at position b . It can be further extended to a 2D-WT by taking a tensor product of two 1D-WT for both x and y directions.

1.2.4 Phase Unwrapping

After analyzing the fringe patterns by using either phase-shifting or transform-based methods, the phase retrieved $\varphi_w(x, y)$ is rather discontinuous where $\varphi_w(x, y) \in (-\pi, \pi]$. In order to recover the continuous phase $\varphi(x, y)$ for surface measurement, we need to unwrap the phase by a process called phase unwrapping. It has been an active topic for research over decades and several algorithms have been proposed to tackle the challenges in phase unwrapping effectively [37, 72–76]

There exist a simple relationship between the two phases:

$$\varphi(x, y) = \varphi_w(x, y) + 2k(x, y)\pi, k(x, y) \in Z \quad (1.21)$$

where the important factor is $k(x, y)$ to unwrap the phase and can be estimated as:

$$k(x_i, y_i) = \text{round} \left[\frac{\varphi(x_{i-1}, y_{i-1}) - \varphi_w(x_i, y_i)}{2\pi} \right] \quad (1.22)$$

where $\text{round}(\cdot)$ rounds a decimal to its nearest integer and we assume that all the neighboring pixels (x_{i-1}, y_{i-1}) have already been unwrapped with the unwrapped phase $\varphi(x_{i-1}, y_{i-1})$ and thus, we use these pixels to unwrap the phase at the current pixel (x_i, y_i) with a wrapped phase $\varphi_w(x_i, y_i)$.

The phase at location (x_i, y_i) can thus, be unwrapped as:

$$\varphi(x_i, y_i) = \varphi_w(x_i, y_i) + \text{round} \left[\frac{\varphi(x_{i-1}, y_{i-1}) - \varphi_w(x_i, y_i)}{2\pi} \right] \cdot 2\pi \quad (1.23)$$

Determining unwrapped phase in this way appears simple, however, the accuracy of unwrapping depends on determining $k(x_i, y_i)$. Since it is dependent on the

previous pixels, hence, if one pixel goes wrong, the phase of all subsequent pixels will then be bad leading to poor quality reconstruction. Hence, phase unwrapping is a challenging task. Besides noise, there could be some true-phase discontinuities during fringe analysis due to the presence of multiple disjoint objects. Hence, temporal phase unwrapping appears to be the most suited approach to tackle true phase discontinuities which assume that the phase is continuous in the temporal domain and the unwrapped phase is thus, obtained by analyzing the phase difference between the consecutive frames [73].

To achieve high-accuracy during measurement, adopting an adequate and efficient phase unwrapping technique is an essential task.

1.3 Digital fringe projection

Due to non-contact and full-field nature of measurements, optical metrology has been extensively employed and studied. There have been an extensive research in this field and several methods have been developed to attain 3D surface measurements namely, “time of flight” [77], “stereo vision” [78–82], “shape from shading” [83], “laser scanning” [84, 85], “shape from focus and defocus” [86, 87], “shape from texture” and “structured light” [88–93]. Among these optical techniques, “stereo vision” is the most widely studied and used technique. It basically tries to replicate the functioning of the human eye and thus, uses two cameras- left and right. It then captures two images with the help of which it tries to find the correspondence between the two same pixel points in the images to obtain 3D world coordinates of that point. In this fashion, it then replicates the whole 3D surface. However, finding a highly matched correspondence is the biggest issue here. By replacing one camera with a digital projector and projecting structured light fringe patterns onto the object surface, fundamentally solves the correspondence problem and thus, leads to the generation of structured light projection systems [37, 94]. Further making use of the digital fringe projection technology and phase analysis of fringe patterns, a technique called Digital fringe Projection (DFP) emerged which is widely used for many years now.

Instead of using an optical interferometry method to generate fringe patterns, nowadays we try to generate the fringe patterns digitally with the help of a computer and project them using a projector, thus this process of fringe projection is termed as Digital fringe projection. The fringe patterns are projected with the help of a “Digital-Light-Processing (DLP) projector” or a “Liquid-Crystal-Display (LCD) projector” [95]. 3D surface can then be measured by incorporating the fringe analysis approach for phase measurement. The primary advantages of DFP are: first, multiple fringe patterns can be generated digitally with ease; second, the shape and the pitch of fringe patterns can be regulated by computer; third, the errors arising from mechanical devices no longer exist here; and finally, experimental setup is quite simple.

1.4 Contributions and organization

1.4.1 Objective

As mentioned in the Section 1.1 and Section 1.2, FPA is an important aspect of all optical measurement techniques and have been thoroughly studied and established for Digital fringe projection profilometry to measure the 3D surface geometry. However, two issues still remain open for further investigation. First, with the rapid demand and use of DFP systems for 3D surface measurements and increasing number of applications, the demand for high accuracy and high-speed measurements is increasing. Second, to incorporate a fringe analysis technique it is essential to first understand which techniques to use under which conditions and whether some analysis techniques can be used interchangeably without sacrificing the accuracy quotient. Thus, there is a requirement to attain high-accuracy and establish a link between some of the well-established fringe analysis techniques in order to realize their physical feasibility. Therefore, in this thesis, the high accuracy requirement has been fulfilled by realizing some of the existing issues in FPA and thus, resolving them effectively, which is detailed in the next chapter; and the physical feasibility and applications of some of the state-of-the-art fringe analysis methods are investigated thoroughly to understand the link between various methods.

1.4.2 Contributions

In lieu of the objectives described above, the contributions of this thesis are summarized below.

- **Analyzing nonlinear gamma errors for carrier fringe pattern demodulation:** A theoretical analysis is done to first, realize the impact of nonlinear gamma errors using Windowed Fourier Ridges (WFR) and later, the effectiveness in resolving the nonlinear gamma error is observed. WFR analyzes the “local-frequency error” (FEE) and “phase-extraction error” (PEE) for nonlinear gamma estimation and latter, resolves this error using error control and performance prediction. Despite being challenging, the peak phase error due to nonlinear gamma can be reduced to be as low as 0.05 rad. Even though a number of investigations have been performed in literature, this work is still the first of its kind to analyze both FEE and PEE for performance evaluation and has been already published as [1].
- **Resolving FPP errors in both spatial and spectral domain:** After a theoretical analysis of WFR for effectively resolving gamma nonlinearity in single carrier fringe pattern, we try to look at the gamma nonlinearity issue in phase-shifting algorithm as well as the uneven background error in single carrier fringe pattern. We propose a new squeezing based technique involving WFR to further remove gamma nonlinearity [2]. On the other side, the presence of uneven background in the fringe pattern sometimes, makes it difficult to analyze the fringe patterns in certain areas leading to poor reconstruction quality. To tackle this issue, in a single carrier fringe pattern, a Windowed Fourier filtering (WFF) based background removal technique is proposed. It uses a mirroring approach to tackle the corner pixels. An ideal window of size (50, 50) is used for effective background removal and this work has been presented in [3].

- **Comparison of WFR and Sampling Moiré (SM):** After resolving the two major issues in fringe analysis namely, nonlinear gamma and the disturbances due to uneven background, a comparative study is then performed to analyze and compare the effectiveness of two state-of-the-art fringe analysis techniques namely, WFR and Sampling Moiré (SM) for single carrier fringe pattern for the first time in literature and this work has been published in [4]. Sampling Moiré attempts to translate a single carrier fringe pattern into multiple phase-shifted Moiré patterns to demodulate phase information, while WFR obtains the phase information in the window-based Fourier domain. Despite appearing two completely different approaches, both simulations and experimental results confirm that these two algorithms are two sides of the same coin and thus, can be used interchangeably in various applications. In the presence of noise, both algorithms give outstanding performance.
- **WFR as a spatial carrier phase shifting algorithm:** A link between WFR and spatial carrier phase-shifting (SCPS) algorithm is proposed for the first time and published in [5]. Since phase-shifting and Fourier transform are the two pillar techniques of fringe analysis, and as Windowed Fourier transform has already been established as the extension of Fourier transform it is crucial to understand the link between the two to select a proper technique for effective fringe analysis. A mathematical as well as simulated analysis is performed and it is thus, observed that WFR is a special case of spatial carrier phase-shifting algorithm and whenever a fringe pattern is demodulated using SCPS, WFR can also demodulate the same fringe pattern with much more efficiency. Thus, both the algorithms can be used interchangeably.

1.4.3 Organization

The rest of this report is organized as follows. Chapter 2 reviews the issues pertaining in fringe analysis techniques for digital fringe projection in the past decades. The details about resolving two of these errors namely, nonlinear gamma and

background removal are then explained in Chapter 3 and Chapter 4. Chapter 5 illustrates some thoughts on comparing two well-established fringe analysis techniques i.e. WFR and SM. Chapter 6 illustrates the investigation of a link between WFR and SCPS in order to understand the usage better. Finally, the conclusion and future work are summarized in Chapter 7.

Chapter 2

Literature review

As mentioned in Section 1.2, due to the advantages of non-destructiveness, high sensitivity, and high accuracy, optical measurement methods have been successfully proposed and applied to various fields such as computer graphics and animations, virtual reality, optical metrology, stress and strain measurement, biomedical imaging etc. The development of modern optical and digital devices and with an increasing demand for high measurement resolution, it is essential to analyze measurement errors existing in the current devices due to various error sources such as noise and thus, incorporate effective fringe analysis techniques to attain high accuracy.

Digital fringe projection system comprises of a projector and camera where projector is used to project some structured light fringe patterns on the object surface as shown in Fig. 2.1 and have been extensively studied in literature. The structured light fringe patterns used for these systems are: coded binary patterns [96], gray code patterns [96, 97], sinusoidal patterns etc. Digital fringe projection is a modification of interferometric fringe projection [98] and is a special case of triangulation based structured light technique.

In this chapter, Section 2.1 introduces the basic concept of DLP projection technology as to how the fringe patterns are projected on the object surface. Section 2.2 discusses some other fringe analysis techniques besides the one talked about in chapter 1 and Section 2.3 gives a detailed information on the prevailing issues in

Figure 2.1: A digital fringe projection profilometry (DFPP) system.

the measurement system using a digital fringe projection and fringe analysis and the existing methods to tackle these issues.

2.1 Fringe Projection Profilometry (FPP)

Digital fringe projection (DFP) profilometry systems take advantage of both the digital fringe projections and phase analysis using fringe analysis techniques. They are also referred to as the FPP systems. Among various interferometric techniques for 3D surface reconstruction, it is one of the efficient techniques due to its simple setup and high accuracy, which has been extensively studied in literature [37, 94, 95]. A typical FPP system is shown in Fig. 2.1. 3D surface measurement using FPP systems involve:

- **Image Projection and Acquisition Unit:** Structured light fringe patterns are first generated digitally with the help of computer and then, are projected using the DLP projection technique onto the object surface. Sinusoidal fringe patterns are most commonly used for these systems due to its simple structure

and high accuracy, which are later recorded as an image using a camera by a process called acquisition.

- **Fringe Analysis unit:** The recorded fringe patterns are sent back to the computer for analysis where the wrapped phase information is extracted using a fringe analysis techniques.
- **Phase Unwrapping unit:** Further, to convert a wrapped phase map to a continuous phase for effective surface measurement, a suitable phase unwrapping technique is adopted.
- **Calibration and Point Cloud generation Unit:** The whole FPP system is calibrated to find a relation between image coordinates and 3D world coordinates and thus, use the unwrapped phase information to determine the height of the object surface to reconstruct a point-cloud.

Among these, the key elements of these 3D profilometry systems are the fringe projection unit and the fringe analysis unit.

Despite being developed for over three decades now, fringe projection for surface topography measurement was first used by Rowe and Welford in 1967 [99]. It proved to be an effective method for contouring which was then difficult by standard interferometric techniques. After that, several FPP techniques have been intensively studied and developed [18, 28, 39, 40, 46–49, 69, 95, 100–107] over the past three decades now. The success of FPP systems is mainly dependent on: the effective design of the fringe pattern being generated [108], method of projecting fringe patterns, a thorough investigation of the errors caused by the devices and their possible correction methods [109–116], developing new fringe analysis techniques for phase retrieval [40, 46–48, 69, 101–103], improving existing fringe analysis methods [104–106], developing effective phase unwrapping techniques [37, 72–76], introducing efficient calibration techniques [117, 118] and finally, investigating the ease with which these systems can be used in real-world applications. Though the measurement process of FPP has several steps, FPP techniques in general are classified majorly on the basis of the type of fringe analysis technique being used.

2.2 Some other Fringe Analysis Techniques

Fringe analysis is a key step in any fringe projection system which determines the overall performance of 3D reconstruction. Phase-shifting [29–36, 38, 56] and Fourier-based methods [37, 39–41, 60–68] are the two pillar techniques developed in literature to extract the phase information accurately, which have been discussed in detail in section 1.2. In this section, we will give a brief information on some other state-of-the-art fringe analysis techniques which will then serve as a base for further research.

2.2.1 Windowed Fourier Transform (WFT)

As discussed in Section 1.2, phase-shifting technique uses multiple fringe patterns which are all processed with a defined phase shift. Fourier transform on the other hand, is a global operator which analyzes the phase in the frequency domain, hence, the signal at one location will impact the signal far off.

In order to make use of both the spatial and spectral aspects and enable localized phase tracking [37], “an in-between technique” called Windowed Fourier transform (WFT) [43, 44] is introduced. A 2D forward and inverse WFT pair is expressed in Eqs. 1.15 and 1.16. A forward WFT analyzes the fringe pattern window-by-window in the spectral domain and the inverse WFT is then performed to retrieve the information back in the spatial domain. Using this WFT principle, two different approaches for fringe pattern demodulation were proposed [26]

The first approach, Windowed Fourier Filtering (WFF) was based on window-by-window filtering mechanism and the second approach, Windowed Fourier Ridges (WFR) is an attempt to avoid the filtering process and directly obtaining the best frequency match from the frequencies of fringe pattern in spectral domain and the computer generated exponential elements in a window-by window manner.

2.2.1.1 Windowed Fourier Filtering (WFF)

WFF algorithm works on a filtering mechanism to analyze a window of fringe patterns at a time. A block diagram to explain the working of WFF algorithm is

Figure 2.2: Working structure of WFF algorithm. Image courtesy [26]

shown in Fig. 2.2. A 1D exponential phase field (EPF) is considered here to keep the analysis simple. A 1D EPF signal is generally expressed as:

$$f(x) = b(x) \exp[j\varphi(x)] \quad (2.1)$$

where $b(x)$ is the amplitude and $\varphi(x)$ is the phase. The Windowed Fourier transform (WFT) of this signal is defined as:

$$Sf(u, \xi_x) = \int_{-\infty}^{\infty} f(x) g_{\xi_x}^*(x - u) dx \quad (2.2)$$

where x and u are the spatial x-axis coordinates, the superscript $*$ is a conjugate operator, ξ_x is the frequency coordinate and $g_{\xi_x}(x)$ is a WFT kernel expressed as:

$$g_{\xi_x}(x) = g(x) \exp(l\xi_x x) \quad (2.3)$$

$$g(x) = \frac{1}{(\pi\sigma_x^2)^{1/4}} \exp\left(\frac{-x^2}{2\sigma_x^2}\right) \quad (2.4)$$

WFT spectrum generally measures the similarity between a segment of the signal and the WFT kernel. An EPF signal usually gives large coefficients due to its similarity to some WFT kernels. On the contrary, the coefficients due to noise or any other error source, is small due to its randomness and less similarity with WFT kernels. Thus, a thresholding approach is used to classify the coefficients: coefficients which are larger than the threshold are considered “good” and those which are smaller are considered “bad” and thus, removed by setting them to zero. This procedure is formulated as:

$$\bar{f}(x) = \frac{\xi_{xi}}{2\pi} \int_{-\pi}^{\pi} \bar{Sf}(x, \xi_x) \otimes g_{\xi_x}(x) \quad (2.5)$$

$$\bar{S}f(u, \xi_x) = \begin{cases} Sf(u, \xi_x), & \text{if } |Sf(u, \xi_x)| \geq \text{thr} \\ 1, & \text{if } |Sf(u, \xi_x)| < \text{thr} \end{cases} \quad (2.6)$$

$$\hat{\varphi}_w = \angle \bar{f}(x), \quad (2.7)$$

$$\hat{\omega}_x(x) = \frac{d\hat{\varphi}}{dx} \quad (2.8)$$

where $\bar{S}f(u, \xi_x)$ is the windowed Fourier spectrum which is thresholded at a given threshold thr , $\bar{f}(x)$ is the filtered fringe pattern after thresholding, $\hat{\varphi}_w$ is the extracted wrapped phase, $\hat{\varphi}$ is the continuous filtered phase and $\hat{\omega}_x(x)$ is the filtered local frequency. This algorithm of phase demodulation is called WFF. The algorithm is first presented for 1D which is then later extended to 2D in later sections.

2.2.1.2 Windowed Fourier Ridges (WFR)

WFR is a WFT based technique which can be used for single fringe pattern demodulation. It requires background intensity to be removed before demodulation. Among several different kernels, the WFR algorithm attempts to look for the kernel that is most similar to a window of fringe pattern around the pixel u . This kernel is then used to characterize the frequency of the fringe pattern at the center of that window. All other remaining pixels of a fringe pattern are analyzed similarly. The WFR algorithm is thus, written as:

$$\hat{\omega}_x(u) = \text{argmax}_{\xi_x} |Sf(u, \xi_x)| \quad (2.9)$$

$$\hat{\varphi}(u) = \angle Sf[u; \hat{\omega}_x(u)] \quad (2.10)$$

$$r(u) = |f[u; \hat{\omega}_x(u)]| \quad (2.11)$$

where $\hat{\omega}_x(u)$ is the estimated local frequency of a fringe pattern at a pixel u , $\hat{\varphi}(u)$ is the estimated wrapped phase and $r(u)$ is the ridge value.

WFR algorithm is similarly extended to 2D where for a 2D WFT spectrum $Sf[u, v; \hat{\omega}_x, \hat{\omega}_y]$, the WFR algorithm is extended as:

$$[\hat{\omega}_x(u, v), \hat{\omega}_y(u, v)] = \text{argmax}_{\xi_x, \xi_y} |Sf(u, v, \xi_x, \xi_y)| \quad (2.12)$$

$$\hat{\varphi}(u, v) = \angle Sf[u, v; \hat{\omega}_x(u, v), \hat{\omega}_y(u, v)], \quad (2.13)$$

$$r(u, v) = |Sf[u, v; \hat{\omega}_x(u, v), \hat{\omega}_y(u, v)]| \quad (2.14)$$

2.2.2 Sampling Moiré (SM)

Sampling Moiré is another “local” fringe analysis method which was first proposed in [119]. In SM, a number of phase-shifted Moiré patterns are created using a 1D single carrier fringe pattern by a process called “down-sampling” and “intensity-interpolation” [119, 120]. A 1D carrier fringe pattern is usually represented as:

$$f(x) = a(x) + b(x)\cos[\omega_{cx}(x) + \varphi(x)] \quad (2.15)$$

where $a(x)$ and $b(x)$ are the background and amplitude of a fringe pattern respectively, ω_{cx} is the carrier frequency in x direction and $\varphi(x)$ is the desired phase for each pixel x .

In SM, a 1D carrier fringe pattern as mentioned in Eq. 2.15 is down-sampled at a regular sampling pitch T . This creates T down-sampled images of smaller size which are phase-shifted in nature. In order to create T -step phase-shifted Moiré fringe patterns from them, each down-sampled image is interpolated by either using a linear interpolation or a high-order interpolation (say bi-cubic). Thus, the interpolated t -th phase-shifted Moiré pattern $f_{m,t}(x)$ as mentioned in [120] can be depicted as:

$$\begin{aligned} f_{m,t}(x) &= a_m(x) + b_m(x)\cos\left[2\pi\left(\frac{1}{P} - \frac{1}{T}\right)x + \varphi(x) + \frac{2\pi t}{T}\right], \\ &= a_m(x) + b_m(x)\cos\left[\varphi_m(x) + \frac{2\pi t}{T}\right], \quad (t = 0, 1, 2, \dots, T-1) \end{aligned} \quad (2.16)$$

where $a_m(x)$ denotes the background, $b_m(x)$ denotes the fringe amplitude of the Moiré fringe pattern, $\varphi_m(x)$ is the phase of that particular Moiré fringe pattern and $2\pi t/T$ is the phase-shift which is introduced to create T phase-shifted Moiré patterns. Firstly, $2\pi x/T$ is subtracted from the carrier frequency in order to make Moiré frequency and later, a phase-shift of amount $2\pi t/T$ is introduced to generate T phase-shifted Moiré fringe patterns. From the above mentioned equation, Moiré phase is then obtained as:

$$\varphi_m(x) = \tan^{-1} \left[\frac{\sum_{t=0}^{T-1} f_{m,t}(x) \sin\left(\frac{2\pi t}{T}\right)}{\sum_{t=0}^{T-1} f_{m,t}(x) \cos\left(\frac{2\pi t}{T}\right)} \right] \quad (2.17)$$

Since earlier $2\pi x/T$ is subtracted from the carrier frequency, by adding it back the actual wrapped phase of the original fringe pattern can be obtained:

$$\varphi_w(x) = \varphi_m(x) + \frac{2\pi x}{T} \quad (2.18)$$

This wrapped phase is discontinuous and hence, an unwrapped phase can be obtained later by adopting an efficient phase unwrapping technique.

2.2.3 Spatial Carrier Phase Shifting (SCPS) Algorithm

Phase-shifting techniques is one of the pillar techniques of fringe analysis and has been intensively studied for several decades. A number of phase-shifting techniques have been introduced among which temporal phase-shifting, spatial phase-shifting and spatial carrier phase shifting are the three main approaches [26, 37]. Section 1.2.2 elaborates the spatial phase-shifting algorithm in detail. Here, we discuss the phase-shifting algorithm for a spatial carrier fringe pattern.

For a carrier fringe pattern as mentioned in Eq. 1.6, phase at any pixel (u, v) can be extracted by interrogating all the adjacent pixels $(u + i, v)$, $-n \leq i \leq n$. These $N = 2n + 1$ pixels being the adjacent pixels are assumed to have the same background intensity, fringe amplitude and phase. A spatial phase-shifted fringe can thus, be formulated as:

$$f(u + i, v) = a(u, v) + b(u, v)\cos[\phi(u, v) + \delta_i], \quad -n \leq i \leq n \quad (2.19)$$

where the phase and phase-shift values respectively are:

$$\phi(u, v) = \varphi(u, v) + \omega_{cx}u, \quad (2.20)$$

$$\delta_i = \omega_{cx}i, \quad -n \leq i \leq n \quad (2.21)$$

The phase is then extracted using a least-squares fitting approach [121, 122]. To do so, Eq. 2.19 can be re-written as:

$$f(u + i, v) = p_1 + p_2\cos\delta_i + p_3\sin\delta_i, \quad -n \leq i \leq n \quad (2.22)$$

with

$$\mathbf{p} = [p_1, p_2, p_3]^T = \left[a(u, v), b(u, v)\cos[\phi(u, v)], -b(u, v)\sin[\phi(u, v)] \right]^T \quad (2.23)$$

where the superscript T denotes a vector transpose. The vector parameter $\mathbf{p}$ is estimated as:

$$E(\mathbf{p}) = \sum_{i=-n}^n [p_1 + p_2\cos\delta_i + p_3\sin\delta_i - f(u + i, v)]^2 \quad (2.24)$$

For $i = 1, 2, 3$, considering $\partial E/\partial p_i = 0$ a matrix form for the above equation is obtained as:

$$\mathbf{A}\mathbf{p} = \mathbf{B} \quad (2.25)$$

where

$$\mathbf{A} = \begin{bmatrix} N & \sum_{i=-n}^n \cos\delta_i & \sum_{i=-n}^n \sin\delta_i \\ \sum_{i=-n}^n \cos\delta_i & \sum_{i=-n}^n \cos^2\delta_i & \sum_{i=-n}^n \cos\delta_i \sin\delta_i \\ \sum_{i=-n}^n \sin\delta_i & \sum_{i=-n}^n \cos\delta_i \sin\delta_i & \sum_{i=-n}^n \sin^2\delta_i \end{bmatrix}, \quad (2.26)$$

$$\mathbf{B} = [\sum_{i=-n}^n f(u + i, v) \quad \sum_{i=-n}^n f(u + i, v)\cos\delta_i \quad \sum_{i=-n}^n f(u + i, v)\sin\delta_i] \quad (2.27)$$

The parameter vector $\mathbf{p}$ is thus, obtained as $\mathbf{p} = \mathbf{A}^{-1}\mathbf{B}$. The phase of a spatial carrier phase-shifting can then be formulated according to Eq. 2.23 as:

$$\phi(u, v) = \tan^{-1}(-p_3/p_2) \quad (2.28)$$

Eqs. 2.25-2.28 denote a least-squares-based phase-shifting algorithm as depicted in [20, 122].

As Eq. 2.21 depicts, the phase shift is linear in this case. Thus, Eq. 2.26 can be simplified as:

$$\mathbf{A} = \begin{bmatrix} N & Nt_N(\omega_{cx}/2) & 0 \\ Nt_N(\omega_{cx}/2) & N/2 + Nt_N(\omega_{cx})/2 & 0 \\ 0 & 0 & N/2 - Nt_N(\omega_{cx})/2 \end{bmatrix} \quad (2.29)$$

where $t_N(\alpha)$ is defined as:

$$t_N(\alpha) = \frac{1}{N} \sum_{i=-n}^n \cos(2\alpha i) = \frac{\sin(N\alpha)}{N\sin(\alpha)} \quad (2.30)$$

For $\omega_{cx} = 2\pi/N$, $t_N(\omega_{cx}/2) = t_N(\omega_{cx}) = 0$, resulting in:

$$\mathbf{A} = \begin{bmatrix} N & 0 & 0 \\ 0 & N/2 & 0 \\ 0 & 0 & N/2 \end{bmatrix} \quad (2.31)$$

Obtaining the parameter vector $\mathbf{p}$ from the above equation, results in the extracted phase as:

$$\phi(u, v) = \tan^{-1} \left[-\frac{\sum_{i=-n}^n f(u+i, v) \sin(\omega_{cx}i)}{\sum_{i=-n}^n f(u+i, v) \cos(\omega_{cx}i)} \right] \quad (2.32)$$

which is widely used for all phase-shifting algorithms.

2.3 Issues in Fringe projection Profilometry

In order to achieve a high-accuracy and high speed 3D measurement system, several advancements have taken place in research over the past ten decades. Since then, there has been an intensive progress in FPP based methods for 3D surface measurement as mentioned in the previous sections. Even today there is a constant development in advanced FPP systems which serve to analyze and tackle the issues in real-world measurements such as the presence of shadows, non-uniform reflectivity of the object surface, low fringe modulations, fringe discontinuities etc. which will be discussed in detail in this section.

All the errors which reduces the accuracy of the 3D measurement can be analyzed in terms of phase errors. Phase error analysis has been done since long in literature and many phase correction algorithms have been devised including look-up table methods, gamma correction, intensity correction etc [123]. Phase errors can generally be classified as systematic errors and random phase errors. Random phase errors are unavoidable and arises mainly due to random phase noise which is removed using filtering or averaging approaches and is described in detail in the following sub-section. Systematic errors on the other hand can be avoided by eliminating the error source or compensating the error by analysis. Gamma non-linearity is a type of systematic errors. The accuracy of measurement is mainly

dependent on systematic errors. These systematic errors can be corrected accurately by estimation using some error-compensation technique to be close to the ideal value. Some of the existing systematic and random phase errors in the FPP systems are discussed here.

2.3.1 Noise

Figure 2.3: Examples of fringe pattern with simulated noises namely, (a) Additive noise, (b) speckle noise with speckle size = 1, and (c) speckle noise with speckle size = 2. Image courtesy [26].

Noise used to be a prominent error source in FPP systems. Noise can be present in the system in two different forms namely, random additive noise which arises due to optical components such as CCD camera, and the speckle noise which arises when the components with rough surfaces are the target objects and the light falling on the object gets scattered to a large amount, leaving big black and white patches as shown in Fig. 2.3. However, several effective denoising methods have been proposed in literature over the years [8, 10, 26, 37, 124–128] which completely eliminate the issue of noise, making it redundant for the DFP systems. Thus, noise is no longer an issue. Hence, apart from noise, some of these issues are still prevailing and the research is being done to resolve those issues.

2.3.2 Edge Discontinuity

Due to sudden intensity changes, sometimes the border of an object cannot be detected clearly, leading to poor and discontinuous edge reconstruction. This issue is known as Edge discontinuity. Solving edge discontinuity is a relatively newer problem. In a fringe pattern, these discontinuities arise due to intermixing of the fringe patterns around the edge of an object due to the presence of same intensities on both sides of the discontinuity. Dealing with such discontinuities is a demanding and urgent task in FPP [129–131]. They usually arise in the transform based FPP techniques, as it uses a single fringe pattern for 3D measurement. Phase-shifting techniques on the other end, do not have any edge discontinuities as the background information is removed automatically when a series of fringe patterns are used to retrieve the phase and hence, the corresponding height distribution.

The fringe patterns in general are affected by a number of factors. A fringe pattern affected by both noise and discontinuity poses a challenging problem to retrieve accurate object boundaries. Discontinuity problem prevailed since a long time, however, it has recently caught attention of researchers [129–132]. To deal with both noise and edge discontinuity simultaneously, Windowed Fourier transform assisted BM3D block matching algorithm [129] was designed to tackle the problem in both regions. However, some area around the edge discontinuity still remain undetected. Hilbert assisted wavelet transform [130] and Shearlet transform [131] reduces the phase extraction errors at the points where phase is discontinuous. Another method as mentioned in [133] combined the Laplace based variance quality map of the derivative of phase with a residue check principle. In this manner, it performed phase unwrapping of the discontinuous objects and thus, proved useful. Beside these, a user interactive method- “Snake assisted quality-guided phase unwrapping” method was proposed by [132] which used a GVF snake model to retrace the object boundary edges and thus, help resolve edge discontinuities.

All the above mentioned methods are used to remove edge discontinuity. However, some of the above mentioned methods [130, 133] do not consider the presence

of noise and hence, could fail in the presence of heavy noise. Recently, a segmentation based method [134] for discontinuous fringe pattern is proposed which deals with both noise and discontinuity of the fringe patterns.

Since, discontinuity problem is not resolved completely despite various advances, the research in this field is a challenging problem. The other method [132] is a user-assisted method which requires user intervention and fails in a fully automatic scenario. Thus, there is still scope to develop new methods to resolve discontinuity problem.

2.3.3 Uneven background

FPP systems aim to achieve accurate surface measurements by reconstructing reliable object points. However, the uneven and slow varying background adds disturbances in the fringe patterns which are unwanted and need to be addressed quickly. The issue of background disturbances is seen in two different ways:

- The presence of uneven background in the captured fringe patterns as shown in Fig 2.4.
- Some invalid background points that do not belong to the object surface are captured by the camera and hence, need to be segmented.

Some invalid regions in the fringe patterns also arise due to shadows captured by the camera, which also leads to irregular surface measurements which is a separate discontinuity and will be discussed in the next subsection.

Disturbances due to uneven background are treated in two different ways: Background noise removal [135–146] and background point detection [147]. The latter is a relatively newer topic of research than the former one, however, several research has been done in both these scenarios to overcome this issue. Since background varies slowly and is considered to be low frequency, hence, using low pass filtering to remove background appears to be a common solution. A major consequence of this is however, around the regions where phase $\varphi(x, y)$ is flat and

Figure 2.4: Examples of a fringe pattern with uneven background: (a) fringe pattern with white and dark background patches, and (b) extracted background [3].

the term $b(x, y)\cos[\varphi(x, y)]$ has low frequency. Around these areas, there could be an overlap of frequency between the background and the fundamental component, thus, using a low-pass filter to remove background might lead to distortion and will not prove effective. Thus, a new approach Empirical Mode Decomposition (EMD) [135, 137, 141] has been proposed as an adaptive time-frequency data analysis method for background removal. EMD decomposes a signal $f(x, y)$ into several harmonic like functions called intrinsic mode functions (IMFs) $c_i(x)$, $1 \leq i \leq N$, and a residual term $r_N(x)$:

$$f(x) = \sum_{i=1}^N c_i(x) + r_N(x) \quad (2.33)$$

EMD keeps on decomposing the signal iteratively until the signal cannot be decomposed any further. The residual is then subtracted and the low-frequency background component is thus, identified and removed.

Even though EMD is useful, it leads to an issue of mode mixing problem, where multiple modes in an IMF signal gets intermixed. It is also defined as an IMF signal which consists of a similar scale signal residing in different IMF

components. Mode mixing often arises due to signal intermittency. To alleviate this problem, several extensions and advancements have been made to EMD such as Bi-dimensional EMD (BEMD) [148], Ensemble EMD (EEMD) [138] which uses mean of an ensemble of trials to define the true IMF component, multidimensional EEMD (MEEMD) and many more [138–145]. The effectiveness of the EEMD leads to the development of some improved methods such as the complementary EEMD [139] and complete EEMD with adaptive noise (CEEMDAN) [149, 150]. These methods tries to resolve the mode mixing issue and the amount of residual noise. These EMD related techniques have been recently applied to FPP systems [145]. However, EEMD based methods require amplitude of the noise to be selected empirically which is quite difficult in practice and also, for high accuracy, the ensemble selected is of large size leading to heavy computation. Thus, inspired by the idea of taking assistance from the noise, researchers have investigated various methods to remove background noise effectively [140, 142]. RPSEMD [144] is an adaptive regenerated sinusoidal method where sinusoidals are iteratively generated at every IMF level and the further decomposition is based on that, which proved to be effective.

Background point detection on other hand, is an essential step which need to be performed in order to achieve accurate 3D measurement. Invalid points which belong to the background first needs to be detected properly and then, removed from the scene to achieve reconstruction of reliable object points. There are several methods proposed in literature to tackle this problem and identify valid points [42, 151]. Background points that do not belong to the objects need to be segmented. To perform background segmentation, modulation values are computed and thresholded. Different thresholds are required to be set according to different measurement conditions. k-means clustering based method has been used to automatically detect the background, object and boundary points between them to perform automatic background removal. This, process proved to be effective and is widely used.

However, in this thesis we mainly focus on removing the uneven background for effective fringe pattern analysis.

2.3.4 High Dynamic Range (HDR) Errors

In real measurements, high dynamic range errors are of two types namely, shadows and highlights. Shadows commonly exist when either multiple objects are present in a scene or due to varying height differences of the object. In such situations, light from the projector can be blocked due to object's shape. The captured fringes thus, contain regions with low intensity pixels termed as shadows, making the phase retrieval difficult in those regions, thus considered as invalid points. [152] used multiple projectors to capture multiple images of an object from all directions at different angles and later combined the information from these images to obtain a shadow-free surface reconstruction. Since a large number of projector-camera systems need to be placed at multiple locations, hence, the overall hardware cost of the profilometry system increased significantly. Thus, using a PSP measurement system, [151] proposed a Gaussian filter based method to smooth the fringe patterns and determine the invalid points using monochromatic characteristic of the wrapped phase. Besides removing shadows, Gaussian filter also blurs the details on the object surface thus causing more distortion. To remove the invalid shadow area, Data modulation and intensity modulation techniques have been employed [153–155]. However, these techniques fail to determine the threshold effectively.

Various transform-based methods are also used to remove HDR errors from FPP systems. [156] used photo-metric stereo to detect and remove these errors. To detect the invalid shadow area, inverse Fourier transform is also used [152]. However, the nature of the deformed pattern is usually non-stationary while Fourier transform is more suitable for stationary signals leading to inaccuracies in 3D measurement. In wavelet transform, shadow boundary and sharp edges of the inspected object will result in peaks. Since no signal is detected in the shadow or glare region, Wavelet spectrum will be zero, thus allowing us to determine the shadow areas accurately. [157].

The invalid areas on the object surface can also arise due to extreme high illuminations referred to as highlights. Because of this, the fringes in the affected areas are missing leading to phase discontinuities and hence, irregular measurements.

Figure 2.5: Examples of a fringe pattern with HDR discontinuity: (a) dark shadow region in a fish model, and (b) intensity glare discontinuity (highlight) in a plastic banana model.

Fig. 2.5 shows an example to further explain the HDR errors. Possible solutions to remove highlights from a captured fringe pattern is to use a multi-view active stereo technique to capture several fringe patterns from different viewpoints [158], using a moving camera [159] or a moving projector[160] which captures images from all direction just nullifying the problem of highlights, polarization filters [161] etc. All these techniques require additional hardware and complicated calibration procedures to solve the issue. Thus, there is a need to devise some other methods to resolve HDR errors which have low hardware cost. [162] employs an unstructured light pattern to reduce global illumination, however, the overall accuracy of 3D measurement is reduced. [163] requires the illumination information for guiding inpainting procedure. [164] requires a single fringe image to perform inpainting. It first detects the highlight region using a Gaussian mixture model and then an

image inpainting approach (based on sparse representation) is used to recover the invalid area. These methods provide better results compared to the methods with expensive hardwares. However, the accuracy of resolving HDR errors is still not at par. Thus, this requires further investigation to resolve HDR errors using some structured light based fringe analysis techniques.

2.3.5 Nonlinear distortions or Gamma Error

Figure 2.6: Simulated 1D fringe pattern is shown. The blue line indicates an ideal 1D sinusoidal fringe pattern. The dashed red line on the other hand, represents the non-sinusoidal fringe pattern due to gamma non-linearity with $\gamma = 2$.

Nonlinear distortions are the main source of error in any FPP system. The nonlinear response, also termed as the nonlinear gamma, deviates a captured fringe pattern from its ideal sinusoidal nature to non-sinusoidal, and influences the overall phase retrieval process and hence, correspondingly reduces the accuracy of 3D measurement. Though non-linearity can be introduced in the system by all of the optical devices, projector is the main source of error. Fig. 2.6 shows a 1D fringe pattern with nonlinear distortion. A fringe pattern with the projector's nonlinear gamma can be modelled as:

$$I_o = a(I_i)^\gamma + b \quad (2.34)$$

where I_o is the output intensity for a given input I_i , a and b are constants and γ is the nonlinear component to be calibrated. It is evident that there is a need to determine γ to analyze and remove the nonlinear effect of the projector. Estimating γ can be done through “harmonic analysis”, “least squares approach” or directly analyzing the phase error by comparing it with the ideal map. However, a majority of projectors do not follow such a simple gamma structure and hence the gamma function is expanded as:

$$I_o = c_0 + c_1 I_i + \dots + c_6 (I_i)^6 + c_7 (I_i)^7 + \dots, \quad (2.35)$$

where $c_0, c_1, \dots, c_6, c_7$ and many more are the constants to be calibrated.

The nonlinear gamma varies with the projector model. The gamma curve of the most recent projectors tend to be much more smoother and closer to its ideal curve. Non-linearity thus, exists in both transform-based profilometry and phase-shifting profilometry, and several gamma error compensation algorithms have been adopted in literature to compensate its effect. In phase-shifting profilometry, these nonlinear errors can be greatly reduced if an efficient error-compensation technique is adopted [111–113, 115, 116, 165–171]. Hibino [112] proposed a method which deals with non-linearity and phase shift errors simultaneously. In the presence of only non-linearity, a minimum of $j+2$ samples are required to remove the effect of higher harmonic components upto j^{th} order. However, when both phase shift errors and non-linearity exist, a minimum of $2j+2$ samples are required for eliminating the effect of harmonics upto j^{th} order. Error compensation algorithms based on averaging technique for reducing both errors was first proposed in [170]. The further extensions of this work [171], showed that the sensitivity of non-linearity decreases with increasing number of fringe patterns. [165] proposed a method to generate arbitrary customized algorithms to use one to one correspondence between an algorithm and a polynomial and hence, generate insensitivity to non-linearity and phase shift miscalibration errors by customizing the number of samples, algorithmic insensitivity to an arbitrary harmonic content etc. Several other phase

shifting algorithms have been proposed in literature [113–115] to reduce the errors due to nonlinear harmonics and phase shift miscalibration simultaneously. Although these error-compensation algorithms are able to suppress non-linearity, they require a large number of fringe patterns, which poses a difficulty to real-time FPP measurement system. An alternative approach is to use a single carrier fringe pattern to compensate measurement errors and at the same time achieve high-speed. Various transform based methods [65, 98, 125] also proved to be effective in compensating measurement errors.

With the development of 3D shape measurement techniques based on DFP, phase shift errors were no longer a problem. Hence, research took a turn to resolve nonlinear errors in the profilometry system. Using a large number of fringe patterns [168, 170] reduces the influence of higher-order harmonics caused by nonlinear gamma, but using more patterns increases the measurement speed, and thus, is not desirable for high-speed measurements. Several error-compensation algorithms using no more than five frames were developed [111, 116, 166, 167, 172–175] to reduce the nonlinear measurement errors. Nonlinear gamma error compensation methods can be broadly classified into two categories: active methods [172, 176–178], which actively modifies the fringe patterns before projecting them; and passive methods of error compensation [111, 116, 166, 167, 172, 179, 180], which passively compensates the nonlinear phase errors after the fringe patterns are projected. Both active and passive methods have been demonstrated successful in reducing nonlinear errors and the effectiveness of the two methods is compared [175].

2.3.5.1 Active methods of nonlinear Gamma calibration

Active methods of gamma calibration modifies the computer generated fringe patterns by pre-coding a gamma value to distort the fringe patterns before projection so that after projecting, the captured fringe patterns attain a close outlook to its sinusoidal behavior. [168] used a linear regression based compensation method which was then later be used to modify the fringe patterns before projection. The inverse function of projector's gamma is determined using a gamma calibration,

which can then be easily pre-coded into the fringe patterns. [176] applied a tone correction to the structured-light illumination (SLI) patterns in order to correct for gamma distortions, before they were projected. Several researchers [172, 173] used projector defocusing as a low-pass filter to process the fringe patterns and hence deal with gamma non-linearity without additional computation for compensation. [172] developed a novel gamma calibration method which used a pixel by pixel approach to compute energy of high-order harmonics. He later developed two other strategies to compensate nonlinear gamma errors with or without gamma calibration. The technique which didnot involve any gamma calibration determined both the gamma value and the phase value when processing the captured fringe patterns and then these parameters are used later to compensate the distortion. Several other active gamma correction methods based on precoding have been discussed [178, 181] later which effectively reduced the gamma non-linearity to a large extent. All these methods require a time-consuming gamma calibration procedure for error compensation despite reducing the phase errors to a large extent. Hence, to speed up the measurement, a Fourier spectrum analysis based gamma correction method was proposed in [178]. Two spatial carrier fringe patterns are projected on the object surface to determine the gamma value and then fringe patterns are pre-coded with the estimated gamma value to reduce phase error due to nonlinear gamma.

2.3.5.2 Passive methods of Error Compensation

Passive phase error compensation methods on the other hand, processes the phase map of the captured fringe patterns. The nonlinear phase errors are then compensated in the phase domain where the phase errors are first analyzed from the gamma curve. Determining the phase error for each phase value of the captured fringe patterns is straightforward and is a general method to obtain phase error is as follows:

- Compute the distorted phase of the captured fringe patterns ϕ^d .

- Compute the reference phase ϕ^o of the fringe patterns using a large amount of phase shifts (such as $N = 20$).
- The phase error is then computed by evaluating the difference between the reference phase and the distorted phase for each pixel (x, y) which is ignored here for simplification as: $\Delta\phi = \phi^d - \phi^o$.

Several error compensation methods have been developed in literature [111–113, 115, 116, 165–169, 171, 178, 181] which have been successful in eliminating nonlinear gamma. [111] used a normalized cumulative histogram to determine the gamma value as well as solving for a high accurate phase distribution using an iterative approach. Another iterative approach was proposed by [166] where phase error models for three-step, four-step and five-step phase shifting algorithms are derived respectively from a fifth order harmonics. The phase was then, optimized by an iterative algorithm. To compensate phase error for triangular phase shifting profilometry, an intensity ratio error compensation method [182] was proposed to reduce the measurement error belonging to gamma non-linearity and image defocus. It uses a minimum of two triangular patterns to measure the 3D surface and hence, a simple intensity ratio method was used. Later, [167, 179] made use of a look up table (LUT) approach where the phase error due to non-linearity is calculated and the phase error is then stored in LUTs for significant phase compensation in real measurements. Following the same idea of look up tables several methods have been proposed over time [111, 183]. The captured fringe patterns as a nonlinear function of ideal sinusoidal fringe patterns can be written as:

$$f(x, y) = a + \sum_{j=1}^p b_j \cos[j\varphi] \quad (2.36)$$

where the components a, b and φ are the same as described in Eq. 2.6 and p is the highest order harmonic present in a fringe pattern. It involves $p + 2$ unknowns which are solved using least-squares approach. The desired phase is then obtained as:

$$\varphi = \tan^{-1} \left[\frac{-\sum_{i=1}^{p+2} f \sin\left((i-1)\frac{2\pi}{p+2}\right)}{\sum_{i=1}^{p+2} f \cos\left((i-1)\frac{2\pi}{p+2}\right)} \right] \quad (2.37)$$

These $p + 2$ phase-shifted fringe images are thus, required to eliminate phase errors. [169] proposed two novel approaches based on both active and passive phase error compensation methods. For this, a universal phase model is designed which can be used for any phase-shifting technique and thus, later incorporating active and passive compensation method.

Though these methods were efficient in removing non linear phase errors however, a high speed error compensation method was still lacking. Various transform based methods are also used for phase error compensation [65, 98, 125] using a single carrier fringe patterns. [172] further develops a relationship between gamma coefficients of a measurement system and the Fourier series coefficients of the captured fringe patterns for error compensation. Fourier transform based technique is a classical approach to demodulate a single carrier fringe pattern [40] and works well when phase distortion is not too complicated. Subsequently some local transform based techniques have been proposed to analyze the carrier fringe pattern, including windowed Fourier ridge (WFR), windowed Fourier filtering (WFF) [125] and wavelet ridges [50]. The sensitivity of these methods to the non-linearity has been investigated in [65]. Simulation results show that ridge methods based on local transforms outperform others, making WFR one of the best choices against non-linearity. Incorporating the concept of window based Fourier transform, S-transform ridge and S-transform filtering method [184, 185], have been developed to eliminate the phase errors caused by non-linearity.

Despite these developments in FPP systems to remove the unwanted gamma non-linearity, a fast and accurate phase error compensation method based on single carrier fringe pattern is still missing. The phase-shifting methods on the other hand, are able to eliminate nonlinear gamma errors to a large extent however, an accurate algorithm using a single fringe pattern to reduce these errors is required. Thus, the concept of gamma non-linearity is further extended in the next chapter to analyse the proposed phase error compensation based on the concept of Windowed Fourier ridge to eliminate the non-sinusoidal errors. The algorithm is theoretically and statistically justified in order to verify its performance.

Chapter 3

Insensitivity of WFR towards nonlinear Gamma distortions

In this chapter, we try to first understand the nonlinear gamma errors in the system and then thoroughly analyze an efficient WFR based technique for resolving these errors [1].

3.1 Introduction

Fringe pattern analysis is one of the major aspects of optical interferometry. Several FPA algorithms for FPP systems have been developed in literature [14–26] that could help alleviate the imperfections introduced in the measurement system. One such example is system non-linearity, which is often unavoidable, annoying and should be suppressed. The nonlinear response, also termed as the gamma effect, deviates a captured fringe pattern from its ideal sinusoidal nature to non-sinusoidal and influences the overall phase retrieval process. Thus, the accuracy of 3D measurement is also reduced. Though non-linearity can be introduced in the system by all of the optical devices, projector is the main source of error. Figure 2.6 shows a 1D fringe pattern with nonlinear distortion.

In phase shifting interferometry, this error can be greatly reduced if an error compensation algorithm is adopted, which has been extensively investigated over the last three decades. [112] proposed a method which deals with non-linearity and

phase shift errors simultaneously. Considering only the presence of non-linearity in the system, a minimum of $j + 2$ samples are required to remove the effect of higher harmonic components up to j^{th} order where the phase shift interval is set to $2\pi/(j + 2)$ rad. When both non-linearity and phase shift miscalibration exist, a minimum number of $2j + 2$ samples are necessary for eliminating the effect of higher harmonics upto j^{th} order. [165] proposed a method to generate arbitrary customized algorithms to use polynomial correspondence with an algorithm and hence, generate insensitivity to non-linearity and phase shift miscalibration errors by customizing the number of samples, algorithmic insensitivity to an arbitrary harmonic content etc. Several other phase shifting algorithms have been proposed in literature [113–115] to reduce the errors due to nonlinear harmonics and phase shift miscalibration simultaneously. Although these error-compensation algorithms are able to suppress non-linearity, they require a large number of fringe patterns, which poses a difficulty to real-time FPP systems. Hence, several error compensation methods using no more than five frames were developed [111, 116, 166, 167, 172–175, 183]. An iterative method [166], look-up table based methods [111, 167, 183] and some other interesting techniques [116, 172–175] have been proposed as effective harmonic suppression solutions.

Another approach for phase retrieval is to use a single carrier fringe pattern, which is even faster in data acquisition. This process is most useful for static or dynamic measurement provided the phase is spatially smooth. Fourier transform based technique is a classical approach to demodulate a single carrier fringe pattern [40] and works well when phase distortion is not too complicated. Subsequently some local transform based techniques have been proposed to analyze the carrier fringe pattern, including windowed Fourier ridge (WFR), windowed Fourier filtering (WFF) [125] and wavelet ridges [49, 69, 104]. The sensitivity of these methods to the non-linearity has been investigated in [65]. Simulation results show that ridge methods based on local transforms outperform others, making WFR one of the best choices against non-linearity. Industrial Application of WFR is analyzed by applying it for phase retrieval to measure water waves [186] and wafer deformation [187].

Despite several attempts to validate the fact that WFR is insensitive to nonlinear distortions through both simulation and industrial applications, a theoretical analysis to understand its qualitative performance is still lacking. One attempt can be found in [26], but it is more or less based on intuition. Another intuitive analysis was carried out for S-transform, a transform similar to wavelet [184]. Thus, in order to justify WFR as an effective solution for non-linearity suppression, it is necessary to have a thorough theoretical analysis which is the main contribution of this chapter as proposed in [1].

3.2 Carrier fringe pattern model in the presence of Nonlinear distortions

A 2D carrier fringe pattern can be generally modeled as defined in Eq. 1.6. We generally use a horizontal carrier frequency (i.e., $\omega_{cy} = 0$). In this section, we only consider a 1D carrier fringe signal as:

$$f(x) = a(x) + b(x)\cos[\varphi(x) + \omega_{cx}x] \quad (3.1)$$

This equation is similar to Eq. 1.6, where $a(x)$ is the background information, $b(x)$ is the amplitude of the fringe pattern and $\omega_{cx}x$ is the carrier frequency in x direction. This can further be expanded as:

$$f(x) = a(x) + c(x)\exp(j\omega_{cx}x) + c^*(x)\exp(-j\omega_{cx}x) \quad (3.2)$$

where $c(x) = \frac{1}{2}b(x)\exp[j\varphi(x)]$, j is the imaginary unit and $*$ is the conjugate operator and here $c^*(x) = \frac{1}{2}b(x)\exp[-j\varphi(x)]$. These three terms appear as peaks around $0, \omega_{cx}$ and $-\omega_{cx}$ in the spectral domain. Thus, the desired fundamental component $c(x)\exp(j\omega_{cx}x)$ can be easily isolated and the phase can be retrieved. To make the phase part simple, we assume that the carrier term ω_{cx} is absorbed into $\varphi(x)$. In the presence of non-linearity, a fringe pattern includes higher order harmonics, which can be expressed as:

$$f(x) = \{a(x) + b(x)\cos[\varphi(x)]\}^\gamma \quad (3.3)$$

$$f(x) = \sum_{k=-\infty}^{\infty} h_k(x) \quad (3.4)$$

where $h_k(x)$ is the k^{th} harmonic, defined as:

$$h_k(x) = b_k(x) \exp\{jk[\varphi(x)]\} = \gamma_k(x) b_1(x) \exp\{jk[\varphi(x)]\} \quad (3.5)$$

where $\gamma_k(x) = b_k(x)/b_1(x)$ determines how big of the component $h_k(x)$ is in relation to $h_1(x)$. $h_k(x)$ is a k -th exponential phase field (EPF). $h_1(x)$ is the desired fundamental component, while the others can be seen as disturbances. When a WFT kernel matches $h_1(x)$, assuming both $h_0(x)$ and $h_1(x)$ are well separated by the carrier frequency, the fundamental component $h_1(x)$ can be characterized and the phase can be extracted easily. However, non-linearity of the system may affect the separation of $h_1(x)$. Eqs. 3.4 and 3.5 represent a 1D carrier fringe pattern model in the presence of gamma non-linearity. Since both $h_{-k}(x)$ and $h_k(x)$ are present in a carrier fringe pattern, hence, the fringe pattern can be modified as:

$$f(x) = h_0(x) + \sum_{k=1}^{\infty} [h_k(x) + h_{-k}(x)] \quad (3.6)$$

where $*$ denotes the conjugation operator and $h_{-k}(x) = h_k^*(x)$. Similarly, we have $\gamma_{-k}(x) = \gamma_k^*(x)$.

An example of a 1D carrier fringe pattern with second order harmonics is depicted in Fig 3.1, where $\omega_{cx} = 0.5$ rad/pixel, $b_0(x) = 128$, $b_{\pm 1}(x) = 51.2$ and $b_{\pm 2}(x) = 12.8$, and the phase is linear with a frequency of 0.1 rad/pixel. If we use Fourier transform (FT) and consider only the components in the right half spectral plane i.e. $h_1(x)$ and $h_2(x)$, all remaining components $h_0(x)$, $h_{-1}(x)$ and $h_{-2}(x)$ are removed. The extracted phase for the above simulation then, has a root mean squared error (RMSE) of 0.178 rad. The RMSE reduces to 0.018 rad when the WFR algorithm is applied to the same fringe pattern, indicating its effectiveness in eliminating nonlinear distortions. However, this solution is still intuitive and hence, a theoretical explanation is required.

$h_1(x)$ is the desired fundamental component, from which phase $\varphi(x)$ is to be extracted. Since the conjugate of $h_1(x)$ also exist i.e. $h_{-1}(x)$, $\gamma_{-1}(x) = \gamma_1^*(x) = 1$. The background component $h_0(x) = b_0(x) = a(x)$ needs to be removed beforehand

for effective phase retrieval by applying a high-pass filter [26] to eliminate the influence of background intensity. As the number of higher order harmonics $h_k(x)$ with $|k| \geq 2$ increases, their amplitudes become smaller (i.e., $b_k(x) < b_1(x)$ or $\gamma_k(x) < 1$) and eventually die out to zero.

Each higher harmonic component contains the phase information. However, as the number of higher harmonic component increases, the corresponding phase of $h_k(x)$ components also become larger by k times and hence becomes $k\varphi(x)$. Similarly, the local frequency (defined by $\omega(x) = d\varphi/dx$) and the local curvature (defined by $c_{xx}(x) = d^2\varphi/dx^2$) also increases by k . Thus, to filter out the k -th harmonic component in the frequency domain, a larger bandwidth (k times larger than the bandwidth of the fundamental component $h_1(x)$) is required. Thus, to depict the nonlinear higher-order harmonics in frequency domain, a Fourier spectrum is presented in Fig. 3.1. Here, Fh_k represents the Fourier spectrum of h_k where Fh_1 is the Fourier transform of h_1 . These additional spectrums in Fig. 3.1 except Fh_1

Figure 3.1: Example to depict the harmonics in the Fourier domain [1].

are disturbances and need to be eliminated for accurate phase extraction. Since the amplitude of higher-order harmonics decreases with the number, hence currently for a carrier fringe pattern, Fh_2 is the most prominent influence on Fh_1 . In some cases, where a defocused binary fringe patterns are used rather than a carrier fringe pattern, Fh_3 is the prominent error source [188]. As can be seen from Fig. 3.1, Fh_0 is the background intensity and thus, needs to be removed. However, in some cases, this background intensity also influence the extraction of fundamental

spectrum Fh_1 and thus, is a disturbance from right side. We also consider the influence of Fh_{-3} , Fh_{-2} , Fh_{-1} , Fh_0 , Fh_2 , and Fh_3 on the fundamental component Fh_1 .

For performance evaluation and to analyze the impact of other harmonics on the fundamental one, we consider $\gamma_{-3} = \gamma_{-2} = \gamma_0 = \gamma_2 = \gamma_3 = 0.1$ and $\gamma_1 = 1$, throughout this chapter.

Extending the same for 2D, a 2D carrier fringe pattern is used here, as mentioned in Eq. 1.6. However, in the presence of non-linearity, the 2D fringe pattern with higher order harmonics can be expressed as:

$$f(x, y) = \sum_{k=-\infty}^{\infty} h_k(x, y) \quad (3.7)$$

where $h_k(x, y)$ is the k^{th} harmonic, defined as:

$$h_k(x, y) = b_k(x, y) \exp\{jk[\varphi(x, y) + \omega_{cx}x + \omega_{cy}y]\} \quad (3.8)$$

$h_1(x, y)$ is the desired fundamental component, while the others can be seen as disturbances. However, non-linearity of the system may affect the separation of $h_1(x)$ the phase can be extracted easily.

3.3 1D Perturbation Analysis

The perturbation analysis was first studied in 1991 and can be found in [189]. It is then later used in [190] to analyze the perturbation due to noise for WFR algorithm. For a complex valued signal or function $h(\Omega_x)$ that depends on a real variable Ω_x as:

$$\Omega_{x0} = \operatorname{argmax}_{\Omega_x} |h(\Omega_x)| \quad (3.9)$$

A perturbation of $h(\Omega_x)$ is denoted as $\delta h(\Omega_x)$, which leads to an error of $\delta\Omega_x$ to Ω_{x0} as:

$$\Omega_{x0} + \delta\Omega_x = \operatorname{argmax}_{\Omega_x} |h(\Omega_x) + \delta h(\Omega_x)| \quad (3.10)$$

which indicates that:

$$\frac{\partial \left[h(\Omega_{x0} + \delta\Omega_x) + \delta h(\Omega_{x0} + \delta\Omega_x) \right]}{\partial \Omega_x} = 0 \quad (3.11)$$

Partial differentiation is then, used because of the dependency of h on other variables. Therefore,

$$\delta h_2(\Omega_x) = h(\Omega_x)h^*(\Omega_x) \quad (3.12)$$

$$\delta h_2(\Omega_x) \approx 2\Re \left[h(\Omega_x)\delta h^*(\Omega_x) \right] \quad (3.13)$$

Using these, Eq. 3.11 can then be expanded as:

$$\frac{\delta h_2(\Omega_{x0} + \delta\Omega_x)}{\partial \Omega_x} + \frac{\partial \delta h_2(\Omega_{x0} + \delta\Omega_x)}{\partial \Omega_x} = 0 \quad (3.14)$$

Approximating the function at the maximum value Ω_{x0} by using Taylor series expansion, this can further be simplified as:

$$\frac{\partial^2 h_2(\Omega_{x0})}{\partial \Omega_x^2} \delta\Omega_x + \frac{\partial \delta h_2(\Omega_{x0})}{\partial \Omega_x^2} = 0 \quad (3.15)$$

Hence, the perturbation $\delta\Omega_x$ can be obtained as:

$$\delta\Omega_x = \frac{-B}{A}, \quad (3.16)$$

$$A = 2\Re \left[h(\Omega_{x0}) \frac{\partial^2 h^*(\Omega_{x0})}{\partial \Omega_x^2} + \frac{\partial h(\Omega_{x0})}{\partial \Omega_x} \frac{\partial h^*(\Omega_{x0})}{\partial \Omega_x} \right], \quad (3.17)$$

$$B = 2\Re \left[h(\Omega_{x0}) \frac{\partial h^*(\Omega_{x0})}{\partial \Omega_x} + \frac{\partial h(\Omega_{x0})}{\partial \Omega_x} \delta h^*(\Omega_{x0}) \right] \quad (3.18)$$

Thus, in order to determine the error, we make use of this perturbation analysis.

3.4 WFR algorithm for a distorted 1D Carrier fringe pattern

Our purpose in this section is to justify the intuitiveness of the WFR algorithm discussed in the previous chapter. We derive approximate expressions for local frequency and phase estimation errors provided the WFR algorithm resolve nonlinear

errors. The analysis uses a first order perturbation analysis which is described in section 3.3. The perturbation analysis is a maxima of random functions, of the kind used in analyzing our sub-optimal algorithm. It is advisable to read it at this point before continuing.

For a distorted signal, if the fundamental component is well separated from other harmonic terms; $h_1(x)$ can be easily retrieved by searching for its peak value. Even when there is some overlapping between the harmonic components, provided the peak of the fundamental component is not affected by higher harmonics, the fundamental component $h_1(x)$ can still be effectively retrieved. However, it is a tedious task for several other algorithms to retrieve the fundamental component in the presence of overlapping. A theoretical explanation for the same is provided here.

In order to analyze the influence of higher order harmonics on fundamental component h_1 for both the frequency and phase estimations, we consider the fringe signal as:

$$f(x) = h_1(x) + h_k(x) \quad (3.19)$$

The phase is assumed to be locally quadratic as:

$$\varphi(x) = \varphi(u) + \omega_x(u)(x - u) + 0.5c_{xx}(u)(x - u)^2 \quad (3.20)$$

where $\omega_x(u)$ is the local frequency, $c_{xx}(u)$ is the local curvature and u is the corresponding pixel. When the WFR algorithm as described in Eqs.1.15-1.17 and Eqs. 3.10-3.12, is applied to the distorted signal $f(x)$, the estimated local frequency $\hat{\omega}_x(u)$ produces an error $\delta\omega_x$. Due to the linearity of WFT,

$$Sf(u; \xi_x) = Sh_1(u; \xi_x) + Sh_k(u; \xi_x) \quad (3.21)$$

where Sh_1 is the windowed Fourier spectra of a distortion free signal $h_1(x)$ and Sh_k represents the perturbation in Sh_1 due to nonlinear distortion. Sh_k is hence, the windowed Fourier spectra of $h_k(x)$. It has been derived in [26] that the WFT

spectrum of $Sh_1(u; \xi_x)$ is as follows:

$$Sh_1(u; \xi_x) = h_1(u) \left(\frac{4\pi\sigma_x^2}{1 + \sigma_x^4 c_{xx}^2} \right)^{1/4} \exp \left[\frac{-\sigma_x^2 [\xi_x - (\omega_x + \omega_{cx})]^2}{2(1 + \sigma_x^4 c_{xx}^2)} - j \frac{\sigma_x^4 c_{xx} [\xi_x - (\omega_x + \omega_{cx})]^2}{2(1 + \sigma_x^4 c_{xx}^2)} + j \frac{1}{2} \arctan(\sigma_x^2 c_{xx}) \right], \quad (3.22)$$

By replacing $h_1(u)$ with $h_k(u)$, $(\omega_x + \omega_{cx})$ with $k(\omega_x + \omega_{cx})$ and c_{xx} with $k c_{xx}$, the WFT spectrum of $Sh_k(u; \xi_x)$ can be readily seen as:

$$Sh_k(u; \xi_x) = h_k(u) \left(\frac{4\pi\sigma_x^2}{1 + k^2 \sigma_x^4 c_{xx}^2} \right)^{1/4} \exp \left[\frac{-\sigma_x^2 [\xi_x - k(\omega_x + \omega_{cx})]^2}{2(1 + k^2 \sigma_x^4 c_{xx}^2)} - j \frac{\sigma_x^4 k c_{xx} [\xi_x - k(\omega_x + \omega_{cx})]^2}{2(1 + k^2 \sigma_x^4 c_{xx}^2)} + j \frac{1}{2} \arctan(k \sigma_x^2 c_{xx}) \right], \quad (3.23)$$

Figure 3.2: Frequency spectrum of a windowed Fourier ridges algorithm in the presence of high-order harmonics [1].

For a perfect estimation case where only the fundamental component $h_1(x)$ is present, the peak or ridge of the component is determined perfectly by WFR at $\omega_x(u)$ frequency (desired local frequency). However, as shown in fig. 3.2, the presence of higher order harmonics influences the fundamental component and thus, the ridge location now shifts to $|Sh_1(u; \xi_x) + Sh_k(u; \xi_x)|$ leading to an error in frequency estimation $\delta\omega_{x, \leftarrow k}(u)$, where the arrow $\leftarrow$ indicates the influence of k -th harmonic. The new frequency thus, estimated is depicted by $\hat{\omega}_{x, \leftarrow k}(u)$. Frequency estimation error (FEE) can then be expressed as:

$$\delta\omega_{x, \leftarrow k}(u) = \hat{\omega}_{x, \leftarrow k}(u) - \omega_x(u) \quad (3.24)$$

Similarly, the extracted phase $\hat{\varphi}_{x,\leftarrow k}(u)$ also has a phase estimation error (PEE) of $\delta\varphi_{x,\leftarrow k}(u)$ as:

$$\delta\varphi_{x,\leftarrow k}(u) = \hat{\varphi}_{x,\leftarrow k}(u) - \varphi_x(u) \quad (3.25)$$

These two errors FEE and PEE thus, need to be quantitatively analyzed in order to understand the impact of higher-order harmonics clearly and thus, devise an efficient technique to eliminate the influence.

3.5 Frequency Estimation Error (FEE) Analysis

Eq. 2.9 is used to estimate the local frequency by using WFR and it further extracts the phase information by Eq. 2.10. Thus, in this section we analyze the local frequency estimation error (FEE). In the Fourier domain, since $Sh_k(u; \xi_x)$ poses disturbance on the fundamental component $Sh_1(u; \xi_x)$, to analyze the influence quantitatively, a well-established perturbation analysis technique is adopted as discussed in section 3.3.

With $Sh_1(u; \xi_x)$ corresponding to $h(\Omega_x)$, $Sh_k(u; \xi_x)$ to $\delta h(\Omega_x)$, ξ_x to Ω_x , ω_x to Ω_{x0} , and $\delta\omega_{x,\leftarrow k}(u)$ to $\delta\Omega_x$, the following terms can be derived from Eq. 3.22 as:

$$h(\Omega_{x0}) = Sh_1[\omega(u)] = \left(\frac{4\pi\sigma_x^2}{1 + \sigma_x^4 c_{xx}^2} \right)^{1/4} b_1(u) \exp[j\varphi(u)] \exp\left[j\frac{1}{2} \arctan(\sigma_x^2 c_{xx})\right] \quad (3.26)$$

$$\frac{\partial h(\Omega_{x0})}{\partial \Omega_x} = \frac{\partial Sh_1[\omega(u)]}{\partial \xi_x} = 0 \quad (3.27)$$

$$\frac{\partial^2 h(\Omega_{x0})}{\partial \Omega_x^2} = \frac{\partial^2 Sh_1[\omega(u)]}{\partial \xi_x^2} = \left(\frac{4\pi\sigma_x^2}{1 + \sigma_x^4 c_{xx}^2} \right)^{1/4} b_1(u) \left(\frac{-\sigma_x^2}{1 + \sigma_x^4 c_{xx}^2} \right) (1 + j\sigma_x^2 c_{xx}) \exp[j\varphi(u)] \exp\left[j\frac{1}{2} \arctan(\sigma_x^2 c_{xx})\right] \quad (3.28)$$

$$\begin{aligned} \delta h(\Omega_{x0}) = Sh_k[\omega(u)] &= \left(\frac{4\pi\sigma_x^2}{1 + k^2\sigma_x^4 c_{xx}^2} \right)^{1/4} b_k(u) \exp[jk\varphi(u)] \\ &\exp\left[\frac{-\sigma_x^2(k-1)^2\omega^2(u)}{2(1 + k^2\sigma_x^4 c_{xx}^2)} - j\frac{k\sigma_x^4 c_{xx}(k-1)^2\omega^2(u)}{2(1 + k^2\sigma_x^4 c_{xx}^2)} + j\frac{1}{2} \arctan(\sigma_x^2 c_{xx}) \right], \end{aligned} \quad (3.29)$$

$$\begin{aligned} \frac{\partial \delta h(\Omega x 0)}{\partial \Omega_x} &= \frac{\partial S h_k[\omega(u)]}{\partial \xi_x} = \left(\frac{4\pi\sigma_x^2}{1 + k^2\sigma_x^4 c_{xx}^2} \right)^{1/4} b_k(u) \\ &\quad \left[\frac{-\sigma_x^2(k-1)^2\omega(u)}{(1 + k^2\sigma_x^4 c_{xx}^2)} + j \frac{k\sigma_x^4 c_{xx}(k-1)^2\omega(u)}{(1 + k^2\sigma_x^4 c_{xx}^2)} \right] \exp[jk\varphi(u)] \\ &\quad \exp \left[\frac{-\sigma_x^2(k-1)^2\omega^2(u)}{2(1 + k^2\sigma_x^4 c_{xx}^2)} - j \frac{k\sigma_x^4 c_{xx}(k-1)^2\omega^2(u)}{2(1 + k^2\sigma_x^4 c_{xx}^2)} + j \frac{1}{2} \arctan(\sigma_x^2 c_{xx}) \right] \end{aligned} \quad (3.30)$$

Corresponding to these equations, the local frequency error or shift $\delta\omega_x(u)$ is derived as:

$$\begin{aligned} \delta\omega_{x,\leftarrow k}(u) &= \frac{-B}{A} = (k-1)\gamma_k\omega_x \frac{(1 + \sigma_x^4 c_{xx}^2)^{5/4}}{(1 + k^2\sigma_x^4 c_{xx}^2)^{3/4}} \exp \left[-\frac{(k-1)^2\sigma_x^2\omega_x^2}{2(1 + k^2\sigma_x^4 c_{xx}^2)} \right] \times \\ &\quad \cos \left[(k-1)\varphi(u) - \frac{k(k-1)^2\sigma_x^4 c_{xx}\omega_x^2}{2(1 + k^2\sigma_x^4 c_{xx}^2) + \frac{3}{2}\arctan(k\sigma_x^2 c_{xx}) - \frac{1}{2}\arctan(\sigma_x^2 c_{xx})} \right] \end{aligned} \quad (3.31)$$

where A and B are described in section 3.3. The maximum FEE is thus, represented as:

$$|\delta\omega_{x,\leftarrow k}(u)|_{\max} = |k-1|\gamma_k\omega_x \frac{(1 + \sigma_x^4 c_{xx}^2)^{5/4}}{(1 + k^2\sigma_x^4 c_{xx}^2)^{3/4}} \times \exp \left[-\frac{(k-1)^2\sigma_x^2\omega_x^2}{2(1 + k^2\sigma_x^4 c_{xx}^2)} \right] \quad (3.32)$$

For quantitative analysis, we consider the window size to be $\sigma_x = |c_{xx}|^{-1/2}$ so that $\sigma_x^4 c_{xx}^2 = 1$ [26]. When h_k harmonics for $k = -1, 0, 2$ and 3 are present, a relation between maximum FEE can be readily observed from Eq. 3.32 as: $|\delta\omega_{x,\leftarrow(-1)}|_{\max} < |\delta\omega_{x,\leftarrow(0)}|_{\max} < |\delta\omega_{x,\leftarrow(3)}|_{\max} < |\delta\omega_{x,\leftarrow(2)}|_{\max}$ for $\omega_x \geq 0.3$.

For the frequency evaluation $\omega_x \geq 0.3$, the fringe period $T = 2\pi/\omega_x \leq 21$ pixels is expected to be true for a general case. Hence, the following FEEs can be obtained:

$$|\delta\omega_x|_{\max} \approx \begin{cases} |\delta\omega_{x,\leftarrow(0)}|_{\max} = 2.4\gamma_0\omega_x \exp(-0.5\sigma_x^2\omega_x^2), & \text{without nonlinearity} \\ |\delta\omega_{x,\leftarrow(2)}|_{\max} = 0.71\gamma_2\omega_x \exp(-0.1\sigma_x^2\omega_x^2), & \text{when } h_2 \text{ dominates} \\ |\delta\omega_{x,\leftarrow(3)}|_{\max} = 0.85\gamma_3\omega_x \exp(-0.2\sigma_x^2\omega_x^2), & \text{when } h_3 \text{ dominates} \end{cases} \quad (3.33)$$

To justify this theoretical result, we try to provide an intuitive explanation here. From Eq. 3.32, it can be observed that FEE error is mainly influenced by

the exponential term where the numerator depicts the distance ($d_k = |k - 1|\omega_x$) of the harmonic component $h_k(x)$ from the fundamental component $h_1(x)$. If the distance is large, it indicates a lower influence on FEE error by $h_k(x)$ since it is far-off from $h_1(x)$. The denominator of the exponent on the other hand, adds an additional k signifying that if a higher order harmonic is present, higher is the influence on FEE. For the case when the two harmonic components have the same distance, the denominator of the FEE i.e. the value of k determines the influence on FEE. Consider a case where $h_0(x)$ and $h_2(x)$ have the same frequency distance. Despite that, $h_0(x)$ has lower influence on FEE than $h_2(x)$ since $k = 0 < k = 2$.

3.5.1 Aliasing Problem

We assume that both $h_k(x)$ and $h_{-k}(x)$ co-exist in a fringe pattern. Even though $h_{-k}(x)$ is at a larger distance from $h_1(x)$ as seen from Figs. 3.1 and 3.2, in this subsection, we would consider a well-known issue called Aliasing and thus, consider the impact of $h_{-k}(x)$ on FEE and PEE both. Hence, to be practical, when the influence of $h_k(x)$ is examined, $h_{-k}(x)$ should also be considered.

Aliasing occurs when the frequency of a harmonic component is greater than the Nyquist sampling frequency i.e. π . For example, considering the local frequency of $h_1(x)$ as $\omega_x > \pi/k$, the frequency of the k -th harmonic component becomes $k\omega_x > \pi$ which is higher than the Nyquist sampling frequency. Hence, the k -th harmonic component is aliased in the frequency domain where the new frequency of the k -th component becomes $k\omega_x - 2\pi$. $h_k(x)$ thus, moves from the right half plane to the left half plane, increasing the distance between the k -th component and the fundamental component, thus, reducing the impact on FEE. On the other hand, $h_{-k}(x)$ is aliased from left-half plane to the right-half plane with the new local frequency of $-k\omega_x + 2\pi$. This frequency is changed from the original frequency of changing from $-k\omega_x < -\pi$, thus, posing high influence on FEE may significantly influence $h_1(x)$.

Now let us consider the influence of $h_{-k}(k \geq 2)$ component on FEE in the presence of aliasing. Utilizing the same derivation for FEE as in Eq. 3.32, the

FEE error due to the aliased component $h_{-k}(k \geq 2)$ is:

$$|\delta\omega_{x,\leftarrow(-k)}|_{\text{alias}} = \gamma_k \cdot (k+1)[\omega_x - 2\pi/(k+1)] \times \frac{(1 + \sigma_x^4 c_{xx}^2)^{5/4}}{(1 + k^2 \sigma_x^4 c_{xx}^2)^{3/4}} \\ \exp \left[-\frac{(k+1)^2 \sigma_x^2 [\omega_x - 2\pi/(k+1)]^2}{2(1 + k^2 \sigma_x^4 c_{xx}^2)} \right], \text{ for } \quad k \geq 2 \quad \text{ and } \quad \omega_x \geq \pi/k \quad (3.34)$$

which corresponding to Eq. 3.33 can be written as:

$$|\delta\omega_x|_{\text{alias}} \approx \begin{cases} |\delta\omega_{x,\leftarrow(-2)}|_{\text{alias}} = 2.1\gamma_{-2}(\omega_x - 2\pi/3) \times \\ \quad \exp(-0.9\sigma_x^2(\omega_x - 2\pi/3)^2), & \text{when } h_2 \text{ dominates} \\ |\delta\omega_{x,\leftarrow(-3)}|_{\text{max}} = 1.7\gamma_{-3}(\omega_x - \pi/2) \times \\ \quad \exp(-0.8\sigma_x^2(\omega_x - \pi/2)^2), & \text{when } h_3 \text{ dominates} \end{cases} \quad (3.35)$$

For a scenario, when FEE due to aliased $h_k(k \geq 2)$ has a tolerance τ (i.e., $|\delta\omega_{x,\leftarrow k}|_{\text{max}} \leq \tau$) and the local frequency becomes $\omega_x \geq (\omega_{x,\leftarrow k})_t$, then in order for FEE error due to $h_{-k}(x)$ to satisfy the same tolerance (i.e., $|\delta\omega_{x,\leftarrow(-k)}|_{\text{max}} \leq \tau$) when $\omega_x \leq (\omega_{x,\leftarrow(-k)})_t$, the local frequency threshold needs to be determined. For the same, the fractional components of the exponential functions in Eqs. 3.32 and 3.34 can be written as:

$$(k+1) \left| (\omega_{x,\leftarrow(-k)})_t - \frac{2\pi}{k+1} \right| = (k-1)(\omega_{x,\leftarrow k})_t, \quad (3.36)$$

from which,

$$(\omega_{x,\leftarrow(-k)})_t = \frac{2\pi}{k+1} - \frac{k-1}{k+1}(\omega_{x,\leftarrow k})_t \quad (3.37)$$

Thus, the accurate range for local frequency can be measured as:

$$\omega_x \in \left[(\omega_{x,\leftarrow k})_t, \frac{2\pi}{k+1} - \frac{k-1}{k+1}(\omega_{x,\leftarrow k})_t \right] \quad (3.38)$$

If only second-order harmonic is present i.e. $k = 2$, the range for local frequency becomes $\omega_x \in [(\omega_{x,\leftarrow 2})_t, \frac{2\pi}{3} - (\omega_{x,\leftarrow 2})_t/3]$. Using Eq. 3.33, the value of $(\omega_{x,\leftarrow 2})_t$ becomes 0.6 in order to remove its influence on FEE. Thus, the range for local frequency in any fringe pattern becomes: $\omega_x \in [0.6, 1.9]$ to eliminate the influence of both $h_2(x)$ and $h_{-2}(x)$ on FEE.

3.6 Phase Extraction Error (PEE) Analysis

PhEE occurs due to: (i) the imperfect frequency estimation due to the influence of higher order harmonics as discussed in Section 3.5; (ii) the overlapping of the harmonic components onto one another; and (iii) the imperfect estimation of curvature c_{xx} .

3.6.1 PhEE Due to the Imperfect Frequency Estimation

Similar to the FEE analysis as mentioned in the previous sub-section, the phase extraction error can be obtained from Eq. 2.10 as:

$$\delta\varphi_{\omega_{x,\leftarrow k}} = -\frac{\sigma_x^4 c_{xx}}{2(1 + \sigma_x^4 c_{xx}^2)} (\delta\omega_{x,\leftarrow k})^2 \quad (3.39)$$

which corresponding to Eq. 3.33 yields:

$$|\delta\varphi_{\omega_x}|_{\max} \approx \begin{cases} |\delta\varphi_{\omega_{x,\leftarrow(0)}}|_{\max} = 1.4\gamma_0^2 \sigma_x^2 \omega_x^2 \exp(-\sigma_x^2 \omega_x^2), & \text{without nonlinearity} \\ |\delta\varphi_{\omega_{x,\leftarrow(2)}}|_{\max} = 0.13\gamma_2^2 \sigma_x^2 \omega_x^2 \exp(-0.2\sigma_x^2 \omega_x^2), & \text{when } h_2 \text{ dominates} \\ |\delta\varphi_{\omega_{x,\leftarrow(3)}}|_{\max} = 0.18\gamma_3^2 \sigma_x^2 \omega_x^2 \exp(-0.4\sigma_x^2 \omega_x^2), & \text{when } h_3 \text{ dominates} \end{cases} \quad (3.40)$$

3.6.2 PhEE Due to Spectrum Overlapping

Due to the presence of multiple harmonic components, overlapping occurs sometimes when the tail of the k -th spectrum ends into the fundamental component. The overlapped spectrum is depicted as:

$$\begin{aligned} Sh(u; \xi_x) &= Sh_1(u; \xi_x) + Sh_k(u; \xi_x) \\ &= Sh_1(u; \xi_x) \left\{ 1 + \frac{|Sh_k|}{|Sh_1|} \exp[j(\angle Sh_k - \angle Sh_1)] \right\} \end{aligned} \quad (3.41)$$

from which, PEE can be represented as:

$$\begin{aligned} \delta\varphi_{h,\leftarrow k} &\approx \tan(\delta\varphi_{h,\leftarrow k}) = \frac{\frac{|Sh_k|}{|Sh_1|} \sin(\angle Sh_k - \angle Sh_1)}{1 + \frac{|Sh_k|}{|Sh_1|} \cos(\angle Sh_k - \angle Sh_1)} \\ &\approx \frac{|Sh_k|}{|Sh_1|} \sin(\angle Sh_k - \angle Sh_1) \end{aligned} \quad (3.42)$$

Maximum phase error thus, becomes:

$$|\delta\varphi_{h,\leftarrow k}|_{\max} = \frac{|Sh_k|}{|Sh_1|} \gamma_k \cdot \left(\frac{1 + \sigma_x^4 c_{xx}^2}{1 + k^2 \sigma_x^4 c_{xx}^2} \right)^{1/4} \exp \left[\frac{-(k-1)^2 \sigma_x^2 \omega_x^2}{2(1 + k^2 \sigma_x^4 c_{xx}^2)} \right] \quad (3.43)$$

which leads to a relationship between PEEs of components $k = 0, 1, 2$ and 3 as:

$$|\delta\varphi_{h,\leftarrow(-1)}|_{\max} < |\delta\varphi_{h,\leftarrow(0)}|_{\max} < |\delta\varphi_{h,\leftarrow(3)}|_{\max} < |\delta\varphi_{h,\leftarrow(2)}|_{\max} \text{ for } \omega_x \geq 0.3 \quad (3.44)$$

and

$$|\delta\varphi_h|_{\max} \approx \begin{cases} |\delta\varphi_{h,\leftarrow(0)}|_{\max} = 1.2\gamma_0 \exp(-0.5\sigma_x^2 \omega_x^2), & \text{without nonlinearity} \\ |\delta\varphi_{h,\leftarrow(2)}|_{\max} = 0.80\gamma_2 \exp(-0.1\sigma_x^2 \omega_x^2), & \text{when } h_2 \text{ dominates} \\ |\delta\varphi_{h,\leftarrow(3)}|_{\max} = 0.67\gamma_3 \exp(-0.2\sigma_x^2 \omega_x^2), & \text{when } h_3 \text{ dominates} \end{cases} \quad (3.45)$$

When comparing the PhEEs represented in Eqs. 3.40 and 3.45, it is observed that overlapping has a large influence on PEE as in Eq. 3.45 and can be used to represent the overall PEE as: $|\delta\varphi|_{\max} \approx |\delta\varphi_h|_{\max}$.

3.6.3 Aliasing Problem

Similar to Eq. 3.43, the PEE error in the presence of an aliased component h_{-k} ($k \geq 2$) becomes:

$$|\delta\varphi_{h,\leftarrow(-k)}|_{\text{alias}} = \gamma_k \cdot \left(\frac{1 + \sigma_x^4 c_{xx}^2}{1 + k^2 \sigma_x^4 c_{xx}^2} \right)^{1/4} \times \frac{(1 + \sigma_x^4 c_{xx}^2)^{5/4}}{(1 + k^2 \sigma_x^4 c_{xx}^2)^{3/4}} \exp \left[-\frac{(k+1)^2 \sigma_x^2 [\omega_x - 2\pi/(k+1)]^2}{2(1 + k^2 \sigma_x^4 c_{xx}^2)} \right] \quad (3.46)$$

and,

$$|\delta\varphi_h|_{\text{alias}} \approx \begin{cases} |\delta\varphi_{h,\leftarrow(-2)}|_{\text{alias}} = 0.80\gamma_2 \exp(-0.9\sigma_x^2 (\omega_x - 2\pi/3)^2), & \text{when } h_2 \text{ dominates} \\ |\delta\varphi_{h,\leftarrow(-3)}|_{\text{alias}} = 0.67\gamma_3 \exp(-0.8\sigma_x^2 (\omega_x - \pi/2)^2), & \text{when } h_3 \text{ dominates} \end{cases} \quad (3.47)$$

3.7 Simulation based Verification

To validate the above mentioned theoretical results with the help of simulations, we design a fringe pattern as:

$$f(x) = 0.5\exp(j\varphi) + 0.05\exp(j2\varphi) \quad (3.48)$$

which only suffers the disturbance of $h_2(x)$ and the phase is simulated as a quadratic phase as in Eq. 3.20 assuming $c_{xx} = 0.01$ and ω_x from 0 to 2π with an increment of $1/(2\pi)$.

WFR is used to process this fringe pattern with a window size $\sigma_x = 1/\sqrt{c_{xx}} = 10$. For each ω_x , different φ_0 values give different FEEs, among which the maximum FEE is recorded. The maximum FEE both by simulation and the theoretical analysis (from Eq. 3.33) shown in Fig. 3.3(a). Similarly, the maximum PhEE from both the simulation and the theoretical analysis (from Eq. 3.45) is shown in Fig. 3.3(b). It is evidently observed that the simulated result perfectly replicates the theoretical result thus, validating our theoretical analysis.

Figure 3.3: Simulated and theoretical results for a simulated fringe pattern are presented: (a) FEE and (b) PhEE. Image courtesy [1].

For a practical fringe pattern:

$$f(x) = 0.05 + \cos(\varphi) + \cos(2\varphi) \quad (3.49)$$

FEE and PEE errors for the co-existence of both $h_k(x)$ and $h_{-k}(x)$ is observed in figure 3.4. However, first the background component 0.5 is removed by a high-pass filter and then later, WFR is used to process the fringe pattern. The previous simulation experiment is then repeated for $k = 2$.

Figure 3.4: Simulation result to show comparison of theoretical and simulated results for a fringe pattern containing fundamental component, second-order harmonic and its conjugate, and the DC component: (a) FEE and (b) PhEE. Image courtesy [1].

From the figure, it can be observed that: (i) simulated result retraces the theoretically obtained results perfectly from $\omega_x \geq 0.3$. However, for lower local frequency $\omega_x < 0.3$, there is a difference between theoretical and simulated results due to large overlap between harmonics; (ii) a large error at frequency $\omega_x = 2\pi/3 \approx 2.1$ can be clearly observed which occurs due to aliasing where $h_{-2}(x)$ completely overlaps with $h_2(x)$; and (iii) $\omega_x \in [0.6, 1.9]$ is validated as a good local frequency range of a fringe pattern.

3.8 Experimental Analysis

This section is devoted to computing the effectiveness of WFR algorithm in resolving nonlinear gamma errors for a real-optical setting. For experimentation, the

author used an FPP system comprising of a “liquid crystal display (LCD) projector (EPSON, EB-1761W, 3LCD, WXGA 1280 by 800 pixels, 2600lm) and a CMOS camera (Imaging Source Co., DFK 72BUC02, 1600 by 1200 pixels @ 13fps)”. The camera is mounted by an 8mm focal-length mega-pixels lens(Computar, M0814-MP). The distance of the object from the camera and projector system is 800 mm where both the camera and projector lay at an angle of about 30 degrees with each other. The experimental fringe pattern captured by the camera of size 800×800 is shown in Fig. 3.5.

Figure 3.5: Fringe pattern generated by an FPP system.

Some experimental analysis conditions are considered as:

- For performance evaluation, we used a section of the fringe pattern of size 400×800 pixels, as indicated in Fig. 3.6(a).
- For WFR to work effectively, it is advised to remove the negative component

and the background bias prior to experimentation so that only the fundamental and other higher harmonic components remain.

Figure 3.6: Experimental result for (a) 400×800 captured fringe pattern (b) 3D phase map using 2D FT algorithm with nonlinear gamma ripples (c) wrapped phase map extracted using 2D WFR algorithm and (d) 3D phase map using WFR algorithm to resolve for nonlinear gamma.

The captured fringe pattern is then processed with a 2D-FT algorithm which shows the ripples in the 3D phase map as shown in Fig. 3.6(b). Windowed Fourier Ridge (WFR) algorithm is then used to process the fringe pattern to resolve the nonlinear gamma errors in the system and the 3D phase map corresponding to it is shown in Fig. 3.6(c). It is quite clear from the figure that WFR algorithm is able to resolve the nonlinear gamma errors effectively leaving behind a smooth surface as desired.

3.9 Discussion on other error sources and other WFT-based demodulation techniques

In this chapter, the insensitivity of WFR towards nonlinear gamma has been clearly stated, theoretically proven and effectively verified by simulations. However, a few other error sources such as noise and fringe model error are discussed in this section

to give a full picture of error distribution. These error sources are then compared with other WFT-based algorithms to complete the analysis by providing a selective approach in choosing the algorithm for error compensation.

3.9.1 Phase Model Revisited

If a fringe pattern contains complex phase distribution say quadratic as indicted in Eq. 3.20, the phase distribution may deviate from its true nature and can lead to large errors. To illustrate this we consider two nonlinear phases: first, circular and quadratic phase [26], expressed as:

$$\varphi(x, y) = 0.0025[(x - N/2)^2 + (y - N/2)^2] \quad (3.50)$$

where $N \times N$ is the size of the image; and the second, peaks phase represented using “MATLAB peaks function” [26] as:

$$z(x, y) = 3(1 - x)^2 \exp[-x^2 - (y + 1)^2] - 10\left(\frac{x}{5} - x^3 - y^3\right) \exp(-x^2 - y^2) - \frac{1}{3} \exp[-(x + 1)^2 - y^2], \quad -3 \leq (x, y) \leq 3 \quad (3.51)$$

The two fringe patterns have an additional carrier frequency of 1.2 and are shown in Figs. 3.7(a) and 3.7(b), respectively. The maximum FEE and PhEE when processed using WFR with different window sizes are shown in Figs. 3.7(c) and 3.7(d), respectively. It can be observed that: first, Both FEE and PhEE are very small for circular and quadratic phases. However, for the peaks phase, these errors increases with the window size. Thus, a smaller window size is recommended to reduce fringe model error. Second, currently FEE is in the same order of PhEE, but it can be larger than PEE, which is surprising but not unexpected.

3.9.2 Nonlinearity versus noise

FEE being susceptible to noise as seen in section 3.9.1, in this section we mainly focus on PhEE. The mean error of PEE in the presence of noise is insignificant hence, the standard can be observed as:

$$\sigma_{\delta\varphi} = \left(\frac{1 + \sigma_x^4 c_{xx}^2}{4\pi\sigma_x^2} \right) \frac{\sigma_n}{\sqrt{2}b_1} \quad (3.52)$$

Figure 3.7: Fringe model with nonlinear phase distribution: (a) circular fringe pattern; (b) peaks fringe pattern; (c) maximum FEEs of (a) and (b); and (d) maximum PhEE of (a) and (b). Image courtesy [1].

Here, the amplitude is specified as b_1 and the standard deviation of noise is adjusted to $\sigma_n/\sqrt{2}$ for a carrier fringe pattern. With a window size $\sigma_x = |c_{xx}|^{-1/2}$ and $\gamma_n = \sigma_n/b_1$, Eq. 3.52 becomes:

$$\sigma_{\delta\varphi} = 0.53\gamma_n\sigma_x^{-1/2} \quad (3.53)$$

Considering the lowest local frequency $\omega_x = 0.6$ in the presence of $h_2(x)$, PEE in reference to Eq. 3.45 due to $h_2(x)$ becomes:

$$|\delta\varphi_h|_{\max} = 0.80\gamma_2\exp(-0.1\sigma_x^2\omega_x^2) \quad (3.54)$$

By equating Eqs. 3.53 and 3.54::

$$\frac{\gamma_n}{\gamma_2} = 1.5\sigma_x^{1/2}\exp(-0.036\sigma_x^2) \quad (3.55)$$

For $\sigma_x = 5$, $\gamma_n/\gamma_2 = 1.4$, indicating that γ_2 and γ_n need to be similar, as making one of them extremely small does not reduce the overall error.

3.9.3 WFR versus WFR2

Figure 3.8: Simulation results for a noiseless and noisy case for both WFR and WFR2 for a fringe pattern with peaks phase: (a) noiseless fringe pattern; (b) noisy fringe pattern; (c) maximum PhEEs of WFR and WFR2 for (a); and (d) maximum PhEEs of WFR and WFR2 for (b). Image courtesy [1].

WFR2 is a 2D version of WFR as mentioned in section 3.4. To examine other error sources, the peaks phase from Fig. 3.7(b) is replicated in Fig. 3.8(a). Model errors after applying WFR and WFR2 on this fringe pattern is shown in Fig.

Figure 3.9: Simulation result to compare the performance of WFR and WFR2: (a) carrier fringe pattern with peaks phase; (b) a fringe pattern without carrier; (c) maximum PhEEs for a fringe pattern shown in (a) with additional nonlinearity; and (d) maximum PhEEs for a fringe pattern shown in (b). Image courtesy [1].

3.8(c). Similarly, in the presence of noise with $\gamma_n = 0.15$, the errors for WFR and WFR2 for a fringe pattern shown in Fig. 3.8(b) are shown in Fig. 3.8(d). In Fig. 3.8(d), both the model error and error due to noise are observed for both WFR and WFR2. It is clear that WFR2 is more effective in resolving noise as compared to WFR.

Figure 3.9(a) shows another example of fringe pattern with peaks phase with additional non-linearity and figure 3.9(b) displays a fringe pattern without carrier. When, WFR and WFR2 are applied to these fringe patterns the performance is shown in figures 3.9(c), and Fig. 3.9(d). From the figures, it is clear that both WFR and WFR2 are insensitive to even higher amounts of nonlinearity.

3.10 Conclusions

WFR algorithm has been considered as an effective solution for eliminating nonlinear distortions for a carrier fringe pattern. In order to justify this both theoretically and with simulations, a local frequency estimation error and the phase extraction error is performed. It is clearly observed that: (i) an effective local frequency range to reduce the influence of nonlinear gamma is observed to be $\omega_x \in [0.6, 1.9]$. (ii) A small window size is preferred for a fringe pattern with model error. (iii) Both WFR and WFR2 are insensitive to noise. (iv) The phase error is small (e.g., less than $2\pi/100 \approx 0.06$ rad, corresponding to $1/100$ pitch of the fringe pattern), even with the presence of different error sources.

Chapter 4

Resolving Phase analysis errors in both spatial and spectral domain

Based on the knowledge developed in chapter 2 and chapter 3, we try to look at the two main error sources- gamma nonlinearity and uneven background, which are resolved in this chapter. Gamma nonlinearity since has already been dealt in carrier fringe patterns, here we will examine the effectiveness of WFR to resolve gamma nonlinearity in phase-shifting approach [2]. Since uneven background is a problem associated with carrier fringe pattern, hence we will try to examine the issue and resolve it using WFF [3].

4.1 WFR based carrier squeezing interferometry

Since WFR appears to be one of the most effective solutions for resolving gamma nonlinearities in the system as proven in chapter 3, here in this section we try to propose a new squeezing based technique involving WFR to further remove gamma nonlinearity and thus, analyze the effectiveness as proposed in [2]. Though WFR when applied directly is efficient in resolving gamma errors, this section is just an additional information to resolve the gamma nonlinearity when multiple phase-shifted fringe patterns are present in the system.

4.1.1 Introduction

In fringe projection profilometry (FPP), accurately performing the 3D measurement has been a major concern over decades. The error due to nonlinearity or gamma is one of the biggest issues. The nonlinearity mainly arises due to the projector. Several phase compensation techniques have been proposed [169, 175] to resolve this nonlinear error and obtain accurate 3D measurement. However, these techniques either require a complex pre-calibration procedure before every measurement or poses stringent conditions to suppress gamma. Hence, there is a need to develop a simple algorithm. In this paper, a carrier squeezing technique together with WFR is proposed [2] for phase retrieval to suppress nonlinearity.

4.1.2 Carrier Squeezing Interferometry (CSI)

In CSI [191–193], N -step phase shifting fringe patterns are captured and then transformed into a single spatial carrier fringe pattern [191]. In detail, an N -step phase shifting fringe equation can be written as:

$$f_i(x, y) = a(x, y) + b(x, y)\cos[\varphi(x, y) + \delta_i] \quad (4.1)$$

where $i = 0, 1, \dots, N - 1$, $\delta_i = 2\pi i/N$ is the phase shift and (x, y) coordinates are $x = 0, 1, \dots, L - 1$ and $y = 0, 1, \dots, L - 1$ respectively. These N fringe patterns of size $L \times L$ are squeezed along y coordinate to obtain a single carrier fringe pattern of size $L \times NL$ as:

$$f_{STF}(x, y') = a(x, y') + b(x, y')\cos[\varphi'(x, y') + 2\pi f_t y'] \quad (4.2)$$

where y changes to y' as $y' = iy + N$ and hence, the phase remains constant for every N pixels as suggested in [191]. Also, $f_t = 1/N$ is the carrier frequency. For a 3-step phase shifting example, the transformation is straightforward; and we re-arrange our three PS fringe patterns to obtain a single fringe pattern $f'(x, y)$ as:

$$\begin{aligned} f'(3x, y) &= f(x, y, -\delta) \\ f'(3x + 1, y) &= f(x, y, 0) \\ f'(3x + 2, y) &= f(x, y, +\delta) \end{aligned} \quad (4.3)$$

where $\delta = 2\pi/3$ and $f(x, y, \pm\delta)$ are the 3-step phase shifted fringe patterns which are converted to a carrier squeezing pattern as shown in fig. 4.1. As seen in Eqs.

Figure 4.1: (a), (b) and (c) represents 3-step phase shifted fringe patterns, and (d) is the carrier squeezed fringe pattern obtained using Eq. 4.3. Image courtesy [191].

4.1 and 4.2, the phase $\varphi'(x, y')$ indicates the same pixel as $\varphi(x, y)$ in the original PSF patterns. In [192], the middle point was selected as the final phase value (of size $L \times L$) when $N = 3$. However, the case when N is even (say $N = 4$) was not explicitly mentioned. Two possible solutions are, averaging the two middle points corresponding to pixels 2 and 3, and averaging all 4 pixels to obtain the final phase as:

$$\hat{\varphi}(x, y) = \frac{\sum_{n=1}^N \hat{\varphi}_n(x, y')}{N} \quad (4.4)$$

where $\hat{\varphi}_n(x, y')$ is the extracted wrapped phase for a CSI pattern and $\hat{\varphi}(x, y)$ is the final extracted phase.

The phase errors by these two solutions are shown in Fig. 4.2, which shows that both are effective. We notice that averaging all pixels also works for all $N = \text{odd}$.

Hence, for simplicity, Eq. 4.3 can be used for all N . This thus, serves as a minor contribution to CSI and as a fundamental work of the proposed WFR processing.

Figure 4.2: Phase errors corresponding to the average of middle 2 pixels (a) and all 4 pixels (b), respectively, for a 4-step phase shifted fringe patterns [2].

4.1.3 WFR for CSI nonlinearity suppression

Due to nonlinearity, the spatial carrier fringe in CSI has harmonics as follows:

$$f_{STF}(x, y') = \sum_{k=0}^{\infty} b_k \{ \cos[k\varphi'(x, yi') + 2k\pi f_t yi'] \} \quad (4.5)$$

Fourier Transform (FT) has been used to process this carrier fringe pattern. By careful selection of the window center and width in the frequency domain, the spectrum lobes due to high order harmonics can be rejected to reduce nonlinearity. However, the manual selection of the window width is troublesome. Furthermore, when the phase distribution is complicated, spectrum lobes will overlap, making the ideal window selection impossible.

In order to reduce the effect of high order harmonics even further, a WFR algorithm is used which can be explained as follows. For a signal $f(x, y)$ which in our case can be expressed as Eq. 4.5, assume that its windowed Fourier spectrum

is $Sf(u, v; \xi_x, \xi_y)$, then its WFR algorithm can be represented as mentioned in Eqs. 2.12-2.14. For the CSI pattern as mentioned in Eq. 4.5, WFR becomes:

$$[\hat{\omega}_x(u, v), \hat{\omega}_y(u, v)] = \underset{\xi_x, \xi_y}{\operatorname{argmax}} |(u, v; \xi_x, \xi_y)|, \quad (4.6)$$

$$\hat{\phi}'(x, y') = \tan^{-1}\{Sf(u, v; \hat{\omega}_x(u, v), \hat{\omega}_y(u, v))\} \quad (4.7)$$

where $[\hat{\omega}_x(u, v), \hat{\omega}_y(u, v)]$ is the estimated local frequency of the fringe pattern at pixel (u, v) and $\hat{\phi}'(x, y')$ is the estimated modulated phase of size $L \times NL$. The final phase $\hat{\phi}(x, y)$ is obtained further by using Eq. 4.4. The WFR algorithm is expected to work better due to the following factors. First, WFR works locally so that the spectrum of a local fringe pattern is simpler. Second, WFR utilizes the ridge (peak) of spectrum which is less sensitive to the spectrum overlapping.

To analyze the effectiveness of WFR algorithm in reducing nonlinearity, we further compute the phase error in comparison to the traditional CSF as shown in Fig 4.3. A original phase function is considered as $\varphi(x, y) = 2 * \text{peaks}(L)$ in simulations where L is the length along both x and y direction. For $\gamma = 1.5$, the WFR algorithm reduces nonlinearity more effectively in comparison to traditional CSF. The RMS phase error of WFR for the same gamma is approx. 3.5 times better than FT in eliminating nonlinear gamma errors. Hence, it proves to be more effective than traditional FT techniques and hence, can be used widely.

After performing the overall analysis, we figured out that the phase $\hat{\phi}(x, y)$ computed using carrier squeezing technique is same as the one calculated using N -step phase-shifting algorithm and both use the same formula as in Eq. 1.10. Since, there is no difference between the phase-shift of the two algorithms and the way phase is computed, carrier squeezing interferometry technique is phase-shifting algorithm itself, deeming carrier squeezing interferometry as a poor and incorrect measurement technique.

4.1.4 Conclusions

First, we have presented a simple phase compensation technique for an N -step phase shifting fringe patterns to resolve the nonlinearity error. N -step phase shift-ing fringe patterns are squeezed to obtain a single carrier fringe pattern. WFR is

Figure 4.3: Phase error of the proposed method corresponding to the traditional CSI for $\gamma = 1.5$ [2].

used as an effective solution to resolve nonlinear distortions in carrier fringe patterns. The extracted phase is then averaged to obtain the final phase of size $L \times L$. The effectiveness of the proposed method is verified by simulation. However, we later figured that carrier squeezing interferometry is phase-shifting algorithm itself limiting the use-case.

4.2 WFF based Background Removal

4.2.1 Introduction

Background in any fringe pattern is defined by:

$$f(x, y) = a(x, y) + b(x, y)\cos(\varphi(x, y)) \quad (4.8)$$

where $a(x, y)$ is defined as the background intensity of a fringe pattern and it is spread across all pixels. Background is usually slow varying and hence, in a spectral domain it is mainly concentrated around the 0 – *th* frequency, and thus is

also called a low-frequency component. DC component or a constant $a(x, y)$ is the most-common way to add background in a fringe pattern. However, sometimes the way in which light is falling on the object surface or the way image is being captured by the camera, this background can easily become uneven, even though it will still remain low-frequency component. FPP systems aim to achieve accurate 3D surface measurements by reconstructing reliable object points. However, the uneven and slow varying background adds disturbances in the fringe patterns which are unwanted and need to be addressed quickly. The issue of background disturbances is seen in two different ways:

- The presence of uneven background in the captured fringe patterns as shown in Fig 2.4.
- Some invalid background points that do not belong to the object surface are captured by the camera and hence, need to be segmented.

Some invalid regions in the fringe patterns also arise due to shadows captured by the camera, which also leads to irregular surface measurements which is a separate discontinuity and will be discussed later in this report.

For a high carrier frequency, phase-modulated signals are spread out in frequency domain and can be easily separated by using FT [144, 145]. However, for a complicated phase, it becomes more difficult to separate the frequency components easily and thus, a large error appears.

Disturbances due to uneven background are treated in two different ways: Background noise removal [135–146] and background point detection [147]. The latter is a relatively newer topic of research than the former one, however, several research has been done in both these scenarios to overcome this issue. Since background varies slowly and is considered to be low frequency, hence, using low pass filtering to remove background appears to be a common solution. A major consequence of this is however, the regions where $\varphi(x, y)$ phase is flat and the term $b(x, y)\cos[\varphi(x, y)]$ has low frequency, there could be an overlap of frequency between the background

and the fundamental component, thus, using a low-pass filter to remove background might lead to distortion and will not prove effective. Thus, a new approach Empirical Mode Decomposition (EMD) [135, 137, 141] has been proposed as an adaptive time-frequency data analysis method for background removal. EMD decomposes a signal $f(x, y)$ into several harmonic like functions called intrinsic mode functions (IMFs) $c_i(x)$, $1 \leq i \leq N$, and a residual term $r_N(x)$ as in Eq. 2.33. EMD keeps on decomposing the signal iteratively until the signal cannot be decomposed any further. The residual is then subtracted and the low-frequency background component is thus, identified and removed.

Even though EMD is useful, it leaves some annoying difficulties behind. One of the major drawbacks of EMD is the mode mixing problem, where multiple modes in an IMF signal gets intermixed. It is also defined as an IMF signal which consists of a similar scale signal residing in different IMF components. Mode mixing often arises due to signal intermittency. To alleviate this problem, several extensions and advancements have been made to EMD such as Bi-dimensional EMD (BEMD) [148], Ensemble EMD (EEMD) [138] which defines the true IMF component as the mean of an ensemble of trials, multidimensional EEMD (MEEMD) and many more [138–145]. The effectiveness of the EEMD lead to some modifications, such as the complementary EEMD [139] and complete EEMD with adaptive noise (CEEMDAN) [149, 150] that aims to reduce the residual noise or spurious modes. These EMD related techniques have been recently applied to FPP systems [145]. However, EEMD based methods require amplitude of the noise to be selected empirically which is quite difficult in practice and also, the ensemble size is required to be large leading to heavy computation. Thus, inspired by the idea of taking assistance from the noise, researchers have investigated various methods to remove background noise effectively [140, 142]. RPSEMD [144] is an adaptive regenerated sinusoidal method where sinusoidals are iteratively generated at every IMF level and the further decomposition is based on that, which proved to be effective.

In this section, the properties of fringe background will be examined. Based on the analysis, we explore the possibility of background removal in the Fourier domain

and the windowed Fourier domain by removing the low frequency components in an appropriate manner, and subsequently, extended Fourier transform method with reasonable padding and windowed Fourier filtering (WFF) method with proper parameter setting are proposed for effective and efficient background removal. This work is presented in [3].

4.2.2 Windowed Fourier Filtering (WFF) for single carrier fringe pattern

Our purpose in this section is to justify the intuitiveness of the WFF algorithm discussed in the previous chapter. A 1D single carrier fringe pattern can be defined as:

$$f(x) = a(x) + b(x)\cos[\varphi(x) + \omega_{cx}x] \quad (4.9)$$

where $a(x)$ is the background intensity, $b(x)$ is the fringe amplitude, $\varphi(x)$ is the phase information and ω_{cx} denotes the carrier frequency; which is same as the Eq. 1.6. A 2D forward and inverse WFT pair is expressed in Eqs. 1.15 and 1.16.

WFF algorithm works on a filtering mechanism to analyze a window of fringe pattern at a time. A block diagram to explain the working of WFF algorithm is shown in figure 2.2. The Windowed Fourier transform (WFT) of this signal defined in Eq. 4.9 is then:

$$Sf(u, \xi_x) = \int_{-\infty}^{\infty} f(x)g_{\xi_x}^*(x - u)dx \quad (4.10)$$

where x and u are the spatial x-axis coordinates, the superscript $*$ is a conjugate operator, ξ_x is the frequency coordinate and $g_{\xi_x}(x)$ is a WFT kernel expressed as:

$$g_{\xi_x}(x) = g(x)\exp(l\xi_x x), \quad (4.11)$$

$$g(x) = \frac{1}{(\pi\sigma_x^2)^{1/4}}\exp\left(\frac{-x^2}{2\sigma_x^2}\right) \quad (4.12)$$

A thresholding approach is used in WFF to classify the coefficients or filter the coefficients: coefficients which are larger than the threshold are considered “good”

and those which are smaller are considered “bad” and thus, removed by setting them to zero. This procedure is formulated as:

$$\bar{f}(x) = \frac{\xi_{xi}}{2\pi} \int_{-\pi}^{\pi} \bar{S}f(x, \xi_x) \otimes g_{\xi_x}(x) \quad (4.13)$$

$$\bar{S}f(u, \xi_x) = \begin{cases} Sf(u, \xi_x), & \text{if } |Sf(u, \xi_x)| \geq \text{thr} \\ 1, & \text{if } |Sf(u, \xi_x)| < \text{thr} \end{cases} \quad (4.14)$$

$$\hat{\varphi}_w = \angle \bar{f}(x) \quad (4.15)$$

$$\hat{\omega}_x(x) = \frac{d\hat{\varphi}}{dx} \quad (4.16)$$

where $\bar{S}f(u, \xi_x)$ denotes the thresholded spectrum, and $\bar{f}(x)$ denotes the filtered fringe pattern, thr denotes a threshold, $\hat{\varphi}_w$ is the extracted wrapped phase, $\hat{\varphi}$ is the filtered phase and $\hat{\omega}_x(x)$ is the filtered local frequency.

The main task is to extract the uneven background by analyzing the fringe pattern in the spectral domain and thus, extracting the background for effective fringe pattern demodulation. To do so, 1D-FT, 1D-WFF algorithm is implemented as follows:

- For a 1D-FT, a 5×5 window is selected for filtering (low-pass filtering). A lower window 3×3 is also tried but it leads to ripples since the window size is small.
- A low threshold is kept to be able to extract the background effectively and then, instead of extracting or filtering the fundamental component [26], we extract the background component around 0 frequency.

4.2.3 Simulation based Observations

In order to analyze the effectiveness of background removal using WFF, a simulated uneven background is constructed as shown in Fig. 4.4(a) and is embedded in the fringe pattern as in Fig. 4.4(b).

1D-WFF and 1D-RPSEMD approaches are then applied to reconstruct the uneven background as shown in Fig. 4.5. It shows the results for background

Figure 4.4: Example of a simulated background: (a) uneven background, and (b) uneven background in a simulated fringe pattern [2].

Figure 4.5: Background retrieval using EMD and WFF as (a) 1D-WFF retrieved background, (b) RPSEMD retrieved background and (c) result for both WFF and RPSEMD for 100th row [2].

retrieval using WFF and the state-of-the-art EMD approach- RPSEMD [145]. It is observed that 1D-WFF fails across the image borders or corner pixels.

Thus, to improve the quality of retrieval around the edges using a WFF algorithm a mirroring approach is used. In this, approach the original image is mirrored on all four sides (east,west, north and south) and also along north-east, south-east, north-west and south-west directions as shown in Fig. 4.6. Applying the same WFF algorithm to extract the low-frequency background component

Figure 4.6: Mirroring technique to replicate the image in all directions around it: (a) Original fringe pattern, and (b) mirrored fringe pattern [2].

on the mirrored image now leads to much smoother and improved quality result around the corners as shown in Fig. 4.7. Once the background is extracted from the fringe pattern, the quality of the extracted background is compared against the original background to test the effectiveness of background removal and it was observed that a PSNR of almost 64.9 db is observed for a window size $(\sigma_x, \sigma_y) = (50, 50)$ which appeared as the most effective window size for background removal. The frequency selection for WFF is also narrowed down since the peak of background component in spectral domain is sharp and thus, the effective frequency selection is down around 0-th frequency by keeping the frequency range as: $[\xi_x l, \xi_x i, \xi_x h] = [-0.05, 0.025, 0.05]$.

4.2.4 Experimental Observations

In this section, the performance of uneven background removal is evaluated and compared with the state-of-the-art RPSEMD algorithm. For experimentation, the FPP system comprised of a “liquid crystal display (LCD) projector (EPSON, EB-1761W, 3LCD, WXGA 1280 by 800 pixels, 2600lm) and a CMOS camera (Imaging Source Co., DFK 72BUC02, 1600 by 1200 pixels @ 13fps)”. The camera is mounted by an 8mm focal-length mega-pixels lens (Computar, M0814-MP). The distance of the object from the camera and projector system is 800 mm where both the camera

Figure 4.7: Background retrieval using EMD, FT and WFF as (a) original background distribution, (b) a row cross-section of background retrieval by EMD, FT and WFF. Image courtesy [2].

and projector lay at an angle of about 30 degrees with each other. The experimental fringe pattern captured by the camera is shown in Fig. 4.8(a).

The captured fringe pattern include uneven background and noise as shown in Fig. 4.8(a). To resolve for uneven background, we perform denoising beforehand to only focus on the uneven background errors. The extracted background using both RPSEMD and the proposed WFF algorithm are shown in Fig. 4.8(b)-(d) respectively. The extracted WFF based background appears blurry as compared to the background extracted using RPSEMD. One particular reason behind this could be that WFF algorithm being a filtering algorithm blurs the details to some extent. In order to further analyze it more clearly, we considered a cross-section (100-th row) marked in yellow in Fig. 4.8(a). The background corresponding to all the algorithms RPSEMD, WFF and FT are shown in Fig. 4.8(e). It is clearly observed that both WFF and RPSEMD depicts almost similar results for the extracted background despite the blurry effect by WFF, which is the most interesting observation of this chapter. The reason behind this is the mode-mixing problem in EMD which is resolved by WFF algorithm.

Figure 4.8: Experimental results for background retrieval using EMD, FT and WFF as (a) captured fringe pattern, (b) EMD based background removal (c) background removal using EMD with median filtering (d) WFF based background removal (e) background for a row cross-section.

4.2.5 Discussion

Findings and observations made in this chapter are summarized here:

- **Theoretical accuracy:** Similar accuracy for both RPSEMD and WFF algorithms has been observed for removal of uneven background. 2D-FT on the other hand, leads to ripples and poor quality.
- **Performance on complicated phase:** For a real experimental scenario where a mix of noises are present, it is advisable to resolve each noise prior to analyzing for uneven background. For this scenario, WFF blurs the details due to the filtering nature of the algorithm. Still, both WFF and RPSEMD

algorithms again show similar results which is the most interesting finding of this chapter.

4.2.6 Conclusions

We have explored the possibility of background removal in the windowed Fourier domain by removing the low frequency components in an appropriate manner, and subsequently, extended Fourier transform method with reasonable padding and windowed Fourier filtering (WFF) method with proper parameter setting are proposed for effective and efficient background removal. We observed that WFF and the state-of-the-art EMD based techniques work comparably and provide similar results for removing background intensity. WFF being a simpler approach than EMD and hence, is more effective. However, WFF due to its filtering nature leads to blurry effect. Despite this fact, it still performs at par with the state-of-the-art RPSEMD algorithm which is a novel finding of this chapter.

Chapter 5

Sampling Moiré and Windowed Fourier Transform (WFT): a comparative study

This chapter revolves around the comparative study of two established fringe analysis techniques namely, Sampling Moiré (SM) and the windowed Fourier transform (WFT). Sampling Moiré uses a single fringe pattern to obtain N phase-shifted Moiré patterns from it, while WFT processes a fringe pattern in a window by window manner to analyze phase information in the frequency domain. Even though, these two methods look different we compare the two techniques to evaluate the phase extraction errors for both a noiseless and a noisy case for phase distributions varying from simple to complex. The phase extraction accuracies for this technique is published as [4] and also mentioned explicitly in this chapter.

5.1 Introduction

The success of all fringe analysis methods is dependent on the phase distribution. If an accurate phase is extracted with ease, the measurement will have high accuracy. Among the several fringe analysis methods mentioned in literature, Sampling Moiré and Windowed Fourier transform techniques are among them. Windowed Fourier transform is a middle technique between spatial and spectral analysis which uses both properties and is effective in demodulating a carrier fringe pattern. Sampling

Moiré (SM) method [119] is another useful technique which is similar to phase-shifting algorithm for phase extraction, as it generates multiple phase-shifted Moiré fringe patterns and thus, use them for fringe demodulation.

FT, WFT, and SM all techniques have their own advantages and are used for varied applications as mentioned in chapter 2. Thus, it is important to have a comparison analysis for all three methods in order to have a more in-depth understanding of these methods. A number of comparative studies have been done in literature for numerous phase analysis techniques. A comprehensive comparison study focusing on 1D-CWT against FT, PSA, and WFT have been reported [194]. Another comparison study involving FT, WFT, and WT for phase extraction from a single fringe pattern is depicted in detail in [65] WFT has been established as a special case of spatial-carrier phase shifting (SCPC) in [5]. However, till date, there is no comparison study which analyzes the accuracy of SM and WFT for phase demodulation for a single carrier fringe pattern. This is thus, a motivation behind this study.

Since SM and WFT are both local fringe analysis techniques, in this chapter we try to compare the performance of the two algorithms against a reference technique 2D-FT. Among two well-established algorithms on WFT: windowed Fourier ridges (WFR) and windowed Fourier filtering (WFF), windowed Fourier ridges (WFR) is used extensively for this study. The performance evaluation is based on a fringe pattern with both linear and nonlinear nature of phase in both noiseless and noisy environments. Since SM processes a fringe pattern in a row-by-row manner, to keep the comparison fair, a 1D-WFR is selected.

The rest of the chapter is organized as follows. Section 5.2 provides the principle of 1D-SM. Section 5.3 sets up the comparison environment and later, the simulation results are observed based on both linear and nonlinear phases. Section 5.4 depicts the computational time efficiency of the two algorithms. Section 5.5 analyzes and compares the performance of both methods for a real-experimental data. Section 5.6 provides further discussion based on comparison study. Finally, Section 5.7 provides concrete conclusions.

5.2 Principle of 1D-Sampling Moire (SM)

Sampling Moiré comprises of two processes to create a number of Moiré fringe patterns from a single image namely, “down-sampling” and “intensity-interpolation” [119, 120] In SM, a 1D carrier fringe pattern as mentioned in Eq. 2.15 is down-sampled at a regular sampling pitch T . This creates T down-sampled images of smaller size which are phase-shifted in nature. In order to create T -step phase-shifted Moiré fringe patterns from them, each down-sampled image is interpolated by either using a linear interpolation or a high-order interpolation (say bi-cubic). We will use both a linear and bi-cubic interpolated SM technique for comparison. Throughout this chapter they will be depicted as 1D-SM (1-order) and 1D-SM (3-order), respectively.

The t -th phase-shifted Moiré fringe pattern, $f_{m,t}(x)$ are considered as described in Eq. 2.16. Firstly, $2\pi x/T$ is subtracted from the carrier frequency in order to make Moiré frequency and later, a phase-shift of amount $2\pi t/T$ is introduced to generate T phase-shifted Moiré fringe patterns. Moiré phase is then obtained as:

$$\varphi_m(x) = \tan^{-1} \left[\frac{\sum_{t=0}^{T-1} f_{m,t}(x) \sin \left(\frac{2\pi t}{T} \right)}{\sum_{t=0}^{T-1} f_{m,t}(x) \cos \left(\frac{2\pi t}{T} \right)} \right] \quad (5.1)$$

This extracted Moiré phase $\varphi_m(x)$ is depicted in figure 5.1 (at right-hand side). Since earlier $2\pi x/T$ is subtracted from the carrier frequency, by adding it back the actual wrapped phase of the original fringe pattern can be obtained:

$$\varphi_{SM}(x) = \varphi_m(x) + \frac{2\pi x}{T} \quad (5.2)$$

WFR algorithm used here has been described in detail in section 2.2.1.2.

5.3 Comparison by simulation

In this section, the performance of 2D-FT, 1D-SM (1-order/3-order) and 1D-WFR for phase demodulation are analyzed and compared with each other. However, before jumping to the comparisons directly, the fringe generation and parameter

Figure 5.1: Working principle of 2D-FT, 1D-SM, and 1D-WFR [4].

selection approaches are discussed first. Nature of the fringe pattern and deducing the presence of noise is an essential element before comparison and thus discussed in the next sub-section. later, comparative study for different noise levels is carried out.

5.3.1 Fringe generation and parameter selection

Figure 5.2: Simulated fringe pattern (256×256) pixels: (a) linear phase with $k = 0$, and nonlinear phases with (b) $k = 0.5$, (c) $k = 1.0$, and (d) $k = 2.0$ are shown, respectively. Image courtesy [4].

The phase for a 2D fringe pattern is simulated using the “MATLAB peaks

function” as:

$$\varphi_{simu}(x) = \frac{2\pi}{P}x + k \cdot \text{peaks}(M) \quad (5.3)$$

where M denotes the size of the fringe pattern $M \times M$ and k suffices for the complexity. In this chapter, we will consider $M = 256$ and $P = 8.1$ pixels. Four different fringe patterns are shown in figure 5.2, where figure 5.2(a) shows a linear fringe pattern when $k = 0$. Complex fringe patterns for $k = 0.5, 1.0$ and 2.0 are shown in figures 5.2(b)-(d).

The three algorithms require some parameter settings as:

- 2D-FT: It is implemented according to [39, 40]. To retrieve the fundamental component, band-pass filter is used. However, the frequency range or bandwidth of the band-pass filter (BPF) is selected to be a window size of 61×61 pixels surrounding the carrier frequency.
- 1D-SM (1-order/3-order): It is implemented according to [120, 195] The sampling pitch $T = 8$ pixels and the grating pitch $P = 8.1$ is considered for both simulations and experiments.
- 1D-WFR: It is implemented according to [26]. Even though it is advised to use a smaller window size for WFR, usually 5 or 10, in this chapter we consider a window size of $\sigma_x = T/2$ is used to make window size comparable to 1D-SM (1-order/3-order). Since, WFR uses a frequency scanning range to search for the ridge location, we set the frequency search range to be $[\xi_{xl}, \xi_{xi}, \xi_{xh}] = [0.685, 0.025, 0.885]$ in a scanning range of $2\pi/T \pm 0.1$.

Once the phase is estimated using all three algorithms, the phase error in comparison to the original phase are computed as:

$$\Delta\varphi(x) = \varphi_{method}(x) - \varphi_{simu}(x), \text{method} = [\text{FT}, \text{SM}, \text{WFR}] \quad (5.4)$$

Since both WFR and SM uses a window by window approach, it is thus advisable to exclude the image border and hence, in the rest of the paper only the central pixels 200×200 will be used for computing phase errors and thus, consequently RMS phase error.

5.3.2 Performance evaluation on noiseless fringe patterns

5.3.2.1 Linear phase

In this sub-section, we consider a fringe pattern with linear phase i.e. $k = 0$ and the fringe pattern is generated corresponding to Eq. 1.6. The phase estimation error for all three methods namely, 2D-FT, 1D-SM (1-order), 1D-SM (3-order) and 1D-WFR are shown in Fig. 5.2(a) and the RMS phase are recorded as 7.6×10^{-3} , 3.35×10^{-5} , 1.64×10^{-9} and 9.57×10^{-15} rad, respectively. A significant error is observed for 2D-FT in comparison to other methods, since the number of fringe in the image are non-integer. Thus FT assumes that the fringe pattern is periodic and repeats itself continuously leading to a large error. Being the local fringe analysis techniques, both 1D-SM and 1D-WFR leads to errors around the pixel boundaries and thus, should only be analyzed for central pixels. 1D-SM (1-order) gives a relatively higher error in comparison to 1D-SM (3-order) and 1D-WFR.

5.3.2.2 Nonlinear phase

A nonlinear fringe pattern with peaks phase with $k = 2.0$ is analyzed in this case and the simulation result for a noiseless case is depicted in Figure 5.3. A clear distinction in the phase error range is seen from the figure where light-pink zone indicates a phase error less than 0.1% of 2π while, the gray zone indicates a phase error less than 1% of 2π . Phase error less than 0.1% of 2π is extremely important for high-precision measurements. With changing complexity of phase i.e. with changing k , 2D-FT is almost invariant and depicts a phase error of 0.0076 rad belonging to 0.12% of 2π . 1D-WFR results in a low phase error of about 4.3×10^{-3} to 4.9×10^{-3} rad, which is 0.07 to 0.08% of 2π . For SM, 1D-SM (3-order) will perform better than 1D-SM (1-order) as indicated in [120, 195] and thus, we compare 1D-SM (3-order) and 1D-WFR. It is thus, observed that the accuracy of the two algorithms is similar for $k \leq 1.0$. For $k > 1.0$, 1D-WFR performs better since WFR searches for a best frequency element by scanning.

In order to do an in-depth analysis, the performance results are also shown for a cross-section of fringe pattern i.e. 64th row for a nonlinear phase with $k = 1.0$,

Figure 5.3: Simulation result for the case of $P = 8.1$ and $T = 8$ for a row cross-section of the fringe patterns. The colored zones depict phase error to be less than 0.1% of 2π , while the above portion represents phase error to be less than 1% as indicated. Image courtesy [4].

as shown in figure 5.4 for all three algorithms. Since the number of fringe patterns is not an integer, 2D-FT shows a periodical phase error. For $P = 8.1$ pixels and $t = 8$, there is a mismatch in the two pitches which leads to periodical phase errors in 1D-SM (1-order) [120] too as shown here. However, phase error of 1D-SM (3-order) is consistent with that of 1D-WFR result. Thus, it can be depicted that for a noiseless fringe pattern, both 1D-SM (3-order) and 1D-WFR perform in a similar manner for both linear and nonlinear phases with high accuracy.

5.3.3 Accuracy for fringe patterns in the presence of noise

After analyzing the accuracies for a noiseless case, to bring the analysis closer to a real-world scenario, we will consider noise in this sub-section. A 2D captured

Figure 5.4: Simulation results for 64th row of a noiseless fringe with $k = 1$: (a) 2D-FT, 1D-SM (1-order), and (b) 1D-SM (3-order), 1D-WFR. Image courtesy [4].

fringe pattern in the presence of noise can be represented as:

$$f(x, y) = a(x, y) + b(x, y)\cos[\varphi(x, y)] + n(x, y) \quad (5.5)$$

where $n(x, y)$ is the additive random noise. In this sub-section we assume that noise has a zero mean and the standard deviation ranges from 0% (noiseless) to 40%.

5.3.3.1 Linear phase

The RMS phase error is computed as an average of 10 test cases for the same noise level. Figure 5.5(a) shows the RMS phase error for a linear fringe pattern with

Figure 5.5: RMS phase errors of 2D-FT, 1D-SM (1-order), 1D-SM 93-order) and 1D-WFR algorithms for different random noise %: (a) linear phase, and (b) non-linear phase. Image courtesy [4].

$k = 0$. The error less than 1% of 2π is indicated in the gray zones while the error less than 0.1% of 2π is indicated by the light-pink zone as depicted in 5.5(a).

Since the noise spreads evenly in the entire spectral domain, for a 2D-FT, effective extraction of fundamental component becomes difficult. However, from figure 5.5(a), it can be clearly seen that FT outperforms all other methods. One possible reason for this could be that since a 2D-FT algorithm is used it uses a large number of pixels for the entire image making the noise suppression effective than the other 1D algorithms which use a relatively lower number of pixels. Since,

interpolation can also add random errors for pixel estimation using neighboring pixels, these errors could be large in high-order interpolation. Thus, 1D-SM (1-order) performs slightly better than 1D-SM (3-order).

5.3.3.2 Nonlinear phase

Noise scenario as mentioned above is replicated here for fringes with $k > 0$ for a peaks phase. An example of a nonlinear fringe pattern with peaks phase for $k = 1.0$ is shown in figure 5.5(b). The results thus, obtained are very similar to that obtained for a linear phase as shown in figure 5.5(a) and 1-D SM (1-order/3-order) and 1D-WFR both show almost identical results. This indicates that the nonlinear nature of phase does not impact the fringe analysis technique much whereas noise plays a major role.

As depicted from figure 5.5, in this case 2D-FT outperforms 1D-SM and 1D-WFR, however, it is less susceptible to noise. 2D-WFR on the other hand, is more tolerant to noise as already mentioned in [65].

To analyze the effect of noise, a fringe pattern with a noise level = 1% is shown in figure 5.6. As shown in figure 5.6(a), 2D-FT leads to some fluctuations and periodical phase error in the horizontal direction. SM (3-order) provides similar results as 1D-WFR and is more effective than 1D-SM (1-order) as shown in Fig. 5.6(b)-(d).

In order to attain an accuracy in the light-pink zone for the nonlinear phase, i.e. accuracy 0.1% of 2π the random noise % should be kept low and thus, restricted.

5.4 Comparison of computational Time

For a real-experimentation computational efficiency is an important part. However, testing the computational efficiency for simulations gives an insight on the overall computational time and the real-experimental time can be related to it. Here, in this section we investigate the computational efficiency of the three algorithms 2D-FT, 1D-SM (1-order) and 1D-WFR whose performance evaluation has already been done in the previous sections.

Figure 5.6: Phase error distributions for 1% noise level for a peak phase with $P = 8.1$ and nonlinear quotient $k = 1.0$: (a) 2D-FT, (b) 1D-SM (1-order), (c) 1D-SM (3-order), and (d) 1D-WFR both in the case of $T = 8$ pixels. Image courtesy [4].

Table 5.1 shows the computational efficiency for the simulated fringe patterns shown in figure 5.5(a) and figure 5.5(b). An average of 10 runs is recorded here. A “Windows 7 PC (64-bit; Xeon CPU E5-1650; 3.2GHz)” and “MATLAB (Ver. R2014b)” system is used to test the computational time. From the results in the table, it can be observed that 2D-FT is the fastest since it is “global” in nature and thus processes the whole image in a go. For 1D-SM, it was already confirmed that 1D-SM (3-order) is 1.5 times slower than 1D-SM (1-order) as mentioned in

[120]. Hence, we only compared 1D-SM (1-order) and 1D-WFR here. Due to the frequency search involved in 1D-WFR, the computational time for it is higher.

Table 5.1: Computational time efficiencies for three methods: 2D-FT, 1D-SM (1-order) and 1D-WFR.

Complexity	Noise	2D-FT	1D-SM(1-order)	1D-WFR
Linear ($k = 0$)	noise (40%)	0.007s	0.023s	0.261s
Nonlinear ($k = 2$)	noiseless (0%)	0.007s	0.022s	0.261s

5.5 Comparison by Experiment

The complete the analysis, a performance evaluation of all the algorithms is done for an experimental fringe pattern from an FPP system [103, 168, 173, 176]. We first describe the experimental setup and analysis conditions are introduced and then, the performance evaluation is shown later.

5.5.1 Setup and Analysis conditions

We used an FPP setup which includes a “liquid crystal display (LCD) projector (EPSON, EB-1761W, 3LCD, WXGA 1280 by 800 pixels, 2600lm) and a CMOS camera (Imaging Source Co., DFK 72BUC02, 1600 by 1200 pixels 13fps)”. The camera is mounted by an “8mm focal-length mega-pixels lens(Computar, M0814-MP). The distance of the object from the camera and projector system is 800 mm where both the camera and projector lay at an angle of about 30 degrees with each other. An experimental fringe pattern is given in figure 5.7.

We used an N -step phase-shifting measurement (PSM), due to its high reliability and high resolution. This will be considered as a reference for comparison of 2D-FT, 1D-SM(1-order/3-order) and 1D-WFR techniques. For the phase-shifted fringe patterns as in Eq. 1.5, the phase extraction process is similar to Eq. 5.1. Here, we analyze the phase gradients of all the methods in the x -direction as the

Figure 5.7: Fringe pattern generated by an FPP system of size 1600×1200 . We only consider the right-hand side portion 400×800 pixels for comparison of PSM, 2D-FT, 1D-SM(1-order/3-order), and 1D-WFR. Image courtesy [4].

accuracy of measurement is dependent on the calibration procedure apart from the phase estimation. The phase gradient is calculated as:

$$\frac{\delta\varphi(x, y)}{\delta x} = \frac{1}{2}[\varphi(x + 1, y) - \varphi(x - 1, y)] \quad (5.6)$$

For analysis, we used $N = 8$ phase shifted fringe patterns one of which is shown in figure 5.8(a). Figure 5.8(b) shows the extracted wrapped phase. The fringe amplitude, $b(x, y)$, is shown in figure 5.8(c) and the phase gradient in figure 5.8(d), which will serve as a reference for performance evaluation of 2D-FT, 1D-SM, and 1D-WFR.

Some other experimental analysis conditions are as follows:

- For performance evaluation, we used a section of the fringe pattern of size 400×800 pixels, as indicated by the green rectangle in figure 5.7.

Figure 5.8: Experimental results for a 8-step PSM: (a) fringe pattern, (b) wrapped phase map, (c) fringe amplitude, and (d) phase-gradient. Image courtesy [4].

- $T = 15$ is considered as an optimal parameter for 1D-SM (1-order/3-order) as the fringe patterns projected by the projector is around 15 pixels.
- For the 1D-WFR to work effectively, it is advised to remove the negative component and the background bias prior to experimentation so that only the fundamental component remains. This helps in identifying the ridge of the fundamental component accurately.

5.5.2 Comparison study for experimental results

The fringe pattern as in figure 5.7 is now evaluated or process by all the methods 2D-FT, 1D-SM, and 1D-WFR and the results are shown in figure 5.9. Figure 5.9(a) shows the frequency spectrum by 2D-FT. Figures 5.9(b) and 5.9(c) show that the Moiré fringe patterns and indicates that the Moiré fringe obtained by 3-order interpolation is better and much more smoother than the 1-order interpolation. Figure 5.9(d) shows the ridge map by 1D-WFR. And finally, figures 5.9(e)-5.9(h) show the phase distributions by 2D-FT, 1D-SM (1-order) and 1D-SM (3-order) and 1D-WFR, respectively.

Figure 5.9: Experimental results for a face image depicting: (a) Fourier spectrum of 2D-FT, (b) Moiré fringes generated by the 1D-SM (1-order), and (c) Moiré fringes generated by the 1D-SM (3-order), (d) ridge map of 1D-WFR; (e)-(h) phase distributions of the four algorithms: 2D-FT, 1D-SM (1-order), 1D-SM (3-order), and 1D-WFR, respectively. Image courtesy [4].

Phase gradients of all the algorithms are shown in figures 5.10(a)-(d). It is clear from this, that 2D-FT leads to some periodic errors and thus, result in poor measurement quality. 1D-SM (1-order/3-order) and 1D-WFR methods, around the smooth face region, provides similar results.

Fig. 5.10(e)-(h) show the phase difference computation between 2D-FT, 1D-SM, 1D-WFR and 8-step PSM respectively. The yellow area indicates the absolute phase difference exceeds 5% due to complicated structure of face. The areas around the noise or eyes where discontinuities and shadows exist, fringe patterns become difficult to detect and thus, all four algorithm give poor results. However, for the portions where face is relatively smoother, all algorithms perform satisfactorily.

A cross-section AA' (as shown in figure 5.8(d) and figure 5.10(c)) representing

Figure 5.10: Phase gradient results obtained for: (a) 2D-FT, (b) 1D-SM (1-order), (c) 1D-SM (3-order), (d) 1D-WFR; (e)-(h) phase differences (in %) between the above mentioned methods and temporal 8-step PSM. Image courtesy [4].

the smooth part of the face, is analyzed by computing phase-gradients for all methods as shown in figure 5.11. It is observed that both 1D-SM (3-order) and 1D-WFR depicts almost identical and better performance in comparison to 8-step PSM, which is the most interesting observation and finding of this chapter. Since 8-step PSM uses only 8 points from 8 fringe patterns in this case, there is a large large standard deviation in phase computation.

5.6 Discussions

Findings and observations made in this chapter are summarized here:

Figure 5.11: Phase gradient evaluation for a cross-section AA' for all methods namely, 8-step PSM, 1D-SM (3-order), and 1D-WFR. Image courtesy [4].

- **Theoretical accuracy:** 2D-FT, 1D-SM, and 1D-WFR being the effective solutions for fringe analysis have been observed to have high theoretical accuracy.
- **Performance on complicated phase:** 2D-FT being a “global operator” leads to poor performance for a complicated phase as well as discontinuities since the information on one part of the image can influence the information far off. The local processing of 1D-SM and 1D-WFR insures that the information at one particular point is only related to the pixels close by and thus, the error does not propagate to other parts of the image.
- **Performance on noise:** In the presence of noise, all three algorithms 2D-FT, 1D-SM and 1D-WFR shows excellent noise suppression.

- **Novel finding:** The most interesting observation of this chapter is, despite being two seemingly different algorithms, the performance of 1D-SM and 1D-WFR is identical.
- **Applications:** 1D-SM has already been applied to various applications such as “strain of various materials” [196, 197], accurate “in-plane displacement” and “structural health monitoring of large-scale structures” [198], or “removal of vibration in phase-shifting digital holography” [199] at a very high speed by effectively using an “optimal sampling pitch” based on prior knowledge. Similarly, WFR has also been applied to various applications as mentioned in [200]

Apart from these, even though a few observations are not emphasized in this chapter, it is still worth-mentioning:

- **Improvement of performance:** Currently since SM uses a fixed frequency, thus, in order to use a scanning frequency for sampling pitch, SM will be expected to be able to perform well for more complicated fringe patterns. Similarly, WFR can process more general fringe patterns.
- **Improvement of speed:** SM using a fixed frequency for sampling pitch, shows high-speed and high accuracy and thus, can be used for a lot of useful practical applications. If such prior-knowledge is given to WFR without performing a frequency scanning, the narrower the frequency band for ridge searching, higher the speed of measurement.
- **Extension to the 2D analysis:** 2D-SM [201] and 2D-WFR [26] are expected to have similar accuracy. 2D algorithms are more robust to noise since they use a large number of pixels and hence useful for several applications.

5.7 Conclusion

Several fringe analysis techniques have been proposed in literature for a single carrier fringe pattern demodulation. Among them, in this chapter we mainly focused on 1D-SM and 1D-WFR to analyze the phase measurement accuracies. These two techniques though seem different and have been applied for different applications. In order to compare them, author first tried to evaluate and analyze their working principles and later compared the performance with respect to the standard 2D-FT. Both the noiseless and noisy environments are considered for comparison. The comparisons are thus, based on: first, theoretical accuracies of both the algorithms for both linear and nonlinear phases are recorded and compared in a noiseless environment. Second, the same procedure is then followed as mentioned in first, for a noisy case in order to compare the performance. Finally, the comparison is based on a real experimental fringe pattern (using an FPP system) to analyze the performance for real measurement to compute both the phase and phase gradient. Computational efficiency of the two algorithms are then further recorded. From these comparisons, it was clearly confirmed that the phase measurement accuracies of 1D-SM and 1D-WFR are quite similar and thus, both of the algorithms can be used for each others applications which has been observed for the first time in literature and is undoubtedly, a novel finding of this chapter. Thus, with this, we encourage the users to use both the 1D-SM and 1D-WFR in their practice, as well as their 2D versions.

Chapter 6

WFR as a spatial carrier phase-shifting

The windowed Fourier ridges (WFR) algorithm is a middle ground between spatial phase-shifting and the spectral Fourier transform techniques. A clear relation between WFR and Fourier transform can already be seen since both the algorithms are transform-based methods. However, the link between WFR and phase-shifting algorithm has not been discovered yet which serves as a motivation for this paper and hence be used to analyze it. This link will prove useful in understanding the different phase extraction techniques in an effective manner [5].

6.1 Introduction

“Pixelwise” phase shifting and “global” Fourier transform being the two pillar techniques of fringe analysis, creates a base for analyzing a fringe pattern in both spatial and spectral domain. With the development of WFR which uses both the spatial and spectral properties for fringe pattern analysis, it serves as a middle or in-between technique among the two. WFT is a natural extension of Fourier transform technique. Since the pixel at one point is closely related to its neighboring pixels and depicts less correlation with the pixels far off, WFT serves as an effective technique for fringe rendering in many applications. However, till date the relationship between WFT and the phase-shifting technique is unclear and has

not been presented yet. With this motivation in mind, in this chapter we try to present an in-depth correlation between WFT and phase-shifting algorithms. This link is useful in understanding different phase extraction techniques in an effective manner and is published in [5].

The content of this chapter is structured as follows: Section 6.2 gives a brief description of a spatial carrier phase-shifting (SCPS) algorithm. Section 6.3 describes the windowed Fourier ridges (WFR) algorithm for a single carrier fringe pattern. Section 6.4 provides the overall discussions related to the established link and finally, Section 6.5 concludes the chapter.

6.2 Spatial Carrier Phase-shifting Technique

Among the three majorly used phase-shifting techniques namely, temporal phase-shifting, spatial phase shifting and spatial carrier phase shifting techniques, in this section, we consider spatial carrier phase shifting (SCPS) technique for analysis. In the case of spatial carrier fringe pattern, the demodulation can be done using either SCPS [37] or WFT and thus, a possible equivalence between them is yet to be explored.

A carrier fringe pattern as mentioned in Eq. 1.6 can be used to extract the phase at any pixel (u, v) with adjacent pixels at $(u + i, v)$, $-n \leq i \leq n$. These $N = 2n + 1$ pixels being the adjacent pixels are assumed to have the same background intensity, fringe amplitude and phase, can be used to modulate a phase-shifted fringe pattern as:

$$f(u + i, v) = a(u, v) + b(u, v)\cos[\varphi(u, v) + \delta_i], -n \leq i \leq n \quad (6.1)$$

where the phase and phase-shift values as mentioned in chapter 2 are:

$$\varphi(u, v) = \varphi_0(u, v) + \omega_{cx}u \quad (6.2)$$

$$\delta_i = \omega_{cx}i, -n \leq i \leq n \quad (6.3)$$

The phase is then extracted using a least-squares fitting approach [121, 122]. To do so, Eq. 6.1 can be re-written as:

$$f(u + i, v) = p_1 + p_2\cos\delta_i + p_3\sin\delta_i, -n \leq i \leq n \quad (6.4)$$

with

$$\mathbf{p} = [p_1, p_2, p_3]^T = [a(u, v), b(u, v)\cos[\phi(u, v)], -b(u, v)\sin[\phi(u, v)]]^T \quad (6.5)$$

where the superscript T denotes a vector transpose. The parameter vector $\mathbf{p}$ can be obtained from Eq. 2.25 as $\mathbf{p} = \mathbf{A}^{-1}\mathbf{B}$, which is used to compute the extracted phase according to Eq. 2.23 as:

$$\phi(u, v) = \tan^{-1} \left[-\frac{\sum_{i=-n}^n f(u+i, v)\sin(\omega_{cx}i)}{\sum_{i=-n}^n f(u+i, v)\cos(\omega_{cx}i)} \right] \quad (6.6)$$

which is widely used for all phase-shifting algorithms.

6.3 WFR as spatial carrier phase shifting algorithm

With the developments in the previous sections and to establish the WFR algorithm as a SCPS algorithm, Eq. 1.15 needs to be observed with following considerations: (i) Since the real computations are discrete, instead of using a continuous WFT, we use a discrete WFT by changing the integration to a summation; (ii) the window function for frequency search is assumed to be a 1-D rectangular window with a width of $N = 2n + 1$ around the center pixel (u, v) to make it similar to the neighboring pixels used in SCPS; (iii) A 1D analysis is performed by ignoring the carrier frequency along y . Eq. 1.15 can thus be expressed as:

$$Sf(u, v, \xi_x) = \sum_{i=-n}^n f(u+i, v)\exp[-j\xi_x(u+i)] \quad (6.7)$$

Consider $c_{xx} = c_{yy} = 0$ to make a constant phase similar to phase-shifting and, the second and third terms on the right-hand side of Eq. 2.13 disappear. By substituting Eq. 6.7 into Eq. 2.13 and replacing ξ_x by $\hat{\omega}_x$, and the phase estimation in relation to Eq. 6.2 can be obtained as:

$$\hat{\varphi}(u, v) = \tan^{-1} \left[-\frac{\sum_{i=-n}^n f(u+i, v)\sin(\hat{\omega}_{cx}i)}{\sum_{i=-n}^n f(u+i, v)\cos(\hat{\omega}_{cx}i)} \right] \quad (6.8)$$

It can be observed that the Eq. 6.8 is almost exactly the same as Eq. 2.32, where instead the global frequency ω_{cx} replaced the local frequency $\hat{\omega}_x$ to deal with local pixels. With this relation, SCPS and WFR algorithms can be related and hence, WFR can be seen a SCPS algorithm.

However, there is a slight difference in the phase estimated by the two techniques. Eq. 2.32 extracts the true-phase $\varphi(u, v)$, while Eq. 6.8 provides an estimated value $\hat{\varphi}(u, v)$. In other words, the SCPS algorithm provides accurate phase extraction by extracting the true-phase of the fringe pattern. On the other hand, WFR algorithm only provides an estimation of the phase which is close to the true phase value. This eventually raises a question to the effectiveness of the WFR estimation as to how good of the estimation does WFR makes. To answer for that, it is can be observed from Eq. 2.32 that $t_N(\alpha)$ decreases with N , and if N is large, $t_N(\omega_x/2) \approx t_N(\omega_x) \approx 0$, making Eq. 6.8 a good estimator of the true-phase. Since there are a total of three-components in a spatial carrier fringe pattern the other two present undesired influences as mentioned in [26] are removed and the extraction process is mainly focused on obtaining the fundamental component. WFR is mainly focused on the analyzing the fundamental component (by analyzing the ridge or peak value of the fundamental component) for effective phase extraction when the other components are present as well, thus, providing a good estimation of the true phase.

To understand and analyze the phase extraction error by WFR algorithm, a straightforward way is to find the estimated phase by substituting Eq. 6.1 into Eq. 6.8 and later, compare it with its nominal value. Before applying WFR for phase extraction, it is suitable to remove the background component in order to avoid its influence i.e. make $a(x, y)$ by applying a high-pass filtering in advance, and thus we assume $a(x, y) = 0$. Later, substituting Eq. 6.1 after background is removed i.e. $a(x, y) = 0$ into Eq. 6.8, WFR based phase estimation can be yielded as:

$$\hat{\varphi}(u, v) = \tan^{-1} \left[\frac{\sin[\varphi(u, v)][1 - t_N(\omega_x)]}{\cos[\varphi(u, v)][1 + t_N(\omega_x)]} \right] \quad (6.9)$$

where we replaced $\hat{\omega}_x$ by ω_x since it is a good estimation of the true frequency. The phase estimation error, $\delta\varphi(u, v) = \delta\varphi_0(u, v) = \hat{\varphi}_0(u, v) - \varphi_0(u, v)$, can also be evaluated as:

$$\delta\varphi(u, v) \approx \tan[\delta\varphi(u, v)] = \frac{\tan[\varphi(\hat{u}, v)] - \tan[\varphi(u, v)]}{1 + \tan[\varphi(\hat{u}, v)]\tan[\varphi(u, v)]} \quad (6.10)$$

$$\frac{-\sin(2\varphi)t_N(\omega_x)}{1 + \cos(2\varphi)t_N(\omega_x)}$$

The extreme points are found at $\cos(2\varphi_0) = -t_N(\omega_x)$ with the extreme error as $\delta\varphi(u, v) = \frac{-\sin(2\varphi)t_N(\omega_x)}{1-t_N(\omega_x)}$ by considering $\delta\varphi(u, v)$ to be zero and thus, the maximum phase error can be obtained as:

$$|\delta\varphi|_{\max} \approx |t_N(\omega_x)| \quad (6.11)$$

6.4 Validation Results

The observed theoretical analysis need to be quantified and validated. The validation is done using both simulations and real-experimental setup as discussed below.

6.4.1 Simulated Analysis

The theoretical result thus, obtained in Eq. 6.11 is then verified by simulations and it is observed that the result is consistent with our simulation as shown in Fig. 6.1. $|\delta\varphi|_{\max}$ with respect to N and ω_x can thus, be easily obtained by using Eq. 6.11. The white portion in figure 6.1(a) shows that $|\delta\varphi|_{\max} \leq 2\pi/100$ and the black portion in the same figure shows otherwise. Similar observations can be made in figure 6.1(b) by changing the threshold to $2\pi/30$. The thresholds $2\pi/100$ and $2\pi/30$ are chosen since they are widely used in the fringe projection community for expected performance. It can thus, be concluded that a low phase extraction error can be achieved using WFR.

Figure 6.1: Simulation to depict WFR phase extraction error. White portion depicts that the errors are less than (a) $2\pi/100$ and (b) $2\pi/30$. Image courtesy [5].

6.4.2 Experimental Result

To justify the analysis for a real-optical setting, a performance evaluation is done for both WFR and SCPS algorithms in comparison to 2D FT algorithm. We used the same experimental setup as described in Sec. 5.5.1 for the projector and camera based FPP system.

Figure 6.2: Experimental wrapped phase for SCPS and WFR algorithms: (a) captured fringe pattern, (b) wrapped phase map using SCPS algorithm and (c) wrapped phase map using WFR algorithm.

For analysis we used a SCPS fringe pattern with $N = 8$ phase-shifted neighbouring pixels which is shown in Fig. 6.2(a). Fig. 6.2(b) shows the wrapped phase using

SCPS algorithm and Fig. 6.2(c) shows the wrapped phase using WFR algorithm. For the WFR algorithm here, it is advised to remove the negative component and the background bias prior to experimentation so that only the fundamental component remains. This helps in identifying the ridge of the fundamental component accurately. The phase difference between the two algorithms is indicated in Fig. 6.3. It can be clearly observed that the maximum phase error between the two algorithms for $N = 8$ is $> 2\pi/100$ and $> 2\pi/30$ since $N = 8$ is relatively small; thus, validating the theoretical analysis. However, if we further increase N the error will further reduce. Hence, WFR can be seen as a spatial carrier phase-shifting algorithm for higher N .

Figure 6.3: Phase difference between SCPS and WFR algorithms with $N = 8$.

6.5 Discussions

WFR is a technique which is based on frequency search in the spectral domain and spatial carrier phase-shifting technique analyzes the phase information with

the help of neighboring pixels. In order for WFR to qualify as a phase-shifting algorithm, it requires a large number of pixels thus, N cannot be small, while the phase-shifting algorithm can work very well with small N too. This is one of the disadvantages of WFR. However, in practice, since we try to involve a large number of pixels thus a larger N , it increases the insensitivity towards noise, and thus, both WFR and phase-shifting can work effectively. This is one major advantage of WFR. WFR uses the local frequency ω_x as the phase-shift value, whereas SCPS uses the global carrier frequency ω_{cx} . Thus, for the case when phase is locally changing, SCPS is not suitable for fringe demodulation, while WFR can still work effectively. For a locally changing phase, some iterative algorithms have been proposed to analyze the fringe demodulation in spatial domain.

Based on the above mentioned advantages of WFR, in a precise manner we can clearly state that:

- There is no assumption for the phase to be locally constant for WFR;
- Insensitivity of WFR towards noise leads to the fact that the estimated phase using WFR is already denoised;
- Due to digital nature of the fringe pattern, there is no phase-shift error;
- WFR is proven as an effective method against high-order harmonics and thus, is insensitive towards them;
- the a Gaussian window is often used in WFR;
- For a quadratic phase, the error due to eastimated phased as performed by WFR is corresponding to the second and third term in Eq. 2.13; and
- WFR as depicted by Eqs. 2.12 and 2.13 is 2D, which can be converted to a one-dimensional or three-dimensional with slight modifications.

Thus, WFR and SCPS can be attempted interchangably for better results. Also, WFR for a fringe with a temporal carrier [26] is assumed to be linked up similar to the temporal phase-shifting technique, however, it is currently not emphasized in this chapter.

6.6 Conclusion

Despite appearing to be different techniques, the link between WFR and SCPS algorithm has been successfully developed where the WFR algorithm for fringe pattern analysis is in itself a phase-shifting algorithm. However, there is a slight difference between the two algorithms is also highlighted here. WFR provides a phase estimation, while SCPS gives the true value. But, both algorithms work equally well in resolving FPP errors and extracting the phase information accurately for surface measurement. Thus despite this difference, these two algorithms are interestingly linked up here. And it can be well understood that Fourier transform, WFT, and phase-shifting technique can be a continuous spectrum in fringe pattern analysis.

Chapter 7

Conclusions and future work

7.1 Conclusions

Optical measurement techniques have been studied and applied for surface measurement, detecting deformation of objects and many more, due to the advantages of non-contact, non-destructiveness, and high accuracy and sensitivity. Fringe pattern analysis (FPA) is an important aspect of all optical measurement techniques. In the past decade, FPA has been thoroughly studied and developed, however, two main problems still exist which needs further investigation. First, in pursuit of 3D measurement of object surfaces which can be readily applied and used in various applications such as virtual reality, animations, precision inspection of optical components and many more, obtaining a high-speed and highly accurate measurement has become a necessity. Second, since a number of fringe analysis methods have been developed over past decades, it has become increasingly difficult to understand, analyze and introspect some best suited applications and conditions where different fringe analysis methods can prove useful and thus, requires a link to be set up between different algorithms for further investigation. These two problems can be solved by investigating some fringe analysis techniques for high-accuracy measurements and establishing a link between different fringe analysis techniques to enable effective use of algorithms with respect to measurement conditions. In recent years, several methods have been developed for high speed and high accuracy 3D surface measurement based on varied principles. Since the spectral information

of a patch of fringe pattern is much simpler to analyze and thus, instead of resolving some of the major error sources in existing FPP systems in spatial and spectral domain separately, we try to make use of both the properties by using a Windowed Fourier transform (WFT). This thesis has provided a systematic study of powering fringe pattern analysis techniques, using the Windowed Fourier transform (WFT).

First, Some of the state-of-the-art fringe analysis techniques are reviewed. Also, some major issues in fringe analysis for FPP systems are inspected such as nonlinear gamma, uneven background, edge discontinuity and high dynamic range, and the existing state-of-the-art techniques for resolving these issues including both the phase-shifting and transform-based methods in the past decades are reviewed. It is found that the accuracy of measurement can be improved by either using a large number of fringe patterns or by iterating the algorithm a number of times thus, leaving a scope for further improvement.

To resolve these issues in fringe analysis, a theoretical analysis for resolving the nonlinear gamma errors using Windowed Fourier Ridges (WFR) for carrier phase demodulation is then proposed. It first, analyzes the local frequency error and phase extraction error for the carrier fringe pattern in order to describe the amount of gamma non-linearity and then, further resolves the error with the help of a performance prediction and error control using WFR. It was observed that despite being challenging it is possible to reduce the peak phase error to be as low as 0.05 rad in the presence of the error source. Thus, WFR is considered as one of the most effective solutions for resolving gamma nonlinearity.

Later, we analyzed gamma nonlinearity in phase-shifted fringe patterns and tried to devise an algorithm to tackle the Nonlinearity errors using WFR by squeezing the N phase-shifted fringe patterns into a single carrier fringe pattern. Further, To tackle the issue of uneven and discontinuous background in a single carrier fringe pattern, a Windowed Fourier filtering (WFF) based background removal technique is proposed. It uses a mirroring approach to tackle the corner pixels. An ideal window of $[50,50]$ is used for effective background removal and nonlinearity suppression.

After discussing two major issues in fringe analysis, a comparative study is then performed to analyze the effectiveness of WFR in comparison to Sampling Moire (SM) for single carrier fringe pattern for the first time. While SM converts a single carrier fringe pattern to multiple phase-shifted Moire patterns to demodulate phase, WFR obtains the phase information in the window based Fourier domain. Despite appearing two completely different approaches, both simulations and experimental results confirm that the two algorithms are two sides of the same coin and thus, can be used interchangeably in various applications. In the presence of noise, both algorithms give outstanding performance.

A link between WFR and spatial carrier phase-shifting (SCPS) algorithm is proposed for the first time. Since phase-shifting and Fourier transform are the two pillar techniques of fringe analysis, and as Windowed Fourier transform has already been established as the extension of Fourier transform it is crucial to understand the link between the two to select a proper technique for effective fringe analysis. It is thus, observed that whenever SCPS is used to analyze a fringe pattern, WFR can always be attempted for better results.

7.2 Future work

In order to develop a fully robust and accurate fringe analysis techniques, some future developments needs to be done; some of which are highlighted in this chapter. As mentioned in section 1.2, the various issues in fringe analysis techniques still prevail which impact the overall performance of 3D surface measurement and thus, needs to be eliminated. In order to develop an accurate 3D measurement system, in near future we wish to work on resolving some other existing issues in the current FPP systems and devising novel and accurate 3D fringe analysis techniques which will result in high accurate measurements.

7.2.1 Further improving the measurements using FPP system

7.2.1.1 Eliminating high dynamic range errors and reconstructing the invalid part

High dynamic range errors appears around the area of the object where over-shadowed highlights or dark shadows appear and fringe patterns become unreadable in these areas. In the absence of fringe pattern, these portions of the object is considered invalid or discontinuous and hence, appear as empty discontinuous points or patches in the wrapped phase or point cloud as shown in Fig 7.1. These invalid regions on the object surface thus, reduces the accuracy of measurement. In order to resolve these high-dynamic errors and thus, improve the accuracy of measurement, several methods have been developed in literature over the past years which is discussed in sections 2.3 of this report. In order to remove these invalid areas, one of the best techniques is to use multiple camera-projector system, to capture images from different angle and use all the information to reconstruct the object surface which will eventually eliminate the shadow and highlight areas [73] completely. However, in doing so, the overall cost of the system becomes too high and syncing all the devices at the same time or with a lag becomes a difficulty. Other important technique [148, 149] is to fill the shadow or highlight areas with the help of the surrounding pixels by extrapolating the pixels to fill the information in these regions. The inpainting based methods to fill the highlight or shadow portions has less hardware requirement and is a good technique to eliminate shadow irregularities. Since the already developed WFF algorithm is a filtering based process and is able to accurately remove the noise hence, applying a WFF algorithm to eliminate high dynamic range can prove to be useful. The preliminary results to remove some small high dynamic patches from the object surface using WFR which eliminates the invalid areas on the object surface highlighted in red effectively as shown in Fig. 7.2. One particular reason for this is that since the invalid regions are small hence, the WFF algorithm considers them as noise and eliminates them effectively. However, there are two questions which need further investigation:

Figure 7.1: Experimental result for high-dynamic range errors where, (a) the captured image of kitty with the red circles marking the highlights and the shadow regions, (b) Corresponding highlights and show regions in the wrapped phase.

- How big the invalid region should be in order for the WFF algorithm to completely remove it?
- If the area is too big, will WFF algorithm can still be used to resolve the issue

Figure 7.2: After applying WFR, both the highlight and shadow regions in the red highlighted portion of the previous image have disapppread giving a smooth 3D surface.

of high dynamic range or not and can its performance then be compared to the existing algorithms?

All these questions are still an uncertainty to our preliminary work based on WFR which will be explored in future.

Besides that, the existing inpainting methods exploit the information of sparsity or wavelet transform to extrapolate the pixels to fill the missing shadow or highlight regions and hence, reconstruct a smooth 3D object surface. However, no method has been developed so far which utilizes the segmentation or clustering approach combined with utilizing the surrounding pixel intensities to extrapolate the missing region. The regions around the shadow or highlight regions can be segmented and clustered to form groups required to extrapolate a particular portion of the invalid region. Simple extrapolation using neighboring causal pixels may lead to irregularities if the shadow area is big enough. But when the area is segmented at every stage while extrapolating, the new pixels generated and segmented will have

a high probability to achieve a uniform 3D surface which is a possibility to further discuss, experiment and work on in future.

7.2.1.2 Removing Edge Discontinuities

Current fringe analysis methods to tackle discontinuity are either automatic or human assisted. Fig 7.3 shows an example of edge discontinuity after WFR algorithm is applied to a noisy wrapped phase as mentioned in [132]. This indicates that the discontinuity arises when the computer is unable to differentiate between the background and the object fringes due to a merge effect generated during capturing. However, in this figure the discontinuity is quite obvious and can be removed to some extent by WFR algorithm. However, for the cases when discontinuity is not obvious, the conventional automatic analysis methods fail to automatically build a clear barrier between two pieces as shown in Fig 7.3. Hence, a semi-automatic technique involving human assistance was proposed [68] to remove the discontinuity accurately and hence, obtain an accurate phase map.

Figure 7.3: Resolving discontinuity: (a) shows a denoised wrapped phase map, (b) Quality map, (c) Path map, and (d) the Unwrapped phase. Image courtesy [132].

Human assisted methods allow the user to mark a few points which the computer cannot detect and with the help of the information obtained by computer and fed by user it successfully reconstructs the object boundary and hence remove discontinuity. Such a technique gives a satisfactory result. However, there is still scope to develop an accurate method to remove edge discontinuity which can be viewed in the following ways:

- **User-interactive method:** A human assisted method could be developed based on the boundary the eyes sees as marking the points on feeding the outline to the computer for an accurate reconstruction of the object surface.

Thus, improvisations in the current method is an easy alternative to increase the accuracy of measurement by selecting the area around discontinuity using a user-interactive based method. This suffices my future work plans for the next 1.5 years and if implemented properly we would be able to design highly accurate fringe analysis techniques for 3D surface measurements which could then later, we developed and used further based on their applications.

7.2.2 High speed 3D real-time FPP system

One the accuracy of the current FPP system is improved, the second task is to improve the speed so that moving objects can be simultaneously reconstructed. The current FPP system is able to reconstruct moving objects at a speed of 30fps however, the acquisition and reconstruction are not yet simultaneous. The main reason behind it is sequential processing of the fringe patterns. In order to improve the speed further, there is a need to parallelize parts of the sequential fringe pattern analysis along with GPU acceleration to be able to achieve a simultaneous acquisition and reconstruction. Also, a much advanced camera should be used with the frame rate of ≥ 30 fps so that high-speed moving objects can also be reconstructed well.

7.2.3 Impact of AI on 3D measurement

With a vast advancements in AI technology represented by deep learning, it is worthwhile to analyze the impact of AI on 3D measurements. Over the past decade, there has been intensive research to use neural networks in particular to analyze fringe patterns [202–209]. Neural networks has its own merits in terms of high-speed and noise resistance and effective approximation. [209] is a first of its kind paper which discusses a framework using deep neural networks to enhance

phase demodulation accuracy from a single fringe pattern in comparison to other conventional fringe analysis techniques. In order to improve the accuracy further, there is a need to explore AI techniques to accurately demodulate fringe patterns and make the 3D measurement system fully automated.

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- Agarwal, N., Wang, C., Wang, H. and Kemaio, Q., “Windowed Fourier Filtering based background intensity removal for fringe enhancement,” presented at *International Conference on Optical and Photonic Engineering* (2018), Shanghai, China.