Atmospheric wind characterisation for power generation

Tieo, Jing Jin

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**ATMOSPHERIC WIND CHARACTERISATION FOR
POWER GENERATION**

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**Interdisciplinary Graduate School
Energy Research Institute @ NTU**

2019

ATMOSPHERIC WIND CHARACTERISATION FOR POWER GENERATION

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INTERDISCIPLINARY GRADUATE SCHOOL

A thesis submitted to the Nanyang Technological University
in partial fulfilment of the requirement for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Asst Prof. Alessandro Romagnoli

Authorship Attribution Statement

This thesis contains material from paper(s) published in the following peer-reviewed journal(s) where I was the first and/or corresponding author.

Chapter 4 is published as J.-J. Tieo, T.-Y. Koh, M. Skote, N. Srikanth. Variance Characteristics of Tropical Radiosonde Winds Using a Vector-Tensor Method. *Energies* **2018**, *11*, 137 (2018). doi:10.3390/en11010137.

The contributions of the co-authors are as follows:

- Prof. Koh Tieh Yong conceived the initial idea and provided the initial direction for the project.
- I wrote and edited the drafts of the manuscript. Prof. Martin Skote and Prof. Koh Tieh Yong revised the manuscript, and Dr. Narasimalu Srikanth reviewed the manuscript.
- I downloaded the relevant radiosonde data from the IGRA database.
- I wrote the MATLAB scripts to analyse the data and carried out the analysis.
- I co-developed the suite of vector-tensor statistics together with Prof. Koh Tieh Yong.
- Prof. Koh Tieh Yong explained the relation between the scalar variance and the mean wind speed and provided the relevant proofs in Appendix C and D.

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The contributions of the co-authors are as follows:

- Dr. Narasimalu Srikanth provided the lidar wind data used in the project.
- I wrote and edited the drafts of the manuscript. Prof. Martin Skote revised the manuscript and Dr. Narasimalu Srikanth reviewed it.

- I wrote the MATLAB scripts needed to carry out the analysis and analysed the results.
- I developed the alternative wind profile models proposed in the study. Prof. Martin Skote suggested the use of an inverse tangent-based function.
- Dr. Narasimalu Srikanth provided the discourse on the lidar method and error included in Section 5.3.2.

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Abstract

Wind energy is one of the main sources of renewable energy currently being exploited on an industrial scale. But despite its prevalence in the extratropical regions of the world, the feasibility of using the wind as an energy source within the deep tropics is only beginning to be discovered. Underpinning the development of wind energy is the accurate characterisation of the atmospheric winds so that assessments on the potential wind power generation can be made. Therefore, there is a need to improve the current knowledge on the characteristics of winds in the tropical atmosphere so as to support future developments of wind energy in the deep tropics.

Two exploratory studies on the statistical characterisation of atmospheric winds in the deep tropics are presented. The first is an investigation on the variance of the upper-air winds in a near-equator monsoon region using radiosonde data. An unconventional vector-tensor method was used that allows the directional characteristics of the variance to be quantified. A suite of statistics was then designed that provided an intuitive interpretation of the geometric properties of the variance. It was found that the size and the anisotropy of the variance vary with height whereas the preferred axis of the variance tends to align with the direction of the mean wind. It was also found that the variance of the scalar wind speed increases with its mean.

In the second study, the boundary-layer wind profile on an offshore island of Singapore was investigated using wind data obtained from a lidar. It was found that the observed wind profiles deviated characteristically from what can be predicted by using the traditional power-law wind profile model. Two alternative wind profile models were then proposed and it was found that the one based on an inverse tangent function was able to give better predictions of the observed boundary-layer profiles. An investigation on the seasonal and diurnal variations of the observed wind speeds was also carried out. It was thus concluded that the tropical wind characteristics can deviate considerably from the well-known established models and rules of thumb.

Lay Summary

Wind energy is one of the main sources of renewable energy currently being exploited on an industrial scale. But despite its prevalence in many parts of the world away from the tropics, the feasibility of using the wind as an energy source near to the equator is only beginning to be discovered. Underpinning the development of wind energy is the ability to accurately determine the trends and patterns in the atmospheric winds so that predictions on the potential wind power output can be made. Therefore, there is a need to improve the current knowledge on the characteristics of winds in the tropical atmosphere so as to support possible future developments of wind energy in the tropics.

Two exploratory studies on characterising atmospheric winds in the tropics using observation data are presented. The first uses wind data obtained from weather balloons to investigate the characteristics of the variations in higher-level winds found in a near-equator monsoonal region. To carry out this work, an unconventional vector-tensor based method was used that allows the directional characteristics of these variations to be uncovered. It was found that, among other notable trends, the axis where the wind encounters its largest variation tends to align with the direction of the prevailing wind even though they need not be so.

The second study uses wind lidar data obtained on an offshore island of Singapore to investigate the manner by which the wind speed changes with height near the surface in the first 200 m of the atmosphere. It was found that the power-law boundary-layer model, which is an established model used to predict wind speeds near the surface, deviated significantly from the observations. An alternative boundary-layer model, which was proposed during the study, was found to give better predictions. The day-to-night and seasonal changes of the observed wind speeds were also investigated. It was concluded that the existing models and rules of thumb widely used in wind estimation might not be suitable for application in the tropics.

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Abbreviations

ABL	Atmospheric Boundary Layer
cdf	Cumulative distribution function
CFD	Computational Fluid Dynamics
ENSO	El Niño-Southern Oscillation
GPS	Global Positioning System
HAWP	High-Altitude Wind Power
IGRA	Integrated Global Radiosonde Archive
IOD	Indian Ocean Dipole
ISO	Intra-Seasonal Oscillation
ITCZ	Inter-Tropical Convergence Zone
MJO	Madden-Julian Oscillation
MOST	Monin-Obukhov similarity theory
MSE	Mean squared error
NLLJ	Nocturnal low-level jet
NOAA	National Oceanic and Atmospheric Administration
NWP	Numerical Weather Prediction
PBL	Planetary Boundary Layer
pdf	Probability distribution function / Probability density function
QC	Quality control
SSE	Sum of squared errors
SST	Sea-surface temperature
UTLS	Upper Troposphere + Lower Stratosphere
WECS	Wind Energy Conversion System
WMO	World Meteorological Organisation
YMC	“Years of the Maritime Continent”

Chapter 1 Introduction

This chapter gives a broad overview on the topic of wind characterisation. It also provides the objectives and structure of this dissertation.

1.1 General Overview

The wind has been used as a source of energy by human societies since antiquity. It serves as the means of propulsion for sailing ships and was harnessed by agrarian settlements to produce useful mechanical work. Today, wind energy represents one of the main sources of alternative, renewable energy in the global fight against anthropogenic climate change brought about by the use of fossil-based fuels. According to the GWEC Global Wind Statistics 2016 [1], the global cumulative installed wind capacity grew by 12.5% in 2016 from 433 GW of the previous year to 487 GW. This in itself is an almost 6.6-fold increase over the 74 GW just 10 years prior in 2006. Underpinning this rapid growth in wind energy is the accurate characterisation of wind behaviour at potential wind energy extraction sites that allows planners and decision makers to assess the economic viability of developments at such sites. Aside from energy generation, the knowledge of wind characteristics is also critical in other industries. In civil engineering, the structural load caused by wind is an important design consideration for bridges and skyscrapers [2]. In aviation, high-altitude winds en route affect flight times and fuel burn [3-7]. On the ground, the wind characteristic around airports is a major factor for flight operations efficiency and safety [8].

In the simplest understanding, wind is simply the result of air mass moving in response to the differences in pressure within the atmosphere. This is in turn driven by the uneven solar heating across the Earth's surface. To understand the complete picture of the atmospheric circulation however, one must also consider more complex effects such as the surface properties and topography, the moisture content in the atmosphere and associated cloud formation, the Coriolis effect induced by the Earth's rotation, et cetera. Aside from the physical mechanisms, another point of consideration is that of the scale, both spatial and

temporal. Because of the stochastic nature of atmospheric motion, statistical methods are almost always used to describe the characteristics of the wind. This makes the results and the inferences from such results subject to the consideration of the frequency and spatial distribution of the measured data. For example, data recorded at a frequency of once a day would not be able to adequately capture phenomena that occur and evolve within the span of a day or less. In a similar sense, a sparsely-spread network of sensors would not be able to capture phenomena of sizes smaller than the gaps between the said sensors.

Current research in the characterisation of the atmosphere can be divided into two types: statistical characterisation using real data and numerical modelling. In statistical characterisation, data collected from field observations are used to determine the trends and patterns exhibited by the atmospheric parameter(s) being investigated. Such studies are typically carried out when the use of real-world data is necessary, such as resource assessment for a new potential wind energy site or for the validation of theoretical or numerical models [9-23]. While the use of real data gives confidence to the “truthfulness” of the observations, it often requires considerable expenditure to acquire the necessary space and observation equipment as well as significant lead time to collect enough data for meaningful analysis. For numerical modelling, results from computer models are used instead. The caveat however, is that these results are not actual observations but are instead model predictions. The most sophisticated class of such models are the numerical weather prediction (NWP) models which contain parameterisations for most of the atmospheric processes and are used for general atmospheric research as well as operational weather forecasting by meteorological agencies [24-29]. These models can create large-scale weather simulations and they solve equations related to physical processes in the atmosphere, such as cloud formation and precipitation, that are not represented in simpler models. For more specialised applications, other types of models are also in use such as the linear wind flow model WAsP [11,21,30-33] and computational fluid dynamics (CFD)-based softwares [32-37] in the case for wind energy resource assessment. The latter solve mass and momentum balance equations for the atmospheric flow and are used to predict the wind field in a localised area.

In the tropics, the mechanisms driving atmospheric flow are somewhat different from that of the higher-latitude extratropics [38,39]. Within the deep tropics (defined here as within 15° latitude from the equator), weather is dominated by strong sunshine, which rapidly heats up the Earth's surface and drives deep convection, low temperature gradient, and a weak Coriolis force. This results in, aside from the obvious higher temperatures, more frequent cloud formation, higher rainfall, and generally lower wind speeds [40] compared to the extratropics. Aside from these, some parts of the tropics also experience unique synoptic-scale features such as the monsoons [39,41]. Despite these differences, the tropical atmosphere remains not as extensively studied compared to the extratropics especially in the area of the characterisation of winds using observation data. This is partly due to the lack of a comprehensive network of observation sensors and available data for the region [42] and also due to lukewarm commercial interests because of the lower wind speeds.

1.2 Objectives and Scope

This dissertation thus seeks to address and contribute to the gap in research by presenting two observation-driven wind characterisation studies for the deep tropics. The first involve the characterisation of high-level winds in the larger Africa-Asian monsoon region using public domain radiosonde data. The second involve the characterisation of the tropical boundary-layer wind profile using observations from a lidar sensor sited on an offshore island of Singapore. The latter would be of particular interest to the budding wind energy community in the deep tropics. Given the differences in the physical mechanisms driving atmospheric flow in the tropics, it is conceivable that the observed wind characteristics would also be different. This notion would be examined as part of the boundary-layer study.

1.3 Hypothesis

It is hypothesised that the wind characteristics in the deep-tropical atmosphere is different from that of the extratropics, and that some existing models for atmospheric winds, which

were mostly developed in the extratropics, will be unable to satisfactorily capture these characteristics.

1.4 Dissertation Structure

The subsequent chapters of this dissertation are as follows: Chapter 2 gives readers a background overview of atmospheric circulation and wind characterisation, including the review of contemporary theories and practices. Chapter 3 describes the statistical tools and techniques employed in the two characterisation studies presented in this dissertation. Chapter 4 presents the radiosonde characterisation study, including all results and discussions. Chapter 5 presents the same for the boundary-layer study. Finally, Chapter 6 strings together the salient conclusions from each chapter and concludes the dissertation overall.

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Chapter 2 Literature Review

This chapter provides the relevant background knowledge necessary to understand the research presented in this dissertation. In the first section, a description of the known circulatory patterns of the atmosphere and contemporary theories are discussed. In the second section, the methods used for characterising the atmospheric winds are described. Lastly, the third section describes current techniques employed for the measurement of wind data.

2.1 The Atmospheric Circulation

2.1.1 Global Circulation

The Earth's atmosphere is not a static body but is instead in a state of constant motion driven by pressure differences within the atmosphere as well as by external forces. The atmospheric winds are simply the result of this motion. Among the external forces driving atmospheric motion, the most important of them would be the Coriolis force, friction, and to a much lesser extent, gravity.

2.1.1.1 Vertical Structure of the Atmosphere

Gravity holds the atmosphere to the Earth's surface and is responsible for the stratification of the atmosphere as a result of hydrostatic balance [1]. While its effects are largely static and it does not contribute significantly to the formation of wind and weather patterns, it is responsible for the large-scale vertical structure of the atmosphere. This vertical structure [2] is diagrammatically shown in Figure 2.1 for an idealised standard atmosphere.

Figure 2.1: The vertical structure and temperature profile of the atmosphere. Adapted from Figure 2.15 of R. G. Barry, R. J. Chorley. *Atmosphere, Weather and Climate*, 8th ed. London: Routledge (2003).

In Figure 2.1, the vertical axes denote the height with the corresponding atmospheric pressure at each height. The horizontal axis denotes temperature with the curve showing the vertical temperature profile. The major vertical layers of the atmosphere are also labelled. Due to the stratification, much of the atmospheric mass is concentrated within the lowest layer of the atmosphere, the troposphere, immediately adjacent to the planetary surface. In fact, the atmospheric density drops to only about 50% of its sea-level value at a mere height of about 6.7 km [3]. Above the troposphere lies the stratosphere where an inversion in the temperature gradient keeps the layer stably stratified. As a consequence of this, atmospheric motion resulting in most of the weather phenomena of interest are capped within the troposphere.

The troposphere can be further subdivided into three layers according to its primary physical effects. The lower troposphere (sometimes also referred to as the planetary boundary layer or PBL) is the lowest layer and lies adjacent to the planetary surface [4]. This layer is strongly affected by the topography of the Earth's surface and also by the surface friction. Most human activities take place in this layer. Above the lower troposphere is the middle troposphere, also known as the free troposphere. This layer lies far away and is unaffected by the boundary effects of the planetary surface or the tropopause (boundary between the troposphere and the stratosphere). Above the middle troposphere and adjacent to the tropopause is the upper troposphere. This layer experiences interactions with air coming from the lower stratosphere and is commonly reported in literature in studies done together with the lower stratosphere [5] in what is known as the UTLS (upper troposphere + lower stratosphere).

2.1.1.2 Meridional Circulation and the Hadley Cell

Within the troposphere, atmospheric motion is primarily driven by pressure differences. This is in turn caused by the uneven temperature distribution across the Earth's surface as a result of the uneven heating by the Sun and differences in the thermal and radiative properties of the surface [6]. The air above a warmer surface heats up more rapidly, becomes less dense, and rises due to buoyancy. This creates a region of low pressure near the surface which the surrounding cooler air flows towards. On the converse, the air above a cooler surface sinks and creates a region of high pressure. This sets up an outflow towards the surrounding low pressure and completes a convection cell. Such a cell is illustrated in Figure 2.2.

On the global scale, the mean meridional (along lines of longitude) circulation is typically idealised in the form of three large convective cells [7] in each hemisphere, as shown by Figure 2.3. The sun's heating is the strongest at the equator, causing air to rise near the equator and resulting in the rising branch of the Hadley cell. The descending branches of the cell occur at about the 30° latitude in both hemispheres where it is commonly referred to as the "subtropical high". At the surface, air moves from the higher pressure near the

subtropical highs towards the lower pressure near the equator. This results in northerly (i.e. coming from the north) winds in the northern hemisphere and southerly winds in the southern hemisphere when near the equator. The Hadley cell is considered a thermally-direct cell as it is driven directly by solar heating with warmer air in the rising branch of the cell. Further from the equator, the thermally-indirect Ferrell cell and the Polar cell complete this global circulation pattern.

Figure 2.2: An idealised structure of convection in the atmosphere. Adapted from Figure 6.7 of R. G. Barry, R. J. Chorley. *Atmosphere, Weather and Climate*, 8th ed. London: Routledge (2003).

Figure 2.3: The three-cell model of the global mean meridional circulation. Reprinted from Figure 7.18 of R. G. Barry, R. J. Chorley. *Atmosphere, Weather and Climate*, 8th ed. London: Routledge (2003).

Figure 2.4: The Inter-Tropical Convergence Zone (ITCZ), showing as a band of clouds near the equator. Reprinted from Meteorological Service Singapore. Learn | Tropical Circulation [online document]. URL: http://www.weather.gov.sg/learn_tropical/ (Accessed April 10, 2019).

The equator-centric picture of the Hadley cell is however only representative of the mean circulation and is subject to seasonal variations. Due to the tilt in the Earth's axis, the latitude with the strongest solar heating shifts between the Tropic of Cancer and the Tropic of Capricorn, passing the equator twice, in an annual cycle. The rising branch of the Hadley cell similarly tracks this movement of the Sun's heating [8]. Around the globe, the point where the surface winds converge onto the rising branch of the Hadley cell is known as the Inter-Tropical Convergence Zone (ITCZ) [9]. The ITCZ can commonly be seen in satellite pictures of the Earth as a band of clouds near the equator, such as in Figure 2.4. Within the zone, there is enhanced cloud formation due to the rising air and consequently, enhanced rainfall that is characteristic of a tropical climate. At the descending branches of the Hadley cell in the subtropical highs, cloud formation is instead suppressed with a consequent

reduction in rainfall resulting in very dry climates [10]. Most of the world's major deserts lie within the subtropical highs.

2.1.1.3 The Coriolis Force and other Effects

Aside from differential pressure, the Coriolis force is also an important driver of atmospheric motion, especially further away from the equator. The Coriolis force is a result of the rotation of the Earth [11]. The rotating reference frame in which atmospheric observations are based on meant that there exists a fictitious force that acts upon moving air masses with a magnitude that is proportional to the speed of the motion. In the case of the Earth, the Coriolis force deflects airflow to the right in the northern hemisphere and to the left in the southern hemisphere. The size of the Coriolis force is also dependent on the latitude. At the equator, the force acts only in the vertical direction and, being orders of magnitude smaller than gravity, is usually inconsequential and neglected. As one moves away from the equator however, the planar component of the force increases and becomes significant in the deflection of atmospheric flow. Mathematically, the magnitude of the Coriolis force acting on per unit mass of air can be calculated using Equation (2.1) where Ω is the angular speed of Earth's rotation, ϕ is the latitude, and u is the wind speed.

Figure 2.5: Cyclonic and anticyclonic flow in the northern hemisphere. The directions are reversed in the southern hemisphere. Adapted from Figure 6.8 of R. G. Barry, R. J. Chorley. *Atmosphere, Weather and Climate*, 8th ed. London: Routledge (2003).

$$|\mathbf{F}_{\text{cor}}| = 2\Omega u \sin \phi \quad (2.1)$$

$$\rho \mathbf{F}_{\text{cor}} - \nabla p = \mathbf{0} \quad (2.2)$$

One central concept in atmospheric physics that arises from this Coriolis deflection is that of the geostrophic wind [12]. Due to the influence of the Coriolis force, atmospheric flows do not move in a straight line directly from high pressure to low pressure. Instead, they are deflected and eventually reach a speed where there is a balance between the pressure forces and the Coriolis force (noting that the Coriolis force is a function of the flow speed). This is the geostrophic wind. Equation (2.2) shows the force balance under geostrophicity, noting that pressure forces act in the negative direction of the pressure gradient ∇p . Such flows flow along isobars (lines of equal pressure) and are responsible for the familiar circular flow patterns of cyclones and anticyclones. Illustrated by Figure 2.5, cyclones contain a low-pressure centre and spin anticlockwise in the northern hemisphere whereas anticyclones contain a high-pressure centre and spin clockwise. The direction of spin is reversed in the southern hemisphere.

On the large scale, the deflection caused by the Coriolis force is what resulted in the three-cell meridional circulation structure, instead of a single cell with rising air near the equator and descending air over the poles. It is also partly responsible for the high-speed jet streams [13] that occur near the interfaces of the cells and which are important to intercontinental air travel.

Closer to the equator, where the effects of the Coriolis force are not as pronounced, the northerly winds in the northern hemisphere and southerly winds in the southern hemisphere are deflected towards the west [14]. These resulting northeasterly and southeasterly winds, illustrated in Figure 2.6, are known to early sailors and are referred to as the “trade winds”.

Figure 2.6: A map of the trade winds. The doldrums are regions of very light and variable winds. Reprinted from Figure 7.13 of R. G. Barry, R. J. Chorley. *Atmosphere, Weather and Climate*, 8th ed. London: Routledge (2003).

It is important to note that the global-scale circulation patterns described are idealised and representative of a mean state of the atmosphere. A real circulation pattern would be subject to variations at different time scales and also be affected by the surface properties and topography of the Earth. For example, it is known that the major elevated regions of the world, such as the Himalayas/Tibetan Plateau and the East African Rift Valley, play a big role in shaping the actual global circulation [15,16]. Another point of consideration is the distribution of land masses. Due to the differences in the heat capacities between the land and the sea, this would set up an uneven temperature distribution on the surface, even at the same latitude. One example of a region that has seen increased scientific interest due to its varied topography and complex landmass distribution is maritime Southeast Asia, sometimes also referred to in literature as the Maritime Continent [17,18]. A more detailed discussion on the other effects perturbing the global circulation is however beyond the scope of this dissertation.

2.1.2 Tropical and Mesoscale Circulation

2.1.2.1 Monsoons

The tropics experience some unique circulation features owing partly to the strong solar heating and a relatively weak Coriolis force as compared to the extratropics [19,20]. One of the most prominent and well-known phenomena in tropical meteorology is the monsoon [21]. Monsoons are caused by seasonal variations of the solar heating on major continental land masses resulting in prevailing winds that switches between opposing directions twice a year. Contrary to popular belief, a monsoon system is not defined by its rainfall even though enhanced rainfall can be a consequence of the monsoon flow. Although the definitions in literature may vary, a commonly-accepted definition of the monsoon is a wind system that consistently switches direction twice a year and that the two directions are the direct opposite of each other.

There are several monsoon systems throughout the world, all of which straddle the equatorial tropics. For the purpose of this discourse, the focus shall be on the Asian-Australian monsoon that is illustrated in Figure 2.7. On the (a) pane of the figure, the circulation pattern for the boreal winter is shown, showing northeasterly winds in much of the Indian subcontinent and southeast Asia, which are driven by the stronger solar heating on the Australian continent as compared to the East Asian land mass. On the (b) pane of the figure, a reversed flow pattern is seen instead for the boreal summer as East Asia heats up more than Australia.

Figure 2.7: An illustration of the Asian-Australian monsoon showing the reversing of wind direction over the Indian subcontinent and Southeast Asia for the two monsoon seasons. Reprinted from Chapter 7: "Circulation of the Atmosphere" [presentation slides]. URL: <https://slideplayer.com/slide/5351633/> (Accessed April 3, 2019).

Aside from the monsoon, other features seen in this part of the world include the Madden-Julian Oscillation (MJO) [22,23], the Indian Ocean Dipole (IOD) [24], and the El Niño-Southern Oscillation (ENSO) [25]. These are “wave-like” disturbances in the zonal (east-west) direction where an anomalously high sea-surface temperature (SST) in a region of equatorial ocean drives enhanced convective activity and this is closed by an anomalously low SST in another region resulting in suppressed convection. Both the ENSO and IOD are irregularly occurring standing waves with the regions of enhanced and suppressed convection alternating between the eastern and western coasts of the respective oceans where they act. ENSO refers to the phenomenon that occurs over the equatorial Pacific Ocean whereas IOD refers to the similar phenomenon observed over the equatorial Indian Ocean. Unlike the ENSO and IOD, the MJO is a more-regularly occurring propagating wave that travels eastwards throughout the equatorial Indian and Pacific Oceans. The MJO is also known as the Intra-Seasonal Oscillation (ISO) due to its 30 to 60 days period between each successive cycle of enhanced and suppressed convection.

2.1.2.2 Land-Sea Breezes

On the smaller scale, land-sea breezes [26] are also a prominent circulation feature in the tropics and they drive significant weather patterns. Although the land-sea breeze is by no means exclusive to the tropical regions, they are more frequently the drivers of the weather due to the more intense solar heating near the equator. Most diurnal weather patterns observed in coastal and islandic conditions can be at least in part attributed to the effects of land-sea breezes.

Figure 2.8: The sea breeze and the land breeze. Reprinted from Figure 6.11 of R. G. Barry, R. J. Chorley. *Atmosphere, Weather and Climate*, 8th ed. London: Routledge (2003).

Land-sea breezes occur near coastlines and are caused by the temperature differences between the land and the sea as a result of their different heat capacities. During the day, the land surface heats up more rapidly as compared to the sea. This causes the atmosphere over the land to also heat up more rapidly, creating a region of lower pressure, and drives a flow from the higher pressure over the sea to the lower pressure over land. This wind, coming from the sea and flowing towards land, is the sea breeze. During the night, a reverse circulation is established instead as the sea surface cools down more slowly as compared

to land and thus maintains a higher temperature. This wind, flowing from the land towards the sea, is the land breeze. The land-sea breeze system is illustrated in Figure 2.8. In some cases, the sea breeze can cause enhanced cloud formation and rainfall over land as a result of the topography [27]. For example, if the coast is adjacent to elevated terrain, this can cause a mechanical uplifting of the sea breeze air. Another example would be that of islands. If an island is of an appropriate size, sea breezes from around the island could reinforce each other resulting in very strong uprising air near the centre of the island.

2.1.3 The Boundary Layer

2.1.3.1 Structure of the Boundary Layer

Figure 2.9: The structure of the atmospheric boundary layer over the course of a day. Adapted from Figure 1.7 of R. B. Stull. *An Introduction to Boundary Layer Meteorology*. Dordrecht: Kluwer Academic Publishers (1988).

The atmospheric boundary layer (ABL), sometimes also referred to as the planetary boundary layer (PBL), is the lowest part of the troposphere and is subject to friction and other interferences from the Earth's surface [4]. This part of the atmosphere is arguably the

most watched as it is where most human activities take place. The height of the ABL can vary considerably from between a few hundred meters to over a kilometer depending on the convective activity within the boundary layer, usually determined by the time of day. Figure 2.9 shows a typical structure [28] of the boundary layer and how it evolves over the course of a day.

The surface layer is the lowest part of the ABL (usually cited as the bottom 10%) and is where turbulent fluxes are assumed to be constant. During the day, solar heating drives convective activity and turbulence generation within the boundary layer, resulting in a mixed layer (also known as a convective or unstable boundary layer) where strong turbulence results in uniform mixing of atmospheric quantities such as momentum and entropy in the vertical direction. During the night, the convective activity dies down and a much reduced stable boundary layer is formed. Above the stable boundary layer lies the residual layer, which is a vestige of the previous day's mixed layer where sporadic turbulence may still occur. In some cases, decoupled from the frictional effects of the surface, a localised jet of high wind speed may be observed in the residual layer. This is known as the nocturnal low-level jet (NLLJ) [29,30].

In some literature, there is also the mention of an Ekman layer [31]. This represents the portion of the ABL where the Coriolis force begins to noticeably deflect the boundary-layer flow. Given the relatively weak Coriolis force in the deep tropics however, this effect is not significant in the context of the scope of this dissertation.

2.1.3.2 Boundary Layer Stability

Stability is an important concept in the study of atmospheric boundary layers. It can be parameterised by the Monin-Obukhov length [32] given by Equation (2.3) for dry air below:

$$L = -\frac{u^*{}^3 \theta}{\kappa g \overline{w'\theta'}} \quad (2.3)$$

where u^* is the friction velocity, θ is the potential temperature, κ is the von Kármán constant, $\overline{w'\theta'}$ is the vertical potential temperature flux from the surface, and g is gravitational acceleration. In an unstable boundary layer, which is usually associated with the day time, the surface heats up the atmosphere, generating turbulence and driving convection. The surface temperature flux in this case is positive and this results in a negative L which tends to zero as the boundary layer becomes increasingly more unstable. In a stable boundary layer however, the surface cools and the temperature flux becomes negative. L is positive in this case. For a neutral boundary layer, the surface temperature flux is zero and L becomes infinite.

Note that the potential temperature θ is used in Equation (2.3) instead of the temperature (one which would be measured by a thermometer). The potential temperature is defined as the temperature an air parcel would have if it is brought adiabatically from a pressure level p to a reference pressure level p_0 , usually at 1000 mb. It can be related to the temperature T by Equation (2.4):

$$\theta = T \left(\frac{p_0}{p} \right)^{\frac{R}{c_p}} \quad (2.4)$$

where R is the gas constant and c_p is the specific heat capacity at constant pressure. The potential temperature is thus a proper measure of entropy and can be used to compare the “temperature” (and thus the buoyant tendency) of the air at different heights neglecting the natural lapse rate (decrease in temperature with height) caused by the hydrostatic balance. By definition, the temperature and potential temperature are the same at the reference level.

While the Monin-Obukhov length is frequently cited due to its appearance in the equations of the logarithmic-law wind profile (which will be discussed in the next section), the stability of the boundary layer can perhaps be more easily inferred by looking at the sign of the potential temperature gradient $d\theta/dz$, where z is the vertical height. This is because temperature can be more easily measured compared to the temperature flux. When the potential temperature increases with height ($d\theta/dz > 0$), the boundary layer has a stable

stratification. Whereas if the potential temperature decreases with height ($d\theta/dz < 0$), the boundary layer is unstable. The boundary layer is neutrally stratified if $d\theta/dz = 0$. The potential temperature gradient thus also appears in some other measures related to the stability of atmospheric boundary layers. Two examples are the Brunt-Väisälä frequency, given by Equation (2.5), and the gradient Richardson number, given by Equation (2.6). In Equations (2.5) to (2.7), u is the magnitude of the horizontal wind velocity and the other terms are as defined for Equation (2.3) above. A dry atmosphere is also assumed.

$$N^2 = \frac{g}{\theta} \frac{d\theta}{dz} \quad (2.5)$$

$$Ri_g = \frac{g}{\theta} \frac{\frac{d\theta}{dz}}{\left(\frac{du}{dz}\right)^2} \quad (2.6)$$

$$Ri_b = \frac{g}{\theta} \frac{(\theta_2 - \theta_1)(z_2 - z_1)}{(u_2 - u_1)^2} \quad (2.7)$$

The significance of the Brunt-Väisälä frequency [33], which carries the same sign as the potential temperature gradient, is that N is the angular frequency at which an air parcel in a stable boundary layer will oscillate at if it is perturbed from its equilibrium position. In an unstable boundary layer, a perturbed air parcel will move away from the equilibrium position. The Richardson number [34], on the other hand, is a dimensionless ratio of the turbulence generation (or suppression) in the atmospheric flow due to vertical buoyancy (i.e. stability) versus the horizontal wind shear. Two formulations of the Richardson number are presented here: the gradient Richardson number by Equation (2.6) and the bulk Richardson number by Equation (2.7). The latter is derived from the former by considering a bulk layer of the atmosphere, bounded by heights indicated by the subscripts 1 and 2 in Equation (2.7), instead of the gradient at a point. While the bulk Richardson number is not typically used as a measure of the boundary layer stability per se, it can be related to the Monin-Obukhov length L [35] and is sometimes used as a proxy [36] to calculate L as the

calculation of Ri_b does not require the measurement of the temperature flux (though it does require measurement of the temperature at two different heights).

2.1.3.3 Boundary Layer Wind Profiles

Similar to the boundary layers in engineering fluid mechanics, wind speed in the ABL is zero near the surface (no-slip condition) and typically assumed to increase monotonically with height. The manner in which the wind speed varies with height is known as the boundary-layer wind profile and the gradient of the profile is known as the wind gradient or wind shear. The shape and magnitude of the boundary-layer wind profile (henceforth shortened to just “boundary-layer profile”) is primarily affected by the intensity of the buoyant convective activity, or lack thereof, in the atmosphere (i.e. its stability), the “roughness” of the underlying surface, and to some extent the wind speed in the free troposphere above the boundary layer [37-39].

Historically, two models have typically been employed to represent the boundary-layer profile: the power law, and the log law. The power-law profile [40] is derived from empirical observations and is based on the notion that the wind speed at all heights can be extrapolated from a known wind speed at a certain reference height. Its profile equation is given below by Equation (2.8):

$$u = u_{\text{ref}} \left(\frac{z}{z_{\text{ref}}} \right)^{\alpha} \quad (2.8)$$

where u is the wind speed to be determined at height z , and u_{ref} is the known wind speed at the reference height z_{ref} . The equation contains only a single parameter, α , that captures all variables that affect the shape of the boundary layer. While the power law is mathematically simple to use, it is widely regarded as usable only in a neutral boundary layer. In such cases, the value of α is commonly cited as $1/7$ (≈ 0.14) [41,42]. Outside of neutral conditions, the value of α can vary widely [43]. Some authors have attempted to express α as an explicit

function of surface roughness and atmospheric stability [44,45] in attempts to improve the utility of the power-law profile. However, such efforts have yet to see wide adoption.

The log-law profile [46], on the other hand, contains parameters for several variables that may affect the shape of the boundary layer. The profile equation, which is fundamentally derived from the law of the wall in the field of turbulent boundary layers [47], is given below by Equation (2.9):

$$u = \frac{u^*}{\kappa} \left[\ln \left(\frac{z}{z_0} \right) + \Psi \left(\frac{z}{L} \right) \right] \quad (2.9)$$

where u is the wind speed to be determined at height z , u^* is the friction velocity, κ is the von Kármán constant, z_0 is the roughness length which parameterises the underlying surface roughness, $\Psi(z/L)$ is the Monin-Obukhov stability correction function with the argument z/L , and L is the Monin-Obukhov length that parameterises the boundary-layer stability. The roughness length is typically assumed to take on a fixed value over land surfaces. However, over water, this roughness length has to be modelled further [48] as it is known to be affected by the wave speed and wave height of the water surface, which are in turn dependent on the wind speed.

The major difficulty in using the log-law profile is the selection of a correct expression and the associated coefficients for the stability correction function. Since the publication of the initial theory in 1954, numerous authors have attempted to refine the expressions for the correction function [32,49-56]. The currently most commonly seen expressions are:

$$\Psi \left(\frac{z}{L} \right) = b \frac{z}{L} ; b = 6 \quad (2.10)$$

$$\Psi \left(\frac{z}{L} \right) = -2 \ln \left[\frac{1+x}{2} \right] - \ln \left[\frac{1+x^2}{2} \right] + 2 \tan^{-1} x - \frac{\pi}{2} ; x = \left(1 - \gamma \frac{z}{L} \right)^{\frac{1}{4}} ; \gamma = 19.3 \quad (2.11)$$

$$\Psi\left(\frac{z}{L}\right) = -\frac{3}{2}\ln\left[\frac{1+x+x^2}{3}\right] + \sqrt{3}\tan^{-1}\left[\frac{1+2x}{\sqrt{3}}\right] - \frac{\pi}{\sqrt{3}} ; x = \left(1 - \gamma\frac{z}{L}\right)^{\frac{1}{3}} ; \gamma = 12. \quad (2.12)$$

Equation (2.10) is meant for a stable boundary layer ($L > 0$) and Equation (2.11) is for an unstable boundary layer ($L < 0$). For extremely unstable conditions, Equation (2.12) may be preferable [36,51,55,56]. The stability correction function is zero and the term drops out of the Equation (2.9) for a neutral boundary layer.

The log-law boundary-layer profile is strictly only valid in the surface layer as it assumes a constant friction velocity and vertical temperature flux throughout the boundary layer [57]. In an attempt to extend and construct a profile model that is valid for the entire height of the boundary layer, Gryning et. al. [58] redesigned the length scale used to derive the original log-law profile and included corrections for the middle and upper boundary layer. In their formulation, the length scale for the middle boundary layer, instead of being proportional to the height as in the case of the log-law surface layer, is nearly a constant. The length scale of the upper boundary layer is instead proportional to the distance to the top of the boundary layer. The redesigned length scale is then an inverse summation of the surface layer, middle boundary layer, and upper boundary layer length scales. Using the new length scale, the authors were able to derive the following new profile equations (2.13), (2.14), and (2.15) for the neutral, stable, and unstable boundary layers respectively,

$$u = \frac{u^*}{\kappa} \left[\ln\left(\frac{z}{z_0}\right) + \frac{z}{L_{\text{MBL}}} - \frac{z}{z_i} \left(\frac{z}{2L_{\text{MBL}}} \right) \right] \quad (2.13)$$

$$u = \frac{u^*}{\kappa} \left[\ln\left(\frac{z}{z_0}\right) + b \frac{z}{L} \left(1 - \frac{z}{2z_i} \right) + \frac{z}{L_{\text{MBL}}} - \frac{z}{z_i} \left(\frac{z}{2L_{\text{MBL}}} \right) \right] \quad (2.14)$$

$$u = \frac{u^*}{\kappa} \left[\ln\left(\frac{z}{z_0}\right) + \Psi\left(\frac{z}{L}\right) + \frac{z}{L_{\text{MBL}}} - \frac{z}{z_i} \left(\frac{z}{2L_{\text{MBL}}} \right) \right] \quad (2.15)$$

where L_{MBL} is the middle boundary layer length scale, z_i is the height of the boundary layer, b is the coefficient for the stable stability correction function given by Equation (2.10), and

the other terms having their usual meanings in relation to the log-law profile equation. Here, the stability correction function $\Psi(z/L)$ in Equation (2.15) follows the form given by Equation (2.12). The new profile relations were then validated against tall-mast observations conducted in Denmark and Germany. From the final figure in their paper, it was shown that the profiles for the stable and unstable boundary layers begun deviating from the traditional stability-corrected log-law profile from a height as low as 40 m. The new profile equations adhered more closely to the observational measurements in these cases. For the neutral boundary layer profile however, the traditional log-law profile adhered more closely with the observations up to a height of about 80 m, beyond which the new profile equation produces the better fit.

2.2 Characterisation of Atmospheric Winds

The observational characterisation of the atmospheric winds is an important step in ascertaining the real wind environment and conditions. Typical heights of wind resource extraction have been increasing over the years and today, wind turbines with hub heights above 80 m (in the United States) and 120 m (Germany) are commonplace [59]. While modern numerical modelling methods are increasingly playing a bigger role in atmospheric characterisation due to its convenience and cost effectiveness [60-64], observational works remain important as only field-collected data is certain to represent the actual wind conditions that are encountered. There are many potential inferences that can be made from the wind data, such as the long-term trends, short-term variations, cyclical patterns at the various timescales, frequency and intensity of extreme wind events, and others. The purpose of the characterisation would then determine the methods that would be employed, the type(s) of measurement equipment used, and the length of the observation period [65,66].

2.2.1 Point Statistic: Mean and Variance

Statistical methods are almost always employed in the characterisation of atmospheric winds due to the perceivably random and unsteady nature of winds. Many techniques and tools are available to researchers, engineers, and meteorologist to infer wind trends and patterns. The simplest and most basic method is to simply calculate the mean and variance (and by extension, the standard deviation) of the wind speed. The mean wind speed represents the average wind speed that can be expected over a period of time whereas the variance is an indication of how far the actual wind speed is likely to deviate from the average. These point statistics are a quick way to obtain a “feel” of the wind trend and are also useful for the comparison of the wind between multiple observation points. They alone however do not provide information regarding the manner in which the wind speed varies nor do they provide any information on the wind direction. Nevertheless, knowing the mean and variance of the wind speed is often necessary for more complex analyses and to use more advanced characterisation techniques.

2.2.2 Probability Distributions for Characterising Winds

For a more robust characterisation of the wind speed, a probability distribution function (pdf) is often used. Using a pdf provides a mathematical equation by which the likelihood of occurrence of different wind speeds can be calculated. It gives further insights to the way that the wind speed varies, as opposed to just a single numerical value in the case of the variance, and allows the risk of extreme wind events to be quantified [67]. Furthermore, the use of an equation that can describe the distribution of all possible wind speeds allows for more robust and accurate quantitative analysis. For example, the power output of a wind turbine can be calculated using the Equation (2.16) below:

$$P = \frac{1}{2} \rho C_P A u^3 \quad (2.16)$$

where ρ is the air density, A is the area swept by the turbine blades, u is the wind speed, and C_P is the power coefficient that measures the efficiency of the energy conversion. In

this case, the expected power output by the turbine over a long period of time is given by Equation (2.17) [68,69]:

$$\bar{P} = \frac{1}{2} \int_{u_{ci}}^{u_{co}} \rho C_P A u^3 f(u) du \quad (2.17)$$

where u_{ci} and u_{co} are the cut-in and cut-out wind speeds of the wind turbine respectively and $f(u)$ is the pdf of the wind speed. The use of the pdf is necessary in this instance to obtain a correct estimation. Simply substituting the mean wind speed into Equation (2.16) is not sufficient as the equation does not vary linearly with the wind speed.

Among the probability distributions found in literature to represent the wind speed distribution, the Weibull distribution [39,41,42,68,70-78] is by far the most commonly used. The pdf of the Weibull distribution is given below by Equation (2.18) and the cumulative distribution function (cdf) by Equation (2.19):

$$f(u) = \frac{d}{c} \left(\frac{u}{c}\right)^{d-1} \exp \left[-\left(\frac{u}{c}\right)^d \right] ; u \geq 0 \quad (2.18)$$

$$F(u) = 1 - \exp \left[-\left(\frac{u}{c}\right)^d \right] ; u \geq 0 \quad (2.19)$$

where $f(u)$ and $F(u)$ are the probability density and the cumulative probability at wind speed u , and d and c are the shape and scale parameters of the Weibull distribution respectively. The parameters control the shape of the distribution and can be calculated by a fair number of methods. They can be directly solved by equating the mean and variance of the observed wind speed with the analytic mean and variance of the Weibull distribution or, if the entire data is known, be evaluated by more robust methods such as least-squares fitting or the maximum likelihood estimation [73,76,79,80]. In the study by Koh et. al. [73], they found that the scale parameter of the Weibull distribution is approximately equal to the mean if the shape parameter is approximately equal to 2. They also found that this is indeed the

case for radiosonde winds in the equatorial tropics and deduced that the shape parameter will be exactly equal to 2 (which would yield the Rayleigh distribution) if the distribution of the wind components are Gaussian. Although the Weibull distribution is the most widely-employed distribution in literature, other distributions have also been suggested such as the log-normal and Rayleigh distributions, and even more complex multi-modal (distributions with more than one “peak” in the pdf) probability distributions [69,80,81].

For the characterisation of the wind direction, the von Mises distribution is commonly used [72,81-83]. It is a circular probability distribution with a wrap-around domain designed to represent angular quantities, such as direction. The pdf of the von Mises distribution is given below by Equation (2.20):

$$f(\phi) = \frac{\exp[\nu \cos(\phi - \xi)]}{2\pi I_0(\nu)} \quad (2.20)$$

where $f(\phi)$ is the probability density at direction ϕ , I_0 is the modified Bessel function of the first kind of order 0, and ξ and ν are the distribution parameters. ξ is also the mean direction. The cdf of the von Mises distribution does not have an analytic form due to its circular characteristic.

While the Weibull and von Mises distributions can be used to characterise the wind speed and the wind direction respectively, they do so for each variable independently. This means that it is not possible to obtain the full picture of the observed wind variations as both the wind speed and the wind direction are related variables that stem from a single quantity: the wind velocity (which is a vector quantity). For instance, even if the most probable wind speed and the most probable wind direction are known, it is still not sufficient, just by the use of the wind speed and wind direction distributions alone, to answer questions like what is the direction at which the highest wind speed occurs or what is the average wind speed in the most probable wind direction. For the purpose of this dissertation, only the two-dimensional horizontal wind is considered.

2.2.3 Wind Roses

To graphically display the full picture of the observed winds, meteorologists use the wind rose [42,72,76,84]. A wind rose is essentially a polar-coordinate histogram with the direction axis divided into several discrete sectors. For each direction sector, the likelihood of occurrence for different ranges of wind speeds are then plotted. An example of a wind rose is shown in Figure 2.10. The wind rose provides an intuitive way to visually display the wind characteristics that includes both its speed and direction. However, it is primarily a presentation tool and does not represent a mathematical means to combine the wind speed and the wind direction.

Figure 2.10: A wind rose showing the climatological wind conditions in Singapore. Reprinted from Meteorological Service Singapore. Climate of Singapore [online document]. URL: <http://www.weather.gov.sg/climate-climate-of-singapore/> (Accessed April 4, 2019).

2.2.4 Joint Probability Distribution Functions

In order to represent the wind speed and the wind direction in a single mathematical formula, various authors have attempted to design and construct a class of probability distribution functions known as the joint probability distribution functions. Also known as bivariate probability distribution functions, the pdf equation of these functions contains two variables that can represent either the wind speed and the wind direction or the two components of the horizontal wind velocity. Thus, these distributions are able to capture more information than what would be possible by just using the wind speed and wind direction distributions independently. They would allow the wind speed and the wind direction to be analysed concurrently, thereby giving a more comprehensive and two-dimensional picture of the wind variations that is akin to that given by a wind rose, but in an analytic form.

An early example of such a joint pdf was derived by McWilliams et. al. [85]. In their work, known as the isotropic Gaussian model, they assumed that the two components of the wind velocity vary according to the Gaussian distribution and that they both have an equal variance σ^2 . The longitudinal component of the wind velocity, which is defined as in the direction of the prevailing wind, has a mean of μ , whereas the perpendicular lateral component is of zero mean. With some mathematical manipulation and transformation to the polar coordinates, it is possible to arrive at this form of the pdf given by Equation (2.21) below:

$$f(u, \phi) = \frac{u}{2\pi\sigma^2} \exp \left[-\frac{\mu^2 + u^2 - 2\mu u \cos\phi}{2\sigma^2} \right] \quad (2.21)$$

where $f(u, \phi)$ is the bivariate probability density at the wind speed u and in the wind direction ϕ as measured from the longitudinal axis. The mean and variance in this case serve as the function's parameters. Nevertheless, the assumption of isotropy in the variance is questionable at best and in fact, it would be shown later in the fourth chapter of this

dissertation that this assumption is generally not true. Various authors have since then developed and proposed other joint probability distribution functions, including generalisations to the isotropic Gaussian model [86] as well as models based on combining the Weibull and von Mises distributions [82,83].

2.2.5 Applications of Wind Characterisation

There are numerous areas of practical applications in which the observational characterisation of atmospheric winds serves an important role. The most obvious example is perhaps that in the field of wind energy. In the planning of wind energy projects, one of the first steps is typically to prospect the wind energy potential of potential wind energy extraction sites [71,74,78,87-92]. This is done through wind resource assessment exercises where meteorological towers mounted with atmospheric sensors for the measurement of wind speed and wind direction, and possibly also other atmospheric variables such as air pressure, temperature, rainfall, and the turbulent and heat fluxes, are deployed on site to collect atmospheric data. For the purpose of wind resource assessment for energy purposes, a minimum of a one-year observation period is usually considered mandatory [65,66]. The data will then be processed and used to derive salient statistics critical to the design and future operations of the potential wind energy site such as the mean wind speed, the distribution of the wind speed, the diurnal and seasonal cycle in the wind speed, the prevailing wind direction, and other significant trends that determine the wind power generation potential at the site or that may cause challenges in future operations. In addition, knowing the wind speed distribution, the turbulence intensity, and the potential for wind gusts and extreme winds at the site is also important for the selection of appropriate wind turbine models that are able to operate safely and efficiently in the expected wind conditions [91,93].

Other industries that see the need to perform wind characterisation activities include construction (civil engineering) and aviation. In the design of tall structures, such as bridges and skyscrapers, engineers require wind and gust information to calculate the required strength of the structure [94]. In aviation, knowledge of the wind strength and wind

direction at airport sites and its surroundings is critical to the alignment of runways and the design of departure and arrival flight procedures that affect the safety and efficiency of airport operations [95]. For example, runways are typically aligned in the direction of the prevailing wind as flying into a headwind reduces the effective ground speed during takeoff and landing approaches. This allows pilots a larger margin of error, which improves safety, and by doing so also reduces the chance of missed approaches during landing, which increases airport throughput. At the higher altitudes, winds at aircraft cruise level affect flight duration and fuel burn [96-100]. They are thus important to an airline's flight scheduling and fuel planning.

Outside of practical industry applications, the observation and characterisation of real winds is also important in supporting fundamental atmospheric research. Major atmospheric phenomena have been discovered or have its understanding enhanced, including tropical phenomena such as the Madden-Julian Oscillation, as a result of direct atmospheric observations [22,24]. Results from field measurements are also important in the validation of theoretical and computational models of the atmosphere [42,54,58,64,101,102]. Therefore, despite the availability of computational alternatives, scientific field observation campaigns continue to be planned and conducted [103,104]. Some recent examples of these include those conducted under the auspices of the "Years of the Maritime Continent" (YMC) international project [105].

In the realm of computational modelling of the atmosphere, observational wind data are often required to construct the initial and boundary conditions of atmospheric models. For the case of global and regional scale numerical weather prediction (NWP) models, these normally take the form of reanalysis datasets (e.g. the ERA-Interim Reanalysis [106]). Reanalyses are model-driven advanced interpolation techniques used to produce regularly-spaced and regular-time-interval model "data" from real-world observation data through a process known as data assimilation [106,107]. They are required because atmospheric models require regularly-spaced gridded data made at a regular time interval for use as the inputs to the initial and boundary conditions in order to run. Real atmospheric observations, however, rarely conform to a uniform grid nor take place at regular time intervals. Aside

from being used as inputs for atmospheric models, reanalysis data are also increasingly being used as a primary data source for studying the trends and variations in the larger atmosphere [108]. As assimilated data directly determine the quality of reanalysis output, the underlying real-world observation data thus indirectly determine the accuracy of atmospheric models as well as the soundness of conclusions derived in studies using reanalysis data.

On the smaller scale, computational fluid dynamics (CFD)-based models are increasingly being used to model the boundary-layer wind conditions found near the surface [61,109-113]. They are now a part of many wind resource assessment exercises as a means to generate initial estimates for the wind energy potential, due to their cost effectiveness and ability to produce results relatively quickly. In order for these CFD-based models to run, they require boundary conditions of the inflow winds [111,114-116]. These are normally extrapolated from estimates of real wind observations. Thus, an accurate representation of the boundary-layer wind profile of the inflow winds is necessary to produce accurate model results. Aside from wind energy, other fields also use CFD-based solvers for atmospheric transport problems such as in air quality management, pollution dispersion modelling, and urban air flow management [117,118].

2.3 Observation of Atmospheric Winds

2.3.1 Surface and Near-Surface Observation

The traditional method to measure the wind speed at the surface is by the use of an anemometer. There are many varieties of anemometers available [119] and they measure the wind speed, and possibly also other atmospheric variables, using various physical principles. The most iconic of all anemometer designs is perhaps that of the three-cup anemometer, shown in Figure 2.11. Three-cup anemometers [119,120] are widely used, even today in the face of more advanced anemometer designs, because they are relatively low cost and require little on-site power to operate. They measure the wind speed by the

rotation of the cups, which are made to spin due to the wind, and this rotation is transferred to a vertical shaft and measured. The wind speed can then be inferred from the speed of the rotation. The primary disadvantages of cup anemometers are that they contain mechanical moving parts, which limits their suitability for long-term deployments in harsh and dusty conditions, and that they are unable to directly measure the wind direction. Due to this, wind vanes are often co-located with cup anemometers to measure the wind direction.

Another type of anemometer commonly used for field observations is the ultrasonic anemometer, shown in Figure 2.12. Ultrasonic anemometers measure the wind speed by emitting and receiving ultrasonic sound waves in between the emitter and receiver arms and then measuring the time of transit of the waves in different directions [119]. They are for some applications superior to the cup anemometers due to the lack of moving parts and their ability to also measure the wind direction (ultrasonic anemometers measure the wind velocity instead of just the speed) and turbulent momentum fluxes [115]. They are however more costly to procure compared to cup anemometers and require significantly more power to operate continuously.

For practical observation of the surface winds, anemometer stations are usually located in flat open terrain such as near airports to minimise obstruction to the wind flow. The instrument itself would be mounted on a mast at the standard height of 10 m above the surface [65,121]. Figure 2.13 shows an example of such an apparatus.

Figure 2.11: A 3-cup anemometer co-located with a wind vane. Reprinted from National Geographic Society. Anemometer [online document]. URL: <https://www.nationalgeographic.org/encyclopedia/anemometer/> (Accessed April 4, 2019).

Figure 2.12: An ultrasonic anemometer. Reprinted from Front: 3D-Ultrasonic Anemometer (=Sonic Anemometer) [online document] in *GEOB 300 Field Visit #2 - Energy Balance Station*. URL: <http://ibis.geog.ubc.ca/courses/geog300/images/field-visit-2/Pages/5.html> (Accessed April 4, 2019).

Figure 2.13: A 10 m-tall mast used to measure surface wind speeds. Note the anemometer mounted at the top of the mast. Retrieved from Wikimedia Commons. URL: https://commons.wikimedia.org/wiki/File:2008-07-09_Eureka_Airport_AWOS_viewed_from_the_south_cropped.jpg (Accessed April 4, 2019).

In wind resource assessment, the measurement of the wind speed at a single height may not be sufficient. In order to obtain better estimates of the boundary-layer wind profile up to the height of typical wind turbines, meteorological towers are often set up to collect atmospheric data from multiple heights at a single location. These towers carry multiple instances of atmospheric instruments to measure the wind speed, the wind direction, and other atmospheric variables at the various heights where data are to be collected, and can reach overall heights in the tens of metres. With the growing size and height of modern wind turbines however, the use of such towers is becoming difficult as the cost of deploying and maintaining meteorological towers scales with the overall height. The tower itself also presents as an obstruction to the wind flow (known as the tower shadow) and that may

affect the accuracy of the measurements [122]. In addition, many meteorological towers are predominantly deployed in the extratropical regions of the world. This means that the results of many fundamental boundary-layer wind research were validated using data only from extratropical boundary layers [54,58,64,123-125].

The use of remote sensing methods offers to be a potentially more convenient and cost-effective alternative to meteorological towers [123,126,127]. As methods such as the sodar and the lidar do not require the erection of tall structures, they are not subject to lengthy construction times and can be deployed more easily. There are currently three major types of remote sensing methods used for the measurement of atmospheric winds: the radar, the sodar, and the lidar. All three methods work by reflecting energy waves off targets in the atmosphere and using the Doppler effect to infer the atmospheric motion. The Doppler effect, which shifts the frequency of waves coming from a moving object relative to an observer, shifts the frequency of the reflected waves and this is used to determine the wind speed and the wind direction.

Sodars [128] measure the wind velocity by reflecting acoustic waves off moving layers of turbulent air in the atmosphere. As some degree of turbulence is required, sodar measurement may not work in a very stable atmosphere. Lidars [128-130], on the other hand, measure the wind velocity by reflecting light (i.e. lasers) off aerosols suspended in the atmosphere. In this case, lidar measurement may not be suitable in places where the atmosphere is very clean and clear. Both sodars and lidars are able to measure the wind velocities at a single location for multiple heights up to a few hundred metres, making them practical alternatives to meteorological towers for the measurement of boundary-layer wind profiles. According to a study by Lang and McKeogh [131], both the sodar and the lidar were able to produce wind speed measurements in agreement to that of a cup anemometer with a correlation of at least 0.97. Given the convenience of using a sodar or a lidar, it is conceivable that they would increasingly be used in places where no meteorological towers already exist or where the construction of such towers would be difficult or impossible (such as for offshore applications).

The ZephIR 300 commercial lidar unit [132], which is used in Chapter 5 of this thesis, is an example of a continuous wave lidar system used for long-term wind measurements at multiple heights within the atmospheric boundary layer. To measure the wind speed and wind direction at a specific height, the lidar focuses a laser beam and sweeps through a conical scanning pattern as depicted by Fig. 2.14. By measuring the frequency shifts in the lidar returns, the system is then able to calculate the component of the wind velocity V_{LOS} in the line of sight of the beam when the beam is pointed at the instantaneous azimuth direction of ϕ_z . Multiple measurements (up to a frequency of 50 Hz) are made within the scanning circle to paint a complete picture of the wind flow. By using these measurements, the Equation (2.22) [130], where θ is the cone angle, can then be solved for the horizontal wind speed u , wind direction ϕ , and the vertical component of the wind velocity w by using a least-squares fit.

Figure 2.14: Schematic diagram of a lidar scanning cone. Reprinted from J. Li, X. Yu. LiDAR technology for wind energy potential assessment: Demonstration and validation at a site around Lake Erie. Energy Conversion and Management 144, 252-261 (2017). doi:10.1016/j.enconman.2017.04.061.

$$V_{LOS} = |u \cos(\phi_z - \phi) \sin \theta + w \cos \theta| \quad (2.22)$$

Figure 2.15: A commercial sodar wind profiler. Reprinted from P. Dvorak. Comparing remote sensing to met towers [online document], 14 October 2015. *Windpower Engineering & Development*. URL: <https://www.windpowerengineering.com/projects/environmental/comparing-remote-sensing-to-met-towers/> (Accessed April 4, 2019).

Figure 2.16: A commercial lidar wind profiler. Retrieved from ZX Lidars. Gallery. URL: <https://www.zxlidars.com/our-wind-lidars/lidar-gallery/> (Accessed April 4, 2019).

Figure 2.17: A doppler weather radar installation near an airport. Retrieved from Wikimedia Commons.

URL:

https://commons.wikimedia.org/wiki/File:Advanced_Radar_for_Meteorological_and_Operational_Research.jpg (Accessed April 4, 2019).

A third major remote sensing method used for the measurement of atmospheric winds is the radar. Doppler radars (or weather radars, to distinguish it from conventional radars) measure the horizontal wind velocity by reflecting microwave beams off precipitation as it falls through the atmosphere. Unlike the sodar and the lidar, they are primarily used to profile winds over a large horizontal region and can reach ranges of up to a few hundred kilometres. Because the presence of precipitation is required, Doppler radars are only able to obtain wind measurements in areas where there is rain or snowfall. They are typically employed in weather advisory roles such as for the detection of wind shear and gust conditions near airports [133,134].

2.3.2 Upper Air Observation

For the measurement of atmospheric winds beyond the boundary layer, the use of ground-based instruments and sensors is no longer viable. The standard and most-commonly used way to obtain atmospheric observations for the upper atmosphere is by the use of a radiosonde [121,135]. Radiosondes are packages of atmospheric instruments sent into the upper atmosphere by high-altitude balloons (i.e. weather balloons) in what is referred to as a sounding. They typically contain instruments to measure the atmospheric pressure, temperature, ozone concentration, and possibly other variables and can ascend up to the edge of space at about 30 km in height. Radiosonde measurements are taken with pressure as the vertical coordinate and the measured variables are typically reported at specific pressure levels.

For the measurement of atmospheric winds, this is achieved by tracking the movement of the balloon as it rises [135,136]. Tracking is normally done via a navigation tool carried onboard the radiosonde such as using the global positioning system GPS, but it can also be carried out from the ground using a theodolite. Radiosondes with such tracking ability to perform wind measurements are sometimes also referred to in literature as a rawinsonde to distinguish it from radiosondes without such ability.

Most national meteorological agencies around the world conduct routine radiosonde soundings to support the collection and archive of long-term climate records. On top of that, radiosondes can also be acquired independently for research purposes [137] to support the needs of individual projects.

Radiosonde soundings are routinely conducted throughout the world and they represent the most long-term meteorological record of the upper atmosphere currently available. Figure 2.19 shows the global distribution of radiosonde stations represented in the Integrated Global Radiosonde Archive (IGRA) database [138], which is a repository of worldwide radiosonde data. Routine record of radiosonde soundings begun as early as in the 1950s and has continued on till this day. The use of radiosondes for the collection of long-term

atmospheric data is governed by certain international norms and conventions prescribed by the World Meteorological Organisation (WMO) [121,138]. During a sounding ascent, atmospheric measurements are made at a set of mandatory or standard pressure levels (of which there are 11 at and below the 100 mb pressure level) as well as at significant pressure levels where there exist scientific interests. Also, radiosonde soundings are typically carried out globally every day at 00:00 UTC for a daily sounding, at 00:00 and 12:00 UTC for a twice daily sounding, and at 00:00, 12:00, 06:00, and 18:00 UTC for a four-times-a-day sounding.

Figure 2.18: A weather balloon carrying a radiosonde on a sounding ascent. Reprinted from Radiosonde Museum of North America. What is a Radiosonde? [online document]. URL: <http://radiosondemuseum.org/what-is-a-radiosonde/> (Accessed April 4, 2019).

Figure 2.19: A map of all radiosonde stations represented in the IGRA database. Reprinted from I. Durre, R. S. Vose, D. B. Wuertz. Overview of the Integrated Global Radiosonde Archive. *Journal of Climate* **19**, 53-68 (2006). doi:10.1175/JCLI3594.1.

Modern alternatives to the radiosonde for wind observation include observations made by aircraft [139] as well as a new generation of lidars capable of measuring winds for up to a few kilometres in height [140]. However, these methods do not replace the radiosonde and it remains the primary and most widely-understood method for the systematic collection of long-term upper air data.

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Chapter 3 Methods

This chapter documents and explains the statistical methods that are employed in this dissertation. To achieve this, it also provides a basic discourse on the fundamentals of statistical analysis.

3.1 Basic Concepts in Statistics

In the study of the atmosphere, statistical methods are typically always used for the characterisation of the atmosphere. This is because of the inherent unsteadiness of atmospheric circulation, thus requiring means to properly characterise and interpret the resulting fluctuations and variability that are present in atmospheric data. Statistics is the branch of mathematics that is concerned with the analysis, characterisation, and interpretation of data. It therefore contains the relevant techniques to analyse and methods to interpret such data in a consistent and quantifiable way. In statistical analysis, variables subject to the noted unsteady fluctuations and variations, for example the wind speed or the wind direction in the case of wind studies, are investigated. The variable, known as a random variable in statistical terminology, is taken to follow a certain probability distribution that provides the probabilistic description of its behaviour. The characteristics of this distribution as well as the quantitative parameters that characterise the distribution, referred to as the population statistics because they represent the true behaviour of the variable, are normally unknown. Therefore, observation data are collected to provide estimates of the actual distribution.

3.1.1 Probability Distribution Functions

A probability distribution function (pdf), also known as a probability density function, is a mathematical function whose integral over an interval provides the probability that a single observation of the random variable will fall within the selected interval [1]. Consider a

continuous random variable U . The probability that a single measurement of U will yield a value between u_a and u_b is given by Equation (3.1) below:

$$P(u_a \leq U \leq u_b) = \int_{u_a}^{u_b} f(u) du \quad (3.1)$$

where $f(u)$ is the pdf of U .

3.1.2 Mean, Variance, and Standard Deviation

There are many ways by which the probability distribution of a random variable can be characterised. The most simple and common way is perhaps by the use of point statistics such as the mean and the variance. The mean of a probability distribution, also known as the expectation or the average, is a measure of its central value [2] and is useful for comparisons of the size or magnitude between similar variables, such as for comparing the wind speed between different locations [3]. Mathematically, the mean μ is obtained from the pdf by Equation (3.2):

$$\mu = \int_{-\infty}^{\infty} u f(u) du. \quad (3.2)$$

The variance, on the other hand, is a measure of the spread of the distribution. It is the mean of the squared deviation of the random variable away from its mean [4]. A larger variance is indicative of an increased likelihood of an observation of the variable occurring further away from its mean value and vice versa. The variance σ^2 is obtained from the pdf by Equation (3.3):

$$\sigma^2 = \int_{-\infty}^{\infty} (u - \mu)^2 f(u) du. \quad (3.3)$$

The standard deviation σ is the square root of the variance. Figure 3.1 shows the effects the mean and the variance have on a probability distribution by showing the plots of different Gaussian distributions with varying means and variances. Note how changing the mean changes the central location of the distribution whereas changing the variance affects how “flat” or “pointy” the distribution is, thus affecting the likelihood of observing the variable further away from its mean.

Figure 3.1: The Gaussian (normal) distribution with different means and variances.

3.1.3 Skewness and Kurtosis

The skewness and the kurtosis are higher moments of a probability distribution function and they give further insights into the shape of the function. Using the same notations as the previous section, the skewness [5] is obtained by Equation (3.4):

$$\text{skewness} = \frac{1}{\sigma^3} \int_{-\infty}^{\infty} (u - \mu)^3 f(u) du. \quad (3.4)$$

The skewness measures the degree of asymmetry in the distribution. For a distribution with positive skewness, more of the distribution’s mass lies towards the larger values of u and

the mean of the distribution is thus larger than its modal value (where the “peak” of the pdf is located). For negative skewness, the reverse is true and the mean is smaller than the mode. A zero skewness indicates a symmetric distribution and the mean in this case is equal to the mode.

The kurtosis of a probability distribution is obtained by Equation (3.5) below:

$$\text{kurtosis} = \frac{1}{\sigma^4} \int_{-\infty}^{\infty} (u - \mu)^4 f(u) du. \quad (3.5)$$

The kurtosis is usually described as a measure of how “fat” the tails of a distribution are. Comparing two probability distribution functions with equal variances but different kurtosis, the one with the higher value for kurtosis would have its tail-end(s) approach zero probability density more slowly and thus has a higher chance of observing an outlier. The kurtosis of the Gaussian distribution is 3.

3.1.4 Cumulative Distribution Functions

Another probability function closely related to the pdf is the cumulative distribution function (cdf) [1]. It is related to the pdf by Equation (3.6):

$$F(u) = P(U \leq u) = \int_{-\infty}^u f(u) du \quad (3.6)$$

where $F(u)$ represents the cdf of the random variable U . The value of the cdf at u gives the probability of the variable being observed at a value less than that of the given argument u (i.e. the area under the pdf from $-\infty$ to u). As such, unlike the pdf, the cdf always increases monotonically with u . The cdf can also be used to determine the probability of the variable taking a value within the interval bounded by u_a and u_b by using the equation:

$$P(u_a \leq U \leq u_b) = F(u_b) - F(u_a). \quad (3.7)$$

3.1.5 Quantiles and Percentiles

Percentiles (and more generally, quantiles) [6] are another group of statistical parameters that can be used to characterise probability distributions. They can be easily obtained from the cdf by considering the relation in Equation (3.8):

$$F(u_p) = P(U \leq u_p) = p \quad (3.8)$$

where u_p is the 100 p -th percentile of the random variable U . The meaning of the 100 p -th percentile is such that 100 p % of all observations of the variable can be expected to have values less than that of the 100 p -th percentile value at u_p . Percentiles are quantiles which divide a probability distribution into 100 equal divisions with a 1% increment in probability per division, hence the name *percent*-tiles. Other commonly used quantiles include the deciles (10 divisions, 10% probability increments) and the quartiles (4 divisions, 25% probability increments). The 50th percentile is also commonly known as the median.

3.1.6 Estimation Using a Sample

In most practical scenarios however, the population statistics such as the mean μ and the variance σ^2 are not normally known and they have to be estimated using observation data. The set of data, which comprises of measurements made for the random variable being investigated, forms a sample from which the sample statistics can be calculated. Sample statistics are analogous to their population counterparts but are derived solely from the use of the observation data, and they are used as estimations for the population statistics. The caveat however is that they do not necessarily represent the true behaviour of the variable (as they are strictly only representative of the sample) and that because different samples of the same variable can give different sample results, the sample statistics are themselves also random variables. This thus gives rise to the uncertainty in the sample statistics.

3.1.6.1 Sample Mean and Variance

Consider a set of n independent measurements for the random variable U , each indicated by its subscript in the form of u_1, u_2, u_3 , through to u_n . The sample mean $\bar{u}$ based on this particular set of measurements is given by Equation (3.9) [7]:

$$\bar{u} = \frac{\sum_{i=1}^n u_i}{n}. \quad (3.9)$$

It can be shown that the sample mean is an unbiased estimator of the population mean. An unbiased sample estimator is one which its expectation is equals to the population value [8], i.e. in this case of the mean: μ . This, in theory, allows the true population value to be recovered when given a very large number of observations.

The sample variance s^2 is also an unbiased estimator of the population variance σ^2 . It is given by Equation (3.10) [9]:

$$s^2 = \frac{\sum_{i=1}^n (u_i - \bar{u})^2}{n - 1}. \quad (3.10)$$

Note that the definition of the sample variance differs from that of the population variance of a discrete random variable by a factor of $n/(n - 1)$. This is due to the Bessel's correction.

3.1.6.2 Mean and Variance of the Sample Mean

Finally, it was already mentioned earlier that the sample statistics themselves are also random variables. As such, they too have their own means and variances (not to be confused with the mean and variance of the main variable U that is under study). For the sample mean, the mean of the sample mean is the population mean, which gives the sample mean its unbiased quality, whereas the variance of the sample mean is given by the Equation (3.11) [7,10] below:

$$\text{var}(\bar{u}) = \frac{\sigma^2}{n} \quad (3.11)$$

where σ^2 is the population variance as before. In practice however, the population variance is often unknown and thus it is estimated by using the sample variance s^2 .

Note that the variance of the sample mean is inversely proportional to n . This implies that as the number of measurements in the sample increases, the spread of the sample mean will reduce and the calculated sample mean will as a result be more likely to be near the true value for the population mean. Knowledge of the variance of a sample statistic is usually necessary for the calculation of its uncertainty [11], with a smaller variance indicating a smaller uncertainty for the obtained sample statistic. The standard deviation (square root of the variance) of a sample statistic is also known as the standard error.

In the event where the individual measurements of the random variable U cannot be assumed to be independent, the actual uncertainty of the sample mean would be larger than otherwise suggested by Equation (3.11). In such cases, an effective sample size n_{eff} would be used in place of the actual number of measurements n . The value of n_{eff} is always smaller than n when the autocorrelation is positive (essentially always the case for atmospheric variables) and it is, assuming a first-order autoregressive time series [12,13] for the measurements of U , given by Equation (3.12):

$$n_{\text{eff}} = n \frac{1 - r_1}{1 + r_1} \quad (3.12)$$

where r_1 is the lag-1 autocorrelation coefficient given by:

$$r_1 = \frac{\sum_{i=1}^{n-1} (u_i - \bar{u}_-)(u_{i+1} - \bar{u}_+)}{[\sum_{i=1}^{n-1} (u_i - \bar{u}_-)^2 \sum_{i=2}^n (u_i - \bar{u}_+)^2]^{1/2}} \quad (3.13)$$

where $\bar{u}_-$ and $\bar{u}_+$ are the sample means of the subsets of the measurements of U with members $[u_1, u_2, \dots \text{ to } u_{n-1}]$ and $[u_2, u_3, \dots \text{ to } u_n]$ respectively. The standard error of the

sample mean where the measurements are not independent of each other will always be larger than that if they are.

3.2 Confidence Intervals

Confidence intervals [8] are constructed as a means to represent the uncertainty of sample statistics. Whereas point estimates such as the mean provide only a singular value to characterise a random variable, constructing a confidence interval allows a range of possible values as well as the confidence level associated with that range of values to be systematically evaluated. Confidence intervals are always quoted with a certain confidence level, usually stated in terms of a percentage. A 95% confidence interval, for example, is one where there is a 95% chance (i.e. probability of 0.95) that the true population value being estimated will be contained within. An alternative way to look at it is that if the 95% confidence intervals from 100 different samples of the same random variable are constructed, then 95 of them can be expected to contain the true population value.

For most of the time in atmospheric studies, the number of measurements obtained for a sample is fairly large. In such cases, the central limit theorem can be invoked and the sample mean (which is itself a random variable as discussed previously) follows an approximate Gaussian distribution [14]. The $100p\%$ confidence interval of the sample mean for the random variable U is then given by the interval below:

$$\left[\bar{u} - z_{\text{cr}} \frac{\sigma}{\sqrt{n}} \quad \bar{u} + z_{\text{cr}} \frac{\sigma}{\sqrt{n}} \right] \quad (3.14)$$

where $\bar{u}$ is the point estimate for the sample mean, $\sigma/\sqrt{n}$ is the standard error of the sample mean, and z_{cr} is the critical value such that the probability:

$$P(Z \leq z_{\text{cr}}) = 1 - \left(\frac{1-p}{2} \right)$$

where Z is the standard Gaussian distribution. As with Equation (3.11), the population standard deviation σ of the variable U is often estimated by using the sample standard deviation s .

The size of the confidence interval for a given confidence level is directly proportional to the standard error of the sample statistic. From this, it can be seen that a larger standard error directly causes a greater uncertainty in the sample estimation. Observe also that when a higher confidence level is used, the critical z value becomes larger and this results in a larger confidence interval. The 100% confidence interval, for example, is a trivial interval as it simply contains all possible values. Therefore, a balance between the desired confidence level and the size of the resultant confidence interval has to be sought in practice.

For a sample with a small sample size however, the sample mean can no longer be approximated by the Gaussian distribution. In such cases, the Student's t distribution is used instead of the Gaussian distribution to obtain the critical value [14]. The $100p\%$ confidence interval is then given by the interval:

$$\left[\bar{u} - t_{\text{cr}} \frac{s}{\sqrt{n}} \quad \bar{u} + t_{\text{cr}} \frac{s}{\sqrt{n}} \right] \quad (3.15)$$

where s is the sample standard deviation as before and t_{cr} is the critical value such that the probability:

$$P(T \leq t_{\text{cr}}) = 1 - \left(\frac{1-p}{2} \right)$$

where T is the Student's t distribution with $n - 1$ degrees of freedom and n is the size of the sample. The t distribution tends towards the standard Gaussian distribution as its degree of freedom increases and is identical to the standard Gaussian at infinite degrees of freedom. The above expression for the small sample case is valid only if the main random variable U is approximately Gaussian distributed.

For even more complex cases where neither the distribution of the sample statistic nor that of the main variable can be known or assumed, resampling methods such as bootstrapping [15,16] may also be attempted. Such methods do not make any assumptions with regards to the distribution or have any preconditions on the sample size. During bootstrapping, the original sample is resampled multiple times, usually in the hundreds or the thousands, thus creating a large sample of the sample mean. This allows the central limit theorem to be invoked and the confidence interval can then be constructed by the use of the interval in Equation (3.14).

3.3 Hypothesis Testing

Hypothesis tests (or statistical hypothesis testing, to distinguish it from the testing of a scientific hypothesis) [17] are a formal method by which one seeks to reject an assumption about a statistic in favour of another. It can be used to test, for example, if the mean of a random variable differs significantly from a certain value. To conduct a hypothesis test, four pieces of information are needed. First, a test statistic with a known or assumed probability distribution has to be defined. Next, the null hypothesis H_0 regarding the test statistic being equal to a certain value, which is the hypothesis the test seeks to reject, is needed. Thirdly, the significance level of the test p has to be decided. p is also the probability that the null hypothesis will be rejected even if it is true. Lastly, an alternate hypothesis H_1 is needed. There are three types of alternate hypotheses and they all reject the null hypothesis in favour of the test statistic being 1) larger (upper-tailed test), 2) smaller (lower-tailed test), or 3) not equal (two-tailed test) to the certain value that is assumed by the null hypothesis.

To perform the test, first the sample value of the test statistic is calculated using the collected data. Next, an interval based on the probability distribution of the test statistic, akin to that of a confidence interval, is constructed such that the probability of the test statistic having a value within the interval should the null hypothesis be true is $1 - p$. This interval is right bounded for an upper-tailed test, left bounded for a lower-tailed test, and

bounded at both ends for a two-tailed test. If the calculated test statistic falls outside the range of the interval, then it can be concluded that the null hypothesis is rejected in favour of the alternate hypothesis at a significance level of $100 \times p\%$. Note that one cannot strictly “accept” a null hypothesis. If the test statistic does not fall within the constructed interval, then it can only be said that there is insufficient evidence to reject it.

While a hypothesis test can in theory be carried out on any test statistic as long as its distribution is known, there are several standard ones that are commonly used. Examples of them include the t test, the χ^2 test for variance, and the F test for equality of variances.

3.3.1 t test for mean

The t test [18] tests the hypothesis that the mean is equal to some predetermined value μ_0 . The test statistic t for the test is given by:

$$t = \frac{\bar{u} - \mu_0}{\frac{s}{\sqrt{n}}} \sim T_{n-1} \quad (3.16)$$

where $\bar{u}$ is the sample mean, s is the sample standard deviation, n is the size of the sample, and that t follows a Student's t distribution with $n - 1$ degrees of freedom.

3.3.2 χ^2 test for variance

The χ^2 test for variance [19] tests if the variance is equal to some predetermined value σ_0^2 . The test statistic χ^2 for the test is given by:

$$\chi^2 = (n - 1) \frac{s^2}{\sigma_0^2} \sim \chi^2_{n-1} \quad (3.17)$$

where s^2 is the sample variance, n is the size of the sample, and that χ^2 follows a χ^2 distribution with $n - 1$ degrees of freedom.

3.3.3 F test for equality of variances

The F test for equality of variances [19] tests if the variances of two random variables are the same, and thus if the ratio between them is equal to 1. The test statistic F for the test is given by:

$$F = \frac{s^2_1}{s^2_2} \sim F_{n_1-1, n_2-1} \quad (3.18)$$

where s^2_1 and s^2_2 are the sample variances of the two variables, n_1 and n_2 are their sample sizes respectively, and that F follows an F distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom.

For the F and the χ^2 tests, a Gaussian distribution has to be assumed for the population(s).

3.4 Least-Squares Regression Analysis and Curve Fitting

Regression analysis is yet another statistical technique used to characterise data. The objective of regression is to determine how a variable (the dependent variable) is related to or controlled by one or more other variables (the independent variables or predictors). To carry out a regression analysis, a function relating the dependent variable to the independent variables (the regression model) is decided upon and the unknown parameters controlling this function are then estimated by the regression. Aside from investigating the statistical relation, regression is also a common technique used to perform curve fitting when the theoretical relation between the dependent and independent variables is known.

The least squares approach is perhaps the most common of regression techniques. The objective of least squares is to minimise the sum of the squared error (SSE) between the data and the regression model when evaluating the regression coefficients. There are two

types of least squares: ordinary least squares and weighted least squares. In weighted least squares [20], the error of each data point is weighted by the inverse of its variance, thus affording less weight to data that are more uncertain. Ordinary least squares is the simple case of the weighted least squares where all the weights are equal.

A generic regression model can be expressed as a function:

$$y = f(\mathbf{x}, \boldsymbol{\beta}) \quad (3.19)$$

where y is the modelled response of the dependent variable, $\mathbf{x}$ is the set of all independent variables, and $\boldsymbol{\beta}$ is the set of all m regression parameters that control the function $f(\cdot)$.

Consider also a set of n data points of the variables: $(y_1, \mathbf{x}_1)$, $(y_2, \mathbf{x}_2)$, $(y_3, \mathbf{x}_3)$, through to $(y_n, \mathbf{x}_n)$. The least-squares regression approach then seeks to find a set of estimates $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ such that the following objective function is minimised:

$$r^2 = \sum_{i=1}^n w_i (y_i - f(\mathbf{x}_i, \boldsymbol{\beta}))^2 \quad (3.20)$$

where w_i is the weight of the i -th data point (all weights are equal to 1 for ordinary least-squares regression).

3.4.1 Linear Least-Squares Regression

A linear regression model [21,22], where y is modelled as a linear function of all the variables in $\mathbf{x}$, is often used when there is no reason a priori to suspect a more complicated function. This is because of its mathematical simplicity and that $\hat{\boldsymbol{\beta}}$ for a linear regression can be solved analytically. The linear model may also be used in cases where the function being modelled takes a form that can be linearised, such as the logarithmic or the exponential functions.

For a simple linear model with only a single independent variable x , its function can be expressed as:

$$y = bx + a \quad (3.21)$$

where b and a are the regression coefficients to be estimated.

The error function to be minimised is then:

$$r^2 = \sum_{i=1}^n w_i (y_i - bx_i - a)^2. \quad (3.22)$$

It can be derived using calculus that the least-squares estimators for the coefficients b and a are:

$$\hat{b} = \frac{\sum_{i=1}^n w_i x_i \sum_{i=1}^n w_i y_i - \sum_{i=1}^n w_i \sum_{i=1}^n w_i x_i y_i}{(\sum_{i=1}^n w_i x_i)^2 - \sum_{i=1}^n w_i \sum_{i=1}^n w_i x_i^2} \quad (3.23)$$

$$\hat{a} = \frac{\sum_{i=1}^n w_i x_i \sum_{i=1}^n w_i x_i y_i - \sum_{i=1}^n w_i x_i^2 \sum_{i=1}^n w_i y_i}{(\sum_{i=1}^n w_i x_i)^2 - \sum_{i=1}^n w_i \sum_{i=1}^n w_i x_i^2}. \quad (3.24)$$

The SSE and the mean of the squared errors (MSE) σ_y^2 in this case are then given by Equation (3.25) and (3.26) respectively:

$$\text{SSE} = \sum_{i=1}^n w_i (y_i - \hat{b}x_i - \hat{a})^2 \quad (3.25)$$

$$\text{MSE} = \sigma_y^2 = \frac{\text{SSE}}{n - 2} \quad (3.26)$$

where the factor $n - 2$ is due to the Bessel's correction as there are two coefficients being estimated. The derivation for Equation (3.23) and (3.24) is provided in Appendix A.

The standard errors σ_b and σ_a for the coefficients b and a can be derived by using the error propagation [23] of the MSE. Their derivation is provided in Appendix B and they are given by:

$$\sigma^2_b = \sum_{j=1}^n \left[\frac{w_j \sum_{i=1}^n w_i x_i - w_j x_j \sum_{i=1}^n w_i}{(\sum_{i=1}^n w_i x_i)^2 - \sum_{i=1}^n w_i \sum_{i=1}^n w_i x_i^2} \right]^2 \left(\frac{\sigma^2_y}{w_j} \right) \quad (3.27)$$

$$\sigma^2_a = \sum_{j=1}^n \left[\frac{w_j x_j \sum_{i=1}^n w_i x_i - w_j \sum_{i=1}^n w_i x_i^2}{(\sum_{i=1}^n w_i x_i)^2 - \sum_{i=1}^n w_i \sum_{i=1}^n w_i x_i^2} \right]^2 \left(\frac{\sigma^2_y}{w_j} \right). \quad (3.28)$$

If the sample distribution of y about a fixed x can be assumed to be approximately Gaussian distributed, then the confidence intervals for the regression coefficients can be constructed by using Equation (3.14).

3.4.2 Non-Linear Least-Squares Regression

For regression models where the relation between y and $\mathbf{x}$ is not linear and cannot be linearised, there are often no closed solutions for the regression coefficients. In such cases, an iterative approach is needed. This is known as a non-linear least-squares regression [24,25].

To carry out the non-linear regression, an initial estimate for all the regression coefficients has to be first made. Successive iterations of the coefficients are then carried out to reduce the SSE until the SSE has been minimised. The resultant coefficients are the non-linear least-squares estimates. Using the same notations defined earlier in Equation (3.19) and (3.20), the SSE and the MSE are then given by:

$$\text{SSE} = \sum_{i=1}^n w_i \left(y_i - f(\mathbf{x}_i, \hat{\boldsymbol{\beta}}) \right)^2 \quad (3.29)$$

$$\text{MSE} = \frac{\text{SSE}}{n - m} \quad (3.30)$$

where $\hat{\boldsymbol{\beta}}$ are the regression coefficients estimated by the iterations.

3.5 Bivariate Mean and Variance

Multivariate statistical methods [26] are used to characterise random variables with multiple covarying components such as vector quantities. An example of such a quantity in atmospheric studies is that of the horizontal wind, which is a 2-dimensional vector. The use of traditional scalar-based statistical methods in this case may be insufficient as was discussed in the previous chapter's Section 2.2. Thus, to perform statistical characterisation of the horizontal wind, one may instead choose to use the bivariate sample statistics.

Consider a sample of n observations of the horizontal wind velocity (or any 2-dimensional random variable): $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, through to $\mathbf{v}_n$, which written in the form of its components is: $(v_{x1}, v_{y1}), (v_{x2}, v_{y2}), (v_{x3}, v_{y3})$, through to (v_{xn}, v_{yn}) . The sample mean $\bar{\mathbf{v}}$ is a vector given by Equation (3.31):

$$\bar{\mathbf{v}} = \frac{\sum_{i=1}^n \mathbf{v}_i}{n} = \begin{bmatrix} \frac{\sum_{i=1}^n v_{xi}}{n} \\ \frac{\sum_{i=1}^n v_{yi}}{n} \end{bmatrix} = \begin{bmatrix} \overline{v_x} \\ \overline{v_y} \end{bmatrix}. \quad (3.31)$$

The physical meaning of the mean vector is very much straightforward. The components of the mean are simply the mean of the components.

The variance, on the other hand, takes the form of a tensor given by Equation (3.32):

$$\text{var}(\mathbf{v}) = \begin{bmatrix} \text{var}(v_x) & \text{cov}(v_x, v_y) \\ \text{cov}(v_x, v_y) & \text{var}(v_y) \end{bmatrix} = \begin{bmatrix} \sigma_x^2 & \sigma_x \sigma_y \\ \sigma_x \sigma_y & \sigma_y^2 \end{bmatrix}. \quad (3.32)$$

The variance tensor consists of the component variances for its diagonal elements and the covariance for its off-diagonal elements. The covariance is a measure of the joint variability of two random variables and is the term responsible for capturing the covariation in the two vector components, in this case the x and y components of the wind velocity. The sum of the diagonal components $\sigma_x^2 + \sigma_y^2$, which is also the sum of the component variances, forms the trace of the tensor. The trace is an invariant property of a matrix and it remains a constant under rotation of the coordinate system.

As the coordinate system rotates, there exists one in which the axes are aligned with the principal directions of the tensor. In this new coordinate system, the covariance terms become zero and one of the component variances is maximised whereas the other one is minimised. The principal directions are important because they give the direction of the maximum and the minimum variance. This maximum and minimum variance are obtained by the diagonalisation of the tensor. An eigenvalue-eigenvector analysis can be performed and the resultant eigenvalues are then the maximum and the minimum variance with their corresponding eigenvectors aligned in the respective principal directions.

The above formulation can be generalised for the case of a 3-dimensional random variable, such as the flow velocity in turbulent engineering flows [27]. The mean is then a 3-dimensional vector and the variance a 3-by-3 tensor.

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Chapter 4 Variance Characteristics of Tropical Radiosonde Winds

In this chapter, an exploratory study on the variance characteristics of upper-air winds in a near-equator monsoon region is presented. The data were obtained from historical radiosonde observations from up to 250 stations within the region of interest for the period between 1954 and 2013. An alternative method based on vector statistics was employed in this study which characterises the mean by a vector and the variance by a tensor. Unlike the conventional approach of using scalar wind speeds, this vector-tensor approach allows the directional properties of the variance to be studied. A suite of statistics to describe the geometric properties of the variance tensor was also developed. These characterise the size of the variance, its degree of anisotropy, and the alignment of the preferred direction (if anisotropy is present) with the direction of the mean wind vector. Through analysis of these statistics, several salient trends were observed for the middle troposphere. It was found that the variance size and anisotropy exhibit significant variation with height whereas the alignment with the mean vector varies with the mean wind magnitude instead. It was also found that the scalar variance increases with mean wind speed.

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4.1 Introduction

The statistical characterisation of the wind is an important step in the planning of major infrastructure projects. In the wind energy sector, such characterisation exercises, referred to as resource assessments, are carried out to assess the wind energy availability and predict prevailing wind patterns at potential sites for new wind power plants [1–4]. This is necessary for project financiers and operators to estimate the power output from the plant and hence calculate the potential returns from the investment. Wind farm designers also rely on this information to optimise the placement of individual wind turbines. In aviation, airport planners use wind information to place and orient runways. Airport runways are typically oriented along the direction of the prevailing wind to minimise crosswinds that can pose a hazard to flight safety. Poorly oriented runways with frequent strong crosswinds also increase the number of aborted landings and reduce the effective capacity of the airport [5]. While there are numerous studies on wind characteristics near the surface, such studies at higher altitudes are rare due to the lack of specific practical applications. However, with recent advances in high altitude wind power (HAWP) technologies [6], characterisation of higher-level winds could take on greater importance. In addition, it is a trivial fact in aviation operations that en-route winds are a major determinant of flight duration and fuel burn [7,8]. Comprehensive knowledge of these winds is thus important to construct better-optimised flight schedules, flight plans, and improve general operation efficiency.

Current methods of wind characterisation found in literature typically involve quantifying the statistics of the wind speed by its mean and variance values. Some studies take it a step further and fit the observed wind speed distribution to prescribed probability distribution functions, with the two-parameter Weibull distribution being the most commonly used [1–3,9–11]. More advanced methods may also include a spectral analysis to determine the frequencies of wind speed fluctuations [1,10]. When wind direction is concerned, the von Mises distribution is commonly used to describe the observed direction distribution [10]. In some cases, wind roses are used to provide a graphical representation of both the wind speed and direction on a single diagram [3,10,11]. While these methods are adequate for most practical applications, they consider the wind speed and direction as separate scalar

quantities. This ignores the fact that wind measurements are actually vectors and that its speed and direction are two components of a single quantity. Even though the wind rose does provide an intuitive picture containing both the wind speed and direction, the representation is inherently graphical and difficult to manipulate mathematically. In this chapter, an alternative method to characterise the wind statistics is proposed that is based on the statistics of vectors. This method is similar to that employed by Koh and Ng [12] in their study on wind errors but is applied here directly onto actual wind measurements. The method is then used in an exploratory study of winds in the near-equator monsoon region using observation data obtained from radiosondes.

Radiosondes are instrument packages launched on weather balloons to collect atmospheric data as the balloon ascends through the atmosphere. They are a primary source of meteorological data, including horizontal wind speed and direction, for the upper atmosphere above the reach of ground-based measurement sensors. Unlike measurements by the ground-based sensors, radiosondes record data at various altitudes based on the pressure rather than at fixed physical heights above the ground. The World Meteorological Organisation (WMO) specifies eleven such pressure levels as mandatory for radiosonde observations. They are, from a lower to a higher level, 1000, 925, 850, 700, 500, 400, 300, 250, 200, 150, and 100 mb. The WMO also standardises two daily launch times at 00:00 and 12:00 UTC. Data from radiosondes are currently mostly used in operational weather forecasting where it serves as a source for data assimilation in numerical weather prediction (NWP) models [13]. Systematic studies on upper-air wind statistics using radiosonde data are rare, however, especially in the tropics. One such study was conducted by Koh et al. [14] where wind speeds in the equatorial monsoon region were investigated.

This chapter seeks to accomplish two objectives. The first is to introduce readers to the alternative vector-based method for characterising wind statistics. This alternative method invokes the mathematics for describing random vectors, with particular focus on the variance which takes on the form of a tensor. The second objective is to present an exploratory study, performed both using the said method and the more conventional approach of using scalar wind speeds, to characterise the upper-air wind variance of the

near-equator monsoon region using radiosonde data. The chapter is structured as follows: the next section describes the proposed vector-based method and lists the wind statistics used in the subsequent radiosonde study. Section 4.3 gives an overview of the data source and the steps taken to process the data. Section 4.4 then provides the results, analysis of the results, and discussion. Section 4.5 derives the relation between the variance tensor and eddy diffusivity. Finally, Section 4.6 concludes the chapter.

4.2 Method

4.2.1 Mean Vector and Variance Tensor

Consider a set of n measurements of horizontal wind velocity $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \dots, \mathbf{v}_n$ with corresponding magnitudes $v_1, v_2, v_3, \dots, v_n$. Each measurement is a two-dimensional vector with components v_x and v_y in the Cartesian x and y direction respectively. The statistics of the scalar wind speed can be characterised by its mean and variance as given by the following well-established equations:

$$\bar{v} = \frac{\sum_{i=1}^n v_i}{n} \quad (4.1)$$

$$\text{var}(v) = \frac{\sum_{i=1}^n (v_i - \bar{v})^2}{n - 1}. \quad (4.2)$$

In the proposed vector-based method, however, the characterisation of the whole velocity vector is sought. The mean and variance in this case, taking the form of a vector and a tensor respectively, are given by these equations:

$$\bar{\mathbf{v}} = \begin{bmatrix} \overline{v_x} \\ \overline{v_y} \end{bmatrix} \quad (4.3)$$

$$\text{var}(\mathbf{v}) = \begin{bmatrix} \text{var}(v_x) & \text{cov}(v_x, v_y) \\ \text{cov}(v_x, v_y) & \text{var}(v_y) \end{bmatrix}. \quad (4.4)$$

Five independent terms, two from the mean vector and three from the variance tensor, are required to characterise the wind velocity as compared to only two for the wind speed. These additional terms capture the information on direction contained in the velocity vector and characterises the measured wind more holistically. Note that the variance tensor is also referred to as the covariance matrix in some texts.

Figure 4.1: Schematic representation of the mean vector and variance tensor. The angles of φ and θ are also shown on the diagram.

Visualising the geometry of the mean vector is simple. Its two independent terms can either be the two components of the vector, as in Equation (4.3), or be stated as its magnitude and direction. Visualising the variance tensor, however, is more abstract. Koh and Ng [12] described such a tensor graphically by means of an ellipse. In their approach, the eigenvalues, λ_{min} and λ_{maj} , and eigenvectors of the tensor matrix are first calculated. An ellipse can then be constructed with the square-root of the eigenvalues being the lengths of the semiaxes, and the said semiaxes being oriented in the direction of the corresponding eigenvectors. When the centre of this ellipse is plotted on the point representing the mean

vector on a 2D Cartesian plane, the perimeter of the ellipse would represent the boundary of one standard deviation away from the mean. An explanatory figure is provided as Figure 4.1 for clarity.

Interested readers are referred to Koh and Ng [12] for more elaboration. One important feature of this ellipse representation, henceforth referred to as the variance ellipse, is the identification of the principal axes of the variance tensor. For a two-dimensional horizontal wind, the principal axes, which are the semiaxes of the variance ellipse, are axis of the maximum and minimum variance.

4.2.2 List of Statistics Used in This Study

The following is the list of the statistics and their associated symbols used in the subsequent wind characterisation study. They can be divided into scalar-based and vector-based statistics. The vector-based statistics are specifically designed in this study to describe the geometric properties of the mean vector and variance tensor as described in the previous subsection.

4.2.2.1 Scalar-Based Statistics

- Mean wind speed, $V_s = \bar{v}$: Mean of the wind speed. This is given by Equation (4.1).
- Scalar variance, $\sigma_s^2 = \text{var}(v)$: Variance of the wind speed. This is given by Equation (4.2).

4.2.2.2 Vector-Based Statistics

- Magnitude of mean velocity, V_t : Magnitude of the mean vector. This is given by:

$$V_t = \sqrt{\bar{v}_x^2 + \bar{v}_y^2} \quad (4.5)$$

- Direction of mean velocity, φ : Direction bearing of the mean vector. $0^\circ \leq \varphi < 360^\circ$, measured clockwise from the cardinal north.
- Trace of the variance tensor, σ_t^2 : This measures the general size of the variance and can be considered analogous to the scalar variance. It is given by Equation (4.6). The two expressions given in Equation (4.6) are equivalent because the trace is an invariant of a tensor. In fact, σ_t is properly the standard deviation of the wind vector and is depicted by the size of the ellipse in Figure 4.1. (Note that $4\sigma_t$ is the perimeter of the rhombus formed by joining the tips of the semi-axes of the ellipse in Figure 4.1.)

$$\sigma_t^2 = \text{var}(v_x) + \text{var}(v_y) = \lambda_{\min} + \lambda_{\max} \quad (4.6)$$

- Eccentricity of the variance tensor, ε : This measures the degree of anisotropy in the variance. Bounded from 0 to 1, $\varepsilon = 0$ indicates equal variance in all directions (variance ellipse is actually a circle) whereas $\varepsilon = 1$ indicates variations only along a particular axis (semiminor axis of the ellipse has zero length). To be distinguished from the conventional definition of eccentricity in the geometry of conic sections, the definition of eccentricity used in this study follows the one proposed by Koh and Ng [12] and is more properly termed “symmetrised eccentricity, ε_s ” by Koh, Wang and Bhatt [15] although in this dissertation it shall be simply called the “eccentricity” for convenience. It is given by:

$$\varepsilon = \varepsilon_s = \frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} + \lambda_{\min}} \quad (4.7)$$

- Orientation of the variance tensor, θ : Direction bearing of the axis of maximum variance. This is a principal axis of the variance tensor and also the semimajor axis of the variance ellipse. Because this statistic measures an axis rather than a specific direction, its value is bounded within a half circle: $0^\circ \leq \theta < 180^\circ$, measured clockwise from the cardinal north.

The three statistics, σ_t^2 , ε , and θ , capture all information in the variance tensor which can be rewritten simply as in Equation (31) of Koh, Wang and Bhatt [15]:

$$\text{var}(\mathbf{v}) \equiv \begin{bmatrix} \text{var}(v_x) & \text{cov}(v_x, v_y) \\ \text{cov}(v_y, v_x) & \text{var}(v_y) \end{bmatrix} = \frac{1}{2} \sigma_t^2 \left\{ \mathbf{I} + \varepsilon \begin{bmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} \right\} \quad (4.8)$$

where $\mathbf{I}$ is the identity tensor.

An additional statistic, η , is introduced that measures the alignment of the mean vector to the axis of maximum variance. It is defined as the absolute dot product of unit vectors in the direction of φ and θ . Mathematically, this can be expressed by:

$$\eta = |\hat{\mathbf{e}}_\varphi \cdot \hat{\mathbf{e}}_\theta| \quad (4.9)$$

where $\hat{\mathbf{e}}$ denotes a unit vector in the direction of the angle given by its subscript. The value of η is bounded from 0 to 1 with $\eta = 1$ indicating perfect alignment between the mean velocity and the axis of maximum variance. For the current study, η is investigated in lieu of φ and θ to explore an observed tendency for the two directions to align.

4.3 Data

4.3.1 Data Source

The radiosonde data used in this study was obtained from the Integrated Global Radiosonde Archive (IGRA). The archive contains records of meteorological data from radiosonde observations taken from upper-air observation stations throughout the world. One useful feature of the data is that it has undergone relevant quality-control (QC) procedures and can be considered free of erroneous readings and outliers [16]. IGRA is maintained by the

United States National Oceanic and Atmospheric Administration (NOAA) and is accessible by the public at no cost.

For the current study, data from stations in a rectangular region bounded by 15° S and 15° N in latitude and 20° W and 155° E in longitude are considered. This represents a near-equator monsoon region which includes the entire longitudinal extent of the areas that experience the Asian and African monsoons. The region encompasses equatorial Africa and Indian Ocean, southern tip of the Indian subcontinent, and the Maritime Continent. There are a total of 250 stations within the selected region. A map showing the location of these stations is provided as Fig. 4.2. Even though records for some stations in IGRA date as far back as 1946, a check revealed that many stations in the current region of interest only began regular observations in 1954. As such, this study only uses data taken from between 1954 and 2013. In addition, the study only considers data collected at the eleven WMO mandatory pressure levels and at the standard radiosonde launch times of 00:00 and 12:00 UTC.

Figure 4.2: Map of the radiosonde stations within the selected near-equator monsoon region that are represented in the IGRA database.

4.3.2 Data Processing

The data for each station, at each pressure level, and for each launch time were processed individually. Individual measurements for wind velocity were grouped into sets with each set corresponding to a particular station, at a particular pressure level, and to a specific launch time. These sets were then further split into four each with each corresponding to

one of the four monsoon or intermonsoon seasons commonly associated with the region. The result is 88 sets of raw observation data for each station in the selected geographic region, with each set of data taken to represent the wind behaviour at a particular combination of pressure level ($\times 11$), season ($\times 4$), and time ($\times 2$: 00:00 and 12:00 UTC). The four seasons mentioned are two monsoon seasons: June to September (JJAS) and December to March (DJFM), and two intermonsoon seasons: April to May (AM) and October to November (ON). In regions that experience monsoon wind patterns, the prevailing wind comes from opposite directions during the two monsoon seasons whereas winds are generally light and variable during the intermonsoon seasons.

In order to ensure the statistical significance of the computed statistics, sets containing too few observation data were discarded from the study. A minimum number of 600 individual measurements was enforced for sets representing the monsoon seasons JJAS and DJFM, and 300 individual measurements for sets representing intermonsoon seasons AM and ON. This is roughly equivalent to the amount of data collected in five years of continuous records. The list of statistics introduced in Section 4.2.2 were then computed for each of the remaining sets of data.

4.3.3 Tests of Significance

An additional layer of significance checks was carried out on the computed statistics to weed out those that do not make physical sense. Firstly, t tests were performed on each set of observations to identify those potentially characterised by a zero mean velocity. The tests were carried out on the wind components in the two cardinal directions separately and at a 95% confidence level (two-tailed) against the null hypotheses $\overline{v_x} = 0$ and $\overline{v_y} = 0$. A set is deemed to be characterised by a zero mean velocity if the null hypothesis cannot be rejected in both directions. In such cases, the direction of the mean velocity ϕ would have little meaning and would therefore be discarded.

Next, χ^2 tests for variance were performed to identify sets characterised by a very small trace of the variance tensor σ^2 . The tests, using a modified test statistic given by:

$$(n - 1) \frac{2\sigma_t^2}{\sigma_0^2} \sim \chi^2_{2(n-1)} \quad (4.10)$$

where n is the number of observations, σ_0^2 is the hypothesised value of the trace, and $2(n - 1)$ is the degree of freedom of the χ^2 distribution, were carried out at a 95% confidence level (one-tailed) against the null hypothesis $\sigma_t^2 < 0.01V_t^2$. A set is deemed to be characterised by a very small trace if the null hypothesis cannot be rejected and its tensor-linked statistics, eccentricity ε and orientation θ , would then be discarded.

Even if the trace is not small, a F test for equality of variances would further be performed to determine if the variance tensor is isotropic by comparing the variance in the two principal directions. The F tests were carried out at a 95% confidence level (two-tailed) against the null hypothesis $\lambda_{\min} = \lambda_{\max}$. A set is deemed to be characterised by an isotropic variance if the null hypothesis cannot be rejected. In such cases, the orientation of the variance tensor θ would make little sense and would be discarded.

Finally, noting that the alignment statistic η is a dot product derived from φ and θ , η would also be discarded when either φ or θ was discarded. A description of these standard tests can be found in most texts on statistics and hypothesis testing such as [17] and has also been summarised in Section 3.3 of the previous chapter. A summary of the tests performed is given in Table 4.1.

Table 4.1: Significance checks conducted on the computed statistics.

Condition	Test, Confidence Level	Null Hypothesis	Action taken if null hypothesis is not rejected
$V_t = 0$	t test (two-tailed), 95%	$\overline{v_x} = 0 ; \overline{v_y} = 0$	Discard φ and η .
$\sigma_t^2 \rightarrow 0$	χ^2 test (upper tail), 95%	$\sigma_t^2 < 0.01V_t^2$	Discard ε , θ , and η .
$\varepsilon = 0$	F test (two-tailed), 95%	$\lambda_{\min} = \lambda_{\max}$	Discard θ and η .

4.4 Results

4.4.1 Qualitative Scatter Analysis

The characteristics of the computed statistics were visualised using scatter plots. Four scatter relations are considered: σ_s^2 vs. V_s , σ_t^2 vs. V_t , ε vs. V_t , and η vs. V_t . For each relation, eleven separate plots were made to describe the scatter at each of the eleven pressure levels considered. Four of these are representatively shown in Figures 4.3 to 4.6: 850 mb (lower troposphere), 500 mb and 250 mb (middle troposphere), and 150 mb (upper troposphere). The complete plots containing all eleven pressure levels are documented in Appendix E. Each scatter point in these plots represents the statistics obtained for a single set of the observation data as described in the previous section. The sets can be of different seasons or launch times but all points on a single plot are from sets of the same pressure level.

A quick inspection of the scatter plots reveals some salient features. The plots of σ_s^2 vs. V_s in Figure 4.3 show a trend of increasing σ_s^2 with increasing V_s . This implies that the scalar variance tends to get larger with higher mean wind speed. Also, the spread of scatter points increases with increasing V_s , giving the impression that the scatter points are fanning out from the origin. This implies a greater variability of the scalar variance at higher mean wind speeds.

As for the plots of σ_t^2 vs. V_t (Figure 4.4), which are the vector equivalent of σ_s^2 vs. V_s , neither σ_t^2 nor its spread seems to be related to V_t . However, the spread of σ_t^2 exhibits a peculiar skewness with high density of scatter points coalescing towards a pseudo lower bound and the density gradually tapering off towards higher values of σ_t^2 . It is noted that this pseudo lower bound appears to move upwards with a higher pressure level. This implies that the trace of the variance tensor tends to become larger with increasing height.

Figure 4.3: Scatters of σ_s^2 vs. V_s for four of the eleven standard pressure levels: (a) 850 mb; (b) 500 mb; (c) 250 mb; (d) 150 mb.

Figure 4.4: Scatters of σ_t^2 vs. V_t for four of the eleven standard pressure levels: (a) 850 mb; (b) 500 mb; (c) 250 mb; (d) 150 mb.

Figure 4.5: Scatters of ϵ vs. V_t for four of the eleven standard pressure levels: (a) 850 mb; (b) 500 mb; (c) 250 mb; (d) 150 mb.

Figure 4.6: Scatters of η vs. V_t for four of the eleven standard pressure levels: (a) 850 mb; (b) 500 mb; (c) 250 mb; (d) 150 mb.

For the plots of ε vs. V_t in Figure 4.5, the spread of ε reduces and appears to tend towards a particular median value of ε as V_t increases. This implies that the degree of anisotropy in the variance tends towards a certain value at higher magnitudes of the mean wind velocity whereas at lower magnitudes, a greater variability of the anisotropy can be expected. It is also noted that the median ε value and the spread are different in each of the plots, suggesting that these characteristics are related to the pressure level.

Finally, for the plots of η vs. V_t in Figure 4.6, the spread of η reduces and η tends sharply towards 1 with increasing V_t . Furthermore, even at low V_t , the spread of η is also highly skewed towards $\eta = 1$. This implies that the direction of the mean wind velocity tends to align with the axis of the maximal variance and especially so at higher magnitudes of mean wind velocity.

4.4.2 Regression Fitting of Percentile Lines

In furthering the analysis, percentile lines were used to quantify the observed scatter patterns. These percentile lines describe how the location and spread of the scatter points change with V_s or V_t , similar to how a standard regression line describe that for the scatter mean. Five percentiles were considered: 10th, 25th (1st quartile), 50th (median), 75th (3rd quartile), and 90th percentile. The percentile lines were obtained using a two-step process. The first involved binning of scatter points in each scatter plot into bins in the horizontal V_s or V_t -axis. For each of the five percentiles considered, a regression point was computed for each bin. The vertical coordinate of this point is the percentile calculated for only the scatter in the particular bin and the horizontal coordinate is the bin centre. In order to ensure statistical significance, regression points calculated from bins with less than five scatter points were discarded. The second step was to obtain the percentile line by fitting a regression line through the regression points from all the bins. For this, a weighted linear least-squares regression analysis was used with the regression points weighted by the number of scatter points in the respective bins.

Two binning schemes were tested in the analysis. A coarser scheme has, for all pressure levels except 150 mb, a bin width of $2 \text{ m}\cdot\text{s}^{-1}$ for V_s or V_t between 0 and $20 \text{ m}\cdot\text{s}^{-1}$ followed by a single bin from 20 to $30 \text{ m}\cdot\text{s}^{-1}$. For the 150 mb pressure level, the scheme has a bin width of $2 \text{ m}\cdot\text{s}^{-1}$ between 0 and $24 \text{ m}\cdot\text{s}^{-1}$ followed by a single bin from 24 to $30 \text{ m}\cdot\text{s}^{-1}$. An inspection of the scatter points showed none of them falling beyond $30 \text{ m}\cdot\text{s}^{-1}$. The finer bin scheme contains smaller bin widths and twice the number of total bins. It has, for all pressure levels other than 150 mb, a bin width of $1 \text{ m}\cdot\text{s}^{-1}$ for V_s or V_t between 0 and $20 \text{ m}\cdot\text{s}^{-1}$ followed by two larger bins from 20 to $25 \text{ m}\cdot\text{s}^{-1}$ and 25 to $30 \text{ m}\cdot\text{s}^{-1}$. For the 150 mb pressure level, this scheme has a bin width of $1 \text{ m}\cdot\text{s}^{-1}$ between 0 and $25 \text{ m}\cdot\text{s}^{-1}$ followed by a single bin from 25 to $30 \text{ m}\cdot\text{s}^{-1}$. It was found that the use of either binning scheme did not alter the conclusions of the study. The results presented hereafter in this chapter are that obtained for the finer scheme.

For the scatters of σ_s^2 vs. V_s , σ_t^2 vs. V_t , and ε vs. V_t , linear relations were assumed between percentiles of σ_s^2 , σ_t^2 , ε and V_s/V_t . The equations of the percentile lines fitted take the form:

$$\sigma_{s,\%|h}^2 = L_{\%|h}V_s + C_{\%|h} \quad (4.11)$$

$$\sigma_{t,\%|h}^2 = M_{\%|h}V_t + D_{\%|h} \quad (4.12)$$

$$\varepsilon_{\%|h} = N_{\%|h}V_t + E_{\%|h} \quad (4.13)$$

In these equations, C , D , E , L , M , and N are fitting coefficients which characterise the percentile lines and that in turn characterise the scatters. Since five percentile lines were fitted for each scatter and eleven scatters, one for each pressure level, were made for each scatter relation, the fitting coefficients obtained can be functions of both the pressure level and the percentile. This is denoted by the subscript $\%|h$: for example, $L_{25\%|500}$ would denote the L coefficient for the 25th percentile line of the 500 mb scatter. The same notation is also used to uniquely identify equations of individual percentile lines as shown by the left-hand-side terms of Equations (4.11) to (4.13). In the linear Equations (4.11) to (4.13), L , M , and N are coefficients of gradient and they give the correlation between the percentile and V_s or V_t . C , D , and E , on the other hand, are axis-intercept coefficients and they give the value of σ_s^2 , σ_t^2 , ε at each percentile when V_s or V_t approaches $0 \text{ m}\cdot\text{s}^{-1}$.

For the scatter relation η vs. V_t , however, the percentile lines that would describe the scatters are clearly not linear. An exponential relation, as given by the form:

$$\eta_{\%|h} = 1 - A_{\%|h}e^{-B_{\%|h}V_t} ; A \geq 0, B \geq 0 \quad (4.14)$$

is proposed instead. Linear fitting of the lines were performed in the logarithmic space using the linearised form of Equation (4.14) as shown by:

$$\ln(1 - \eta_{\%|h}) = B_{\%|h}(-V_t) + \ln A_{\%|h}. \quad (4.15)$$

Here, B is the gradient coefficient which indicates how sharply the percentile line tends towards $\eta = 1$ whereas A is the intercept coefficient with $1 - A$ being the value of η at each percentile as V_t approaches $0 \text{ m}\cdot\text{s}^{-1}$. If $A = 1$ ($\ln A = 0$), the percentile line passes through the origin.

4.4.3 Vertical Profiles of Fitting Coefficients

The results for the fitting coefficients L , M , N , C , D , E , A , and B are plotted in Figures 4.7 to 4.14. As separate scatters were created for each individual pressure level, vertical profiles of the fitting coefficients could be obtained and plotted for each of the percentiles considered. In the plots, the pressure level is plotted on the horizontal axis from a lower level to a higher level while the value of the respective fitting coefficient is plotted on the vertical axis. The results obtained for the different percentiles are represented by markers of different colours. On top of the point-estimated values obtained from the regression analysis, the 98% confidence intervals for each estimate are also given as denoted by the co-located error bars.

In Figure 4.7, the point estimates for the coefficient L are positive at all pressure levels except 1000 mb and 925 mb. Even at these two levels, the confidence intervals obtained do not exclude the possibility of L being positive. Two further features were observed based on an impressionistic observation of the plot that also takes into account the error bars. First, the value of L at each pressure level appears to increase with higher percentiles. This implies that the gradient of the percentile lines in the σ_s^2 vs. V_s scatters increase with the percentile and this is consistent with the earlier qualitative observation that the scatter points appear to fan out from the origin. Next, it was also observed that the value of L for each percentile could be the same regardless of the pressure level. This possibly suggests a constant vertical profile.

Figure 4.7: Results and vertical profile for the coefficient L at the five percentiles: 10%, 25%, 50%, 75%, and 90%. The error bars shown are for the 98% confidence interval.

Figure 4.8: Results and vertical profile for the coefficient C at the five percentiles: 10%, 25%, 50%, 75%, and 90%. The error bars shown are for the 98% confidence interval.

Figure 4.9: Results and vertical profile for the coefficient M at the five percentiles: 10%, 25%, 50%, 75%, and 90%. The error bars shown are for the 98% confidence interval.

Figure 4.10: Results and vertical profile for the coefficient D at the five percentiles: 10%, 25%, 50%, 75%, and 90%. The error bars shown are for the 98% confidence interval.

Figure 4.11: Results and vertical profile for the coefficient N at the five percentiles: 10%, 25%, 50%, 75%, and 90%. The error bars shown are for the 98% confidence interval.

Figure 4.12: Results and vertical profile for the coefficient E at the five percentiles: 10%, 25%, 50%, 75%, and 90%. The error bars shown are for the 98% confidence interval.

Figure 4.13: Results and vertical profile for $\ln A$ at the five percentiles: 10%, 25%, 50%, 75%, and 90%. The error bars shown are for the 98% confidence interval.

Figure 4.14: Results and vertical profile for the coefficient B at the five percentiles: 10%, 25%, 50%, 75%, and 90%. The error bars shown are for the 98% confidence interval. Where applicable, the error bars are truncated at $B = 0$ as B is defined to be positive.

In Figure 4.8, the point estimates for coefficient C are scattered about the $C = 0$ line. A similar impressionistic observation of the plot suggests that the value of C could be zero as many of the error bars do contain the zero line. This implies that the percentile lines in the σ_s^2 vs. V_s scatters pass through the origin and this is again in line with the earlier qualitative observation.

For the gradient coefficients M and N in Figures 4.9 and 4.11, respectively, the plots are similar to C in that the point estimates also scatter about zero and that many of the error bars contain the zero line. This implies that the scatters of σ_t^2 and ε do not correlate significantly with V_t . Also, it was observed specifically in Figure 4.11 that the point estimates for N at the 1000 mb and 925 mb pressure levels are more negative as compared to the other pressure levels.

In Figures 4.10 and 4.12, the plots of coefficients D and E show non-trivial vertical profiles with salient features. This is notable as unlike the other coefficients, the error bars here are generally smaller compared to the overall trends exhibited by the point estimates and this excludes the possibility of a constant vertical profile. The coefficient D in Figure 4.10 shows an increasing trend towards higher pressure levels while the coefficient E in Figure 4.12 shows a gradual decreasing trend up to about the 200 mb pressure level followed by a sharper increasing trend above it.

For the scatter relation η vs. V_t , it was observed in Figure 4.13 that the point-estimated values for $\ln A$ (and thus also for A) at each pressure level decrease with higher percentiles. This means that the η -axis intercept of the percentile lines, given by $1 - A$, increase with the percentile. The observation is expected and in fact necessary for the percentile lines to stack in their proper order as V_t approaches $0 \text{ m}\cdot\text{s}^{-1}$.

One other feature, noted based on an impressionistic observation of Figure 4.13, is that the value of $\ln A$ for each percentile appears to be the same regardless of the pressure level. This implies that the η -axis intercept of each percentile is the same for all pressure levels.

Finally, in Figure 4.14, an impressionistic observation of the plot suggests that coefficient B could be a non-zero constant with a same value regardless of the percentile or pressure level. This, together with the observations made of A , would imply that the scatters of η vs. V_t are characteristically similar for all the pressure levels.

4.4.4 Simplifying Relations for L, M, N, C, A , and B .

Using the impressionistic observations described above, the following simplifying relations for the fitting coefficients L, M, N, C, A , and B are hypothesised. No such hypothesis was made for the coefficients D and E . In the Equations (4.16) and (4.20) below, the dropping of h in the subscript of the right-hand-side terms is to denote that the coefficient is hypothesised to not depend on the pressure level but is still dependent on the percentile.

$$L_{\%|h} = L_{\%} \quad (4.16)$$

$$M_{\%|h} = 0 \quad (4.17)$$

$$N_{\%|h} = 0 \quad (4.18)$$

$$C_{\%|h} = 0 \quad (4.19)$$

$$A_{\%|h} = A_{\%} \quad (4.20)$$

$$B_{\%|h} = \text{constant} \quad (4.21)$$

To examine the validity of the hypothesised relations, the following test was carried out. Consider a single result of a fitting coefficient. Assuming the hypothesised value to be true, the probability of the $100p\%$ confidence interval ($p = 0.98$ in the plots shown above) not including the hypothesised value is $1 - p$. Now, further consider the 55 instances of results, corresponding to the five considered percentiles and the eleven pressure levels, for each fitting coefficient. The probability that k or more of them do not include the hypothesised value can be modelled by the binomial distribution and is given by:

$$P(K \geq k) = \sum_{i=k}^n \binom{n}{i} (1-p)^i p^{n-i} \quad (4.22)$$

where K is the random variable denoting the number of confidence intervals not including the hypothesised value and n is the total number of instances of results ($n = 55$ in this case). Using this logic, if the value of k observed from the results is such that the probability $P(K \geq k)$ is less than the significance value $1 - p$, it can be concluded that there is sufficient evidence that the hypothesised value is not correct. Otherwise, even though rigorous statistical theory would say no conclusions can be made, for the purpose of this work it is taken that the hypothesised value is sound. Note that the hypothesised values for

coefficients L , A , and B are not explicit. For these cases, k was obtained by selecting a hypothesised value for the coefficient that minimises k .

Table 4.2: Observed values of k considering all eleven pressure levels.

$n = 55; p = 0.98. k_{\text{crit}} = 5$	
Coefficient	k
L	8
M	7
N	3
C	9
A	2
B	6

Table 4.3: Observed values of k considering only the middle troposphere between the 700 mb and 200 mb pressure levels.

$n = 30; p = 0.98. k_{\text{crit}} = 4$	
Coefficient	k
L	0
M	0
N	1
C	2
A	0
B	3

Table 4.2 shows the results for k for the six fitting coefficients with hypothesised relations. Also given is the value of k_{crit} , the critical value of k such that the hypothesised relation is rejected if $k \geq k_{\text{crit}}$. The results show that the hypothesised relations for coefficients L , M , C , and B are not supported. On closer investigation, however, it was realised that most instances with non-conforming confidence intervals are for either the higher levels at or above 150 mb, levels near the tropopause, or the lower levels at or below 850 mb, levels within the surface boundary layer or affected by topography. Removing these levels, Table

4.3 shows the results for k considering only the mid-tropospheric levels of 700 mb to 200 mb (with consequent reduction of n from 55 to 30). These new results show that the hypothesised relations for all the coefficients are sound when considering only the middle troposphere.

4.4.5 Implications and Discussion

Assuming the proposed relations given in Equations (4.16) to (4.21) are correct, some inferences can be made regarding the characteristics of σ_s^2 with respect to V_s and σ_t^2 , ε , and η with respect to V_t . Equations (4.23) to (4.26) in the following paragraphs show Equations (4.11) to (4.14) rewritten to reflect the simplified relations.

$$\sigma_{s,\%}^2 = L_{\%} V_s \quad (4.23)$$

For σ_s^2 , the coefficient L says that the percentile lines at each percentile have positive gradients and are the same for all pressure levels. Furthermore, the coefficient C suggests that all these lines pass through the origin. This implies two things. First, the scalar variance can be expected to increase with the mean wind speed and notably, for the inference to make physical sense, zero variance is expected for a no wind condition (i.e., zero mean wind speed, noting that wind speed is always positive or zero). Second, the scatters of σ_s^2 vs. V_s are characteristically similar for all pressure levels. This implies that the scalar variance, while dependent on the wind speed, does not depend on the pressure level.

$$\sigma_{t,\%|h}^2 = D_{\%|h} \quad (4.24)$$

The characteristics of σ_t^2 is, in contrast, quite the opposite. The coefficient M taking zero value implies that the trace of the variance does not depend on the magnitude of the mean wind. Also, the coefficient D shows a clear vertical profile in Figure 4.10 that indicates dependence on the pressure level. Since $M=0$, D , rather than only being an intercept value, can be regarded as indicative of the value of σ_t^2 at each percentile regardless of V_t . The vertical profile of D observed is such that it implies the standard deviation (the size of the

ellipse in Figure 4.1) grows larger with height until near the tropopause level. This trend could easily be mistaken for an increasing trend with the wind magnitude as the magnitude of the mean wind velocity too increases with height. However, the scatter analysis showed that the dependency is on the pressure level and not the wind velocity.

$$\varepsilon_{\%|h} = E_{\%|h} \quad (4.25)$$

ε shows similar characteristics to σ_t^2 in that it does not depend on the magnitude of the mean wind but is dependent on the pressure level. The vertical profile of E in Figure 4.12 shows that the spread of the eccentricity reduces with height as the higher percentile lines approach the lower percentile lines which remain around the same value up to about the 200 mb pressure level. In this sense, the wind variance gets more isotropic with height. But upon nearing the tropopause (at pressures lower than 200 mb), the eccentricity seems to rise while keeping the same spread, denoting increasing anisotropy in the wind variance.

$$\eta_{\%} = 1 - A_{\%}e^{-BV_t} \quad (4.26)$$

For η , Equations (4.20) and (4.21) for coefficients A and B further the description of η by saying that the scatters of η vs. V_t are characteristically identical for all pressure levels. Moreover, the constant value of B across percentile lines lends fundamental support that the spread of η tightens as the mean wind gets stronger and implies a growing tendency for the largest variance to occur in the direction of the mean wind as the mean wind strengthens. This last point is not a logical necessity but seems to be an interesting result of fundamental wind dynamics.

Table 4.4 and Figure 4.15 summarise the characteristics of σ_s^2 , σ_t^2 , ε , and η on their dependencies on V_s , V_t , and pressure level.

Table 4.4: Summary of the characteristics of σ_s^2 , σ_t^2 , ε , and η with respect to V_s , V_t and pressure level.

Variance Statistic	Dependency on:		
	V_s	V_t	Pressure Level, h
σ_s^2	Yes	No	No
σ_t^2	No	Yes	Yes
ε	No	Yes	Yes
η	Yes	No	No

Figure 4.15: Qualitative illustrations of the observed trends summarised in Table 4.4. The black lines represent the median value whereas the shaded grey areas depict the spread. In (c) and (d), the areas of darker shade with the solid median line represent the mid-troposphere between the 700 mb and 200 mb pressure levels where the results in Table 4.4 were shown to be valid. (a) σ_s^2 vs. V_s ; (b) η vs. V_t ; (c) σ_t^2 vs. pressure level; (d) ε vs. pressure level.

The increase of the scalar variance with mean wind speed can be explained by the other three results in Table 4.4. Appendix C proves the following identity:

$$V_s^2 + \sigma_s^2 \equiv V_t^2 + \sigma_t^2 \quad (4.27)$$

which implies, using the result in Table 4.4 that σ_t^2 is independent of V_t ,

$$\frac{d(V_s^2)}{d(V_t^2)} \left\{ 1 + \frac{d(\sigma_s^2)}{d(V_s^2)} \right\} = 1. \quad (4.28)$$

Thus, σ_s^2 increases with V_s ($d(\sigma_s^2)/d(V_s^2) > 0$) if and only if V_s^2 increases more slowly than V_t^2 ($d(V_s^2)/d(V_t^2) < 1$) as the mean wind strengthens (V_t increases). Appendix D proves that this is indeed so for the case of small variations compared to the mean wind. Here, a consistent, heuristic explanation is offered for the general case. As the mean wind gets stronger, the size and anisotropy of the wind variations (σ_t^2 and ε) remain constant but there is greater alignment of the direction of wind variations with the direction of the mean wind, as Table 4.4 says. Such alignment leads to more effective variation in the magnitude of the wind vector and so σ_s^2 increases. With the constraint set by the identity in equation (4.27), V_s^2 must increase more slowly than V_t^2 , or otherwise, decrease. The latter case of decreasing V_s^2 occurs when there are frequent, large enough variations compared to the mean wind speed: from Equation (4.28), negative $d(V_s^2)/d(V_t^2)$ implies that $d(\sigma_s^2)/d(V_s^2) = (\sigma_s/V_s) d\sigma_s/dV_s < -1$ and so $\sigma_s/V_s > |dV_s/d\sigma_s|$. However, our result in Table 4.4 that σ_s^2 increases with V_s and hence V_s^2 is explained by the former case, where the wind variations are sufficiently small most of the time, i.e., qualitatively more like the case proven in Appendix D.

Some caveats are to be noted when considering the trends inferred above. Firstly, the proposed simplifying relations were shown to be valid only for the mid-troposphere between the 700 mb and 200 mb pressure levels. This means that the trends described are strictly valid only within this same part of the atmosphere. While no specific physical

mechanisms will be proposed in this dissertation to explain the discrepancy at the lower and the higher levels, these excluded levels represent two unique parts of the troposphere. The lower levels at 850 mb and below is within the planetary boundary layer and are affected by the mountainous terrain found in parts of the monsoonal tropics (e.g., Sumatra, Borneo, East Africa). The obstructions caused by topography as well as friction against the planetary surface can cause distortions to wind flow. Similarly, at the higher levels of 150 mb and 100 mb pressure, distortions to the flow dynamics may also happen due to outflow from deep convection near the tropopause. The second caveat is that the results are applicable only within the specified near-equatorial monsoon region for which the radiosonde data for this study have been obtained. As scatter plots were used to derive the trends and implications, the inferred trends can only be considered as an aggregate for the entire geographic region being studied and that the trends observed at specific individual locations may be considerably different.

4.5 Relation between Wind Variance and Eddy Diffusivity

Consider the momentum equations (i.e., Navier-Stokes equations) for the atmosphere written with pressure as the vertical coordinate. At synoptic scales, the atmosphere can be approximated to be in hydrostatic balance and vertical advection terms are thus dropped. The resulting equations are in the form used in quasi-geostrophic approximation [18,19] and are given by:

$$\frac{D_g \mathbf{v}_g}{Dt} = -\nabla \psi + \mathbf{F} \quad (4.29)$$

where the horizontal wind $\mathbf{v}$ has been approximated by the (non-divergent) geostrophic wind $\mathbf{v}_g$, D_g/Dt represents the material derivative following the geostrophic motion, $\mathbf{F}$ denotes body forces per unit mass including the Coriolis force, and ψ is the geopotential which is defined as:

$$\psi(z_h) = \int_0^{z_h} g \, dz$$

where g is the gravitational acceleration and z is the physical height (in metres). The geopotential is a measure of the potential energy per unit mass due to gravity.

Note that at planetary scales over climatological time-scale, $\mathbf{v}_g$ can be thought of as a stochastic vector. Applying Reynolds averaging and considering only the resultant covariance terms, the following is obtained:

$$\left(\frac{D_g \overline{\mathbf{v}_g}}{Dt} \right)_{\text{Reynolds}} = \nabla \cdot \text{var}(\mathbf{v}) = \nabla \cdot \mathbf{X} \quad (4.30)$$

where $\mathbf{X} = \text{var}(\mathbf{v})$ is introduced to simplify the notations. Equation (4.30) is identical to the Reynolds-averaged Navier-Stokes equations with the exception that partial derivatives are evaluated holding the pressure constant instead of vertical height. The right-hand-side term of the equation accounts for the contributions to the total acceleration of the mean wind due to the stochastic fluctuations of the wind vector and $\mathbf{X}$ is analogous to the Reynolds stress [20] in turbulent flows.

Rewriting the covariance terms using the framework below:

$$\mathbf{X} = \mathbf{J} \nabla \overline{\mathbf{v}_g} \quad (4.31)$$

where $\mathbf{J}$ is a matrix of coefficients and $\nabla \overline{\mathbf{v}_g}$ is the deformation rate of the mean geostrophic flow. The diffusivity matrix is then defined as the symmetric part of $\mathbf{J}$:

$$\mathbf{K} = \frac{1}{2} (\mathbf{J} + \mathbf{J}^T). \quad (4.32)$$

$\mathbf{K}$ is responsible for the diffusion of momentum by eddy motion. Thus, combining Equations (4.31) and (4.32) and assuming $\nabla \overline{\mathbf{v}_g}$ is invertible, the relation between the eddy diffusivity tensor and the velocity variance tensor is obtained in Equation (4.33):

$$\mathbf{K} = \frac{1}{2} \left[\mathbf{X} \nabla \overline{\mathbf{v}_g}^{-1} + (\nabla \overline{\mathbf{v}_g})^{-1} \mathbf{X} \right]. \quad (4.33)$$

The significance of Equation (4.33) is that from the velocity covariance and mean flow, it is possible to diagnose the eddy diffusivity and this may aid the prognostic modelling of the mean flow over long time-scales or the theoretical understanding of the eddy momentum transport in the atmosphere.

4.6 Conclusions

In this chapter, an alternative method was proposed for the statistical analysis and qualitative characterisation of atmospheric winds. This method is based on the statistics of vectors and features additional terms, in the form of a vector for the mean and a tensor for the variance, to more holistically describe the wind statistics. Several statistical measures based on the geometric interpretation of the variance tensor were also designed in this study. They are namely the trace of the variance tensor (the square-root of which is the standard deviation of the vector wind), the symmetrised eccentricity, and also the alignment measure η . Unlike the more traditional scalar analysis, these statistical measures, each with its own physical interpretation, allow the directional properties of the variance to be captured and analysed. The above methods were applied in an exploratory study of radiosonde wind data from a near-equator monsoon region. Through the use of scatter analysis, several trends were identified:

1. The scalar variance increases with the mean wind speed but is not affected by the pressure level.

2. The standard deviation and the anisotropy of the wind variations (measured by σ_t and ε) are not dependent on the magnitude of the mean wind vector (V_t) but show clear dependence on the pressure level.
3. The axis of maximal variance tends to align with the direction of the mean wind at stronger mean wind velocity but this behaviour is not affected by the pressure level.

Vector-based statistics, being more comprehensive summarising wind variance information, can provide insight into the scalar-based statistics. In particular, increasing alignment of maximal wind variations in the direction of the mean wind vector provides an explanation for the observed increase in scalar variance with mean wind speed.

It was also noted that the trends observed in this study appear valid only for the middle troposphere. While the study is primarily statistical in nature and no physical explanations were given for the observed trends, or lack thereof, some speculations were made with regards to the lower and the upper parts of the troposphere. Nevertheless, the results of this work can serve as a basis for future investigations into the physical mechanisms behind the observed trends and the discrepancy in the lower and upper troposphere. It is also hoped that this work would spark further interest in the use of vector-based approaches in the statistical characterisation of winds.

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Chapter 5 The Boundary Layer Wind Profile on an Island Offshore of Singapore

In this study, the ability of the power-law wind model to reproduce a tropical vertical wind profile is investigated. Data from a one-year observation on an offshore island of Singapore, obtained using a lidar, were used and it was found that the power-law model profiles are unable to capture the pertinent features of the observed wind profiles. Two alternative profile models, one based on the free convection limit scaling and the other based on the arctangent function, were introduced and tested, and it was found that the latter was superior in predicting the measured profiles. A systematic study of the seasonal and diurnal variations of the model parameters is presented and some practical implications of the results are also discussed.

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5.1 Introduction

While the use of wind energy as a renewable energy source has seen substantial growth in many parts of the world [1,2], its potential remains relatively untapped in the deep tropics. There are two main reasons for this. First, wind speeds in the tropics, particularly near the equator between the 15° N and 15° S latitudes, are generally low and do not present a strong case for the deployment of contemporary large wind turbines. Second, the abundance of solar radiation in the tropics also means that solar energy is the more readily-accessible and thus preferred form of renewable energy in the region. Despite this, recent advances in the development of low-speed wind turbines are making the prospect of wind energy more appealing in the tropics. Developments in the area of small wind turbines for urban and microgrid applications are also opening up new opportunities where wind energy systems can be deployed [3,4]. As such, the growth of wind energy in the region can be expected to accelerate in the near future.

Aside from wind energy, the knowledge of the wind characteristics is also important in other areas such as for the calculation of wind loads on buildings and for the planning of airport infrastructure and terminal flight procedures. The depiction of local winds is also essential as a boundary condition for the numerical mapping of wind resources and pollution dispersion [5,6]. Most current methods and models used in the prediction of wind characteristics were developed based on experiments and data conducted in extra-tropical locations [7-10]. Unlike at the higher latitudes, atmospheric circulation in the tropics is primarily driven by direct solar heating. Geostrophic winds, which dominate much of extra-tropical weather, are almost non-existent due to the weak Coriolis force near the equator [11]. Many parts of the tropics also experience seasonal monsoon winds which change direction twice a year. In light of the different mechanisms that drive tropical weather, it is important to re-examine the validity of some of the methods and assumptions that current wind assessment studies rely on.

One particular wind characteristic of interest to wind engineers and atmospheric scientists is the vertical wind shear profile: the variation of wind speed with height. The conventional

wisdom is that the higher you go, the stronger the winds. This characteristic is embodied in the widely-known and mathematically simple power-law profile [12] which predicts monotonically increasing wind speed with height. However, the power-law profile model does not explicitly account for the effects of atmospheric stability and surface roughness [13]. A more rigorous model which does account for these effects is the log-law profile with stability corrections first proposed by Monin and Obukhov in 1954 [14] and sometimes referred to as the Monin-Obukhov similarity theory (MOST). While the log-law model does explicitly account for atmospheric stability and surface roughness by accepting the relevant parameterisations as variables in the profile equation, the correct forms of the stability correction functions remain an open question for discussion and active research [7,15-17]. As of yet, no correction functions are universally applicable for all atmospheric conditions. Despite the inherent shortcomings of the power-law and log-law profile models, they are widely used for practical wind assessment applications [18-21] and the power-law profile is particularly convenient as a first-pass approximation for estimating wind speeds at various heights. The aim, in the present work, is to construct an alternative for the power-law model which is valid in tropical regions while retaining its simplicity, thus distinguishing it from the more common MOST-based approach.

This chapter presents a data-driven study of the vertical wind shear profiles over an offshore island of Singapore. Singapore, located on the southern tip of the Malay Peninsula in Southeast Asia and at 1° north of the equator, experiences tropical weather patterns and is also affected by the Asian monsoon. The goal is to demonstrate that the power-law profile model does not adequately capture the major features of the observed wind shear profiles and propose alternative empirical models that can better capture the shape of the observed profiles. The chapter is structured as follows: The next section provides a description of the existing and proposed wind shear profile models. Section 5.3 gives a description of the observation site and its meteorology, as well as the steps taken to process and analyse the data. Section 5.4 presents the results and discussion. Finally, Section 5.5 concludes the chapter.

5.2 Wind shear profile models

5.2.1 Existing wind shear profile models

5.2.1.1 Power-law profile

The power-law profile model [12] is a well-known vertical wind shear profile that allows the wind speeds at different heights to be easily extrapolated from a single measurement at some reference height, usually at 10 m. The profile is given by:

$$u = u_{\text{ref}} \left(\frac{z}{z_{\text{ref}}} \right)^{\alpha} \quad (5.1)$$

where u and z denote the wind speed and vertical height respectively and α is the shear coefficient. The term u_{ref} denotes the measured reference wind speed at the reference height z_{ref} . The profile has the advantage of being simple to manipulate mathematically and it contains only a single parameter, α , which controls the shape of the profile curve. A commonly-cited value for α is $1/7$ (≈ 0.14) but various experiments in literature have uncovered differing values of α for different atmospheric and surface conditions [22-25]. The main drawback of the power-law profile is its inability to explicitly account for these varying conditions, namely in the form of the atmospheric stability and surface roughness. Even though there have been research conducted into casting α as an explicit function of atmospheric stability and surface roughness [26,27], such methods are not widely adopted.

5.2.1.2 Log-law profile

An alternative to the power-law profile that does account for the surface roughness is the log-law profile model. It is commonly stated together with a stability correction term which corrects the base log-law profile for different atmospheric stabilities. The profile equation is given by:

$$u = \frac{u^*}{\kappa} \left[\ln \left(\frac{z}{z_0} \right) + \Psi \left(\frac{z}{L} \right) \right] \quad (5.2)$$

where z_0 is the roughness length, which parameterises the surface roughness, and Ψ is the stability correction function which takes on the argument z/L where L is the Monin-Obukhov length, a parameterisation of atmospheric stability. u^* and κ are the friction velocity and von Kármán constant, respectively. Unlike the power-law profile, the log-law profile accounts explicitly for the surface roughness and atmospheric stability through the parameters z_0 and L . However, this comes at a price of much increased complexity in working with the profile equation. To fully define the profile, four parameters or constants have to be determined or assumed and an appropriate form of the stability correction function, which itself contains additional empirical constant(s), has to be selected. The effects of the parameters on the profile shape are also less obvious compared to the effects of α on the power-law profile shape. To that end, Holtslag et. al. investigated the effects of z_0 and L on the log-law profile shape and reported the results in their recent paper [28]. They found that varying z_0 results in a change in the predicted wind speeds, essentially a translation of the profile curve along the speed-axis, but does not change the profile shape. Varying L changes the wind shear with the wind speed increasing more quickly with height under a stable condition as compared to neutral, whereas the wind speed increases more slowly under an unstable condition.

5.2.1.3 Other profile models

In the effort to achieve better predictions of the observed vertical wind profiles obtained in experimental studies, several authors have proposed modifications to the basic wind profile models described above. One example is Gryning et. al. [8] who attempted to construct a wind profile that can produce more accurate predictions of wind speeds throughout the entire atmospheric boundary layer, up to a height of 1000 m. The authors were particularly interested in correcting the profile for heights above the surface layer where the performance of the log-law profile is known to be poor. By modifying the length scale used in the derivation of the original log-law profile, they were able to derive new profile

equations for atmospheres with neutral, stable, and unstable stratification. A similar attempt to obtain an extended boundary-layer profile, valid only for the neutral stratification, was also attempted earlier by Deaves and Harris [29]. In their so-called Deaves & Harris model, a correction function in the form of a parabolic function is added to the log-law wind profile. By introducing the boundary-layer height as an additional parameter, the resulting profile equation represents a more realistic bounded boundary layer as opposed to the infinite boundary layer modelled by the power-law or log-law profile models.

Another study that investigated boundary-layer wind profiles was reported by Grachev et. al. [30] who re-examined some of the assumptions used in deriving the wind profile equations, specifically when applied to an unstably-stratified atmosphere. By using revised parameterisations more appropriate for a highly-convective boundary layer, they obtained expressions which suggest that the wind speed in a convective atmospheric boundary layer scales with the $-1/3$ -power of vertical height. It was concluded that the new resultant profiles were a better match with the experimental observations. Interestingly, the $1/3$ -exponent also appears in some suggested forms of the log-law stability correction function for an unstable convective atmosphere [31].

5.2.2 Proposed wind shear profile models

The main impetus of this study is to assess the suitability of the power-law wind profile when applied in the tropics. Aside from the power-law model (Eqn. 5.1), two alternative wind profile models are introduced here that can better fit the wind profiles obtained from the observation data. The two alternative profile models will be referred to as the $-1/3$ -exponent model and the arctangent ('arctan') model.

5.2.2.1 $-1/3$ -exponent model

The $-1/3$ -exponent model was inspired by the expressions derived in Grachev et. al. [30] which suggest that the wind speed u scales with the $-1/3$ -power of vertical height z (i.e. $z^{-1/3}$).

^{1/3}) in the free convection limit. Specifically, the Equations (15) and (17) obtained in the paper, which represent convectively-unstable conditions, can both be easily rearranged to the form given by Equation (5.3) below. Hence, the profile equation for the -1/3-exponent model is:

$$u = u_\beta - \beta z^{-\frac{1}{3}}, \beta > 0 \quad (5.3)$$

with the parameters u_β and β . Unlike the power-law and log-law profiles, where the wind speed can increase without bounds as height increases, the wind speed in the -1/3-exponent model profile tends towards a finite value as the height increases. In this case, the parameter u_β determines this limiting wind speed and β determines how quickly the profile converges towards the limit. Another feature of the -1/3-exponent model profile is that the zero wind speed occurs at some non-zero height above the surface. This makes it in some sense relatable to the log-law profile in that both contain a roughness layer.

5.2.2.2 Arctangent model

The profile equation of the arctan model, which was inspired solely by the shape of the observed wind profiles, is:

$$u = u_\tau \left(\frac{2}{\pi} \right) \tan^{-1}(\tau z) , \tau > 0 \quad (5.4)$$

where u_τ and τ are the parameters. Similar to the -1/3-exponent model, the wind speed in the arctan model is also bounded. However, unlike the -1/3-exponent model, zero wind speed occurs on the surface at $z = 0$. The parameter u_τ determines the limiting wind speed of the profile whereas τ determines how quickly the profile converges towards that speed with increasing height.

5.3 Data and methodology

5.3.1 Site description

Figure 5.1: A map showing the location of Pulau Bukom (Bukom Island) off the southwestern coast of Singapore. The location on the island where the lidar was situated is marked by the yellow point. (Source of base map: Google Maps)

Wind data for this current study were collected on Pulau Bukom, an island off the southwest coast of Singapore, by means of a ZephIR 300 lidar unit. The island, whose location is shown on the map in Figure 5.1, is about 4.5 km across and primarily houses an oil refinery and storage facility. Its terrain is entirely flat, and the surface comprises of man-made structures such as oil tanks and refinery buildings. The lidar was physically situated on the southeastern tip of the island at the coast, away from the main refinery plant.

Located at 1° N latitude, Singapore experiences typical tropical weather patterns characterised by strong sunshine, high rainfall, and relatively constant temperature throughout the year. The country also experiences the Asian monsoon which dominates the local wind pattern. There are two main monsoon seasons. During the southwest monsoon (JJAS), which lasts from June to September, winds predominantly come from the south.

As for the northeast monsoon (DJFM), which lasts from December to March, the winds switch to the opposite direction and predominantly come from the northeast. Winds during the northeast monsoon are also typically stronger than during the southwest monsoon. During the intervening inter-monsoon seasons which last from April to May (AM) and from October to November (ON), the winds are weaker than during the monsoon seasons and their direction is variable. A check with the collected wind direction data confirms that the monsoon characteristics of the wind were captured. This is shown in Figure 5.2 where the percentages of wind data recorded in each of four direction quadrants (northeast, southeast, southwest, and northwest) are shown for each of the monsoon and inter-monsoon seasons. The definitions for ‘night’ and ‘day’ in the figure are explained in Section 5.3.3.

With regards to the temperature, Singapore experiences its warmest months during the southwest monsoon when the daytime temperature is typically above 30 °C. The coolest months are the northeast monsoon months of December and January when monsoon surges from the Greater Asian continent can sometimes cause the temperature to fall below 25 °C.

Another phenomenon experienced at the observation location is the diurnal variation of the wind owing to the coastal geography. Figure 5.3 shows a plot of the hourly wind speed at the height of 10 m averaged over the entire period of the collected data. It shows a clear diurnal variation of the wind speed.

Figure 5.2: Percentage of 38-m wind data recorded in each of four direction quadrants for the seasons AM, JJAS, ON, and DJFM. The four direction quadrants are: northeast (NE 0°-90°), southeast (SE 90°-180°), southwest (SW 180°-270°), and northwest (NW 270°-360°).

5.3.2 Experiment and Data Measurement

Lidar (LIght Detection And Ranging) [32,33] is a remote sensing device that was first developed in 1970s using gas laser but has further matured using solid state sources and telecommunication-based components. They operate with lasers of wavelength in the order of microns and operates based on the interaction of laser with the natural aerosols in the atmosphere (such as particles of dust, droplets, etc.) which results in back scattering of small fraction to the receiver. The motion of the particles helps measure wind speed and direction through the Doppler shift effect. This frequency shift is quantified by interacting

the return signal with the original beam and identifying the beats at different frequency on a photodetector. Lasers offers the new remote sensing opportunity compared to conventional tall wind masts that requires site permitting, working at height and increased uncertainty of the data's reliability in complex sites and inadequate wind mast heights.

In the present study, ZephIR 300 lidar commercial model was used to measure wind characteristics from 10 m up to 200 m at a sampling rate of 50 Hz. A Campbell Scientific CR800 datalogger is connected to the ZephIR 300, which transmits data over modem. The overall device is a mobile unit that can be handled and deployed easily in remote places. In the present study, firstly the lidar was deployed on the shores of Pulau Bukom at the coordinates 1.2241° N, 103.7788° E. Secondly, wherever possible, IEA recommended practices for deploying and operating the lidar profiler and data quality control have been followed [34]. The lidar profiler was calibrated by the manufacturer and the calibration accuracy was validated against the measurements from a met mast. The calibration uncertainty ranged less than 0.1% and overall variation in calibration of less than $\pm 0.5\%$ is assured.

5.3.3 Data processing

Figure 5.3: Averaged hourly wind speed (at 98% confidence interval) at 10 m showing a clear diurnal cycle. Times shown are in local time (UTC +8).

Data were collected for an almost one-year period from 21 November 2014 to 5 November 2015. Aside from a short break that occurred from 15 to 17 March 2015 due to equipment maintenance, there are no breaks in the data time series. Wind speed data were recorded at 11 heights of between 10 m and 200 m to form a vertical wind profile and these were logged at a rate of approximately 3 times per minute by the lidar datalogger. The heights at which the wind speeds were measured are: 10, 20, 30, 38, 50, 60, 75, 90, 120, 150, and 200 m. The recorded wind profiles were then scrutinised and those with missing entries at any of the measurement heights were discarded. The valid profiles were then sorted according to the season and the local time (UTC +8) at which they were recorded. For the seasonal analysis, the four monsoon and inter-monsoon seasons described earlier (AM, JJAS, ON, DJFM) were used. For the diurnal analysis, the profiles were classified as either a daytime 'day' profile or a night-time 'night' profile. Using Figure 5.3 as a reference, the night period is defined as between 2200 and 1000 hrs and the day period between 1000 and 2200 hrs so as to equally divide the amplitude of the wind speed cycle into two halves. The sorted profiles for each season and diurnal period were then averaged to obtain a single mean profile for use in the subsequent analyses.

5.3.4 Parameter determination and model error calculations

In order to assess the suitability of each of the profile models in their ability to reproduce the real averaged wind profile, a set of model parameters must first be determined by utilising the lidar observations. The errors between the model profile based on the parameters and the measured profile were then used as a gauge of the suitability of the model. A smaller error implies that the modelled profile matches more closely with the observed wind profile and thus can better predict the real-world condition. Qualitative observations of the modelled profiles versus the observed wind profiles were also made. For the power-law profile, curve fitting was performed by means of the ordinary least-squares linear regression method applied on the logarithm of Equation (5.1), which yields a linear expression. Hence, the parameter α can be determined from the regression.

For the other two profile models, namely the $-1/3$ -exponent model and the arctan model, curve fitting was achieved by non-linear regression directly on the profile Equations (5.3) and (5.4) using an iterative least-squares approach with the initial guesses for the parameters: u_β , $u_\tau = 1 \text{ m s}^{-1}$ and β , $\tau = 0.1$.

The errors for the respective model profiles were accounted for by the sums of the squared errors (SSEs). These were evaluated by first squaring at each observation height the difference of the wind speed between the observed wind profile and that predicted by the model. The differences were then summed up for all 11 of the observation heights. For the sake of clarity, the terms SSE_{pow} , $\text{SSE}_{1/3}$, and SSE_{tan} are used to denote the SSEs for the power-law profile, the $-1/3$ -exponent profile, and the arctan profile, respectively.

5.4 Results and discussion

5.4.1 Results for power-law profile

Figure 5.4 is a plot of the values of α obtained from the curve fitting. These values are also tabulated in Table 5.1. Figure 5.5 shows the observed vertical wind profiles and the power-law profiles for the four seasons (AM, JJAS, ON, and DJFM) and the two diurnal time periods, night and day. The observed profile and the corresponding model profile were plotted on top of each other to allow qualitative comparisons of their shapes.

Figure 5.4: Plot of the fitted values of α . The upper line is obtained from the night profiles whereas the lower line is from the day profiles.

Table 5.1: Tabulated values for α .

	α	
	night	day
AM	0.097	0.061
JJAS	0.098	0.070
ON	0.117	0.072
DJFM	0.129	0.093

Figure 5.5: Plots of the observed wind profiles and the fitted power-law profiles for the four seasons (AM, JJAS, ON, and DJFM) and the two diurnal time periods (night and day). The solid

lines with square markers are the observed profile curves with the markers at the observation heights. The dashed lines are the fitted power-law curves.

From Figure 5.4, α varies seasonally with the smallest value obtained in AM. This then increases through the seasons of JJAS and ON, and the largest value was obtained in DJFM. The seasonal variation is the same for both the night and day profiles. α also shows a clear diurnal variation with the values obtained from the night profiles being consistently larger than that obtained from the corresponding day profiles. Interestingly, the only value of α that came close to the commonly-cited value of $1/7$ is that from the DJFM night profile. This is also the largest value of α obtained in this study. While α alone cannot be considered as a reliable indicator of the atmospheric stability, the DJFM night profile does correspond to a mean atmospheric condition with the lowest temperature among all of the profiles. This might suggest that the observed tropical atmosphere is inherently unstably stratified and that a neutral stability assumption cannot realistically apply. The values of α are tabulated in Table 5.1.

From Figure 5.5, it can be seen that the fitted power-law curves were unable to fully capture the shape of the observed wind profiles. The observed wind profiles all exhibit a 'knee' at the height of about 20 m to 40 m where the wind gradient (rate of increase of the wind speed with respect to height) decreases sharply. Above the knee, the wind speed plateaus rapidly and reaches a near constant at the height of about 60 m. This behaviour of the observed profiles is in contrast to that displayed by the power-law curves where, in line with the shape of the power-law function, the decrease in the wind gradient is smooth and gradual. The result is the under-prediction of wind speeds by the fitted power-law profiles at the lower heights and an over-prediction at the upper heights. While the observed errors in wind speed appear to be small, this, together with the generally low wind speeds, can result in fairly large errors for some applications such as in the prediction of wind power output where the power scales with the cube of wind speed. For example at the 200 m height, the error in power prediction for DJFM day (observed: 5.79 m s^{-1} vs. power law: 6.13 m s^{-1}) would be about 19%. Even the relatively small error in wind speed for JJAS day (observed: 4.96 m s^{-1} vs. power law: 5.10 m s^{-1}) would result in an error of about 9%.

Another observation made specific to the observed night profiles was the existence of an inflection point at about 100 m above which the wind gradient starts to increase. The reason for this is unclear but one can assume that this has to do with the more stably stratified (or less unstably stratified) atmosphere associated with the night-time conditions. While this feature allows the power-law profiles to more closely fit the observed wind profiles, it does not change that the fundamental shape of the profile was not captured.

It can also be inferred from Figure 5.5 the seasonal and diurnal variations of the observed wind speed. The highest wind speeds were recorded in the DJFM season, followed by JJAS, and then followed by the inter-monsoon seasons. This is consistent with the known seasonal variation of the wind speed described earlier in Section 5.3.1. Diurnally, it can be observed that the wind speeds were higher during the day as compared to during the night.

5.4.2 Results for -1/3-exponent model

Figure 5.6: Plots of the fitted values of parameters (a) u_β and (b) β .

Table 5.2: Tabulated values for the parameters u_β and β .

	u_β		β	
	night	day	night	day
AM	4.43	4.98	3.43	2.74
JJAS	5.29	5.60	4.12	3.39
ON	4.04	4.45	3.58	2.80
DJFM	6.55	6.88	6.19	5.21

Figures 5.6(a) and 5.6(b) and Table 5.2 show the curve-fitted values of parameters u_β and β for the -1/3-exponent profile model. From Figure 5.6(a), it can be observed that u_β has the largest value in DJFM, followed by JJAS, and then followed by the inter-monsoon seasons AM and ON. Noting that u_β is the wind speed a fitted profile converges to and thus is a proxy of the observed wind speeds, this implies that the observed wind speeds were the highest during DJFM, followed by JJAS, and then by the inter-monsoon seasons. This is consistent with the known seasonal variation of the wind speed as described earlier. Diurnally, the observed values of u_β were larger for the day profiles as compared to the night profiles. This implies that the wind speeds were higher during the day than during the night. This is in agreement with the diurnal variation of the wind speed observed in the power-law results.

The parameter β is observed in Figure 5.6(b) to have the largest value in DJFM, followed by JJAS. The smallest values were obtained in the inter-monsoon seasons and both seasons yielded similar values. Recall that β controls the sharpness of the knee and thus determines how quickly a fitted profile converges towards u_β , with a smaller β indicating faster convergence with height. This implies that the fitted profiles converged most rapidly during the inter-monsoon seasons, followed by JJAS, and then by DJFM, which was the slowest. It is interesting to note that the seasonal variation of β is the same as u_β but is not at all correlated to that of α or τ (τ is described in the next section.). This might be explained as an artifact of the fact that the -1/3-exponent model is different from the power-law and arctan models in that the wind speed in the -1/3-exponent profile does not go to zero at height $z = 0$. Diurnally, β was observed to be larger for the night profiles as compared to

the day profiles. This implies that the fitted profiles converge faster during the day than during the night. Since the inflection point in the observed night profiles has the effect of extending the knee of a fitted profile and thus making it more blunt, this diurnal result is thus expected.

5.4.3 Results for arctan model

Figure 5.7: Plots of the fitted values of parameters (a) u_τ and (b) τ .

Table 5.3: Tabulated values for the parameters u_τ and τ .

	u_τ		τ	
	night	day	night	day
AM	3.80	4.48	0.205	0.299
JJAS	4.53	4.97	0.202	0.278
ON	3.39	3.95	0.169	0.244
DJFM	5.40	5.94	0.159	0.204

Figure 5.7(a) and Figure 5.7(b) and Table 5.3 show the curve-fitted values of parameters u_τ and τ for the arctan profile model. From Figure 5.7(a), it can be observed that u_τ has the largest value in DJFM, followed by JJAS, and then followed by the inter-monsoon seasons AM and ON. Noting that u_τ is the wind speed a fitted arctan profile converges to and thus

is an indicator of the observed wind speeds, this implies that the observed wind speeds were the highest during DJFM, followed by JJAS, and then by the inter-monsoon seasons. This is again consistent with the known seasonal variation of the wind speed as described earlier. Diurnally, the observed values of u_τ were, similar to u_β , larger for the day profiles as compared to the night profiles. This implies that the wind speeds were higher during the day than during the night. This is in agreement with the diurnal variation of the wind speed observed in the power-law results and it also matches the -1/3-exponent model results.

The parameter τ is observed in Figure 5.7(b) to have the largest value in AM. This then decreased through the seasons JJAS and ON, and the smallest value was obtained in DJFM. Recall that τ , analogous to β , controls the sharpness of the knee and thus determines how quickly a fitted arctan profile converges towards u_τ , with a larger τ indicating faster convergence with height. This implies that the fitted profiles converged most rapidly during AM and most slowly during DJFM. The seasonal variation is, aside from both having the slowest convergence during DJFM, not the same as the one inferred from β . Diurnally, τ was observed to be larger for the day profiles as compared to the night profiles. This implies that the fitted arctan profiles converge faster during the day than during the night. The diurnal variation is in agreement with that inferred from β and could be attributed to the same effect of the inflection point.

One other interesting observation made was the apparent inverse correlation that exists between τ and α which is evident when comparing Figures 5.4 and 5.7(b). From Figure 5.4, α is larger for the night profiles when compared to the day profiles but in Figure 5.7(b), τ is smaller for the night profiles. Also, the value of α increases through the seasons from AM to DJFM but in the case of τ , it decreases. The reason for this relationship can be attributed to how the fitted profile curves attempt to capture the shape of the observed wind profiles. For the case of a power-law fit, a smaller α is indicative of a sharper decrease of the wind gradient with height whereas for the case of an arctan model fit, this would translate into a sharper knee and result in a larger value of τ .

5.4.4 Results and comparison of the SSEs

Figure 5.8: Comparison of the sum-of-squared errors (SSEs) of the three profile models.

Figure 5.9: Plots of the observed wind profiles and the three fitted profile models (power law, -1/3-exponent, and arctan) for the four seasons (AM, JJAS, ON, and DJFM) and the two diurnal

time periods (night and day). The solid lines with square markers are the observed profile curves with the markers at the observation heights. The dashed lines are the three fitted model curves.

Figure 5.8 shows the comparison plot of the SSEs for the three profile models: power-law, $-1/3$ -exponent, and arctan. Figure 5.9 shows the comparison between the actual model curves with the observed wind profiles (akin to Figure 5.5 but with all profile models plotted) for the four seasons and the two diurnal time periods. The errors appear to be the largest during the DJFM season as can be seen clearly in Figure 5.8. One may also note that the errors vary seasonally and diurnally in a similar manner for all the three profile models, except for the arctan model during DJFM. For all profile models, the night and day errors are similar during AM, while during JJAS the error is larger during the night. During ON, however, the trend reverses and the day error becomes larger. During DJFM, the day error remains larger than the night error for the power-law and $-1/3$ -exponent profile models, while the night error becomes the larger one for the arctan model. This is interesting as the observed DJFM night wind profile has the most pronounced inflection point among all the profiles and is perhaps the closest match to the profile of a traditional power-law curve. It explains why the power-law curve for DJFM night achieved a smaller error compared to the corresponding day curve while the arctan model, which have a much sharper drop in the wind gradient with height, demonstrates the opposite. Overall, as evident in Figure 5.9, the $-1/3$ -exponent and arctan models (for the day profiles at least) still suffer the same issue of overprediction at higher heights and underprediction at lower heights that was observed with the power-law profile model. However, they are able to achieve smaller fitting errors because the shapes of their profile curves are able to more closely match that of the observed wind profiles.

Comparing the errors of the three profile models, it is clear that the arctan model is the best performing model, followed by the $-1/3$ -exponent model, and then lastly the power law. All the errors observed for the power-law model were larger than the corresponding errors for the $-1/3$ -exponent model and these were in turn larger than that for the arctan model. Even the $-1/3$ -exponent profile model, which was based on the free convection theory, was unable to outperform the arctan profile model, which was inspired solely by the shape of

the observed wind profiles. It does thus appear that the real wind profiles observed in this study have wind gradients that decrease so rapidly with height that the arctan profile model resulted in better predictions.

5.4.5 Directional properties of u_τ and τ

Figure 5.10: The parameter u_τ in the four directional quadrants for seasons (a) AM, (b) JJAS, (c) ON, and (d) DJFM. The four quadrants are: northeast (NE 0° - 90°), southeast (SE 90° - 180°), southwest (SW 180° - 270°), and northwest (NW 270° - 360°).

Figure 5.11: The parameter τ in the four directional quadrants for seasons (a) AM, (b) JJAS, (c) ON, and (d) DJFM. The four quadrants are: northeast (NE 0°-90°), southeast (SE 90°-180°), southwest (SW 180°-270°), and northwest (NW 270°-360°).

After concluding that the arctan model is superior in the previous section, its profile parameters were then investigated further. Figure 5.10 shows the values of u_τ obtained when considering winds coming from only one of four directional quadrants, as defined by the wind direction measured by lidar at the 38 m height. The quadrants are: northeast (NE), southeast (SE), southwest (SW), and northwest (NW). It can be seen for the DJFM season in Figure 5.10(d) that the largest u_τ was recorded in the NE quadrant whereas the smallest was in the SW quadrant. Noting again that u_τ is a proxy of the average wind speed, it can be deduced that the strongest winds during DJFM come from the northeastern quadrant. This is expected as that is the direction of the prevailing northeast monsoon winds during

DJFM. For JJAS, in Figure 5.10(b), the strongest winds can be seen to come from the SE quadrant, followed by SW, and with the weakest winds from the NE quadrant. While there appears to be a contradiction with the prevailing wind direction during the southwest monsoon, it is known that the southwest monsoon winds in Singapore during JJAS tend to be more southerly rather than southwesterly [35]. Thus, the results simply show that the southerly winds with easterly components tend to be stronger compared to those with westerly components.

The results obtained for the inter-monsoon seasons are less obvious as one would expect given the variable wind direction during the two inter-monsoon seasons. For AM (Figure 5.10a), the strongest winds come from the SE and NE quadrants, which apparently show the transition from the NE-dominant winds during DJFM to the SE-dominant winds during JJAS. For ON (Figure 5.10c) however, such transition characteristics are absent. The strongest winds come from the NW quadrant during the day hours but come from the SE quadrant instead during the night hours. Compared to the seasonal influence, the diurnal influence on the directional variations of u_τ is also not apparent. In almost all cases, with the exception of NW quadrant during AM, SE quadrant during ON, and SW quadrant during DJFM, the day values of u_τ are larger than the corresponding night values. This is the same trend observed in the non-directional analysis reported in the Section 5.4.3. While the exception cases may be seen as the direction-specific diurnal influences, the fact that they occur in different quadrants during different seasons meant that no meaningful conclusions can be drawn. It is observed that the directional variation of u_τ is seasonally influenced, most notably for JJAS and DJFM where the quadrant with the largest value of u_τ coincide with the direction of the prevailing wind. While this itself does not preclude the existence of non-seasonal direction-only influences, the lack of a consistence pattern of variation across the different seasons implies that such effects are overshadowed by the seasonal influence. As such, it can be concluded that the seasonal influence on the directional variations of u_τ is dominant.

From Figure 5.11, which shows the directional results for τ , it can be observed that (with the exception of DJFM night) all the seasons follow the same trend: the largest τ in the SW

quadrant, followed by the easterly quadrants, and then lastly the smallest τ in the NW quadrant. Recall that a larger τ indicates a sharper knee. This implies that the profile of winds coming from the southwestern quadrant has the sharpest knee whereas that coming from the northwestern quadrant has the bluntest. The consistency of the results across the different seasons and night/day gives confidence to the observation and it can be inferred that τ is influenced by properties of the atmospheric flow that vary with direction but remain constant throughout the year in each direction. A strong possibility for such a property is the surface roughness. The reason as to why the DJFM night result does not follow the trend may be attributed to the very-pronounced inflection point noted in the DJFM night profile distorting the curve-fit results. In all, it was shown here that the variation of average wind speed with direction is predominantly influenced by seasonal changes whereas that of the sharpness of the profile knee is controlled by the directional properties, possibly the surface roughness.

5.4.6 Observed profiles of the wind direction

Fig. 5.12 shows the observed variation in the prevailing wind direction with height (i.e. the wind direction profile) for the four seasons and the two diurnal time periods. The wind directions are obtained by taking the vector mean of the wind velocity at each height and then using the direction of the resultant mean vector. Following meteorological conventions, the wind directions reported here are the direction where the wind is coming from, measured clockwise from the cardinal north.

Figure 5.12: Plots of the observed variation in the prevailing wind direction with height for the four seasons (AM, JJAS, ON, and DJFM) and the two diurnal time periods (night and day).

Directions are measured in degrees clockwise from the cardinal north. The square markers denote the observation heights.

The observed wind direction profiles can be characterised by the direction of the prevailing wind as well as by the sense of the wind veering with height. Considering first the main monsoon seasons of JJAS and DJFM, it can be seen that the prevailing wind is predominantly southwesterly during JJAS and northeasterly during DJFM. This is in line with the known seasonal trend in the wind direction represented in Fig. 5.2. Further, it can also be seen Furthermore, in most cases that the wind veers overall towards the left with increasing height with the exception of the JJAS night profile, which veers towards the right. The DJFM night profile also exhibits a peculiar characteristic where the wind initially veers towards the left, but the veering then reverses with further increase in height. For the inter-monsoon seasons, both day profiles exhibit similar characteristics to each other and they are also similar to the JJAS day profile in terms of the prevailing wind direction and the sense of the veering. The night profiles, however, are different from each other with the AM night profile being more similar to JJAS night whereas the ON night profile is more similar to DJFM night. Overall, the range of the veering observed within the lowest 200 m of the boundary layer ranges from between 4° to 12° with the most pronounced veering occurring within the lowest 40 m.

5.4.7 Observed profiles of the wind speed variance

Fig. 5.13 shows the variation of the observed variance in the wind speed with height for the four seasons and the two diurnal time periods. It can be seen that the shape of the variance profiles roughly follows that of the wind speed profiles in that the wind speed variance varies with height in the same manner as the wind speed. Salient features in the wind speed profiles, such as the ‘knee’, the plateauing of the curve above the knee, as well as the ‘inflection’, are also observed in the variance profiles.

Figure 5.13: Plots of the observed variance in the wind speed with height for the four seasons (AM, JJAS, ON, and DJFM) and the two diurnal time periods (night and day). The square markers denote the observation heights.

With regards to the numerical values, the variance observed in the seasons AM, JJAS, and ON are similar to each other, ranging from between 1.5 and 2.5 $\text{m}^2 \text{s}^{-2}$ (standard deviation of 1.2 to 1.6 m s^{-1}) during the night and between 2.0 and 3.5 $\text{m}^2 \text{s}^{-2}$ (standard deviation: 1.4 to 1.9 m s^{-1}) during the day. This is despite the fact that the seasonal mean wind speed during JJAS is higher than the two inter-monsoon seasons. Noting that the wind speed variance varies with the wind speed with height and is also larger during the day where the wind speed is higher, this suggests that the variance is largely contributed by local-scale perturbances and not so much by the larger-scale seasonal winds. However, this trend no longer holds during DJFM where a larger variance of between 2.5 and 3.5 $\text{m}^2 \text{s}^{-2}$ (standard deviation: 1.6 to 1.9 m s^{-1}) during the night and between 3.5 and 5.0 $\text{m}^2 \text{s}^{-2}$ (standard deviation: 1.9 to 2.2 m s^{-1}) during the day was observed. This is due to a larger contribution to the variance by the seasonal winds unique to the DJFM monsoon and could be attributed to the phenomenon known as monsoon surges (or cold surges) [36] where the wind speed may increase for up to days at a time accompanied by prolonged rainfall.

5.4.8 Practical implications

The results obtained for the observed wind profiles have practical implications for the application of wind energy in the deep tropics. It was noted that the observed profiles exhibit a knee where the wind gradient drops sharply across it and that the wind speed above the knee rapidly plateaus. Wind speed increases above the 60 m height were small in most cases. While the observed night profiles also displayed an inflection point at above 100 m where the wind gradient increases again, wind speeds in the night profiles were lower compared to the day profile of the same season. Thus, unlike in the extra-tropics where wind turbine towers are built to immense heights to capture the higher wind speeds, towers of heights above 60 m might be unnecessary in the tropics and would not result in significantly higher power generation. Also, the wind speeds observed were low as expected, with the highest mean wind speed at 90 m being only 5.7 m s^{-1} , obtained for the DJFM day profile. This means that wind turbines designed specifically for low wind speed applications will be desirable or even necessary.

Figure 5.14: Plot of the observed lidar wind profile and the three predicted model profiles (power law, -1/3-exponent, and arctan) for DJFM day.

It was also demonstrated that the arctan model is the superior model in predicting the tropical vertical wind profile. Figure 5.14 shows the observed lidar profile plotted together with the three model profiles for DJFM day and serves to illustrate this point. Unlike the power law, which is a scaling model with a single parameter, the arctan profile model has two parameters that completely determine the profile without any extrapolation. This gives us some insights into the nature of the observed wind profiles. First, the wind speed that the profile converges to above the knee (and parameterised by u_r) is independent of the wind speeds measured at any of the lower heights. This means that the value of this definitive wind speed must be determined separately through measurement at a significant height above the ground. The traditional method of extrapolating wind speed from a lower height no longer apply and the wind speed measured near the surface, such as at 10 m, is not used for predicting wind speeds at typical wind turbine hub heights. Second, two wind speed measurements are required to fully determine the profile. This second measurement

must be made within the knee of the profile and can be used to determine the parameter (in this case, τ) which controls its sharpness.

In practice however, it is still possible to construct an arctan model profile, through mathematical manipulation, from wind speed measurements at two separate heights near the surface. In such an approach, the sharpness of the knee is first inferred from the two measurement points and the value of τ calculated by iteration. The parameter u_τ can then be determined using either of the measurement points given the known value of τ . This approach is not ideal as it can be seen in Figure 5.14 that the discrepancy between the model profile and the observed profile is the largest at heights near the surface within the knee and would thus make a relatively poor basis for the construction of a model profile. Nevertheless, the arctan model still outperforms the power-law model at these heights and the constructed arctan model profile would likely give more accurate wind speed predictions over traditional power-law extrapolation.

5.5 Conclusion

From this study, it was found that the fitted power-law profiles were unable to capture the shape of the observed wind profiles. This is due to a sharp drop in the wind gradient across a knee in the observed profiles and the subsequent plateauing of the wind speed above the knee. The power-law curves were unable to capture the sharp decrease in the wind gradient and the shape of the curves also did not accommodate the feature of a plateauing wind speed. With regards to the wind shear coefficient α , the values obtained in this study were always lower than the commonly-cited value of $1/7$. In light of these observations, it was suggested that there is likely little value in having wind turbines in the tropics above the height of 60 m. It was also suggested that the power-law model should not be used as a basis for wind speed prediction in the tropics.

Next, two alternative profile models were introduced that were able to better fit the observed wind profiles. They are the $-1/3$ -exponent model and the arctan model. A

systematic investigation of the seasonal and diurnal characteristics of their parameters was reported and by comparison of the sum-of-squared errors, it was determined that the arctan profile model was able to produce the best predictions. This is likely due to its ability to better capture the sharp decrease in the wind gradient across the knee. The result also has the implication that the wind speed above the knee must be measured separately and cannot be extrapolated from a lower height, although a procedure was described where two near-surface measurements can be used as a proxy.

While the work reported here is certainly not the definitive word on the matter, it was shown that there are significant discrepancies between the behaviour of the tropical and extra-tropical atmospheres within the boundary layer. Thus, more research into the tropical atmospheric boundary layer is needed to achieve a more comprehensive understanding of its characteristics and attain greater confidence in the prediction of wind speeds for practical applications. The introduction of the arctan model in the present work will enhance the confidence in using a simple estimation procedure for tropical regions, similar to the power-law for the mid-latitude regions.

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Chapter 6 Implications, Conclusion, and Future Work

This chapter strings together the salient findings from Chapters 4 and 5 and concludes the overall dissertation. It also provides a list of unanswered questions stemming from the research performed and related opportunities for future investigations. It is assumed that readers have read the earlier chapters of this dissertation, particularly the Chapters 4 and 5, prior to this chapter. Some initial works on a modified bivariate probability distribution model as well as wind-induced variance in aircraft flight duration using the variance tensor formulation, both of which were not developed further, are also documented in this chapter.

6.1 Implications for Wind Energy and Wind Resource Assessment

In the preceding chapters of this dissertation, several salient discoveries were made with regards to the wind characteristics of the tropical atmosphere. It was found in Chapter 4 that the variance of the wind in the tropical middle troposphere possesses certain large-scale trends in relation to the mean wind. Whereas at the boundary layer, a study conducted in Singapore and presented in Chapter 5 showed a boundary-layer profile shape that is different from one that can be predicted by the traditional power-law wind profile model.

6.1.1 Practical Implications

In the boundary layer, it was found that the wind speed plateaus rapidly with height, forming a “knee”, and becomes a near constant above the height of about 60 m. The observed boundary-layer profiles deviate significantly from the predictions made using the power-law profile model because the power-law curve is unable to capture this rapid decrease and plateauing of the wind speed with height across the knee. It was also found

that the wind speed during the day is substantially higher than during the night, suggesting potentially higher power generation during the day. While the night-time boundary-layer profile does show an “inflection” above which the wind speed increases with height again, the substantially lower wind speed during the night meant that the night-time wind speeds remain lower than the corresponding day-time wind speeds, even at heights above the inflection, for the 200 m boundary layer that was studied.

There are two main implications for the observed shape of the wind profile. First, the power-law wind model is a poor representation of the actual boundary-layer profile for such a tropical boundary layer and should not be used for the estimation or extrapolation of wind speeds. Second, the potential returns in terms of increased power generation with the increasing height of a wind turbine beyond the height where the wind speed plateaus (i.e. about 60 m in this case) might be small. As such, there may be few justifications for building wind turbines taller than that at a tropical location such as Singapore. This is in contrast to the established thumb of rule in extratropical wind energy where wind turbines are built to immense heights because an increased power generation is always expected with increasing height. A further note was also made with regards to the generally low average wind speeds that were observed. Given that the cut-in wind speed for contemporary large wind turbines is about 3 m s^{-1} [1], new wind turbines designed specifically for low wind speed applications might be necessary to fully take advantage of the available wind resource.

The second major finding of the boundary layer study is that the proposed arctan model, based on the inverse tangent function, is able to provide a better representation of the observed wind profiles. This is because the inverse tangent function is able to better capture the rapid reduction and the plateauing characteristic of the wind speed with height and this makes the arctan model superior to the traditional power-law model for the estimation of boundary-layer wind speeds in the deep tropics.

The implication of the arctan model is that two independent parameters are required to define the wind profile, one of which is representative of the limiting wind speed found

above the knee. While it is mathematically possible to obtain estimates of both parameters by using wind speed measurements near the surface, it was also observed that the discrepancy between the observed wind profile and the model prediction is the largest near the surface. Thus, it is recommended that one of the measurements be made at heights above the knee where the wind speed is already a near constant. This approach is different from the power-law model where wind speeds at heights can be extrapolated from near-surface measurements and spells potentially more costly resource assessment requirements in the tropics, thus opening up potential opportunities for remote-sensing technologies such as the lidar.

Further up in the middle troposphere, the vector-based statistics designed in Chapter 4 using a geometric representation of the variance tensor form a set of new statistical measures, each with an intuitive physical interpretation, that can be used by future studies to describe the statistical characteristics of the horizontal wind velocity. Consider a hypothetical future where advancements in high-altitude wind power (HAWP) [2,3] have allowed the regular tapping of mid-tropospheric winds as an energy resource. The discovery that the scalar variance scales with the scalar mean of the wind speed meant that an increased variability in power generation can be expected at sites where the average wind speed is higher. Further, the discovery that the axis of the maximum variance tends to align with the direction of the prevailing wind also carries the implication that the largest variation in the wind speed is going to be in the direction of the strongest winds. This has potential impacts on the structural and fatigue design [4,5] of HAWP wind energy systems.

6.1.2 Modelling of Atmospheric Winds

In the numerical modelling of the atmosphere, the arctan boundary-layer profile model can provide a new basis for the reconstruction of the boundary-layer wind profiles that are used as boundary conditions in CFD-based wind models [6-8]. It is objectively superior compared to the power-law profile model in the deep tropics and offers some degree of mathematical simplicity over the log-law model while maintaining fair accuracy. For more advanced modelling on the larger scale or for the long term, the vector-based statistics

presented in Chapter 4 offer as a suite of intuitive diagnostics that can be used to assess model performance by comparing the model results with real-world observations.

Figure 6.1: The arctan model parameter τ in the four directional quadrants for seasons (a) AM, (b) JJAS, (c) ON, and (d) DJFM. The four quadrants are: northeast (NE 0° - 90°), southeast (SE 90° - 180°), southwest (SW 180° - 270°), and northwest (NW 270° - 360°).

In Chapter 5, it was found, using a directional analysis with four quadrants, that the wind speed (or more specifically, the parameter u_r) in Singapore is dominantly influenced by the monsoonal patterns that dominate the large-scale atmospheric flow in the region. On the other hand, the “sharpness” of the knee (controlled by the parameter τ) was suggested to be influenced by the surface roughness. If this is true, it will be an apparent contradiction to the log-law profile model where the decrease in the wind shear with height is determined

by the atmospheric stability and not the surface roughness (the surface roughness affects the wind speed without changing the shear) [9]. This is not to say that the log-law model is invalidated however. Figure 6.1 reproduces here the Figure 5.11 found in Chapter 5 where the directional results for τ were plotted. It can be seen that the value of τ for each quadrant is always the largest in either the AM or JJAS season and it is always the smallest in the DJFM season. Given that the boundary layer stability is affected by the surface temperature and that the temperature in Singapore varies seasonally (with the coolest temperature in DJFM), it can thus be concluded that the sharpness of the profile knee can indeed be affected by the boundary layer stability – just as suggested by the log-law model. As to whether the wind speed can be affected by the surface roughness as suggested by the log law, the results obtained do not support a conclusive answer as the values of u_τ are dominated by the monsoonal patterns as mentioned earlier.

6.1.3 A Bivariate Gaussian Probability Distribution Model based on the Principal Directions of the Variance

For the statistical modelling of the horizontal wind vector, the geometric approach introduced and applied on the variance tensor allows the principal directions of the variance to be uncovered. This knowledge can be useful, for example, in the construction of a bivariate probability distribution model to describe the wind velocity vector. Existing models such as the anisotropic Gaussian model [10] use a coordinate system where one axis is in the direction of the prevailing wind whereas the other is perpendicular to it. This however does not allow the full variance of the wind vector to be represented and thus also does not capture the full extent of the velocity distribution. The major principal axis of the variance does not necessarily align with the direction of the prevailing wind even though it was found that there is a tendency for it to do so.

A revised Gaussian model for the random vector $\mathbf{v}$, using the principal directions of the variance (denoted by the subscripts ‘min’ and ‘maj’) as its coordinate system, can therefore be proposed as:

$$\mathbf{v} = \begin{bmatrix} v_{\min} \\ v_{\max} \end{bmatrix} \sim \begin{bmatrix} \mathcal{N}(\overline{v}_{\min}, \lambda_{\min}) \\ \mathcal{N}(\overline{v}_{\max}, \lambda_{\max}) \end{bmatrix} \quad (6.1)$$

where $\overline{v}_{\min}$ and $\overline{v}_{\max}$ are the component means in the said principal directions and $\lambda_{\min}$ and $\lambda_{\max}$ are the corresponding eigenvalues (which are also the component variances). $\mathcal{N}$ denotes that the vector components each follow a Gaussian (normal) distribution. The proposed distribution model captures the maximum and the minimum variance of the velocity vector and allows the full velocity distribution to be represented without sacrificing the representation of the mean vector.

6.2 Uncertainty in Aircraft Flight Duration due to Cruise Winds

The time needed for an aircraft to fly from one location to another is affected by a variety of factors. One of which that the airlines and aircraft operators have no control over is the wind speed at the en-route cruise altitude(s). The variability in the wind gives rise to uncertainty in the flight times and this is an important consideration for aircraft scheduling and the timetabling of flights. It also affects the fuel consumption. By using the variance tensor method, the uncertainty in the time of flight due to variations in the wind can be quantified.

There are three speeds (or more accurately, velocities) of interest when it comes to air navigation. They are the true airspeed $\mathbf{V}_A$, the ground speed $\mathbf{V}_G$, and the wind speed $\mathbf{V}_W$, and they are related by Equation (6.2):

$$\mathbf{V}_G = \mathbf{V}_A + \mathbf{V}_W \quad (6.2)$$

The true airspeed is the speed of the aircraft through the air and it has a known magnitude determined by operational conditions. The ground speed, on the other hand, is the speed of the aircraft over the ground and is what that determines the time of flight. It is by definition

always in the direction of the flight. Lastly, the wind speed is the random horizontal wind vector.

By taking only the component of the speeds that is in the direction of the flight and noting that the component of the ground speed that is perpendicular to the direction of flight is zero, it can be obtained that:

$$V_{G,y} = V_G = \sqrt{(V_A^2 - V_{W,x}^2)} + V_{W,y} \quad (6.3)$$

where the subscript y denotes the component in the direction of the flight, the subscript x denotes the component perpendicular to it, and V_A is the (magnitude of the) true airspeed.

If one assumes that the x component of the wind is small compared to V_A , Equation (6.3) can then be simplified to Equation (6.4):

$$V_G = V_A + V_{W,y} \quad (6.4)$$

It can be noted from Equation (6.4) that the ground speed is boosted if the wind is in the direction of the flight (a tailwind) but the ground speed is reduced if the wind is against this direction (a headwind).

The mean time of flight $\bar{T}$ between two points is then determined by Equation (6.5):

$$\bar{T} = \frac{d}{V_G} = \frac{d}{V_A + \overline{V_{W,y}}} \quad (6.5)$$

where d is the distance between the two points. Note that the term $V_{W,y}$ is a random variable which makes the time of flight T also a random variable and therefore subject to uncertainty.

The variance of $V_{W,y}$ can be determined from the tensor relation given by the Equation (4.8) in Chapter 4 and is:

$$\text{var}(V_{W,y}) = \sigma^2_{W,y} = \frac{1}{2} \sigma^2_t (1 + \varepsilon \cos 2\zeta) \quad (6.6)$$

where ζ is the angle between the major principal axis of the variance and the direction of the flight. All other symbols are as per defined in Chapter 4. By error propagation, the standard deviation in the time of flight σ_T can then be derived and it is given by Equation (6.7):

$$\sigma_T = \bar{T} \frac{\sigma_{W,y}}{V_{W,y}}. \quad (6.7)$$

The formulation described above is simple and does not take into account the numerous considerations that affect real-world flight times [11-12]. Nevertheless, it can still serve as a tool by which the uncertainty in the flight duration due to the wind variability can be quantified for applications such as in the numerical optimisation of flight paths [13,14].

6.3 Conclusion

In this dissertation, two exploratory studies on the statistical characterisation of atmospheric winds in the deep tropics were presented. The first, reported in Chapter 4, employed an alternative vector-tensor approach to investigate the variance in the upper-air winds over a near-equator monsoon region by the use of radiosonde data. This alternative approach allows the directional characteristics of the variance to be elucidated through the analysis of the variance tensor and thus allows a more holistic characterisation of the observed winds. A suite of parameters, each with an intuitive interpretation on the directional characteristic of the variance, was then designed using a geometric representation of the variance tensor. Through the analysis that was presented in said

chapter, it was found that salient relations exist between the variance and the mean wind as well as between the variance and the pressure level for within the middle troposphere.

In the second study that was reported in Chapter 5, wind data recorded by a lidar that was sited on an offshore island of Singapore were used to investigate the characteristics of the tropical boundary-layer wind profile. It was found that the traditional power-law wind profile model is unable to provide a good representation of the observed wind profiles because the shape of the power-law curve cannot adequately capture some key features in the shape of the observed profiles. Alternative wind profile models were then proposed and it was determined that the arctan model, which is based on the inverse tangent function, is able to provide a much improved representation. The diurnal and seasonal variations in the parameters of the proposed models were also investigated. Finally, some practical implications of the findings relating to the optimal height of wind turbines and resource assessment practices in the tropics were then discussed.

The dissertation provides new insights into the characteristics of atmospheric winds in the deep tropics. The use of the variance tensor methodology, accompanied by the suite of intuitive parameters designed from the geometric representation of the tensor, represents an unconventional way of looking at the wind variance and this allows for potentially new trends in the wind characteristics to be uncovered, as was demonstrated in Chapter 4 with the near-equator monsoon region. The suite of designed parameters can also be used for similar future studies. At the boundary layer, the discovery of the atypical shape of the boundary-layer wind profile, which was not well represented by the traditional power-law wind model, calls into question the validity of existing wind profile models, many of which were developed using empirical evidences from extratropical boundary layers, when they are applied in the deep tropics. The proposal of the arctan model can also serve as a basis for the development of an even more accurate model for the tropical boundary-layer wind profile. Through the work carried out in this dissertation, new characteristics of the atmospheric winds not previously documented for a deep-tropical atmosphere were discovered and it was shown, in particular by the results presented in Chapter 5, that these characteristics can deviate considerably from its extratropical equivalent. Therefore,

appropriate care is recommended when attempting to apply established models and rules of thumb in the estimation and prediction of atmospheric winds in the deep tropics.

6.4 Unanswered Questions and Suggestions for Future Work

The work presented in this dissertation is by no means an extensive treatment on tropical wind characteristics as such a treatment would be beyond what could be realistically accomplished within a single dissertation. The results presented, however, do uncover new questions regarding the behaviour of winds in a tropical atmosphere. A major gap present in this dissertation is the lack of concrete physical explanations for the observed characteristics of the atmospheric winds. While some informed speculations were made, the main focus of the dissertation was on statistical characterisation and thus the physical mechanisms driving the observed wind characteristics were not thoroughly investigated. The following is a list of unanswered questions, and suggested approaches to address them if possible, that arose from the studies performed.

From the radiosonde (variance tensor) study in Chapter 4:

- What is the mechanism causing the preferred axis of the variance to align with the direction of the mean wind? This is an important result as it allows the directionality of the variance in the middle troposphere to be inferred from the mean wind. The alignment can be the result of particular atmospheric phenomena found in the region of the atmosphere being studied or it could also be a fundamental consequence of the fluid dynamics governing free atmospheric flow.
- What are the reasons causing the variance characteristics in the lower and the upper troposphere to not exhibit the trends that were uncovered for the middle troposphere?
- Do the middle tropospheric winds in the extratropics also exhibit the same relations between the variance and the mean wind? It would be interesting to know if the trends

uncovered apply to other parts of the world or if they are something that is unique to the monsoonal deep tropics.

From the boundary-layer study in Chapter 5:

- Is there another function other than the inverse tangent function that can even better represent the boundary-layer wind profile while retaining the mathematical simplicity? It was noted that the arctan model predictions are the poorest within the knee of the profile. As such, a better model for predicting the wind speeds within the knee is desirable if the estimation of near-surface wind speeds is sought.
- What is the source and physical significance of the “inflection” that was observed in the night-time (‘am’) wind profiles? It is clear that the inflection is the result of the diurnal differences in the structure of the observed boundary layer but its presence has, for many parts, been neglected in the study. Thus, this begs the questions of what are the physical effects that caused it and how can the arctan wind profile model be modified to accommodate its presence.
- Is the log-law wind profile model able to capture the shape of the observed boundary-layer wind profiles? The log-law wind model is an alternative to the power-law wind profile model that explicitly takes into account the effects of the surface roughness and the atmospheric stability on the profile shape. Its ability to reproduce the observed boundary-layer wind profiles is thus also of interest. Such an investigation would additionally require data for the momentum and temperature fluxes (or the temperature gradient in lieu of the temperature flux [15]) to be recorded in concurrence with the wind speed and the wind direction so that the relevant terms in the log-law function can be calculated. These additional data were however not available for the study presented here.
- Do the boundary-layer wind profiles of other locations in the deep tropics also demonstrate the same characteristics as those that were observed in Singapore? As the

study only represented one single location in the deep tropics (an offshore island of Singapore), similar studies on the boundary layers of other deep-tropical locations are needed in order to ascertain if the observed wind profiles are characteristic of all tropical boundary-layer wind profiles, just applicable for the Maritime Continent, or if they are specifically unique to Singapore.

- Lastly, the observed seasonal and diurnal variations of the wind speed can also be used to investigate further questions regarding the operational aspects of using wind power in Singapore, such as:
 - 1) What is the error in the power prediction if the power-law wind profile had been used to estimate the wind speed? and
 - 2) How does the seasonal and diurnal intermittency [16-18] of power generation, caused by the said variations in the wind speed, impact the practical deployment of local wind energy?

Such follow-on investigations interface the results of the wind speed study with actual power generation and would require the analysis of further information such as the wind turbine performance (through the selection of a wind turbine model) as well as data on the seasonal and diurnal patterns of the domestic power consumption.

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APPENDIX A

The objective of the linear least-squares regression is to minimise the weighted error function given in Equation (3.18). This is achieved by differentiating the function with respect to the coefficients to be determined and then equating the results to zero.

$$\frac{\partial(r^2)}{\partial b} = 2 \sum_{i=1}^n w_i (y_i - bx_i - a)(-x_i) = 0 \quad (\text{A.1})$$

$$\frac{\partial(r^2)}{\partial a} = 2 \sum_{i=1}^n w_i (y_i - bx_i - a)(-1) = 0 \quad (\text{A.2})$$

Solving the Equations (A.1) and (A.2) simultaneously, the expressions for the least-squares estimators of coefficients b and a are obtained in Equations (3.19) and (3.20) respectively.

APPENDIX B

The Equations (3.19) and (3.20) can be rewritten in the form:

$$\hat{b} = \sum_{j=1}^n \frac{w_j \sum_{i=1}^n w_i x_i - w_j x_j \sum_{i=1}^n w_i}{(\sum_{i=1}^n w_i x_i)^2 - \sum_{i=1}^n w_i \sum_{i=1}^n w_i x_i^2} y_j \quad (\text{B.1})$$

$$\hat{a} = \sum_{j=1}^n \frac{w_j x_j \sum_{i=1}^n w_i x_i - w_j \sum_{i=1}^n w_i x_i^2}{(\sum_{i=1}^n w_i x_i)^2 - \sum_{i=1}^n w_i \sum_{i=1}^n w_i x_i^2} y_j. \quad (\text{B.2})$$

The standard error of the j -th measurement y_j can also be inferred from the MSE and is given by:

$$\sigma_{y_j} = \frac{\sigma_y}{\sqrt{w_j}}. \quad (\text{B. 3})$$

By propagating the error of y_j in the sums in Equations (B.1) and (B.2), the standard errors of the regression coefficients, σ_b and σ_a , are obtained in Equations (3.23) and (3.24) respectively.

APPENDIX C

Consider a random wind vector $\mathbf{v}$ with mean component $\bar{\mathbf{v}}$ and fluctuating component $\mathbf{v}'$. For simplicity of notation, the y -direction is defined to be in the direction of $\bar{\mathbf{v}}$. Thus, the components of $\bar{\mathbf{v}}$ in the x and y directions are 0 and V_t respectively. The x and y components of $\mathbf{v}'$ are denoted as v_x' and v_y' . By definition:

$$|\mathbf{v}|^2 = (0 + v_x')^2 + (V_t + v_y')^2 = V_t^2 + v_x'^2 + v_y'^2 + 2V_tv_y' \quad (\text{C. 1})$$

Therefore,

$$\overline{|\mathbf{v}|^2} = V_t^2 + \overline{v_x'^2} + \overline{v_y'^2} + 2V_t\overline{v_y'} = V_t^2 + \sigma_t^2 \quad (\text{C. 2})$$

using Equation (4.6) and noting that $\overline{v_y'} = 0$ as the mean of fluctuating components is by definition zero.

Also, consider that the magnitude of $\mathbf{v}$ is the scalar wind speed. One can write:

$$|\mathbf{v}| = V_s + |\mathbf{v}'| \quad (\text{C. 3})$$

where $|\mathbf{v}'|$ denotes the fluctuating component of the wind speed. Therefore,

$$\overline{|\mathbf{v}|^2} = V_s^2 + \overline{|\mathbf{v}'|^2} + 2V_s\overline{|\mathbf{v}'|} = V_s^2 + \sigma_s^2. \quad (\text{C. 4})$$

Combining Equations (C.2) and (C.4), the identity given by Equation (4.27) is thus proven.

APPENDIX D

For small variances, applying Taylor series expansion on Equation (C.1) up to and including the second order terms, the following is obtained:

$$|\mathbf{v}| \approx V_t \left(1 + \frac{v_y'}{V_t} + \frac{v_x'^2}{2V_t^2} \right)$$

Therefore, using the definition of V_s :

$$V_s = \overline{|\mathbf{v}|} \approx V_t + \frac{\overline{v_x'^2}}{2V_t}. \quad (\text{D.1})$$

Also, from Equation (4.8):

$$\overline{v_x'^2} = \text{var}(v_x) = \frac{1}{2} \sigma_t^2 (1 - \varepsilon \cos 2\theta)$$

Note that from Table 4.4, σ_t^2 and ε are independent of V_t and that θ approaches 0 as V_t increases. It can therefore be deduced:

$$\frac{d\overline{v_x'^2}}{dV_t} < 0. \quad (\text{D.2})$$

By differentiating Equation (D.1) with respect to V_t , substituting for $\overline{v_x'^2}$ with the help of Equation (D.1) again, and applying the inequality from Equation (D.2), the following can be obtained:

$$\frac{dV_s}{dV_t} \approx 1 - \frac{V_s - V_t}{V_t} + \frac{1}{2V_t} \frac{d\overline{v_x'^2}}{dV_t} < 2 - \frac{V_s}{V_t} \quad (\text{D.3})$$

This implies that:

$$\frac{d(V_s^2)}{d(V_t^2)} = \frac{V_s}{V_t} \frac{dV_s}{dV_t} < \frac{V_s}{V_t} \left(2 - \frac{V_s}{V_t} \right) = 1. \quad (\text{D.4})$$

The last step used the fact that the quadratic function of V_s/V_t has a maximum value of 1.

Therefore, for small variances, it is shown here that $d(V_s^2)/d(V_t^2) < 1$.

APPENDIX E

ϵ vs. V_t

η vs. V_t