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2019

Chi, Y., Liu, L., Guo, J., Song, G., Yuen, C., & Guan, Y. L. (2019). Variable-rate coding with constant BER for NOMA via multilevel IRA coding. *IEEE Transactions on Vehicular Technology*, 68(5), 5149-5153. doi:10.1109/TVT.2019.2904625

<https://hdl.handle.net/10356/139384>

<https://doi.org/10.1109/TVT.2019.2904625>

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Variable-Rate Coding with Constant BER for NOMA via Multilevel IRA Coding

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Abstract—A variable-rate coding scheme is proposed for an uplink NOMA system in this paper. Specifically, each user employs a multilevel irregular repeat-accumulate (IRA) coding scheme, which consists of multiple common component IRA encoders. The variable-rate coding scheme is realized by flexibly adjusting the number of component IRA encoders allocated to each user. We design this common component IRA encoder based on extrinsic information transfer analysis to achieve an overall good multi-user decoding performance. The advantages of our proposed scheme are: 1) simple implementation of rate variation, 2) flexible design of multilevel IRA coding, i.e., the well-designed component IRA encoders does not need to be re-designed when any user changes its rate, 3) fair decoding convergence of each user, 4) capacity-approaching performance.

Index Terms—NOMA, variable-rate, multilevel coding

I. INTRODUCTION

NOMA is a promising air technique to support massive data transmissions in the future wireless communication [1]–[4]. In many existing works [3]–[11], authors assume that all users have identical transmission rates to simplify the system designs and analyses. However, in practice, different users usually have different kinds of communication contents, such as text, voice, and video [12], [13], which require different transmission rates. Therefore, variable-rate code design is critical for practical NOMA systems.

From information theory [14], each point in the capacity region of an uplink NOMA system can be achieved by using successive interference cancellation (SIC) and time sharing. However, SIC relies on unequal power allocation, which is quite sensitive to availability and accuracy of channel state information (CSI) at transmitter side. Considering channel perturbation and limited feedback in real communication systems, it is difficult to obtain the accurate CSI, which significantly deteriorates the decoding performance. To deal with this problem, works [15]–[18] focus on rate-allocation schemes with joint multi-user decoding.

This work has been supported by the National Natural Science Foundation of China (No. 61802296, 61750110529), China Postdoctoral Science Foundation Grant (No. 2017M620438), and also supported by the ISN State Key Laboratory. The associate editor coordinating the review of this paper and approving it for publication was Prof. Z. Ding. (Corresponding author: Guanghui Song.)

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For NOMA systems with few users, work [15] considered two-user rate allocation problem with turbo encoding and a theoretical achievable rate region is derived. For up to 12-user NOMA systems [16], [17], users are divided into groups and an individual channel code is designed for each group based on a multi-dimensional extrinsic information transfer (EXIT) analysis. However, for systems with large number of users and various rate requirements, the rate-allocation schemes in [15]–[17] are too complex to realize. Work [18] considered rate allocation for up to 64 users with the assumption of each user employing a simple repetition code. Note that works [16]–[18] only considered single receive antenna systems without channel code optimization. To the best of our knowledge, relatively few works considered variable-rate code optimization for uplink NOMA systems.

In this paper, we propose a variable-rate coding scheme for the uplink NOMA system. In the transmitters, each user employs a multilevel irregular-repeat accumulate (IRA) code, which consists of multiple common component IRA encoders. The rate variation is achieved by flexibly adjusting the number of component IRA encoders allocated for users. We design this common component IRA encoder based on EXIT analysis to achieve an overall good multi-user decoding performance. The major contributions of this paper are summarized as follows.

- 1) By designing the common component IRA encoder with the goal of maximizing the sum transmission rate, the proposed variable-rate scheme is easily scalable, i.e., the different transmission rates of each user can be achieved by adjusting the number of allocated component IRA encoders.
- 2) The well-designed component IRA encoder does not need to be re-designed when any user changes his code rate.
- 3) We show that the proposed variable-rate coding scheme can enable fair decoding convergence of each user and achieve capacity-approaching performance.

Different from previous rate-allocation schemes [15]–[17], the novelty of the proposed scheme is that each user adopts completely the same component encoders rather than different encoders in [15]–[17]. The proposed scheme just needs to allocate as many component encoders as necessary for each user, depending on its rate requirement, which makes the rate allocation more flexible.

II. SYSTEM MODEL

Figure 1 illustrates an uplink variable-rate NOMA system, including K single-antenna users and a base station (BS) equipped with N_r receive antennas. In the K -user transmitters,

Fig. 1. System model of a K -user variable-rate NOMA system. ENC/DEC: component encoder/decoder. π/π^{-1} : interleaver/de-interleaver. S/P and P/S: serial-to-parallel and parallel-to-serial converters. SCM: superposition coded modulator. ESE: elementary signal estimator.

information sequence $\mathbf{u}_k$ of user k , $k = 1, \dots, K$, is encoded by an M_k -level encoder. Specifically, $\mathbf{u}_k$ is transformed into M_k parallel subsequences through a serial-to-parallel converter (S/P), which are encoded and permuted by a component encoder (ENC) and an interleaver $\pi_{k,m}$, $m = 1, \dots, M_k$, respectively. The interleaver output $c_k^1, \dots, c_k^{M_k}$ are superposed by a superposition coded modulator (SCM) to generate symbol sequence $\mathbf{x}_k = [x_{k,1}, \dots, x_{k,N}]^T$, which is sent to the channel via one transmit antenna. When each user employs a binary phase-shift keying (BPSK) modulation, we have $x_{k,t} = \sum_{m=1}^{M_k} (1 - 2c_{k,t}^m) = \sum_{i=1}^{M_k} d_{k,t}^m$, $t = 1, \dots, N$, where $d_{k,t}^m = 1 - 2c_{k,t}^m$ is a BPSK symbol.

Since in above transmission strategy, each user employs a superposition coded modulator with a single transmit antenna, we call it an SCM single-antenna strategy. An alternation is to transmit user- k 's message via multiple antennas, that is, each component encoder or several component encoders in group is associated with an independent transmit antenna. The signals of different component encoders associated with the same antenna are superimposed by SCM while the signals sent from different antennas are superimposed in the air. We call it multi-antenna transmission strategy. The later economizes on SCM but use more antennas. We will show that in fast fading channel, the two strategies have almost the same decoding performance.

Let $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_K]^T$ be the K users' signal matrix with size $K \times N$. Assuming that all the users' signals arrive at BS synchronously, the received signal vector $\mathbf{y}_j = (y_{j,1}, y_{j,2}, \dots, y_{j,N})$ at the j -th receive antenna is written by

$$\mathbf{y}_j = \mathbf{h}_j \mathbf{X} + \mathbf{z}_j \quad (1)$$

where $\mathbf{h}_j = [h_{1,j}, h_{2,j}, \dots, h_{K,j}]$ is the channel gain; $\mathbf{z}_j \sim \mathcal{N}(0, \sigma_n^2 \mathbf{I}_{N_r})$ is an additive Gaussian noise vector; and $\mathbf{I}_{N_r}$ is an $N_r \times N_r$ identity matrix. Here, we assume that $\mathbf{h}_j$ is available at BS but unavailable at the transmitters and consider BPSK modulation with real channel gain and noise. For complex-modulated system with a complex channel gain, Eq. (1) can be rewritten in a complex form accordingly.

Since all component encoders are the same and the code rates are assumed as R , transmission rate of user k is $R_k = M_k R$ and sum rate is $R_{\text{sum}} = R \sum_{k=1}^K M_k = MR$, where $M = \sum_{k=1}^K M_k$ is the total number of component encoders.

Note that the multilevel coding is first used here to implement variable-rate NOMA transmission.

III. ITERATIVE DECODING AND EXIT ANALYSIS

As shown in Fig. 1, the iterative receiver at BS consists of an elementary signal estimation (ESE) [5]–[7] and a bank of component decoders. The whole decoding is realized based on the principle of belief-propagation (BP) algorithm. Specifically, based on received signal $\mathbf{y}_j$, $j = 1, 2, \dots, N_r$, ESE calculates a soft estimation, which is a log likelihood ratio (LLR), for each coded bit of each component code of each user. The LLR is used as *a priori* message for DEC. After DEC processing, a more reliable message is fed back to ESE for the next round of processing. After a preset maximum number of iteration, a hard decision is made at the DEC to reformulate each user's information.

Now we present the processing of ESE. The input of ESE $\tilde{\mathbf{l}}_{k,t}^{m,\text{ESE}} = (\tilde{l}_{k,1}^{m,\text{ESE}}, \dots, \tilde{l}_{k,N}^{m,\text{ESE}})$ is *a priori* LLR of $(d_{k,1}^{m,\text{ESE}}, \dots, d_{k,N}^{m,\text{ESE}})$ and its output is a refined extrinsic LLR $\mathbf{l}_{k,t}^{m,\text{ESE}} = (l_{k,1}^{m,\text{ESE}}, \dots, l_{k,N}^{m,\text{ESE}})$. We consider the estimation for $d_{k,t}^m$, $k = 1, \dots, K$, $i = 1, \dots, M_k$, $t = 1, \dots, N$. We aim to calculate the extrinsic LLR $l_{k,t}^m$ of $d_{k,t}^m$ based on *a priori* LLR $\tilde{l}_{k,t}^m$. Since $d_{k,t}^m$ is received at all the N_r antennas, based on each of the receive signal $y_{j,t}$, $j = 1, 2, \dots, N_r$, we can calculate an LLR of $d_{k,t}^m$. These LLRs from N_r antenna are combined to form the final LLR of $d_{k,t}^m$. From Eq. (1), we have

$$\begin{aligned} y_{j,t} &= \sum_{k=1}^K h_{k,j} x_{k,t} + z_{j,t} = \sum_{k=1}^K h_{k,j} \sum_{m=1}^{M_k} d_{k,t}^m + z_{j,t} \\ &= h_{k,j} d_{k,t}^m + \eta_{k,t,j}^m \end{aligned} \quad (2)$$

where $\eta_{k,t,j}^m = h_{k,j} \sum_{m' \neq m} d_{k,t}^{m'} + \sum_{k' \neq k} h_{k',j} \sum_{m=1}^{M_{k'}} d_{k',t}^{m'} + z_{j,t}$ is the interference of $d_{k,t}^m$, where the first term is caused by SCM and the second term is caused by NOMA transmission.

Since $\eta_{k,t,j}^m$ contains the summation of multiple independent terms, as the user number K is large, from the central limit theorem we approximately regard it as Gaussian with $\eta_{k,t,j}^m \sim \mathcal{N}(\mathbb{E}[\eta_{k,t,j}^m], \text{Var}[\eta_{k,t,j}^m])$ [5]–[7]. The receive signal is hence characterised as a conditional Gaussian distribution with

$$\Pr(y_{j,t} | d_{k,t}^m = \pm 1) = C \exp \left(-\frac{(y_{j,t} \mp h_{k,j} - \mathbb{E}[\eta_{k,t,j}^m])^2}{2\text{Var}[\eta_{k,t,j}^m]} \right)$$

where C is a constant. The LLR based on the received signal of the j -th antenna is calculated as

$$l_{k,t,j}^{m,\text{ESE}} = \log \frac{\Pr(y_{j,t} | d_{k,t}^m = 1)}{\Pr(y_{j,t} | d_{k,t}^m = -1)} = \frac{2h_{k,j}(y_{j,t} - \mathbb{E}[\eta_{k,t,j}^m])}{\text{Var}[\eta_{k,t,j}^m]}$$

where $\mathbb{E}[\eta_{k,t,j}^m]$ and $\text{Var}[\eta_{k,t,j}^m]$ are calculated as

$$\begin{aligned} \mathbb{E}[\eta_{k,t,j}^m] &= h_{k,j} \sum_{m' \neq m} \mathbb{E}[d_{k,t}^{m'}] + \sum_{k' \neq k} h_{k',j} \sum_{m=1}^{M_{k'}} \mathbb{E}[d_{k',t}^m] \\ \text{Var}[\eta_{k,t,j}^m] &= h_{k,j}^2 \sum_{m' \neq m} \text{Var}[d_{k,t}^{m'}] + \sum_{k' \neq k} h_{k',j}^2 \sum_{m=1}^{M_{k'}} \text{Var}[d_{k',t}^m] \end{aligned}$$

where $\mathbb{E}[d_{k,t}^m] = \tanh(\frac{\tilde{l}_{k,t,j}^{m,\text{ESE}}}{2})$, $\text{Var}[d_{k,t}^m] = 1 - \tanh^2(\frac{\tilde{l}_{k,t,j}^{m,\text{ESE}}}{2})$ are calculated from *a priori* LLR [5]–[7].

Fig. 2. EXIT curves of ESE over variable-rate NOMA with $M = 48$, $N_r = 8$, and $\sigma_n^2 = \{5, 10, 15, 20, 25, 30\}$.

By combining the LLRs from N_r antennas, we obtain the final extrinsic LLR of $d_{k,t}^m$ output from ESE to DEC

$$l_{k,t}^{m,\text{ESE}} = \sum_{j=1}^{N_r} l_{k,t,j}^{m,\text{ESE}}. \quad (3)$$

Although the computation complexity of ESE is very low, near optimal performance is achieved [5]–[7]. The joint multi-user decoding is used effectively in code-domain NOMA [3]–[7], which does not rely on different power allocation for each user as in power-domain NOMA [19]–[21]. Here we only considered real signal detection, the principle can be extended to complex signal detection with complex channel gain and noise straightforwardly [7] [8].

To facilitate the code design, we should analyze the EXIT property of ESE. Since all component encoding and decoding processes are similar, this EXIT can be characterise as a univariate function, denoted by $I_E = T_{\text{ESE}}(I_A, M, N_r, \sigma_n^2)$, where $I_A = I(d_{k,t}^{m,\text{ESE}}; \tilde{l}_{k,t}^{m,\text{ESE}})$ and $I_E = I(d_{k,t}^{m,\text{ESE}}; l_{k,t}^{m,\text{ESE}})$ are the mutual information between LLR of ESE input and output. Note that this EXIT function does not rely on the number of component encoders (rate) of each user, but is only related to the total number of component encoders M , which makes our code design very simple. Since similar analysis, for single receive antenna, is given by [5]–[7], due to space limitation, we only provide some numerical results in Fig. 2. Fig. 2 illustrates that the EXIT curves of ESE over variable-rate NOMA with $M = 48$, $N_r = 8$, and $\sigma_n^2 = \{5, 10, 15, 20, 25, 30\}$ where fast Rayleigh fading channel is considered. According to EXIT matching theory [3], the objective of code design is to find a code whose EXIT matches with that of ESE. Since the EXIT curve for the point-to-point channel is a horizontal curve, codes designed for the proposed system should be different from that of point-to-point channel.

IV. COMPONENT CODE DESIGN

In this section, we design the common component code for all users independent of their individual rate requirements.

As referred above, the objective of code design is to find a code with the maximum rate and in the meanwhile its EXIT perfectly matches with that of ESE. The use of a common

component encoder for each user significantly simplifies our code design

Here, we design a kind of repetition-aided irregular repeat-accumulate (IRA) code extended from [7], [8]. It is a parallel concatenation between a rate- $1/q$ repetition code and a non-systematic IRA code. The IRA code is specified by its variable node degree distributions $\lambda = \{\lambda_i, i = 2, 3, \dots, d_{\max}\}$ and combiner rate α , where λ_i is the fraction of number of edges that connects to degree- i variable nodes and α is associated to the degree of check nodes in the factor graph of an IRA code [7], [8], [22]. The rate of nonsystematic IRA code is written as $\alpha \sum_i \lambda_i / i$ [7], [8], [22]. The rate of the parallel concatenation between a nonsystematic IRA code and a rate $1/q$ code is hence $\frac{\alpha \sum_i \lambda_i / i}{q \alpha \sum_i \lambda_i / i + 1}$. Let $I_E = T_{\text{IRA}}(I_A, q, \lambda, \alpha)$ be the EXIT function of the employed component code, which can be derived based on analysis in [7], [8]. The code optimization is realized by solving:

$$\begin{aligned} \text{Max} \quad & R_{\text{sum}} = \frac{M \alpha \sum_i \lambda_i / i}{q \alpha \sum_i \lambda_i / i + 1} \\ \text{s. t.} \quad & \sum_i \lambda_i = 1, \quad 0 \leq \lambda_i \leq 1, \quad \alpha \geq 1, \\ & T_{\text{IRA}}(T_{\text{ESE}}(I, M, N_r, \sigma_n^2), q, \lambda, \alpha) > I, \quad 0 \leq I < 1 \quad (4) \end{aligned}$$

where Eq. (4) guarantees the mutual information increasing for each decoding iteration, which is the success condition of iterative decoding when the iteration number approaches infinity. For fixed parameters of M, N_r, σ_n^2 , this optimization problem is solved by a standard differential evolution algorithm [23] to find the optimal q, α , and λ .

As a result, M well-designed component IRA encoders can be allocated to K users depending on their rate requirements, where $M > K$. The total number of optimal rate pairs allocated to users is $\binom{M-1}{K-1}$, which can satisfy various rate requirements of all users.

For example, we consider a special case of NOMA, i.e., 2-user AWGN NOMA system, to show the maximum achievable sum rate pairs and the capacity region. We fix number of receive antenna $N_r=1$, total number of component code $M=10$, noise variance $\sigma_n^2 = 6.55$, and run the code optimization procedure to obtain a rate-0.1 IRA code with parameters in Table I. The system sum rate is $R_{\text{sum}} = MR = 1$. Note that the gap between decoding threshold, require $\text{SNR} = \frac{M}{\sigma_n^2}$ for successful decoding, and the corresponding two-user Shannon limit (S.L.) is only 0.08 dB. The rate variation is realized by flexibly adjusting the values of M_1 and M_2 , i.e., the number of component IRA encoders allocated to the two users. As shown in Fig. 3, there are $\binom{10-1}{2-1} = 9$ optimal rate pairs available which are near the boundary of the two-user capacity region.

TABLE I
OPTIMIZED COMPONENT IRA CODE FOR A 2-USER AWGN NOMA SYSTEM ($N_r = 1$), WHERE THE COMPONENT CODE RATE $R = 0.1$ AND THE NUMBER OF COMPONENT ENCODERS IS $M = 10$.

q	α	λ_2	λ_3	λ_{30}	SNR_{dB}	S.L.(dB)
5	1	0.16229	0.300876	0.536834	4.85	4.77

Fig. 3. Rate allocation for 2-user NOMA system, where $N_r = 1$, $M = M_1 + M_2 = 10$, $R = 0.1$, and $R_{\text{sum}} = 1$, with near capacity boundary performance.

Fig. 4. BER curves of different rate allocations $(M_1, M_2) = \{(2, 8), (4, 6)\}$ for a 2-user NOMA over AWGN channel, where the rate-0.1 IRA code with parameters in Table I is employed as the component code.

V. NUMERICAL RESULTS

The above EXIT analysis and code optimization are based on the assumptions of infinite code length and iterations. In this section, we provide the numerical results of finite length simulations to verify the validation of the proposed system. In all of our simulations, interleavers employed in the multilevel encoder are randomly generated. The information lengths input to each component encoder is fixed as 4096 for two-user NOMA and 2048 for multiple-user NOMA. The maximum iteration number for decoding is 250.

We first consider 2-user AWGN NOMA system with com-

Fig. 5. BERs comparisons between rate allocations of the proposed coding scheme and repetition coding scheme in [18] over 32-user (left figure) and 40-user (right figure) NOMA in fast Rayleigh fading channel, where $N_r = 16$, the parameters of rate-0.05 IRA component code is given in Table II, and the rate allocation profile is given in Table III.

ponent code designed in Table I. We simulated the rate pairs of $(M_1, M_2) = \{(2, 8), (4, 6)\}$ in Fig. 4. The gaps between the BER curves at 10^{-4} of two users and the corresponding S.L. are about 1.74 dB for both cases. Moreover, the BER curve of user 1 overlaps that of user 2, which verifies the performance fairness for both users.

To validate the generalization of the proposed scheme, we consider the rate variations for 32-user and 40-user NOMA systems over a complex fast Rayleigh fading channel, where the number of receive antennas is $N_r = 16$. QPSK modulation is employed by each user and the elements of $\mathbf{h}_j, j = 1, 2, \dots, N_r$, obey i.i.d. complex Gaussian $\mathcal{CN}(0, 1)$. In order to compare with the repetition coding scheme in [18], we set M as 55 in 32-user case and 60 in 40-user case. Since the total number of component encoders are similar, we design a rate-0.05 component IRA code for the case of 32 users with $M = 55$ and we also apply this code to the scenario of 40 users with $M = 60$. By running the code optimization procedure we obtain the parameters in Table II. Similar as in [18], we divide the users into several groups, where the users in each group possess the same code rate. According to [18], 32 users are divided into 3 groups with 11, 1, 20 users in each group and 40 users are divided into 4 groups with 6, 1, 4, 29 users in each group. The rate allocation profiles are given in Table III, which shows that the code rate of each group as well as the

TABLE II
OPTIMIZED COMPONENT IRA CODE FOR A 32-USER NOMA SYSTEM, WHERE THE COMPONENT CODE RATE $R = 0.05$ AND THE NUMBER OF COMPONENT ENCODERS IS $M = 55$.

q	α	λ_3	λ_{10}	λ_{30}	λ_{80}	λ_{100}	λ_{500}	SNR _{dB}	S.L.(dB)
2	2	0.047452	0.03851	0.128527	0.112509	0.134463	0.538539	-5.55	-5.47

TABLE III
COMPARISONS OF THE PROPOSED MULTILEVEL IRA CODING SCHEME WITH REPETITION CODING SCHEME [18] OVER 32 AND 40-USER NOMA.

K	Coding Scheme	R_{sum}	User no. per group / User code rate / no. of allocated IRA encoders or repetition length [18]
32	proposed multilevel IRA coding	5.5	11/0.3/3, 1/0.2/2, 20/0.1/1
	repetition coding [18]	4	11/0.25/8, 1/0.166/12, 20/0.054/37
40	proposed multilevel IRA coding	6	6/0.4/4, 1/0.3/3, 4/0.1/1, 29/0.1/1
	repetition coding [18]	5	6/0.4/5, 1/0.28/7, 4/0.072/28, 29/0.069/29

Fig. 6. BERs comparisons between SCM single-antenna, dual-antenna, and multi-antenna transmission strategies over 32-user (left figure) and 40-user (right figure) NOMA over fast Rayleigh fading channel, where the dual-antenna transmission strategy denotes that each user employs 2 transmit antennas, the multi-antenna transmission strategy denotes that the number of transmit antennas of each user equals to the number of allocated component encoders in Table III, and the number of receive antenna is $N_r = 16$. The employed IRA component code is given in Table II.

sum rate of our scheme is higher than that of the repetition coding scheme [18].

Fig. 5 shows that the proposed scheme has 4 ~ 4.5 dB gain over the best performance among users in [18]. Note that the proposed scheme achieves the constant BER performance for both users regardless the rate allocation profile, while in [18], the difference between the best users and the worst users can be more than 7 dB. Moreover, our coding scheme achieves a near S. L. performance with a gap of only 1.47 ~ 1.57 dB.

Note that the SCM single-antenna transmission strategy should have similar decoding performance as the multi-antenna transmission strategy over fast Rayleigh fading channel. In the multi-antenna case, several component encoders in group is associated with an independent transmit antenna, and the number of required antennas depends on our grouping profile of these component encoders. Here we consider an extreme case of each component encoder being associated with an independent transmit antenna. Thus, the number of transmit antennas of each user should equal to the number of component encoders. We compare these two transmission strategies over 32 and 40-user NOMA with the component code given in Table II and the rate allocation given in Table III. Fig. 6 shows that the multi-antenna transmission strategy has about 0.5 dB performance gain over the single-antenna strategy. The gain is due to that each of its component codewords in multi-antenna strategy experiences an independent channel fading which leads to larger diversity. Moreover, we consider the more practical cases that each user has 2 transmit antennas and SCM is performed at each antenna, i.e., dual-antenna transmission strategy. Fig. 6 shows that the performances of dual-antenna transmission are very close to those of multi-antenna transmission, which illustrates practicability of the proposed scheme.

VI. CONCLUSION

We proposed a very efficient variable-rate multi-level IRA coding scheme for the uplink NOMA. Since the component IRA encoder was designed independent of any user's rate requirement, the rate variation could be realized flexibly by

allocating different number of component IRA encoders to users. Numerical results verified that the proposed multilevel IRA coding scheme can also enable fair decoding performance for all users and achieve capacity-approaching performance.

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