Manipulating Cherenkov radiation by artificial structures

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Manipulating Cherenkov Radiation
By
Artificial Structures

HU HAO

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2019

Statement of Originality

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Authorship Attribution Statement

This thesis contains material from 1 paper from papers accepted at conferences in which I am listed as an author.

Chapter 4 is published as H. Hu, X. Lin, D. Liu, P. Genevet, B. Zhang, and Y. Luo, "Probe the ultimate nonlocal limit of 'threshold-free' Cherenkov radiation," in *Conference on Lasers and Electro-Optics (CLEO)*, FTh4M.8 (2019)

The contributions of the co-authors are as follows:

- Prof. Luo Yu provided the initial project direction and edited the manuscript drafts.
- I prepared the manuscript drts. The manuscript was revised by Prof. Luo Yu, Prof. Zhang Baile, Prof. Patrice Genevet and Mr. Liu Dongjue.
- I completed derivations of expressions, calculations of physical quantities and productions of figures, following the suggestions by Dr. Lin Xiao and Prof. Luo Yu.

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Summary

Since its first experimental observation by P. A. Cherenkov in 1934, Cherenkov radiation has attracted widespread attention in both the academia and the industry in the past decades. It is well-known that Cherenkov radiation arises when a charged particle moves at a velocity exceeding the velocity threshold, i.e., the phase velocity of light in the medium. The radiation direction of the moving charged particle is intimately relevant to the particle velocity. Owing to these properties, Cherenkov radiation is widely applied in the fields of electron microscopy, medical therapy and astronomy telescope. Even with such great achievement, Cherenkov radiation still has a broad space for the development in the future. Because electromagnetic features of Cherenkov radiation are strongly dependent on the surrounding environment, replacing natural materials by artificial structures provides enormous freedoms to manipulate Cherenkov radiation and promote its applications.

Photonic crystals and metamaterials are two main classes of artificial structures. Photonic crystals are periodically structured media, whose structural dimension is proportional to the wavelength of light. The periodicity of photonic crystals is electromagnetic analogue to the atomic lattice of natural crystals. This periodic dielectric contrast experienced by the propagating light produces the photonic bandgaps of photonic crystals, where the photon transmission is prohibited. Based on the Bloch-Floquet theorem, we can artificially engineer photonic bandgaps with material constituents and structural symmetries. In the end, we are able to flexibly engineer electromagnetic states of photonic crystals. Photonic crystals have demonstrated great potentials for novel applications in optical guiding, optical trapping and optical filtering. However, when the structural dimension scales down to the subwavelength regime, the artificial structures become metamaterials. Metamaterials can be treated as

homogeneous media, described within Maxwell Garnett theory. Historically, the ideas of metamaterials were initially proposed by V. Veselago, aiming to realize the negative refraction. Recent years witnessed the powerful functionalities of metamaterials, besides of the negative refraction. For example, metamaterials allow us to arbitrarily control and direct the electromagnetic fields by employing the method of transformation optics. Thanks to its ability to access and manipulate the near field, metamaterials are rapidly developing in the fields including hyperlens, invisible cloak and biosensing.

In this thesis, we focus on the study of Cherenkov radiation in the one-dimensional artificial structures, including one-dimensional photonic crystals and one-dimensional metamaterials. The methods and conclusions can be extended to the two-dimensional or three-dimensional artificial structures. Below we present our three main projects on manipulating Cherenkov radiation by artificial structures.

For conventional Cherenkov detectors such as the ring-imaging Cherenkov detector, the detectable momenta of charged particles are restricted by finite refractive indices of natural transparent materials. In the first work, we proposed the Brewster Cherenkov detector, which enables us to identify charged particles in a wider range of momenta than ever reported. We firstly designed a Brewster photonic crystal based on one-dimensional photonic crystals. Making the use of photonic bandgaps and the Brewster effect, the Brewster photonic crystal shows wide controllability of the dispersion-less effective refractive index. Then we studied the interplay between the charged particle and the Brewster photonic crystal. By controlling the effective refractive index of the Brewster photonic crystal, we can manipulate the Cherenkov angle at the arbitrary momentum.

Homogeneous hyperbolic metamaterials are generally believed to eliminate the threshold velocity of Cherenkov radiation. Thus, Cherenkov radiation in hyperbolic

metamaterials forms an important basis of threshold-free free-electron sources. However, electromagnetic features of Cherenkov radiation in hyperbolic materials with nonlocal corrections still remain elusive. In our second work, we explore the nonlocal regime of Cherenkov radiation in the hyperbolic metamaterial constructed by a periodic metallodielectric structure. We found that the spatial dispersion from the finite structural size and nonlocal electron screening in metals give rise to the Cherenkov threshold. The value of the threshold velocity is dependent on both the structural dimensions and the Fermi velocities of metals. Even though the nonlocal effects deteriorate the performance of Cherenkov sources in small-energy regimes, the additional longitudinal modes arising from the spatial nonlocality can modify and enhance the photon emission at epsilon-near-zero frequencies. Our results provide new insights to guide future experiments in low-threshold free-electron sources.

Because refractive indices of natural transparent materials are finitely large, generally Cherenkov detectors are able to identify charged particles only in several gigaelectronvolt range. It still lacks an efficient way to identify charged particles in small-energy regimes. In the third work, we proposed a new type of Cherenkov detector based on longitudinal modes arising from the spatial nonlocality. We theoretically predicted that the moving charged particle on the metallic grating can produce strong longitudinal modes induced by the nonlocal electron screening in metals, only when the structural dimension is comparable to the atomistic length. Our results indicate that longitudinal Cherenkov photons has very small threshold velocity. Meanwhile, the radiation angle of longitudinal modes shows extremely high sensitivity to the particle velocity especially in small-energy regimes. This work extends the working regime of previous Cherenkov detectors below 1 GeV/ c with enhanced sensitivities.

In all, our work not only enriches the theory of Cherenkov radiation in the artificial structures, but also improves the performance of photonic devices based on Cherenkov radiation, including Cherenkov detectors and Cherenkov sources. In addition, our work provides a theoretical guidance for the experimental study of Cherenkov radiation in artificial structures in the future.

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1

Introduction

1.1 Background

1.1.1 Introduction of Cherenkov radiation

Cherenkov radiation refers to a physical phenomenon that charged particles emit coherent photons when they transverse a medium. Early in 1889, O. Heaviside firstly theoretically predicted that Cherenkov radiation from a moving charged particle has a conical wave front, i.e., the Cherenkov cone [1]. Specifically, the Cherenkov angle is related to the particle velocity by

$$\cos(\theta_c) = \frac{c}{nv}, \quad (1.1)$$

where θ_c is the Cherenkov angle between the electric field and the particle trajectory, n is the refractive index of the medium, v is the particle velocity and c is light velocity in the vacuum. It is until 1934 that P. A. Cherenkov firstly experimentally observed

Cherenkov radiation under the supervision of S. Vavilov [2]. Soon after the discovery of Cherenkov radiation by P. A. Cherenkov, Cherenkov's colleagues I. Tamm and I. Frank theoretically interpreted Cherenkov radiation in 1937 [3]. In honor of their pioneering contributions to Cherenkov radiation, P. A. Cherenkov, I. Tamm and I. Frank shared the Nobel Prize in 1958. According to the classical theory of Cherenkov radiation, Cherenkov radiation arises only when the particle velocity is larger than the threshold velocity, i.e., the phase velocity of light in the medium

$$v_{th} = \frac{c}{n}. \quad (1.2)$$

Accordingly, the threshold energy of a charged particle with the rest mass m_0 is calculated as

$$E_{th} = m_0 c^2 \left[\left(1 - \frac{1}{n^2} \right)^{-1/2} - 1 \right]. \quad (1.3)$$

In the framework of classical electrodynamics, the physical interpretation of Cherenkov radiation is that wavelets from points along the particle trajectory undergo destructively interference with the particle velocity below the threshold velocity v_{th} . However, when the particle velocity exceeds the threshold velocity, wavelets from these points will interfere constructively at the Cherenkov angle θ_c .

After the extensive research for decades, Cherenkov radiation has demonstrated broad applications in many fields. In the field of particle physics, Cherenkov radiation is fundamentally important for the particle identification. In terms of the strategies of the particle identification, Cherenkov detectors are classified into three groups. The first class of the Cherenkov detector is named as threshold Cherenkov counter [4]. The principle of the threshold Cherenkov counter relies on the threshold condition in Cherenkov radiation. The threshold energy of Cherenkov radiation is dependent on both the refractive indices of the medium and the particle rest masses. With the proper medium, we are able to discriminate between charged particles by judging whether

Cherenkov radiation is produced or not. The second class of the Cherenkov detector is the ring-imaging Cherenkov detector (RICH) [5]. At a certain momentum, the Cherenkov angles will vary according to the particle rest masses. By accurately measuring the Cherenkov angles with photodetectors, we can identify charged particles with different masses. The third class of the Cherenkov detector is the time-of-propagation Cherenkov detector (TOP) [6]. For the TOP, Cherenkov photons are produced inside a quartz waveguide. The large refractive index of the quartz results in the total refraction of Cherenkov photons at the boundary of the quartz waveguide. Because the different charged particles emit photons at different angles, corresponding Cherenkov photons experience different optical length inside the quartz waveguide. Therefore, by precisely measuring the arrival time of Cherenkov photons at the end of the waveguide, the charged particles could be identified. As a consequence of the rapid development of various Cherenkov detectors, scientists discovered the J-particle [7], the anti-proton [8] and so on continuously. Moreover, in the field of astrophysics, Cherenkov radiation is used for identifying the cosmic ray. Researchers collect Cherenkov radiation from the cosmic ray entering the atmosphere with the imaging air Cherenkov telescopes (IACT) [9]. The properties of the cosmic ray are confirmed by analyzing the radiation angle of cosmic ray at a certain energy. Plus, in the field of biomedicine, Cherenkov radiation is widely applied on the everyday radiation therapy of tissue and the biological fluorophore imaging [10-13].

1.1.2 Review of Cherenkov radiation in artificial structures

Recently, artificial structures obtain extensive attention because they provide new freedoms for us to tailor the light-matter interaction [14]. The artificial structures

are novel materials, which comprise metallic or dielectric structures. When the structural dimension is comparable with light wavelength of interest, the artificial structures are photonic crystals. The photonic crystals are the photonic analog to the bandgap materials in the electronics. However, if the structural dimension is much smaller than the wavelength of interest, the artificial structures become metamaterials, which can be treated as anisotropic homogeneous media. These artificial structures contains a great amount of extraordinary properties, including negative or zero refractive indices [15], giant optical chirality or optical nonlinearity [16, 17], broadband angular selectivity and photonic topology [18].

It is fundamentally interesting to explore how the moving charged particle interacts with artificial structures. By tailoring the geometry of artificial structures, we can artificially control the electromagnetic characteristics of Cherenkov radiation. In the end, the extraordinary emission properties would promote applications of Cherenkov radiation including Cherenkov sources and Cherenkov detectors (Fig. 1.1).

Extensive research has been involved to improve Cherenkov sources with artificial structures [19-21]. In the first, artificial structures enable to reduce or eliminate the energy threshold of Cherenkov radiation, while high-energy charged particles are required to produce Cherenkov radiation in conventional materials. In 1953, S. Smith and E. Purcell experimentally observed the coherent radiation from the electron moving close to the periodic grating [22]. This phenomenon is then named as Smith-Purcell radiation. Theoretically, the threshold velocity of Smith-Purcell radiation can be zero. The central wavelength λ , the particle velocity β and the Cherenkov angle θ_c of Smith-Purcell radiation are related to the grating pitch a by the formula as [23],

$$\lambda = \frac{a}{m} \left(\frac{1}{\beta} - \cos\theta_c \right), \quad (1.4)$$

where m is the diffraction order. Recently, the technical availability of the nanoscale gratings enhances the electron-light-matter interaction, leading to the observation of the Smith-Purcell radiation from the electron with the low energy of 1.5 eV [24]. In parallel to the Smith-Purcell radiation, researchers also theoretically predicted threshold-free Cherenkov radiation in the two-dimensional photonic crystals [25] and nanowire array [26]. However, the complicated fabrication of these systems prohibits the experimental observation of threshold-free Cherenkov radiation. Only until 2017, Y. Huang experimentally reported a compact threshold-free Cherenkov source based on the hyperbolic metamaterial. In their work, the moving electron emits Cherenkov radiation in the hyperbolic metamaterial when its energy is as small as 0.25 eV [27]. Secondly, polarizations of Cherenkov radiation can be tailored by artificial structures. Previous studies showed that the degeneracy of the circular polarized states of Cherenkov radiation could be lift when the moving charged particles travel inside the chiral medium [28, 29]. In other words, the left circular polarization and right circular polarization of Cherenkov radiation have distinct Cherenkov angles and threshold energies. However, the natural materials have too weak chirality to effectively separate two polarization states. Therefore, researchers studied Cherenkov radiation from the moving charged particle in the helical metagrating, which is an artificial chiral structure [30]. In their work, the polarization states of Cherenkov radiation can be well controlled by tuning the handedness of the helical structure. In addition, artificial structures dramatically enhance Cherenkov radiation, while the radiation efficiency of the moving charged particle in natural materials is relatively small. Recently, researchers found that Cherenkov radiation can be enhanced by surface plasmons, owing to their extremely large photonic density of states [26, 31]. It is well-known that the surface plasmons exist at the boundary between the metal and dielectric. Due to the large loss of the metal, the

quality factor (Q-factor) of the metal-based system is suppressed, leading to the finite enhancement of Cherenkov radiation. To overcome the influence of the loss in the metal-based artificial structures, S. Marin enhanced Cherenkov radiation with the bound states in the continuum (BIC) [32, 33]. The Q-factor of the symmetric-protected BIC is theoretically infinite in an all-dielectric photonic crystals, due to the free radiative loss and non-radiative loss. Therefore, the coupling of moving charged particles with BIC could result in extremely high radiation efficiency of Cherenkov radiation.

Artificial structures also offer broad space to improve Cherenkov detectors [34, 35]. First of all, artificial structures enhance the sensitivity of Cherenkov detectors. In history, RICH is commonly used for particle identification owing to its high accuracy. As mentioned before, the principle of RICH is based on the Eq. (1.1). According to this formula, the sensitivity of RICH is relevant to the refractive indices of radiators that the charged particle traverses. Because of the limited natural transparent materials, RICH only works on several specific momentum range, beyond which RICH becomes insensitive. For example, silica aerogel ($n \sim 1.03$) is used to discriminate particles in the momentum range up to ~ 10 GeV/c; C_4F_{10} ($n = 1.004$) for particles in the range of $10 \sim 40$ GeV/c; CF_4 ($n = 1.0005$) for particles in the range of $16 \sim 100$ GeV/c. Recently, X. Lin *et.al.* proposed resonance transition radiation to address this problem of the sensitivity [36]. In their work, the constructive interference of the resonance transition radiation excited in all-dielectric photonic crystals leads to the effective Cherenkov radiation. With the proper selection of dielectrics, this mechanism enables us to control Cherenkov angles in high sensitivity in any desired momentum range. Secondly, artificial structures update the detection scheme of Cherenkov radiation for Cherenkov detectors. Conventionally, the Cherenkov angle is limited within the range from 0° to 90° . The forward Cherenkov radiation is usually detected by photodetectors array

arranged in the front of the particle trajectory. This detection scheme causes a mixture between photons and particles. As a consequence, high-energy particles are highly likely to bombard and destroy the photodetectors. To solve the problem, we must resort to artificial structures. In 1967, V. Veselago hypothesized the negative-index metamaterials, where both the effective permittivity and permeability are negative [37]. In such metamaterials, the wave propagation follows the left-handed rule, resulting in the negative refractive index. Recent work studied Cherenkov radiation in the negative-index metamaterials. It has been experimentally demonstrated that the Cherenkov angle is flipped toward the backward direction in the negative-index metamaterials, facilitating the experimental detection of the Cherenkov emission [38]. Besides negative-index metamaterials, reversed Cherenkov radiation is also observed in hyperbolic metamaterials [27] and metal-insulator-metal waveguides [39], where the effective refractive indices are negative. Furthermore, nonreciprocal Cherenkov radiation is a hot topic currently. The nonreciprocal Cherenkov radiation enables us to detect the particle velocity and moving direction simultaneously. Nonreciprocal Cherenkov radiation has been predicted in the artificial structures such as the Weyl semimetals [40], the photonic topological insulators or two-dimensional materials with a drift-current [41, 42].

Figure 1. 1 Route map of Cherenkov radiation. The appearance of artificial structures improves conventional Cherenkov devices. In the direction of Cherenkov sources, artificial structures enable Cherenkov radiation to be threshold-less [27], efficiency-enhanced [31], and polarization-tunable [30]. In the direction of Cherenkov detectors, the propose of reversed Cherenkov radiation [38], resonance transition radiation [36] and nonreciprocal Cherenkov radiation [42] helps improve conventional Cherenkov detectors. The further research direction of Cherenkov radiation would be the quantum Cherenkov radiation.

1.2 Motivation and objectives

In spite of the abundant progress and achievement of Cherenkov radiation in the artificial structures, the relevant theories are far from completion. For example, the

homogeneous description of metamaterials is not accurate enough for the study of Cherenkov radiation in the metamaterials. The spatial nonlocality including the spatial dispersion from the finite structural size and nonlocal electron screening in metals could have appreciable influence on the behaviors of Cherenkov radiation under specific conditions. What kind of the influence of the nonlocal effects may bring about and how much each of them contributes to the influence still remain elusive. Therefore, it is necessary for us to establish a complete theory which can give a more accurate description of Cherenkov radiation in the artificial structures. A complete theory of Cherenkov radiation in the artificial structures could guide future experiments in this area.

At the same time, the performances of many photonic devices based on Cherenkov radiation still have a broad space for the improvement. For example, while the resonance transition radiation enables us to control Cherenkov angles in the wide momentum, it operates only at the resonant frequency which enlarges the difficulty of experiments. Thus, the exploration of new mechanisms to further improve the performance of Cherenkov radiation-based photonic devices is challengeable but desirable for us. In this thesis, we would like to uncover the extraordinary behaviors of Cherenkov radiation in the artificial structures, understand the physics behind the phenomena and finally make use of them with the aim to further improve the performance of photonic devices based on Cherenkov radiation.

1.3 Major contribution of the Thesis

In the thesis, we developed a complete theory, which gives an accurate description of Cherenkov radiation in the artificial structures (both one-dimensional

photonic crystals and one-dimensional metamaterials). Within the framework of our theory, we have made several major achievements in the thesis listed as below:

1. Finite natural transparent materials throughout the visible wavelength range brings long-standing scientific challenge for the particle identification at arbitrary momentum with Cherenkov radiation. While the resonance transition radiation lifts this fundamental limit, the single-frequency response of this new type of Cherenkov detector prevents itself from experimental verification and industrial commercialization. In this work, we theoretically proposed Brewster Cherenkov radiation from a charged particle moving nearby the Brewster photonic crystal. The Brewster photonic crystal not only offers a wide controllability of effective refractive index, but also shows minor dispersion in the waveband of interest. Therefore, we are able to detect moving charged particles in any desired momentum range without the restriction of natural transparent materials, simply by the means of measuring the position of Cherenkov pulses formed by the Brewster photonic crystal. This discovery not only updates previous strategies for the particle identification, but also facilitates the next generation of Cherenkov detectors.
2. Cherenkov radiation is generally believed to arise when a charged particle moves at a velocity larger than the phase velocity of light in the surrounding medium. Counterintuitively, recent experiments showed that the extremely large density of states in hyperbolic metamaterials enables Cherenkov radiation without any lower-bound velocity cutoff. Here, we investigated the radiation of a moving charged particle in hyperbolic metamaterials consisting of periodic metallodielectric structures. Our results indicate that Cherenkov radiation in hyperbolic metamaterials is strongly nonlocal: (1) the interplay between inhomogeneous structure and nonlocal electron screening results in a threshold energy below which photon

generation is prohibited; (2) the additional longitudinal wave resulting from nonlocal spatial dispersion can strongly modify and enhance photon emission at epsilon-near-zero frequencies. Our study not only completes the theory of ‘threshold-free’ Cherenkov radiation, but also provides new insights to guide future experiments in this and relevant fields.

3. Cherenkov radiation is widely used for the particle identification, exploiting the velocity-dependent Cherenkov angles. Previously, Cherenkov detectors are designed to identify the charged particles only in the high-energy regime, restricted by the finite value of refractive indices in natural transparent materials. It still remains challengeable to measure slow-moving particles using Cherenkov radiation. In this work, we found the quantum-sized metallic grating supports longitudinal Cherenkov photons, enabling to identify the charged particle in the low-energy regime. Longitudinal Cherenkov photons arise from the nonlocal electron screening in metals, when the moving charged particle interacts with the ultrathin metallic grating. Our results show that longitudinal Cherenkov photons exist in a wide range of velocities, i.e., from the Fermi velocity in metals to the light velocity in the vacuum. More importantly, the radiation angle of longitudinal modes exhibits strong sensitivity to the particle velocity especially in low-energy regimes. Therefore, Cherenkov detectors based on longitudinal modes may open a new door to measure the particle velocities or identify unknown particles below 1 GeV/ c in high sensitivity.

1.4 Organization of the Thesis

This thesis is organized as follows:

Chapter 1 is an introduction part of the thesis. We began with the history and applications of Cherenkov radiation. Then we reviewed the development of Cherenkov radiation in the artificial structures in the past few decades. Furthermore, we cleared the motivation, objectives and major contribution of this thesis.

In Chapter 2, we first presented the theoretical description of Cherenkov radiation in the framework of the classical electrodynamics. Then, we comprehensively discussed Cherenkov radiation in one-dimensional photonic crystals. The transfer matrix method was introduced to analytically solve for the electromagnetic fields generated by the moving charged particles in the photonic crystals. To identify the optical responses of different polarizations, we separately derived expressions to calculate the s -polarized field and p -polarized field of Cherenkov radiation respectively. In the next part, we comprehensively discussed Cherenkov radiation in the metamaterials. We started with the introduction of the effective medium theory and hydrodynamic Drude model. For comparison, the expressions for the local field and nonlocal field of Cherenkov radiation in the metal-based metamaterials were derived, respectively.

In Chapter 3, we proposed the Brewster Cherenkov detector, with the aim to improve the performance of previous Cherenkov detectors. The Brewster Cherenkov detector is based on the Brewster photonic crystal. In the beginning, we demonstrated how to design the Brewster photonic crystal and how the Brewster Cherenkov detector works. To emphasize its advantages, the sensitivity of the Brewster Cherenkov detector was detailed discussed. Then, we explored the measurement strategy and system error for the Brewster Cherenkov detector. In the end, we discussed how the polarization states of Cherenkov radiation influence the performance of the Brewster Cherenkov detector.

In Chapter 4, we investigated how the nonlocal effects influence Cherenkov radiation in the hyperbolic metamaterial. We first discussed how the spatial dispersion from the finite structural size and the nonlocal electron screening in the hyperbolic metamaterials independently affect the Cherenkov threshold. Secondly, we demonstrated the impact of these nonlocal effects on the radiation spectrum of the moving charged particle. Finally, we discovered longitudinal Cherenkov photons in the hyperbolic metamaterial and explained its physical origin.

In Chapter 5, we pointed out that longitudinal Cherenkov photons from the interaction between the charged particle and ultrathin metallic grating can be used for slow-moving particle identification. We firstly explained the origin and production condition of longitudinal Cherenkov photons. Then, the advantages of longitudinal Cherenkov photons are revealed through the isofrequency study. In the end, we presented an example of slow-moving particle identification with longitudinal modes.

In Chapter 6, we finalized the thesis by concluding our achievements in this project, illustrating the significance of these achievements, and giving recommendations for our future work.

2

Theory of Cherenkov radiation in artificial structures

2.1 Classical theory of Cherenkov radiation

Let's consider a particle with the charge of q_0 travelling in a homogeneous isotropic medium along z -direction as shown in Fig. 2.1. Assuming that the energy loss due to the radiation of the charged particle is negligible, the motion of the charged particle becomes uniform. Therefore the current produced by the charged particle is equivalent to [43]

$$\bar{J}^q(\bar{r}, t) = \hat{z}q_0v\delta(x)\delta(y)\delta(z - vt), \quad (2.1)$$

where v is the particle velocity. On the other hand, the equivalent current $\bar{J}^q(\bar{r}, t)$ can be expressed by the sum of its Fourier components $\bar{J}^q(\bar{k}, \omega)$ as:

$$\bar{J}^q(\bar{r}, t) = \int dt \int d\bar{k}_\rho \bar{J}^q(\bar{k}, \omega) e^{i(\bar{k}_\rho \cdot \bar{\rho} - \omega t)}, \quad (2.2)$$

where $\bar{k}_\rho = \hat{x}k_\rho \cos\alpha + \hat{y}k_\rho \sin\alpha$, and $\bar{\rho} = \hat{x}\rho \cos\theta + \hat{y}\rho \sin\theta$. And the Fourier components of the equivalent current density are derived by the inverse Fourier transformation as

$$\bar{J}^q(\bar{k}, \omega) = \hat{z} \frac{q_0}{(2\pi)^3} e^{ik_z z}, \quad (2.3)$$

where $k_z = \frac{\omega}{v}$.

Figure 2. 1 Schematic diagram of Cherenkov radiation. A charged particle with the velocity of $\bar{v}$ is traveling along z-direction. Above the threshold velocity, the equi-phase plane of the emission from the charged particle forms the Cherenkov cone. The Cherenkov angle θ_c is defined as the angle between the electric field and the z-axis.

In the framework of the classical electrodynamics, the electromagnetic field emitted by the charged particle is determined by Maxwell equations as,

$$\bar{k} \times \bar{E}(\bar{k}, \omega) = \omega \mu_0 \bar{H}(\bar{k}, \omega), \quad (2.4)$$

$$\bar{k} \times \bar{H}(\bar{k}, \omega) = -\omega \epsilon_0 \epsilon_r \bar{E}(\bar{k}, \omega) - i \bar{J}^q(\bar{k}, \omega), \quad (2.5)$$

where ϵ_r is the dielectric constant of the surrounding medium. Therefore, we get the wave equation for the moving charged particle in the isotropic medium as

$$\bar{k} \times \bar{k} \times \bar{E}(\bar{k}, \omega) + \epsilon_r k_0^2 \bar{E}(\bar{k}, \omega) = -\hat{z} i \omega \mu_0 \bar{J}^q(\bar{k}, \omega). \quad (2.6)$$

This expression can also be rewritten in the form of the matrix in the cartesian coordination as following:

$$\begin{bmatrix} -k_y^2 - k_z^2 + \epsilon_r k_0^2 & k_x k_y & k_x k_z \\ k_x k_y & -k_x^2 - k_z^2 + \epsilon_r k_0^2 & k_y k_z \\ k_x k_z & k_y k_z & -k_x^2 - k_y^2 + \epsilon_r k_0^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -i \omega \mu_0 J^q \end{bmatrix}. \quad (2.7)$$

After simple transformations of the matrix, we get

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \frac{-i \omega \mu_0 J^q}{\epsilon_r k_0^2 (k_\rho^2 - k_{\rho 0}^2)} \begin{bmatrix} k_x k_z \\ k_y k_z \\ -k_{\rho 0}^2 \end{bmatrix}, \quad (2.8)$$

where $k_{\rho 0} = \sqrt{\epsilon_r k_0^2 - k_z^2}$ is the eigenvalue of the wavevector k_ρ . Thus, we figure out the expression of E_z in the domain of the wavevector and frequency as

$$E_z(\bar{k}, \omega) = \frac{i q_0 e^{i k_z z}}{(2\pi)^3 \omega \epsilon_0 \epsilon_r} \frac{k_{\rho 0}^2}{(k_\rho^2 - k_{\rho 0}^2)}. \quad (2.9)$$

And the E_z can be transformed into the domain of the real space as

$$E_z(\bar{r}, \omega) = \frac{i q_0 e^{i k_z z}}{(2\pi)^3 \omega \epsilon_0 \epsilon_r} \int d\bar{k}_\rho \frac{k_{\rho 0}^2}{(k_\rho^2 - k_{\rho 0}^2)} e^{i \bar{k}_\rho \cdot \bar{\rho}}, \quad (2.10)$$

which is equivalent to the expression in the cylinder coordination (ρ, φ, z) as

$$\begin{aligned} E_z(\bar{r}, \omega) &= \int_0^{+\infty} k_\rho dk_\rho \int_0^{2\pi} e^{i k_\rho \rho \cos(\theta - \alpha)} d\theta \left[\frac{i q e^{i k_z z}}{(2\pi)^3 \omega \epsilon_0 \epsilon_r} \frac{k_{\rho 0}^2}{(k_\rho^2 - k_{\rho 0}^2)} \right] = \\ &= \int_0^{+\infty} k_\rho dk_\rho 2\pi J_0(k_\rho \rho) \left[\frac{i q e^{i k_z z}}{(2\pi)^3 \omega \epsilon_0 \epsilon_r} \frac{k_{\rho 0}^2}{(k_\rho^2 - k_{\rho 0}^2)} \right], \end{aligned} \quad (2.11)$$

where J_0 is the 0^{th} order Bessel function [44]. Since the 0^{th} order Bessel function J_0 is related to the 0^{th} order Hankel function of the first kind $H_0^{(1)}$ by

$$J_0(k_\rho \rho) = \frac{1}{2} \left(H_0^{(1)}(k_\rho \rho) - H_0^{(1)}(-k_\rho \rho) \right), \quad (2.12)$$

the $E_z(\bar{r}, \omega)$ is simplified to be

$$E_z(\bar{r}, \omega) = \frac{iq_0 e^{ik_z z}}{8\pi^2 \omega \epsilon_0 \epsilon_r} \int_{-\infty}^{+\infty} dk_\rho k_\rho H_0^{(1)}(k_\rho \rho) \left[\frac{k_{\rho 0}^2}{(k_\rho^2 - k_{\rho 0}^2)} \right]. \quad (2.13)$$

Applying the Residue theory, we can remove the pole inside the integral expression as

$$\int_{-\infty}^{+\infty} dk_\rho k_\rho H_0^{(1)}(k_\rho \rho) \left[\frac{k_{\rho 0}^2}{(k_\rho^2 - k_{\rho 0}^2)} \right] = \pi i k_{\rho 0}^2 H_0^{(1)}(k_{\rho 0} \rho). \quad (2.14)$$

Therefore, the final expression of the E_z can be written in the domain of the real space and frequency as:

$$E_z(\bar{r}, \omega) = \frac{-q_0 e^{ik_z z}}{8\pi \omega \epsilon_0 \epsilon_r} k_{\rho 0}^2 H_0^{(1)}(k_{\rho 0} \rho). \quad (2.15)$$

In the end, E_z can be transformed into the time domain as

$$E_z(\bar{r}, t) = \int_{-\infty}^{+\infty} d\omega e^{-i\omega t} \frac{-q_0 e^{ik_z z}}{8\pi \omega \epsilon_0 \epsilon_r} k_{\rho 0}^2 H_0^{(1)}(k_{\rho 0} \rho). \quad (2.16)$$

Applying Gauss's law ($\nabla \cdot \bar{E} = 0$) and Faraday's law ($\nabla \times \bar{E} = i\omega \mu_0 \bar{H}$), we can easily find out the other two field components of Cherenkov radiation in the isotropic medium as:

$$E_\rho(\bar{r}, t) = \int_{-\infty}^{+\infty} d\omega e^{-i\omega t} \frac{iq_0 k_z k_\rho e^{ik_z z}}{8\pi \omega \epsilon_0 \epsilon_r} H_1^{(1)}(k_\rho \rho), \quad (2.17)$$

$$H_\varphi(\bar{r}, t) = \int_{-\infty}^{+\infty} d\omega e^{-i\omega t} \frac{iq_0 k_\rho e^{ik_z z}}{8\pi} H_1^{(1)}(k_\rho \rho). \quad (2.18)$$

2.2 Cherenkov radiation in photonic crystals

2.2.1 Transfer matrix method

Transfer matrix method is widely used to calculate the transmission and reflection coefficients in layered structures or other relevant systems[45]. Below we detailed present the method of the transfer matrix. We start by introducing the cascading method and combining method for scattering matrices. Let us consider a pair of

scattering matrices $\mathbf{U}$ and $\mathbf{V}$. They connect the arbitrary vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{F}$ by the following relations,

$$\begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} = \begin{bmatrix} \mathbf{U}_{11} & \mathbf{U}_{12} \\ \mathbf{U}_{21} & \mathbf{U}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{C} \\ \mathbf{D} \end{bmatrix}, \quad (2.19)$$

$$\begin{bmatrix} \mathbf{D} \\ \mathbf{E} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{11} & \mathbf{V}_{12} \\ \mathbf{V}_{21} & \mathbf{V}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{B} \\ \mathbf{F} \end{bmatrix}. \quad (2.20)$$

After simple matrix derivations, the above relations can be transformed into [46]

$$\begin{bmatrix} \mathbf{A} \\ \mathbf{E} \end{bmatrix} = \begin{bmatrix} \mathbf{U}_{11} + \mathbf{U}_{12}(\mathbf{1} - \mathbf{V}_{11}\mathbf{U}_{22})^{-1}\mathbf{V}_{11}\mathbf{U}_{21} & \mathbf{U}_{12}(\mathbf{1} - \mathbf{V}_{11}\mathbf{U}_{22})^{-1}\mathbf{V}_{12} \\ \mathbf{V}_{21}(\mathbf{1} - \mathbf{U}_{22}\mathbf{V}_{11})^{-1}\mathbf{U}_{21} & \mathbf{V}_{22} + \mathbf{V}_{21}(\mathbf{1} - \mathbf{U}_{22}\mathbf{V}_{11})^{-1}\mathbf{U}_{22}\mathbf{V}_{12} \end{bmatrix} \begin{bmatrix} \mathbf{C} \\ \mathbf{F} \end{bmatrix}, \quad (2.21)$$

$$\begin{bmatrix} \mathbf{B} \\ \mathbf{D} \end{bmatrix} = \begin{bmatrix} (\mathbf{1} - \mathbf{U}_{22}\mathbf{V}_{11})^{-1}\mathbf{U}_{21} & (\mathbf{1} - \mathbf{U}_{22}\mathbf{V}_{11})^{-1}\mathbf{U}_{22}\mathbf{V}_{12} \\ (\mathbf{1} - \mathbf{V}_{11}\mathbf{U}_{22})^{-1}\mathbf{V}_{11}\mathbf{U}_{21} & (\mathbf{1} - \mathbf{V}_{11}\mathbf{U}_{22})^{-1}\mathbf{V}_{12} \end{bmatrix} \begin{bmatrix} \mathbf{C} \\ \mathbf{F} \end{bmatrix}. \quad (2.22)$$

Next, we consider a layered structure with the total number of regions g as shown in the Fig. 2.2, where $\mathbf{T}_j^\pm$ are transmission coefficients and $\mathbf{R}_j^\pm$ are reflection coefficients in the region j .

Figure 2. 2 Schematic diagram of the transfer matrix method. $\mathbf{T}_j^\pm$ are transmission coefficients and $\mathbf{R}_j^\pm$ are reflection coefficients. The total region number of the layered structure is g .

We assume that the transmission and reflection coefficients are connected by calculated scattering matrices $\mathbf{S}$ as

$$\begin{aligned} \begin{bmatrix} \mathbf{R}_0^- \\ \mathbf{T}_1^+ \end{bmatrix} &= \mathbf{S}_1 \begin{bmatrix} \mathbf{T}_0^- \\ \mathbf{R}_1^+ \end{bmatrix}, \begin{bmatrix} \mathbf{R}_1^- \\ \mathbf{T}_2^+ \end{bmatrix} = \mathbf{S}_2 \begin{bmatrix} \mathbf{T}_1^- \\ \mathbf{R}_2^+ \end{bmatrix}, \dots, \begin{bmatrix} \mathbf{R}_{j-1}^- \\ \mathbf{T}_j^+ \end{bmatrix} = \mathbf{S}_{2j-1} \begin{bmatrix} \mathbf{T}_{j-1}^- \\ \mathbf{R}_j^+ \end{bmatrix}, \begin{bmatrix} \mathbf{R}_j^- \\ \mathbf{T}_{j+1}^+ \end{bmatrix} = \mathbf{S}_{2j} \begin{bmatrix} \mathbf{T}_j^- \\ \mathbf{R}_{j+1}^+ \end{bmatrix}, \dots, \\ \begin{bmatrix} \mathbf{R}_{g-1}^- \\ \mathbf{T}_g^+ \end{bmatrix} &= \mathbf{S}_{2g-1} \begin{bmatrix} \mathbf{T}_{g-1}^- \\ \mathbf{R}_g^+ \end{bmatrix}, \begin{bmatrix} \mathbf{R}_g^- \\ \mathbf{T}_{g+1}^+ \end{bmatrix} = \mathbf{S}_{2g} \begin{bmatrix} \mathbf{T}_g^- \\ \mathbf{R}_{g+1}^+ \end{bmatrix}. \end{aligned} \quad (2.23)$$

With resort to the relation Eq. (2.21), we are able to find out cascading matrices $\mathbf{H}$ so that all the coefficients are connected with the amplitude of the incidence $\mathbf{T}_0^-$ by $\mathbf{H}$ as

$$\begin{aligned} \begin{bmatrix} \mathbf{R}_0^- \\ \mathbf{T}_1^+ \end{bmatrix} &= \mathbf{H}_1 \begin{bmatrix} \mathbf{T}_0^- \\ \mathbf{R}_1^+ \end{bmatrix}, \begin{bmatrix} \mathbf{R}_1^- \\ \mathbf{T}_2^+ \end{bmatrix} = \mathbf{H}_2 \begin{bmatrix} \mathbf{T}_1^- \\ \mathbf{R}_2^+ \end{bmatrix}, \dots, \begin{bmatrix} \mathbf{R}_{j-1}^- \\ \mathbf{T}_j^+ \end{bmatrix} = \mathbf{H}_{2j-1} \begin{bmatrix} \mathbf{T}_{j-1}^- \\ \mathbf{R}_j^+ \end{bmatrix}, \begin{bmatrix} \mathbf{R}_j^- \\ \mathbf{T}_{j+1}^+ \end{bmatrix} = \mathbf{H}_{2j} \begin{bmatrix} \mathbf{T}_j^- \\ \mathbf{R}_{j+1}^+ \end{bmatrix}, \dots, \\ \begin{bmatrix} \mathbf{R}_0^- \\ \mathbf{T}_g^+ \end{bmatrix} &= \mathbf{H}_{2g-1} \begin{bmatrix} \mathbf{T}_0^- \\ \mathbf{R}_g^+ \end{bmatrix}. \end{aligned} \quad (2.24)$$

Applying similar procedures, all the coefficients are connected with the amplitude of the reflection from the infinity $\mathbf{R}_j^-$ by cascading matrices $\mathbf{G}$ as

$$\begin{aligned} \begin{bmatrix} \mathbf{R}_g^+ \\ \mathbf{T}_g^- \end{bmatrix} &= \mathbf{G}_1 \begin{bmatrix} \mathbf{T}_g^+ \\ \mathbf{R}_g^- \end{bmatrix}, \begin{bmatrix} \mathbf{R}_{g-1}^+ \\ \mathbf{T}_{g-1}^- \end{bmatrix} = \mathbf{G}_2 \begin{bmatrix} \mathbf{T}_{g-1}^+ \\ \mathbf{R}_{g-1}^- \end{bmatrix}, \dots, \begin{bmatrix} \mathbf{R}_j^+ \\ \mathbf{T}_j^- \end{bmatrix} = \mathbf{G}_{2g-2j+1} \begin{bmatrix} \mathbf{T}_j^+ \\ \mathbf{R}_j^- \end{bmatrix}, \begin{bmatrix} \mathbf{R}_{j-1}^+ \\ \mathbf{T}_{j-1}^- \end{bmatrix} = \\ \mathbf{G}_{2g-2j+2} \begin{bmatrix} \mathbf{T}_{j-1}^+ \\ \mathbf{R}_{j-1}^- \end{bmatrix}, \dots, \begin{bmatrix} \mathbf{R}_0^+ \\ \mathbf{T}_0^- \end{bmatrix} &= \mathbf{G}_{2g-1} \begin{bmatrix} \mathbf{T}_0^+ \\ \mathbf{R}_0^- \end{bmatrix}. \end{aligned} \quad (2.25)$$

Then, $\begin{bmatrix} \mathbf{R}_0^- \\ \mathbf{T}_1^+ \end{bmatrix} = \mathbf{H}_{2j-1} \begin{bmatrix} \mathbf{T}_0^- \\ \mathbf{R}_1^+ \end{bmatrix}$ and $\begin{bmatrix} \mathbf{R}_j^- \\ \mathbf{T}_g^+ \end{bmatrix} = \mathbf{G}_{2g-2j+1} \begin{bmatrix} \mathbf{T}_j^- \\ \mathbf{R}_g^+ \end{bmatrix}$ can be combined by combining matrices $\mathbf{I}$ using Eq. (2.22) as

$$\begin{bmatrix} \mathbf{T}_j^+ \\ \mathbf{R}_j^+ \end{bmatrix} = \mathbf{I}_{2j-1} \begin{bmatrix} \mathbf{T}_0^- \\ \mathbf{R}_g^- \end{bmatrix}. \quad (2.26)$$

While $\begin{bmatrix} \mathbf{R}_0^- \\ \mathbf{T}_j^- \end{bmatrix} = \mathbf{H}_{2j} \begin{bmatrix} \mathbf{T}_0^- \\ \mathbf{R}_j^- \end{bmatrix}$ and $\begin{bmatrix} \mathbf{R}_j^- \\ \mathbf{T}_g^- \end{bmatrix} = \mathbf{G}_{2g-2j} \begin{bmatrix} \mathbf{T}_j^- \\ \mathbf{R}_g^- \end{bmatrix}$ are combined by $\mathbf{I}$ as

$$\begin{bmatrix} \mathbf{T}_j^- \\ \mathbf{R}_j^- \end{bmatrix} = \mathbf{I}_{2j} \begin{bmatrix} \mathbf{T}_0^- \\ \mathbf{R}_g^- \end{bmatrix}. \quad (2.27)$$

In the end, as long as we know the amplitude of the incidence $\mathbf{T}_0^-$ and the reflection from the infinity $\mathbf{R}_g^-$, all the coefficients can be determined.

2.2.2 *P-polarized field of Cherenkov radiation*

Now we consider a charged particle moving along z -direction, with its trajectory near the top interface of one-dimensional photonic crystals as shown in Fig. 2.3. The distribution of the electromagnetic field can be analytically solved out using transfer matrix method. It is notably that our method is not limited to the periodic structure.

Figure 2. 3 Schematic diagram of Cherenkov radiation in one-dimensional photonic crystals. The charged particle is traveling along z -direction, with its trajectory near the top interface of the photonic crystal. The photonic crystal is alternating by two dielectrics periodically.

To generalize this situation, we introduce a scalar potential ϕ_0 . Since the electromagnetic field emitted by the charged particle is a transverse magnetic wave, the electric and magnetic fields take the form as [47, 48],

$$\vec{E}(\vec{r}, \omega) = \frac{i}{\omega \epsilon_0 \epsilon_{r1}} \nabla \times \nabla \times \hat{z} \phi_0, \quad (2.28)$$

$$\vec{H}(\vec{r}, \omega) = \nabla \times \hat{z} \phi_0. \quad (2.29)$$

Recalling the expressions of field components of Cherenkov radiation in the isotropic medium (Eq. (2.16-2.18)), the scalar potential ϕ_0 of the moving charged particle can be calculated as [39]

$$\phi_0 = \frac{iq_0}{8\pi} H_0^{(1)}(k_\rho \rho) e^{ik_z z} = \int_{-\infty}^{+\infty} dk_x X_0 e^{ik_x x + ik_{y1}|y| + ik_z z}, \quad (2.30)$$

where $k_{y1} = \sqrt{\epsilon_{r1} k_0^2 - k_x^2 - k_z^2}$, and $X_0 = \frac{iq_0}{8\pi^2 k_{y1}}$.

The charged particles can generate Cherenkov radiation, when the particle velocity is greater than the threshold velocity in the surrounding medium. Cherenkov radiation emitted by the charged particle can be tailored by the photonic crystal. In fact, Cherenkov radiation from the charged particle comprises both the p -polarized component and s -polarized component. Because the photonic crystal generally has drastically different optical responses for these two polarization states, it is necessary for us to study the p -polarized field and s -polarized field of Cherenkov radiation on the photonic crystal independently.

For the extraction of the p -polarized field, we calculate the field component E_y derived from the potential ϕ_0 of the source with Eq. (2.28) as

$$E_y = \frac{i}{\omega \epsilon} \frac{\partial^2 \phi_0}{\partial y \partial z} = -\frac{i}{\omega \epsilon_0} \int_{-\infty}^{+\infty} dk_x \text{sgn}(y) X_0 \frac{k_y k_z}{\epsilon_{r1}} e^{ik_x x + ik_{y1}|y| + ik_z z}. \quad (2.31)$$

Under the situation that the charged particle is moving on the top of the layered structure, the potential ϕ_{pj} on the region j is

$$\phi_{pj} = \int_{-\infty}^{+\infty} dk_x (T_{pj}^+ e^{ik_{yj}(y-y_j)} + R_{pj}^+ e^{-ik_{yj}(y-y_j)}) e^{ik_x x + ik_z z}, \quad (2.32)$$

or

$$\phi_{pj} = \int_{-\infty}^{+\infty} dk_x (T_{pj}^+ e^{ik_{yj}(y-y_j)} + R_{pj}^- e^{-ik_{yj}(y-y_{j+1})}) e^{ik_x x + ik_z z}. \quad (2.33)$$

Therefore, the field component E_y in the region j is obtained as

$$E_{yj} = \frac{i}{\omega \epsilon_0} \int_{-\infty}^{+\infty} dk_x \left(\frac{k_{yj} k_z}{\epsilon_{rj}} \left(-T_{pj}^+ e^{ik_{yj}(y-y_j)} + R_{pj}^- e^{-ik_{yj}(y-y_{j+1})} \right) \right) e^{ik_x x + ik_z z}. \quad (2.34)$$

We apply Gauss's law ($\nabla \cdot \bar{E} = 0$) and Faraday's law ($\nabla \times \bar{E} = i\omega\mu_0\bar{H}$), to calculate the field components $E_{\parallel j}$ and $H_{\perp j}$ of the p -polarized field in the x - z plane as

$$E_{\parallel j} = \frac{i}{\omega\epsilon_0} \int_{-\infty}^{+\infty} dk_x \left(\frac{k_{yj}^2 k_z}{\epsilon_{rj} k_{\parallel j}} (T_{pj}^+ e^{ik_{yj}(y-y_j)} + R_{pj}^- e^{-ik_{yj}(y-y_{j+1})}) \right) e^{ik_x x + ik_z z}, \quad (2.35)$$

$$H_{\perp j} = i \int_{-\infty}^{+\infty} dk_x \left(\frac{k_{yj} k_z}{k_{\parallel j}} (T_{pj}^+ e^{ik_{yj}(y-y_j)} - R_{pj}^- e^{-ik_{yj}(y-y_{j+1})}) \right) e^{ik_x x + ik_z z}, \quad (2.36)$$

where $E_{\parallel j}$ is the in-plane electric field component, $H_{\perp j}$ is the out-of-plane magnetic field, and the in-plane wavevector $\bar{k}_{\parallel j} = k_x \hat{x} + k_y \hat{y}$.

In order to determine the reflection and transmission coefficients in each region of the photonic crystal, we need to calculate scattering matrices. At the boundary $y = y_j$ in the photonic crystal, we have $\mathbf{T}_{j-1}^- = T_{pj-1}^-$, $\mathbf{R}_{j-1}^- = R_{pj-1}^-$, $\mathbf{T}_j^+ = T_{pj}^+$ and $\mathbf{R}_j^+ = R_{pj}^+$. By matching boundary conditions,

$$\hat{n} \times (\bar{H}_{j-1}(y_j) - \bar{H}_j(y_j)) = 0, \quad (2.37)$$

$$\hat{n} \times (\bar{E}_{j-1}(y_j) - \bar{E}_j(y_j)) = 0, \quad (2.38)$$

we have

$$\frac{k_{yj-1}^2}{\epsilon_{rj-1} k_{\parallel j-1}} T_{pj-1}^- + \frac{k_{yj-1}^2}{\epsilon_{rj-1} k_{\parallel j-1}} R_{pj-1}^- = \frac{k_{yj}^2}{\epsilon_{rj} k_{\parallel j}} T_{pj}^+ + \frac{k_{yj}^2}{\epsilon_{rj} k_{\parallel j}} R_{pj}^+, \quad (2.39)$$

$$\frac{k_{yj-1}}{k_{\parallel j-1}} T_{pj-1}^- - \frac{k_{yj-1}}{k_{\parallel j-1}} R_{pj-1}^- = \frac{k_{yj}}{k_{\parallel j}} T_{pj}^+ - \frac{k_{yj}}{k_{\parallel j}} R_{pj}^+. \quad (2.40)$$

The scattering matrix at the boundary is calculated by Eq. (2.39 and 2.40) as

$$\mathbf{S}_{1j} = \frac{1}{\frac{k_{yj-1}}{\epsilon_{rj-1}} + \frac{k_{yj}}{\epsilon_{rj}}} \begin{bmatrix} \frac{k_{yj}}{\epsilon_{rj}} - \frac{k_{yj-1}}{\epsilon_{rj-1}} & 2 \frac{k_{yj}}{\epsilon_{rj}} \frac{k_{\parallel j-1} k_{yj}}{k_{\parallel j} k_{yj-1}} \\ 2 \frac{k_{yj-1}}{\epsilon_{rj-1}} \frac{k_{\parallel j} k_{yj-1}}{k_{\parallel j-1} k_{yj}} & \frac{k_{yj-1}}{\epsilon_{rj-1}} - \frac{k_{yj}}{\epsilon_{rj}} \end{bmatrix}. \quad (2.41)$$

Inside the region j , we have $\mathbf{T}_j^\pm = T_{pj}^\pm$ and $\mathbf{R}_j^\pm = R_{pj}^\pm$. The scattering matrix in the region j can be easily obtained as

$$\mathbf{S}_{2j} = \begin{bmatrix} 0 & e^{ik_{yj} h_j} \\ e^{ik_{yj} h_j} & 0 \end{bmatrix}, \quad (2.42)$$

where the thickness of the region j is $h_j = y_{j+1} - y_j$.

Given the condition that $\mathbf{T}_1^+ = X_0$ and $\mathbf{R}_g^- = 0$, all the transmission and reflection coefficients $(T_{p1}^\pm, T_{p2}^\pm, \dots, T_{pg}^\pm$ and $R_{p1}^\pm, R_{p2}^\pm, \dots, R_{pg}^\pm)$ are calculated out with transfer matrix method. In the end, the field component E_y in the region j is expressed by

$$E_{yj} = \frac{i}{\omega\epsilon_0} \int_{-\infty}^{+\infty} dk_x \left(\frac{k_{yj}k_z}{\epsilon_{rj}} \left(-T_{pj}^+ e^{ik_{yj}(y-y_j)} + R_{pj}^- e^{-ik_{yj}(y-y_{j+1})} \right) \right) e^{ik_x x + ik_z z}. \quad (2.43)$$

2.2.3 *S-polarized field of Cherenkov radiation*

With the similar procedures for the p -polarized field of Cherenkov radiation, we can figure out the s -polarized field of Cherenkov radiation. For extraction of the s -polarized field, we calculate the field component H_y derived from the potential ϕ_0 of the source with Eq. (2.29) as

$$H_y = -\frac{\partial \phi_0}{\partial x} = i \int_{-\infty}^{+\infty} dk_x (-k_x) X_0 e^{ik_x x + ik_{y1}|y| + ik_z z}. \quad (2.44)$$

Since the potential ϕ_{sj} in the region j takes the form as

$$\phi_{sj} = \int_{-\infty}^{+\infty} dk_x \left(T_{sj}^+ e^{ik_{yj}(y-y_j)} + R_{sj}^- e^{-ik_{yj}(y-y_{j+1})} \right) e^{ik_x x + ik_z z}, \quad (2.45)$$

the field component H_y in the region j is expressed as

$$H_{yj} = i \int_{-\infty}^{+\infty} dk_x \left((-k_x) \left(T_{sj}^+ e^{ik_{yj}(y-y_j)} + R_{sj}^- e^{-ik_{yj}(y-y_{j+1})} \right) \right) e^{ik_x x + ik_z z}. \quad (2.46)$$

Now we apply Gauss's law ($\nabla \cdot \bar{H} = 0$) and Ampere's law ($\nabla \times \bar{H} = i\omega\epsilon_0\epsilon_{rj}\bar{E}$) to determine the field components $E_{\parallel j}$ and $H_{\perp j}$ of the s -polarized light in the region j ,

$$H_{\parallel j} = i \int_{-\infty}^{+\infty} dk_x \left(\frac{k_x k_{yj}}{k_{\parallel j}} \left(T_{sj}^+ e^{ik_{yj}(y-y_j)} - R_{sj}^- e^{-ik_{yj}(y-y_{j+1})} \right) \right) e^{ik_x x + ik_z z}, \quad (2.47)$$

$$E_{\perp j} = \frac{ik_0^2}{\omega\epsilon_0} \int_{-\infty}^{+\infty} dk_x \left(-\frac{k_x}{k_{\parallel j}} \left(T_{sj}^+ e^{ik_{yj}(y-y_j)} + R_{sj}^- e^{-ik_{yj}(y-y_{j+1})} \right) \right) e^{ik_x x + ik_z z}, \quad (2.48)$$

where $H_{\parallel j}$ is the in-plane magnetic field and $E_{\perp j}$ is the out-of-plane electric field component.

At the boundary of $y = y_j$ in the photonic crystal, we have $\mathbf{T}_{j-1}^- = T_{sj-1}^-$, $\mathbf{R}_{j-1}^- = R_{sj-1}^-$, $\mathbf{T}_j^+ = T_{sj}^+$ and $\mathbf{R}_j^+ = R_{sj}^+$. By matching the boundary conditions (Eq. (2.37 and 2.38)), we have the relations as followings

$$k_{yj-1}T_{sj-1}^- - k_{yj-1}R_{sj-1}^- = k_{yj}T_{sj}^+ - k_{yj}R_{sj}^+, \quad (2.49)$$

$$T_{sj-1}^- - R_{sj-1}^- = T_{sj}^+ - R_{sj}^+. \quad (2.50)$$

Determined by Eq. (2.49 and 2.50), the associated scattering matrix at the boundary is

$$\mathbf{S}_{3j} = \frac{1}{k_{yj-1} + k_{yj}} \begin{bmatrix} k_{yj-1} - k_{yj} & 2k_{yj} \\ 2k_{yj-1} & k_{yj} - k_{yj-1} \end{bmatrix}. \quad (2.51)$$

Inside the region j of the layered structure, we have $\mathbf{T}_j^\pm = T_{sj}^\pm$ and $\mathbf{R}_j^\pm = R_{sj}^\pm$. The scattering matrix in the region j is obtained as

$$\mathbf{S}_{4j} = \begin{bmatrix} 0 & e^{ik_{yj}h_j} \\ e^{ik_{yj}h_j} & 0 \end{bmatrix}. \quad (2.52)$$

Under the conditions that $\mathbf{T}_1^+ = X_0$ and $\mathbf{R}_g^- = 0$, we can calculate the transmission and reflection coefficients ($T_{s1}^\pm, T_{s2}^\pm, \dots, T_{sg}^\pm$ and $R_{s1}^\pm, R_{s2}^\pm, \dots, R_{sg}^\pm$) with the transfer matrix method. In the end, the magnetic component H_y of Cherenkov radiation in the region j becomes

$$H_{yj} = i \int_{-\infty}^{+\infty} dk_x \left((-k_x) \left(T_{sj}^+ e^{ik_{yj}(y-y_j)} + R_{sj}^- e^{-ik_{yj}(y-y_{j+1})} \right) \right) e^{ik_x x + ik_z z}. \quad (2.53)$$

2.3 Cherenkov radiation in metamaterials

2.3.1 Effective medium theory and its limitations

As an important member in the family of artificial structures, one-dimensional metamaterials alternating by two subwavelength materials has obtained extensive

attentions for decades [49-53]. The functions of one-dimensional metamaterials are various, including the negative reflection [54, 55], beam splitting [56], topological transition [57-59], invisible cloaking [60, 61], emission enhancement [62, 63] and optical magnification [64, 65]. It is desirable for us to build up a systematic theory, which can effectively describe the optical responses of one-dimensional metamaterials.

Effective medium theory is widely applied to homogenize the periodic metamaterials. Generally, this method can give an effective evaluation for the optical responses of metamaterials [66]. Here, we present the effective medium description of metamaterials of a layered structure in deep subwavelength regime as shown in Fig. 2.4. In the effective medium theory, the effective permittivity of the layered structure has the tensor form as [67, 68]

$$\bar{\bar{\epsilon}}_{eff} = \begin{pmatrix} \epsilon_{r\parallel} & 0 & 0 \\ 0 & \epsilon_{r\perp} & 0 \\ 0 & 0 & \epsilon_{r\parallel} \end{pmatrix}. \quad (2.54)$$

Maxwell Garnet approximation gives the values of the in-plane permittivity and out-of-plane permittivity as [67, 68]

$$\epsilon_{r\parallel} = \frac{\epsilon_1 d_1 + \epsilon_2 d_2}{d_1 + d_2}, \quad (2.55)$$

$$\epsilon_{r\perp} = \frac{\epsilon_1 \epsilon_2 (d_1 + d_2)}{\epsilon_1 d_2 + \epsilon_2 d_1}. \quad (2.56)$$

Figure 2. 4 Schematic diagram of the effective medium theory. The metamaterial is alternating by two isotropic materials with the permittivity of ϵ_{r1} and ϵ_{r2} . The thicknesses of materials are d_1 and d_2 respectively. The period of the metamaterial is $P = d_1 + d_2$. After homogenization of the metamaterial with the effective medium theory, the metamaterial is equivalent to a homogeneous anisotropic material with the permittivity of $\bar{\epsilon}_{eff}$.

The eigenmode equation of the transverse magnetic field in the effective medium theory is given by [68]

$$\frac{k_x^2 + k_z^2}{\epsilon_{r\perp}} + \frac{k_y^2}{\epsilon_{r\parallel}} = k_0^2, \quad (2.57)$$

where k_0 is the wavevector in the vacuum. Typically, as suggested by the effective medium theory, there are three types of eigenfrequency contours in the reciprocal space, as shown in Fig. 2.3. For all-dielectric metamaterials, their isofrequency contours are always elliptical. However, for the metal-based metamaterials, their isofrequency contours could be ellipse, type I hyperboloid or type II hyperboloid, dependent on the operating wavelength. For the isofrequency contour with hyperboloid shape, the in-plane permittivity and out-of-plane permittivity of the metamaterial have the opposite sign. The metamaterials with the hyperboloid-shaped isofrequency contours are named as hyperbolic metamaterials [68].

Figure 2. 5 Isofrequency contour of metamaterials with the homogeneous description. In the reciprocal space, there are three types of isofrequency contours for layered metamaterials: i) elliptical dispersion for $\epsilon_{r||} > 0$ and $\epsilon_{r\perp} > 0$; type I hyperbolic dispersion for $\epsilon_{r||} > 0$ and $\epsilon_{r\perp} < 0$; type II hyperbolic dispersion for $\epsilon_{r||} < 0$ and $\epsilon_{r\perp} > 0$.

Even though the effective medium theory can accurately describe the optical responses of metamaterials in most cases, recent theoretical and experimental studies revealed that the effective medium theory is no longer valid in some specific cases. First of all, the effective medium theory breaks down when the structural dimension becomes comparable with the wavelength, i.e., $P \sim \lambda$ [66]. In such situation, the metamaterials are transformed into the photonic crystals. To be specific, the propagating bulk Bloch modes experience the details of the layered structure. The periodic dielectric contrast results in the formation of photonic bandgaps, which can not be captured by the effective medium theory [69]. Secondly, the effective medium theory is invalid when the bulk Bloch modes with large wavevectors are excited. The bulk Bloch modes with large wavevectors could also see the finite structural size of metamaterials even in the subwavelength regime, leading to the strong spatial dispersion of optical fields. Previous study showed that the distortion and truncation of the isofrequency contour are the

manifestations of the spatial dispersion from the finite structural size [70]. The deviation of the isofrequency contour modifies the optical responses of metamaterials as predicted by the effective medium theory. The bulk Bloch modes with large wavevectors generally come from the coupling between the large-wavevector surface plasmons on the boundaries of metals in the layered structure. Thus the spatial dispersion from the finite structural size should be considered when we study the bulk Bloch modes with large wavevectors in the metal-based metamaterials such as hyperbolic metamaterials. Moreover, the validity of the effective medium theory is lost when the nonlocal electron screening in metals is strong in metal-based metamaterials [71]. The nonlocal electron screening refers to the spatial dispersion of the surface charge density on the boundaries of metals [72, 73]. The nonlocal electron screening is appreciable when structural dimension of metal is comparable with the Tomas-Fermi screening length λ_{TF} (only a few Å) [74]. As a consequence of the nonlocal electron screening, the optical responses of metamaterials become extremely nonlocal and the isofrequency contours are further deviated from the effective medium predictions.

2.3.2 *Hydrodynamic Drude model*

The incidence would induce the polarization charged at the metallodielectric boundary. In the local theory, the surface charge density perfectly resides at the boundary as an impulse function. However, such an assumption is unphysical. The Pauli exclusion principle forbids two fermions from occupying one same quantum state. This results in the quantum repulsion between polarization charges. Consequently, the surface charge density is spatially spreading over the boundary between the metal and dielectric, i.e., the electron spill-out and finite penetration [75], as shown in the Fig. 2.6. In the end, the optical response of nonlocal metals becomes wavevector dependent. Such

an effect is known as the nonlocal electron screening in metals [76-79]. When the structural dimension of metals is much larger than the Tomas-Fermi screening length $\lambda_{TF} = \frac{v_F}{\omega_p}$, the nonlocal electron screening is generally neglected. Here, v_F is the Fermi velocity and ω_p is the bulk plasma frequency. However, as the size of metals becomes comparable with λ_{TF} , the quantum repulsions are so strong that nonlocal electron screening must be taken into account [80, 81]. With the demand for the high density of integration and the availability of the nanofabrication technology, the photonic devices are scaling down to the atomistic level [82, 83]. Such small dimensions of nanostructures unavoidably give rise to strong nonlocal electron screening in metals. To effectively guide future experiments, inclusion of nonlocal electron screening into the theoretical modeling of metamaterials is fundamentally meaningful. In the past few years, researchers have developed several complicated full-quantum approaches, such as the time-dependent density functional theory, which model atomic-length nanostructures in an extremely high precision [84-87]. However, restricted by their high demand for computational sources, these methods are unrealistic to model nanostructures comprising atoms over hundreds [88]. As a semiclassical theory, hydrodynamic Drude model describes the spatial nonlocality of electrons in the form of the first order perturbation term of the charge density from quantum repulsions, meanwhile this method remains economic for computational sources [89, 90]. The simplicity of the hydrodynamic Drude model contributes to in-depth study of metamaterials below the nanoscale [91-97].

Figure 2. 6 Schematic diagram of the nonlocal electron screening in metals. In the local theory, the surface charge density only resides at the surface of metals as an impulse function. However, in the nonlocal theory, the surface charge density has the spatial dispersion over the surface of metals, i.e., electron spill-out and finite penetration.

Below we introduce the hydrodynamic Drude model in details. We start with the generalized momentum equation, derived from the quantum mechanical Hamiltonian in the metallic system [96, 98],

$$m_0 \left(\frac{\partial}{\partial t} + \bar{v}_e \cdot \nabla \right) \bar{v} = -q_0 (\bar{E} + \bar{v}_e \times \bar{B}) - m_0 \gamma \bar{v}_e - \nabla \left(\frac{\delta g[n]}{\delta n} \right), \quad (2.58)$$

where m_0 is the electron mass, q_0 is the element charge, γ is the collision frequency, n is the electron density, $\bar{v}_e$ is the velocity of the electron, and $g[n]$ is the internal energy functional of the electron fluid. On the left of Eq. (2.58), the terms of $\frac{\partial}{\partial t}$ and $\bar{v}_e \cdot \nabla$ represent the accelerated velocity and nonlinearity, respectively. On the right of Eq. (2.58), the terms of $q_0 (\bar{E} + \bar{v}_e \times \bar{B})$, $m_0 \gamma \bar{v}_e$ and $\nabla \left(\frac{\delta g[n]}{\delta n} \right)$ represent Lorenz force in the external static magnetic field, damping motion of the electrons and functional derivation, respectively. Under the assumption that the nonlinearity is negligible and the external magnetic field is absent, Eq. (2.58) can be simplified as

$$m_0 \frac{\partial \bar{v}}{\partial t} = -q_0 \bar{E} - m_0 \gamma \bar{v}_e - \nabla \left(\frac{\delta g[n]}{\delta n} \right). \quad (2.59)$$

Neglecting the exchange and correlation effects of the internal energy functional, we have

$$\nabla \left(\frac{\delta g[n]}{\delta n} \right) \approx m_0 \beta^2 \frac{\nabla n_1}{n_0}, \quad (2.60)$$

where the nonlocal parameter β is proportional to the Fermi velocity v_F as $\beta = \sqrt{\frac{3}{5}} v_F$. n_0 is the equilibrium density and n_1 is first-order term of its perturbation. Thus, the total charge density is $n = n_0 + n_1$. In addition, the continuity of the electron fluid suggests that

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \bar{v}_e) = 0. \quad (2.61)$$

With Eq. (2.59-2.61), we can derive the hydrodynamic current equations in the domain of the frequency and wavevector as

$$-\beta^2 \bar{k} \left(\bar{k} \cdot \bar{J}(\bar{k}, \omega) \right) + \omega(\omega + i\gamma) \bar{J}(\bar{k}, \omega) = i\omega \omega_p^2 \epsilon_0 \bar{E}(\bar{k}, \omega). \quad (2.62)$$

In the nonlocal theory, the quantum repulsions between free electrons give rise to the additional longitudinal electric field. Thus, the nonlocal field is the superposition of the longitudinal electric field and transverse field. For the longitudinal electric field, $\bar{J}(\bar{k}_L, \omega) \parallel k_L$, where k_L is the wavevector of the longitudinal electric field. Accordingly, the nonlocal free-electron response can be obtained from Eq. (2.62) as [99, 100],

$$\epsilon_L(\bar{k}_L, \omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma) - \beta^2 \bar{k}_L^2}. \quad (2.63)$$

Meanwhile, since the permittivity of the longitudinal electric field is determined by the dispersion relation as

$$\epsilon_L(\bar{k}_L, \omega) = 0, \quad (2.64)$$

the wavevector of the longitudinal electric field is determined by

$$k_L = \sqrt{\frac{\omega(\omega+i\gamma)-\omega_p^2}{\beta^2}}. \quad (2.65)$$

For the transverse field, $\bar{J}(\bar{k}_T, \omega) \perp k_T$, where k_T is the wavevector of the transverse field. Under the condition that $\bar{J}(\bar{k}_T, \omega) \perp k_T$, the hydrodynamic Drude model is simplified to the classic Drude model as

$$\epsilon_T(\bar{k}_T, \omega) = 1 - \frac{\omega_p^2}{\omega(\omega+i\gamma)}. \quad (2.66)$$

Thus, in the nonlocal theory, the free-electron response of the transverse field remains same as the one in the local theory.

2.3.3 Local field of Cherenkov radiation

Here we consider a charged particle with the velocity $\bar{v}$ moving along z -axis on top of metal-based metamaterials as shown in Fig. 2.7. In this subsection, we analytically solve for Cherenkov radiation on metal-based metamaterials without the consideration of the nonlocal electron screening in metals.

Figure 2. 7 Schematic diagram of Cherenkov radiation on metal-based metamaterials. The charged particle is traveling along z -direction, with its trajectory near the top

interface of the metamaterial. The metamaterial is alternating by the metal and dielectric periodically.

The source potential from the moving charged particle is given by Eq. (2.30). The potential of transverse fields inside the region j is the superposition of the transverse magnetic (TM) field and transverse electric (TE) field,

$$\phi_{TMj} = \int_{-\infty}^{+\infty} dk_x (T_{TMj}^+ e^{ik_{Tyj}(y-y_j)} + R_{TMj}^- e^{-ik_{Tyj}(y-y_{j+1})}) e^{ik_x x + ik_z z}, \quad (2.67)$$

$$\phi_{TEj} = \frac{1}{\omega \epsilon_0} \int_{-\infty}^{+\infty} dk_x (T_{TEj}^+ e^{ik_{Tyj}(y-y_j)} + R_{TEj}^- e^{-ik_{Tyj}(y-y_{j+1})}) e^{ik_x x + ik_z z}, \quad (2.68)$$

where $k_{Tyj} = \sqrt{\epsilon_{rj} k_0^2 - k_x^2 - k_z^2}$.

The electric field and magnetic field can be determined from the associated potentials by the formulas as [47]:

$$H_T = \nabla \times (\hat{z} \phi_{TMj}) + \frac{i}{\omega \mu_0 \mu_{rj}} \nabla \times \nabla \times (\hat{z} \phi_{TEj}), \quad (2.69)$$

$$E_T = \frac{i}{\omega \epsilon_0 \epsilon_{rj}} \nabla \times \nabla \times (\hat{z} \phi_{TMj}) - \nabla \times (\hat{z} \phi_{TEj}). \quad (2.70)$$

Thus, we can calculate each transverse field component in the region j of the layered structure listed as followings:

$$H_{Txj} = i \int_{-\infty}^{+\infty} dk_x \left(k_{Tyj} (T_{TMj}^+ e^{ik_{Tyj}(y-y_j)} - R_{TMj}^- e^{-ik_{Tyj}(y-y_{j+1})}) - \frac{k_x k_z}{\mu_{rj} k_0^2} (T_{TEj}^+ e^{ik_{Tyj}(y-y_j)} + R_{TEj}^- e^{-ik_{Tyj}(y-y_{j+1})}) \right) e^{ik_x x + ik_z z}, \quad (2.71)$$

$$H_{Tyj} = i \int_{-\infty}^{+\infty} dk_x \left(-k_x (T_{TMj}^+ e^{ik_{Tyj}(y-y_j)} + R_{TMj}^- e^{-ik_{Tyj}(y-y_{j+1})}) - \frac{k_{Tyj} k_z}{\mu_{rj} k_0^2} (T_{TEj}^+ e^{ik_{Tyj}(y-y_j)} - R_{TEj}^- e^{-ik_{Tyj}(y-y_{j+1})}) \right) e^{ik_x x + ik_z z}, \quad (2.72)$$

$$H_{Tzj} = i \int_{-\infty}^{+\infty} dk_x \left(\frac{k_x^2 + k_{Tyj}^2}{\mu_{rj} k_0^2} \left(T_{TEj}^+ e^{ik_{Tyj}(y-y_j)} + R_{TEj}^- e^{-ik_{Tyj}(y-y_{j+1})} \right) \right) e^{ik_x x + ik_z z}, \quad (2.73)$$

$$E_{Txj} = \frac{i}{\omega \epsilon_0} \int_{-\infty}^{+\infty} dk_x \left(-\frac{k_x k_z}{\epsilon_{rj}} \left(T_{TMj}^+ e^{ik_{Tyj}(y-y_j)} + R_{TMj}^- e^{-ik_{Tyj}(y-y_{j+1})} \right) - \right. \\ \left. k_{Tyj} \left(T_{TEj}^+ e^{ik_{Tyj}(y-y_j)} - R_{TEj}^- e^{-ik_{Tyj}(y-y_{j+1})} \right) \right) e^{ik_x x + ik_z z}, \quad (2.74)$$

$$E_{Tyj} = \frac{i}{\omega \epsilon_0} \int_{-\infty}^{+\infty} dk_x \left(-\frac{k_{Tyj} k_z}{\epsilon_{rj}} \left(T_{TMj}^+ e^{ik_{Tyj}(y-y_j)} - R_{TMj}^- e^{-ik_{Tyj}(y-y_{j+1})} \right) + \right. \\ \left. k_x \left(T_{TEj}^+ e^{ik_{Tyj}(y-y_j)} + R_{TEj}^- e^{-ik_{Tyj}(y-y_{j+1})} \right) \right) e^{ik_x x + ik_z z}, \quad (2.75)$$

$$E_{Tzj} = \frac{i}{\omega \epsilon_0} \int_{-\infty}^{+\infty} dk_x \left(\frac{k_x^2 + k_{Tyj}^2}{\epsilon_{rj}} \left(T_{TMj}^+ e^{ik_{Tyj}(y-y_j)} + R_{TMj}^- e^{-ik_{Tyj}(y-y_{j+1})} \right) \right) e^{ik_x x + ik_z z}. \quad (2.76)$$

Similarly, the scattering matrices need to be retrieved to determine the transmission and reflection coefficients. At the boundary $y = y_j$ between the dielectric and metal without nonlocal electron screening, we have $\mathbf{T}_{j-1}^- = [T_{TMj-1}^- \quad T_{TEj-1}^-]^\tau$, $\mathbf{R}_{j-1}^- = [R_{TMj-1}^- \quad R_{TEj-1}^-]^\tau$, $\mathbf{T}_j^+ = [T_{TMj}^+ \quad T_{TEj}^+]^\tau$, $\mathbf{R}_j^+ = [R_{TMj}^+ \quad R_{TEj}^+]^\tau$. The boundary conditions for transverse fields remain same as Eq. (2.37 and 2.38). Thus, we obtain the scattering matrix at the boundary $y = y_j$ as

$$\mathbf{S}_{5j} = \mathbf{M}_1^{-1} \cdot \mathbf{M}_2, \quad (2.77)$$

where

$$\mathbf{M}_1 = \begin{bmatrix} -k_{Tyj-1} & -\frac{k_x k_z}{\mu_{rj-1} k_0^2} & -k_{Tyj} & \frac{k_x k_z}{\mu_{rj} k_0^2} \\ 0 & \frac{k_x^2 + k_{Tyj-1}^2}{\mu_{rj-1} k_0^2} & 0 & -\frac{k_x^2 + k_{Tyj}^2}{\mu_{rj} k_0^2} \\ -\frac{k_x k_z}{\epsilon_{rj-1}} & k_{Tyj-1} & \frac{k_x k_z}{\epsilon_{rj}} & k_{Tyj} \\ \frac{k_x^2 + k_{Tyj-1}^2}{\epsilon_{rj-1}} & 0 & -\frac{k_x^2 + k_{Tyj}^2}{\epsilon_{rj}} & 0 \end{bmatrix},$$

$$\mathbf{M}_2 = \begin{bmatrix} -k_{Tyj-1} & \frac{k_x k_z}{\mu_{rj-1} k_0^2} & -k_{Tyj} & -\frac{k_x k_z}{\mu_{rj} k_0^2} \\ 0 & -\frac{k_x^2 + k_{Tyj-1}^2}{\mu_{rj-1} k_0^2} & 0 & \frac{k_x^2 + k_{Tyj}^2}{\mu_{rj} k_0^2} \\ \frac{k_x k_z}{\epsilon_{rj-1}} & k_{Tyj-1} & -\frac{k_x k_z}{\epsilon_{rj}} & k_{Tyj} \\ -\frac{k_x^2 + k_{Tyj-1}^2}{\epsilon_{rj-1}} & 0 & \frac{k_x^2 + k_{Tyj}^2}{\epsilon_{rj}} & 0 \end{bmatrix}.$$

Inside the region j , we have $\mathbf{T}_j^\pm = [T_{TMj}^\pm \quad T_{TEj}^\pm]^\tau$ and $\mathbf{R}_j^\pm = [R_{TMj}^\pm \quad R_{TEj}^\pm]^\tau$. The scattering matrix is easy to be calculated as

$$\mathbf{S}_{6j} = \begin{bmatrix} 0 & 0 & e^{ik_{Tyj}h_j} & 0 \\ 0 & 0 & 0 & e^{ik_{Tyj}h_j} \\ e^{ik_{Tyj}h_j} & 0 & 0 & 0 \\ 0 & e^{ik_{Tyj}h_j} & 0 & 0 \end{bmatrix}. \quad (2.78)$$

Knowing that $\mathbf{T}_0^- = [X_0 \quad 0]^\tau$ and $\mathbf{R}_g^- = [0 \quad 0]^\tau$, we are able to figure out the transmission and reflection coefficients of transverse fields.

2.3.4 Nonlocal field of Cherenkov radiation

As mentioned before, the nonlocal field of Cherenkov radiation is the superposition of transverse fields and longitudinal electric field. Here, we introduce an additional scalar potential ϕ_L for the longitudinal electric field in the region of metals,

$$\phi_{Lj} = \frac{1}{\omega \epsilon_0} \int_{-\infty}^{+\infty} dk_x (T_{Lj}^+ e^{ik_{Lyj}(y-y_j)} + R_{Lj}^- e^{-ik_{Lyj}(y-y_{j+1})}) e^{ik_x x + ik_z z}, \quad (2.79)$$

where $k_{Lyj} = \sqrt{k_{Lj}^2 - k_x^2 - k_z^2}$ and the k_L is given by Eq. (2.63). Longitudinal electric

field is the gradient of its potential as

$$E_L = \nabla \phi_{Lj}. \quad (2.80)$$

Finally, the longitudinal field components take the form as

$$E_{Lxj} = \frac{i}{\omega \epsilon_0} \int_{-\infty}^{+\infty} dk_x \left(k_x (T_{Lj}^+ e^{ik_{Lyj}(y-y_j)} + R_{Lj}^- e^{-ik_{Lyj}(y-y_{j+1})}) \right) e^{ik_x x + ik_z z}, \quad (2.81)$$

$$E_{Lyj} = \frac{i}{\omega \epsilon_0} \int_{-\infty}^{+\infty} dk_x \left(-k_{Ly}^j (T_{Lj}^+ e^{ik_{Lyj}(y-y_j)} - R_{Lj}^- e^{-ik_{Lyj}(y-y_{j+1})}) \right) e^{ik_x x + ik_z z}, \quad (2.82)$$

$$E_{Lzj} = \frac{i}{\omega \epsilon_0} \int_{-\infty}^{+\infty} dk_x \left(k_z (T_{Lj}^+ e^{ik_{Lyj}(y-y_j)} + R_{Lj}^- e^{-ik_{Lyj}(y-y_{j+1})}) \right) e^{ik_x x + ik_z z}. \quad (2.83)$$

In addition to the conditional boundary, i.e., the continuity of tangent field components, the continuity of the normal component of electric field is applied to calculate the amplitudes of transverse fields and longitudinal electric field as

$$\hat{n} \cdot (\bar{E}_{j-1}(y_j) - \bar{E}_j(y_j)) = 0. \quad (2.84)$$

The inclusion of the longitudinal electric field modifies the scattering matrices at interfaces between the metal and dielectric. Meanwhile the scattering matrices are dependent on the transition direction of the field. When the field transfers from the dielectric to the metal, we have $\mathbf{T}_{j-1}^- = [T_{TMj-1}^- \quad T_{TEj-1}^-]^\tau$, $\mathbf{R}_{j-1}^- = [R_{TMj-1}^- \quad R_{TEj-1}^-]^\tau$, $\mathbf{T}_j^+ = [T_{TMj}^+ \quad T_{TEj}^+ \quad T_{Lj}^+]^\tau$ and $\mathbf{R}_j^- = [R_{TMj}^+ \quad R_{TEj}^+ \quad R_{Lj}^+]^\tau$. The scattering matrix derived from boundary conditions is

$$\mathbf{S}_{7j} = \mathbf{M}_3^{-1} \cdot \mathbf{M}_4, \quad (2.85)$$

where

$$\mathbf{M}_3 = \begin{bmatrix} -k_{Tyj-1} & -\frac{k_x k_z}{\mu_{rj-1} k_0^2} & -k_{Tyj} & \frac{k_x k_z}{\mu_{rj} k_0^2} & 0 \\ 0 & \frac{k_x^2 + k_{Tyj-1}^2}{\mu_{rj-1} k_0^2} & 0 & -\frac{k_x^2 + k_{Tyj}^2}{\mu_{rj} k_0^2} & 0 \\ -\frac{k_x k_z}{\epsilon_{rj-1}} & k_{Tyj-1} & \frac{k_x k_z}{\epsilon_{rj}} & k_{Tyj} & -k_x \\ \frac{k_x^2 + k_{Tyj-1}^2}{\epsilon_{rj-1}} & 0 & -\frac{k_x^2 + k_{Tyj}^2}{\epsilon_{rj}} & 0 & -k_z \\ \frac{k_{Tyj-1} k_z}{\epsilon_{rj-1}} & k_x & \frac{k_{Tyj} k_z}{\epsilon_{rj}} & -k_x & -k_{Lyj} \end{bmatrix},$$

$$\mathbf{M}_4 = \begin{bmatrix} -k_{Tyj-1} & \frac{k_x k_z}{\mu_{rj-1} k_0^2} & -k_{Tyj} & -\frac{k_x k_z}{\mu_{rj} k_0^2} & 0 \\ 0 & -\frac{k_x^2 + k_{Tyj-1}^2}{\mu_{rj-1} k_0^2} & 0 & \frac{k_x^2 + k_{Tyj}^2}{\mu_{rj} k_0^2} & 0 \\ \frac{k_x k_z}{\epsilon_{rj-1}} & k_{Tyj-1} & -\frac{k_x k_z}{\epsilon_{rj}} & k_{Tyj} & k_x \\ -\frac{k_x^2 + k_{Tyj-1}^2}{\epsilon_{rj-1}} & 0 & \frac{k_x^2 + k_{Tyj}^2}{\epsilon_{rj}} & 0 & k_z \\ \frac{k_{Tyj-1} k_z}{\epsilon_{rj-1}} & -k_x & \frac{k_{Tyj} k_z}{\epsilon_{rj}} & k_x & -k_{Lyj} \end{bmatrix}.$$

However, when the field transfers from the metal to the dielectric, we have $\mathbf{T}_{j-1}^- =$

$$[T_{TMj-1}^- \quad T_{TEj-1}^- \quad T_{Lj-1}^-]^\tau, \mathbf{R}_j^- = [R_{TMj-1}^- \quad R_{TEj-1}^- \quad R_{Lj-1}^-]^\tau, \mathbf{T}_j^+ = [T_{TMj}^+ \quad T_{TEj}^+]^\tau,$$

$\mathbf{R}_j^+ = [R_{TMj}^+ \quad R_{TEj}^+]^\tau$. Accordingly, the scattering matrix becomes

$$\mathbf{S}_{8j} = \mathbf{M}_5^{-1} \cdot \mathbf{M}_6, \quad (2.86)$$

where

$$\mathbf{M}_5 = \begin{bmatrix} -k_{Tyj-1} & -\frac{k_x k_z}{\mu_{rj-1} k_0^2} & 0 & -k_{Tyj} & \frac{k_x k_z}{\mu_{rj} k_0^2} \\ 0 & \frac{k_x^2 + k_{Tyj-1}^2}{\mu_{rj-1} k_0^2} & 0 & 0 & -\frac{k_x^2 + k_{Tyj}^2}{\mu_{rj} k_0^2} \\ -\frac{k_x k_z}{\epsilon_{rj-1}} & k_{Tyj-1} & k_x & \frac{k_x k_z}{\epsilon_{rj}} & k_{Tyj} \\ \frac{k_x^2 + k_{Tyj-1}^2}{\epsilon_{rj-1}} & 0 & k_z & -\frac{k_x^2 + k_{Tyj}^2}{\epsilon_{rj}} & 0 \\ \frac{k_{Tyj-1} k_z}{\epsilon_{rj-1}} & k_x & -k_{Tyj} & \frac{k_{Tyj} k_z}{\epsilon_{rj}} & -k_x \end{bmatrix},$$

$$\mathbf{M}_6 = \begin{bmatrix} -k_{Tyj-1} & \frac{k_x k_z}{\mu_{rj-1} k_0^2} & 0 & -k_{Tyj} & -\frac{k_x k_z}{\mu_{rj} k_0^2} \\ 0 & -\frac{k_x^2 + k_{Tyj-1}^2}{\mu_{rj-1} k_0^2} & 0 & 0 & \frac{k_x^2 + k_{Tyj}^2}{\mu_{rj} k_0^2} \\ \frac{k_x k_z}{\epsilon_{rj-1}} & k_{Tyj-1} & -k_x & -\frac{k_x k_z}{\epsilon_{rj}} & k_{Tyj} \\ -\frac{k_x^2 + k_{Tyj-1}^2}{\epsilon_{rj-1}} & 0 & -k_z & \frac{k_x^2 + k_{Tyj}^2}{\epsilon_{rj}} & 0 \\ \frac{k_{Tyj-1} k_z}{\epsilon_{rj-1}} & -k_x & -k_{Tyj} & \frac{k_{Tyj} k_z}{\epsilon_{rj}} & k_x \end{bmatrix}.$$

Inside the dielectric of the region j , the scattering matrix remains same as Eq. (2.74),

while inside the metal of the region j , we have $\mathbf{T}_j^\pm = [T_{TMj}^\pm \quad T_{TEj}^\pm \quad T_{Lj}^\pm]^\tau$, $\mathbf{R}_j^\pm =$

$[R_{TMj}^\pm \quad R_{TEj}^\pm \quad R_{Lj}^\pm]^\tau$ with

$$\mathbf{S}_{9j} = \begin{bmatrix} 0 & 0 & 0 & e^{ik_{Tyj}h_j} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{ik_{Tyj}h_j} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{ik_{Lyj}h_j} \\ e^{ik_{Tyj}h_j} & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{ik_{Tyj}h_j} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{ik_{Lyj}h_j} & 0 & 0 & 0 \end{bmatrix}. \quad (2.87)$$

With the preparations of all the scattering matrices, we can determine the transmission and reflections coefficients for the transverse field and longitudinal electric field with the transfer matrix method. In the end, the nonlocal field of Cherenkov radiation is the superposition of the transverse electric field, transverse magnetic field and longitudinal electric field.

3

Identifying high-energy particles with the Brewster Cherenkov detector

3.1 Background and motivation

Cherenkov radiation has wide applications in the field of particle physics. For example, the ring-imaging Cherenkov detector (RICH) is used to identify the moving charged particles by measuring Cherenkov angles [101]. However, previous Cherenkov detectors have a limited momentum range to discriminate charged particles. The working momentum range for previous Cherenkov detectors is constrained by limited refractive indices of natural transparent materials. For example, to identify the charged particles in the momentum around teraelectronvolt, we need a transparent medium with the refractive index below 1.000001. However, such a material is unavailable in the nature. In the end, we have to resort to artificial structures, e.g., photonic crystals, to break such limitations by natural transparent materials. Recent work theoretically

reported that the resonance transition radiation in the one-dimensional photonic crystals enables us to control Cherenkov angles in high sensitivity in any desired range of momenta [36]. However, for this strategy, the sensitivity of the proposed Cherenkov detector is stringently dependent on the frequency, reducing the corresponding working bandwidth. Therefore, it naturally forces researchers to look for a non-resonant mechanism, which not only enables us to effectively control Cherenkov angles in a wide momentum range but also maintains the high sensitivity in a broad range of frequencies.

In this work, we theoretically proposed a new mechanism of Cherenkov radiation, i.e., Brewster Cherenkov radiation to address above problems. Brewster Cherenkov radiation is produced when the moving charged particle interacts with the Brewster photonic crystal. The Brewster Cherenkov detector makes use of Brewster Cherenkov radiation to discriminate charged particles. Our study shows that the Brewster photonic crystal has wide controllability of its effective refractive index. Thus the Brewster Cherenkov detector allows the particle identification in a much wider momentum range than ever before. More importantly, the Brewster Cherenkov detector works in a broad range of frequencies, facilitated by photonic bandgaps and Brewster effect. In addition, the Brewster Cherenkov detector is immune to the fabrication defects.

3.2 Brewster Cherenkov detector

3.2.1 Demonstration of the Brewster Cherenkov detector

We begin our analysis by considering a charged particle travelling along z -axis in a background environment with the permittivity of ϵ_1 . The equi-phase surface of the photon emission from the charged particle forms the Cherenkov cone, when the particle

velocity is beyond the threshold velocity, i.e., $v > v_{th} = \frac{c}{\sqrt{\epsilon_1}}$. If a Brewster photonic crystal is inserted below the particle trajectory, the Cherenkov cone would be modulated by the Brewster photonic crystal. After proper design, the Brewster photonic crystal is transparent to Cherenkov photons only at specific incidence angle in a broad range of frequencies. In the end, the spatial modulation of Cherenkov radiation leaves a pair of Cherenkov pulses in the detection plane under the Brewster photonic crystal as shown in Fig. 3.1(a). The configuration of the Brewster photonic crystal is shown as Fig. 3.1(b). The Brewster photonic crystal is constructed by a series of individual photonic crystals, whose pitches are ranging from P_0 to P_M . All the photonic crystals are made of two transparent dielectrics with different permittivities.

At a certain momentum, charged particles with different rest masses generate Cherenkov pulses with different shapes. The shape of Cherenkov pulses is characterized by the Brewster Cherenkov angle θ_{BCR} , which refers to the angle between the Brewster wavevector $\bar{k}_B$ and the particle trajectory (seeing the inset of Fig. 3.2). The Brewster wavevector is given by $k_B = n_{eff}k_0$, where n_{eff} is the effective refractive index of the Brewster photonic crystal. To clearly show the shape of Cherenkov pulses, we plot the time-domain field distribution of Cherenkov pulses in the detection plane as shown in Figure 3.2. When the particle velocity is $v = 0.93c$, a pair of Cherenkov pulses are symmetrically distributed along x -axis. Moreover, the in-plane wavevector $\bar{k}_{||} = \bar{k}_B$ is normal to the wave front of each Cherenkov pulse. To discriminate charged particles, we have to determine the shape of Cherenkov pulses in the detection plane.

We identify charged particles by measuring the lateral position of Cherenkov pulses. The Cherenkov pulse with certain Brewster Cherenkov angle has a determined position along the x direction in the detection plane. By measuring the lateral position of Cherenkov pulses with the photodetector array along x -direction in the detection

plane, we can infer the shape of Cherenkov pulses. Using this method, we can therefore determine the particle velocities and the corresponding particle masses at a given momentum. In the end, the unknown particles can be identified with the proposed Brewster Cherenkov detector.

Figure 3. 1 Schematic diagram of the Brewster Cherenkov detector. (a) Three-dimensional demonstration of the Brewster Cherenkov detector. The Cherenkov cone is modulated by the Brewster photonic crystal, leaving a pair of Cherenkov pulses in the detection plane at $|y| = y_0$. The distance between two Cherenkov pulses is denoted as $2|x_{BCR}|$. (b) Construction of Brewster photonic crystal. The Brewster Photonic crystal is composed of M stacks of individual photonic crystals. Each individual photonic crystal is made of two dielectrics with the pitch of $P_i = d_{i1} + d_{i2}$, where d_{i1} and d_{i2} are thicknesses of two dielectrics, respectively. The pitch number of each individual

photonic crystal is N . The pitch of the i^{th} photonic crystal is $P_i = P_0 \times r^{i-1}$, where $r = \sqrt[M]{P_M/P_0}$ and P_0 (P_M) is the pitch of the first (last) stack of the photonic crystal.

Figure 3. 2 Field distribution of Cherenkov pulses in the time domain. The distribution of the field component E_y is plotted when the particle velocity $v = 0.93c$ at the detection plane $y_0 = 2.3$ mm. The inset is the partial enlarged view of Cherenkov pulses. The Brewster wavevector k_B is normal to the equi-phase surface as shown clearly in the inset. The Brewster Cherenkov angle θ_{BCR} is defined as the angle between the in-plane wavevector $\bar{k}_{||} = \bar{k}_B$ and the particle trajectory, where $\bar{k}_B$ is the Brewster wavevector.

3.2.2 Brewster photonic crystal

The effective refractive index of the Brewster photonic crystal is controllable and dispersion-less in a broad range of frequencies. The transmission of the Brewster photonic crystal is dependent on both the incidence angle and the incidence polarization

[102]. Considering the p -polarized field of Cherenkov radiation, the effective refractive index of the Brewster photonic crystal can be efficiently controlled by properly choosing dielectrics with permittivities of ϵ_{r1} and ϵ_{r2} (Fig. 3.3). We can tune the effective refractive index to the value, which is unavailable for natural transparent materials. For example, the Brewster photonic crystal made of SiO_2 and Al_2O_3 has $n_{eff} = 1.13$ in the whole visible wavelength range, which is unavailable for natural materials. In addition, we can even achieve the n_{eff} arbitrarily close to the unity, much smaller than the refractive indices of gasses and aerogels.

Figure 3. 3 Engineering the effective refractive index with Brewster effect. The ratio of the relative permittivity $\epsilon_{r2}/\epsilon_{r1}$ determines the Brewster angle $\theta_{Brewster}$. The dispersion-less effective refractive index n_{eff} is controlled by $\theta_{Brewster}$ and ϵ_{r2} . The n_{eff} can easily reach the value that is unavailable in natural material including the one infinitely close to the unity $n_{eff} \rightarrow 1$. For example, Brewster photonic crystal made of

SiO₂ and Al₂O₃ has an effective refractive index $n_{eff} = 1.13$, where $\epsilon_{r1} = 2.18$ is the permittivity of SiO₂ and $\epsilon_{r2} = 3.07$ is the permittivity of Al₂O₃. However, the refractive index of 1.13 is unavailable for natural transparent materials.

Fig. 3.4 shows the principal of the Brewster photonic crystal. An individual periodic photonic crystal has a Brewster angle satisfying $\tan\theta_{Brewster} = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}$ or a Brewster wavevector satisfying $k_B = \sqrt{\epsilon_{r1}} \sin \theta_{Brewster}$. Only at the Brewster wavevector, the transmission of the *p*-polarized incidence is 100%. Off the Brewster wavevector, the incidence is partially reflected. Meanwhile, the periodic dielectric contrast in the photonic crystal forms photonic bandgaps off the Brewster wavevector. Incidence within the photonic bandgap undergoes 100% reflection. Fig. 3.4(a,b) shows that the location of the photonic bandgap is intimately relevant to the pitch of the photonic crystal. The Brewster photonic crystal is created by stacking photonic crystals with different pitches together. The spectrum of the Brewster photonic crystal shows that the *p*-polarized field of Cherenkov radiation remains 100% transmission at k_B but has little transmission off k_B in a broad range of frequencies (Fig. 3.4(c)). This is because the incidence off k_B is all blocked by photonic bandgaps of individual photonic crystals. Thus, we can define the effective refractive index of the Brewster photonic crystal as

$$n_{eff} = \left| \frac{\bar{k}_B}{k_0} \right| = \sqrt{\frac{\epsilon_{r1}\epsilon_{r2}}{\epsilon_{r1} + \epsilon_{r2}}}. \quad (3.1)$$

Knowing the particle velocity v , the Cherenkov pulse emitted at the Brewster angle θ_B has $k_x = \pm \sqrt{n_{eff}^2 k_0^2 - \left(\frac{\omega}{v}\right)^2}$ and $\cos \theta_{BCR} = \frac{k_z}{|\bar{k}_B|} = \frac{c}{n_{eff}v}$. It is notable that both the wavevector component k_x and the Brewster Cherenkov angle θ_{BCR} are solely

dependent on the particle velocity v , but are irrelevant to the wavelength λ (Fig. 3.4(d-i)). Thus, it is feasible to measure the particle velocity or identify unknown particles with the Brewster photonic crystal in the broad range of frequencies.

Figure 3. 4 Transmission spectra of the Brewster photonic crystal. (a,b) The transmission spectra of single periodic photonic crystals with the pitch P of 200 nm and 250 nm, respectively. (c) The transmission spectrum of the Brewster photonic crystal in Fig. 3.1(b), where $P_0 = 120$ nm and $P_M = 370$ nm. (e)-(f) The transmission spectra as the function of k_x and λ when the particle velocity v/c is 0.95, 0.97 and 0.99, respectively. (g)-(i) The transmission spectra as the function of the in-plane angle θ and the particle velocity v/c , when the wavelength λ is 0.5 μm , 0.6 μm and 0.7 μm , respectively.

3.2.3 Sensitivity of the Brewster Cherenkov detector

There are two factors that restrict the sensitivity of the Brewster Cherenkov detector. On the one hand, similar to the conventional Cherenkov detector, the sensitivity of the Brewster Cherenkov detector is restricted by the effective refractive index (Fig. 3.5). For a designed Brewster photonic crystal with the effective refractive index n_{eff} , the condition to produce the Cherenkov pulse is that the particle velocity is larger than the Brewster Cherenkov threshold $\beta_{th} = \frac{1}{n_{eff}}$. The sensitivity of the Brewster Cherenkov angle θ_{BCR} on the particle velocity v is considerable only when β is slightly larger than the β_{th} , i.e., $\beta \gtrsim \beta_{th}$. And the Brewster Cherenkov detector becomes insensitive to identify charged particles when v becomes much larger than β_{th} in Fig. 3.5. In other word, to identify charged particles in teraelectronvolt range, we would require an effective refractive index n_{eff} very close to the unity.

Figure 3. 5 The Brewster Cherenkov angle versus the particle velocity. The relation between the particle velocity $\beta = v/c$ and the Brewster Cherenkov angle θ_{BCR} , with

respect to different effective refractive indices n_{eff} . The dashed line represents the correspond Cherenkov threshold of the swift charged particle. β_{th} is the corresponding Brewster Cherenkov threshold.

Thanks to the wide controllability of the effective refractive index of the Brewster photonic crystal, the Brewster Cherenkov angle can be controlled in an extended momentum range. As shown in Fig. 3.6, we design the Brewster Cherenkov detector with effective refractive indices n_{eff} of 1.13 and 1.00000013. Particularly, when $n_{eff} = 1.13$, the Brewster Cherenkov detector can identify unknown particles in 1~10 GeV/c. More importantly, when $n_{eff} = 1.00000013$, the Brewster Cherenkov detector can identify particles in 2~5 TeV/c, in which previous Cherenkov detectors become insensitive to the particle masses.

Figure 3. 6 Performance of the Brewster Cherenkov detector with different effective refractive indices. By engineering the effective refractive index n_{eff} , the Brewster Cherenkov detector can identify charged particles at an extended momentum range. The Brewster Cherenkov detector with n_{eff} of 1.13 in (a) and 1.00000013 in (b) could

successfully identify charged particles in the momentum range of 1~10 GeV/ c and 2~5 TeV/ c , respectively.

On the other hand, the sensitivity of the Brewster Cherenkov detector is also determined by the finite size of the Brewster photonic crystal. When both the stack number of photonic crystals M and the pitch number of an individual photonic crystal N approach to infinity, the Brewster photonic crystal only allows the transmission of the p -polarized field of Cherenkov radiation with $\bar{k}_{\parallel} = \bar{k}_B$. However, the values of M and N are finite in a realistic Brewster photonic crystal. The finite size of the Brewster photonic crystal ensures that the incidence with the wavevector off $\bar{k}_B$ has partial transmission. In the end, the wavevector of Cherenkov pulses is actually spreading as $\Delta\bar{k}_{\parallel}$ over $\bar{k}_B$. The wavevector spreading extent $\Delta\bar{k}_{\parallel}$ broadens the Brewster Cherenkov angle θ_{BCR} as $\Delta\theta_{\text{BCR}}$. As a consequence, the θ_{BCR} becomes insensitive to the small variation of the particle velocity v . As shown in Fig. 3.7, the only way to enhance the sensitivity limited by the finite size of the Brewster photonic crystal is to increase the M and N .

Figure 3. 7 Evolution of transmission spectra with the increase of the size of the Brewster photonic crystal. (a,b) The transmission spectra as the function of the in-plane wavevector $k_{||}$ and the wavelength λ . (c,d) The transmission spectra as the function of k_x and λ , where the particle velocity $v/c = 0.99$. (e,f) The transmission spectra as the function of the in-plane angle θ and the particle velocity v/c when $\lambda = 500$ nm. In (a), (c) and (e), $M = 100$ and $N = 20$. In (b), (d) and (f), $M = 200$ and $N = 50$. M is the stack number of photonic crystals and N is the pitch number of an individual photonic crystal.

3.2.4 Evaluating position of Cherenkov pulses and measurement error

We probe the shape of Cherenkov pulses indirectly by measuring the lateral position of Cherenkov pulses in the detection plane. Applying the ray tracing method, the lateral position of Cherenkov pulses $\left| \frac{x_{\text{BCR}}}{y_0} \right|$ is related to the Brewster Cherenkov angle θ_{BCR} by the expression as

$$\left| \frac{x_{\text{BCR}}}{y_0} \right| = \frac{n_{\text{eff}}}{\sqrt{\epsilon_1 - n_{\text{eff}}^2}} \sin(\theta_{\text{BCR}}) + \Delta_{\text{int}}(y_0). \quad (3.2)$$

The $\Delta_{\text{int}}(y_0)$ is the lateral displacement caused by the internal refraction of the Brewster photonic crystal, which is calculated as

$$\Delta_{\text{int}}(y_0) = \frac{\Delta x_{\text{BCR}}}{y_0} = - \left(\frac{\sqrt{\frac{n_{\text{eff}}^2 - c^2}{v}}}{\sqrt{\epsilon_{r1} - n_{\text{eff}}^2}} - \frac{\sqrt{\frac{n_{\text{eff}}^2 - c^2}{v}}}{\sqrt{\epsilon_{r2} - n_{\text{eff}}^2}} \right) \frac{y_2}{y_0}, \quad (3.3)$$

where the y_2 is the total thickness of the dielectric with ϵ_{r2} in the Brewster photonic crystal. Replacing $\sin(\theta_{\text{BCR}})$ by $\sqrt{1 - \frac{c^2}{n_{\text{eff}}^2 v^2}}$, we formulize the relation between the lateral position of Cherenkov pulses $\left| \frac{x_{\text{BCR}}}{y_0} \right|$ and the particle velocity v as

$$\left| \frac{x_{\text{BCR}}}{y_0} \right| = \frac{\sqrt{n_{\text{eff}}^2 - \frac{c^2}{v^2}}}{\sqrt{\epsilon_1 - n_{\text{eff}}^2}} + \Delta_{\text{int}}(y_0). \quad (3.4)$$

With Eq. (3.4), we can theoretically predict the lateral position of Cherenkov pulses from charged particles in any momentum as shown in Fig. 3.8. At the momentum of 1.8 or 2.2 GeV/c, the predictions from the ray tracing method are identical to the results in Fig. 3.9, which reflects the actual positions of Cherenkov pulses in the detection plane.

Figure 3. 8 Lateral position of Cherenkov pulses predicted by the ray tracing method. The Brewster photonic crystal is constructed by Al_2O_3 and SiO_2 , with the effective

refractive index $n_{eff} = 1.13$ in the visible wavelength range. The lateral position of Cherenkov pulses varies for different charged particles at a given momentum.

Figure 3. 9 Lateral positions of Cherenkov pulses from numerical results. The time-domain intensity distribution of Cherenkov pulses in the detection plane, when the momenta are (a) 1.8 GeV/c and (b) 2.2 GeV/c, respectively. Here, the detection plane is placed at $y_0 = 2.3 \mu\text{m}$.

In the actual measurement, we probe Cherenkov pulses in the far field for the particle identification. In the limit that $y_0 \gg h$, the lateral position is proportional to $\sin(\theta_{\text{BCR}})$ as

$$\lim_{y_0 \rightarrow \infty} \left| \frac{x_{\text{BCR}}}{y_0} \right| = \frac{n_{\text{eff}}}{\sqrt{\epsilon_1 - n_{\text{eff}}^2}} \sin(\theta_{\text{BCR}}), \quad (3.5)$$

where h is the thickness of the Brewster photonic crystal. However, when the y_0 is finitely large, the system error mainly comes from the internal refraction in the Brewster photonic crystal. As shown in Fig. 3.10, when $y_0 = 2.3 \mu\text{m}$, there is an obvious deviation $\Delta_{\text{int}}(y_0)$ between the realistic lateral position of Cherenkov pulses and the one predicted by Eq. 3.5. However, when $y_0 = 23 \mu\text{m}$, the thickness h of the Brewster photonic crystal becomes negligible compared with y_0 . In the end, the lateral displacement from the internal refraction in the Brewster photonic crystal becomes relatively minor. In Fig. 3.11, the calculated lateral positions with considering $\Delta_{\text{int}}(y_0)$ and without considering $\Delta_{\text{int}}(y_0)$ show a convergent tendency with the increase of y_0 , indicating the reduction of the system error from the internal refraction in the Brewster photonic crystal.

Figure 3. 10 Lateral displacement $\Delta_{int}(y_0)$ caused by the internal refraction in the Brewster photonic crystal. (a) and (c) show the lateral position of Cherenkov pulses as the function of the Brewster Cherenkov angle θ_{BCR} . (b) and (d) show the lateral position of Cherenkov pulses as the function of the momentum p of proton. The detection plane is placed at $y_0 = 2.3 \mu\text{m}$ for (a) and (b), and $y_0 = 23 \mu\text{m}$ for (c) and (d). The dashed line represents the predicted position of Cherenkov pulses without considering the $\Delta_{int}(y_0)$. The solid line represents the predicted position of Cherenkov pulses considering the $\Delta_{int}(y_0)$.

Figure 3. 11 Lateral position of Cherenkov pulses versus the distance between the particle trajectory and the detection plane y_0 . The dashed line represents the predicted position of Cherenkov pulses without considering the $\Delta_{int}(y_0)$. The solid line represents the predicted position of Cherenkov pulses considering the $\Delta_{int}(y_0)$. Here, the studied swift charged particle is proton.

3.2.5 Polarization discussion

So far, we have only discussed the p -polarized field of Cherenkov radiation without the s -polarized field of Cherenkov radiation. Fig. 3.12(a) indicates that the s -polarized field of Cherenkov radiation is non-negligible and even becomes dominant when the incidence angle θ_i approaches to 90° . How the s -polarized field of Cherenkov radiation contributes to the transmission spectrum of Brewster Cherenkov radiation still remains elusive. Thus, we calculated the transmission spectrum for the s -polarized field and found that the s -polarized field has little transmission in the studied range of wavelengths (Fig. 3.12(b)). Unlike the p -polarized field, s -polarized field does not have Brewster effect. As a result, the transmission of s -polarized Cherenkov photons is disabled by multiple photonic bandgaps in the visible wavelength range.

Figure 3.12 S -polarized field of Brewster Cherenkov radiation. (a) The proportion of p -polarized field in Cherenkov radiation at different incidence angle θ_i of Cherenkov photon. I_p and I_s are intensities of p -polarized field and s -polarized field of Cherenkov radiation, respectively. (b) The transmission spectrum of the Brewster photonic crystal for the s -polarized Cherenkov photon.

3.3 Conclusion

To conclude, we presented a novel mechanism – Brewster Cherenkov radiation to identify moving charged particles. The Brewster Cherenkov radiation arises from the interaction between the charged particle and the Brewster photonic crystal. Our study shows that the effective refractive index of the Brewster photonic crystal can be flexibly

engineered. The large freedoms of the Brewster photonic crystal enable us to sensitively discriminate charged particles in a much wider momentum range than ever before. For example, by engineering the effective refractive index of the Brewster photonic crystal very close to the unity, we can efficiently identify charged particles in the teraelectronvolt momentum, in which previous Cherenkov detectors generally could not operate. Meanwhile, the Brewster Cherenkov detector could work in a wide range of frequencies, because the effective refractive index of the Brewster photonic crystal has minor dispersion. This facilitates the experimental measurement of Cherenkov pulses in the detection plane. Moreover, the Brewster Cherenkov detector is robust to the fabrication defects, since Brewster effect is irrelevant to the thickness of each dielectric. Last but not least, we update the measurement strategies of Cherenkov radiation by probing Cherenkov pulses below the particle trajectory. Our method would prevent the photodetectors from the bombardment by high-energy particles. The proposal of the Brewster Cherenkov detector not only enriches the family of Cherenkov detectors, but also accelerates the development of particle physics.

4

Revealing the nonlocal regime of ‘threshold-free’ Cherenkov radiation

4.1 Background and motivation

According to Frank and Tamm’s theory, the threshold condition for the production of Cherenkov photons in a transparent medium is given by $v_{th} = c/n$, where n is the refractive index of the transparent medium and c is the light velocity in the free space [43]. Due to the finite value of n in natural materials, especially for transparent dielectrics in the optical regime ($n < 4$), the threshold energy for Cherenkov radiation is generally in the order of mega-electron-volt. This inevitably limits the potential applications of Cherenkov radiation in low-energy regimes, such as radiation sources from low-energy particles [103]. Driven by the demands for compact free-electron light sources without any threshold velocity, researchers show substantial interest in the manipulation of Cherenkov radiation with artificial materials [24, 25, 36, 104, 105].

Surprisingly, the Cherenkov threshold completely vanishes in a broad range of frequencies when a moving charged particle interacts with hyperbolic metamaterials, experimentally reported by Liu et al. [27]. Hyperbolic metamaterials refer to the extremely anisotropic media with the isofrequency surface open hyperboloid-shaped, where the in-plane permittivity has an opposite sign with the out-of-plane permittivity [67, 68]. Recently, hyperbolic metamaterials are rapidly developing in many potential fields such as super-resolution imaging [64, 65], ultrasensitive biosensing [106, 107] and lifetime engineering [62, 108]. The widespread applications of hyperbolic metamaterials are facilitated by the existence of bulk Bloch modes with large wavevectors, which also helps explain the elimination of the Cherenkov threshold in such configuration. Theoretically, an indefinite medium with the hyperbolic dispersion has unbounded wavevectors of bulk Bloch modes, contributing to the infinite photonic density of states over a broad range of frequencies [109]. The extremely high photonic density of states ensures the increased number of radiative decay channels, overcoming non-radiative decay routes of the charged particle such as phonons [108]. As a consequence of ultrafast radiative decay channels, hyperbolic metamaterials could efficiently extract Cherenkov photons emitted from the charged particle. With the dramatical enhancement of the spontaneous emission by the charged particle, the Cherenkov threshold is disappearing in hyperbolic metamaterials.

It is ascertained that Cherenkov radiation is threshold-free in homogeneous hyperbolic metamaterials, while a comprehensive theory predicting its behaviors in a realistic situation is still missing. In fact, realistic hyperbolic metamaterials are usually made by periodic subwavelength metal planes or metal rods in a dielectric host material [68]. Previous work revealed that the spatial dispersion from the finite structural size and the nonlocal electron screening in metals are ubiquitous effects in realistic

hyperbolic metamaterials [74, 110]. The spatial nonlocality gives rise to a truncation of large wavevectors of bulk Block modes, leading to the divergence of the photonic density of states [70]. It can be anticipated that the finite photonic density of states will finally induce a lower-bound velocity cutoff for Cherenkov radiation in realistic hyperbolic metamaterials. How much these nonlocal effects contribute to the Cherenkov threshold and modify other behaviors of Cherenkov radiation in the hyperbolic metamaterials still remains elusive.

In present paper, we theoretically explored Cherenkov radiation in the hyperbolic metamaterials with nonlocal corrections and observed the Cherenkov threshold existing in a broad range of frequencies. We discussed the impacts of the spatial dispersion from the finite structural size and the nonlocal electron screening in metals on the Cherenkov threshold, when the structural dimension is extending from the classically described regime to the quantum size regime. Remarkably, the nonlocal effects ensure that the threshold energy is always larger than or equal to $\sqrt{3/5}v_F$ (i.e., $v_{\text{th}} \geq \sqrt{3/5}v_F$), where v_F is the Fermi velocity. Besides, we surprisingly found the emergence of longitudinal Cherenkov photons at the epsilon-near-zero frequencies in the periodic layered structure. Even though the nonzero Cherenkov threshold deteriorates the performance of free-electron sources in low-energy regimes, strong longitudinal Cherenkov photons instead indicate potential applications such as compact monochromatic Cherenkov sources. Taken together, our work consolidates the claim of nonzero Cherenkov threshold in Frank and Tamm's theory, and provides a theoretical guidance for the research of potential applications of Cherenkov radiation in low-energy regimes.

4.2 Nonlocal regime of ‘threshold-free’ Cherenkov radiation

4.2.1 Homogeneous description of the metamaterial

To begin with, we consider a hyperbolic metamaterial constructed by the periodic metallodielectric bilayers with a subwavelength unit cell, where the thicknesses of the metal and dielectric slabs are d_1 and d_2 , respectively. The pitch of the unit cell is $P = d_1 + d_2$, with $d_1/P = 0.4$ and $P \ll \lambda$, where the studied wavelength range is $\lambda \in [0.1 \text{ } 10] \text{ } \mu\text{m}$. The relative permittivities of the metal and dielectric are denoted as ϵ_{r1} and ϵ_{r2} , respectively.

In the homogeneous description of the realistic periodic structure. the effective permittivity of the homogeneous medium has the diagonal tensor form as $\bar{\epsilon}_{eff} = \{\epsilon_{\parallel} \quad \epsilon_{\perp} \quad \epsilon_{\parallel}\}$. The Maxwell Garnett approximation gives values of the effective in-plane permittivity $\epsilon_{\parallel}$ and the effective out-of-plane permittivity $\epsilon_{\perp}$ by Eq. (2.55 and 2.56).

As a concrete example, we select silver (Ag) as the metal with ϵ_{r1} and silicon nitride (Si_3N_4) as the dielectric with $\epsilon_{r2} = 4$. Here, $\epsilon_{r1} = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$, where $\omega_p = 8 \text{ eV}$ and $\gamma = 0.032 \text{ eV}$ [39, 111]. The dispersion curves of $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ are shown as Fig. 4.1. From dispersion curves, we classify the whole wavelength bands into four regions as followings, i.e., (i). $\lambda < 0.16 \mu\text{m}$: elliptical dispersion relation; (ii). $0.16 \mu\text{m} < \lambda < 0.3 \mu\text{m}$: type I hyperbolic dispersion relation; (iii). $0.3 \mu\text{m} < \lambda < 0.4 \mu\text{m}$: elliptical dispersion relation; (iv). $\lambda > 0.4 \mu\text{m}$: type II hyperbolic dispersion relation. Especially, the epsilon-near-zero region in the given configuration refer to the wavelength band $\lambda \in [0.36, 0.41] \text{ } \mu\text{m}$. In the hyperbolic region, the Cherenkov threshold is supposed to vanish in the homogeneous description.

Figure 4. 1 Effective permittivity of the metamaterial with the homogeneous description. $\epsilon_{||}$ is the effective in-plane permittivity and $\epsilon_{\perp}$ is the effective out-of-plane permittivity. Below 0.16 μm and between 0.3~0.4 μm , the homogeneous medium exhibits elliptical dispersion. Between 0.16~0.3 μm , the homogeneous medium becomes type I hyperbolic metamaterial. Above 0.4 μm , the homogeneous medium becomes type II hyperbolic metamaterial.

4.2.2 Cherenkov threshold induced by the spatial dispersion from the finite structural size

Without loss of generality, we consider that a charge moves with a velocity of $\bar{v}$ in the air, with its trajectory parallel to and close to the top surface of a hyperbolic metamaterial, as displayed schematically in Fig. 4.2(a).

We begin our analysis by studying Cherenkov radiation in a hyperbolic metamaterial in the classically described regime where the size of metals d_1 is far more than the Tomas-Fermi screening length λ_F , i.e., $d_1 \gg \lambda_{TF}$. The spatial extent of the surface charge density is in the order of $\lambda_{TF} \approx \frac{\beta}{\omega_p}$, which is generally a few Å [112].

Here the nonlocal parameter $\beta = \sqrt{\frac{3}{5}}v_F$, where v_F is the electron Fermi velocity. Due to the negligible spatial extent of surface charge density with respect to the geometric size, radiation fields by the swift charge in the classic layered structure and in the layered structure with nonlocal screening show identical contour maps. Thus, the spatial dispersion from the finite structural size is the dominant mechanism that affects the Cherenkov radiation.

To investigate the influence of the spatial dispersion from the finite structural size on Cherenkov radiation, we take $d_1 = 10$ nm and $P = 25$ nm. Figures 4.2(b-e) show the distribution of radiation fields in the time domain. Cherenkov radiation appears in the homogeneous hyperbolic metamaterial for arbitrary value of the charge velocity (Figs. 4.2(b, d)). In contrast, with the consideration of the spatial dispersion of the finite structural size, the emergence of Cherenkov radiation becomes dependent on the charge velocity [Figs. 4.2(c, e)]. Below we denote the threshold velocity at each wavelength as $v_{th}(\lambda)$ and the minimum value of all Cherenkov thresholds in the studied wavelength range as $\min(v_{th}(\lambda))$. Then it can be clearly seen that Cherenkov radiation into the bulk hyperbolic metamaterial appears when $v > \min(v_{th}(\lambda))$ (Fig. 4.2(c)), but disappears when $v < \min(v_{th}(\lambda))$ (Fig. 4.2(e)), indicating the existence of the Cherenkov threshold. In addition to the Cherenkov threshold, the surface plasmons at the interface between the air and hyperbolic metamaterial also show up when taking into account the spatial dispersion from the finite structural size (Figs. 4.2(c, e)). We note that when v is sufficiently large in Figs. 4.2(b, c), Cherenkov radiation in the homogeneous hyperbolic metamaterial effectively characterizes the field distributions inside the layered structure. However, when v is close to or smaller than $\min(v_{th}(\lambda))$, the spatial dispersion from the finite structural size prohibits the excitation of bulk Bloch modes with large wavevectors. In the end, the homogenous description of the hyperbolic

metamaterial fails to correctly model the realistic situation (Figs. 4.2(d, e)). From Fig. 4.2, it is reasonable to argue that even if the hyperbolic metamaterial has deep-subwavelength unit cells with $P \ll \lambda$, the spatial dispersion from the finite structural size plays a nontrivial role to give rise to a nonzero Cherenkov threshold and to determine the features of Cherenkov radiation when the charge velocity is on the verge of the threshold velocity.

Figure 4. 2 Influence of the spatial dispersion from the finite structural size on the Cherenkov threshold. (a) Structural schematic. A swift electron moves with a velocity of $\vec{v} = \hat{z}v$ and its trajectory parallel to the interface of the air and hyperbolic metamaterial. The hyperbolic metamaterial is constructed by a periodic layered structure, with its unit cell composed by a metal slab (e.g. silver, with a thickness of d_1) and a dielectric slab (e.g. Si_3N_4 , with a thickness of d_2). The pitch of unit cell is $P =$

$d_1 + d_2$ ($P \leq 50$ nm), which is much smaller than the interested wavelength λ in the free space ($\lambda \in [0.1 \text{ } 10] \text{ } \mu\text{m}$). For all studied cases here and below, we keep $d_1/P = 0.4$ unchanged. (b-e) Field distribution of Cherenkov radiation in the time domain. The realistic layered structure in (a) is adopted for (c, e), while it is replaced by a homogeneous medium with the effective permittivity of $\bar{\epsilon}_r = \{\epsilon_{||} \ \epsilon_{\perp} \ \epsilon_{||}\}$ for (b, d). The nonlocality will give rise to a wavelength-dependent threshold velocity, $v_{th}(\lambda)$ for the emergence of Cherenkov radiation. As a result, with the studied wavelength range of $\lambda \in [0.1 \text{ } 10] \text{ } \mu\text{m}$, Cherenkov radiation appears only when $v = v_1 > \min(v_{th}(\lambda))$ [see (c) for example], but disappears if $v = v_2 < \min(v_{th}(\lambda))$. Other parameters setup in (b-e) are: $P = 25$ nm, $v_1 = 0.2c$ and $v_2 = 0.05c$, where c is the speed of light in the free space.

4.2.3 Cherenkov threshold induced by the nonlocal electron screening in metals

Next we proceed to analyze the influenced of nonlocal electron screening in metals on the Cherenkov threshold. The nonlocal electron screening refers to spatial spreading of induced surface charge densities over boundaries of metals, originating from the quantum repulsion of electrons or the electron gas pressure. As a manifestation of the nonlocal electron screening, the spreading length is comparable with the Tomas-Fermi wavelength $\lambda_{TF} = \frac{\beta}{\omega_p}$, where the nonlocal parameter β is generally around $c/300$ for realistic metals (the value could be even one order of magnitude smaller for semiconductors). Therefore, nonlocal electron screening would have significant impact

on the electromagnetic responses of nanostructure when the size of nonlocal metal is comparable to λ_{TF} (at the atomistic length level).

Figure 4.3 studies Cherenkov radiation in a hyperbolic metamaterial within the quantum size regime, where the nonlocal electron screening in metals is non-negligible, i.e., when d_1 is comparable to λ_{TF} . Figure 4.3(a) shows the Cherenkov threshold $v_{th}(\lambda_0)$ at a single wavelength (e.g., $\lambda_0 = 1 \mu\text{m}$) as a function of the pitch P of unit cell for the hyperbolic metamaterial, at different values of β . When P decreases, $v_{th}(\lambda_0)$ decreases in Fig. 4.3(a). Particularly, when P goes down to be comparable to λ_{TF} (e.g., $P < 1 \text{ nm}$), the nonlocal response of the hyperbolic metamaterial mainly arises from the nonlocal electron screening in metals, and $v_{th}(\lambda_0)$ approaches to β . To be specific, if $\beta = 0$ (neglecting the nonlocal electron screening), $v_{th}(\lambda_0)$ intuitively vanishes or becomes zero for ultimately small P , leading to the so-called “threshold-free” Cherenkov radiation. However, nonlocal electron screening modifies the radiation fields by the swift charge in the homogeneous limit by creating effective thicknesses at each boundary of metals. The finite effective thicknesses regularize the singularity of the photonic density of states, determining the ultimate bound of the threshold velocity. Thus, if $\beta > 0$ (the nonlocal electron screening in metals is taken into consideration), the threshold velocity $v_{th}(\lambda_0)$ is always nonzero for arbitrary value of P and theoretically we have $\lim_{P \rightarrow 0} v_{th}(\lambda_0) = \beta$. When P is sufficiently large (e.g., $P > 10 \text{ nm}$ in Fig. 4.3(a)), the nonlocal response of the hyperbolic metamaterial mainly comes from the spatial dispersion induced from the finite structural size, and as a result, Fig. 4.3(a) shows a convergent tendency of $v_{th}(\lambda_0)$ for different values of β . When P is finitely large (e.g., $1 \text{ nm} < P < 10 \text{ nm}$ in Fig. 4.3(a)), the value of $v_{th}(\lambda_0)$ is determined by the nonlocality induced both by the spatial dispersion from the finite structural size and by the nonlocal electron screening in metals.

Figures 4.3(b-e) show the influence of the nonlocal electron screening in metals on Cherenkov radiation in the time domain. To highlight the nonlocality from the nonlocal electron screening in metals, we set $d_1 = 0.8 \text{ nm}$ and $P = 2 \text{ nm}$ for the hyperbolic metamaterials, and choose $v = 0.015c$ in Fig. 4.3(b-e). The field of Cherenkov radiation shows up inside the hyperbolic metamaterial if $v > \beta$ ($\beta_1 = 0$ in Fig. 4.3(c) and $\beta_2 = 1/300c$ in Fig. 4.3(d)), but disappears if $v < \beta$ ($\beta_3 = 1/30c$ in Fig. 4.3(e)). Therefore, when considering the nonlocal electron screening in metals, there is always a nonzero threshold for Cherenkov radiation in the hyperbolic metamaterial. Moreover, there will be no Cherenkov radiation of surface plasmons at the interface between the air and hyperbolic metamaterial when the nonlocal effects are mainly induced by the nonlocal electron screening in metals (Fig. 4.3(e)). This is different from Fig. 4.2(e), which has Cherenkov radiation of surface plasmons when the nonlocal effects are mainly induced by the spatial dispersion from the finite structural size (Fig. 4.2(e)). In addition, the emitted fields of Cherenkov radiation in Figs. 4.3(c, d) are similar to the emitted field in the homogeneous medium in Fig. 4.3(b), except the difference inside the triangular regions highlighted in Figs. 4.3(b-d). The difference can be understood as follows: since $v_{\text{th}}(\lambda)$ is highly dependent on both β and λ , the spectrum of Cherenkov radiation in hyperbolic metamaterials will change if β varies (see details in Fig. 4.5).

Figure 4. 3 Influence of the nonlocal electron screening in metals on the Cherenkov threshold. (a) $v_{th}(\lambda_0)$ at a specific wavelength λ_0 as a function of the pitch P of unit cell in the hyperbolic metamaterial, at different nonlocal parameters $\beta = \sqrt{3/5} v_F$. The Fermi velocity v_F varies for different metals. For conceptual illustration, $\lambda_0 = 1 \mu\text{m}$ is used, and the value of β/c changes from 0 (i.e., without the consideration of the nonlocal electron screening in metals), 1/300 to 1/30. Due to the nonlocal electron screening in metals, we have $v_{th}(\lambda_0) \geq \beta$, When P decreases to zero, $v_{th}(\lambda_0)$ approaches to β in theory. (b-e) Field distribution of Cherenkov radiation in the time domain. The realistic layered structure in Fig. 4.2(a) is adopted for (c-e), while it is replaced by a homogeneous medium with the effective permittivity of $\bar{\epsilon}_r = \{\epsilon_{||} \epsilon_{\perp} \epsilon_{||}\}$ for (b). The Cherenkov radiation appears when $v_3 > \beta$ (see (c, d) for example), but disappears when $v_3 < \beta$ (see (e) for example). Other parameters are $P = 2 \text{ nm}$ and $v = v_3 = 0.015c$. When $P < 10 \text{ nm}$, the nonlocality induced by the nonlocal electron screening in metals becomes non-negligible.

Above results can also be understood with resort to isofrequency contour study. Below we present the eigenmode equations for the three models, i.e., homogeneous

hyperbolic metamaterial, classic layered structure and layered structure with nonlocal screening.

- i) For the homogeneous hyperbolic metamaterial, the eigenmode equation is given by [68]

$$\frac{k_x^2 + k_z^2}{\varepsilon_\perp} + \frac{k_y^2}{\varepsilon_\parallel} = k_0^2, \quad (4.1)$$

- ii) For the classic layered structure, the eigenmode equation is expressed as [113]

$$\cos(k_y P) = \cos(k_y^m d_1) \cos(k_y^d d_2) - \frac{1}{2} \left(\frac{\varepsilon_m k_y^d}{\varepsilon_d k_y^m} + \frac{\varepsilon_d k_y^m}{\varepsilon_m k_y^d} \right) \sin(k_y^m d_1) \sin(k_y^d d_2), \quad (4.2)$$

where $k_y^d = \sqrt{\varepsilon_d k_0^2 - k_x^2 - k_z^2}$ and $k_y^m = \sqrt{\varepsilon_m k_0^2 - k_x^2 - k_z^2}$.

- iii) For the layered structure with nonlocal screening, the eigenmode equation becomes [74]

$$\begin{aligned} \cos(k_y P) = & \left(\cos(k_y^d d_2) \left(k_{Ly}^m \cos(k_{Ty}^m d_1) \sin(k_{Ly}^m d_1) - \right. \right. \\ & \left. \frac{k_z(w_d - w_m)}{z_m} \right) \sin(k_{Ty}^m d_1) \cos(k_{Ly}^m d_1) + \sin(k_y^d d_2) \left(\frac{k_z(w_d - w_m)}{z_d} \left(1 - \right. \right. \\ & \left. \left. \cos(k_{Ty}^m d_1) \cos(k_{Ly}^m d_1) \right) \right) - \frac{1}{2} \left(\frac{k_z^2}{k_{Ly}^m} \frac{(w_d - w_m)^2}{z_d z_m} + k_{Ly}^m \left(\frac{z_d}{z_m} + \right. \right. \\ & \left. \left. \frac{z_m}{z_d} \right) \sin(k_{Ty}^m d_1) \sin(k_{Ly}^m d_1) \right) \right) \cdot \left(k_{Ly}^m \sin(k_{Ly}^m d_1) - \frac{k_z(w_d - w_m)}{z_m} \sin(k_{Ty}^m d_1) \right)^{-1}, \end{aligned} \quad (4.3)$$

where $z_m = \frac{k_{Ty}^m}{k_0 \varepsilon_m}$, $z_d = \frac{k_y^d}{k_0 \varepsilon_d}$, $w_m = \frac{k_z}{k_0 \varepsilon_m}$ and $w_d = \frac{k_z}{k_0}$. The y-components of wavevectors of the transverse field and longitudinal field in the region of metals are

$k_{Ty}^m = \sqrt{\epsilon_m k_0^2 - k_x^2 - k_z^2}$ and $k_{Ly}^m = \sqrt{k_L^2 - k_x^2 - k_z^2}$, respectively; here the wavevector of the longitudinal electric field is determined by Eq. (2.65).

Figure 4.4 shows the evolution of the isofrequency contours when period is narrowing from 50 nm to 2 nm in the wavelength $\lambda_0 = 1 \mu\text{m}$. Clearly, homogeneous hyperbolic metamaterials support infinite wavevector bulk Bloch modes, while in the layered structure, the maximum wavevector for the bulk Bloch modes is $k_y^{cf} = \pi/P$, so called the wavevector cutoff. When $k_y > k_y^{cf}$, the decoupling of the surface plasmons at each metallodielectric interface prevents the formation of the bulk Bloch modes. The k_y^{cf} leads to the k_z^{max} , determining the threshold velocity in the layer structure as $v_{th} = \frac{\omega}{k_z^{max}}$.

In the classically described regime (e.g., $P = 50 \text{ nm}$), the isofrequency contours of the classic layered structure and the layered structure with nonlocal screening coincide with each other. But they are different from the isofrequency contour of the homogeneous hyperbolic metamaterial, especially when the wavevector is sufficiently large. This indicates that the spatial dispersion from the finite structural size is dominant while the nonlocal electron screening in metals is minor in the classically described regime. With the reduce of the pitch, the wavevector cut off k_y^{cf} becomes larger, leading to the larger k_z^{max} . Thus the threshold velocity is decreasing gradually. Within the quantum size regime (e.g., $P=2 \text{ nm}$), the isofrequency contour of the layered structure with nonlocal screening shows smaller k_z^{max} at $k_y = k_y^{cf}$, compared with the isofrequency contour of the classic layered structure. Thus, the nonlocal electron screening in metals becomes a non-negligible factor to affect bulk Bloch modes with large wavevectors, thereby the threshold velocity.

Figure 4.4 Isofrequency contours of the metamaterial at the wavelength of $\lambda_0 = 1 \mu\text{m}$. The x -component of the wavevector $k_x = 0$. For the red dashed line, the realistic layered structure is replaced by a homogeneous medium with the effective permittivity $\bar{\epsilon}_r = \{\epsilon_{\parallel} \quad \epsilon_{\perp} \quad \epsilon_{\parallel}\}$. For the solid lines, the nonlocal parameters β/c of the metal are varied as 0, 1/300 and 1/30, respectively. Pitches P of the layered structure are selected as (a) 50 nm, (b) 10 nm and (c) 2 nm, respectively.

Figure 4.5 shows that the threshold velocity as a function of the wavelength in classically described regime and quantum size regime (i.e. $P=25 \text{ nm}$ and $P=2 \text{ nm}$), respectively. It is clearly to see that the threshold velocity is dependent on the wavelength for a realistic layered structure. Here, we only focus on the $\min(v_{th}(\lambda))$, in the studied wavelength range. When the pitch of the layered structure is 25 nm, the nonlocality induced by the nonlocal electron screening in metals is negligible and the Cherenkov threshold is mainly determined by the spatial dispersion from the finite structural size. If the electron velocity $v = v_1 > \min(v_{th}(\lambda))$, we can observe the field of Cherenkov radiation (Fig. 4.2(c)). While if $v = v_2 < \min(v_{th}(\lambda))$, the field of Cherenkov radiation vanish (Fig. 4.2(e)). When the pitch of the layered structure is 2 nm, the nonlocal electron screening in metals becomes appreciable and the Cherenkov threshold in the homogeneous limit is β . When $\beta = \beta_1 = 0$ (without considering nonlocal electron screening in metals) and $\beta = \beta_2$, $v = v_3 > \min(v_{th}(\lambda))$. Thus, we

can observe the emitted field of Cherenkov radiation (Figs. 4.3(c) and (d)). However, if $\beta = \beta_3$, $v = v_3 < \min(v_{th}(\lambda))$. In the end, Cherenkov radiation disappears in the periodic layered structure (Fig. 4.3(e)).

Figure 4. 5 Threshold velocity as the function of the wavelength. The solid line represents the threshold velocity in the classic layered structure (or $\beta = 0$). The hollow circle and the hollow diamond mark the threshold velocity in the layered structure with β of 1/300 and 1/30, respectively. In the panel (a), the pitch P of the layered structure is 25 nm. Here the dashed line donates the charge velocity, set as $v_1=0.2c$ and $v_2=0.05c$, respectively. In the panel (b), the pitch P of the layered structure is 2 nm, where the charge velocity is set as $v_3 = 0.015c$.

4.2.4 Impact of the spatial nonlocality on the radiation spectrum of a swift charge

Spatial nonlocality in hyperbolic metamaterials not only has a strong impact on the Cherenkov threshold, but also changes the dissipated power of a swift charge. Figure 4.6 shows the influence of nonlocal effects on the dissipated power of the swift charge in a hyperbolic metamaterial. The dissipated power is the integration of the power

spectral density G over the whole range of frequencies as $P_0 = \int_0^{+\infty} G \cdot d\omega$. Here, the power spectral density by the moving charge over a length Δl is computed using the power dissipation formula as [39, 40]

$$G = \lim_{\rho \rightarrow 0} \frac{\pi \rho \Delta l}{\Delta t} \text{Re}(\bar{J}^{q*} \cdot \bar{E}^R) = \frac{q_0 v}{2\pi \omega \varepsilon_0} \text{Im}[\int_0^{+\infty} dk_x (k_x^2 + k_{Ty1}^2) R_{TM1}^-], \quad (4.4)$$

where the $\bar{E}^R$ is the radiation field of the swift charge and R_{TM1}^- is the reflection coefficient of the transverse magnetic field in the top surface of the layered structure.

As Fig. 4.6 shows very clearly, the spatial dispersion from the finite structural size plays a dominant role in determining the dissipated power, while the nonlocal electron screening in metals affects the dissipated power only when P is small (e.g., $P = 2$ nm). Cherenkov radiation in the layered structure has a larger dissipated power than the emitted power in the homogeneous hyperbolic metamaterial when the velocity is relatively large (e.g., $v > 0.02c$ when $P = 2$ nm), because the surface plasmons in the interface between the air and layered structure provide an additional radiative decay channel. When v is sufficiently small or close to the Fermi velocity v_F (e.g., $v < 0.02c$ when $P = 2$ nm), the dissipated power dramatically drops due to the elimination of radiative decay channels by bulk Bloch modes with large wavevectors. In addition, when v is sufficiently large (e.g., $v > 0.2c$) along with the sufficiently small P ($P < 2$ nm), the homogeneous description of the hyperbolic metamaterial can well describe the dissipated power of Cherenkov radiation in the realistic scenario.

Figure 4. 6 Impact of the spatial nonlocality on the radiation spectrum of a swift charge. For comparison, we treat the periodic layered structure in Fig. 1(a) in three different ways. For the red dashed line, the layer structure is replaced by a homogeneous medium. For the solid lines, only the spatial dispersion from the finite structural size is considered by using the classic layered structure or setting the nonlocal parameter $\beta = 0$. For the data indicated by hollow symbols, both the nonlocal electron screening in metals and the spatial dispersion from the finite structural size are taken into consideration, using the layered structure with nonlocal screening. The dissipated power calculated in the homogeneous medium is independent of P , since we set d_1/P to be a constant. The nonlocality induced by the nonlocal electron screening in metals becomes non-negligible only when P is sufficiently small (e.g., $P < 10$ nm). When P is large, the dissipated power of Cherenkov radiation becomes highly sensitive to the value of P or the spatial dispersion from the finite structural size.

4.2.5 Longitudinal Cherenkov photons induced by the spatial dispersion from the finite structural size

In addition to the appearance of the Cherenkov threshold and the modification of the radiation spectrum, we surprisingly found that the longitudinal modes can be efficiently excited by a swift charge in the periodic metallodielectric structure at epsilon-near-zero frequencies. Here the epsilon-near-zero frequency refers to the frequency at which the hyperbolic metamaterial has the effective in-plane permittivity close to zero (Fig. 4.1). The excitation of longitudinal modes is intimately related to the spatial nonlocality [72, 73, 110]. This can be clearly seen from the comparison of analytical field plots between Fig. 4.7(a) (with homogeneous description) and Fig. 4.7(b) (with nonlocal description), at an epsilon-near-zero frequency. For the emitted longitudinal modes in Fig. 4.7(b), they propagate along a direction almost normal to and away from the interface between the air and hyperbolic metamaterial. Fig. 4.7(c) shows the power spectral density of the emitted longitudinal modes as a function of the wavelength and the charge velocity. Although the longitudinal modes in the hyperbolic metamaterial exist in a relatively wide wavelength range, the swift charge with a determined velocity, theoretically, can only excite a longitudinal mode at a specific wavelength, if we artificially neglect the loss. Due to the existence of the realistic loss, a narrow bandwidth of longitudinal modes can be excited at a specific particle velocity in Fig. 4.7(c). Figure 4.7(d) shows the power spectral density at an epsilon-near-zero frequency (e.g., the corresponding wavelength is $\lambda = 395$ nm) as a function of the charge velocity. From Fig. 4.7(d), it is reasonable to argue that longitudinal Cherenkov photons emerge only when the spatial dispersion from the finite structural size is strong, but is irrelevant to the nonlocal electron screening in metals. Meanwhile, as a clear manifestation of the spatial dispersion from the finite structural size, the emitted power of longitudinal modes can be comparable to the emitted power of transverse modes in Fig. 4.7(d). This might facilitate the potential experimental observations.

Figure 4. 7 Longitudinal Cherenkov photons at epsilon-near-zero frequencies. (a, b) Field distribution of emitted waves at an epsilon-near-zero frequency (the corresponding wavelength in the free space is $\lambda = 0.395 \mu\text{m}$). The realistic layered structure in Fig. 4.2(a) is adopted for (b) with the consideration of nonlocal effects, while it is treated as a homogeneous medium with the effective permittivity of $\bar{\bar{\epsilon}}_r = \{\epsilon_{\parallel} \quad \epsilon_{\perp} \quad \epsilon_{\parallel}\}$ for (a). At $\lambda = 0.395 \mu\text{m}$, $\epsilon_{\parallel}$ is close to zero. (c) Power spectral density G as a function of wavelength λ and particle velocity v . For (a-c), $P = 25 \text{ nm}$. (d) Power spectrum density G versus the charge velocity at a single wavelength (e.g., $\lambda = 0.395 \mu\text{m}$). For comparison, we treat the periodic layered structure in Fig. 4.2(a) with three different ways in (d), same as Fig. 4.6. The longitudinal mode emerges only when the spatial dispersion from the finite structural size is dominant (e.g., when $P = 25 \text{ nm}$ in (b, d)).

In addition, from the isofrequency contour as shown in Fig. 4.8(a), we also find that the nonlocal effects give rise to the emergence of the longitudinal near-zero frequencies in the layered structure with the pitch of 25 nm (classically

described regime). While for the layered structure with $P = 2$ nm (quantum size regime), the longitudinal mode is disappearing, indicating that this longitudinal mode is brought by the spatial dispersion from the finite structural size, but is irrelevant to the nonlocal electron screening in metals.

Figure 4. 8 Isofrequency contours of the metamaterial at the wavelength of $\lambda_0 = 0.395$ μm . The x -component of the wavevector $k_x = 0$. For the red dashed line, the realistic layered structure is replaced by a homogeneous medium with the effective permittivity $\bar{\epsilon}_r = \{\epsilon_{\parallel} \quad \epsilon_{\perp} \quad \epsilon_{\parallel}\}$. For the solid lines, the nonlocal parameters β/c of the metal are varied as 0 and 1/300, respectively. Pitches P of the layered structure are selected as (a) 25 nm and (b) 2 nm, respectively.

4.3 Conclusion

In our theoretical study, we clarified the transition of “threshold-free” Cherenkov radiation between from the classically described regime and to the quantum size regime, with detailed analysis and comparison of field distribution in three models, i.e., homogeneous hyperbolic metamaterial, classic layered structure and layered structure with nonlocal screening. We stressed that the Cherenkov threshold exists for

in all the dimensional scales of a realistic hyperbolic metamaterial with all the dimensional scales. The spatial dispersion from the finite structural size is the underlying mechanism that prevents the elimination of the Cherenkov threshold from the elimination when the structural dimension is in the classical described regime. While in the quantum size regime, the dominance of the nonlocal electron screening in metals ensures that the threshold velocity is always larger than or equal to $\sqrt{\frac{3}{5}}v_F$ in metal-based hyperbolic metamaterials. Apart from the Cherenkov threshold, these nonlocal effects also have a strong influence on other electromagnetic features of Cherenkov radiation in the hyperbolic metamaterials, including the radiation spectrum when the particle velocity is close to the Cherenkov threshold, and the counterintuitive appearance of longitudinal Cherenkov photons at epsilon-near-zero frequencies.

It can be concluded that Cherenkov radiation by a swift charge contains extraordinary phenomena with the inclusion of nonlocal effects into the nearby nanostructured media. These corrections are necessary for the precise prediction of Cherenkov radiation in artificial structures, especially when structures possess singularity or atomic-level thickness, such as the graphene-based metamaterials [114-118]. The platforms proposed here thus are fundamentally significant for exploring and regulating the interplays between the moving charged particle and artificial structures with a wide variety of geometries and materials. Our work not only enriches the theory of Cherenkov radiation in the artificial structures, but also provides a theoretical guidance for the experimental design of compact free-electron sources in the future.

5

Tailoring radiation angles of slow-moving particles with spatial nonlocality

5.1 Background and motivation

The classical theory of Cherenkov radiation suggests that the Cherenkov angle θ_c is intimately related to the particle velocity v as $\cos(\theta_c) = v_{th}/v$, where $v_{th} = c/n$ is the threshold velocity (i.e., the phase velocity of light in the medium with the refractive index n). Exploiting this relation, scientists promote potential applications of Cherenkov radiation including the detection of cosmic rays [119] and the sensing of biomolecules [12]. However, Cherenkov radiation in conventional media generally has high threshold energies up to several megaelectronvolt, because of finitely large refractive indices of natural transparent materials. In the end, Cherenkov detectors in conventional setups enable to discriminate charged particles only in high-energy regimes [7, 36, 120, 121].

The resourceful metamaterials provides enormous freedoms to engineer the electromagnetic characteristics of Cherenkov radiation [26, 122-124]. Making use of

metamaterials, scientists observe counter-intuitive Cherenkov radiations such as reversed Cherenkov radiation and threshold-free Cherenkov radiation continuously [25, 38, 39, 125]. Remarkably, recent experiment showed that Cherenkov radiation could be generated in the hyperbolic metamaterials by charged particles without any threshold velocity [27], accounting for extremely large mode refractive indices in the hyperbolic metamaterials. While Cherenkov radiation in the hyperbolic metamaterials is promising for achieving low-threshold free-electron sources, it is not an ideal strategy for identifying the slow-moving charged particles. Owing to the existence of the asymptote in the hyperboloid-shaped isofrequency contour of hyperbolic metamaterials, the corresponding Cherenkov angle becomes very insensitive to the small particle velocity. It still has been a topic of interest for us to efficiently identify the charged particle in low-energy regimes.

In this work, we found that longitudinal modes excited by moving charged particles on the ultrathin metallic grating, can be exploited to identify charged particles in low-energy regimes. In the quantum-sized structure, the nonlocal electron screening in metals causes strong quantum electron repulsions, manifested as the oscillations of longitudinal waves [80, 99, 100]. Coupling between longitudinal waves in each metal slab gives rise to the formation of longitudinal modes below the bulk plasmon wavelength λ_p . Our results indicate that longitudinal Cherenkov photons not only possess small threshold velocity, but also shows great sensitivity to the particle velocity especially in small-energy regimes.

5.2 Longitudinal Cherenkov photons for slow-moving particle identification

5.2.1 Origin and properties of longitudinal Cherenkov photons

Without the loss of generality, we consider a charged particle with the velocity of $\bar{v}$ moves in the air. The particle trajectory is parallel to and close to the top surface of the metallic grating, as shown in Fig. 5.1. The thickness of metals is d_1 , the thickness of gaps is d_2 , and the pitch of the grating is P .

To begin with, we demonstrate that longitudinal Cherenkov photons can be generated by the moving charged particle, only when the nonlocal electron screening in metals is non-negligible. As a concrete example, we select the silver (Ag) as the metal with $v_F = 1.36 \times 10^6$ m/s. The thickness of the metal is $d_1 = 2$ nm, and the pitch of the metallic grating is $P = 6$ nm. Fig. 5.2 plots the calculated E_z field distribution of Cherenkov radiation and the energy loss spectrum G . For comparison, Fig. 5.2(a) and (c) neglect the nonlocal electron screening in metals with the local description of free-electron response, while Fig. 5.2(b) and (d) consider the nonlocal electron screening in metals with hydrodynamic description. We can clearly observe the appearance of longitudinal modes below the bulk plasmon wavelength $\lambda_p = 0.137$ μm when the spatial nonlocality is considered (Fig. 2(b)). To be specifically, Cherenkov radiation with 1st order longitudinal modes is generated in the nonlocal metallic grating by the charged particle with the velocity $v = 0.5c$, when the wavelength is $\lambda = 0.129$ μm . However, in the local metallic grating, the radiation field of the charged particle undergoes evanescent decay into the metamaterials (Fig. 5.2(a) and (b)). In Fig. 5.2(c) and (d), the power spectral density G shows three resonances below the bulk plasmon wavelength λ_p in the nonlocal metallic grating, while they are absent in the local

metallic grating. These three resonances correspond to 1st order longitudinal mode and other two high-order longitudinal modes, respectively. Both of them can be excited by the moving charged particle simultaneously. Interestingly, owing to the giant mode refractive index, the threshold velocity of longitudinal modes is even smaller than that of transverse modes above the λ_p (the hyperbolic dispersion region).

Figure 5. 1 Schematic diagram of Cherenkov radiation on the metallic grating. A charged particle with the velocity of v_0 moves on the top of the metallic grating along the z -direction, generating longitudinal Cherenkov photons into the metamaterial. The thicknesses of the metal slab and the gap width are d_1 and d_2 , respectively. The pitch of the metamaterial is $P = d_1 + d_2$. The nonlocal free-electron response of the metal $\epsilon(\omega, \bar{k})$ is given with hydrodynamic Drude model. We denote the radiation angle of longitudinal modes as θ_r , i.e., the angle between the Poynting power flow and the y -axis. Cherenkov radiation is forward when $\theta_r > 0$ and Cherenkov radiation is backward when $\theta_r < 0$.

Figure 5. 2 The calculated E_z field distribution in the frequency domain, emitted by a charged particle moving on the local metallic grating (a) and the nonlocal metallic grating (b). Here the particle velocity is $v = 0.5c$ and the wavelength of interest is $\lambda = 0.129 \mu\text{m}$. The energy loss spectrum G from the charged particle as the function of the particle velocity and the wavelength, in local metallic grating (c) and the nonlocal metallic grating (d). Here L mode and T mode represent the longitudinal mode and transverse mode respectively.

The radiation angles of longitudinal modes θ_r are extremely sensitive to the variation of the particle velocity v_0 . As shown in Fig. 5.3(a), when the particle velocity is $v = 0.03c$, the radiation angle of the first-order longitudinal mode is $\theta_r = 30^\circ$. With the increase of the particle velocity, the radiation direction of the charged particle is flipped from the forward direction to the backward direction. As the v approaches to the light velocity in the vacuum c , the radiation angle decreases to be -50° monotonously. The large variation range of radiation angles allows us to measure the particle velocity

from $0.03c$ to c in high sensitivity. While radiation angles of high-order longitudinal modes are sensitive to the particle velocity in small-energy regimes, the potential working region actually becomes smaller, compared with the radiation angle of the 1st order longitudinal mode. Above the Cherenkov threshold, with the increase of the particle velocity, the radiation angles are no longer monotonously decreasing, but bend over at $v = 0.5c$ for the 2nd longitudinal modes and $v = 0.37c$ for the 3rd longitudinal modes (Fig. 5.3(b) and (c)). Thus, Cherenkov radiations with these high-order longitudinal modes work in a narrower range of velocities for the particle identification. Meanwhile, the emission intensities of high-order longitudinal modes are smaller than that of 1st longitudinal mode. Therefore, Cherenkov radiation with the 1st longitudinal mode outweighs Cherenkov radiation with high-order longitudinal modes by the large working region and intensive photon emission.

Figure 5. 3 Radiation angles of longitudinal modes θ_r and the energy loss spectrum G versus the particle velocity v/c . For comparison, we study 1st, 2nd and 3rd longitudinal mode. The wavelengths λ of interest for (a)-(c) are 129 nm, 117 nm and 104 nm, respectively. Longitudinal Cherenkov photons can identify charged particles in a wide range of velocities, so called the working region. Here, the effective working regions are highlighted with the green color.

5.2.2 Isofrequency contour study of longitudinal modes

The small velocity threshold and high sensitivity to particle velocity of longitudinal Cherenkov photons account for the exotic isofrequency contours at the resonant frequencies of longitudinal modes in the metallic grating. As shown in Fig. 5.4(a), the isofrequency contour of longitudinal modes shows a maximum achievable wavevector $\max(k_z)$. In fact, $\max(k_z)$ is jointly affected by the spatial dispersion from the finite structural size and the nonlocal electron screening in metals. However, in our quantum-sized metallic grating, the spatial dispersion from the finite structural size is negligible. Therefore, the nonlocal screening in metals is the main factor to limit the $\max(k_z)$, ensuring the threshold velocity v_{th} comparable to the Fermi velocity v_F . In the end, the working region of Cherenkov detectors based on longitudinal modes is ranging from the Fermi velocity in metals to the light velocity in free space. In addition, the radiation angle θ_r becomes sensitive to the particle velocity in a lossy metallic grating. In the lossless case (Fig. 5.4(a)), the isofrequency contour of longitudinal modes is flatten in a wide range of wavevectors k_z . In this range, the radiation directions of longitudinal modes are normally downward, independent on the k_z . Therefore, longitudinal Cherenkov photons are quite insensitive to the particle velocity in the lossless metallic grating. However, the situation becomes completely different when the loss is involved in the metallic grating. As shown in Fig. 5.4(b), the existence of the loss in metals makes the isofrequency contours of longitudinal modes smoother than that in the lossless case. As a consequence, the θ_r becomes highly dependent on the wavevector k_z . For example, the isofrequency contour of 1st order longitudinal modes shows that longitudinal Cherenkov photons is emitted forward when $\frac{k_z P}{\pi} > 2$ (low-

energy regimes), but backward when $\frac{k_z P}{\pi} < 2$ (high-energy regimes). Finally, the broad variation range of radiation angles allows longitudinal Cherenkov photons to discriminate charged particle in high sensitivity in the working region.

Figure 5. 4 Isofrequency contours of longitudinal modes in the metallic grating. Here, we consider the lossless case (a) and the lossy case (b), respectively. For comparison, we study three longitudinal modes with different orders as indicated in Fig. 5.3.

5.2.3 Example for the slow-moving particle identification

Finally, we show the feasibility of identifying charged particles in low-energy regimes with longitudinal Cherenkov photons. Similar to conventional Cherenkov detectors, charged particles are identified by measuring corresponding radiation angles at a given momentum. The momentum of the charged particle with a rest mass of m_0 is

$$p = \gamma m_0 v, \text{ where } \gamma = \left(1 - \left(\frac{v}{c}\right)^2\right)^{-1/2}. \text{ The particle velocity varies, according to}$$

different rest masses of charged particles at a given momentum. Thus corresponding radiation angles change for different charged particles. Since the radiation angle of longitudinal modes is extremely sensitive to the small particle velocity, charged particles can be easily discriminated in the range of momenta from 0.02 GeV/c to 1 GeV/c. Fig.

5.5 plots the radiation angle θ_r as the function of momentum p for three kinds of charged particles, i.e., proton, pion and kaon. When $p = 0.05 \text{ GeV}/c$, radiation angles θ_r of longitudinal modes by proton, pion and kaon are -19° , -27° and -50° , respectively. The giant differences between radiation angles for different charged particles ensure that Cherenkov detector based on longitudinal modes is highly efficient for the particle identification in low-energy regimes.

Figure 5. 5 Radiation angles θ_r as the function of the momentum p . For comparison, we study three types of charged particles, i.e., pion, kaon and proton. Here, the rest masses of pion, kaon and proton are 140, 493 and 938 MeV/c^2 , respectively. Notably, when the momentum of the charged particle is 0.1 GeV/c , the θ_r for the pion, kaon and proton are -50° , -27° and -19° , respectively.

5.3 Conclusion

To conclude, we proposed a new type of charged particle detection technique, based on longitudinal Cherenkov photons. The longitudinal modes are intrinsically induced by the nonlocal electron screening in metals. Thus longitudinal Cherenkov

photons is strong only when the structural dimension of the metallic grating is in the order of Tomas-Fermi screening length λ_{TF} . Compared with previous avenues, Cherenkov detector based on longitudinal modes enables us to identify charged particles efficiently in a wide range of velocities. The sensitivity of such detector could be especially appreciable in low-energy regimes. This work not only enriches the theory of Cherenkov radiation, but also bridges the research gap of the slow-moving particle identification.

In addition, scientists generally believed that the nonlocal electron screening in metals should deteriorate the performance of plasmonic systems. However, this work indicates another potential application of the spatial nonlocality (i.e., in the field of Cherenkov radiation), ever since its contribution to the nonlinear processes is investigated by Y. Luo *et.al* [126].

6

Conclusion and recommendations

6.1 Conclusion

This thesis focuses on manipulating Cherenkov radiation by artificial structures including photonic crystals and metamaterials. The chapter 1 gives a comprehensive introduction of Cherenkov radiation and extensively reviews Cherenkov radiation in artificial structures. Previous work on this topic show that tailoring Cherenkov radiation with artificial structures promotes many applications of Cherenkov radiation. Meanwhile, we pointed out that even with such resourceful achievements, Cherenkov radiation in artificial structures still has a wide space for the development. In the Chapter 2, we analytically developed a complete theory of Cherenkov radiation in one-dimensional photonic crystals and one-dimensional metamaterials. In the Chapter 3-5, we systematically explored the features and applications of Cherenkov radiation manipulated by artificial structures with our established theory.

Chapter 3 studies Cherenkov radiation in the Brewster photonic crystal. Brewster photonic crystal breaks the limitation by the finite refractive indices of natural transparent materials. Thus, the Brewster Cherenkov angle can be controlled in the wide range of momenta. Secondly, due to the small dispersion of the Brewster photonic crystal, the Brewster Cherenkov detector can identify charged particle in a broad range of frequencies. The Brewster Cherenkov detector is fundamentally important for the particle identification in wide energy range.

Chapter 4 revisits Cherenkov radiation in the hyperbolic metamaterial. It is generally believed that Cherenkov radiation is threshold-free in hyperbolic metamaterials, which is important for achieving the threshold-free Cherenkov sources. However, the spatial dispersion from the finite structural size and the nonlocal electron screening in metals disprove the claim of threshold-free Cherenkov radiation. We stressed that while the Cherenkov threshold exists in the realistic hyperbolic materials, the longitudinal modes brought by the finite structural size can be exploited as monochromatic sources.

Chapter 5 investigates Cherenkov radiation in the metallic grating. Another longitudinal modes induced by the nonlocal electron screening in metals can be generated by moving charged particles, only when the structural dimension of the metallic grating is sufficiently small. Our results show that radiation angle of longitudinal modes is extremely sensitive to the particle velocity especially in low-energy regimes. This finding improves techniques of slow-moving particle identification.

6.2 Recommendations for future research

There are several points that worth further exploration in this research direction. Photodetector is used to measure time-domain signals. In Chapter 3, we only discussed Brewster Cherenkov radiation in the visible wavelength range. Beyond the visible wavelength range, the incidence with the wavevector off the Brewster wavevector may have partial transmission in the Brewster photonic crystal, because of the finite photonic bandgaps. These leaky fields become the noises for Cherenkov pulses in the visible wavelength range. How these noises influence the Cherenkov pulses remains illusive. Thus, it would be meaningful for us the explore the influence brought by these noises. In addition, it is desirable for us to design a bandpass filter for the Brewster Cherenkov detector to suppress these noises. This would facilitate the experimental realization of the Brewster Cherenkov detector.

As an extension for Chapter 3 and Chapter 4, we would like to observe the influence of the spatial nonlocality on Cherenkov radiation in the experiments. The nonlocal electron screening in metals is strong only when the structural dimension is approaching the Tomas-Fermi wavelength $\lambda_{TF} = \frac{v_F}{\omega_p}$. The Fermi velocity v_F for metals is generally in the order of 10^6 m/s. The plasma frequency ω_p for metals is generally in the order of 10^{15} rad/s. As a result, we can estimate that $\lambda_{TF} \approx 1$ nm for metals. Thus, to observe the spatial nonlocality from the nonlocal electron screening in metals, the structural dimension should be at the atomistic level. However, such quantum-sized structures are still difficult to be fabricated with the state of art nanotechnology. Semiconductors such as Indium antimonide (InSb) has much smaller carrier density, leading to the ω_p in the order of 10^{13} rad/s [127]. Thus, the nonlocal charge screening could be appreciable in a semiconductor particle or slab with the size around 100 nm. Therefore, it is an opportunity for us to experimentally explore the influence of the

spatial nonlocality on Cherenkov radiation in hyperbolic metamaterials or gratings made of semiconductors.

The best advantage of free electron radiation sources is that they have extremely broad wavelength range, facilitating the broadband tunable light source. However, the photon extraction efficiency of free electron radiation sources is still rather low, compared to the conventional on-chip light sources, such as silicon-based light sources and III-IV semiconductor light sources. Recent attentions are drawn to the stimulated quantum Cherenkov effect, which opens up an opportunity to dramatically enhance the emission efficiency of swift electrons [128]. Moreover, the quantum description of Cherenkov radiation enables us to shape electron wavefunctions flexibly, providing significant guidance for the design of novel free electron radiation sources. Therefore, it would be a good research direction to enrich the theory of Cherenkov radiation in the framework of quantum electrodynamics.

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