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A Closed-Loop Analytic Filtering Scheme of Capacitor Voltage Ripple in Multilevel Cascaded H-Bridge Converters

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Abstract—This paper proposes a closed-loop analytic filtering scheme for the low-frequency capacitor voltage ripple in cascaded H-bridge converters. Although the technique can be applied to many cascaded converter applications, this paper focuses on the low-capacitance cascaded H-bridge static compensator. Filtering of capacitor voltage signals is particularly important in this application, due to the presence of large low-frequency harmonic voltage ripple. The main novelty of the proposed algorithm in this paper is the estimation of the capacitance value of each H-bridge. The closed-loop capacitance estimation method reduces steady-state error in estimated voltage ripple magnitude and phase, which is otherwise present in the available open-loop filters in the literature. Furthermore, fast dynamic response is achieved compared to filtering schemes based on lowpass and bandstop filters. Therefore, the proposed solution optimises the trade-off between filtering accuracy and transient response. It also mitigates parametric uncertainties, time delays, and harmonic-contamination in the outer voltage control loop. Furthermore, the estimated capacitance is also useful to track the state of health of the dc-link capacitors. Experimental results on a seven-level 1-kVA cascaded H-bridge prototype are presented to demonstrate the comparative performance of the proposed filtering scheme and traditional approaches.

I. INTRODUCTION

Static compensators (StatComs) play an important role in the reliable operation of modern transmission lines [1], [2]. The cascaded H-bridge (CHB) multilevel converter is the leading technology for medium/high power StatComs, due to its modularity, superior efficiency, excellent harmonic performance, and fast dynamic response [3], [4]. The CHB converter is constructed from three identical single-phase converter arms, like the one shown in Fig. 1(a) [5], [6], hence, the instantaneous power flowing through each phase-arm contains twice-fundamental frequency oscillations [7]. Typically, the amplitude of the low-frequency voltage ripple, induced in the dc-link capacitors, is limited to less than $\pm 10\%$ of the nominal dc voltage, as an approximate rule of thumb. This is achieved by utilising large dc capacitors. Film or ceramic capacitors are the preferred technology in modern high-power converter applications due to the low reliability of high power-density electrolytic capacitors [8]–[10]. However, a substantial increase in the converter size and weight occurs due to the larger volume per Farad of film and ceramic capacitors compared to electrolytic capacitors.

Fig. 1. CHB power converter arm. (a) Circuit diagram and (b) control block diagram.

To overcome the aforementioned trade-off in terms of compactness, cost, and reliability, future StatComs are desired to have lower capacitance values. This was the driving force for the recently introduced low-capacitance StatCom (LC-StatCom) [11], [12], which is able to reduce the total capacitance by more than 50% by allowing large low-frequency voltage ripple on the dc-link capacitors. However, this results in two main differences with respect to a conventional StatCom device, namely, 1) a lower ‘electrical inertia’ within the converter (i.e., the capacitor voltages are more sensitive to transient currents), and 2) the existence of a smaller margin between the cluster voltage (sum of capacitor voltages in one converter phase-arm) and the absolute value of the reference converter voltage. These features result in new challenges for the control implementation in the LC-StatCom.

The feasibility of a cascade control scheme with multiple feedback loops to control the LC-StatCom was confirmed in [11], [12]. This control structure is illustrated in Fig. 1(b) using a simplified control block diagram. The square-sum-

capacitor-voltages in the power converter arm is denoted as $z_{clus} = \sum_{j=1}^n v_{Cj}^2$, where v_{Cj} is the capacitor voltage in the j^{th} H-bridge, where $j \in \{1, 2, \dots, n\}$ is the H-bridge index in the converter arm, as shown in Fig. 1(a). The aim of the outer voltage loop is to regulate the dc component of the square-sum-capacitor-voltages to the reference $z_{clus,0}^*$. This aim is effectively achieved by calculating the required active power component of the arm current, $\hat{i}_d^*$, which together with its reactive counterpart, $\hat{i}_q^*$, constitute the current reference for the inner current loop. When using this type of cascade control structure based on the classical dq -rotating frame [13] to control LC-StatComs, the outer voltage loop requires a relatively high bandwidth, in order to overcome the aforementioned control challenges. However, filtering schemes proposed in the literature (i.e., low-pass filters [14], bandstop filters [15], or moving average filters (MAF) [16]) require a reduction in bandwidth of the outer voltage loop due to their insufficient transient response in LC-StatCom applications.

To solve this issue, an open-loop observer of the low-frequency capacitor voltage ripple was proposed in [17], which uses the concept of voltage ripple cancellation [18]. Some other studies have also proposed accurate analytical equations to estimate the capacitor voltages in modular multi-level converters [19]–[22]. Differently from [18], the estimated ripple component proposed in [17] is more suitable for large low-frequency voltage ripples, as it does not assume a pure sinusoidal behavior on the capacitor voltages. The algorithm achieves a negligible delay in filtering the capacitor voltages, through using an analytic formula to estimate the low-frequency voltage ripple. Improved transient response was demonstrated in comparison with a conventional second-order notch filter. However, the open-loop nature of the aforementioned model-based observers lead to steady-state oscillations in the filtered signal due to estimation errors, which occur in the baseband region of the frequency spectrum and are independent of the number of voltage levels. These estimation errors occur due to the inherent discrepancies between the actual system parameters and those used in the observer model. It should be remarked that these estimation errors propagate throughout the entire cascade control scheme, thus deteriorating the overall performance of the converter. This phenomena is exacerbated in the LC-StatCom case due to the greater sensitivity to capacitance uncertainty of the low-frequency capacitor voltage ripple magnitude [17].

The main drawback of the solution in [17] is the assumption of a fixed capacitance value for all the series-connected H-bridges in the amplitude estimation of the low-frequency voltage ripple. This creates two main problems: 1) The output from the cluster voltage controller, which is the current reference for the inner current loop (as shown in the control block diagram of Fig. 1(b)), becomes distorted, which results in unnecessary harmonic currents that increase losses and decrease waveform quality. This manifests due to the inherent tolerance in the capacitance values, which can be as high as $\pm 20\%$. 2) The model does not consider variations in the capacitance due to aging, operating conditions, and/or failures.

Given the above context, this paper proposes a closed-loop model-based observer that estimates the actual system capacitance and corrects phase angle detection errors. These model corrections are made based on the error between the measured square-sum-capacitor-voltages and its estimated low-frequency component. The estimation error is processed through an adaptive feedback loop that decomposes it in order to obtain information of the capacitance and phase angle deviations. Indeed, this allows to suitably update the analytical equation used in the observer model, thus avoiding steady-state estimation error. The main novelty of the proposed filtering scheme in this paper is that it optimises the existing trade-off between filtering accuracy and transient response. Furthermore, an attractive extra feature of the proposed closed-loop observer is the condition monitoring capability of the dc-link capacitor of each H-bridge in the converter arm, which is of particular interest in order to determine its health status and forecast possible future failures. Hence, the proposed closed-loop observer also enhances the converter reliability and minimizes its failure cost.

This paper is a more complete iteration of work originally presented in [23], which demonstrated the improved filtering performance of the proposed closed-loop observer in comparison with [17]. The results in [23] also illustrated the viability of using higher bandwidth controllers in the outer voltage loop, compared to the conventional approach based on a second-order generalized integrator (SOGI) configured as a notch filter [15]. The key contributions in this paper, compared to [23], are:

- Using the estimation of the capacitance to calculate the amplitude of the low-frequency voltage ripple, instead of direct estimation of the ripple amplitude in [23]. This improvement results in superior transient response of the proposed algorithm in this paper;
- Estimation of the capacitance value of each H-bridge also allows tracking of the state of health of capacitors. Having this instantaneous indicator enhances the overall system reliability. Accordingly, preventive maintenance actions can be taken when the differences between capacitors are relatively large, hence avoiding the sudden failure of the power converter;
- Experimental confirmation of the proposed approach. The feasibility and improved performance of the proposed filtering approach is confirmed by experiments on a seven-level 1-kVA single-phase CHB-based LC-StatCom laboratory prototype. Effects of mismatches between nominal parameters used in the model and practical system parameters are also investigated;
- The operation of the proposed closed-loop observer throughout the system is analysed in order to describe conditions to limit control interactions and instability issues.

The remaining sections of the paper are organized as follows. Section II presents the mathematical model to estimate the low-frequency ripple of the square of capacitor voltages. The estimation error that arises by using an open-loop observer is

shown in Section III. This estimation error is used in Section IV to formulate the proposed closed-loop observer. In Section V, experimental results obtained from the filtering schemes studied in this paper are analysed, and Section VI concludes the paper.

II. BACKGROUND - MATHEMATICAL MODEL FOR THE OBSERVER

In this section, a comprehensive analysis of the oscillating instantaneous power in the converter arm is presented. Based on this analysis, a model is derived, which allows to estimate the low-frequency ripple on the square value of capacitor voltages. This model is used in the next sections to design the proposed closed-loop observer. It should be noted that square-sum-capacitor-voltages has a sinusoidal ripple, however, the sum-capacitor-voltages does not have a sinusoidal ripple [17], [19]. Hence, square-sum-capacitor-voltages is utilised in the proposed closed-loop observer. Accordingly, its filtered signal is used in the converter outer voltage loop, as shown in the control block diagram of Fig. 1(b).

A general circuit representation of the CHB converter arm is depicted in Fig. 1(a), which can be considered as a multilevel pulse-width modulated (PWM) voltage source, behind the reactance of the filtering inductor L . Assuming lossless H-bridges, the instantaneous power flowing at the dc-side of the j^{th} H-bridge is equal to the instantaneous power on its ac-side,

$$v_{Cj}(t) C_j \frac{dv_{Cj}(t)}{dt} = -v_{outj}(t) i_L(t), \quad (1)$$

where $\forall j \in \{1, 2, \dots, n\}$.

The variable v_{outj} is the instantaneous voltage at the ac-side of the j^{th} H-bridge. The parameter C_j denotes the H-bridge capacitance value, and i_L is the current for the n series-connected H-bridges in the converter arm.

If the converter voltage is evenly distributed among the n H-bridges (i.e., $v_{outj} = v_{out}/n$, $\forall j \in \{1, 2, \dots, n\}$), (1) can be reformulated in terms of the time derivative of the square-sum-capacitor-voltages as follows:

$$\frac{dz_{clus}(t)}{dt} = -\frac{2v_{out}(t) i_L(t)}{nC_{eq}}. \quad (2)$$

The parameter C_{eq} is the equivalent converter arm capacitance, correspondent to the series-connection of the per-H-bridge capacitances (i.e., $1/C_{eq} = \sum_{j=1}^n 1/C_j$), which is known within a specified tolerance.

By solving the differential equation in (2), the following general expression for the square-sum-capacitor-voltages is derived:

$$z_{clus}(t) = z_{clus,0} + z_{clus,2}(t) + d(t), \quad (3)$$

in which,

$$z_{clus,0} = z_{clus}(0) - \frac{\hat{v}_{out}\hat{i}_L}{2\omega_g n C_{eq}} \sin(\alpha_v + \varphi), \quad (4)$$

$$z_{clus,2}(t) = \frac{\hat{v}_{out}\hat{i}_L}{2\omega_g n C_{eq}} \sin(2\omega_g t + \alpha_v + \varphi), \quad (5)$$

$$d(t) = -\frac{\hat{v}_{out}\hat{i}_L}{nC_{eq}} \cos(\alpha_v - \varphi)t. \quad (6)$$

Harmonic contents of the PWM converter voltage and arm current are neglected. This is a reasonable assumption when utilising either a sufficiently high switching frequency and/or large number of voltage levels. The case where the grid voltage has undesired harmonics is discussed in the Appendix.

The grid voltage, v_g , is chosen as the reference phasor with the angular frequency of ω_g . Parameters $\hat{v}_{out}$ and α_v represent the instantaneous amplitude and angle of the converter output voltage, respectively. When dealing with predominantly reactive power, the angle α_v tends to be relatively small, determined by the losses and the operating point [19]. Similarly, $\hat{i}_L = \sqrt{\hat{i}_d^2 + \hat{i}_q^2}$ and $\varphi = \text{atan2}(\hat{i}_q, \hat{i}_d)$ are the magnitude (amplitude conservative dq transformation) and angle of the arm current, respectively. The arm current is defined as flowing from the converter to the grid (see Fig. 1(a)). Consequently, the arm current will be lagging the grid voltage during the capacitive operation, whilst leading it during inductive mode.

It is observed in (3)-(6) that the square-sum-capacitor-voltages is composed by a dc term, $z_{clus,0}$, a sinusoidal term at twice the fundamental frequency, $z_{clus,2}$, and a nonperiodic term, d . The parameter $z_{clus}(0)$ denotes the initial condition. In steady-state operation (i.e., capacitor voltages are periodic waveforms), the nonperiodic disturbance term (6) is considered balanced out by the real power loss in the power converter arm. In other words, the injected current into the grid is not purely reactive (i.e., $\varphi = \alpha_v \pm \pi/2$ rad). Therefore, the instantaneous power flowing through the converter arm, which is proportional to dz_{clus}/dt , does not contain a dc component. Otherwise, it will imply a continuous increase or decrease in the stored energy in the capacitors. In such a case, (2) becomes:

$$\frac{dz_{clus}(t)}{dt} = \mp \frac{\hat{v}_{out}\hat{i}_L}{nC_{eq}} \sin(2\omega_g t + 2\alpha_v). \quad (7)$$

The outer voltage control loop, which is explained in detail in the next section, is responsible for regulating the dc component of the square-sum-capacitor-voltages, $z_{clus,0}$, to a desired reference value, $z_{clus,0}^*$. Fast regulation of the capacitor voltages in the LC-StatCom is essential due to the low amount of energy stored within the converter arm, which makes the capacitor voltages very sensitive to transient reactive power conditions. Therefore, the sinusoidal component in the measured square-sum-capacitor-voltages, $z_{clus,2}$, must be filtered with a negligible time delay, so that $z_{clus,0}$ can be quickly and accurately regulated during transients. To achieve this aim, a novel filtering scheme is proposed in Section IV.

Fig. 2. LC-StatCom cascade control scheme with an open-loop observer of low-frequency capacitor voltage ripples [11].

III. DERIVATION OF OPEN-LOOP OBSERVER ERROR

This section briefly describes the control scheme of a single-phase LC-StatCom, which is based on the cascade connection of three decoupled stages. This control scheme was developed in [11], [17], [24], and is depicted in Fig. 2. The filtering approach used in the aforementioned references is further analysed in this section. This section also identifies the limitations of this open-loop model-based observer, thereby providing further motivation for the proposed closed-loop version in this paper, which will be introduced in the next section.

The heart of the control scheme is the current controller, which was implemented using a deadbeat controller to facilitate fast dynamic performance [25], [26]. The main reference to the system is the required reactive power level, $\hat{i}_q^*$, to be injected to or absorbed from the grid. The reference for the dc component of the square-sum-capacitor-voltages, $z_{clus,0}^*$, is a function of the reactive power in the LC-StatCom. Its value is updated by the ‘capacitor voltage limiter’ block to keep the maximum in the capacitor voltages regulated to the constant value $V_{C,max}$ at any reactive power level, as the LC-StatCom operation principle requires [11]. The aim of the outer voltage loop is to regulate the dc component of the cluster voltage to its reference value. This aim is effectively achieved by calculating the required active power component of the arm current, $\hat{i}_d^*$.

The three-phase CHB-based converter consists of three single-phase CHB power converter arms, like the one shown in Fig. 1(a), connected either in delta or star configuration. Therefore, each phase-arm is exposed to second-order power oscillations, which translates into low-frequency voltage ripple on the capacitors. In a balanced-three-phase system, the $2\pi/3$ rad phase difference among the square-sum-capacitor-voltages of each power converter arm leads to a natural cancellation of the low-frequency voltage ripple component when adding them together. However, this is only true for a CHB StatCom operating under balanced conditions, if there is any unbalance / negative sequence component in the grid voltage and/or the injected StatCom current, then the oscillatory components of the three cluster voltages will not sum to zero, and therefore the proposed closed-loop observer is applicable. Furthermore,

the closed-loop observer can also be utilised to estimate each cluster voltage, which is necessary in the control loop responsible for evenly distributing capacitor energy between the phases [3], [6], [14].

Expression (5) is utilised as a model to provide an estimate of $z_{clus,2}$, which is denoted as $z_{clus,2}^{est}$. For this purpose, as the practical system parameters are estimated, the following assumptions are made:

- Proper controller operation (i.e., $v_{out} \approx v_{out}^*$, and $i_L \approx i_L^*$), where the converter voltage reference, v_{out}^* , is adjusted by the current controller in order to guarantee the tracking of the reference current, i_L^* .
- The capacitance value is assumed as a nominal value, C_0 , throughout all the series-connected H-bridges within the power converter arm (i.e., $C_{eq} = C_0/n$).

Therefore, the open-loop estimate for $z_{clus,2}$ is given by:

$$z_{clus,2}^{est}(t) = A_0 \sin(2\omega_g t + \theta_0). \quad (8)$$

The variables A_0 and θ_0 provide instantaneous estimates of the amplitude and phase of the sinusoidal signal of interest (5), respectively, and they are defined as:

$$A_0 = \frac{\hat{v}_{out}^* \hat{i}_L^*}{2\omega_g C_0}, \quad (9)$$

$$\theta_0 = 2\alpha_v^* \pm \frac{\pi}{2}. \quad (10)$$

The amplitude and phase of v_{out}^* can be easily obtained by introducing a quarter of a fundamental period delay function (i.e., $e^{-sT_g/4}$ in the frequency domain, with $T_g = 2\pi/\omega_g$), which determines the quadrature component of v_{out}^* , followed by the $\alpha\beta$ to dq transformation [13], [27]. On the other hand, the amplitude and phase of i_L^* can be calculated with a negligible delay since its dq components are known within the control scheme (see Fig. 2).

The filtered square-sum-capacitor-voltages is defined as $z_{clus}^f = z_{clus} - z_{clus,2}^{est}$. It is also assumed that $A = A_0 + \Delta A$

Fig. 3. LC-StatCom outer voltage control loop with the proposed closed-loop observer strategy.

and $\theta = \theta_0 + \Delta\theta$, due to discrepancies between the practical system parameters (A and θ) and the utilised model parameters (A_0 and θ_0). Therefore, the filtered square-sum-capacitor-voltages is

$$z_{clus}^f(t) = z_{clus,0} + (A_0 + \Delta A) \sin(2\omega_g t + \theta_0 + \Delta\theta) - A_0 \sin(2\omega_g t + \theta_0), \quad (11)$$

where ΔA and $\Delta\theta$ represent the unknown deviations due to estimation errors and parametric uncertainty.

Applying the angle difference trigonometric identity and rearranging, and defining $\psi = 2\omega_g t + \theta$ to simplify the notation, the rectangular form of the filtered square-sum-capacitor-voltages corresponds to:

$$z_{clus}^f(t) = z_{clus,0} + \underbrace{[A_0(1 - \cos \Delta\theta) + \Delta A]}_{\epsilon_d} \sin(\psi) + \underbrace{(A_0 \sin \Delta\theta)}_{\epsilon_q} \cos(\psi), \quad (12)$$

where ϵ_d and ϵ_q represent the direct and quadrature components of the estimation error.

Equation (12) states that discrepancies between estimates and measurements lead to an estimation error in the filtered signal, which can be viewed as two orthogonal phasors rotating at twice the fundamental frequency. It is noted that the d - and q -components of the estimation error are coupled by the difference in the phase, $\Delta\theta$. This is mainly because a sinusoid is a nonlinear function of its phase. If $\Delta\theta = 0$ and $\Delta A \neq 0$, only ϵ_d should be compensated; whereas if $\Delta\theta \neq 0$ and $\Delta A = 0$, both ϵ_d and ϵ_q should be compensated. Therefore, the filtered signal may be polluted during the steady-state operation when using an open-loop observer, thus affecting the overall cascade control scheme.

The presented context motivates the study of a closed-loop observer that brings the estimation error in (12) to zero in the

steady-state, which represents the main contribution of this paper.

IV. PROPOSED CLOSED-LOOP OBSERVER

In the previous section, the necessity of a closed-loop observer for the low-frequency capacitor voltage ripple has been demonstrated. In this section, a closed-loop version to account for any discrepancies between the practical system parameters (A and θ) and the nominal parameters used in the model (A_0 and θ_0) is presented. The filtered signal, z_{clus}^f , which contains information about the differences between estimates and measurements, is fed back to accordingly update the observer model (as depicted in Fig. 3) so that estimation errors are brought towards zero as time progresses. The details of the proposed closed-loop observer are explained in the following subsections.

A. Closed-Loop Observer Based on Amplitude Estimation Method (AEM)

The idea behind the proposed closed-loop observer is to directly use the information provided by the estimation errors, ϵ_d and ϵ_q , as they disclose how well the estimated signal matches the measured one. The filtered square-sum-capacitor-voltages, z_{clus}^f , consists of some undesired and nonfiltered harmonic components when using an open-loop observer. Therefore, feeding back z_{clus}^f represents a potential alternative to update the observer model, as depicted in Fig. 3. As mentioned above, the aim is to bring the estimation error towards zero in the steady-state (i.e., $\epsilon_d \rightarrow 0$ and $\epsilon_q \rightarrow 0$ as $t \rightarrow \infty$), which requires a certain value for the unknown deviations (i.e., $\Delta A \rightarrow (A - A_0)$ and $\Delta\theta \rightarrow (\theta - \theta_0)$ as $t \rightarrow \infty$).

By linearising the dq components of the estimation error in (12), the corresponding relationships between estimation errors and deviations can be derived as,

$$\epsilon_d \approx \Delta A, \quad (13)$$

$$\epsilon_q \approx A_0 \Delta\theta, \quad (14)$$

where the Maclaurin series up to the first term of $\Delta\theta$ is utilised (i.e., $\sin \Delta\theta \approx \Delta\theta$, and $\cos \Delta\theta \approx 1$). It is worth noting that the previous linearisation allows decoupling of the dq components of the estimation error. The previous approximation holds because the angle detection is based on mature phase-locked loop (PLL) circuits, and then, the unknown variable $\Delta\theta$ can be considered very small. The previous result represents a rough estimation of the required ΔA and $\Delta\theta$ to update the observer model. Nevertheless, it gives intuition about how the deviations in amplitude estimation and phase angle detection affect the different components of the estimation error. Therefore, to achieve optimal performance of the proposed adaptive feedback loop, a control loop for the dq components of the estimation error is needed so that A and θ are correctly estimated. For this purpose, two independent proportional-integral (PI) controllers in the synchronous frame are proposed to process ϵ_d and ϵ_q in order to determine ΔA and $\Delta\theta$, respectively, as:

$$\Delta A = k_{pa}\epsilon_d + k_{ia} \int \epsilon_d dt, \quad (15)$$

$$\Delta\theta = \frac{k_{pa}}{A_0}\epsilon_q + \frac{k_{ia}}{A_0} \int \epsilon_q dt. \quad (16)$$

The parameters k_{pa} and k_{ia} are the proportional and integral gains of the PI controllers used in the proposed adaptive feedback loop, respectively. The main criteria for choosing these parameters are explained at the end of this Section, where the stability of the proposed closed-loop observer and its operation throughout the control system, are analysed.

As a conclusion, if the filtered square-sum-capacitor-voltages, z_{clus}^f , contains some second-order harmonic contamination, which is quantified by ϵ_d and ϵ_q , the responsible ΔA and $\Delta\theta$ variables can be determined to update the model and bring the second-harmonic content in z_{clus}^f to zero in steady-state. Therefore, the closed-loop estimate for $z_{clus,2}$ when using the AEM is given by:

$$z_{clus,2}^{est}(t) = (A_0 + \Delta A) \sin(2\omega_g t + \theta_0 + \Delta\theta). \quad (17)$$

As it can be observed in Fig. 3, a unitary midband gain bandpass filter (with $G_{\omega_r}(s)$ as transfer function), tuned at $\omega_r = 2\omega_g$, is used to extract the unwanted second-order frequency component from z_{clus}^f . Subsequently, this signal and its quadrature signal are used to derive the estimation errors, ϵ_d and ϵ_q . An eighth of the grid-voltage period delay function is used to generate the quadrature component required for the $\alpha\beta$ to dq transformation. This is because the grid angular frequency is used to obtain the in-quadrature signal of a signal of twice the grid frequency. The use of the bandpass filter is to attenuate the effect of the superimposed dc component,

$z_{clus,0}$, and other undesired components, such as noise, in the determination of ϵ_d and ϵ_q . The Park transformation synchronised with the angle ψ is utilised to obtain the envelope (or ϵ_d and ϵ_q) of the sinusoidal signals ϵ_α and ϵ_β :

$$\begin{bmatrix} \epsilon_d \\ \epsilon_q \end{bmatrix} = \begin{bmatrix} \sin(\psi) & -\cos(\psi) \\ \cos(\psi) & \sin(\psi) \end{bmatrix} \begin{bmatrix} \epsilon_\alpha \\ \epsilon_\beta \end{bmatrix} \approx \begin{bmatrix} \Delta A \\ A_0 \Delta\theta \end{bmatrix}. \quad (18)$$

Most importantly, and differently than in conventional approaches based on second-order filters [15], the bandpass filter here is inserted in parallel with the outer voltage loop, as depicted in Fig. 3. This offers the possibility of avoiding interactions between the two control loops (i.e., outer voltage loop and proposed adaptive feedback loop), thus easing the design of the cluster voltage controller and not deteriorating the fast dynamic response provided by the open-loop observer in [17]. The necessary conditions that guarantee that the proposed closed-loop observer has no effect on the transient response, which constitutes one of the main advantages of the proposed solution, are now explained.

B. Closed-Loop Observer Based on Capacitance Estimation Method (CEM)

The estimated phase of the square-sum-capacitor-voltages, θ_0 , represents a reliable quantity, because it is determined based on mature PLL circuits. On the other hand, the fact of assuming a nominal capacitance value equal to C_0 for each and every H-bridge to estimate the amplitude A_0 , represents a considerable source of error in practical systems. In these systems, tolerances, operating conditions, component aging, and sudden failures, affect the equivalent capacitance value in the converter arm. Moreover, the observer model in (8) is also highly sensitive to the capacitance value, specially in LC-StatCom applications.

It can be appreciated that under proper controller operation, the main source of variation in the amplitude estimation in (9) is due to capacitance variations. In this subsection, another method for estimating the low-frequency capacitor voltage ripple, based on using the parametric uncertainty in the capacitance value, denoted as ΔC , is presented. For this purpose, the capacitance estimation is defined as:

$$C = C_0 + \Delta C. \quad (19)$$

In order to derive the parametric uncertainty in the capacitance value, the following expression can be derived from (5) and (9):

$$\frac{A}{A_0} \approx \frac{C_0}{C}, \quad (20)$$

and, using $A = A_0 + \Delta A$ and (19), results in:

$$\Delta C = -\frac{\Delta A}{A_0 + \Delta A} C_0. \quad (21)$$

Therefore, in this new case, the proposed closed-loop estimate for $z_{clus,2}$ when using the CEM is:

TABLE I
PARAMETERS OF THE EXPERIMENTAL SETUP

Definition	Symbol	Value
Grid Voltage Nominal Amplitude	$\hat{V}_g$	$200\sqrt{2}$ V (1 p.u.)
Nominal Power	S_g	1 kVA (1 p.u.)
Number of H-Bridges per Arm	n	3
Grid Frequency	f_g	50 Hz
Peak Capacitor Voltage Reference	$V_{C,max}$	113 V
Nominal Capacitance Value per H-Bridge	C_0	0.18 mF (0.442 p.u.)
Interface Filter Inductance	L	5 mH (0.039 p.u.)

$$z_{clus,2}^{est}(t) = \frac{\hat{v}_{out}^* \hat{i}_L^*}{2\omega_g(C_0 + \Delta C)} \sin(2\omega_g t + \theta_0 + \Delta\theta), \quad (22)$$

which is equivalent to the AEM in (17), but offers promising features that are discussed below.

It should be noted that the effective capacitance value in the converter arm, $C = C_0 + \Delta C$, is mostly constant under normal operation (no failure) for different reactive power levels, as the voltage drop across the interface inductance is typically small. Furthermore, the amplitude of the low-frequency voltage ripple is directly proportional to the reactive power level (5). Therefore, the CEM-based observer provides a superior response in the regulation of $z_{clus,0}$, when a change in the reactive power command occurs, than when using the AEM-based one. Moreover, the proposed adaptive feedback loop to estimate C can be slow, because it only needs to adjust for the capacitance, which changes very slowly over time.

The following reasoning/numerical example aims to justify the superiority of the proposed CEM-based observer:

Example #: Consider that at $t = t_1$, the reactive power command changes from $\hat{i}_q^* = 0.5$ p.u. to $\hat{i}_q^* = -1$ p.u. (i.e., from half inductive to full capacitive operation). If using the AEM, a possible scenario just before the transient, assuming steady-state, could be that the estimated amplitude is $A_0 = 0.45$ p.u., and hence a deviation in the amplitude of $\Delta A = 0.05$ p.u. is mitigating the discrepancies. When the transient takes place, the estimated amplitude changes instantly to $A_0 = 0.90$ p.u., as it is directly proportional to $\hat{i}_q^*$ magnitude, but ΔA remains unchanged due to the limited bandwidth of the proposed adaptive feedback loop. Therefore, $A_0 + \Delta A \neq 1$ p.u. at the transient instant. On the other hand, if using the CEM described in this subsection, the converter arm capacitance, C , is known before the transient due to the proposed adaptive feedback loop. Therefore, the low-frequency voltage ripple amplitude determined by (22) matches the measured amplitude, i.e., $A_0 + \Delta A = \frac{\hat{v}_{out}^* |\hat{i}_q^*|}{2\omega_g(C_0 + \Delta C)} = 0.5$ p.u. When the transient occurs, again the amplitude in (22) changes instantly with $|\hat{i}_q^*|$, but

TABLE II
CONTROL PARAMETERS

Definition	Symbol	Value
Proportional Gain of the Cluster Voltage Controller	k_{pz}	$3.3C_0$
Integral Gain of the Cluster Voltage Controller	k_{iz}	$55.4C_0$
Proportional Gain of the Adaptive Loop Controller	k_{pa}	1
Integral Gain of the Adaptive Loop Controller	k_{ia}	20
Bandwidth of the Cluster Voltage Loop	f_z	150 Hz
Bandwidth of the Current Loop	f_i	1.5 kHz
Damping Ratio of the Bandpass Filter Used in the Adaptive Loop	ζ	0.1
Center Frequency of the Bandpass Filter	f_r	100 Hz
Sampling Frequency	f_s	15 kHz

now, the previously calculated ΔC is kept constant, as the converter capacitance is mostly independent of the injected StatCom current $\hat{i}_q^*$. In this situation, $\frac{\hat{v}_{out}^* |\hat{i}_q^*|}{2\omega_g(C_0 + \Delta C)} = 1$ p.u. is obtained at the transient instant. This ideal operation of the proposed closed-loop observer based on CEM is corroborated by an experimental test in Section V.

It is important to highlight that both in the AEM and in the CEM, 1) the amplitude estimation changes instantly with $|\hat{i}_q^*|$, 2) the sign of the estimated sinusoid also changes instantly with the sign of $\hat{i}_q^*$, and 3) the previously calculated $\Delta\theta$ can be kept constant during the transient, as it only corrects the PLL angle detection accuracy.

In order to verify the theory presented in this section, the proposed closed-loop filtering scheme illustrated in Fig. 3 has been implemented in the MATLAB/Simulink environment, and perturbations in the capacitance (amplitude) and phase angle used in the observer model have been introduced. The parameters of the proposed closed-loop observer are given in Tables I and II. This test is depicted in Fig. 4, when a step change from $C_0 = 180 \mu\text{F}$ to $1.5C_0 = 270 \mu\text{F}$ is conducted at time $t = 0.2$ s, and a step change from $\theta_0 = -90^\circ$ to $1.25\theta_0 = -112.5^\circ$ is applied at $t = 0.7$ s. When the capacitance value, used in the observer model, is suddenly increased, the d component of the estimation error, ϵ_d , increases, and the proposed adaptive loop generates a ΔC in the opposite direction in order to bring the estimation error to zero. Similarly, when the phase angle is increased in magnitude, now the q component of the estimation error, ϵ_q , increases, and the adaptive loop accordingly generates a $\Delta\theta$ to mitigate the error. This event, apart from proving the theory developed in this section, also demonstrates that the

Fig. 4. Proposed closed-loop observer performance under capacitance and phase angle disturbances. Evolution of (a) the dq components of the estimation error, (b) the parametric uncertainty in the capacitance value, and (c) the parametric uncertainty in the phase angle detection.

actual capacitance value, C , can be tracked by the proposed closed-loop observer. It can also be observed how the ϵ_d and ϵ_q components are slightly coupled.

C. Stability of the Proposed Closed-Loop Observer and its Operation Throughout the Control System

In this subsection, the nested control loop composed of the outer capacitor voltage loop integrated with the proposed closed-loop observer is analysed in detail in order to describe the conditions that limit control interactions and instability issues.

A block diagram of the outer voltage loop is shown in Fig. 5, in which $C_z(s)$ is the PI-based cluster voltage controller

that carries out the dc voltage regulation, and $G_{zd}(s)$ and $G_{i,cl}(s)$ constitute the converter plant model to be controlled. The former models the dc-side, while the latter refers to the closed-loop current control, which includes the connection inductance. Usually, a filtering scheme, with $H_f(s)$ as its equivalent transfer function, is implemented for attenuating (from the control point of view) the twice fundamental frequency component of the square-sum-capacitor-voltages. Otherwise, this component produces three times fundamental component in the arm current reference, when performing the frame transformation operation $i_L^* = \hat{i}_d^* \sin(\omega_g t) + \hat{i}_q^* \cos(\omega_g t)$ (see Fig. 2). The equivalent closed-loop transfer function of the outer voltage loop is

$$G_{z,cl}(s) = \frac{\tilde{Z}_{clus,0}(s)}{\tilde{Z}_{clus}^*(s)} = \frac{L_z(s)}{1 + L_z(s)}, \quad (23)$$

where

$$L_z(s) \approx -H_f(s)C_z(s)G_{zd}(s), \quad (24)$$

represents the loop gain of the outer voltage loop, which assumes, at the frequency range of the outer voltage loop design, the inner current loop as a unity gain due to its much higher bandwidth. The dc-side is modelled as

$$G_{zd}(s) = \frac{\tilde{Z}_{clus,0}(s)}{\tilde{I}_d(s)} \approx -\frac{\hat{V}_g}{sC_0}, \quad (25)$$

which is independent of the operating point, as discussed in [24]. Parameter $\hat{V}_g$ is the nominal grid voltage amplitude.

A detailed block diagram representation of the proposed closed-loop filtering scheme, shown in Fig. 3, is depicted in Fig. 6(a). However, as the phase calculation of the square-sum-capacitor-voltages is determined using mature PLL circuits, the uncertainty in the phase angle detection can be assumed negligible (i.e., $\Delta\theta \approx 0$). This assumption allows to further simplify the block diagram to the one shown in Fig. 6(b). Here, as detailed in [28], a PI regulator used in a synchronously rotating reference frame working with dc components, corresponds with a resonant controller in the stationary frame.

Based on the previous discussion, the proposed closed-loop observer can be modelled as follows:

$$H_f(s) = \frac{1}{1 + L_a(s)}, \quad (26)$$

with

$$L_a(s) \approx G_{\omega_r}(s) \left(k_{pa} + \frac{2k_{ia}s}{s^2 + \omega_r^2} \right). \quad (27)$$

Therefore, the proposed closed-loop observer will be input-output stable if the roots of the characteristic equation, i.e., $1 + L_a(s) = 0$, are in the left half of the complex plane, which is always true (following Routh–Hurwitz criterion, for example) provided the controller parameters are all chosen positive (i.e., $k_{pa} \geq 0$, $k_{ia} \geq 0$). To choose the controller parameters of the proposed closed-loop observer, the rough rule of ten times slower than the outer voltage loop is adopted to limit interactions between the nested loops. Once the

Fig. 5. Block diagram of the outer capacitor voltage control loop.

Fig. 6. Block diagram of (a) the proposed closed-loop observer using PI regulators in a synchronous frame, and (b) its equivalent resonant controller in a stationary frame.

 Fig. 7. Bode diagrams of the system's loop gain, $L_z(j\omega)$, when using the proposed closed-loop observer different methods (i.e., amplitude estimation method (AEM) and capacitance estimation method (CEM)).

stability of the proposed closed-loop observer has been proven, its interaction throughout the system operation is addressed.

It should be noted that at frequencies where $L_a(s)$ is small in magnitude (i.e., $\|L_a(j\omega)\| \ll 1$), the equivalent transfer function $H_f(s)$ is approximately equal to 1, thus ensuring decoupling between the outer voltage loop and the proposed closed-loop observer. However, $L_a(s)$ presents infinite gain at the frequency ω_r due to the equivalent resonant controller used in the adaptive loop, which provides the proposed closed-

loop observer with an input-output notch filter characteristic. Nevertheless, as the proposed closed-loop observer based on CEM mainly needs to adjust for the capacitance uncertainty, which does not change with the system operating point, $\hat{i}_q^*$, the estimated capacitance, $C_0 + \Delta C$, can remain constant during transient events. Indeed, this implies that the proposed closed-loop observer has no effect on the transient response of the control loop, as in [17], which allows the implementation of a fast voltage control loop. In contrast, if using the AEM instead of the CEM, the operation of the proposed closed-loop observer will affect the system operation.

As the proposed closed-loop observer based on CEM does not affect the system operation during transient events, the closed-loop transfer function of the outer voltage loop in (23) can be approximated as

$$G_{z,cl}(s) \approx \frac{\hat{V}_g k_{pz} (k_{pz}s + k_{iz})}{C_0 s^2 + \hat{V}_g k_{pz}s + \hat{V}_g k_{iz}}, \quad (28)$$

which corresponds with a second-order system, whose closed-loop poles can be adjusted by the cluster voltage controller parameters, namely k_{pz} and k_{iz} .

The parameters of the outer voltage control loop are given in Table II, which provide a phase margin of 90° and a bandwidth of 150 Hz when using the CEM. Fig. 7 depicts the frequency response of the system loop gain, $L_z(j\omega)$, in the case of the

Fig. 8. Experimental setup.

CEM, and when using the AEM instead. As it can be observed, the main difference between both Bode diagrams is the high attenuation of the magnitude at 100 Hz due to the notch filter characteristic of the proposed closed-loop observer. Moreover, the cut-off frequency is reduced from 150 Hz to 90 Hz, and the phase-margin from 90° to 60° , if maintaining the same controller parameters.

V. EXPERIMENTAL RESULTS

With the purpose of illustrating the feasibility and performance of the proposed closed-loop observer, experimental results are obtained on a 1-kVA scaled-down CHB converter arm with three H-bridges (see Fig. 8). In this converter, an interface inductor L is connected between the converter phase-arm positive terminal and the grid voltage. The grid voltage is provided by a GE&EL 15 kVA CINERGIA grid emulator. IMPERIX PEH2015 H-bridge modules are used, which are connected in series to form the seven-level converter. The parameters of the scaled-down LC-StatCom prototype and its control are given in Tables I and II, respectively. It can be appreciated that the size of the capacitors C_0 is relatively small, which results in approximately 70% (of the nominal dc component) low-frequency voltage ripple at the rated power. The control schemes depicted in Figs. 2 and 3 are implemented using a BOOMBOX RCP board from IMPERIX. Phase-shifted carrier PWM (PSC-PWM) is implemented in the controller, which generates a constant switching firing pulse pattern in the converter.

Three different filtering schemes are tested, namely, the conventional SOGI with notch configuration as in [15], an open-loop model-based observer as in [17], and the proposed closed-loop observer (both the AEM and the CEM). The various features of the different filtering schemes are evaluated on the LC-StatCom CHB converter with three different test cases, namely: *Case I*: Steady-state operation, *Case II*: Transient response under a step change in the reactive power reference, and *Case III*: Operation under a sudden change in the capacitance value. The total-harmonic-distortion (THD) of the injected current into the grid is also shown for the three different filtering approaches.

Case I: The steady-state filtering effectiveness and sensitivity under the presence of mismatches between the practical sys-

Fig. 9. **Case I:** Experimental waveforms showing the steady-state performance when (a) the open-loop observer is used, (b) the error in phase estimation is corrected, and (c) the errors in both phase and capacitance estimation are corrected using the proposed closed-loop observer.

tem parameters and the utilised model parameters is analysed in this case. Fig. 9 shows the steady-state performances of the proposed solution in comparison with the open-loop method. A 20% mismatch in both, the utilised model capacitance, C_0 , and estimated phase angle, θ_0 , has been considered in order to corroborate the effectiveness of the proposed closed-loop observer in mitigating estimation errors, thus overcoming the problems presented in Section III when using an open-loop observer. As it can be noted, the filtered signal, z_{clus}^f , contains large second-order harmonic contamination when using the open-loop observer (Fig. 9(a)), which corroborates the theoretical result obtained in (12). Fig. 9(b) shows how the estimation error in phase is compensated when closing the

proposed adaptive feedback loop that eliminates the estimation error ϵ_q (14). For this purpose, the PI controller that processes ϵ_q , in Fig. (3), generates a value for the variable $\Delta\theta$ to update the observer model. Finally, the parametric uncertainty in the capacitance can be compensated by closing the loop that eliminates the estimation error ϵ_d (13). This can be observed in Fig. 9(c), where the proposed closed-loop approach based on CEM (22) is implemented. In this case, a filtered signal, almost harmonic-free, is obtained, which translates into a current reference without extra undesired harmonic components. In this case, the injected current into the grid will have a better quality (THD), as will be highlighted at the end of this section. It is noted that these undesired oscillations occur in the baseband region of the frequency spectrum and are independent of the number of voltage levels, i.e., even with a high number of CHBs, these oscillations are present within the control structure and therefore they still significantly deteriorate the converter efficiency and its harmonic performance. The effectiveness of the proposed closed-loop observer to mitigate both phase and amplitude estimation errors has been validated in this test case. It is verified that the proposed observer is able to effectively filter out the large low-frequency voltage ripples on the capacitors.

Case II: The transient performance of the proposed closed-loop observer is compared against the SOGI method, and results are illustrated in Fig. 10. The step change in reactive power is from full to 33% capacitive operation at time $t = 0.02$ s. 30% mismatch in the observer model capacitance has been considered.

In Fig. 10(a), the transient response of the SOGI is illustrated, where it is evident how the bandpass characteristic of the SOGI introduces a large delay in the outer voltage loop transient response. Indeed, this leads to an underdamped response with high settling time and oscillations when a high bandwidth cluster voltage controller is used. Therefore, a reduction in the bandwidth of the outer voltage loop is required due to stability issues. Specifically, the parameters of the cluster voltage controller were reduced three times with respect to those in Table II, to obtain a steady-state performance similar to that of the proposed scheme.

The differences in the transient response when directly estimating the low-frequency voltage ripple amplitude, as in (17), and when estimating it indirectly through the capacitance value, as in (22), are depicted in Figs. 10(b) and (c), respectively. As it can be observed, when trying to estimate the amplitude directly, its dependency with the current reference magnitude causes initial exponentially decreasing oscillations. This is because the proposed adaptive feedback loop has to update the value of ΔA to compensate for the mismatches in the amplitude. On the other hand, when using the CEM, there is no significant initial transient, because the capacitance value estimated previously to the transient, namely $C_0 + \Delta C$, is kept constant during the step change in the current reference due to its independence from the operating point. Otherwise, an initial transient would also appear because the parametric uncertainty in the capacitance value, ΔC , is calculated based on ΔA , as per in (21). As it can be seen, the transient performance of the

Fig. 10. **Case II:** Experimental waveforms during a step reactive power change from 100% capacitive operation to 33% capacitive operation when using (a) the SOGI with a lower bandwidth outer voltage loop, (b) the proposed closed-loop observer with the AEM, and (c) the proposed closed-loop observer with the CEM.

proposed closed-loop observer is extremely fast in comparison with that obtained when using the conventional SOGI.

In this case, it has been validated that the dynamic performance under a reactive power transient is enhanced when using the proposed closed-loop observer based on CEM. This allows the use of higher bandwidth outer voltage control loops, which leads to a faster regulation of the capacitor voltages. This feature makes it useful in applications with

Fig. 11. **Case III:** Experimental waveforms showing the capacitance estimation capability of the proposed observer under a sudden change in the capacitance value used in the model from C_0 to $1.3C_0$.

TABLE III
PERFORMANCE COMPARISON OF THE PROPOSED CLOSED-LOOP OBSERVER, OPEN-LOOP OBSERVER, AND SOGI FILTERING SCHEMES WHEN THERE ARE 20% MISMATCHES BOTH IN CAPACITANCE AND PHASE ESTIMATION

Filtering Technique	%THD Injected Current at 0.5 p.u.	%THD Injected Current at 1.0 p.u.
Open-Loop Model-Based Observer [17]	12.4	11.4
Proposed Closed-Loop Model-Based Observer	8.3	6.1
SOGI Configured as a Notch-Filter [15]	8.2	6.0

low ‘electrical inertia’, like the LC-StatCom system. The improved performance of the proposed scheme avoids the increase in maximum capacitor voltage after the transient (and therefore also the increase in voltage applied across the H-bridge switching devices), which is experienced in the SOGI approach.

Case III: The capacitance estimation capability of the proposed closed-loop observer, when the capacitance value

suddenly changes, is evaluated in this case and the results are shown in Fig. 11. For the sake of simplicity, a step change in the capacitance value, which is used in the observer model, is studied instead of directly changing the capacitance in the converter. A step change from $C_0 = 180 \mu\text{F}$ to $1.3C_0 = 234 \mu\text{F}$ is conducted at time $t = 0.02 \text{ s}$. As it can be observed, the capacitance value, initially used in the observer model ($C_0 = 180 \mu\text{F}$), does not strictly match the present one within the power converter ($C \approx 190 \mu\text{F}$). Therefore, a certain parametric uncertainty in the capacitance is correcting the discrepancies, i.e., $\Delta C \approx 10 \mu\text{F}$. When the capacitance value, used in the model, is suddenly increased, the adaptive loop generates a ΔC in the opposite direction. This is because in this illustrative test, the actual system capacitance C has not really changed (i.e., a discrepancy is emulated between the nominal capacitance value, used in the model, and the actual capacitance value within the power converter). As a consequence, the proposed closed-loop observer reestablishes the value of ΔC in order to obtain the real capacitance value of the converter arm ($C \approx 190 \mu\text{F}$). This event demonstrates that the actual capacitance value C can be tracked regardless of any error between modelled and actual C value, i.e., convergence and correct control operation are proven even in the presence of large mismatches.

Finally, Table III shows the performance comparison of the three studied filter schemes in terms of THD of the injected current into the grid. As in *Case I*, 20% mismatch in both the nominal capacitance value and phase angle value used in the observer model, has been adopted. As expected, the worst case is when using an open-loop observer approach, because the estimation errors due to mismatches are not compensated. On the other hand, when using the proposed closed-loop observer, the THD is significantly improved. It should be highlighted that even though the proposed observer is a model-based filter, it exhibits a very similar steady-state performance compared to those obtained by using a transfer-function-based filter, such as the SOGI.

VI. CONCLUSION

In this paper, a closed-loop analytic filtering scheme of the low-frequency voltage ripple on the capacitors of a CHB multilevel converter-based LC-StatCom has been proposed. The proposed solution optimises the trade-off between filtering accuracy and transient response. A harmonic-free filtered signal is obtained in the steady-state. This is thanks to the closed-loop design of the observer, which updates its mathematical model over time by feeding back information regarding the mismatches between the practical system parameters and the utilised model parameters. The extremely fast transient response is achieved because the proposed analytic solution introduces a negligible delay in the outer voltage control loop. An additional salient feature of the proposed solution is that it provides information regarding the state of health of capacitors, which can be used to forecast failures in the converter. Experimental results on a scaled-down LC-StatCom prototype have shown the excellent performance of

the proposed strategy, compared to the available algorithms in the literature.

APPENDIX

In this Appendix, the capability of the proposed filtering scheme to deal with the case when the grid voltage has undesired harmonics is addressed.

Even though the grid voltage is distorted, a fundamental frequency current for reactive power compensation purposes is considered. The interaction between the odd voltage harmonics and the fundamental injected current will cause even harmonic components on the square-sum-capacitor voltages z_{clus} . It is important to highlight that the phase shift parameter $\varphi = \alpha_v \pm \pi/2$ is not affected by the harmonics.

The open-loop estimate for $z_{clus,2}$ presents the same form as in (8). However, the instantaneous estimates of the amplitude and phase are now equal to:

$$A_0 = \frac{\hat{i}_L^*}{2\omega_g C_0} H_0, \quad (29)$$

$$\theta_0 = -\vartheta_0 + \frac{\pi}{2}, \quad (30)$$

where intermediate variables H_0 and ϑ_0 are calculated as $H_0 = \sqrt{a^2 + b^2}$ and $\vartheta_0 = \text{atan2}(b, a)$, respectively, with:

$$\begin{aligned} a &= \hat{v}_{out}^* \sin(\alpha_v^* + \varphi^*) - \hat{v}_{g,3} \sin(\phi_3 - \varphi^*) \\ b &= \hat{v}_{out}^* \cos(\alpha_v^* + \varphi^*) - \hat{v}_{g,3} \cos(\phi_3 - \varphi^*). \end{aligned} \quad (31)$$

The amplitude and phase of the fundamental component of v_{out}^* (i.e., $\hat{v}_{out}^*$ and α_v^*), together with the amplitude and phase of all the voltage harmonics (i.e., $\hat{v}_{g,h}$ and ϕ_h , with h as harmonic index), can be easily obtained by using multifrequency PLLs [29]–[31].

A similar process can be carried out to estimate all the other even harmonics present in the square-sum-capacitor-voltages. However, this is not strictly necessary when using the proposed CEM, since all amplitudes of the even harmonics in z_{clus} are inversely proportional to the capacitance value.

Parametric uncertainty in the capacitance will lead to a filtered signal z_{clus}^f polluted with even harmonics. However, the second harmonic will still be the predominant one (the amplitude of the distorting harmonics $\hat{v}_{g,h}$ are considered low enough), and therefore it is the chosen one to be extracted for its processing.

Therefore, the proposed closed-loop estimate for the ac components in z_{clus} , when using the CEM, is:

$$z_{clus,2}^{est} = \frac{\hat{i}_L^*}{2\omega_g (C_0 + \Delta C)} H_0 \sin(2\omega_g t + \theta_0 + \Delta\theta), \quad (32)$$

$$\begin{aligned} z_{clus,h}^{est} &= \frac{\hat{v}_{g,3} \hat{i}_L^*}{4\omega_g (C_0 + \Delta C)} \sin(4\omega_g t + \phi_3 + \varphi^*) + \sum_{h=5}^{\infty} \frac{\hat{v}_{g,h} \hat{i}_L^*}{\omega_g (C_0 + \Delta C)} \\ &\times \left[\frac{\sin((h+1)\omega_g t + \phi_h + \varphi^*)}{h+1} - \frac{\sin((h-1)\omega_g t + \phi_h - \varphi^*)}{h-1} \right]. \end{aligned} \quad (33)$$

It is important to note that using an adaptive loop for each even harmonic component in z_{clus}^f is possible (and necessary if using the AEM). However, when using the CEM this will only increase the complexity of the scheme without adding relevant benefits, as 1) the capacitance uncertainty ΔC can be determined by a single adaptive loop that processes the predominant harmonic component in z_{clus}^f , and 2) the angle and magnitude of each voltage harmonic (ϕ_h and $\hat{v}_{g,h}$) can be accurately tracked by multifrequency PLL approaches.

REFERENCES

- [1] J. Dixon, L. Moran, J. Rodriguez, and R. Domke, "Reactive power compensation technologies: state-of-the-art review," *Proc. of the IEEE*, vol. 93, no. 12, pp. 2144–2164, Dec. 2005.
- [2] B. Singh, R. Saha, A. Chandra, and K. Al-Haddad, "Static synchronous compensators (STATCOM): A review," *IET Power Electronics*, vol. 2, no. 4, pp. 297–324, Jul. 2009.
- [3] J. I. Leon, S. Vazquez, and L. G. Franquelo, "Multilevel converters: Control and modulation techniques for their operation and industrial applications," *Proc. of the IEEE*, vol. 105, no. 11, pp. 2066–2081, Nov. 2017.
- [4] K. Sano and M. Takasaki, "A transformerless D-STATCOM based on a multivoltage cascade converter requiring no dc sources," *IEEE Trans. Power Electron.*, vol. 27, no. 6, pp. 2783–2795, Jun. 2012.
- [5] H. Akagi, "Classification, terminology, and application of the modular multilevel cascade converter (MMCC)," *IEEE Trans. Power Electron.*, vol. 26, no. 11, pp. 3119–3130, Nov. 2011.
- [6] H. D. Tafti, A. I. Maswood, G. Konstantinou, C. D. Townsend, P. Acuna, and J. Pou, "Flexible control of photovoltaic grid-connected cascaded H-bridge converters during unbalanced voltage sags," *IEEE Trans. Ind. Electron.*, vol. 65, no. 8, pp. 6229–6238, Aug. 2018.
- [7] H. Akagi, E. H. Watanabe, and M. Aredes, *Instantaneous power theory and applications to power conditioning*, vol. 62. John Wiley & Sons, 2017.
- [8] H. Wang and F. Blaabjerg, "Reliability of capacitors for DC-link applications in power electronic converters: An overview," *IEEE Trans. Ind. Appl.*, vol. 50, no. 5, pp. 3569–3578, Sep. 2014.
- [9] B. Karanayil, V. G. Agelidis, and J. Pou, "Evaluation of dc-link decoupling using electrolytic or polypropylene film capacitors in three-phase grid-connected photovoltaic inverters," in *Proc. IECON 2013 - 39th Annual Conference of the IEEE Industrial Electronics Society*, pp. 6980–6986, Nov. 2013.
- [10] R. Maheshwari, S. Munk-Nielsen, and S. Busquets-Monge, "Design of neutral-point voltage controller of a three-level NPC inverter with small DC-link capacitors," *IEEE Trans. Ind. Electron.*, vol. 60, no. 5, pp. 1861–1871, May 2013.
- [11] G. Farivar, C. D. Townsend, B. Hredzak, J. Pou, and V. G. Agelidis, "Low-capacitance cascaded H-bridge multilevel StatCom," *IEEE Trans. Power Electron.*, vol. 32, no. 3, pp. 1744–1754, Mar. 2017.
- [12] T. Isobe, D. Shiojima, K. Kato, Y. R. R. Hernandez, and R. Shimada, "Full-bridge reactive power compensator with minimized-equipped capacitor and its application to static VAR compensator," *IEEE Trans. Power Electron.*, vol. 31, no. 1, pp. 224–234, Jan. 2016.
- [13] R. H. Park, "Two-reaction theory of synchronous machines generalized method of analysis - Part I," *Transactions of the American Institute of Electrical Engineers*, vol. 48, no. 3, pp. 716–727, 1929.
- [14] H. Akagi, S. Inoue, and T. Yoshii, "Control and performance of a transformerless cascade PWM STATCOM with star configuration," *IEEE Trans. Ind. Appl.*, vol. 43, no. 4, pp. 1041–1049, Jul. 2007.
- [15] T. Isobe, L. Zhang, H. Tadano, J. A. Suul, and M. Molinas, "Control of DC-capacitor peak voltage in reduced capacitance single-phase S-TATCOM," in *Proc. IEEE 17th Workshop on Control and Modeling for Power Electron. (COMPEL)*, pp. 1–8, Jun. 2016.
- [16] K. Fujii and R. W. De Doncker, "A novel DC-link voltage control of PWM-switched cascade cell multi-level inverter applied to STATCOM," in *Proc. Fourtieth IAS Annual Meeting. Conference Record of the 2005*

Industry Applications Conference, 2005., vol. 2, pp. 961–967 Vol. 2, Oct. 2005.

- [17] G. Farivar, B. Hredzak, and V. G. Agelidis, “Reduced-capacitance thin-film H-bridge multilevel STATCOM control utilizing an analytic filtering scheme,” *IEEE Trans. Ind. Electron.*, vol. 62, no. 10, pp. 6457–6468, Oct. 2015.
- [18] N. A. Ninad and L. A. C. Lopes, “Operation of single-phase grid-connected inverters with large dc bus voltage ripple,” in *Proc. IEEE Canada Electrical Power Conference*, pp. 172–176, Oct. 2007.
- [19] C. Townsend, T. J. Summers, and R. E. Betz, “Comparison of modulation strategies for a cascaded H-bridge StatCom - Part 1: Theoretical background,” in *Proc. IEEE Ind. Electron. Conf. (IECON)*, pp. 2019–2024, Nov. 2010.
- [20] M. Vasiladiotis, N. Cherix, and A. Rufer, “Accurate capacitor voltage ripple estimation and current control considerations for grid-connected modular multilevel converters,” *IEEE Trans. Power Electron.*, vol. 29, no. 9, pp. 4568–4579, Sep. 2014.
- [21] L. Angquist, A. Antonopoulos, D. Siemaszko, K. Ilves, M. Vasiladiotis, and H. Nee, “Open-loop control of modular multilevel converters using estimation of stored energy,” *IEEE Trans. Ind. Appl.*, vol. 47, no. 6, pp. 2516–2524, Nov. 2011.
- [22] K. Ilves, A. Antonopoulos, L. Harnefors, S. Norrga, L. Angquist, and H. Nee, “Capacitor voltage ripple shaping in modular multilevel converters allowing for operating region extension,” in *Proc. IEEE Ind. Electron. Conf. (IECON)*, pp. 4403–4408, Nov. 2011.
- [23] E. Rodriguez, N. Beniwal, G. Farivar, C. D. Townsend, S. Vazquez, and J. Pou, “Adaptive filtering scheme for a low-capacitance StatCom,” in *Proc. IEEE 4th Southern Power Electronics Conference (SPEC)*, pp. 1–7, Dec. 2018.
- [24] G. Farivar, B. Hredzak, and V. G. Agelidis, “Decoupled control system for cascaded H-bridge multilevel converter based STATCOM,” *IEEE Trans. Ind. Electron.*, vol. 63, no. 1, pp. 322–331, Jan. 2016.
- [25] M. Odavic, V. Biagini, P. Zanchetta, M. Sumner, and M. Degano, “One-sample-period-ahead predictive current control for high-performance active shunt power filters,” *IET Power Electronics*, vol. 4, no. 4, pp. 414–423, Apr. 2011.
- [26] R. E. Betz, C. Townsend, T. J. Summers, and G. Mirzaeva, “Simultaneous estimation of sequence and harmonic components for StatCom applications,” in *Proc. International Power Electronics and Motion Control Conference (EPE-PEMC)*, pp. 47–54, Sep. 2010.
- [27] E. Clarke, *Circuit analysis of AC power systems*, vol. 1. Wiley, 1943.
- [28] D. N. Zmood, D. G. Holmes, and G. H. Bode, “Frequency-domain analysis of three-phase linear current regulators,” *IEEE Trans. Ind. Appl.*, vol. 37, no. 2, pp. 601–610, Mar. 2001.
- [29] S. Vazquez, J. A. Sanchez, M. R. Reyes, J. I. Leon, and J. M. Carrasco, “Adaptive vectorial filter for grid synchronization of power converters under unbalanced and/or distorted grid conditions,” *IEEE Trans. Ind. Electron.*, vol. 61, no. 3, pp. 1355–1367, Mar. 2014.
- [30] P. Rodriguez, A. Luna, I. Candela, R. Mual, R. Teodorescu, and F. Blaabjerg, “Multiresonant frequency-locked loop for grid synchronization of power converters under distorted grid conditions,” *IEEE Trans. Ind. Electron.*, vol. 58, no. 1, pp. 127–138, Jan. 2011.
- [31] P. Rodriguez, J. Pou, J. Bergas, J. I. Candela, R. P. Burgos, and D. Boroyevich, “Decoupled double synchronous reference frame PLL for power converters control,” *IEEE Trans. Power Electron.*, vol. 22, no. 2, pp. 584–592, Mar. 2007.