An investigation into the design for rideability of small wheel single-track bicycles and e-scooters

Paudel, Milan

2019

Paudel, M. (2019). An investigation into the design for rideability of small wheel single-track bicycles and e-scooters. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/140129>

<https://doi.org/10.32657/10356/140129>

This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License (CC BY-NC 4.0).

Downloaded on 26 Apr 2023 22:09:34 SGT

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

SINGAPORE

An Investigation into the Design for Rideability of Small Wheel Single-track Bicycles and E-scooters

MILAN PAUDEL

SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

2019

*An Investigation into the Design for Rideability of Small Wheel
Single-track Bicycles and E-scooters*

MILAN PAUDEL
2019

An Investigation into the Design for Rideability of Small Wheel Single-track Bicycles and E-scooters

MILAN PAUDEL

SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

A thesis submitted to the Nanyang Technological University in partial
fulfilment of the requirement for the degree of

Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarized materials, and has not been submitted for a higher degree to any other University or Institution.

30-July-2019

.....

Date

.....

Milan Paudel

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

30- July-2019

.....
Date

.....
Assoc. Prof. Yap Fook Fah

School of Mechanical Aerospace Engineering
Nanyang Technological University, Singapore

Authorship Attribution Statement

This thesis contains material from 2 papers published in peer-reviewed journals where I am the first author.

Chapter 3 contains a portion of the journal paper published as Milan Paudel and Fook Fah Yap “Development of an improved design methodology and front steering design guideline for small wheel bicycles for better stability and performance”, Journal of Sports Engineering and Technology.

The contributions of the co-authors are as follows:

- Prof. Fook Fah Yap provided the initial project direction and edited the manuscript drafts.
- I performed the analysis and wrote the manuscript draft.
- The manuscript was reviewed and revised by Prof. Fook Fah Yap.

Chapter 4 and 5 contain the major portions from the following journal publications:

The first journal is published as Milan Paudel, Fook Fah Yap, Anil K. Bastola “Safety assessment of personal mobility devices with different wheel size based on their dynamic stability performance”, International Journal of Sustainable Design, 2020.

The second journal is published as Milan Paudel, Fook Fah Yap “Development of an improved design methodology and front steering design guideline for small wheel bicycles for better stability and performance”, Journal of Sports Engineering and Technology, 2020.

The contributions of the co-authors are as follows:

- Prof. Fook Fah Yap provided the initial project direction and edited manuscripts.
- Anil K. Bastola and I designed the simulation in MATLAB Simulink to perform the oscillation analysis of the single-track vehicle.
- I performed analytical, experimental works and analysed the data.
- I prepare the manuscript drafts. The manuscript was revised by Prof. Fook Fah Yap and Anil K. Bastola.

30- July-2019

.....

Date

.....

Milan Paudel

Abstract

Over the years, bicycles evolved as one of the most efficient means of transportation. A plethora of bicycles design is available in the market. However, the principal designs and most notably the wheel size has changed little ever since the evolution of the safety bicycles. The bicycles with bigger wheels (26-inch - 29-inch in diameter) have been the most common design and remained dominant over a long period. However, the desire for compact and portable means of transportation led to the invention of the small wheels and small-wheeled personal mobility devices (PMDs) like folding bicycles, kick scooters, e-scooters, Segway and so on. In recent decades, rapid urbanisation has created one of the biggest challenges in the transportation system: the first- and last-mile problem. Today, many modern cities, including Singapore, are restructuring their transport infrastructure to allow greater use of these bicycles and e-scooters to smoothen the first and last mile problem in urban mobility. Small wheel (≤ 20 -inch diameter) bicycles and e-scooters are becoming increasingly popular in such modern cities because of their compactness, manoeuvrability and portability. However, the rideability (i.e. stability and handling) of these small wheelers has often been questioned. It is a well-known fact that the small wheel bicycles and e-scooters do not ride as good as its big wheel counterparts and their ride is often described as “less-stable”, “wobbly” and “twitchy”. Most believed that small wheelers are inherently unstable because of the smaller wheel size. Nonetheless, there has not been any comprehensive investigation to quantitatively compare the performance of the different wheel sized PMDs and suggest possible solutions to improve the current designs of small wheelers. On top of that, the growing popularity and the advent of the micro-mobility in last few years has also brought forth an alarming increase in PMDs related accidents and injuries. Therefore, there is a rising concern about the safety of the riders who use these small wheelers.

A comprehensive investigation on the performance of the different wheel sized PMDs is the major objective of this study. The design philosophies, methodologies and available guidelines were analysed for different wheel sized PMDs. The rideability of 20-inch, 16-inch and 12-inch bicycles and 8-inch e-scooters have been studied and compared with that of 26-inch referenced big wheel bicycle based on the dynamic self-stability and handling characteristics. The inherent stability characteristics have been analysed based on the eigenvalues, self-stability velocity range and, the steer and lean oscillation behaviour. Furthermore, the obstacle rollover tendency and steady-state

turning performances were considered for handling analysis. Finally, an improved and cost-effective methodology has been established to enhance the performance of small wheel PMDs by incorporating the dynamic models into the design process. Simple front steering design guidelines were developed using the improved design methodology. Prototype small wheel bicycles were fabricated, and their performance was compared analytically and experimentally to the current designs of big and small wheel bicycles.

The analysis of current design practice revealed that most of the available design guidelines were developed for big wheel bicycles, and no specific design guidelines were found for designing the small wheelers. Therefore, a strong influence of big wheel bicycles' design was observed in the designs of small wheel PMDs. Furthermore, the study has successfully confirmed that the current designs of the small wheel PMDs are indeed less stable and difficult to ride at normal speed. The analysis suggested that the available design guidelines for big wheel bicycles do not translate directly to the small wheelers and the performance discrepancy in big wheelers and small wheelers is not because of the smaller wheel sized, as per se but, lack of adequate design guidelines for small wheelers that resulted in lesser stability and poor readability. A good agreement was observed between the experimental and the analytical results for prototype bicycles developed using proposed design guidelines and the current bicycle designs. Therefore, the proposed design guidelines were successfully validated, both analytically and experimentally. Lastly, it has been successfully shown that the current designs of small wheel PMDs are yet to be optimised and performance could be improved by carefully tuning their design configurations such as front steering geometry and mass distribution.

Acknowledgements

Undertaking this PhD has been a remarkable experience for me, and it would not be possible to complete the journey without the constant support, encouragement and the guidance of many people.

First and foremost, I would like to extend the highest appreciation for the constant support and guidance from my Supervisor, Prof. Yap Fook Fah. I am very grateful to learn from his experience in bicycle design and its dynamics. I am indebted to his sound advice to improve my analytical and critical thinking as well as my academic writing skills. His encouragements during the difficult time helped me to maintain my enthusiasm for my research work.

Beside my supervisors, I would like to thank my Technical Advisory Committee (TAC) members, Prof. Ng Heong Wah and Prof. Wong Yiik Diew for evaluating my research progress and providing valuable suggestions during this PhD study.

I would also like to extend a special thanks to my close friend Anil K. Bastola. We started our journey of higher studies together. I am very indebted his endless encouragement, advice and support throughout the PhD journey. From concept development, experiment designs and proofreads of manuscripts to the FIFA competitions, and all the moments, I am very appreciative to him.

Thanks to the School of Mechanical and Aerospace Engineering, NTU for the research facilities. I am thankful for the financial support from the SINGA scholarship by the Singapore government for my PhD study. I am also thankful to the Institute for Sports Research (ISR) at NTU for providing me with a cubicle. I am thankful to the technical staff, Mr Terry, for his help in experimental setup design and Mr Poon and Mrs Hali, for providing me with the measurement instruments.

I am very grateful to my wife, Geet, who has been a constant source of immense love, support and encouragement in my life. Her presence, love, support, suggestions and the delicious food have provided me with the strength to overcome the difficult period in this challenging journey. I am also thankful to my parents for the unconditional encouragement and support throughout my academic career and personal life. I would like to dedicate this thesis to my wife and my parents.

I would also like to remember Shirish for the delicious food and thank her for organizing the wonderful vacation trip for my wife and me on our anniversary. Lastly, I would like to remember my friends Purna, Rakesh, Arjun, Rinoj, Arun, Elvin, and all the friends who helped me directly or indirectly in this wonderful journey.

Table of Contents

STATEMENT OF ORIGINALITY	I
SUPERVISOR DECLARATION STATEMENT	III
AUTHORSHIP ATTRIBUTION STATEMENT	V
ABSTRACT.....	VII
ACKNOWLEDGEMENTS.....	IX
TABLE OF CONTENTS.....	XI
LIST OF FIGURES.....	XV
LIST OF TABLES	XIX
LIST OF NOTATIONS AND ABBREVIATIONS.....	XXI
CHAPTER 1. INTRODUCTION	1
1.1 MOTIVATION	5
1.2 OBJECTIVES.....	7
1.3 SCOPE.....	7
1.4 STRUCTURE OF THE THESIS	8
CHAPTER 2. LITERATURE REVIEW	11
2.1 A BRIEF HISTORY OF BICYCLE AND E-SCOOTERS	11
2.1.1 Evolution of Folding Bicycles	12
2.1.2 Evolution of Small Wheel and Folding Bicycles	13
2.1.3 Evolution of Small Wheel E-scooters	15
2.2 DYNAMICS OF SINGLE-TRACK VEHICLES: BICYCLE AND E-SCOOTERS	16

2.2.1 Benchmarked Equations for Bicycle	20
2.2.2 Extension 1: Inclusion of Rigid Rider in Bicycle Model.....	28
2.2.3 Extension 2: Inclusion of the Rider's Upper Body Lean Movement.....	30
2.2.4 Extension 3: Inclusion of the Tire Dynamics	35
2.2.5 Extension 4: Inclusion of the Front Frame Flexibility.....	39
2.2.6 Extension 5: Inclusion of Acceleration and Braking	42
2.3 HANDLING ASPECT OF THE SINGLE-TRACK VEHICLE	45
2.3.1 Handling from the Subjective Aspect	46
2.3.2 Handling Analysis from Mathematical Approach	49
2.3.3 Handling Aspect from an Experimental Approach.....	52
2.4 SUMMARY OF CHAPTER 2	53
CHAPTER 3. CURRENT DESIGNS OF THE PMDs	55
3.1 METHODOLOGY	55
3.2 DESIGN PHILOSOPHIES AND AVAILABLE DESIGN GUIDELINES.....	56
3.2.1 Current Design Philosophies and Guidelines for Bicycles	56
3.2.2 Current Design Philosophies and Guidelines for E-scooters	65
3.3 AVAILABLE DESIGN GUIDELINES VS CURRENT DESIGN PRACTICE.....	66
3.3.1 Current Designs of Big Wheel Bicycles	66
3.3.2 Current Designs of Small Wheel Bicycles.....	71
3.3.3 Current Designs of Small Wheel E-scooters	75
3.4 SUMMARY OF CHAPTER 3	77
CHAPTER 4. PERFORMANCE COMPARISON OF CURRENT DESIGNS OF PMDs	79
4.1 METHODOLOGY	80
4.1.1 Measurement of Geometrical Parameters	80
4.1.2 Mass Measurement	81
4.1.3 Centre of Mass Location and Moment of Inertia.....	81
4.2 STABILITY PERFORMANCE COMPARISON OF CURRENT DESIGN OF PMDs	83
4.2.1 Self-Stability Analysis: Benchmarked Model with the Rigid Rider.....	83
4.2.2 Self-Stability Analysis: Rider Lean Model.....	98
4.2.3 Effect of Acceleration and Deceleration.....	101
4.3 HANDLING PERFORMANCE COMPARISON OF CURRENT PMDs	105
4.3.1 Steady-State Turning	105

4.3.2 Obstacle Interaction	107
4.4 SUMMARY OF CHAPTER 4.....	108
CHAPTER 5. IMPROVED GUIDELINE FORMULATION FOR THE CURRENT DESIGN OF PMDs.....	109
5.1 IMPROVED METHODOLOGY	109
5.2 OBSERVATION ON THE PARAMETRIC VARIATION	113
5.3 DESIGN GUIDELINE FORMULATION	116
5.3.1 Design Guideline Formulation for Small Wheel Bicycles.....	116
5.3.2 Design Guideline Formulation for 8-inch Wheel E-scooter	123
5.4 FRAME DESIGN RECOMMENDATIONS	129
5.5 VERIFICATION OF THE DEVELOPED DESIGN GUIDELINE	131
5.5.1 Analytical Verification of the Developed Guidelines	131
5.5.2 Experimental Verification of Developed Guidelines	136
5.6 SUMMARY OF CHAPTER 5.....	149
CHAPTER 6. CONCLUSIONS	151
6.1 CONCLUSIONS	151
6.1 FUTURE WORK.....	155
REFERENCES.....	157
APPENDIX.....	169

List of Figures

Figure 1-1: Principal components of single track PMDs: (a) e-scooter, (b) bicycle.	3
Figure 2-1: Chronological development of bicycle design.	12
Figure 2-2: Evolution of early folding or separable bicycles.	13
Figure 2-3: Small wheel bicycles and folding designs.	14
Figure 2-4: Whipple/Carvallo bicycle model.	20
Figure 2-5: Geometrical relation between front and rear contact points.	22
Figure 2-6: Generalized angular rotation and forces on the bicycle system.	23
Figure 2-7: Typical solution of the benchmarked bicycle model [81]	27
Figure 2-8: Combined bicycle rider system and grid points [95].	29
Figure 2-9: Solution of characteristic equations for a bicycle with and without a rider.	30
Figure 2-10: Illustration of bicycle model with rider's lean relative to the rear frame.	31
Figure 2-11: Experimental set-up to find rider's compliance [96]	32
Figure 2-12: Comparison of benchmark model with rider lean extension model (a) comparison between real part of eigenvalues (b) comparison between imaginary part of eigenvalues.	34
Figure 2-13: Rider's upper body lean model with no spring and damping element at pivot point (a) real part of eigenvalues (b) imaginary part of eigenvalues.	35
Figure 2-14: Forces and moments acting on a wheel.	36
Figure 2-15: Extension to the Benchmark model including tire dynamic model (a) unstable wobble mode at high speed (b) comparison of capsize, castering and weave mode (c) comparison of imaginary part of eigenvalues.	39
Figure 2-16: Bicycle model with flexible front frame.	40
Figure 2-17: Comparison between benchmark model and extended model with front frame flexibility (a) real part of eigenvalues (b) imaginary part of eigenvalues.	42
Figure 2-18: Eigenvalues comparison for a bicycle under deceleration and acceleration, and descending and ascending a hill [114].	43
Figure 2-19: Effect of acceleration and deceleration on stability.	44
Figure 2-20: Geometrical parameters that influence the handling characteristic of bicycles...	46
Figure 2-21: Schematic diagram of obstacle wheel interaction. (a) the angle of attack (b) rolling over an obstacle.	51
Figure 3-1: Application of CAD software to obtain design parameters of 'Turn bicycle'.	56

Figure 3-2: Rider's anthropomorphic parameters related to ergonomics or fit.	59
Figure 3-3: ISO recommended front steering design criteria.	63
Figure 3-4: Bottom bracket height and crank arm length for bicycles.	67
Figure 3-5: Seat tube angle distribution for bicycles.	68
Figure 3-6: Comparison of available front frame design guidelines and current bicycle design.	69
Figure 3-7: Comparison of overall design between small wheel bicycle and big wheel bicycle.	71
Figure 3-8: Variation of trail with wheel size, head tube angle and fork offset.	72
Figure 3-9: Comparison of available front wheel assembly design guidelines translated for small wheeler and current small wheel bicycle designs in the market: (a) 20-inch wheel bicycle, (b) 16-inch wheel bicycle.	73
Figure 3-10: Wheel size vs overall weight of the e-scooters.	76
Figure 3-11: Location of the saddle for sample e-scooter designs.	76
Figure 3-12: Head tube angle vs trail for e-scooters.	77
Figure 4-1: Front fork rake or offset measurement for accurate trail calculation.	81
Figure 4-2: Experimental approach to determine the COM location and MOI of the frame. ..	82
Figure 4-3: Typical designs of PMDs: (a) 26-inch big wheel bicycle, (b) 20-inch small wheel bicycle and, (c) 8-inch small wheel e-scooter.	85
Figure 4-4: Vehicle-rider models: (a) rider on a bicycle representing hands-free riding and, (b) rider on e-scooter representing standing posture.	86
Figure 4-5: Eigenvalues and the self-stable velocity range for different wheel sized PMDs. .	87
Figure 4-6: A comparison between the imaginary parts of eigenvalues for different wheel sized PMDs.	88
Figure 4-7: Steer and lean response of a typical big wheel bicycle, small wheel bicycle and e-scooter to lateral perturbation: a, b, and c are the steer angle response and d, e, and f are the lean response angle at velocities 4.5 m/s, 3.5 m/s, and 2.5 m/s respectively.	89
Figure 4-8: Stability performance comparison of current big wheel bicycles (BWB) and small wheel bicycles (SWB) over the dominant frame design range: (a) 20-inch wheel bicycle, (b) 16-inch wheel bicycle.	92
Figure 4-9: Seating and standing postures on e-scooters.	95
Figure 4-10: Eigenvalues and self-stable velocity comparison between the standing and seating posture on a small wheel e-scooter.	96
Figure 4-11: Self-stable speed range over the most common design range for small wheel e-scooters: (a) seating posture (b) standing posture.	97

Figure 4-12: Comparison of weave critical speed between seating and standing posture on e-scooters in reference to the legislated riding speed range.	98
Figure 4-13: Eigenvalues and self-stability comparison between typical big and small wheel bicycles based on extended rider lean model.	100
Figure 4-14: Eigenvalues and self-stability comparison for seating posture and standing posture on e-scooters based on extended rider lean model and considering the pivotal connection between vehicle-rider system.....	101
Figure 4-15: Shifted weight at the front wheel-ground contact point due to deceleration.	102
Figure 4-16: Effect of deceleration (a, b, c, d) and acceleration (e, f, g, h) on eigenvalues and self-stability of different wheel sized PMDs.	103
Figure 4-17: Effect of acceleration and deceleration on self-stable speed range relative to constant speed case performed for different wheel sized PMDs.....	105
Figure 4-18: Comparison of handlebar torque by rider to maintain steady-state turning: (a) at constant turning radius of 4 m and (b) at constant turning speed of 4.5 m/s.....	105
Figure 4-19: Effect of obstacle-wheel interaction for different sizes of wheels (a) Time to reach climb an obstacle of height 4 cm (b) The vertical velocity of wheels while climbing the obstacle of height 4 cm.....	107
Figure 4-20: A comparison of horizontal force required to overcome the obstacles for different size of wheels.	108
Figure 5-1: Conventional design methodology for big wheel bicycle, small wheel bicycles and small wheel e-scooters.	110
Figure 5-2: Improved philosophy for PMD design.	111
Figure 5-3: Improved design methodology to design small wheel PMDs.	112
Figure 5-4: Effect of design parameters on self-stable velocity range.	114
Figure 5-5: Self-stable speed range for steeper head tube angles and broad trail range: (a) 20-inch wheel bicycle and, (b) 16-inch wheel bicycle.	118
Figure 5-6: (a) wheel flop factor of current designs of big and small wheel bicycles, (b) wheel flop factor range for improved 20-inch wheel bicycle designs and, (c) wheel flop factor for improved 16-inch wheel bicycles.....	120
Figure 5-7: Typical designs of 12-inch wheel bicycles.	121
Figure 5-8: Self-stable speed range for 12-inch small wheel bicycle.....	122
Figure 5-9: A general guideline to translate big wheel bicycle's steering design to a minimum head tube angle and trail values for small wheelers.	123
Figure 5-10: Typical design of e-scooter with straight head tube and steering stem.	124
Figure 5-11: The self-stability speed range for e-scooter considered a standing rider.....	124
Figure 5-12: The self-stability speed range for e-scooter considered a seating rider.	125

Figure 5-13: Modified front steering geometrical design: (a) illustration of steering stem and front fork and (b) effect if varying steering stem angle keeping head tube angle constant. ...	126
Figure 5-14: Shift in the COM position per degree change in steering stem angle with respect to head tube angle.	126
Figure 5-15: Effect of steering stem angle variation on the self-stable speed range for the seating and the standing postures.....	127
Figure 5-16: Effect of frame flexibility on self-stability and wobble mode.	130
Figure 5-17: Effect of the distance between the twisting axis and ground contact point on self-stability and wobble mode.	131
Figure 5-18: The eigenvalues and self-stable velocity comparison between the improved designs of small wheel bicycles and referenced big wheel bicycle.	132
Figure 5-19: Eigenvalues and eigenvalues comparison for improved designs of e-scooter and referenced big wheel bicycle.....	133
Figure 5-20: Steer and lean responses of a typical big wheel bicycle, an improved small wheel bicycle and, an improved e-scooter to lateral perturbation: a, b, and c are the steer angle response and d, e, and f are the lean response angle at velocities 4.5, 3.5, and 2.5 m/s respectively.	135
Figure 5-21: Effect of acceleration and deceleration on self-stable speed range for improved design of PMDs.....	135
Figure 5-22: Comparison of handlebar torque by rider to maintain steady-state turning in improved PMDS: (a) at constant turning radius of 4 m and (b) at constant turning speed of 4.5 m/s.....	136
Figure 5-23: Prototype 20-inch small wheel bicycles.....	137
Figure 5-24: Schematic representation of the experimental area	138
Figure 5-25: Hands-free riding experiments.	139
Figure 5-26: Experimental minimum hands-free speed for different wheel sized bicycle models.....	144
Figure 5-27: Curve fitting for a typical racing bike's steering oscillation characteristic at different speeds. (a) speed= 10 km/h (b) speed= 12.6 km/h and (c) speed = 14.5 km/h.	146
Figure 5-28: Steering oscillation characteristics of typical bicycles and improved bicycles at different speeds. (a) speed =10 km/h, (b) speed=12.6 km/h and (c) speed=14.5 km/h.	148

List of Tables

Table 2-1: 25 design parameters for benchmarked bicycle model.	21
Table 3-1: Bicycle rider's fit parameter values for an adult rider from different recommended guidelines.	61
Table 3-2: Head tube angle and trail values for 91 samples big wheel bicycle.....	70
Table 3-3: Design parameter range for most of the bicycle and e-scooter parameters.....	78
Table 4-1: Ergonomic design parameters for a typical bicycle and e-scooter design.	84
Table 4-2: Time to reach the ± 20 -degrees deviation in lean angle and steer angle for typical designs different wheel sized PMDs.	90
Table 4-3: Centre of mass and moment of inertia properties comparison between the standing and seating posture on small wheel e-scooters.....	95
Table 5-1: The eigenvalues trend for increasing the design parameter.	115
Table 5-2: Allowable trail range for each head tube angle obtained for 20-inch and 16-inch small wheel bicycles that gives comparable self-stability range as the big wheel bicycles. ...	118
Table 5-3: Allowable trail range for each head tube angle identified for 12-inch small wheel bicycle.	122
Table 5-4: Allowable trail range for each head tube angle identified for small wheel e-scooters considering a standing rider.	124
Table 5-5: Allowable front steering design parameter range for standing posture on e-scooters having steeper steering stem angle than head tube angle.	128
Table 5-6: Allowable front steering design parameter range for standing posture on e-scooters having steeper head tube angle than steering stem angle.	129
Table 5-7: Allowable front steering design parameter range for seating posture on e-scooters having steeper head tube angle than steering stem angle.	129
Table 5-8: Major design parameters for prototype bicycles.	133
Table 5-9: Major design parameters for prototype e-scooters.	134
Table 5-10: Time to reach the ± 20 -degrees deviation in lean angle and steer angle for typical designs different wheel sized PMDs.	135
Table 5-11: Subjective performance evaluation of typical bicycle designs and improved prototype bicycles	140
Table 5-12: Comparison between the theoretical minimum speed and experimental threshold hands-free speed.....	145
Table 5-13: Average time for steering angle to reach the 20-degrees deviation.	147

Table 5-14: Comparison of real part of eigenvalues from theoretical model and experiments at 14.5 km/h	148
---	-----

List of Notations and Abbreviations

Notation	Definition
XYZ	Global coordinate axes with the origin at the rear wheel-ground contact point
g	Acceleration due to gravity
P	Rear-wheel ground contact point
Q	Front-wheel ground contact point
m_B	Mass of rear frame (with rider) in benchmark bicycle model
m_H	Mass of front frame in benchmark bicycle model
m_R	Mass of rear wheel
m_F	Mass of front wheel
m_T	The total mass of the vehicle rider system
c	Trail
w	wheelbase
$\emptyset$	Lean angle
δ	Steering angle
ψ	Yaw angle
λ	Steering axis angle measured from vertical
$T_{\emptyset}$	External lean torque
T_{δ}	External steer torque
$I_{xx} \ I_{xy} \ I_{xz}$	Moment of inertias
S_F, S_R	Gyrostatic coefficients
G_{R1}, G_{R2}	Vertical ground reaction force at rear and front wheel to ground contact points
F_{y1}, F_{R2}	Horizontal ground reaction force at rear and front wheel to ground contact points
u_A	Front assembly COM location ahead of the steering axis
μ	The ratio of the mechanical trail to the wheelbase
$\lambda_{si}, i = 1, 2, 3, ..$	Eigenvalues
m_r	Mass of rider's upper body
ϕ_r	Rider lean angle

s_r	Height of rider's COM measured from the rider-vehicle pivot point
$k_{\phi r}, C_{\phi r}$	Stiffness and damping coefficient between rider and vehicle at the pivot point
γ	Chamber angle
α	Slip angle
σ	Tire relaxation length
β	Front steering twisting axis
a	Acceleration
i_f	Inversion velocity

Abbreviation	Definitions
PMD	Personal mobility devices
COM	Centre of mass
MOI	Moment of inertia
BB	Bottom bracket
CAD	Computer-aided design
ABCC	Association of British cycling council
Sp	Jones' stability criteria
STA	Seat tube angle
HTA/Hta	Headtube angle
Sta	Steering stem angle
BWB	Big wheel bicycle
SWB	Small wheel bicycle

Chapter 1. Introduction

The history of personal mobility started along with the invention of the bicycle about 200 years ago. Von Drais, in 1817, developed a “running machine” using two inline wheels in a wooden tandem [1]. Technically, a bicycle is referred to as a single-track, human-powered vehicle having two wheels, one behind another. Over the centuries, bicycles endured various modifications in terms of designs, materials and technologies. A number of inventions and innovations as related but not limited to drive chain, calliper brakes, pneumatic tires and improvements in the material science to make lightweight but stronger frame transferred bicycle into a much safer and stable vehicle around 1890 [2]. That bicycle was well known as “safety bicycle”. The safety bicycle closely resembles modern day's bicycles. The safety bicycles are the ancestor to today's motorcycles, bicycles and e-scooters.

The popularity of bicycles gradually increased after the invention of safety bicycles. Although bicycles are not the best option for long-distance travel, they have been widely accepted for short-distance commuting. Besides its use for transportation, bicycles have emerged as a primary choice for recreation, general fitness, sports and so on. Due to the low cost, less maintenance and multi-purpose use, the bicycle is becoming one of the big global business industries [3]. Bicycles are tremendously efficient vehicles in terms of energy needed to travel a certain distance [4]. The rising cost of fuel, increasing awareness of personal fitness, reducing the price for Li-ion battery, and the new concept of sharing the mobility devices are the major driving force for personal mobility devices (PMDs). Here, the PMDs are defined as the mode of transportations that can carry one or two passengers for short-distance commuting. Such PMDs include bicycles, e-bikes, e-scooters and so on. These PMDs are environmentally friendly and impose significantly less burden on the environment. Therefore, the global market and demand trend for the PMDs are expected to increase sharply in the near future.

A plethora of bicycle design is currently available in the market; for example mountain bike, racing bikes, road bikes, touring bikes etc. The frame shape could be different for different bicycles. Nonetheless, the principal design and most notably the wheel size has changed little since the evolution of the Diamond frame safety bicycle. Most of the adult bicycle use 26-inches to 29-inches diameter wheels and are also known as a big wheel bicycle. The bigger wheel bicycles became dominant over a long period. The reason

being, the big wheels roll more distance per cadence compared to small wheels and provide a comfortable ride in uneven terrain, bumps and small road depressions. Despite the dominance of larger wheels, the desire for compact and portable bicycles led to the invention of smaller wheels and small wheel bicycles. The idea of small wheels has been proven to be a key factor for today's small wheel PMDs like kick scooters, e-scooters and Segways. The recent trend of PMDs is indeed dominated by the small wheelers. A plethora of small wheel e-scooter and bicycle designs are also emerging in the market lately.

Because of urbanisation, people are migrating towards cities, and places are becoming crowded. In such a scenario, consumer preference is often towards compact and portable devices. In recent years, the popularity and demand for PMDs such as bicycles, e-bikes and e-scooters are increasing sharply and consequently sales rate of the bicycles in major countries and cities of Europe and Asia [5-7]. The invention of efficient multi-speed gear and battery-powered devices assists riders in propulsion. Furthermore, smooth roads, improved suspension mechanism, and tire provide a comfortable ride. Thus, the major disadvantages of the small wheelers have already been overcome with the advancement in engineering and manufacturing technologies, and distance and comfort are not an issue. The use of small wheels allows manufacturers to design for compactness and portability. The portability and compactness of small wheel PMDs make them very suitable for riding in narrow bike lanes and pavements. Also, the foldable design allows storage in limited space inside the office and small apartments. Furthermore, to reduce congestion and environmental-related problem caused by the fossil fuel-powered mobility devices, such as car and motorcycles, countries including Netherlands, Germany, Singapore and more are constructing their transportation infrastructure to allow greater use of bicycles and small wheel PMDs. However, as the number of bicycle and e-scooters increases, the risk associated also increases. With the increasing number of mobility devices, the number of accidents causing fatality and injuries are also increasing [8-10]. An alarming rate of e-scooter related accidents has been reported in recent years [10-12]. Therefore, one of the current needs is to investigate the dynamic performance of the small wheel PMDs concerning safety and rideability.

It is a well-known fact that the small wheel bicycles and scooters do not ride as well as its big wheel counterpart. Therefore, despite having the various advantages over the big

wheelers, the rideability of the small wheelers have always been questioned. The lesser rideability of smaller wheels has been reported in some articles [13, 14] and talks [15] and also evident from the recent reports on accidents [10]. The ride of the small wheelers is often described as “less stable”, “wobbly” and “twitchy”. The difference in the riding performance can be appreciated with hands-free riding at normal riding speed; about 15 -17 km/h. It is relatively easy to perform the hands-free riding in big wheel bicycles, but it becomes increasingly difficult with the reducing wheel size. E-scooters in standing posture are virtually impossible to perform hands-free riding. It is interesting to notice that despite of having similar principle design, shown in Figure 1.1, big wheel bicycles are easy and stable to ride hands-free at normal speed, whereas it is difficult to do so on its descendant small wheel PMDs.

Figure 1-1: Principal components of single track PMDs: (a) e-scooter, (b) bicycle.

The design of all single track two-wheeled vehicles is almost the same with two inline wheels, one front steering assembly containing the fork and handlebar and a rear frame where the rider can rest. However, their performance is markedly different. Currently, there is very little information in the published literature that scientifically investigates the reasons for the difference in ride characteristics between the different wheel sized electric scooters and bicycles. Most people attribute the lower stability to the smaller wheel size. However, there is virtually no literature available thus far that investigated the performance discrepancy of small wheel bicycles and e-scooters and suggested potential design improvements scientifically and quantitatively. To address this discrepancy in the bicycles and e-scooters’ performances, it is essential to understand the fundamentals of the design approach, the essence of their dynamic behaviour and working of a vehicle-rider system.

The fundamental technique in balancing a single-track vehicle is to steer in the direction of fall. In other words, if the rider falls towards the right side, he must be steered towards

the right to restore the balance [16]. The current steering geometry and mechanism of the bicycle are so elegant that they have the ability to steer the bicycle into the direction of fall automatically. As a result, a rider without his/her hands on the handlebar or even a riderless bicycle can remain upright when pushed forward with sufficient speed. The hands-free riding is a qualitative approach to examine the inherent stability of bicycle design. In addition, hands-free riding is also a good test for balancing and handling [17]. For stable design configuration, riders do not have to provide excessive manual control to ride PMDs because a part of required steering control is provided by design itself. As the small wheel bicycles and e-scooters are perceived less stable compared to the big wheel bicycles, the current need is to investigate the possible design modifications on current small wheel PMDs in order to improve their performance which is the function of overall mass, rider's position and geometrical configurations.

Through years of design modifications and experiences, the effects of front design parameters such as variations in head tube angle and trail are qualitatively known. But, in order to improve the design and compare with the big wheel bicycle, a quantitative approach is required for evaluating the effects of each parametric variations. The recent progress in understanding the bicycle-rider system dynamics in recent decade allows us to quantify the performance of any bicycle design configuration in terms of the self-stability [18]. Thus, there is a possibility to improve the stability performance of the small wheel PMDs by appropriately selecting the overall position, the mass distribution and most importantly, the front geometry. The design configuration determines the performance of the single-track vehicles. Therefore, the quantitative analysis of design configurations could provide a deep insight into current design methodology and performance. Furthermore, there are many guidelines to select the ergonomic parameters appropriately or to maintain the rider's fit. However, the front steering is still a mystery in bicycles and poorly understood. The precise guidelines for selecting appropriate steering geometry is very limited. On top of this, the available guidelines were developed for big wheel bicycles, and unfortunately, no such design guidelines have been developed for small wheel bicycles and e-scooters.

Till date, most of the PMDs have been designed with the empirical methods based on experiences. The main aim of this research is to explore the different aspects in current PMDs design approach and scientifically and quantitatively compare their performance in relation to the safety, stability and handling. By taking advantage of the standard

mathematical models, the understanding of the single-track vehicle dynamics can be extended to improve the current design of PMDs.

1.1 Motivation

One of the major issues in today's transportation service is the first- and last-mile problem. Sometimes, the transportation stations such as bus stations, train stations or ferry stations might not be within the accessible distance. Therefore, passengers may often find it difficult to travel in public transportations which result in the first- and last-mile problem. This leads to severe congestion, pollution and urban sprawl. Therefore, the lack of first- and last-mile transportation options has created inconvenience to the use of public transportation. There are many proposed solutions, such as service provided by Grab, Lyft, and Uber, to smoothen the first and last mile difficulties. However, such solutions do not solve the problem of traffic congestion on the road. Furthermore, fossil fuel-driven vehicles have environmental concerns. The PMDs, on the other hand, is believed to be the next big thing in transportation. Therefore, many cities and countries like Singapore, China, USA, Netherlands are developing and improving their transportation policies to allow greater use of PMDs. Research has shown that the drivers in the USA takes about 900 million trip per day and half of them are less than 5 miles [19]. The PMDs such as bicycles and small wheel e-scooters have emerged as one of the potential solutions to end such severe congestions, control pollution and urban sprawl [20]. Moreover, the price of gasoline is increasing every year. Therefore, many crowded cities and countries are also motivating people to use environmentally friendly PMDs like bicycles and e-scooters for short-distance commuting.

In recent years, the number of small-wheeled mobility devices is increasing rapidly. In addition, because of overcrowding and availability of limited space, consumer preference is often towards compact and portable devices. The use of small wheels allows manufacturers to design for compactness and portability. The portability and compactness of small wheel bicycles make them very suitable for riding in narrow bike lanes and pavements. Also, the foldable design allows storage in limited space inside the office and small city apartments.

As the use of small wheelers in cities increases, the risk associated with accidents also increases. The number of accidents involving the PMDs is increasing over the last

decades [15, 21-23]. The accidents related to PMDs can be categorised into two parts: (i) single-vehicle accident and (ii) double vehicle accidents. A double vehicle accident refers to an accident where the vehicle is hit or collided with other vehicles. Whereas single vehicles accidents do not involve other vehicles. The single-vehicle accidents generally involve falling off from the vehicles and loss of control when encountered with obstacles like potholes or road bumps. A large part of bicycles and scooters accidents are the single-vehicle accidents because of the falling off from and poor handlebar controllability [24, 25]. On average, 17 % of the fatal injuries to the cyclist are reported to be single-vehicle accidents. Likewise, about 60% to 95 % of the cyclists admitted to hospital is found to be injured in single-vehicle accidents [23]. Schepers and Wolt [26] found that an encounter with small obstacle accounted for 12%, loss of control because of an uneven surface such as bumps, and potholes accounted for 7 %, loss of control at low speed accounted for 16%, and abrupt steering maneuverer accounted for 13 % of the bicycle-related accidents. Many riders have reported the small wheel e-scooters being very unstable compared to normal big wheel bicycles. Such modes of accidents can easily be correlated with dynamic stability performance of PMDs. The major concern here is both the rideability and the safety of riders. The safety aspects of the PMDs have two parts: (i) rider's ability and (ii) vehicle dynamics stability. The rider's ability is related to his/her skill and behaviour to ride safely. The latter is the intrinsic property of such two-wheeled inline vehicles. That is, by the virtue of the dynamics of the vehicle itself, it can remain upright above a certain threshold velocity without any external control. This property is often referred to as self-stability of the vehicle. A vehicle in the self-stable state could easily resist small perturbations to the rider-vehicle system caused by sideways winds or small obstacles. If a vehicle has good dynamic stability, it can assist riders with better control of vehicles and thus riding becomes safer.

The stability and handling aspect of small wheel PMDs is yet to be included in the design procedure. Most of the small wheel PMDs are designed either by taking reference to the designs of big wheel bicycles or using the experience and trial and error method. The design aspect of such small wheel PMDs is still a matter of craftsmanship rather than proper engineering. There are only limited number of guidelines to select proper design configurations, especially the front steering design to ensure the performance in terms of stability and handling. However, all the design guidelines were developed for big

wheel bicycles and, unfortunately, there are no guidelines for small wheel bicycles and e-scooters. As a result, small wheel PMD designers are likely to rely on the same guidelines developed for big wheel bicycles. But then again, the same guidelines have not yielded a similar performance as the big wheel bicycles. The answer to this question is still unknown. Furthermore, it is also still unclear whether the performance of the small wheel PMDs could be improved and, if yes, how to improve it. No research has been reported that compares the performance of different wheel sized PMDs and provide possible suggestions to make them better rideable and safer. Thus, the research focus is to understand the current design of different wheel sized PMDs and develop the relevant design guidelines that could improve the current small wheel bicycle designs for better stability and performance. A validated mathematical model for uncontrolled bicycle along with other extensions to this model have been used as tools to evaluate the performance of the various wheel sized PMDs. It is expected that the outcome of this research could transform the current design practice of small wheel PMDs in future, making it more stable and safer.

1.2 Objectives

This study is designed to assess the performance of the different wheel-sized personal mobility devices (PMDs) and improve the design accordingly. The specific objectives of this study are:

- To explore the design principles, methodologies, and available guidelines used in current designs of personal mobility devices.
- To compare the stability and handling performance of different wheel sized PMDs.
- To investigate the effect of various design parameters on stability and handlings performance of PMDs.
- To investigate the potential improvements by incorporating dynamics models into the conventional design practice to enhance the performance of small wheel bicycles and scooters.
- To develop design guidelines for small wheel PMDs for better performance and safety.

1.3 Scope

The scope of this research is:

- The focus has been given to the most common wheel sizes for personal mobility bicycles. For bicycles, the most common wheel size; 26-inch, 20-inch, 16-inch and 12-inch have been selected. While for e-scooters, 8-inch wheel size is selected.
- The validated mathematical models have been used as a tool to compare the performance of different wheel-sized PMDs.
- The self-stability and lateral perturbation, steady-state turning, and obstacle rollover tendency analysis will be used to evaluate and compare the performance of the PMDs.
- The front steering guideline development for different wheel sized PMDs have been given a major priority. The guidelines related to ergonomics has not been the scope of this study.

1.4 Structure of the Thesis

The thesis is divided into 6 chapters. Each chapter presents a chronological development of the research work, starting from the problem identification, motivations, literature reviews, detail analytical works and experimental verifications. A short synopsis for each chapter is presented below:

Chapter 1: Introduction

This chapter briefly presents a brief introduction to the research motivation and current research gap. This chapter clearly states the research questions, the objectives set to address the research questions, the future implication of the research and limitations.

Chapter 2: Literature Review

This chapter is sub-divided into three sections. The first section discusses a brief history of bicycles. It presents a chronological development of different bicycles models and scooters till the date. The second section presents a chronological discussion on the field of single-track vehicle dynamics, especially focusing on bicycles. It also discusses a number of methods and dynamic models to quantitatively and qualitatively access the stability of single-track vehicles under various conditions. The last section discusses the handling characteristics of single-track vehicles. Moreover, different quantitative and qualitative approaches to evaluate the handling perception of current designs of PMDs are also presented.

Chapter 3: Current Design Approach of PMDS

This chapter presents an in-depth analysis of the design approach of different wheel sized PMDs. The available design guidelines, tables and empirical formula have been presented and compared with the PMDs of different brands and models. The chapter also investigates the influence of traditional big wheel bicycles design guidelines to the current design of small wheel PMDs. Finally, this chapter identifies the most common range of design parameters used in the current design of different wheel sized PMDs.

Chapter 4: Performance Comparison of Current Designs of PMDs

This chapter presents a quantitative analysis of the current design of different wheel sized PMDs. It investigates the performance of current designs of PMDs over the most common range of design parameters. Different mathematical models have been used to quantitatively compare the performance of small wheel PMDs with big wheel bicycles. The main aim of this chapter is to investigate if there is a difference in performance on current designs of PMDs and, if yes, identify the major cause for it.

Chapter 5: Performance Improvement and Guideline Formulation

This chapter presents an improved design methodology by incorporating the dynamics into the design process. It also presents guidelines to improve the performance of the current designs of small wheel PMDs. The proposed guidelines have also been compared with the available current design guidelines. The main focus has been given to the development of front steering design guidelines. Analytical and experimental verification of the proposed guidelines is also presented.

Chapter 6: Conclusion and Future Works

This chapter summarises the overall research findings, draws conclusions and finally suggest future recommendation based on the overall findings.

Chapter 2. Literature Review

This chapter is sub-divided into three sections. The first section presents a chronological development of safety bicycles, folding bicycles, small wheel adult bicycles, and e-scooters. The second section presents the dynamics of single-track vehicles. It also includes various mathematical models and developments in the field of single-track vehicle dynamics especially focusing on bicycles and e-scooters. The third section presents the handling aspect of single-track vehicles. Methods to assess stability and handling aspect of the single-track vehicles are also discussed.

2.1 A Brief History of Bicycle and E-scooters

The history of bicycle begun with the invention of a “the running machine” using two inline wheels in tandem by Karl von Drais in 1817. It was the first human means of transportation using two inline wheels [2]. The first bicycle had no mechanism for mechanical propulsion [27]. This invention pioneered the concept of human-powered two-wheel vehicles. MacMillan added the mechanical propelling mechanism in 1839. Later, in 1966 and 1968, Lallement and Michaux modified the propulsion mechanism by using cranks and pedals at the front wheel for direct drive, known as “Velocipede” or “Bone-Shakers”. By late 1960s wheels were modified with tangent-tension spokes and the front wheels were made as large as 1.5 meters in radius in order to increase the distance coverage per unit crank rotation. In 1870, “Velocipede” was changed into the “High-wheeler” or the “Ordinary” or the “Penny-farthing” with the front wheel much larger than the rear wheel [2]. Despite the rising popularity, the high wheelers were considered unsafe, and the possibility of accidents and injuries were high because of difficulties in mounting and dismounting [28]. Therefore, designers searched for the other alternative driving mechanisms to increase the travelling distance and improve safety. Levers and cranks, belts and pulleys, rotary driveshafts, and chains and sprockets were some of the tried mechanism for the indirect drive [27]. Among the tried solutions, the chain drive became dominant. After 1885, the chain drive on the rear wheel became the most popular type of the drive [2]. Concurrently, inventions like calliper brake, pneumatic tires, and multiple gears were also present from 1876 to 1888 [1]. With such inventions, the era of “safety bicycle” begun in 1890. The riders’ seating position was shifted towards the rear wheel after realizing that it would be safer to put riders in between two wheels. The wheel sizes were reduced, and two equal-sized wheels were

used. The bicycles were easier to ride with the choice of multiple gear ratios and improvement in metallurgy also enabled the production of light but strong frames. The safety bicycles closely resemble today's modern bicycle designs. Various improvements related to transmission, braking, saddles, pedals and handlebar and, suspension system led to the evolution of racing bicycles, touring bicycles, mountain bikes and small wheel bicycles. The motorcycles and e-scooters are also the descendants of safety bicycles. Figure 2.1 shows a chronological evolution of bicycle in the timeline.

Figure 2-1: Chronological development of bicycle design.

2.1.1 Evolution of Folding Bicycles

The evolution of foldable bicycles was instigated by the desire for compact and portable bicycles. W. Grout [29], in 1878, is often credited for the invention of first folding bicycles. The bicycle was the high wheeler that can be folded and packed into a bag. Latta, in 1887, patented a foldable bicycle design for a rear-drive safety bicycle with 28-inch wheel size. Similarly, M.B. Ryan, in 1893, patented a folding Dimond frame safety bicycles [27]. When rear-drive safety bicycles started to become popular in the market, the British army also considered using them in military purpose. The folding bikes had an advantage to the soldiers that they could easily carry their bike on their back while travelling. The early military folding bike is believed to be developed by a lieutenant in the Belgian army, but Capitaine Gerard's French military bike in 1896 attracted

significant military attention [27]. Following the French trend, many other countries adopted folding bikes for military purpose [27].

Figure 2-2: Evolution of early folding or separable bicycles.

The early so-called folding bicycles are often described as the separable bicycle rather than the true folding bicycles. The frame or other components like handlebar or wheels were detachable for compact packaging and carrying. The mass production of folding military bicycles using small wheels known as Fonger's military folder is dated back to 1909. Only a few folding bikes were developed during the second world war until the late 1930s [27]. A.J. Marcelin, in 1939, designed Le Petit Bi, using 16-inch semi-balloon tires [2]. Although, the first design of Le Petit Bi did not have a folding frame, the later version, in 1944, had a folding frame.

2.1.2 Evolution of Small Wheel and Folding Bicycles

It was interesting to notice that high wheeler bicycles also used a small 16-inch rear wheel. As early as 1894, a pneumatic wheel company in London marketed a rear-drive safety bicycle using 18-inch spoke less and rimless wheels. In 1897, a 20-inch tension spoke pneumatic wheel bicycle was designed on a Diamond frame by a German company known as Nitsche and Kausmann [27].

A noticeable person who advocated for small wheel bicycles was a French cyclist tourist Paul de Vivie, also known as Velocio around the 1920s [30]. Small wheels in that period were based on the idea of reducing the wheel's diameters and increasing the cross-

sectional area so that the air pressure would remain the same a big wheel. The wider tires had the advantage on uneven terrain. The next person who advocated for the small wheel bicycle was Alex Moulton. Unlike Velocio, Moulton, in the early 1960s introduced small size and high-pressure tires. Moulton bicycles also incorporated suspension system to eliminate the rough ride due to the high-pressure wheels. The Moulton bikes had also changed the conventional frame design with their modified “F frame” and later “spaceframe” [31]. A similar small wheel bicycle known as ‘Shopper’ was later designed by Raleigh using the 20-inch wheel. However, none of the Moulton and Raleigh was folding bicycles.

The next step in the field of small wheel bicycle was stimulated by the desire to design compact portable bicycles so that it would be convenient to carry the bicycles or store in a limited space. Although the concept of folding bicycles had been in practice since the time of high-wheeler, the breakthrough design “Bickerton Portables” was launched in 1970 by the aeronautical engineer H. Bickerton [32]. Inspired by the Bickerton portables, A. Ritchie [33] started designing the folding bicycles called “Brompton” using multiple hinges since 1976. Concurrently, the American inventor David Hon applied a patent for a folding bicycle and started production by founding manufacturing company called “Dahon” in 1982 [34]. Some of the early small wheel bicycles and folding designs are shown in Figure 2.3. Moreover, some noticeable early small wheel folding bicycles are “Stick” by Shoji Iwai in 1974, “Strida” by Mark Sanders in 1987 and “Zoombike” by Richard Sapper 1998 [27].

Figure 2-3: Small wheel bicycles and folding designs.

Today, more than 175 bikes manufacturers are producing the small wheel and folding bicycles [29]. Among which, some of the bicycles are designed for compactness, some are designed for high-performance, and some are designed in between the compact and high-performance bicycle. A few renowned names in the small wheel bicycle industries are Moulton, Dahon, Brompton, Bike Friday, Bickerton portables, Bike Friday, Airnimals, Birdy. Most of the current small wheel bicycles have 20-inch or 16-inch wheel. However, some recent electric bike models have even smaller wheel size about 14-12 inches.

2.1.3 Evolution of Small Wheel E-scooters

After the popularity of safety bicycles, the inventors tried various alternatives to extend the use of bicycles. One of the noticeable improvements was the propulsion assisted system. In 1895, an American inventor Ogden Bolton Jr. filed a patent for an electric motor bicycle [35]. The idea was to use six brush poles, a DC collector and a hub motor mounted on the rear wheel [36]. Hosea W. Libbey patented another electric bicycle using a double electric motor in 1895 [37]. Later in 1896, a British bicycle manufacturer exhibited the first electric tandem bicycle using lead-acid batteries [38]. In the early 1900s, Ajax motor company produced electric scooters, but that became unsuccessful [39]. In 1919, British agricultural machinery maker Ransomes, Sims and Jefferies developed a prototype electric motorcycle [39]. Unfortunately, the vehicle was not used on the road as it suffered several malfunctioning. On the other hand, gas-powered Autoped were becoming successful in the US and Germany until 1922. Furthermore, those Autoped also had a foldable handlebar. In 1919, ABC motorcycle released a stand-up scooter called “Skootamota” and was hugely adopted [38]. Thus, the success of scooter attracted many companies in the market. In 1974, an electric scooter called “Charger” was released by Auranthic Corp. In 1996, an electric scooter called Scoot’Elec was mass-produced by Peugeot. The Scoot’Elec was powered by Ni-Cd batteries [38].

The commercialization of Lithium-Ion batteries in the early 1990s was a turning point for the e-bikes and e-scooters. The batteries were very efficient in terms of energy per weight and output current compared to the previously existed batteries. Furthermore, they could be charged fast [39]. Thus, the improvement of the existing design of e-scooters flourished rapidly with the latest technologies in electric motors and efficient batteries. The invention of Li-ion battery allowed inventors to design lightweight and portable vehicles. In 2009, Myway started a project to design electric scooters which

later became “Inokim”, one of the leading e-scooter manufacturers [40]. Most of the current e-scooters use 8-10 inches small wheels and have a foldable design. Thus, use of the foldable frame, Li-ion battery and small wheel helped e-scooters evolve as one of the most prominent PMDs in recent years. Today, many e-scooters brands like Dualtron, Xioami, Mobot, Inokim etc. are available in the market. Furthermore, they are also being used as share vehicles in many modern cities and countries. Brands like Lime, Bird, Telepod are flourishing globally in the field of share micro-mobility.

2.2 Dynamics of Single-track Vehicles: Bicycle and E-scooters

Bicycle is an intriguing invention in history. Its unique behaviour of its self-stability caught the attention of many riders, renown scientists and researchers from the past. The self-stability can be defined as the intrinsic property of the vehicle to remain balanced without any external control. When bicycles or any single-track vehicles achieve certain threshold forward speed, the vehicles can compensate for certain perturbations and have the ability to maintain upright forward motion without any external control. Furthermore, at speed greater than 15 km/h, many riders had experiences that the bicycles could easily be ridden without hands on the handlebar and controlled using a slight lean of the body. But the concrete and scientific explanations to describe such behaviour of bicycles were not available in the early days. Some researchers explained the self-stable phenomenon by comparing the bicycle motion to the rolling hoop [41-44]. The major ideas to explain the self-stability were based on the gyroscopic effect of the spinning wheels [41, 42, 45-48], trail and front steering axis tilt [48-53] and the front frame centre of mass (COM) location [54-56].

Whipple [57], in 1899, initiated the mathematical layout to describe the unique behaviour of the bicycle by dividing bicycle into four different structurally symmetrical components: - a rear wheel, a front wheel, a rear frame where the rider is allowed to attach rigidly and a front frame including fork and handlebar. The rigid body dynamics was used to illustrate a bicycle moving with an appropriate speed shows self-stability. Similar mathematical equations were also described by Carvallo [58], in 1899, for the motion of the bicycle. Since then, a number of researchers have endeavoured to derive the correct mathematical model to describe the motion of a bicycle to an acceptable level of accuracy.

Sommerfeld and Klein [59], in 1903, also derived the linearized equation of motion for a basic bicycle model using similar assumptions to the Whipple [57] and Carvallo [58] model. The bicycle model included mass and inertial properties of the front assembly and wheels. They explored the effect of wheel gyroscopic action on the stability of bicycles. However, errors have been found in their derivation, and it has also been proven that gyroscopic effects are not sufficient to ensure the self-stability as a bicycle with its handlebar locked will not be self-stable.

Bower [60], in 1915, also derived the linearized equation of motion for the simplified bicycle model. Yet again, the focus was given to the gyroscopic effects. Wheel inertial properties and front trail were included, but the steering axis was chosen to be vertical [61]. Furthermore, the derivation; (i) ignored the product of inertia terms, (ii) ignored the shifting of the centre of mass location due to steering and, (iii) incorrectly used the centrifugal force. Pearsall [62], in 1922, tried to extend the Bower's work to find the cause of wobble. However, the work contained errors and did not agree with Bower's equations [63].

Timoshenko and Young [64], in 1948, derived a nonlinear (large-angle) lean equation for a simplified basic bicycle model. The basic model had a point mass in the rear part of the bicycle, and a steering angle assumed to be controlled by the rider. The centrifugal force developed by a bicycle when moving on a circular path was attributed to overcome the couple due to gravity which tends to make bicycle fall. But the model did not consider various parameters such as trail, head angle, the mass of the wheel, and centre of mass and inertia properties of the front frame. Furthermore, the model also failed to explain the fact that a riderless bicycle can be in the self-stable state.

Herfkens [63], in 1949, derived the equation of motion for bicycle using the Whipple model and present the effect of front-wheel inertia and other design parameters on the stable forward speed range. Döhring [65], in 1957, used Newton's laws to derive linearized equations of motion for motorcycles and motor-scooters. Collins [55] studied a basic bicycle model using Euler's equations for four rigid bodies [61]. The linearized governing equations were presented in terms of coupled second-order ordinary differential equations (O.D.E) defined for lean and steer angles. Singh [66], in 1964, added the effect of tire side slip to the Collins' model and rederived the governing equations. However, Singh's equations were not compatible with the Collins' equations

[63]. Neĭmark and Fufaev [67] followed Döhring [65] method to the derived equation of motion for the bicycle using Lagrange's equations with the non-holonomic system. However, an error was made while computing the potential energy equations, the rise and fall of the bicycle when steered were not considered. Singh and Goel [68], in 1971, also take reference of Döhring [65] model to analyse a Rajdoot motor scooter using basic bicycle model. They also added steer damping to the basic bicycle model [61].

Jones [47] is popular for his work to build unrideable bicycles. The potential energy model was used to derive the stability of the bicycles. The positive trail and angular momentum of the front wheel were found important factors contributing to the hand-free riding ability and the self-stability. However, the general equation, as reported by Jones, has recently been proven to be incorrect as it only considers potential energy, whereas bicycle is a dynamic system having some forward speed.

Sharp [69] used the Lagrangian approach to derive linearized equations of motion for a motorcycle incorporating wheel side slip. Major eigenmodes: weave, wobble and capsize modes were also identified (please refer to Section 2.2.1, 2.2.3 and 2.2.4 for the modes). Sharp further extended the model including a different tire model [70]. Ronald [71] also derived non-linear equations of motion including tire deformation. However, the steering rate angle was not considered while defining the side-slip angle for the front wheel. Weir [72], in 1972, derived equations of motion for the basic bicycle using a Newtonian approach in PhD dissertation. The equations had a good agreement with Sharp [69]. Singh and Goel [73] extended their previous investigation of Rajdoot study and presented a 12th order mathematical model. The equations were similar to that of Sharp's model.

Adiele [74], in 1979, derived non-linear equations of motion for the basic bicycle with tire side slip using Kane's method. Although the equations were also found comparable to Sharp's [69], several sign errors were found, and front mass was assumed to be zero, which is practically incorrect [63]. Based on a similar argument as Timoshenko and Young [64], Lowell and McKell [75], in 1982, also developed a simplified equation including the steering inertia and front-wheel gyroscope but still failed to include front mass and steering axis tilt.

Hand [61], in 1988, presented the equation of motion for a simplified bicycle in his master thesis. Lagrangian approach was adopted and holonomic, and non-holonomic

rolling constraints were also considered to find the non-linear equations of motion. The equations were then linearized to a vertical position. The equations agreed with the Papadopoulos [76] although they were derived using a different approach. Papadopoulos used Newtonian method to derive the equations of motion for a bicycle. Sharp and Limebeer [77] used the multibody dynamics software called “AutoSim [78]” to derive the equations of motion for motorcycles. Similar equations for a motorcycle with 11 degrees of freedom was described by Cossalter and Lot [79], in 2002. The model was extended to include front and rear suspensions, tire model and pitching motion.

In 2004, Meijaard [80], independently wrote a derivation of the linearized equation of a bicycle model. Later, in 2007, Meijaard et al. [81], after a careful review of the equations derived from the past, benchmarked the equation for balance and steer of a bicycle. Åström et al. [82] also published equations of motion for a bicycle. The derivation was largely based on Papadopoulos [76] and Hand [61]. The equations were similar to the Benchmarked equations of motion [81]. Similarly, Moore [83], used Kane’s method of virtual work to derived the equation of motion of a bicycle in PhD thesis. The results were matched with the benchmarked model and Basu-Mandal [84]. Lipp [85], also derived the equation of motion using Lagrange’s approach. The derivation differs from the Hands [61] in the process of linearization and selection of the generalized coordinate system. A fully nonlinear Lagrangian expression was derived using Euler transformation angles to give the complex relationships between reference frames before expanding to the second order for linearization purposes.

For more than 150 years, researchers and scientists have been interested in understanding the dynamics of bicycles or single-track vehicle in general. The equations have been developed using various approaches such as the Lagrangian method, Newtonian Method and Kane’s method. Furthermore, some of the researches have also simplified the bicycle model and presented simplified equations of motion for bicycles. It is evident that each model and result has help researchers enrich the understanding of the single-track bicycle behaviours. The study of bicycle dynamics has progressed from the early explanation of bicycle motion in relation to the rolling hoop leading to the concept of centrifugal force, gyroscopic effects, the positive trail for self-stability and finally to the correct sets of benchmarked equations of motion for basic bicycle model. Different unstable and stable modes like weave, capsize and castering have been identified and experimentally validated [86]. Various extensions of the benchmarked

models have been reported in recent years. The benchmark equations and extensions are discussed in the subsequent subsections.

2.2.1 Benchmarked Equations for Bicycle

The dynamics of the single-track vehicles include interconnecting parts and various physical phenomena; thus, the equations of motion and the analysis become complex. Researchers have described the complex phenomenon considering a minimum set of parameters in the system with some reasonable assumption and yet preserving the overall dynamic properties of the system. Papadopoulos [76], Hand [61], Sharp [87], Meijaard [80, 81], Moore [83], Lipp [85] have used different approaches as mention in Section 2.2, nonetheless, equations of motion converge to a very similar set of generalized equations. This section provides an overview of the linearized equation for lean and steers of an uncontrolled bicycle following the derivation of Meijaard et al. [81].

Figure 2-4: Whipple/Carvallo bicycle model.

The basic bicycle model is defined as a system of four rigid bodies that consist of a rear wheel, a front wheel, a rear frame where riders can be attached rigidly and a front frame including handlebar and fork is depicted in Figure 2-4. This basic bicycle model is also called the Whipple/Carvallo bicycle model. This model assumes the bicycle is symmetrical about the vertical plane, i.e. left and right symmetry. The wheels are modelled to have knife-edge rolling contact with the ground. The four rigid bodies are interconnected with the revolute joints and friction in the connection points are

neglected. The equations of motions are defined using 25 bicycle-related parameters as listed in Table 2-1.

Table 2-1: 25 design parameters for benchmarked bicycle model.

Parameters	Symbol
Wheelbase	w
Trail	c
Head angle	$90-\lambda$
Forward speed	v
Rear wheel R	
Radius	r_R
Mass	m_R
Mass moment of inertia	$I_{Rxx}; I_{Ryy}$
Rear frame (including rigid rider)	
Centre of mass	(x_B, z_B)
Mass	m_B
Mass moment of inertia	$\begin{bmatrix} I_{Bxx} & 0 & I_{Bxz} \\ 0 & I_{Byy}; & 0 \\ I_{Bxz} & 0 & I_{Bzz}; \end{bmatrix}$
Front handlebar and fork assembly H	
Centre of mass	(x_H, z_H) or (u, v)
Mass	m_H
Mass moment of inertia	$\begin{bmatrix} I_{Hxx} & 0 & I_{Hxz} \\ 0 & I_{Byy}; & 0 \\ I_{Hxz} & 0 & I_{Hzz}; \end{bmatrix}$
Front wheel F	
Radius	r_F
Mass	m_F
Mass moment of inertia	$I_{Fxx}; I_{Fyy}$

The system has both holonomic and non-holonomic constraints. Holonomic constraints are those which restricts motion in certain places and contains only geometrical coordinates. Whereas, non-holonomic constraints have the derivative terms involved and restrict the motion in a certain direction. The condition that both the wheels touch the ground provides two holonomic constraints in the normal direction. Furthermore, assuming rolling without slip gives two additional non-holonomic constraints in the longitudinal and lateral direction for each wheel. Thus, the system has two holonomic and total of four non-holonomic constraints.

The front contact point and the rear contact point have the geometric relationship including the steering angle (δ), yaw angle (ψ), wheelbase (w), trail (c), and steering axis angle (λ).

Figure 2-5: Geometrical relation between front and rear contact points.

When the bicycle is not steered ($\psi = 0$) as shown in Figure 2-5 (a), the front contact point is defined by:

$$Y_Q = Y_P + w \sin \psi \quad (2.1)$$

Similarly, when the bicycle is not yawed (as shown in Figure 2-5 (b), the front contact point is defined by:

$$Y_Q = Y_P - c\delta \cos \lambda \quad (2.2)$$

Combining Equation (2.1) and Equation (2.2) gives the front contact point position Equation (2.3) for a yawed and steered bicycle.

$$Y_Q = Y_P + w \sin \psi - c\delta \cos \lambda \quad (2.3)$$

Assuming rigid knife-edge wheels, the kinematic constraints for the bicycle system are defined as:

Holonomic constraints: wheels touch the ground plane

$$Z_P = 0 \quad (2.4)$$

$$Z_Q = 0 \quad (2.5)$$

Non-Holonomic constraints: wheels rolling without slip

$$\dot{X}_P = r_R * (-\dot{\theta} \cos \psi) \approx -R \dot{\theta} \approx u \quad (2.6)$$

$$\dot{Y}_P = r_R * (-\dot{\theta} \sin \psi) \approx -R \dot{\theta} \sin \psi \approx u * \sin \psi \quad (2.7)$$

$$\dot{X}_Q = r_F \cos(\psi + \delta \cos \lambda) (-\dot{\theta}) \approx u \quad (2.8)$$

$$\dot{Y}_Q = r_F \sin(\psi + \delta \cos \lambda) (-\dot{\theta}) \approx u * (\sin \psi + \delta \cos \lambda) \quad (2.9)$$

Solving Eq. (2.3), Eq. (2.7) and (2.9) yields Eq. (2.10) and Eq. (2.11) and again differentiating Eq. (2.3) and using Eq. (2.9) yields Eq. (2.12).

$$\dot{\psi} = (u\delta + c\dot{\delta}) * \cos\lambda/w \quad (2.10)$$

$$\ddot{\psi} = (vu + c\ddot{\delta}) * \cos\lambda/w \quad (2.11)$$

$$\ddot{Y}_p = (u^2\dot{\delta} + uc\ddot{\delta}) * \cos\lambda/w \quad (2.12)$$

The moment balance approach can be used to derive the equations of motion for a particular axis of interest. The generalised coordinate system and forces acting on the system are shown in Figure 2-6. The terms m_b , m_f , and m_T are the masses of the rear frame including rider, front frame assembly including handlebar and fork, and the total mass of the system including a rear frame, front frame and the mass of the wheels. G_{Rz} and G_{Fz} are the ground reaction forces in the z-direction at the points where wheels touch the ground. Similarly, F_{y1} and F_{y2} are the ground reaction force in the y-direction to prevent sideslip of the wheels. The total centre of mass is represented by (x_T, z_T) and centre of mass location for front assembly (front frame + wheel) is represented by (x_A, z_A) . Other intermediate variables are defined as:

$$\text{COM of front frame ahead of the steering axis } (u_A) = (x_A - w - c)\cos\lambda - z_A \sin\lambda$$

$$\text{Ratio of mechanical trail to wheelbase } (\mu) = c \cos\lambda/w$$

$$\text{Gyrostatic coefficients } (S_R, S_F, S_T,) = \left(\frac{I_{Ryy}}{r_R}, \frac{I_{Fyy}}{r_F}, \frac{I_{Ryy}}{r_R} + \frac{I_{Fyy}}{r_F} \right)$$

$$\text{Static moment } (S_A) = m_A u_A + \mu m_T x_T$$

Figure 2-6: Generalized angular rotation and forces on the bicycle system.

The first equation is the equation of motion for the total bicycle rider system about lateral direction.

$$m_T \ddot{Y}_p + m_T x_t \ddot{\psi} - m_T z_T \ddot{\phi} + m_A u_A \ddot{\delta} = F_{y1} + F_{y2} \quad (2.13)$$

The first term on the left-hand side represents the lateral acceleration of total bicycle due to the lateral acceleration of the rear contact point. Similarly, remaining terms on the left-hand side represent the acceleration in y-direction due to yawing and leaning of the whole bike and steering of the front assembly, respectively. The terms on the right-hand side represent the forces on the y-direction.

The second equation is the lean equation of motion for the whole bicycle about the X-axis.

$$-m_T \ddot{Y}_p z_T + I_{Txx} \ddot{\phi} + I_{Txx} \ddot{\psi} + I_{A\lambda x} \ddot{\delta} + u S_T \dot{\psi} + u S_F \dot{\delta} \cos \lambda = T_\phi - g m_T z_t \phi + g S_A \delta \quad (2.14)$$

The first term on the left-hand side represents the moment to accelerate the centre of mass of the whole bike in the Y direction. The second and third terms represent the moment required for lean and yawing angular acceleration of the whole bicycle about rear contact point. The fourth term represents a similar lateral moment for the front steering assembly. The fifth term and sixth terms represent the gyroscopic moment responsible for yawing the whole bike about the vertical axis and the gyroscopic precession of the front wheel about steering axis respectively. The first term on the right-hand side is the external moment applied to the rear frame. The second term is the lean moment from the gravitational force acting on the centre of mass of total bicycle rider system. The last term represents the gravitational moment considering the effects of sideways displacement of the front contact point and the centre of mass location when steered.

The third equation of motion is the yaw moment balance for the bicycle-rider system about the Z-axis.

$$m_T \ddot{Y}_p x_T + I_{Txx} \ddot{\phi} + I_{Tzz} \ddot{\psi} + I_{A\lambda z} \ddot{\delta} - u S_T \dot{\phi} - u S_F \dot{\delta} \sin \lambda = w F_{y1} \quad (2.15)$$

The last equation of motion is the steer moment balance for a front assembly about the steering axis.

$$m_A \ddot{Y}_p u_A + I_{A\lambda x} \ddot{\phi} + I_{A\lambda z} \ddot{\psi} + I_{A\lambda\lambda} \ddot{\delta} + u S_F (\dot{\psi} \sin \lambda - \dot{\phi} \cos \lambda) = T_\delta - c F_{y1} \cos \lambda + g (\phi + \delta \sin \lambda) S_A \quad (2.16)$$

The first four terms on the left-hand side have a similar explanation as in previous equations about their respective axis. The last term in the left-hand side represents the gyroscopic moment for precession. The first term on the right-hand side represents the external handlebar steering torque. The second term represents the lateral force at the front wheel contact point. The last terms represent the moment about the steering axis due to the front reaction force and the gravity acting on the centre of mass of front assembly.

The lateral force term F_{y1} can be eliminated by solving the yaw equation; Equation (2.15) and steer equation; Equation (2.16). Furthermore, the kinematic constraints Equation (2.10-2.12) can be substituted to Equations (2.14-2.16). After simplifications, the resulting equations become two coupled second-order, constant coefficient, ordinary differential equations defined for lean angle (Φ) and steer angle (δ). The first equation is called the lean equation, and the second equation is called the steer equation. The two equations of motion can also be represented in the matrix form as:

$$M\ddot{q} + uC_1\dot{q} + [gK_0 + u^2K_2]q = f \quad (2.17)$$

Where, $q = [\Phi, \delta]^T$ and forcing terms $f = [T_\Phi, T_\delta]^T$ are lean torque and steer torque. In the case of self-stability in hands-free riding, $f = 0$. The coefficients M , C_1 , K_0 , and K_2 are the mass matrix, damping like matrix and stiffness matrix. The stiffness matrix is the sum of terms containing gravitational acceleration and term which is quadratic in velocity. The coefficients are 2X2 matrixes defined in terms of 25 design parameters which can be categorized into geometric and mass parameters. Equation (2.17) is known as the linearized benchmarked equations for the uncontrolled motion of a bicycle.

Based on Equation (2.17), assuming the solution of the form $e^{\lambda t}$, the characteristic equation of motion is defined as:

$$\det (M\lambda^2 + uC_1\lambda + [gK_0 + u^2K_2]) = 0 \quad (2.18)$$

Once the characteristic equation of motion is formulated, there are many possible ways to analyse the stability of the design configuration. Based on the characteristic equation, the self-stability of the bicycle model can be computed. A few well-known techniques to analyse and assess the self-stability are:

- Routh-Hurwitz criterion

- Eigenvalues and eigenvector analysis

2.2.1.1 Routh-Hurwitz Criterion

The Routh-Hurwitz criterion is a mathematical test to predict if the characteristic equations of a system have negative roots without solving them. The negative roots are associated with the stable state, and positive roots are associated with the unstable state. After simplification, Equation (2.18) results in a fourth-order polynomial equation in λ , whose roots are eigenvalues λ_i ,

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0 \quad (2.19)$$

Routh-Hurwitz criterion allows determining the stability of the system based on coefficients of the polynomial equation. The Routh stability criteria state that for all eigenvalues satisfying the quartic characteristic equation to have a negative real part, all polynomial coefficients A, B, C, D, E and the Routh determinant $X = BCD - ADD - EBB$ must have the same sign. The coefficients (A to E) are a complex expression in 25 design parameters. In this particular case of a basic bicycle, all coefficients, including the Routh determinant, must have positive sign [88]. A typical example of Routh-Hurwitz criteria to analyse bicycle dynamics can be found in the master thesis by Hand [61], and supplementary materials for two-mass skate bicycle [89]. However, the Routh-Hurwitz criteria do not provide any means to compare different designs. It does indicate whether a design is a stable configuration but does not give the exact measure of how stable is a particular configuration relative to other [61].

2.2.1.2 Eigenvalues and Eigenvector Analysis

The characteristics Equation (2.18) can be used to determine eigenvalues and the eigenvectors of the system at a particular speed. Eigenvalues are the roots of the characteristic equations representing the natural frequencies and damping ratios, and the eigenvectors are the mode shape associated with the eigenvalues. Since the characteristic equations of motion for the benchmarked bicycle model is a fourth-order polynomial, it yields four eigenvalues at a particular forward speed and the same number of eigenvectors. The eigenvalues can be used to predict the stability of the system. If any eigenvalues have positive real parts, the system will be unstable when disturbed. Complex eigenvalues represent the oscillation, and the real parts determine whether the oscillation will die or increase with time. For a system to be stable, it must have all the

real part of eigenvalues negative. A typical solution to a benchmarked bicycle is shown in Figure 2-7.

Figure 2-7: Typical solution of the benchmarked bicycle model [81]

The horizontal axis represents the forward velocity, and the vertical axis represents the computed eigenvalues using the characteristic equation, Equation (2.18). At zero velocity, four eigenvalues are represented by λ_{s1} , λ_{s2} , λ_{s3} and λ_{s4} , respectively. The eigenvalues change with the forward velocity. The four eigenvalues could all be real or two real and two conjugate imaginary numbers. Depending on the nature of eigenvalues, they represent different mode or eigenvectors. The important eigenvectors are shown in Figure 2-7. The castering is an always stable mode of bicycle dominated by a steer. The castering mode is associated with the large, negative real eigenvalues. This represents the tendency of the steering to align with the rear frame [88]. The weave is the complex oscillatory behaviour of the bicycle about the steering axis. It represents a slow 0 – 4 Hz sinusoidal oscillation of the bicycle about headed direction. The bicycle oscillated between leaning left and steering right and vice-versa. The real part of the eigenvalues represents the damping, and imaginary part represents the natural frequency of the oscillation. Weave mode is unstable at low speed and becomes stable with an increase in velocity. At high speed, the weave can become unstable again (which is well a known phenomenon in fast or heavily loaded motorcycles). At last, capsize mode represents non-oscillatory falling off the bicycle dominated by lean. The capsize mode has a smaller (positive or negative) real eigenvalues. The capsize mode is stable at low speed and becomes unstable at a higher speed. A capsizing bicycle leans progressively into a tightening spiral with steer and leans both increasing proportionally

as it falls over [81]. The self-stable region is bounded by a region where all eigenvalues have negative real parts, as highlighted in Figure 2-7.

Unlike the Routh-Hurwitz criteria, eigenvalue and eigenvector analysis allow the comparison between various design configurations. It is possible to determine which configuration is mathematically more stable by comparing the eigenvalues of the particular set of configurations. Therefore, the eigenvalues and eigenmode analysis are one of the most widely used techniques in single-track vehicle dynamics.

2.2.2 Extension 1: Inclusion of Rigid Rider in Bicycle Model

The benchmarked bicycle model allows a rigid rider on the rear frame of the bicycle. An addition of a rigid rider makes the model more realistic. Compare to other heavy single-track vehicles like motorcycles and scooters, bicycles are very lightweight vehicles. The rider-vehicle weight ratio for bicycle is very high, unlike motorcycles and scooters. The mass related parameters such as weight, seating posture, the centre of mass and corresponding moment of inertia of rider dominate the mass related parameters of the bicycle and have a vital influence on the stability and rideability of the bicycle. Therefore, an added rider on the bicycle's rear frame helps improve the accuracy of performance evaluation. However, benchmarked bicycle model does not allow the rider to move relative to the rear frame plane. In addition, restricting the out of the plane motion of a rigid rider relative to the rear frame reduces the mathematical complexity. It neglects the possibilities of rider's feedback control. Therefore, this model could be viewed as the open-loop system without any feedback control from the rider and helps to analyze the inherent ability of a bicycle for self-stability. Thus, this model is very suitable to analyze the ability to ride hands-free on any bicycle design. However, modelling a rider onto the bicycle a challenging task and require merging the bicycle geometrical parameters to rider's anthropometric parameters.

Researchers have proposed various methods to find the mass, centre of mass, and moment of inertia of human body segments [90-94]. Most of the early research was based on cadaver studies. One of the methods used is the water displacement method [91, 94]. In this method, the cadaver body or the living human body is immersed in the water and volume of displaced water was measured. Masses of the segments are used to be calculated assuming the body segment has uniform density, and the density for the cadaver also represents the living human body. The centre of mass of the body segments

can be determined using static balance methods. The moment of inertia can be determined by measuring the time period of the oscillation of either the body segments or the cast replica of the body segments from compound pendulum experiments [91]. Another approach for determining the rider's parameters is the mathematical modelling of the rider. The accuracy of this method depends on the number of anthropomorphic measurements taken and the geometrical shapes used.

Figure 2-8: Combined bicycle rider system and grid points [95].

The simplest method for mathematically modelling a rider on to the bicycle with acceptable accuracy was proposed by Moore et al. [95]. The head was represented by a sphere, torso by a rectangular block, and hands and legs by cylinders. Once the human body was represented by standard geometries, the centre of mass and moment of inertia of the whole body can be calculated using the respective formulae. To model the bicycle rider system, the posture of the rider on the bicycle is essential to be mimicked. For this, basic bicycle geometry and the fourteen human anthropomorphic parameters were considered. The weight of each body segment was assumed to be proportional to the overall weight of the body. The bicycle-rider system was defined by the 31 grid points in 3D space, as shown in Figure 2-8.

Figure 2-9: Solution of characteristic equations for a bicycle with and without a rider.

The major difference between the benchmarked bicycle model with and without a rider is the mass related parameters and the reaction forces on wheels. The centre of mass generally shifts backwards and upwards compared to the riderless bicycle. Similarly, the inertial parameters also increased [95]. Rider's property plays a dominant role in the dynamic performance of the bicycles and other personal mobility vehicles like e-scooters. However, the equations of motion and the characteristic equation do not change by adding the rigid rider on the rear frame. Nonetheless, the results will change. Figure 2-9 shows the eigenvalues and eigenvectors analysis for a bicycle model with and without a rider (please refer to Appendix A-3 (improved 20-inch wheel I and bike without rider for data set)).

It is clearly evident that the added rider has a dominant role in dynamic stability performance. The self-stability shifted towards the higher velocity, and the self-stable range also broaden. Therefore, it is logically necessary to include a rider on the vehicle while analysing the dynamic performance of different designs of bicycles and e-scooters.

2.2.3 Extension 2: Inclusion of the Rider's Upper Body Lean Movement

The benchmark equations are based on Whipple and Carvallo model which restricts the rider body motion relative to the rear frame. That is, the rider body is assumed to be an extension of the bicycle rear frame and cannot move laterally out of the plane of the

bicycle rear frame. This neglects any possibility of rider upper body lean control on the bicycle. However, in a real scenario, the rider's body is not the rigid extension to the rear frame as assumed in the benchmarked bicycle model. The rider uses his/her upper body to control the bicycle motion, especially in hands-free riding. Furthermore, the rider's body vibrates when the vehicle is oscillating. This vibration might occur even without any voluntary control from the rider [96]. Thus, it is important to consider the rider's body motion relative to the mainframe to make a realistic bicycle model.

Figure 2-10: Illustration of bicycle model with rider's lean relative to the rear frame.

Sharp [97] used multibody software to develop a model by allowing the rider's body to move laterally with respect to the rear frame. This increases the degree of freedom of the system by one. The added degree of freedom (DOF) represents the lean of the upper body of the rider. The lower body is modelled to be rigidly attached to the rear frame. The upper body and the lower body of the rider is assumed to be connected by torsional spring and damper elements. The elastic and damping properties of rider act to oppose the lean motion of the rider's upper body relative to the rear frame. The similar method has been used to analyse the effect of rider's upper body lean in bicycles and motorcycle by Schwab et al. [98], Doria and Tognazzo [96], Lai et al. [99], Haudum [100], Plöchl et al. [101]. The structure of the governing equations of motion is identical to the benchmark equations, but the number of equations and degree of freedom increases by one. In addition, a study carried out by Cain et al. [102] demonstrated that the rider's lean angle and bicycle roll angle are negatively correlated.

A typical representation of the bicycle model with the rider's upper body lean is shown in Figure 2-10. Here, hands-off the handlebar posture of the rider is shown. The upper part of the rider consists of torso, hands and head. Likewise, the lower part of the rider consists of legs. The rider's upper body is separated from the rear frame and can lean sideways relative to the rear frame of the bicycle. However, the lower part of the rider is still assumed to be rigidly attached to the rear frame of the bicycle. The two parts of the rider are connected by a spring element with a stiffness coefficient of K_{ϕ_r} and a damping element having a damping coefficient of C_{ϕ_r} . Such coefficients depend on riding posture and can be evaluated experimentally. A typical approach is to shake the rider's body by mock-up two-wheel vehicle set-up and measure the response on the body parts. An example of the test setup is shown in Figure 2-11. Nishimi et al. [103] experimentally determined the stiffness and damping coefficient for Motorcycle. Similarly, the rider's properties for the bicycle-rider system is experimentally identified by Doria et al. [96, 104].

Figure 2-11: Experimental set-up to find rider's compliance [96]

The equations of motion for the rider's lean extension to the benchmark equations are:
Equation of motion in lateral direction Y

$$m_T \ddot{Y}_p + m_T x_T \ddot{\psi} - m_T z_T \ddot{\phi} + m_r s_r \ddot{\phi}_r + m_A u_A \ddot{\delta} = F_{y1} + F_{y2} \quad (2.20)$$

Equation of motion for Yaw direction ψ

$$m_T \ddot{y}_P x_T + I_{Tzz} \ddot{\psi} + I_{Txx} \ddot{\phi} + I_{r\phi rz} \ddot{\phi}_r + I_{A\lambda z} \ddot{\delta} - u S_T \dot{\phi} - u S_F \dot{\delta} \sin \lambda = w F_{y1} \quad (2.21)$$

Equation of motion for roll direction ϕ

$$-m_T \ddot{y}_P z_T + I_{Tzx} \ddot{\psi} + I_{Txx} \ddot{\phi} + I_{r\phi rx} \ddot{\phi}_r + I_{A\lambda x} \ddot{\delta} + u S_T \dot{\psi} - m_r s_r g \phi_r + u S_F \dot{\delta} \cos \lambda + g m_T Z_t \phi - g S_A \delta = T_\phi \quad (2.22)$$

Equation of motion for the rider's upper body roll motion ϕ_r

$$m_r s_r \ddot{y}_P + I_{r\phi rz} \ddot{\psi} + I_{r\phi rx} \ddot{\phi} + I_{r\phi r\phi r} \ddot{\phi}_r + C_{\phi r} \dot{\phi}_r - m_r s_r g \phi + (K_{\phi r} - m_r s_r g) \phi_r - C_\delta \sin \lambda \dot{\delta} - K_\delta \sin \lambda \delta = T_{\phi r} \quad (2.23)$$

Equation of motion for steering motion δ

$$m_A \ddot{y}_P u_A + I_{A\lambda x} \ddot{\phi} + I_{A\lambda z} \ddot{\psi} + I_{A\lambda \lambda} \ddot{\delta} + u S_F \dot{\psi} \sin \lambda - u S_F \dot{\phi} \cos \lambda - g S_A \phi + (K_\delta - g S_A \delta \sin \lambda) + C_\delta \dot{\delta} = T_\delta - c F_{y1} \cos \lambda \quad (2.24)$$

Solving Equation (2.21) and Equation (2.24) eliminates the lateral tire forces and the required equations will be reduced to three for lean motion, rider's upper body motion and steering motion. The equations in the matrix form can be represented as:

$$M \ddot{q} + (C_0 + u C_1) \dot{q} + [K_0 + u^2 K_2] q = f \quad (2.25)$$

Where $q_e = (\phi, \phi_r, \delta)$ represents the degree of freedom in roll, upper body lean and steering direction. Similarly, $f_e = (T_\phi, T_{\phi_r}, T_\delta)$ represents the external forces on the system. For self-stability and hands-free riding, $f_e = 0$. The extra matrix appeared to represent the torsional spring and damper components that relate the joint between rider's upper body and lower body. Furthermore, K_0 is also modified for the same reason. Equation (2.25) can be solved to obtain the eigenvalues and eigenvectors.

Figure 2-12 shows the comparison between the benchmarked model and extended rider lean mode for a forward velocity up to 20 m/s. Figure 2-12 (a) shows the results for the real part of eigenvalues analysis and Figure 2-12 (b) shows the results for the imaginary part of eigenvalues analysis. Rider's compliance properties, stiffness coefficient and damping coefficients were chosen 760 Nm/rad and 34 Nms/rad, respectively [100] [101]. An extra mode appeared representing the motion of the rider's upper body. This

rider lean mode has imaginary eigenvalues. Thus, results support the general observations that the rider's body also vibrates or oscillates when the rider is in oscillating vehicles. The frequency of oscillation of the rider's upper body is higher than the weave mode oscillation of the bicycle. The frequency corresponding to the rider's lean is obtained about 2 Hz. It should be noted that the frequency largely depends on the stiffness and damping coefficients. For more realistic values as measured by Doria et al. [96], the frequency for rider's lean is obtained at about 1.5Hz. With the chosen values of the rider's compliance properties, the lean mode appeared to be stable as eigenvalues have negative real parts. Furthermore, allowing rider's upper body to lean relative to the rear frame reduced the weave damping and the natural frequency compared to that of the benchmarked model at higher velocity.

Figure 2-12: Comparison of benchmark model with rider lean extension model (a) comparison between real part of eigenvalues (b) comparison between imaginary part of eigenvalues.

The self-stability range started at identical velocity for the selected rider's compliance values. A lower value of the rider's compliance suggests a poor connection between the upper body and the lower body at pivot points and higher values represent a firm connection between the upper and lower body parts. A typical comparison case no proper connection between rider's upper body and lower body i.e. $K_{\phi r} = 0 \text{ Nm/rad}$ and $C_{\phi r} = 0 \text{ Nms/rad}$ is shown in Figure 2-13. When there was no spring and damper element in the model, the overall system became unstable. The dynamic also changed, as the rider lean mode does not have the imaginary eigenvalues. The eigenvalues for this unstable upper body lean mode have only real parts. Therefore, there is no oscillating

motion. The motion is like the capsize mode. The upper body always tends to fall to either side (left-right) as there is no element to restrain the upper body of the rider. Thus, the system is always unstable in the absence of a proper connection between the upper and lower part of the rider's body. However, low speed weave mode and capsize mode are almost similar to the benchmark model. On the other hand, when the stiffness and damping coefficients were increased, the connection becomes more rigid, and results became closer to the benchmark model.

Figure 2-13: Rider's upper body lean model with no spring and damping element at pivot point (a) real part of eigenvalues (b) imaginary part of eigenvalues.

2.2.4 Extension 3: Inclusion of the Tire Dynamics

The tire is an important component of the bicycle system. It helps to reduce the vibration due to uneven terrain and maintain the required adherence with the road. There are various forces and moments acting on the tire due to the deformation of the tire on the ground are shown in Figure 2-14. The forces and moments are the functions of the forward velocity (u), chamber angle (γ), slip angle (α) and reaction load (F_z) acting on the tire itself.

The longitudinal force and the reaction force acting on the tire depends on the velocity. However, for a constant velocity case and assuming no longitudinal slip, longitudinal force and change in reaction force on the tire can be neglected. The remaining force is the lateral force (F_y) and the moments (M_x, F_z). The tire-related properties such as

stiffness, slip angles, relation lengths, lateral forces etc. for both motorcycles and bicycles have been measured experimentally [105-107].

Figure 2-14: Forces and moments acting on a wheel.

The nonlinear model of the lateral force on the tire was presented by Pacejka model or magic tire model [108]. For small slip and camber angles, the non-linear relationship between slip angles, camber angles and lateral force is nearly linear [56]. The linear relationship for the lateral forces and moments are defined as:

Lateral forces:

$$F_{yi} = c_{F\alpha i} \alpha_i + c_{F\gamma i} \gamma_i \quad (2.26)$$

Self-aligning moments

$$M_{zi} = -c_{M\alpha i} \alpha_i + c_{M\gamma i} \gamma_i \quad (2.27)$$

Overtopping couples

$$M_{xi} = -c_{MX\gamma i} \gamma_i \quad (2.28)$$

i=r, f; where, f=front wheel, r= rear wheel

Where $c_{F\alpha i}$, $c_{F\gamma i}$, $c_{M\alpha i}$, $c_{M\gamma i}$, $c_{MX\gamma i}$, α_i , and γ_i are called the cornering stiffness, camber stiffness, coefficients of self-aligning moments, coefficients of overtopping couples and transient side-slip angle and the camber angle. This tire lateral force does not appear instantaneously. The wheel needs to move a certain distance depending on the cornering and lateral stiffness of the tire for the lateral force to develop. There is a lag between the appearance of the lateral force and given camber and slip angle related to the tire relaxation length (σ). The relaxation length represents the distance a tire should cover for the lateral force to reach 63 % of the steady-state [56]. This transient behaviour of the tire is defined by a first-order differential equation with relaxation length as:

$$\frac{\sigma_{\alpha i}}{u} \dot{\alpha}_i + \alpha_i = \alpha_i \quad (2.29)$$

$$\frac{\sigma_{\gamma_i}}{u} \dot{\gamma} + \dot{\gamma}_i = \gamma_i \quad (2.30)$$

Sharp [69, 70] included the tire dynamics to study the stability and control of the motorcycles. The analysed used a tire with (i) infinitely stiff property, (ii) tire side slip and (iii) tire with side slip and camber with relaxation length. The found the third model to be more accurate than the others. Similarly, Cossalter et al. [109] developed a multibody software called FastBike [110] for simulating motorcycle dynamics. Meijaard and Schwab [111] extended the benchmark model to include lateral forces. Sharp [87, 97] also studied bicycle dynamics, including tire lateral forces and moments. A simple model of tire with only lateral forces and relaxation length was used by Souh [112] to study the parametric effects on bicycle self-stability.

The kinematic constraints explained in 2.2.2 also changes when non-holonomic constraints are replaced with the lateral forces. The relations are re-defined as:

Rear-wheel side slip angle:

$$\alpha_r = -v/u \quad (2.31)$$

Rear-wheel camber angle:

$$\gamma_r = \phi \quad (2.32)$$

Front-wheel camber angle:

$$\gamma_f = \phi + \delta \sin \lambda \quad (2.33)$$

Yaw rate:

$$\dot{\psi} = \frac{u}{w} (\alpha_r - \alpha_f - \delta \cos \lambda) + \frac{c \cos \lambda}{w} \dot{\delta} \quad (2.34)$$

Yaw acceleration:

$$\ddot{\psi} = \frac{u}{w} (\dot{\alpha}_r - \dot{\alpha}_f - \dot{\delta} \cos \lambda) + \frac{c \cos \lambda}{w} \ddot{\delta} \quad (2.35)$$

Lateral acceleration:

$$\dot{Y} = u(\dot{\psi} - \dot{\alpha}_r) = u \left(\frac{u}{w} (\alpha_r - \alpha_f - \delta \cos \lambda) + \frac{c \cos \lambda}{w} \dot{\delta} \right) - u \dot{\alpha}_r \quad (2.36)$$

The equations of motion, including tire lateral dynamics and moments, are as follows:

Equation of motion for lateral direction

$$m_T \ddot{Y}_p + m_T x_T \ddot{\psi} - m_T z_T \ddot{\phi} + m_r s_r \ddot{\phi}_r + m_A u_A \ddot{\delta} = F_{y1} + F_{y2} \quad (2.37)$$

Equation of motion for Yaw direction ψ

$$\begin{aligned} m_T \ddot{Y}_p x_T + I_{Tzz} \ddot{\psi} + I_{Txz} \ddot{\phi} + I_{r\phi rz} \ddot{\phi}_r + I_{A\lambda z} \ddot{\delta} - u S_T \dot{\phi} \\ - u S_F \dot{\delta} \sin \lambda = w F_{y1} + M_{z1} + M_{z2} \end{aligned} \quad (2.38)$$

Equation of motion for roll direction ϕ

$$\begin{aligned}
& -m_T \ddot{Y}_P Z_T + I_{T_{xz}} \ddot{\psi} + I_{T_{xx}} \ddot{\phi} + I_{r\phi_{rx}} \ddot{\phi}_r + I_{A\lambda x} \ddot{\delta} + u S_T \dot{\psi} - m_r s_r g \phi_r \\
& + u S_F \dot{\delta} \cos \lambda + g m_T Z_t \phi - g S_A \delta = T_\phi + M_{X1} + M_{X2}
\end{aligned} \tag{2.39}$$

Equation of motion for the rider's upper body roll motion ϕ_r

$$\begin{aligned}
& m_r s_r \ddot{Y}_P + I_{r\phi_{rz}} \ddot{\psi} + I_{r\phi_{rx}} \ddot{\phi} + I_{r\phi_r\phi_r} \ddot{\phi}_r + C_{\phi_r} \dot{\phi}_r - m_r s_r g \phi \\
& + (K_{\phi_r} - m_r s_r g) \phi_r - C_\delta \sin \lambda \dot{\delta} - K_\delta \sin \lambda \delta \\
& = T_{\phi_r} - M_\delta \sin \lambda
\end{aligned} \tag{2.40}$$

Equation of motion for steering motion δ

$$\begin{aligned}
& m_A \ddot{Y}_P u_A + I_{A\lambda x} \ddot{\phi} + I_{A\lambda z} \ddot{\psi} + I_{A\lambda\lambda} \ddot{\delta} + u S_F \dot{\psi} \sin \lambda - u S_F \dot{\phi} \cos \lambda) - \\
& g S_A \phi + (K_\delta - g S_A \delta \sin \lambda) + C_\delta \dot{\delta} = T_\delta - c F_{y1} \cos \lambda + M_{X1} \sin \lambda + \\
& M_{Z1} \cos \lambda
\end{aligned} \tag{2.41}$$

Equations (2.26-2.36) can be substituted into equations of motion, Equation (2.37-2.41), to include the tire dynamics. After rearrangements and collecting similar terms, the equations of motion can be represented into matrix form as:

$$M \ddot{q} + \left(C_1 * u + \frac{C_2}{u} \right) \dot{q} + (K_0 * g + K_2 * u^2 + K_t) = f \tag{2.42}$$

Where C_2 and K_t matrices include tire-related properties.

Figure 2-15 shows a typical example of extended benchmark model, including tire dynamics. An extra oscillating mode appeared when tire dynamics is considered. The new oscillating mode is called the wobble mode. The wobble mode is characterized by the rapid oscillation of the front frame. For motorcycle, the frequency of oscillation generally is about 9-14 cycle/sec [56]. At very high speed, this mode causes rapid oscillation of the front steering and is often ends with an unpleasant experience. The addition of tire dynamics affected the eigenmodes at higher velocities. At lower to normal riding velocity up to 7 m/s, the effect of the tire dynamic model was found insignificant. Furthermore, the wobble mode is stable at low to normal riding velocity and unstable at very high velocity. The frequency of the wobble mode in bicycle was found to be in between 6-8 Hz. Increasing the camber stiffness was found to improve the wobble mode damping. The effect of cornering stiffness is more distinct than that of

camber stiffness. The wobble mode stability at higher velocity also improves with increased cornering stiffness [112].

Figure 2-15: Extension to the Benchmark model including tire dynamic model (a) unstable wobble mode at high speed (b) comparison of capsize, castering and weave mode (c) comparison of imaginary part of eigenvalues.

2.2.5 Extension 4: Inclusion of the Front Frame Flexibility

The benchmark model assumes a perfectly rigid bicycle frame. However, in the rear scenario, the front frame tends to twist from the head tube angle because of the frame flexibility. Thus, frame flexibility is one of the important extensions to the benchmark model to make the analysis more realistic. The dynamics stability of motorcycles and bicycles considering flexible front frame have been studied by Sharp [87, 97], Doria and Melo [113], and Plöchl et al. [101].

A general approach to include the frame flexibility is analogous to the addition of rider lean to the rear frame. Here, the front frame is divided into two parts: (i) lower fork and (ii) upper handlebar and stem tube. The lower and upper part of the front frame is

modelled to be connected with a spring and damper element having stiffness coefficient and damping coefficient.

Figure 2-16: Bicycle model with flexible front frame.

A typical schematic diagram of the bicycle with a frame flexibility model is shown in Figure 2-16. Here, the twisting axis of the front frame is perpendicular to the steering axis and S_s distance from the centre of mass location of the combined lower part of the front frame and front wheel. Furthermore, the twisting axis is at S_c distance from the front wheel contact points on the ground. The S_s and S_c distance depends on the fork length of the bicycle. The flexible front frame consideration also modifies the constraints equations of the benchmark bicycle model as:

$$\dot{\psi} = \frac{(u\delta + c\dot{\delta}) * \cos\lambda}{w} + \frac{S_c\dot{\beta}}{w} - \frac{u \sin\lambda \beta}{w} \quad (2.43)$$

$$\ddot{\psi} = \frac{(u\delta + c\dot{\delta}) * \cos\lambda}{w} + \frac{S_c\ddot{\beta}}{w} - \frac{u \sin\lambda \dot{\beta}}{w} \quad (2.44)$$

$$\ddot{Y}_p = u \left(\frac{(u\delta + c\dot{\delta}) * \cos\lambda}{w} + \frac{S_c\dot{\beta}}{w} - \frac{u \sin\lambda \beta}{w} \right) \quad (2.45)$$

Allowing the front frame to twist about the twisting axis β adds one extra degree of freedom into the equations of motion. The equations of motion for different directions are:

Equation of motion for lateral direction Y

$$m_T \ddot{Y}_p + m_T x_T \ddot{\psi} - m_T z_T \ddot{\phi} + m_r s_r \ddot{\phi}_r + m_A u_A \ddot{\delta} = F_{y1} + F_{y2} \quad (2.46)$$

Equation of motion for Yaw direction ψ

$$m_T \ddot{Y}_P x_T + I_{Tzz} \ddot{\psi} + I_{Txx} \ddot{\phi} + I_{r\phi rz} \ddot{\phi}_r + I_{s\beta z} \ddot{\beta} + I_{A\lambda z} \ddot{\delta} - u S_T \dot{\phi} - u S_F \dot{\delta} \sin \lambda - u S_F \dot{\beta} \cos \lambda = w F_{y1} \quad (2.47)$$

Equation of motion for roll direction ϕ

$$\begin{aligned} -m_T \ddot{Y}_P z_T + I_{Txx} \ddot{\psi} + I_{Txx} \ddot{\phi} + I_{r\phi rx} \ddot{\phi}_r + I_{A\lambda x} \ddot{\delta} + I_{s\beta x} \ddot{\beta} - u S_F \dot{\beta} \sin \lambda \\ + u S_T \dot{\psi} - m_r s_r g \phi_r + u S_F \dot{\delta} \cos \lambda + g m_T Z_t \phi - g S_A \delta \\ - g S_\beta \beta = T_\phi \end{aligned} \quad (2.48)$$

Equation of motion for the rider's upper body roll motion ϕ_r

$$\begin{aligned} m_r s_r \ddot{Y}_P + I_{r\phi rz} \ddot{\psi} + I_{r\phi rx} \ddot{\phi} + I_{r\phi r\phi r} \ddot{\phi}_r + C_{\phi r} \dot{\phi}_r - m_r s_r g \phi \\ + (K_{\phi r} - m_r s_r g) \phi_r - C_\delta \sin \lambda \dot{\delta} - K_\delta \sin \lambda \delta = T_{\phi r} \end{aligned} \quad (2.49)$$

Equation of motion for steering motion δ

$$\begin{aligned} m_A \ddot{Y}_P u_A + I_{A\lambda x} \ddot{\phi} + I_{A\lambda z} \ddot{\psi} + I_{A\lambda \lambda} \ddot{\delta} + I_{s\beta \lambda} \ddot{\beta} + u S_F \dot{\psi} \sin \lambda - u S_F \dot{\phi} \cos \lambda - \\ g S_A \phi + (K_\delta - g S_A \delta \sin \lambda) + C_\delta \dot{\delta} - u S_F \dot{\beta} - g S_\beta \sin \lambda \beta = T_\delta - c F_{y1} \cos \lambda \end{aligned} \quad (2.50)$$

Equation of motion for front frame twisting motion β

$$\begin{aligned} -m_s \ddot{Y}_P S_s + I_{s\beta \lambda x} \ddot{\phi} + I_{s\beta \lambda z} \ddot{\psi} + I_{s\beta \lambda \lambda} \ddot{\delta} + I_{s\beta \beta} \ddot{\beta} + u S_F \dot{\psi} \cos \lambda - \\ u S_F \dot{\phi} \sin \lambda - g S_\beta \phi + u S_F \dot{\delta} - g S_\beta \sin \lambda \delta + c_\beta \dot{\beta} + (k_\beta - g S_\beta \cos \lambda) \beta = \\ T_\delta - c F_{y1} \cos \lambda \end{aligned} \quad (2.51)$$

The intermediate variable S_β is defined as:

$$S_\beta = \frac{m_T z_T}{w} s_c - m_s S_s \quad (2.52)$$

Substituting Equation (2.43-2.45) and Equation (2.52) on equations of motions, Equations (2.47-2.51), and collecting the similar terms, the equations of motion can be represented into matrix form as:

$$M \ddot{q} + (C_1 * u + C_0) \dot{q} + (K_0 * g + K_2 * u^2 + K_1) = \quad (2.53)$$

Equation (2.53) is defined for $q_e = (\phi, \phi_r, \delta, \beta)$ and can be solved for eigenvalues and eigenmodes. A typical example of the eigenvalues and eigenmode analysis is shown in Figure 2-17.

Figure 2-17: Comparison between benchmark model and extended model with front frame flexibility
(a) real part of eigenvalues (b) imaginary part of eigenvalues.

In contrast to the high-velocity wobble instability due to tire dynamics, the wobble instability appeared at a relatively lower speed. The wobble mode eigenvalues are highly dependent on the stiffness and the damping of the front frame. The unstable wobble mode can be stabilized by selecting proper fork design and materials having good stiffness and damping properties. Furthermore, including frame flexibility also affected the weave and capsize mode. Weave mode is modified significantly at higher speed, and capsize mode is slightly modified, as shown in Figure 2-17. However, the weave critical speed remained almost the same as the benchmark model.

2.2.6 Extension 5: Inclusion of Acceleration and Braking

The acceleration and deceleration are two important aspects of vehicle driving. Accelerating and decelerating vehicle shifts the weight towards the rear and front wheel, respectively. The shifting of the weight depends on the centre of mass of the overall bicycle rider system and wheelbase. However, the benchmark model considers a constant speed of the bicycle and the eigenvalues and eigenvector are calculated for constant speed scenario. Under acceleration conditions, the speed of the vehicle varies, and the equation of motion becomes a time-varying equation [114]. A common approach to simplify this scenario is to recognize the inertial forces associated with acceleration but ignore any speed variation [87, 114-116]. A close approximation is achieved by considering the constant speed of the vehicle on the inclined road surface [114], shown

in Figure 2-18. An acceleration (a) on a flat surface is equivalent to riding a bicycle on an incline slip of $\sin^{-1}(a/g)$ degrees at a constant speed.

Figure 2-18: Eigenvalues comparison for a bicycle under deceleration and acceleration, and descending and ascending a hill [114].

The change in normal reaction force on the wheel due to acceleration (a) is given by:

$$\text{Front-wheel} \quad N_f = \frac{g * M_t X_t}{w} - \frac{a * M_t h_t}{w} \quad (2.54)$$

$$\text{Rear-wheel} \quad N_r = \frac{g * M_t (w - X_t)}{w} + \frac{a * M_t h_t}{w} \quad (2.55)$$

Where h_t is the height of the centre of mass of the combined bicycle rider system. Due to the shift of the weight when accelerating and decelerating, the moment balance equations also changes to account for the change in normal reaction force. Some additional terms appear on the Equations (2.14)-(2.16) as [114]:

- In roll direction ϕ : $-a * \frac{Sx * c \cos \lambda}{w} * \delta$
- In yaw direction ψ : $a * \phi * Sx - a \delta m a * u a$
- In steering direction δ : $-a * \phi * \mu * Sx - a \delta (m a * u a * \cos(\lambda) + \mu * Sx * \sin(\lambda))$

Substituting the additional terms in Equations (2.14-2.16) and applying the kinematic constraints, Equation (2.10-2.12), yields extended benchmark equations can be obtained. The matrix form of extended benchmarked equations, including acceleration and deceleration, is:

$$M \ddot{q} + u C_1 \dot{q} + [K_0 + a K_1 + u^2 K_2] q = f \quad (2.56)$$

Where, K_1 is the coefficient matrix representing the effect of acceleration and deceleration. The remaining coefficient matrices are the same as in the benchmark model.

Figure 2-19: Effect of acceleration and deceleration on stability.

Figure 2-19 shows the effect of acceleration and deceleration on bicycle self-stability. It was found that an acceleration improved the weave damping, however, reduced the self-stability range. On the other hand, the deceleration increased the self-stable range. Nonetheless, the self-stability shifted towards the higher velocity. Furthermore, the weave mode damping also reduces with the deceleration. Therefore, neither accelerating nor decelerating seems to improve the self-stability of the vehicle compared to the benchmarked model.

The section presents various extensions to the benchmarked bicycle model in order to improve the dynamic performance analysis. Each extension makes the analysis one step closure to the real scenario. However, it was noticed that, for low and constant velocity case, the results for different extensions, for example, rider lean extension, tire lateral force-extension, front frame flexibility and acceleration and deceleration extension are almost similar. The effect of the extension becomes more pronounced at higher velocity region. Therefore, for normal riding at low velocity, the benchmark analysis could be used to analyse the performance. However, in some special case, for example, in scooters, it is recommended to use the rider lean extension and acceleration and deceleration model because of the mass distribution and dynamics of the system change for riding in a standing and seating posture. The abovementioned models are extensively

used in this research to compare the performance of the different designs of personal mobility devices and develop the design guidelines.

2.3 Handling Aspect of the Single-Track Vehicle

The dynamic performance of single-track vehicles is often described based on their manoeuvrability, handling and stability [18]. Stability aspect of the vehicle performance has already been discussed in the previous Section 2.2. Manoeuvrability is defined as the performance measurement of the vehicle related to its ability to do time-dependent manoeuvres. Similarly, handling is defined as the ability of the vehicle to perform complex assigned tasks in relation to the effort required for different vehicles to perform those specific tasks. Correlations between the handling perception and control effort suggested that less effort is related to the better handling of the vehicle [117]. According to the Foale [118], apart from overall geometry and weight distribution, rider's physical and mental response is directly related to handlings or manoeuvrability of the system, thus making ergonomic an important factor. Manoeuvrability and handling aspects of the vehicles describe the ability of vehicles and riders to execute different manoeuvres and task and comparing how difficult or easy a vehicle is for the rider to perform those tasks. Although handling assessment includes a strong subjective involvement, it is often characterised by the vehicle response to some control actions, which in turn is closely related to the dynamic of uncontrolled vehicles [119]. Therefore, handling is also related to the stability and control characteristics of the vehicle in a complex way. The geometrical parameters and rider's position play a vital role in the handling of the single-track vehicles. Furthermore, better handling vehicles are considered good as they assist riders to make quick manoeuvring, better steering and control. The handling analysis is very matured in aircraft research; however, research on handling qualities and assessment is very limited in motorcycles and bicycles. However, there is no scientific research on handling aspect comparison for modern personal mobility devices with different wheel sizes.

This section reviews the handling aspects of personal mobility vehicles, especially bicycles and e-scooters. The section is subdivided into three sub-sections. The first section reviews the general understanding of the effects of different bicycles and e-scooters design parameters on handling from the subjective aspect of different researchers, elite riders and frame manufacturers. The second section reviews different mathematical approach to analyse the handling characteristics of the single-track

vehicles. Finally, the last section reviews different experimental approach adopted by researchers to measure the handling of the vehicles.

2.3.1 Handling from the Subjective Aspect

Figure 2-20: Geometrical parameters that influence the handling characteristic of bicycles

The geometry serves to alter the handling of the vehicles [120]. An easy vehicle elevates the rider's enjoyment by providing easy control. A typical annotation of a bicycle with geometrical parameters that influence the handling quality of the bicycles is shown in Figure 2-20. The influence of different design parameters is discussed here.

A. Head tube angle

Head tube angle dictates how much effort is required to perform certain manoeuvres. In general, when the head tube angle is steeper, the vehicle becomes easier to turn and make it possible to execute sharp turns [120]. The manoeuvrability of the bike generally increases with the increase in head tube angle. However, a very steep head tube angle also have a tendency to be oversteered and are prone to a sudden change in direction when encountered with small obstacles [121].

B. Trail

A bicycle is said to have positive trail if the point of the steering projection on the ground is ahead of the front wheel ground contact point. The positive trail acts as a caster and helps to maintain the front frame centred. It measures the extent to which the front wheel of the vehicle follows the steering axis, like the caster of the shopping cart [120]. On the other hand, the positive trail also makes it possible to auto-steer in the direction of lean for self-balancing. For each lean angle, there is a corresponding equilibrium steering turn angle which is a function of trail distance [122]. It is believed that a shorter trail

makes steering more sensitive and lighter and less stable and twitchier. On the other hand, the longer trail increases the tendency of the front frame to remain centred make steering sluggish.

C. Stem length and handlebar width

The stem length and handlebar width determines the size of the steering arc and change the steering response of the vehicles [121]. Shorter stem length and handlebar width make small steering arc and vice versa. The smaller arc makes the handlebar more sensitive and responsive, whereas greater arc makes steering more sluggish and less responsive. Furthermore, short stem provides a relaxed posture and weight of the overall system is pushed back, making steering assembly lighter. On the other hand, shorter handlebar improves manoeuvrability, but sometimes it also gives rise to twitchiness when the shorter handlebar is used in combination with shorter stem length. It is believed that wider handlebar helps to stabilize the rider and generally handlebar width are selected based on the rider's shoulder width.

D. Bottom bracket height

The bottom bracket height does not influence the steering of the vehicle directly. However, it determines the position of the rider and affects the centre of gravity of the overall vehicle-rider system [121]. Generally, changing the bottom bracket height alters the centre of mass of bicycle rider system, and stability is also affected. [123]. Since the crank arm is attached to the bottom bracket, the stretching and bending of the rider's leg largely depend on the position of the bottom bracket. On the other hand, a lower bottom bracket makes the vehicle more agile while turning [124]

E. Seat tube angle and seat tube length

Seat tube angle along with seat tube length is a crucial parameter that determines the position of the saddle and governs the ergonomics. For normal road bicycle, seat tube angle between 72-76 degrees are considered most effective for optimal pedalling performance [125] and seat tube length is determined based on the rider's anthropomorphic measurement, especially the inseam length. Furthermore, it has been found that making the seat tube angle more steep results in kinematic changes at ankle and hip decreasing the cardiorespiratory demand thus increasing the power efficiency when utilizing the aero bars [126].

F. Stack and reach

Reach and the stack are the horizontal and vertical distance measured from the bottom bracket to the top of the head tube, respectively. These distances are used as a standard to compare the size of the different bicycle. The selection of reach or stack depends on the rider's anthropomorphic parameters. The reach and the stacked are correlated with the bicycle fit by the stack to reach (STR) ration [127]. Generally, the ratio less than 1.35 represents a very stretched out riding position, whereas the ratio greater than 1.5 represents an upright position. The range between 1.4 and 1.5 is aimed by many frame builders [128].

G. Chainstay length

Chainstay length is the distance between rear axle to the bottom bracket (BB). The longer chainstay length provides better traction on the road and also provide better quality ride by absorbing the vibrations caused by the terrain [121, 129, 130]. However, longer chainstay length is not desirable for cornering and avoiding the obstacles. A shorter chainstay length is considered easier to avoid the obstacles as it brings the rear wheel closer to the bike's centre and manoeuvrability becomes easier. Furthermore, a shorter stay length is also good for immediate power delivery to the rear wheel [131]. The chainstay length is also correlated to the wheelbase. Usually, longer chainstay length yields longer wheelbase and vice versa.

H. Wheelbase and weight distribution

Wheelbase largely influence the cornering ability of the vehicles. The curvature of the path when steered is also proportional to wheelbase [56]. Therefore, a shorter wheelbase is required for quick and easy turn, whereas, longer wheelbase is believed to stabilise the vehicle [131] [47]. The wheelbase also influences the weight distribution on wheels. Furthermore, the shifting of the weight during braking and accelerating is a strong function of the wheelbase. Longer wheelbase decreases the amount of weight shift and reduces the effect of braking and accelerating [124].

On the other hand, designers should be more careful while positioning the rider and selecting an appropriate weight distribution ratio. As a rule of thumb, it is usually suggested that a 45/55 (front/rear) split ratio is ideal for a road bike, but this will be subject to the intrinsic steering response of the bike [132] [133].

2.3.2 Handling Analysis from Mathematical Approach

Handling is an important but very complex aspect of the vehicle design. It includes a strong subjective perception of the rider. Previous section 2.2.1.1 presented a brief review on qualitative analysis of different parameters and their effect on handling from the subjective perspective of riders and frame designers. However, quantitative approaches are necessary for performance comparison between the different vehicles. Handling is usually associated with the vehicle's response to the control action or sudden disturbances. Furthermore, handling is also viewed in conjunction with the dynamics of uncontrolled vehicles [119]. Many researchers have been working on defining the handling aspects of the rider vehicle system using different approaches. Most of the approaches take account of the control torque required to perform certain manoeuvres. This chapter reviews some of the well-practised methods to mathematically quantify the handling performance of the single-track vehicles.

2.3.2.1 Steady-State Turning

The steady-state turning test is one of the simplest, efficient and extensively used tools to access the handling of the single-track vehicles. Fu [134] developed and tested a steady-state model for motorcycles by measuring steering angle and lean angle. Cossalter et al. [135-137] used non-linear equations and analysed the steady-state turning of the motorcycles based on the aligning and dis-aligning steering moments. Bortoluzzi et al. [138, 139] used an instrumented motorcycle to validate the steady-state model developed by Cossalter et al. [135]. Basu-Mandal et al. [140] used the benchmarked bicycle equation to studied steady-state turning behaviour of a bicycle. Paterson and Hubbard [141] used a benchmarked bicycle model to study the relation between steering torque and velocity required for steady-state turn and highlighted the importance of steering in bicycle control, and its dependence on the design configuration. Further analysis using benchmarked study has been presented by Meijaard et al. [89]. Furthermore, Schwab et al. [119] also used benchmarked bicycle model to compare the handling performance of recumbent bicycles. Patterson [142] has developed his own design model to theoretically measure the steering torque based on lateral force, velocity and location of the centre of mass. More recently, Cain and Perkins [143] studied the steady-state turning both by using the benchmarked model as well as developing their own model [144]. It is clear from the literature review that the most adopted approach in the steady-state turning analysis is to measure the steering torque

and relate them to either lean angle or steer angle or turning radius. The steady-state turning involves a feeling of rider's related to the steering torque necessary to follow the required path with given forward velocity.

For steady-state turning, lean and steering angular rate and acceleration are zero. Assuming there is no external lean torque, the remaining terms are handlebar torque, velocity, lean angle and steer angle. The steering angle and handlebar torque follow the constant speed and the lean angle.

The benchmarked bicycle equations can be modified for steady-state turning analysis. The lean equation and steer equation for steady-state turning is defined as

Lean equation: (2.57)

$$g * K_{0\phi\phi}\phi + (v^2 * K_{2\phi\delta} + g * K_{0\phi\delta})\delta = 0$$

Steer equation: (2.58)

$$g * K_{0\phi\delta}\phi + (v^2 * K_{2\delta\delta} + (g * K_{0\delta\delta}))\delta = T_\delta$$

Furthermore, the steer angle could be replaced with turning radius as:

$$\text{turning radius (Rc)} = w/(\cos \lambda * \delta) \quad (2.59)$$

The torque or moment could be divided into two parts:

- Aligning moment: Moments that help to align the front assembly with the rear frame
- Dis-aligning moment: Moments that help to rotate the front assembly away from the rear frame

The first term on the left-hand side of Equation (2.58), represent the dis-aligning moment due to the weight of components and reaction forces. When a bicycle is leaned, it exerts a moment around the steering axis to steer the front towards the direction of lean. The second term represents the aligning moment. This aligning moment is due to yaw gyroscopic effect and centrifugal force while turning. The third term represents the dis-aligning moment due to the weight of components and reaction force when steered. The right-hand side represents the required torque to main the steady-state turning. When dis-aligning moment dominates, there is a torque around the steering that tends to increase the steering angle. Thus, the rider has to provide aligning moment to maintain a constant radius of the turn. Similarly, when aligning moment dominates, there is a

torque around the steering that tends to decrease the steering angle. Thus, the rider has to provide a dis-aligning moment to maintain the constant radius of the turn.

2.3.2.2 Riding over the Bumps (Angle of Attack)

Any obstacles such as potholes, road hump or steps try to instabilities the vehicle. Furthermore, the ability of a vehicle to roll over an obstacle depends on the radius of the wheel and the height of the obstacle. Furthermore, whenever a wheel encounters an obstacle and starts to roll over the obstacle, the rider experiences an unpleasant jerk on the handlebar, which decreases the handling quality of the vehicle.

Figure 2-21: Schematic diagram of obstacle wheel interaction. (a) the angle of attack (b) rolling over an obstacle.

One of the most important terminologies for vehicle-obstacle interaction is the angle of attack. If the angle of attack is small, the wheel can easily roll over the obstacles and vice versa. For the same obstacle height, the angle of attack increases with a decrease in the wheel size. Thus, the angle of attack is the function of wheel size (r) and the obstacle height (h). Figure 2-21 (a) depicts the angle of attack (ϵ) for a wheel with radius r . Furthermore, if the angle of attack is small, impact force also becomes small and rider experience less jerk on the handlebar. A simplified schematic diagram of a wheel of radius r rolling over an obstacle of height h is shown in Figure 2-21 (b). The analysis here is based on the quasi-steady state response. The wheel is assumed to have a constant velocity, and the tire properties and deformations have not been considered in order to simplify the analysis.

When the wheel starts to roll over, after time t , the wheel moves ΔX distance horizontally, and it moves ΔY distance vertically. When the wheel travels a distance X horizontally, it moves a distance h vertically to the top of the obstacle. The motion of the wheel centre can be obtained by simple kinematic analysis. At any time ' t ', the

vertical distance travelled by the wheel, assuming constant speed v , is given by Equation (2.60).

$$\Delta Y = h - r + \sqrt{r^2 - (X - vt)^2} \quad (2.60)$$

The derivative of Equation (2.60) gives the upward travelling velocity of the wheel while rolling over an obstacle. Again, performing the double derivative of the upward travelling velocity gives the jerking feeling at the handlebar. Furthermore, the force required to overcome the obstacle is given by:

$$F = mg \sqrt{(2rh - h^2)/(r - h)} \quad (2.61)$$

2.3.3 Handling Aspect from an Experimental Approach

As mentioned in Section 2.3, handling is also associated with the subjective perception of the riders. To examine the handling aspect of single-track vehicles, different experiments have been performed by various researchers or designers. There are usually two types of experiments that are usually conducted. The first category of experiments aims at identifying the effects and sensitivity of the design parameter variation on handling behaviour. The second category aims at comparing the performance of different designs of vehicles. This includes different riding experiments on existing vehicle models and ranking their performance based on the rider's perception. One of the major objectives these experiments is to ensure the rideability and safety of the vehicle and rider under various real scenarios.

Rice and Roland [145] experimentally investigated the effect of nine design parameters by conducting different experiments like straight-line tracking, obstacle avoidance and, slalom. Godthelp and Buist [146] also did similar experiments by changing wheelbase, trail, inertial properties of front frame and wheels. The study was further extended to high and low-speed performance evaluation of four different style bicycles. Jones [47] was also famous for his extensive experiments on unrideable bicycles in a quest to discover the factors that affect the rideability of the bicycle. Mortimer et al. [147] conducted six different tasks to examine the manoeuvrability of different handlebar configurations. The six tasks were riding on a circle, the figure "8", lane change, straight-line tracking, cornering and slalom. Papadopoulos [148] also suggested a few tests to examine the handling behaviour of bicycles. According to Papadopoulos [148], riding on a straight line and in curve with no hands-on handlebar help examine the

stability performance of bicycles. In addition, comparing the minimum speed for hands-free riding provides a qualitative measure of the stability and handling performance of bicycles. Furthermore, travelling through a given radius or curve at various speed and observing the steering torque help to analyze the effect of speed and turning radius on the direction of steering torque. Paine [149] conducted braking, stability, manoeuvrability and top speed performance related experiments on small wheel scooters and big wheel bicycle to analyze the safety concerns.

In addition to the qualitative approaches, a few handling indexes such as roll index, Koch index, roll transfer function have also been formulated especially for motorcycle handling test [56]. Similarly, Davol and Owen [150] have proposed indexes like roll authority, yaw authority and, control authority based on the simplified bicycle model. However, the accuracy of such a model is not known. On the other hand, using more advance mathematical model, Moore [83] has developed handling quality Metric (HQM) for bicycles.

2.4 Summary of Chapter 2

This chapter presented a brief history of different wheel sized single-track personal mobility devices (PMDs) Thereafter, it presented various mathematical models to analyze the performance of these singe track vehicles in terms of stability and handling. Apart from mathematical models, experimental methods that have been used to examine the stability and handling performance were also discussed. Although most of the presented models and analysis methods presented here focus on typical big wheel bicycles and motorcycles, it is also applicable to small wheel PMDs like folding bicycles and e-scooters. The presented mathematical models for self-stability and handling can be used to scientifically and quantitatively predict and compare the performance of existing personal mobility devices with different wheel size. Since there has not been any research that compares the performance of the different wheel sized PMDs in a systematic and scientific way, the presented model can be used as a for validating rider's' perception of small wheelers. However, the models require appropriate adjustments before applying to the small wheel folding bicycles and e-scooters. For example, while analyzing the extended rider lean model to personal mobility devices, the pivot point between rider and vehicle should be carefully selected to represent the actual scenario.

Chapter 3. Current Designs of the PMDs

The main objective of this chapter is to understand the current design approach of the PMDs. The available design philosophies, methodology, and guidelines have been analyzed. Furthermore, the influence of big wheel bicycle designs on small wheelers has also been discussed. The analysis also provides important information on the most common range of design parameters used in current designs of PMDs.

This Chapter is divided into 3 sub-sections. The first section briefly describes the methodology adopted to collect the necessary data. The second section presents design philosophies from renowned frame builders and available design guidelines such as empirical formulae or charts to select appropriate frame design parameters. The third section analyses the current design of PMDs with respect to the available design guidelines.

3.1 Methodology

The design philosophies and available guidelines have been obtained by reviewing various design handbooks and related websites from many frame builders and bicycle manufacturers. Most of the frame builders and manufacturers share their design philosophies to help novel designs to take their first step in frame building. In addition, many frame builders usually propose design guidelines in terms of empirical formula or design charts based on the years of manufacturing and riding experiences. The design philosophies and design guidelines have a strong influence on the selection of the frame design parameters.

In this study, three methods have been used to collect the design parameter values for different wheel sized PMDs. The first method is to contact the manufacturers or search the design parameters values via manufacturers' websites or collect information from published research articles. However, all manufacturers do not reveal details of their designs to prevent possible infringements. Nonetheless, the information about major dimensional parameters such as wheelbase, wheel size, overall length, weight, and saddle height are usually provided. Therefore, the second method makes use of the provided dimensions and CAD software to obtain the required design parameters. An image of the PMDs with major dimensions was imported in SolidWorks [151], and the photo was adjusted to match the known dimensions without distorting the aspect ratio of the image. Once the image was adjusted, the remaining design parameters can be

obtained. An example is shown in Figure 3-1. Similar approach has also been used to collect the required data for small wheel bicycles, folding bicycles and e-scooters. The third method is to measure the design parameters or available PMDs in laboratory using standard tools.

Figure 3-1: Application of CAD software to obtain design parameters of 'Turn bicycle'.

3.2 Design Philosophies and Available Design Guidelines

This section presents a few bicycle design philosophies and available design guidelines from some of the renowned frame builders.

3.2.1 Current Design Philosophies and Guidelines for Bicycles

Bicycles have a long history of more than 200 years, and the design philosophies and guidelines related to the bicycles are believed to be optimized. As a result, the philosophy and guidelines have remained almost stagnant over time. Therefore, the current design practice can also be referred to as conventional design practice. The current design of the bicycle is a result of the design philosophies of various frame designers and manufacturers with years of modifications and experiences in frame design. Some of the design philosophies from the well-known bicycle frame builders are as follow.

- **Paul Proteus** [152]:
 - “The first step in building your frame is to design one that will meet your needs and at the same time, fit your body.”
- **Richard P. Talbot** [153]:

- “Establish the geometry that will fit the cyclist’s anatomy in such a way to facilitate the use of his muscles in the most efficient manner possible for the purpose of propulsion.”
 - “Develop a design that will best fulfil the specific use to which will be put while wasting a minimum of the cyclist’s energy.”
- **Michael and Denise** [154]:
 - “Most of the costume frame builder use the rider’s body specification for the proper design of the frame.”
- **Association of British cycling coaches (ABCC)** [155]:
 - “In order to make effective use of your fitness, the riding position on your road bike must be correct. This involves the optimum setting of your three points of contact with the bike: the pedals, the saddle, and the handlebars.”
- **De long’s Guide to bicycles and bicycling** [156]
 - “You and your bicycles are a matched team. Your bicycles must be carefully fitted to your body dimensions for the most enjoyment and best results. Not only your legs, but the length of the arm, body, and feet must be considered.”
- **Rolo bicycle** [157]
 - “Basic comfort and handling need to be addressed through frame geometry rather than special components at a later stage. Once the basic geometry has been established to drive both the fit and handling, only then we can tackle the ride aspects of bike design, i.e. structural engineering challenges.”
- **Ken Luther** [158]
 - “A superbly fit cyclists want to wiring maximum advantage from his conditioning by riding a bicycle that is scaled precisely to his dimensions.”
- **Richard’s 21st Century Bicycle Book** [159]
 - “Enjoying a bike and being able to get most out of it requires a good fit: the right frame size and crank length, with correct placement of handlebars, saddle, controls, and shoe cleats if used.”

A common conclusion that can be drawn from the abovementioned design philosophies is that all the bicycle frame builders give priority to the intended purpose of the bicycle and the rider's fit. Here, the term 'rider's fit' relates the ergonomics of the riders on the bicycle. A common approach is to measure the rider's anthropomorphic parameters as an input and use sets of empirical formulae or charts or graphs to size the bicycle frame. Most of the manufacturers have their own optimized sets of empirical formulae or charts to determine the perfect fit for the riders. The identification of optimized set requires years of modifications, trials, and experimentations. If manufacturers or frame builders want a new set of design parameters, they must repeat the same procedure, which greatly increases the investments and overall manufacturing costs. Therefore, once manufacturers or frame builders find the so-called "optimised sets", they stick to the same design parameters. For example, British frame builder "Condor Cycle" prefers three different measurements: inside leg measurement, torso length and arm length for designing a customized frame. Seat tube size is adjusted by subtracting 9-inch from inseam measurement, and top tube size is adjusted using torso length. Cinelli Cino &C. used the distance from head of the femur to the floor as the principle measurement in frame design [133].

This section revisits some of the bicycle design parameters presented in Section 2.3 which affect the rider's comfort and the handling of the bicycles. However, the parameters will be discussed based on available design guidelines to understand how designers appropriately select their dimensions. Here, the parameters are divided into two categories: (i) rider's fit parameters and (ii) steering geometry parameters.

3.2.1.1 Rider's Fit Guidelines for Bicycle

Rider's fit parameters help to position a rider on the vehicle which best suits for his/her andromorphic structure. The rider's fit governs the ergonomics or the comfort in the rider-vehicle system. A best-fit vehicle not only improves the ergonomics or comfort while riding, it also elevates the enjoyment of the riding. The major design parameters that affect the rider's comfort are bottom bracket height, seat tube length and seat tube angle, saddle to handlebar length, reach and stack, chainstay length, top tube length, and weight. The selection of appropriate design values depends on the rider's anthropomorphic measurements. Figure 3-2 shows a typical example of rider body parameters. Hereafter, the design guidelines will be discussed with respect to the annotations used in Figure 3-2.

- A: Overall height
 B: Sternal Notch Height
 C: Inseam length
 D: Arm length
 E: Torso length
 F: shoulder width
 ua: Upper arm length
 la: Lower arm length
 ul: upper leg length
 ll: Lower leg length

Figure 3-2: Rider's anthropomorphic parameters related to ergonomics or fit.

3.2.1.1.1 Bottom Bracket

Bottom bracket heights are usually selected based on the crank length and intended purpose of use [153]. The intended purpose of use helps determines the ground clearance. Usually, racing bikes required more ground clearance than a road bike to allow pedalling while cornering. On the other hand, the crank length is selected based on the measurements of the rider's anthropomorphic parameters.

BikeCAD [160] relates the crank length with inseam length as:

$$\text{Crank arm length (CAL)} = 0.21 * C \text{ (mm)} \quad (3.1)$$

Bike dynamics [161] suggests the following empirical formula for selecting crank length:

$$\text{Crank arm length (CAL)} = 0.097 * A \text{ (mm)} \quad (3.2)$$

Machinehead-Software [162] relates crank length to inseam length as:

$$\text{Crank arm length (CAL)} = 1.25 * \text{inseam length in cm} + 65 \text{ (mm)} \quad (3.3)$$

The bottom bracket height is the summation of crank length and the ground clearance. Generally, the crank length varies from 162.5 mm to 180 mm. Considering the appropriate ground clearance, the bottom bracket height ranges from 9.5 inches (240 mm) to 12 inches (300 mm) [153].

3.2.1.1.2 Seat Tube Angle and Seat Tube Length

Seat tube angles and seat tube length are usually selected based on the inseam length measurement. There are also some empirical formulae to calculate the closest seat tube angle and seat tube length based on rider's body specification.

ABCC [155] recommends the following empirical formula for seat tube length and seat tube angle:

$$\text{Seat tube length} = \frac{2}{3} * C \text{ (mm)} \quad (3.4)$$

$$\text{Seat tube angle } (\alpha_{st}) = \cos^{-1}\left(\frac{0.264 * \text{thigh length}}{ll}\right) \quad (3.5)$$

BikeCAD [160] also relates seat tube length to inseam length of the rider as:

$$\text{Seat tube length} = 0.65 * C \text{ (mm)} \quad (3.6)$$

Paterek [163] formula is a bit complex to use. It relates inseam length (C), bottom bracket height (BBh), Crotch clearance (CC: 4 cm for touring bike, 6 cm for a racing bike) and seat tube angle (α_{st}) to select appropriate values of seat tube length. The relation is defined with respect to an intermediate parameter, IP as:

$$IP = C - (BBh + CC) \quad (7)$$

$$\text{Seat Tube Length (STL)} = IP [\sec (90 - \alpha_{st})] \quad (8)$$

Other important design parameters are the saddle height from the bottom bracket and saddle height off the ground. The empirical formula recommended by BikeCAD [160] for saddle height is:

$$\text{Saddle height (SH)} = 0.883 * C \quad (3.9)$$

Saddle height off the ground is the function of bottom bracket height, saddle height and seat tube angle which are related as:

$$\text{Saddle height off ground (SHG)} = BBh + SH * \sin (\alpha_{st}) \quad (3.10)$$

Apart from empirical formulae, frame builders have also developed design charts or tables to help select appropriate design dimension [153, 164]. Like empirical formulae, these design charts are also based on rider's anthropomorphic parameters.

3.2.1.1.3 Reach and Stack

Reach and stack determines the posture of the rider. The ratio between stack and reach is an important metric to most of the designers. Ideally, the ratio of stack to reach in bike should be constant to serve all riders. However, most of the manufacturers were found inconsistent in their stack to reach ratio; nonetheless, it was also found that most of the manufacturers have stack to reach ratio ranging from 1.4 to 1.5 [157].

3.2.1.1.4 Saddle to Handlebar Length

Saddle to handlebar length depends on top tube length and stem length. The saddle to handlebar length sometimes is viewed as the rider's compartment. There are two approaches to select set the saddle to handlebar length. The first approach is to select the top tube length and stem length. The combination of top tube length and stem length gives saddle to handlebar length. The second approach is to use rider's measurements directly to set saddle to handlebar length. In some cases, the appropriate stem length can also be selected by combining both the first and second approaches.

A few available empirical formulae for selecting saddle to handlebar length are presented here.

Paterek [163] recommends two formulae for a saddle to handlebar length as:

$$\text{Saddle to handlebar length (S - H)} = la + \text{comfort range (0 - 25 mm)} + 150 \text{ mm (in mm)} \quad (3.11)$$

$$\text{Saddle to handlebar length (S - H)} = (ua + la + E)/2.3 \text{ (in mm)} \quad (3.12)$$

BikeCAD [160] empirical formula also includes a rider's inseam length. The relationship is defined as:

$$\text{Rider's compartment} = 0.523 * (B - C + D) \quad (3.13)$$

As mentioned earlier in this section, saddle to handlebar length is a function of top tube length. Top tube length is generally selected based on the torso and arm length of the rider. One recommended empirical formula to select top tube length is [165]:

$$\text{Top tube length (TTL)} = 0.5 * (E + D) - 6 \text{ (in inches)} \quad (3.14)$$

Paterek [163] also recommends top tube length based on the summation of the torso and arm length. Chain reaction cycles [166] uses total height and inseam length of a rider to choose appropriate top tube length for a bicycle.

Table 3-1: Bicycle rider's fit parameter values for an adult rider from different recommended guidelines.

Crank arm length (mm)	Seat tube length (mm)	Seat tube angle (degrees)	Saddle to handlebar reach (mm)	Top tube length (mm)
170[160], 70 [161]	546[167],533 [160]	74 [167], 74 [163]	570 [163], 570 [163]	555 [160]
167.5 [162]	533[163],540 [164]	74 [160]	675 [160]	562 [153]

A typical illustration of the abovementioned guidelines for a 175 cm tall rider is given in Table 3-1. Although the approach and selection of input rider's parameters for rider's

fit may differ, strong agreements can be observed on their outcomes. As a result, parameters like the head tube length, seat-tube angle, trail, rake, chainstay length, bottom bracket height, etc., are maintained within a close range. These ranges are considered good for ergonomics, stability and handling, and all frame builders have been bound within this range. These ranges are still considered in today's design practice.

3.2.1.2 Front Steering Geometry Guidelines for Bicycle

Unlike the rider's fit parameters selection guidelines, the steering geometry is poorly understood [157]. Furthermore, bicycle stability, balancing and handling are strongly correlated with the steering geometry in a very complex way. The parameters for front steering geometry such as head tube angle and trail are selected mostly based on the experiences or trial and error methods. The general perception is that the shallower head tube angle and the longer trail [167] makes bicycle stable in the straight ride [168] but at the same time, less responsive to handlebar control. On the other hand, a steeper head tube angle and shorter trail give bicycles a nervous feeling. As a rule of thumb, the head tube angle is maintained in the range of 70-73 degrees and trail distance may vary from 50 mm to 65 mm. Most of the frame builder suggest trail about 56 mm to be an ideal value better stability and agility performance [132, 169-171]. However, there are a few guidelines for designing the front frame geometry and are discussed follow:

3.2.1.2.1 ISO Requirement for Bicycle Safety

The standardization of the bicycle "*Cycles — Safety requirements for bicycles*" has been prepared by Technical Committee ISO/TC 149 [172]. The aim is to make bicycles as safe as possible and ensure the stability of the bicycles. Part2: Requirements for city and trekking, young adult, mountain and racing bicycles (BS EN ISO 4210-2:2015), recommended the following design criteria for bicycle's front steering geometries:

- a) the steering head angle be not more than 75-degrees and not less than 65 degrees in relation to the ground line, and
- b) the steering axis intersects a line perpendicular to the ground line, drawn through the wheel centre, at a point not lower than 15 % and not higher than 60 % of the wheel radius when measured from the ground line.

Figure 3-3: ISO recommended front steering design criteria.

A simple geometrical relation can be derived for fork offset (f) and trail (c) in terms of wheel radius (R) and head tube angle (λ) from Figure 19.

$$\text{Fork offset } (f) = R * (1 - K) * \cos (\lambda) \quad (3.15)$$

$$\text{Trail } (c) = R * K * \tan (\sigma) \quad (3.16)$$

where ‘d’ indicates the distance measured from the ground line to a point at which the steering axis intersects the vertical line drawn from the wheel centre. The distance is measured in terms of wheel radius (R) in such a way that maximum and minimum allowed distance at which steering axis could intersect the vertical line from wheel centre are $0.6R$ and $0.15R$ respectively (i.e. $K=0.15$ to 0.6). The trail is longer if the point of intersection is at its highest point and vice versa. It should be noted that the ISO recommendation for bicycle safety does not allow negative fork-offset in the bicycle. This might be the reason for backward bending of front forks in many road bicycles or shifting of the front fork crown from headtube in many mountain bikes. Interestingly, this ISO recommendation has remained the same for more than 30 years.

3.2.1.2.2 Davison’s Formula

Davison’s formula suggests the same trail and fork rake value for the bicycle. For each head tube angle, the recommended fork offset and the trail by Davison are given by [53].

$$\text{Fork Offset } (f) = R \tan(\sigma/2) \quad (3.17)$$

$$\text{Trail } (c) = (R \sin \sigma - R \tan \sigma / 2) / \cos \sigma \quad (3.18)$$

3.2.1.2.3 Jones Stability Criteria

David Jones is very famous for his work on unrideable bicycles. He modified bicycles and carried out various experiments to understand the dynamics of the bicycle. He

successfully demonstrated that gyroscopic effect is not required for hands-free riding and highlighted the importance of front steering trail. Using his mathematical model, he computed the lines of constant stability ($d^2H/d\alpha dl$) relating the head tube angle and front projection [47]. A relation was derived for front fork (f) relating the wheel size, head tube angle and stability parameters from Figure 5 presented in his Jones paper as [53]:

$$\text{Front fork } (f) = R \cos \sigma (0.36(SP) + 0.183\sigma) \quad (3.19)$$

Where SP is the Jones stability parameter value, and the value is negative for a stable bicycle. According to Jones, the stability parameter value for a normal and racing bicycle is about -5. Jones stability criterion was one of the widely recommended guidelines for front steering design of bicycles. However, the criterion has recently been proven to be incomplete [16].

3.2.1.2.4 Schwinn/ Calspan Stability Index

Calspan Corporation prepared reports on the simplified dynamic stability of a bicycle for Schwinn Bicycle corporation in 1974 [173] and 1976 [174]. The report identified a specific value of the velocity at which steering torque control requirement is theoretically zero. At this velocity, the gyroscopic moment is said to be just balanced by the torque from steering geometry effect. This velocity was called the “Inversion velocity (V_I)”. The bicycle trail could be calculated as [53]:

$$\text{Trail } (c) = (V_I^2 * i_F * \cos \sigma) / (Z_F * l * R) \quad (3.20)$$

Where Z_F , l , R , and i_F are the vertical force on the front wheel, wheelbase, wheel radius and moment of inertia of front-wheel about its spin axis, respectively. A typical value of inversion velocity for a standard Schwinn suburban model was reported to be 17.5 mph.

3.2.1.2.5 Patterson Control Model

Bill Patterson [142] in the book “Lords of the Chainring” proposed a bicycle model, also called the Patterson control model. According to the Patterson control model, the minimum trail can be calculated as:

$$\text{Minimum trail } (c_{min}) = K5 * (B/M)(1/h^2 + 1/K_x^2) \quad (3.21)$$

Where, variables $K5$, B , h , M , and K_x are the steering feel constant, the horizontal position of the centre of gravity measured from the rear axle, the height of the centre of

the gravity from the ground, the mass of bicycle and rider and, the radius of gyration through the centre of gravity respectively. The recommended value of steering feels constant K_5 is 1.2 kg m^2 for light steering and 2.4 kg m^2 for heavier steering feel. The radius of gyration for a rider in Dimond frame was estimated about 0.36 m. However, it should be noted that the Patterson formula is mostly applied in recumbent bicycles.

3.2.2 Current Design Philosophies and Guidelines for E-scooters

Being the recent descendant of the safety bicycles, it is difficult to find the frame design philosophies specifically for e-scooters. However, it would be reasonable to assume that the available guidelines and bicycle design approaches have influenced frame design of the e-scooters. One of the supporting pieces of evidence is the backward inclined steering axis in most e-scooters for producing a caster effect. In recent years, e-scooters are becoming very popular as personal mobility devices (PMDs). However, there is no literature related to the frame design of the small wheel e-scooters. Most of the scooter manufacturer's focus is on battery, its efficiency and overall foldability of the device. So far, no guidelines have been found for the scooter frame design.

Here, designs of different e-scooters on the market have been analysed to understand their frame design philosophies and priorities. The e-scooters are designed for short commuting. Therefore, the design philosophies of e-scooters include portability, compactness, foldability and weight as the major design aspects. It is evident because most of the e-scooters designs are foldable in nature and their overall length is relatively shorter than the typical bicycles. The frame designers seem to focus on different methods of folding using innovative hinges to facilitate easy and quick folding and improve portability of the e-scooters. Furthermore, the weight of the e-scooters depends majorly on the weight of the hub-motor wheels and batteries.

Other design parameters such as ground clearance, deck length, deck width, saddle positions, saddle height and handlebar height are found to be the priorities for frame designers. These design parameters are related to the rider's fit like in bicycle designs. On the other hand, the head tube or steering angle and trail distances are major contributors to the steering geometry and handling of the vehicle. Owing to the lack of available design guidelines, the only available option to understand the current design of e-scooters is to analyse existing e-scooters on the market and collect data by measuring their relevant dimensions.

3.3 Available Design Guidelines Vs Current Design Practice

Section 3.2 described the design philosophies and guidelines for bicycles and e-scooters. This section compares the current design of various PMDs with available design guidelines. The main objective of this section is to understand the relationship between the current design of PMDs and available design guideline. This section is further divided into two sub-sections. The first section analyzes the current design of big wheel bicycles and compares them with the available design guidelines. A similar analysis is also done for small wheel bicycles having the wheel size of 20-inch and 16-inch in diameter. This section also compares current designs of big wheel bicycles and small wheel bicycles to analyze the influence of big wheel bicycles on small wheel bicycles. Lastly, the second section presents a similar analysis for e-scooters. Current e-scooter designs were analysed to understand the most commonly used range of design parameters.

3.3.1 Current Designs of Big Wheel Bicycles

Big wheel bicycles refer to bicycles having wheel size more than 20-inches. The most common range of wheel diameters in big wheel bicycles is in between 26-inches to 29-inches. A total number of 111 big wheel bicycle designs have been analyzed. Among 111 bicycle data, 91 bicycle data were retrieved from the PhD thesis of J. Prince [175] which included 30 Tour de France 2013. A remaining 20 bicycle data were from various online sources. Rider's fit parameters and front steering geometry parameters were obtained from collected bicycles.

The rider's fit design parameters such as top tube length handlebar reach, reach and stack, seat tube length, and saddle length are a direct function of rider's anthropomorphic measurements. On the other hand, parameters like bottom bracket height, seat tube angles are less dependent on the rider's anthropomorphic measurement. For example, the bottom bracket height not only depends on crank length but also on ground clearance. And the ground clearance for a bicycle depends on the intended purpose. Similarly, the rider's position on the saddle can be fitted by selecting the seat tube angle and adjusting the seat tube length and saddle length. Apart from the rider's fit design parameters, head tube angles and trail values were also collected to study the front geometry design of the big wheel bicycles.

3.3.1.1 Bottom Bracket Height Analysis

Figure 3-4: Bottom bracket height and crank arm length for bicycles.

Figure 3-4 shows a general trend for the big wheel bicycle bottom bracket heights and crank lengths. The frame sizes; small, mediums and large are categorized based on the rider's body measurements. A total number of 20 big wheel bicycles from 8 different brands have been analysed to collect the data for bottom bracket height and crank arm length. Designers use shorter crank arm length for small size and longer crank arm length for large size frame. The most common range of crank length was found to be in between 170 mm and 175 mm. However, some bicycles models also found to have 165 mm crank arm length. On the other hand, the bottom bracket height was found to be in between 260 mm to 285 mm. Nonetheless, the most common range of bottom bracket height was from 260 mm to 275 mm. The selection of the bottom bracket height depends on the purpose of the bike. For example, mountain bikes have greater bottom bracket as they are ridden on uneven terrain than a normal road bike or racing bike.

3.3.1.2 Seat Tube Angle Analysis

Seat tube angle data from a total number of 91 big wheel bicycle retrieved from J. Prince [175] was analyzed. Figure 3-5 shows the distribution of the seat tube angles for 91 different designs of big wheel bicycles. It was found that the seat tube angle ranges from 70-degrees to 75-degrees. However, almost 91 % of the seat tube angle was found to be in between 72-75 degrees. Furthermore, seat tube angles in between 73-74 degrees were found to be the prominent choice for the designers as this range covers almost 56 % of the total sample data. Furthermore, the mean and mode of the sample data are found to be 72.86 and 73.5, respectively.

Figure 3-5: Seat tube angle distribution for bicycles.

3.3.1.3 Front Steering Design Analysis

The head tube angle and trail collectively represent the steering geometry. This section analyses the head tube and trail of the current design of big wheel bicycles. To understand the current design of the bicycles and available design guidelines, front steering geometries, head tube angle and trail, of 91 big wheel bicycles from different brands including 30 Tour de France bicycles were analysed. Thus, the selected bicycle sample represented a wide range of current big wheel bicycle designs. The data were retrieved from the PhD thesis of J. Prince [175].

Figure 3-6 shows the graphical representation of the available guidelines, as presented in Section 3.2.1.2 along with the 91 different big wheel bicycle's data. The ISO recommendation for the bicycle safety provided a broad range of permissible bicycle head tube and trail. The shallower head tube angles are associated with greater trail range as compared to the steeper head tube angles. The trail range shrunk, and the permissible trail value also reduced with the steeper head tube angle. For example, the recommended trail value for a 70-degree head tube angle ranged from 19 mm to 73 mm. Likewise, for 74-degree, it ranged from 15 mm to 61 mm. Therefore, shallower head tube angles were likely to be used with longer trail and vice versa. The Davison's recommended trail values were closer to the maximum trail values as recommended by ISO requirements. Unlike the other design front geometry design guidelines, Jones' stability criteria, Schwinn/Calspan criteria and Patterson model are based on the mathematical model giving due consideration to the dynamics of the bicycle. These criteria suggested approximately constant trail values.

Figure 3-6: Comparison of available front frame design guidelines and current bicycle design.

The Jones' criteria and Schwinn/Calspan criteria for big wheel bicycles were close to each other and resided within the ISO recommended criteria. The recommended values of the trail would change in accordance with the stability parameter index (SP) in Jones' criteria and inversion speed V_I in Schwinn/Calspan criteria. Similarly, for constant weight distribution and handling feel constant, the Patterson formula yielded a constant trail value. The trail value from the Patterson formula is much lower than the trail value from other guidelines. The reason could be that the Patterson formula is mainly applicable to recumbent bicycles. The mass distribution and structural design of these bicycles are very different from normal bicycles.

All the big wheel bicycles were well within the ISO recommended criteria. However, bicycle front design parameters were closer to the ISO upper limit, i.e. maximum allowable trail for a particular head tube angle. This supports the argument on current bicycle design practice that the longer trail makes the bicycle more stable. However, it was found that very few bicycles are designed with a shallower head tube angle and much longer trail. This is because cyclists prefer stability as well as good handling. It was already an accepted fact that the shallower head tube angle and the longer trail make the bicycle a bit sluggish and reduce the agility which is not desired by many cyclists [47, 176]. In fact, all the above-mentioned front steering guidelines are found to corroborate with current bicycle design practice for a typical range of head tube angle 71-74 degrees and trail about 55 mm to 65 mm.

The bicycle front geometries were further analysed for a better insight into current design practices as well as to identify the most common design parameters. Table 3-2

presents a detailed breakdown of 91 big wheel bicycles' data. It was found that very few bicycles are designed with a shallower head tube angle and much longer trail. This is because cyclists prefer stability as well as good handling. It was already an accepted fact that shallower head tube angle and the longer trail make the bicycle a bit sluggish which is not desired by many cyclists. Many elite cyclists and frame builders recommend 73-degree head tube angle and 56 mm trail for an ideal ride which is more evident from the mean and model values of the sample from Table 3-2.

Table 3-2: Head tube angle and trail values for 91 samples big wheel bicycle.

		Trail range (mm)						
		40-45	45-50	50-55	55-60	60-65	65-70	70-75
Head tube angle (degrees)	<70	0	0	0	0	0	0	0
	70-71	0	0	0	0	1	0	0
	71-72	0	0	0	4	2	5	1
	72-73	0	0	0	9	10	0	0
	73-74	2	1	15	32	0	0	0
	74-75	0	3	6	0	0	0	0
	>75	0	0	0	0	0	0	0
<i>Head tube angle (Mean): 72.8 degrees</i>					<i>Head tube angle (Mode): 73.0 degrees</i>			
<i>Trail (mean): 56.5 mm</i>					<i>Trail (mode): 58 mm</i>			

The breakdown of the collected bicycle data revealed the following:

- The head tube angle ranged from 71-degrees to less than 74-degrees included almost 90% of the bicycles.
- The trail ranged from 50 mm to 65 mm included almost 86 % of the bicycles.
- The head tube angle in the range of 73-74 degrees included almost 55 % of the bicycle designs.
- The trail ranges from 50 mm to 60 mm covered almost 72 % of the bicycle designs.
- The 73-degree head tube angle and trail range of 50 mm to 60 mm included almost 51 % of the bicycle designs. Thus, this head tube angle and trail could be considered as dominating frame design geometry.
- The most repeated head tube angles are about 73-degrees, and trail values are 56-58 mm.

It is clear from the analysis that the majority of the current bicycle designs uses 73-degrees head tube angle and 56-58 mm trail as their design reference frame geometry. The reason could be years of modification, test rides and experience that settled the front geometry design to an optimal head tube angle about 73-degree and 56-58 mm trail for

good stability and handling performance [177]. Therefore, it was reasonable to use big wheel bicycle with a 73-degree head tube angle and 56 mm trail as a reference frame design in the analysis. The performance of this big wheel bicycle will be compared with the performance of the other small wheel bicycle designs.

3.3.2 Current Designs of Small Wheel Bicycles

This section examines how the above-mentioned guidelines are changed for reduced wheel size. Furthermore, this section also highlights the influence of big wheel bicycle design guidelines on small wheel bicycle designs when the wheel size is reduced.

Figure 3-7: Comparison of overall design between small wheel bicycle and big wheel bicycle.

When a big wheel bicycle is replaced with a small wheel, the overall height of the bicycle is lowered. In other words, the bottom bracket height, saddle height and the handlebar height are reduced. This significantly distorts the ergonomics of the rider. Ergonomics in bicycle design is an important aspect every designer seeks to maintain. Therefore, both the big wheel bicycles and small wheel bicycles are designed with similar ergonomic parameters in order to provide the same level of comfort. Figure 3-7 shows a common big wheel bicycle and 20-inch Moulton bicycle design. It can be noticed that the position of the saddle, bottom bracket height, handlebar height and saddle to handlebar reach of 20-inch Moulton bicycle are almost identical to that of the big wheel bicycle. The ergonomic parameters are often treated as the constraints in bicycle design.

On the other hand, the reduction in the wheel size results in the front wheel having (i) smaller rotational inertia, (ii) smaller gyroscopic effect and (iii) smaller front fork trail. The effect of smaller rotational inertia and gyroscopic effects were known to reduce the stability of the bicycle [56, 82, 97, 178]. Trying to increase these factors leads to an

increase in wheel mass. As a result, the overall bicycle becomes heavy. As per the conventional design practice, it appears that only the trail could practically be changed in order to improve the stability. The trail distance is a function of wheel radius, head tube angle and fork offset or rake. The trail reduces drastically with the reduction in the wheel size. For example, a big wheel bicycle with 26-inch wheel size, 73-degree head tube angle and 45 mm fork offset has 56 mm trail. If the 26-inch wheel is replaced by the 20-inch wheel on the same frame, the trail reduces from 56 mm to 27 mm. Similarly, if the 26-inch wheel is replaced with a 16-inch wheel, the trail reduces to 16 mm. To achieve a longer front fork trail, following options are available: (i) head tube angle could be shallower, (ii) front fork offset or rake could be reduced and (iii) a combination of (i) and (ii) as shown in Figure 3-8.

Figure 3-8: Variation of trail with wheel size, head tube angle and fork offset.

Figure 3-9 shows the front steering design parameters, trail and head tube angle, of twenty 20-inch wheel bicycles and seven 16-inch wheel bicycles from different brands and models. Here, Patterson's formula is not included as it is mainly recommended for recumbent bicycles rather than the normal bicycle. The models include some of the well-established small wheel brands like Brompton, Bike Friday, Dawes etc. The head tube angle and trail values for most of the bicycles were verified from the manufacturers. However, for a few bicycle models, the values were obtained by adjusting the photograph of the bicycle frames in CAD software as explained in Section 3.1. The information about the steering design of small wheel bicycles was helpful to know the range of front design parameters used in small wheel bicycle and to understand how the front geometry selection is influenced by the big wheel design practice and the guidelines.

(a)

(b)

Figure 3-9: Comparison of available front wheel assembly design guidelines translated for small wheeler and current small wheel bicycle designs in the market: (a) 20-inch wheel bicycle, (b) 16-inch wheel bicycle.

It was very difficult to understand to what extent the manufacturers use those guidelines for small wheel bicycle design from Figure 3-9. However, it was evident that most of the manufacturers prefer the combination of reducing head tube angle and fork offset to achieve the desired trail length. The reason for preferring the combination of head tube angle and fork offset to achieve the desired trail could be the influence of conventional big wheel bicycle design practice. In a small wheel bicycle, if head tube angle and fork offset are changed individually to achieve the desired trail, it could result in a very shallow head tube angle or very short or negative fork offset. However, either of a very shallow head tube angle and a very short or negative fork offset is not preferred in conventional bicycle design. Therefore, the only good possibility for maintaining a desired trail length is to use a combination of the change in head tube angle and fork

offset. The details of the 27 small wheel bicycle brands and specifications are listed in Appendix A-1. The following observations were made with respect to Figure 3-9.

- Like big wheel bicycle designs, the head tube angle and trail values of most of the small wheel bicycles were close to the ISO maximum permissible trail range.
- The head tube angles of all bicycles were well within the ISO permissible range. However, the trail distance of some of the bicycles was found greater than the ISO maximum permissible range. This showed the influence of conventional bicycle design practice that suggests longer trail provides more stability in the bicycle.
- The head tube angles for most of the small wheel bicycles were less than 73 degrees.
- Most of the small wheel bicycles had head tube angles in the range of 70 to 72 degrees.
- The trail range from 35 to 55 mm includes 77 % of the sample data for small wheel bicycle designs.
- The minimum, maximum, mean and median value for the trail is 29.5, 72, 46 and 44 respectively.

Thus, reducing the wheel size affects ergonomic as well as front steering design aspects in a bicycle. Furthermore, when these design parameters are treated as design constraints, the steering geometry and overall design of the bicycle interlink with each other. For example, to increase the front fork trail, the head tube angle is reduced. As a result, the wheelbase increases. This was also reflected in Figure 3-7. Although the overall length of a small wheel bicycle is shorter than that of a big wheel bicycle, the wheelbase could be longer than that of a typical big wheel bicycle.

It is clear from the analysis that the small wheel designs are indeed influenced by the big wheel bicycle designs and are even designed with a similar approach. The trail distance is considered as the direct indicator of stability and handling and to improve the stability performance designers increase the trail distance by making head tube angle shallower and reducing the fork offset. David Hon [179], the founder of Dahon bicycle, also suggested the influence big wheel bicycles on the small wheel bicycle designs. According to David Hon [179], “the traditional bike principles should be adhered to. The construction of various types of bicycle has evolved over a century and is generally considered optimised based on ergonomics and physics. Folding bicycles and E-folding bicycles are bicycles, thus they should follow the traditional technology where

interactions with the rider and the road are concerned”. This statement suggests that small wheel bicycles are also designed following the same conventional design practice adopted for big wheel bicycles. Furthermore, the trail values recommended by Dave Moulton [122] is also around 60 mm which is very much closer to the trail values used for big wheel bicycle. Therefore, it also supports the concept of the longer trail to improve the stability as well as provide evidence for the influence of big wheel bicycles on parameter value selection for small wheel bicycles.

3.3.3 Current Designs of Small Wheel E-scooters

The most common range of design parameters in current e-scooter designs was identified by collecting 28 e-scooters from different brands and models. Among 28 e-scooters, ten e-scooters had 10-inch wheels and remaining had 8-inch wheels. These wheel sizes are the most prominent wheel size for e-scooters. Like bicycles, parameters such as overall weight, handlebar height, position of saddle and, ground clearance are related to the rider’s fit or ergonomic parameters. Whereas, the head tube angle and trail are two steering design parameters of the e-scooters that are vital for handling and stability. The rider’s fit and steering geometry parameters have been analysed based on the collected e-scooter data.

3.3.3.1 Overall Weight Analysis

The overall vehicle weight of the scooters mostly depends on the weight of the battery and motorized wheel. In general, bigger wheels are more powerful and hefty compared to a smaller wheel. The weight of the overall e-scooter with respect to the wheel size is shown in Figure 3-10. The weight of e-scooters was found to vary from 10 Kg to 30 Kg. However, the range of weight from 10 Kg to 20 Kg was found to be the more common range for e-scooter weight. E-scooters with dual hub motor wheel may weigh up to 30 Kg. However, most of the e-scooters in the current market have only one hub motor wheel.

Figure 3-10: Wheel size vs overall weight of the e-scooters.

3.3.3.2 Position of Saddle Analysis

The position of saddle top was measured from the rear wheel contact point. Among 28 e-scooters, 11 e-scooters also had the option to mount the saddle on the deck. This option allows users to ride with two different postures (i) standing posture and, (ii) seating posture. For seating posture, the location of the saddle determines the rider's position. The range of saddle position on the e-scooters is presented in Figure 3-11. The horizontal position and vertical position of the saddle are represented by X-axis and Y-axis respectively.

Figure 3-11: Location of the saddle for sample e-scooter designs.

The horizontal and vertical positions of the saddle were found to range from 70 mm to 190 mm and 550 mm to 730 mm respectively. The main reason for such dispersed data for the vertical position is due to the telescopic seat post that allows riders to adjust the vertical height of the saddle. However, the most common range for the horizontal and vertical position was found between 130 mm and 180 mm (67 %) and 680 mm and 730 mm (72 %) respectively. Therefore, 150 mm horizontal (mean:134 mm) and 700 mm

(mean: 698 mm) vertical position can be taken as the reference saddle position for e-scooters.

3.3.3.3 Steering Geometry Design Analysis

Figure 3-12: Head tube angle vs trail for e-scooters.

The front steering design parameters, head tube angle and trail, are plotted in Figure 3-12. It was clear that the scooters are designed with the much steeper head tube angle ranging from 74 degrees to 84 degrees. Similarly, the trail distance was found to range from 10 mm to 30 mm (96%). Like bicycle designs, the shallower head tube angles have a longer trail and vice versa. The most common trail range was found between 15 mm and 25 mm (65 %) for 8-inch wheel scooters and 20 mm and 30 mm (70%) for 10-inch wheel scooters. Similarly, the most common range of head tube angle was found to be in between 75-degrees and 80 degrees (77 %).

3.4 Summary of Chapter 3

This chapter has reviewed different design philosophies and guidelines for designing bicycles and e-scooters. In order to make the analysis easier to follow, the design parameters were divided into two categories: (i) rider's fit, and (ii) steering geometry. The available guidelines and current design practice for rider's fit were found to be in good agreement for bicycles. Furthermore, similar rider's fit guidelines were also found to be adopted by the small wheel bicycle designers. The concept of rider's fit was also applicable to the e-scooter designs. However, all the rider's fit parameters for bicycles may not be directly applicable in e-scooters.

Table 3-3: Design parameter range for most of the bicycle and e-scooter parameters

Bicycle Parameters	General Range	E-scooter parameters	General range
HTA	72-74 degrees (76 %)	HTA	75-80 degrees (77 %)
Trail	50-60 mm (73 %)	Trail	15-30 mm (85 %)
Stem length	60-130 mm	Handlebar height	1000-1200 mm (80 %)
Bottom bracket height	240-300 mm	Ground clearance	100-200 mm
Seat tube angle	72-75 degrees (91 %)	Saddle position (x)	70-190 mm (100 %)
Stack to reach ratio	1.4-1.5	Saddle position (y)	550-750 mm (100 %)
Chainstay length	400-465 mm		
Weight distribution	Front /rear =45/55		

The identified most common range for some of the vital design parameters are listed in Table 3-3. A good agreement was also found between the steering design of big wheel bicycles and available design guidelines. However, when the same guidelines were applied to small wheelers, there was no good agreement like the big wheel bicycles. Nonetheless, a strong influence of big wheel bicycle design practice was observed for small wheel bicycles. Furthermore, e-scooter designs have much steeper head tube angle and very short trail distance compared to bicycles. However, a lack of proper and scientific guidelines for front steering design was noticed.

Chapter 4. Performance Comparison of Current Designs of PMDs

As described in Chapter 3, a strong influence of big wheel bicycles on the current designs of small wheel PMDs is observed. Despite having similar rider's fit guidelines, design philosophies and principal design geometry, smaller wheel bicycles and e-scooters are perceived to be less stable and less rideable compared to their big wheel counterparts. However, there is virtually no published literature that scientifically investigates the reasons for this difference in the ride characteristics between the small wheelers and the big wheelers. Moore and Hubbard [178] studied the effect of wheel radius on the self-stability of a bicycle using the benchmarked equations. Nakagawa, et al. [180] studied the dynamics of bicycle focusing on small wheel bicycle. The bicycle stability was examined based on "Japan Industrial Standards Bicycle Durability Test Method" taking account of the stability time. The stability time is defined as "the time till the lateral translation of a bicycle reaches ± 0.25 m from the centre line, and roll angle reaches $\pm 20^\circ$ from an upright position. The stability time for the big wheel was found to be considerably greater than that of small wheel indicating the influence of the wheel size on bicycle dynamics. The authors have also suggested that varying the head tube angle and increasing the moment of inertia of the wheel improves the stability of the small wheel bicycle. However, to increase the moment of inertia, the mass of the wheel was increased from 0.62 kg to 2.62 kg, making the overall system even heavier. Other factors such as the trail, wheelbase, and the rider position, which might play a key role in maintaining stability with a minimal increase in wheel mass and overall bicycle were ignored. Kazuki et al. [181] compared the stability of 8-inch wheel bicycle having 23 mm negative trail using the benchmarked equations (see 2.2.1, Chapter 2). The results presented are in some way biased because the small wheel PMDs usually have a short but mostly positive trail. Furthermore, M. Paine [149] used the experimental approach with different bicycle and scooters models and conducted stability tests by riding the PMDs over the various obstacle and observing the steering behaviour. The scooters were concluded to be the least stable among the various big wheel and small wheel bicycles.

A substantial comparison among the different wheel sized PMDs including the typical big wheel bicycle, the similar-looking small wheel bicycle and the small wheel e-scooter has not been reported yet. The investigations, so far, have hinted at the lesser stability

of the small wheelers, but have not identified the reasons behind the lesser stability or any possible suggestions and guidelines to improve the current design.

In this chapter, the performance of a typical big wheel, small wheel bicycles and small wheel e-scooters are compared using the various mathematical models as described in Chapter 2. Both the self-stability and the handling performance of the PMDs have been analysed and compared. The typical designs for the current PMDs were selected and analysed within the most common range of the design parameters, as identified in Chapter 3. The overall chapter is divided into three sections. The first section describes the methodology used to measure the necessary vehicle-rider related parameter and tools to compare the performance of PMDs having different wheel size. The second section presents the stability performance comparison using the various mathematical models. Finally, the third section presents the handling performance comparison for the current designs of PMDs.

4.1 Methodology

Each mathematical model is defined using a set of vehicle design parameters and system properties. The parameters include geometrical parameters, mass related parameters and properties of the rider-vehicle systems. The accuracy of the results depends on the use of the appropriate mathematical model as well as the measurement of the required parameters. Kooijman et al. [86], Moore et al. [95], and J. Prince [175] have highlighted appropriate methods to determine the mass related parameters experimentally. This section describes the methodology adopted to obtain the necessary parameters required for different mathematical models.

4.1.1 Measurement of Geometrical Parameters

The head tube angle, trail and wheelbase are major geometrical parameters that are directly involved in mathematical models. Whereas parameters like bottom bracket height, saddle height, seat tube angle, handlebar height, wheel diameters and wheel drop out determines the rider's seating posture and affect the overall centre of mass and the mass related inertial properties.

The geometrical parameters are relatively easy to measure. The vehicles should be on the levelled ground, and the tires should be inflated to a recommended pressure. The required angles can be measured using the electronic level instruments, and the length-

related measurement can be measured using the rulers. Furthermore, the accurate measurement of the trail can be achieved by measuring the fork offset, as shown in Figure 4-1. Thereafter, the actual trail can be computed by relating the fork offset with wheel size and head tube angle.

Figure 4-1: Front fork rake or offset measurement for accurate trail calculation.

4.1.2 Mass Measurement

Mathematical models often divide the single-track vehicles into four different components: rear wheel, front wheel, rear frame and front frame. An accurate digital weighing machine can be used to measure the weight of each component.

4.1.3 Centre of Mass Location and Moment of Inertia

The mathematical models assume vertical symmetry of the vehicle-rider system. Therefore, the location of the centre of mass (COM) can be defined by two coordinates (X and Z) and inertia tensor matrices containing Y-axis are zero.

Wheels have COM located at its geometrical centre and the moment of inertia (MOI) properties can be determined using a compound pendulum and torsional pendulum experiments. However, determining the COM and the MOI for the front frame and the rear frame are relatively tedious work. One of the potential approaches to determine the COM and the MOI of the vehicle-rider system is to model the vehicle's components on CAD software and extract the mass related information. Another approach is to measure the mass related parameters experimentally. The first approach serves best for designing new vehicles and analysing their performance in an early stage before manufacturing. The latter method suits best to analyse the designs of the existing vehicles because it is difficult to model butted tubes and complex shapes of the tubes accurately.

A torsional pendulum experimental set-up was used to locate the centre of mass of the frames as well as to determine the MOI of the existing bicycle frame. The principle is:

a loosely held object always orient itself such that the line of suspension passes through its centre of mass. The experiments were repeated three times by hanging the frame at 3 different orientation, and the photographs were taken each time. The photographs were later drafted in a single frame without distorting the aspect ratio. The intersection of the three extended lines of suspension from three photographs gave the COM location of the frames, as shown in Figure 4-2. The same experimental set-up was used to determine the MOI of the frames. The frame was then securely held and perturbed slightly. The time period of oscillation was recorded for each three orientation. Those three different orientations gave three moments of inertias (I_1 , I_2 , and I_3) for the frames about the torsional axis. Furthermore, the angle of frame rotation for each orientation (α_1 , α_2 , and α_3) was also measured. The MOI about the global axis was obtained from the MOI about the torsional axis by solving Equation 4.2.

To analyse the combined vehicle-rider system, the rider body was mathematically modelled using the simple geometrical shapes. The mass-related properties of the rider from the mathematical model was combined with the mass-related properties of the vehicle's frame following the procedure explained by Moore et al. [95].

Figure 4-2: Experimental approach to determine the COM location and MOI of the frame.

$$I = K * \left(\frac{T}{2\pi} \right)^2 \quad (4.1)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \cos^2 \alpha_1 & -2 \sin \alpha_1 \cos \alpha_1 & \sin^2 \alpha_1 \\ \cos^2 \alpha_2 & -2 \sin \alpha_2 \cos \alpha_2 & \sin^2 \alpha_2 \\ \cos^2 \alpha_3 & -2 \sin \alpha_3 \cos \alpha_3 & \sin^2 \alpha_3 \end{bmatrix} \begin{bmatrix} I_{xx} \\ I_{yy} \\ I_{zz} \end{bmatrix} \quad (4.2)$$

Furthermore, the MOI of the wheel about the diametrical axis was computed from a torsional pendulum experiment using Equation 4.1, and that about wheel axle was computed from compound pendulum experiment using Equation 4.3.

$$I = \left(\frac{T}{2\pi} \right)^2 * Mg * l - M * l^2 \quad (4.3)$$

where

M = mass of the wheel

g = gravity

l = pendulum length/ wheel inner radius

4.2 Stability Performance Comparison of Current Design of PMDs

In this section, the benchmarked model has been used as a baseline mathematical model to obtain the eigenvalues and eigenmodes and, thus, compare the self-stability. However, the extended benchmarked models have also been used in accordance to represent scenario and to make the analysis more realistic. For example, rider lean extension in Section 2.2.3 should be used to analyse the standing and seating posture on the e-scooter to include the effect connection between the vehicle and rider. Furthermore, this section compares the performance of PMDs within the most common range of design parameters, as presented in Table 3-3 in Chapter 3.

4.2.1 Self-Stability Analysis: Benchmarked Model with the Rigid Rider

A rigid rider was considered at the rear frame of the vehicle. The connection between the vehicle and the rider is assumed to be perfectly rigid. This represents an ideal case where there is no feedback from the rider to the vehicle control. The model, as mentioned earlier in Section 2.2, is very suitable to analyse the inherent stability property of the vehicle. In the actual scenario, this analysis closely resembles the ability of the vehicles to be ridden hands-free. The analysis is divided into three sub-sections. The first section presents a case study for the performance comparison of big wheel bicycles with small wheel bicycles and small wheel e-scooters. The second and third sections present a more generalised analysis for the self-stability of different wheel sized PMDs.

4.2.1.1 Self-Stability Performance: A Case Study of Typical Designs of PMDs

A typical design of 26-inch wheel bicycle, 20-inch small wheel bicycle and 8-inch e-scooter have been considered to present a preliminary case study on the eigenvalues and

self-stability comparison. The design of the single-track vehicles can be quantitatively compared based on their eigenvalues. In general, the more negative is the eigenvalues, the more stable is the vehicles.

The selected ergonomic parameters for both bicycles and e-scooters are listed in Table 4-1. The ergonomics parameters for the big wheel bicycles and the small wheel bicycles are selected considering a typical rider of 183 cm height and 72 Kg weight. Here, the typical design of big wheel bicycle having 73-degree head tube angle and 56 mm trail was also referred to as the reference big wheel bicycle as this combination of steering geometry covered more than 50% of the sample designs considered in Chapter 3.

Table 4-1: Ergonomic design parameters for a typical bicycle and e-scooter design.

Bicycle design		E-scooter design	
Ergonomic parameters	Values	Ergonomic parameters	Values
Bottom bracket height	0.267 m	Ground clearance	0.17 m
Reach	0.375-0.4 m	Handlebar to ground height	1.170 m
Stack	0.55-0.59 m	Weight (e-scooter)	About 15 kg
Saddle height from the ground	0.952 m	Deck length	0.530m
Saddle to handlebar reach	0.527 m		
Handlebar to ground height	0.899 m		

Based on the current design practice, the small wheel bicycle manufacturers seek to maintain a similar trail as that of the big wheel bicycles and the same level of ergonomics. Therefore, the manufactures reduce the head tube angle and fork-offset to achieve a similar trail value. For this reason, most of the small wheel bicycles have shallower head tube angles between 71 and 72 degrees which consequently increases the wheelbase. The wheelbase of Brompton and Moulton bike can be taken as examples to a case in point. In general, Brompton and Moulton bikes have 1045 mm and 1060 mm wheelbase respectively, which are longer than the typical road bikes' wheelbase. In this analysis, the typical design of a small wheel bicycle had 71.5-degrees head tube angle and 56 mm trail. Furthermore, a typical 8-inch wheel scooter having a 75-degree head tube angle and a 30 mm trail (Tracker 2000 from [149]) was selected as a baseline model. The schematics of the typical design of 26-inch big wheel bicycles, 20-inch small wheel bicycle and 8-inch e-scooter are shown in Figure 4-3.

Figure 4-3: Typical designs of PMDs: (a) 26-inch big wheel bicycle, (b) 20-inch small wheel bicycle and, (c) 8-inch small wheel e-scooter.

A rigid rider was added to the bicycle and e-scooter models, as shown in Figure 4-3. The rider posture was modelled to best represent the hands-free riding on the bicycle and typical standing riding posture on the e-scooters. Thereafter, the mass related parameters were extracted from the combined vehicle-rider system. The detail of the design parameters for different wheel sized PMDs are presented in Appendix A-2.

Figure 4-4: Vehicle-rider models: (a) rider on a bicycle representing hands-free riding and, (b) rider on e-scooter representing standing posture.

4.2.1.1.1 Eigenvalues Analysis for the Current Design of PMDs

For a system to be stable, all the eigenvalues must have negative real parts. Figure 4-5 presents the calculated real parts of eigenvalues and associated eigenmodes with respect to the forward speed for big wheel bicycle, small wheel bicycle and e-scooter. The weave mode has conjugate imaginary eigenvalues in the form of $a + ib$. Where a is the real part and ib is the imaginary part. Thus, it represents the oscillating behaviour of the bicycle dominated by the steering angle. Initially, the real parts of the eigenvalues for the weave mode are positive. This means, when a bicycle is perturbed laterally, it will oscillate, and the oscillation increases continuously with time moving the system away from the equilibrium. Thus, a bicycle is not self-stable at low speed and requires the rider's continuous control. The weave eigenvalues gradually become less positive as forward speed increases and cross the zero reference line to become negative. The speed at which the weave eigenvalue crosses the zero line is called the 'weave critical speed'. Similarly, the real parts of eigenvalues for the capsize mode are initially negative. Moreover, with an increase in forward speed, it crosses the zero reference line to become positive. The speed at which the real part of the capsize eigenvalue crosses the zero line is called the 'capsize critical speed'. The self-stable speed region is bounded in between these two critical speeds. For the typical big wheel bicycle, the self-stable region was computed to be between the weave critical speed of 4.71 m/s and the capsize critical speed of 7.63 m/s as indicated by the blue arrowhead.

Figure 4-5: Eigenvalues and the self-stable velocity range for different wheel sized PMDs.

The overall self-stable region of the small wheel bicycle shifted towards the right. The changed wheel diameter, associated wheel MOI, head angle and the wheelbase are responsible for this rightward shifting of the self-stable region. The small wheel bicycle showed self-stability from 5.41 m/s to 9.36 m/s as indicated by the black arrowhead. Similarly, the electric scooter was self-stable from 6.21 m/s to 16.53 m/s as indicated by red arrowhead. Although the small wheeler exhibited a broad self-stability region as compared to the big wheel bicycle, the broad region appeared at a higher speed. Thus, a rider on small wheeler should gain relatively higher speed as compared to the big wheel bicycle to enter the self-stable region. This region further shifts towards the higher speed with the reduction in wheel mass, diameter, and associated MOI. This result provides one of the compelling evidence for perceiving small wheel bicycle and scooter less stable than their big wheel at normal riding speed. This is because, during the normal riding speed less than 20 km/h, small wheel PMDs are in an unstable mode and, thus, required higher degrees of manual control to be ridden. Furthermore, the weave eigenvalues for small wheel bicycle and e-scooters were more positive at lower speed up to 6 m/s compared to the weave eigenvalues for big wheel bicycle. The higher degree of positivity in the eigenvalues also suggests that the small wheel bicycle and e-scooter design are more unstable at low speed compared to the big wheel bicycles.

In addition to the real part of the eigenvalue analysis, the imaginary part of the eigenvalues associated with the weave modes is also presented in Figure 4-6. The imaginary part of the eigenvalues is associated with the frequency of oscillation of the

system. For typical big wheel and small wheel bicycle design, the imaginary parts of eigenvalues were almost similar for the low-speed region up to 5.5 m/s. However, for e-scooter, the imaginary eigenvalues were higher. This indicates that the small wheel e-scooter oscillates with a higher frequency than that of small wheel bicycles and big wheel bicycles. The higher frequency of oscillation and more positive eigenvalues could attribute to twitchier or nervous rider perception on e-scooter.

Figure 4-6: A comparison between the imaginary parts of eigenvalues for different wheel sized PMDs.

4.2.1.1.2 Lateral Perturbation Test

As discussed in Section 3.1.1, one of the explanations for the difficulties associated with the small wheel bicycle is the requirement of relatively higher speed for the self-stability. To get further insight into this problem, a simulation, based on the linearized equation of motion, Equation (2.17), was modelled to perform lateral perturbation test. An impulse lean torque of magnitude 50 Nm was applied to replicate a sudden perturbation in the vehicle-rider system at time $t = 1$ sec. Then, the leaning and steering response at different velocities were analysed. It should be noted that the applied 50 Nm scales the leaning and steering response. For consistency purpose, the same 50 Nm impulse torque was applied to big wheel bicycles, small wheel bicycles and e-scooters. The main purpose of this analysis was to observe the leaning and steering behaviour of the big wheel and the small wheel PMDs when imposed to a small obstacle, potholes or sideways winds. Small obstacles, potholes or sideways winds could perturb the vehicle laterally. The perturbation makes the vehicle oscillate about the steering axis. In the self-stable region, these vehicles have an ability to resist these small perturbations and maintain straight upright motion irrespective of small disturbance and the high-speed

capsize instability is very marginal, which could easily be controlled. Therefore, priority was given to the weave instability at low forward speed. Three different forward speeds: 4.5 m/s, 3.5 m/s and 2.5 m/s were chosen at which the PMDs are unstable. The initial conditions were defined as $(\Phi, \delta) = (0,0)$ and $(\dot{\Phi}, \dot{\delta}) = (0,0)$. These speeds represent the region where the weave mode is unstable for most of the PMDs. The simulation was carried out for time $t = 20$ sec, and the criteria for comparison was set as the time till the lean and steer angle reached $\pm 20^\circ$ deviation from the straight upright condition.

Figure 4-7: Steer and lean response of a typical big wheel bicycle, small wheel bicycle and e-scooter to lateral perturbation: a, b, and c are the steer angle response and d, e, and f are the lean response angle at velocities 4.5 m/s, 3.5 m/s, and 2.5 m/s respectively.

As the system is linear, the steer and lean response are governed by the respective eigenvalues. The steer and the lean response of two bicycles and a scooter at selected speeds are as shown in Figure 4-7. Figure 4-7 (a, d) represent the sinusoidal oscillation of the bicycle and remaining Figure 4-7 (b, c, e and, f) are a small portion of the oscillation. The oscillation amplitude increased sharply with the increase in time, and this amplitude increment was even faster with the decrease in the speed. This observation can be correlated with the general experience that the rider must provide more handlebar control at low speed and the need for manual control increases as the riding speed decreases.

Table 4-2 summarises the time taken for both bicycles and scooter to reach $\pm 20^\circ$ margin in steer and lean angle. For each speed, the small wheelers reached the criteria faster than big wheel bicycle. This suggested that the oscillation amplitude increased much faster for small wheelers. Since PMDs were unstable at these speeds, the rider must manually control the bicycles continuously. However, as summarised in Table 4-2, riders should provide more control to the small wheel PMDs because of the higher increment rate of the oscillation amplitude. This makes small wheeled PMDs difficult to ride at a slower speed. Furthermore, the scooter reached the criteria faster than that of the small wheel bicycle. That means, amidst the big wheel bicycle, small wheel bicycle and, scooter, the scooters required greater control to ride at lower speeds. This result strongly corroborates the subjective perception of riders. Thus, a sharp growth in oscillation amplitudes for low velocities provides yet another explanation for the difficulties associated with the small wheel bicycles and scooters.

Table 4-2: Time to reach the ± 20 -degrees deviation in lean angle and steer angle for typical designs different wheel sized PMDs.

Speed m/s(km/h)	Time for Lean angle criteria (sec)			Time for Steer angle criteria (sec)		
	Big wheel	Small wheel	E-scooter	Big wheel	Small wheel	E-scooter
4.5 (16.2)	20+	8.20	7.4	20+	9.40	8.75
3.5(12.6)	6.15	4.81	4.4	6.38	4.98	4.6
2.5 (9)	4.10	3.60	3.15	3.88	3.68	3.28

The case study presented here has revealed an interesting observation on the performance of small wheel PMDs. It was found that the small wheel PMDs required higher forward speed to be self-stable, and the oscillation amplitude increases much faster at low speed. The preliminary results from the case study strongly corroborated with the rider's perception of small wheel PMDs. Based on the presented case study, the

current design of small wheel PMDs are indeed less stable, twitchy and difficult to ride. Nonetheless, the analysis presented here is just a case study based on typical design big wheel bicycle and small wheel PMDs. Therefore, a more generalised analysis has been performed in the next section. Since ergonomic parameters were to be treated as constraints, the main focus was given to the front steering design parameters. The next section extends the current self-stability performance analysis. The performance of big wheel bicycles was compared with small wheel bicycles and small wheel e-scooters separately.

4.2.1.2 Self-Stability Comparison: Big Wheel Vs Small Wheel Bicycles

In this section, a more generalised performance comparison between big wheel bicycle and small wheel bicycles have been presented. The big wheel bicycles with standard 26-inch (700c) wheel size have been compared with the standard 20-inch and 16-inch wheel bicycles. The most common range of the big wheel design bicycle design parameter i.e. head tube angle range from 71 degrees to 74 degrees and trail range from 50 mm to 60 mm has been considered. As small wheel bicycles have shallower head tube angle, the most common range of head tube angle, i.e. 70-73 degrees was considered, and the trail was varied from 35 mm to 55 mm for each head tube angle. It should be mentioned that the big wheel bicycles having head tube angle 73-degrees and 56 mm trail was used as the reference steering geometry. A typical rider's fit design parameter values are presented in Table 4-1, and the same parameters are used in the analysis. The main purpose of this analysis was to understand whether there is any combination of front steering design parameters in the current small wheel bicycle designs that correspond to the performance of the big wheel bicycle.

The benchmarked bicycle model was used to analyse the stability of both the big wheel bicycles and the small wheel bicycles. This self-stable range varies with the bicycle design configuration such as wheelbase, head tube angle, trail and bicycle rider mass distributions. However, as the ergonomic constraints for an adult rider were considered in this analysis, the position of the rider could be assumed almost constant. In addition, when the ergonomic parameters are treated as constraints, parameters such as wheel size, head tube angle, trail, and wheelbase become interlinked to each other. For example, reducing the wheel size reduces the trail. Therefore, the small wheel bicycle manufactures usually go for the shallower head tube angles (less than 73-degrees) increasing the wheelbase. In addition to this, the ergonomic parameters such as bottom

bracket, saddle to handlebar reach, reach, and stack of the bicycle are also affected. In this study, these interlink between the different design parameters and the ergonomics of the rider has been considered.

Figure 4-8: Stability performance comparison of current big wheel bicycles (BWB) and small wheel bicycles (SWB) over the dominant frame design range: (a) 20-inch wheel bicycle, (b) 16-inch wheel bicycle.

Parameters such as the head tube angle and trail were varied in small steps, and the self-stable velocity range was computed. Figure 4-8 shows the self-stability speed of the big wheel and small wheel bicycle for each combination of head tube angle and trail over the selected region of head tube angle and trail. For each self-stable bicycle configurations, there exists a range bounded by two above mentioned critical speeds.

When design configuration changes, the critical speeds and the self-stable range also changes. When the calculated critical speeds are joined, it forms two lines. The lower lines correspond to the weave critical speeds, and the upper lines correspond to capsize critical speeds. Here, the self-stable speed ranges of big wheel bicycles were compared with that of small wheel bicycles. A clear distinction in the self-stable range was observed for big wheel and small wheel bicycles. This distinction was more pronounced for 16-inch wheel bicycle. The overall self-stability region shifted towards the higher speed range when the wheel size was reduced.

Firstly, the performance of the big wheel bicycle was compared with the small wheel bicycle with shallower head tube angles. For example, the self-stability velocity range for 73-degrees head tube angle and trail range from 50 mm to 60 mm is compared with the self-stable speed range of small wheel bicycles having head tube angles less than 73 degrees, i.e. 70-degrees to 72-degrees. The main aim was to search for a combination of head tube angle and trail values for which small wheel bicycles would exhibit self-stability range as that of a big wheel bicycle. In other words, the weave critical speed should be equal to or lower than the reference weave speed. Similarly, the capsize critical speed should be equal to or higher than the reference capsize speed. The acceptable regions are shown in Figure 4-8. If both critical velocities fall within the acceptable region, the combination of head tube angle and trail could be accepted. For example, the self-stability range (4.71 m/s - 7.63 m/s) of referenced big wheel bicycle was compared with 20-inch wheel bicycle with 72 degrees head tube angle and 45 mm trail. The self-stability range for the small wheel bicycle's steering geometry was 4.75 m/s (weave critical speed) to 7.67 m/s (capsize critical speed). Here, the capsize critical speed of the 20-inch wheel bicycle is higher than that of the referenced big wheel bicycle. Thus, the capsize speed was within the acceptable region. Nonetheless, the weave critical speed was not equal to or lower than the reference weave critical value. Therefore, the combination of 72-degrees head tube angle and 45 mm trail was rejected as it could not provide comparable self-stability as that of the referenced big wheel bicycle.

A similar analysis was performed for 71-degrees, 72-degrees, 73-degrees and 74-degrees head tube angles for big wheel bicycle and compared their performance with small wheel bicycles. It was also interesting to know that even if the small wheel bicycles had the same trail as that of the big wheel bicycle (56 mm), the self-stability

region was not comparable. A 20-inch small wheel bicycle with 71.5 degrees head tube and 16-inch wheel bicycle with 71 degrees head tube angle with the same trail of 56 mm had the self-stability range of 5.51 m/s - 9.36 m/s and 6.12 m/s - 11.15 m/s respectively. The self-stability of the big wheel and small wheel bicycle designed with the same steering geometry (73-degrees head tube angle and 56 mm trail) was also compared. The performance was still not comparable.

For small wheel bicycle, the study could not find any combination of front geometry parameters when shallower head tube angles in between 70 to 72 degrees were considered. It was found that the current designs small wheel bicycle needed to achieve a higher speed to become self-stable and are likely to be unstable at the normal riding speed of 15 km/h -17 km/h, thus requires a higher handlebar control from the rider. The self-stable range further shifted towards the higher speed when wheel size was reduced. Thus, as the wheel size reduces, the required speed of a bicycle to become self-stable increases. Therefore, more physical input and rider's handlebar control are required making it difficult to ride. The result obtained from the generalised comparison of big wheel bicycles and small wheel bicycles strongly supports the finding from the case study presented in Section 4.2.1.1. The result corroborates the rider's subjective perception that the ride of small wheelers is less stable, difficult and nervous at normal riding speed. At the same time, the results also confirmed that the difficulties in riding small wheel bicycles increase as wheel size reduces.

The intriguing results clearly showed that the current designs of small wheel bicycles with shallower head tube are not as stable as that of the big wheel bicycles. In other words, the current design practice of increasing the trail by reducing the head tube angle and fork offset did not yield a similar self-stability as that of the big wheel bicycle. In fact, reducing the head tube angle and increasing the trail shifted the self-stable range further towards the higher speed. Most importantly, the results also showed that the use of trail value as an indicator of stability is not adequate. Although the practice has worked well for big wheel bicycles, it does not translate directly when it comes to small wheel bicycle design and performance.

4.2.1.3 Stability Comparison: Big Wheel Bicycles Vs E-scooters

A similar analysis was performed for small wheel e-scooters. The analysis, presented in this section, is based on the typical design of e-scooter having an 8-inch wheel. Like

bicycle-rider system modelling, scooter-rider model was developed. In most of the e-scooters, riders stand on the deck while riding. However, some scooters have an option to retrofit the seat post and saddle. This does not change the overall design. Nonetheless, it allows riders to be seated while driving. Thus, there are two distinct postures that a rider can adopt while riding an e-scooters, shown in Figure 4-9. Therefore, the self-stable analysis presented in the case study was extended further by considering both the riding models and comparing their self-stability characteristics. Apart from the ergonomic parameters as presented in Table 4-1, the position of the saddle for seating posture was selected at 150 mm ahead and 700 mm high from the rear wheel contact point.

Figure 4-9: Seating and standing postures on e-scooters.

The major difference between two modes of riding is the location of the overall centre of mass and moment of inertia which in turn also change the reaction forces on each wheel. A typical example of mass related parameters for seating and standing posture are given in Table 4-3.

Table 4-3: Centre of mass and moment of inertia properties comparison between the standing and seating posture on small wheel e-scooters.

Parameters	Standing posture	Seating posture
Location of COM of rear frame (X, Z), m	(0.34, -1.04)	(0.28, -0.77)
MOI of the rear frame, kg-m ²		
$\begin{bmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_{yy} & 0 \\ I_{xz} & 0 & I_{zz} \end{bmatrix}$	$\begin{bmatrix} 19.21 & 0 & 0.037 \\ 0 & I_{yy} & 0 \\ 0.037 & 0 & 1.78 \end{bmatrix}$	$\begin{bmatrix} 9.85 & 0 & -2.80 \\ 0 & I_{yy} & 0 \\ -2.80 & 0 & 3.81 \end{bmatrix}$

The centre of mass location for standing posture is ahead and at a higher position than that of seating posture. However, the noticeable difference was observed for the inertia property distribution. The moments of inertia about the X-axis for standing posture is almost twice that of the inertia for the seating posture. This difference in mass distribution affects the overall vehicle-rider system. The result for self-stable speed range computed from the benchmarked model for both the standing and seating postures on a typical baseline model e-scooters is shown in Figure 4-10.

Figure 4-10: Eigenvalues and self-stable velocity comparison between the standing and seating posture on a small wheel e-scooter.

A generalized analysis was performed by varying the steering design parameters over their most common range of e-scooters. The head tube angle and trail were varied from 75-degrees to 80-degrees and 15mm to 30 mm respectively. The self-stability range for each combination of steering design over the selected range was computed and compared against the big wheel bicycle designs. The computed self-stable velocity region for both e-scooters and big wheel bicycles are shown in Figure 4-11. It should be noted that the ergonomic parameters were treated as constraints. Figure 4-11 compares the self-stable speed range of the e-scooters with the big wheel bicycles.

The results showed that the self-stability speed range comparison follows a similar trend as that of small wheel bicycles. The self-stable speed range shifted further towards the higher speed region for e-scooters. Considering the legislation on riding speed of 15-20 km/h for different personal mobility devices on shared paths, only the big wheel bicycles were found self-stable in that speed range. E-scooters were mostly found to be unstable over the range of 15-20 km/h. Furthermore, the self-stability speed range not only

differed from the big wheel bicycles, but a noticeable difference was also observed between the standing posture and seating posture. The low-speed stability, i.e. the weave critical speed, was found greatly influenced by the difference in mass distribution between two types of riding postures. Figure 4-12 depicts the difference in low-speed stability between the standing and seating posture of the e-scooter rider.

Figure 4-11: Self-stable speed range over the most common design range for small wheel e-scooters: (a) seating posture (b) standing posture.

The shaded portion in Figure 4-12 represents the most common range of legislated speed range (15km/h - 20 km/h) for e-scooters on a shared path, pavements or road. The legislation on speed limit was implemented to force riders to reduce their riding speed for safety concerns. The legislation itself can be considered good because riding at a lower speed helps to prevent the severity of the accidents for both riders and surroundings. However, current designs of e-scooters and imposed legislation do not seem to fit each other. It was clear that the e-scooters in general are, in fact, unstable at the prescribed speed limit and requires more rider input to ride the e-scooters. An

unstable e-scooter can easily throw riders off the e-scooters if appropriate control input is not provided by the riders. In other words, the requirement of more rider control to balance the e-scooters at low speed makes them more rider dependent and vulnerable to external disturbance.

Figure 4-12: Comparison of weave critical speed between seating and standing posture on e-scooters in reference to the legislated riding speed range.

Figure 4-12 also depicts the difference in self-stability speed for standing and seating postures. The standing posture on the e-scooters was found to have better low-speed self-stability. The results highlighted the influence of the rider's mass distribution on the vehicle-rider system. Furthermore, it also supported the fact that the rider's mass related parameters dominate the mass related parameters in such lightweight and compact PMDs. However, the results were very counterintuitive because the standing posture was found to be more stable than that of the seating posture despite having higher COM position. One of the possible explanations could be the assumption of the benchmarked model. Here, the rider is modelled rigidly attached to the e-scooter deck. Therefore, the connection between the rider and the vehicle was also assumed to be rigid. The analysis could be improved by considering a flexible connection between vehicle and rider that allows the rider to move laterally with respect to the vehicle's rear frame. Thus, the self-stability analysis of e-scooters was further analysed using extended rider's lean model in the subsequent Section 4.2.2.

4.2.2 Self-Stability Analysis: Rider Lean Model

The rider itself and the connection between the vehicle and rider are assumed to be rigid in the benchmarked model. Therefore, it does not allow the rider's movement relative to the rear frame. However, in a real scenario, the rider can move relative to the rear

frame and even apply some lean input to control the steering behaviour. Therefore, improved vehicle-rider model is used in this section to get more accurate insight on the dynamic stability of bicycles and e-scooters.

The details of the mathematical model allowing rider's body lean are presented in Section 2.2.3. For seating posture, only upper body parts, i.e. head, torso and arm can move relative to the vehicles frame and the lower body parts, i.e. legs can be modelled as components that are rigidly attached to the vehicle's frame. Similarly, for standing posture, the whole rider body can move laterally relative to the e-scooter deck. In addition, the rigidity of the connection between the vehicle and rider also varies depending on riding posture. For example, hands-free riding with seating posture on either bicycles or e-scooters has a relatively more rigid vehicle-rider connection than the standing posture on the e-scooters. In the case, where the rider is seating on the saddle, the stiffness coefficient and damping coefficient that defines the connection between the vehicle and rider are chosen as 760 Nm/rad and 34 Nms/red, respectively as used by Sharp [87]. However, the standing posture on e-scooter does not have a strong connection between the rider feet and e-scooter's deck, therefore, lower values of stiffness coefficient (100 Nm/rad) and damping coefficient (8 Nms/rad) are used. The analysis using the rider lean mode is further divided into two sub-sections. The first section compares the performance of big wheel bicycle and small wheel bicycle using the rider lean model. The second section presents a similar analysis between seating and standing postures on e-scooter.

4.2.2.1 Big Wheel Bicycle Vs Small Wheel Bicycle

Figure 4-13 shows the eigenvalues comparison for big wheel and small wheel bicycle using extended rider lean model. The nature of the eigenvalues obtained from extended rider lean mode at low speed was similar to that obtained from the benchmarked model except for an additional rider lean mode. The self-stability of typical big wheel bicycle was found in between 4.71-7.62 m/s. Similarly, the self-stability of typical 20-inch wheel bicycle was found to be 5.42 – 9.43 m/s. It should be noted that, although the rider's lean relative to the real frame was allowed in the mathematical model, no control input from the rider had been added. Therefore, the analysis is still a similar passive rider on the bicycle. However, it suggested that there should be a proper connection between the rear frame and rider for self-stability or in order to maintain the balance.

Figure 4-13: Eigenvalues and self-stability comparison between typical big and small wheel bicycles based on extended rider lean model.

4.2.2.2 Seating Posture Vs Standing Posture on E-Scooter

Figure 4-14 shows the analysis of the eigenvalues for the seating and standing postures on e-scooter based on their respective spring and damping coefficients. There was no significant change in the self-stability range of the seating posture on e-scooter. This is because there is an adequate connection between the rider and vehicle when the rider seats on the saddle. On the other hand, the e-scooter-rider system with the rider standing on the deck was not found self-stable at all. Here, it should be noted that the rider is a passive rider who is not allowed to provide any control input such as handlebar control or lean control. Because of the lack of proper connection between the e-scooter deck and the rider's feet, the vehicle-rider system is more likely to be unstable. Since the vehicle-rider system itself was found to be unstable for standing posture on e-scooter, this posture requires more handlebar control. Therefore, it can be concluded that the seating posture is more comfortable, stable and safer for the riders. However, it was also found that the results presented in Section 4.2.1 for the standing posture of e-scooter would be valid if there is a proper connection between the rider and the vehicle. For example, if the e-scooter had a long seat tube and saddle to support the rider on the same standing posture, it will improve the connection between the vehicle and rider. In this case, the analysis will have similar results as that is presented in 4.2.1.

Figure 4-14: Eigenvalues and self-stability comparison for seating posture and standing posture on e-scooters based on extended rider lean model and considering the pivotal connection between vehicle-rider system.

4.2.3 Effect of Acceleration and Deceleration

Riders frequently need to accelerate or decelerate depending on the riding scenario like mounting on the vehicle or dismounting from the vehicle, traffic lights obstacle avoidance and so on. Accelerating or decelerating a vehicle causes the overall weight of the system to shift towards the rear wheel contact point or the front wheel contact point respectively. Figure 4-15 shows the shifting of the weight towards the front wheel contact point when bicycles and e-scooters are decelerated or braked.

The seating posture on e-scooter had a low COM. Therefore, the weight shift during deceleration was also small. Despite having a similar vertical position of the COM in standing posture on e-scooters and bicycles, there was a distinct difference in the weight shift. It can be noticed that the weight shift for bicycle systems was similar to that of seating posture on e-scooter, which can be explained on the basis of longer wheelbase compared to e-scooters. The weight shift is maximum for standing posture on e-scooters.

Figure 4-15: Shifted weight at the front wheel-ground contact point due to deceleration.

In the real scenario where riders might have to apply hard brakes, the deceleration could be very high. In such cases, the standing posture on the e-scooters could be more dangerous because it highly increases the chance of flipping over the vehicle. The maximum allowable deceleration to prevent the flipping over can be calculated as [56]:

$$a_{max} \leq \frac{w - x_T}{h} g \quad 4.4$$

Where w = wheelbase, x_T = horizontal location of COM and h = vertical location of COM of the entire system.

The maximum allowable deceleration to prevent the flipping over in standing posture was calculated about 5 m/s^2 which is much lower compared to seating posture (6.7 m/s^2) and bicycle riding (7.5 m/s^2). An experiment conducted by Diana Ser [182] found that riding an e-scooter at a speed of 15 km/h on the standing posture requires a stoppage distance of 1.6m (5.42 m/s^2) to 3.2 m (2.7 m/s^2). If the rider fails to adapt with the hard braking by leaning backwards, the chances of flipping over the e-scooters are high. Furthermore, the stoppage distance corresponding to seating posture on e-scooters and bicycles based on maximum allowable deceleration was 1.3 m and 1.15 m . This showed that the standing posture on e-scooters requires a greater distance to avoid collision and prevent from flipping over the vehicle than riding a bicycle or e-scooters with seating posture.

Figure 4-16: Effect of deceleration (a, b, c, d) and acceleration (e, f, g, h) on eigenvalues and self-stability of different wheel sized PMDs.

Figure 4-16 shows the self-stability plot comparing constant speed case with deceleration and acceleration. Again, the typical 26-inch big wheel bicycle, typical 20-inch small wheel bicycle and 8-inch e-scooter including standing and seating postures have been considered. It was observed that the deceleration shifted the self-stable region towards the higher speed region. This means, if a rider is travelling downhill applying the brake to maintain the constant speed, the rider should maintain a relatively higher speed than the constant speed that is required for self-stability on a flat surface. Using the analogy that riding a vehicle downhill at a constant speed is similar to the motion of a decelerating vehicle on a flat surface if a rider is decelerating on a flat surface, the vehicles are likely to be even more unstable making it difficult to ride. Most importantly, the damping of the weave mode is decreased under deceleration.

The deceleration was found to destabilise the weave mode. The real part of the eigenvalues becomes more positive with the deceleration. It suggested that when the single-track vehicles encountered a sudden obstacle and required a quick deceleration, the tendency of the vehicles to oscillate or weave increases and chances of losing control would also increase. On the other hand, the effect of acceleration is just opposite to that of the deceleration. Acceleration was found to improve the weave and capsize damping. However, the self-stability range was found to shrink drastically with the acceleration. The analysis found an acceleration greater than 3 m/s^2 could make the self-stable region disappear completely losing its self-stability property. Therefore, hefty acceleration on the single-track PMDs could cause the sudden loss of the intrinsic self-stability property, which could make them difficult to balance or demand more and cautious rider control to avoid any unwanted accidents. The effects of the acceleration and deceleration on vehicle's stability were found to have a similar trend.

In order to compare the relative effect of the acceleration and deceleration on self-stability velocity range of different wheel sized PMDs, the values for acceleration and deceleration were varied from 0 m/s^2 to 2.5 m/s^2 and self-stable range for each acceleration and decelerations were computed and compared. The relative change in self-stability range for 26-inch big wheel bicycle, 20-inch small wheel bicycle and 8-inch e-scooter considering standing and seating postures is shown in Figure 4-17. The results showed that e-scooters are more sensitive to the acceleration and decelerations as the change on self-stability range per unit change (slopes of the curve) in acceleration or deceleration was found higher for e-scooters than bicycles.

Figure 4-17: Effect of acceleration and deceleration on self-stable speed range relative to constant speed case performed for different wheel sized PMDs.

4.3 Handling Performance Comparison of Current PMDs

In this section, handling performance of different wheel sized PMDs are compared based on mathematical models. The torque required to perform steady-state turning at various velocity and turning radius was evaluated. Also, the ability of the vehicles to roll over the obstacles such as bumps were discussed.

4.3.1 Steady-State Turning

Figure 4-18: Comparison of handlebar torque by rider to maintain steady-state turning: (a) at constant turning radius of 4 m and (b) at constant turning speed of 4.5 m/s.

Figure 4-18 shows the steering torque required at the handlebar to maintain the steady-state turning at (i) a constant turning radius of 4 m and various velocity, and (ii) a constant velocity of 4.5 m/s at a various radius of turning. The lesser steering torque suggests greater manoeuvring capability. The ground reaction forces and the lateral forces provide dominating dis-aligning moment (positive) at low speed. This dis-aligning moment tends to oversteer the vehicles towards the turn. Therefore, to counter

the oversteering and maintain the constant radius of curvature on steady-state turning, the rider needs to provide aligning torque (negative) at low speed. As the riding velocity increases, the aligning torque required at the handlebar also decreases because the gyroscopic torque counters the dis-aligning torque. At very high speed, aligning torque could dominate because of the higher gyroscopic torque and centrifugal torque. Therefore, riders might have to apply dis-aligning torque to maintain the constant radius of curvature at a higher speed. Since the angular spin momentum decreases with the reduction of wheel size, the aligning torque provided by the gyroscopic effect is also small for small wheelers. As a result, the handlebar torque required for big wheel bicycle transit from negative to positive much earlier than the 20-inch, 16-inch bicycles and 8-inch e-scooters, as shown in Figure 4-18 (a).

The steering torque required for various turning radius at a normal riding speed of 4.5 m/s, see Figure 4-18 (b), revealed that the riders need to provide aligning torque to maintain the steady-state turning for the current design of PMDs. In fact, a little negative torque is required at the handlebar for good handling feel [56]. E-scooters were found to require the least handlebar steady-state torque compared to bicycles. This is attributed to the shorter wheelbase, trail distance, and location of rider near to the rear wheel which makes the handlebar much lighter and reduces the handlebar torque. Interestingly, current designs of small wheel bicycles were found to require greater steering torque compared to the typical big wheel bicycle. However, it should be noted that most of the current designs of small wheel adult bicycles (20-inch and 16-inch) have a longer wheelbase. For the same radius of turning, the longer wheelbase requires more steering angle and therefore, more steering torque. Moreover, the smaller wheel does not have sufficient gyroscopic effect as the big wheel bicycles, and dis-aligning torque becomes more pronounced for small wheel bicycles.

4.3.2 Obstacle Interaction

Figure 4-19: Effect of obstacle-wheel interaction for different sizes of wheels (a) Time to reach climb an obstacle of height 4 cm (b) The vertical velocity of wheels while climbing the obstacle of height 4 cm.

Figure 4-19 (a) shows the time required for different wheel sized PMDs to reach the top of an obstacle of height 4 cm with a forward velocity of 5 m/s using the kinematic model, Equation (2.60), as presented in Section 2.3.2.2. For a given obstacle height and forward velocity, the time required to overcome an obstacle depends on the horizontal distance from the wheel contact point on the obstacle to the wheel contact point on the ground. The bigger wheel must travel longer horizontal distance before reaching the top. Therefore, the time required to reach the top of the obstacle is also greater than the smaller wheels. As a result, the upward velocity of the wheel becomes higher for the small wheel as it travels the same obstacle height at a much shorter time, as shown in Figure 4-19 (b). Therefore, small wheelers have more jerk feel at the handlebar compared to the bigger wheelers.

Furthermore, the minimum horizontal force required to pull a wheel over an obstacle of different height is shown in Figure 4-20. The bigger wheel requires less pulling force to overcome the obstacle. This force is negatively correlated with the wheel to obstacle ratio. Since bigger wheels have a lower “wheel to obstacle height” ratio, it requires less force and rolling over the obstacle becomes smoother. On the other hand, small wheelers require more pulling force and rolling over the obstacle becomes more difficult. Furthermore, the possibility of riders being thrown over the handlebar is also higher on small wheelers, especially on e-scooters when encountered with uneven surface or obstacles.

Figure 4-20: A comparison of horizontal force required to overcome the obstacles for different size of wheels.

4.4 Summary of Chapter 4

This chapter objectively and quantitatively compared the performance of current designs of PMDs. The self-stable velocity range, eigenvalues, and handlebar torque to perform steady-state turning and response of a vehicle against obstacles were analysed. The self-stability analysis has confirmed that the current designs of small wheelers are indeed less stable and difficult to ride at normal riding speed. Furthermore, lateral perturbation tests for low to moderate speed (2.5-4.5 m/s) in 26-inch big wheel bicycle, 20-inch small wheel bicycle and 8-inch small wheel e-scooter have confirmed that the rate of increment in oscillation amplitude was greater for small wheelers compared to the big wheelers. Also, a good connection between the vehicle and rider was found to be essential for any single-track vehicle rider system to be self-stable. Because of the poor pivot connection between the e-scooter's deck and the rider's feet, the standing posture on e-scooters are likely to be unstable and required more handlebar control.

The self-stability speed range for e-scooters was also found to be more sensitive to the acceleration and deceleration. Finally, the handling performance of the PMDs was analysed. E-scooters were found to be easier to manoeuvre compared to bicycles. The current designs of 20-inch and 16-inch small wheel adult bicycles were found to require more handlebar torque to maintain steady-state turning. Small wheelers were also found to be more vulnerable when encountering an obstacle as they give more jerking feel at the handlebar and require more force to overcome an obstacle.

Chapter 5. Improved Guideline Formulation for the Current Design of PMDs

Small wheelers have been shown to be less stable, more difficult to ride at normal speed, and more vulnerable against obstacles, acceleration, and braking condition. Therefore, there is a need to improve the dynamic performance of the current design of PMDs for better rideability and safety.

This chapter is divided into 3 subsections. The first section discusses the improved design philosophy and the proposed design methodology. The second section presents improved design guidelines for small wheelers and e-scooters. The final section presents the experimental analysis to verify the proposed design guidelines.

5.1 Improved Methodology

A general procedure that is followed for the design of bicycles and e-scooters is represented in the flow chart in Figure 5-1. The design procedure starts with the selection of the basic geometric parameters for manufacturing. Apart from the selection of the geometric parameters, the design components related to the comfort or the ergonomics such as the suspension, brake types, saddles are also selected. Thereafter, the performance of the vehicle is evaluated using various riding tests. If the results from the test rides are not satisfactory, the whole procedure has to be repeated. In order to obtain a single set of satisfactory design parameters, designers usually have to make several design modifications, develop prototypes and evaluate the performance. Therefore, the overall design process is slow, costly, and rely mainly on subjective assessments. For the same reason, many bicycles designers use a single set of design parameters. For example, the front steering design of Brompton bicycle has a 73-degree head tube angle and a 41 mm trail. Bike Friday bicycle generally have a 73-degree head tube angle and a 35 mm trail. Therefore, the current design methodology of the PMDs does not facilitate the designers to change the set of design parameters and predict the performance of the design configuration before manufacturing.

In this chapter, the conventional design procedure is modified by incorporating the well-defined and validated mathematical models into the design procedure. The ergonomics and handling, the self-stability characteristics and portability have been included in the

bicycle design objectives. Figure 5-2 illustrates the adopted design philosophy in order to improve the performance of small wheelers.

Figure 5-1: Conventional design methodology for big wheel bicycle, small wheel bicycles and small wheel e-scooters.

The small wheelers are designed for portability, compactness and easy handling. Therefore, the weight and foldability are essential aspects of PMDs. Most of the well-known small wheel bicycle including some foldable designs have the frame made of steel or aluminium and weigh around 10 Kg to 12 Kg. Similarly, e-scooters usually weigh around 15 Kg. Therefore, the weight limit is recommended to be not more than 12 Kg for bicycles and 15 Kg e-scooters. Also, the ergonomic parameters should not be distorted to maintain a similar comfort level as that of the current design of PMDs.

A vehicle should be responsive enough for easy manoeuvring in a crowded space or for cornering. At the same time, the response should not be twitchy or nervous during cornering or when encountering small disturbances like potholes and bumps. The latter aspect is related to the self-stability of the PMD. As explained earlier in Chapter 4, self-stability is the intrinsic characteristic of a single-track vehicle design configuration.

Since small wheel PMDs are now known to be less stable, it is required to improve the design by using the mathematical models as a tool to achieve better self-stability.

Figure 5-2: Improved philosophy for PMD design.

An improved methodology has been developed for small wheel PMDs and is shown in Figure 5-3. The proposed methodology is an iterative process based on “the design by rules” concept. This concept is based on the set of formulae that are intended to help designers. For the proposed improved design methodology, the required formulae for mathematical analysis and selecting appropriate initial design parameters have been presented in Chapter 2 and 3. The following general procedures are recommended designers to apply the proposed design methodology.

1. The methodology starts with the selection of the basic frame design parameters that govern the ergonomics and the handling aspects. The selection of basic design parameters is based on the rider’s anthropomorphic measurements, past data and experiences. The formulae mentioned in Section 3.3 can be used to select appropriate sets of design parameters. A basic vehicle model can be developed from selected ergonomic and handling parameters.
2. The frame for small wheel PMDs can be modelled using CAD software like Solidworks [151] and CATIA [183]. After that, the required mass related parameters such as mass, the COM and MOI can be obtained.
3. The self-stability of the design configuration can be computed using an appropriate mathematical model. Different mathematical models like the

benchmarked model and its extensions can be used appropriately to compute the eigenvalues and self-stability speed region.

4. The self-stability speed thus obtained can be compared with the reference self-stability range for big wheel bicycle. A general procedure to compare the eigenvalues and the self-stable region is explained in Section 4.2.
5. If the computed self-stability range is not comparable to the reference self-stable range, the front steering parameters can be incrementally changed. The required combination of the head tube angle and the trail could be found after a few iterations.
6. If the current sets of parameters are not satisfactory, a few further iterations could be performed to get the next sets of parameters which would still exhibit similar self-stable region. In this way, different sets of parameters could be obtained which exhibit similar stability and ergonomics.

Figure 5-3: Improved design methodology to design small wheel PMDs.

The improved methodology and mathematical tool allowed design parameters to be evaluated in the early design phase. Unlike the conventional method, this new methodology provides the flexibility for the frame builders to evaluate different designs without having to fabricate many prototypes. Hence, the methodology is more cost-effective and scientific. Thus, frame builders do not have to rely on a limited set of design parameters.

5.2 Observation on the Parametric Variation

The improved design methodology uses an iterative process by varying the design parameters. A parametric variation analysis provides direction whether a particular design parameter should be increased or decreased to match the reference self-stability. The variation in design parameters shows their effects by shifting the self-stable range either towards low speed (leftward shifting) or higher speed (rightward shifting) region. The shifting trend of the self-stability region is helpful to either increase or decrease the parameter.

The effect of the head tube angle and the trail variation has already been discussed in Chapter 4. It has been found that decreasing the front fork trail and increasing the head tube angle shifted the self-stability region towards the low-speed region. Nevertheless, the self-stability range became narrower as it shifted towards the low-speed region. The effects of parameters such as wheelbase, the centre of mass location for the rear frame and front frame and, front frame mass are shown in Figure 5-4.

The self-stable region shifted towards the low-speed region while slightly Reducing the wheelbase. However, the self-stable speed range remained almost the same. The effect of wheelbase change was not as pronounced as for head tube and trail. Furthermore, increasing the front frame mass shifts the self-stability region towards the higher velocity region. Therefore, the weight of the front, as well as the rear frame, should not be increased too much because the overall system becomes heavy and more cumbersome to ride at low speed.

Figure 5-4: Effect of design parameters on self-stable velocity range.

The rider's position was varied from 0.23 m up to 0.31 m horizontally, and changes in MOI were also considered. An increase in the horizontal distance of COM of rear frame shifted the self-stable region towards the higher velocity. The effect was found to be more pronounced in capsizes mode compared to the weave mode, as a result, the self-stability region also broadened as it shifted towards the higher velocity. An analysis suggested that increasing the vertical distance for the rear frame COM tends to reduce the self-stability range slightly. However, the effects of varying the vertical distance were almost negligible as compared to the effects due to the variation in horizontal distance. A similar observation was obtained while shifting the front centre of mass ahead and upward along the steering axis, the self-stability region shifted towards higher velocity.

Table 5-1: The eigenvalues trend for increasing the design parameter.

Parameters		Weave			Capsize			Castering
		Left	Self-stable	Right	Left	Self-stable	Right	
Rear frame COM	X	+Ve ↑	-Ve ↑	-Ve ↑	-Ve ↑	-Ve ↑	+Ve ↓	-Ve ↑
	Y	+Ve ↓	-Ve ↓	-Ve ↓	-Ve ↓	-Ve ↓	+Ve ↑	-Ve ↓
Head tube angle		+Ve ↓	-Ve ↑	-Ve ↑	-Ve ↓	-Ve ↓	+Ve ↓	-Ve ↑
Trail		+Ve ↑	-Ve ↓	-Ve ↓	-Ve ↑	-Ve ↑	+Ve ↓	-Ve ↑
Wheel radius and MOI (mass constant)		+Ve ↓	-Ve ↑	-Ve ↑	-Ve ↓	-Ve ↓	+Ve ↑	-Ve ↓
Mass and MOI of wheel (radius constant)		+Ve ↓	-Ve ↓	-Ve ↓	-Ve ↓	-Ve ↓	+Ve ↑	-Ve ↓

The trend of the eigenvalues variation for change in design parameters has also been analysed. The eigenvalues are very important not only for self-stability prediction but also for comparing the bicycle performance. If the real parts of the eigenvalues are more negative, the oscillation decays much faster. On the other hand, if the real parts are less positive, the growth rate of PMD's oscillation is also minimum. Therefore, major design parameters were varied, and the trend for the eigenvalues was analysed. The velocity region is divided into "left" region representing unstable low-velocity, "self-stable" region representing self-stable velocity region and "right" region representing the high-velocity unstable region. Table 5-1 shows the effect of increasing each parameter and their effect on eigenvalues. The upward arrow (↑) represents an increase in positivity or negativity of the eigenvalues with an increase in design parameters, and downward arrow (↓) represents a decrease in positivity or negativity of the real part of eigenvalues with an increase in design parameters. Increasing the head tube angle and reducing trail was found to improve the weave mode by reducing the positivity at low speed and increasing the negativity of the eigenvalues at self-stable speed. Similarly, increasing the MOI of the wheel and shifting the position of rear frame COM improv the weave mode eigenvalues.

Since the self-stability of the current small wheelers starts relatively at a higher speed and eigenvalues are more positive up to typical riding speed of 15-17 Km/h. Thus, it is required to shift the overall self-stability towards the lower velocity region and reduce

the positivity of the eigenvalues to improve the performance of the current design of PMDs. The parametric variation and the trend analysis help to appropriately vary the design parameter to match the self-stability of the big wheel bicycle.

5.3 Design Guideline Formulation

This section presents the improved design guidelines for small wheel PMDs. Again, the big wheel bicycle having a 73-degrees head tube angle and a 56 mm trail is considered as the reference bicycle. It has a self-stable speed range of 4.71 m/s (17.0 km/h) to 7.63 m/s (27.5 km/h). The main focus has been given to the guideline formulation of the steering geometry. The analysis performed in Chapter 4 has been further extended over a wider range of head tube angle and trail. The front steering guidelines have been developed for 20-inch, 16-inch and 12-inch small wheel bicycles and 8-inch small wheel e-scooters.

5.3.1 Design Guideline Formulation for Small Wheel Bicycles

Since small wheel bicycles with shallower head tube angles (70-73 degrees) do not result in similar stability as that of big wheel bicycles, thus, the analysis here was started from 74-degree head tube angle and 35 mm trail for 20-inch, 16-inch and 12-inch small wheel bicycles. The associated changes in wheelbase were considered accordingly. The computed self-stability was compared with the referenced self-stability range for big wheel bicycle using a similar method, as explained in Chapter 4. The weave critical speed of the improved small wheel bicycles should be equal or lower, and the capsize critical speed should be equal or higher than that of referenced big wheel bicycle. The criteria ensure that the improved small wheel vehicles have at least a similar self-stable speed range as that of the big wheel bicycles. When computed self-stability was not comparable to the reference self-stability range, the trail was increased by 1 mm. This iterative process was continued until a set of head tube angle and trail were obtained, which could result in a comparable self-stability. The process was continued for each head tube angle and over the trail range in order to find the multiple sets of front steering design parameters.

5.3.1.1 Improved Design Guideline for 20-inch and 16-inch Wheel Bicycle

Figure 5-5 shows the self-stable region computed for steeper head tube angle greater than 74-degrees and a broader range of trail from 35 mm to 85 mm. The referenced self-stable region is indicated by the shaded region. Considering the steeper head tube angle, the self-stability region shifted towards the lower velocity region. This allowed identifying some combinations of the head tube angle and trail which resulted in a comparable self-stability range as that of referenced big wheel bicycle. It was interesting to know that 45 mm trail and 74-degrees head tube angle are nominal values for 20-inch wheel bicycle to exhibit referenced big wheel bicycle self-stability. Similarly, the nominal values for the 16-inch wheel bicycles are 24 mm trail and 76-degrees head tube angle. The nominal values might vary slightly depending on the rider's sitting position. Nonetheless, the rider's position on a bicycle is almost constant for ergonomic reasons. Here, the COM of the rider on the rear frame is maintained at 28-29 cm from the rear wheel axle. The analysis provided strong evidence that increasing the head tube angle helps improve the stability of the small wheel bicycles. Furthermore, the allowable range of the trail increased when head tube angles were made steeper. For example, 20-inch wheel bicycle with 74-degrees head tube angle and trail ranging from 45-48 mm showed the comparable self-stability and range increased to 45-59 mm for 77-degrees. The results were contrary to the general trend obtained from the ISO recommended criteria; Figure 3-6. Furthermore, it was found that the range of self-stable speed broadened when a steeper head tube angle was used with the longer trail. The range of allowable trail for each head tube angle is presented in Table 5-2.

Figure 5-5: Self-stable speed range for steeper head tube angles and broad trail range: (a) 20-inch wheel bicycle and, (b) 16-inch wheel bicycle.

Table 5-2: Allowable trail range for each head tube angle obtained for 20-inch and 16-inch small wheel bicycles that gives comparable self-stability range as the big wheel bicycles.

20-inch wheel bicycle			16-inch wheel bicycle		
Head tube angle (degree)	Allowable trail range (mm)		Head tube angle (degree)	Allowable trail range (mm)	
	Minimum	Maximum		Minimum	Maximum
72	-	-	75	-	-
73	-	-	76	24	25
74	45	48	77	24	26.5
75	45	52	78	24	28.5
76	45	56	79	24	30
77	45	59	80	24.5	32
78	45	63	81	24.5	33.5
79	45	67	82	24.5	35.5
80	45	71	83	24	37
81	45	75	84	24	39
82	44	79	85	23.5	40.5

The identified sets of the head tube angles and the trail were further analysed based on the “wheel flop factor”. The wheel-flop factor relates the head tube angle and trail as:

$$\text{Wheel flop (f)} = c * \sin \lambda \cos \lambda \quad (5-1)$$

Where ‘c’ is the geometrical trail, and ‘ λ ’ is the head tube angle measured from the vertical axis. The wheel flop refers to the phenomena of lowering of the bicycle front while turning the handlebar [17]. Because wheel flop involves the lowering of the front end of a bicycle, the handlebar continues to turn under the influence of the gravitational force in the search for the equilibrium point. Therefore, depending on the amount of wheel flop, the handlebar tends to turn more than expected. If the wheel flop is too much, it tends to veer off and weave even the slightest provocation makes it difficult to ride in a straight line [14]. Decreasing the trail and increasing the head tube angle decreases the wheel flop. In addition, the rotational inertia of the wheel also has a greater influence on the wheel flop. Wheel with a greater moment of inertia counters the wheel flop effect and helps to stabilize the bicycle. Therefore, the wheel-flop is usually viewed in conjunction with bicycle stability and handling.

Figure 5-7 shows the wheel-flop for current designs of the big wheel and small wheel bicycles and recommended guidelines for 20-inch and 16-inch small wheel bicycles. Since the improved guidelines for small wheel bicycles suggested a range of trail values for each head tube angle, there are two lines representing the wheel-flop factor. The lower limit corresponds to the lower limiting value of the trail, and upper limit corresponds to the higher limiting value of the trail.

Figure 5-6: (a) wheel flop factor of current designs of big and small wheel bicycles, (b) wheel flop factor range for improved 20-inch wheel bicycle designs and, (c) wheel flop factor for improved 16-inch wheel bicycles.

A clear distinction for the wheel-flop factor was observed for the current designs of big wheel bicycles and small wheel bicycles. Big wheel bicycles have higher wheel-flop compared to the small wheel bicycles. As small wheels have less wheel rotational inertia, the tendency to counter the wheel-flop effect is also small. Therefore, if small wheel bicycles are designed with much higher wheel flop factor, the wheel rotational inertia cannot counter the wheel flop phenomenon and the chances of veering off or weaving the small wheels bicycle, even with the slightest provocation are higher. Therefore, it is very reasonable to design small wheel bicycles with less wheel flop factor. Since the current designs of small wheel bicycles are not as stable and good handling as the big wheel bicycles, it can be suggested the wheel flop factors used for small wheelers are not optimised.

Figure 5-6 (b) and (c) show the wheel-flop factor based on the improved front design guideline. The major difference between the current designs of small wheel bicycle and

the improved design guideline is the minimum allowable head tube angle. Furthermore, allowable wheel-flop factor regions for improved small wheel bicycles were also lower than that of current small wheel bicycles. The recommended design region can be used to identify the trail value to be used with a particular head tube angle that yields similar self-stability as the big wheel bicycles.

5.3.1.2 Improved Design Guideline for 12-inch Wheel Bicycle

Electrically assisted 12-inch small wheel bicycles are becoming popular in many cities, including Singapore. Typical design of such 12-inch wheel bicycles is shown in Figure 5-7. The bikes have a short wheelbase compared to 16-inch, 20-inch or 26-inch wheel bicycles. The typical range of the wheelbase may vary from 700 mm to 800 mm. Since the 12-inch bikes have very small wheels, battery-powered hub motors are used to help in propulsion. The hub motors are commonly used at the rear wheel. The batteries and the controller are usually placed inside the top tube to protect it from the outside environment and to balance the weight distribution. The weight of the hub motor may vary from 3.5 Kg to 4 Kg. The front wheel with no hub motor may weigh from 1.7 Kg to 2 Kg. A typical bike including the battery, controller and wheels may weigh as light as 12 Kg to 15 Kg.

Figure 5-7: Typical designs of 12-inch wheel bicycles.

A similar analysis has been performed for 12-inch wheel bicycles. A typical design of a 12-inch wheel bicycle was selected as the baseline model and the front steering design guideline was developed accordingly. The head tube angle was again varied from 74-degrees to 87-degrees and trail was varied from 10 mm to 40 mm. The computed self-stable speed range from the benchmarked model is shown in Figure 5-8. The identified sets of the head tube angle and the trail are listed in Table 5-3. It was found that the minimum head tube angle and the trail values were 77-degrees and 19 mm respectively. Like 20-inch and 16-inch small wheel bicycles, the trail range for 12-inch wheel also increased when head tube angle became steeper.

Figure 5-8: Self-stable speed range for 12-inch small wheel bicycle.

Table 5-3: Allowable trail range for each head tube angle identified for 12-inch small wheel bicycle.

12-inch wheel bicycle		
Head tube angle (degree)	Allowable trail range (mm)	
	Minimum	Maximum
77	19	19
78	19	20
79	19	21.5
80	19	23
81	19	24
82	19	25
83	19	26
84	19	27
85	19	28
86	18	29
87	17	30

It was noticed that there exists a minimum trail value and head tube angle for each small wheel (20-inch, 16-inch and 12-inch) bicycle that corresponded to the self-stability of reference big wheel bicycle (73-head tube angle and 56 mm trail). Therefore, the analysis was further extended. Three head tube angles, 71-degrees, 72-degrees and 73-degrees and three trail values 50 mm, 55 mm and 60 mm were selected for big wheel bicycles, and corresponding steering designs parameters were searched for small wheelers. In this study, nine sets of head tube angles and trail were considered for big wheel bicycles. The results were very surprising. For each trail, there exists a minimum trail value corresponding to particular wheel size. For big wheel bicycle with 50 mm trail, the corresponding minimum values of the trail for 20-inch, 16-inch and 12-inch small wheel bicycles were found to be 40.5 mm, 21 mm and 17.5 mm trail, respectively. Similarly, for 60 mm trail, the corresponding minimum values were 48 mm, 26 mm and

20 mm. Most importantly, the ratio of the referenced trail for big wheel bicycle to the minimum trail for small wheel bicycles was almost constant. Furthermore, the trend for the head tube angle was consistent with the previous finding that the smaller wheel bicycles should be steeper head tube angle than the referenced big wheel bicycles. In general, for each 4-inches reduction in wheel size, the head tube angle should be increased by 1-1.5 degrees to obtain the minimum allowable head tube angle. A general guideline was developed to translate the steering design of big wheel bicycles to small wheel bicycles, as shown in Figure 5-9.

Figure 5-9: A general guideline to translate big wheel bicycle's steering design to a minimum head tube angle and trail values for small wheelers.

5.3.2 Design Guideline Formulation for 8-inch Wheel E-scooter

This section analyzes the possibilities of developing an improved guideline that ensure the self-stability of the current design of e-scooters for both standing and seating postures. This section is further divided into three subsections. The first section presents the possible design improvements for standing posture. The second section deals with the seating posture. The third section proposes the common guidelines that are valid for both the standing and seating postures.

5.3.2.1 Improved Design Guideline for Standing Posture on E-scooter

The self-stable range for the standing rider's posture on e-scooters was analysed by varying the head tube angle from 75-degrees to 83-degrees and trail from 0 mm to 30 mm. A typical design of e-scooter was considered where the steering stem is in line with the head tube angle. Such steering design is very common for the current e-scooters and a few examples are shown in Figure 5-10.

Figure 5-10: Typical design of e-scooter with straight head tube and steering stem.

The computed self-stability for 8-inch small wheel scooters considering a standing rider is shown in Figure 5-11. The minimum head tube angle and trail was found to be 75-degrees and 7 mm. For each variation in head tube angle and trail, the associated change in wheelbase was also considered. The identified range of trail for each head tube angles is listed in Table 5-4.

Figure 5-11: The self-stability speed range for e-scooter considered a standing rider.

Table 5-4: Allowable trail range for each head tube angle identified for small wheel e-scooters considering a standing rider.

Head tube angle (degree)	Trail for standing posture (mm)
75	7-9.5
76	7-10
77	7-10.5
78	7-11.5
79	7-12.5
80	7-13.5
81	7-14.5
82	7-15.5
83	7-16.5

5.3.2.2 Improved Design Guideline for Seating Posture on E-scooter

A similar analysis was performed for 8-inch e-scooter, considering a seated rider. The head tube angle was varied from 75-degrees to 83 degrees, and the trail was varied from

0 mm to 30 mm. The computed self-stable speed range using the benchmarked model is shown in Figure 5-12.

Figure 5-12: The self-stability speed range for e-scooter considered a seating rider.

A combination of head tube angle and trail over the selected design range was searched in such a way that would result in a similar self-stability as referenced big wheel bicycle. Unfortunately, no set of head tube angle and trail was found that yielded similar self-stable speed range as the reference big wheel bicycle.

5.3.2.3 Improved Design Guideline for both Standing and Seated Postures

Herein, the main objective is to identify the common design set of parameters for both standing and seated rider postures on e-scooters. The position of the saddle, handlebar height, deck ground clearance, and overall weight of the e-scooters cannot be changed abruptly while trying to improve the self-stability. Moreover, it has already been found that the front geometrical parameters only are not adequate to improve the self-stability of e-scooter with the seated rider. Therefore, the front steering of the e-scooter was divided into two parts, as shown in Figure 5-13 (a). The lower part (red colour) is the front fork, and the upper part (blue colour) is the steering stem, including the handlebar. The advantage of the methodology is that it allowed varying both the COM location as well as the geometrical parameters simultaneously, as illustrated in Figure 5-13 (b). The head tube angle and trail could be varied from the lower part and location of COM of the overall front steering frame could be influenced by varying the steering stem angle.

Figure 5-13: Modified front steering geometrical design: (a) illustration of steering stem and front fork and (b) effect if varying steering stem angle keeping head tube angle constant.

The previous two sections have already covered the analysis of e-scooter designs where steering stem angle is equal to the head tube angle. Therefore, the two remaining possibilities are:

Category 1: Head tube angle is less than the steering stem angle

Category 2: Head tube angle is greater than the steering stem angle

Figure 5-14: Shift in the COM position per degree change in steering stem angle with respect to head tube angle.

The COM of the front frame and associated inertial properties changes with the variation in the head tube angle and the steering stem angle. The shift in the centre of mass location per degree change in the steering stem angle with respect to the head tube angle (steering axis) is shown in Figure 5-14. The region near origin represents the steering design with a head tube angle and steering stem angle being equal and has the centre of mass location very close to the steering axis. Category 1 shifts the COM of overall front

assembly further ahead (+ve) of the steering axis. Similarly, category 2 shifts the COM of overall front assembly behind (-ve) the steering axis. Each degree changes in steering stem with respect to the head tube angle resulted in approximately 9 mm horizontal shift of the overall front steering assembly's COM.

Figure 5-15: Effect of steering stem angle variation on the self-stable speed range for the seating and the standing postures.

The effect of shifting the location of the front assembly COM is shown in Figure 5-15. The front assembly design with head tube angle (HTA) 79-degrees and steering stem angle (STA) 83-degrees has the centre of mass almost 3.8 cm ahead of the steering axis. Similarly, the location of the COM of front assembly for 79-degrees head tube angle (HTA) and same steering stem angle (STA) was approximately 4 mm ahead of the steering axis (very close to the steering axis). Finally, for 79-degrees head tube angle (HTA) and 75-degrees steering stem angle (STA) was almost 2.9 cm behind the steering axis. It was found that shifting the COM of front steering assembly forward shifts the self-stability range towards the higher velocity range and vice versa. Furthermore, the self-stability range broadened when the centre of mass of front frame assembly was shifted forward and vice versa. The results were also consistent with the parametric variation performed for bicycle design.

Category 1: Head tube angle is less than the steering stem angle

For each head tube angle, the steering stem angle was varied up to 83-degrees such that the head tube angle is less than the steering stem angle. For the head tube angle of 79-degrees, the stem angle from 80-degrees to 83-degrees was considered. Similarly, for 75-degrees head tube angle, steering stem angle from 76-degrees to 83-degrees was considered. When head tube angle is less than the steering stem angle, the COM of the

front steering assembly is generally ahead of the steering axis. The self-stability range for each combination of head tube angle, steering stem angle and trail was computed for both the standing and seating posture. The results for standing posture on e-scooter are presented in Table 5-5.

Table 5-5: Allowable front steering design parameter range for standing posture on e-scooters having steeper steering stem angle than head tube angle.

Head tube angle (degree)	Allowable trail range for similar self-stability as big wheel bicycle (mm)							
	Steering stem angle (degree)							
	76	77	78	79	80	81	82	83
75	7-8	6-6.5	5.5	No combination of head tube angle, steering stem angle and trail yielded a similar				
76		6.5-8.5	6-7					
77			6.5-9.5	6-8	5.5-6	self-stability as a big wheel bicycle		
78				6.5-10	6-8.5			
79					6.5-11	6-9	5.5-7.5	5-5.5
80						6.5-12	6-10	5.5-8
81							6.5-12.5	6-10.5
82								6.5-13.5

The result showed that shifting the front frame COM further ahead of the steering axis will have a negative effect on the dynamic self-stability. Shifting the COM forward of the steering axis shifted the self-stability region further towards the higher speed region. As a result, the allowable trail range is also decreased. It was also found that the immoderate forward shifting of the front frame COM location could make it difficult to obtain a similar self-stability as the big wheelers.

The analysis was also performed for the seating posture on e-scooter. Unfortunately, no combination of the head tube angle, steering stem angle and trail resulted in a similar self-stability as a typical big wheel bicycle. Thus, shifting the COM forward could not provide any single set of the front design parameters that could result in a similar self-stable speed range for both the standing and seating postures as a typical big wheel bicycle.

Category 2: Head tube angle is greater than the steering stem angle

This category generally has the location of COM of front assembly generally behind the steering axis. Again, a detailed analysis was performed by varying the head tube angle, steering stem angle and trail. Steering stem angles were maintained less than the head tube angle. The self-stable speed range was computed for both standing and seating posture on e-scooter. The results for the standing posture on e-scooter are presented in Table 5-6.

Table 5-6: Allowable front steering design parameter range for standing posture on e-scooters having steeper head tube angle than steering stem angle.

Head tube angle (degree)	Allowable trail range for similar self-stability as big wheel bicycle (mm)							
	Steering stem angle (degree)							
	75	76	77	78	79	80	81	82
76	7.5-11							
77	8-13.5	7.5-12						
78	8.5-16	8-14.5	7.5-13					
79	9-18.5	8.5-17	8-15.5	7.5-14				
80	9.5-21	9-20	8.5-18.5	8-17	7.5-15			
81	10-23.5	9.5-22.5	9-21	8.5-19.5	8-18	7.5-16.5		
82	10.5-26.5	10-25	9.5-24	9-22.5	8.5-21	8-19.5	7.5-17.5	
83	11-30	10.5-28.5	10-27	9.5-25.5	9-24	8.5-23.5	8-20.5	7.5-18.5

Finally, shifting the COM of front frame assembly behind the steering axis by using shallower steering stem angle than the head tube angle, sets of front design configuration were identified for seating posture on e-scooter which yielded a similar self-stability as the referenced big wheel bicycle. Fortunately, the identified front design configuration for seating posture was within the design configuration identified for the standing posture. Therefore, the design guidelines listed in Table 5-7 also represented the common range of the front steering design parameter that is valid for both the standing and seating postures on e-scooters.

Table 5-7: Allowable front steering design parameter range for seating posture on e-scooters having steeper head tube angle than steering stem angle.

Head tube angle (degree)	Allowable trail range for similar self-stability as big wheel bicycle (mm)							
	Steering stem angle (degree)							
	75	76	77	78	79	80	81	82
76	-							
77	-							
78	10	-	-					
79	11-11.5	10-10.5	9.5	-				
80	11.5-12.5	11-11.5	10-10.5	9.5	-			
81	12-13.5	11.5-13	11-12	10-11	9.5-10	9		
82	12.5-15	12-14.5	11.5-13.5	10.5-12.5	10-11.5	9.5-10.5	9	
83	13-16.5	12.5-15.5	12-14.5	11.5-13.5	10.5-12.5	10-11.5	9.5-10	9-9.5

5.4 Frame Design Recommendations

Beside proper steering geometry design, there are other aspects in the PMDs design which contribute to improving the rideability and safety. One of the major aspects that affect the dynamics of the PMDs is the fork length and the stiffness of the frames and selected materials. The front fork length and the stiffness of the frame are crucial to prevent wobbling of the vehicle. The wobbling phenomenon has already been introduced in Chapter 2. Although both tire dynamics and frame flexibility are

responsible for wobble mode oscillation in single-track vehicles, the effect of tire dynamics becomes more pronounced only at very high speed whereas the frame stiffness property can give rise to the wobble mode at normal riding speed. Therefore, the focus was given to the effect of frame design on the dynamics of the PMDs.

Figure 5-16: Effect of frame flexibility on self-stability and wobble mode.

Figure 5-16 shows eigenvalues computed using front frame flexibility extension model to illustrate the effect of frame compliance on the wobble mode. A flexible frame with lower values for frame stiffness and damping can experience wobble instabilities at very low to moderate speed. On the other hand, the stiffer frame having good damping properties and have the ability to suppress the wobble instabilities at low to moderate speed. Many small wheel bicycles and e-scooters, nowadays, do not have a Diamond frame. Generally, the small wheel bicycles are designed with a single top tube as the mainframe component and head tube, and front frame are directly connected to it. Such a frame could easily be twisted if not design with proper care. Therefore, such frames should be designed carefully to improve stiffness characteristics. Furthermore, a stiffer frame design not only improves the wobble mode eigenvalues but also shifts the wobble critical speed further towards the higher speed region. This ensures that the rider will not have unpleasant experience due to wobbling of the front frame and help to improve the riding safety.

Another front frame design parameter that affects the wobble mode is the distance between the twisting axis or hinge from the ground contact along the steering axis, S_C ,

as shown in Figure 5-17. This distance depends on the front fork length. Longer the fork length, the longer will be the value of S_C , and the clearance between the wheel and the fork hinge also increases. The result from eigenvalue analysis, however, showed a negative effect of increasing the hinge distance from the ground or the fork length. The damping of the wobble mode decreased when the S_C distance was increased. The results suggested that the fork length should not be made unnecessarily long because the longer fork length tends to make PMDs wobble at low speed.

Figure 5-17: Effect of the distance between the twisting axis and ground contact point on self-stability and wobble mode.

5.5 Verification of the Developed Design Guideline

The performance of different wheel sized PMDs designed using the improved methodology and proposed guidelines have been compared against the referenced big wheel bicycle. The self-stability analysis, the effect of acceleration and deceleration, lateral perturbation test and handling test were revisited to confirm the improvements analytically. The guidelines were further verified with various riding experiments on prototype bicycles.

5.5.1 Analytical Verification of the Developed Guidelines

Prototypes were developed for bicycles and e-scooters based on the identified front frame design criteria. The geometrical parameters were used to model the prototype in

Solidworks [151], and the mass related parameters were obtained. Thereafter, different mathematical models have been used to compare the performance of improved designs of PMDs with the referenced big wheel bicycles. The results were also compared with conventional designs of PMDs to verify the performance improvement.

5.5.1.1 Self-Stability Analysis for Improved Designs of Small Wheel PMDs

Two 20-inch wheel bicycles having head tube angles of 74.5-degrees and 77-degrees and, the trail of 45 mm and 56 mm, a 16-inch wheel having 81-degrees head tube angle and 29 mm trail and a 12-inch bicycle design had a head tube angle of 79-degrees and trail of 20 mm were modelled in Solidworks [151]. The design configurations were randomly selected from the identified sets of design criteria. The prototype bicycles have unusual front steering geometries compared to the current small wheel bicycle design practice. The associated eigenvalues and computed self-stability speed range are shown in Figure 5-18. Furthermore, Table 5-8 listed the major design parameters for each bicycle designs and detail parameters are listed in Appendix A-3.

Figure 5-18: The eigenvalues and self-stable velocity comparison between the improved designs of small wheel bicycles and referenced big wheel bicycle.

The stability speed range of improved PMDs was comparable to reference big wheel bicycle. Furthermore, the self-stable speed range was also wider and started from a relatively lower speed than that of the referenced big wheel bicycle. The weave

eigenvalues at the unstable low velocities are less positive compared to those of big wheel bicycle, and the eigenvalues within the self-stable region are more negative with an increase in velocity. This suggests that the improved bicycles would be easy to control and more stable at low speeds and with-in the self-stable region.

Table 5-8: Major design parameters for prototype bicycles.

Parameters	26" wheel	20" wheel I	20" wheel II	16" wheel	12" wheel
Head tube angle (degree)	73	74.5	77	82	79
Trail (mm)	56	45	56	29	20
Rake (mm)	45	22	0	0	9
Seat tube angle (degree)	74	73	74	74	83
Chain stay length (mm)	405	415	405	415	206
Saddle height (mm)	952	952	952	952	850
Handlebar height (mm)	899	899	899	899	950
Stack to reach ratio	1.44	1.48	1.40	1.46	1.48
Wheelbase (mm)	990	977	950	884	776
Overall weight (Kg)	≈ 10	≈ 10	≈ 10	≈ 10	≈ 12
Self-stable region (m/s)	4.71-7.63	4.61-7.68	4.64-8.52	4.44-8.37	4.61-7.82

Figure 5-19 shows a similar analysis for the improved designs of e-scooters. Eigenvalues and self-stable speed ranges for both the standing and seating postures are presented. Again, weave eigenvalues for improved designs of e-scooters are less positive compared to referenced big wheel bicycle and self-stability starts at a lower speed and range beyond the big wheel bicycle. The detail parameters are listed in Appendix A-4.

Figure 5-19: Eigenvalues and eigenvalues comparison for improved designs of e-scooter and referenced big wheel bicycle.

Table 5-9: Major design parameters for prototype e-scooters.

Parameters	HTA>STA (seating)	HTA>STA (standing)	HTA=STA
Head tube angle (HTA) (degree)	81	81	79
Trail (mm)	10	10	10
Steering stem angle (STA) (degree)	78	78	79
Handlebar height (mm)	1140	1140	1122
Wheelbase (mm)	827	827	842
Overall weight (Kg)	≈15	≈15	≈15
Saddle position vertical (mm)	700	-	-
Saddle position horizontal (mm)	150	-	-
Self-stable range (m/s)	4.49-7.65	3.88-8.51	4.47-9.12

The oscillation characteristics of a typical big wheel (26-inch) bicycle improved small wheel (8-inch) e-scooter (standing posture), and small wheel (20-inch) bicycle over the selected speeds are presented in Figure 5-20. Time for the improved small wheel PMDs till lean and steer angle reached the set criteria was increased as presented in Table 5-10. Thus, it is clear that the growth of oscillation amplitude at three different velocities were much reduced for the improved bicycle and e-scooters. This attenuation of the oscillation amplitude should improve the riding experience at low velocities at the same time provides easy steering control.

Figure 5-20: Steer and lean responses of a typical big wheel bicycle, an improved small wheel bicycle and, an improved e-scooter to lateral perturbation: a, b, and c are the steer angle response and d, e, and f are the lean response angle at velocities 4.5, 3.5, and 2.5 m/s respectively.

Table 5-10: Time to reach the ± 20 -degrees deviation in lean angle and steer angle for typical designs different wheel sized PMDs.

Speed m/s(km/h)	Time for Lean angle criteria (sec)			Time for Steer angle criteria (sec)		
	Big wheel	Small wheel	Scooter	Big wheel	Small wheel	Scooter
4.5 (16.2)	20+	20+	20+	20+	20+	20+
3.5(12.6)	6.15	6.3	7.5	6.35	6.46	7.7
2.5 (9)	4.10	4.62	4.38	3.88	3.95	4.10

5.5.1.2 Acceleration and Deceleration Analysis

Figure 5-21: Effect of acceleration and deceleration on self-stable speed range for improved design of PMDs.

The effects of the acceleration and deceleration on the self-stable speed range for improved small wheel bicycles and e-scooters were evaluated as shown in Figure 5-21. It was found that the effect of acceleration and deceleration on stability has been improved. The change in the self-stability range has been reduced for small wheelers as compared to the typical designs. The most noticeable change was observed in standing posture on e-scooter. The effect of acceleration and deceleration on its self-stability was

made similar to that of seating posture. Using the shorter trail significantly reduces the effect of weight shift on the front wheel and minimise the effect of acceleration and deceleration on the self-stability. The sensitivity of 20-inch wheel bicycle, e-scooter with standing rider and seating riders was reduced by 24 %, 29 % and 20 %, respectively.

5.5.1.4 Handling Analysis

The results for the steady-state turning for improved bicycles and e-scooters and, referenced bicycle are shown in Figure 5-22. The handlebar torque required for steady-state turning for low to moderate riding speed was reduced and was negative suggesting that an aligning handlebar torque is required for maintaining the steady-state turn. Since the improved bicycles have much steeper head tube angle, the wheelbase is relatively shorter than current designs of small wheel adult bicycles and the referenced big wheel bicycle, the handlebar torque required to maintain the steady-state turning at a constant radius of turn has been reduced in small wheel bicycles. Furthermore, the steering torque was also reduced for improved e-scooters.

Figure 5-22: Comparison of handlebar torque by rider to maintain steady-state turning in improved PMDs: (a) at constant turning radius of 4 m and (b) at constant turning speed of 4.5 m/s.

5.5.2 Experimental Verification of Developed Guidelines

This section presents the verification of the proposed design guidelines by various riding experiments on prototype bicycles. Two prototype 20-inch small wheel bicycles frame were fabricated. The CAD drawings of the fabricated bicycles are shown in Figure 5-23.

Different types of bicycles were used to compare the performance against the prototype bicycle designs. The experimental bicycles included big wheel mountain bicycle, racing bicycles, share bikes (Ofo-bike, Mo-bike), and small wheel bicycles (Moulton,

Brompton). A total of 15 riders participated in the riding experiments. The participants were asked to ride hands-free and perform various task and provide subjective assessments. Among the 15 Participants, 8 participants used to ride bicycles on a regular basis and possessed some skills with hands-free riding, 3 participants had not tried hands-free riding for a long time, and 4 participants were riding hands-free for the first time. The height and weight of the participants are given in Appendix A-5.

Figure 5-23: Prototype 20-inch small wheel bicycles.

The riding experiments were divided into three categories. The first category included the experiments that were designed to evaluate the performance of the typical big wheel and small wheel bicycles subjectively based on the rider's perception. The second category included the experiments that identify the self-stable speed for typical big wheel and small wheel bicycles and improved small wheel bicycles. Finally, the third category of the experiments analysed the steering behaviour of typical big and small wheel and improved small wheel bicycles at various low speed.

5.5.2.1 Subjective Analysis

To evaluate the overall performance of the typical bicycles and improved bicycles, various riding tasks were performed, and comments regarding the performance of the bicycles were collected from the participants. The controlled experiments were conducted. The control parameter in the experiments was the wheel size. All the experiments are performed at the same designated area and riders are asked to ride hands off the handlebar (i.e. hands-free) by maintaining a similar riding posture. The only deviation or the change in the experiments for each rider is the bicycles with different wheel sizes. All the experiments were performed on the flat road.

Figure 5-24: Schematic representation of the experimental area

One lap of the riding track was estimated to be 368 m, and 114 m track was used for straight-line riding. The experimental sites had four corners having outside turning radius of 11-12 m and an inner turning radius of 5-6 m. The experimental track was divided into two lanes, as shown in Figure 5-24. Furthermore, there were some drain covers which were treated as the obstacles and participants were requested to avoid them using their upper body lean and examine the response of the bicycles.

Each participant was given approximately 5 minutes of time to practice hands-free riding. Thereafter, the bike riding experiments were performed at various conditions such as riding hands-free on a straight line, hands-free turning, hands-free riding over the bump, and the hands-free riding with obstacle avoiding. It should be mentioned that the order of bicycles was randomised for each rider. The participants were asked to maintain a similar riding posture, as shown in Figure 5-25 or make the upper body as rigid as possible to minimise any control action from the upper body and hands. Participants were also asked to maintain a constant riding speed by pedalling at a constant rate. A lightweight magnetic speed sensor was attached on the handlebar to facilitate the rider to see real-time riding speed. A few questionnaires were prepared for the participants to provide their subjective perceptions. The questionnaires are listed as follow.

1. Is it possible to ride bicycles hands-free?

2. How is the response of the bicycle on hands-free turning?
3. How is the response of the bicycle while riding over the bump?
4. How responsive is the bike steering when controlling the bicycle with your upper body lean only?
5. Can you avoid obstacles or change the lane by guiding the bike using your upper body lean?

Figure 5-25: Hands-free riding experiments.

Once a rider completed a set of experiments, the comments regarding the abovementioned questionnaires were collected. Some of the common comments for each bicycle are presented in Table 5-11.

Table 5-11: Subjective performance evaluation of typical bicycle designs and improved prototype bicycles

Bicycles	Comments
<p>Racing bicycle (26-inch)</p>	<ul style="list-style-type: none"> • Hands-free riding in a straight-line riding is possible but required a higher speed. However, the hands-free turning was felt difficult by most of the riders, and it required a greater turning radius. • Hands-free riding over the bump was found easy at high speed. • Leaning to steer was possible, and participants could change the lane using upper body control, but quick obstacle avoidance was found difficult.
<p>Montague bicycle (28-inch)</p>	<ul style="list-style-type: none"> • The straight-line hands-free riding was found to be very easy and can be performed at relatively lower speed compared to a racing bicycle. However, the turning required more weight shift and greater radius of turn. This could be because of the heavy front steering assembly compared to a racing bicycle. • Riding over the bump was found easy compared to a racing bicycle. • Lane changing was found easier, but quick turning for obstacle avoidance was difficult.
<p>Mountain bicycle (26-inch)</p>	<ul style="list-style-type: none"> • The straight-line hands-free riding and turning were found to be very easy and can be performed at lower speed compared to racing bicycles. The steering responds to rider's lean was faster than the racing and Montague bicycle. • Riding over the bump was no difficult for mountain bicycles. • Lane changing and obstacle avoidance performance was found better than the Montague and racing bicycles.

<p>Ofo bicycle (26-inch)</p>	<ul style="list-style-type: none"> • Both straight-line riding and turning were difficult. Straight-line riding required unexpectedly higher speed. The hands-free turning was difficult even at a higher speed. The bicycles were very heavy, and the steering was not responsive to the lean. • The riding over the bump was not performed considering the risk associated with riding hands-free at a higher speed for such unstable bicycle. • Lane change and obstacle avoidance were difficult.
<p>Mo bike (24-inch)</p>	<ul style="list-style-type: none"> • Both straight-line riding and turning were difficult. Straight-line riding required unexpectedly higher speed. The hands-free turning was difficult even at a higher speed as that of Ofo bicycle. The bicycles were very heavy, and the steering was not responsive to the lean. • The riding over the bump was not performed considering the risk associated with riding hands-free at a higher speed for such unstable bicycle. • Lane change and obstacle avoidance were difficult.
<p>Moulton bicycle (20-inch)</p>	<ul style="list-style-type: none"> • The straight-line riding was possible but required higher velocity compared to racing bike, Montague bike and mountain bike. Participants who tried hands-free for the first time could not turn the bicycle around the corner. For those who are either regular or had the experience of hands-free riding also felt difficulties in turning the bicycle. The quick turning near the inner turning radius was very difficult as it required a relatively greater turning radius. • The hands-free riding over the bump could be performed at very high speed but handlebar tends to veer off when rolling over the bump. • The steering was felt very sensitive towards the slightest lean of the rider's body and most of the time it

	<p>tended to oversteer. Therefore, it was difficult to avoid the obstacle with quick body lean control. However, the rider can change the lean but, again, it required a longer distance.</p>
<p>Brompton bicycle (16-inch)</p>	<ul style="list-style-type: none"> Surprisingly, the bicycle was very difficult to perform both hands-free straight-line riding and turning. This bicycle was a folding small wheel bicycle. To facilitate the folding mechanism, the brake and gear shifting wires were routed in a specific manner which was found to restrict the free movement of the steering. Since the hands-free riding was found extremely difficult in this bicycle, no further hands-free experiments were conducted. However, the steering was felt nervous or twitchy even riding at a normal speed with hands on the handlebar.
<p>The improved bicycle I (20-inch)</p>	<ul style="list-style-type: none"> Hands-free riding in the straight-line was found to be very easy and could be performed at relatively lower speed compared to big wheel bicycles and small wheel bicycles used in this study. The hands-free turning was also felt easy to perform and could be performed at a relatively lower speed compared to other bicycles. The steering was very responsive towards the rider's lean, thus, did not have to make large weight shift to perform the cornering. Furthermore, quick turning around the inner radius of turn was also possible. The bicycle can comfortably be rolled over the bump without any control on the handlebar. Furthermore, the steering did not veer off while rolling over the obstacle. The bicycle was very quick and easy to change the lanes—furthermore, the quick response of the steering allowed to avoid the obstacle easily.

<p>Improved bicycle II (20-inch)</p>	<ul style="list-style-type: none"> • Both Hands-free riding in the straight-line and hands-free turning was also felt comfortably easy to perform and could be performed at relatively lower speed compared to other bicycles. As long the rider did not use body lean or weight shift, the bicycle can continue to ride in a straight line. The steering was very responsive towards the rider's lean, thus, did not have to make large weight shift to perform the cornering. Furthermore, because of the shorter wheelbase, the quick turning around the inner radius of turn was also possible with an improved bicycle. • The bicycle can comfortably be rolled over the bump without any control on the handlebar. Furthermore, the steering did not veer off while rolling over the obstacle. • The bicycle was very quick and easy to change the lanes. Furthermore, the quick response of the steering allowed to avoid the obstacle easily.
--	--

The results are consistent with the general perception of the riders and the results presented in Chapter 4. The typical big wheel bicycles were found easy to ride hands-free especially in straight-line at relatively lower speed compared to small wheel bicycles. Moreover, the hands-free turning was also possible, and steering behaviour was found much balanced compared to the current design of small wheel adult bicycles. Surprisingly, the share bikes were found much difficult to ride hands-free and unstable despite having bigger wheels. This finding also supported the fact that bicycle frame design plays an important role in the overall riding performance and only a properly designed frame can improve the stability, rideability and overall safety of the vehicle-rider system. In contrast, the properly designed small wheel prototype bicycles were found to be much easier to ride hands-free, and overall rideability was also perceived to be improved.

5.5.2.2 Minimum Hands-Free Riding Experiment

Bicycles are self-stable within a certain range of forward speed. The range, as explained in Section 2.2.1.2, is bounded by two critical speeds called the weave critical speed and

capsize critical speed. The weave critical speed is the lower limit from which the bicycle becomes self-stable. The hands-free riding is possible only when the vehicle-rider system is at a self-stable state. In other words, the threshold speed at which a rider can perform hands-free riding should be equal to the weave critical speed. Therefore, for each selected bicycle, the threshold speeds at which participants can perform hands-free were obtained.

A bicycle speedometer (Cateye Velo wireless) was used to record minimum speed required to perform the hands-free riding. Riders started from low speed; around 12 km/h. When the rider maintains a relatively constant speed, he/she would try to perform hands-free riding. If unsuccessful, riders would repeat the procedure with increased speed. The threshold speeds were recorded just when bicycles were felt easy to ride hands-free. In this way, the threshold speeds for experimental bicycles were obtained. The results for threshold speed obtained for big wheel bicycles, share bicycles, small wheel bicycle and improved small wheel bicycles are shown in Figure 5-26.

Figure 5-26: Experimental minimum hands-free speed for different wheel sized bicycle models.

The results provided strong evidence that typical big wheel bicycles (mountain, Montague, racing) can be ridden hands-free at a lower speed compared to the typical small wheel bicycle (Moulton). The minimum speed at which rider could ride hands-free also depends on the weight of the vehicle-rider system and the skills. Since the weight of each participant was not exactly the same, and riding skills also differed, the minimum speed at which they rode the bicycles were also different. However, analysing the participants having similar hands-free riding skill, a general trend was found that heavier participant required relatively higher speed compared to the lightweight

participant. Similarly, participants who tried the hands-free riding for the first time also required higher speed compared to participants who were a regular cyclist or had some past experience on hands-free riding. However, the trend of threshold speed required to perform hands-free riding was found similar for all the participants. The typical big wheel bicycle was found to be easy for hands-free riding than typical small wheel bicycle. On the other hand, the improved bicycles have lower threshold speed compared to the conventional big wheel and small wheel bicycles.

The experimental results for bicycles were also compared with the theoretical weave critical speed computed from the benchmarked model. The theoretical values and experimental values are based on participants having 75 Kg weight, 177 cm height (Rider 1) and 52 Kg weight, 163 cm height (Rider 2) are presented in Table 5-12. All anthropomorphic measurements were taken for these two participants, and weave critical speed was computed using the benchmarked model for vehicle-rider model.

Table 5-12: Comparison between the theoretical minimum speed and experimental threshold hands-free speed.

Bicycles	Wheel size	Rider 1 (75 Kg)		Rider 2 (52 Kg)	
		Theoretical weave speed (km/h)	Experimental threshold hands-free speed (km/h)	Theoretical weave speed (km/h)	Experimental threshold hands-free speed (km/h)
Big wheel	26-inch	16.99	16.50	15.73	15.50
Montague	28-inch	17.13	16.30	16.05	15.50
Racing	26-inch	17.50	17.50	16.18	16.00
Moulton	20-inch	19.17	19.00	18.00	18.00
Improved I	20-inch	16.63	16.00	15.69	15.40
Improved II	20-inch	16.70	15.80	15.51	15.20

As can be observed from Table 5-12, the experimental values were very close to the theoretical values. However, the experimental values were found slightly lower than the theoretical values. This difference can be attributed to the fact that the theoretical values were based on the rigid body model with no external control from the rider, whereas, in real experiments, participants did not provide handlebar control, but there might exist slight control input from the upper body movement of the rider that enabled participants to ride at lower speed than the theoretical speed. However, the participants were requested to minimise their upper body movements, as a result, the maximum percentage difference between the theoretical and experimental values was found to be less than 5%.

5.2.2.3 Oscillation Analysis

The steering behaviours of four bicycles were analysed in order to compare their performance at low speed. The four bicycles were a 26-inch racing bicycle, 20-inch Moulton bicycles and two improved prototypes 20-inch small wheel bicycles. In this experiment, each bicycle was ridden in a straight line without hands on the handlebar. Three different riding speeds (10 ± 0.2 km/h; 12.6 ± 0.2 km/h and 14.5 ± 0.2 km/h) were selected. This experiment was performed by a 52 Kg rider for whom all the selected riding speeds fall in the unstable region. The major requirement in this experiment was the sensory device should not obstruct the free movement of the steering. A mobile application called “Sensor Kinetics” has been used to record the steering angles while riding. Sensor kinetics measures the rotational angles based on the accelerometer and the gyroscopic sensor used in the mobile. The mobile was securely mounted on the handlebar stem near the steering axis to minimise the effect of its weight. The main advantage of using this mobile app was that it did not obstruct the handlebar rotation.

Figure 5-27: Curve fitting for a typical racing bike’s steering oscillation characteristic at different speeds. (a) speed= 10 km/h (b) speed= 12.6 km/h and (c) speed = 14.5 km/h.

For each bicycle, five sets of steering angles were collected while riding in a straight-ahead direction. Since all bicycles were in weave unstable region, the steering oscillated about the direction of travel. Because of the unstable nature, the oscillation amplitude was increasing with time. Such unstable weaving behaviour of the bicycle can be fitted using an exponentially increasing sinusoidal oscillation. The following exponential equation was used to fit the steering data.

$$\delta = Ae^{bt} \sin(\omega t + \phi) \quad (5-1)$$

Where, A= amplitude coefficient, b= damping coefficient, ω = angular speed and ϕ = phase angle. The curves were fitted using the least square error method. A typical result obtained for racing bicycle is shown in Figure 5-27.

The experimental results corroborated with the results obtained from the mathematical model as obtained in Section 4.2.1.1.2. It was evident that the lower riding speeds have a greater oscillation increment rate compared to the higher speed. The similar analysis was performed for Moulton bicycle (20-inch small wheel bicycle) and two prototype bicycles, developed in this study, by curve fitting the steering oscillation data. The steering oscillation characteristics for each bicycle is presented in Figure 5-28.

The incremental rate of steering oscillation was found to be maximum for 20-inch wheel Moulton bicycle. The big wheel bicycle had relatively smaller incremental rate compared to Moulton bicycles. The prototype bicycles had the least incremental rate in the oscillation as predicted by the analytical results as presented in Chapter 4. The time for bicycle steering angle to cross the 20-degrees margin was calculated from each set of data points. The prototype bicycles required a relatively long time to reach the 20-degrees margin compared to the racing bicycle and the Moulting bicycle. This also confirms the smaller incremental rate for improved bicycles. The average time taken by each bicycle to cross the 20-degrees margin at a different velocity is presented in Table 5-13.

Table 5-13: Average time for steering angle to reach the 20-degrees deviation.

Speed (km/h)	Time to reach ± 20 -degree margin for each bicycle (sec)			
	Moulton	Racing	Prototype I	Prototype II
10	1.51	2.31	4.16	4.27
12.6	2.42	4.18	4.87	6.36
14.5	4.65	8.37	10+	10+

Figure 5-28: Steering oscillation characteristics of typical bicycles and improved bicycles at different speeds. (a) speed = 10 km/h, (b) speed = 12.6 km/h and (c) speed = 14.5 km/h.

Table 5-14: Comparison of real part of eigenvalues from theoretical model and experiments at 14.5 km/h

Bikes	Real parts of eigenvalues at 14.5 km/h	
	Theoretical	Experimental
Racing	0.4	0.28
Moulton	0.45	0.39
Improved I	0.3	0.24
Improved II	0.2	0.15

The different between the theoretical and experimental values is due to the influence of the rider's body lean control. The theoretical model assumes no external control from the rider's body movements whereas involuntary rider's lean control cannot be eliminated in actual hands-free experiments, especially below weave critical speeds. For

the same reason, the experimental minimum hands-free riding speed is lower than the predicted minimum self-stability speed for each bike, Table 5-13. However, the experimental results have shown a similar trend as predicted by the analytical results. The subjective analysis, hands-free minimum speed analysis and the steering oscillation analysis results have confirmed that the current designs of small wheel bicycles are less stable, difficult to ride and requires more handlebar control to maintain the balance compared to the conventional big wheel bicycles. On the other hand, the improved designs of small wheel bicycles based on the proposed steering design guidelines have been found easy to ride, require less control input and can be ridden hands-free relatively at a lower speed than current designs of small and big wheel bicycles. Thus, the experimental results have also confirmed the performance improvement in the prototype small wheel bicycles.

5.6 Summary of Chapter 5

This section presented a modified design philosophy and an improved and cost-effective methodology for designing small wheel bicycles and e-scooters. The proposed methodology incorporates a mathematical model for dynamic evaluation of the frame design in the early stage of the design. Using the proposed methodology, frame design guidelines have been developed especially focusing on the front steering geometry. The primary objective of the proposed design guidelines was to make the small wheel PMDs at least as stable as the big wheel bicycle.

The chapter also presented the theoretical analysis on the performance of improved PMDs based on proposed guidelines and compared with referenced big wheel bicycles. Prototype bicycles were fabricated by selecting the front steering design parameters based on the proposed design guidelines. Various experiments were conducted to validate the improvements of the prototype bicycles compared to the conventional big wheel and small wheel bicycles.

The experimental observations agreed with the mathematical analysis and, thus, validated the improved methodology and proposed front design steering guidelines. The results provided evidence that the design of the current small wheel bicycles could be improved for better stability and performance by carefully selecting the front design parameters. It also showed that the difference in performance between different big

wheel bicycle and small wheel bicycle was not because of the smaller wheel size but due to the improper selection of design parameters and configurations.

Chapter 6. Conclusions

This chapter summarises the overall research findings, draws conclusions and suggests recommendations based on the overall findings.

6.1 Conclusions

This study is focused on the performance evaluation of the current design of small wheel single-track PMDs in the search for possible design modifications to improve their stability and handling performance. Many riders perceived small wheelers difficult to ride and unstable and the rideability is always questioned. With the increasing number of PMDs related accidents, the overall performance of these PMDs should be improved to elevate the rider's safety and the rideability. This chapter presents the summary research work carried out to meet the objectives followed by the conclusions drawn from the results obtained from the comprehensive studies.

A total of 111 big wheel bicycles, 27 small wheel bicycles and 28 e-scooters designs were considered for data collection. The collected data were compared against the available design guidelines for the frame design. The coherence between the available design guidelines and the collected data were analysed. The most common design range for major design parameters was identified for different wheel sized bicycles and e-scooters. Thereafter, the dynamic performance of current designs of PMDs was quantitatively analysed using validated benchmarked models and some extended benchmarked model. The analysis was performed by comparing the eigenvalues, and the self-stable speed region for current designs of different wheel sized PMDs. The analysis confirmed the general perception for the small wheelers of being less stable, twitchy or nervous and difficult to ride at normal riding speed. Therefore, the current design philosophies and methodology were modified by incorporating the dynamic analysis into the design process. Moreover, the front steering design and its complex effects on the dynamics of single-track PMDs were poorly understood. To facilitate the novel designers with minimum technical knowledge, the front steering design guidelines have been proposed for 20-inch, 16-inch, 12-inch wheel bicycles and 8-inch wheel e-scooters. Various experiments were conducted on the prototype bicycles having unconventional front steering designs based on the proposed guidelines.

The overall research work presented in this thesis can be divided into three sections. First, analysis of the current design philosophies and guidelines to understand the overall design methodology of single-track PMDs. Second, evaluation of the dynamic self-stability and handling performance of the current design of PMDs. Third, the development of improved methodology and guidelines in order to enhance the stability and handling performance of small wheel PMDs and experimental verification of the proposed guidelines.

For current designs of big wheel bicycles, small wheel bicycles and e-scooters, the following conclusions were drawn based on their design philosophies, methodologies and design approaches.

- The current design philosophies are driven by two aspects. The first is the intended purpose and the second is rider's fit.
- A strong agreement was observed among the various design guidelines for rider's fit. Furthermore, the rider's fit guidelines and parameter selection approach as used for big wheel bicycles have a strong influence on small wheel adult bicycles and e-scooter designs.
- Although rider's fit parameters are much better established and well understood, the coupling effect between the front frame design parameters (such as head tube and trail length) and wheel size on stability is not well understood. There are only a few guidelines developed from the years of design experiences on big wheel bicycles.
- A strong agreement was found between the available guidelines and current designs of big wheel bicycles. Most of the front steering designs were found to be influenced by the concept of the longer trail for better self-stability.
- The most common range of head tube angle is found to be in between 73-74 degrees, and trail range is found to be 50-60 mm. The most commonly recommended sets of front design parameter are 73 degrees head tube angle and 56 mm trail.
- Owing to the lack of proper design guidelines for small wheel bicycles, the design approaches are found to be strongly influenced by the concept of a longer trail to achieve better stability. Therefore, head tube angles are relatively shallower compared to big wheel bicycles.

- E-scooters are designed with much steeper head tube angles ranging from 75 degrees to 80 degrees and shorter wheelbase compared to typical big and small wheel bicycles. The trail value typically ranges from 15 mm to 30 mm.

The following conclusions were drawn from the mathematical analysis performed to compare the stability and handling performance of different wheel sized PMDs.

- Thus, the results obtained from the mathematical models corroborated the subjective perception of the riders on small wheelers.
- The current designs of small wheel bicycles and e-scooters require higher speed to become self-stable. At normal riding speed of 15-17 km/h, the small wheel PMDs were found to have no self-stability.
- The self-stable speed shifted further towards the higher speed, and the difference between the self-stable speed region became significantly wider for PMDs with decreasing wheel size compared to the big wheel bicycles.
- The current design practice of making shallower head tube angle to increase the trail values for small wheel bicycles could not yield similar stability as big wheel bicycles.
- At lower speed, the amplitude of oscillation for small wheel PMDs was found to increase much faster than the big wheel bicycles. Therefore, when vehicle encountered any obstacle, the tendency of the vehicle to veer off is greater in smaller wheel sized PMDs.
- The weave eigenvalues for smaller wheel sized PMDs were found more positive for lower to moderate riding speed. This suggested that small wheelers require more handlebar control to balance the vehicle compared to big wheel bicycles.
- The proper connection at the pivot point between the rider and the vehicle was found very crucial for self-stability in the rider-vehicle system. A poor connection in standing posture on e-scooter was found likely to make the riding unstable and more vulnerable at the in acceleration and deceleration compared to the seating posture.
- Small wheel e-scooters were found more sensitive towards the acceleration and deceleration. The stability of the standing posture on e-scooter was affected the most because of the greater weight shift on the wheels when accelerated and decelerated.

- Small wheel bicycles with shallower head tube angle, longer trail and wheelbase were found to require more handlebar aligning torque to maintain the steady-state turning at lower speed compared to typical big wheel bicycles.
- The compact design of e-scooters required much lesser handlebar torque which makes them more manoeuvrable compared to bicycles.

The conclusions drawn from Chapter 5 on the development of the improved guidelines and experimental verification are:

- The current designs of small wheel bicycles and e-scooters are not well optimized for its rideability and safety, and improvement could be achieved by properly selecting the design parameters.
- The discrepancy in the dynamic performance of small wheelers is not only because of the smaller wheel size and reduced spin inertia. In fact, the information that we currently have for small wheel PMDs design is not adequate to ensure stability and performance.
- In general, small wheelers should be designed with a steeper head tube angle and trail should be selected accordingly. This approach is the opposite of the current design approach. Furthermore, it was also found that for each set of front steering design configuration for big wheel bicycle, there exists a minimum head tube angle and trail value that can result in a similar self-stability as referenced big wheel bicycles.
- Small wheel PMDs should be designed with shorter front fork length to ensure the wobble stability at a moderate speed.
- Finally, the riding experiments on typical big wheel bicycle, small wheel bicycles and improved small wheel bicycle corroborated the results obtained from the mathematical models.
- The current design of small wheel bicycle was found difficult to ride hands-free at lower speed compared to big wheel bicycle, whereas, improved bicycles, developed in this study, could easily be ridden hands-free at lower speed and oscillation increment was also lower compared to the current designs of big and small wheelers.

It was clearly shown that a proper modification of the frame could improve the stability and handling performance of the current designs of small wheel PMDs. The

experiments on prototype bicycles and mathematical analysis also agreed which validates the proposed guidelines for designing the small wheel PMDs. Therefore, the study could contribute to improving the future generation of small wheel PMDs by making them easy rideable and safer. Moreover, it is believed that the improved designs of stable and safer PMDs could help reduce the number of accidents and injuries associated with them.

6.1 Future Work

Although a comprehensive analysis has been presented in this study, there are still some of the areas to further investigate in future.

- **Rider control models**

The study presented here was mainly focused on the open-loop stability of the vehicle-rider system. The open-loop analysis is one of the important tools to evaluate the inherent characteristics of the vehicle designs. Particularly, in this study, where the performance of different designs of PMDs was to be evaluated, the open-loop stability was best-suited for the analysis. However, in real life riders hardly ride these single-track vehicles hands off the handlebar. The vehicles are constantly controlled by the rider's input based on the external sensory. There is very limited research in the rider control model on bicycles. Extending the rider control model on different PMDs and comparing with the big wheel bicycles could also provide more insight into the performance discrepancy between the bigger wheel vehicles and small wheelers. Furthermore, as the uses of e-scooters are booming in recent time, and thus, injuries associated with e-scooters are also increasing at an alarming rate. Therefore, a detail investigation on rider control model for e-scooters and other small wheel bicycles could help to make such vehicle safer and easy rideable to the public.

- **Detail experiments on e-scooters**

This study is mainly based on the analytical approach for comparing the performance of different wheel sized PMDs. The mathematical models used in this study were the validated models from previous studies, and it is a good approach to use validated models for obtaining accurate results. The study has also presented experimental studies that have successfully validated the analytical results. However, the experiments were carried out only for prototype bicycles. It was difficult to perform

a similar set of experiments on e-scooters as they were found extremely unstable at normal riding speed and rider's safety concerns were very high. Therefore, the experiments on e-scooters have not been considered in the present study. Although it is valid to postulate that results based on bicycles are also valid for e-scooters owing the similar principle design as the bicycles, the research can be extended in future by experimental analysis on e-scooters to validate the analytical results. However, it would require a very safe and secure experimental set-up. One of the interesting investigations on e-scooters could be to analyze the difference in the rigidity of the connection between the rider and e-scooter for standing and seating posture.

- **Handling models for PMDs**

Handling is vastly associated with subjective perception; therefore, it simply increases the complexity of the analysis. The handling aspect for single-track PMDs like bicycles and e-scooters designs are still at its armature stage compared to aeroplane and motorcycle designs. Only steady-state turning and obstacle rollover tendency have been evaluated in this study. The handling behaviour of the current designs of PMDs could be further investigated using various indices and methods such as handling quality matrices, acceleration index, Koch Index, lane change roll index etc. In addition, new indices could also be developed especially for small wheel bicycles and e-scooters to ensure better handling in conjunction with stability.

- **PMDs design Software**

There are various design software codes available for evaluating the performance of the motorcycle. However, there is hardly any software for the detailed design analysis of the bicycle and the e-scooters. The only software available for evaluating the performance of bicycles is Jbike6 [184]. But this software also requires highly technical knowledge. On the other hand, there are few bicycle design software, but this software only helps with geometrical design and do not provide a performance analysis of the design configuration. Therefore, a software that provides flexibility to modify the configuration as per the rider's fit, model a rider on to it and evaluates the detail results performance could be developed to help novel designers.

References

- [1] D. Mozer. "Chronology of the Growth of Bicycling and the Development of Bicycle Technology." International bicycle fund. <http://www.ibike.org/library/history-timeline.htm> (accessed May 8, 2016).
- [2] D. G. Wilson and J. Papadopoulos, "A short history of bicycling," in *Bicycling science*: MIT press, 2004.
- [3] worldometers. "Bicycles produced." <http://www.worldometers.info/bicycles/> (accessed October 9, 2016).
- [4] S. Wilson, "Bicycle technology," *Scientific American*, vol. 228, pp. 81-91, 1973.
- [5] B. europe. "Growth on All Fronts for EU Bike Industry in 2014." <http://www.bike-eu.com/sales-trends/nieuws/2015/11/growth-on-all-fronts-for-eu-bike-industry-in-2014-10124768> (accessed January 2, 2016).
- [6] "Global folding bikes market." Transparency market research <https://www.transparencymarketresearch.com/pressrelease/folding-bikes-market.htm> (accessed 2019).
- [7] "Dahon doubling production capacity with new factory, and expanding distribution ", ed: Dahon, 2018.
- [8] A. C. Scholten, S. Polinder, M. J. Panneman, E. F. Van Beeck, and J. A. Haagsma, "Incidence and costs of bicycle-related traumatic brain injuries in the Netherlands," *Accident Analysis & Prevention*, vol. 81, pp. 51-60, 2015.
- [9] M. J. Thompson and F. P. Rivara, "Bicycle-related injuries," *American family physician*, vol. 63, no. 10, pp. 2007-2013, 2001.
- [10] A. Azad, "That electric scooter might be fun. It also might be deadly," in *CNN*, ed, 2018.
- [11] S. Chapman, C. Webber, and M. O'Meara, "Scooter injuries in children," *Journal of paediatrics and child health*, vol. 37, no. 6, pp. 567-570, 2001.
- [12] S. Jayson, "Injuries prompt CDC investigation into e-scooters," ed: CNN, 2019.
- [13] J. Forester;, "Report on stability of the Dahon bicycle," vol. 2017, ed, 1989.
- [14] J. Heine. Tracking Straight. *advenchercycling* [Mechanical Advantage]. Available: https://www.adventurecycling.org/default/assets/resources/20140601_MechanicalAdvantageTrail_Heine.pdf
- [15] A. L. Schwab. Why bicycles do not fall. Available: <https://www.youtube.com/watch?v=2Y4mbT3ozcA&t=771s>.
- [16] J. Kooijman, J. Meijaard, J. M. Papadopoulos, A. Ruina, and A. Schwab, "A bicycle can be self-stable without gyroscopic or caster effects," *Science*, vol. 332, no. 6027, pp. 339-342, 2011.
- [17] J. Heine. (2005) Front-end geometry for different speeds, loads and tire sizes. *Bicycle Quarterly* 8.

- [18] A. L. Schwab and J. Meijaard, "A review on bicycle dynamics and rider control," *Vehicle System Dynamics*, vol. 51, no. 7, pp. 1059-1090, 2013.
- [19] R. Liu and R. Parthasarathy, "Segway Human Transporter (HT): potential opportunities and challenges for transportation systems," in *82nd Annual Meeting of the Transportation Research Board, Washington, DC*, 2003.
- [20] R. Dowling, J. D. Irwin, I. J. Faulks, and R. Howitt, "Use of personal mobility devices for first-and-last mile travel: The Macquarie-Ryde trial," in *Proceedings of the 2015 Australasian Road Safety Conference*, 2015.
- [21] M. Siman-Tov, D. H. Jaffe, K. Peleg, and I. T. Group, "Bicycle injuries: a matter of mechanism and age," *Accident Analysis & Prevention*, vol. 44, no. 1, pp. 135-139, 2012.
- [22] W. Weijermars, N. Bos, and H. L. Stipdonk, "Serious road injuries in the Netherlands dissected," *Traffic injury prevention*, vol. 17, no. 1, pp. 73-79, 2016.
- [23] P. Schepers *et al.*, "An international review of the frequency of single-bicycle crashes (SBCs) and their relation to bicycle modal share," *Injury prevention*, pp. injuryprev-2013-040964, 2014.
- [24] P. Schepers *et al.*, "An international review of the frequency of single-bicycle crashes (SBCs) and their relation to bicycle modal share," *Injury prevention*, vol. 21, no. e1, pp. e138-e143, 2015.
- [25] S. Boufous, L. de Rome, T. Senserrick, and R. Q. Ivers, "Single-versus multi-vehicle bicycle road crashes in Victoria, Australia," *Injury prevention*, vol. 19, no. 5, pp. 358-362, 2013.
- [26] P. Schepers and K. K. Wolt, "Single-bicycle crash types and characteristics," *Cycling Research International*, vol. 2, no. 1, pp. 119-135, 2012.
- [27] T. Hadland and H.-E. Lessing, *Bicycle design: An illustrated history*. Mit Press, 2014.
- [28] L. M. Neal. "The Bicycle's Deadly History." <https://allthatsinteresting.com/bicycle-history/2> (accessed 2018).
- [29] "The history of the folding bike." The folding cyclist. <http://www.foldingcyclist.com/folding-bike-history.html> (accessed Feb 17, 2017).
- [30] T. Hadland, "Small wheels for adults bicycles," in *Hadland's Blog* vol. 2017, ed, 2012.
- [31] M. b. company. "Heritage " <http://www.moultonbicycles.co.uk/heritage.html> (accessed Feb 25, 2017).
- [32] "The Bickerton Story " Bickerton Portables <http://www.bickertonportables.co.uk/heritage/the-bickerton-story/> (accessed 2016).
- [33] "Brompton History " Brompton Bike. <https://www.brompton.com/about-us/history> (accessed 2016).
- [34] H.-E. L. Tony Hadland, "Small-wheeled bicycles," in *Bicycle design: An illustrated History* The MIT Press, 2014, pp. 447-471.
- [35] E. Salmeron-Manzano and F. Manzano-Agugliaro, "The electric bicycle: Worldwide research trends," *Energies*, vol. 11, no. 7, p. 1894, 2018.

- [36] J. OGDEN BOLTON, "Electric bicycle " US Patent Appl. US552271A, 1895.
- [37] A. A. Parker and A. Alan, "The electric power assisted bicycle: a clean vehicle to reduce oil dependence and enhance the mobility of the elderly," in *International Conference on Sustainability Engineering and Science, Auckland, New Zealand*, 2004.
- [38] "ELECTRIC SCOOTER: ORIGINS, HISTORY AND EVOLUTION." Madcharge. <https://www.madcharge.com/electric-scooter-origins-history-and-evolution/> (accessed.
- [39] J. Griffin. "Electric Scooter History: The Early Story." Best electric ride. <https://www.bestelectricride.com/electric-scooter-history/> (accessed.
- [40] "Inokim history " Inokim e-scooter <https://inokim.com/about-us/#history> (accessed.
- [41] J. F. Bottomley, *The Velocipede: Its Past, Its Present, and Its Future*. By JFB [ie Joseph Firth Bottomley]... With 25 [sic] Illustrations. Simpkin, Marshall & Company, 1869.
- [42] Anonymous, *The world of wonders; a record of things wonderful in nature, science, and art*. London, Cassell, Petler, and Galpin, 1869.
- [43] Anonymous, *Velocipedes*. . Van Nostrand's eclectic engineering magazine, I, 1969.
- [44] R. K. Philp, *Best of Everything*. JB Lippincott and Company, 1870.
- [45] W. Thomson, "Nature Series. Popular Lectures and Addresses.(A lecture delivered at the Institution of Civil Engineers, May 3, 1883) Vol. 1," ed: Macmillan and Co London:, 1889.
- [46] A. Gray, *A treatise on gyrostatics and rotational motion*. Macmillan, 1918.
- [47] D. E. Jones, "The stability of the bicycle," *Physics today*, vol. 23, no. 4, pp. 34-40, 1970.
- [48] Anonymous, *Amateur work, illustrated, volume 1*. Ward, Lock and Co., 1881.
- [49] A. Doyle, "The essential human contribution to bicycle riding," *Training, human decision making, and control*, pp. 351-370, 1988.
- [50] R. P. Scott, *Cycling art, energy and locomotion*. Рипол Классик, 1889.
- [51] A. Davison, "Upright frames and steering," ed: Cycling, 1935.
- [52] A. Sharp, *Bicycles & Tricycles: An Elementary Treatise on Their Design and Construction, with Examples and Tables*. Longmans, Green, 1896.
- [53] R. Rice, "A note on design criteria for bicycle stability in terms of front end geometry," *Internal memorandum, Calspan, Buffalo, NY*, 1974.
- [54] E. Doehring, "Über die Stabilität und die Lenkkräfte von Einspurfahrzeugen," Technische Hochschule Braunschweig, 1953.
- [55] R. N. Collins, "A mathematical analysis of the stability of two wheeled vehicles," 1963.
- [56] V. Cossalter, *Motorcycle dynamics*. Lulu. com, 2006.
- [57] F. J. Whipple, "The stability of the motion of a bicycle," *Quarterly Journal of Pure and Applied Mathematics*, vol. 30, no. 120, pp. 312-321, 1899.

- [58] E. Carvallo, "Théorie du mouvement du monocycle et de la bicyclette," 1901.
- [59] F. Klein and A. Sommerfeld, "Über die Theorie des Kreisels (Teubner-Verlag, Leipzig)," 1910.
- [60] G. S. Bower, "Steering and stability of single-track vehicles," *The Automobile Engineer*, vol. 5, pp. 280-283, 1915.
- [61] R. S. Hand, *Comparisons and stability analysis of linearized equations of motion for a basic bicycle model*. Cornell University, May, 1988.
- [62] R. Pearsall, "The stability of a bicycle," in *Proc. Instn Automobile Engrs*, 1922, vol. 17, no. 1, pp. 395-404.
- [63] A. L. Schwab. "History of bicycle steer and dynamics equations." Delft University (accessed).
- [64] S. P. Timoshenko and D. H. Young, "Advanced dynamics," 1948.
- [65] E. Döhring, *Stability of single-track vehicles*. 1957.
- [66] D. Singh, *Advanced Concepts of the Stability of Two-Wheeled Vehicles: Application of Mathematical Analysis to Actual Vehicles*. University of Wisconsin--Madison, 1964.
- [67] J. I. Neimark and N. A. Fufaev, "Dynamics of nonholonomic systems, Translations of Mathematical Monographs, vol. 33," *American Mathematical Society, Providence, Rhode Island*, vol. 518, pp. 65-70, 1972.
- [68] D. Singh and V. Goel, "Stability of Rajdoot scooter," SAE Technical Paper, 0148-7191, 1971.
- [69] R. S. Sharp, "The stability and control of motorcycles," *Journal of mechanical engineering science*, vol. 13, no. 5, pp. 316-329, 1971.
- [70] R. S. Sharp, "The dynamics of single track vehicles," *Vehicle system dynamics*, vol. 5, no. 1-2, pp. 67-77, 1976.
- [71] R. D. Roland and D. E. Massing, *A digital computer simulation of bicycle dynamics*. Cornell Aeronautical Laboratory, 1971.
- [72] D. H. Weir, "MOTORCYCLE HANDLING DYNAMICS AND RIDER CONTROL AND THE EFFECT OF DESIGN CONFIGURATION ON RESPONSE AND PERFORMANCE," 1973.
- [73] D. Singh and V. Goel, "Stability of single track vehicles," *Vehicle System Dynamics*, vol. 4, no. 2-3, pp. 116-120, 1975.
- [74] C. Adiele, "Two wheeled vehicle design," *Sibley School of Mechanical and Aerospace Engineering, Cornell University, MSD-79-03, July*, 1979.
- [75] J. Lowell and H. McKell, "The stability of bicycles," *American Journal of Physics*, vol. 50, no. 12, pp. 1106-1112, 1982.
- [76] J. M. Papadopoulos, "Bicycle steering dynamics and self-stability: a summary report on work in progress," *Cornell Bicycle Research Project, Cornell University*, pp. 1-23, 1987.

- [77] R. S. Sharp and D. J. Limebeer, "A motorcycle model for stability and control analysis," *Multibody system dynamics*, vol. 6, no. 2, pp. 123-142, 2001.
- [78] B. SIM. Mechanical Simulation. <https://www.carsim.com/products/bikesim/index.php> (accessed 2018).
- [79] V. Cossalter and R. Lot, "A motorcycle multi-body model for real time simulations based on the natural coordinates approach," *Vehicle system dynamics*, vol. 37, no. 6, pp. 423-447, 2002.
- [80] J. Meijaard, "Derivation of the linearized equations for an uncontrolled bicycle," Internal report, University of Nottingham, UK, 2004.
- [81] J. P. Meijaard, J. M. Papadopoulos, A. Ruina, and A. L. Schwab, "Linearized dynamics equations for the balance and steer of a bicycle: a benchmark and review," in *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 2007, vol. 463, no. 2084: The Royal Society, pp. 1955-1982.
- [82] K. J. Astrom, R. E. Klein, and A. Lennartsson, "Bicycle dynamics and control: adapted bicycles for education and research," *IEEE Control Systems Magazine*, vol. 25, no. 4, pp. 26-47, 2005.
- [83] J. K. Moore, "Human control of a bicycle," PhD, University of California Davis, 2012.
- [84] P. Basu-Mandal, "Studies on the dynamics and stability of bicycles," 2009.
- [85] G. M. Lipp, "Single-track vehicle dynamics and stability," DUKE UNIVERSITY, 2014.
- [86] J. Kooijman, A. Schwab, and J. Meijaard, "Experimental validation of a model of an uncontrolled bicycle," *Multibody System Dynamics*, vol. 19, no. 1-2, pp. 115-132, 2008.
- [87] R. S. Sharp, "On the stability and control of the bicycle," *Applied mechanics reviews*, vol. 61, no. 6, p. 060803, 2008.
- [88] P. Meijaard, "Supporting Online Text Material (SOM Chapters 1-11)," *Science Magazine*, vol. 332, no. 6027, pp. 339-342, 2011.
- [89] J. P. Meijaard, J. M. Papadopoulos, A. Ruina, and A. L. Schwab, "Supporting Online Text Material (SOM Chapters 1-11)," *Science Magazine*, vol. 332, no. 6027, pp. 339-342, 2011.
- [90] W. T. Dempster, "Space requirements of the seated operator, geometrical, Kinematic, and mechanical aspects of the body with special reference to the limbs," 1955.
- [91] R. C. Rudolfs Drillis, Maurice Bluestein, *Body Segment Parameters: A Survey of Measurement Techniques*. Artificial Limbs, 1964.
- [92] C. E. C. R.F. Chandler, J.T. McConville, H.M. Reyonlds and J.W. Young, "Investigation of inertial properties of the human body," 1975.
- [93] W. S. P. Robertson, "A modern take on the theoretical modelling of inertial properties of a human body for biomechanical simulations," presented at the 20th International Congress on Modelling and Simulation, Adelaide, Australia,, Dec 1-6, 2013.

- [94] D. J. Pearsall, "The study of human body signet parameters in biomechanics. An historical review and current status report," *Sports medicine*, vol. 18, no. 2, pp. 126-140, 1994.
- [95] J. K. Moore, M. Hubbard, J. Kooijman, and A. Schwab, "A method for estimating physical properties of a combined bicycle and rider," in *ASME 2009 international design engineering technical conferences and computers and information in engineering conference*, 2009: American Society of Mechanical Engineers, pp. 2011-2020.
- [96] A. Doria and M. Tognazzo, "The influence of the dynamic response of the rider's body on the open-loop stability of a bicycle," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 228, no. 17, pp. 3116-3132, 2014.
- [97] D. J. Limebeer and R. S. Sharp, "Bicycles, motorcycles, and models," *IEEE Control Systems Magazine*, vol. 26, no. 5, pp. 34-61, 2006.
- [98] A. Schwab, J. Meijaard, and J. Kooijman, "Lateral dynamics of a bicycle with a passive rider model: stability and controllability," *Vehicle system dynamics*, vol. 50, no. 8, pp. 1209-1224, 2012.
- [99] H.-C. Lai, J.-S. Liu, D. Lee, and L.-S. Wang, "Design parameters study on the stability and perception of riding comfort of the electrical motorcycles under rider leaning," *Mechatronics*, vol. 13, no. 1, pp. 49-76, 2003.
- [100] M. Haudum, "Entwurf und Simulation eines Fahrermodells zum Lenken und Stabilisieren von Fahrrädern " Master, Wien, Techn. Univ, 2012.
- [101] M. Plöchl, J. Edelmann, B. Angrosch, and C. Ott, "On the wobble mode of a bicycle," *Vehicle system dynamics*, vol. 50, no. 3, pp. 415-429, 2012.
- [102] S. M. Cain, J. A. Ashton-Miller, and N. C. Perkins, "On the skill of balancing while riding a bicycle," *PLoS one*, vol. 11, no. 2, p. e0149340, 2016.
- [103] T. Nishimi, A. Aoki, and T. Katayama, "Analysis of straight running stability of motorcycles," SAE Technical Paper, 1985.
- [104] A. Doria, M. Tognazzo, and V. Cossalter, "The response of the rider's body to roll oscillations of two wheeled vehicles; experimental tests and biomechanical models," *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 227, no. 4, pp. 561-576, 2013.
- [105] P. Hewson, "Method for estimating tire cornering stiffness from basic tire information," *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 219, no. 12, pp. 1407-1412, 2005.
- [106] A. Dressel and A. Rahman, "Measuring sideslip and camber characteristics of bicycle tires," *Vehicle system dynamics*, vol. 50, no. 8, pp. 1365-1378, 2012.
- [107] A. Doria, M. Tognazzo, G. Cusimano, V. Bulsink, A. Cooke, and B. Koopman, "Identification of the mechanical properties of bicycle tires for modelling of bicycle dynamics," *Vehicle system dynamics*, vol. 51, no. 3, pp. 405-420, 2013.

- [108] H. B. Pacejka and E. Bakker, "The magic formula tire model," *Vehicle system dynamics*, vol. 21, no. S1, pp. 1-18, 1992.
- [109] V. Cossalter, R. Lot, and F. Maggio, "A multibody code for motorcycle handling and stability analysis with validation and examples of application," SAE Technical Paper, 0148-7191, 2003.
- [110] FASTBIKE. Dynamotion
<http://www.dynamotion.it/eng/products/software/fastbike.htm> (accessed 2018).
- [111] J. Meijaard and A. Schwab, "Linearized equations for an extended bicycle model," in *III European Conference on Computational Mechanics*, 2006: Springer, pp. 772-772.
- [112] B. Souh, "Influence of tire side forces on bicycle self-stability," *Journal of mechanical science and technology*, vol. 29, no. 8, pp. 3131-3140, 2015.
- [113] A. Doria and S. D. Roa Melo, "On the influence of tire and structural properties on the stability of bicycles," *Vehicle system dynamics*, vol. 56, no. 6, pp. 947-966, 2018.
- [114] D. J. Limebeer and A. Sharma, "Burst oscillations in the accelerating bicycle," *Journal of Applied Mechanics*, vol. 77, no. 6, p. 061012, 2010.
- [115] V. Cossalter, R. Lot, and F. Maggio, "On the braking behavior of motorcycles," *SAE transactions*, pp. 1274-1280, 2004.
- [116] D. J. Limebeer and A. Sharma, "The dynamics of the accelerating bicycle," in *2008 3rd International Symposium on Communications, Control and Signal Processing*, 2008: IEEE, pp. 237-242.
- [117] J. Kooijman and A. Schwab, "A review on handling aspects in bicycle and motorcycle control," in *ASME 2011 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, 2011: American Society of Mechanical Engineers, pp. 597-607.
- [118] T. Foale, *Motorcycle handling and chassis design: the art and science*. Tony Foale, 2006.
- [119] A. Schwab, J. Kooijman, and J. Nieuwendijk, "On the design of a recumbent bicycle with a perspective on handling qualities," in *ASME 2012 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, 2012: American Society of Mechanical Engineers, pp. 303-308.
- [120] K. Anderson, "An Introduction to Bicycle Geometry and Handling," ed.
- [121] Cyclingtips. "The geometry of bike handling." <http://cyclingtips.com/2011/02/the-geometry-of-bike-handling/> (accessed July 25, 2016).
- [122] D. Moulton, "The ideal handling bicycle," in *Dave Moulton's blog*, ed, 2010.
- [123] A. Cyclery. "Tech Talk: How frame geometry affects a bike's handling." <https://reviews.mtbr.com/tech-talk-how-frame-geometry-affects-a-bikes-handling> (accessed 2018).

- [124] S. Stott. "The ultimate guide to bike geometry and handling." Bikeradar. <https://www.bikeradar.com/features/the-ultimate-guide-to-bike-geometry-and-handling/> (accessed 2018).
- [125] M. D. Ricard, P. Hills-Meyer, M. G. Miller, and T. J. Michael, "The effects of bicycle frame geometry on muscle activation and power during a Wingate anaerobic test," *Journal of Sports Science and Medicine*, vol. 5, no. 1, pp. 25-32, 2006.
- [126] D. P. Heil, A. R. Wilcox, and C. M. Quinn, "Cardiorespiratory responses to seat-tube angle variation during steady-state cycling," *Medicine and Science in Sports and Exercise*, vol. 27, no. 5, pp. 730-735, 1995.
- [127] N. Bodman. "Biking 101: Bike Fit & Geometry—Part 1." Blister. <https://blisterreview.com/gear-reviews/mountain-bike-reviews/bike-fit-and-geometry-part-1> (accessed 2019).
- [128] R. Olds, "Bike Sizing: Stack And Reach," in *Velovoice*, ed, 2015.
- [129] C. Maddocks. "Frame Geometry: Part 2 – Trail Handling." <http://www.singletracks.com/blog/mtb-gear/frame-geometry-part-2-trail-handling/> (accessed August 5, 2016).
- [130] T. Paterek, *The Paterek manual*. Kermesse Distributors Inc.
- [131] M. asdfs. "Geometry, an In Depth Explanation." <http://www.pinkbike.com/news/bike-frame-geometry-2009.html> (accessed August 6, 2016).
- [132] C. DESIGN. "Geometry of Bike Handling." CALFEE DESIGN. <http://calfeedesign.com/tech-papers/geometry-of-bike-handling/> (accessed July 25, 2016).
- [133] D. M. d. I. R. Michael J Koilin, *The custom bicycle: Buying, setting up, and riding the quality bicycle*. Rodale Press Emmaus, Pa., 1979.
- [134] H. Fu, "Fundamental characteristics of single-track vehicles in steady turning," *Bulletin of JSME*, vol. 9, no. 34, pp. 284-293, 1966.
- [135] V. Cossalter, A. Doria, and R. Lot, "Steady turning of two-wheeled vehicles," *Vehicle system dynamics*, vol. 31, no. 3, pp. 157-181, 1999.
- [136] V. Cossalter, R. Lot, and M. Peretto, "Steady turning of motorcycles," *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 221, no. 11, pp. 1343-1356, 2007.
- [137] V. Cossalter, R. Lot, and M. Massaro, "An advanced multibody code for handling and stability analysis of motorcycles," *Meccanica*, vol. 46, no. 5, pp. 943-958, 2011.
- [138] D. Bortoluzzi, R. Lot, and N. Ruffo, "Motorcycle steady turning: the significance of geometry and inertia," in *proc. of 7th ATA International Conference & Exhibition*, 2001.
- [139] D. Bortoluzzi, A. Doria, R. Lot, and L. Fabbri, "Experimental investigation and simulation of motorcycle turning performance," in *SAFETY ENVIRONMENT FUTURE III-PROCEEDINGS OF THE 2000 INTERNATIONAL MOTORCYCLE CONFERENCE*, 2000.

- [140] P. Basu-Mandal, A. Chatterjee, and J. M. Papadopoulos, "Hands-free circular motions of a benchmark bicycle," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 463, no. 2084, pp. 1983-2003, 2007.
- [141] D. L. Peterson and M. Hubbard, "General steady turning of a benchmark bicycle model," in *ASME 2009 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, 2009: American Society of Mechanical Engineers, pp. 1961-1970.
- [142] W. Patterson, *The Lords of the Chainring*. William Patterson, 2004.
- [143] S. M. Cain and N. C. Perkins, "Comparison of experimental data to a model for bicycle steady-state turning," *Vehicle system dynamics*, vol. 50, no. 8, pp. 1341-1364, 2012.
- [144] S. Cain and N. Perkins, "Comparison of a bicycle steady-state turning model to experimental data," in *Proceedings of Bicycle and Motorcycle Dynamics Conference, TU Delft*, available at: <http://www.bicycle.tudelft.nl/ProceedingsBMD2010/papers/cain2010comparison.pdf> (last access: 3 January 2019), 2010.
- [145] R. Rice and R. Roland, "An evaluation of the performance and handling qualities of bicycles. Buffalo (NY): Cornell Aeronautical Laboratory," *Inc., Report CAL NO. VJ-2888-K*, 1970.
- [146] J. Godthelp and M. Buist, "Stability and manoeuvrability characteristics of single track vehicles," 1975.
- [147] R. G. Mortimer, P. A. Domas, and R. E. Dewar, "The relationship of bicycle manoeuvrability to handlebar configuration," *Applied ergonomics*, vol. 7, no. 4, pp. 213-219, 1976.
- [148] J. M. Papadopoulos, "A single-step method of aligning a bicycle frame so no handlebar torque is required for straight-line riding," Cornell University, 1988.
- [149] M. Paine, "ANALYSIS OF RELATIVE SAFETY PERFORMANCE OF BICYCLES AND SCOOTERS," Vehicle Design and Research Pty Limited, Australia 2001.
- [150] Andrew Davol and F. Owen, "Model of a bicycle from handling qualities considerations" California Polytechnic State University
- [151] Dassault Systemes. [Online]. Available: <https://www.solidworks.com/>
- [152] P. Proteus, *The Proteus framebuilding book: A guide for the novice bicycle framebuilder* Proteus Design Inc. , 1975.
- [153] R. P. Talbot, *Designing and Building Your Own Frameset: An Illustrated Guide for the Amateur Bicycle Builder*. Manet Guild, 1984.
- [154] M. J. Koilin; and D. M. d. I. Rosa;, *The custom bicycle: Buying, setting up, and riding the quality bicycle*. Rodale Press Emmaus, Pa., 1979.
- [155] M. Firth. "How To Set Up Your Road Bike." Association of British Cycling Coaches <http://www.abcc.co.uk/how-to-set-up-your-road-bike/> (accessed March, 2017).

- [156] F. DeLong, *DeLong's guide to bicycles & bicycling: the art & science*. Chilton Book Co., 1974.
- [157] Anonymous. (2015) Elementary aspects of road bicycle design, engineering and fabrication. *Rolo motion devices*
- [158] J. Kossak, *Bicycle Frames: A Close-up Look*. World Publications, 1975.
- [159] R. Ballantine, *Richard's 21st Century Bicycle Book*. Pan, 2000.
- [160] BikeCad. "BikeCad: Bicycle design software." <https://www.bikecad.ca/> (accessed.
- [161] Mike. "Crank Arm Lengths." BikeDynamics: Bike fitting specialists. <https://bikedynamics.co.uk/FitGuidecranks.htm> (accessed 2017).
- [162] N. Jones. "Optimum Bicycle Crank Length." Machinehead Programing Team. http://www.machinehead-software.co.uk/bike/cranks/cyclist_crank_length_calculator.html (accessed.
- [163] T. Paterek, *The Paterek Manual for bicycle framebuilders*. Framebuilders' Guild, 1985.
- [164] Anonymous. "Bike sizing recommendations " Pedal force <http://pedalforce.com/online/bikefit.php?osCsid=36ce8f5b72684df8ace64942986a9d46> (accessed 2018).
- [165] Anonymous. "6 Easy Ways to Get the Proper Size of Road Bikes for You." FireX. <https://thexfire.com/bike-sizing-tips/> (accessed.
- [166] Anonymous. What Frame Size Do You Need? [Online] Available: <https://www.chainreactioncycles.com/mobile/browse/mobileSizeChartInfo.jsp?productId=prod161705>
- [167] M. Firth. "How to set up your road bike." Association of British cycling coaches. <http://www.abcc.co.uk/how-to-set-up-your-road-bike/> (accessed Mar 10, 2017).
- [168] S. Brown. "Sheldon Brown's bicycle Glossary." https://www.sheldonbrown.com/gloss_tp-z.html (accessed 2017).
- [169] R. Novović, "Bicycle frame geometry," in *Bike Gremlin* vol. 2016, ed.
- [170] T. Kellogg, "Trail and its effects," in *Specturm* vol. 2017, ed.
- [171] T. John, "Buyer's guide: bicycle geometry," in *Road cycling UK*, ed, 2014.
- [172] *Cycle-safety requirements for bicycles*, B. Standard, 2015.
- [173] R. Rice, "Bicycle dynamics, simplified steady state response characteristics and stability indices," Report ZN-5431-V-1, June, Calspan, Buffalo, NY, 1974.
- [174] R. Rice, "Bicycle dynamics—simplified dynamic stability analysis," Report ZN-5921-V-2, September, Calspan, Buffalo, NY, 1976.
- [175] J. Prince, "An investigation into bicycle performance and design," Auckland University of Technology, 2014.

- [176] C. Kvale; and J. Corbett. "A fresh look at steering geometry." Chris Kvale Cycle. <http://www.chriskvalecycles.com/steering-geometry.html> (accessed 2019).
- [177] T. John. "Buyer's guide: bicycle geometry, Head angle, fork rake and trail." RoadCyclingUK. <https://roadcyclinguk.com/gear/buyers-guide-bicycle-geometry.html/4> (accessed.
- [178] J. Moore and M. Hubbard, "Parametric study of bicycle stability (P207)," in *The engineering of sport 7*: Springer, 2009, pp. 311-318.
- [179] D. T. Hon, *Folding bicycles: A treatise*. Dahon bicycle, 2016.
- [180] C. Nakagawa, S. Takehara, and Y. Suda, "Dynamic Analysis and Driving Experiment of a Bicycle Focusing on Small Wheels," *Journal of Mechanical Systems for Transportation and Logistics*, vol. 5, no. 1, pp. 43-57, 2012.
- [181] T. Kazuki, N. Chihiro, S. Atsuhiko, and I. Tomohiro, "Basic study on upright stability of a small wheel bicycle with steer-by-wire system," *The Japan Society of Mechanical Engineers*, vol. 82, no. 837, pp. 1-13, 2016.
- [182] D. Ng; and K. Lau, "Why being hit by an e-scooter can be deadly - and a call to ban them from footpaths," in *CNA insider*, ed: CNA, 2018.
- [183] Dassault systems [Online]. Available: <https://www.3ds.com/products-services/catia/>
- [184] *Jbike6*. (2003).

Appendix

Appendix A-1: Steering designs of current small wheel bicycles.

S. N	Bicycle brand/ model	Wheel size	Head tube angle	Trail
1	Giant Expressway	20"	71**	48.9*
2	Giant Halfway	20"	72**	52*
3	Bickerton Portable	20"	71.58*	51*
4	Bike Friday Pocket rocket	20"	73**	35**
5	Pacific bicycle Birdy	20"	72.5**	56**
6	Seattle cycle Bruke 20	20"	73**	58**
7	B'Twin	20"	73*	43*
8	Citizen bicycle	20"	73*	42*
9	Critical bicycle	20"	71.6*	41.1*
10	Dewes bicycle	20"	71**	72**
11	Kross bicycle	20"	71.5*	30*
12	Moulton Jubilee	20"	70.76*	52.6*
13	Moulton	20"	72***	41***
14	Raleigh	20"	73**	45.5**
15	Speedone	20"	70**	51.4**
16	Swift folders	20"	72.09*	35.3*
17	Tern verge 10	20"	71.5*	45*
18	Vurm	20"	73*	33.4*
19	Origami Cricket 7	20"	67.7**	58.42**
20	Origami Gazelle	20"	69.8**	40.64**
21	Brompton	16"	73**	41**
22	Seattle cycle Bruke 16	16"	70**	58**
23	Peugeot	16"	70**	48.4**
24	Atala	16"	70.88*	43.5*
25	Beixo Crosstown	16"	72**	30**
26	IXI	16"	69*	43.7*
27	Bike Friday Pakit	16"	73**	35**

*The design information was obtained by properly adjusting the picture of the bicycle in CAD software.

**The design information was obtained from manufacturers.

***The design information was measured in Lab.

Appendix A-2: Benchmarked parameters for typical designs of PMDs.

Parameters	Symbol (unit)	Typical Big wheel bicycle	Typical Small wheel bicycle	Typical E-scooter
Wheelbase	W (m)	0.990	1.020	0.800
Trail	C (m)	0.056	0.056	0.03
Head angle	λ (degrees)	73	71.5	75
Forward speed	V (m/s)	-	-	-
Rear-wheel R				
Radius	r_R (m)	0.337	0.243	0.1016
Mass	m_R (kg)	1.773	1.752	1.189
Mass moment of inertia	$I_{Rxx}; I_{Ryy}$ (kg-m ²)	0.4791; 0.09116	0.0220; 0.0438	0.0022,0.004
Rear body and frame assembly B				
Centre of mass	x_B, z_B (m)	0.28635; 1.0283	0.28635; 1.0283	0.3985,1.0549
Mass (including rider)	m_B (kg)	75.6204	75.6204	80 (considering battery weight)
Mass moment of inertia	$I_{Bxx}; I_{Bxz}; I_{Bzz}$ (kg-m ²)	9.111; 2.212;2.488	- 9.111; -2.212; 2.4880	17.444; 0.2187; 1.8514
Front handlebar and fork assembly H				
Centre of mass	$u; v$ (m)	0.032; 0.75	0.032; 0.75	0.0034;0.754
Mass	m_H (kg)	1.7847	1.7847	2.2893
Mass moment of inertia	$I_{Hxx}; I_{Hxz}; I_{Hzz}$ (kg-m ²)	0.0848; -0.006; 0.0276	0.0838; -0.0075; 0.0279	0.3362; 0.05518; 0.02199
Front-wheel F				
Radius	r_F (m)	0.337	0.243	0.1016
Mass	m_F (kg)	1.181	1.38	1.189
Mass moment of inertia	$I_{Fxx}; I_{Fyy}$ (kg-m ²)	0.04626; 0.0898	0.0215; 0.0426	0.0022,0.004

Angles are in degrees

Distances are in meter

Masses are in Kg

Moment of inertias are in Kg-m²

Appendix A-3: Benchmarked design parameters for improved bicycles.

Parameters	Symbol	Big wheel bicycle	Improved 20-inch wheel I	Improved 20-inch wheel II	Improved 16-inch wheel	Improved 12-inch wheel
Wheelbase	W	0.990	0.9772	0.950	0.884	0.776
Trail	C	0.056	0.045	0.056	0.029	0.02
Head angle	λ	73.0	74.5	77.0	82	79
Forward speed	V	-	-	-	-	-
Rear wheel R						
Radius	r_R	0.337	0.243	0.243	0.191	0.1448
Mass	m_R	1.773	1.752	1.752	2.594	3.7 (hub motor)
Mass moment of inertia	$I_{Rxx};$ I_{Ryy}	0.4791; 0.0911;	0.0220; 0.0438;	0.0220; 0.0438;	0.0145; 0.0224;	0.009; 0.016;
Rear body and frame assembly B						
Centre of mass	$x_B;$ $z_B;$	0.2863; -1.0283	0.2898; -0.0248	0.2913; -1.0240;	0.2975; -1.0283;	0.2365; 0.9289;
Mass	m_B	75.6204	75.9675	75.9846	75.6250	76.98
Mass moment of inertia	$I_{Bxx};$ $I_{Bxz};$ $I_{Bzz};$	9.111; -2.212; 2.488;	9.468; -2.078; 2.583;	9.482; -2.020; 2.554;	9.111; -1.199, 2.4793;	8.984; -1.3371; 2.643;
Front handlebar and fork assembly H						
Centre of mass	$u;$ $v;$	0.032; 0.75;	0.0368; 0.6848;	0.02273; 0.6842;	0.028; 0.68;	0.002; 0.657;
Mass	m_H	1.7847	1.70	1.70	2.192	1.559
Mass moment of inertia	$I_{Hxx};$ $I_{Hxz};$ $I_{Hzz};$	0.0848; -0.006; 0.0276	0.1358; -0.0160; 0.0158	0.1353; -0.0107; 0.0135	0.1128; -0.0223; 0.0137;	0.1275; -0.0011; 0.0092;
Front wheel F						
Radius	r_F	0.337	0.243	0.243	0.191	0.1448
Mass	m_F	1.181	1.181	1.181	1.021	1.7
Mass moment of inertia	$I_{Fxx};$ $I_{Fyy};$	0.04626; 0.0898;	0.0271; 0.0506;	0.0271; 0.0506;	0.0144; 0.0229;	0.0061; 0.0108;

Angles are in degrees

Distances are in meter

Masses are in Kg

Moment of inertias are in Kg-m²

Parameters	Symbol (unit)	Bicycle with-out rider
Wheelbase	W (m)	0.9772
Trail	C (m)	0.045
Head angle	λ (degrees)	74.5
Forward speed	V (m/s)	-
Rear-wheel R		
Radius	r_R (m)	0.243
Mass	m_R (kg)	1.752
Mass moment of inertia	$I_{Rxx}; I_{Ryy}$ (kg-m ²)	0.0220; 0.0438;
Rear body and frame assembly B		
Centre of mass	x_B, z_B (m)	0.33; -0.43
Mass (including rider)	m_B (kg)	3.9675
Mass moment of inertia	$I_{Bxx}; I_{Bxz}; I_{Bzz};$	0.243; -0.04988; 0.2349
Front handlebar and fork assembly H		
Centre of mass	$u; v$ (m)	0.0368; 0.6848;
Mass	m_H (kg)	1.70
Mass moment of inertia	$I_{Hxx}; I_{Hxz}; I_{Hzz};$ (kg-m ²)	0.1358; -0.0160; 0.0158
Front-wheel F		
Radius	r_F (m)	0.243
Mass	m_F (kg)	1.181
Mass moment of inertia	$I_{Fxx}; I_{Fyy}$ (kg-m ²)	0.0271; 0.0506

Angles are in degrees

Distances are in meter

Masses are in Kg

Moment of inertias are in Kg-m²

Appendix A-4: Benchmarked design parameters for improved e-scooters.

Parameters	Symbol	Big wheel bicycle	Improved e-scooter (Standing)	Improved e-scooter (Seating)	Improved e-scooter Standing
Wheelbase	W	0.990	0.827	0.827	0.842
Trail	C	0.056	0.01	0.01	0.01
Head angle	λ	73.0	81	81	79
Forward speed	V	-	-	-	-
Rear-wheel R					
Radius	r_R	0.337	0.1016	0.1016	0.1016
Mass	m_R	1.773	3.27	3.27	3.27
Mass moment of inertia	$I_{Rxx};$	0.4791;	0.003;	0.003;	0.003;
	I_{Ryy}	0.0911;	0.006;	0.006;	0.006;
Rear body and frame assembly B					
Centre of mass	$x_B;$	0.2863;	0.3461;	0.2828;	0.3468;
	$z_B;$	-1.0283	-1.035;	-0.7758;	-1.0355;
Mass	m_B	75.6204	81	81.5	81
Mass moment of inertia	$I_{Bxx};$	9.111;	19.2105;	9.970;	19.2133;
	$I_{Bxz};$	-2.212;	0.0472;	-2.691;	0.0620;
	$I_{Bzz};$	2.488;	1.7921;	3.899	1.8033;
Front handlebar and fork assembly H					
Centre of mass	$u;$	0.032;	-0.0206;	-0.0206;	0.0046;
	$v;$	0.75;	0.8502;	0.8502;	0.8509;
Mass	m_H	1.7847	1.9867	1.9867	1.9867
Mass moment of inertia	$I_{Hxx};$	0.0848;	0.27992;	0.27992;	0.2877;
	$I_{Hxz};$	-0.006;	-0.05068;	-0.05068;	-0.04912;
	$I_{Hzz};$	0.0276	0.02152;	0.02152;	0.02037;
Front-wheel F					
Radius	r_F	0.337	0.1016	0.1016	0.1016
Mass	m_F	1.181	1.189	1.189	1.189
Mass moment of inertia	$I_{Fxx};$	0.04626;	0.0022;	0.0022;	0.0022;
	$I_{Fyy};$	0.0898;	0.004;	0.004;	0.004;

Angles are in degrees

Distances are in meter

Masses are in Kg

Moment of inertias are in Kg-m²

Appendix A-5: Details of the participants

Riders	Skill Level	Weight (Kg)	Height (cm)
Rider 1	A	74.5	180
Rider 2	A	62	177
Rider 3	S	65	165
Rider 4	S	73	180
Rider 5	A	81	171
Rider 6	S*	75	177
Rider 7	S*	75	173
Rider 8	S	70	177
Rider 9	S	52	163
Rider 10	S	70	171
Rider 11	S	65	174
Rider 12	S	80	183
Rider 13	S	60	165
Rider 14	A	69	174
Rider 15	S*	70	174

Skill Level

A= Amateur riders with no previous experience for hands-free riding

S= Skilled rider who rides bicycles regularly and have experience for hands-free riding

S*= Skilled but had not tried hands-free riding for a long time