Edge and curvilinear structures detection on medical images via unsupervised, adaptive and deep learning approaches

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Edge and Curvilinear Structures Detection on Medical Images via Unsupervised, Adaptive and Deep Learning Approaches

DHIMAS ARIEF DHARMAWAN

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

2020

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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DHIMAS ARIEF DHARMAWAN

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Assoc. Prof. Ng Boon Poh

Authorship Attribution Statement

This thesis contains material from four papers published in the following peer-reviewed journals and from two papers accepted at conferences in which I am listed as an author.

Chapter 4 is published as:

[1] D. A. Dharmawan and B. P. Ng, “A new two-dimensional matched filter based on the modified Chebyshev type I function for retinal vessels detection,” *2017 39th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)*, Seogwipo, 2017, pp. 369-372. DOI: 10.1109/EMBC.2017.8036839.

The contributions of the co-authors are as follows:

- A/Prof Ng Boon Poh provided the initial project direction, suggested the use of the Dolph-Chebyshev function and edited the manuscript drafts.
- I initiated the idea and further developed it into a concrete idea and performed all the experiment and the laboratory work at the School of Electrical and Electronic Engineering.
- I analysed the data collected during the experiments and presenting them in the paper properly.

[2] D. A. Dharmawan and B. P. Ng, “Segmenting Retinal Vessels with a Multi-scale Modified Dolph-Chebyshev Type I Function Matched Filter,” *2018 4th International Conference on Science and Technology (ICST)*, Yogyakarta, 2018, pp. 1-4. DOI: 10.1109/ICSTC.2018.8528648.

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- I analysed the data collected during the experiments and presenting them in the paper properly.

[3] D. A. Dharmawan, B. P. Ng and S. Rahardja, “A Modified Dolph-Chebyshev Type II Function Matched Filter for Retinal Vessels Segmentation,” *Symmetry*, 2018, 10, 257.

The contributions of the co-authors are as follows:

- A/Prof Ng Boon Poh provided the initial project direction, suggested the use of the Dolph-Chebyshev function and edited the manuscript drafts.
- I prepared the manuscript drafts. The manuscript was revised by Prof Susanto Rahardja.
- I initiated the idea and further developed it into a concrete idea and performed all the experiment and the laboratory work at the School of Electrical and Electronic Engineering.
- I analysed the data collected during the experiments and presenting them in the paper properly.
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[4] D. A. Dharmawan, B. P. Ng and N. Borijindargoon, "Design of Optimal Adaptive Filters for Two-Dimensional Filamentary Structures Segmentation," *IEEE Signal Processing Letters*, vol. 26, no. 10, pp. 1511-1515, 2019.

The contributions of the co-authors are as follows:

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- I initiated the idea and further developed it into a concrete idea with Narong Borijindargoon and performed all the experiment and the laboratory work at the School of Electrical and Electronic Engineering.
- I and Narong Borijindargoon analysed the data collected during the experiments and presenting them in the paper properly.
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- A/Prof Ng Boon Poh provided the initial project direction and edited the manuscript drafts.
- I prepared the manuscript drafts. The manuscript was revised by Prof Susanto Rahardja.
- I initiated the idea and further developed it into a concrete idea and performed all the experiment and the laboratory work at the School of Electrical and Electronic Engineering.
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- A/Prof Ng Boon Poh provided the initial project direction and edited the manuscript drafts.
- I prepared the manuscript drafts. The manuscript was revised by Prof Susanto Rahardja and Li Di.
- I initiated the idea and further developed it into a concrete idea with Li Di and performed all the experiment and the laboratory work at the School of Marine Science and Technology, Northwestern Polytechnical University, Xian, China.
- I and Li Di analysed the data collected during the experiments and presenting them in the paper properly.
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“If I had one hour to save the world, I would spend 55 minutes defining the problem and only five minutes finding the solution.”

—Einstein, Albert

To my dear family

Abstract

Computer-aided-diagnosis (CAD) systems are very useful to help doctors in detecting various human diseases. To build a CAD system, several computer vision algorithms are required, particularly to handle object detection and segmentation tasks automatically. To develop object detection algorithms, edges and curvilinear structures detection tasks are typically required. However, performing these tasks manually is tedious, time-consuming and prone to human errors. In this thesis, we design computer algorithms for edge and curvilinear structures detection, particularly for the application of optical disc boundary and retinal vessel detection from fundus images. The algorithms are developed based on the mathematical function that can closely represent the edge and curvilinear structures behaviours. The algorithms can detect edge and curvilinear structures under an unsupervised framework and they also allow an implementation with a deep learning architecture. This provides meaningful insight for robust edge and curvilinear structures detection algorithms developments on other image modalities.

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List of Abbreviations

Chapter 1

CAD	Computer Aided Diagnosis
OD	Optic Disc

Chapter 2

CHT	Circular Hough Transform
CNN	Convolutional Neural Network
CRF	Conditional Random Field
FOV	Field of View
GMM	Gaussian Mixture Model
HOG	Histogram of Oriented Gradient
KNN	K-Nearest Neighbor
RF	Random Forest
SOSVM	Structured Output Support Vector Machine
SVM	Support Vector Machine

Chapter 3

RMSE	Root Mean Square Error

Chapter 4

CLAHE	Contrast Limited Adaptive Histogram Equalization
DoG	Difference of Gaussian
GCV	Generalized Cross-Validation

GPU	Graphics Processing Unit
MDCF-I	Modified Dolph-Chebyshev type I Function
MDCF-II	Modified Dolph-Chebyshev type II Function

Chapter 5

ODC	Optic Disc Centre point
ROI	Region of Interest

Chapter 6

ANN	Artificial Neural Network
LoG	Laplacian of Gaussian
ReLu	Rectified Linear Unit

List of Symbols

Chapter 2

$\nabla_x (\cdot)$	the gradient in the x -direction
$\nabla_y (\cdot)$	the gradient in the y -direction
$\nabla (\cdot)$	the gradient magnitude
θ	the gradient orientation
λ	the wavelength of the sinusoidal carrier
γ	the ellipticity of the Gabor filter
E_{snake}	the snake energy in the Snake model
E_{int}	the internal energy in the Snake model
E_{ext}	the external energy in the Snake model
E_{cont}	the continuity energy in the Snake model
E_{curv}	the curvature energy in the Snake model
E_{cons}	the constraint energy in the Snake model
E_{img}	the image energy in the Snake model
$\mathbf{H}$	the Hessian matrix
R	the ridgeness measure
λ_1	the first eigenvalue of the Hessian matrix
λ_2	the second eigenvalue of the Hessian matrix
$\ \cdot\ _F$	the Frobenius norm
$S_{\mathbf{H}}(x, y; \sigma)$	the vesselness measure
$\beta_{\mathbf{H}}$	the Hessian matrix magnitude sensitivity
c	the sensitivity of the ridgeness measure
AOL	the Area Overlap
DSC	the Dice coefficient
TPF	the True Positive Fraction
FPF	the False Positive Fraction

Se	the Sensitivity
Sp	the Specificity
Acc	the Accuracy

Chapter 3

ω_c	the Dolph-Chebyshev function main-lobe width
M	the length of the Dolph-Chebyshev function
a	the weighting coefficient
P	the optimum set of the Dolph-Chebyshev function parameters
Q	the optimum set of the single Gaussian function parameters
R	the optimum set of the double Gaussian function parameters
l_m	the length of the motion blur window
θ_m	the orientation of the motion blur window
σ_G	the standard deviation of the Gaussian noise
d	the density of the Salt and pepper noise
σ_S	the standard deviation of the Speckle noise
Re	the Recall
Pr	the Precision
F	the F-measure

Chapter 4

$V(x, y)$	the proposed MDCF-I matched filter kernel
$W(x, y)$	the proposed MDCF-II matched filter kernel
T_h	the threshold value used for exudates detection
T_B	the threshold value used to binarize the matched filter response
T_L	the threshold value used for the length filtering
$R_{i,j}(x, y)$	the matched filter response at a certain scale and orientation
$R_{max,j}$	the matched filter response at all possible orientations and certain scale
$R_{combined}$	the matched filter response at all possible scales and orientations
w_j	the weighting coefficient for the matched filter response at a certain scale
$R_{std,j}$	the normalized matched filter response at a certain scale
$\mathbf{X}$	the retinal image as the input of the proposed adaptive approach
$\mathbf{Y}$	the ground truth of the retinal image
$\mathbf{Z}_k$	the k^{th} patch extracted from $\mathbf{X}$

$\text{vec}(\cdot)$	the matrix vectorization operator
$\mathbf{C}_t$	the t^{th} overlapping patch extracted from $\mathbf{X}$
$\mathbf{F}_{ov}$	the mapping function
$\mathbf{B}_{ov}$	the set of training patches
$\mathbf{I}$	the identity matrix
λ_r	the regularization parameter
$\mathbf{O}$	the segmentation output of the adaptive approach
$F1$	the F1-score
G	the G-mean
MCC	the Matthews Correlation Coefficient

Chapter 5

r_{OD}	the estimated optic disc radius
D_{OD}	the average diameter of the human optic disc
A_{FOV}	the imaging area of the field of view
N_{FOV}	the number of pixels within the field of view
r_{min}	the minimum radius used in the Circular Hough Transform
r_{max}	the maximum radius used in the Circular Hough Transform
(P_c, r)	the set of centre points and radii from the Circular Hough Transform
I_{MFC}	the optic disc boundary candidate
I_{OD}	the binary mask for the optic disc area
$Sc(\%)$	the percentage of optic discs being correctly localized on a certain database

Chapter 6

α	the Dolph-Chebyshev function side-lobe level
f_b	the retinal images with enhanced blood vessel details
p	the patch size
Ppv	the positive predictive value
$\widetilde{Ppv}$	the estimated positive predictive value
$\widetilde{F1}$	the estimated F1-score
$\widetilde{G}$	the estimated G-mean
$\widetilde{MCC}$	the estimated Matthews Correlation Coefficient
AUC	the Area Under Curve
C	the connectivity measure

A	the area measure
L	the length measure
$f(C, A, L)$	the product of connectivity, area and length measures
$\min(.)$	the minimum operator
C_t	the contrast of blood vessels to the background

Chapter 1

Introduction

1.1 Background and Motivation

In recent years, medical image analysis has become an important research field as it provides useful information regarding various structures of human bodies and tissues. The information is commonly used by doctors to make clinical diagnosis and intervention. Furthermore, when computer vision algorithms are used to

FIGURE 1.1: The existing CAD research [1].

FIGURE 1.2: CAD of glaucoma using colour fundus images [2].

handle the clinical processes automatically, the development of computer-aided diagnosis (CAD) systems can be realized [9–11]. Research on CAD systems has been of interest and the existing CAD systems can be divided into seven clusters as illustrated in Figure 1.1 [1].

The main characteristic of a CAD system is the use of several computer vision algorithms to fully automatize all the clinical processes. Figure 1.2 shows the steps involved in the development of a computer-aided glaucoma diagnosis system. As illustrated in the figure, detecting the optic disc (OD) boundary and subsequently extracting the OD area play an important role in the development of the system. They provide meaningful information regarding the object, i.e., its shape, size and structure. The information is then processed to obtain the diagnosis result.

Besides OD boundary detection, extracting blood vessels from fundus images (see Figure 1.3 for the illustrations) is another example of the object detection task that is required in the development of a computer aided retinal diseases, i.e., diabetic retinopathy diagnosis system. Nevertheless, this task and also the OD boundary detection task cannot be handled by humans easily, particularly when the detection tasks deal with some challenging situations, which include varied complexities of optical disc boundaries and vascular structures, images with low-contrast and the presence of some distractors near the objects of interest. Therefore, the development of computer-algorithms for optical disc boundary and retinal vessel detection on fundus images is required. Therefore, computer-based algorithms for OD boundary and retinal blood vessel detection on fundus images are required. Note that

FIGURE 1.3: (a) Extracting the boundaries of optical discs [3] and (b) blood vessel segmentation [4] on fundus images are object detection tasks that are required in the development of a CAD system for retinal disease detection such as glaucoma and diabetic retinopathy.

algorithms should be able to detect the objects in a relatively short time. Moreover, the computational complexity of the algorithms should be considered.

In the literature, a lot of algorithms for OD boundary and retinal vessel detection on fundus images have been developed. They can be divided into two groups that follow unsupervised and supervised approaches. In the unsupervised group, the detection tasks are commonly carried out by introducing some models that represent the characteristics of the objects of interest. For instance, Gaussian function-based models are widely used to describe the blood vessel intensity profiles and many edge detectors are also developed based on the Gaussian function. However, a Gaussian function is known to provide excessive blurring effects on the structures being detected. Hence, the developed models may not be able to provide appropriate representations for the edge and curvilinear structures that are typically recognized through the abrupt changes in the pixel intensity distributions. This drawback will propagate when supervised algorithms are developed with the Gaussian-based convolution filters act as the hand-crafted features of optical disc boundary and blood vessel structures. On the other hand, although deep learning has been widely adopted to handle image segmentation tasks and has provided a reliable performance, the resulted algorithms are prone to high computational complexity, such that they may not be suitable for practical applications.

In this thesis, the main focus is to design algorithms for edge and curvilinear structures detection in medical images, particularly for the applications of optical

disc boundary and retinal vessel detection on fundus images, respectively. The algorithms are developed based on the mathematical function which can closely represent the behaviour of edge and curvilinear structures. The algorithms, on one hand, can perform edge and curvilinear structures detection under an unsupervised framework and on the other hand they allow an implementation with deep learning architectures. This provides meaningful insight into the development of robust edge and curvilinear structures detection algorithms on other image modalities.

1.2 Objectives

The overall objective of this thesis is to design algorithms for edge and curvilinear structures detection in medical images. In particular, the algorithms are developed for the applications of optical disc boundary and retinal vessel detection on fundus images. In view of the overall objective, the specific objectives are as follows:

- (a) To design a flexible model for edge structures representation. The model is designed based on the Dolph-Chebyshev function and the model parameters are estimated under a least-square framework. The profiles of edge structures extracted from a set of images with various levels of blur and noise are utilised to validate the proposed models.
- (b) To construct three curvilinear structure detection algorithms. The first two models are designed based on the validated edge model while the third algorithm is developed under an optimisation framework. The designed algorithms are applied to detect retinal blood vessel on colour fundus images and the performance is compared with several published methods.
- (c) To propose an edge detection algorithm, particularly for optical disc boundary detection on colour fundus images. The edge detection algorithm is developed by tailoring the curvilinear structure detection algorithm. The performance of the proposed algorithm is compared with current-state-of-the-art methods.
- (d) To develop a deep retinal vessel segmentation framework that consists of a new enhancement technique and a CNN architecture. The framework is proposed to increase the robustness of current deep learning-based blood

vessel detection methods in such a way that it can deal with imbalanced training data and challenging situations such as detecting low-contrast micro-vessels, vessels with central reflex and vessels in the presence of pathologies.

1.3 Main Contributions

This section highlights the main contributions of the works described in this thesis, which are as follows:

- (a) A flexible model for edge representation is developed based on the Dolph-Chebyshev function. The parameters of the model are estimated under a least-square framework and the model's validity is shown using the profiles of edge structures extracted from a set of images with various levels of blur and noise. In the experiment, the proposed model outperforms the well-known Gaussian-based models in terms of root-mean-square-error.
- (b) Three curvilinear structure detection algorithms with an application on retinal blood vessel detection on colour fundus images are developed. The first two algorithms are designed based on the validated edge structures model while the third algorithm is developed under an optimisation framework. The proposed algorithms produce a better performance than most of the compared algorithms. This suggests adapting the proposed algorithms for curvilinear structure detection for practical applications.
- (c) The curvilinear structure detection algorithm is tailored for edge detection applications, particularly for optical disc boundary detection on colour fundus images. The superiority of the proposed algorithm over current-state-of-the-art methods suggests that they will be useful for real applications.
- (d) A new algorithm for retinal blood vessel detection on colour fundus images is proposed by incorporating the curvilinear structure detection algorithm and a deep learning model. The proposed algorithm is useful to increase the performance of current deep learning-based blood vessel detection methods, particularly when dealing with imbalanced training data and challenging situations such as detecting low-contrast micro-vessels, vessels with central

reflex and vessels in the presence of pathologies. The superiority of the proposed algorithm offers a meaningful solution for the development of robust curvilinear structures detection algorithms on other image modalities.

1.4 Outline of the Thesis

The remaining chapters of this thesis are organized in the following manner.

Chapter 2 describes the underlying idea of edge and curvilinear structures detection in digital imagery. We then review some existing edge and curvilinear structures detection algorithms, particularly that are applied for optical disc boundary and blood vessel detection on fundus images. Finally, we also list and discuss several indicators commonly used to measure the performance of the existing algorithms.

Chapter 3 describes the proposed flexible model for edge representation in digital imagery. The model is developed based on the Dolph-Chebyshev function and the edge model parameters are estimated under a least-square framework. Extensive experiments using edge profiles with various blur and noise levels are reported to demonstrate the validity of the proposed model. Quantitative analysis in terms of root-mean-square-error is provided to show the advantages of the proposed model over the Gaussian-based models. Also, a procedure for edge detection in two-dimensional images using the proposed model is derived and it is shown that our edge detector performs better than the compared algorithms.

Chapter 4 presents three curvilinear structure detection algorithms with an application on retinal blood vessel detection on colour fundus images. The first two algorithms are designed based on the validated edge model and the third algorithm is developed under an optimisation framework. The proposed algorithms are validated using fundus images from the publicly available databases, the DRIVE and STARE databases. Quantitative comparisons with other methods show the advantages of the proposed algorithms over the reported counterparts.

Chapter 5 describes a new edge detection algorithm, which is developed by tailoring the curvilinear structure detection algorithms. The edge detection algorithm is applied to perform detection and segmentation of optical discs on colour fundus images. Experimental results indicate the superiority of the proposed algorithm

over current-state-of-the-art methods. This suggests that the algorithm will be beneficial for practical applications.

Chapter 6 presents a new framework for deep retinal vessel segmentation. The proposed framework is beneficial to increase the performance of current deep learning-based blood vessel detection methods, particularly when dealing with imbalanced training data and challenging situations such as detecting low-contrast microvessels, vessels with central reflex and vessels in the presence of pathologies. The superiority of the proposed algorithm offers a meaningful solution for the development of robust curvilinear structures detection algorithms on other image modalities.

Chapter 7 draws concluding remarks of this thesis work and provides the recommendations for future works.

Chapter 2

Literature Review and Theoretical Basis

This chapter begins by providing the underlying idea of edge and curvilinear structures detection in digital imagery. We then review some existing edge and curvilinear structures detection algorithms, particularly those being applied for optical disc boundary and blood vessel detection on fundus images. Finally, we also list and discuss several indicators commonly used to measure the performance of the existing algorithms.

2.1 Edge and Curvilinear Structures Detection: Feature Descriptors

2.1.1 Image Gradient Operators for Edge Detection

The underlying idea of detecting edges in a wide variety of images is by looking at abrupt changes on the pixel intensity distributions. One of the common ways to locate abrupt changes on the neighbouring pixels is by calculating the gradient of the corresponding pixels. Image gradient is not only useful to locate edges but also beneficial to approximate the local orientation and the shape of these structures. In general, image gradient is calculated as the intensity difference of the adjacent pixels or through a convolution operation of the image with a gradient operator.

Calculating the intensity difference of neighbouring pixels is a basic operation in estimating image gradient. In general, the intensity difference of neighbouring pixels on image $\mathbf{A}$ can be calculated through one of the central difference and intermediate difference approaches. Using the central difference approach, the gradient of image $\mathbf{A}$ in the x and y -directions ($\nabla_x \mathbf{A}$ and $\nabla_y \mathbf{A}$) can be expressed as follows:

$$\nabla_x \mathbf{A} = \frac{A(x+1, y) - A(x-1, y)}{2}, \quad (2.1)$$

and

$$\nabla_y \mathbf{A} = \frac{A(x, y+1) - A(x, y-1)}{2}. \quad (2.2)$$

On the other hand, the intermediate difference approach describes the gradient of image $\mathbf{A}$ in the x and y -directions as follows:

$$\nabla_x \mathbf{A} = \frac{A(x+1, y) - A(x, y)}{2}, \quad (2.3)$$

and

$$\nabla_y \mathbf{A} = \frac{A(x, y+1) - A(x, y)}{2}. \quad (2.4)$$

The formulas for both the central difference and intermediate difference approaches can also be expressed as convolution operations of image $\mathbf{A}$ and the gradient operators $\mathbf{CG}$ and $\mathbf{IG}$, respectively, where:

$$\mathbf{CG}_x = \frac{1}{2} \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}, \quad (2.5)$$

and

$$\mathbf{IG}_x = \frac{1}{2} \begin{bmatrix} -1 & 1 \end{bmatrix}. \quad (2.6)$$

Note that the gradient operators in the y -direction for the two approaches are the transpose of the corresponding operators in the x -direction.

Besides the central difference and intermediate difference gradient operators, there are other popular gradient operators used for image gradient estimation such as Sobel [12], Prewitt [13], and Canny [14] operators. Each of the first two operators is a 3×3 separable and integer-valued operator, which is used to detect image gradient in the x and y -directions. On the other hand, Canny [14] describes the Canny operator as the directional gradient of a Gaussian function characterized by a certain value of standard deviation σ . Mathematically, these three operators are

FIGURE 2.1: The estimate gradient magnitudes of (a) an image of a brick wall and a bike rack using the (b) central difference, (c) intermediate difference, (d) Sobel, (e) Prewitt and (f) Canny ($\sigma = 1$) operators.

consecutively given by:

$$SG_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, \quad SG_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}, \quad (2.7)$$

$$PG_x = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}, \quad PG_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}, \quad (2.8)$$

and

$$CaG_x(x, y) = -\frac{x}{\sigma^2 \sqrt{2\pi}} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right), \quad (2.9)$$

$$CaG_y(x, y) = -\frac{y}{\sigma^2 \sqrt{2\pi}} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right).$$

SG , PG and CaG are for the Sobel, Prewitt and Canny operators, respectively. Given the gradient of image $\mathbf{A}$ in the x and y -directions, the gradient magnitude

$(\nabla \mathbf{A})$ and orientation (θ) can be obtained as follows:

$$\nabla \mathbf{A} = \sqrt{\nabla_x \mathbf{A}^2 + \nabla_y \mathbf{A}^2}, \quad (2.10)$$

$$\theta = \tan^{-1} \left(\frac{\nabla_y \mathbf{A}}{\nabla_x \mathbf{A}} \right). \quad (2.11)$$

Figure 2.1 shows the estimate gradient magnitudes of an image calculated using the above-described operators.

2.1.2 Convolution Filtering for Curvilinear Structures Detection

The idea of applying a convolution operation to locate edges in an image can be adapted to perform curvilinear structures detection. In this respect, the convolution operation involves an elongated and rotatable tube-like filter to take account the geometric characteristics of curvilinear structures. Among the existing filters, Gabor filter banks are popularly employed for curvilinear structures detection. A Gabor filter kernel can be calculated as an isotropic Gaussian kernel modulated using a sinusoidal wave $g_{\lambda,\sigma,\theta,\phi}(x, y)$:

$$g_{\lambda,\sigma,\theta,\phi}(x, y) = \exp \left(-\frac{\tilde{x}^2 + \gamma^2 \tilde{y}^2}{2\sigma^2} \right) \cos \left(2\pi \frac{\tilde{x}}{\lambda} + \phi \right), \quad (2.12)$$

where

$$\tilde{x} = x \cos \theta + y \sin \theta, \quad (2.13)$$

$$\tilde{y} = -x \sin \theta + y \cos \theta, \quad (2.14)$$

γ is the spatial ratio, which determines the ellipticity of the curvilinear structure, λ is the wavelength, θ is the preferred orientation, σ determines the width of the curvilinear structure and ϕ is the phase offset that corresponds to the symmetry of $g_{\lambda,\sigma,\theta,\phi}(x, y)$. The response of the Gabor filter to an input image $f(x, y)$ can then be computed as a convolution operation (denoted as ‘ $*$ ’) of $g_{\lambda,\sigma,\theta,\phi}(x, y)$ and $f(x, y)$:

$$r_{\lambda,\sigma,\theta,\phi}(x, y) = (f * g_{\lambda,\sigma,\theta,\phi})(x, y). \quad (2.15)$$

Figure 2.2 shows the responses of the Gabor filter banks for different values of parameters.

FIGURE 2.2: Curvilinear structures detection on a Chinese character image using Gabor filter banks with $\gamma = 0.5$, (a) $\lambda = 25$ and (b) $\lambda = 30$. From left to right: original image, $\theta = 0^\circ$, $\theta = 45^\circ$, $\theta = 90^\circ$ and $\theta = 135^\circ$.

2.2 Edge and Curvilinear Structures Detection on Medical Images

The concept of edge and curvilinear structures detection described in the previous section has instigated the development of various algorithms for edge and curvilinear structures detection on medical images. The existing algorithms vary in those exploiting low-level image features such as the gradient of the image, considering the image morphological profiles and those inspired by visual information processing in the human brain, where most of the algorithms involve various learning strategies. In this section, we review some existing algorithms for edge and curvilinear structures detection on medical images. Specifically, we discuss several current algorithms for optical disc boundary detection and blood vessel segmentation on colour fundus images.

2.2.1 Optical Disc Boundary Detection

Automatic optic disc (OD) segmentation in fundus images plays an important role in computerized retinal disease diagnosis systems, such as glaucoma and diabetic retinopathy. Also, OD segmentation often serves as a pre-processing step in other retinal structure segmentation. For instance, the OD location is used to reduce the

number of false positives in exudate segmentation since both the OD and exudate have similar grey-levels. Also, the relatively constant distance between the OD and the macula is beneficial in finding the location of the macula [15]. Therefore, many algorithms for automatic OD segmentation have been published and some of the existing algorithms are discussed as follows.

2.2.1.1 The OD Appearance-based Approaches

In this group of approaches, OD boundaries are detected with the means of the OD appearance, i.e., the OD emerges as a bright and circular object. To capture bright regions on fundus images, several methods such as morphological operation, edge detection and automatic thresholding are used while the circularity of the OD is commonly detected using the Circular Hough Transform (CHT). These two features are commonly merged to obtain the final boundary of the OD.

The work in Aquino et al. [15] detects the OD boundary via morphological and edge detection approaches. The OD region of interest is initially determined as the region that consists of pixels with high grey-level values (since OD usually appears brighter than the surrounding) and at the same time as the region that contains major vascular trees. The blood vessels in the region of interest are then eliminated with the help of morphological operations. The initial boundary map of the OD is detected using the Prewitt edge detector. Finally, the edge map is refined using the CHT.

In Morales et al. [16], the OD boundary is detected with the means of mathematical morphology. Specifically, the stochastic watershed transformation is applied to the pre-processed image to obtain the OD boundary. The pre-processing steps that aim to provide images with distinguishable retinal structures are carried out by applying PCA, non-uniform illumination correction and image in-painting.

The work in Marin et al. [17] uses an automatic thresholding approach to detect the OD candidates. In particular, the thresholding approach is applied to the image pre-processed using some morphological operations. The OD boundary is then obtained by applying the CHT on the image containing detected OD candidates. The application of the CHT aims to capture bright regions with a circular shape.

More recently, a thresholding principle is also utilised by Zahoor and Fraz [18] as a part of their OD boundary detection framework. The thresholding approach is applied on the sub-image that has been converted from the Cartesian coordinate to the Polar coordinate and pre-processed using successive morphological operations, namely erosion by reconstruction and dilation by reconstruction. The contour of the OD boundary is then obtained with the aid of CHT and ellipse fitting.

2.2.1.2 Active Contour Modelling-based Approaches

Active contour modelling, also known as snakes, is an image processing technique that is beneficial to describe the boundary of objects within images. Snakes work by first defining a curve model and the defined model will iteratively evolve and finally fit the object boundary. The fitting problem can be posed as an energy minimization task under a certain constraint. In particular, given a snakes model that is defined by a set of n points v_i , where $i = 0, 1, \dots, n - 1$, the snakes will calculate and minimize the snakes energy (E_{snake}) given by:

$$E_{\text{snake}} = \int_0^1 (E_{\text{int}}(v(s)) + E_{\text{ext}}(v(s))) ds. \quad (2.16)$$

E_{int} and E_{ext} are the internal and external energies, where the former controls the deformation of the snakes model (determines the continuity and the smoothness of the model) while the latter handles the contour fitting of the object, respectively. Mathematically, these two energy terms can be expressed as follows:

$$E_{\text{int}} = E_{\text{cont}} + E_{\text{curv}}, \quad (2.17)$$

and

$$E_{\text{ext}} = E_{\text{cons}} + E_{\text{img}}, \quad (2.18)$$

where E_{cont} and E_{curv} correspond to the continuity and the smoothness of the contour while E_{cons} and E_{img} denote the constraint energy and the image energy, respectively. The image energy is generally composed of some features of the image such as lines, edges and terminations while the constraint energy comes from the constraint defined by the user.

Active contour modelling has been widely used for OD boundary detection as it can provide reliable shape and size of the optic disc boundary. However, the presence

of other objects in the optical disc surrounding such as blood vessels may affect the boundary detection process. Hence, some pre-processing steps are commonly applied to the images, such that only the OD appears in the region of interest. The pre-processing steps also aim at providing the initial maps of OD boundaries. In this respect, the above-mentioned OD appearance-based approaches are commonly utilised as the pre-processing steps.

The work by Mary et al. [19] proposes an OD boundary detection algorithm based on the CHT and active contour modelling. The blood vessels are initially removed from the red channel of fundus images using the multi-orientation morphological line operator. The processed red channel is then converted to a binary image and the CHT is utilised to obtain the initial contour of the OD boundary. Finally, given the initial contour, the gradient-based active contour approach is employed to obtain the final contour of the OD.

In the work by Dai et al. [20], a modified active contour approach is introduced for the OD boundary detection problem. The approach works by minimizing three energy terms namely the phase-based boundary energy, PCA-based shape energy and region energy. The initial OD boundary used in the energy minimization process is obtained using the CHT.

2.2.1.3 Shallow Learning

In the OD boundary detection problem, learning strategies, both shallow and deep learning, are commonly employed to avoid false positive detection. In general, this occurs on bright regions that do not belong to the OD (such as exudates) due to the difficulty in finding an appropriate threshold value. In addition, false positives can be due to the blood vessels since the employed edge detection method cannot distinguish edges from curvilinear structures. In the algorithms that employ shallow learning strategies, hand-crafted features are extracted from retinal images to represent the OD characteristic and subsequently become the input of supervised classifiers.

The work by Roychowdhury et al. [3] characterises the OD using 21 features that consists of one Vessel-Sum feature, two combination features, four-pixel intensity-based features and 14 structural features. The generated features are then classified using the Gaussian Mixture Model (GMM) classifier. In [21], the orientation

change of the OD boundary is characterised using the histogram of oriented gradient (HOG) features. In particular, the features are extracted from local patches, each contains different parts of the OD. A non-linear function is then formed to map the extracted features to the corresponding ground truth of the OD boundary. Finally, the obtained function is used to perform OD boundary detection on images from the test set.

Another example of OD boundary detection methods based on shallow learning is the work by Rehman et al. [22]. In this work, highly discriminative features such as statistical-based, texton histogram-based and fractal-based features are utilised to represent the OD. Four benchmark classifiers: Support Vector Machine (SVM), AdaBoost, Random Forest (RF) and RusBoost are employed to obtain the OD boundary and the performance of each classifier is then compared. More recently, to develop a glaucoma screening system based on the optic cup to disc ratio, Mohamed et al. [23] performs OD segmentation by initially representing OD using statistical pixel-level features and subsequently classifying the features using SVM.

2.2.1.4 Deep Learning

In recent years, deep neural networks have shown outstanding performance on supervised learning problems. Among all types of these networks, convolutional neural networks (CNNs) are widely used in many visual recognition tasks, i.e., image classification, segmentation and detection. The first CNN used for a medical purpose is introduced by Ronneberger et al. [24], which is called U-Net. Specifically, U-Net is used for cell segmentation and it outperforms all previous networks with only 30 training images. The limited training images issue is addressed by applying excessive data augmentation. Also, the concatenation of the high and low-resolution features in U-Net allows the network to produce a more precise output.

The success of U-Net on medical image segmentation has instigated the development of deep learning-based algorithms for OD boundary detection. For instance, Yu et al. [25] adapts the ResNet-34 model as the down-sampling part of the U-Net model, while for the up-sampling part, the original U-Net architecture is retained.

To enrich the training patterns of the OD boundary, data augmentation techniques, i.e., rotation, upside-down flipping and left-right flipping are employed.

A multi-stage deep convolutional neural network based on the modified U-Net is also employed by Wang et al. [26] to perform OD boundary detection. In their work, false detection is addressed by applying a coarse-to-fine segmentation strategy. In particular, the network is separately trained on colour fundus images and the grey-scale vessel density maps. The obtained results are integrated with the mean of an overlap technique aiming at a local image patch identification. Finally, the optic disc is segmented by training the U-net architecture using the obtained patches in the previous step.

2.2.2 Retinal Blood Vessel Segmentation

2.2.2.1 Two-Dimensional Matched Filter

In Chaudhuri et al. [27], a two-dimensional filter is designed to perform curvilinear structures detection. In particular, the matched filter is employed to perform retinal vessel detection on colour fundus images. The shape of the filter compared to the Gabor function is quite similar, however, the matched filter can be designed in a more straightforward way, i.e., by manipulating a single Gaussian function as follows:

$$K'(x, y) = K(x, y) - \overline{K(x, y)}, \quad (2.19)$$

where

$$K(x, y) = \exp\left(-\frac{x'^2}{2\sigma^2}\right), \quad (2.20)$$

$$|x'| \leq 3\sigma, \quad (2.21)$$

$$0 \leq y' < L, \quad (2.22)$$

$$x' = x \cos \theta + y \sin \theta, \quad (2.23)$$

$$y' = -x \sin \theta + y \cos \theta. \quad (2.24)$$

The variables L , σ and θ are the approximated length, width and preferred orientation of blood vessels.

The work by Chaudhuri et al. [27] has inspired the development of other matched filters for curvilinear structures detection that offer better performance. For instance, Al-Rawi et al. [28] performs a heuristic search to optimize the values of L and T , where T is the length of the matched filter window. Later, Zolfagharnasab and Naghsh-nilchi [29] and Singh and Srivastava [30] use the Cauchy and Gumbel probability functions, respectively, to replace the Gaussian function in designing the matched filter window.

2.2.2.2 Hessian Matrix Analysis

The Hessian matrix $\mathbf{H}$ analysis is a useful technique that can be used to describe the local curvature of a multi-variable function. It consists of second-order partial derivatives of the described function. The multi-variable functions that can be described by the Hessian matrix include an image $\mathbf{A}$, where its Hessian matrix is given by:

$$\mathbf{H}_{\mathbf{A}} = \begin{bmatrix} \nabla_x^2 \mathbf{A} & \nabla_x \nabla_y \mathbf{A} \\ \nabla_y \nabla_x \mathbf{A} & \nabla_y^2 \mathbf{A} \end{bmatrix}. \quad (2.25)$$

Note that a Gaussian filter with a certain standard deviation σ is usually applied to image $\mathbf{A}$ prior to the calculation of its partial derivatives. The useful property of the Hessian matrix is on its eigenvalues configurations, where each of them reflects a particular local structure of the image. Specifically, given λ_1 and λ_2 are the eigenvalues of the Hessian matrix $\mathbf{H}_{\mathbf{A}}$ and $|\lambda_1| \leq |\lambda_2|$, we may describe the local structure of image $\mathbf{A}$ according to Table 2.1.

The useful property of the Hessian matrix has been widely investigated by many researchers for curvilinear structures detection on medical images. In particular, Hessian matrix analysis has been broadly used for blood vessel segmentation on colour fundus images. Frangi et al. [31] introduce a ridgeness measure as the

TABLE 2.1: The image local structure-based on the eigenvalues of the Hessian matrix. (L: low, H+: high positive and H–: high negative.)

λ_1	λ_2	Image local structure
L	L	Noise
L	H–	Bright curvilinear structure
L	H+	Dark curvilinear structure
H+	H+	Bright blob structure
H–	H–	Dark blob structure

ratio of the eigenvalues $R = \lambda_1/\lambda_2$ with an aim to differentiate blood vessels from the background. In addition, Frangi et al. [31] also considers the magnitude of the eigenvalues, which is calculated as the Frobenius norm of the Hessian matrix $\|\mathbf{H}_A\|_F = \sqrt{\lambda_1^2 + \lambda_2^2}$, to determine the presence of blood vessels. Frangi et al. [31] then combine these two measures into a single indicator, namely the vesselness measure to determine the likelihood of pixels being blood vessels as:

$$S_{\mathbf{H}}(x, y; \sigma) = \begin{cases} 0, & \lambda_2 > 0, \\ \exp\left(-\frac{R(x, y; \sigma)}{2\beta_{\mathbf{H}}^2}\right) \left(1 - \exp\left(-\frac{\|\mathbf{H}_A\|_F(x, y; \sigma)}{2c^2}\right)\right), & \text{otherwise} \end{cases}, \quad (2.26)$$

where parameters $\beta_{\mathbf{H}}$ and c govern the sensitivity of the R and $\|\mathbf{H}_A\|_F$ measures, respectively. In addition, the parameter σ can be manipulated to set the preferred width of the blood vessels.

Although the vesselness measure introduced by Frangi et al. [31] is simple to be used, it produces less desirable detection results particularly in terms of continuous vascular tree and sharp vessel edges. Also, it is prone to produce artefacts on the detection result when it deals with images containing significant pathologies. Therefore, several attempts have been made to produce better Hessian matrix analysis-based curvilinear structures detection algorithms. For instance, Annunziata et al. [32] apply an image in-painting technique on colour fundus images before the blood vessel detection step using the Frangi et al. [31]'s approach. This aims to remove the presence of bright lesions such as exudates that may appear on the images and lead to artefacts on the vessel detection result. As a result, the effect of exudates on the vessel detection result can be suppressed.

2.2.2.3 Shallow Learning

Various learning strategies (shallow learning and deep learning) have been applied to build supervised curvilinear structures detection algorithms. Essentially, shallow and deep learning strategies differ on how features for curvilinear structures are formed. On the shallow learning group, hand-crafted features are commonly extracted from fundus images and fed to some supervised classifiers such as AdaBoost classifier, SVM and K-Nearest Neighbor (K-NN). On the other hand, deep learning algorithms are able to automatically extract rich representative features

through a lot of neurons in each layer. As a result, they tend to achieve better performance than the shallow learning algorithms.

Although producing less satisfactory performance than the deep learning algorithms, shallow learning strategies are still widely applied, particularly for those avoiding huge computational complexity. In this respect, a set of features for curvilinear structures should be formed, where the results of the above-described unsupervised curvilinear detection techniques are commonly used as the features. For instance, Gabor filter banks responses for different values of parameters are used as the features of curvilinear structures in the works by Soares et al. [33] while matched filter responses are utilised by Sofka and Stewart [34] to characterise the blood vessel profiles.

Another example of supervised algorithms is the work by Orlando et al. [35], which model the segmentation job as an energy minimization problem in a conditional random field (CRF). Several filter responses are used as the vessels features while a structured output support vector machine (SOSVM) is utilised in the CRF learning process. More recently, Wang et al. [36] combine a set of blood vessel features with a cascade classifier. Unlike most other supervised methods where only the green channel of fundus images is used, the features in this work are extracted from all the channels (red, green and blue) of fundus images. To be specific, 100 different features are extracted and used as the input of the cascade classifier.

2.2.2.4 Deep Learning

An example of deep learning algorithms for blood vessel segmentation is the work by Li et al. [37] that model the blood vessels segmentation task as a cross-modality data transfer problem. The colour retinal image becomes the first modality while the second modality is the corresponding vessel map. In Fu et al. [38], the blood vessel segmentation is performed with the means of a CNN with a side output layer and also a CRF layer. The structure prediction strategy applied in the fully connected CNN by Liskowski and Krawiec [39] is also an important development as it is able to produce noticeable improvements in the segmentation performance.

One of the challenging situations in retinal vessel segmentation is to retain thin blood vessels in the segmentation result. Although deep learning algorithms can

detect vessels at different widths, segmentation of thin vessels is commonly considered as less important than the thick vessels. This is because most algorithms are trained with the means of pixel-to-pixel matching between every obtained probability map and the corresponding ground truth. Therefore, in Zhou et al. [40], CNN and combo filters are used to generate discriminative features (unary potentials) and thin vessels enhanced images (pairwise potentials). The two potentials are used by the dense CRF model to achieve the final blood vessel segmentation.

An attempt is also made by Yan et al. [41] to overcome the thin vessels segmentation problem, where two loss functions, namely segment-wise and pixel-wise losses are employed to train the CNN architecture. In particular, the application of joint loss functions will make the network to learn the importance between thick and thin vessels more equally. Another important finding by the method in Yan et al. [41] is that it has also been tested on high-resolution fundus images.

2.3 Performance Comparison of Existing Algorithms

2.3.1 Optical Disc Boundary Detection

2.3.1.1 Databases

The **MESSIDOR database** [42] contains 1200 fundus colour images that are acquired by the Hôpital Lariboisière Paris, the Faculté de Médecine St. Etienne and the LaTIM– CHU de Brest (France) using a Topcon TRC NW6 non-mydratic retinograph with a 45° FOV. The images are provided in TIFF format with 1440×960, 2240×1488, or 2304×1536 pixels size. 660 images are from patients with the diabetic retinopathy complication and 540 images correspond to patients without the illness. For performance evaluation of an algorithm, the references for OD centre location and boundary of the 1200 images are provided by the University of Huelva, Spain [15] and are publicly available on <http://www.uhu.es/retinopathy>.

DRIONS-DB [43] is a public database that is developed for the evaluation of OD segmentation from digital retinal images algorithms. The database contains 110 colour digital retinal images. The images are acquired using a colour fundus

camera, approximately centred on the OD and digitised using an HP-PhotoSmart-S20 high-resolution scanner with a resolution of 600x400, 8 bits/pixel. The images in the database are collected from patients with glaucoma (23.1%) and hypertension (76.9%). For the evaluation of an OD segmentation algorithm, the references for the OD boundary of the 110 images are also provided.

RIM-ONE is an open database for the evaluation of OD segmentation algorithms that are developed by Fumero et al. [44]. The database consists of 169 fundus images, each is acquired using a fundus camera Nidek AFC-210 with a body of a Canon EOS 5D Mark II of 21.1 megapixels. Each image in this database initially had a size of a full fundus image and subsequently is manually cropped to a region of interest centred at the OD. The images are collected from patients with normal eyes condition (118 images), early glaucoma (12 images), moderate glaucoma (14 images), deep glaucoma (14 images) and ocular hypertension (11 images). For the evaluation of an OD segmentation algorithm, the references for the OD centre and boundary of the 169 images are also provided.

2.3.1.2 Indicators

Several indicators have been used to evaluate the performance of existing optical disc boundary detection algorithms. These include Area Overlap (*AOL*), Dice coefficient (*DSC*), Accuracy (*Acc*), True Positive Fraction (*TPF*) and False Positive Fraction (*FPF*). These indicators are described and defined as follows:

1. *AOL* refers to the ratio of intersecting area between the segmented and actual OD.

$$AOL = TP / (TP + FN + FP). \quad (2.27)$$

A higher *AOL* value corresponds to a better segmentation result.

2. *DSC* describes the similarity degree between the segmented and actual OD.

$$DSC = 2TP / (2TP + FN + FP). \quad (2.28)$$

Perfect segmentation is achieved when *DSC* is equal to 1.

3. Acc denotes the ratio of correctly classified OD and non-OD regions to the total number of pixels in the image.

$$Acc = (TP + TN) / (TP + TN + FP + FN). \quad (2.29)$$

A higher Acc score denotes a better segmentation result.

4. TPF corresponds to the ratio of correctly detected OD pixels to the actual OD pixels.

$$TPF = (TP) / (TP + FN). \quad (2.30)$$

A higher TPF value refers to a better segmentation result.

5. FPF denotes the ratio of falsely detected non-OD pixels to the actual non-OD pixels.

$$FPF = (FP) / (TN + FP). \quad (2.31)$$

A lower FPF value denotes a better segmentation result.

The variables TP , TN , FP and FN stand for true positive, true negative, false positive and false negative, respectively. Note that each indicator is calculated based on the area surrounded by the detected optical disc boundary.

In Table 2.2, we summarize the performance of the discussed optical disc boundary detection algorithms on fundus images from several databases. The performance is measured in terms of AOL , DSC , Acc , TPF and FPF . The best score of each indicator is listed in bold. In the table, it is observed that the deep learning-based algorithm by Yu et al. [25] achieves the best performance among the other methods on the RIM-ONE database. On the other hand, the shallow learning-based approach by Li et al. [21] and the active contour-based approach by Dai et al. [20] outperform the compared methods on the MESSIDOR and DRIONS-DB databases, respectively. As pointed out by Wang et al. [26], they have a problem with the GPU memory limitation, such that unlike in other methods, the training images in Wang et al. [26] should be down-sampled to the size of 256×256 . Consequently, this lead to a less satisfactory performance than the other methods.

The superiority of the method in [25] indicates that the non-linear property of the deep learning architecture allows it to easily learn complicated shapes of OD boundaries. In addition, it also has the capability to differentiate the OD from

TABLE 2.2: Performance of OD boundary detection algorithms on fundus images from several databases

Database: MESSIDOR						
Authors	Approach	<i>AOL</i>	<i>DSC</i>	<i>Acc</i>	<i>TPF</i>	<i>FPF</i>
Aquino et al. [15]	OD appearance	0.86	—	—	—	—
Morales et al. [16]	OD appearance	0.823	0.895	0.995	0.930	0.004
Marin et al. [17]	OD appearance	0.87	0.92	—	—	—
Zahoor and Fraz [18]	OD appearance	0.844	0.904	0.992	0.889	0.003
Dai et al. [20]	Active contour	0.910	—	—	—	—
Roychowdhury et al. [3]	Shallow learning	0.837	—	0.996	0.904	0.002
Li et al. [21]	Shallow learning	0.913	—	—	—	—
Rehman et al. [22]	Shallow learning	0.747	0.851	0.988	0.953	0.012
Wang et al. [26]	Deep learning	0.907	0.949	0.972	0.948	—
Database: DRIONS-DB						
Authors	Approach	<i>AOL</i>	<i>DSC</i>	<i>Acc</i>	<i>TPF</i>	<i>FPF</i>
Morales et al. [16]	OD appearance	0.842	0.908	0.993	0.928	0.004
Zahoor and Fraz [18]	OD appearance	0.886	0.938	0.999	0.938	0.002
Dai et al. [20]	Active contour	0.908	—	—	—	—
Li et al. [21]	Shallow learning	0.853	—	—	—	—
Rehman et al. [22]	Shallow learning	0.821	0.899	0.993	0.948	0.006
Wang et al. [26]	Deep learning	0.885	0.936	0.960	0.924	—
Database: RIM-ONE						
Authors	Approach	<i>AOL</i>	<i>DSC</i>	<i>Acc</i>	<i>TPF</i>	<i>FPF</i>
Zahoor and Fraz [18]	OD appearance	0.748	0.849	0.975	0.911	0.019
Li et al. [21]	Shallow learning	0.869	—	—	—	—
Rehman et al. [22]	Shallow learning	0.747	0.851	0.988	0.953	0.012
Mohamed et al. [23]	Shallow learning	—	0.94	—	—	—
Yu et al. [25]	Deep learning	0.907	0.949	—	—	—

other bright regions such as exudates that commonly become the false positive in the detection problem. This property may not be available for the shallow learning methods and also unsupervised methods as hand-crafted features used by these groups of algorithms may only be able to represent particular shapes of OD boundaries. However, on the other side, the method in Wang et al. [26] shows that a deep learning-based algorithm may encounter a hardware limitation problem, such that the application of a deep learning model will not guarantee satisfactory performance. Hence, such algorithms may not be suitable for practical applications.

2.3.2 Retinal Blood Vessel Segmentation

2.3.2.1 Databases

The **DRIVE database** [4] contains 40 fundus images captured during a screening study with 400 subjects. The images are grouped into the training and testing sets, each consists of 20 images. Each image in this database has a size of 565×584 pixels. Two sets of manually annotated images are provided. The first set is utilised as the ground truth.

The **STARE database** [45] consists of 20 fundus images which are offered for public use by the University of California, San Diego. Each image in this database has a size of 700×605 pixels. In this database, 10 images are from healthy subjects while the rest contain numerous pathologies. Two manual segmentation sets are provided and the first set is considered as the ground truth.

The **HRF database** [46] is the result of a collaborative work conducted to support studies on algorithms for automatic vessels segmentation from fundus images. This database contains 45 high-resolution fundus images, which are divided into the healthy, diabetic retinopathy and glaucomatous sets. Every set comprises 15 fundus images, each with a size of 3504×2336 pixels. A binary gold standard vessel segmentation image is provided for each image and it is used as the ground truth.

2.3.2.2 Indicators

The existing algorithms for retinal blood vessel segmentation in the literature are typically evaluated in terms of several measures such as Sensitivity (Se), Specificity (Sp) and Accuracy (Acc). Most of the algorithms calculate the indicators only for the region inside the field of view (FOV). The performance indicators are described and can be calculated as follows:

1. Se (known also as TPF) shows the capability of a particular algorithm to correctly classify vessel pixels as blood vessels.

$$Se = TP / (TP + FN) . \quad (2.32)$$

2. Sp (known also as $1 - FPF$) is an indicator that measures the ability of an algorithm to correctly detect non-vessel structures.

$$Sp = TN / (TN + FP). \quad (2.33)$$

3. Acc denotes the ratio of correctly classified vessel and non-vessel structures to the total number of pixels in the FOV.

$$Acc = (TP + TN) / (TP + TN + FP + FN). \quad (2.34)$$

Acc value ranges from 0 to 1, with 1 indicates perfect segmentation.

TP , TN , FP and FN are the numbers of true positive, true negative, false positive and false negative, respectively.

In Table 2.3, we list the performance of the discussed blood vessel detection algorithms on the DRIVE, STARE and HRF databases. In the table, it is observed that to achieve a reliable performance on the blood vessel detection problem, a huge number of features are required. Hence, algorithms that are based on the deep learning approaches typically offer better performance than the algorithms that use other approaches. This includes the algorithm in Wang et al. [36], which utilise 100 different features in the classification problem. In addition, unlike other algorithms that typically use only the green channel of fundus images, the use of all channels (red, green and blue) of fundus images in the feature extraction step by Wang et al. [36] may also lead to that better performance than other shallow learning-based methods.

It should be pointed in Table 2.3 that only a few algorithms developed and tested on the HRF database. One possibility is due to the much larger size of the images in this database, compared to those in the DRIVE and STARE databases. Hence, a GPU memory limitation may be the primary issue. This conforms to the use of the down-sampling techniques on the images from the HRF database by the algorithms in Orlando et al. [35], Zhou et al. [40] and Yan et al. [41]. As a result of using this technique, they are not able to achieve similar superior performance as that produced on the DRIVE and STARE databases. One reason is due to the information loss caused by the down-sampling technique, such that the learning strategies cannot build appropriate models for the vessel and non-vessel structures.

TABLE 2.3: Performance of blood vessel segmentation algorithms on the DRIVE database

Database: DRIVE				
Authors	Method	<i>Se</i>	<i>Sp</i>	<i>Acc</i>
Chaudhuri et al. [27]	Matched filter	0.6326	0.9064	0.8709
Al-Rawi et al. [28]	Matched filter	0.5993	0.9553	0.9096
Zolfagharnasab and Naghsh-nilchi [29]	Matched filter	0.6239	0.9714	0.9269
Singh and Srivastava [30]	Matched filter	0.7594	0.9708	0.9522*
Frangi et al. [31]	Hessian matrix analysis	0.746	0.972	0.942
Soares et al. [33]	Shallow learning	0.7332	0.9782	0.9466
Sofka and Stewart [34]	Shallow learning	0.609	0.982	0.935
Orlando et al. [35]	Shallow learning	0.7897	0.9684	—
Wang et al. [36]	Shallow learning	0.7648	0.9817	0.9541
Li et al. [37]	Deep learning	0.7569	0.9816	0.9527
Fu et al. [38]	Deep learning	0.7603	—	0.9523
Liskowski and Krawiec [39]	Deep learning	0.7750	0.9795	0.9518
Zhou et al. [40]	Deep learning	0.8078	0.9674	0.9469
Yan et al. [41]	Deep learning	0.7653	0.9818	0.9542
*Different FOV masks are used.				
Database: STARE				
Authors	Method	<i>Se</i>	<i>Sp</i>	<i>Acc</i>
Singh and Srivastava [30]	Matched filter	0.7939	0.9376	0.9270
Frangi et al. [31]	Hessian matrix analysis	0.754	0.974	0.950
Annunziata et al. [32]	Hessian matrix analysis	0.7128	0.9836	0.9562
Soares et al. [33]	Shallow learning	0.717	0.981	0.947
Sofka and Stewart [34]	Shallow learning	0.565	0.981	0.924
Orlando et al. [35]	Shallow learning	0.7680	0.9738	—
Wang et al. [36]	Shallow learning	0.7523	0.9885	0.9640
Li et al. [37]	Deep learning	0.7726	0.9844	0.9628
Fu et al. [38]	Deep learning	0.7412	—	0.9585
Liskowski and Krawiec [39]	Deep learning	0.7766	0.9854	0.9638
Zhou et al. [40]	Deep learning	0.8065	0.9761	0.9585
Yan et al. [41]	Deep learning	0.7581	0.9846	0.9612
Database: HRF				
Authors	Method	<i>Se</i>	<i>Sp</i>	<i>Acc</i>
Orlando et al. [35]	Shallow learning	0.7874	0.9584	—
Zhou et al. [40]	Deep learning	0.8015	0.9699	0.9544
Yan et al. [41]	Deep learning	0.7881	0.9592	0.9437

Another important issue regarding the evaluation procedure is the use of *Acc* as a performance indicator since it is observed in Table 2.3 that the value of *Acc* heavily relies on the *Sp* value. When an algorithm achieves a very high *Sp* value, it will also achieve a very high *Acc* score, and vice-versa. Therefore, using *Acc* as the indicator will lead to a misleading justification, i.e., an algorithm may achieve a high *Acc*, but in reality, it is only able to detect a small number of blood vessels. One possibility to overcome this shortcoming is to replace *Acc* with other indicators like F1-score (*F1*, known also as Dice coefficient) and Matthews correlation coefficient (*MCC*) [35].

2.4 Summary

In this chapter, the underlying idea of edge and curvilinear structures detection in digital imagery has been presented. In addition, some existing edge and curvilinear structures detection algorithms, particularly those being applied for optical disc boundary and retinal vessel detection on fundus images have been reviewed. The existing algorithms vary in those exploiting low-level image features, those considering the image morphological profiles and those involving learning strategies. Among the presented algorithms, those applying deep learning strategies are commonly better than algorithms that using other approaches. However, the high computational complexity may become the drawback of such algorithms, particularly those developed and tested on high-resolution images. In addition, the performance of most algorithms, particularly those for blood vessel detection have not been properly assessed. To overcome these shortcomings, developing new algorithms for optical disc boundary and retinal vessel detection on fundus images that do not lead to high computational complexity and evaluating their performance using more appropriate performance indicators are necessary and meaningful.

Chapter 3

A Flexible Model for Edge Representation in Digital Imagery

3.1 Introduction

Edges are regarded as essential features in many applications of image processing and computer vision. These include object detection [47, 48], image segmentation [49, 50], biological image processing [51–54] and image fusion [55, 56]. By detecting edges, the analysis of images can be simplified as the amount of data to be processed is drastically reduced [14]. Therefore, it is desirable to develop a flexible model for edge analysis that fits a wide variety of edges characteristics.

A lot of research work has been done to model edge profiles. Among them, the Gaussian function is widely used to develop the edge models [5, 6, 57] as it can capture the amount of blur in edge profiles. Moreover, the use of two Gaussian functions in Seo [7] to model the dark-side and light-side brightness of edge profiles is an important development. Nevertheless, these edge models do not take into account other essential edge features in the edge modelling mechanism. In practice, edges of digital images are not only blurred but also noisy. The noise level depends on the electronic components of the imaging device [58]. Edges which contain a particular level of noise may not appear as an ideal step or ramp. Instead, several gutters and humps may emerge on the edge intensity profile. Consequently, the obtained models may not be optimum for a wide variety of edges in real images.

To capture the gutters and humps while modelling the edge, one may develop an edge model based on the linear combination of some Gaussian functions (more than two functions as in Seo [7]). However, the number of parameters required to be estimated will increase and this will inevitably lead to higher computational complexity. Furthermore, the estimation of the model parameters will be prone to be trapped in a local minimum. To deal with such a problem, this chapter proposes a flexible model for edge representation. In summary, the contributions of this chapter are as follows:

1. A flexible model for edge profiles is developed based on the Dolph-Chebyshev function. The main lobe of this function will capture the edge blur level while the amounts of gutters and humps will be recovered by its side-lobe response.
2. The validity of the proposed model is experimentally examined using edge profiles extracted from images with various blur and noise levels. Specifically, the parameters of the proposed model are estimated under a least-square framework.
3. The edge model is compared with the Gaussian-based models in Hagara and Kulla [6] and Seo [7] and it is observed that the proposed model consistently provides better estimates for all tested edge profiles than the compared methods.
4. A procedure to perform edge detection on two-dimensional images using the proposed model is presented. The Gaussian-based models are also used to detect edges under the framework and it is observed that the proposed model produces better detection performance than the Gaussian-based models.

3.2 Edge Modelling with the Dolph-Chebyshev Function

In accordance with other works [5, 6, 57], we mathematically describe the light intensity of edge profile at the location x as the convolution (denoted as ‘*’) of a

step function with any blur function $f(x)$:

$$\begin{aligned} G(x) &\approx h + k * f(x) \\ &\approx h + k \int_{-\infty}^x f(t) dt, \end{aligned} \quad (3.1)$$

while the corresponding slope profile can be modelled as follows:

$$\begin{aligned} g(x) &\approx \nabla (h + k * f(x)) \\ &\approx \nabla (k) * f(x) \\ &\approx f(x). \end{aligned} \quad (3.2)$$

In these works [5, 6, 57], the Gaussian function is used to represent $f(x)$. However, the Gaussian-function cannot effectively capture the gutters and humps, which commonly appear around edges in real images. Therefore, the solution in Ye et al. [5], Tabatabai and Mitchell [57] and Hagara and Kulla [6] cannot be generalized for a wide variety of edge profiles. Figure 3.1(b) shows an example of edge profiles extracted from the real image in Figure 3.1(a). The corresponding slope profile is provided in Figure 3.1(c). It is observed that the slope profile exhibits many humps and gutters on its side-lobes and the monotonous side-lobe response of the Gaussian function clearly cannot fit the humps and gutters (Figure 3.1(d)).

To overcome the above-mentioned drawbacks, we propose a more appropriate representation for $f(x)$ based on the Dolph-Chebyshev function:

$$f(x, a, \beta) = a \cos \left(M \cos^{-1} (\beta \cos (\pi x / M)) \right), \quad (3.3)$$

where

$$\beta = \sec \left(\frac{\pi \omega_c}{M} \right). \quad (3.4)$$

The parameter M is the length of edge profiles to be represented while a is the weighting coefficient. The desired level of blur on $f(x)$ can be obtained by manipulating the value of ω_c . This parameter also corresponds to the optimum level of humps and gutters on the side-lobe. With appropriate parameters values, the proposed model (shown in Figure 3.1(e)) will fit the slope profile in Figure 3.1(c) more precisely.

(a)

(b)

(c)

(d)

(e)

FIGURE 3.1: (a) An example of real images, (b) edge profile extracted along the red arrow in (a), (c) corresponding slope profile for the edge profile in (b), (d) the $f(x)$ model in Ye et al. [5] and (e) the proposed $f(x)$ model. The green contour on the image in (a) denotes the edge locations. The values of $a = 6.2$, $\sigma = 0.9$ and $x' = -0.5$ are used for the model in (d) while $a = 0.2$, $\omega_c = 1.3$, and $x' = -0.5$ are used for the model in (e).

3.3 Estimation of Proposed Edge Model Parameters

In accordance with the work in Seo [7], the proposed model parameters are estimated by fitting the slope of each edge profile $g(x)$ using the function $f(x)$ (see (3.2)). Since the collected edge and slope profiles (the collection procedure is discussed in Section 3.4) may not be centred at zero on the horizontal axis, we need to quantify the centre displacement in our fitting scenario. Hence, we modify the proposed blur models in (3.3) and (3.4) as follows:

$$f(x, x', a, \beta) = a \cos \left(M \cos^{-1} (\beta \cos (\pi (x - x') / M)) \right), \quad (3.5)$$

where

$$\beta = \sec \left(\frac{\pi \omega_c}{M} \right), \quad (3.6)$$

and x' is the centre displacement of $f(x)$.

The fitting scenario attempts to find the optimum set of parameters $P = \{x', a, \omega_c\}$, within a neighbourhood N , which minimise the mean-square-error between the slope profile $g(x)$ and the proposed slope model $f(x, P)$:

$$\underset{P}{\text{minimise}} \ ||f(x, P) - g(x)||_2^2, \quad (3.7)$$

subject to $P \in N$,

where

$$N = \left\{ \begin{array}{l} -\infty \leq x' \leq \infty, \\ -\infty \leq a \leq \infty, \\ 0 \leq \omega_c \leq \infty, \end{array} \right\}. \quad (3.8)$$

It is important to note that the expression in (3.7) is a constrained non-linear optimisation problem. Therefore, we solve it using the Trust-Region-Reflective Least Squares technique [59, 60].

3.4 Collection of Edge Profiles

The validity of the proposed model is verified using edge profiles extracted from a set of images with varying levels of blur and noise. The set contains 64 images generated by adding different levels of blur and noise to a synthetic image. The synthetic image has a size of 400×400 pixels and contains four squares arranged as in Figure 3.2. The brightness value of the darker squares is 0.3 while that of the lighter squares is 0.7.

FIGURE 3.2: The synthetic image used to generate a set of images with different levels of blur and noise.

To generate the images set, the synthetic image is modified with different levels of blur and noise. The detailed blur and noise levels used in this chapter are summarized in Table 3.1 and Table 3.2, respectively. The underlying idea of adding various levels of blur and noise to the synthetic image is to generate a set of images with characteristics as close as to that of real images. Therefore, edges extracted from these images will behave as edges in real images. In Table 3.1, the parameters

TABLE 3.1: Detailed blur types and levels applied to the synthetic image.

No	Type of blur	Parameter
1	Gaussian blur	$\sigma = 0.5$
2	Gaussian blur	$\sigma = 1$
3	Motion blur	$l_m = 4$ and $\theta_m = 45$
4	Motion blur	$l_m = 6$ and $\theta_m = 90$

TABLE 3.2: Detailed noise types applied to the synthetic image.

No	Type of noise	Parameter
1	Gaussian noise	$\sigma_G = 0.02$
2	Gaussian noise	$\sigma_G = 0.04$
3	Gaussian noise	$\sigma_G = 0.06$
4	Gaussian noise	$\sigma_G = 0.08$
5	Gaussian noise	$\sigma_G = 0.1$
6	Poisson noise	—
7	Salt and pepper noise	$d = 0.1$
8	Salt and pepper noise	$d = 0.2$
9	Salt and pepper noise	$d = 0.3$
10	Salt and pepper noise	$d = 0.4$
11	Salt and pepper noise	$d = 0.5$
12	Speckle noise	$\sigma_S = 0.1$
13	Speckle noise	$\sigma_S = 0.2$
14	Speckle noise	$\sigma_S = 0.3$
15	Speckle noise	$\sigma_S = 0.4$
16	Speckle noise	$\sigma_S = 0.5$

l_m and θ_m are the length and angle of the motion blur window while the parameters σ_G , d and σ_S in Table 3.2 are the standard deviation of the Gaussian noise, the Salt and pepper noise density and the standard deviation of the Speckle noise, respectively. Some examples of the images with different levels of blur and noise are depicted in Figure 3.3.

The mechanism for collecting the edge profiles is illustrated in Figure 3.4. Edge profiles are extracted along the normal directions marked by red arrows in Figure 3.4(a). When collecting the edge profiles, we set the length of each red arrow (M) to 17. Note that each edge profile is collected in an ascending manner (dark-side brightness on the left and light-side brightness on the right). Then, the average of these profiles is calculated, producing a mean profile for each image. Figure 3.4(b) shows an example of the mean profile. Finally, the slope of the mean profile is obtained. Figure 3.4(c) shows the slope profile for the image in Figure 3.4(a).

FIGURE 3.3: Some examples of the images with different levels of blur and noise. A Gaussian blur ($\sigma = 0.5$) and a motion blur ($l = 4$, $\theta = 45$) are applied to the synthetic image to produce images in rows 1 and 2, respectively. Different types of noise are applied to each image: (a) Gaussian noise, (b) Poisson noise, (c) Salt and pepper noise and (d) Speckle noise.

3.5 Validation of the Proposed Model

To show the quality of the proposed model, we also fit the slope profiles using the Gaussian-based models in Hagara and Kulla [6] and Seo [7] and compare the obtained results with those of the proposed model. The model in Hagara and Kulla [6] can be calculated as follows:

$$f(x, x', a, \sigma) = \frac{a}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x')^2}{2\sigma^2}},$$

where σ corresponds to blur parameter used to model the blur level of edge profiles, x' is the centre displacement of edge profiles and a is the weighting coefficient. The fitting scenario attempts to find the optimum set of parameters $Q = \{x', a, \sigma\}$, which minimise the mean-square-error between $s(x)$ and $f(x, Q)$:

$$\underset{Q}{\text{minimise}} \|f(x, Q) - g(x)\|_2^2, \quad (3.9)$$

subject to $Q \in S$,

(a)

(b)

(c)

FIGURE 3.4: The mechanism for collecting the edge profiles. (a) The red arrows are some examples of the guideline for edge profiles collection; (b) and (c) The average of edge and slope profiles for the image in (a), respectively.

where

$$S = \left\{ \begin{array}{l} -\infty \leq x' \leq \infty, \\ 0 \leq a \leq \infty, \\ 0 < \sigma \leq \infty, \end{array} \right\}. \quad (3.10)$$

On the other hand, the model in Seo [7] can be expressed as follows:

$$\begin{aligned} f(x, x_1, x_2, a_1, a_2, \sigma_1, \sigma_2) = & -\frac{a_1}{\sigma_1 \sqrt{2\pi}} e^{-\frac{(x-x_1)^2}{2\sigma_1^2}} \\ & + \frac{a_2}{\sigma_2 \sqrt{2\pi}} e^{-\frac{(x-x_2)^2}{2\sigma_2^2}}, \end{aligned} \quad (3.11)$$

where σ_1 and σ_2 correspond to the dark-side and light-side blur parameters, respectively. Unlike in Seo [7], where the values of a_1 and a_2 are initially determined and later fixed for all experiments that follow, in our experiment, we include a_1 and a_2 as the parameters to be estimated in the fitting scenario. Therefore, the optimum set of parameters $R = \{x_1, x_2, a_1, a_2, \sigma_1, \sigma_2\}$, which minimise the mean-square-error between $s(x)$ and $f(x, R)$ can be obtained as follows:

$$\begin{aligned} & \underset{R}{\text{minimise}} \|f(x, R) - g(x)\|_2^2, \\ & \text{subject to } R \in T, \end{aligned} \quad (3.12)$$

where

$$T = \left\{ \begin{array}{l} -\infty \leq x_1 \leq \infty, \\ -\infty \leq x_2 \leq \infty, \\ 0 \leq a_1 \leq \infty, \\ 0 \leq a_2 \leq \infty, \\ 0 < \sigma_1 \leq \infty, \\ 0 < \sigma_2 \leq \infty, \end{array} \right\}. \quad (3.13)$$

The fitting problems in (3.9) and (3.10) as well as (3.12) and (3.13) are then solved using the Trust-Region-Reflective Least Squares technique [59, 60].

Figure 3.5 depicts the root-mean-square (RMSE) graphs of the three models on different blur levels. Each graph reports the RMSE of the three models for a single blur level and various types of noise as listed in Table 3.2. Figure 3.5(a) to Figure 3.5(d), consecutively, indicate the RMSE graphs for the blur levels listed in Table 3.1. Note that A, B and C denote our proposed model, the model in Hagara

FIGURE 3.5: RMSE graphs of the three models on different types of blur and noise. The graphs in (a) and (b) are for the Gaussian blurs with $\sigma = 0.5$ and $\sigma = 1$ while the graphs in (c) and (d) are for the Motion blurs with $\{l_m = 6, \theta_m = 45\}$ and $\{l_m = 7, \theta_m = 90\}$, respectively. Note that each graph reports the results of experiments with different types of noise as listed in Table 3.2.

and Kulla [6] and the model in Seo [7], respectively. It is observed in Figure 3.5 that the proposed model (A) consistently produces the lowest RMSE among the compared methods (B and C) on all edge profiles. This suggests that our proposed model fits the edge profiles characteristics; the blur level of the edges and the amounts of gutters and humps, better than the compared models.

It is important to note in Figure 3.5 that no noticeable increment on the RMSE produced by the proposed model when the different varieties of noise are applied. On the other hand, the other two models seem to produce higher RMSE when the level of noise increases. This is because the two models cannot fully capture the amounts of gutters and humps around the edges extracted from images with high levels of noise. Note that using a higher-order function as a model, such as the model proposed in Seo [7], will only increase the number of parameters and

the difficulty of the fitting problem to be solved. Consequently, in many cases, the model in Seo [7] achieves higher RMSE than that of the model in Hagara and Kulla [6].

Besides the RMSE, the computational cost of each edge model in performing edge modelling becomes another critical aspect of assessment. In this respect, the proposed model requires approximately 1.94 seconds to fit an edge slope profile while performing the same task, the models in Hagara and Kulla [6] and Seo [7] need about 1.33 and 2.11 seconds, respectively. It is interesting to note that to produce a better edge model, the proposed model only requires a slightly higher computational cost than that of the model in Hagara and Kulla [6]. It should also be noted that increasing the order of functions used in developing the edge model, e.g., the model proposed in Seo [7], will not only lead to a higher RMSE but also a higher computational cost.

3.6 Edge Detection using Our Proposed Model

In the previous sections, we have shown that our proposed model can provide close approximations for edge profiles and the parameters of our proposed model can be estimated by fitting the slope profiles $g(x)$ using our proposed function $f(x)$. With the same strategy, we can characterize the location of an edge in a two-dimensional image. Given an image $I(x, y)$, we may obtain the location of each edge by looking at some peaks of the following expression:

$$\begin{aligned} g(x, y) &= \nabla (I(x, y) * f(x, y)) \\ &= \nabla (f(x, y)) * I(x, y). \end{aligned} \quad (3.14)$$

Note that the response of $g(x, y)$ will be maximum if $[x, y]$ corresponds to the edge location. At this location, we also have:

$$\begin{aligned} \nabla (g(x, y)) &= \nabla (\nabla (f(x, y))) * I(x, y) \\ &= \nabla^2 (f(x, y)) * I(x, y) \\ &= 0. \end{aligned} \quad (3.15)$$

The expression in (3.15) attempts to detect the edge locations by finding zero-crossings in the convolution of the image $I(x, y)$ and the second-order derivative (∇^2) of the edge model $f(x, y)$.

Note that the expression in (3.15) requires a two-dimensional model $f(x, y)$. To meet this requirement, we need to form a circularly symmetric two-dimensional Dolph-Chebyshev function as follows:

$$f(x, y) = \cos \left(M \cos^{-1} \left(\beta \cos \left(\pi \sqrt{x^2 + y^2} / M \right) \right) \right), \quad (3.16)$$

where

$$\beta = \sec \left(\frac{\pi \omega_c}{M} \right). \quad (3.17)$$

Figure 3.6 shows an example of the proposed two-dimensional Dolph-Chebyshev function for $\omega_c = 2$ and $M = 13$.

FIGURE 3.6: The proposed two-dimensional Dolph-Chebyshev function for $\omega_c = 2$ and $M = 13$.

We evaluate the validity of the proposed two-dimensional model on edge detection using images from the RuG dataset [61]. This dataset is developed by the University of Groningen and it contains 40 images, each has a size of 512×512 . Each image in this dataset is equipped with a ground truth image depicting the edge of objects within the image. Some examples of the images in this dataset together with the corresponding ground truth are depicted in Figure 3.7(a) and Figure 3.7(b), respectively.

TABLE 3.3: Parameters values used in the simulation for the proposed model, the model in Hagara and Kulla [6] and the model in Seo [7].

Model	Parameter
Proposed model	$a = 1, \omega_c = 2$ and $M^* = 13$
Model in Hagara and Kulla [6]	$a = 1, \sigma = 2$ and $M^* = 13$
Model in Seo [7]	$a_1 = 1, a_2 = 1, \sigma_1 = 3.2, \sigma_2 = 2$ and $M^* = 13$

* M —The filter length and width

FIGURE 3.7: Examples of edge detection results on images from the RuG data set. (a) original images. (b) the corresponding ground truth, (c) the results of our proposed model, (d) the results of the model in Hagara and Kulla [6] and (e) the results of the model in Seo [7].

In the simulation, we also use the two-dimensional expansions of the models in Hagara and Kulla [6] and Seo [7] to represent $f(x, y)$ and compare the results with those of the proposed model. In the simulation, we set the parameters values for our model and also the models in Hagara and Kulla [6] and Seo [7] according to Table 3.3. Edge detection results obtained using our proposed model, the model in Hagara and Kulla [6] and the model in Seo [7] are depicted in Figure 3.7(c) to Figure 3.7(e), respectively.

We use three indicators to measure the performance of an edge detection method, namely Recall (Re), Precision (Pr) and F-Measure (F). These indicators can be calculated as follows:

$$Re = \frac{TP}{TP + FN}, \quad (3.18)$$

$$Pr = \frac{TP}{TP + FP}, \quad (3.19)$$

$$F = \frac{2 \times Pr \times Re}{Pr + Re}, \quad (3.20)$$

where TP , TN , FP and FN stand for the number of true positive, true negative, false positive and false negative, respectively. As suggested by [62], F can be used as a single indicator in the result comparison since it incorporates both Re and Pr in its calculation process. Therefore, we only compare the performance of the above-mentioned methods in terms of F as summarized by the scatter plot in Figure 3.8. The names of the images are shown on the x -axis.

It is observed in Figure 3.8 that our proposed model provides better detection results than the models in Hagara and Kulla [6] and Seo [7] for most images in the database (see images in the top row of Figure 3.7 for an illustration). This conforms to the experimental results in the previous sections. Although on some images such as ‘bear₃’, ‘bear₈’, ‘buffalo’ and ‘gnu’, other methods produce slightly higher F scores than our proposed model’s (see images on the bottom row of Figure 3.7 for an illustration), in general, our proposed model is still better than the models by Hagara and Kulla [6] and Seo [7]. In particular, a right-tailed paired-samples t -test applied to the F scores produced by the three models indicates that our proposed model outperforms the other two models by ($t(39) = 6.23, p < 1.243 \times 10^{-7}$) and ($t(39) = 7.085, p < 8.190 \times 10^{-9}$), respectively.

FIGURE 3.8: F-measure scores achieved by our proposed model and other algorithms for the images of the RuG data set. Blue squares ($\square$), red triangles ($\triangle$) and yellow circles ($\circ$) are for our proposed model, the model in Hagara and Kulla [6] and the model in Seo [7], respectively.

To further show the generalization ability of the proposed method, we have conducted more experiments and subsequently we discuss the results on images from the BSDS300 dataset [63]. Note that these images have also been widely used to test edge detection methods. The additional experiments show that our edge detector is better than the detectors built on top of the models in Hagara and Kulla [6] and Seo [7]. The superiority of our detector suggests the validity and robustness of our proposed model, and it may be beneficial to use it in the development of more sophisticated edge detectors. In particular, a right-tailed paired-samples *t-test* applied to the F scores produced by the three models indicates that our proposed model outperforms the other two models by ($t(299) = 7.462, p < 4.675 \times 10^{-13}$) and ($t(299) = 23.406, p < 7.308 \times 10^{-70}$), respectively.

Although the proposed model achieves better results than the compared models, developing an edge detector based on the Laplacian of particular edge model may provide less satisfactory results when the developed edge detector is applied on images heavily corrupted with noise [58]. To remedy this problem, we may use more sophisticated mechanisms in developing an edge detector, e.g., the mechanisms in Long and Li [64] and Yang et al. [65]. In this respect, the proposed edge model may replace the Gaussian function as the basis of the edge detector.

3.7 Summary

A flexible model for edge representation in digital imagery has been presented. The model is developed based on the Dolph-Chebyshev function aiming to recover some important edge characteristics. The parameters of the proposed model are estimated under a least-square framework. The validity of the proposed model is shown experimentally using edge profiles extracted from images with various blur and noise levels. The model is compared with the single and double Gaussian-based models and it is observed that the proposed model consistently produces the lowest root-mean-square-error (RMSE) on all tested edge profiles. Finally, a procedure for edge detection in two-dimensional images using the proposed model is derived and it is shown that our edge detector performs better than the compared algorithms. With such a performance, the proposed model is beneficial to be used in practical applications.

Chapter 4

Design of Filters for Blood Vessel Detection from Colour Fundus Images

4.1 Introduction

Retinal blood vessels analysis is beneficial for non-invasive detection of several eye-related diseases, such as diabetic retinopathy, hypertension and vein occlusion [66]. Such analysis requires segmented retinal blood vessels which are commonly provided by ophthalmologists. However, the ophthalmologist usually performs retinal vessels segmentation manually. This manual segmentation is prone to errors and is time-consuming, particularly for blood vessels with complicated structures. Hence, it is worthwhile to construct an automatic and robust algorithm for retinal vessels segmentation.

Several algorithms of retinal vessels segmentation have been developed to assist ophthalmologists in assessing retinal vessels' locations. The methods can be divided into two classes, viz. unsupervised and supervised approaches. Unsupervised approaches which include filter-based [27, 32, 46, 67], tracking-based [4] and iterative-based [68, 69] methods perform retinal vessel segmentation by either locally or entirely approximating retinal vessels pixels without any prior knowledge and labelled ground truth. On the contrary, given a set of features vectors and labelled ground truths, several supervised classifiers such as Gaussian Mixture Model

[33, 70, 71], cross-modality learning [37], support vector machine [35] and convolutional neural networks [38–40] map fundus image pixels into a vessel or non-vessel class.

Most of the existing unsupervised methods are unable to provide desired results on some challenging situations, such as segmenting thin vessels and detecting vessels in the presence of pathologies [67]. These pathologies are commonly diabetic retinopathy landmarks, which include bright lesions (exudates or cotton wall spots) and dark lesions (microaneurysms or haemorrhages) [68]. On the contrary, although supervised methods usually offer more reliable vessels segmentation results, they typically require a large number of training samples and are prone to heavy computational complexities.

In this chapter, we design new filters for retinal vessels segmentation. In particular, the contributions of this chapter are as follows:

1. Multi-scale and orientation matched filter banks are developed based on the modified Dolph-Chebyshev function.
2. A new method to incorporate the matched filter bank’s responses is proposed, such that the multi-scale concept allows the matched filters to detect retinal vessels in all possible widths.
3. A new framework to design optimal adaptive filters that can work with various types of vessel structures is proposed. The filters act as mapping functions that map a retinal image patch containing vessel and non-vessel structures to the segmentation output.
4. The performance of the proposed filters is evaluated using fundus images from the DRIVE [4] and STARE [45] databases. Specifically, quantitative measures such as sensitivity (Se), specificity (Sp), F-1 score ($F1$), G-mean (G) and Matthew correlation’s coefficient (MCC) are used for performance evaluation.

4.2 Databases

The **DRIVE database** contains 40 fundus images captured during a screening study with 400 subjects. The images are grouped into the training and testing sets, each consists of 20 images. Each image in this database has a size of 565×584 pixels. Two sets of manually annotated images are provided. The first set is utilised as the ground truth.

The **STARE database** consists of 20 fundus images which are offered for public use by the University of California, San Diego. Each image in this database has a size of 700×605 pixels. In this database, 10 images are from healthy subjects while the rest contain numerous pathologies. Two manual segmentation sets are provided and the first set is considered as the ground truth.

4.3 Blood Vessel Detection using the Modified Dolph-Chebyshev type II Function (MDCF-II) Matched Filter

4.3.1 Design of the MDCF-II Matched Filter

The idea of using a matched filter to detect retinal vessels is to design a suitable model for intensity profiles of retinal vessels' cross-sections [27]. Some examples of these intensity profiles are depicted in Figure 4.1. The top row of Figure 4.1 indicates some excerpts of retinal vessels with thin (a), medium (b) and thick (c) widths while the bottom row shows the plots of the corresponding pixel intensity profiles versus distances in pixels from the centre of blood vessels. The red, green and blue lines (indicated with arrows) in the first row of Figure 4.1 are the guide-lines to obtain the cross-sections of thin, medium and thick vessels, respectively. The lengths of the three lines are represented by the pixel length depicted in the corresponding figures on the second row.

As can be observed in Figure 4.1, each retinal vessel's intensity profile has an inverted Gaussian-like curve. This leads to a decision of utilizing an inverted Gaussian function in a conventional matched filter [27] to approximate intensity profiles

FIGURE 4.1: Some examples of retinal vessels' cross-sections (top row) and their corresponding intensity profiles (bottom row) with (a) thin, (b) medium and (c) thick widths.

of retinal vessels' cross-sections. However, each retinal vessel's intensity profile is measured with a particular distance from the centre of blood vessels while the Gaussian function ($G(x)$) used in Chaudhuri et al. [27] has an infinite range on the x -axis ($-\infty \leq x \leq \infty$). Given the infinite range, $G(x)$ is required to be truncated as it is used to approximate the intensity profiles. Unfortunately, the truncation will lead to information losses, which makes the truncated $G(x)$ less-suitable to approximate the intensity profiles of retinal vessels.

From Figure 4.1, we may observe that the intensity profile of the blood vessels cross-section behaves like an inverted slope profile of an edge as presented in Chapter 3. This is because the blood vessel can be regarded as a bright-dark-bright tubular structure where its intensity profile behaves similarly to the inverted slop profile of an edge or curvilinear structure [58]. This also becomes the reason that edges can be detected by looking for peaks in the first derivative or the gradient of images. Given the Dolph-Chebyshev function has successfully outperformed the Gaussian function in edge modelling and detection (See Chapter 3), it may be beneficial to use it to model the blood vessel intensity profile and subsequently use it to build a blood vessel detector.

The Dolph-Chebyshev function is introduced by Dolph [72] to design an optimal

directional antenna array. Mathematically, the Dolph-Chebyshev function used in [72] can be written as follows:

$$U(x) = \frac{\cos \{M \cos^{-1}[\beta \cos(\pi x/M)]\}}{\cosh[M \cosh^{-1}(\beta)]}, \quad (4.1)$$

where

$$\beta = \sec \left(\frac{\pi \omega_c}{M} \right). \quad (4.2)$$

The parameter M is the function length and ω_c is a parameter which controls the width of the main-lobe. An example of $U(x)$ is shown in Figure 4.2.

FIGURE 4.2: An example of the Dolph-Chebyshev function on $-M \leq x \leq M$ with $M = 42$ and $\omega_c = 7$.

From Figure 4.2, it can be observed that $U(x)$ behaves like an inverted graph of the retinal vessels' intensity profile. Moreover, $U(x)$ is a periodic function with $x \in [-\frac{M-1}{2}, \frac{M-1}{2}]$, such that a truncation on that range of x will not cause any information loss. Hence, an inverted $U(x)$ is more suitable than the Gaussian function to model the retinal vessels' intensity profile. However, the sharp peak of $U(x)$ does not match the rounded vertices of the retinal vessels' intensity profiles, particularly for those in Figure 4.1(b) and Figure 4.1(c). As a result, implementing the original Dolph-Chebyshev function as the basis function of a matched filter

kernel may lead to an undesirable performance. Here, we propose the following modified Dolph-Chebyshev function:

$$W(x) = \begin{cases} - \left| \frac{\cos\{M \cos^{-1}[\beta \cos(\pi x \gamma)]\}}{\cosh[M \cosh^{-1}(\beta)]} \right|, & |A(x)| \leq 1 \\ -1, & \text{otherwise} \end{cases}, \quad (4.3)$$

$$W'(x) = W(x) - \overline{W(x)}, \quad (4.4)$$

where

$$A(x) = \beta \cos(\pi x \gamma), \quad (4.5)$$

$$\beta = \sec\left(\frac{\pi \omega_c}{M}\right), \quad (4.6)$$

$$\gamma = 1/(M + 1), \quad (4.7)$$

and $\overline{W(x)}$ is the mean of $W(x)$. For the relations in (4.3)–(4.7), x is defined on $x \in \left[-\frac{(M-1)}{2}, \frac{(M-1)}{2}\right]$. $W(x)$ in (4.3) is proposed to have a passband ($-\omega_c \leq x \leq \omega_c$) with less-pointed vertices. Some plots of $W(x)$ are shown in the second row of Figure 4.3. Then, by borrowing the term from the Chebyshev type II filter [73], we call the relations in (4.3)–(4.7) as the modified Dolph-Chebyshev type II function (MDCF-II) since the passband does not exhibit ripples.

We have conducted an experimental setup to show that among other existing functions, such as the Gaussian and the Difference of Gaussian (DoG) as used in Chaudhuri et al. [27] and Xiao et al. [74], the MDCF-II provides a better model for the intensity profiles of retinal vessels' cross-sections. In the experiment, the MDCF-II and other existing functions are convolved with the intensity profiles of the retinal vessels' cross-sections in Figure 4.1. We implement the functions with some values of scale (ω_c in the MDCF-II and σ in the Gaussian and DoG functions), i.e., 3, 9 and 15 to approximate the thin, medium and thick vessels while the length (M) of each function is set to six times the scale's value. The results for the thin, medium and thick vessels are shown in Figure 4.3(a) to Figure 4.3(c), respectively.

As depicted in the last row of Figure 4.3, the MDCF-II achieves the highest convolutional response and therefore would provide the best model for the thin, medium and thick vessels. To perform retinal vessel detection, the proposed matched filter is expanded in a 2-D form as follows:

FIGURE 4.3: Convolution performance (fifth row) of intensity profiles of (a) thin; (b) medium; and (b) thick retinal vessels' cross-sections (first row) with MDCF-IIs (second row), Gaussian functions (third row) and DOG functions (fourth row).

$$W_{i,j}(x, y) = \begin{cases} - \left| \frac{\cos\{M \cos^{-1}[\beta_j \cos(\pi u \gamma)]\}}{\cosh[M \cosh^{-1}(\beta_j)]} \right|, & |A_{i,j}(x, y)| \leq 1 \\ -1, & \text{otherwise} \end{cases}, \quad (4.8)$$

where

$$A_{i,j}(x, y) = \beta_j \cos(\pi u \gamma), \quad (4.9)$$

$$\begin{bmatrix} u & v \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{bmatrix}. \quad (4.10)$$

$\begin{bmatrix} x & y \end{bmatrix}$ is the original coordinate system while $\begin{bmatrix} u & v \end{bmatrix}$ is the corresponding coordinate system rotated at an angle (θ_i) . The u and v points are defined in the neighborhood $N = \{(u, v) | -M/2 < u < M/2, 0 < v \leq T\}$. The kernel $W'_i(x, y)$ is calculated at several angles to detect retinal vessels at all possible orientations. Finally, the mean of the template W_i ($\overline{W}_i$) is subtracted from the template W_i to have a zero-mean template W'_i as follows:

$$W'_{i,j}(x, y) = W_{i,j}(x, y) - \overline{W}_{i,j}, \quad (4.11)$$

where

$$\overline{W}_{i,j} = \sum_{\vec{p}_i \in N} W_{i,j}(x, y) / S, \quad (4.12)$$

and S is the number of pixels in the neighbourhood N .

4.3.2 Pre-Processing and Post-Processing Phases

Our algorithm for retinal vessels segmentation includes the main phase which is the retinal vessel detection using the proposed matched filter as well as a pre-processing and a post-processing phases. A schematic diagram for the proposed algorithm is shown in Figure 4.4. The pre-processing phase consists of green channel (I_{green}) extraction, field of view (FOV) extension and background normalization. I_{green} is chosen since it presents the best vessels to the background contrast among other channels. Then, the FOV is extended using the method in Soares et al. [33] to remove a sharp change between the FOV and the region beyond the FOV.

The background normalization step is beneficial in removing exudates which may appear in the fundus image as well as in improving the vessels to background contrast. The exudates are removed using the in-painting technique in Annunziata

FIGURE 4.4: A schematic diagram for the proposed algorithm.

et al. [32] while the vessels contrast is enhanced using contrast-limited histogram equalization (CLAHE). In using the in-painting method, we need to define the kernel size of the median filter for the DRIVE database since those are not specified by Annunziata et al. [32]. The size is determined based on the one used in the STARE database. Also, we propose an adaptive thresholding method to further

improve the performance of exudates detection in Annunziata et al. [32] as follows:

$$T_h = \begin{cases} T_{h,OTSU}, & T_{h,OTSU} \geq \bar{I} \\ 165, & \text{otherwise} \end{cases}. \quad (4.13)$$

$T_{h,OTSU}$ is a threshold value for the image I obtained using the Otsu's method [75] while $\bar{I}$ is the average intensity of I . We have conducted an experiment and found that the constant threshold value of 165 as suggested in Annunziata et al. [32] is optimum for the situation where $Th_{OTSU} \geq \bar{I}$.

The post-processing phase is required to obtain final images of segmented blood vessels. This phase comprises of vessels candidates' contrast enhancement, thresholding, vessel's edge refinement and length filtering. The contrast of the retinal vessel candidates is improved using a linear contrast stretching technique. The thresholding aims to convert the detected retinal vessels into a binary image using a threshold value T_B . In this study, T_B for each image is calculated using second-order entropy-based thresholding [76]. The entropy-based thresholding method has two advantages. First, it can take into account the spatial distribution of grey-levels retinal vessels, which are not independent of each other. Moreover, the algorithm can preserve the spatial structures in the binary vessel image [76].

As a result of thresholding, there are some edges not belonging to vessels but emerge in the surrounding since we apply a global threshold value to binarize all retinal vessels. To address this problem, we follow a morphological-based elimination strategy used by Miri and Mahloojifar [77]. Finally, a length filter with a threshold value T_L is used to remove the remaining undesirable objects. T_L for the DRIVE database is set to be equal to 250 [78] while the value for the STARE database is determined based on that for the DRIVE database ($T_L = 250$). The procedure to set T_L for the STARE database will be discussed in the next section.

4.3.3 Retinal Vessels Detection Phase

The retinal vessels are detected by convolving the pre-processed image with the matched filter kernel at a particular scale $\omega_{c,j}$ and orientation θ_i to get a vessel

image $R_{i,j}$ as follows:

$$R_{i,j}(x, y) = I(x, y) * W'_{i,j}(x, y). \quad (4.14)$$

We utilise twelve θ values ($\theta = \{0^\circ, 15^\circ, 30^\circ, \dots, 165^\circ\}$) in calculating $W'_i(x, y)$. Subsequently, the retinal vessel image at a particular scale $R_{\max,j}$, is obtained by taking the maximum response of $R_{1,j}$ to $R_{12,j}$:

$$R_{\max,j} = \max(R_{1,j}, R_{2,j}, \dots, R_{12,j}). \quad (4.15)$$

Having obtained $R_{\max,j}$ at all scales ($j = 1, 2, \dots, J$), we use a new method which is a modification from that in Nguyen et al. [67] to combine $R_{\max,1}$ to $R_{\max,J}$ as follows:

$$R_{\text{combined}} = \sum_{j=1}^J \left(w_j R_{\text{std},j} + \frac{1}{J+1} I_{\text{std}} \right), \quad (4.16)$$

where

$$w_j = \frac{\exp(-\tau(\omega_{c,j} - \tilde{\omega}_{c,j}))}{1 - \exp(-\tau(\omega_{c,j} - \tilde{\omega}_{c,j}))}, \quad (4.17)$$

$$R_{\text{std},j} = \frac{R_{\max,j} - \overline{R_{\max,j}}}{\sigma_{R_{\max,j}}}. \quad (4.18)$$

R_{combined} , $R_{\text{std},j}$, $\overline{R_{\max,j}}$ and $\sigma_{R_{\max,j}}$ are the combined matched filter's responses, the normalized matched filter's response, the average intensity of $R_{\max,j}$ and the standard deviation of $R_{\max,j}$, respectively. The normalization aims to enhance the contrast of vessels to background but without changing the distribution of the pixels' intensity values [67].

It is interesting to note that we have a parameter τ which controls the weight (w_j) for each $R_{\text{std},j}$. This parameter has a value of $\tau \geq 0$. Figure 4.5 depicts an example plot of w_j values calculated using several values of τ . In the figure, it is observed that a larger weight for $R_{\text{std},j}$, where $\omega_{c,j} < \text{median}(\omega_c)$ can be obtained by using a larger value τ , i.e., $\tau = 1.5$. On the contrary, a larger weight for $R_{\max,j}$, where $\omega_{c,j} > \text{median}(\omega_c)$ can be obtained using a smaller value of τ , i.e., $\tau = 0.5$. Hence, a suitable value of τ for the images from a particular database is required, such that a simple global thresholding technique can be applied to segment retinal vessels with all possible widths simultaneously.

FIGURE 4.5: An example plot of w_j calculated using $\tau = 0.5, 1$ and 1.5 .

4.3.4 Parameter Setting

In this section, we present the procedures to set the parameters' values of the median filter, matched filter and length filter. The unassigned parameters' values on a particular database are calculated based on the values of the corresponding parameters specified earlier on another database. For instance, the kernel size of the median filter on the STARE database which is taken from the work of Annunziata et al. [32] is used to determine the corresponding parameter's value on the DRIVE database. On the contrary, $T_L = 250$ for the length filter on the DRIVE database which is adapted from Ardizzone et al. [78] is utilised to specify the T_L 's values on the STARE database. Those parameters' values can be calculated as follows:

$$P_A = P_B \times s_{A,B}, \quad (4.19)$$

where

$$s_{A,B} = 0.5 \times \left(\frac{X_A}{X_B} + \frac{Y_A}{Y_B} \right). \quad (4.20)$$

The variables X_A and Y_A are the length and width of an image from the database A while $s_{A,B}$ is the size ratio of the image from databases A and B . The variable P_A is the parameter's value for particular database A needs to be determined. Suppose P_A is $T_{L,A}$, then A refers to the STARE database, while B is for the DRIVE database. On the other hand, P_A is the kernel size of the median filter and A and B represent the DRIVE and STARE databases as we want to define the kernel size of the median filter on the DRIVE database.

The parameters' values of the proposed matched filter are determined by adopting those in the conventional matched filter [27] with several modifications. In accordance with the work in Chaudhuri et al. [27], we start defining the values on the DRIVE database while the values for the STARE database can be calculated based on those for the DRIVE database. In MDCF-II, ω_c is the most important parameter since it governs the main lobe width and at the same time, it is related to retinal vessels' widths. To set the ω_c value, we follow the similar procedure as in Chaudhuri et al. [27], in which the average width of retinal vessels is used to set the main lobe width of the Gaussian function (σ).

Besides ω_c , we have to define the values of m , t , M and T . m and t are the length and width of the proposed matched filter kernel. A series of experiments have been conducted to find those values using $\omega_c = 2$ (since $\sigma = 2$ in Chaudhuri et al. [27]). From the experiment, we found that $m = 17$, $t = 16$, $M = 12$ and $T = 8$ provided better performance than $m = 16$, $t = 15$, $M = 13$ and $T = 9$ as used in Chaudhuri et al. [27]. Based on the assigned values of ω_c , m , t , M and T , we are able to define the ratios of M , T and t to ω_c as r_M , r_T and r_t , which are equal to 6, 4 and 8, respectively. Subsequently, we define a relation of m and t as follows:

$$m = t + 1. \quad (4.21)$$

We utilise the procedure in setting the ω_c 's values, the ratios and the expression in (4.21) to set the parameters' values of our matched filter bank on the DRIVE and STARE databases.

Another important parameter needs to be set is the τ value for each database. It is interesting to note that segmenting all types of vessels at the same time is much desirable. For this purpose, the τ value for each database is determined based on the distribution of vessels widths of the image from each database. For instance, if

the average radii of the blood vessels ($\bar{\omega}_c$) on a particular database is larger than $\frac{\omega_{c,min} + \omega_{c,max}}{2}$, then the majority of vessels pixels are on the medium and thick blood vessels. On the other hand, $\bar{\omega}_c \leq \frac{\omega_{c,min} + \omega_{c,max}}{2}$ indicates that the corresponding database contains a larger amount of thin and medium blood vessels pixels than those of thick vasculatures. The values of $\omega_{c,min}$, $\bar{\omega}_c$ and $\omega_{c,max}$ for images from the DRIVE and STARE databases are 1, 3 and 5. This indicates that the majority of the vessels' pixels in the DRIVE and STARE databases belong to thin and medium blood vessels.

4.4 Blood Vessel Detection using the Modified Dolph-Chebyshev type I Function (MDCF-I) Matched Filter

4.4.1 Design of the MDCF-I Matched Filter

The name of the MDCF-I matched filter comes from the fact that the Dolph-Chebyshev function used in the following formulations will exhibit ripples in its mainlobe. Therefore, by borrowing the term from the Chebyshev type I filter [73], we call the matched filter proposed below as the modified Dolph-Chebyshev type I function (MDCF-I) matched filter. The MDCF-I matched filter kernel at a particular scale $\omega_{c,j}$ and orientation θ_i can be calculated as follows:

$$V_{i,j}(x, y) = \begin{cases} |\cos \{M \cos^{-1}[\beta_j \cos(\pi u \gamma)]\}|, & |A_{i,j}(x, y)| \leq 1 \\ |\cosh[M \cosh^{-1}(\beta_j)]|, & \text{otherwise} \end{cases}, \quad (4.22)$$

$$V'_{i,j}(x, y) = V_{i,j}(x, y) - m_{i,j}, \quad (4.23)$$

where

$$A_{i,j}(x, y) = \beta_j \cos(\pi u \gamma), \quad (4.24)$$

$$\beta_j = \cosh[(1/M) \cosh^{-1}(10^{\alpha_j})], \quad (4.25)$$

$$\gamma = \frac{1}{M+1}, \quad (4.26)$$

$$\alpha = \log \left\{ \cosh \left[(M-1) \cosh^{-1} \left(\sec \left(\frac{\pi \omega_{d,j}}{M} \right) \right) \right] \right\}, \quad (4.27)$$

$$\omega_{d,j} = \frac{M+1}{2} - 0.75 \times \omega_{c,j}, \quad (4.28)$$

$$\bar{p}_i = \begin{bmatrix} u & v \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{bmatrix}, \quad (4.29)$$

$$N = \{(u, v) | 0 \leq u \leq M, 0 < v \leq T\}, \quad (4.30)$$

$$m_{i,j} = \sum_{\bar{p}_i \in N} V_{i,j}(x, y) / B. \quad (4.31)$$

$W'_{i,j}$ has a length and width of L and P , respectively. The parameters $\omega_{c,j}$ and T correspond to the width and length of the vessel segment while M is the order of the MDCF-I. The values of M , T , L , and P can be calculated as $6\omega_{c,j} + 1$, $4\omega_{c,j} + 1$, $8\omega_{c,j}$, and $8\omega_{c,j} - 1$, respectively. Unlike the MDCF-II, the mainlobe of the MDCF-I

4.4.2 Retinal Vessels Detection Phase

To detect blood vessels in all possible widths, there is a need to calculate the MDCF-I matched filter kernel at several values of ω_c . In this work, we consider $\omega_c = \{1, 2, 3, 4, 5\}$ for images both from the DRIVE and STARE databases. In addition, we calculate the MDCF-I matched filter kernel at several orientations, i.e., $\theta = \{0^\circ, 15^\circ, \dots, 165^\circ\}$.

The obtained kernel at particular scale and orientation ($V'_{i,j}(x, y)$) is convolved with the pre-processed image. Then, the image containing single width blood vessels with all possible orientation ($R_{max,j}(x, y)$) is calculated as follows:

$$R_{max,j}(x, y) = \max_{1 \leq i \leq 12} (R_{i,j}(x, y)). \quad (4.32)$$

$R_{i,j}(x, y)$ is the matched filter response at particular orientation i and scale j . Finally, the image with detected vessels at all possible widths ($R_{combined}$) can be calculated as follows:

$$R_{combined} = \sum_{j=1}^5 a_j R_j(x, y), \quad (4.33)$$

where

$$a_j = 1 - \frac{1}{1 + e^{-j+3}}. \quad (4.34)$$

4.4.3 Pre-processing and Post-processing Phases

The image pre-processing phase consists of three steps: green channel extraction, vessels contrast enhancement and background normalization. The green channel is chosen due to its highest contrast among the other channels. Thereafter, the contrast of vessels is improved using contrast limited adaptive histogram equalization (CLAHE). CLAHE is carried out with the clip level and the number of intervals of 0.014 and 256, respectively. These values are used for both the DRIVE and STARE databases.

We only perform the background normalization step on images from the STARE database, with an intention to remove lesion pixels. Using this step, we try to replace grey-levels of lesion pixels with those of background pixels. Hence, a background image (I_b) is initially obtained by applying a morphological opening operation with a disc structure element of 25 in radius on the green channel. Then, the grey-levels of lesion pixels are replaced with the maximum intensity of I_b . The result of CLAHE in the DRIVE database and the result of the background normalization step in the STARE database are further called as pre-processed images and are used as the basis for the subsequent stage.

The other additional phase that is required for this study is the post-processing phase. It contains four steps: enhancement of retinal vessel candidates contrast, thresholding, masking, and length filtering. The contrast of vessels candidates ($R_{combined}$) is improved using the contrast stretching technique. Then, a binary mask for $R_{combined}$ (I_{bv}) is obtained using a threshold value T_B . To eliminate the boundary of the field of view (FOV), the masking step is carried out by multiplying the FOV mask to I_{bv} . The FOV mask is generated by applying a small threshold value to the green channel image, followed by morphological erosion. Finally, a length filter with a threshold value T_L is used to reduce undesirable noises in I_{bv} . For the DRIVE database, $T_B = 0.29$ and $T_L = 250$ are used while $T_B = 0.345$ and $T_L = 300$ are chosen for the STARE database.

4.5 Design of Optimal Adaptive Filters for Blood Vessel Segmentation

4.5.1 Data Preparation

We utilise retinal images from the **DRIVE** [4] and **STARE** [45] datasets to develop and evaluate the proposed framework. For the DRIVE dataset, we use the training-testing set division made by Staal et al. [4] to develop and evaluate the proposed framework. On the other hand, since there is no analogous division of the training and testing sets on the STARE dataset, we consider the procedure used in Gu et al. [79] on the separation of the training and testing sets. Before being used in the training and testing phases, the colour images are firstly processed using some enhancement methods. First, the green channel of each image is extracted. The contrast limited adaptive histogram equalization (CLAHE) is then used to improve the contrast of blood vessels on the retinal image. Finally, gamma adjustment ($\gamma = 0.95$) is applied to the result obtained from CLAHE to smooth out possible spurious noises.

The regularization framework accepts training patches extracted from the pre-processed images and the manually labelled images as the input-output pairs, respectively. The input patches are then partitioned into multiple clusters according to their local structures. The K-Means method with the cosine distance is used to handle this task. In this respect, the values of some parameters need to be defined. First, the number of training patches is essential. Most of the supervised methods require a lot of samples to learn the blood vessel structures. For instances, the work in Soares et al. [33] uses a million samples to train the Gaussian mixture model classifier. Moreover, a larger amount of samples will increase performance. However, a vast amount of data will lead to high computational complexity. To avoid such an issue, we consider a total of 600,000 training patches. In the section that follows, we will demonstrate that the proposed framework, with less training samples, can outperform most of the compared methods.

The second important parameter is the patches sizes. In most of the filtering-based approaches [4, 27, 45, 80], the filter size is approximately six times the average vessel calibres. Therefore, the patches sizes for the DRIVE and STARE datasets are set to 9×9 and 13×13 , respectively. These patches sizes are proved to provide better

performance than other compared methods. For implementation on images with different resolutions, we can also consider the above-mentioned ratio to determine the appropriate patch sizes.

Finally, the number of clusters used by the K-Means is also crucial. Partitioning patches into a large number of clusters may be beneficial since it enables the framework to learn a vast amount of patterns. However, more patches, as well as longer computational time, are required for the K-Means algorithm to converge. In other words, the number of clusters should be large enough to cover the amount of distinct vessel and non-vessel structures while not causing a heavy computational burden. Therefore, we determine the number of clusters as follows. First, similar to most of the filtering-based approaches, several filters are needed to detect straight vascular structures with twelve different orientations and five distinct calibres. Then, other vessel structures such as bifurcations and branches are considered. As pointed out by Azzopardi and Petkov [81], a retinal image contains approximately a hundred distinct branches/bifurcations. Therefore, a total of 360 clusters are used. The first half of the clusters are used to cover vessel structures while the rest are used to cover the non-vessel structures.

4.5.2 Design of Optimal Adaptive Filters

As mentioned previously, we attempt to design new optimal adaptive filters which map the retinal image to its corresponding vessel map. Particularly, given a retinal image $\mathbf{X} \in R^{S \times Q}$, the blood vessel segmentation output $\mathbf{Y} \in R^{S \times Q}$ can be obtained by convolving $\mathbf{X}$ with the optimal filter $\mathbf{F} \in R^{M \times N}$ as follows:

$$\begin{aligned} y(i, j) &= (x * f)(i, j) \\ &= \sum_m \sum_n f(m, n) x(i - m + 1, j - n + 1), \end{aligned} \quad (4.35)$$

where $|m| \leq M/2$, and $|n| \leq N/2$. The parameters M and N denote length and width of the filter $\mathbf{F}$ while S and Q are the length and width of both the image $\mathbf{X}$ and $\mathbf{Y}$ respectively.

Suppose $\mathbf{Z}_k \in R^{M \times N}$ is the k^{th} patch of $\mathbf{X}$ which is covered by $\mathbf{F}$ during the convolution in (4.35), and $\mathbf{F}$ is a symmetric matrix, the convolution process in (4.35) can be expressed as an inner product between the vectorized $\mathbf{Z}_k$ and $\mathbf{F}$

matrices as follows:

$$y(i, j) = \text{vec}(\mathbf{Z}_k)^T \text{vec}(\mathbf{F}), \quad (4.36)$$

where $\text{vec}(\cdot)$ denotes the vectorization operator of a matrix. The segmentation output $\mathbf{Y}$ can now be obtained by the following expression:

$$\mathbf{Y} = \text{vec}^{-1} \left(\left[\text{vec}(\mathbf{Z}_1), \dots, \text{vec}(\mathbf{Z}_K) \right]^T \cdot \text{vec}(\mathbf{F}) \right), \quad (4.37)$$

where $K = S \times Q$ is the number of pixels in $\mathbf{Y}$, and $\text{vec}^{-1}(\cdot)$ is an inverse operator of $\text{vec}(\cdot)$ such that $\text{vec}^{-1}(\text{vec}(\mathbf{X})) = \mathbf{X}$.

To improve the readability, from now on, we will refer $\left[\text{vec}(\mathbf{Z}_1), \dots, \text{vec}(\mathbf{Z}_K) \right]^T$ as $\mathbf{B} \in R^{MN \times K}$ and $\text{vec}(\mathbf{F})$ as $\mathbf{f} \in R^{MN \times 1}$ respectively. The expression in (4.37) can now be re-written as follows:

$$\mathbf{y} = \mathbf{B}^T \mathbf{f}, \quad (4.38)$$

where $\mathbf{y} \in R^{K \times 1} = \text{vec}(\mathbf{Y})$. Suppose a set of the training patches and the corresponding vessel maps (target vectors) on a particular cluster are given, we may construct $\mathbf{B}$ and $\mathbf{y}$, accordingly.

Note that the matrices $\mathbf{B}$ in some scenarios are not full column rank, and hence the formulation in (4.38) may cause over-fitting. Thus, a regularization framework is required. The filters coefficients $\mathbf{f}$ can be obtained under the regularization framework as follows:

$$\min_{\mathbf{f}} \{ ||(\mathbf{B})^T \mathbf{f} - \mathbf{y}||_2^2 + \lambda_r^2 ||\mathbf{f}||_2^2 \}, \quad (4.39)$$

which has a close form solution expressible as

$$\mathbf{f} = (\mathbf{B}\mathbf{B}^T + \lambda_r^2 \mathbf{I})^{-1} \mathbf{B}\mathbf{y}, \quad (4.40)$$

where λ_r is a regularization parameter (hyper-parameter), and $\mathbf{I} \in R^{MN \times MN}$ is an identity matrix. λ_r acts as a tuning parameter which balances the model-data fitness and the prior that is enforced onto the solution space. Since the size of $\mathbf{B}$ matrix corresponds to each cluster may vary due to the K-Means process on the extracted patches, the optimum value of λ_r for one particular cluster may be different from the other clusters'. To ensure the stability of the regularization

framework, in this letter, the value of λ_r for each cluster is optimised through a generalized cross-validation (GCV) method [82].

4.5.3 Employing Overlapping Patches to Design Optimal Adaptive Filters

Using overlapping patches in image segmentation is beneficial to smooth the segmentation output [83]. In this case, each pixel of the filter response $y(i, j)$ is covered by several patches. To obtain the final output of this pixel, a reconstruction process is applied; the final output is considered as the average of the filters' responses from the supporting patches. To use the overlap technique, several overlapping patches $\mathbf{C}_t \in R^{M \times N}$ are extracted from the manually labelled images. Following the vectorization procedure introduced earlier, we can construct a new target matrix for the segmentation process $\mathbf{Y}_{ov} \in R^{D \times MN}$ as follows:

$$\mathbf{Y}_{ov} = [\text{vec}(\mathbf{C}_1), \dots, \text{vec}(\mathbf{C}_D)]^T, \quad (4.41)$$

where D is the number of training samples for a certain cluster. The solution $\mathbf{F}_{ov} \in R^{MN \times MN}$ for a given $\mathbf{Y}_{ov}$ and $\mathbf{B}_{ov} \in R^{MN \times D}$ can be obtained from the following expression:

$$\mathbf{F}_{ov} = (\mathbf{B}_{ov}\mathbf{B}_{ov}^T + \lambda_r^2 \mathbf{I})^{-1} \mathbf{B}_{ov} \mathbf{Y}_{ov}, \quad (4.42)$$

where

$$\mathbf{B}_{ov} = [\text{vec}(\mathbf{Z}_1), \dots, \text{vec}(\mathbf{Z}_D)]^T. \quad (4.43)$$

4.5.4 Segmentation on the Test Set

Blood vessel segmentation of an image from the test set is completed by first calculating the filtering result for each i^{th} cluster, $\mathbf{Y}_{ov,i}$ as follows:

$$\mathbf{Y}_{ov,i} = \mathbf{B}_{ov,i}^T \mathbf{F}_{ov,i}, \quad (4.44)$$

where $\mathbf{B}_{ov,i}$ contains patches that belong to the i^{th} cluster and $\mathbf{F}_{ov,i}$ is the optimum filter kernel for the corresponding cluster. Note that the patches are formed for

each pixel in the image. To ensure that the patches in $\mathbf{B}_{ov,i}$ share similar features, the cosine distance between the extracted patches and the centre of each cluster is measured. The centre of each cluster is obtained from the patch clustering process. The filtering result for the image, $\mathbf{Y}_{out} \in R^{S \times Q}$ can be obtained through a reconstruction process on $\mathbf{Y}_{OV}$, where $\mathbf{Y}_{OV} = [\mathbf{Y}_{ov,1}, \mathbf{Y}_{ov,2}, \dots, \mathbf{Y}_{ov,360}]$. The pixel values in $\mathbf{Y}_{out}$ are then mapped to their probability of being vessels or not, $\mathbf{O} \in R^{S \times Q}$, by the following expression:

$$\mathbf{O} = \frac{1}{1 + \exp(-\mathbf{Y}_{out})}. \quad (4.45)$$

Each element value of the matrix $\mathbf{O}$ can also be considered as a probability map. To strengthen the blood vessel response in $\mathbf{O}$, two useful post-processing steps, namely image normalization [67] and contrast adjustment are applied. The final binary image is obtained through a thresholding step, where the threshold value is calculated using the Otsu method [75].

4.6 Experimental Results and Discussion

4.6.1 Vessel Segmentation Performance

The proposed filters are developed on a computer with i-7 core processor and 8GB RAM. Figure 4.6(a) to Figure 4.6(e) show the original colour fundus images, the corresponding vessel maps, the results of MDCF-I, the results of MDCF-II and the results of the optimal adaptive filter, respectively. The top and bottom rows of Figure 4.6 are for the images from the DRIVE and STARE databases, respectively. In the figure, it is observed that our filters can detect blood vessels with a wide variety of widths, including the thin vessels.

The performance of our filters is evaluated in terms of sensitivity (Se), specificity (Sp), F1-Score ($F1$), G-mean (G) and Matthews correlation coefficient (MCC) for the region inside FOV. Although accuracy (Acc) is widely used to quantify the performance of existing methods, (Acc) alone, however, is not sufficient for quantifying vessels segmentation efficacy since it too much reflects Sp and moreover, majority of retinal pixels are not vessels type. Hence, we use $F1$, G and MCC since they are more suitable for imbalanced class ratios [35]. The five measures can

FIGURE 4.6: (a) Original colour fundus images, (b) corresponding vessel maps, (c) results of MDCF-I, (d) results of MDCF-II and (e) results of the optimal adaptive filters. Top and bottom rows: images from the DRIVE and STARE databases.

be calculated as follows:

$$Se = \frac{TP}{TP + FN}, \quad (4.46)$$

$$Sp = \frac{TN}{TN + FP}, \quad (4.47)$$

$$F1 = \frac{2 \times TP}{2 \times TP + FP + FN}, \quad (4.48)$$

$$G = \sqrt{Se \times Sp}, \quad (4.49)$$

$$MCC = \frac{TP/N - S \times P}{\sqrt{P \times S \times (1 - S) \times (1 - P)}}, \quad (4.50)$$

where N is the number of pixels within an image, $S = (TP + FN)/N$ and $P = (TP + FP)/N$.

The performance of our filters and other methods on the DRIVE and STARE databases are presented in Table 4.1 and Table 4.2, respectively. In Table 4.1, the highest value on each quantitative measure for the unsupervised methods is listed in bold while that for the supervised methods is marked with a dagger ($\dagger$). It is observed from Table 4.1 and Table 4.2 that our filters outperform the compared unsupervised methods in terms of $F1$, G and MCC . Also, our filters are able to achieve higher Se scores than most of the unsupervised methods, except for the method in Nergiz and Akin [84]. However, the method in Nergiz and Akin [84] is prone to a huge number of false positives (on exudates and optical disc boundaries), which is indicated by its lowest Sp scores compared to other presented methods.

TABLE 4.1: Performance of Vessel Segmentation Methods on the DRIVE database.

Method	Se	Sp	$F1$	G	MCC
Unsupervised					
Nguyen et al. [67]	0.742	0.970	0.762	0.848	0.728
Odstcilik et al. [46]	0.706	0.969	—	0.827	—
Roychowdhury et al. [68]	0.739	0.978	—	0.850	—
Zhao et al. [69]	0.742	0.982	—	0.854	—
Nergiz and Akin [84]	0.812	0.934	—	—	—
MDCF-I	0.769	0.966	0.769	0.862	0.736
MDCF-II	0.748	0.978	0.786	0.856	0.758
Supervised					
Dai et al. [71]	0.736	0.972	—	0.846	—
Roychowdhury et al. [70]	0.725	0.983 [†]	—	0.844	—
Li et al. [37]	0.757	0.982	—	0.862	—
Liskowski and Krawiec [39]	0.775	0.979	—	0.871	—
Orlando et al. [35]	0.790	0.968	0.786	0.874	0.756
Fu et al. [38]	0.760	—	—	—	—
Zhou et al. [40]	0.808 [†]	0.967	0.794 [†]	0.883 [†]	0.766 [†]
Optimal Adaptive Filter	0.778	0.972	0.791	0.870	0.762

[†] The best value for supervised methods.

TABLE 4.2: Performance of Vessel Segmentation Methods on the STARE database.

Method	Se	Sp	$F1$	G	MCC
Unsupervised					
Nguyen et al. [67]	0.785	0.950	0.704	0.862	0.674
Odstcilik et al. [46]	0.785	0.951	—	0.864	—
Roychowdhury et al. [68]	0.732	0.984	—	0.849	—
Zhao et al. [69]	0.780	0.978	—	0.873	—
Annunziata et al. [32]	0.713	0.984	0.768	0.838	—
Nergiz and Akin [84]	0.813	0.944	—	—	—
MDCF-I	0.757	0.973	0.759	0.858	0.713
MDCF-II	0.793	0.973	0.780	0.877	0.756
Supervised					
Dai et al. [71]	0.777	0.955	—	0.861	—
Roychowdhury et al. [70]	0.772	0.973	—	0.867	—
Li et al. [37]	0.773	0.984	—	0.872	—
Liskowski and Krawiec [39]	0.777	0.985 [†]	—	0.878	—
Orlando et al. [35]	0.768	0.974	0.764	0.863	0.742
Fu et al. [38]	0.741	—	—	—	—
Zhou et al. [40]	0.806 [†]	0.976	0.802 [†]	0.886 [†]	0.783 [†]
Optimal Adaptive Filter	0.797	0.976	0.798	0.882	0.776

[†] The best value for supervised methods.

FIGURE 4.7: Computational costs of the proposed filters and their counterparts.

This is undesirable, particularly when it is applied on fundus images with many pathological signs.

To further show the efficacy of our filters, we also evaluate our filters in terms of computational costs. Figure 4.7 shows the computational costs of our filters. The figure also shows the computational costs of other methods in the literature. Note that the computational costs of the techniques by Nguyen et al. [67] and Orlando et al. [35] are calculated by re-implementing their algorithms on our machine. From the figure, we may observe that the computational times of our unsupervised filters are comparable to those of Nguyen et al. [67]. At the same time, the computational costs of the optimum adaptive filter are similar to those of Orlando et al. [35].

Although the methods in Li et al. [37], Liskowski and Krawiec [39] and Zhou et al. [40] achieved relatively better performance than ours, these methods have high computational complexities, indicated by the use of expensive graphics processing units (GPUs) to develop the techniques. To avoid these problems, we may train deep learning-based algorithms off-line. However, the high degree of inter dataset variation makes the learning architectures and procedures need long training times to provide robust generalization on unseen data. Large screening facilities may be able to invest in high-end computing resources. However, not all clinical facilities have such capabilities. The long training times and computational resource

TABLE 4.3: Segmentation Performance on images with pathologies from the STARE dataset.

Method	<i>AUC</i>	<i>MCC</i>
Roychowdhury et al. [68]	0.964	—
Roychowdhury et al. [70]	0.960	—
MDCF-I	0.958	0.707
MDCF-II	0.962	0.722
Optimal Adaptive Filter	0.966	0.727

limitations will make training from a vast number of new patients data impractical. Consequently, this will limit their applications in real-time eye-related diseases screening and diagnosis.

4.6.2 Abnormal Images Analysis

It is interesting to note that our achieved Se and G scores are better than those of some supervised methods (the methods in Dai et al. [71], Roychowdhury et al. [70] and Fu et al. [38]). Our filters, particularly the optimal adaptive filter, are also more robust than those supervised methods as they perform better on the STARE database which contains images with many pathological indications. In Table 4.3, we present the performance of our filters on images with pathologies from the STARE database, measured in terms of area under the ROC curve (AUC) and MCC . For benchmarking, we include the performance of the methods in Roychowdhury et al. [70] and Roychowdhury et al. [68] in Table 4.3. In the table, we can observe that the results of MDCF matched filters are comparable to those of the methods in Roychowdhury et al. [68] and Roychowdhury et al. [70] while the optimal adaptive filter performs the best among the compared algorithms.

4.6.3 Peripapillary Vessel Segmentation

As have been mentioned earlier, retinal vessel analysis is beneficial for the detection of several eye-related diseases. These include glaucoma, PDR, retinal vein occlusion, and cilio-retinal artery occlusion that can cause changes in the blood vessel structure, mainly in the peripapillary region. Hence, it is essential to evaluate the performance of a retinal vessel segmentation in extracting the peripapillary blood vessels. To perform this task, we create a mask for the peripapillary region

TABLE 4.4: Vessel Segmentation Performance on Peripapillary Regions.

Method	<i>Acc</i>	
	DRIVE database	STARE database
Roychowdhury et al. [68]	0.920	0.818
Roychowdhury et al. [70]	0.915	0.832
MDCF-I	0.912	0.888
MDCF-II	0.914	0.895
Optimal Adaptive Filter	0.920	0.907

of each image from the DRIVE and STARE databases centred at the optic nerve head with the radius of 1–optic disc diameter (1–DD). The optic nerve head location and optic disc diameter for each image are taken from the work in [70]. The evaluation results of our algorithms together with those of the methods in Roychowdhury et al. [70] and Roychowdhury et al. [68] are presented in Table 4.4.

In Table 4.4, we observe that for the DRIVE database, our algorithms are comparable to the methods in Roychowdhury et al. [68] and Roychowdhury et al. [70]. In contrast, on the STARE database, our filters, particularly the optimal adaptive filter provides better peripapillary vessel segmentation performance than the compared methods. Hence, our algorithms will be more useful for applications of retinal-related diseases diagnosis.

4.7 Summary

In this chapter, we have designed three filters for blood vessel detection. The first two filters are designed under an unsupervised framework while the rest is developed under a supervised scenario. The performance of the proposed filters has been evaluated using fundus images from the DRIVE and STARE databases. In the experiment, the proposed filters, particularly the optimal adaptive filter can outperform most of the compared methods. Although loosing from the deep learning methods, our proposed filters provide a more practical and realizable solution, unlike the deep learning methods, no expensive GPU needed to develop our filters. Finally, the experimental results on abnormal images and on peripapillary vessel segmentation suggest that the proposed algorithms will be more useful for applications of eye-related diseases diagnosis.

Chapter 5

A New Optic Disc Segmentation Method using a Modified Dolph-Chebyshev Matched Filter

5.1 Introduction

Automatic optic disc (OD) segmentation on fundus images plays an important role in computerized retinal disease diagnosis systems, such as glaucoma and diabetic retinopathy. In addition, OD segmentation often serves as a preprocessing step in other fundus structure segmentation. For instance, the OD location is used to reduce the number of false positives in exudates segmentation since both the OD and exudates have similar grey-levels. Also, the relatively constant distance between the OD and the macula is beneficial in finding the location of the macula [15]. Although OD segmentation is useful, it is, however, a challenging task due to variations in the colour, shape and size of the OD. Moreover, uneven contrast around the OD boundary and outgoing vessels from the OD's origin are significant distractors in the OD segmentation process.

Many algorithms for automatic OD segmentation have been published. They can be divided into groups which perform template-based, deformable model-based, morphological-based, and supervised-based methods. In the first three groups, the OD boundary is detected based on the OD appearance, i.e., the OD emerges as a bright and circular object. Hence, several techniques, such as thresholding in

Zahoor and Fraz [18] and edge detection in Aquino et al. [15] are initially employed to extract bright objects in fundus images and the Circular Hough Transform (CHT) [85][86] is then employed to obtain the final OD boundary. However, the result of thresholding which is expected to be the OD candidates may belong to regions other than the OD, such as exudates while the vessels' edges may appear in the result of edge detection. In addition, applying CHT to finish the segmentation process may not lead to a precise solution since the shape of the OD boundary is not always a perfect circle.

To avoid false bright regions and vessels edges, a supervised based method is proposed by Fan et al. [87]. In this method, the OD boundary candidates are extracted using structured learning. Then, the extracted candidates are thresholded and fitted using CHT. Employing structured learning to detect the OD boundary candidates may be useful in avoiding false bright regions or vascular edges. However, the obtained final OD boundary is a perfect circle, and it does not intuitively match the shape of the OD.

In this chapter, a new algorithm for automatic OD segmentation is proposed. The algorithm utilises a modified Dolph-Chebyshev type I function (MDCF-I) matched filter to detect the OD boundary candidates and CHT as well as spline fitting techniques to obtain the final OD boundary. The matched filter, CHT and spline fitting work on a particular region of interest (ROI). The OD centre location, which is initially detected using three independent methods: template-based, vessels density map-based, and maximum entropy-based methods, serves as the origin of the ROI.

The performance of the proposed algorithm is evaluated using 1200 fundus images from the MESSIDOR dataset [42]. Specifically, quantitative measures such as Area Overlap (*AOL*), Dices coefficient (*DSC*), Accuracy (*Acc*), True Positive Fraction (*TPF*) and False Positive Fraction (*FPF*) are used for performance evaluation. In addition, the percentage of images where the proposed algorithm successfully locates the OD is also considered. From the experiment, the proposed algorithm successfully locates 1192 ODs out of 1200 fundus images. Moreover, the proposed algorithm has superior performance in terms of *AOL*, *DSC*, *Acc*, *TPF*, and *FPF*.

5.2 Proposed OD Segmentation Method

The proposed OD segmentation algorithm consists of three main phases: OD size estimation, OD localization and OD boundary detection. The OD size is required to be estimated since it is beneficial in two subsequent phases. The OD localization phase aims to provide an initial OD centre location which is utilised in developing the region of interest (ROI). Finally, some steps in the last phase are applied to the obtained ROI to detect the OD boundary. The details of the three steps are presented as follows.

5.2.1 OD Size Estimation

In this work, the OD radius is estimated using the procedure and equations described in Yu et al. [88] as follows:

$$r_{OD} = \sqrt{\frac{(D_{OD}/2)^2}{f_{img}}}, \quad (5.1)$$

where

$$f_{img} = \frac{A_{FOV}}{N_{FOV}}. \quad (5.2)$$

A_{FOV} is the imaging area of a certain field of view (FOV) while D_{OD} is the average diameter of the human OD. These parameters are defined to be equal to 124.8 mm² and 1.85 mm, respectively [88]. On the other hand, N_{FOV} is the number of pixels within the FOV.

5.2.2 OD Localization

To obtain a reliable centre location of the OD (ODC), we use three independent OD detection methods. Each method provides its own location of the OD. The final ODC is determined by taking into account the locations of the three ODC candidates with respect to their average location (centroid). For this purpose, the centroid of the three candidates is initially calculated. Then, the distance between each ODC candidate and the centroid is measured. Finally, a statistical selection method described in Grubbs [89] is applied to remove the ODC candidate with

an unreasonably large distance to the centroid. A description of the three OD detection methods is illustrated in Figure 5.1 and is presented as follows.

5.2.2.1 Template-based method

This method locates the ODC candidate based on the shape and grey-levels of the OD. For this purpose, a binary template as described in Yu et al. [88] is created. The obtained template is convolved with I_p to obtain the filtered image (I_f) which contains ODC candidates. I_p is the first principle component of the red, green, and lightness channels of the fundus image. Then, the pixel with the maximum response on I_f is defined as the ODC pixel (see Figure 5.1(b) and Figure 5.1(g), indicated by a red cross '×').

5.2.2.2 Vessels density map-based method

This method works based on the fact that the vascular tree in the OD's vicinity usually has a higher density than that of vascular trees in other locations. To measure the vessels density, the vessels are firstly detected from the green channel of the fundus image using the method in Jerman et al. [90]. The detected vessels are thresholded using the Otsu method [75] and the length filter is then applied to the result of thresholding to get the final vessel image (I_v). Finally, the vessels density map (I_{VDM}) is defined as the number of vessels pixels on I_v calculated using an $r_{OD} \times 0.5r_{OD}$ centred window. The location of the maximum response in I_{VDM} corresponds to the ODC candidate using this method (see Figure 5.1(c) and Figure 5.1(h), indicated by a green cross '×').

5.2.2.3 Maximum entropy-based method

The bright pixels of the OD and the dark vessels in the surrounding cause the maximum grey-level variations to occur within the OD. In this method, the obtained ODC candidate is the location where the maximum grey-levels variation is achieved. The variation is statistically measured in terms of the entropy using a $0.5r_{OD} \times 0.5r_{OD}$ centred window. Two examples of the ODC candidate rendered by this method are depicted in Figure 5.1(d) and Figure 5.1(i), indicated by a blue cross '×'.

From Figure 5.1(a) to Figure 5.1(e) it can be observed that the three independent methods provide ODC candidates in relatively similar locations. Hence, the final ODC is calculated as the average of the three candidates. On the other hand, the second example (Figure 5.1(f) to Figure 5.1(j)) indicates that the ODC candidate obtained by the template-based method is relatively too far from the centroid. Thus, this ODC candidate is discarded by the selection method and the final ODC is defined as the centroid of the other two ODC candidates. The final ODCs of the first and second examples are shown in Figure 5.1(e) and Figure 5.1(j), which are indicated by yellow crosses 'x'.

FIGURE 5.1: ODC detection: (a) and (f) are original images. (b) and (g) show ODC candidates produced by the template-based method. (c) and (h) depict ODC candidates rendered by the vessel density map-based method. (d) and (i) show ODC candidate obtained by the maximum entropy-based method. (e) and (j) depict final ODCs (magenta cross) for images in (a) and (f), respectively.

5.2.3 OD Boundary Detection

The OD boundary detection phase is performed on the ROI extracted from I_p . The ROI that includes the OD is produced by cropping I_p with the final ODC as the centre of the ROI. In Yu et al. [88], the size of the ROI is set to be equal to $6r_{OD} \times 6r_{OD}$. By using this way, the search space of the OD boundary could be reduced. As a result, the presence of distractors and artefacts is reduced, such that the robustness of the steps in the OD boundary detection phase could be increased. In addition, performing the OD boundary detection phase only on the ROI will reduce the computational complexity.

5.2.3.1 Elimination of Blood Vessels

As have been mentioned above, we create the ROI on I_p as it contains the most beneficial contrast and information of the OD. However, I_p often contains a number of blood vessels in the surrounding of the OD due to the presence of these vessels in the green and lightness images. These are the potential distractors in the OD boundary detection phase. To remove these distractors, the vessels in the ROI are inpainted using the method in Morales et al. [16]. Subsequently, the inpainted ROI is smoothed out using a median filter with a kernel size of the average width of the blood vessels (see Figure 5.2(c)).

5.2.3.2 Detection of OD Boundary Candidates

The OD boundary candidates are detected by convolving the result of the median filter with 12 kernels of the MDCF-I matched filter. Each kernel corresponds to the OD boundary at a particular orientation (θ). In this work, we utilise $\theta = \{0^\circ, 15^\circ, \dots, 165^\circ\}$. The MDCF-I matched filter is chosen as it is observed in the previous chapter that the filter is not only able to detect blood vessels but also some parts of the OD boundary. Thus, with appropriate pre-processing techniques, the MDCF-I matched filter can be employed to detect the OD boundary candidates. The MDCF-I matched filter kernel can be expressed as follows:

$$W_i(x, y) = \begin{cases} |\cos \{M \cos^{-1}[\beta \cos(\pi u \gamma)]\}|, & |A(x, y)| \leq 1 \\ |\cosh[M \cosh^{-1}(\beta)]|, & \text{otherwise} \end{cases}, \quad (5.3)$$

$$V'_i(x, y) = W_i(x, y) - m_i, \quad (5.4)$$

where

$$A(x, y) = \beta \cos(\pi u \gamma), \quad (5.5)$$

$$\beta = \cosh[(1/M) \cosh^{-1}(10^\alpha)], \quad (5.6)$$

$$\gamma = \frac{1}{M+1}, \quad (5.7)$$

$$\alpha = \log \left\{ \cosh \left[(M-1) \cosh^{-1} \left(\sec \left(\frac{\pi \omega_d}{M} \right) \right) \right] \right\}, \quad (5.8)$$

$$\omega_d = \frac{M+1}{2} - 0.75 \times \omega_c, \quad (5.9)$$

$$\bar{p}_i = \begin{bmatrix} u & v \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{bmatrix}, \quad (5.10)$$

$$N = \{(u, v) | 0 \leq u \leq M, 0 < v \leq T\}, \quad (5.11)$$

$$m_i = \sum_{\bar{p}_i \in N} W_i(x, y) / B. \quad (5.12)$$

The proposed matched filter kernel V'_i has a length and width of L and P , respectively. Based on the conducted empirical study, the value of ω_c is set to be $r_{OD}/40$. In addition, we define the values of M , T , L , and P to be equal to $6.5\omega_c$, $4.5\omega_c$, $8\omega_c + 1$, and $8\omega_c$, respectively.

After calculating the matched filter responses for each orientation (I_{MF_1} to $I_{MF_{12}}$), the complete OD boundary candidates ($I_{MF, \max}$) is obtained by taking the maximum response of I_{MF_1} to $I_{MF_{12}}$. Subsequently, an image normalization technique as described in Nguyen et al. [67] is applied to $I_{MF, \max}$ with an intention to improve the $I_{MF, \max}$ contrast. As depicted in Figure 5.2(d), the normalized $I_{MF, \max}$ (we call it I_{MFS}) contains information on edges, particularly on the location with the grey-levels variation. Given that the blood vessels have been previously erased, the edges emerging in the I_{MFS} correspond to the OD boundary candidates. However, the matched filter produces an OD reduction of ω_c pixels in all directions. This will be considered at the end of the processing.

5.2.3.3 Selection of OD Boundary Candidates

Although the inpainting technique has been applied to erase the blood vessels, on I_{MFS} , there are some edges not belonging to the OD boundary but emerge in the surrounding due to the unsmooth background. To address this problem, we apply the CHT to I_{MFS} to extract the edges which form a circular pattern. The CHT finds circular patterns within an image with a particular range of radii ($r_{\min} \leq r \leq r_{\max}$) as follows [85][86]:

$$(P_c, r) = \text{CHT}(I_{MFS}, r_{\min}, r_{\max}), \quad (5.13)$$

where $P_c = (i_c, j_c)$ and r are the centre point and radius of the circular shape with the highest score in the accumulator matrix produced by the CHT. To obtain a satisfactory result of the OD boundary, $r_{\min}$ and $r_{\max}$ are set to be equal to

$0.7r_{OD}$ and $1.6r_{OD}$, respectively. Then, every pixel on I_{MFS} which are more than $1.2r$ far away from P_c is erased. This is done to eliminate detected edges which are too far from P_c and to preserve edges which in fact are the OD boundary but located outside the obtained circle by the CHT (the red circle in Figure 5.2(e)). On the contrary, to remove the optic cup boundary which may emerge inside the OD boundary, we eliminate every pixel on I_{MFS} within the range of $\{P_c, P_c + 0.7r\}$. Mathematically, these operations can be expressed as follows:

$$I_{MFC}(x, y) = \begin{cases} I_{MFS}(x, y) & 0.7r < d \leq 1.2r \\ 0 & \text{otherwise} \end{cases}, \quad (5.14)$$

where

$$d = \sqrt{(i_c - x)^2 + (j_c - y)^2}, \quad (5.15)$$

and I_{MFC} is the final OD boundary candidate. A thresholding process based on the second-order entropy method is then performed on the I_{MFC} to obtain the binary mask for the OD boundary (I_{BM}) as depicted in Figure 5.2(f).

FIGURE 5.2: Illustration of the process for the OD boundary detection phase: (a) A color ROI including an OD, (b) an ROI extracted from I_p , (c) a pre-processed image, (d) a standardized MDCF-I matched filter response, (e) a detected circle by the CHT, (f) a binary mask for OD boundary candidates, (g) a binary mask for OD boundary candidates in its polar form, (h) a fitted binary mask for OD boundary candidates, (i) an estimate for OD boundary and (j) a final OD boundary imposed on the color ROI.

5.2.3.4 Obtaining Final OD Boundary

This step aims to provide a reliable shape of the OD boundary. In addition, this step helps to connect disjointed parts of the OD boundary as illustrated in Figure 5.2, image f. For these purposes, the detected OD boundary is fitted using a spline fitting method [91]. Prior to this, I_{BM} is transformed into a polar form (Figure 5.2(g)) since in this form, the OD boundary has an opened rather than a closed contour as in the rectangular form, such that the fitting process could be done more effectively. The fitted equation is used to obtain an estimate of the OD boundary (Figure 5.2(h)). Then, the fitted OD boundary in the rectangular form is obtained by transforming the image in Figure 5.2(h), back to their rectangular form as illustrated in Figure 5.2(i).

As earlier commented, the matched filter applied in (3)–(11) shrinks the OD by ω_c pixels. Thus, the OD boundary in Figure 5.2(i) must be corrected as follows. First, a binary mask for the OD (I_{od}) is created by filling the holes on the OD boundary in Figure 5.2(i). Then, a morphological dilation operation is applied on I_{od} using a disc structure element with ω_c of radius to obtain the final OD binary mask (I_{OD}) which has a larger area than I_{od} . At last, the final OD boundary is produced by extracting the boundary of I_{OD} . An example of the final OD boundary imposed on the colour ROI is depicted in Figure 5.2(j).

5.3 Experimental Results and Discussions

5.3.1 Database

In this work, 1200 fundus colour images from the publicly available MESSIDOR dataset [42] are utilised to test the performance of the proposed algorithm. The images are acquired by the Hôpital Lariboisière Paris, the Faculté de Médecine St. Etienne and the LaTIM–CHU de Brest (France) using a Topcon TRC NW6 non-mydratic retinograph with a 45° FOV. The images are provided in TIFF format with 1440×960, 2240×1488, or 2304×1536 pixels size. 660 images are from patients with the diabetic retinopathy complication and 540 images correspond to patients without the illness. For performance evaluation of the proposed algorithm, the

references for OD centre location and boundary of the 1200 images are provided by Aquino et al. [15] and are publicly available on <http://www.uhu.es/retinopathy>.

5.3.2 Metrics

The results of our proposed algorithm are evaluated in terms of several metrics: *AOL*, *DSC*, *Acc*, *TPF*, and *FPF*. In addition, the percentage of images where the proposed algorithm successfully locates the OD (*Sc*) is also considered. These measures are described and defined as follows:

1. *AOL* refers to the ratio of intersecting area between the segmented and actual OD.

$$AOL = TP / (TP + FN + FP). \quad (5.16)$$

A higher *AOL* value corresponds to a better segmentation result.

2. *DSC* describes the similarity degree between the segmented and actual OD.

$$DSC = 2TP / (2TP + FN + FP). \quad (5.17)$$

Perfect segmentation is achieved when *DSC* is equal to 1.

3. *Acc* denotes the ratio of correctly classified pixels to the total number of pixels in the image.

$$Acc = (TP + TN) / (TP + TN + FP + FN). \quad (5.18)$$

A higher *Acc* score denotes a better segmentation result.

4. *TPF* corresponds to the ratio of correctly detected OD pixels to the actual OD pixels.

$$TPF = (TP) / (TP + FN). \quad (5.19)$$

A higher *TPF* value refers to a better segmentation result.

5. *FPF* denotes the ratio of falsely detected non-OD pixels to the actual non-OD pixels.

$$FPF = (FP) / (TN + FP). \quad (5.20)$$

A lower *FPF* value denotes a better segmentation result.

6. Sc (%) represents the percentage of images in a particular dataset where the proposed algorithm correctly locates the OD. The obtained OD location is considered to be a correct location if it is inside the OD boundary in the reference standard [16][88].

5.3.3 OD Segmentation Results and Performance Evaluation

5.3.3.1 OD Localization

Some examples of ODCs produced using our OD localization method are depicted in Figure 5.3, where our ODCs are indicated by red crosses '×' while green circles denote corresponding references for OD boundaries. It is observed from Figure 5.3 that the proposed OD localization method performs well on most of the cases. Specifically, the method successfully locates 1192 ODs out of 1200 images (99.33%). It should be noted that some examples in Figure 5.3 show the efficacy of our method on several challenging cases, which include ODs that are not visible (images e and f) and the presence of peripapillary atrophies that are larger than the OD (Figure 5.3(g)). The use of vessels density map and maximum entropy-based methods helps our algorithm to perform well in these two cases.

FIGURE 5.3: Some results of the proposed OD localization method where green circles are the ground truths for OD boundaries and red crosses are the detected ODCs: (a)-(g) successful cases and (h)-(j) unsuccessful cases.

There are 8 images in which the algorithm is not able to locate the ODs. This is caused by several factors, such as severe peripapillary atrophies, which completely overlay the OD (two images, an example is illustrated in Figure 5.3(h)), the OD is not visible (three images, an example is illustrated in Figure 5.3(i)), and the presence of exudate clusters (three images, two examples are illustrated in Figure 5.3(j)). In these cases, all of our OD localization methods provide high responses on relatively similar false positions, such that the final ODCs are not in the correct locations.

5.3.3.2 OD Boundary Detection

The proposed method for OD boundary detection is applied using the default values of the parameters as mentioned in the previous sections. Figure 5.4 shows some examples of the OD boundaries obtained by the proposed method (red circles) compared to the corresponding ground truths (green circles). It is important to note that our method can detect the OD boundaries accurately for most given cases. In particular, the proposed method also performs well on some difficult cases, which include ODs that are not visible (Figure 5.4(e) and Figure 5.4(f)) and cases with the presence of large myelinated nerve fibres adjacent to the OD (Figure 5.4(g)). The use of the spline fitting technique helps our method to provide reliable shapes of the OD boundaries, which highly match the ground truths.

In Figure 5.4(h) to Figure 5.4(j), it is observed that there are some cases, in which our method is not able to provide accurate shapes of the OD boundary. In general, we can divide the failure cases into some categories as follows:

- A detection result with an ideal FPF value but a low TPF value.
- A detection result with an ideal TPF value but a high FPF value.
- A detection result with a low TPF value and a high FPF value.

In the first category, the obtained OD boundaries are included in the ground truths even though some regions within the ground truths are not covered by the detected OD boundaries (Figure 5.4(h)). On the contrary, the second category corresponds to the obtained OD boundaries completely overlaying the ground truths but the obvious deviation is spotted (Figure 5.4(i)). For the last category, the detected OD

FIGURE 5.4: Some results of the proposed OD boundary detection method where green circles are the ground truths for OD boundaries and red circles are the detected OD boundaries: (a)-(g) accurate OD boundaries and (h)-(j) inaccurate OD boundaries.

boundaries are either unable to be included or completely being uncovered by the ground truths. An extreme situation belonging to the third category may happen, in which the detected OD boundaries and ground truths do not intersect at all (see Figure 5.4(j)). In this situation, the proposed method yields a non-ideal (zero) TPF value and a very high FPF value.

To assess the performance of the proposed OD boundary detection method independently from OD localization, we exclude the images, where our OD localization method fails to locate the OD from the performance evaluation process. The performance of the proposed OD boundary method with this scenario is available in Table 5.1 and indicated by an asterisk (*). It should be noted that with correct initial OD locations, our method provides better values of all metrics used. It indicates that the proposed OD boundary method may provide higher performance as implemented on ROIs produced using reliable initial OD locations.

5.3.4 Comparison with Other Algorithms

In this work, we compare our OD segmentation method to several available algorithms that can be divided into three groups, namely unsupervised, shallow learning and deep learning-based algorithms. In the first group, methods that are proposed in Aquino et al. [15], Morales et al. [16], Marin et al. [17], Zahoor and Fraz [18]

and Yu et al. [88] are chosen to represent the unsupervised group. To compare with the group performing shallow learning-based methods, we select the methods introduced by Roychowdhury et al. [3] and Fan et al. [87] are selected. Finally the algorithms by Wang et al. [26], Lim et al. [92] and Sadhukhan et al. [93] are selected to represent the deep-learning based algorithms. Table 5.1 summarizes the obtained metrics using our method together with those of other competing methods. The best score of each indicator in the unsupervised group is listed in bold while those in the remaining groups are marked with dagger ([†]) and double dagger ([‡]), respectively.

TABLE 5.1: Performance of OD Segmentation Algorithms on Images from the MESSIDOR dataset

Method	<i>AOL</i>	<i>DSC</i>	<i>Acc</i>	<i>TPF</i>	<i>FPF</i>	<i>Sc</i> (%)
Unsupervised Method						
Aquino et al. [15]	0.86	—	—	—	—	98.83
Morales et al. [16]	0.823	0.895	0.995	0.930	0.004	—
Marin et al. [17]	0.87	0.92	—	—	—	99.75
Zahoor and Fraz [18]	0.844	—	0.992	0.889	0.003	—
Yu et al. [88]	0.83	—	—	—	—	99.08
Proposed Algorithm	0.889	0.937	0.999	0.942	0.001	99.33
Proposed Algorithm*	0.895	0.941	0.999	0.948	0.001	—
Shallow Learning						
Roychowdhury et al. [3]	0.837	—	0.996 [†]	0.904	0.002 [†]	100 [†]
Fan et al. [87]	0.864 [†]	0.920 [†]	0.977	0.921 [†]	0.011	—
Deep Learning						
Lim et al. [92]	0.888	—	—	—	—	—
Wang et al. [26]	0.907	0.949 [‡]	0.972	0.948	—	99.9
Sadhukhan et al. [93]	0.91 [‡]	—	—	—	—	99.92 [‡]

As indicated in Table 5.1, the proposed algorithm is able to correctly locate the ODs in a larger number of fundus images than most competing methods. It indicates that three proposed independent OD localization methods are able to address each own limitations on most of the OD segmentation cases. Although the methods in Marin et al. [17] and Roychowdhury et al. [3] achieved better scores for *Sc*, our algorithm significantly outperforms them as well as the other competing methods in terms of the remaining indicators. It indicates that the MDCF-I matched filter is shown to be a suitable detector for the OD boundary, even compared with the well-known Canny edge detector used in Aquino et al. [15] and the supervised edge detector utilised in Fan et al. [87]. Furthermore, the use of spline fitting helps the proposed algorithm to capture the irregular shapes of the OD boundaries better

than other algorithms utilizing CHT alone [15, 87] or employing other approaches [3, 16–18, 88]. It is also interesting to note that our proposed method could outperform the algorithm in Lim et al. [92] and is comparable to the algorithms in Wang et al. [26] and Sadhukhan et al. [93].

A more comprehensive comparison is provided in Table 5.2, where the obtained *AOL* values using the proposed algorithm are compared with those produced using the methods in Aquino et al. [15], Cheng et al. [94] and Fan et al. [87]. The comparison is performed in terms of percentage of images per *AOL* intervals ($AOL > 0.95$, $AOL > 0.90$, $AOL > 0.85$, $AOL > 0.80$, and $AOL > 0.70$). As a result, more than 90% of the *AOL* values produced by our algorithm are higher than 0.80, which is better than those in Aquino et al. [15], in which only 84% of the *AOL* values are higher than 0.80. In addition, it is interesting to note that the uses of super-pixel classification in Cheng et al. [94] as well as structured learning in Fan et al. [87] only improve the number of images with $AOL > 0.80$ both by 2% and they are still not better than ours.

TABLE 5.2: Comparisons of the Proposed with Other Edge-based Algorithms in Terms of Percentage of Images per *AOL* Intervals

Method	<i>AOL</i> >				
	0.95	0.90	0.85	0.80	0.70
Aquino et al. [15]	7%	46%	73%	84%	93%
Cheng et al. [94]	8%	51%	76%	86%	92%
Fan et al. [87]	7%	51%	78%	86%	93%
Proposed Algorithm	15%	68%	86%	92%	96%

5.4 Summary

In this chapter, a new algorithm for automatic optic disc segmentation in fundus images is proposed. The proposed algorithm utilises a modified Dolph-Chebyshev type I function matched filter to detect the OD boundary candidates and the Circular Hough Transform as well as spline fitting to obtain the final optic disc boundary. The matched filter, Circular Hough Transform, and spline fitting work on a particular region of interest. The optic disc centre, which is located using three independent methods: template-based, vessels density map-based, and maximum entropy-based methods, serves as the centre of the region of interest. The

performance of the proposed algorithm is evaluated using 1200 fundus images from the MESSIDOR dataset. In the experiments, the proposed algorithm outperforms all competing methods in five major performance indicators, i.e., Area Overlap, Dices coefficient, Accuracy, True Positive Fraction and False Positive Fraction.

Chapter 6

Retinal Vessel Segmentation using a New Matched Filter-based Enhancement Technique and Deep Learning

6.1 Introduction

The blood vessels condition of human eyes is an important indicator for the diagnosis of several retinal diseases. To extract such information, usually ophthalmologists first perform segmentation from fundus images and this is done manually which is tedious and time-consuming. Thus, it is desirable to have an automatic blood vessels segmentation algorithm which is in addition to the above-mentioned, further able to deal with challenging situations such as segmentation of vessels in the presence of pathologies, segmentation of micro-vessels with low-contrast and segmentation of vessels with central reflex [41].

Many retinal vessels segmentation algorithms have been proposed in the past. Essentially, they can be classified into supervised and unsupervised algorithms. The unsupervised algorithms perform segmentation with limited or without any learning procedure. For instance, in the work by Chaudhuri et al. [27], a template for grey-levels profiles of vessels' cross-sections is designed based on the Gaussian function. Another representative is the work by Nguyen et al. [67], where line detectors

are employed to accumulate the presence of vessels in every pixel. In the work by Azzopardi et al. [80], several shifted filter responses (COSFIRE) are utilised to detect blood vessels.

In supervised algorithms, segmentation is performed by feeding several features representing vessels and non-vessels structures to a supervised classifier such as a support vector machine (SVM) and an artificial neural network (ANN). An example of supervised algorithms is the work by Orlando et al. [35], which models the segmentation job as an energy minimization problem in a conditional random field (CRF). Several filter responses are used as the vessels features while a structured output support vector machine (SOSVM) is utilised in the CRF learning process. In Li et al. [37], the blood vessels segmentation task is modelled as a cross-modality data transfer problem. The colour retinal image becomes the first modality while the second modality is the corresponding vessel map. More recently, due to the superiority of convolutional neural networks (CNNs) in supervised segmentation, several CNN based retinal vessel segmentation methods have also been proposed [8, 95, 96].

In general, supervised algorithms are better than unsupervised ones, particularly for those that utilise convolutional neural network (CNN) architectures. However, there are at least two drawbacks of the above CNN based algorithms [8, 95, 96]. First, most of the CNN based algorithms do not consider the highly imbalanced ratio between the vessel and non-vessel pixels. Without a proper strategy, the imbalanced ratio will guide CNN models to be less sensitive to the minority class (vessel pixels). Second, the CNN based algorithms are trained by pixel-to-pixel matching between every obtained probability map and the corresponding ground truth. Nevertheless, due to the highly imbalanced thick and thin vessels ratio, segmentation of thin vessels will be considered as less important than the thick vessels [41]. Such algorithms will inevitably face difficulty when dealing with the above-mentioned challenging situations, particularly for segmentation of micro-vessels.

To overcome the problems, one approach is to use deeper CNN architectures, which inevitably will lead to higher complexity. In this chapter, a new algorithm for retinal vessels segmentation on fundus images is proposed. The algorithm firstly highlights the blood vessels using a new enhancement method before sending the

images to a CNN architecture, derived from U-Net [97]. In particular, five main contributions of this chapter can be summarised as follows:

1. The proposed blood vessel segmentation algorithm combines a new blood vessel enhancement method and a CNN structure, derived from U-Net [97]. The enhancement method is based on the multi-scale and orientation modified Dolph-Chebyshev type I function (MDCF-I) matched filter. The matched filter is modified such that it can be used to detect blood vessels with all possible calibres. Also, we improve the U-Net architecture in Ronneberger et al. [97] such that it provides some advantages over the former and other CNN structures used for blood vessel segmentation. The objective of applying such a new enhancement method is to strengthen the blood vessel features such that the CNN model can learn without suffering from the highly imbalanced ratio between vessel/non-vessel pixels and between thick/thin vessels.
2. The performance of the proposed algorithm is evaluated on fundus images from the DRIVE [4] and STARE [45] databases, as well as high-resolution fundus images from the HRF [46] database. Several quantitative indicators such as Sensitivity (Se), Specificity (Sp), Positive predictive value (Ppv), F1-score ($F1$), G-mean (G) and Mathews correlation coefficient (MCC) are utilised for performance evaluation. Experimental results show that our algorithm outperforms current state-of-the-art methods in terms of Se , $F1$, G and MCC .
3. The segmentation output obtained using the proposed algorithm is also evaluated in terms of the connectivity-area-length ($f(C, A, L)$) score. This score measures the amount of vessel tree features preserved in the segmentation output [98]. We compare the $f(C, A, L)$ score obtained using the proposed algorithm with those state-of-the-art methods' and experimental results show that our score is better than the competing methods.
4. We conduct robustness analysis to emphasize the advantages of the proposed algorithm over U-Net alone and the state-of-the-art methods. In particular, the proposed algorithm is consistently better than U-Net + pre-processing method in Dasgupta and Singh [8] in several challenging cases, namely vessels with central reflexes, thin vessel structures and vessels in the presence of pathologies.

5. The results of the experiments suggest that the proposed algorithm is robust and has the potential to be used for segmentation of vessel-like structures on other image modalities. Also, since the proposed enhancement method is independent of deep learning architectures, it can be easily applied to other deep learning models.

6.2 Methodology

6.2.1 Databases

The **DRIVE database** contains 40 fundus images captured during a screening study with 400 subjects. The images are grouped into the training and testing sets, each consists of 20 images. Each image in this database has a size of 565×584 pixels. Two sets of manually annotated images are provided. The first set is utilised as the ground truth.

The **STARE database** consists of 20 fundus images which are offered for public use by the University of California, San Diego. Each image in this database has a size of 700×605 pixels. In this database, 10 images are from healthy subjects while the rest contain numerous pathologies. Two manual segmentation sets are provided and the first set is considered as the ground truth.

The **HRF database** is the result of a collaborative work conducted to support studies on algorithms for automatic vessels segmentation from fundus images. This database contains 45 high-resolution fundus images, which are divided into the healthy, diabetic retinopathy and glaucomatous sets. Every set comprises 15 fundus images, each with a size of 3504×2336 pixels. A binary gold standard vessel segmentation image is provided for each image and it is used as the ground truth. For the training purpose, since images from this database are approximately four times larger than those from the other two databases, the images and their corresponding ground truths are down-sampled by a factor of 4. The images are down-sampled after they are pre-processed using the new method that is described in Section 6.2.2. Therefore, they have a size of 876×584 pixels. Then, the generated segmentation results are up-sampled before it is compared with the original ground truths.

For the DRIVE and HRF databases, masking images for the field of view (FOV) are provided by the authors [4][46]. On the other hand, since the FOV mask for each image from the STARE database is not publicly provided either by the author [45] or other published algorithms, we obtain the FOV mask using a new method as follows. First, the FOV border (I_{FOVB}) is detected using the Laplacian of Gaussian (LoG) filter. We have conducted an experimental setup and found that $\sigma = 2$ and a kernel size of 13×13 pixels provide the best LoG response. Subsequently, the contrast of I_{FOVB} is increased using the contrast stretching technique. A threshold value which is obtained using the Otsu's method [75] is applied to the resulting image to have a binary mask for the FOV border (I_B). A binary mask for the FOV (I_{FOV}) is obtained by filling the region surrounded by I_B . As I_{FOV} has some regions which lie outside the expected FOV, we subtract I_B from I_{FOV} to have a final mask for the FOV.

To test the quality of the obtained FOV masks, we use them in performance evaluation of the second manual segmentation set from the STARE database. In this case, the performance is measured only for the area inside the FOV mask and the first manual segmentation set is used as the ground truth. In accordance with the work in Li et al. [37], we use Se , Sp and accuracy (Acc) as the performance indicators. The results are compared with those reported in Yan et al. [41] and Li et al. [37] (see Table 6.1). As can be seen in Table 6.1, the results measured using our FOV masks are the same as those presented in Yan et al. [41] and Li et al. [37]. This indicates that our FOV masks are reliable to be used for performance evaluation of blood vessels segmentation methods developed using the images from the STARE database.

TABLE 6.1: Quantitative Results of Manual Segmentation from the STARE Database Measured using Different FOV Masks

Method of FOV Mask	Se	Sp	Acc
Yan et al. [41]	—	—	0.9349
Li et al. [37]	0.8952	0.9384	0.9349
Our Method	0.8952	0.9384	0.9349

Examples of fundus images from the three databases together with their corresponding FOV masks and manual segmentation images are shown in Figure 6.1. Figure 6.1(a) shows examples of fundus images while the corresponding FOV masks and manual segmentation results are depicted in Figure 6.1(b) and Figure 6.1(c).

FIGURE 6.1: Examples of (a) colour images, (b) FOV masks and corresponding manually annotated images from the DRIVE (top), STARE (middle) and HRF (bottom) databases.

The top, middle and bottom rows of Figure 6.1 are for images from the DRIVE, STARE and HRF databases, respectively.

6.2.2 The New Blood Vessel Enhancement Method

We propose a new enhancement method for blood vessels based on the following expression:

$$f_{en} = f_{CLAHE} - f_b, \quad (6.1)$$

where f_{en} is the enhanced image, while f_{CLAHE} and f_b correspond to the grey channel of the fundus image processed using CLAHE and the image with blood

vessels details, respectively. The expression in (6.1) attempts to minimise the grey-levels of the blood vessels such that they can be easily distinguished from the background. In this study, f_b is detected using the MDCF-I matched filter. The MDCF-I matched filter kernel can be calculated using the following expressions:

$$W_{i,j}(x, y) = \begin{cases} |\cos \{M \cos^{-1}[\beta \cos(\pi u \gamma)]\}|, & |A(x, y)| \leq 1 \\ |\cosh[M \cosh^{-1}(\beta)]|, & \text{otherwise} \end{cases}, \quad (6.2)$$

$$W'_{i,j}(x, y) = W_{i,j}(x, y) - m_{i,j}, \quad (6.3)$$

where

$$A(x, y) = \beta \cos(\pi u \gamma), \quad (6.4)$$

$$\beta = \cosh[(1/M) \cosh^{-1}(10^\alpha)], \quad (6.5)$$

$$\alpha = \log \left\{ \cosh[(M-1) \cosh^{-1}(\sec(\pi \omega_{c,j}/M))] \right\}, \quad (6.6)$$

$$\omega_{c,j} = \frac{M+1}{2} - 0.75 \times \omega_{d,j}, \quad (6.7)$$

$$\gamma = 1/(M+1), \quad (6.8)$$

$$\bar{p}_i = \begin{bmatrix} u & v \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{bmatrix}, \quad (6.9)$$

$$m_{i,j} = \sum_{\bar{p}_i \in N} W_{i,j}(x, y) / S. \quad (6.10)$$

The parameter $\omega_{d,j}$ is the scale of the matched filter and corresponds to a certain vessels width in the fundus image. The values of u and v lie on the neighbourhood N , such that $N = \{(u, v) | 0 \leq u < M, 0 < v \leq T\}$. The variable S is the number of pixels in the neighbourhood N .

To ensure that f_b consists of blood vessels in all possible orientations and widths, we calculate the MDCF-I matched filter kernel at some orientations ($\theta = \{0, 15^\circ, \dots, 165^\circ\}$) and scales. The 15° of increment has been widely used and proved to provide sufficient enhancement effects [27][80][99]. The kernel obtained at a particular scale and orientation ($W_{i,j}(x, y)$) is convolved with f_{CLAHE} . Then, the image containing single-width blood vessels with all possible orientation ($R_j(x, y)$) is calculated as follows:

$$R_j(x, y) = \max_{1 \leq i \leq 12} (R_{i,j}(x, y)), \quad (6.11)$$

FIGURE 6.2: Examples of (a) f_{CLAHE} , (b) f_b and (c) f_{en} for images from the DRIVE (top), STARE (middle) and HRF (bottom) databases.

where $R_{i,j}(x, y)$ is the matched filter response at particular orientation and scale. Finally, f_b can be obtained using the following expression:

$$f_b = \frac{1}{n} \sum_{k=1}^K R_j(x, y). \quad (6.12)$$

The variable k is the number of scales used. We consider $K = 5$ and $\omega_d = \{1, 1.5, \dots, 3\}$ for the DRIVE and STARE databases while for the HRF database, $K = 7$ and $\omega_d = \{5, 5.5, \dots, 9\}$ are chosen. Examples of f , f_b and f_{en} are shown in Figure 6.2(a) to Figure 6.2(c). The top, middle and bottom rows of Figure 6.2 are for images from the DRIVE, STARE and HRF databases, respectively.

6.2.3 The Network Architecture

The CNN structure used in this work is derived from U-Net [97]. To be used for blood vessels segmentation, some modifications and adjustments to the U-Net architecture are done. First, the structure in Ronneberger et al. [97] used images with a full size as the input while we take relatively smaller patches with a size of $p \times p$ as the input. In this study, we consider $p = 48$ which is reasonable to preserve sufficient information on vessel features while maintaining computational complexities. The patches are selected both from inside and outside the FOV. Examples of positive (vessel) and negative (non-vessel) patches are shown in Figure 6.3(a), together with their corresponding labels which are depicted in Figure 6.3(b). The top, middle and bottom rows of Figure 6.3 are for images from the DRIVE, STARE and HRF databases, respectively.

FIGURE 6.3: Examples of (a) patches and (b) corresponding labels for images from the DRIVE (top), STARE (middle) and HRF (bottom) databases.

Due to the relatively small size of the input patches, we cannot use too many down-sampling operations. This is to ensure that the input of each convolutional layer contains sufficient information on vessel features. Thus, we use only two rather than four down-sampling and up-sampling operations as suggested in Ronneberger et al. [97]. Following the default configuration of the original U-Net, down-sampling and up-sampling operations are placed after each two successive convolutional layers. To reduce over-fitting problems, we utilise a dropout layer between two successive convolutional layers. In addition, unlike in Ronneberger et al. [97], each down-sampled image in our network is not cropped.

In Figure 6.4, we draw the flow diagram of the proposed algorithm which includes the network architecture. As can be observed in Figure 6.4, the network consists of two parts. The left part is called the contracting path while the right part is the expansive path. The contracting part consists of 3 blocks: each of the first and second blocks has two typical 3×3 convolutional layers while the third block contains a 3×3 convolutional layer. Each convolutional layer is followed by a ReLU and dropout. Every block in this path is connected using a 2×2 max-pooling layer with the stride of 2 for down-sampling. Down-sampling is beneficial to reduce the network computational complexity. At each down-sampling step, the number of feature channels is doubled while its size is halved.

The expansive path also comprises 3 blocks. The first block contains a 3×3 convolutional layer while each of the second and third blocks consists of two typical 3×3 convolutional layers. Like in the contracting path, each convolutional layer in the expansive part is also followed by a rectified linear unit (ReLU) and a dropout. To recover the original input size, each block is connected with a 2×2 up-pooling layer, which also halves the number of feature channels. To combine the local information and global features, the outputs of the first block in the contracting path and the second block of the expansive path are concatenated. In addition, the output of the second block in the contracting unit is concatenated with the output of the first block in the expansive unit. At the end of the architecture, a 1×1 convolutional layer and a soft-max classifier are utilised to map the multi-channel features to the preferred class. In total, the network consists of 11 convolutional layers. Note that the cross-entropy loss function is used to train the network.

FIGURE 6.4: The flow diagram of the proposed algorithm.

It is important to note that the network used in this work has at least three advantages over other CNN structures utilised for blood vessels segmentation. First, unlike the structures in Feng et al. [95] and Dasgupta and Singh [8], no fully connected layer is used in our architecture. The output of the network is the prediction for every pixel on the input patch rather than the centre pixel of the patches. Hence, a heavy computational burden can be avoided. Secondly, the concatenation operation allows the network to learn both from local and global features. This leads to a better performance of the network. Finally, since the network is based on U-Net, it can be trained with a relatively small number of patches yet with comparable or better performance. This is beneficial in performing supervised medical image segmentation as the number of training images is commonly limited.

6.2.4 Network Training and Testing

The patches which are used to train and test the network are extracted from images within the training and testing sets, respectively. We follow the division of fundus images into the training and testing sets for the DRIVE database. However, there is no similar division made originally on the STARE and HRF databases. Hence, the training/testing procedure on the STARE database is carried out using the leave-one-out cross-validation which is also previously used in Yan et al. [41] and Li et al. [37]. On the other hand, for the HRF database, we do not adopt the training/testing procedure used in other supervised methods, i.e., Yan et al. [41] and Orlando et al. [35] as there is no rationale on the division of their training and testing sets. Alternatively, we propose a new training/testing strategy based on the 5-fold cross-validation: the training/testing cycle is repeated 5 times, and in each iteration 36 (12 images are taken from each of the healthy, diabetic retinopathy and glaucomatous sets) of 45 fundus images form the training set while the rest (9 images: 3 images are taken from each of the sets) are utilised as the testing images. The 5-fold cross-validation is selected as the base of the training/testing strategy to provide enough training patches while maintaining the computational complexity. The total numbers of training patches for the DRIVE, STARE and HRF databases are 190,000, 180,500 and 342,000, respectively. The training patches for each database are used to train the network for 100 epochs.

6.2.5 Experimental Setup

The experiments in this work are implemented in two hardware configurations, namely a CPU with Intel Xeon E5-2696 v4 processors and a GPU with an NVIDIA Tesla P40 card. The CPU is needed to carry out the pre-processing steps while the training and testing procedures of the CNN model are implemented on the GPU under the Keras and Tensorflow framework. Our machine requires about five hours to complete the training phase on the DRIVE database. On the STARE and HRF databases, the training phases on each fold take approximately six hours.

6.3 Experimental Results and Discussion

The experiments are done on the proposed algorithm and on the CNN with the conventional pre-processing method (CNN + pre-processing method in Dasgupta and Singh [8]). Note that both the proposed algorithm and the CNN + pre-processing method in Dasgupta and Singh [8] use the same network architecture. In the following sub-sections, we analyse the performance achieved by the proposed algorithm, by CNN + pre-processing method in Dasgupta and Singh [8] and by state-of-the-art algorithms.

6.3.1 Performance Evaluation and Comparison of Retinal Vessel Segmentation Algorithms

The class for each pixel is determined using the default setting: positive class (vessel) is considered if the output of the classifier is greater than a threshold value; otherwise the pixel is classified as a non-vessel (negative decision). The threshold value is calculated using Otsu's method [75]. To evaluate the proposed algorithm, we firstly conduct experiments on fundus images from the DRIVE, STARE and HRF databases. The proposed algorithm takes approximately 4, 6 and 7 seconds to fully segment each image from the DRIVE, STARE and HRF databases, respectively.

Figure 6.5(a) shows some examples of original images. Probability maps generated by the proposed algorithm and CNN + pre-processing method in Dasgupta

FIGURE 6.5: Examples of (a) original images, (b) probability maps generated by the proposed algorithm, (c) corresponding binary masks for images in (b), (d) probability maps generated by the CNN + pre-processing method in Dasgupta and Singh [8], (e) corresponding binary masks for images in (d) and (f) corresponding manual annotations. From left to right: images from the DRIVE, STARE and HRF databases.

and Singh [8] are illustrated in Figure 6.5(b) and Figure 6.5(d), respectively. Figure 6.5(c) and Figure 6.5(e) show the binary masks for the images in Figure 6.5(b) and Figure 6.5(d), respectively. In addition, the corresponding manual annotations are depicted in Figure 6.5(f). The top, middle and bottom rows of Figure 6.5 are for fundus images from the DRIVE, STARE and HRF databases, respectively. Note that the probability maps in Figure 6.5(b) and Figure 6.5(d) are plotted in colour for better clarity. In the figures, it is observed that most fine vessels are preserved in the results of the proposed algorithm.

The proposed algorithm is evaluated in terms of Se , Sp , Ppv , $F1$, G and MCC . All indicators are measured only for the region inside the FOV. The performance indicators can be calculated as follows:

$$Se = \frac{TP}{TP + FN}, \quad (6.13)$$

$$Sp = \frac{TN}{TN + FP}, \quad (6.14)$$

$$Ppv = \frac{TP}{TP + FP}, \quad (6.15)$$

$$F1 = \frac{2 \times Ppv \times Se}{Ppv + Se}, \quad (6.16)$$

$$G = \sqrt{Se \times Sp}, \quad (6.17)$$

$$MCC = \frac{TP/A - S \times P}{\sqrt{P \times S \times (1 - S) \times (1 - P)}}, \quad (6.18)$$

where A is the number of pixels within the FOV, $S = (TP + FN)/A$ and $P = (TP + FP)/A$. TP , TN , FP and FN are the numbers of true positive, true negative, false positive and false negative, respectively.

Se (known also as Recall) reflects the ability of a certain algorithm to correctly detect blood vessels while Sp is a metric which measures the capability of the algorithm on correctly detecting non-vessel structures. Ppv indicates the ratio of pixels detected as vessels that are truly vessels. We do not use Acc as it is sensitive to highly imbalanced distributions of the vessel and non-vessel pixels. Alternatively, $F1$, G and MCC are included as they are more suitable for the imbalanced class ratio. $F1$ is the harmonic mean of Se and Ppv , which has a property to show an appropriate trade-off of the two scores. It has a range value of 0 (the lowest value) and 1 (perfect segmentation). Likewise, G is an indicator which quantifies

the balance between Se and Sp . It returns a value between 0 and 1. Finally, MCC is a correlation coefficient between the segmentation output of the algorithm and ground truth. Its value ranges from -1 to 1, with 1 indicating a perfect matching of the prediction and ground truth, 0 reflecting a random prediction and -1 representing a total disagreement between the prediction and ground truth [35].

In Table 6.2 and Table 6.3 we report quantitative results of the experiments on the DRIVE and STARE databases, together with the results of recently published methods. The highest score of each indicator is listed in bold. Note that the reported Se and Sp scores in Table 6.2 and Table 6.3 are the average over the testing images' of a certain database. Since most of the authors of other works did not report the average scores other than Se and Sp , their average scores of other metrics such as Positive predictive value ($\widetilde{Ppv}$), F1-score ($\widetilde{F1}$), G-mean ($\widetilde{G}$) and Matthews correlation coefficient ($\widetilde{MCC}$) are approximated based on the given Se and Sp scores in the corresponding work:

$$\widetilde{Ppv} = \frac{\widetilde{TP}}{\widetilde{V}}, \quad (6.19)$$

$$\widetilde{F1} = \frac{2 \times Se \times \widetilde{Ppv}}{Se + \widetilde{Ppv}}, \quad (6.20)$$

$$\widetilde{G} = \sqrt{Se \times Sp}, \quad (6.21)$$

$$\widetilde{MCC} = \frac{\widetilde{TP}/\widetilde{A} - \widetilde{S} \times \widetilde{P}}{\sqrt{\widetilde{P} \times \widetilde{S} \times (1 - \widetilde{S}) \times (1 - \widetilde{P})}}, \quad (6.22)$$

where

$$\widetilde{TP} = Se \times \widetilde{V}, \quad (6.23)$$

$$\widetilde{FP} = (1 - Sp) \times \widetilde{BG}, \quad (6.24)$$

$$\widetilde{S} = \widetilde{V}/\widetilde{A}, \quad (6.25)$$

$$\widetilde{P} = \frac{\widetilde{TP} + \widetilde{FP}}{\widetilde{A}}, \quad (6.26)$$

$$\widetilde{A} = \widetilde{V} + \widetilde{BG}. \quad (6.27)$$

The variables $\widetilde{TP}$ and $\widetilde{TN}$ are the total numbers of true positive and true negative over the testing images from a certain database while $\widetilde{FP}$ and $\widetilde{FN}$ are the corresponding total numbers of false positive and false negative, respectively. In

addition, $\widetilde{V}$ and $\widetilde{BG}$ are the total numbers of vessels and background pixels (only those inside the FOV masks) in the testing images of a certain database.

TABLE 6.2: Performance of blood vessel segmentation algorithms on the DRIVE database

Method	Se	Sp	$\widetilde{Ppv}$	$\widetilde{F1}$	$\widetilde{G}$	$\widetilde{MCC}$
2nd Human Observer	0.7760	0.9724	0.8040	0.7897	0.8687	0.7614
Azzopardi et al. [80]	0.7655	0.9704	0.7904	0.7778	0.8619	0.7475
Li et al. [37]	0.7569	0.9816	0.8571	0.8039	0.8620	0.7793
Dasgupta and Singh [8]	0.7691	0.9801	0.8498	0.8074	0.8682	0.7819
Orlando et al. [35]	0.7897	0.9684	0.7847	0.7872	0.8745	0.7575
Brancati et al. [96]	0.7820	0.9760	0.8262	0.8034	0.8736	0.7776
Yan et al. [41]	0.7653	0.9818	0.8598	0.8098	0.8668	0.7871
Hu et al. [100]	0.7772	0.9793	0.8452	0.8098	0.8724	0.7851
Yu et al. [101]	0.7643	0.9803	0.8494	0.8046	0.8656	0.7801
Feng et al. [102]	0.7625	0.9809	0.8534	0.8054	0.8648	0.7819
CNN + pre-processing method in Dasgupta and Singh [8]	0.7975	0.9792	0.8481	0.8220	0.8837	0.7990
MDCF-I	0.7690	0.9660	0.7674	0.7682	0.8620	0.7357
MDCF-II	0.7480	0.9780	0.8289	0.7861	0.8560	0.7595
Optimal Adaptive Filter	0.7780	0.9720	0.8038	0.7907	0.8700	0.7623
Proposed Algorithm	0.8314	0.9726	0.8157	0.8235	0.8992	0.7991

Note that the expressions in (6.15) to (6.18) are used to calculate the actual scores of Ppv , $F1$, G and MCC for each fundus image while the expressions in (6.19) to (6.27) are utilised to approximate the average scores of Ppv , $F1$, G and MCC which are not reported by most of the authors of the compared methods. Hence, there will inevitably be margins between the average and the approximate scores. For a fair comparison, the reported scores of positive predictive value, F1-score, G-mean and Matthews correlation coefficient for the proposed algorithm and CNN + pre-processing method in Dasgupta and Singh [8] in Table 6.2 and Table 6.3 are also calculated using (6.19) to (6.27).

It is observed from Table 6.2 and Table 6.3 that with the proposed blood vessel enhancement method, the network attains noticeable better values of Se , $F1$, G and MCC than CNN + pre-processing method in Dasgupta and Singh [8]. In addition, the proposed algorithm consistently outperforms most state-of-the-art methods in those four indicators. Note that significant improvements of Se scores, particularly on the DRIVE database from current state-of-the-art methods indicate that the

TABLE 6.3: Performance of blood vessel segmentation algorithms on the STARE database

Method	Se	Sp	$\widetilde{Ppv}$	$\widetilde{F1}$	$\widetilde{G}$	$\widetilde{MCC}$
2nd Human Observer	0.8952	0.9384	0.6270	0.7375	0.9165	0.7156
Azzopardi et al. [80]	0.7716	0.9701	0.7490	0.7601	0.8652	0.7320
Annunziata et al. [32]	0.7128	0.9836	0.8341	0.7687	0.8373	0.7471
Li et al. [37]	0.7726	0.9844	0.8514	0.8100	0.8721	0.7920
Orlando et al. [35]	0.7680	0.9738	0.7722	0.7701	0.8648	0.7436
Yan et al. [41]	0.7581	0.9846	0.8506	0.8017	0.8640	0.7818
Hu et al. [100]	0.7543	0.9814	0.8243	0.7877	0.8604	0.7654
Yu et al. [101]	0.7837	0.9787	0.8097	0.7965	0.8758	0.7736
Feng et al. [102]	0.7709	0.9848	0.8544	0.8105	0.8713	0.7911
CNN + pre-processing method in Dasgupta and Singh [8]	0.7872	0.9828	0.8411	0.8133	0.8796	0.7930
MDCF-I	0.7570	0.9730	0.7609	0.7687	0.8580	0.7309
MDCF-II	0.7926	0.9730	0.7693	0.7810	0.8770	0.7554
Optimal Adaptive Filter	0.7967	0.9765	0.7969	0.7972	0.8820	0.7734
Proposed Algorithm	0.7924	0.9827	0.8412	0.8161	0.8824	0.7959

proposed algorithm is more able to detect a greater number of blood vessels. In particular, this shows that the proposed enhancement method successfully highlights blood vessels with different widths. Thus, features learned by the network are more distinguishable to classify vessel pixels from non-vessel pixels.

It is interesting to note that we achieve relatively lower scores of Sp and Ppv than other competing methods. In the blood vessel segmentation task, Sp denotes the ability of the algorithm to correctly detect the non-vessel class, and it usually decreases when the segmentation result has a great number of false positives. Likewise, low scores of Ppv are caused by a high number of false positives in the segmentation result. Although achieving high scores on both Se and Sp score is desirable, there will usually be a trade-off between the scores. In this case, we may refer to the achieved scores of $F1$, G and MCC . As suggested by [35], these indicators can be used to show the appropriate trade-off of Se and Sp scores. Given that we achieve higher scores on $F1$, G and MCC , it is worth to mention that our Sp and Ppv scores are acceptable although they are lower than those from other methods.

The results of additional experiments conducted on high-resolution fundus images

TABLE 6.4: Performance of Blood Vessel Segmentation Algorithms on the HRF Database

Method	Se	Sp	$\widetilde{Ppv}$	$\widetilde{F1}$	$\widetilde{G}$	$\widetilde{MCC}$
Orlando et al. [35]	0.7874	0.9584	0.6599	0.7180	0.8687	0.6896
Yan et al. [41]	0.7881	0.9592	0.6647	0.7211	0.8695	0.6939
CNN + pre-processing method in Dasgupta and Singh [8]	0.7485	0.9808	0.7998	0.7733	0.8568	0.7514
Proposed Algorithm	0.8136	0.9770	0.7838	0.7984	0.8916	0.7775

from the HRF database are listed in Table 6.4, together with those from other algorithms. The highest score of each indicator is listed in bold. Like in Table 6.2, the Se and Sp scores of other methods are taken from the original paper while the Ppv , $F1$, G and MCC scores are approximated using (6.19) to (6.27). For a fair comparison with the methods in Yan et al. [41] and Orlando et al. [35], the metric scores of CNN + pre-processing method in Dasgupta and Singh [8] and the proposed algorithm reported in Table 6.4 are calculated only based on the last 10 images of each healthy, diabetic retinopathy and glaucomatous sets (30 images altogether). It can be observed that the proposed algorithm is better than the CNN + pre-processing method in Dasgupta and Singh [8] in terms of four indicators (Se , $F1$, G and MCC). It should also be noted that the proposed algorithm is better than all competing methods, including the CNN-based method in Yan et al. [41]. This is important as performing retinal vessels segmentation on high-resolution fundus images is more challenging [46].

To highlight the advantages of the proposed algorithm specifically over the CNN + pre-processing method in Dasgupta and Singh [8], we plot the ROC curves of the proposed algorithm and CNN + pre-processing method in Dasgupta and Singh [8] in Figure 6.6. Figure 6.6(a) to Figure 6.6(c) show the ROC curves for the DRIVE, STARE and HRF databases, respectively. Note that the ROC curves for the HRF database are based on the segmentation results of the last 10 images of each healthy, diabetic retinopathy and glaucomatous sets (30 images altogether). In addition, we calculate the area under curve (AUC) scores for the proposed algorithm and the CNN + pre-processing method in Dasgupta and Singh [8] on all databases. As a result, the proposed algorithm consistently maintains higher AUC scores than those of CNN + pre-processing method in Dasgupta and Singh [8] on all databases.

Specifically, the new enhancement method helps the proposed algorithm to achieve significant improvements in the segmentation results on high-resolution images.

6.3.2 Connectivity, Area and Length in Vessels Segmentation

In this part, the segmentation results of the proposed algorithm are evaluated in terms of the connectivity (C), area (A) and length (L) [98]. Suppose S and S_G are the segmentation result and the corresponding ground truth, C measures the fragmentation degree between S and S_G . Since the vascular tree is a connected component, accurate vascular segmentation is expected to have only a small number of connected components (1 is a perfect condition). This metric penalizes fragmented segmentation outputs by comparing the number of connected components in S and S_G with regard to the total number of vessel pixels in S_G :

$$C = 1 - \min \left(1, \frac{|\#_C(S_G) - \#_C(S)|}{\#(S_G)} \right), \quad (6.28)$$

where $\min(\cdot)$ denotes the minimum operation, $\#_C(S_G)$ and $\#_C(S)$ are the number of connected components in S and S_G , respectively, and $\#(S_G)$ is the cardinality of S_G .

The second metric, A quantifies the degree of overlapping area between S and S_G , which can be defined as:

$$A = \frac{\#((\delta_\alpha(S) \cap S_G) \cup (\delta_\alpha(S_G) \cap S))}{\#(S \cup S_G)}. \quad (6.29)$$

The functions δ_α and δ_β are morphological dilations using disc structure elements of α and β in radii.

The third metric, L measures the degree of coincidence between S and S_G in terms of the total length of the vascular tree:

$$L = \frac{\#((\varphi(S) \cap \delta_\beta(S_G)) \cup (\varphi(S_G) \cap \delta_\beta(S)))}{\#(\varphi(S) \cup \varphi(S_G))}, \quad (6.30)$$

where φ is homo-topic skeletonization while $\cap$ and $\cup$ denote the set intersection and union operators, respectively. Finally, a global measure $f(C, A, L)$ can be

(a)

(b)

(c)

FIGURE 6.6: ROC curves for the proposed algorithm and CNN + pre-processing method in Dasgupta and Singh [8] on the (a) DRIVE, (b) STARE and (c) HRF databases.

TABLE 6.5: C , A , L and $f(C, A, L)$ Scores of Blood Vessel Segmentation Algorithms on the DRIVE, STARE and HRF Databases

Method	C	A	L	$f(C, A, L)$
DRIVE				
2nd Human Observer	1	0.9398	0.9347	0.8784
Sheng et al. [104]	0.9998	0.9006	0.8988	0.8092
CNN + pre-processing method in Dasgupta and Singh [8]	0.9998	0.9338	0.9287	0.8670
Proposed Algorithm	0.9997	0.9421	0.9379	0.8834
STARE				
2nd Human Observer	0.9998	0.8476	0.8600	0.7288
Sheng et al. [104]	0.9996	0.8242	0.8382	0.6906
CNN + pre-processing method in Dasgupta and Singh [8]	0.9998	0.8928	0.9042	0.8099
Proposed Algorithm	0.9998	0.8977	0.9087	0.8181
HRF				
CNN + pre-processing method in Dasgupta and Singh [8]	1	0.7961	0.8594	0.6866
Proposed Algorithm	1	0.8466	0.8977	0.7601

expressed as the product of C , A and L as follows:

$$f(C, A, L) = C \times A \times L. \quad (6.31)$$

In their work, Gegúndez-Arias et al. [98] has shown that the connectivity, area and length metrics provide the best correspondence with human perception in retinal vessel segmentation when compared to other quality evaluation functions such as Jaccard coefficient, Dice coefficient, Kappa coefficient, average symmetric contour distance, root-mean-square symmetric contour distance and maximum symmetric contour distance [98]. In addition, according to Yan et al. [103], the $f(C, A, L)$ metric is important to quantify thick and thin vessels more equally.

The top, middle and bottom parts of Table 6.5 show the performance evaluation of the proposed algorithm on the DRIVE, STARE and HRF databases, respectively. The performance of CNN + pre-processing method in Dasgupta and Singh [8] and that of the method in Sheng et al. [104] are also provided for comparison purpose. It is observed from Table 6.5 that the proposed algorithm consistently maintains the best $f(C, A, L)$ scores among the compared methods. This provides another perspective for demonstrating the efficacy of the proposed algorithm.

FIGURE 6.7: Examples of segmentation results on (a) micro-vessels, (b) vessels with central reflex and (c) vessels in the presence of pathologies. From left to right are retinal image patches, segmentation results using the proposed algorithm, segmentation results using CNN + pre-processing method in Dasgupta and Singh [8] and manual annotations. Images in the first column are processed using CLAHE only for better visualization.

6.4 Robustness Analysis

As mentioned earlier, a robust retinal vessels segmentation method should be able to deal with some challenging cases, which include 1) segmentation of micro-vessels; 2) segmentation of vessels with central reflex, and 3) vessels segmentation in the presence of pathologies. The effectiveness of the proposed algorithm in dealing with the above situations is discussed in this section.

6.4.1 Segmentation of micro-vessels with Low Contrast

The capability of our algorithm in detecting micro-vessels is shown in Figure 6.7(a). In this case, micro-vessels with very low contrast (shown in the first row of Figure 6.7(a)) and micro-vessels surrounding the macular (shown in the second row of Figure 6.7(a)) are selected from the HRF database. The first, second, third and fourth columns of Figure 6.7(a) are for the retinal image patches with micro-vessels, segmentation results of the proposed algorithm, segmentation results of the CNN + pre-processing method in Dasgupta and Singh [8] and corresponding manual annotations. Some regions with micro-vessels are marked by red circles. It is observed that the MDCF-I matched filter helps the proposed algorithm to correctly detect low contrast micro-vessels which are not detected by the CNN + pre-processing method in Dasgupta and Singh [8]. Similarly, although on the grey channel of a retinal image, the macular has similar grey-level values to those of blood vessels, a larger number of micro-vessels near the macular are successfully classified as vessels by the proposed algorithm than those using the CNN + pre-processing method in Dasgupta and Singh [8].

6.4.2 Segmentation of Vessels with Central Reflex

To indicate the effectiveness of the proposed algorithm to detect blood vessels with central reflex, we have selected two patches containing vessels with a central reflex as depicted in the first and second rows of Figure 6.7(b), respectively. The first, second, third and fourth columns of Figure 6.7(b) are for the retinal image patches with micro-vessels, segmentation results of the proposed algorithm, segmentation results of the CNN + pre-processing method in Dasgupta and Singh

[8] and corresponding manual annotations. As observed in Figure 6.7(b), the proposed algorithm correctly identifies vessels which are undetected by the CNN + pre-processing method in Dasgupta and Singh [8] (See regions with the red circles).

6.4.3 Vessels Segmentation in the Presence of Pathologies

It has been mentioned earlier that the presence of pathologies like microaneurysms and haemorrhages can make segmentation of blood vessels to become more challenging. The results of the proposed algorithm and those using the CNN + pre-processing method in Dasgupta and Singh [8] on images with pathologies are shown in Figure 6.7(c). The first, second, third and fourth columns of Figure 6.7(c) are for the retinal image patches with pathologies, segmentation results of the proposed algorithm, segmentation results of the CNN + pre-processing method in Dasgupta and Singh [8] and corresponding manual annotations. Note that the first and second rows of Figure 6.7(c) are for images with bright and dark lesions, respectively. It is observed that the segmentation outputs obtained using the proposed algorithm preserves more blood vessels than those obtained using CNN + pre-processing method in Dasgupta and Singh [8] (red circles) while the false positives due to bright and dark lesions are relatively comparable (green circles).

To emphasize the ability of the proposed algorithm for vessel segmentation in the presence of pathologies, the reported performance indicators for the STARE database in Table 6.3 are separated into the healthy and pathological cases and the results are presented in Table 6.6. In Table 6.6, we also provide the performance of the work in Yan et al. [41] as well as the filters presented in Chapter 4 on the healthy and pathological cases. The top and bottom parts of Table 6.6 report the average performance indicators of the algorithms tested on the healthy and pathological images, respectively. Note that in Table 6.6, we present the actual scores of Se , Sp , Ppv , $F1$, G and MCC for the filters in Chapter 4, the CNN + pre-processing method in Dasgupta and Singh [8], the proposed algorithm and the method in Yan et al. [41]. For the healthy case, our proposed algorithm outperforms the compared methods in all indicators. On the pathological images, the proposed algorithm achieves the best scores of $F1$, G and MCC among the compared methods.

TABLE 6.6: Performance of Blood Vessel Segmentation Algorithms on Healthy and Pathological Images from the STARE Database

Method	Se	Sp	Ppv	$F1$	G	MCC
Healthy						
Yan et al. [41]	0.8457	0.9828	0.8594	0.8474	0.9116	0.8312
CNN + pre-processing method in Dasgupta and Singh [8]	0.8449	0.9823	0.8614	0.8531	0.9110	0.8320
MDCF-I	0.7943	0.9739	—	0.7962	0.8795	0.7699
MDCF-II	0.8414	0.9716	—	0.8140	0.9037	0.7908
Optimal Adaptive Filter	0.8305	0.9749	—	0.8208	0.8991	0.7979
Proposed Algorithm	0.8458	0.9853	0.8669	0.8562	0.9129	0.8368
Pathological						
Yan et al. [41]	0.6705	0.9864	0.8500	0.7307	0.8313	0.7242
CNN + pre-processing method in Dasgupta and Singh [8]	0.7296	0.9833	0.8196	0.7720	0.8470	0.7492
MDCF-I	0.7185	0.9753	—	0.7328	0.8371	0.7064
MDCF-II	0.7437	0.9752	—	0.7468	0.8513	0.7220
Optimal Adaptive Filter	0.7629	0.9782	—	0.7707	0.8629	0.7505
Proposed Algorithm	0.7390	0.9816	0.8157	0.7755	0.8517	0.7516

TABLE 6.7: Performance of Blood Vessel Segmentation Algorithms on Each Set of the HRF Database

Method	Se	Sp	Ppv	$F1$	G	MCC
Healthy						
CNN + pre-processing method in Dasgupta and Singh [8]	0.7661	0.9888	0.8922	0.8229	0.8699	0.8074
Proposed Algorithm	0.8207	0.9877	0.8891	0.8526	0.9001	0.8373
Diabetic Retinopathy						
CNN + pre-processing method in Dasgupta and Singh [8]	0.7193	0.9762	0.7495	0.7304	0.8366	0.7075
Proposed Algorithm	0.7918	0.9707	0.7268	0.7548	0.8756	0.7328
Glaucomatous						
CNN + pre-processing method in Dasgupta and Singh [8]	0.7602	0.9774	0.7532	0.7558	0.8617	0.7341
Proposed Algorithm	0.8284	0.9725	0.7322	0.7768	0.8974	0.7573

We also categorize the results of CNN + pre-processing method in Dasgupta and Singh [8] and the proposed algorithm on the HRF database into three cases: the healthy, diabetic retinopathy and glaucomatous, and present them in the top, middle and bottom parts of Table 6.7, respectively. Note that in Table 6.7, we present the actual scores of Se , Sp , Ppv , $F1$, G and MCC for both CNN + pre-processing method in Dasgupta and Singh [8] and the proposed algorithm since we conduct the experiments on the two approaches. The results in Table 6.7 show that we consistently maintain higher scores of Se , $F1$, G and MCC in all cases. It indicates that the proposed algorithm is robust and will be more useful for real applications.

6.5 The Effect of Our Enhancement Method on Retinal Images

In the previous sections, we have demonstrated that the proposed retinal vessel segmentation algorithm could outperform state-of-the-art methods, mainly due to the application of the proposed enhancement technique. In this respect, it is essential to investigate the performance of our enhancement method independently from the segmentation framework. For this purpose, we have conducted several experiments on the proposed enhancement method using different combinations of the values of its parameters. In particular, we perform the tests on several combinations of ω_c values since they play a crucial role in the enhancement process. Since the ω_c values correspond to the vessel radius (V_r), we may obtain the possible combinations of the ω_c values by applying ${}^N C_K = \frac{N!}{K!(N-K)!}$ operations to the set of vessel radii, where N and K denote the numbers of vessel radii and ω_c values, respectively. The qualities of the enhanced images are then measured in terms of the contrast of blood vessels to the background (C_t) that can be expressed as follows [105]:

$$C_t = \left| \frac{Y - G}{Y + G} \right|, \quad (6.32)$$

where Y and G are the average intensities of the blood vessels and background, lying inside the FOV, respectively. The value of C_t ranges from 0 to 1. A perfect enhancement result is obtained when Y equals to 0 and G equals to 1 (C_t equals to 1). On the other hand, $Y = G$ ($C_t = 0$) refers to the worst contrast of blood vessels to the background.

(a)

(b)

FIGURE 6.8: (a) The C_t values obtained on images from the test set of the DRIVE database. (b) The average C_t values across all images.

We conduct the experiments only on the DRIVE database as our segmentation algorithm achieves results with a similar trend on the DRIVE, STARE and HRF databases. According to the work of Al-Rawi et al. [28], the blood vessels in the DRIVE database range from 2 to 10 pixels of diameters. Hence, we may define the set of vessel radii to $V_r = \{1, 1.5, \dots, 5\}$. It is worth noting that for each implementation, our enhancement method requires five different ω_c values ($K = 5$). Hence, choosing five ω_c values from nine possible vessel radii ($N = 9$), i.e., ${}^9C_5$ will provide 126 possible combinations of ω_c values. The beginning parts of the combinations will be dominated with small values of ω_c like 1, 1.5 and 2. On the other hand, the last few sets of the combinations will contain larger values of ω_c , such as 4, 4.5 and 5. The contrast values obtained from the implementations of our enhancement method on 20 images from the test set of the DRIVE database are presented in Figure 6.8. From the figure, we may observe that the proposed enhancement method offers improvements in the quality of the images. In particular, the increases range from 50.62% to 60.35%, with the average and standard deviation of 58.06% and 1.72%, respectively.

6.6 Summary

In this chapter, we have presented a new algorithm for retinal vessels segmentation. The algorithm uses a new blood vessels enhancement method and combines it with a convolutional neural network structure derived from U-Net. Experimental results on publicly available databases indicate that the proposed algorithm outperforms state-of-the-art algorithms in four major indicators. Specifically, the proposed algorithm performs well in challenging situations such as detecting low-contrast micro-vessels, vessels with central reflex and vessels in the presence of lesions. The robustness of the proposed algorithm, particularly on high-resolution fundus images indicates that our algorithm will be useful for practical applications. In addition, there is a possibility to further exploit the proposed algorithm for vessel-like structures segmentation in other medical applications.

Chapter 7

Conclusions and Recommendations for Future Work

7.1 Conclusions

This thesis has described the design of algorithms for edge and curvilinear structures detection in medical images, particularly for the applications of optical disc boundary and retinal vessel detection on fundus images, respectively. The algorithms are developed based on the mathematical function which can closely represent the behaviour of edge and curvilinear structures. The algorithms, on one hand, can perform edge and curvilinear structures detection under an unsupervised framework and on the other hand allow an implementation with deep learning architectures. This provides meaningful insight into the development of robust edge and curvilinear structures detection algorithms on other image modalities. Detailed works that have been conducted are explained as follows:

First, a flexible model for edge representation in digital imagery has been proposed. The model is developed based on the Dolph-Chebyshev function and the edge model parameters are estimated under a least-square framework. Extensive experiments using edge profiles with various blur and noise levels are reported to demonstrate the validity of the proposed model. Quantitative analysis in terms of root-mean-square-error (RMSE) is provided to show the advantages of the proposed model

over the Gaussian-based models. Also, a procedure for edge detection in two-dimensional images using the proposed model is derived and it is shown that our edge detector performs better than the Gaussian-based models. In particular, a right-tailed paired-samples *t-test* applied to the F-measures produced by the tested models indicates that our proposed model outperforms the other two models by $(t(39) = 6.23, p = 1.243 \times 10^{-7})$ and $(t(39) = 7.085, p = 8.190 \times 10^{-9})$, respectively.

Second, three curvilinear structure detection algorithms with an application on retinal blood vessel detection on colour fundus images have been designed. The first two algorithms are designed based on the validated edge model and the third algorithm is developed under an optimisation framework. The proposed algorithms are validated using fundus images from the publicly available databases, the DRIVE and STARE databases. Quantitative comparisons with other methods show an improvement on the Matthews Correlation Coefficient of 5% over the reported counterparts.

Third, a new edge detection algorithm has been developed by tailoring the curvilinear structure detection algorithms. The edge detection algorithm is applied to perform detection and segmentation of optical discs on colour fundus images. In the experimental parts, the proposed algorithm successfully locates the optic discs in 1192 out of 1200 fundus images. Furthermore, the proposed algorithm has the ratio of intersecting area between the segmented optic discs and the ground truth of 0.9 on more than 60% of the total images. The superiority of the proposed algorithm over current-state-of-the-art methods suggests that the algorithm will be beneficial for practical applications.

Finally, a new framework of deep retinal vessel detection algorithm has been proposed. The proposed framework is beneficial to increase the performance of current deep learning-based blood vessel detection methods, particularly when dealing with imbalanced training data and challenging situations such as detecting low-contrast micro-vessels, vessels with central reflex and vessels in the presence of pathologies. In particular, noticeable improvements are achieved in terms of Sensitivity and Matthews Correlation Coefficient. Also, the enhancement method used in our framework is independent of the deep learning architecture, such that it can be easily combined with other deep learning architectures. The superiority and the flexibility of the proposed algorithm offer a meaningful solution for the development of robust curvilinear structures detection algorithms on other image modalities.

7.2 Recommendations for Future Work

This thesis has presented several algorithms for optical disc boundary and retinal vessel detection on fundus images. Future work will aim to improve the performance of the presented algorithms, applying our algorithms for detection of abnormalities in fundus images and adapting the algorithms on other image modalities.

7.2.1 Development of a New Framework for Deep Optical Disc Detection and Segmentation

In Chapter 6, we have proposed a new framework for deep retinal vessel detection. Before feeding fundus images to a deep learning architecture, the algorithm strengthens the blood vessel features using the proposed enhancement method, such that although the amount of vessel structures in the training patches is much smaller than the background, the deep learning architecture is still able to learn meaningful features of blood vessels. A similar implementation could also be developed to solve the OD boundary detection problem. In this respect, the extracted OD boundary using the algorithm in Chapter 5 may be used as the initial contour learned by a deep learning model.

7.2.2 Neovascularizations Detection using the Proposed Algorithms

Fundus image analysis is beneficial for non-invasive detection of several eye-related diseases, including diabetic retinopathy. In general, diabetic retinopathy occurs in two stages, the non-proliferative diabetic retinopathy (NPDR) and proliferative diabetic retinopathy (PDR). In the first stage, patients may not have any warning symptoms. In contrast, in the second stage, abnormal blood vessels (neovascularization) emerge, and when they burst and bleed, the patients may lose their vision. To avoid this problem, early detection, mainly using a computer algorithm, is required [106]. Neovascularizations in fundus images are commonly characterized as thin and highly unregulated vessels [107]. In this respect, the proposed blood vessel segmentation algorithm presented in Chapter 6 have offered a satisfactory performance in detecting small and fine vessels. The algorithm would be beneficial

for neovascularization detection. Moreover, for the detection of neovascularization within the optical disc, our optical disc detection algorithm would be useful to locate the OD location.

7.2.3 Generalization of the Proposed Algorithms for Edge and Curvilinear Structures Detectors

In the previous chapters, we have demonstrated the efficacy of our proposed OD boundary and blood vessel detection algorithms compared to the state-of-the-art methods. In this respect, it is worthwhile to adopt the proposed algorithms for applications on other image modalities. In particular, blood vessel detection algorithms can be tailored to detect other curvilinear structures like cracks on roads and walls, DNA filaments and wrinkles on human faces. On the other hand, the MDCF-I matched filter that has been used as the edge detector in the OD boundary detection problem can be generalized as a detector for edge structures in a wide variety of images.

List of Author's Awards and Publications

Awards

- **Best Paper Awards**, D. A. Dharmawan and B. P. Ng, "Segmenting Retinal Vessels with a Multi-scale Modified Dolph-Chebyshev Type I Function Matched Filter," *2018 4th International Conference on Science and Technology (ICST)*, Yogyakarta, 2018, pp. 1-4.

Journal Articles

- **D. A. Dharmawan**, B. P. Ng, S. Rahardja, "A Modified Dolph-Chebyshev Type II Function Matched Filter for Retinal Vessels Segmentation," *Symmetry*, 2018, 10, 257.
- **D. A. Dharmawan**, D. Li, B. P. Ng and S. Rahardja, "A New Hybrid Algorithm for Retinal Vessels Segmentation on Fundus Images," *IEEE Access*, vol. 7, pp. 41885-41896, 2019.
- **D. A. Dharmawan**, B. P. Ng and N. Borijindargoon, "Design of Optimal Adaptive Filters for Two-Dimensional Filamentary Structures Segmentation," *IEEE Signal Processing Letters*, vol. 26, no. 10, pp. 1511-1515, 2019.
- **D. A. Dharmawan**, B. P. Ng, and S. Rahardja, "A new optic disc segmentation method using a modified Dolph-Chebyshev matched filter," *Biomedical Signal Processing and Control*, vol. 59, 2020.

- **D. A. Dharmawan** and B. P. Ng, "A flexible model for edge representation in digital imagery," Under review by *Signal Processing*.

Conference Proceedings

- **D. A. Dharmawan** and B. P. Ng, "A new two-dimensional matched filter based on the modified Chebyshev type I function for retinal vessels detection," *2017 39th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)*, Seogwipo, 2017, pp. 369-372.
- **D. A. Dharmawan** and B. P. Ng, "Segmenting Retinal Vessels with a Multi-scale Modified Dolph-Chebyshev Type I Function Matched Filter," *2018 4th International Conference on Science and Technology (ICST)*, Yogyakarta, 2018, pp. 1-4.
- D. Li, **D. A. Dharmawan**, B. P. Ng, and S. Rahardja, "Residual U-Net for Retinal Vessel Segmentation," *2019 IEEE International Conference on Image Processing (ICIP)*, 2019, pp. 1425-1429.

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