Real-time flight scheduling for optimal air traffic flow management

Gammana Guruge, Nadeesha Sandamali

2020

Gammana Guruge, N. S. (2020). Real-time flight scheduling for optimal air traffic flow management. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/140363>

<https://doi.org/10.32657/10356/140363>

This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License (CC BY-NC 4.0).

Downloaded on 26 Apr 2023 15:00:31 SGT

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

SINGAPORE

**REAL-TIME FLIGHT SCHEDULING FOR
OPTIMAL AIR TRAFFIC FLOW
MANAGEMENT**

**GAMMANA GURUGE NADEESHA SANDAMALI
SCHOOL OF ELECTRICAL & ELECTRONIC
ENGINEERING**

2020

**REAL-TIME FLIGHT SCHEDULING FOR
OPTIMAL AIR TRAFFIC FLOW
MANAGEMENT**

**GAMMANA GURUGE NADEESHA
SANDAMALI**

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2020

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

08-01-20

Date

GAMMANA GURUGE NADEESHA
SANDAMALI

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

08-01-20

Date

ASSOC. PROF. SU RONG

Author Attribution Statement

This thesis contains material from three journal papers, in which one of them is published and other two are under review, and three conference papers, in all of which I was the first and the corresponding author.

A preliminary version of the work in Chapter 3 is published as G. G. N. Sandamali, R. Su, Y. Zhang, and Q. Li, “Flight Routing and Scheduling with Departure Uncertainties in Air Traffic Flow Management,” 13th IEEE International Conference on Control & Automation (ICCA), 2017, pp. 301-306.

doi: 10.1109/ICCA.2017.8003077.

The contributions of the co-authors are as follows:

- I reviewed the literature, came up with the new model formulation, prepared the manuscript, and performed simulations.
- A/Prof. Su Rong provided the initial research direction, guided me through the process, and contributed with suggestions.
- Yicheng Zhang participated in the initial brainstorming discussions, revised the manuscript, and provided several technical directions.

A major part of Chapter 3 is published as G. G. N. Sandamali, R. Su and Y. Zhang, “Flight Routing and Scheduling Under Departure and En-route Speed Uncertainty,” IEEE Transactions on Intelligent Transportation Systems, 2019, doi: 10.1109/TITS.2019.2907058.

The contributions of the co-authors are as follows:

- I reviewed the literature, conceived and analyzed the idea, co-developed the mathematical model with Prof. Su Rong, prepared the manuscript draft, developed the simulation platform, and conducted the experiments.
- A/Prof. Su Rong reviewed the idea, continuously guided me through the process, provided feedback on the formulation, and revised the manuscript and the response letters.
- Yicheng Zhang contributed with suggestions and edited the manuscript draft.

Another part of the work in Chapter 3 is published as G. G. N. Sandamali, R. Su,

and Y. Zhang, “Short-term En-route Air Traffic Flow Management Under Departure and Wind Uncertainties with a Heuristic and Greedy Solution Approach”, 2019 American Control Conference (ACC), Philadelphia, USA, 2019, pp. 2121-2126.

- I reviewed the literature, developed the idea, prepared the manuscript draft, conducted the simulation experiments, and presented the paper.
- A/Prof. Su Rong assisted in developing the model, provided valuable feedback, and revised the manuscript.
- Yicheng Zhang revised the manuscript and provided feedback on the model.

The Chapter 4 is under review as G. G. N. Sandamali, R. Su, K. L. K. Sudheera, and Y. Zhang, “A Safety-Aware Real-time Air Traffic Flow Management Model Under Demand and Capacity Uncertainties,” IEEE Transactions on Intelligent Transportation Systems, 2019.

- I reviewed the literature, developed the idea, formulated the models and solution mechanisms, prepared the manuscript draft, and conducted the simulation experiments.
- A/Prof. Su Rong supervised the work, provided valuable comments and suggestions, helped in removing assumptions, and revised the manuscript and the response letters.
- K. L. K. Sudheera contributed in developing the model and helped in experiments.
- Yicheng Zhang provided valuable feedback and revised the manuscript.

The major part of Chapter 5 is under review as G. G. N. Sandamali, R. Su, K. L. K. Sudheera, Y. Zhang and Y. Zhang “Two-stage Scalable Air Traffic Flow Management Model Under Uncertainty,” IEEE Transactions on Intelligent Transportation Systems.

The contributions of the co-authors are as follows:

- I extended the idea from the previous Journal paper to enhance the scalability, developed the mathematical model, prepared the manuscript draft, and conducted the simulation experiments.
- A/Prof. Su Rong provided feedback, supervised the work, and revised the

manuscript.

- K. L. K. Sudheera contributed with suggestions, helped in simulations and revised the manuscript.
- Yicheng Zhang helped in developing the flow-based model.
- Yi Zhang provided valuable feedback on the formulation and revised the manuscript.

Another part of the work in Chapter 5 is under review as G. G. N. Sandamali, R. Su, and K. L. K. Sudheera, “Flow-based Flight Routing and Scheduling under Uncertainty”, 21st IFAC World Congress.

- I reviewed the literature, developed the idea, prepared the manuscript draft, and conducted the simulation experiments.
- A/Prof. Su Rong provided valuable comments and feedback.
- K. L. K. Sudheera helped in developing the model and revised the manuscript.

08-01-20

Date

GAMMANA GURUGE NADEESHA
SANDAMALI

Acknowledgement

I would like to sincerely thank my supervisor, Prof. Su Rong, for his aspiring guidance and support throughout my PhD study. I'm deeply honored for selecting me as a PhD student under his supervision. I must specially thank him for continuously monitoring the progress and being very supportive with invaluable constructive ideas and helpful comments. I am also sincerely grateful to Prof. Jan H. van Schuppen, Prof. Peter B. Luh, Prof. Christos Cassandras, and Prof. Rahul Mangharam for their helpful comments and advice during their visits to Nanyang Technological University (NTU).

I would also like to sincerely thank the Director, Prof. Vu N. Duong and the Deputy Director, Mr. Mohamed Faisal Bin Mohamed Salleh of Air Traffic Management Research Institute (ATMRI) for the financial and technical support during my PhD study. I would like to thank all the staff and students in ATMRI for their friendship and cooperation. I must also thank Mr. Jean Frank of the AirTOP support team for continuously answering and helping for inquiries related to AirTOP simulations.

I also really appreciate and thank Dr. Yicheng Zhang for his enormous support, valuable advice, and constructive ideas, especially in the first two years of my PhD study. It helped to have a clear vision on the new research domain. I am also thankful to the members of our research group, especially to Dr. Li Qing, Ms. Zhang Yi, and Ms. Zhu Yuting for being supportive and friendly during my studies.

I should also acknowledge and thank the thesis advisory committee, qualifying examination panel and in advance the thesis examiners for their valuable comments and suggestions in improving the quality of the thesis. Recognition should also go to NTU Research Scholarship for the financial support.

A massive thank should go to my lovely husband, Kushan, for always being there

ACKNOWLEDGEMENT

for me as a major support pillar, by encouraging and keeping me up during this time. Also, I should thank my parents and siblings. Even though we live miles away, they wholeheartedly supported throughout this journey.

Finally, I would like to express my sincere gratitude towards each individual who has directly or indirectly helped me in reaching my goals.

Thank you.

Abstract

Inherent uncertainties of the air transportation system can induce unexpected anomalies in its operations such as deviations in flight schedules, sudden imbalances of demands and capacities, higher workload for Air Traffic Controllers (ATCos), etc. Current Air Traffic Flow Management (ATFM) models rarely consider uncertainty in their algorithms, and generally focus on minimizing the flight delays under deterministic constraints. Thus, to bridge this gap, the thesis proposes several frameworks for en-route ATFM, while scrutinizing different uncertainty components present in air transportation system. Generally, uncertainty exists in two major forms, i.e., capacity uncertainty and demand uncertainty, which result in the stochastic behavior of the Air Traffic Network (ATN).

Demand uncertainty is one critical form, which has an adverse effect on ATFM. It is mainly due to the deviation in departure time and aircraft speed from their scheduled values, and can further lead to an unexpected arrival of aircraft at certain routes at unscheduled times, causing unanticipated demands for those routes. The uncertainty of demand imposes several difficulties in air transportation systems, including higher workloads for air traffic controllers, higher delays, travel costs, as well as safety risks. Thus, in the initial part of the thesis, we propose a preliminary flight routing and scheduling scheme, while considering both departure and speed uncertainties present in ATN. Following robust optimization, we ensure that capacity violations are eliminated from the system while maintaining the required in-trail separation between aircraft even under the uncertainty of demand.

Similar to the demand, the uncertainty of capacity is also another critical form of uncertainty present in air transportation system, and it is primarily due to unexpected weather situations, emergency conditions, controller workload variations, etc. Thus, in this thesis, we further propose a framework for en-route ATFM, while scrutinizing uncertainties in en-route capacity and demand and their imbalance, via a chance constraint-based probabilistic approach. The proposed framework

plays a key role in ensuring the safety of the overall air transportation system in terms of maintaining the safety separation between flights and constraining the capacity of the sectors as well. Moreover, a flight level assignment and a speed optimization scheme are proposed based on the Base of Aircraft Data (BADA) of the European Organization for the Safety of Air Navigation (EUROCONTROL) with the objective of minimizing the fuel consumption. The model further minimizes the overall expected delay of the network using the control actions of ground holding, speed control, rerouting, and flight cancellations. At the implementation stage, two phases of ATFM, i.e., the pre-tactical phase and tactical phase, are considered, in which the former focuses on generating optimal trajectories, and the latter focuses on real-time updates of flight plans.

However, due to the flight-by-flight structure, the above models scale poorly for large-scale systems. To overcome this drawback, especially anticipating the high volume of air traffic in the future, we finally present a two-stage architecture, in which the first stage scrutinizes the behavior of a set of flights as a flow, while the second stage decomposes them into individual flight plans. Due to the flow-based modeling structure at the first stage, it enhances the scalability while taking into account constraints of the link, sector and airport capacities and also the waypoint flow rate. The second stage ensures that the flights are safely separated with the required in-trail separation, thus, enhances the safety of the overall system.

Acknowledging the importance of computational complexity along with the optimality, several solution approaches are proposed for above optimization problems, which are primarily base on the concept of maximum independent set, in which the overall problem is decomposed into a set of subproblems based on their independence, and further optimized parallelly resembling a greedy approach, yet with a tighter closeness to the global optimality. Furthermore, each ATFM framework is evaluated in terms of flight delays and possible conflicts with simplified and realistic case studies.

Table of Contents

Acknowledgement	i
Abstract	iii
List of Figures	x
List of Tables	xi
List of Algorithms	xiii
List of Abbreviations	xv
1 Introduction	1
1.1 Background	2
1.1.1 Air Traffic Flow Management	2
1.1.2 Uncertainty Types in air transportation system	4
1.1.3 Layout of the Air Traffic Network (ATN)	7
1.2 Motivation	8
1.3 Objectives and Contributions	11
1.3.1 ATFM under Demand Uncertainty	11
1.3.2 ATFM under Demand and Capacity Uncertainty	11
1.3.3 Two-stage Scalable ATFM Model under Uncertainty	12
1.3.4 Minimization of Computational Complexity	12
1.4 Outline of the Thesis	13
2 Literature Review	15
2.1 Formulation Structure	15
2.2 Deterministic ATFM Models	18
2.3 Consideration of Uncertainty	20
2.4 Computational Performance and Scalability	23

2.5	Comparison and Summary	25
3	Flight Routing and Scheduling Under Demand Uncertainty	27
3.1	Introduction	27
3.2	ATFM System Model	28
3.2.1	General Overview of Air Traffic Network	28
3.2.2	Convolution of Departure time and Speed Uncertainties . . .	31
3.3	Flight Routing and Scheduling Problem Formulation	36
3.3.1	Notations	36
3.3.2	Decision Variables	37
3.3.3	Constraints	37
3.3.4	Linearizing the Constraints	45
3.3.5	Objective Function	46
3.4	A Greedy Strategy with Parallel Computation and MISs	47
3.5	Results and Discussions	50
3.5.1	Case Study I : Simulation of a Simplified ATN	50
3.5.2	Case Study II : Simulation of ASEAN Air Traffic Network .	58
3.6	Summary	60
4	Two-phase ATFM under Demand and Capacity Uncertainties	61
4.1	Introduction	61
4.2	ATFM model	62
4.2.1	Notations	62
4.2.2	Constraints	64
4.2.3	Objective Function	80
4.3	Short Tactical Scheduling and Solution Mechanism	81
4.3.1	Short Tactical Scheduling Procedure	81
4.3.2	Solution Mechanism : Preprocessing followed by Greedy Strategy with Maximum Weighted Independent Sets (MWIS)	84
4.4	Simulation Results	86
4.4.1	Simulation Study I: Experiment with a Simplified ATN . . .	86
4.4.2	Simulation study II: Experiment with the ASEAN ATN . . .	86
4.4.3	Experimental Results of Simulation Studies I and II	87
4.5	Summary	94
5	Two-stage Scalable ATFM Model Under Uncertainty	99
5.1	Introduction	99
5.2	Stage I Formulation:- Flow-based ATFM	100

TABLE OF CONTENTS

5.2.1	Notations	100
5.2.2	Constraints	101
5.2.3	Stage I - Objective Function	109
5.2.4	Solution Mechanism : Greedy Strategy with MWIS	111
5.3	Stage II Formulation:- Flight-by-flight-based ATFM	112
5.3.1	Notations	112
5.3.2	Flights decomposition	112
5.3.3	Constraints	114
5.3.4	Stage II - Objective function	118
5.4	Experimental Results	119
5.4.1	Case Study I (CS I) - Grid Network	119
5.4.2	Case Study II (CS II) - ASEAN Network	123
5.4.3	Case Study III - ASEAN Network Simulation on AirTOp	125
5.5	Summary	128
6	Conclusions and Future Work	129
6.1	Conclusions	129
6.2	Future Work	131
	Author's Publications	135
	Bibliography	137

List of Figures

1.1	ATM planning and ATFM phases [1]	3
1.2	Representation of uncertainties of weather and controller workload.	5
1.3	Uncertainty of departure time of flights from Changi airport, Singapore on 31st of August 2018(5-11pm) [2] (several extreme outliers are removed)	6
1.4	Speed (yellow) and altitude (blue) traces of a flight from Ninoy Aquino Airport, Manila to John F. Kennedy Airport, New York on 6th October 2019 taken from FlightRadar24 [2]	6
1.5	En-route Air Traffic Network (extracted from, https://skyvector.com/ [3])	8
3.1	Simplified air traffic network	29
3.2	Illustrative distributions of departure time and speed uncertainties	31
3.3	Departure time uncertainty distribution	32
3.4	An illustration of the uncertainties of departure time and speed variation	32
3.5	Generation of arrival time distributions and time bounds	34
3.6	Possible scenarios of flight entering a link	39
3.7	Time axis representation of flight overtake	42
3.8	Illustration of a bottleneck link	49
3.9	The simulated simplified version of the ASEAN map	51
3.10	Capacity violation and latency profiles of Model III against considered uncertainty.	55
3.11	Average delay profiles of fixed (II) and free (III) routing.	56
3.12	The simulated version of the ASEAN map	59
4.1	Speed Distribution in two segments [4]	66
4.2	Three possible cases of merging at waypoint w	69
4.3	An illustration of a flight overtake scenario	74

4.4	An example of a PTF file for the Airbus A306 aircraft [5]	76
4.5	Representation of aircraft weight, cruise fuel consumption and travel time in a link	78
4.6	Reference figures to describe the cruising and climbing stages	79
4.7	Representation of aircraft weight, climbing height and rate of climb	79
4.8	Time axis representation of flights in two scheduling periods	83
4.9	Simplified ATN for the simulation study I	87
4.10	Demand and capacity of sectors with respect to time in simulation study I and II.	89
4.11	In-trail separation violations and latency profiles against α	90
4.12	Piecewise approximation for the cumulative distribution function. .	95
5.1	Simplified ATN model	101
5.2	Departure coefficient envelop.	110
5.3	Simulated grid network - Case Study I	120
5.4	Radar controller workload variation (Legend: Pink - Total work- load, Purple - Monitoring workload , Blue - Radio telephony work- load , Green - Conflict resolution workload, Red - Level clearance workload)	127
6.1	Encapsulation of different features into a single ATFM framework. .	131

List of Tables

2.1	Summary of selected literature (Note that the solution time are not comparable in these models due to their different computational environments)	16
3.1	Features of the three models	53
3.2	Number of capacity violations for three models	54
3.3	Experimental results for three models	55
3.4	Network parameter values and results of Model III (Proposed)	57
3.5	Normal and double traffic solution time of ASEAN network	59
4.1	Table of Notations	63
4.2	DPDs of the three sectors in the ASEAN map	88
4.3	Pre-tactical and tactical phase performance matrices	91
4.4	Solution time performances	92
4.5	Comparative Cost calculations (in USD)	92
5.1	CS I - Calculations of violations and delays against chance constraint probability	121
5.2	CS I - Comparison of MIS approach with ILP results	122
5.3	CS II - Comparison between ILP and MIS solutions	124
5.4	CS II - Normal, double and triple traffic results	124
5.5	S	125
5.6	Results comparison in AirTOp simulation	128

List of Algorithms

1	Greedy approach - MIS and parallel computation	47
2	Generation of dependency matrix	48
3	Greedy approach with MWIS	85
4	Flow decomposition into flights	113

List of Abbreviations

ADP	ATFM Daily Plan
AO	Airline Operator
ASEAN	Association of Southeast Asian Nations
ATC	Air Traffic Control
ATCo	Air Traffic Controller
ATFM	Air Traffic Flow Management
ATM	Air Traffic Management
ATFMP	Air Traffic Flow Management Problem
ATFMRP	Air Traffic Flow Management Rerouting Problem
ATN	Air Traffic Network
ATS	Air Traffic Service
BADA	Base of Aircraft Data
CAGR	Compound Annual Growth Rate
CDF	Cumulative Distribution Function
CDRP	Conflict Detection and Resolution Problem
CS	Case Study
DEM	Deterministic Equivalent Model
DPD	Discrete Probability Distribution
ETMS	Enhanced Traffic Management System

LIST OF ALGORITHMS

LIST OF ALGORITHMS

FAA	Federal Aviation Administration
FIR	Flight Information Region
FL	Flight Level
FRA	Free Route Airspace
IATA	International Air Transport Association
ICAO	International Civil Aviation Organization
IFR	Instrument Flight Rule
ILP	Integer Linear Programming
IP	Integer Programming
IQP	Integer Quadratic Programming
KK	Kota Kinabalu
KL	Kuala Lumpur
LP	Linear Programming
MAGHP	Multi-Airport Ground-Holding Problem
MILP	Mixed Integer Linear Programming
MIQP	Mixed Integer Quadratic Programming
MIS	Maximum Independent Set
MWIS	Maximum Weighted Independent Set
NAS	National Airspace System
OD	Origin-Destination
PBD	Poisson Binomial Distribution
PDF	Probability Density Function
PTF	Performance Table File
QP	Quadratic Programming
ROC	Rate Of Climb

LIST OF ALGORITHMS

LIST OF ALGORITHMS

ROD	Rate Of Descend
RMP	Restricted Master Problem
RVSM	Reduced Vertical Separation Minima
SAGHP	Single-Airport Ground-Holding Problem
SESAR	Single European Sky Airspace Research
SIN	Singapore
TAS	True Air Speed
TMI	Traffic Management Initiative
USA	United States of America
VFR	Visual Flight Rule

Chapter 1

Introduction

With the advancement in modern transportation technologies, the number of air travellers grows rapidly. The International Air Transport Association (IATA) has predicted that the passenger count will nearly be doubled to 8.2 billion by 2037 based on a 3.5% average Compound Annual Growth Rate (CAGR) [6]. Further, IATA revealed that the biggest growth of demand would be expected from the Asia-Pacific region over the next 20 years, with more than half the total number of new air travellers.

With the above commonly agreed projections on air travel, safety plays a crucial role in air transportation systems due to its calamitous effects on human lives. Yet, there has been a considerable amount of safety-related accidents and incidents in the history of air travel. According to the International Civil Aviation Organization (ICAO) harmonized safety analysis, 10% of accidents were en-route accidents, and a majority of 79% of accidents involved passenger flights out of the accidents recorded in 2013 [7]. In July-2002, Bashkirian Airlines Tupolev TU-154 collided with Boeing 757 cargo airplane and it killed 71 people [8]. The airspace was controlled from Zurich by only one Air Traffic Controller (ATCo) and the ATCo failed to recognize the problem in time, which resulted in a violation of safety distance and finally a mid-air collision. In general, the lack of separation, miscommunication between ATCos and pilots, higher demand of traffic, deviation from the original flight plan due to uncertainties are considered as the main causes for the safety risks.

With the soaring air travel growth, it is important to provide an efficient and safe journey for passengers, while generating financial profits for the aviation industry

and minimizing environmental footprints. In order to achieve these expectations, a proper Air Traffic Flow Management (ATFM) process is indeed required.

1.1 Background

1.1.1 Air Traffic Flow Management

Air Traffic Flow Management is a service established to optimize traffic flows, with the objectives of managing a safe, orderly and expeditious flow of air traffic, while utilizing ATCo capacity. The detailed objectives of ATFM can be listed as [9],

- Effective management of demand and capacity with optimal use of airspace.
- Enhancement of safety with reduced workload, while utilizing ATCo capacities.
- Minimization of all cost related functions such as delay, non-optimum flight level, fuel consumption, etc.

ATFM consists of three phases, i.e., strategic, pre-tactical and tactical phases, respectively as shown in Fig. 1.1 [10]. The strategic phase involves with planning, research and coordination activities, and evaluations of demands and capacities, which are typically carried out two to six months in advance. Depending on the updated demand and the revised capacity of the system, an ATFM Daily Plan (ADP) is prepared the day before the actual flight takes off. A pre-tactical plan is a fine-tuned strategic plan. Although the ADP is being prepared the day before the actual date of flying, the planning and coordination activities are carried out six days before the actual flight in order to manage the available capacity in the best possible way. Tactical phase decisions are taken on the day of actual flight operations while considering the actual traffic and system capacities. Depending on the current capacity and the demand, the traffic management decisions and control actions are applied in order to minimize the delay as well as the cost of transportation. Depending on the actual conditions, the implemented regulations will be updated, which, in some occasions, may include flight cancellation. The roles and functionalities at each phase are illustrated in Fig. 1.1 in detail.

With the rapid growth of travel demands, ATFM services need to be more effective and efficient, especially considering that the capacity of the airspace is considerably saturated. Thus, the importance of ATFM can never be under-estimated. Moreover, ATFM is essential in achieving optimal schedules of traffic flows. With

Figure 1.1: ATM planning and ATFM phases [1]

the unpredictability of the en-route and airport capacities, it is more challenging to optimally handle the traffic flows, while satisfying ATFM constraints.

The following control actions are generally used in ATFM to optimize the traffic flows in order to match the demands with capacities.

- **Ground holding:** The most well-known and economically viable control action is the use of ground holding. In this case an aircraft will wait on the ground by absorbing expected delays in the air until it gets clearance to takeoff. This will help to minimize the possible airborne delay.
- **Airborne speed control:** Airborne speed can be controlled to impose an airborne flight delay. This may lead the flights to fly in either higher or lower speed than the optimal speed. More critically, if an airborne delay is imposed, it will lead to extra fuel consumption.
- **Rerouting:** Generally, the ATCos assign the optimal/shortest path for each aircraft. But, there can be situations, where the optimal path is congested or not available. In such cases, ATCos tend to allocate several alternative paths (or routes) to ensure an on-time arrival. Several possible situations, where airlines may have to deviate from the original path are listed below,
 - In situations, where a segment/part of the original flight plan is either not available or blocked, for example due to poor weather conditions, airlines need to use an alternative path to reach the destination.
 - There may be some situations, where certain flights have to experience a considerable ground delay to reach the destination in the original path due to the congestion along the path. But if there is any other alternative path to be used (with enough capacity), then the airline may tend to use a rerouting path to maintain on-time arrival.
- **Flight cancellation:** If a flight cannot be routed in the airspace within its permissible bounds of ground or airborne delay, the flight will be canceled.

1.1.2 Uncertainty Types in air transportation system

The quality of the flow management process is significantly impacted by unanticipated factors and events. Basically, there are two types of uncertainties existing in an air transportation systems, i.e., capacity uncertainty and demand uncertainty, that cause the stochastic behavior of air transportation system [11]. Capacity un-

CH. 1. INTRODUCTION

1.1. BACKGROUND

certainty is the unpredictability of capacities of sectors, links, airports and runways mainly due to weather fluctuations, controller workload variations, emergency situations, etc.

Figure 1.2: Representation of uncertainties of weather and controller workload.

A weather prediction graph obtained from the FlightAware tool [12] is shown in Fig. 1.2a which represents the possibilities for different weather conditions such as rain, snow, storm in the US airspace with a 6-hour prediction horizon. It indicates the chances of adverse weather situations, which directly influence the capacity of the airspace. The graph in Fig. 1.2b is from an experiment performed by the Federal Aviation Administration (FAA) [13], on controller workload using full-performance levels of controller volunteers. It illustrates the increment of controller workloads with respect to the increment of air traffic and, further, the workload ratings among different teams are varying due to the different capabilities of each controller team. Depending on the air traffic controllers' efficiencies, the handling capacities may vary significantly, as shown in Fig. 1.2b. This variation of airspace capacity due to weather fluctuations and variations of controller workloads can make air transportation system unreliable and unsafe, as reflected in flight delays, more stresses and workloads on ATCos, more rerouting decisions, etc.. Thus, an ATFM process needs to consider the uncertainty of capacity for better utilization of the airspace to manage aircraft safely in such unforeseen situations.

The uncertainty of (air space) demands is mainly caused by the deviation of aircraft departure time from its scheduled time and the deviation of speed from its optimal aircraft speed. Therefore, demand uncertainty eventually results in a time deviation between the actual time and the scheduled time. The bar graph in Fig.

1.3 is plotted using the deviation of the actual take-off time from the scheduled time for flights departing from Changi airport, Singapore for a duration of 6 hours. It clearly shows the dispersion of the actual departure times from the scheduled departure times, which also causes subsequent en-route delays and unexpected arrivals to sectors/air routes. Similarly, en-route flight times can also be deviated from expected times due to the wind effect on flight speed. The graph in Fig. 1.4 highlights the variation of the speed of a flight from Ninoy Aquino Airport, Manila to John F. Kennedy Airport, New York on 6th October 2019. The cruising speed of the flight has drastically changed with lots of ups and downs, even at the same altitude levels.

Figure 1.3: Uncertainty of departure time of flights from Changi airport, Singapore on 31st of August 2018(5-11pm) [2] (several extreme outliers are removed)

Figure 1.4: Speed (yellow) and altitude (blue) traces of a flight from Ninoy Aquino Airport, Manila to John F. Kennedy Airport, New York on 6th October 2019 taken from FlightRadar24 [2]

Both departure and speed deviations can incur unexpected en-route traffic in several sectors/links creating an imbalance between capacity and demand. Due

to this reason, ATCos may have to handle additional workloads than expected within their areas of control, which can also cause high safety risks. The way air transportation system handles or reacts to uncertainty depends on the underlying system model and regulations. However, general practice is to make fine adjustments based on the developing uncertainty events such as bad weather conditions at an operational level. These adjustments maybe enough to make the system going under the current air traffic volume. However, with the future predicted air traffic volume, this approach might not be feasible. Moreover, these adjustments can, and generally will, cause for non-optimal performances. Hence, it is imperatively important to anticipate the effects of inherent uncertainties at the scheduling phase in a more proactive manner for an effective and safe flow management process.

1.1.3 Layout of the Air Traffic Network (ATN)

The air traffic network mainly consists of airports, links, control points and boundaries as depicted in Fig. 1.5. Waypoints are the fixed reference points in the airspace to indicate the location of the aircraft for the radar navigation. Two adjacent waypoints are connected by a directional air route/link. With a safe vertical separation, the air link can be considered as a bidirectional route, allowing flights to travel in both directions. The whole airspace is divided into Flight Information Regions (FIR), and they are governed by the respective controlling authorities to ensure that the air traffic services are adequately provided within their FIRs. Furthermore, FIRs are divided into sectors to control the traffic flows and to guide the aircraft safely.

The flight path is a directional track between its origin and destination starting from the departure fix, following a set of waypoints, links and finally reaching the arrival fix and then, the arrival airport. The flight path is constrained by capacity limitations and several other restrictions, which produces the final flight plan.

The concept of Flight Level (FL) divides the airspace into a set of vertical layers with a safe separation of altitude to fully utilize the existing en-route capacity. A program called Reduced Vertical Separation Minima (RVSM) was initiated by ICAO to reduce the standard vertical separation from 2000ft to 1000ft, which doubled the capacity of the network above FL290 [14]. Other than the vertical separation imposed by the flight levels, aircraft need to keep a safe separation in lateral and longitudinal directions to prevent collisions either by distance or time

Figure 1.5: En-route Air Traffic Network (extracted from, <https://skyvector.com/> [3])

or visually. In the Visual Flight Rule (VFR) operation, the flight crew will use a "see and avoid" mechanism to keep the safe separation [15]. The Instrument Flight Rule (IFR) operation can be radar or non-radar separation depending on the position of the aircraft, i.e., above land with radar coverage or above oceans or above undeveloped land masses with no radar coverage.

1.2 Motivation

Many researchers have proposed different frameworks for air traffic flow management in different perspectives. Each has pros and cons in terms of scalability, applicability, safety, computational complexities, accuracy, etc. The main incentive of this work is to propose a comprehensive ATFM framework, which is responsive to real-time network conditions, enhances the safety¹, addresses detailed flight attributes, lower in complexity, scalable (catering for the future traffic growth), and more importantly, makes it closer to the ground truth. In order to develop that platform, we address both pre-tactical and tactical phases of ATFM. The term 'Safety' refers to the number of in-trail separation violations and sector capacity violations in the context of this thesis (lesser the number, safer the ATN)

¹T

From a formulation point of view, the existing literature follows either a flight-by-flight or flow-based modelling framework. The flight-by-flight framework considers individual flights; thus, they provide detailed flight plan information. In contrast, flow-based models consider the behavior of a group of aircraft as a flow relinquishing the descriptive flight information, but they are more efficient in large-scale systems due to the involvement of fewer decision variables than that of a flight-by-flight framework. With these pros and cons, this thesis first proposes a framework for flight routing and scheduling based on the flight-by-flight concept, while providing descriptive information on flights. To enhance the scalability, later, this thesis proposes a two-stage framework, in which the first stage scrutinizes the behavior of a set of flights as a flow, while the second stage decomposes them into individual flight plans. Due to the two-stage architecture, the proposed model is not computationally weak in solving large-scale systems, and further, it can also generate descriptive flight plans.

A time-based optimization model can be either continuous time or discrete time, depending on their formulation structure and the application. In discrete time models, the planning horizon is partitioned into small time intervals, and they provide decisions at one point, say the start of the time slots. Thus, it may cause errors depending on the size of the discrete time slot allowing capacity violations, unutilized airspace, delays, etc. In contrast, continuous time models provide exact time and location information, thus, more accurate. In this thesis, we first start with a continuous time model with a flight-by-flight formulation structure, while taking flights as the reference points. But, when the model is developed into a flow-based structure, we consider discrete time formulation. Especially, in our last framework with two-stage architecture, we first take discrete time intervals to generate flow results, and then, use continuous time formulation, when the flows are decomposed into individual flights, for the sake of providing more accurate flight information.

Safety is crucial in air transportation since it directly affects human lives. Thus, maintaining a safe separation is very important in ATFM to minimize possible flight conflicts. Both miles-in-trail and minutes-in-trail separation guarantee the head-and-tail distance between two consecutive flights [16]. In literature, only a few have considered in-trail separation in their models [17], [18] while others allow it to handle by the ATCos, which in turn incurs a high workload/stress on ATCos. In our frameworks, we ensure that flights keep minutes-in-trail safety separation throughout the flight time. Similar to the separation requirement, it

is also important to consider that the aircraft are well packed in en-route sectors without exceeding their handling capabilities. Thus, we also constrain the sector capacity in our later frameworks, while enhancing the overall safety of ATN. It eventually minimizes conflicts and violations as well as controller workloads.

Each air traffic network always experiences inherent uncertainties, which generate unexpected anomalies such as huge flight delays, cancellations, conflicts, violations, stress and higher workload for ATCos, etc. Thus, deterministic models cannot ensure a safe and trustworthy trip for passengers since they consider an uncertainty free ATN, which is unrealistic. At a operational level, air transportation system reacts to uncertainty in a reactive manner by adjusting the flight plans according to uncertain events. In the literature, several models consider uncertainty, yet, limit to a specific form as either demand or capacity. Alternatively, we consider both demand and capacity uncertainties cooperatively in flight routing and scheduling to minimize possible unexpected situations and to enhance the safety of passengers. Hence, we formulate models with the consideration of uncertainties, while minimizing the gap with the reality. These frameworks can ensure that the constraints are satisfied with a certain level of confidence, even under uncertainty conditions.

Generally, the predictions of uncertain variables are less accurate when the prediction horizon is long. Therefore, it is important to recurrently update the flight plans based on more accurate predictions through a real-time ATFM process. Pertaining to that, we consider two phases as pretactical and tactical, in which the former focuses on generating optimal trajectories, while the latter focuses on real-time updates of flight plans. The results generated from the pre-tactical phase is served as a reference for the tactical phase operation. In addition, at the tactical phase, real-time traffic demands and actual network conditions are taken into account when the flight plans are fine tuned for better results.

The major computational barrier in a flight-by-flight formulation is the involvement of a very large number of decision variables since they consider each flight as an individual entity. Thus, with a complex formulation structure, this can be even a serious challenge. Moreover, when there are logical condition based constraints, integer relaxation methodologies cannot be directly applied to release the auxiliary Boolean variables introduced in mixed-logic constraints. Therefore, we decompose the overall problem into several independent sets using Maximum Independent Sets (MISs), and then the sub problems are solved in an iterative and greedy manner.

1.3 Objectives and Contributions

This thesis primarily focuses on four main aspects of ATFM, which helps to enhance the flight routing and scheduling process.

1.3.1 ATFM under Demand Uncertainty

As depicted in the motivation, demand uncertainty is one critical form of uncertainty present in air transportation system. Thus, in the first technical part of the thesis, we introduce a novel pre-tactical phase flight routing and scheduling model, while scrutinizing the effect of demand uncertainty.

As a preliminary work, we only consider the effect of departure time uncertainty, which is one main component of demand uncertainty, and follow a robust optimization approach. Later, we modify the model by considering the demand uncertainty due to both departure time and aircraft speed deviation in details. The ATFM problem is formulated as a Mixed Integer Quadratic Programming (MIQP) problem with the objective of minimizing expected total delay of the system while maintaining required in-trail separation between aircraft even under demand uncertainty. In this framework, we use a probability distribution based approach to tackle the demand uncertainty effect in flight scheduling. Further, following robust optimization, we ensure that the link capacity violations (in terms of separation violations) are eliminated from the system. Additionally, we use an optimal flight level assignment method and speed assignment strategy to minimize the system delay and to fully utilize the system capacity. This framework is structured based on the flight-by-flight concept in continuous time, thus, it generates detailed flight plans accurately.

1.3.2 ATFM under Demand and Capacity Uncertainty

In this part of the work, we build on previous models and consider both demand and capacity uncertainties in the flight scheduling process. In terms of formulations, a Mixed Integer Linear Programming (MILP) model is developed including both sector capacity and in-trail separation constraints, which is vital for air transportation safety.

Moreover, the flight level assignment and speed optimization schemes are further developed, based on the Base of Aircraft Data (BADA) of the European Organization for the Safety of Air Navigation (commonly known as EUROCONTROL)

with the objective of minimizing the fuel consumption. Furthermore, a two-phase framework is presented, including the pre-tactical and tactical phases of ATFM, in which, the pre-tactical phase aims for finding the BADA based fuel optimal and feasible trajectories, whereas, the tactical phase fine-tunes the flight times on top of the provided trajectories. More importantly, via a real-time flight scheduling scheme which incorporates real-time updates, the adverse effects incurred from the incorporation of inherent uncertainties on the flight delays, are minimized.

1.3.3 Two-stage Scalable ATFM Model under Uncertainty

In this work, our main intention is to overcome the major drawback of the above frameworks in pre-tactical phase, which is the scalability issue due to the flight-by-flight-based formulation structure. Hence, we start to move into a flow-based formulation structure, but still leveraging the pros of a flight-by-flight model. For that, we develop a comprehensive ATFM model with a two-stage architecture involving both flow and flight-by-flight concepts. The flow-based modeling structure at stage I enhances the scalability, while constraining the link, sector and airport capacities and also the waypoint flow rates at a macroscopic level. The stage II model ensures that the flights are safely separated with the required in-trail separation at a microscopic level, thus, this model enhances the safety of the overall system. Further, the stage I formulation scrutinizes the flow and capacity uncertainty and the stage II formulation scrutinizes the demand uncertainty of flights. The stage I and II problems are formulated as an Integer Linear Programming (ILP) model and an MILP model, respectively.

1.3.4 Minimization of Computational Complexity

Due to the comprehensive formulation structures and the associated computational complexity in flight-by-flight structure, the overall problem cannot be directly solved in a centralized way by using optimization solvers such as Gurobi [19], CPLEX [20]. Thus, we implement a greedy strategy [21] by decomposing the problem into a set of MISs and then solve in an iterative manner. We further reduce the solution time by using a preprocessing mechanism, which essentially removes all unnecessary variables and shrinks the feasibility region by tightly setting the bounds. This approach can generate results with a reasonable optimality gap in a limited time, thus, the proposed frameworks are applicable in real-time.

1.4 Outline of the Thesis

The rest of the thesis is structured as follows.

Chapter 2: Literature Review

In this chapter, we survey and review existing related studies. Moreover, we expect to differentiate state-of-the-art studies in respective domains and identify the prevailing research gaps in them.

Chapter 3: Flight Routing and Scheduling Under Departure and En-Route Speed Uncertainty

This chapter proposes a framework for the pre-tactical phase of ATFM, mainly addressing the effects of departure time and en-route speed uncertainties, which lead to demand uncertainty in flow management. The framework aims to maintain the safety separation throughout the flight time, while ensuring the safety of ATN. The solution mechanism of MISs is also introduced in this chapter.

Chapter 4: A Real-Time ATFM Model Under Demand and Capacity Uncertainties

This chapter focuses on the scalability of the ATFM model and considers both aforementioned uncertainty types in a two-phase architecture, including pre-tactical and tactical phases.

Chapter 5: Two-stage Scalable ATFM Model Under Uncertainty

This chapter proposes an ATFM model in pre-tactical phase, while considering both aforementioned uncertainty types with a two-stage architecture. The stage I model proposes a flow-based model, while stage II decompose it into a flight-by-flight level.

Chapter 6: Conclusion and Future works

We conclude the thesis in this chapter by summarizing the key contributions and drawing possible future directives.

Chapter 2

Literature Review

In this chapter, we make an extensive review of the existing literature in ATFM. The literature review is classified as follows: 1) Formulation structure - depict different mathematical formulation methods used to model the flow management problem with their pros and cons, 2) Deterministic models - describe ATFM models which considered uncertainty free ATN, 3) Uncertainty models - discuss stochastic models with their consideration of different uncertainty types, 4) Computational performance and scalability - describe the solution methods and the applicability of the frameworks, 5) Comparison - analyze the state-of-the-art models comparatively with our work and summarized in Table 2.1. Note that, only complete en-route ATFM models that are comparable to our model, specifically optimization-based models for flight scheduling, are included in Table 2.1.

A detailed survey about the existing flow management optimization models can be found in [29], covering the main domains of Single-Airport Ground-Holding Problem (SAGHP), Multi-Airport Ground-Holding Problem (MAGHP), Air Traffic Flow Management Problem (ATFMP), Air Traffic Flow Management Rerouting Problem (ATFMRP), and ATFMRP with uncertainty.

2.1 Formulation Structure

In terms of modeling, the existing literature consists of Lagrangian models, aggregate models, and Eulerian models [30]. Lagrangian models are structured with the flight-by-flight concept [18], [22], [23], [31], [32], [24], [25], in which, they generate individual flight profiles. Aggregate and Eulerian models use the flow-based modeling structure, in which, they generate the behavior of a group of aircraft

Table 2.1: Summary of selected literature (Note that the solution time are not comparable in these models due to their different computational environments)

Ref. No.	Model type	Time	Uncertainty	Control actions	Number of flights; airports; sec-tors/links*	Run time; Horizon	Solution strategy	Fuel	ET
[17]	f-b-f	Con	No	G,A,R,S	48;n/a;192*	<1h;n/a	Sequential strategies	✓	x
[22]	f-b-f	Dis	No	G,A	1002;18;305	>8h;8h	LP relaxations	x	x
[23]	f-b-f	Dis	No	G,A,R	6745;30;145	10m;8h	CPLEX Branch-and-bound	x	x
[24]	f-b-f	Dis	Capacity, Demand	G,A,R,S,C	425;45;40(27 Sc)	13m;32*	LP relaxations, CPLEX branch-and-cut	x	x
[25]	f-b-f	Dis	No	G,A,R,S,C	6475;30;144	6.5m;12h	LP relaxations	x	x
[26]	flow	Dis	No	G,A,R,S,L	166 OD pairs;43;484*	2m;4h	Lagrangian relaxation	x	x
[27]	flow	Dis	No	G,A,R,S,L	1164;45;484*	9m;24h	Hierarchical approach	x	x
[28]	f-b-f	Dis	Capacity	G,A,R,C	17500;370;375(5-25 Sc)	5-20m;24h	Column generation	x	x

Notations: f-b-f - flight-by-flight model, Dis - Discrete time, Con - Continuous time, h-hours, m- minutes, n/a - not available, Sc - Scenarios, LP - Linear Programming, MIP - Mixed Integer Programming, Fuel - Fuel optimization, FL - Flight level assignment, MIS - Maximum Independent Sets, G - Ground holding, A - Airborne holding, R - Retrouting, S - Speed control, C - Cancellations, L - landing to a different airport, * - 32 discrete time steps (couldn't find the step duration).

as a flow [30], [33], [34]. Although these models scale well with the number of aircraft, they are unable to provide detailed information about each aircraft. On contrary, Lagrangian models are built on the flight-by-flight basis, which incorporates flight trajectories of individual flights. Hence, Lagrangian models provide more details about individual flights than aggregate and Eulerian models, but in terms of computational performance for large-scale systems, they are generally inferior.

In [22] and [23], they formulate the problem as a 0-1 Integer Programming (IP) model, where the main decision variable is Boolean to indicate whether a flight arrives in a sector at a particular time or not. Their model covers takeoff, cruising and landing phases with the control actions of ground holding, rerouting, airborne holding, and speed control. However, they have only constrained the traversing time within sectors, thus, lack of information on flight trajectories, link capacity constraints, and link traversing times. Furthermore, their formulation structure limits the applicability of their model with those constraints due to the involvement of a very large number of Boolean variables. Many researchers have used the formulation structure of 0-1 IP to explore further directions in ATFM and Air Traffic Control (ATC), which are shown in [32], [24], [25], [35], [36], [37], etc. Most of these works follow the formulation structure used in [23], [31], which is the so-called BLO (Bertsimas, Lulli and Odoni) model.

In [38], authors have proposed a unified approach for airport flow management including runway sequence selection, runway assignment, and flight routing to runways, withing terminal areas, and near-terminal areas. The formulation is mainly based on 0-1 IP structure and proposed as a two-phase approach to overcome the model comprehensiveness by taking out the key bottleneck as a phase 1 problem, which is the runway selection and flight assignment.

Akgunduz et al. [17] propose an en-route flight planning formulation with the objectives of minimizing the cost due to earliness, tardiness, and fuel consumption of flights. Their formulation is based on a flight-by-flight approach in continuous time. Further, they guarantee the minimum separation between aircraft to avoid head-on collision of flights. Although the authors guarantee a conflicts free network due to the violation of minimum separation, still there is a possibility for conflicts due to capacity violations since their method does not constrain the capacity of sectors in their model. The work proposed in [39] is a flight-by-flight optimization model, assuming that there is an input of traffic flow rate at each link with their associated time information. It schedules flights based on the flow input and

generates the sequence of nodes with their timestamps along the flight path.

In contrast, [26], [40], [41], and, [42] have used the basic concept of cell transmission to describe the flow dynamics in details. A multicommodity Eulerian-Lagrangian large-capacity cell transmission model is proposed in [40] for ATFM, using the basic idea of cell transmission models. Zhang et al. [26], [41], [27] have proposed models similar to [40], but with a much richer set of constraints. A comprehensive discrete time formulation is presented in [26], while dividing each link into segments based on the distance an aircraft travels within a discrete time slot. The concept of segmentation has overcome some of the limitations in flow models, but with an increase in the computational complexity due to the partitioning of links into further small segments. In their later work, they have modified this model by minimizing the use of segmentation in [41]. Moreover, the existing models [26], [41] have used incremental strategies to decompose the obtained flow results into individual flights in a sequence manner.

In addition to complete ATFM models, there are several works, which attempt to count the traffic at sectors/centers. An Eulerian model is developed by Bayen et al. [34] to predict the traffic count of the National Airspace System (NAS), which has also validated using the real data of the Enhanced Traffic Management System (ETMS). To predict the traffic count in the centers of the United States, linear time-varying system models are proposed in [30] and [43] as aggregate models. The generated traffic counts then need to be balanced against the system capacity using traffic flow management strategies.

2.2 Deterministic ATFM Models

In the early stages, researchers have mostly assumed uncertainty free ATN, thus their formulations are based on deterministic conditions. In the previous topic under ‘Formulation Structure’, we described some of them, and here, we expect to explore more deterministic models from the existing literature.

Following the formulation structure in [23], Agustin et al. [25] propose a comprehensive mixed 0-1 formulation to solve a deterministic ATFM problem including rerouting, flight cancellations, flight continuation, and the consideration of bounds and penalty for deviations, and aircraft speed. The main difference of their work with the state-of-the-art model in [23] is that their formulation is applied on arcs/links, thus propose a very detailed ATFM model. Furthermore, as

they highlight from different test cases, the Linear Programming (LP) relaxation has given the optimal solution directly even without executing the branch and bound phase or appending additional cuts, due to the tightness of their model. The computational performance of this model is listed in Table 2.1.

A two-level hierarchical architecture is proposed by Dell’Olmo and Lulli in [18] by dividing the problem into two levels as a network model and an airway model. At the initial level, the problem is modeled as an integer multi-commodity network dynamic model to compute the optimal air traffic flows on every arc/links in the network with the objective of minimizing delays and speed deviations. The first level constraints are used to route flights between their Origin-Destination (OD) pairs using a 0-1 formulation structure. The second level represents the single airway and it computes an optimal conflict-free path for each aircraft by safely separating flights.

Following the formulation structure in [31], Churchill et al. [32] propose a strategic level flow management model, including additional features of airspace volumes in place of sectors and a comprehensive formulation structure for delay propagation in comparison to the state-of-the-art model. In their experiments, they have illustrated that the flights are strategically delayed and rerouted, while responding to the airspace capacity changes.

Conflict Detection and Resolution Problem (CDRP) is another major direction to consider in the flow management process. A mixed 0-1 model is developed in [44], [45] for the USA FAA, and they have performed a conflict risk analysis for each pair of aircraft in all sectors. The CDRP mainly attempts to find the best trajectory and flight conditions to avoid conflicts with others, with the help of altitude changes, trajectory changes, speed adjustments, and maintaining separations [46], [47], [48].

Lulli and Odoni propose a deterministic ATFM model for the European airspace in [49], involving ground and airborne holding strategies while assuming the flights are traversing through each sector with the same speed. In this work, they have highlighted the main differences between ATFM procedures in the USA and Europe. As mentioned, the critical challenge is the utilization of sector capacities in European airspace in comparison to the USA airspace. They performed four different case studies with several network structures and sector capacities to illustrate how the airborne and ground delays are assigned and how the delays are spread among flights depending on the conditions. Their experiments have illus-

trated that, in some circumstances, allocating a more expensive air holding delay is much better than a ground holding delay in terms of total delay and cost.

In addition to the above described deterministic models, more works can be found in the literature for SAGHP [50], [51], for MAGHP [36], [52], and [53], [54] for ATFM and ATFMRP.

2.3 Consideration of Uncertainty

The common feature of the above described deterministic models is that the capacity of airports/sectors is considered to be known at the start of the planning horizon and the flight timings are expected to be in-line with the plan. But, in reality, this is hardly true since there can be variations of capacities and demands due to unforeseen situations.

In recent literature, researchers have explored different forms of uncertainties in ATFM. Most researchers have proposed flow management models mostly, including the uncertainties of en-route sector capacity and/or airport capacity, and only a few have considered demand uncertainty in ATFM. As stated in [55], [56], during the September-October 2002 period, only 17% of the USA 111 ground delay programs had run according to the initial plan, and most of the plans had diverged due to unforeseen situations. This clearly emphasizes the importance of consideration of uncertainty in flow management to minimize unnecessary costs, delays, and extra work/operations.

In [57], the author has acknowledged the different attributes in which air transportation system uncertainty resides, such as take-off time, travel time, maintenance, and weather conditions. However, their main focus is on increasing the accuracy of the trajectory predictions considering the global perspective for a better optimized ATN. In the existing literature, there are works which estimate the airport and airspace capacity under uncertainty conditions as well as their consideration in flow management, while the framework proposed in [55], [58], [59], [60], [61], [62] are for airport and [63], [64], [65], [66], [67], [68], [28] are for en-route airspace.

A strategic level stochastic programming model is proposed in [59], by considering the effect of airport capacity uncertainty in the arrival and departure traffic. The uncertainty due to weather conditions is scrutinized and used as a set of scenarios with their associated probabilities. Their formulation is based on forming a strat-

egy that is optimal in average for the set of airport capacity scenarios, thus, they have used the concept of expected capacity of the airport to tackle the uncertainty. In [55], an aggregate stochastic programming model is proposed incorporating airport capacity uncertainties, trade-off between airport arrivals and departures, and their interactions by representing the uncertainty as a set of scenarios.

In [65], the authors use a comprehensive methodology to develop an en-route weather-capacity model. First, they generate scenarios based on weather forecasts to represent the evolution of convective weather in spatial and temporal dimensions, thus focusing on the pre-tactical phase. Then, they develop an algorithm to resolve conflicts and to generate conflict-free trajectories within the airspace, shifting the focus slightly towards the tactical phase. Finally, they perform Monte Carlo simulations of traffic flows over weather patterns and derive the probability distribution of capacity over time. They further develop a stochastic optimization algorithm to determine the sector volumes based on the generated probabilistic capacity predictions.

A chance-constrained optimization approach is proposed in [66] based on the deterministic formulation structure of the cell transmission model in [42]. The stochastic capacities are constrained for sectors, departure, and arrival airports in discrete time. They further propose a polynomial approximation-based approach with a distributed computing structure to solve large-scale chance-constrained optimization problems while reducing the computational complexity. The framework proposed in [67] also uses a chance constraint optimization approach by taking the uncertainty of sector capacity into consideration. In their study, the probability of exceeding the sector capacity has set to be no greater than a given percentage.

Nilim and Ghaoui [68] have proposed a stochastic dynamic programming model taking convective weather into consideration. When the original path experiences severe weather conditions, a dynamic routing strategy is proposed without completely avoiding the obscured zones. A very large-scale framework is developed in [28], with the consideration of capacity uncertainty using probabilistic scenario trees for nation-scale examples from the United States.

Even though the above models scrutinize the uncertainty of capacity and its different forms, none of them have explicitly considered the uncertainty of demand in flow management.

However, several studies have acknowledged and attempted to quantify the inherent demand uncertainty present in air transportation system. In [69], the authors

have identified and quantified both capacity uncertainty due to weather conditions, and the demand uncertainty due to deviations in time. Furthermore, they have proposed a method to measure the expected number of violations (demand exceeding capacity) in a given schedule. More importantly, they have highlighted the increase in the number of violations when the considered uncertainty in ATFM decreases. Similarly, Wanke et al. [70] have acknowledged the uncertainty in sector demand prediction and have quantified it under operational conditions to improve the performance in traffic flow management. The sector demand prediction error is statistically modeled based on the empirical measurement to achieve the uncertainty quantification. Several regression models are analysed by Gilbo and Smith [11] considering different factors which affect the demand in sectors and airports, such as adjacent intervals, look ahead time, airport specific coefficient and active and proposed flights. In later research [71], they have proposed two regression models to improve the performance of ETMS in terms of traffic demand predictions by considering the effect of demand uncertainty. Moreover, in [72], they have investigated the impact the ground delay imposes on arrival demand at airports. The ground delays applied on flights at origin airports postpone the arrival times of the corresponding flights at the arrival airports, inducing demand uncertainties for the respective airports.

A multi-commodity network dynamic flow model under the uncertainties of airport departure and arrival capacities, airspace sector capacity and the flight demand is proposed in [24]. This work is an extension of their previous work [25] and the uncertainty of capacity is represented as a set of scenarios. Although this work considers the uncertainty on flight demand by the airline companies, they have not explicitly considered the demand uncertainty due to flight time deviations. The Deterministic Equivalent Model (DEM) is a mixed 0-1 model, which is similar to [22], with full recourse for a risk neutral situation. They have come up with multiple objective terms to be included in the objective function, depending on the goal of the user, such as the expected cost of cancellations, rerouting, deviation of the arrival period, air travel time and the flight duration from the scheduled values. The computational performance of this model is listed in Table 2.1.

Although the above literature highlights the importance of demand uncertainty, none of them have incorporated it in ATFM. The above-discussed literature has used different stochastic programming approaches, including an average strategy with expected values, scenario tree-based approach, and chance constraint to handle inherent capacity uncertainty in airports or en-route sectors. Besides, there are

more methodologies for stochastic programming problems such as risk-averse measures, conditional value-at-risk, excess probabilities, etc., for more methodologies, see [24] and references therein.

One of the major drawbacks in the scenario tree approach is the exponential increase of solution time with the increase of scenarios considered, thus, the high computational complexity in expanding over large-scale scenario sets. Such a formulation difficulty can be overcome through a chance constraint-based formulation, but still, there is a certain level of probability for constraints to be violated depending on the selection of chance constraint values.

In addition to the above described literature, more works can be found in [73], [74] for SAGHP and [37] for MAGHP with uncertainty considerations.

2.4 Computational Performance and Scalability

One of the main drawbacks in ATFM related research is the computational complexity in solving large-scale realistic problems. Such computing difficulties can be overcome through a proper formulation structure as well as through an efficient solving mechanism. Comparatively, ATFM models that consider the effect of uncertainty have higher computational complexities due to their logical condition-based modeling [17] and/or multiple stochastic scenario-based modeling structures [28].

From the existing literature on ATFM, [22], [23], [41], [28], and [75] have considered in solving the large-scale en-route ATFM problems. Although the run times are not directly comparable due to different network and computing environments, here, we have numerically illustrated their network configurations and solution times to layout a baseline. A problem with 30 airports, 145 sectors, and 6475 flights is solved within 10 min in [23], 3419 flight paths in a network with 284 sectors are solved within 21 min in [75], and a problem with 206 OD pairs, 45 airports, 484 air-routes with 1164 flights is solved within 9 min in [41]. Further, the deterministic model in [28] has solved an ATFM problem with 17,500 flights in a network of 370 airports, and 375 sectors in 5 min.

In order to lower the computational complexities, researchers have used different solution strategies, as described below.

An LP relaxation is directly applied in [22] to solve their 0-1 model and has found that, mostly, the results are integral and improved in comparison to the LP relaxed

solutions of [36]. In the early stages, researchers have mostly used 0-1 IP structure in flow management models and have solved directly via LP relaxation. In cases where the solution of LP relaxation is not very strong, branch and bound has been used to generate integral results [23], [76].

A distributed approach is applied in [26] by first relaxing all integer decision variables in the Integer Quadratic Programming (IQP) problem into real numbers, since a standard Quadratic Programming (QP) problem is relatively easier to solve. Furthermore, they partition the whole network into sub-networks, and then, by applying the Lagrangian relaxation approach they solve the Lagrangian dual problem through a standard iterative subgradient method. Finally, a heuristic approach called forward-backward propagation algorithm is used to obtain the integer solution for the original problem. In the comparison of their results with the centralized approach results, they have highlighted that the largest gap difference is 7.8% for a problem with 1800 flights in a grid network of 4-by-4. To further reduce the computational complexity, a link based en-route air traffic system model is proposed in [41], and a hierarchical architecture is carried out in a bottom-up manner by relying on an incremental strategy. Finally, a heuristic algorithm has used to obtain individual flight plans from the resulting flows and schedules.

In [28], they have used the column generation algorithm with parallel computation to solve a problem with 17,500 flights in a few minutes. In order to solve their integer master problem, first, they have relaxed and converted it to a linear programming problem. In column generation, it solves a Restricted Master Problem (RMP), which is based on a subset of variables and new variables are brought into the RMP as needed. This process has terminated when no more variables can be added or when the required optimality gap or time limit is reached. In order to generate the integer solutions, they have applied a heuristic approach after solving the RMP.

There are other ways to model the ATFM problem and consider uncertainty, such as scenario based decision trees, Markov chain, and some primitive ways such as the use of expected values. However, considering uncertainty variables as random variables gives us the opportunity capture its entire effect more profoundly through their inherent distributions, and moreover, it allows the problem to model as a linear optimization problem. Markov chain or decision trees only capture a certain possible scenarios of the uncertainty variables, and thus, not complete. In order to capture more events, we need have more branches or states as in [28], and it

will significantly increase the computational complexity.

2.5 Comparison and Summary

In comparison to the current literature on flight-by-flight models, we solve a large-scale air traffic problem in a competitive time, while providing detailed flight plan information. Moreover, the proposed frameworks consider the inherent uncertainties of both demand and capacity in the flow management process, which are missing in the state-of-the-art models in [22], [23], [28].

Secondly, most of the above-described literature such as [22], [23], [26] and [41] are discrete time models in which the planning horizon is partitioned into a set of time intervals, and the decisions are provided/taken at the start of each time slot. Thus, it can cause significant inaccuracies depending on the size of the discrete time slot increasing chances of capacity violations, un-utilized airspace, delays, etc.. But, most of our proposed frameworks are continuous time models, thus, provide the exact time and location information. Even in cases, where discrete time slots are considered, the final outcome is transformed into a continuous time domain. Furthermore, to enhance the practicality, we also introduce flight level assignment and speed optimization features, while optimizing fuel consumption, which is rare in the existing literature of ATFM.

In terms of safety, the proposed frameworks uphold several safety related aspects such as reduction in inter-flight conflicts, capacity violations and, eventually, the controller workload. Aforementioned safety risks are reduced by constraining the sector capacity and maintaining the in-trail separation between flights. In fact, the latter constraint is rarely considered in the existing ATFM models.

Overall, in comparison to the existing literature of ATFM, our models scrutinize a diverse set of air traffic related attributes, prioritizing uncertainty influenced safety, while enjoying a (relatively) low computational complexity, in a real-time ATFM. A summary of selected literature is listed in Table 2.1.

Chapter 3

Flight Routing and Scheduling Under Demand Uncertainty

3.1 Introduction

In this chapter, we develop a novel flight routing and scheduling framework, while considering two main factors of demand uncertainty, i.e., departure time and en-route speed uncertainties present in ATN. As stated in the introduction, the uncertainty in demand could lead to unpredictable arrivals of aircraft into en-route sectors, thus, leading to a higher workload for controllers, more rerouting decisions as well as flight delays. Even though the demand uncertainty is analyzed in the literature, generally, it has not been included in the ATFM process. Hence, the fundamental objective of this chapter is to develop a framework, which considers the uncertainty of demand, while minimizing unexpected flight deviations and enhancing the safety of ATN. To the best of our knowledge, this is the first piece of research work that has addressed the demand uncertainty due to time deviations and in-trail separation between aircraft.

Further, in this chapter, we explain our solution method, which is a greedy strategy with the problem decomposed into a set of maximum independent sets to reduce the computational complexity in solving a large-scale ATFM problem. The results of this chapter are published in [77] and [78]. In [77], we consider the effect of departure time deviation in detail with a robust optimization approach and [78] considers two main factors of departure time and aircraft speed deviations in the process of flight routing and scheduling.

The main contributions of this chapter can be listed as follows.

1. A flight routing and scheduling model, which takes the demand uncertainty in the forms of departure time deviation and speed deviation, into account.
2. Elimination of capacity violations due to demand uncertainty from the system.
3. Compliance of the minimum safety separation requirement between flights throughout the entire flight duration.
4. Optimal assignments of flight levels, speeds and fuel consumptions.
5. A solution mechanism to overcome the computational complexity in large-scale networks.

The rest of the chapter is organized as follows. Section 3.2 presents a general overview of the network and insights on the proposed model. Section 3.3 and 3.4 introduce the proposed ATFM model and the computational improvement strategy, respectively. Section 3.5 discusses the results, and finally, section 3.6 summarizes the chapter.

3.2 ATFM System Model

3.2.1 General Overview of Air Traffic Network

The en-route ATN is a directed graph of waypoints and air links, which is denoted by, $G = (V, E)$, where V is a disjoint union of waypoints and airports, and E denotes the edges of the graph, i.e. links/ air routes. A simplified ATN is depicted in Fig. 3.1. Waypoints are the fixed located reference points in the airspace to indicate the location of the aircraft for the radar navigation. Two adjacent waypoints are connected by a directional air route/link. With a safe vertical separation, the air link can be considered as a bi-directional route, allowing flights to travel in both directions. The airport is represented as a node with a departure link (I) and an arrival link (I'). The departure link of an airport is the direct connection from the airport to a departure fix at a cruising level, and an arrival link is at the end, from an arrival fix to an airport. The flight path is a directional track between its origin and destination starting from the departure link, following a set of waypoints, links, and finally reaching the arrival airport. It is constrained by capacity limitations and several other restrictions which produce the final flight plan.

Figure 3.1: Simplified air traffic network

The concept of flight level divides the airspace into a set of vertical layers with a safe separation of altitude. In this model, we optimize the flight level assignments to fully utilize the existing en-route capacity and to minimize the average flight delay. Other than the vertical separation imposed by the flight levels, aircraft need to keep a safe separation in lateral and longitudinal directions to prevent collisions either by distance or time or visually. Here, we use the separation in terms of time to prevent collisions and capacity violations.

We divide the set of flights into two categories, as long haul and short haul, depending on the total flight duration since the flight type is important in optimizing the speed, flight level assignment based fuel consumption. Short haul flights only use the lowest flight level due to their short flight duration. For long haul flights this model assigns flight levels with the aim of saving the amount of consumed fuel. There are several factors influencing the consumption of fuel. At initial links, as

the aircraft weight is very high (due to high fuel weight), it will burn a lot of fuel if the aircraft ascend to a higher flight level. However, as the fuel is consumed the aircraft weight gradually decreases, and thus, can be ascended to a higher flight level.

Depending on the aircraft type, they have a special parameter called the Mach number which represent their optimal cruising speed [79]. The Mach number depends on the speed of the sound and the speed of the aircraft relative to the air. With an increase of altitude, the speed of sound declines since the temperature decreases. Hence, the pilot needs to fly at a lower speed to maintain a constant Mach number when it increases the altitude. Here, we calculate the optimum speed at each flight level depending on the Mach number and minimize the deviation of the actual speed from the optimal speed.

Additionally, we make the following assumptions to realize a low complexity model.

1. As the main focus of this model is on demand uncertainty, the capacity of the network is assumed to be known in advance.¹
2. The speed between two consecutive waypoints (in a link) is considered in average, and thus, a constant. However, it can vary in different links.
3. Uncertainty distributions of departure time and cruising speeds are considered as independent from each other.²he uncertainty of aircraft speed is mainly caused by weather conditions, together with some secondary conditions such as pilot's behavior, engine efficiency, etc. Since this model can be applied with short horizons in real-time (as the solution time is very low), the uncertainty due to weather prediction error, so as their correlations can be considered as small. Thus, the secondary conditions, which are fairly independent, dominate the speed uncertainty.
4. The uncertainties in ascending and descending phases are neglected as its effect is negligible in comparing to the cruising time.

¹Although the network capacity is time-varying, mainly due to weather fluctuations, the forecasting data is available in advance for a pre-estimation.

²T

3.2.2 Convolution of Departure time and Speed Uncertainties

In this approach, we consider both departure and en-route speed uncertainty of each aircraft. The model is generally formulated with respect to α (area of the *un-considered* uncertainty), in which the value is left for the users to decide. However, we encourage the selection of a low percentage for α . With $\alpha = 0$, the model considers the aircraft's maximum possible time period of stay within the route which is obtained using the lower bound of entry time and the upper bound of exit time. Example departure (left) and speed (right) uncertainty distribution graphs are shown in Fig. 3.2. The x-axis represents the difference of *actual value* – *scheduled value*. The left and right hand sides from zero indicate early and late departures in the left graph, respectively, while speed lower and higher than optimal speed, are indicated in the right graph, respectively.

Figure 3.2: Illustrative distributions of departure time and speed uncertainties

The departure time and speed deviations are statistically estimated using the previous flight records of scheduled departure time, actual departure time, actual speed at each route, and optimal cruising speed. Using data sets of deviation and their frequencies, the best-fitted probability distributions can be determined, as shown in Fig. 3.3. Then, the Probability Density Functions (PDFs) are obtained based on the distribution parameters. Finally, the corresponding PDFs are transformed and convoluted as described below to obtain the required link arrival time distributions.

Figure 3.3: Departure time uncertainty distribution

Let's consider an example with a departure link I_1 and five consecutive identical downstream links in flight f_1 's path, as illustrated in Fig. 3.4. Since the uncertainty in the ascending phase is neglected, the departure link I_1 does not possess any uncertainty distribution. Therefore, when the flight f_1 enters the link j at waypoint w_1 , it has the same distribution as the departure time uncertainty. But, the arrival distribution of flight f_1 at link $j + 1$ is different due to the uncertainty associated with the speed in link j . The arrival distribution at link $j + 1$ depends on both uncertainty distribution at waypoint w_1 and speed distribution of link j .

Figure 3.4: An illustration of the uncertainties of departure time and speed variation

Let the random variable representing the arrival time of flight f_1 at waypoint w_1 and w_2 be T_{w_1} and T_{w_2} , respectively, and flight f_1 's speed in link j be V (Z -reciprocal of V). L_j is the length of link j . Thus, T_{w_2} can be derived as,

$$T_{w_2} = T_{w_1} + \frac{L_j}{V} = T_{w_1} + L_j Z \quad (3.1)$$

If we have the PDF of speed uncertainty distribution as $\varphi(v)$, the PDF of the

random variable Z can be derived by following the PDF transformation techniques in [80]. However, first, it is required to modify the definition of Z to suit the transformation theorem as follows. So that its value exists for all the corresponding V values as follows.

$$Z = \begin{cases} \frac{1}{V}, & V \neq 0 \\ 0, & \text{otherwise} \end{cases} \quad (3.2a)$$

With Z defined for all the corresponding V values, the PDF transformation can be employed as in (3.2b).

$$G(z) = \frac{1}{Z^2} \varphi\left(\frac{1}{Z}\right) \quad (3.2b)$$

where $G(z)$ is the PDF of the reciprocal of the speed.

In (3.1), $L_j Z$ refers to the travel time of flight f_1 in link j . Let the PDF of travel time be $X(z)$ and it can be derived by following the same PDF transformation technique in [80] as,

$$X(z) = \frac{1}{|L_j|} G\left(\frac{Z}{L_j}\right) \quad (3.2c)$$

It should be noted that we can estimate the PDF of travel time purely based on the statistical data as well. However, it requires several additional data of each link, such as link length and scheduled travel time.

In (3.1), T_{w_1} 's uncertainty is same as the departure time uncertainty, and let's refer to it as $D(t)$. The sum of two independent random variables (T_{w_1} , $L_j Z$) in (3.1) implies the convolution of their individual distributions under the assumption of independent uncertainty distributions. The convolution will give the arrival distribution of flight f_1 at link $j + 1$. In the same way, the arrival distribution at waypoint w_3 can obtain using the convolution of the distribution at w_2 (the one derived above) with the travel time distribution of link $j + 1$. In this way, the arrival time uncertainty at each link can be derived using the propagated arrival distribution of the previous link and the travel time uncertainty distribution of the considered link.

We illustrate the arrival time estimation via an example with respect to Fig. 3.4 as follows. Let the departure time uncertainty ($D(t)$) be a normal distribution

with a mean of 18.69 min and standard deviation of 8.577 min and travel time uncertainty ($X(z)$) also be a normal distribution with a mean of 0 and standard deviation of 2.4 min Then, the arrival time at $j + 5$ link can be stated as in (3.3).

As the departure distribution and all the link travel time distributions are normal distributions, the resulting convoluted arrival time distribution at waypoint w_4 ($I^{j+5}(t)$) will also be a normal distribution as shown in Fig. 3.5. The mean and the standard deviation of the distribution can be calculated as follows.

Figure 3.5: Generation of arrival time distributions and time bounds

$$T_{w_4} = T_{w_1} + 5 * L_j Z \quad (3.3)$$

$$E[T_{w_4}] = E[T_{w_1}] + 5.E[T_{travel}] \quad (3.4a)$$

$$E[T_{w_4}] = E[T_{w_1}] + 5.E[T_{travel}] \quad (3.4b)$$

$$E[T_{w_4}] = 18.69 \text{ min} \quad (3.4c)$$

$$\sigma^2[T_{w_4}] = \sigma^2[T_{w_1}] + 5.\sigma^2[T_{travel}] \quad (3.4d)$$

$$\sigma[T_{w_4}] = 10.12 \text{ min} \quad (3.4e)$$

where T_{travel} refers to the travel time in a single link.

As mentioned earlier, we consider the link arrival time as an interval with respect to a $(1 - \alpha)$ portion of the arrival time distribution. For a general arrival time distribution, these bounds $(\Delta^-(f_i, j, \alpha), \Delta^+(f_i, j, \alpha)$ - considered maximum early and arrival times, respectively) can be estimated as follows.

$$\int_{\mu - \Delta^-(f_i, j, \alpha)}^{\mu} \Gamma^j(t) dt = \frac{1 - \alpha}{2} \quad (3.5a)$$

$$\int_{\mu}^{\mu + \Delta^+(f_i, j, \alpha)} \Gamma^j(t) dt = \frac{1 - \alpha}{2} \quad (3.5b)$$

where μ refers to the mean of the arrival time distribution.

As per our example, these bounds can be calculated by converting the normal distribution into a standard normal distribution and looking up the corresponding **Z** table values. Let's assume, $\alpha = 0.1$, then,

$$\frac{\Delta^+(f_1, j + 5, \alpha) - \mu}{\sigma} = Z_{table}(\alpha) \quad (3.6a)$$

$$\Delta^+(f_1, j + 5, 0.1) = 35.34 \text{ min.} \quad (3.6b)$$

$$\frac{\Delta^-(f_1, j + 5, \alpha) - \mu}{\sigma} = -Z_{table}(\alpha) \quad (3.7a)$$

$$\Delta^-(f_1, j + 5, 0.1) = 2.04 \text{ min.} \quad (3.7b)$$

3.3 Flight Routing and Scheduling Problem Formulation

In this section, we formulate our ATFM model under demand uncertainty. The proposed model anticipates an effect of $(1 - \alpha)$ percentage of demand uncertainty, and optimize under the given constraints. Thus, it is different from conventional stochastic programming models where multiple cases are considered with respective probabilities. A design motivation is taken from packet routing and scheduling in communication network [81–84].

3.3.1 Notations

Here, we list the notations that we use in this model.

- $G = (V, E)$ - The directed graph consisting a set of vertices (V) and edges (E). Waypoints, $W \subseteq V$ and airports, $A \subseteq V$ are considered as nodes and links are the directed connections between nodes.
- J - Set of all en-route links excluding departure and arrival links. $j = (w, w') \in J \subseteq W \times W$ denotes any en-route link from waypoint w to w' . $I = (a, v) \in E \subseteq A \times W$ and $I' = (v', a') \in E \subseteq W \times A$ represent the departure and arrival link. a and a' denotes the departure and arrival airport, respectively. All $j \in J$ has the properties of L_j - length of j , C_j - capacity of j and $D_j^{f_i}$ - downstream links connected with link j of f_i 's path.
- J^{f_i} - $J^{f_i} \subseteq J$ contains all routes that flight f_i can take, and $J^{f_i}(a, j) \subseteq J^{f_i}$ consists of routes from airport a to the considered link j that flight f_i can take.
- F - Set of all flights, $f_i \in F$. F has the subsets of F_S and F_L denoting short and long haul flights, respectively.
- H - Set of all FLs, $h \in H$. H_S and H_L denote the sets of FLs of short and long haul flights, respectively.
- s^{f_i}, τ^{f_i} - Scheduled and desirable departure time of f_i .
- $Y_{j,h}^{f_i}$ - If the flight level h of route j has been selected for f_i 's routing this variable will be 1, otherwise zero.
- $T_{arr}^{f_i}, T_{sch}^{f_i}$ - Expected and scheduled arrival time of f_i .

- $Z_{j,h}^{f_i}$ -Reciprocal of the speed, $v_{j,h}^{f_i}$ of f_i in link j at FL h .
- $v_{opt,h}^{f_i}$ - Calculated optimal speed of flight f_i at FL h , $Z_{opt,h}^{f_i}$ -it's reciprocal.
- T_{min} - Minimum required in-trail time separation.
- $\tilde{T}_{low,j,h}^{f_i}$, $\tilde{T}_{up,j,h}^{f_i}$ - Lower and upper bounds of entry time of f_i to link j at flight level h , respectively.
- $\ddot{T}_{low,j,h}^{f_i}$, $\ddot{T}_{up,j,h}^{f_i}$ - Lower and upper bounds of exit time of f_i from link j at flight level h , respectively.
- $t_{a,j}^{f_i}$ - Total travel time of f_i from airport a to link j .
- $\tilde{t}_{j,h}^{f_i}$ - Deterministic travel time of f_i in link j at FL h .
- $\Delta^-(f_i, j, \alpha)$, $\Delta^+(f_i, j, \alpha)$ - Maximum early and late arrival time deviation at link j (with respect to α), respectively.
- $W_0^{f_i}$, W^{f_i} - Initial weight, End of journey weight of f_i .
- $\tilde{w}_{j,h}^{f_i}$ - Weight depreciation factor of f_i in link j at FL h .

3.3.2 Decision Variables

A binary decision variable, $Y_{j,h}^{f_i}$ is used to indicate the selected routing path of flights.

$$Y_{j,h}^{f_i} = \begin{cases} 1; & \text{if flight } f_i \text{ passes route } j \text{ at flight level } h \\ 0; & \text{otherwise} \end{cases}$$

Further, we introduce two modified versions of the above decision variable since there is no consideration of flight levels for the departure link and arrival link as, $Y_I^{f_i}, Y_{I'}^{f_i}$. The second decision variable τ^{f_i} indicates the desired time value at which the flight should take-off from its departure airport. Instead of $v_{j,h}^{f_i}$, we use the reciprocal of the speed, $Z_{j,h}^{f_i}$ as the third decision variable to prevent the difficulty in using $1/v_{j,h}^{f_i}$.

3.3.3 Constraints

We use constraints for flight routing, link capacity, flight overtake, flight level assignment, and aircraft speed adjustment as described below.

Flight routing constraint: The routing constraint is used to dispatch the flights from their origin to the destination. Here, inequality constraint (3.8a) is used to allow flight cancellation. The departure and arrival links will be zero if the flight is cancelled. In all routed flights, (3.8a) fix the departure and arrival routes as,

$$(\forall f_i \in F)(I, I' \in E) \quad Y_I^{f_i} \leq 1 \text{ and } Y_{I'}^{f_i} \leq 1 \quad (3.8a)$$

Equation (3.8b) and (3.8c) are to select the next immediate link after passing link j . It is one of the downstream link connected with link j of flight f_i 's path. Depending on the aircraft type,

For short haul flights $f_i \in F_S$,

$$\sum_{h \in H_S} Y_{j,h}^{f_i} = \sum_{j' \in D_j^{f_i}, h' \in H_S} Y_{j',h'}^{f_i} \quad (3.8b)$$

For long haul flights $f_i \in F_L$,

$$\sum_{h \in H_L} Y_{j,h}^{f_i} = \sum_{j' \in D_j^{f_i}, h' \in H_L} Y_{j',h'}^{f_i} \quad (3.8c)$$

The expected arrival time of flight f_i through the selected path is calculated as in (3.8d).

$$T_{arr}^{f_i} = \tau^{f_i} + \sum_{j \in J^{f_i}, h \in H} (L_j Y_{j,h}^{f_i} Z_{j,h}^{f_i}) + \sum_{j \in J^{f_i}, h \in H} t_r^{f_i}(j, j') \quad (3.8d)$$

Link capacity constraint: In this model, we have not considered the capacity of sectors, instead we consider the link capacity, which is reflected by the head-and-tail separation. If the factors such as controller workload, traffic demand, and weather variations are assumed to be saturated, then the link capacity is a reflection of the capacity of the sector in which the link belongs. Both miles-in-trail and minutes-in-trail constraints guarantee the separation requirement of head-and-tail. In this model, we use minutes-in-trail separation to avoid capacity violations. The two strategies described below help to keep the minimum time separation between consecutive aircraft.

In this model, we constrain the aircraft to maintain a minimum time separation

of T_{min} when they enter to the link as well as when they exit the link. Further, this separation requirement can be modified based on the estimated link/sector capacities, thus, takes the varying nature of capacity into consideration as well. If a user wants to ensure that there is no capacity violations ($\alpha = 0$), the maximal time window during which an aircraft can safely enter the link, should be considered. As illustrated in Fig. 3.6 scenario 1, if flight f_i enters to the link first and f_k second, then the gap between the lower bound of entry time of flight f_k ($\tilde{T}_{low,j,h}^{f_k}$), and the upper bound of entry time of flight f_i ($\tilde{T}_{up,j,h}^{f_i}$), will give the minimum possible time separation between these two flights. This should be higher than T_{min} to minimize the capacity violation (with $\alpha = 0$, no capacity violations) within the link j , even under uncertainty. Equation (3.9) makes the connection between the selected path (Y) and the time of flight arrival to the links. $\tilde{T}_{low,j,h}^{f_k}$ and $\tilde{T}_{up,j,h}^{f_i}$ are given by,

$$\tilde{T}_{low,j,h}^{f_k} = \tau^{f_k} + t_{a,j}^{f_k} - \Delta^-(f_k, j, \alpha) \quad (3.9a)$$

$$\tilde{T}_{up,j,h}^{f_i} = \tau^{f_i} + t_{a,j}^{f_i} + \Delta^+(f_i, j, \alpha) \quad (3.9b)$$

where,

$$t_{a,j}^{f_k} = \sum_{j \in J^{f_k}(a,j), h \in H} (L_j Y_{j,h}^{f_k} Z_{j,h}^{f_k}) + \sum_{j \in J^{f_i}, h \in H} t_r^{f_i}(j, j') \quad (3.9c)$$

Figure 3.6: Possible scenarios of flight entering a link

In the same way, for the case of flight f_k enters first and f_i second, then the

minimum time separation should be considered between the lower bound of entry time of flight f_i ($\tilde{T}_{low,j,h}^{f_i}$), and the upper bound of entry time of flight f_k ($\tilde{T}_{up,j,h}^{f_k}$), as shown in Fig. 3.6 scenario 2. **Either** one of these two conditions should satisfy to keep the time separation between two aircraft when they enter to the link, as in (3.10).

$$\text{either } \tilde{T}_{low,j,h}^{f_k} - \tilde{T}_{up,j,h}^{f_i} \geq T_{min} \text{ or } \tilde{T}_{low,j,h}^{f_i} - \tilde{T}_{up,j,h}^{f_k} \geq T_{min} \quad (3.10)$$

The logical condition (3.10) is converted into linear constraints by introducing a binary auxiliary variable $B(f_k, f_i, j)$ and a sufficiently large upper bound M_1 [85].

$$\tilde{T}_{low,j,h}^{f_k} - \tilde{T}_{up,j,h}^{f_i} \geq T_{min} - M_1 B(f_k, f_i, j) \quad (3.11a)$$

$$\tilde{T}_{low,j,h}^{f_i} - \tilde{T}_{up,j,h}^{f_k} \geq T_{min} - M_1 (1 - B(f_k, f_i, j)) \quad (3.11b)$$

The term M_1 should be chosen in such a way that the left side of the constraints (3.11a) and (3.11b) are always greater than $T_{min} - M_1$. So in all the cases, depending on the value of $B(f_k, f_i, j)$, either one of the constraint (3.11a) or (3.11b) will be imposed while the other one is weakened due to big M_1 .

Moreover, this time separation should only be applied for the flights at the same flight level. If the two aircraft are in two different flight levels, they are already separated by the altitude, so that they do not require to hold this minutes-in-trail separation. Therefore, the linear constraints (3.11a) and (3.11b) are modified further as they will only apply to the flights in the same flight levels. Constraints (3.12a) and (3.12b) will be same as the constraints (3.11a) and (3.11b) when $Y_{j,h}^{f_i} = Y_{j,h}^{f_k} = 1$ which means that flights f_i and f_k are in the same route, same flight level. Only in such cases, this time separation constraint will hold. For the cases where $Y_{j,h}^{f_i} \neq Y_{j,h}^{f_k}$ either M_2 or M_3 will make both constraints (3.12a) and (3.12b) weak since they become non-binding. Moreover, when $Y_{j,h}^{f_i} = Y_{j,h}^{f_k} = 0$ the constraint will be more weakened due to the large values of M_2 and M_3 .

For all $f_k, f_i \in F, j \in J, h \in H$,

$$\tilde{T}_{low,j,h}^{f_k} - \tilde{T}_{up,j,h}^{f_i} \geq T_{min} - M_1 B(f_k, f_i, j) - M_2 (1 - Y_{j,h}^{f_k}) - M_3 (1 - Y_{j,h}^{f_i}) \quad (3.12a)$$

$$\tilde{T}_{low,j,h}^{f_i} - \tilde{T}_{up,j,h}^{f_k} \geq T_{min} - M_1(1 - B(f_k, f_i, j)) - M_2(1 - Y_{j,h}^{f_k}) - M_3(1 - Y_{j,h}^{f_i}) \quad (3.12b)$$

where $M_2, M_3 \gg M_1$

As both constraints (3.12a) and (3.12b) only guarantee the separation of T_{min} at the time flights enter the link, the constraints (3.13a) and (3.13b) will make the same separation when they depart from the link. If two flights consecutively enter the same link in the same flight level, then we only require to check the in-trail separation at the start of the link as the exit of the previous link is identical to the start of the next link. However, there could be situations where one of these flights changes into a different flight level during the transition of links. It is vital to check the in-trail separation in these scenarios as the flights may not keep the minimum required separation at the exit. The constraint of time separation by T_{min} at the time of exit holds if and only if the inequalities (3.13a) and (3.13b) both hold,

$$\ddot{T}_{low,j,h}^{f_k} - \ddot{T}_{up,j,h}^{f_i} \geq T_{min} - M_1 B(f_k, f_i, j) - M_2(1 - Y_{j,h}^{f_k}) - M_3(1 - Y_{j,h}^{f_i}) \quad (3.13a)$$

$$\ddot{T}_{low,j,h}^{f_i} - \ddot{T}_{up,j,h}^{f_k} \geq T_{min} - M_1(1 - B(f_k, f_i, j)) - M_2(1 - Y_{j,h}^{f_k}) - M_3(1 - Y_{j,h}^{f_i}) \quad (3.13b)$$

where $j' \in D_j^{f_k}$

$$\ddot{T}_{low,j,h}^{f_k} = \tau^{f_k} + t_{a,j}^{f_k} + L_j Y_{j,h}^{f_k} Z_{j,h}^{f_k} - \Delta^-(f_k, j', \alpha) \quad (3.14a)$$

$$\ddot{T}_{up,j,h}^{f_k} = \tau^{f_k} + t_{a,j}^{f_k} + L_j Y_{j,h}^{f_k} Z_{j,h}^{f_k} + \Delta^+(f_k, j', \alpha) \quad (3.14b)$$

A robust optimization approach is presented in [77] by setting $\alpha = 0$ in the link capacity constraint. Thus, it considers the maximum time window of a flight in a link, ensuring zero link capacity violations.

Constraint to eliminate flight overtake: Although link capacity constraints guarantee the time separation of flights at entry and exit, they do not consider the flight sequence. If the succeeding aircraft has a higher speed than the preceding aircraft when they enter to the link, there is a possibility for a flight overtake. To

avoid such situations, we use a constraint to eliminate flight overtake.

The Fig. 3.7 illustrates a flight overtaking scenario while keeping minimum time separation of T_{min} at entry and exit times. Points p_1 and p_2 denote the deterministic entry time and exit time of flight f_k , respectively. Points p_3 and p_4 denote the deterministic entry time and exit time of flight f_i , respectively. Hence, flight f_k enters first and flight f_i enters afterwards with a T_{min} separation, but f_i exit the link before f_k still with a T_{min} separation. Thus, according to Fig. 3.7, if the deterministic link travel time difference of two flights is higher than $2T_{min} + \Delta^+(f_k, j, \alpha) + \Delta^-(f_i, j, \alpha) + \Delta^+(f_i, j, \alpha) + \Delta^-(f_k, j, \alpha)$, it means that flight overtake is possible. Hence, the constraint to avoid flight overtake is given as in (3.15).

Let $T_0 = 2T_{min} + \Delta^+(f_k, j, \alpha) + \Delta^-(f_i, j, \alpha) + \Delta^+(f_i, j, \alpha) + \Delta^-(f_k, j, \alpha)$,

Figure 3.7: Time axis representation of flight overtake

$$-T_0 < \tilde{t}_{j,h}^{f_k} - \tilde{t}_{j,h}^{f_i} < T_0 \quad (3.15)$$

where $\tilde{t}_{j,h}^{f_k} = L_j Y_{j,h}^{f_k} Z_{j,h}^{f_k}$

Furthermore, this flight overtake constraint should only be applied if both aircrafts enter the link j at flight level h . Hence, the above constraint is modified using a big M as,

$$-T_0 - M(1 - Y_{j,h}^{f_k}) < \tilde{t}_{j,h}^{f_k} - \tilde{t}_{j,h}^{f_i} < T_0 + M(1 - Y_{j,h}^{f_i}) \quad (3.16)$$

Constraints for optimal flight level assignment: In terms of fuel consumption, it is always economical to use higher flight levels, as in these levels, the fuel consumption rate is low due to the low thrust produce by the engine as the air density reduces with the increase of altitude. However, if the short haul flights use higher flight levels, it will be economically disadvantageous since ascending and descending phases burn extra fuel than flying at the same level. Therefore, we

only assign the lowest flight level for the short haul flights. We have modeled this under the routing section by opening short haul flights only to the lowest flight level. Therefore effectively,

For all $f_i \in F_S$, $j \in J$, $h \in H_L - H_S$,

$$Y_{j,h}^{f_i} = 0 \quad (3.17)$$

Due to the longer flight time of long haul flights, they change their flight level considering a few factors. Yet, the long haul flights also need to use a lower flight level first because of the initial high weight of aircraft mainly due to the full fuel tank. With the consumption of fuel, the aircraft weight will start to decline, so that they can fly into higher flight levels. Here, we use a weight assignment strategy to allow long haul flights to first select a lower flight level and as the flight time goes on incline to higher flight levels, so that the lower flight levels are available for the short haul flights.

With the travel time, the initial weight of the aircraft will reduce according to (3.18) and we use this term in our objective function to assign flight levels optimally as it maximizes the W^{f_i} which is the remaining weight of the aircraft at the end of the journey.

$$W^{f_i} = W_0^{f_i} - \sum_{j \in J^{f_i}, h \in H} (L_j Y_{j,h}^{f_i} Z_{j,h}^{f_i}) * \tilde{w}_{j,h}^{f_i} \quad (3.18)$$

Our aim is to assign flight levels so as to maximize the total remaining weight W^{f_i} . We use below described weight assignment strategies to optimally assign weights to the flight levels by dividing the whole flight time into multiple stages and assign different weight reduction factors as,

- During the initial flight stages, to keep the heavy flight in lower levels, we assign weight reduction factors as, $\tilde{w}_{j,1}^{f_i} < \tilde{w}_{j,2}^{f_i} < \dots < \tilde{w}_{j,h}^{f_i}$. (altitude of FL increase with h)
- At later and final stages, we motivate flights to move into higher flight levels, by adjusting weight reduction factors as, $\tilde{w}_{j,1}^{f_i} > \tilde{w}_{j,2}^{f_i} > \dots > \tilde{w}_{j,h}^{f_i}$.

In general practice, flights use a continuous descent approach in which the arriving aircraft descends continuously up to a possible extent. It has also benefited in terms of reducing fuel consumption, aircraft noise and emissions. Hence, this model will not assign FLs after the arrival fix. When long haul flights change

the altitude, (3.19) is used to calculate the transition time between FLs. For all $f_i \in F_L$, $j \in J$, $h \in H_L$,

$$t_r^{f_i}(j, j') = \max(0, (\frac{h^+ Y_{j', h^+}^{f_i} - h Y_{j, h}^{f_i}}{v_{j, h}^{f_i}}) + (\frac{h Y_{j', h}^{f_i} - h^+ Y_{j, h^+}^{f_i}}{-v_{j, h^+}^{f_i}})) \quad (3.19)$$

The altitude of $h^+ > h$ and $j' \in D_j^{f_i}$. Here, we considered that the ascending or descending speed will be same as the previous link speed. If the aircraft ascends to a higher FL or descends to a lower FL, the ‘max’ function will return the corresponding transition time and if the aircraft travel in the same FL then it will return zero.

$$\text{Let's take, } f(.) = (\frac{h^+ Y_{j', h^+}^{f_i} - h Y_{j, h}^{f_i}}{v_{j, h}^{f_i}}) + (\frac{h Y_{j', h}^{f_i} - h^+ Y_{j, h^+}^{f_i}}{-v_{j, h^+}^{f_i}})$$

To convert this ‘max’ function into linear constraints, we introduce an auxiliary boolean variable as $\delta(t_r(j, j'))$ which satisfies, $[f(.) \geq 0] \rightarrow [\delta(t_r(j, j')) = 1]$, $[f(.) < 0] \rightarrow [\delta(t_r(j, j')) = 0]$. These logical conditions are converted into linear constraints using mixed logics as in [85]. Then, the transition time of flight f_i when it travels from link j to j' is,

$$t_r^{f_i}(j, j') = f(.)\delta(t_r(j, j')) \quad (3.20)$$

Constraint to optimize the speed of the flight: Aerodynamics have designated a special parameter called the Mach number to define the ratio of the speed of a object to the speed of sound as follows [86],

$$\text{Mach number} = \frac{\text{object's speed relative to the medium}}{\text{speed of sound}} \quad (3.21)$$

Airliners define their optimum speed as a Mach number, for example, Boeing 777 has a Mach number 0.84 and Airbus 330 has a Mach number 0.82. Here, our objective is to adjust the speed of the flight as it can maintain its optimal Mach number throughout the flight time. Let m_s and m_l be the Mach numbers of short and long haul flights, respectively, and v_h be the speed of sound at altitude level h . According to (3.21), the True Air Speed (TAS) of long haul flights at flight level h can be calculated as *True air speed* = $m_l v_h$.

Here, we consider the true airspeed is same as ground speed. $v_{opt, h}^{f_i}$ is the calculated optimal speed of flight f_i at flight level h . Equation (3.22) is used in the objective

function to minimize the deviation of actual speed from the optimal speed.

$$\text{Speed deviation} = \sum_{j \in J^{f_i}, h \in H} (Z_{j,h}^{f_i} - Z_{opt,h}^{f_i})^2 \quad (3.22)$$

3.3.4 Linearizing the Constraints

From the above constraints, (3.8d), (3.9a), (3.9b), (3.13a), (3.14a), (3.14b), and (3.18) are non-linear due to the term $Y_{j,h}^{f_i} Z_{j,h}^{f_i}$. We use mixed integer constraints to linearize them as follows [87], [88], [?]. The term, $Y_{j,h}^{f_i} Z_{j,h}^{f_i}$ is replaced by an auxiliary real variable $U_{j,h}^{f_i} \triangleq Y_{j,h}^{f_i} Z_{j,h}^{f_i}$, which satisfies,

$$[Y_{j,h}^{f_i} = 0] \rightarrow [U_{j,h}^{f_i} = 0] \quad (3.23a)$$

$$[Y_{j,h}^{f_i} = 1] \rightarrow [U_{j,h}^{f_i} = Z_{j,h}^{f_i}] \quad (3.23b)$$

Hence, $U_{j,h}^{f_i} = Y_{j,h}^{f_i} Z_{j,h}^{f_i}$ is equivalent to,

$$U_{j,h}^{f_i} \leq M Y_{j,h}^{f_i} \quad (3.23c)$$

$$U_{j,h}^{f_i} \geq m Y_{j,h}^{f_i} \quad (3.23d)$$

$$U_{j,h}^{f_i} \leq Z_{j,h}^{f_i} - m(1 - Y_{j,h}^{f_i}) \quad (3.23e)$$

$$U_{j,h}^{f_i} \geq Z_{j,h}^{f_i} - M(1 - Y_{j,h}^{f_i}) \quad (3.23f)$$

where,

$$M \triangleq \max_{j \in J^{f_i}, h \in H, f_i \in F} Z_{j,h}^{f_i} \text{ and } m \triangleq \min_{j \in J^{f_i}, h \in H, f_i \in F} Z_{j,h}^{f_i}.$$

Constraint (3.8d), (3.9a), (3.9b), (3.13a), (3.14a), (3.14b), and (3.18) are modified using the new auxiliary real variable, $U_{j,h}^{f_i}$ to convert them into the linear form as listed in the summary. Similarly, (3.20) is also linearized using mixed logic conversion.

3.3.5 Objective Function

Our primary objective is to minimize the expected total delay time of each aircraft. Minimizing the departure time deviation and the arrival time deviation help to minimize the total expected delay time of each aircraft. Moreover, it is important for both airlines and passengers that the aircraft fly in the optimal speed to avoid unnecessary disturbances. In the same way, airlines expect to save fuel cost at the end of the journey. This further helps to reduce the carbon emission into the environment as well. Cancellation of flights is extremely costly for both passengers and airlines. Hence, it should be minimized with a higher priority. Based on these factors, the objective function is formulated as,

$$\min \sum_{f_i \in F} \left\{ C_D(\tau^{f_i} - s^{f_i})^2 + C_A(T_{arr}^{f_i} - T_{sch}^{f_i})^2 + C_Z \sum_{j \in J^{f_i}, h \in H} (Z_{j,h}^{f_i} - Z_{opt,h}^{f_i})^2 - C_W W^{f_i} + C_M \sum_{f_i \in F} (1 - Y_I^{f_i}) \right\} \quad (3.24)$$

The first two terms denote the total departure and arrival deviation, and the third term is to minimize the speed deviation from the optimal speed. The fourth term of weight maximization will help for a smooth flight level assignment. The last term is used to minimize the flight cancellations. The highest cost coefficient value is assigned to the flight cancellation term, (C_M) as it is vital to minimize the number of cancelled flights. The coefficient of departure and arrival deviation are weighted as $C_D \ll C_A$ since on-time arrival is more crucial in air transportation. The remaining weights in the objective function are selected based on a priority (preceding) basis. As our primary focus is to minimize flight delay, we assign very high coefficient values to C_A and C_D . Speed optimization and flight level optimization have decrementing priorities, respectively, and thus $C_Z > C_W$. Overall, $C_M \gg C_A \gg C_D \gg C_Z \gg C_W$.

The proposed model is a MIQP problem with continuous and binary decision variables, a quadratic cost function and a set of linear constraints, which can be summarized as,

minimize (3.24)

subject to (3.8a), (3.8b), (3.8c), (3.8d), (3.12), (3.13), (3.16), (3.17), (3.18).

3.4 A Greedy Strategy with Parallel Computation and MISs

The computational bottleneck of our approach is tied with the involvement of logical conditions in the link capacity constraint. Since the proposed model is a continuous time model, we used flight-by-flight basis to interpret the capacity constraint, which forced the use of *either or* constraints. The mathematical interpretation of *either or* conditions introduced auxiliary boolean variables, which reduced the computational efficiency of the model. We cannot use linear relaxation mechanisms directly to reduce the complexity since the constraints (3.12a), (3.12b), (3.13a) and (3.13b) are only valid if and only if the auxiliary variables hold the binary status. Therefore, we decompose the problem into several independent sets using MISs [89] algorithms and schedule them using a greedy strategy with parallel computation to reduce the computational complexity. The overall solution process is shown in Algorithm 1.

Algorithm 1 Greedy approach - MIS and parallel computation

- Step 1:** Convert the Air Traffic Network into a graph, $G' = (V', E')$. ▷ V' - Flight set, E' - Flight dependency set.
- 1: Flight dependency matrix generation process is shown in Algorithm 2.
- Step 2:** Search and obtain all maximum independent sets in the graph. For every two flights in a MIS, there is no edge connection which represent the independence of the two flights. Let $S_1, S_2, \dots, S_n$ denote the MISs of the network.
- Step 3:** Initialize set, Fixed = $\{\}$. ▷ Empty set, as none of the flights are routed yet
- Step 4:** **while** (Fixed $\neq V'$) ▷ From S_1 to S_n
- 1: Route and Schedule S_i on the **Fixed** flight set
- 2: Each flight in S_i is routed parallelly as they are independent from each other
- 3: Fixed $\leftarrow$ (Fixed $\cup S_i$) ▷ Accumulate fixed flights
- Step 5:** **Fixed** set contains all the routed and scheduled flights.
-

As the initial step, ATN is converted into a weighted graph denoted as $G' = (V', E')$. Here, (V') refers to the set of flights and (E') refers to their interaction (dependency). For an example, if two flights' coordinated position, and time overlap with each other, then there exists an edge in between those two vertices (flights). A weight is assigned to each node (i.e. for flights) depending on their scheduled flight time. Higher priority is given to early scheduled flights. The weighted graph is built by considering all possible trajectories in both space and time. A simplified dependency matrix generation process is shown in Algorithm

2.

Algorithm 2 Generation of dependency matrix

Input: $\triangleright n_F$
 - number of flights, n_N - number of links, $L[n_N]$ - set of link lengths, $T^d[n_F]$ - set of departure times, $R^{u,l}[n_N][n_F]$ - set of upstream links in the longest path, $R^{u,s}[n_N][n_F]$ - set of upstream links in the shortest path, $V^{min|max}[n_N][n_F]$ - set of min|max speeds on links, $\Delta^{-|+}[n_F]$ - set of considered maximum early (−) and late (+) departure time uncertainty limits
Output: $dep_matrix[n_F][n_F]$ $\triangleright$ flight dependency matrix (1-if two flights are dependent, 0-otherwise)
 1: $dep_matrix \leftarrow zeros;$ $\triangleright$ start with an empty matrix
 2: **for** ($f_k = 1 \rightarrow n_F$) **do**
 3: **for** ($f_i = f_k + 1 \rightarrow n_F$) **do**
 4: $r \leftarrow \text{getBottleneckLink}(f_k, f_i);$ ³
 $\triangleright$ The lower bound entry time of flight f to link r
 5: $\tilde{T}_{low,r}^f \leftarrow T^d[f] - \Delta^-[f] + \sum_{j \in R^{u,s}[r][f]} \frac{L[j]}{V^{max}[j][f]}$
 $\triangleright$ The upper bound exit time of flight f from link r
 $T^d[f] + \Delta^+[f] +$
 6: $\tilde{T}_{up,r}^f \leftarrow \left[\sum_{j \in R^{u,l}[r][f]} \frac{L[j]}{V^{min}[j][f]} \right] + \frac{L[r]}{V^{min}[r][f]}$
 7: **if** (flight f_k 's and f_i 's maximum possible existence on r overlaps) **then**
 8: $dep_matrix[f_k][f_i] \leftarrow 1;$
 9: **else**
 10: $dep_matrix[f_k][f_i] \leftarrow 0;$
 11: **end if**
 12: **end for**
 13: **end for**
 14: $dep_matrix[i][j] \leftarrow dep_matrix[j][i], \forall i > j$ $\triangleright$ Copy upper triangle of dep_matrix over lower triangle

In step 2, we extract all MISs from the graph by following the work of [89]. The process starts with an empty flight set (Fixed). Then, at each iteration, we route and schedule MISs on the network. The first flight set (largest MIS) is optimally routed in ATN, as none of the flights have routed at the first iteration. In the second iteration, we optimize the second largest MIS using the proposed model on top of the fixed flights (from the previous iteration). The flights in each set are optimized in parallel on the fixed network, as they are independent, and thus no effect on each other. The protocol continues until all the flights are scheduled.

³Bottleneck link is the last common link in the considered flights' flight plans as shown in Fig. 3.8. At this link, uncertainties will be highest and has the highest potential for an overlap.

Figure 3.8: Illustration of a bottleneck link

It should be noted that at each simulation run, only a single flight is routed and scheduled in the system. The prevailing constraints are applied on both already fixed flights and current scheduling flights, while only the scheduling flight is included in the objective function. Moreover, from the second iteration onwards, we limit the inclusion of the auxiliary binary variable in the capacity constraint (3.12) by removing the *either or* condition (3.10) on several flight pairs. As the flight arrival times of the fixed flights are known, we can accurately predict the precedence of the flight arrival to a link. Uncertainty is only associated with the scheduling flight of that iteration, yet, with its known arrival time bounds (earliest arrival time and most delayed arrival time), the flight arrival precedence can be accurately predicted on most flights. Thus, we do not require to impose both *either or* conditions, rather use one of them based on their arrival order as in (3.25).

$$\text{Capacity Constraint} = \begin{cases} \tilde{T}_{low,j,h}^{f_k} - \tilde{T}_{up,j,h}^{f_i} \geq T_{min}; & \text{if } f_i \text{ enters before } f_k \\ \tilde{T}_{low,j,h}^{f_i} - \tilde{T}_{up,j,h}^{f_k} \geq T_{min}; & \text{otherwise} \end{cases} \quad (3.25)$$

Note: The proposed computational improvement strategy is highly efficient in terms of computation time as only a single flight is optimized at each time. The performance dominantly lies in the density of air traffic, not on the number of aircraft. If the network is dense, i.e. flights are scheduled tightly, the dependency matrix would also be dense, as more flight pairs would meet in their time-based trajectories. This would lead to a greater number of MISs, and ostensibly, to higher computation time. In contrast, if the ATN is sparse, the dependency matrix would

also be sparse, and it will lead to a lower number of MISs. Regardless of the network density, the MIS approach will converge to a solution, as we have enabled the flight cancellation option in the model. When the number of MISs is high, the flights in the last sets will find an already congested network, and thus, a higher probability exists for them to cancel.

In the case of a dense network, one possible improvement is to have a relaxed definition on the dependency, i.e. instead of defining a flight pair as dependent if their trajectories overlap at any point, consider the number of overlaps and set a threshold on which they are considered as dependent. This will help to have a lesser number of MISs, and overall lower solution time.

3.5 Results and Discussions

In this section, we comparatively evaluate the performance of this model and the computational improvement strategy.

3.5.1 Case Study I : Simulation of a Simplified ATN

3.5.1.1 Simulation network

To validate the effectiveness of the proposed model, we used a simplified version of the Association of Southeast Asian Nations (ASEAN) network map, with 2 departure airports, 4 arrival airports, 19 waypoints, and 47 links, as shown in Fig. 3.9. The Changi airport - WSSS (notation 1) and Penang airport - PEN/WMKP (notation 2) are the two departure airports and all other 4 airports, Phuket, Bangkok, Ho Chi Minh City and Kota Kinabalu in notations of 3, 4, 5 and 6 as the arrival airports. The average distance between waypoints is 234 NM.

Two flight levels are considered for en-route links as FL1 and FL2 with altitude of 20,000 ft and 30,000 ft, respectively. In the simulation, we consider zero transition time in changes of flight levels. The flights which have a flight time of lower than 4 hours are considered as short haul flights (40 flights) and all other flights are considered as long haul flights (40 flights). This consists of 10 flights per each airport pair, with an average flight time of 1.6 hours. A minimum time separation of 6 min is used in this simulation.

The number of decision variables to generate 80 flight plans for the network shown in Fig. 3.9 is 44980, among them, 36540 are Boolean variables and all others

Figure 3.9: The simulated simplified version of the ASEAN map

are continuous variables. The optimization problem consists of 43081 inequality constraints and 4921 equality constraints. Short haul flights only use FL1 due to their short flight time and long haul flight use both FL1 and FL2 depending on the weight assignment strategy we proposed in Section 3.3.

3.5.1.2 Calculation of optimal cruising speed

Mach number 0.78 is used for short haul flights (A320) and Mach number 0.85 is used for long haul flights (A380) [90] for the calculation of the nominal cruising speed. Moreover, based on the statistics [91], the speed of sound at altitude 20,000 ft and 30,000 ft are taken as 316 ms^{-1} (614 *knots*) and 303.1 ms^{-1} (589 *knots*), respectively. Using (3.21), TAS at FL1 and FL2 for long haul flights are calculated as 521.9 *knots* and 500.65 *knots*, respectively. Similarly, for short haul flights at FL1, TAS is calculated as 478.92 *knots*. Here, we consider the ground speed is same as the above calculated true airspeed. As the proposed model uses the reciprocal of the speed in the formulations, the optimal values of $Z_{opt,h}^{fi}$ for long

haul flights at two different flight levels of FL1 and FL2 are given by $1.916 * 10^{-3}$ and $1.997 * 10^{-3}$. The $Z_{opt,h}^{f_i}$ of short haul flights (FL1) is computed as $2.088 * 10^{-3}$.

3.5.1.3 Uncertainty distributions of departure time and en-route speed

The two distributions in Fig. 3.3 imply that there is a low probability for early departures than late departures due to the comparatively short tail on the left side. We consider a time slot of 5 min as $[-2 \text{ min}, 3 \text{ min}]$ i.e. early departure offset of 2 min and late departure offset of 3 min from the scheduled time as *on-time* departure, since the current practice is allowed to obey this time slot for the departures. A maximum lower bound of 2 min and maximum upper bound of 15 min from the on-time departure slot are considered for early and late departures, respectively. It should be noted that here we take α as 0 in the simulation, as our primary focus is to eliminate capacity violations due to demand uncertainty completely from the system. So, the maximum early and late departure time uncertainties considered are 4 min and 18 min, respectively. For simplicity, we consider that each en-route link has the same speed uncertainty distribution with a maximum deviation of 10 knots from the nominal speed in either sides.

3.5.1.4 Experimental results

The MIQP Gurobi solver [19] integrated with MATLAB on a PC with an Intel Core (TM) i7-4770 @3.40GHz CPU and an 8GB of RAM, is used in the experiments. We use below described following three models to validate the proposed model. The features of the three models are listed in Table 3.1.

Model I : Fixed Routing, Deterministic

In this case, we change this model in such a way that the flights can only use the optimal path for its routing from the departure airport to the arrival airport. So that, each OD pair consists of only one route (path), which is the shortest path. To convert the model into a deterministic flight scheduling scheme, we set the considered maximum early arrival and late arrival time deviation values to zero, i.e. $\Delta^-(f_i, j, \alpha), \Delta^+(f_i, j, \alpha) = 0$. For the comparison purpose, we further removed the speed deviation minimization and flight level optimization in this case.

Model II : Fixed Routing, Stochastic

Table 3.1: Features of the three models

Features\Models	I	II	III
Departure time and speed uncertainties		✓	✓
Free routing			✓
Speed optimization			✓
Flight level optimization			✓
Flight cancellation	✓	✓	✓

In this case, we use the same routing structure as in model 1, but with actual values for $\Delta^-(f_i, j, 0)$ and $\Delta^+(f_i, j, 0)$ based on departure time and speed uncertainty distributions. Here, we use the maximum lower and upper bound of uncertainties ($\alpha = 0$) to eliminate capacity violation completely. Moreover, model II doesn't consider the speed and flight level optimization.

Model III : Free Routing, Stochastic, Flight Level and Speed Optimization (Proposed)

This case represents the proposed model, which includes free routing, demand uncertainty, speed optimization, and flight level optimization. We use the routing strategy, as explained in Section 3.3 so that the flights will use rerouting paths to minimize the overall delay if the optimal path is congested. This case accounts for the maximum departure time and speed uncertainties as in model II to cope with the demand uncertainty.

Comparative Analysis 1 : Link capacity violations (Conflicts)

The focus of this analysis is to evaluate the importance of demand uncertainty in ATFM. As mentioned earlier, the inherent demand uncertainty can lead to an unexpected arrival of aircraft in the en-routes causing conflicts. To quantify conflicts, we use capacity violations as the performance metric which indicates the situations where minimum separation requirement between flights are not maintained. The following two simulation procedures evaluate the resistance of the tested model against demand uncertainty. First, flight plans are generated using above three models and then, tested for capacity violations using below two procedures.

Procedure 1 : Random departure time deviations and random speed variations are generated within the uncertain limits and embedded into the model generated flight plans.

Procedure 2 : Uncertainties are injected in to the flight plans assuming that the flights will fly in the worst case scenarios, i.e. with the maximum possible uncertainties in departure time and aircraft speed.

The number of capacity violations is analyzed with the new generated flight plans from the above two procedures for all three models, and the results are tabulated in Table 3.2. Four bottleneck links are selected for the analysis as routes from waypoint 3 to 1, 6 to 2, 11 to 10 and 15 to 18.

Table 3.2: Number of capacity violations for three models

Procedures\Models	Model I	Model II	Model III
Procedure 1	23	0	0
Procedure 2	94	0	0

Note : No. of all possible violation cases at the entry and exit of the routes.

According to the capacity violation results, as expected, both stochastic models (II and III), eliminated all possible safety-related capacity violations from the system even under worst-case uncertainty scenarios⁴ote that we have not considered or injected uncertainty related to ascending and descending phases in this work, and thus, in real situation, with these uncertainties, a few number of violations can be expected. Whereas, the deterministic model (model I), displays a serious amount of capacity violation situations (23 for random and 94 under worst cases). These possible capacity violations dramatically increase the controllers' workload and could lead to severe mid-air collisions. As it can be seen from the numbers, this is a serious safety issue in ATFM, which current deterministic models do not address. It is imperatively vital to consider the effect of demand uncertainty in ATFM models, especially, as the effects of demand uncertainty continue to rise dangerously with the expected future growth in ATN [92].

The results highlight the importance of the proposed model, mainly in terms of safety, as it eliminates all safety-related effects from the network due to the consideration of the total uncertainty span in the model. However, this improvement comes at the expense of delay, which we discuss in the next subsection.

Comparative Analysis 2 : Performance evaluation

In this subsection, we evaluated the performance of all three models above in terms of average departure and arrival delays which are the average absolute deviations from their scheduled times, number of cancellations, total remaining aircraft

⁴N

weight (reflection of fuel consumption) at the end of the journey, and average speed deviation from the nominal cruising speed. The results are tabulated in Table 3.3.

Table 3.3: Experimental results for three models

Model	Average departure delay	Average arrival delay	Number of cancellations	Total remaining weight	Average speed deviation
Model I	1.05 min	1.04 min	0	17175	20.21 knots
Model II	10.07 min	10.38 min	2	16100	20.66 knots
Model III	8.31 min	7.71 min	1	26540	15.21 knots

Moreover, we plot the behaviour of capacity violations for the link between way-points 6 and 2 (calculated at both entry and exit points of the link) and average delays with respect to the considered values of $\Delta^-(f_i, j, 0)$ and $\Delta^+(f_i, j, 0)$ for model III and illustrated in Fig. 3.10. The latency distributions of model II and III are also comparatively evaluated for the same uncertainty percentages and graphically illustrated in Fig. 3.11.

Figure 3.10: Capacity violation and latency profiles of Model III against considered uncertainty.

Figure 3.11: Average delay profiles of fixed (II) and free (III) routing.

According to Table 3.2, as expected, the model I displayed the lowest latency results in both departure and arrival flight times. The underlying reason is that the flights are considered as a point in both time and space without considering their departure time and speed uncertainties. Whereas, in both stochastic cases (model II and III), the delay is significantly increased, due to the consideration of demand uncertainty, in which the flight's existence is stretched over time and space horizons based on their maximum uncertainty limits. However, our proposed system (model III), exhibits a lower delay profile than model II, as we additionally incorporate free routing in our system, allowing flights more flexibility to balance off the departure and airborne delays. Furthermore, there exists several flight cancellations in stochastic cases as shown in Table 3.3, as the consideration of maximum possible uncertainty imposes a larger separation between flights and a higher competition for air space.

The effects of the proposed sub-optimizations of deviation of speed and flight level based fuel consumption are distinctly emphasized by the comparative results in Table 3.2. Evidently, model III has displayed the minimum average speed deviation. This is solely due to the fact that, unlike in other models, the proposed

model penalizes any deviation of speed from the corresponding nominal speed in the objective function. The retained total value of weight is an inverse representation of the fuel burning rate. As mentioned earlier, the amount of fuel consumed can be saved by a smooth incrementation of flight levels. The proposed model rewards such flight level assignments, and thus, a higher retainment of weight at the end of the journey are displayed as per Table 3.2.

The Fig. 3.10 and 3.11 depict the importance in consideration of demand uncertainty. According to Fig. 3.10, when the considered uncertainty range increases, the number of possible capacity violations per route decreases but, at the expense of increasing latency, highlighting a trade-off. However, as per Fig. 3.11, the delay increment in the proposed model (III) is limited by the use of free routing due to the availability of multiple routes, in comparison to the fixed routing model (II).

Comparative Analysis 3 : Computational time

Here, we analyse the computational scalability of the proposed MIS and parallel computing based greedy approach. The proposed ATFM model is first solved without any alteration as a global problem via the Gurobi MIQP solver (*MIQP*). Then, the same problem is decomposed into multiple MISs and solved following the approach introduced in the previous section (*MIS*). The results are tabulated in Table 3.4.

Table 3.4: Network parameter values and results of Model III (Proposed)

Parameters	MIQP	MIS
Number of MIS sets	1	13
Solution time	800 s*	2.2 s
Average departure delay	8.31 min	9.67 min
Average arrival delay	7.71 min	14.98 min
No. of cancellations	1	3
Remaining fuel weight	26540	25840
Average speed deviation	15.21knots	10.77knots

* - The solution process terminate with the optimality gap of 1.26%.

According to the results, it can be seen that the optimal method struggles to reach a 0.01 % (default termination gap of Gurobi) optimality gap even after 800 seconds. The massive stack of boolean variables in the model, which escalate the computational complexity, can be stated as the root cause for the very high solution time. On contrary, the proposed MIS based decomposed method has the ability to find a good enough feasible solution, which is close to the solution gained

from the original method, except for the arrival delay. We observed that the flights which are scheduled to be optimized in latter MIS sets struggled to maintain on-time arrival, as in those iterations, all the previously fixed flights already own the airspace in both time and space-wise. This is one of the drawbacks of this approach and requires further research. On the positive side, the proposed computational improvement method has been able to solve the problem in just 2.2 seconds, which is nearly 363 times improvement with respect to the conventional method. The significant edge in the proposed method lies in the parallel optimization of the MIS. Since the flights in a single set are independent of each other, it takes a very small time to optimize a single iteration. In fact, the solution time of the proposed method mostly depends on the number of iterations.

3.5.2 Case Study II : Simulation of ASEAN Air Traffic Network

In this case study, we apply the proposed ATFM model with and without the computational improvement strategy in a real ATN. Our intention here is to analyse the feasibility of the proposed method, especially the significance of the computational improvement strategy in a real-world scenario.

3.5.2.1 Network map

We considered Singapore (SIN), Kuala Lumpur (KL), and Kota Kinabalu (KK) FIRs of the ASEAN region as shown in Fig. 3.12. A total of 103 OD pairs are included. Among them, 28 OD pairs are in KL FIR, 29 OD pairs are in KK FIR and 46 OD pairs are traversing through these 3 FIRs. The simulation conducted for 2 traffic sets as normal and double traffic. A total of 695 flights is tested for the case study of normal traffic as departing and arriving flights. The ATN consists of 275 air links, 78 waypoints and 25 airports within these 3 FIRs.

Two flight levels are considered for en-route links as FL1 and FL2 with altitudes of 20,000 ft, and 35,000 ft, respectively. From the set of normal traffic, 384 flights are short haul and others are long haul flights. The number of decision variables to generate 695 flight plans for the network shown in Fig. 3.12 are 983,211, among them 896,512 are Boolean and all others are continuous variables. The optimization problem consists of 1,939,645 inequality and 67,277 equality constraints. Short haul flights only use FL1 due to their short flight time and long haul flights can use both flight levels depending on the assignment strategy. The number of

Figure 3.12: The simulated version of the ASEAN map

MISs generated in solving the normal, and double traffic scenarios are 35, and 72.

3.5.2.2 Experimental results

The computational times of the proposed model with MIQP Gurobi solver (without any alterations) and the proposed MIS based solution approach for the two traffic cases are tabulated in Table 3.5. The MIQP Gurobi solver takes a very long time to solve the optimal large-scale realistic air traffic scheduling problem, therefore, we terminated the solution process at the 10th hour using the Gurobi parameter tuning feature. The optimality gaps at the termination point are 3.5% and 18.25% for the normal and double traffics, respectively. But, the proposed computational improvement strategy solves the problem in few seconds emphasizing the computational efficiency of our approach for a large-scale realistic flight scheduling problem, even under the consideration of uncertainty.

Table 3.5: Normal and double traffic solution time of ASEAN network

Case Study	MIQP Gurobi	MIS
695 flights	10 hrs*	71 s
1390 flights	10 hrs**	180 s

1) * - Solution process terminate using a time limit. Optimality gap is 3.5%.

2) ** - Solution process terminate using a time limit. Optimality gap is 18.25%.

Our proposed solution approach solves the sub-optimization problem associated with each iteration up to the best optimal solution, regardless of the scale of the

network. Also, as the flights in a single iteration are independent from each other, the solution time of a single iteration is minimal. In terms of the quality of the results, our proposed solution approach generates results in an optimality gap of **0.08%** with respect to the results obtained from the MIQP Gurobi solver at the termination point in normal traffic. This proves that the proposed approach provides nearly optimal results in several seconds compared to the unaltered original model in large-scale problems.

As the proposed computational improvement strategy drastically brings down the solution time, the proposed ATFM model can be applied in real-time at the tactical phase with smaller prediction horizons. At the tactical phase, the weather prediction data are considered as more accurate than at the pre-tactical phase due to the general fact that, closer to the target prediction scenario in time, more the information available about the scenario, and thus, higher the accuracy in prediction. Therefore, the speed uncertainty caused by weather conditions which possibly contain a correlation effect will have a comparatively less impact than the speed uncertainty due to factors such as pilot behaviour, engine efficiency, which are fairly independent. In fact, a real-time ATFM model is more resilient against the uncertainties in air transportation system as the optimization problem is repeatedly executed in response to the changes in the network.

3.6 Summary

In this chapter, we proposed an ATFM model while scrutinizing demand uncertainty due to departure time and speed deviations. The proposed model addresses the pre-tactical phase ATFM, which focuses on generating optimal trajectories considering the 24-hour time horizon in the scheduling process. The MIQP model minimizes the deviations of departure time, arrival time, speed and fuel consumption of individual flights. The safety of the overall system is ensured by maintaining the in-trail separation between flights throughout the flight time. Due to the high complexity of the formulation structure, a greedy strategy with the problem decomposed into MISs is used to solve large-scale problems.

Chapter 4

Two-phase ATFM under Demand and Capacity Uncertainties

4.1 Introduction

Similar to demand uncertainty, the uncertainty of capacity is also a critical factor in the flow management process due to its adverse effects such as capacity violations, conflicts, safety risks, delays, cancellations, etc. None of the existing works has considered both demand and capacity uncertainty in a cooperative manner in ATFM. With this motivation, in this chapter, we consider both demand and capacity uncertainties in the flight routing and scheduling process. The proposed framework plays a key role in ensuring the safety of air transportation system in terms of maintaining the safety separation between flights and furthermore, constraining the capacity of sectors as well.

In comparison to the model presented in the previous chapter, here, we use a chance-constraint-based probabilistic approach to tackle the above uncertainty types. Moreover, a flight level assignment and a speed optimization scheme are proposed based on BADA of EUROCONTROL with the objective of minimizing the fuel consumption. The model further minimizes the overall expected delay of the system using the control actions of ground holding, speed control, rerouting, and flight cancellations. At the implementation stage, two phases of ATFM as pre-tactical and tactical are considered, in which the former focuses on generating the optimal trajectories and the latter focuses on real-time updates of flight plans. The solution approach presented in the previous chapter, which is the greedy strategy with MISs, is used in this chapter, but, further reduces the computational

complexity by shrinking the feasibility region as a preprocessing mechanism.

The results of this chapter are published in [93] and [94]. The main contributions of this chapter can be listed as follows.

1. An ATFM model, which takes the capacity uncertainty, demand uncertainty and their imbalance into account.
2. A Two-phase framework for ATFM addressing pre-tactical and tactical phases.
3. Reduction in capacity violations due to uncertainties of capacity and demand from the system.
4. Guarantee of the safety separation between flights through the entire flight durations, while complying with the sector capacity.
5. A flight level assignment strategy based on the optimization of fuel consumption using BADA from EUROCONTROL.
6. A computationally effective solution process which can be applied in tactical ATFM.

The rest of the chapter is organized as follows. Section 4.2 presents an ATFM model related to the pre-tactical phase in detail. Section 4.3 describes the implementation details of the tactical phase, followed by the solution mechanism. The results of two case studies are shown in section 4.4 and section 4.5 concludes the chapter. The appendix represents the linearization of logical constraints.

4.2 ATFM model

In this section, we present an ATFM problem formulation with respect to the pre-tactical phase of air transportation system.

4.2.1 Notations

We list the parameters and the decision variables used in this model in Table 4.1 below.

CH. 4. TWO-PHASE ATFM UNDER DEMAND AND CAPACITY UNCERTAINTIES

4.2. ATFM MODEL

Table 4.1: Table of Notations

Parameter	Definition
$G = (V, E)$	Graph of vertices (V) and edges (E), where V is a disjoint union of waypoints ($W \subseteq V$) and airports ($A \subseteq V$) and E is the set of links, $j \in J \subseteq E$.
F	Set of flights, $f_k \in F$.
H	Set of flight levels, $h \in H$.
L_j	Length of link j .
l, l'	Departure and arrival links, respectively.
T_{min}	Minimum in-trail time separation between aircraft.
$Y_{j,h}^{f_k}$	This Boolean variable takes 1, if f_k selects route j at FL h .
$X_{w,h}^{f_k}$	This Boolean variable takes 1, if f_k is traversing through waypoint w at FL h .
$v_{j,h}^{f_k}$	Speed of f_k in route j at FL h . The reciprocal is $Z_{j,h}^{f_k}$.
$t_{dep,sch}^{f_k}$	Scheduled departure time of f_k ($dep \rightarrow arr$ for arrival).
$t_{dep,act}^{f_k}$	Actual departure time of f_k ($dep \rightarrow arr$ for arrival).
$T_{arr,w,h}^{f_k}$	Deterministic arrival time of f_k at w through FL h .
$T_{arr,s}^{f_k}$	Deterministic arrival time of f_k to sector s .
$T_{ext,s}^{f_k}$	Deterministic exit time of f_k from sector s .
$\underline{T}_{arr,s}^{f_k}$	Lower bound of entry time of f_k to sector s .
$\bar{T}_{arr,s}^{f_k}$	Upper bound of entry time of f_k to sector s .
$J_w^{f_k}(U)$	Set of upstream air routes connected with waypoint w (or j for link) in the flight f_k 's path. $U \rightarrow D$ for downstream.
$\tilde{T}_{w,h}^{f_k}$	Stochastic traversing time of f_k at w through FL h .
$\Delta_{arr,w}^{f_k}$	Uncertainty of arrival time of f_k at waypoint w .
$\tilde{N}_s^{f_k}(t)$	Stochastic volume of sector s , when f_k enters.
$N_s^{f_k}$	Deterministic volume of sector s , when f_k enters.
$\Delta_{N_s^{f_k}}(t)$	Volume uncertainty of sector s , when f_k enters.
$\tilde{C}_s$	Stochastic capacity of sector s .
W^{f_k}	Remaining weight of f_k at the arrival airport.
$W_w^{f_k}$	Weight of f_k at waypoint w .
$J_{a,\omega'}^{f_k}$	Set of routes from airport a to waypoint ω' that f_k can take.
$t_{j,h}^{f_k}$	Travel time of f_k in route j at FL h .
$\tau_a^{f_i,f_k}$	Turnaround time of f_i to prepare for the next flight f_k at airport a .
$\mathcal{F}^C(f_k, h, low)$	Cruising fuel rates(kg/min) of f_k at low weight at FL h ($low \rightarrow nom$ for nominal weight, $low \rightarrow hi$ for high weight).
$\gamma_{h,h'}^{ROC}(f_k, low)$	Rate of climb from FL h to h' of f_k at low weight ($low \rightarrow nom$ for nominal weight, $low \rightarrow hi$ for high weight).
$\mathcal{F}^C(f_k, h)$	Fuel rate in climbing phase for the nominal weight at FL h (kg/min).
$\gamma_{h,h'}^{ROD}(f_k)$	Rate of descend for the nominal weight at FL h (fpm-feet per minute)
$\mathcal{F}^D(f_k, h)$	Fuel rate in descending phase for the nominal weight at FL h (kg/min)
$\mathcal{V}_h^{f_k}$	True airspeed at each flight level (Knots)
$F_j(f_k, h)$	Cruise fuel consumption of f_k at FL h in link j
$F_w^{f_k}(h, h')$	Fuel consumption of f_k to increase the altitude from h to h' .
$\mathcal{O}_w^{f_k}(h, h')$	This Boolean variable takes 1, if f_k climbs/descends from h to h' at w' .
$T_{dep,late}^{f_k}$	Departure lateness of f_k ($dep \rightarrow arr$ for arrival).
$T_{dep,early}^{f_k}$	Departure earliness of f_k ($dep \rightarrow arr$ for arrival).
$\mu_w^{f_k}, \sigma_w^{f_k}$	Mean and standard deviation of the arrival time uncertainty of f_k at waypoint w , respectively.

4.2.2 Constraints

4.2.2.1 Flight Routing Constraints

The constraint in (4.1) builds the routing path by allowing it to select one of the downstream paths from the set, $J_j^{f_k}(D)$ with respect to its current position j . This flight dispatching process will start from the departure link (l), and then following a set of intermediate links, finally reaches the arrival link (l'), similar to the routing constraint in chapter 3.

$$(\forall f_k \in F)(\forall j \in J) \sum_{h \in H} Y_{j,h}^{f_k} = \sum_{j' \in J_j^{f_k}(D), h' \in H} Y_{j',h'}^{f_k} \quad (4.1)$$

As the model allows flight cancellations, the above constraints are modified at the departure and arrival links as, $Y_{l,h}^{f_k} \leq 1$ and $Y_{l',h}^{f_k} \leq 1$. However, we highly penalize such actions in the objective function.

4.2.2.2 Timing Constraints

Let $(w, w') \in W$ be two consecutive waypoints, where j is the link between them.

- Deterministic arrival time at waypoint w' is the summation of the time at w and the travel time on link j .

$$T_{arr,w'}^{f_k} = T_{arr,w}^{f_k} + \sum_{h \in H} L_j Y_{j,h}^{f_k} Z_{j,h}^{f_k} \quad (4.2a)$$

- Deterministic arrival/exit time to/from a sector (s),

$$T_{arr,s}^{f_k} = t_{dep,act}^{f_k} + \sum_{j' \in J_{a,\omega'}^{f_k}, h \in H} L_{j'} Y_{j',h}^{f_k} Z_{j',h}^{f_k} \quad (4.2b)$$

$$T_{ext,s}^{f_k} = T_{arr,s}^{f_k} + \sum_{j' \in J_{\omega'',\omega'}^{f_k}, h \in H} L_{j'} Y_{j',h}^{f_k} Z_{j',h}^{f_k} \quad (4.2c)$$

where, ω', ω'' are the entry and exit waypoints of flight f_k at sector s , respectively.

- Expected arrival time at an airport (a'),

$$t_{arr,act}^{f_k} = t_{dep,act}^{f_k} + \sum_{j' \in J_{a,a'}^{f_k}, h \in H} L_{j'} Y_{j',h}^{f_k} Z_{j',h}^{f_k} \quad (4.3)$$

- Arrival time uncertainty at a waypoint (w),

Based on the previous flight records of actual and scheduled arrival time of flight at waypoints, the best fitted normal distribution is obtained and represented as $\Delta_{arr,w}^{f_k} \sim N(\mu_w^{f_k}, \sigma_w^{f_k})$. The arrival time uncertainty at a waypoint will consist of its departure time uncertainty and travel time uncertainty along its path.

Although we separate demand and capacity uncertainties as two separate forms, the adverse weather not only affects the capacity uncertainty, but also can cause fluctuations in speed thus, influences the demand uncertainty as well. But, due to the consideration of the uncertainty of time in this model, the flight arrival time uncertainty at any waypoint consists of mainly its departure time uncertainty, speed uncertainty and other minor forms of time uncertainties. In other words, when the model considers the arrival time uncertainty at an en-route waypoint, it consists of all types of uncertain factors, which eventually caused the time deviation, including the speed deviation due to weather conditions as well.

Note:- In order to have a low complexity system model with a linear formulation structure and to make our analysis easily comprehensible without being distracted by technical treatment on complex distributions, we propose this model based on normal distributions. Note that most of the real-world distributions can be approximated by a normal distribution. Thus, if the actual distribution follows a different type of distribution other than normal, our intention is to approximate them by a normal distribution, and then, follow the same procedure as proposed in our model. For an example, there are normal approximation processes for most of the well-known distributions such as Binomial distributions, Poisson distribution, Weibull distribution, Poisson binomial distributions, etc...

Moreover, the study in [4] highlights that the travel time is determined by the link length and the speed of the aircraft traversing it. Further, they have conducted an experiment to analyze the distribution of speed from over 300,000 radar data during one week and have found that the aircraft speed follows a normal distribution as illustrated in Fig. 4.1. Moreover, we conducted an experiment for Singapore Changi airport to determine the departure time distribution using data for 24 hours and found that it can also be represented by a normal distribution as shown in Fig. 3.3a.

Figure 4.1: Speed Distribution in two segments [4]

4.2.2.3 In-trail Separation Constraint

The current literature generally emphasizes on constraining the sector capacity, as scrutinizing the in-trail separation takes a toll on the computational complexity. Yet, several models have considered the in-trail separation [17], absent to acknowledge the uncertainty of time. On contrary, we incorporate the effect of flight arrival time uncertainty in keeping the safety separation between them.

In this model, we use a probabilistic approach called chance constraint [95] to meet the separation requirement between flights with a given reliability level. As stated in chapter 3, if we consider two flights f_k and f_i entering a specific waypoint, there are two possible scenarios as flight f_i reaches the waypoint first and f_k next (scenario 1) and vice versa (scenario 2). We will first explain the chance constraint formulation for scenario 1 in detail and the constraint for scenario 2 will follow the same procedure. In scenario 1, the separation between the entry time of flight f_k ($\tilde{T}_{w,h}^{f_k}$) and the entry time of flight f_i ($\tilde{T}_{w,h}^{f_i}$) need to be higher than T_{min} to keep the head-and-tail distance. However, as emphasized in the introduction, the entry times of flights at the respective waypoint could be deviated due to departure and

travel time uncertainties. Therefore, following chance constraint, we constrain the probability of separation between stochastic traversing times of two flights to be higher than T_{min} at a α reliability level, as in (4.4).

$$P\left(\tilde{T}_{w,h}^{f_k} - \tilde{T}_{w,h}^{f_i} \geq T_{min}\right) \geq \alpha \quad (4.4)$$

For an instance, if $\alpha = 0.9$, then the time separation between flight f_k and f_i will hold for 90% of time. Furthermore, the above probabilistic constraint is converted into a linear interpretation as follows.

$$P\left((T_{arr,w,h}^{f_k} + \Delta_{arr,w}^{f_k}) - (T_{arr,w,h}^{f_i} + \Delta_{arr,w}^{f_i}) \geq T_{min}\right) \geq \alpha \quad (4.5a)$$

Let $\Delta_{arr,w}^{f_i} - \Delta_{arr,w}^{f_k} = b_{arr}(w, f_i, f_k)$

$$P\left(b_{arr}(w, f_i, f_k) \leq T_{arr,w,h}^{f_k} - T_{arr,w,h}^{f_i} - T_{min}\right) \geq \alpha \quad (4.5b)$$

Then, the random variable $b_{arr}(w, f_i, f_k)$ can be represented as, $b_{arr}(w, f_i, f_k) \sim N(\mu_w^{f_i} - \mu_w^{f_k}, \sqrt{\sigma_w^{f_i^2} + \sigma_w^{f_k^2}})$ [95]. Using Cumulative Distribution Function (CDF) ($F(\cdot)$) [95], the probabilistic constraint in (4.5b) is converted to a linear constraint as follows.

$$F_{b_{arr}(w, f_i, f_k)}(T_{arr,w,h}^{f_k} - T_{arr,w,h}^{f_i} - T_{min}) \geq \alpha \quad (4.6a)$$

$$T_{arr,w,h}^{f_k} - T_{arr,w,h}^{f_i} - T_{min} \geq F_{b_{arr}(w, f_i, f_k)}^{-1}(\alpha) \quad (4.6b)$$

By substituting $F_{b_{arr}(w, f_i, f_k)}^{-1}(\alpha)$ from $B_\alpha(w, f_i, f_k)$,

$$T_{arr,w,h}^{f_k} - T_{arr,w,h}^{f_i} - T_{min} \geq B_\alpha(w, f_i, f_k) \quad (4.6c)$$

The values of $B_\alpha(w, f_i, f_k)$ (from this point onward, simply referred as B_α) can be obtained from the cumulative distribution function [96] (let $\mu_i = \mu_w^{f_i} - \mu_w^{f_k}$, $\sigma_i = \sqrt{\sigma_w^{f_i^2} + \sigma_w^{f_k^2}}$, for $B_\alpha \geq \mu_i$),

$$F_b(B_\alpha) = \frac{1}{\sigma_i \sqrt{2\pi}} \int_{-\infty}^{B_\alpha} e^{-(b_{arr} - \mu_i)^2 / (2\sigma_i^2)} db_{arr} \quad (4.7a)$$

$$F_b(B_\alpha) = \alpha = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{B_\alpha - \mu_i}{\sqrt{2}\sigma_i} \right) \right] \quad (4.7b)$$

Hence, the values of B_α can be calculated as in (4.8), using the table of error function values.

$$B_\alpha = \sqrt{2}\sigma_i \operatorname{erf}^{-1}(2\alpha - 1) + \mu_i \quad (4.8)$$

For an instance, $\operatorname{erf}^{-1}(2\alpha - 1) = 0.9$, when $\alpha = 0.9$, thus $B_\alpha = 1.27\sigma_i + \mu_i$. The above approach of converting a probabilistic constraint into a linear constraint can be followed for other distributions such as uniform, gamma, etc., as well [66], [95] and thus, does not lose generality. If scenario 2 happens, following the same steps, the formulation for the separation constraint is obtained as follows.

$$T_{arr,w,h}^{f_i} - T_{arr,w,h}^{f_k} - T_{min} \geq B_\alpha(w, f_k, f_i) \quad (4.9)$$

Depending on the order of flights f_i and f_k , either one of the constraints in (4.6) or (4.9) should guarantee the minimum time separation between flights with a probability of α .

$$\begin{aligned} \text{either } T_{arr,w,h}^{f_k} - T_{arr,w,h}^{f_i} - T_{min} \geq B_\alpha(w, f_i, f_k) \text{ or} \\ T_{arr,w,h}^{f_i} - T_{arr,w,h}^{f_k} - T_{min} \geq B_\alpha(w, f_k, f_i) \end{aligned} \quad (4.10)$$

Repeating the similar steps used in chapter 3, the logical condition ‘either or’ is converted into linear constraints using an auxiliary Boolean variable δ_{f_k, f_i}^w , which takes the value 1 if and only if flight f_k is ahead of flight f_i , and a sufficiently large upper bound M . The value of M is chosen in such a way that, when one constraint is imposed, then the other constraint is to be weakened due to big M [85].

$$T_{arr,w,h}^{f_k} - T_{arr,w,h}^{f_i} - T_{min} \geq B_\alpha(w, f_i, f_k) - M\delta_{f_k, f_i}^w \quad (4.11a)$$

$$T_{arr,w,h}^{f_i} - T_{arr,w,h}^{f_k} - T_{min} \geq B_\alpha(w, f_k, f_i) - M(1 - \delta_{f_k, f_i}^w) \quad (4.11b)$$

Moreover, when several air routes meet at a waypoint, there are possibilities for merging conflicts, if two or more aircraft reach the waypoint without having the safety separation among them. Given that a flight only changes the FL at a way-

point, we can identify three possible patterns of flights going through waypoints as shown in Fig. 4.2.

Figure 4.2: Three possible cases of merging at waypoint w

1. f_1 - go through the waypoint at the same FL
2. f_2 - descend to a lower FL after passing the waypoint
3. f_3 - ascend to a higher FL after passing the waypoint

Here we introduce an additional Boolean variable, $X_{w,h}^{f_k}$ to indicate the flight f_k 's trajectory through the waypoint w at FL h , using any of the above mentioned patterns. The variable $X_{w,h}^{f_k}$ takes the value 1, if f_k passes through waypoint w .

$$X_{w,h}^{f_k} = \max \left\{ \sum_{j \in J_w^{f_k}(U)} Y_{j,h}^{f_k}, \sum_{j' \in J_w^{f_k}(D)} Y_{j',h}^{f_k} \right\} \quad (4.12)$$

The ‘**max**’ condition helps to take into the account both incoming and outgoing flights through the waypoint w . The respective linear constraints are derived in proposition 1 in appendix.

Then, we modify the constraints (4.11a), (4.11b) as (4.13a), (4.13b) to apply the time separation only for the flights that are scheduled to go through the considered waypoint via the respective FL (i.e., $X_{w,h}^{f_k} = X_{w,h}^{f_i} = 1$).

$$T_{arr,w,h}^{f_k} - T_{arr,w,h}^{f_i} - T_{min} \geq B_\alpha(w, f_i, f_k) - M\delta_{f_k, f_i}^w - M(1 - X_{w,h}^{f_k}) - M(1 - X_{w,h}^{f_i}) \quad (4.13a)$$

$$T_{arr,w,h}^{f_i} - T_{arr,w,h}^{f_k} - T_{min} \geq B_\alpha(w, f_k, f_i) - M(1 - \delta_{f_k, f_i}^w) - M(1 - X_{w,h}^{f_k}) - M(1 - X_{w,h}^{f_i}) \quad (4.13b)$$

4.2.2.4 Sector Capacity Constraint

The en-route sector capacity is defined as the maximum number of aircraft within the considered sector that can be handled by the ATCo in a given time period, while allowing an acceptable level of controller workload [97]. The capacity of airspace sectors in a specified time period mainly depends on the ATCo workload, spatial safety separation requirements, and weather conditions. Due to the unpredictable behavior of the controller workload and weather fluctuations, the sector capacity exhibits a level of uncertainty. Therefore, for an accurate flight routing and scheduling process the effect of both airspace capacity uncertainty and demand uncertainty should be considered.

The sector demand/volume (number of aircraft in the sector) will only be increased when a new aircraft enters the sector. Thus, we only evaluate the sector demand at flight entry times. The condition in (4.14) needs to be satisfied to count f_i for the volume of sector s at the time f_k enters it. Simply, f_i should enter the sector s before f_k enters and leave after f_k enters.

$$T_{arr,s}^{f_k} \geq T_{arr,s}^{f_i} \quad \text{and} \quad T_{arr,s}^{f_k} \leq T_{ext,s}^{f_i} \quad (4.14)$$

To count the sector occupancy, we introduce two Boolean variables δ_{f_k, f_i}^s and $\bar{\delta}_{f_k, f_i}^s$ to capture the flight pairs that satisfy the following conditions. The linear interpretation is derived in proposition 2 in the appendix.

$$[T_{arr,s}^{f_k} \geq T_{arr,s}^{f_i}] \longleftrightarrow [\delta_{f_k, f_i}^s = 1] \quad (4.15a)$$

$$[T_{arr,s}^{f_k} \leq T_{ext,s}^{f_i}] \longleftrightarrow [\bar{\delta}_{f_k, f_i}^s = 1] \quad (4.15b)$$

Then, the sector volume, $N_s^{f_k}$ can be calculated as in (4.16).

$$N_s^{f_k} = \sum_{f_i \in F} \delta_{f_k, f_i}^s * \bar{\delta}_{f_k, f_i}^s \quad (4.16)$$

As flights' arrival and exit times at a sector are subjected to uncertainty, similarly, the sector volume also exhibits an uncertainty. At a macroscopic level,

our approach here is to evaluate the sector capacity-demand imbalance and its probability at a considered time, for an instance, at the deterministic entry time of a reference flight f_k ($T_{arr,s}^{f_k}$), and integrate it over the uncertainty domain of the flight f_k 's arrival time, as depicted in (4.17a). Further, we apply a chance-constraint-based probabilistic approach to assert a certain reliability level γ , to the stochastic constraint as follows.

$$\int_{-\infty}^{\infty} P\left(\tilde{N}_s^{f_k}(t) \leq \tilde{C}_s - 1, T_{arr,s}^{f_k} = t\right) dt \geq \gamma \quad (4.17a)$$

$$\int_{-\infty}^{\infty} P\left(\tilde{N}_s^{f_k}(t) \leq \tilde{C}_s - 1 \mid T_{arr,s}^{f_k} = t\right) f(t) dt \geq \gamma \quad (4.17b)$$

$\tilde{N}_s^{f_k}(t)$ can be represented using deterministic and uncertain components as, $\tilde{N}_s^{f_k}(t) = N_s^{f_k}(t) + \Delta_{N_s^{f_k}}(t)$. The stochastic sector volume apparently possesses a Poisson Binomial Distribution (PBD) that is approximated by a normal distribution as shown in procedure 1.

$$\int_{-\infty}^{\infty} P\left(N_s^{f_k}(t) + \Delta_{N_s^{f_k}}(t) \leq \tilde{C}_s - 1 \mid T_{arr,s}^{f_k} = t\right) f(t) dt \geq \gamma \quad (4.18a)$$

Let $Z = \tilde{C}_s - 1 - \Delta_{N_s^{f_k}}(t)$. Here, $\tilde{C}_s$ is a discrete random variable and $\Delta_{N_s^{f_k}}(t)$ is a continuous random variable, and thus, the cumulative distribution function of Z can be derived using the convolution as follows [98].

$$F_Z(z) = \sum_c F_{\Delta_N}(c - 1 - z) \cdot P_{\tilde{C}_s}(c) \quad (4.18b)$$

where $F_{\Delta_N}(\cdot)$ is the cumulative distribution function of $\Delta_{N_s^{f_k}}(t)$ and $P_{\tilde{C}_s}(c)$ is the discrete probability at $\tilde{C}_s = c$. Apparently, $F_Z(z)$ is a sum of scaled cumulative normal distributions.

$$\int_{-\infty}^{\infty} P\left(N_s^{f_k}(t) \leq Z \mid T_{arr,s}^{f_k} = t\right) f(t) dt \geq \gamma \quad (4.18c)$$

$$\int_{-\infty}^{\infty} \left[1 - P\left(Z \leq N_s^{f_k}(t) \mid T_{arr,s}^{f_k} = t\right)\right] f(t) dt \geq \gamma \quad (4.18d)$$

$$\int_{-\infty}^{\infty} f(t)dt = 1,$$

$$1 - \int_{-\infty}^{\infty} F_Z(N_s^{f_k}(t))f(t)dt \geq \gamma \quad (4.18e)$$

$$\int_{-\infty}^{\infty} F_Z(N_s^{f_k}(t))f(t)dt \leq (1 - \gamma) \quad (4.18f)$$

By substituting (4.18b), we can get,

$$\int_{-\infty}^{\infty} \left[\sum_c F_{\Delta_N}(c - 1 - N_s^{f_k}(t)).P_{\tilde{C}_s}(c) \right] f(t)dt \leq (1 - \gamma) \quad (4.18g)$$

As we are unaware about the relationship between flight f_k 's arrival time, i.e., t in this case, and the corresponding sector volume $N_s^{f_k}(t)$, we evaluate the above integral at several discrete points as a simplified approach to solve the aforementioned chance-constraint-based constraint. It should be noted that higher the discrete points, higher the accuracy, but in expense of complexity, and thus, a trade-off to be considered at the implementation.

$$\begin{aligned} & \sum_c \left[F_{\Delta_N}(c - 1 - N_s^{f_k}(t_1)).P_{\tilde{C}_s}(c) \right] f(t_1) + \dots + \\ & \sum_c \left[F_{\Delta_N}(c - 1 - N_s^{f_k}(t_2)).P_{\tilde{C}_s}(c) \right] f(t_2) + \dots + \\ & \sum_c \left[F_{\Delta_N}(c - 1 - N_s^{f_k}(t_n)).P_{\tilde{C}_s}(c) \right] f(t_n) \leq (1 - \gamma) \end{aligned} \quad (4.18h)$$

The linear approximation process of $F_{\Delta_N}(\cdot)$ is described in proposition 3 in appendix.

Procedure 1: Generation of the sector volume distributions

1. To obtain the distribution of $\tilde{N}_s^{f_k}$, we use the previous flight records of actual sector entry, exit times of flight f_k and f_i s. As the sector volume is evaluated with respect to the arrival time of f_k at the sector (with a fix reference point), only the uncertainty in arrival times of f_i s' are considered, thus, the formulation (4.15) is modified as follows.

$$[T_{arr,s}^{f_k} \geq T_{arr,s}^{f_i} + \Delta_{arr,s}^{f_i}] \longleftrightarrow [\delta_{f_k,f_i}^s = 1] \quad (4.19a)$$

$$[T_{arr,s}^{f_k} \leq T_{ext,s}^{f_i} + \Delta_{ext,s}^{f_i}] \longleftrightarrow [\bar{\delta}_{f_k,f_i}^s = 1] \quad (4.19b)$$

2. The probability of flight f_i found inside the sector (i.e., $p(f_k, f_i)$) at the deterministic entry time of flight f_k , which can be also called as ‘success probability’ is defined with respect to (4.20a) and (4.20b). Here, $F(\cdot)$ represents the cumulative distribution function.

$$p(f_k, f_i) = P(\delta_{f_k,f_i}^s = 1) * P(\bar{\delta}_{f_k,f_i}^s = 1) \quad (4.20a)$$

$$p(f_k, f_i) = F_{\Delta_{arr,s}^{f_i}}(T_{arr,s}^{f_k} - T_{arr,s}^{f_i}) * [1 - F_{\Delta_{ext,s}^{f_i}}(T_{arr,s}^{f_k} - T_{ext,s}^{f_i})] \quad (4.20b)$$

3. If we consider all other flights, i.e., f_i s, the event of each flight found inside the respective sector can be treated as a Bernoulli trial, and apparently with different success probabilities. Thus, the sector volume exhibits a PBD¹. As PBD is driven by a very complex PDF and a CDF, especially because of the added complexity when encountered with other distributions, we approximate it to a normal distribution based on the central limit theorem [99] as follows.

$$F_N(k) \approx \phi\left(\frac{k + 0.5 - \mu}{\sigma}\right) \quad (4.21)$$

where $\phi(\cdot)$ is the CDF of the standard normal distribution, and $\mu = \sum_{\forall f_i} p(f_k, f_i)$ and $\sigma = [\sum_{\forall f_i} p(f_k, f_i)(1 - p(f_k, f_i))]^{1/2}$.

4. Applying differentiation to obtain the PDF [95],

$$\frac{d}{dk} F_N(k) = \frac{d}{dk} \left[A\left(\frac{k + 0.5 - \mu}{\sigma}\right) \right] \quad (4.22a)$$

$$\frac{d}{dk} F_N(k) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{k - (\mu - 0.5)}{\sigma}\right)^2/2} \quad (4.22b)$$

$$f_N(k) \sim N(\mu - 0.5, \sigma) \quad (4.22c)$$

¹As we use the actual flight times in (4.19), the flight arrival times are already composed of the correlated delays due to other flights/factors. Hence, the Bernoulli trials are considered as independent.

Thus, the Poisson binomial random variable $\tilde{N}_s^{f_k}$ can be approximated into a normal distribution with mean $\mu - 0.5$ and standard deviation σ .

5. The pure uncertain component can be extracted from the stochastic sector volume as per, $\Delta_{N_s^{f_k}} = \tilde{N}_s^{f_k} - N_s^{f_k}$. For that, first we evaluate the deterministic sector volume, $N_s^{f_k}$, based on the scheduled arrival/exit times of the above same flight set using (4.14) - (4.16). Then the PDF of $\Delta_{N_s^{f_k}}$ can be computed as, $\Delta_{N_s^{f_k}} \sim N(\mu_N, \sigma_N)$, where $\mu_N = \mu - 0.5 - N_s^{f_k}$ and $\sigma_N = \sigma$.

4.2.2.5 Flight overtake elimination constraint

This constraint is similar to the flight overtake constraint in chapter 3 and further modified according to the formulation structure of this model. We use this constraint to maintain the order of flights throughout a given link, and thus, the flight sequence of entering and leaving a link is the same. A flight overtaking scenario is represented in Fig. 4.3. The flight f_k enters the link first and f_i next keeping a minimum separation of T_{min} at waypoint w . Yet, when they leave the link j at waypoint w' , the sequence of flights is reversed, but still keeps the safety separation, which represents a flight overtaking scenario. Hence, we apply the constraint (4.23) to avoid such situations, which simply makes use of the fact that, if the deterministic travel time difference between f_k and f_i (i.e. $t_{j,h}^{f_k} - t_{j,h}^{f_i}$) is less than $t(\phi)$, a flight overtaking is not possible ($t(\phi) = 2T_{min} + B_\alpha(w, f_k, f_i) + B_\alpha(w', f_i, f_k)$). Also, this constraint should only be enforced if both flights enter the link at the same FL, which is emphasized by the path variable $Y_{j,h}^{f_k}$ with big M_ϕ . (Here, $t_{j,h}^{f_k} = L_j Y_{j,h}^{f_k} Z_{j,h}^{f_k}$).

$$-t(\phi) - M_\phi(1 - Y_{j,h}^{f_k}) < t_{j,h}^{f_k} - t_{j,h}^{f_i} < t(\phi) + M_\phi(1 - Y_{j,h}^{f_i}) \quad (4.23)$$

Figure 4.3: An illustration of a flight overtake scenario

4.2.2.6 Flight connectivity constraint

In this model, we consider aircraft with multiple legs, going through sequences of destination airports. In such cases, the aircraft requires a turnaround time to prepare for the next flight. It needs time to refuel, cargo unloading and loading, cleaning, passenger boarding, etc.. For a given flight, let the first leg be f_i , and the second leg be f_k . Then, the two constraints in (4.24a) and (4.24b) state that flight f_k cannot take-off until flight f_i reaches the airport (i.e. $Y_{j'}^{f_i} = 1$) and spend a turnaround time of $\tau_a^{f_i, f_k}$ to prepare for the next flight [22]. Here, J and J' are the departure links of flight f_k and f_i , respectively.

$$Y_J^{f_k} - Y_{J'}^{f_i} \leq 0 \quad (4.24a)$$

$$t_{arr,act}^{f_i} + \tau_a^{f_i, f_k} \leq t_{dep,act}^{f_k} \quad (4.24b)$$

4.2.2.7 Aircraft speed optimization

We use this constraint to maintain a smooth speed variation in adjacent links. Let j and j' be two consecutive links in flight f_k 's path. The constraint (4.25) bounds the speed variation from the current link j to the next link j' to a tight speed bound of ϵ , limiting drastic speed variations. $h, h' \in H$,

$$Z_{j,h}^{f_k} - \epsilon < Z_{j',h'}^{f_k} < Z_{j,h}^{f_k} + \epsilon \quad (4.25)$$

4.2.2.8 Flight level assignment based on BADA fuel consumption data

In this model, we propose a flight level assignment scheme based on the consumption of fuel at each altitude level together with their flight speed. The EUROCONTROL experimental center has developed a database based on the best aircraft performance data, which can be used in trajectory simulation, Air Traffic Management (ATM) research, strategic planning, etc. [5]. Here, we use Performance Table Files (PTFs) of BADA, which contain the cruise, climb and descent performances (fuel consumption, speed, climb/descent rates) at various flight levels for different aircraft types [100]. Thus, we could realistically generate the aircraft's behavior throughout the flight envelop including all the phases of flight. BADA PTFs provide fuel and speed data for different aircraft types under the following categories. An example PTF file for Airbus A306 is shown in Fig. 4.4.

CH. 4. TWO-PHASE ATFM UNDER DEMAND AND CAPACITY UNCERTAINTIES

4.2. ATFM MODEL

BADA PERFORMANCE FILE										Apr 23 2002	
AC/Type: A306__											
Source OPF File:										Mar 26 2002	
Source APF file:										Mar 26 2002	
Speeds:		CAS (LO/HI)		Mach	Mass Levels [kg]				Temperature: ISA		
climb		- 250/300		0.79	low		- 104400				
cruise		- 250/310		0.79	nominal		- 140000		Max Alt. [ft]: 41000		
descent		- 250/280		0.79	high		- 171700				
=====											
FL	CRUISE				CLIMB				DESCENT		
	TAS	fuel			TAS	ROCD		fuel	TAS	ROCD	fuel
	[kts]	[kg/min]			[kts]	[fpm]		[kg/min]	[kts]	[fpm]	[kg/min]
		lo	nom	hi		lo	nom	hi		nom	nom
=====											
0					157	2210	1990	1620	270.3	131	760 97.2
5					158	2190	1970	1600	267.3	132	780 96.1
10					159	2170	1950	1570	264.3	138	800 95.0
15					166	2290	2030	1650	261.5	149	850 94.0
20					167	2270	2010	1620	258.5	181	1020 31.0
30	230	61.2	81.4	104.3	190	2750	2360	1920	253.0	230	1360 25.0
40	233	61.2	81.4	104.4	225	3350	2780	2270	247.7	233	1380 24.5
60	272	65.9	81.7	99.6	272	4210	3070	2370	236.8	240	1410 23.3
80	280	65.8	81.7	99.7	280	4040	2930	2230	225.7	280	1550 22.1
100	289	65.8	81.7	99.8	289	3860	2780	2090	214.8	289	1590 20.9
120	297	65.7	81.7	99.8	356	3820	2800	2170	204.8	332	1880 19.8
140	306	65.6	81.7	99.9	366	3590	2610	2000	194.3	342	1920 18.6
160	389	82.4	93.1	105.3	377	3360	2410	1820	184.1	353	1960 17.4
180	401	82.1	92.9	105.1	388	3120	2220	1650	174.2	363	2000 16.2
200	413	81.7	92.6	104.9	400	2880	2020	1470	164.5	375	2040 15.1
220	425	81.3	92.3	104.7	412	2630	1810	1290	155.0	386	2080 13.9
240	438	80.9	91.9	104.5	425	2380	1610	1100	145.8	398	2120 12.7
260	452	80.4	91.6	104.3	438	2130	1400	920	136.9	411	2160 11.6
280	466	79.9	91.2	104.1	452	1880	1200	730	128.1	424	2200 10.4
290	468	78.4	90.1	103.4	459	1760	1090	640	123.9	431	2220 9.8
310	464	74.3	87.0	101.5	464	2200	1290	660	115.4	444	2250 8.6
330	459	70.6	84.7	100.6	459	1950	1050	420	107.2	459	2290 7.4
350	455	67.6	83.0	97.9	455	1700	810	170	99.2	455	3150 6.3
370	453	65.1	82.0	90.3	453	1320	510	0	91.6	453	2850 5.1
390	453	63.2	81.9	83.0	453	1080	260	0	84.1	453	2850 3.9
410	453	61.9	75.9	75.9	453	830	10	0	77.0	453	2880 2.8
=====											

Figure 4.4: An example of a PTF file for the Airbus A306 aircraft [5]

1. Cruising fuel rates for three different weight categories, low, nominal and high at different flight levels (kg/min) :-
 $\mathcal{F}^C(f_k, h, low), \mathcal{F}^C(f_k, h, nom), \mathcal{F}^C(f_k, h, hi)$
2. Rate Of Climb (ROC) for three different weight categories low, nominal and high at different flight levels (fpm-feet per minute) :-
 $\gamma_{h,h'}^{ROC}(f_k, low), \gamma_{h,h'}^{ROC}(f_k, nom), \gamma_{h,h'}^{ROC}(f_k, hi)$
3. Fuel rate in climbing phase for the nominal weight at different flight levels (kg/min):- $\mathcal{F}^C(f_k, h)$
4. Rate Of Descend (ROD) for the nominal weight at different flight levels (fpm-feet per minute) :- $\gamma_{h,h'}^{ROD}(f_k)$
5. Fuel rate in descending phase for the nominal weight at different flight levels (kg/min):- $\mathcal{F}^D(f_k, h)$
6. True airspeed at each flight level (Knots):- $\mathcal{V}_h^{f_k}$

The objective is to assign aircraft to specific flight levels so that the total fuel consumption during the flight will be minimized. The remaining fuel at the end of the journey is the reduction of fuel burn in cruising, ascending and descending phases from the initial take-off weight. We describe the fuel burn amount when the flight is in cruising and climbing/descending phases in sub topics separately.

Calculation of fuel consumption at the cruise phase

BADA provides fuel consumption data when a flight is in cruising phase for three reference weight levels as low, nominal and high, at each FL. Thus, the fuel rate at these three weight levels are interpolated to obtain a general expression for the relationship between aircraft weight with the fuel rate for a considered FL as shown in Fig. 4.5a.

Let's consider two consecutive waypoints w and w' (the link between them as j). When an aircraft flies from waypoint w to w' along the link j , the weight of the aircraft reduces as the fuel burns. By integrating the fuel rate over time for the duration of travel (refer Fig. 4.5b), the value of $F_j(f_k, h)$, which is the fuel consumption of f_k in link j at FL h can be obtained as in (4.26).

$$F_j(f_k, h) = \int_0^{L_j/\mathcal{V}_h^{f_k}} \left((m_h W_w^{f_k} + c_h) + \frac{(m_h W_w^{f_k} + c_h) - (m_h W_{w'}^{f_k} + c_h)}{L_j/\mathcal{V}_h^{f_k}} t \right) dt \quad (4.26a)$$

Figure 4.5: Representation of aircraft weight, cruise fuel consumption and travel time in a link

$$F_j(f_k, h) = \frac{L_j}{\mathcal{V}_h^{f_k}} \left[\frac{m_h(W_w^{f_k} + W_{w'}^{f_k})}{2} + c_h \right] \quad (4.26b)$$

where m_h and c_h are the slope and intersection values obtained from the graph in Fig. 4.5b.

The formulation in (4.27) calculates the remaining weight of f_k at waypoint w' (i.e. $W_{w'}^{f_k}$) with its weight at waypoint w (i.e. $W_w^{f_k}$) as a reference.

$$W_{w'}^{f_k} = W_w^{f_k} - \sum_{h \in H} Y_{j,h}^{f_k} F_j(f_k, h) \quad (4.27)$$

Estimation of fuel consumption in climbing/descending phases

Here, we first separate the end waypoint of the considered link, j into two virtual sub waypoints as w' and w'' as illustrated in Fig. 4.6a. As shown in Fig. 4.7a, BADA provides ROC for three weight categories of low, nominal and high at each FL². From them, we can obtain the graph in Fig. 4.7b. The area under the curve in Fig. 4.7b gives the time to climb from altitude h to h' .

Then, the value of $F_{w'}^{f_k}(h, h')$, which is the consumed amount of fuel for flight f_k

²For the cases, where the flight descend from a higher flight level to a lower level, BADA has only provided the ROD and fuel rate for a reference mass level, as aircraft weight is less influential in descending.

Figure 4.6: Reference figures to describe the cruising and climbing stages

Figure 4.7: Representation of aircraft weight, climbing height and rate of climb

to increase the altitude from h to h' is given by,

$$F_{w''}^{f_k}(h, h') = H(h, h') \left[\frac{m'_h(W_{w'}^{f_k} + W_{w''}^{f_k})}{2} + c'_h \right] \mathcal{F}^C(f_k, h) \quad (4.28)$$

where m'_h and c'_h are the slope and intersection values obtained from the graph in Fig. 4.7a.

The weight of flight f_k after it increases the altitude from h to h' (from sub waypoint w' to w'') is given by (4.29).

$$W_{w''}^{f_k} = W_{w'}^{f_k} - \sum_{h \in H} \sum_{h' \in H} \mathcal{O}_{w'}^{f_k}(h, h') F_{w'}^{f_k}(h, h') \quad (4.29)$$

where $\mathcal{O}_{w'}^{f_k}(h, h') = \sum_{j \in J_w^{f_k}(U)} Y_{j,h}^{f_k} \sum_{j' \in J_w^{f_k}(D)} Y_{j',h'}^{f_k}$ with the linear form is repre-

sented under the proposition 4 in the appendix.

$$\mathcal{O}_{w'}^{f_k}(h, h') = \begin{cases} 1; & \text{if } f_k \text{ climbs/descends from } h \text{ to } h' \text{ at } w' \\ 0; & \text{otherwise} \end{cases}$$

For the cases, where there are multiple paths to reach a waypoint as shown in Fig. 4.6b, the formulation in (4.27) is slightly modified as in (4.30). According to Fig. 4.6b, a flight can reach the waypoint w_4 either following the path through w_2 or w_3 . In such situations, in order to identify the previous reference waypoint through which the considered flight traveled, we use the minimum function as in (4.30), which is presented in Fig. 4.6b as a reference, for better comprehension.

$$W_{w_4}^{f_k} = \min\{W_{w_2}^{f_k}, W_{w_3}^{f_k}\} - \sum_{j \in J_{w_4}^{f_k}(U), h \in H} Y_{j,h}^{f_k} F_j(f_k, h) \quad (4.30)$$

The above cruising and ascending/descending fuel consumption calculations are repeatedly applied for subsequent waypoints along the flight path and finally, we can calculate the remaining fuel level at the destination airport (W^{f_k}). In the objective function, we maximize the remaining fuel amount at the arrival airport to minimize the fuel consumption during the flight in order to reduce the airline's cost for fuel and furthermore, to reduce the CO_2 emission to the environment.

4.2.3 Objective Function

The overall objective of this model is to minimize the total flight cancellations, total expected delay cost and the fuel consumption cost while optimizing the en-route flight speed with increased safety of the overall system.

$$\min \sum_{f_k \in F} \left\{ \phi_{d1} T_{dep,late}^{f_k} + \phi_{d2} T_{dep,early}^{f_k} - \phi_w W^{f_k} + \phi_{a1} T_{arr,late}^{f_k} + \phi_{a2} T_{arr,early}^{f_k} + \phi_c^{f_k} (1 - Y_I^{f_k}) \right\} \quad (4.31)$$

where,

$$t_{dep,act}^{f_k} - t_{dep,sch}^{f_k} = T_{dep,late}^{f_k} - T_{dep,early}^{f_k} \quad (4.32a)$$

$$t_{arr,act}^{f_k} - t_{arr,sch}^{f_k} = T_{arr,late}^{f_k} - T_{arr,early}^{f_k} \quad (4.32b)$$

The equalities in (4.32) together with minimization of those terms in the objective, offload the respective time deviations to corresponding late or early variables. Moreover, we bound the departure and arrival times with lower and upper bounds to limit their variation within the permissible bound. The third term maximizes the remaining amount of fuel at the end of the journey while optimizing the speed of the flight. The last term is to minimize the cost of flight cancellations.

The departure time coefficients are selected as, $\phi_{d_1} \gg \phi_{d_2}$ to minimize the late departures with higher priority. If all passengers are on board and everything is set, the flight can leave the gate, allowing early departures. In regards to the arrival time coefficients, $\phi_{a_1} \gg \phi_{a_2}$ which minimize late arrivals with higher priority. Mostly, almost all passengers prefer to arrive early to their destination. If the aircraft can reach the arrival airport before its planned time, but, within the permissible bound of early arrival (without incurring any fine for the airlines), we truly do not want to minimize it with high cost. Overall, the coefficients are related as, $\phi_c^{fk} \gg \phi_{a_1} \gg \phi_{d_1} \gg \phi_{a_2}, \phi_{d_2} \gg \phi_w$, with the highest cost comes from flight cancellations. The proposed MILP problem can be summarized as,

minimize (4.31)

subject to (5.1), (4.13), (4.18), (4.23), (4.24), (4.25).

4.3 Short Tactical Scheduling and Solution Mechanism

4.3.1 Short Tactical Scheduling Procedure

The formulations presented in the previous section correspond to the pre-tactical phase and they provide a comprehensive analysis of feasibility and optimized trajectories for flights in advance. In the tactical phase, which takes place on the actual day of operation, we fine tune the provided flight plans based on the updated network conditions and air traffic. ATFM events are associated with less uncertainty at the tactical phase in comparison to the pre-tactical phase due to the general fact that, closer the target prediction scenario in time, more the information available about the scenario, higher the accuracy in prediction. For instance, the weather prediction data are considered as more accurate and moreover, the departed flights are only associated with travel time uncertainty, and the departure time is deterministically known. Thus, the tactical phase can generate more

accurate flight plans with lower prediction errors. We make use of this advantage and propose a short tactical flight scheduling model for the tactical phase with a shortened horizon, which is to be executed periodically with a sliding horizon window.

In this model, several modifications, essentially simplifications, are made to the above formulations to adjust it accordingly for a short-term ATFM. However, the flight plans generated from the pre-tactical phase are not massively disrupted in the tactical phase. In fact, the BADA based flight level assignment is removed, and the flights are incentivized to use optimal trajectories generated at the pre-tactical phase. We do that by penalizing any deviation in the trajectory variables (let $\bar{Y}_{j,h}^{f_k}$ be the set containing all trajectory variables whose value is 1 from the results of the pre-tactical phase) in the objective function. Thus, the new objective function can be depicted as in (4.33). (ϕ_t - cost coefficient of flight deviations)

$$\min \sum_{f_k \in F} \left\{ \phi_{d_1} T_{dep,late}^{f_k} + \phi_{d_2} T_{dep,early}^{f_k} + \phi_{a_1} T_{arr,late}^{f_k} + \phi_{a_2} T_{arr,early}^{f_k} + \phi_t \sum_{\bar{y}_{j,h}^{f_k} \in \bar{Y}_{j,h}^{f_k}} (1 - \bar{y}_{j,h}^{f_k}) \right\} \quad (4.33)$$

Hence, in the tactical phase, the main influential control actions are ground delay, and airborne delay. However, the flights can still be canceled in situations such as extreme weather conditions, high congestion, etc. The effects of $\Delta_{arr,w}^{f_k}$ and $\Delta_{arr,w}^{f_i}$ components in (4.5) will be reduced, resulting in smaller $B_\alpha(w, f_k, f_i)$ values in (4.13), and so as the components of $\Delta_{arr,s}^{f_i}$ and $\Delta_{ext,s}^{f_i}$ in (4.19). In addition to the above modifications in the formulations, we propose to have several operational modifications as well, which are depicted in Procedure 2, to adapt the current model in a short tactical process with short and sliding horizons.

Procedure 2: Short tactical flight scheduling procedure

1. Let n be the scheduling period and F_n be the set of the original flights to be scheduled in the n^{th} period with the following condition satisfied.

$$T_n^- < t_{dep,sch}^{f_i} \leq T_n^+$$

Here, T_n^- and T_n^+ indicate the starting and end times of the scheduling period, respectively.

2. Further, $F_{n_1} \subseteq F_n$ and $F_{n_2} \subseteq F_n$ refer to the subsets of immediate, n_1 and future, n_2 flights, which satisfy:

Figure 4.8: Time axis representation of flights in two scheduling periods

$$F_{n_j} = \{\forall f_i \in F_n | T_{n_j}^- < t_{dep,sch}^{f_i} \leq T_{n_j}^+\}$$

- However, the flights optimized in a scheduling period not only consist of F_n , rather an accumulation of F_n and F_{n-1}^{fwd} , in which latter refers to the **set of flights brought forward** from the previous scheduling period of $n - 1$. Thus, the set of flights to be optimized in the n^{th} iteration, $\bar{F}_n$ can be represented as,

$$\bar{F}_n = F_n \cup F_{n-1}^{fwd} \text{ with,}$$

$$\bar{F}_{n_1} = \{\forall f_i \in \bar{F}_n | t_{dep,sch}^{f_i} \leq T_{n_1}^+\} \text{ and } \bar{F}_{n_2} = \{\forall f_i \in \bar{F}_n | T_{n_1}^- < t_{dep,sch}^{f_i} \leq T_{n_2}^+\}$$

- The results of the n^{th} iteration consist of following.

$\bar{F}_n^C$: Flights canceled during the scheduling period n , $\bar{F}_n^L$: Flights landing before $T_{n_1}^+$, $\bar{F}_n^A$: Flights remaining on the air at $T_{n_1}^+$, $\bar{F}_n^R$: Rest of the flights [$\bar{F}_n^R = \bar{F}_n - (\bar{F}_n^C + \bar{F}_n^L + \bar{F}_n^A)$]

Note: In the first scheduling period, $\bar{F}_n = F_n$. As a higher priority is given to the immediate flights, $\bar{F}_{n_1}$ and $\bar{F}_{n_2}$ will be optimized sequentially.

- From the results, the flights taken to the next scheduling period, F_{n-1}^{fwd} , are the flights which are not scheduled in the previous iteration, and possible to schedule in the next iteration within their departure bounds.

$$F_{n-1}^{fwd} = \{\forall f \in \{\bar{F}_{n-1}^C + \bar{F}_{n-1}^R\} | t_{dep,sch}^{f_i} + \lambda^f \geq T_{n_1}^+\}$$

where λ^f is the permissible bound for the late departure from its scheduled take-off time.

- In a general n^{th} iteration ($n > 1$),

- Airborne flights from the previous iteration are injected into their last known locations (taken from radar data).
- The hourly updated ATN data are coupled with the $\bar{F}_n$ flights.

- The constraints are applied for the airborne flights ($\bar{F}_{(n-1)}^A$) and flight of this iteration ($\bar{F}_n$).
- The objective function is applied only for the ($\bar{F}_n$).

4.3.2 Solution Mechanism : Preprocessing followed by Greedy Strategy with Maximum Weighted Independent Sets (MWIS)

Major computational barriers of this model are also the involvement of logical conditions and the flight-by-flight formulation structure. The formulations structure, even though increases the complexity, provides valuable detailed flight information in comparison to other macro models. Thus, we use a simple heuristic preprocessing mechanism followed by a MWIS based greedy approach to minimize the computational complexity of the proposed model. Hence, it can be applied at the tactical phase even in large-scale systems. Getting the solution time to a reasonable level is vital, especially in the short tactical phase, as the model is to be repeatedly executed with real-time updates.

Basically, the preprocessing mechanism removes all unnecessary variables, fix all deterministically known variables, and tightly bounds the scope of the remaining variables based on the network configuration. For an example, flights do not go through all the links, and waypoints in the network, and thus, the variables related them can be removed. Also, some flight pairs are scheduled so far apart, we can accurately estimate their entering orders to links and waypoints, hence, Boolean variables related to these pairs in in-trail separation (4.13) and sector capacity constraints (4.35) can be deterministically estimated and fixed. In fact, we were able to remove and fix around 67% of Boolean variables in the model using the preprocessing method. Furthermore, tightly setting the variable bounds, effectively shrinks the feasible region of the problem, hence, considerably improves the solution process.

Then, the resulting model is solved using a greedy approach with the problem decomposed into a MWIS. The operational procedure is summarized in the Algorithm 3, which is a modified version of algorithm 1 in chapter 3.

Algorithm 3 Greedy approach with MWIS

Step 1: Convert the ATN into a weighted graph, $G_0 = (V_0, E_0)$. $\triangleright V_0$ - Flight set, E_0 - Flight dependency set.

Flights are selected as nodes in the graph, and they are weighted based on their scheduled departure time and short/long haul status as shown below. Flights with higher weights are given priority.

$$\omega^f = \omega_{dep} \cdot (T_{max} - t_{dep,sch}^f) + \omega_{sh} \cdot I_{sh}^f + \omega_{lh} \cdot I_{lh}^f$$

where I_{sh}^f, I_{lh}^f represent the flight f 's short/long haul status, respectively. $\omega_{dep}, \omega_{sh}, \omega_{lh}$ are the weights assigned for the factors of departure time, short haul, long haul, respectively. $\omega_{dep} > \omega_{sh} > \omega_{lh}$. T_{max} is a constant.

An edge between two nodes, i.e., between two flights, is formed if the two flights are dependent according to the definition 1.

Definition 1: Two flights are considered as dependent, if their trajectories are to be overlapped or their time separation is to be less than T_{min} , in any time period, which includes uncertainty influenced worst case scenarios.

Step 2: Search and obtain all MISs in the graph [101]. In order to find consecutive MISs, nodes from the previously extracted MISs are removed from the graph with all their edges. For every two flights in a MIS, there is no edge connection which represents the independence of the two flights. Let $S_1, S_2, ..S_n$ denote the MISs of the network.

Step 3: Initial set, Fixed = ϕ . $\triangleright$ An empty set. However, in the real-time process, an initial set may contain on-air flights.

Step 4: while (Fixed $\neq V_0$) $\triangleright$ From S_1 to S_n

1: Schedule (optimize) S_i on the **Fixed** flight set $\triangleright$ As all flights in a S_i are independent from each other, they essentially can be optimized in parallel, cutting down the solution time of a single iteration drastically.

2: Fixed $\leftarrow$ (Fixed $\cup S_i$) $\triangleright$ Accumulate fixed flights

Step 5: **Fixed** set contains, the routed and scheduled flights.

4.4 Simulation Results

In this section, we evaluate the performance and the real-time applicability of our proposed ATFM framework. The results are taken with two simulation networks, and the platform as MATLAB.

4.4.1 Simulation Study I: Experiment with a Simplified ATN

We use a similar ATN used in chapter 3 with some modifications (Fig. 4.9) to validate the proposed model and to analyze the performance of the model in detail as shown in Fig. 4.9. The network consists of 2 departure airports (Changi, Penang), 4 arrival airports (Bangkok, Phuket, Ho Chi Minh City, Kota Kinabalu), 19 waypoints, 47 air routes and two sectors as S_1 and S_2 . A total of 80 flights with an average flight time of 1.6 hours is considered with 40 of them as short haul (flight time < 4 hours [102]) and the rest as long haul. A minimum time separation of 6 min is entertained. Three flight levels are used with altitudes of 29,000ft (290FL), 33,000ft (330FL) and 37,000ft (370FL). Further, we use BADA data for the fuel rate at cruise and climb/descent phases, rate of climb/descent and aircraft speed data related to three long haul aircraft types of A320, A319, B738 and one short haul aircraft type of ATR72.

4.4.2 Simulation study II: Experiment with the ASEAN ATN

In this case study, we use the real air traffic network and data of the ASEAN region to assess the feasibility of the proposed model in a realistic scenario. As stated in chapter 3, the network consists of three FIRs of Singapore, Kuala Lumpur and Kota Kinabalu as shown in Fig. 3.12 with 103 OD pairs. Two traffic conditions referred to as normal and double are considered in the simulation with 695 flights and 1390 flights, respectively. The normal traffic is the existing actual traffic in the above 3 FIRs, whereas double traffic is generated by doubling the normal traffic with a 15min span. The considered airspace consists of 275 air routes, 78 waypoints, 25 airports. For simplicity, the FIR boundaries are considered as the sector boundaries, thus, three sectors are considered. Three flight levels with altitudes of 25,000ft (250FL), 32,000ft (320FL) and 40,000ft (400FL) are considered. Among 695 flights, 384 flights are short haul flights and the rest are

Figure 4.9: Simplified ATN for the simulation study I

long haul flights.

4.4.3 Experimental Results of Simulation Studies I and II

4.4.3.1 Sector capacity violations

Here, our intention is to illustrate the importance of the proposed model in reducing uncertainty influenced sector capacity violations, in air transportation system. For that, we run the model at the pre-tactical phase described in section 4.2 while considering both demand and capacity uncertainties (propose model), and also, without considering the uncertainty (reference model). The chance constraint probability γ in (4.17) is taken as 0.8.

In simulation study I, we take the capacity variation of both sectors as 6, 8, 10, 12 and 14 with associated probabilities of 0.05, 0.1, 0.2, 0.4 and 0.25, respectively. In the reference model, we set the sector capacity to the ‘maximum capacity’ of 14. Similarly, we conduct the experiments for the real network in Fig. 3.12 with the Discrete Probability Distributions (DPDs) shown in Table 4.2 for the same uncertainty settings.

Table 4.2: DPDs of the three sectors in the ASEAN map

(a) DPD of Sector 1 (S_1 - KL)						(b) DPD of Sector 2 (S_2 - SIN)					
Cap.	18	19	20	22	24	Cap.	13	14	15	16	18
$P(S_1)$	0.05	0.1	0.3	0.4	0.15	$P(S_2)$	0.05	0.1	0.4	0.3	0.15

(c) DPD of Sector 3 (S_3 - KK)						
Cap.	14	15	16	17	18	19
$P(S_3)$	0.05	0.05	0.1	0.4	0.2	0.2

The results are illustrated in Fig. 4.10 for the two simulation studies. For a better explanation, we have marked two extreme capacity possibilities (minimum and maximum capacity) to visually depict the possible sector capacity violations. The demand-capacity imbalance is only analyzed for the sector 1 in the simulation study I (Fig. 4.10a) as the sector 1 facilitates most of the traffic.

As mentioned above, both capacity and demand are subjected to uncertainty. The sector capacity can vary in between the minimum and maximum levels, owing to various situations such as weather conditions, and similarly, the number of flights in the sector can vary owing to departure and travel time uncertainties. From the results, it is evident that the sector demand of the reference model fluctuates more closer to the maximum capacity than the proposed model. Thus, when the sector capacity drops due to unforeseen reasons, this can lead to severe demand-capacity imbalances. In terms of numerical figures, in Fig. 4.10a, if the sector capacity drops to the minimum, the reference model will undergo 22 violations, whereas, the proposed model will go through 15. For an instance, if the sector capacity drops to its mid value 10, then the proposed model is still free of violations, yet, the reference model will sustain 6. Similarly, in Fig. 4.10b-4.10d, if sector capacities drop to 20, 14, and 17 in three respective sectors, the proposed model will not incur any violations, whereas the reference model will sustain a significant amount of violations, in terms of numerical figure, an aggregation of 31. This becomes severe, if the sector capacities are dropped to their minimums. However, probabilities of such realizations are very less. In fact, we can estimate the expected number of violations in three sectors considering the respective probabilities in Table 4.2, to be 1.4, 0.05, 1.85 for the proposed model, whereas the same values are much higher, 13.5, 3.9, 7.2, for the reference model. The above numerical figures clearly indicate the higher possibility for capacity violations in deterministic ATFM models, and moreover, comparatively highlight the importance of handling demand-capacity imbalance while scrutinizing the uncertainty.

Figure 4.10: Demand and capacity of sectors with respect to time in simulation study I and II.

4.4.3.2 In-trail separation violations

Maintaining the sector capacity does not necessarily guarantee a safe ATN. As emphasized in the introduction, departure and travel time uncertainties can lead to violations in in-trail separations between flights, which is a major safety issue. Thus, the focus of this analysis is to highlight the importance of considerations of the above uncertainties in ATFM. Here, we consider that the departure time uncertainty is a normal distribution with, $\Delta_{dep}^{f_k} \sim N(6.9min, 2.7min)$ and the travel time uncertainty (per nautical mile) is a normal distribution with, $\Delta_{trv}^{f_k} \sim N(0, 0.15s)$. The value of α in (4.5) is varied between 0 and 1 to evaluate the in-trail separation violations and average departure and arrival delays. The results are taken at the pre-tactical phase and plotted in Fig. 4.11 for both simulation studies. As expected, the number of separation violations is decreased with the increase of α , while sacrificing the departure and arrival delays. According to the results, a balanced value for α can be stated as 80%, as drastic slopes are visible

in both departure and arrival delays, as soon as α moves up from 80%. This highlights the trade off between the separation violations and the latency profiles.

Figure 4.11: In-trail separation violations and latency profiles against α

When a higher probability value is assigned for α , the model forces the flights to keep a much higher separation ($> T_{min}$), so that, even under uncertainty, the chances of separation being lower than T_{min} is very less. Apparently, this affects negatively on flight delays. At the pre-tactical phase, the presence of uncertainty is very large, as most of the events, including flight departure, are almost stochastic. However, at the tactical-phase, as certain events such as departure time of departed flights, are deterministic, the above negative effect can be minimized by means of the proposed short tactical approach formulated in section 4.3. Through the short tactical approach, the flight plans are modified based on the updated network conditions and uncertainty, thus, minimizes unnecessary delays. This allows to use a higher chance constraint probability to ensure that the constraints are satisfied with higher probabilities and with lower flight delays. In the next topic, we experimentally depict this achievement.

4.4.3.3 Pre-tactical and short tactical performance comparison

In Table 4.3, we have listed the improvement of the tactical phase model in comparison to the pre-tactical phase model in terms of average flight delay, which is the average absolute deviation from the scheduled values and the number of cancellations.

The short tactical average delay values and the number of cancellations in Table 4.3 are always less than the pre-tactical phase values. As the pre-tactical phase is

Table 4.3: Pre-tactical and tactical phase performance matrices

No. of Flights	Scheduling Stage	Performance parameters		
		Departure Delay	Arrival Delay	Number of Cancellations
80	Pre-tactical	2.14 min	1.42 min	0
	Short tactical	1.23 min	0.75 min	0
695	Pre-tactical	4.35 min	4.04 min	4
	Short tactical	3.26 min	2.41 min	0
1390	Pre-tactical	11.01 min	11.12 min	184*
	Short tactical	4.69 min	1.92 min	76*

* - The reason for the higher number of cancellations is the strict bound of departure time within a window of 2 hour.

executed a couple of days prior to the actual operation, and also, since a higher prediction horizon is entertained, the uncertainty associated with the events remains at high, compelling the model to treat them as stochastic variables, and thus, a degraded performance. However, with the proposed short-horizon procedure, the real-time updates of the network such as flight departed times, weather conditions can be incorporated, and thus, more deterministic events. Moreover, the remaining stochastic events will also have a lower prediction error. Thus, it allows to occupy the airspace for more aircraft, while reducing flight delays as well as cancellations as experimentally shown by the results in Table 4.3.

4.4.3.4 Computational complexity

Here, we analyze the computational time of the models in pre-tactical and tactical phases, and also, in short tactical, to validate the proposed model's applicability in a real environment. By means of 'pre-tactical', we refer to the model introduced in section 4.2. The term, 'tactical' refers the model with a 24 prediction horizon. The term 'short tactical' points to the model with a short prediction horizon and a sliding window approach, which is repeatedly applied with real operation. The MILP Gurobi solver [19] integrated with MATLAB on a PC with an Intel Core (TM) i7-4770 @3.40GHz CPU and an 8GB of RAM, is used in the experiments, and the results are tabulated in Table 4.4.

From the results, we can see that the MILP Gurobi solver cannot solve the global problem even after 3 hours due to the high computational complexity in both pre-tactical and tactical phases. However, our proposed solution scheme including the pre-solving mechanism and the MWIS approach has managed to reduce the computational burden and its enhanced output is visible from the results tabulated

Table 4.4: Solution time performances

Number of Flights	Pre-tactical		Tactical		short tactical
	T_{org}	T_{MIS}°	T_{org}	T_{MIS}°	T_{avg}°
695	*	4.9 min	*	1.7 min	0.7 min
1390	*	39.8 min	*	7.7 min	3.1 min

* - cannot solve even after 3 hours. T_{org} - without any computational improvement, T_{MIS} - with the pre-solving part and MWIS approach, T_{avg} - Average iteration time (solution time of a window), $^\circ$ - without matrix generation time.

under T_{MIS} . For the case of normal traffic, the problem can be solved in a few minutes in both pre-tactical and tactical phases. The pre-tactical phase solution time is comparatively high since it solves the complex ATFM problem formulated in section 4.2. However, at the tactical phase, the computational complexity is reduced since it only needs to fine tune the flight times on top of the optimal trajectories obtained from the pre-tactical phase. Moreover, the average iteration solution time is less than one minute in the normal traffic scenario and 3.1 min in the double traffic scenario, which emphasis how quickly the flight plans are updated in real-time.

4.4.3.5 Cost calculation

In this subsection, we perform a cost comparison between the proposed short tactical model (named as ‘Short tactical’) and a Primary model (named as ‘Primary pre-tactical’) to evaluate the financial improvements yielded by the proposed ATFM framework. The Primary model is constructed by excluding the BADA based flight level assignment, fuel optimization, uncertainty consideration and the short tactical scheduling process. The calculated cost values are tabulated in Table 4.5.

Table 4.5: Comparative Cost calculations (in USD)

Cost Factors	695 flights		1390 flights	
	Primary pre-tactical	Short tactical	Primary pre-tactical	Short tactical
Passenger cost*	48,978	34,952	502,392	230,953
Airlines cost	34,488	4,160	467,021	140,874
Cancellation cost**	0	0	553,920	438,520
Fuel cost	1,007,201	946,481	1,700,961	1,517,458

* - Assuming the flights are occupied up to 2/3 of the aircraft’s maximum passenger capacity.

** - These values only include the losses in tickets.

air transportation system is associated with numerous stakeholders, and the events taken place in air transportation system directly affect them. In an ATFM perspective, the main stakeholders can be shortlisted to passengers and Airline Operators (AOs). The main parameters that affect can be listed to, flight delays, cancellations, fuel cost, and ground operational costs. Passengers are mostly sensitive to the flight delays, to be more specific, arrival delays. A passenger's value of time has been addressed, and pointed to a 26.7 USD/hour in [103]. Thus, we estimate the 'passenger cost' with respect to arrival delay and the above numerical figure as a reference. Similarly, AOs are also financially struck by the flight delays. For them, both departure and arrival delays are important, as departure delays can also lead to more gate/ground operational charges. Yet, to get a numerical representation, we only estimate the cost due to arrival delays. According to [103], the cost due to arrival delay has a piecewise linear representation, and the cost of first 15 min is essentially zero. After 15 min, the cost is aggregated to a 28.6 USD/hour/passenger, which is mainly dictated by the expenses due to missing connecting flights. The highest cost associated with air transportation system is the flight cancellation cost. As per [104], the calculated cancellation cost only includes the losses in tickets, which is in average 5770 USD per each canceled flight. However, flight cancellations can incur many additional charges for AOs including rescheduling crews, transportation, hotel costs, etc., which can be accumulated even up to 40,000 USD depending on the flight type [104]. The last factor is the 'fuel cost', which is calculated as $(initial\ weight - remaining\ weight) * (per\ kg\ fuel\ price)$ [105].

Based on the calculated cost values, it can be clearly seen that all the cost values in the proposed model are significantly less than the cost values in the basic model. In percentage, the proposed model has 8.7% and 23% of cost saving (excluding the passenger cost since it does not reflect a financial expense) in normal and double traffic, respectively, in comparison to the basic model. This emphasizes the financial impacts of consideration of flight level assignment, fuel optimization and short tactical scheduling in ATFM.

The main intention of this work is to propose a comprehensive ATFM model with a lower computational complexity to ensure a real-time response, while attaining a good approximation of reality. The above validated features of the model generally align with the main objectives (listed below) of implementing an ATFM service according to the ICAO [106].

- Reduce ground and en-route delays

- Maximize capacity and optimize the flow of air traffic
- Provide an informed choice between departure delay, re-routing and/or flight level selection
- Provide improved solutions around predicted severe weather
- Balance the demand and capacity of ATC sectors
- Determine the necessity for an airspace/ground delay program and other Traffic Management Initiatives (TMIs) and enact them

4.5 Summary

The proposed model in this chapter depicts a comprehensive ATFM model which considers both demand and capacity uncertainties. The two-phase architecture, including pre-tactical and tactical phases, addresses two main ATFM phases, while the former generates feasible trajectories for flights with optimal fuel consumption, whereas the latter fine-tunes them further with the real-time updates of the network. In terms of safety, this model outperforms the model presented in the previous chapter due to the consideration of both sector capacity and in-trail separation constraints. An improved version of the MIS approach is used together with a heuristic preprocessing mechanism to overcome the computational complexity. The experiment results clearly emphasize the performance improvement and the real-time applicability of this model.

Appendix - Linearization of Logical Conditions

Proposition 1: The linear conversion of the ‘max’ function in (4.12) can be interpreted as follows.

$$X_{w,h}^{f_k} \geq \sum_{j \in J_w^{f_k}(U)} Y_{j,h}^{f_k} \quad (4.34a)$$

$$X_{w,h}^{f_k} \geq \sum_{j' \in J_w^{f_k}(D)} Y_{j',h}^{f_k} \quad (4.34b)$$

$$X_{w,h}^{f_k} \leq \sum_{j \in J_w^{f_k}(U)} Y_{j,h}^{f_k} + \sum_{j' \in J_w^{f_k}(D)} Y_{j',h}^{f_k} \quad (4.34c)$$

Proposition 2: The ‘if and only if’ conditions in (4.15) are linearly interpreted using mixed logic conversion [87]. Here, M is a sufficiently big integer and m is a small integer. ϵ is the machine precision, beyond which the constraint is violated.

$$\epsilon + (m - \epsilon)\underline{\delta}_{f_k, f_i}^s \leq T_{arr, s}^{f_i} - T_{arr, s}^{f_k} \leq M(1 - \underline{\delta}_{f_k, f_i}^s) \quad (4.35a)$$

$$\epsilon + (m - \epsilon)\bar{\delta}_{f_k, f_i}^s \leq T_{arr, s}^{f_k} - T_{ext, s}^{f_i} \leq M(1 - \bar{\delta}_{f_k, f_i}^s) \quad (4.35b)$$

Proposition 3: A cumulative normal distribution can be approximated to a set of piecewise linear functions as shown in Fig. (4.12) [107]. This can be applied to the distribution of the uncertain component of the sector volume, which is F_{Δ_N} in (4.18h). Let $\vartheta_{CN}(t) = c - 1 - N_s^{f_k}(t)$.

Figure 4.12: Piecewise approximation for the cumulative distribution function.

$$[\vartheta_{CN}(t) \leq N_1] \longleftrightarrow [\delta_1 = 1] \quad (4.36a)$$

$$[\vartheta_{CN}(t) \leq N_2] \longleftrightarrow [\delta_2 = 1] \quad (4.36b)$$

$$[\vartheta_{CN}(t) \leq N_3] \longleftrightarrow [\delta_3 = 1] \quad (4.36c)$$

where δ_1 , δ_2 , and δ_3 are three boolean variables and will be triggered based on the $\vartheta_{CN}(t)$ value. The linear conversion of three conditions in (4.36) is similar to the proposition 2.

$$\begin{aligned} F_{\Delta_N}(\vartheta_{CN}(t)) = \max \Big\{ & (a_1\vartheta_{CN}(t) + b_1) - M(1 - \delta_1), \\ & (a_2\vartheta_{CN}(t) + b_2) - M(1 - \delta_2 + \delta_1), \\ & (a_3\vartheta_{CN}(t) + b_3) - M(1 - \delta_3 + \delta_2), \\ & (a_4\vartheta_{CN}(t) + b_4) - M(1 - 1 + \delta_3) \Big\} \end{aligned} \quad (4.37)$$

Depending on the value of $\vartheta_{CN}(t)$, only the corresponding piecewise segment will be assigned to the cumulative distribution through the ‘max’ function in (4.37), while other terms will be submerged due to the big M . A modified version of the proposition 1 can be used to linearize the max function in (4.37), as the parameters in (4.37) are continuous unlike in proposition 1. Thus, instead of (4.34c), the output of the max function, F_{Δ_N} , should be minimized in the objective function.

Proposition 4: A multiplication of two boolean variables can be linearized using mixed logic conversions. We present the formulations with respect to the term $\mathcal{O}_w^{f_k}(h, h')$ in (4.29). The same formulations apply for (4.16) as well.

$$- \sum_{j \in J_w^{f_k}(U)} Y_{j,h}^{f_k} + \mathcal{O}_w^{f_k}(h, h') \leq 0 \quad (4.38a)$$

$$- \sum_{j' \in J_w^{f_k}(D)} Y_{j',h'}^{f_k} + \mathcal{O}_w^{f_k}(h, h') \leq 0 \quad (4.38b)$$

$$\sum_{j \in J_w^{f_k}(U)} Y_{j,h}^{f_k} + \sum_{j' \in J_w^{f_k}(D)} Y_{j',h'}^{f_k} - \mathcal{O}_w^{f_k}(h, h') \leq 1 \quad (4.38c)$$

Proposition 5: The linear conversion of the ‘min’ function in (4.30) is as follows. Let $W_{min}^{f_k}(w_2, w_3) = \min\{W_{w_2}^{f_k}, W_{w_3}^{f_k}\}$.

$$W_{min}^{f_k}(w_2, w_3) \leq W_{w_2}^{f_k} \quad (4.39a)$$

$$W_{min}^{f_k}(w_2, w_3) \leq W_{w_3}^{f_k} \quad (4.39b)$$

Here, $W_{min}^{f_k}(w_2, w_3)$ should be maximized in the objective function, in order to accurately execute the ‘min’ function. But, instead, we maximize the final remaining weight of aircraft in order to minimize the number of terms used in the objective function.

Chapter 5

Two-stage Scalable ATFM Model Under Uncertainty

5.1 Introduction

The previous two chapters have introduced frameworks for flight routing and scheduling under different uncertainty conditions using a flight-by-flight formulation structure. Owing to the major drawback of high computational complexity in the previous models, this chapter proposes a novel framework to enhance scalability in the flow management process. To this end, this chapter depicts a two-stage architecture, in which the first stage uses a flow-based formulation structure, while at the second stage, the resulting flows are transformed into individual flight plans.

The uniqueness of this framework is in its ability to solve large-scale ATFM problems efficiently, thus, enhancing the scalability, while still maintaining unique features in a flight-by-flight structure. Moreover, in this framework, we consider flow uncertainty and capacity uncertainty at the first stage and demand uncertainty at the second stage.

The contributions of this chapter are listed below.

1. An ATFM model, which takes en-route and airport uncertainties into account, while minimizing violations and conflicts due to uncertainties present in the system.
2. A two-stage model in which the first stage generates flow values and the second stage decomposes them into individual flight plans, enhancing scala-

bility.

3. Reduction in safety risks by constraining the sector capacity and ensuring the separation among flights.
4. An efficient solution mechanism, which can be applied in solving realistic large-scale ATFM problems.

The remainder of the chapter is structured as follows. Section 5.2 depicts the proposed stage I ATFM models in detail and the solution mechanism. Section 5.3 presents the stage II formulation. The experimental results are illustrated in section 5.4, and finally, section 5.5 concludes the chapter.

5.2 Stage I Formulation:- Flow-based ATFM

At stage I, flow values are generated incorporating the uncertainty of demand. Also, flows are to be constrained mainly by sector, link, and waypoint capacities while incorporating the effect of capacity uncertainty.

In this section, we describe the constraints and the objective function of our optimization problem at stage I in detail.

5.2.1 Notations

Here, we list the main notations used in the stage I flow-based model.

- L_j - Length of link j .
- O_p, ϕ - Set of OD pairs and aircraft types, respectively. $o \in O_p, \lambda \in \phi$.
- $\underline{V}^\lambda, \bar{V}^\lambda$ - Maximum and minimum cruising speed of aircraft type λ .
- T_{min}, L_{min} - Required minimum in-trail time and distance separation, separately.
- T_{PH}, t_Δ - Prediction horizon and the duration of a discrete time slot, respectively. t denotes the time index, which is an integer such that $t \in T$, where T is the total number of sampling points in the prediction horizon and $T_{PH} = t_\Delta * T$.
- $F_j^{o,\lambda}(t), \tilde{F}_j^{o,\lambda}(t)$ - Deterministic and stochastic flows with OD pair o and type λ in link j at time interval t , respectively. $j \rightarrow w$ for waypoint. A flow is defined with respect to the OD pair and flight type, thus, a tuple of (o, λ) .

- $\Delta_{F_j}(t)$ - Uncertainty of flow (deviation from the deterministic value) in link j at time interval t .
- $F_{j,in}^{o,\lambda}(t), F_{j,out}^{o,\lambda}(t)$ - The number of incoming and outgoing flights with OD pair o and type λ in link j at time t . $j \rightarrow d$ for the departure link and $j \rightarrow a$ for the arrival link, $d \in J_D, a \in J_A$ where J_D and J_A are the set of all departure and arrival links.
- $F_{j,j'}^{o,\lambda}(t)$ - The number of aircraft entering from link j to j' with OD pair o and type λ at time t .
- $\tilde{C}_D(t), \tilde{C}_A(t)$ - Stochastic departure airport (D) and arrival airport (A) capacities at time t , respectively.
- $V_s(t), \tilde{V}_s(t)$ - Deterministic and stochastic volume of sector s at time t .
- $\tilde{C}_s(t)$ - Stochastic capacity of sector s at time t .
- $J_j^o(U), J_j^o(D)$ - Set of upstream and downstream links connected with link j for the OD pair o , respectively. $j \rightarrow w$ for waypoint.

5.2.2 Constraints

5.2.2.1 Flow dynamic constraints

These constraints are used to describe the relationship of link volume with the incoming/outgoing flows from/to adjacent links. Basically, the incoming flow to a link is the summation of flows from its upstream links, and outgoing flow should be the summation of flows towards the downstream links during a considered time interval (refer to Fig. 5.1, d - departure link, a - arrival link, $w \in W$ - waypoints, $j \in J$ - air routes/links, h - holding link).

Figure 5.1: Simplified ATN model

$$(\forall j \in J - J_A)(\forall t \in T)(o \in O_p)(\lambda \in \phi) F_{j,in}^{o,\lambda}(t) = \sum_{j'' \in J_j^o(U)} F_{j'',j}^{o,\lambda}(t) \quad (5.1a)$$

$$(\forall j \in J - J_A^o(U))(\forall t \in T)(o \in O_p)(\lambda \in \phi) F_{j,out}^{o,\lambda}(t) = \sum_{j' \in J_j^o(D)} F_{j,j'}^{o,\lambda}(t) \quad (5.1b)$$

The above two constraints are only applied to en-route links, while arrival links are modified, including holding patterns in (5.15a) and (5.15b).

The link volume at time interval $t + \Delta t$ can be derived as,

$$F_j^{o,\lambda}(t + \Delta t) = F_j^{o,\lambda}(t) + \left[F_{j,in}^{o,\lambda}(t) - F_{j,out}^{o,\lambda}(t) \right] \quad (5.1c)$$

5.2.2.2 Link capacity constraint

Although a sector capacity constraint limits the sector volume, it cannot ensure that the flights are well packed within a link. Thus, we use this constraint to restrict the link volume depending on the link length and minimum in-trail separation between flights.

$$(\forall t \in T)(\forall j \in J) \sum_{o \in O_p, \lambda \in \phi} \tilde{F}_j^{o,\lambda}(t) \leq \frac{L_j}{L_{min}} \quad (5.2a)$$

Let, $\tilde{F}_j(t) = \sum_{o \in O_p, \lambda \in \phi} \tilde{F}_j^{o,\lambda}(t)$.

As mentioned before, the number of flights in a link can vary from scheduled values due to the inherent demand uncertainty. Thus, based on the previous flight records of actual link arrival and exit times, the distribution of $\tilde{F}_j(t)$ is obtained and approximated by a normal distribution. Let $\tilde{F}_j(t) \sim N(\mu_{\tilde{F}_j}(t), \sigma_{\tilde{F}_j}(t))$ and $C_j = \frac{L_j}{L_{min}}$. By applying the chance constraint with probability β ,

$$P(\tilde{F}_j(t) \leq C_j) \geq \beta \quad (5.2b)$$

A chance constraint approach will certify that a given constraint will be sustained with a considered probability, in this case, β , thus different from a regular constraint where it will sustain all the time. By separating the deterministic and uncertain components, $\tilde{F}_j(t)$ can be written as, $\tilde{F}_j(t) = F_j(t) + \Delta_{F_j}(t)$.

$$P(\Delta_{F_j}(t) \leq C_j - F_j(t)) \geq \beta \quad (5.2c)$$

The above probabilistic constraint can be converted into a linear constraint using the CDF as follows.

$$F_{\Delta_{F_j}(t)}(C_j - F_j(t)) \geq \beta \quad (5.2d)$$

$$C_j - F_j(t) \geq F_{\Delta_{F_j}(t)}^{-1}(\beta) \quad (5.2e)$$

$$C_j - F_j(t) \geq \zeta_{\Delta_{F_j}(t)} \quad (5.2f)$$

The CDF of a normal distribution can be written as in (5.2g) (For $\zeta_{\Delta_{F_j}(t)} \geq \mu_{\Delta_{F_j}(t)}$) [96].

$$\frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\zeta_{\Delta_{F_j}(t)} - \mu_{\Delta_{F_j}(t)}}{\sqrt{2}\sigma_{\Delta_{F_j}(t)}} \right) \right) \geq \beta \quad (5.2g)$$

The constraint in (5.2f) can be rewritten as follows.

$$F_j(t) \leq C_j - \mu_{\Delta_{F_j}(t)} - \sqrt{2}\sigma_{\Delta_{F_j}(t)} \operatorname{erf}^{-1}(2\beta - 1) \quad (5.2h)$$

$\mu_{\Delta_{F_j}(t)}$ and $\sigma_{\Delta_{F_j}(t)}$ are derived as described below.

The link flow consists of the deterministic and uncertain components such that $\tilde{F}_j(t) = F_j(t) + \Delta_{F_j}(t)$. The deterministic link flow can be determined purely based on the scheduled link arrival/exit times of flights. Then, the distribution of the link flow uncertainty can be derived as, $\Delta_{F_j}(t) \sim N(\mu_{\Delta_{F_j}(t)}, \sigma_{\Delta_{F_j}(t)})$ with $\mu_{\Delta_{F_j}(t)} = \mu_{\tilde{F}_j}(t) - F_j(t)$ and $\sigma_{\Delta_{F_j}(t)} = \sigma_{\tilde{F}_j}(t)$.

5.2.2.3 Sector capacity constraint

The deterministic sector volume at time t is the summation of link volumes within the sector at the respective time, thus,

$$(\forall t \in T) \quad V_s(t) = \sum_{j \in J_s, o \in O_p, \lambda \in \phi} F_j^{o,\lambda}(t) \quad (5.3a)$$

The handling capability of a sector mainly depends on the ATCo workload and weather conditions at a given time interval. Due to the stochastic nature of the above factors, the sector capacity is an uncertain parameter, thus, we use a chance-

constraint-based approach to constraint the sector flow by the sector capacity as described below. Simply, the sector volume should be lower than sector capacity.

$$\tilde{V}_s(t) \leq \tilde{C}_s(t) \quad (5.3b)$$

Applying a chance constraint probability of β_S ,

$$P(\tilde{V}_s(t) \leq \tilde{C}_s(t)) \geq \beta_S \quad (5.3c)$$

$$P(V_s(t) + \Delta_{V_s}(t) \leq C_s(t) + \Delta_{C_s}(t)) \geq \beta_S \quad (5.3d)$$

Let $\Delta_{V_s, C_s}(t) = \Delta_{V_s}(t) - \Delta_{C_s}(t)$

$$P(\Delta_{V_s, C_s}(t) \leq C_s(t) - V_s(t)) \geq \beta_S \quad (5.3e)$$

Following the same steps in (5.2d)-(5.2h), the linear interpretation of (5.3e) can be derived as follows.

$$V_s(t) \leq C_s(t) - \mu_{\Delta_{V_s, C_s}}(t) - \sqrt{2}\sigma_{\Delta_{V_s, C_s}}(t)erf^{-1}(2\beta_S - 1) \quad (5.3f)$$

The steps in deriving mean and standard deviation of $\Delta_{V_s}(t)$ are described in Procedure 1 below.

Using the previous sector capacity data, the best fitted normal distribution is obtained with mean and standard deviation of $\mu_{\Delta_{C_s}}(t)$ and $\sigma_{\Delta_{C_s}}(t)$. Then, the distribution of $\Delta_{V_s, C_s}(t)$ is derived with mean and standard deviation of $\mu_{\Delta_{V_s, C_s}}(t) = \mu_{\Delta_{V_s}}(t) - \mu_{\Delta_{C_s}}(t)$ and $\sigma_{\Delta_{V_s, C_s}}(t) = \sqrt{\sigma_{\Delta_{V_s}}(t)^2 + \sigma_{\Delta_{C_s}}(t)^2}$.

Procedure 1: Generation of sector flow distributions

- i. Based on the link volume values obtained previously, the sector volume at time t can be estimated as follows. J_s represents the set of all links within sector s .

$$\tilde{V}_s(t) = \sum_{j \in J_s, o \in O_p, \lambda \in \phi} \tilde{F}_j^{o, \lambda}(t) = \sum_{j \in J_s} \tilde{F}_j(t) \quad (5.4)$$

where $\tilde{F}_j(t)$ possesses a normal distribution such that, $\tilde{F}_j(t) \sim N(\mu_{\tilde{F}_j}(t), \sigma_{\tilde{F}_j}(t))$.

- ii. Let's take two adjacent links j and j' , then, $\tilde{F}_j(t)$ and $\tilde{F}_{j'}(t)$ are jointly normal or bivariate normal as they can be expressed as a sum of two inde-

pendent components [108] such that, $\tilde{F}_j(t) = \Delta_{F_j}(t) + \mu_{F_j}(t)$ and $\tilde{F}_{j'}(t) = \Delta_{F_{j'}}(t) + \mu_{F_{j'}}(t)$.

- iii. The sum of two jointly normal random variables is still normally distributed with the mean and standard deviation given by [109],

$$\text{mean} = \mu_{\tilde{F}_j}(t) + \mu_{\tilde{F}_{j'}}(t),$$

$$\text{standard deviation} = \sqrt{[\sigma_{\tilde{F}_j}(t)]^2 + [\sigma_{\tilde{F}_{j'}}(t)]^2 + 2Cov[j, j'](t)}$$

where $\mu_{\tilde{F}_j}(t), \sigma_{\tilde{F}_j}(t)$ are the mean and the standard deviation of $\tilde{F}_j(t)$, respectively. $Cov[j, j'](t)$ is the covariance of the link flow distributions of j and j' and it is given by (5.5), assuming equal probabilities in the sample space n [109].

$$Cov[j, j'](t) = \frac{\sum_{\forall n} (\tilde{F}_j(t) - \mu_{\tilde{F}_j}(t))(\tilde{F}_{j'}(t) - \mu_{\tilde{F}_{j'}}(t))}{n - 1} \quad (5.5)$$

- iv. Since a sector consists of multiple air routes, the sector flow follows a normal distribution as illustrated below.

$$\tilde{V}_s(t) \sim N\left(\sum_{j \in J_s} \mu_{\tilde{F}_j}(t), \sqrt{\sum_{j, j' \in J_s} [\sigma_{\tilde{F}_j}(t)]^2 + 2Cov[j, j'](t)}\right) \quad (5.6)$$

Note: Here, we only consider the covariance of the adjacent links, assuming zero correlation between non-adjacent links.

- v. To obtain the distribution of the $\Delta_{V_s}(t)$ which is the uncertain component of the sector flow, (5.6) is further modified as follows. The deterministic value, $V_s(t)$ can be obtained purely based on the scheduled flow values.

$$\Delta_{V_s}(t) \sim N\left(\sum_{j \in J_s} [\mu_{\tilde{F}_j}(t)] - V_s(t), \sqrt{\sum_{j, j' \in J_s} \left[\sigma_{\tilde{F}_j}(t)^2 + 2Cov[j, j'](t) \right]}\right) \quad (5.7)$$

5.2.2.4 Airport capacity constraint

An airport consists of capacitated elements such as gates, taxiways, runways, near-terminal airspace, etc. In this work, we consider all of them as an aggregated unit, and the departure and arrival flow rates are constrained to be less than the airport capacity under uncertainty.

The outgoing flow rate of the departure link (outgoing flights from the airport) should not be bigger than the departure airport capacity. By applying the chance constraint with probability β_D ,

$$P\left[\sum_{o \in O_p, \lambda \in \phi} \tilde{F}_{d,out}^{o,\lambda}(t) \leq \tilde{C}_D(t)\right] \geq \beta_D \quad (5.8a)$$

Let $\tilde{F}_{d,out} = \sum_{o \in O_p, \lambda \in \phi} \tilde{F}_{d,out}^{o,\lambda}$. The deterministic and the uncertain components can be separated as in (5.8b).

$$P[F_{d,out}(t) + \Delta_{F_{d,out}}(t) \leq C_D(t) + \Delta_{C_D}(t)] \geq \beta_D \quad (5.8b)$$

Here, the uncertainty in flow, $\Delta_{F_{d,out}}(t)$ is solely owing to the uncertainty of demand due to departure time uncertainty of the airport. Further, $\Delta_{C_D}(t)$ is the capacity uncertainty of the departure airport.

Let $\Delta_{F_{d,C_D}}(t) = \Delta_{F_{d,out}}(t) - \Delta_{C_D}(t)$,

$$P[\Delta_{F_{d,C_D}}(t) \leq C_D(t) - F_{d,out}(t)] \geq \beta_D \quad (5.8c)$$

By applying the steps in (5.2d)-(5.2g),

$$F_{d,out}(t) \leq C_D(t) - \mu_D - \sqrt{2}\sigma_D \text{erf}^{-1}(2\beta_D - 1) \quad (5.8d)$$

where μ_D and σ_D are the mean and the standard deviation of $\Delta_{F_{d,C_D}}(t)$.

Similarly, at the arrival airport the incoming flow rate at the arrival link (arriving flights to the airport) should be less than the handling capacity of the arrival airport. The arrival airport capacity constraint with a chance constraint probability of β_A ,

$$P\left[\sum_{o \in O_p, \lambda \in \phi} \tilde{F}_{a,in}^{o,\lambda}(t) \leq \tilde{C}_A(t)\right] \geq \beta_A \quad (5.9a)$$

Let $\tilde{F}_{a,in} = \sum_{o \in O_p, \lambda \in \phi} \tilde{F}_{a,in}^{o,\lambda}$. By separating the deterministic and the uncertain components,

$$P[F_{a,in}(t) + \Delta_{F_{a,in}}(t) \leq C_A(t) + \Delta_{C_A}(t)] \geq \beta_A \quad (5.9b)$$

Let $\Delta_{F_a, C_A}(t) = \Delta_{F_a, in}(t) - \Delta_{C_A}(t)$,

$$P[\Delta_{F_a, C_A}(t) \leq C_A(t) - F_{a, in}(t)] \geq \beta_A \quad (5.9c)$$

By applying the steps in (5.2d)-(5.2g),

$$F_{a, in}(t) \leq C_A(t) - \mu_A - \sqrt{2}\sigma_A \text{erf}^{-1}(2\beta_A - 1) \quad (5.9d)$$

where μ_A and σ_A are the mean and the standard deviation of $\Delta_{F_a, C_A}(t)$.

5.2.2.5 Flow shift constraint

Depending on the maximum and minimum cruising speeds of the aircraft type, the flow dynamics are constrained by the minimum and maximum link traversing times, respectively. The link traversing time is bounded between $[L_j/(\bar{V}^\lambda + \Delta_v^{+, \lambda}), L_j/(\underline{V}^\lambda - \Delta_v^{-, \lambda})]$. Here, $\Delta_v^{+, \lambda}$ and $\Delta_v^{-, \lambda}$ denote the upper and lower bounds of the speed uncertainty, respectively. We use ceiling and floor functions together with the duration of the discrete time slot as follows to divide this range into discrete time slots.

$$\left\lceil \frac{L_j}{(\bar{V}^\lambda + \Delta_v^{+, \lambda})t_\Delta} \right\rceil \leq T_n^\lambda(j) \leq \left\lfloor \frac{L_j}{(\underline{V}^\lambda - \Delta_v^{-, \lambda})t_\Delta} \right\rfloor \quad (5.10a)$$

where $\lceil \cdot \rceil$ and $\lfloor \cdot \rfloor$ denotes ceiling and floor functions, respectively. $T_n^\lambda(j)$ is an integer representing total traversing time in discrete points in the range of $[\underline{T}_n^\lambda(j), \bar{T}_n^\lambda(j)]$, where $\underline{T}_n^\lambda(j)$ and $\bar{T}_n^\lambda(j)$ represent lower and upper bounds as per (5.10a).

For short links, flows may pass through several links within one discrete time step. In such cases, the shift constraint is applied for aggregated links. For long links (with a wide range of traversing time), we partitioned them into several link segments in such a way that there is a unique shift of time from $\underline{T}_n^\lambda(j)$ to $\bar{T}_n^\lambda(j)$ (i.e. $\underline{T}_n^\lambda(j) = \bar{T}_n^\lambda(j)$ and let's take it as $n(j)$). Thus, there will not be any flow overtake scenarios, since there is no any possibility to overtake the first entered flow by a subsequently entered flow.

$$F_{i, in}^{o, \lambda}(t) = F_{i, out}^{o, \lambda}(t + n(j)) \quad (5.10b)$$

$$F_{i, out}^{o, \lambda}(t) \leq F_i^{o, \lambda}(t) \quad (5.10c)$$

An incoming flow to a link should exit the link in a subsequent time step as per (5.10b). Further, the outgoing flow of the link at time t should not be higher than the volume of the link at time t as per (5.10c).

5.2.2.6 Waypoint flow rate constraint

When multiple air routes merge and diverge at a waypoint, there are possibilities for merging flow conflicts if they exceed the time based capacity at the waypoint (i.e. merging without safe separation). Thus, we use the following constraint to limit the traffic at merging waypoints. The sum of flow rates at a waypoint at a given time, $F_w(t)$ is the summation of flows traversing from upstream links to downstream links, which is given in (5.11a). (refer to Fig. 5.1)

$$(\forall t \in T) \quad F_w(t) = \sum_{j'' \in J_w^o(U), j \in J_w^o(D), o \in O_p, \lambda \in \phi} F_{j'', j}^{o, \lambda}(t) \quad (5.11a)$$

The waypoint flow rate is constrained by the capacity of the waypoint, which depends on the minimum time separation between two flights and the duration of the discrete time slot.

$$(\forall t \in T)(\forall w \in W) \quad \tilde{F}_w(t) \leq \frac{t_\Delta}{T_{min}} \quad (5.11b)$$

Applying a chance constraint with a probability of β_W ,

$$P(\tilde{F}_w(t) \leq \frac{t_\Delta}{T_{min}}) \geq \beta_W \quad (5.12)$$

$$P(\Delta_{F_w}(t) \leq t_\Delta/T_{min} - F_w(t)) \geq \beta_W \quad (5.13)$$

and following the steps in (5.2d)-(5.2h), the linear representation of (5.13) can be derived as follows.

$$F_w(t) \leq t_\Delta/T_{min} - \mu_{\Delta_{F_w}}(t) - \sqrt{2}\sigma_{\Delta_{F_w}}(t)\text{erf}^{-1}(2\beta_W - 1) \quad (5.14)$$

Based on the historical data of waypoint arrival time, the uncertainty distribution of the flow rate at waypoints can be obtained, thus the values of $\mu_{\Delta_{F_w}}(t)$ and $\sigma_{\Delta_{F_w}}(t)$ can be derived.

5.2.2.7 Applying holding patterns

Holding patterns are one of the control actions for delaying en-route flights, which occurs mostly in the airspace closer to the arrival airport. It delays the aircraft that has arrived at the destination airport, but the landing is not possible due to congestion at the airport, bad weather conditions, runway unavailability, etc.

We implement this control action at stage I, by controlling the arrival flow as follows. (please note that the holding link h is not considered as an upstream link of the arrival link a , refer to Fig. 5.1)

$$F_{a,in}^{o,\lambda}(t) = \sum_{i \in J_a(U^o)} F_{i,a}^{o,\lambda}(t) + F_{h,a}^{o,\lambda}(t) \quad (5.15a)$$

$$(\forall i \in J_a(U^o)) F_{i,out}^{o,\lambda}(t) = F_{i,a}^{o,\lambda}(t) + F_{i,h}^{o,\lambda}(t) \quad (5.15b)$$

To implement multiple holding patterns,

$$F_{h,out}^{o,\lambda}(t) = F_{h,in}^{o,\lambda}(t) + F_{h,a}^{o,\lambda}(t) \quad (5.15c)$$

These holding links are also constrained by the link capacity constraint similar to (5.2).

5.2.2.8 Flow upper bounds

An upper bound for both departure and arrival flow rates are implemented since the actual flow should not be bigger than the scheduled flow rate. The constraints in (5.16a) and (5.16b) bound the upper limits of the departure and arrival rates, respectively.

$$(o \in O_p)(\lambda \in \phi) \sum_{t \in T} F_{d,out}^{o,\lambda}(t) \leq \sum_{t \in T} \gamma_{d,out}^{o,\lambda}(t) \quad (5.16a)$$

$$(o \in O_p)(\lambda \in \phi) \sum_{t \in T} F_{a,in}^{o,\lambda}(t) \leq \sum_{t \in T} \gamma_{a,in}^{o,\lambda}(t) \quad (5.16b)$$

5.2.3 Stage I - Objective Function

At this stage, our objective is to dispatch and receive all flows in their respective time slots. The first and second terms in (3.24) penalize the departure and arrival

flow deviations, respectively. The cost coefficients are gradually decreased with the deviation of the flow from the scheduled time, as illustrated in Fig. (5.2). So that, if the flows (departure/arrival) further deviate from the optimal time, the amount of cost removed from the objective would be less, thus, flows will always attempt to dispatch or arrive at their optimal time slots or more closer to the optimal ones. $\underline{t}_d^{o,\lambda}, \bar{t}_d^{o,\lambda}$ are the minimum and maximum allowable departure time limits and $\underline{t}_a^{o,\lambda}, \bar{t}_a^{o,\lambda}$ are the respective values for the arrival time limits. M_d and M_a are two big constants. If the coefficient envelops of two flows are overlapped, then we take the maximum value for the coefficient as highlighted in Fig. 5.2.

In the third term, we force the total flight flow to be scheduled without incurring any cancellations.

$$\begin{aligned} \min \sum_{d \in J_D, o \in O_p, \lambda \in \phi} \left\{ \sum_{\tilde{t} \in T_d, t \in t_d} C_D (M_d - c_d (\tilde{t} - t)^2) [\gamma_{d,out}^{o,\lambda}(\tilde{t}) - F_{d,out}^{o,\lambda}(t)] \right. \\ \left. + \sum_{\tilde{t} \in T_a, t \in t_a} C_A (M_a - c_a (\tilde{t} - t)^2) [\gamma_{a,in}^{o,\lambda}(\tilde{t}) - F_{a,in}^{o,\lambda}(t)] \right\} \quad (5.17) \\ - \sum_{a \in J_A, o \in O_p, \lambda \in \phi} C_C^{o,\lambda} \left(\sum_{t \in t_a} F_{a,in}^{o,\lambda}(t) - \sum_{t \in t_a} \gamma_{a,in}^{o,\lambda}(t) \right) \end{aligned}$$

where $T_d = \{\tilde{t} \in T | \gamma_{d,out}^{o,\lambda}(\tilde{t}) > 0\}$, $T_a = \{\tilde{t} \in T | \gamma_{a,in}^{o,\lambda}(\tilde{t}) > 0\}$ and $t_d = \{\tilde{t} - \underline{t}_d^{o,\lambda} : \tilde{t} + \bar{t}_d^{o,\lambda}\}$, $t_a = \{\tilde{t} - \underline{t}_a^{o,\lambda} : \tilde{t} + \bar{t}_a^{o,\lambda}\}$.

Figure 5.2: Departure coefficient envelop.

$\gamma_{d,out}^{o,\lambda}(\tilde{t})$ and $\gamma_{a,in}^{o,\lambda}(\tilde{t})$ denote the scheduled departure and arrival flow rates, respectively. C_D and C_A are the departure and arrival flow rate coefficients. As one of the major concern is to dispatch all the flights without any cancellations, $C_C^{o,\lambda}$ is assigned a higher weight. This forces all the flights to be scheduled even in deviated time slots. c_d and c_a respectively dictate the degradation of coefficient envelop for departure and arrival delay.

The above formulated integer linear programming problem at stage I can be summarized as,

minimize (5.17)

subject to (5.1), (5.2h), (5.3a), (5.3f), (5.8d), (5.9d), (5.10), (5.11a), (5.14) (5.15), (5.16).

5.2.4 Solution Mechanism : Greedy Strategy with MWIS

The main computational barrier of this model is in solving the stage I problem due to the involvement of a large number of decision variables and constraints. Due to the complexity of stage I problem, a modified version of the maximum independent set based solution mechanism is used to decompose the overall problem into a set of sub-problems and solved in an iterative manner to reduce the computational complexity. The steps of stage I solution mechanism are described below.

First, the stage I problem is converted into a weighted graph of $G = (V, E)$, where V and E respectively denote the set of nodes and edges in the weighted graph. Time spanned sub-flows¹ are considered as the set of nodes, whereas their space-time dependency is represented by the edges. Nodes, in this case, sub-flows, are weighted depending on their priority in the departure time. Flow pairs are considered as dependent if any of their trajectories are overlapped in terms of time. From the resulting adjacency matrix (graph), set of all possible MISs are iteratively extracted [101] by removing the nodes in the previous MISs from the adjacency matrix. In fact, a MIS reflects a set of sub-flows in the network, which can be independently scheduled without any conflicts, thus, faster.

Let $S_1, S_2, \dots, S_n$ denote the MISs of the stage I problem. The solution process starts with the initial set S_1 , and ostensibly, all the sub-flows in S_1 are scheduled in the optimal trajectories. Then, the subsequent MISs ($S_2, S_3, \dots$) are iteratively scheduled on the previously fixed network. Here, as the sub-flows in an independent set are conflict free, the solution time of a such set is minimal. This process continues iteratively until the last MIS, which is S_n . The proposed solution approach is a greedy approach, as the underlying motivation is to aggressively schedule maximum possible set of flows. Yet, with the concept of Independence, we are able to force the optimality gap closer to the global optimality.

¹If two sets of flights with the same od pair and aircraft type (theoretically belongs to the same flow) are spanned across in the time domain, they are considered as two sub-flows.

5.3 Stage II Formulation:- Flight-by-flight-based ATFM

5.3.1 Notations

Here, we list the notations specific to the flight-by-flight model.

- F - Set of flights, $f_k \in F$ with the times $t_{dep}^{f_k}, \hat{t}_{dep}^{f_k}$ representing actual and scheduled departure times, $t_{arr}^{f_k}, \hat{t}_{arr}^{f_k}$ representing actual and scheduled arrival times. t denotes the time index, which is an integer such that $t \in T$, where T is the total number of sampling points in the prediction horizon and $T_{PH} = t_{\Delta} * T$.
- $Y_j^{f_k}$ - This Boolean takes 1, if flight f_k selects link j .
- $\chi_w^{f_k}$ - This Boolean takes 1, if flight f_k passes through waypoint w .
- $\tilde{t}_w^{f_k}, t_w^{f_k}$ - Stochastic and deterministic traversing times of flight f_k at waypoint w , respectively.
- $\Delta_w^{f_k}$ - Arrival time uncertainty of flight f_k at waypoint w .
- $Z_j^{f_k}$ - The reciprocal of the speed of f_k in link j ($V_j^{f_k}$).

5.3.2 Flights decomposition

At this stage, based on stage I results, the flow values are decomposed into individual flights as described in Algorithm 4.

The stage I result essentially indicates the number of aircraft from each OD pair and aircraft type traversing in links at discrete time steps. Thus, the intention here is to assign flights to each flow, with the induction of minimum possible flight delays.

This procedure starts from a departure, moves through the intermediate network, and completes at an arrival. In the first phase, the flows departing from the departure links are assigned to the respective flights, as depicted in Algorithm 1. The basic idea is to assign flights incrementally based on the flow departure time, with an exception on cases where a set of flights is canceled. When the resulting total flow is less than the expected outgoing flow, a set of flights has to be canceled in stage II. The selection of the above set of flights are subjected to a wide range of permutations, and thus, here we explore all possible permutations

Algorithm 4 Flow decomposition into flights

Inputs: $F_{d,out}^{o,\lambda}(t), \gamma_{d,out}^{o,\lambda}(t)$ for all t, j, o, λ, d

```

1: for each  $p \in (o, \lambda)$  do
2:    $f_{act}^p \leftarrow \sum_{\forall t} F_{d,out}^{o,\lambda}(t)$   $\triangleright f_{act}^p$ - total departed flights
3:    $f_{sch}^p \leftarrow \sum_{\forall t} \gamma_{d,out}^{o,\lambda}(t)$   $\triangleright f_{act}^p$ - total scheduled flights
4:    $n_c^p \leftarrow f_{sch}^p - f_{act}^p$   $\triangleright n_c^p$  - number of canceled flights
5:    $S^p \leftarrow \text{getAllPermutations}(n_c^p, f_{sch}^p)$   $\triangleright S^p$  - collection of sets of all
      flight profiles excluding cancellations
6:   PermDelay  $\leftarrow []$ 
7:   for each  $s \in S^p$  do
8:      $T_{delay} \leftarrow 0$   $\triangleright T_{delay}$ - Total delay
9:     for  $t \in T$  do  $\triangleright t$  - discrete time steps
10:      - Assign  $f_k \in s$  departure slot to  $t$  unless  $f_k$  is already assigned or
        sum of  $f_k$ 's at  $t$  has already reached  $F_{d,out}^p(t)$ 
11:       $t_{delay}^{f_k} \leftarrow |\hat{t}^{f_k} - t|$   $\triangleright$  scheduled departure time slot of  $f_k$ 
12:       $T_{delay} \leftarrow T_{delay} + t_{delay}^{f_k}$   $\triangleright$  calculate total delay for all  $f_k \in s$ 
13:    end for
14:    PermDelay  $\leftarrow [\text{PermDelay } T_{delay}]$ 
15:  end for
16:   $pm_{min} \leftarrow \text{selectPermutation}(\text{PermDelay})$   $\triangleright$  Select the permutation with
      the least total delay
17:  for  $t \in T$  do
18:    - Schedule  $f_k \in pm_{min}$  at  $t$  unless  $f_k$  is already assigned or sum of flights
      at  $t$  has already reached  $F_{d,out}^p(t)$ 
19:     $Y_d^{f_k} \leftarrow 1$   $\triangleright$  fix departure link
20:     $t_0 + (t - 1)t_{\Delta} - \frac{t_{\Delta}}{2} \leq t_{w'}^{f_k} \leq t_0 + (t - 1)t_{\Delta} + \frac{t_{\Delta}}{2}$   $\triangleright t_0$  - time
      offset to match with the initial time slot,  $w'$  - departure fix,  $t_{\Delta}$  - length of
      a discrete time slot
21:  end for
22: end for
23: At intermediate stages, the flight trajectory (path and time) is traced based
    on the flow dynamic constraints and link shift constraints.

```

($getAllPermutations(n_c^p, f_{sch}^p)$) of flight cancellations, and select the one with the minimum total delay (pm_{min}). Subsequently, the flights, excluding the canceled, are incrementally assigned to the departure link at the corresponding times. Here, we assume that the flights in the permutations sets are incrementally sorted based on their scheduled departure times.

In the intermediate phase, the departed flights are followed in the respective downstream links and time slots. As, we can get an evaluation on the relationship of $F_{j,in}^{o,\lambda}(t)$ and $F_{j,out}^{o,\lambda}(t)$ in terms of time via (5.10b), the flights can be traced from the departure link to the arrival link.

Finally, the above flights are fixed to the selected paths ($Y_j^{f_k}$) and further, their arrival times at corresponding waypoints are also bounded to the respective time slots as in line 20 in Algorithm 1. Now, the optimization problem at stage II simplifies to a problem of scheduling flight times inside a given time slot at each waypoint without violating the uncertainty embedded in-trail separation.

5.3.3 Constraints

5.3.3.1 In-trail Separation Constraint

Although stage I constraint the sector and link capacities, they cannot guarantee that flights maintain the required head-tail separation between them. Thus, we use these constraints at stage II, when we decompose the link flows into individual flights.

Prerequisite: Let the entry and exit waypoints of link j be w and w' . Then, the traversing time at waypoint w' can be derived as follows.(refer to Fig. 5.1)

$$t_{w'}^{f_i} = t_w^{f_i} + L_j Y_j^{f_i} Z_j^{f_i} \quad (5.18)$$

Since stage II follows a flight-by-flight ATFM process, there are two scenarios for each flight pairs depending on their entering order to the link, as stated in the previous chapters. Let scenario 1 be a situation where flight f_i enters first and f_k second and scenario 2 be its opposite. We use a similar approach as in chapter 3 and 4 to handle these scenarios. With the use of logical condition ‘either or’, the in-trail separation constraint can be defined as in (5.19). Here, the entry time variables are taken as stochastic, since stage I does not consist of an in-trail

separation constraint, thus at this stage, the flow values should be decomposed into individual flights under the uncertainty of demand.

$$\text{either } \tilde{t}_w^{f_k} - \tilde{t}_w^{f_i} \geq T_{min} \text{ or } \tilde{t}_w^{f_i} - \tilde{t}_w^{f_k} \geq T_{min} \quad (5.19)$$

If we only take the left side of the ‘either or’ condition (scenario 1), the stochastic variables can be replaced using the deterministic components and uncertain components as follows. Further, we allow the in-trail separation between flight f_i and f_k to hold with a probability of γ .

$$P\left((t_w^{f_k} + \Delta_w^{f_k}) - (t_w^{f_i} + \Delta_w^{f_i}) \geq T_{min}\right) \geq \gamma \quad (5.20a)$$

Let $\psi_w(f_i, f_k) = \Delta_w^{f_i} - \Delta_w^{f_k}$,

$$P\left(\psi_w(f_i, f_k) \leq t_w^{f_k} - t_w^{f_i} - T_{min}\right) \geq \gamma \quad (5.20b)$$

Let $\Delta_w^{f_i} \sim N(\mu_w^{f_i}, \sigma_w^{f_i})$ and $\Delta_w^{f_k} \sim N(\mu_w^{f_k}, \sigma_w^{f_k})$. Then, the random variable $\psi_w(f_i, f_k)$ can be represented as a normal distribution with mean $\mu_w^{f_i} - \mu_w^{f_k}$ and standard deviation $\sqrt{\sigma_w^{f_i}^2 + \sigma_w^{f_k}^2}$.

When obtaining the arrival time uncertainty distributions at waypoints ($\Delta_w^{f_i}$), first, we partition the airspace into several blocks depending on the predictions of wind, such that an area with a similar wind pattern is considered as one sector and further referred to as a ‘weather sector’. Within the weather sector, we consider the uncertainty distribution of speed to be the same due to the similar wind conditions². With this sectorization approach, the uncertainty distributions of two adjacent sectors’ can be considered as being independent and we also assume that each link only belongs to one sector. However, these weather sectors are different from the sectors used in the stage I model.

Then, the values $\mu_w^{f_i}$ and $\sigma_w^{f_i}$ (w is within weather sector s) can be derived as follows. Here, w' is the entering waypoint to the weather sector s , $J(w', w)$ is the set of links from waypoint w' to w , μ_s and σ_s are the mean and the standard deviation of the demand uncertainty (travel time uncertainty) distribution of a

²Aircraft speed is mainly affected by surrounding wind condition and several secondary factors such as pilot behaviour, engine efficiency etc. However, the effect of wind dominates the secondary factors, thus the speed uncertainty highly depends on wind uncertainty.

link in sector s .

$$\mu_w^{f_i} = \mu_{w'}^{f_i} + \sum_{j \in J(w', w)} L_j \mu_s \quad (5.21a)$$

$$(\sigma_w^{f_i})^2 = (\sigma_{w'}^{f_i})^2 + \sum_{j \in J(w', w)} L_j^2 (\sigma_s)^2 \quad (5.21b)$$

Note: The arrival time uncertainty distribution at waypoint w consists of flight's departure time uncertainty and aggregated travel time uncertainty of links from airport to waypoint w .

The constraint in (5.20b) is converted to a linear constraint using CDF as in (5.22).

$$F_{\psi_w(f_i, f_k)}(t_w^{f_k} - t_w^{f_i} - T_{min}) \geq \gamma \quad (5.22a)$$

$$t_w^{f_k} - t_w^{f_i} - T_{min} \geq F_{\psi_w(f_i, f_k)}^{-1}(\gamma) \quad (5.22b)$$

$$t_w^{f_k} - t_w^{f_i} - T_{min} \geq \zeta_w(f_i, f_k) \quad (5.22c)$$

Following the steps in (5.2g) and (5.2h), the value of $\zeta_w(f_i, f_k)$ can be obtained as follows.

$$F_{\psi_w(f_i, f_k)}(\zeta_w(f_i, f_k)) = \gamma = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\zeta_w(f_i, f_k) - \tilde{\mu}_w}{\sqrt{2}\tilde{\sigma}_w} \right) \right] \quad (5.22d)$$

where $\tilde{\mu}_w = \mu_w^{f_i} - \mu_w^{f_k}$ and $\tilde{\sigma}_w = \sqrt{\sigma_w^{f_i}^2 + \sigma_w^{f_k}^2}$.

$$\zeta_w(f_i, f_k) = \sqrt{2}\tilde{\sigma}_w \operatorname{erf}^{-1}(2\gamma - 1) + \tilde{\mu}_w \quad (5.22e)$$

Then, the constraint in (5.22c) can be rewritten as follows.

$$t_w^{f_k} - t_w^{f_i} \geq \sqrt{2}\tilde{\sigma}_w \operatorname{erf}^{-1}(2\gamma - 1) + \tilde{\mu}_w + T_{min} \quad (5.23)$$

In the same way, the linear constraint in (5.24) can be obtained following the steps in (5.20) - (5.23) to obtain the linear representation of the right side of the 'either

or' condition in (5.19) (scenario 2).

$$t_w^{f_i} - t_w^{f_k} \geq \sqrt{2}\hat{\sigma}_w \operatorname{erf}^{-1}(2\gamma - 1) + \hat{\mu}_w + T_{min} \quad (5.24)$$

where $\hat{\mu}_w = -\tilde{\mu}_w$ and $\hat{\sigma}_w = \tilde{\sigma}_w$.

By applying the mixed logic conversion to obtain the linear constraints of the logical condition in (5.19) with a sufficiently large upper bound M_1 [85], [110], [?],

$$t_w^{f_k} - t_w^{f_i} \geq \sqrt{2}\tilde{\sigma}_w \operatorname{erf}^{-1}(2\gamma - 1) + \tilde{\mu}_w + T_{min} - M_1\delta_{f_k, f_i}^w \quad (5.25a)$$

$$t_w^{f_i} - t_w^{f_k} \geq \sqrt{2}\hat{\sigma}_w \operatorname{erf}^{-1}(2\gamma - 1) + \hat{\mu}_w + T_{min} - M_1(1 - \delta_{f_k, f_i}^w) \quad (5.25b)$$

Where δ_{f_k, f_i}^w represents the flight entering order of f_k and f_i at w .

When multiple links are merged at a waypoint, there are possibilities for conflicts at the waypoint, if two or more flights reach the waypoint without holding the in-trail separation. In this model, we avoid such merging conflicts by applying the in-trail separation constraint at waypoints with the help of variable $\chi_w^{f_i}$, similar to the separation constraint in chapter 4.

$$\chi_w^{f_i} = \sum_{j \in U_w^{f_i}} Y_j^{f_i} \quad (5.26)$$

where $U_w^{f_i}$ is the set of upstream links connected with waypoint w in flight f_i 's path.

To avoid merging conflicts at waypoints, the constraints in (5.25) are modified as follows.

$$t_w^{f_k} - t_w^{f_i} \geq \sqrt{2}\tilde{\sigma}_w \operatorname{erf}^{-1}(2\gamma - 1) + \tilde{\mu}_w + T_{min} - M_1\delta_{f_k, f_i}^w - M_2(1 - \chi_w^{f_i}) - M_3(1 - \chi_w^{f_k}) \quad (5.27a)$$

$$t_w^{f_i} - t_w^{f_k} \geq \sqrt{2}\hat{\sigma}_w \operatorname{erf}^{-1}(2\gamma - 1) + \hat{\mu}_w + T_{min} - M_1(1 - \delta_{f_k, f_i}^w) - M_2(1 - \chi_w^{f_i}) - M_3(1 - \chi_w^{f_k}) \quad (5.27b)$$

5.3.3.2 En-route speed optimization

A smooth speed transition between adjacent links is required mainly for fuel-optimal travel and for the comfort of passengers and flight crew. Similar to the speed constraint in chapter 4, the constraint in (5.28) is also bounded the flight speed at link j , by allowing a slight adjustment of ϵ from the speed of the adjacent link j' , thus a smooth speed transition.

$$Z_{j'}^{f_i} - \epsilon < Z_j^{f_i} < Z_{j'}^{f_i} + \epsilon \quad (5.28)$$

5.3.4 Stage II - Objective function

At this stage, our main objective is to minimize the overall expected delay of ATN by minimizing the departure and arrival delay of each flight. The first two terms minimize the departure late and early deviations, respectively. The third and fourth terms minimize the late and early arrival deviations, respectively. At this stage, we do not need a cancellation option, as we only decompose flows into flights with a fine-tuning on timing. Rather than formulating as a quadratic objective, here, we transform it into a linear representation as follows.

$$\min \sum_{f_k \in F} \left\{ \phi_1 t_{dep+}^{f_k} + \phi_2 t_{dep-}^{f_k} + \phi_3 t_{arr+}^{f_k} + \phi_4 t_{arr-}^{f_k} \right\} \quad (5.29)$$

where,

$$t_{dep}^{f_k} - \hat{t}_{dep}^{f_k} = t_{dep+}^{f_k} - t_{dep-}^{f_k} \quad (5.30a)$$

$$t_{arr}^{f_k} - \hat{t}_{arr}^{f_k} = t_{arr+}^{f_k} - t_{arr-}^{f_k} \quad (5.30b)$$

$t_{dep+}^{f_k}, t_{dep-}^{f_k}$ and $t_{arr+}^{f_k}, t_{arr-}^{f_k}$ are the amounts of departure lateness, earliness and arrival lateness, earliness of flight f_k , respectively.

The arrival time of flight f_k at the arrival airport is derived as (including holding time as well),

$$t_{arr}^{f_k} = t_{dep}^{f_k} + \sum_{j \in J_{a,a'}^{f_k}} L_j Y_j^{f_k} Z_j^{f_k} \quad (5.31)$$

where $J_{a,a'}^{f_k}$ is the set of all possible links of f_k 's routing, starting from its origin

(a) to the destination (a').

The solution process of stage II MILP problem is less time consuming since it only requires to decompose the stage II flow results into individual flight plans under in-trail separation constraints. Thus, the stage II problem is solved directly via the MILP Gurobi solver.

5.4 Experimental Results

We use two case studies consisting of an example grid network and an actual ASEAN air traffic network to evaluate the performance and the real-time applicability of our model. We use MATLAB as the software platform with ILP Gurobi solver [19] on a PC with an Intel Core (TM) i5-8250U @1.80GHz CPU and an 8GB of RAM. We also simulated our flight plans in a commercial platform called AirTop (Air traffic optimization) [111] to test them in real conditions.

5.4.1 Case Study I (CS I) - Grid Network

The grid network shown in Fig. 5.3, which consists of 92 air routes, 8 departure airports, 8 arrival airports, and 44 waypoints, is used in this case study. The airspace is partitioned into 4 sectors, with each sector consisting of 15 en-route links. We consider 24 OD pairs with 3 flights from each, thus, a total of 72 flights with a time horizon of 10 hours. Further, the discrete time interval is taken as 10 min in this case study. The scheduled departure and arrival flows are tightly spanned to generate a congested network. The primary purpose of this case study is to analyze and validate the performance of the proposed model in detail under different scenarios.

For the uncertainty parameters at stage I, we considered the stochastic link flow as a normal distribution with, $\tilde{F}_j(t) \sim N(6, 1.288)$, and then following the steps in procedure 1, the uncertain component of the sector volume is derived as $\Delta_{V_s}(t) \sim N(0, 4.5538)$. The uncertainty of sector capacity is taken as, $\Delta_{C_s}(t) \sim N(0, 1.035)$, and then, the distribution of $\Delta_{V_s, C_s}(t)$ is estimated as, $\Delta_{V_s, C_s}(t) \sim N(0, 4.6624)$. The deterministic sector capacity was set as 12. Further, the waypoint flow rate uncertainty is taken as, $\Delta_{F_w}(t) \sim N(0, 0.288)$ and the airport related uncertainties are taken as, $\Delta_{F_d, C_D}(t) = \Delta_{F_a, C_A}(t) \sim N(0, 1)$.

At stage II, a minimum time separation of $T_{min} = 5 \text{ min}$ is applied. For the time uncertainties, we considered that the departure time uncertainty and the travel

Figure 5.3: Simulated grid network - Case Study I

time uncertainty are normally distributed. The mean and standard deviation are taken as 6.9 min and 2.7 min, respectively for the departure uncertainty and 0 and 0.15 s for the travel time uncertainty (per nautical mile).

5.4.1.1 Evaluation of en-route violations and delay performances with respect to the chance constraint probability

Here, we intend to analyze the safety assurance of the proposed model with respect to the selection of chance constraint probability. The sector capacity violations are calculated for all four sectors, and similarly, the in-trail separation violations are evaluated in terms of the number of flight pairs. To avoid anomalies in the results due to the selection of extreme cutoff bounds, we considered chance constraint probabilities of 95% and 5% instead of 100% and 0%, respectively. The results are tabulated in Table 5.1.

Further, we obtain a complete deterministic model by removing all uncertain com-

Table 5.1: CS I - Calculations of violations and delays against chance constraint probability

Chance constraint probability	Sector capacity violations	In-trail separation violations	Departure delay (min)	Arrival delay (min)	Cancellations
5%	23	82	4.36	9.62	0
20%	22	77	4.3683	9.6219	0
40%	20	73	4.4790	10.4949	0
60%	18	44	9.9026	10.5752	0
80%	10	34	9.2178	12.9666	0
95%	3	6	34.0571	38.2433	6

ponents. The deterministic model has 19 sector capacity violations and 71 in-trail separation violations, which highlights a serious safety related issue in ATFM. Thus, it emphasizes the importance of the consideration of both en-route capacity uncertainty and demand uncertainty in flow management. Further, the results show that the violation counts in 5%, 20%, and 40% are higher than the deterministic model. Therefore, this encourages users to select a chance constraint higher than 50% to get better solutions than the deterministic model.

As expected, we can see that the sector capacity and in-trail separation violations are decreasing with the increase of the chance constraint probability. It has the lowest number of violations when the constraints are sustained with a 95% of probability. Ostensibly, when a user selects a higher value for the chance constraint probability, it ensure a safer en-route ATN with a lower controller workload.

However, on the down-side, it is visibly evident that the flight delays have increased when the constraints are sustained with higher probabilities. The reason is that, flights have to stretch over a greater span of time to maintain the required safe separation, while constraining under uncertain capacities, when the chance constraint probability is high. However, the proposed model ensures higher safety while sacrificing flight times in comparison to the results of the deterministic model. As per the results, it can be concluded that a chance constraint probability closer to 80% would be ideal for the considered network, as it has minimized the possible conflicts with a competitive performance in terms of flight delays and cancellations.

5.4.1.2 Comparison of the solution time

Here, we compare the computational performance of the proposed solution approach (*MIS*) with respect to the global optimal solution obtained from the ILP Gurobi solver (*ILP*). A numerical comparison is depicted in Table 5.2.

Table 5.2: CS I - Comparison of MIS approach with ILP results

Parameters	ILP	MIS
Number of MISs	1	15
Stage I solution time	30.045 min	5.023 min
Stage II solution time	0.57 sec	
Total solution time	30.05 min	5.03 min
Total cost	$-6.8149 * 10^5$	$-5.92591 * 10^5$
Optimality gap	13.04%	

The stage I problem is solved using the MIS mechanism and the stage II problem did not require any additional solution mechanism as it is already a simplified problem thus, solved purely with the MILP Gurobi solver. The stage I flow-based problem is decomposed into 15 MISs and then solved as sub problems. From the results, we can see that the MIS approach only takes around 5 min to solve the problem, while the ILP Gurobi solver takes over 30 min to reach the global optimality. In both cases, stage II problem takes only around 0.6 sec to solve the flight-by-flight problem. Thus, in terms of the total solution time, the MIS approach is 6 times faster³.

In the table, a cost metric is calculated as the 'Total cost' based on the cost of flight delays and cancellations to evaluate the quality of the solution as follows.

$$\begin{aligned}
\text{Total cost}^4 = & (\text{departure cost} * \text{departure delay}) + \\
& (\text{arrival cost} * \text{arrival delay}) - \\
& (\text{cancellation cost} * [\text{Total number of flights} - \text{canceled flights}])
\end{aligned}
\tag{5.32}$$

where **departure cost**, **arrival cost** and **cancellation cost** refer to the values that are used in the objective function (5.17).

³It should be noted that the case study I network employed a tight schedule of flights, thus, a relatively high solution time is forced in comparison to existing literature.

Based on the values of the above performance metric, results from the MIS approach had an optimality gap of 13.04% with the ILP Gurobi optimal results, which clearly emphasizes that the proposed approach can generate a good enough result, closer to the optimal in a considerably limited time. The main reason for the above optimality gap is that, the MIS approach resembles a greedy approach, where sets of independent flows, with largest sets at first, are iteratively scheduled. Therefore, the flights in later iterations may have to be scheduled in an already congested network, as they are scheduled on top of the previously scheduled flights. Thus, flights may cancel, if they cannot be scheduled within their permissible bounds. Still, the MIS approach brings the solution time drastically down with a reasonable optimality gap.

5.4.2 Case Study II (CS II) - ASEAN Network

In case study II, we use a part of the ASEAN air traffic network shown in Fig. 3.12 to assess the feasibility of our model in a realistic large-scale system. As stated in the previous chapters, the network consists of 275 air routes, 78 waypoints, 25 airports, and 103 OD pairs. There are 695 flights within these 3 FIR regions according to the actual flight plan details. For the simplicity, the sector boundaries are considered as same with FIR boundaries, thus, the airspace is partitioned into 3 sectors. Here, we use the same parameter settings used in the case study I.

5.4.2.1 Comparison between the solutions of global optimal and MIS approach

In this experiment, a normal traffic with 695 flights is used to compare the results obtained from the ILP Gurobi solver and the MIS approach. First, we solved the global problem without any alteration using the ILP Gurobi solver and referred it as ‘ILP’ in Table 5.3. Then, the same problem is solved using the MIS approach after decomposing the stage I problem into a set of independent flows and referred it as ‘MIS’ in the table. Similar to the case study I, we do not use any computational method to solve the stage II problem.

According to the results, the ‘ILP’ approach found it difficult to solve the stage I problem even at 30 minutes. Thus, we set a time limit and terminated the solving process in 30 minutes and the results had an optimality gap of 0.87% with the global optimal. But, the proposed MIS approach took only 3.2 minutes to solve the stage I problem with an optimality gap difference of 1.76% with the results of

Table 5.3: CS II - Comparison between ILP and MIS solutions

Parameters	ILP	MIS [°]
Number of MISs	1	9
Stage I solution time	30 min*	3.2 min
Stage II solution time	~ 12 sec	
Total solution time	30.2 min	3.4 min
Average departure delay	19.43 min	19.61 min
Average arrival delay	9.34 min	9.61 min
No. of cancellations	9	21
Performance metric	$-6.8505 * 10^6$	$-6.7302 * 10^6$
Optimality gap difference	1.76%	

* - terminated using a time limit and the optimality gap was 0.87%. ° - without matrix generation time.

‘ILP’. However, the number of cancellations in the MIS approach is higher than the ‘ILP’ results, due to its iterative procedure, which congest the network for the flights in later iterations, as depicted under the case study I.

5.4.2.2 Comparison of solution time

In this experiment, we generate results for three traffic cases with 695 (single), 1390 (double), and 2085 (triple) flights and the results are depicted in Table 5.4. The intention here is to illustrate the computational feasibility of our model in different traffic scenarios by congesting the same en-route network.

Table 5.4: CS II - Normal, double and triple traffic results

Parameters	695 flights		1390 flights		2085 flights	
	<i>ILP</i>	<i>MIS</i> [°]	<i>ILP</i>	<i>MIS</i> [°]	<i>ILP</i>	<i>MIS</i> [°]
Stage I - Solution time	30 min*	3.4 min	30 min [°]	6.9 min	30 min*	11.8 min
Stage II - Solution time	~ 12 sec		~ 14 sec		~ 19 sec	
Optimality gap difference	1.76%		12.0%		7.95%	

* - terminated using time limit and the optimality gap was 0.87%. ° - terminated using time limit and the optimality gap was 0.39%, * - terminated using time limit and the optimality gap was 0.02%, ° - without matrix generation time.

In all three traffic scenarios, the ILP Gurobi solver takes considerable times to

solve the problems. In both double and triple traffic scenarios, the stage I problem could not be solved even in 30 min time. Hence, we set a time limit at 30 min and terminated the solving process. However, the results at those points are not the global optimal, yet, very close to the optimal solutions. When the aforementioned problems are solved using the proposed MIS approach, it only takes a few minutes to solve the same stage I problem. For an instance, in the tripled traffic case with 2085 flights, the MIS approach only needed 11.8 min to solve the stage I problem, and results have an optimality gap of 7.95% with the results at 30 min terminated point. Table 5.5 illustrates a comparison between the results in chapter 4 with the results of this model. As shown in the table the model presented in chapter 4 takes 39.8 min to solve the doubled traffic problem with 1390 flights due to its flight-by-flight structure. But, with these two-stage approach, the same traffic takes only 6.9 min to solve, hence, approximately 5 times faster than the flight-by-flight model in chapter 4.

Table 5.5: S

olution time comparison with Chapter 4 results

Number of Flights	Chapter 4 model		Chapter 5 model	
	T_{org}	T_{MIS}°	T_{org}	T_{MIS}°
695	★	4.9 min	30 min*	3.4 min
1390	★	39.8 min	30 min $^{\diamond}$	6.9 min

★ - cannot solve even after 3 hours, T_{org} - without any computational improvement, T_{MIS} - with the pre-solving part and MWIS approach, $^{\circ}$ - without matrix generation time, * - terminated using time limit and the optimality gap was 0.87%, $^{\diamond}$ - terminated using time limit and the optimality gap was 0.39%

This experimental study illustrates that the proposed two-stage ATFM structure upheld the scalability via its flow-based approach at stage I. It is clear from stage I solution time that, even when the traffic is doubled or tripled, the solution time only increases by few minutes. This is mainly due to the fact that, in our flow-based approach, the number of decision variables is independent from the traffic, unlike in a flight-by-flight-based model.

5.4.3 Case Study III - ASEAN Network Simulation on AirTop

To validate our model in real conditions and to highlight the importance of scrutinizing uncertainty in ATFM, we used AirTop [111] fast-time simulation tool, which is a commercial software platform used in air traffic assessment and model-

ing. As illustrated in Fig. 3.12, we built an ATN of 3 FIRs, including waypoints, ATC routes, ATC sectors, airports, runways, etc. The flight plan data extracted from Flightradar24 [2] tool are used here as in the case study II.

A real air traffic network is built in AirTOp by setting up different uncertainty conditions. We set up in-trail separation at waypoints by enabling *NetQueue.Rate* feature, whereas, sector capacities are set up as a *time-varying capacity* to reflect the capacity uncertainty. Further, departure time uncertainties are embedded for each flight plan and wind areas are set up to reflect the speed uncertainties of flights. To reduce the complexity, we only set tight bounds for the capacity of the Singapore sector with a deterministic capacity of 4 and uncertainty of $\Delta_{V_s, C_s}(t) \sim N(0, 1)$. Other settings in AirTOp are configured similar to case study II.

First, we solved the problem using Matlab for the same settings and obtained the flight plans in two different scenarios as follows.

Scenario I :- Deterministic model - removed all uncertain components and obtained results for a complete deterministic model

Scenario II :- Stochastic model - obtained results for the model proposed in Section 5.2 and 5.3 with a chance constraint probability of 80%.

The flight plans obtained in both scenarios are simulated in the AirTOp platform under realistic environment settings, which is composed of radar controllers, network managers, aircraft operator centers, etc. The results are tabulated in Table 5.6 for the two scenarios. The stochastic model displayed an average delay reduction of 41.6% in comparison to the delay of the deterministic model while sustaining a conflict reduction of 22.8%. This highlights the importance of uncertainty consideration in air traffic flow management. Moreover, the charts in Fig. 5.4 are obtained from AirTOp at the end of the simulation, which exhibits the controller workload of the Singapore sector for both deterministic and stochastic scenarios. The controller workload of the stochastic model is lower than the workload of the deterministic model, which shows the effectiveness of the proposed model. As we have already considered uncertain circumstances in advance to a certain level in our proposed model, the effect of uncertainty, when encountered in a realistic environment such as AirTOp, is minimum. However, in a deterministic approach, when the uncertain events are realized at the deployment, the effect could be devastated as comparatively shown by the extent of conflict, delay, and controller workload in this experimental study.

CH. 5. TWO-STAGE SCALABLE ATFM MODEL UNDER UNCERTAINTY

5.4. EXPERIMENTAL RESULTS

(a) Controller workload - Scenario I (Deterministic Model)

(b) Controller workload - Scenario II (Stochastic Model)

Figure 5.4: Radar controller workload variation (Legend: Pink - Total workload, Purple - Monitoring workload , Blue - Radio telephony workload , Green - Conflict resolution workload, Red - Level clearance workload)

Table 5.6: Results comparison in AirTOp simulation

Parameters \ Scenarios	Scenario I	Scenario II
Average delay time	84.3 min	49.2 min
Delay reduction of Scenario II with respect to Scenario I	41.6%	
Conflict reduction of Scenario II with respect to Scenario I	22.8%	

5.5 Summary

In this chapter, we have proposed a comprehensive ATFM framework for the pre-tactical phase with a two-stage architecture involving both flow and flight-by-flight concepts under different uncertainty conditions. The stage I model captures constraints of link, airport and sector capacities, and the flow rates at waypoints. At each discrete time step, stage I generates flow results for each link. The resulting flow values are input to the stage II model in order to safely separate flights and to make a smooth speed transition at adjacent links, while minimizing departure and arrival delays of individual flights. The stage I model is solved via an updated version of the MIS approach for flows and the stage II model is solved directly via SMILP Gurobi solver due to its low complexity. The experimental case studies have illustrated the improvements and applicability of this model in realistic scenarios, in comparison to the models presented in previous chapters.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this thesis, we have proposed several frameworks for flight routing and scheduling under different uncertainty conditions.

First, we have presented a preliminary flight routing and scheduling model while considering the stochastic nature of demand due to the uncertainty of departure time and speed deviations for the pre-tactical phase. The problem was formulated as an MIQP problem with the objective of minimizing the deviation from the original flight plan, which is quantified in terms of departure deviation, arrival deviation as well as speed deviation. Moreover, the proposed model minimized fuel consumption by using an optimal flight level assignment strategy. Most importantly, the model was formulated to eliminate the in-trail separation violations from air transportation system by ensuring the minimum safety separation requirement between flights even under uncertainty. To overcome the computational complexity involved in trajectory-based modeling, we introduced a greedy strategy with parallel computation and maximum independent sets. The experimental results verified the computational advantage of the proposed method to solve realistic large-scale ATFM problems even under the consideration of demand uncertainty in a limited time.

The above framework was modeled under several assumptions in order to have a low complexity model. Among them, the independent assumption of uncertainty distributions and the assumption of deterministic sector capacity form a gap with the real ATN. Thus, later on, we focused on removing these assumptions to achieve

a more realistic representation of an ATFM model under uncertainty.

First, we dropped the assumption of the deterministically known sector capacities and acknowledged the uncertainties exist in airspace capacity. Moreover a comprehensive study on ATFM was proposed while scrutinizing both demand and capacity uncertainties in ATN. A two-phase framework was proposed, including the pre-tactical and tactical phases of ATFM, in which, the pre-tactical phase aimed for finding the BADA based fuel optimal and feasible trajectories, whereas, the tactical phase fine tuned the flight times on top of the provided trajectories. More importantly, via a tactical flight scheduling scheme which incorporates real-time updates, the adverse effects incurred from the inclusion of inherent uncertainties on the flight delays, were minimized. In terms of formulation, an MILP model was developed including both sector capacity and in-trail separation constraints, which is vital for the air transportation safety. The computation complexity was reduced by a heuristic preprocessing mechanism followed by the MWIS approach. With several simulation studies, we showed the performance improvements, enhancement of safety, real-time applicability, and also financial benefits of the model.

Identifying the trade-offs between flight-by-flight and flow-based formulation structures, we further modified our framework into a two-stage architecture for the pre-tactical phase, incorporating pros of both flow and flight-by-flight concepts. The flow-based modeling structure at stage I, enhanced the scalability while constraining the link, sector and airport capacities and also the waypoint flow rates. The stage II model ensured that the flights are safely separated with the required in-trail separation and thus, it maintained the safety of the overall system. Furthermore, stage I formulation scrutinized the flow and capacity uncertainties, and the stage II formulation scrutinized the demand uncertainty of flights. Through realistic experimental studies, we comparatively showed the enhanced performance of the proposed framework in terms of safety assurance, violation/conflicts reduction, scalability and also real-time applicability.

In summary, this thesis stands out by scrutinizing the effect of inherent uncertainty, real-time applicability, and a diverse set of air transportation system attributes with priority on the safety, in a set of comprehensive ATFM frameworks, yet still with a real-time acceptable complexity. Therefore, it provides meaningful contributions in the flow management process with respect to the existing literature. We have encapsulated different features from the proposed models into a single ATFM framework in Fig. 6.1.

Figure 6.1: Encapsulation of different features into a single ATFM framework.

6.2 Future Work

In this thesis, we have scrutinized various forms of uncertainty existing in air transportation system and incorporated them in ATFM. Furthermore, computational issues are handled to a certain extent via improved solution mechanisms and layered formulation structures.

Still, many foreseeable improvements, directions, and applications are possible in related to ATFM. Here, we discuss a few directions with respect to the content of this thesis.

- Incorporating diverse perspectives of stakeholders

In this thesis, our approach on ATFM is from a general stand-point. However, there are multiple stakeholders in ATFM, which are passengers, airline operators, and network managers/ATCos. They have diverse expectations from/on ATFM. Airline passengers are the ultimate entity that air transportation system serves. From their perspective, the main disruptions in air transportation system are flight delays and cancellations. According to the literature [112] and our daily experiences, the passengers mostly concerns about the arrival delays over departure delays. In terms of airline operators, the most important delay component is also the arrival delay as it causes missing flight compensation, extra fuel costs,

gate/airport charges, ground handling charges, etc. However, according to [112], the cost associated with the arrival delay does not linearly increase. Furthermore, the network operator generally focuses on a smooth operation in air transportation system with low safety risks. In terms of ATCos, their workload generally depends on the congestion/number of aircraft in the network.

The above expectations from stakeholders generally arise at the tactical phase. However, in order to fulfill them, decisions have to be made starting from the pre-tactical phase. These diverse perspectives and expectations naturally lead to game-theoretic situations, where stakeholders/actors cooperatively or non cooperatively compete for their shares. A possible straightforward improvement for the proposed framework is to consider the above factors/perspectives of different stakeholders and formulate as multiple objectives in the flow management process. Furthermore, a Pareto frontier can be generated to get better results for the overall problem.

- Analysis of correlation among aircraft flows in adjacent links/sectors

When we obtained the sector flow distribution as the summation of link flows, we considered the correlation among adjacent links, in theoretical terms, since a variation of flow at one link could change the flow of its adjacent links. However, we didn't conduct experiments to evaluate the adjacent links' flow correlation due to the lack of real data. An analysis can be conducted if link volume data are available at discrete time steps under different airspace conditions. One possible future work is to perform an extensive historical data analysis to evaluate such correlations and to modify the model further towards reality. This work falls into the pre-tactical phase, as the flow routing and scheduling is done prior to the actual operation.

- Further reduction of computational complexity for very large-scale problems

In the proposed frameworks, we have reduced the computational complexity by a certain level via decomposing the large problem into a set of sub problems, which can be solved in parallel and following a two-stage formulation architecture. Further improvements can be carried out in future work to reduce the complexity, especially to solve a very large-scale network with more than 10,000 flights, multiple FIR regions, while retaining detailed flight information under the consideration of uncertainty. Although a centralized approach generates the optimal results, when the problem expands into a scale of tens of FIRs, a distributed solving mechanism is required to solve the problem efficiently. A possible approach would be to solve

local FIRs separately in parallel and to generate the global solution possibly following a distributed computational approach such as the Lagrangian multiplier method [26]. Naturally, the computational complexity issue needs to be addressed at both pre-tactical and tactical phases.

- Estimation of demand-capacity correlation

In this thesis, we considered demand and capacity uncertainties separately in the formulation. Airport and/or airspace capacity variations due to adverse weather conditions and ATCo workload variations are categorized as capacity uncertainty. Uncertainty factors such as departure time deviations, speed variations, pilot or engine efficiency variations are compounded into the time deviation of the flight from the scheduled time, and it is categorized as the demand uncertainty. But the adverse weather conditions can also cause fluctuations in speed, thus, influence the demand uncertainty.

Therefore, it can be argued that the demand uncertainty and capacity uncertainty are not mutually exclusive, rather slightly dependent based on the common factors such as weather conditions as discussed above. Hence, if the dependency and/or correlation between capacity and demand can be analyzed/quantified, the proposed frameworks can be further modified including the correlation components. The estimation of demand-capacity correlation should be performed on the previous data, and thus, does not concern the ATFM operation.

- Modifying the model with the concept of Free Route Airspace (FRA)

The FRA concept is introduced by the Single European Sky Airspace Research (SESAR) Program [113], [114], so that flights can freely route between a defined entry point and an exit point. Depending on the availability of the airspace, the trajectory may be planned directly from one point to the other or through intermediate turning points (navigational aids, published navigational points or points with specified coordinates). With this new concept of FRA, flights are not routed on the defined ATC route, rather they use the airspace freely under the control of ATCos. Hence, it poses a number of challenges in both pre-tactical and tactical phases, to route flights safely and clearly, there are more research challenges/openings with this topic. It will be even more challenging to route flights in an FRA even under uncertainty conditions. Therefore, as a future work, the proposed frameworks can be developed further by adopting the new FRA concept.

Author's Publications

Journal Papers

1. **G. G. N. Sandamali**, R. Su and Y. Zhang, "Flight Routing and Scheduling Under Departure and En-route Speed Uncertainty," in *IEEE Transactions on Intelligent Transportation Systems*, 2019, doi: 10.1109/TITS.2019.2907058.
2. **G. G. N. Sandamali**, R. Su, K. L. K. Sudheera, and Y. Zhang, "A Safety-Aware Real-time Air Traffic Flow Management Model Under Demand and Capacity Uncertainties," *IEEE Transactions on Intelligent Transportation Systems*, (Under Major Revision).
3. **G. G. N. Sandamali**, R. Su, K.L.K. Sudheera, Y. Zhang and Y. Zhang, "Two-stage Scalable Air Traffic Flow Management Model Under Uncertainty," in *IEEE Transactions on Intelligent Transportation Systems* (Under Major Revision).
4. Y. Zhang, R. Su, **G. G. N. Sandamali**, Y. Zhang, C. G. Cassandras and L. Xie, "A Hierarchical Heuristic Approach for Solving Air Traffic Scheduling and Routing Problem with a Novel Air Traffic Model," in *IEEE Transactions on Intelligent Transportation Systems*, doi: 10.1109/TITS.2018.2874235.
5. Y. Zhang, R. Su, Y. Zhang, and **G. G. N. Sandamali**, "A multi-bus dispatching strategy with loading process," in *IEEE Transactions on Intelligent Transportation Systems* (Under Major Revision).

Conference Papers

1. **G. G. N. Sandamali**, R. Su, Y. Zhang, and Q. Li, "Flight Routing and Scheduling with Departure Uncertainties in Air Traffic Flow Management," in *2017 13th IEEE International Conference on Control & Automa-*

- tion (*ICCA*), Ohrid, 2017, pp. 301-306.
2. **G. G. N. Sandamali**, R. Su, and Y. Zhang, “Short-term En-route Air Traffic Flow Management Under Departure and Wind Uncertainties with a Heuristic and Greedy Solution Approach,” in *2019 American Control Conference (ACC)*, Philadelphia, USA, 2019, pp. 2121-2126.
 3. **G. G. N. Sandamali**, R. Su, and K.L.K. Sudheera, “Flow-based Flight Routing and Scheduling under Uncertainty,” in *21st IFAC World Congress* (Accepted).
 4. Y. Zhang, R. Su, **G.G.N. Sandamali**, Y. Zhang and C.G. Cassandras. A Hierarchical Approach for Air Traffic Routing and Scheduling, in *IEEE 56th Annual Conference on Decision and Control (CDC)*, Melbourne, VIC, 2017, pp. 6277-6282.

Bibliography

- [1] International Civil Aviation Organization, “ATFM Post-Operations Analysis Recommended Framework.” [Online]. Available: https://www.icao.int/APAC/APAC-RSO/ATFMIRSWG/ATFM-IR-SWG_4th%2021-22%20November%202019/05-1%20Post-Ops%20Framework_v2.2.pdf
- [2] “Flightradar24,” Accessed: Aug. 31, 2018. [Online]. Available: <https://www.flightradar24.com/>
- [3] “Skyvector,” Accessed: Aug. 18, 2017. [Online]. Available: <https://skyvector.com/>
- [4] D. Chen, M. Hu, H. Zhang, J. Yin, and K. Han, “A network based dynamic air traffic flow model for en route airspace system traffic flow optimization,” *Transportation Research Part E: Logistics and Transportation Review*, vol. 106, pp. 1–19, 2017.
- [5] EUROCONTROL, “User Manual For the Base Of Aircraft Data (BADA) Revision 3.14,” Tech. Rep., 2017, EEC Technical/Scientific Report No. 17/05/29-143.
- [6] IATA Forecast Predicts 8.2 billion Air Travelers in 2037. Accessed: May. 04, 2019. [Online]. Available: <https://www.iata.org/pressroom/pr/Pages/2018-10-24-02.aspx>
- [7] International Civil Aviation Organization, “A Coordinated, Risk-based Approach to Improving Global Aviation Safety : ICAO Safety Report 2014,” Tech. Rep., 2014.
- [8] “Mid-air collision inquiry begins,” Accessed: Sep. 11, 2017. [Online]. Available: <http://edition.cnn.com/2002/WORLD/europe/07/02/midair.crash/>

- [9] E. E. Khoury, "The need for ATFM/CDM," in *ACAO/ICAO ATFM Workshop*, 17-18 March 2019, Casablanca, Morocco. [Online]. Available: <https://www.icao.int/MID/Documents/2019/ACAO-ICAO%20ATFM%20Workshop/1.2-%20ATFM%20ICAO%20Prespective%20-%20Elie.pdf>
- [10] "Manual of Standards - Air Traffic Services," Report, Version 2.5, February 2017, Civil Aviation Authority of Singapore.
- [11] S. Smith and E. Gilbo, "Analysis of uncertainty in ETMS aggregate demand predictions," 2005, volpe National Transportation Systems Center, Report no. VNTSC-ATMS-05-05.
- [12] "Flightaware," Accessed: Oct. 18, 2018. [Online]. Available: <https://flightaware.com/resources/weathermaps/>
- [13] S. Hah, B. Willems, and R. Phillips, "The effect of air traffic increase on controller workload," *Proceedings of the Human Factors and Ergonomics Society Annual Meeting*, vol. 50, no. 1, pp. 50–54, 2006.
- [14] "A Brief History of RVSM," April 2010, ATM section, Air Navigation Bureau, ICAO HQ. [Online]. Available: <https://www.icao.int/ESAF/Documents/RVSM/2010/afi%20riss/presentations.pdf>
- [15] "Scoping Improvements to 'See And Avoid' for General Aviation (SISA)," April 2010, Research Project EASA.2011/07, European Aviation Safety Agency. [Online]. Available: <https://www.easa.europa.eu/sites/default/files/dfu/safety-and-research-research-projects-docs-general-aviation-Final-Report-EASA.2011.07.pdf>
- [16] Asia/Pacific Air Traffic Flow Management Steering Group, (ATFM/SG), "Asia/Pacific Regional Air Traffic Flow Management Concept Of Operations," International Civil Aviation Organization, Report, 2015. [Online]. Available: <https://www.icao.int/APAC/Documents/edocs/Regional%20ATFM%20Concept%20of%20Operations.pdf>
- [17] A. Akgunduz, B. Jaumard, and G. Moeini, "Deconflicted air-traffic planning with speed-dependent fuel-consumption formulation," *IEEE Transactions on Intelligent Transportation Systems*, vol. 19, no. 6, pp. 1890–1901, 2018.
- [18] P. Dell'Olmo and G. Lulli, "A new hierarchical architecture for air traffic management: Optimisation of airway capacity in a free flight scenario,"

BIBLIOGRAPHY

- European Journal of Operational Research*, vol. 144, no. 1, pp. 179–193, 2003.
- [19] “Gurobi Optimizer Reference Manual, Gurobi Optimization, Inc.,” Tech. Rep., 2015, houston, TX, USA.
- [20] “IBM ILOG CPLEX Optimization Studio, CPLEX User’s Manual,” Tech. Rep. [Online]. Available: https://www.ibm.com/support/knowledgecenter/SSSA5P_12.7.0/ilog.odms.studio.help/pdf/usrcplex.pdf
- [21] Y. Liu, Z. Zhang, E. K. P. Chong, and A. Pezeshki, “Performance bounds for the k-batch greedy strategy in optimization problems with curvature,” in *2016 American Control Conference (ACC)*, Conference Proceedings, pp. 7177–7182.
- [22] D. Bertsimas and S. S. Patterson, “The air traffic flow management problem with enroute capacities,” *Oper. Res.*, vol. 46, no. 3, pp. 406–422, 1998.
- [23] D. Bertsimas, G. Lulli, and A. Odoni, “An integer optimization approach to large-scale air traffic flow management,” *Operations Research*, vol. 59, no. 1, pp. 211–227, 2011.
- [24] A. Agustin, A. Alonso-Ayuso, L. F. Escudero, and C. Pizarro, “On air traffic flow management with rerouting. part ii: Stochastic case,” *European Journal of Operational Research*, vol. 219, no. 1, pp. 167–177, 2012.
- [25] —, “On air traffic flow management with rerouting. part i: Deterministic case,” *European Journal of Operational Research*, vol. 219, no. 1, pp. 156–166, 2012.
- [26] Y. Zhang, R. Su, Q. Li, C. G. Cassandras, and L. Xie, “Distributed flight routing and scheduling for air traffic flow management,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 18, pp. 2681–2692, 2017.
- [27] Y. Zhang, R. Su, G. G. N. Sandamali, Y. Zhang, and C. G. Cassandras, “A hierarchical approach for air traffic routing and scheduling,” in *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, Dec 2017, pp. 6277–6282.
- [28] H. Balakrishnan and B. G. Chandran, “Optimal large-scale air traffic flow management,” 2014, Unpublished Work.

- [29] A. Agustin, A. Alonso-Ayuso, L. F. Escudero, and C. Pizarro, “Mathematical optimization models for air traffic flow management: A review,” *Stud. Inform. Univ.*, vol. 8, pp. 141–184, 2010.
- [30] B. Sridhar and P. Menon, “Comparison of linear dynamic models for air traffic flow management,” in *IFAC World Congress*, Conference Proceedings, pp. 1962–1962.
- [31] D. Bertsimas, G. Lulli, and A. Odoni, “The air traffic flow management problem: An integer optimization approach,” in *Proceedings of the 13th International Conference on Integer Programming and Combinatorial Optimization*, ser. IPCO’08. Berlin, Heidelberg: Springer-Verlag, 2008, pp. 34–46.
- [32] A. M. Churchill, D. J. Lovell, and M. O. Ball, “Evaluating a new formulation for large-scale traffic flow management,” in *8th ATM Seminar*, 2009, Napa, California, USA.
- [33] A. M. Bayen, R. L. Raffard, and C. J. Tomlin, “Eulerian network model of air traffic flow in congested areas,” in *American Control Conference, 2004. Proceedings of the 2004*, vol. 6. IEEE, 2004, pp. 5520–5526.
- [34] —, “Adjoint-based control of a new eulerian network model of air traffic flow,” *Control Systems Technology, IEEE Transactions on*, vol. 14, no. 5, pp. 804–818, 2006.
- [35] D. Bertsimas and S. S. Patterson, “The traffic flow management rerouting problem in air traffic control: A dynamic network flow approach,” *Transportation Science*, vol. 34, no. 3, pp. 239–255, 2000.
- [36] P. B. Vranas, D. J. Bertsimas, and A. R. Odoni, “The multi-airport ground-holding problem in air traffic control,” *Operations Research*, vol. 42, no. 2, pp. 249–261, 1994.
- [37] P. B. M. Vranas, D. Bertsimas, and A. R. Odoni, “Dynamic ground-holding policies for a network of airports,” *Transportation Science*, vol. 28, no. 4, pp. 275–291, 1994.
- [38] D. Bertsimas and M. Frankovich, “Air traffic flow management at airports: A unified optimization approach,” in *10th ATM Seminar*, 2013, Chicago, IL.
- [39] Q. Li, Y. Zhang, and R. Su, “A flow-based flight scheduler for en-route air traffic management,” *IFAC-PapersOnLine*, vol. 49, no. 3, pp. 353–358, 2016.

- [40] D. Sun and A. M. Bayen, “Multicommodity eulerian-lagrangian large-capacity cell transmission model for en route traffic,” *Journal of Guidance, Control, and Dynamics*, vol. 31, no. 3, pp. 616–628, 2008. [Online]. Available: <http://dx.doi.org/10.2514/1.31717>
- [41] Y. Zhang, R. Su, G. G. N. Sandamali, Y. Zhang, C. G. Cassandras, and L. Xie, “A hierarchical heuristic approach for solving air traffic scheduling and routing problem with a novel air traffic model,” *IEEE Transactions on Intelligent Transportation Systems*, pp. 1–14, 2018.
- [42] Y. Cao and D. Sun, “Link transmission model for air traffic flow management,” *Journal of Guidance, Control, and Dynamics*, vol. 34, no. 5, pp. 1342–1351, 2011.
- [43] B. Sridhar, T. Soni, K. Sheth, and G. Chatterji, “Aggregate flow model for air-traffic management,” *Journal of Guidance, Control, and Dynamics*, vol. 29, no. 4, pp. 992–997, 2006. [Online]. Available: <http://arc.aiaa.org/doi/abs/10.2514/1.10989>
- [44] H. D. Sherali, R. W. Staats, and A. A. Trani, “An airspace planning and collaborative decision-making model: Part i—probabilistic conflicts, workload, and equity considerations,” *Transportation Science*, vol. 37, no. 4, pp. 434–456, 2003.
- [45] —, “An airspace-planning and collaborative decision-making model: Part ii—cost model, data considerations, and computations,” *Transportation Science*, vol. 40, no. 2, pp. 147–164, 2006.
- [46] A. Alonso-Ayuso, L. F. Escudero, P. Olasso, and C. Pizarro, “Conflict avoidance: 0-1 linear models for conflict detection and resolution,” *TOP*, vol. 21, pp. 485–504, 2013.
- [47] A. Alonso-Ayuso, L. F. Escudero, and F. J. Martín-Campo, “Collision avoidance in air traffic management: A mixed-integer linear optimization approach,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 12, no. 1, pp. 47–57, March 2011.
- [48] A. Alonso-Ayuso, L. F. Escudero, and F. J. Martín-Campo, “A mixed 0–1 nonlinear optimization model and algorithmic approach for the collision avoidance in atm: Velocity changes through a time horizon,” *Computers and Operations Research*, vol. 39, no. 12, pp. 3136 – 3146, 2012.

- [49] G. Lulli and A. Odoni, “The European Air Traffic Flow Management Problem,” *Transportation Science*, vol. 41, no. 4, pp. 431–443, 2007. [Online]. Available: <https://pubsonline.informs.org/doi/abs/10.1287/trsc.1070.0214>
- [50] “Strategic flow management for air traffic control,” *Operations Research*, vol. 41, no. 1, pp. 138–152, Feb. 1993.
- [51] G. Andreatta, A. R. Odoni, and O. Richetta, “Models for the ground holding problem,” in *Large Scale Computation and Information Processing in Air Traffic Control*, L. Bianco and A. R. Odoni, Eds. Springer Berlin Heidelberg, Conference Proceedings, pp. 125–168.
- [52] E. P. Gilbo, “Airport capacity: representation, estimation, optimization,” *IEEE Transactions on Control Systems Technology*, vol. 1, no. 3, pp. 144–154, Sep. 1993.
- [53] M. Zhengping, C. Deguang, and C. Peng, “Dynamic network flow model for short-term air traffic flow management,” *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, vol. 34, no. 3, pp. 351–358, 2004.
- [54] P. A. Dmochowski and J. Skorupski, “Air traffic smoothness. a new look at the air traffic flow management,” *Transportation Research Procedia*, vol. 28, pp. 127 – 132, 2017, iNAIR 2017.
- [55] G. Andreatta, P. Dell’Olmo, and G. Lulli, “An aggregate stochastic programming model for air traffic flow management,” *European Journal of Operational Research*, vol. 215, no. 3, pp. 697–704, 2011.
- [56] M. Ball and G. Lulli, “Ground delay programs: Optimizing over the included flight set based on distance,” *Air Traffic Control Quarterly* 12(1), 2004.
- [57] G. M. Caron, “Optimization and uncertainty handling in air traffic management,” Thesis, 2014, Université Paris-Sud 11.
- [58] R. Kicinger, J.-T. Chen, M. Steiner, and J. Pinto, “Airport capacity prediction with explicit consideration of weather forecast uncertainty,” *Journal of Air Transportation*, vol. 24, no. 1, pp. 18–28, 2016.
- [59] E. P. Gilbo, “Optimization of air traffic management strategies at airports with uncertainty in airport capacity,” *IFAC Proceedings Volumes*, vol. 30, no. 8, pp. 35 – 40, 1997, 8th IFAC/IFIP/IFORS Symposium on Transportation Systems 1997 (TS ’97), Chania, Greece, 16-18 June.

- [60] G. Roberts, K. Grundmann, C. Devlin, M. Fulgenzi, V. Sacro, S. Stapleton, and S. Szurgyi, “Airport arrival and departure management under weather uncertainty,” in *Proceedings of the 2004 IEEE Systems and Information Engineering Design Symposium, 2004.*, April, 2004, pp. 131–138.
- [61] O. Richetta and A. R. Odoni, “Solving optimally the static ground-holding policy problem in air traffic control,” *Transportation Science*, vol. 27, no. 3, pp. 228–238, 1993.
- [62] ———, “Dynamic solution to the ground-holding problem in air traffic control,” *Transportation Research Part A: Policy and Practice*, vol. 28, no. 3, pp. 167–185, 1994.
- [63] G. Hunter and K. Ramamoorthy, “National airspace strategies for future demand scenarios in inclement weather,” in *24th Digital Avionics Systems Conference*, vol. 1, Oct 2005, pp. 3.E.2–31.
- [64] S. Zobell, C. Wanke, and L. Song, “Probabilistic airspace congestion management,” in *12th Conference on Aviation Range and Aerospace Meteorology*, 2006, Atlanta, GA.
- [65] J.-P. Clarke, S. Solak, Y.-H. Chang, L. Ren, and A. E. Vela, “Air traffic flow management in the presence of uncertainty,” in *8th ATM Seminar*, 2009, Napa, California, USA.
- [66] J. Chen, L. Chen, and D. Sun, “Air traffic flow management under uncertainty using chance-constrained optimization,” *Transportation Research Part B: Methodological*, vol. 102, pp. 124–141, 2017.
- [67] G. Clare and A. Richards, “Air traffic flow management under uncertainty: Application of chance constraints,” in *Proceedings of the 2Nd International Conference on Application and Theory of Automation in Command and Control Systems*, ser. ATACCS ’12. Toulouse, France, France: IRIT Press, 2012, pp. 20–26.
- [68] A. Nilim and L. E. Ghaoui, “Algorithms for air traffic flow management under stochastic environments,” in *Proceedings of the 2004 American Control Conference*, vol. 4, Conference Proceedings, pp. 3429–3434 vol.4.
- [69] J. Rios and A. Morando, “The value of reduced uncertainty in air traffic flow management,” in *AIAA Guidance, Navigation, and Control Conference*, pp. 1–14, Portland, Oregon.

- [70] C. Wanke, M. Callahan, D. Greenbaum, and A. Masalonis, *Measuring Uncertainty in Airspace Demand Predictions for Traffic Flow Management Applications*, ser. Guidance, Navigation, and Control and Co-located Conferences. American Institute of Aeronautics and Astronautics, 2003.
- [71] E. Gilbo and S. Smith, *A New Model to Improve Aggregate Air Traffic Demand Predictions*, ser. Guidance, Navigation, and Control and Co-located Conferences. American Institute of Aeronautics and Astronautics, 2007.
- [72] M. Ball, T. Vossen, and R. Hoffman, “Analysis of demand uncertainty effects in ground delay programs,” in *4th ATM Seminar*, 2001, Conference Proceedings, Santa Fe, NM, USA.
- [73] O. Richetta, “Optimal algorithms and a remarkably efficient heuristic for the ground-holding problem in air traffic control,” *Operations Research*, vol. 43, no. 5, pp. 758–770, 1995. [Online]. Available: <https://pubsonline.informs.org/doi/abs/10.1287/opre.43.5.758>
- [74] M. O. Ball, R. L. Hoffman, A. Odoni, and R. Rifkin, “The static stochastic ground holding problem with aggregate demands,” 1999, NEXTOR (National Center of Excellence in Aviation Operations Research).
- [75] P. Wei, Y. Cao, and D. Sun, “Total unimodularity and decomposition method for large-scale air traffic cell transmission model,” *Transportation Research Part B: Methodological*, vol. 53, pp. 1 – 16, 2013.
- [76] K. S. Lindsay, E. A. Boyd, and R. Burlingame, “Traffic flow management modeling with the time assignment model,” *Air Traffic Control Quarterly*, vol. 1, no. 3, pp. 255–276, 1993.
- [77] G. G. N. Sandamali, R. Su, Y. Zhang, and Q. Li, “Flight routing and scheduling with departure uncertainties in air traffic flow management,” in *2017 13th IEEE International Conference on Control Automation (ICCA)*, July 2017, pp. 301–306.
- [78] G. G. N. Sandamali, R. Su, and Y. Zhang, “Flight routing and scheduling under departure and en route speed uncertainty,” *IEEE Transactions on Intelligent Transportation Systems*, pp. 1–14, 2019.
- [79] Mach Number. Accessed: Nov. 15, 2017. [Online]. Available: https://www.skybrary.aero/index.php/Mach_Number

BIBLIOGRAPHY

- [80] R. C. Mittelhammer, *Mathematical Statistics for Economics and Business*, 2nd ed., Springer : New York, 2013.
- [81] K. S. Kalupahana Liyanage, M. Ma, and P. H. J. Chong, “Connectivity aware tribrid routing framework for a generalized software defined vehicular network,” *Computer Networks*, vol. 152, pp. 167–177, 2019.
- [82] K. Sudheera K.L., M. Ma, and P. H. J. Chong, “Efficient flow instantiation via source routing in software defined vehicular networks,” in *2017 IEEE 86th Vehicular Technology Conference (VTC-Fall)*, 2017, pp. 1–5.
- [83] K. L. K. Sudheera, M. Ma, and P. H. J. Chong, “Link stability based hybrid routing protocol for software defined vehicular networks,” in *2018 IEEE International Conference on Communications (ICC)*, 2018, pp. 1–6.
- [84] K. S. K. Liyanage, M. Ma, and P. H. J. Chong, “Connectivity aware tribrid routing framework for a generalized software defined vehicular network,” *Computer Networks*, vol. 152, pp. 167 – 177, 2019.
- [85] L. Magatao, “Mixed integer linear programming and constraint logic programming: Towards a unified modeling framework,” Ph.D. dissertation, The Federal Center of Technological Education of Parana, Brazil, 2005.
- [86] Environmental Technical Manual, Volume I, Procedures for the Noise Certification of Aircraft, International Civil Aviation Organization, Report, 2015. [Online]. Available: <https://www.icao.int/environmental-protection/Documents/SGAR.2015.ETM.Vol.1.pdf>
- [87] A. Bemporad and M. Morari, “Control of systems integrating logic, dynamics, and constraints,” *Automatica*, vol. 35, no. 3, pp. 407–427, 1999.
- [88] K. L. K. Sudheer, M. Ma, and P. H. J. Chong, “Link dynamics based packet routing framework for software defined vehicular networks,” in *GLOBECOM 2017 - 2017 IEEE Global Communications Conference*, Dec 2017, pp. 1–6.
- [89] J. M. Robson, “Algorithms for maximum independent sets,” *journal of Algorithms*, pp. 425–440, Sep 1986.
- [90] AIRBUS. Accessed: Dec. 12, 2018. [Online]. Available: <https://www.airbus.com/>
- [91] Getting to grips with aircraft performance, Customer Services, Flight Operation Support & Line Assistance, AIRBUS. Leiden, Netherlands 2002.

- [92] IATA Forecast Passenger Demand to Double Over 20 Years. Accessed: May. 04, 2019. [Online]. Available: <https://www.iata.org/pressroom/pr/Pages/2018-10-24-02.aspx>
- [93] G. G. Nadeesha Sandamali, R. Su, and Y. Zhang, “Short-term en route air traffic flow management under departure and wind uncertainties with a heuristic and greedy solution approach,” in *2019 American Control Conference (ACC)*, July 2019, pp. 2121–2126.
- [94] G. G. N. Sandamali, R. Su, and Y. Zhang, “A safety-aware real-time air traffic flow management model under demand and capacity uncertainties,” *IEEE Transactions on Intelligent Transportation Systems*, under Major Revision.
- [95] S. Vajda, *Probabilistic Programming*. Academic Press, New York and London, 1972.
- [96] J. M. Linhart, “Algorithm 885: Computing the logarithm of the normal distribution,” *ACM Trans. Math. Softw.*, vol. 35, no. 3, pp. 1–10, 2008.
- [97] A. Majumdar, W. Y. Ochieng, G. McAuley, J. Michel Lenzi, and C. Lepadatu, *The Factors Affecting Airspace Capacity in Europe: A Cross-Sectional Time-Series Analysis Using Simulated Controller Workload Data*, 2004, vol. 57, pp. 385 – 405.
- [98] J. Tsitsiklis, *6.041 Probabilistic Systems Analysis and Applied Probability*, Massachusetts Institute of Technology: MIT OpenCourseWare, Fall 2010.
- [99] Y. Hong, “On computing the distribution function for the poisson binomial distribution,” *Computational Statistics and Data Analysis*, vol. 59, pp. 41–51, 2013.
- [100] [dataset]EUROCONTROL, “Aircraft Performance Summary Tables for the Base of Aircraft Data (BADA) Revision 3.14,” Tech. Rep., 2017, EEC Technical/Scientific Report No. 17/05/29-145.
- [101] R. Olmi. Heuristic Algorithm for finding Maximum Independent Set. Accessed: Dec. 23, 2018. [Online]. Available: <https://www.mathworks.com/matlabcentral/fileexchange/28470-heuristic-algorithm-for-finding-maximum-independent-set>
- [102] Airlines.net, Define long haul. Accessed: Sep. 20, 2018. [Online]. Available: <http://www.airliners.net/aviationforums/>

BIBLIOGRAPHY

- [103] A. Cook, G. Tanner, and S. Anderson, “Evaluating the true cost to airlines of one minute of airborne or ground delay : Final report,” Report, 17 February 2004.
- [104] THE FISCAL TIMES. Accessed: Feb. 03, 2019. [Online]. Available: <http://www.thefiscaltimes.com/2015/03/03/Badr-Weather-Flight-Cancellations-Cost-Americans-24-Billion>
- [105] Jet Fuel Price Monitor. Accessed: Feb. 02, 2019. [Online]. Available: <https://www.iata.org/publications/economics/fuel-monitor/Pages/index.aspx>
- [106] ICAO, “ATFM – Air Traffic Flow Management,” Tech. Rep., 2016, ICAO Doc 9971, Part ii, Chapter 1.
- [107] R. Rossi, S. A. Tarim, S. Prestwich, and B. Hnich, “Piecewise linear lower and upper bounds for the standard normal first order loss function,” *Applied Mathematics and Computation*, vol. 231, pp. 489–502, 2014.
- [108] D. P. Bertsekas and J. N. Tsitsiklis, *Introduction to Probability*, 1st ed. Athena Scientific, 2002.
- [109] M. D. Ugarte, A. F. Militino, and A. T. Arnholt, *Probability and Statistics with R*, 2nd ed. Chapman & Hall/CRC, 2015.
- [110] K. L. K. Sudheera, M. Ma, and P. H. J. Chong, “Link stability based optimized routing framework for software defined vehicular networks,” *IEEE Transactions on Vehicular Technology*, vol. 68, no. 3, pp. 2934–2945, March 2019.
- [111] “Airtopsoft,” Accessed: Sep. 20, 2019. [Online]. Available: <http://airtopsoft.com/>
- [112] A. Cook, G. Tanner, and S. Anderson, “Evaluating the true cost to airlines of one minute of airborne or ground delay : Final report,” Report, 17 February 2004 2004.
- [113] Free Route Airspace (FRA) in ICAO EUR Region. Concept, Publication, Implementation, FAQs, March, 2018. [Online]. Available: <https://www.icao.int/MID/Documents/2018/ACAC-ICAO%20Civ-Mil%20WS/13-%20Session%202.2-1.pdf>

- [114] C. A. Nava Gaxiola, C. Barrado, P. Royo, and E. Pastor, “Assessment of the north european free route airspace deployment,” *Journal of Air Transport Management*, vol. 73, pp. 113–119, 2018.