Modeling and stability of grid-connected power converters with virtual inertia control

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**MODELING AND STABILITY OF GRID-CONNECTED
POWER CONVERTERS WITH VIRTUAL INERTIA
CONTROL**

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SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING

2020

**MODELING AND STABILITY OF GRID-CONNECTED
POWER CONVERTERS WITH VIRTUAL INERTIA
CONTROL**

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University

in partial fulfillment of the requirement for the degree of

Master of Engineering

2020

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarized materials, and has not been submitted for a higher degree to any other University or Institution.

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Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Authorship Attribution Statement

This thesis contains material from 2 papers published in the following peer-reviewed journals where I was the first and/or corresponding author.

Chapter 3 is published as **H. Yang**, J. Fang, and Y. Tang, “On the Stability of Virtual Inertia Control Implemented by Grid-Connected Power Converters with Delay Effects”, in *ECCE*, Baltimore, MD, Sep. 2019, pp. 2881-2888 and **H. Yang**, J. Fang, and Y. Tang, "Exploration of Time-Delay Effect on the Stability of Grid -Connected Power Converters with Virtual Inertia", in *ICPE-2019 ECCE Asia*, Bexco, Busan, Korea, May. 2019, pp. 2573-2578.

The contributions of the co-authors are as follows:

The original idea was proposed by Dr. Fang Jingyang and he also helped me revised the manuscript drafts. I finished the theoretical analysis and experimental verifications.

Part of chapter 4 is submitted as **H. Yang**, Y. Tang, “Sequence Impedance Modeling and Analysis of Three-Phase DC-link Voltage-Controlled Converters,” in *Proc. ICPE 2020-ECCE Asia*, Nanjing, China, 31, May–3, Jun 2020, submitted.

I finished all the literature review, theoretical analysis and simulation verifications of chapter 4.

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Summary

Due to the ever-increasing power demands and a desire for carbon footprint reduction, conventional fossil fuel-based energy generations are gradually replaced by renewable energy sources (RESs), e.g., wind power and solar photovoltaics. However, the lack of inertia contributions from RESs, which are essentially power electronic converters in replacement of synchronous generators, will challenge the frequency control and stability. To resolve this problem, the disturbed virtual inertia provided by grid-connected power converters is attracting growing attention due to its effectiveness and simplicity. However, although being practical for system inertia improvement, the grid-connected power converters with virtual inertia control also bring instability concerns. Therefore, this thesis mainly focuses on the analysis about the modeling as well as the stability of grid-connected power converters with virtual inertia control.

First of all, this thesis focuses on the system performance and stability under islanding operation modes. Specifically, unlike synchronous inertia, frequency measurements and DC-link voltage regulations are necessitated for virtual inertia implementations. As a consequence, the delay effects brought by these dynamics might cause instability concerns. Two different cases are studied in this part: one is the moving average filter-based virtual inertia control and the other one is the centralized virtual inertia control. To fill the research gap, this thesis presents a detailed analysis of the effects of delay on the frequency regulation system. With loop gains and Bode diagrams, it is revealed that the system loop gain shows a negative relationship between the inertia ratio and stability margins, indicating that a high virtual inertia level can bring instability issues into a single area power system. To tackle this instability issue, this paper proposes a modified virtual inertia control to mitigate the phase lag and improve system stability. For verification, the experimental results are presented, which are consistent with the theoretical analysis.

This thesis also investigates the modeling and stability of power converters with virtual inertia control in the grid-connected modes. To fully pinpointing the grid-converter interactions, exploring the sources of resonances, and verifying the mirror frequency coupling effects, the sequence impedance models are introduced and adopted in this part. As the basic block for the inertia emulation, the sequence impedance expressions of a

DC-link voltage-controlled converter are analyzed and derived firstly. It is found that the system would generate more mirror-frequency components in the rectifier mode, while more the same frequency components in the inverter mode. Next, as shown in Chapter 4, the system impedance magnitudes decrease greatly as the virtual inertia gain increases, indicating that the grid-connected power converter would become extensively sensitive to grid voltage perturbations. As a verification, the simulation results show that the virtual inertia control would totally distort the output currents due to the impedance reductions. Additionally, in the presence of the grid impedance, the system stability is evaluated with the impedance-based stability criterion. It is found out that the system closed-loop poles would drift to the right-half-plane as virtual inertia gain increases. Simulation results are also provided, which are consistent with the theoretical analysis.

Overall, this thesis mainly focuses on the modeling and stability of grid-connected power converters with virtual inertia control, whether in islanding modes or grid-connected modes. Both system-level and converter-level stabilities are discussed. As the renewable integration trend continues, new challenges and opportunities will be introduced by the virtual inertia control. For future works, the impacts of mirror-frequency impedance matrixes on the system stability and the design of advanced virtual inertia controllers are worthy of further investigations.

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List of Abbreviations

HVDC	High Voltage Direct Current
RES	Renewable Energy Sources
RoCoF	Rate of Change of Frequency
UFLS	Under Frequency Load Shedding
FLL	Frequency-Locked-Loop
PLL	Phase-Locked-Loop
PV	Photovoltaic
MPP	Maximum Power Point
MPPT	Maximum Power Point Tracking
DFIG	Double-Fed Induction Generator
VSG	Virtual Synchronous Generator
VSM	Virtual Synchronous Machine
ESS	Energy Storage System
GCC	Grid-Connected Converter
DES	Distributed Energy Storage
MAF	Moving Average Filter
FNET	Frequency Monitoring Network
FDR	Frequency Disturbance Recorder
PI	Proportional Integral
PM	Phase Margin
GM	Gain Margin
RHP	Right-Half Plane
PCC	Point of Common Coupling
VSC	Voltage Source Converter
SR	Synchronous Frame

Chapter 1 Introduction

This chapter starts with the introduction of the global grid transformation. As this trend continues, the power electronics-based RESs gradually supplant the conventional power resources. However, due to the lack of inertia contributions from RESs, the ability to maintain the grid frequency and its rate of change within the allowable range is becoming more challenging. Although being effective for inertia enhancement, the virtual inertia provided by GCCs would bring potential instability issues. Therefore, this thesis targets to investigate and explore the modeling and stability of GCCs with virtual inertia control. The expositions of the research motivation, objectives, major contributions and thesis organization are provided as follows.

1.1 Motivation

Due to the ever-increasing power demands and a desire for carbon footprint reduction, conventional fossil fuel-based energy generations are gradually replaced by RESs [1, 2]. For illustration, the global capacity and annual additions of RESs, e.g., solar PV and wind power, are shown in Fig. 1-1 [3], where an increasing penetration level of RESs can be observed. Specifically, the worldwide total RES capacity was only around 67 GW in 2012, and it increased to around 181 GW in 2018. Additionally, as shown in Fig. 1-2 [3],

Fig. 1-1. Global renewable power capacity level, 2012-2018 (Renewables 2018 Global Status Report [Online]. Available: <http://www.ren21.net>).

the share of RESs in global power capacity also increased rapidly, which accounted for nearly 33% of the total power capacity in 2018.

Fig. 1-2. Global renewable power penetration level, 2012-2018 (Renewables 2018 Global Status Report [Online]. Available: <http://www.ren21.net>).

Commonly, as the interfaces between RESs and power grids, GCCs play an indispensable and dominant role in injecting high-quality power, guaranteeing safety operation and providing grid support [4]. For instance, Fig. 1-3 shows the schematic diagram of a PV generation system, which contains PV panels, a boost DC/DC converter, a DC/AC inverter and the power grid. For the purpose of power inversion, the PV panel output is firstly boosted into a higher voltage and then transferred into an AC voltage for grid synchronization and connection.

Fig. 1-3. Schematic diagram of a PV generation system.

Similarly, the schematic diagram of a wind power generation system is shown in Fig. 1-4. As seen, a back-to-back topology converter is adopted as the electrical network interface, which improves power exchange abilities between wind turbines and power grids.

Fig. 1-4. Schematic diagram of a wind generation system.

However, it turns out that the power produced by RESs suffers from several salient shortcomings, precluding it from being adopted as the dominant part in a power system. One drawback lies in its intermittency. Specifically, the outputs of PV generations and wind generations are greatly subject to the solar irradiances, the ambient temperature and wind speeds, respectively. Therefore, to mitigate the impacts of weather conditions and smooth the output variations, energy storage devices, such as batteries and flywheels, are usually adopted as energy buffers in parallel to the RESs [5]. Another issue lies in the lack of inertia contributions. As the RESs normally operate at the maximum power tracking (MPPT) mode without inertial responses to the frequency changes [6], a high renewable energy penetration level imposes unprecedented challenges to the power system stable operation.

Power system inertia is an inherent property of synchronous generators, and it is of importance in terms of the grid frequency control and stability. In conventional power systems, the grid frequency is linked to the electrical angular speed of synchronous generators. During a frequency event, the imbalance between power generation and load demand changes the generator rotor speed. In this case, the kinetic energy stored in rotating masses of generator will release (or absorb) to the grid to compensate the partial power mismatch. This effect is defined as the power system synchronous inertia [7, 8]. However, as mentioned before, this mechanism changes as non-synchronous energy generations, e.g., PV and wind power, gradually replace synchronous generation technologies. As such, the grid frequency becomes greatly sensitive to the load variations without sufficient synchronous inertia, and thus it should be tightly regulated to avoid the frequency instabilities. In occurrence of frequency drop, Under-Frequency Load

Shedding (UFLS) is a common technique to prevent undesirable accidents. When the grid frequency is near the threshold, partial overloads or non-critical loads will be removed to compensate the active power mismatch, thus guaranteeing the system stability [9].

The frequency drop, as well as its changing rate, i.e., the rate of change of frequency (RoCoF) should be kept below the limits defined by grid codes at all costs. Otherwise, improper frequency control may cause undesirable load shedding or even system blackouts [10]. Fig. 1-5 shows a transient frequency response curve under a load change event during the primary frequency control period, where the impacts of various inertia constants H on the frequency regulation can be observed. It can be seen that sufficient system inertia helps reduce both frequency nadir and RoCoF, thereby improving the frequency stability.

Fig. 1-5. Frequency response curves under a load change event.

The low inertia level has already challenged the frequency stability in small-scale power systems. To give an example, a blackout event was recorded in South Australia (SA) on 28th Sep 2016 [11]. It is reported that the main drivers behind the blackout are attributed to the high renewable energy penetration level in SA and extreme weather events, i.e., wind intermittency. The Supervisory Control and Data Acquisition (SCADA) data reported that nearly 48% of total demand was supported by the wind generation

immediately prior to the event. The lack of synchronous inertia and the loss of generation collectively led to a serious frequency drop as well as the system blackout eventually.

To prevent undesirable events, various standards are prescribed for frequency control, as tabulated in Table I [12, 13]. Overall, at the edge of future more-electronics power systems, more works are expected in terms of the grid frequency control and stability.

Table I. International grid standards on frequency control.

Country/Interconnection	Nominal frequency (Hz)	Permissible Derivation
Eastern Interconnection (US)	60	± 0.05 Hz
Western Interconnection (US)	60	± 0.144 Hz
Singapore	50	± 0.2 Hz
Europe	50	± 0.2 Hz
Nordic countries	50	± 0.1 Hz
India	50	+0.2 / -0.5 Hz

1.3 Objectives

As will be comprehensively detailed in the next chapter, numerous approaches have been proposed to address the low system inertia issue [13]. Among them, distributed virtual inertia provided by grid-connected power converters is attracting growing attention due to its effectiveness and simplicity. This inertia emulation is realized by linking the grid frequency and the voltage reference of DC-link capacitors/ultracapacitors [14].

Although being practical for system inertia enhancement, the grid-connected power converters with virtual inertia control can bring potential instability issues under certain conditions. Specifically, the dynamics of grid frequency measurements and DC-link voltage loops are ignored as unit gains in [14], which is valid in the presence of sufficient synchronous inertia. However, this simplification might hide some instability concerns

for an islanded low inertia power system. For the disturbed virtual inertia control, grid frequency signals are measured through phase lock loops (PLLs), whose dynamics should be taken into account. As an example, a moving average filter-based phase lock loop (MAF-PLL) shows a slow response to frequency variations [15, 16]. Also, the transfer delay-based PLL (TD-PLL) introduces a one-quarter of a period delay to acquire an orthogonal signal [17]. Additionally, to avoid asymmetric power injections, centralized virtual inertia control is also showing great promise in the future. However, the impacts of communication delays on the system stability should also be explored. In [18], the author analyzed electric vehicles with centralized virtual inertia control and the instability issues brought by communication delays.

Moreover, even under the grid-connected operation mode, it is reported that the virtual inertia control may destabilize three-phase GCCs in the presence of grid impedance [19]. Notably, the grid voltages are assumed ideal in the existing virtual inertia control [14], and thus the implications of grid imbalance are neglected. Overall, as the grid frequency and the DC-link voltage reference are directly coupled with the virtual inertia controller, the grid-converter interactions become much more complex, and thereby extensive research should be carried out. To fill the aforementioned research gaps, thesis targets to fully investigate the modeling and stability of GCCs with virtual inertia control.

1.3 Major Contribution of the Thesis

This thesis mainly analyzes the modeling and stability of GCCs with virtual inertia control. Its major contributions can be summarized as follows:

- (1) The impacts of both distributed and centralized virtual inertia implementations on an islanded single-area power system have been explored. Furthermore, the potential instability concerns due to the high virtual inertia level and voltage loop / PLL dynamics have been addressed by the proposed modified virtual inertia control.

- (2) Based on harmonic linearization, the sequence impedance models of GCCs with virtual inertia control in the grid-connected operation mode are analyzed and derived in this thesis. The mirror-frequency effects arising from the nonlinear behaviors of power converters have been successfully predicted by the derived impedance expressions.
- (3) The effects of grid imbalance on the GCCs with virtual inertia control are investigated, and the theoretical analysis and simulation results indicate that the virtual inertia control will distort the phase currents due to its impedance magnitude reduction. Moreover, through the impedance-based stability criterion, this thesis has also confirmed the potential instability issues for GCCs with the virtual inertia control when they are connected to weak power grids.
- (4) The future research works on the system stability and inertia enhancement have been pointed out.

1.4 Organization of the Thesis

The following of the thesis is organized as follows:

Chapter 2 introduces the basic concept of the synchronous inertia and reviews the existing inertia enhancement methods.

Chapter 3 discusses the fundamental modeling of a GCC with virtual inertia control and its stability in the islanded operation mode. A small-scale power system is configured firstly. Then, the impacts of delay effects caused by DC-link voltage loop, MAF-PLL and centralized control on the system loop gain are analyzed. Modified virtual inertia controls are proposed subsequently for the stability improvement. At last, experimental verifications are provided, which are consistent with the theoretical analysis.

Chapter 4 studies the impedance model of a GCC with virtual inertia control and its stability in the grid-connected mode. The DC-link voltage ripples and PLL dynamics are

also taken into consideration for a complete model. The mirror-frequency effects are successfully predicted by the derived impedance expressions. Finally, the system performance under nonideal grid conditions are investigated and presented.

Chapter 5 concludes the thesis and suggests some possible future research works.

Chapter 2 Review of Power System Inertia

As demonstrated before, it is of importance to keep sufficient system inertia, especially for more-electronics power systems. Hence, numerous solutions have been proposed to cope with the challenge. This chapter gives a brief review and introduction of power system inertia and its existing enhancement methods.

2.1 Existing Inertia Enhancement Methods

2.1.1 Synchronous Generators

The most straightforward way to rise system inertia is through adopting synchronous generators in a large scale. The inertia constant H for a synchronous generator is defined as [7]

$$H = \frac{E_{\text{gen}}}{VA_{\text{rated}}} = \frac{J\omega_{\text{ref}}^2}{2VA_{\text{rated}}}, \quad (2-1)$$

where E_{gen} represents the kinetic energy stored in the rotor of the generating units, VA_{rated} is the system base power, and J refers to the moment of inertia. The typical inertia constants for gas-fired generators, coal-fired generators are 5 s and 3.5 s, respectively [7]. However, as discussed before, conventional fossil fuel-based energy generations are the main contributors to greenhouse gas emissions. Alternatively, running multiple synchronous generators at partial loads stands as another potential approach, even in the presence of a high RES penetration level, but it will impose heavier financial burdens on the total costs of ownership.

Synchronous condensers, which are essentially synchronous generators without prime movers and are commonly used to balance the reactive power as well as the grid voltage, can be adopted for inertia enhancement. It is reported that the inertia constant for a synchronous condenser is around 2.1 s [20]. However, the high operating cost has

become the dominant concern in terms of its large-scale deployments, precluding them from being widely adopted.

2.1.2 Wind Energy Generations

Wind power represents a potential inertia source, and it has already been utilized for grid frequency regulation support. The inertia constant of commercial fixed speed wind turbines (rated above 1 MW) usually ranges 3~5 s [21]. For illustration, a typical schematic diagram of a double fed induction generator-based (DFIG-based) wind power generator is shown in Fig. 2-1.

Fig. 2-1. Schematic of a DFIG-based wind generation system.

As seen, the rotor of DFIG is connected to the grid through a back-to-back converter, while the stator is directly connected to the grid [13]. Although the wind turbine's rotating mass is decoupled from the grid by static power electronic devices, the inertia emulation can be realized by proper control designs. In [21, 22], the electromagnetic torque is linked to the grid frequency deviation as well as the RoCoF signal to emulate inertia and support primary frequency control. For clarity, Fig. 2-2 shows the corresponding control scheme, where the inertia controller is adopted as a differential controller, which enables the torque reference $\Delta T_{c_ref}^*$ changes with the RoCoF signal for the delivery of inertial responses. In practical, the wind farms in Hydro-Quebec and Ontario have already utilized the wind turbines for grid frequency support and inertia enhancement [23].

(a) Control structure of the generator-side converter.

(b) Control structure for inertia emulation.

Fig. 2-2. Control architecture of the wind turbine for inertia emulation.

Alternatively, to enable wind power to change with the grid frequency variations, an additional term proportional to the RoCoF is introduced into the active power reference, which intends to emulate the swing equation of a synchronous generator [24, 25]. Although the wind plants with frequency support functions have been employed in industrial applications [26], the speed recovery processes will distort the expected inertial responses, making the emulated inertia different from the synchronous inertia [27, 28]. This phenomenon may even cause a recurring frequency dip or rotor stall.

2.1.3 Energy Storage Systems

As one of the most influential contributors in future energy systems, energy storage system (ESS) also offers possibilities for inertia emulation. In [29], the kinetic energy stored in a flywheel is utilized to support system inertia by regulating the rotate speed of the flywheel in proportion to the grid frequency. [27] uses wind power together with a flywheel to maintain the power reserve while providing virtual inertia. Moreover, another promising method to improve system inertia is through battery energy systems [30, 31].

This inertia emulation is realized by proportionally linking the active power reference and the RoCoF signal, which is similar to some control schemes adopted in wind turbines, as mentioned before. For the purpose of fast and robust inertia emulation, the RoCoF signal detection and tracking technologies have been designed and detailed in [32, 33]. Besides, an ultracapacitor is also capable of system inertia enhancement owing to its high power density and long lifetime. Fig. 2-3 shows the schematic diagram of a typical ultracapacitor storage system with virtual inertia control [34], where v_{es} represents the ultracapacitor voltage. As seen, v_{es} is boosted through a DC-DC converter for the grid connection. Notice that the virtual inertia controller regulates v_{es} to follow the grid frequency variations, and simultaneously the grid-side converter aims to keep the DC-link voltage constant.

Fig. 2-3. Schematic of an ultracapacitor energy storage system with virtual inertia control.

Virtual Synchronous Generators (VSGs) are growing increasingly attractive due to the ongoing trend of grid transformations [35, 36]. The fundamental idea behind this concept is to emulate the essential behaviors of synchronous generators, including the droop mechanism and the inertial characteristic. As compared with the conventional synchronous generators, the inertia provided by VSGs can be modified dynamically

during frequency events. In [37], a battery/ultracapacitor hybrid ESS is implemented as a VSG, where the ultracapacitor is controlled to take care of high-frequency power fluctuations, e.g., inertial responses, while the constant power components are attributed to the battery. The experimental results show the effectiveness of the proposed VSG.

2.1.4 Grid-Connected Converters

As the key building blocks of future environment-friendly power systems, GCCs are expected to grow in scope and importance in coming decades due to the large-scale adoption and deployment of RESs. Fig. 2-4 illustrates the applications of GCCs in modern power systems.

Fig. 2-4. Applications of GCCs in more-electronics power systems.

Because GCCs normally necessitate DC-link capacitors for voltage support and harmonic filtering, the DC-link capacitors are found to be another promising energy source for inertia emulation [14]. Specifically, by directly linking the grid frequency and the voltage reference of DC-link capacitors, the DC-link capacitors of power converters act as energy buffers and inertia suppliers. This virtual inertia provided by grid-connected

Fig. 2-5. Virtual inertia coefficient H_p versus the dc-link capacitance C_{dc} , dc-link voltage V_{dc} , and voltage variation ΔV_{dc_max} ($\Delta f_{r_max} = 0.2 \text{ Hz}$, $f_{ref} = 50 \text{ Hz}$, and $V_{A_rated} = 1 \text{ kVA}$).

power converters has been introduced as another promising approach to improve power system inertia due to its effectiveness and simplicity. Fig. 2-5 shows the relationships between the virtual inertia constant and the system parameters, i.e., the DC-link capacitance C_{dc} , DC-link voltage reference V_{dc_ref} , and its maximum allowable change ΔV_{dc_max} [14]. In view of no need for RoCoF signal detection and excessive hardware changes, this effective solution has been extended to modular multilevel converters (MMCs) [38].

2.2 Summary

Various works have been attached to the improvement of the system inertia and frequency stability. The most straightforward way is to employ more synchronous generators. However, it may impose excessive financial burdens on the total costs of ownership. Alternatively, as the renewable energy has been drawing more and more attention due to its environment-friendly features, the grid frequency support provided by wind turbines serves as a proven and adopted approach for inertia enhancement in more-electronics power systems. Also, the virtual inertia offered by energy storage systems also holds promise in the future. Recently, the distributed virtual inertia provided by grid-connected power converters has been introduced as an effective approach to

improve the power system inertia, with minimized or even no hardware change. Consequently, extensive researches are expected as the trend of renewable integration proceeds.

Chapter 3 Modeling and Stability of Grid-Connected Converters with Virtual Inertia Control in the Islanded-Mode

This chapter focuses on the system-level stability of an islanded single-area power system with virtual inertia implementations. It starts with the system configuration and the introduction of virtual inertia control. Then, the impacts of delay effects, which are caused by the DC-link voltage regulation and the grid frequency detection, are analyzed in the following. Through the mathematical derivations and Bode diagrams, we identify that when the total virtual inertia is close to or exceeds the synchronous inertia, the phase lag introduced by the delay effects can destabilize the system. Accordingly, modified virtual inertia controls are proposed to address the instability issue. At last, experimental verifications are provided, which are consistent with the theoretical analysis.

3.1 System Configuration

A single-area power system consisting of a synchronous generator (SG), a frequency-dependent load (i.e. the motor load), a frequency-independent resistive load, and the power electronic-interfaced generator and load, is shown in Fig. 3-1.

Fig. 3-1. Single-area power system (PM is a prime mover; SG means a synchronous generator; M designates a motor load; RG denotes a renewable generator).

As seen, P_m designates the input mechanical power of the SG. $P_e = (P_d + P_l)$ denotes the power absorbed by conventional loads, and P_{gcc} represents the power consumed by the power electronic-interfaced generator and load through GCCs, which is equivalent to power absorbed by rectifier-mode power converters P_{con} minus the power generated from inverter-mode power converters P_{gen} . Assuming that P_{gcc} is a constant, one can derive the following swing equation to quantify the relationships among the aforementioned variables:

$$\Delta P_m - \Delta P_l - D\Delta f_r = 2H \frac{d\Delta f_r}{dt}, \quad (3-1)$$

where the prefix Δ refers to the perturbed quantity. f_r signifies the grid frequency, D is the load damping coefficient, and H is the system inertia constant, which plays an important role on the frequency stability of modern power systems. In essence, the objective of inertia emulation is to increase the value of H . The frequency regulation framework of this system is shown in Fig. 3-2 [7].

Fig. 3-2. Block diagram of the frequency regulation framework.

where T_G , T_{RH} , T_{CH} , F_{HP} , and R are the coefficients of speed governor and reheat turbine [7]. Specifically, T_G is denoted as the speed governor coefficient; T_{RH} stands for the reheater time constant; T_{CH} is the main inlet time constant; F_{HP} represents the turbine HP coefficient; R equals the droop coefficient. Additionally, the load disturbance, load reference change and grid frequency change are expressed as ΔP_{l_pu} , ΔP_{ref_pu} and Δf_{r_pu} , respectively. As seen, the variables are shown in the per-unit forms. The system parameters are listed in Table II.

Table II. Parameters of the synchronous generator frequency regulation framework.

Description	Symbol	Value
Frequency-droop coefficient	R	0.05
Speed governor coefficient	T_G	0.1 s
Turbine HP coefficient	F_{HP}	0.3 s
Time constant of reheater	T_{RH}	7.0 s
Time constant of main inlet	T_{CH}	0.2 s
Load damping coefficient	D	1.0
Inertia constant	H	3.0 s
Rated frequency	f_{ref}	50 Hz
Power rating	VA_{rated}	500 VA

3.2 Virtual Inertia Control

3.2.1 Introduction

For alleviating the adverse effect of inertia reduction, the electrical energy stored in capacitors or ultracapacitors have been utilized to contribute inertia like synchronous generators in the case of frequency events.

To clarify the concept of virtual inertia control, a mapping between SGs and capacitors is illustrated in Fig. 3-3, where H_{cap} is referred to as the inertia constant of capacitors. The inertia constants H and H_{cap} are defined as the ratio of the kinetic energy ($J\omega_{0m}^2 / 2$) and electrical energy ($C_{\text{dc}}V_{\text{dc_ref}}^2 / 2$) to the rated power VA_{base} , respectively [14], where ω_{0m} represents the rated rotor mechanical speed. It is worth mentioning that the rotor speed ω_r equals 2π multiplied by the frequency f_r for synchronous generators with one pair of poles.

Based on above discussions, the mapping between rotor speed ω_r and capacitor voltage v_{dc} can be observed. This mapping indicates that a capacitor has the potential to emulate

Fig. 3-3. Mapping between SGs and capacitors.

inertia when ω_r and v_{dc} are proportionally linked. Consequently, the virtual inertia control is designed as a proportional controller relating the DC-link voltage to the grid frequency through a gain K_{fv} [14], which is

$$\Delta v_{dc} = K_{fv} \Delta f_r, \quad (3-2)$$

where Δv_{dc} and Δf_r represent the DC-link voltage change and grid frequency change, respectively. The constant K_{fv} is defined as the virtual inertia controller gain. Therefore, the electrical energy stored in capacitors can be used for system inertia improvements. Notice that the frequency measurement and DC-link voltage regulation are ignored here. As such, the equivalent virtual inertia constant is

$$H_c = H_{cap} K_{fv}. \quad (3-3)$$

The total virtual inertia constant H_v can be calculated as the sum of the virtual inertia constants of all inertia emulation units, derived as

$$H_v = \sum_{i=1}^n H_{c_i}, \quad (3-4)$$

where n is the number of existing virtual inertia generation units, and H_{ci} represents the virtual inertia constant of each unit. As discussed in [14], with the increased penetration level of renewable energy sources, it is necessary to emulate a large amount of virtual inertia, even exceeding the synchronous inertia.

3.2.2 Implementation

Fig. 3-4 details the implementation of virtual inertia control, where the control structure is made of two parts – a virtual inertia controller and a cascaded voltage/current controller. The virtual inertia controller yields a change of the DC-link voltage reference Δv_{dc_ref} as the grid frequency changes for inertia emulation, while the voltage/current controller simply regulates the DC-link voltage v_{dc} to follow its reference $v_{dc_ref} + \Delta v_{dc_ref}$ [14]. As such, the electrical energy stored in the capacitance is linked to the grid frequency change for inertia emulation.

Fig. 3-4. Schematic of a GCC equipped with virtual inertia control.

The control scheme of a grid-connected power converter with virtual inertia control is shown in Fig. 3-5. As can be seen, all variables are based on the synchronous dq -frame, and the coupling effect between d - and q -axis has been ignored. Additionally, a conventional double-loop controller is used, and the proportional-integral (PI) controllers are implemented for both voltage and current regulation.

$$H_v(s) = K_{vp} + \frac{K_{vi}}{s}, \quad (3-5)$$

$$H_c(s) = K_{cp} + \frac{K_{ci}}{s}. \quad (3-6)$$

Fig. 3-5. Block diagram of the virtual inertia control.

Meanwhile, the system plant transfer function is derived as following, where L represents the total output inductance.

$$G_{\text{plant}}(s) = \frac{1}{Ls}. \quad (3-7)$$

The effects of reference computations and pulse updates are simplified as a first-order lag [39]

$$G_d = \frac{1}{T_d s + 1}, \quad (3-8)$$

where $T_d = 1.5 / f_s$, and f_s is equal to the sampling frequency. Moreover, the regulation of DC-link voltage v_{dc} is through the control of d -axis current i_{cd} . Referring to the power balance between the AC-side and DC-side, the small-signal transfer function from i_{cd} to v_{dc} is derived as [19]

$$G_{i_v} = \frac{-3V_d}{2v_{dc_ref} C_{dc}s}, \quad (3-9)$$

where V_d is the rated value of the grid voltage amplitude, and v_{dc_ref} represents the reference value of v_{dc} . Therefore, the transfer function from the grid frequency change Δf_r to the DC-link voltage change Δv_{dc_ref} can be derived as $K_{fv} G_f(s) G_{clv}(s)$, where $G_f(s)$

represents frequency measurement dynamic. Additionally, the DC-link voltage regulation transfer function $G_{clv}(s)$ is expressed as

$$G_{clv}(s) = G_{clv_pu}(s) = \frac{v_{dc_pu}(s)}{\Delta v_{dc_ref_pu}(s)} = \frac{v_{dc}(s)}{\Delta v_{dc_ref}(s)}. \quad (3-10)$$

According to [1], the response time of (3-10) normally ranges from 0.01 s to 0.1 s. The parameters of the GCC are listed in Table III.

Table III. Parameters of the grid-connected power converter.

Description	Symbol	Value
DC-link voltage reference	V_{dc_ref}	250 V
Total inductance	L	2 mH
Grid voltage amplitude	V_d	$60\sqrt{2}$ V
Sampling/switching frequency	f_s / f_{sw}	1
Current proportional gain	K_{cp}	15 V/A
Current integral gain	K_{ci}	300 V/(A·s)
Voltage proportional gain	K_{vp}	0.2 A/V
Voltage integral gain	K_{vi}	2 A/(V·s)

3.3 Frequency Measurement Dynamics

As seen in Fig. 3-5, the transfer function $G_f(s)$ represents the grid frequency measurement and tracking, which is simplified as a unit gain in [14]. However, its dynamic should not be omitted, especially for an islanded single-area power system. The delay effects brought by this process can even destabilize the system stability. To show as examples, two typical frequency detection cases, i.e., MAF-PLL and centralized control, are briefly introduced in the following.

3.3.1 Moving Average Filter-Based PLL

For GCCs, PLLs are commonly used for grid synchronizations and frequency measurements. The small-signal model of a synchronous frame PLL (SR-PLL) indeed represents a negative-feedback system [40], which commonly contains a loop filter and an integrator. When the grid condition is unbalanced/distorted, a moving average filter (MAF) cascaded with a PI controller is usually adopted as the loop filter for harmonic rejections. The modeling and analysis for a MAF-PLL have been discussed in [15], whose small-signal block diagram is shown in Fig. 3-6.

Fig. 3-6. Block diagram of the small-signal model of a MAF-PLL.

In Fig. 3-6, all variables are described in per-unit forms. Specifically, ω_{r_pu} , V_{d_pu} and θ_{r_pu} denote the angular frequency, magnitude and phase of the grid voltage (fundamental positive sequence component), respectively. ω_{rm_pu} and θ_{rm_pu} are the MAF-PLL angular frequency and phase angle, respectively. D_{pu} represents harmonic interference. $PI_{pu}(s)$ is a proportional-integral controller, given by

$$PI_{pu}(s) = K_{pll_p_pu} + \frac{K_{pll_i_pu}}{s}. \quad (3-11)$$

The transfer function of a MAF in the s -domain can be expressed as

$$MAF(s) = \frac{1 - e^{-T_w s}}{T_w s}, \quad (3-12)$$

where T_w represents the MAF window width, and it is often set as 0.01 s or 0.02 s [16], which indicates that time equals its window width is required for the MAF to get the steady-state condition [16]. Other types of PLLs also possess a similar slow response [17, 41], but only the MAF-PLL is analyzed in this thesis for simplification of analysis. According to [15], The MAF-PLL can achieve optimum performance when

$$2/T_w = b\omega_c, \quad (3-13)$$

$$K_{\text{pll_p_pu}} = \omega_c / V_{\text{l_pu}}, \quad (3-14)$$

$$K_{\text{pll_i_pu}} = \omega_c^2 / bV_{\text{l_pu}}, \quad (3-15)$$

where ω_c represents the MAF-PLL crossover frequency, and b is a constant needed to be tuned. Consequently, the closed-loop transfer function of the MAF-PLL is expressed as

$$G_{\text{MAF-PLL_pu}}(s) = \frac{\omega_{\text{rm_pu}}}{\omega_{\text{r_pu}}} = \frac{f_{\text{rm_pu}}}{f_{\text{r_pu}}} = \frac{\Delta f_{\text{rm_pu}}}{\Delta f_{\text{r_pu}}}. \quad (3-16)$$

In this thesis, T_w is set as 0.02 s to compensate the DC offset, and b is designed as 2.4 for fast transient response and a sufficient stability margin [15].

3.3.2 Centralized Virtual Inertia Control

Due to the low cost and high accuracy dynamic frequency measurements, the wide-area frequency monitoring network (FNET) has been deployed rapidly in the U.S. among those years [42, 43]. As a major component of the FNET, a frequency disturbance recorder (FDR) performs the calculated frequency accuracy around ± 0.0005 Hz, which is even better than some commercial PMUs [42]. The structure block diagram of an FDR is shown in Fig. 3-7 [43],

Fig. 3-7. Block diagram of the FDR architecture.

The wide area, quasi real time, GPS synchronized frequency measurement provided by FDRs has been widely applied in power systems [44]. The data can be used to

- Improve Flexible AC Transmission Systems (FACTS) / energy storage system control.
- Control and coordinate the wide area Power System Stabilizer (PSS).
- Control and coordinate the distribution generation.

Therefore, it is reasonable to expect GCCs will emulate inertia based on the FDR output frequency. However, as explained in [44], the influence brought by the signal communication delays should not be ignored. The worst case delay can be above 150 ms, even with fiberoptic cables. As such, the transfer function of a centralized virtual inertia control loop is modeled as

$$G_{\text{cen_pu}}(s) = \frac{f_{\text{rm_pu}}}{f_{\text{r_pu}}} = \frac{\Delta f_{\text{rm_pu}}}{\Delta f_{\text{r_pu}}} = e^{-\tau s} G_{\text{filter}}(s), \quad (3-17)$$

where the communication delay is expressed as $e^{-\tau s}$, and τ refers to the delay time. $G_{\text{filter}}(s)$ corresponds to the low-pass filter in the FDR, whose bandwidth is around 100 Hz [45].

3.4 Stability Analysis

3.4.1 System Loop Gain

If there are n inertia emulation units in the single-area power system, the system frequency regulation framework is changed as Fig. 3-8.

Fig. 3-8. Block diagram of the frequency regulation framework with virtual inertia implementations.

As seen, each inertia emulation unit is mapped into a red transfer path in Fig. 3-8. Specifically, in each path, the GCC detects the per unit grid frequency change Δf_{r_pu} and then absorbs the power $\Delta P_{gcc_pu_i}$. In the transfer path from Δf_{r_pu} to $\Delta P_{gcc_pu_i}$, there are four blocks, namely $G_{f_pu_i}(s)$, K_{fv_i} , $G_{clv_pu_i}(s)$ and $2H_{cap_i}s$, modeling the grid frequency measurement, virtual inertia control gain, DC-link voltage regulation and power output, respectively.

As discussed before, for each inertia emulation unit $H_{c_i} = K_{fv_i}H_{cap_i}$, where H_{c_i} represents the equivalent virtual inertia coefficient, K_{fv_i} is the proportional gain of the virtual inertia controller and H_{cap_i} equals the capacitance inertia coefficient. Fig. 3-8 can be further changed into Fig. 3-9, where $G(s)$ and $H(s)$ represent the system forward gain and feedback gain, respectively.

Fig. 3-9. Simplified block diagram of frequency regulation framework with virtual inertia.

Specifically, $G(s)$ is derived from Fig. 3-8 and Fig. 3-9 as

$$G(s) = \frac{R(1 + sT_{RH})(1 + sT_{CH})(1 + sT_G)}{(2Hs + D)(1 + sT_G)(1 + sT_{CH})(1 + sT_{RH})R + sF_{HP}T_{RH} + 1}. \quad (3-18)$$

Notice that $G(s)$ indeed represents the grid frequency regulation transfer function without the virtual inertia. Additionally, the feedback loop transfer function $H(s)$ can be derived as

$$H(s) = \sum_{i=1}^n 2H_{c_i} s G_{clv_pu_i}(s) G_{f_pu_i}(s). \quad (3-19)$$

For simplification of analysis, we assume all the inertia emulation units take the same frequency measurements $G_{f_pu}(s)$ and the same voltage regulation loops $G_{clv_pu}(s)$. Hence, according to (3-4), $H(s)$ is rewritten as

$$H(s) = 2H_v s G_{clv_pu}(s) G_{f_pu}(s). \quad (3-20)$$

Consequently, the system loop gain $G(s)H(s)$ is derived as

$$G(s)H(s) = \frac{2H_v R s G_{clv_pu}(s) G_{f_pu}(s) (1 + sT_{RH})(1 + sT_{CH})(1 + sT_G)}{(2Hs + D)(1 + sT_G)(1 + sT_{CH})(1 + sT_{RH})R + sF_{HP}T_{RH} + 1}. \quad (3-21)$$

Suppose the dynamics of DC-link voltage regulation and frequency measurements are ignored, i.e. $G_{clv_pu}(s) = G_{f_pu}(s) = 1$, the system loop gain is expressed as

$$G(s)H_{non}(s) = \frac{2H_v R s (1 + sT_{RH})(1 + sT_{CH})(1 + sT_G)}{(2Hs + D)(1 + sT_G)(1 + sT_{CH})(1 + sT_{RH})R + sF_{HP}T_{RH} + 1}. \quad (3-22)$$

Assuming $H_v = 0.5H$, the Bode diagram of the system loop gain $G(s)H_{\text{non}}(s)$ is shown in Fig. 3-10.

Fig. 3-10. Bode diagram of the loop gain $G(s)H_{\text{non}}(s)$ without delay effects.

As seen, the system loop gain will approach to a proportional gain as the frequency band increases, which can be validated as

$$\lim_{\omega \rightarrow \infty} G(j\omega)H_{\text{non}}(j\omega) = \frac{H_v}{H}. \quad (3-23)$$

(3-23) and Fig. 3-10 collectively indicate that the frequency regulation without the delay effects will always be stable, regardless of the virtual inertia. For clarity, as there is no delay effect, the system loop gain will approach to the ratio of total virtual inertia to system synchronous inertia H_v / H in the middle/high-frequency band.

On the other hand, when the dynamics of the DC-link voltage loop and MAF-PLL are considered, the system loop gain $G(s)H(s)$ is shown in Fig. 3-11. Notice that because both $G_{f_pu}(s)$ and $G_{clv_pu}(s)$ indeed represent low pass filters, they will not affect the system loop gain in the low-frequency band (within their bandwidths). Accordingly, there is almost no difference between $G(s)H(s)$ and $G(s)H_{\text{non}}(s)$ in the low-frequency band, as observed in Fig. 8. However, things are changed in the mid-frequency band. The loop gain without delay effects $G(s)H_{\text{non}}(s)$ approaches to the constant gain H_v / H , as analyzed before, but the gain of $G(s)H(s)$ is attenuated due to the low pass filter

Fig. 3-11. Bode diagram of the loop gain $G(s)H_{\text{non}}(s)$ and $G(s)H(s)$.

characteristics of $G_{f_pu}(s)$ and $G_{clv_pll}(s)$. Additional phase lag is also brought by the delay effects, which may cause instability concerns. It should be noted that these discussions are based on the fact that the dynamics of PLLs and DC-link voltage loops are much faster than the speed governor and turbine [14].

Because the stability is usually determined by the gain margin and the phase margin of the cross-frequency in the mid-frequency band, the expression of the system loop gain $G(s)H(s)$ in the mid-frequency band can be simplified as

$$\begin{aligned}
 G(s)H(s) &= G(s)H_{\text{non}}(s)G_{f_pu}(s)G_{clv_pu}(s) \\
 &\approx \frac{H_v}{H} G_{f_pu}(s)G_{clv_pu}(s).
 \end{aligned} \tag{3-24}$$

As can be seen, the system loop gain in the mid-frequency band is only impacted by H_v / H , $G_{f_pu}(s)$ and $G_{clv_pu}(s)$. (3-24) can greatly simplify the stability analysis, as shown in the following.

3.4.2 Virtual Inertia < Synchronous Inertia

In this case, H_v is assumed sufficiently smaller than H , e.g., $H_v / H = 0.5$. The Bode plots of (3-21) with MAF-PLL-based ($G_{f_pu}(s) = G_{\text{MAF-PLL_pu}}(s)$) and centralized ($G_{f_pu}(s) = G_{\text{cen_pu}}(s)$) virtual inertia control are shown in Fig. 3-12.

Fig. 3-12. Bode diagram of the loop gain $G(s)H(s)$ with $H_v / H = 0.5$.

As seen in Fig. 3-12, because both gain plots are below the 0-dB line in the Bode diagram, the system will always be stable. This condition can be equivalently expressed as

$$\|G(s)H(s)\|_{\infty} < 1. \quad (3-25)$$

(3-25) indicates that the infinite norm of $G(s)H(s)$ is smaller than 1, in this case.

3.4.3 Virtual Inertia $\approx$ Synchronous Inertia

As the total inertia is almost equal to the synchronous inertia, potential instability concerns appear. To give an example, when $H_v / H = 1$ in this case, the Bode plots are shown in Fig. 3-13.

Fig. 3-13. Bode diagram of the loop gain $G(s)H(s)$ with $H_v / H = 1$.

As seen, the gain plots have exceeded the 0-dB line. Specifically, the MAF-PLL-based virtual inertia control scheme possesses a positive stability margin, and the centralized virtual inertia control scheme is stable when the delay time equals 50 ms. However, if the delay time increases to 100 ms, a negative gain margin, indicating an unstable system, appears.

Note that the impacts of the voltage loop are also of importance in this case. The Bode plots of $G(s)H(s)$ with various integral gain K_{vi} are shown in Fig. 3-14.

Fig. 3-14. Bode diagram of the loop gain $G(s)H(s)$ with various K_{vi} and $H_v / H = 1$.

As K_{vi} reduces to zero, the maximum crossover frequency also reduces, and in turn the system becomes stable.

3.4.4 Virtual Inertia > Synchronous Inertia

As increasing RESs supplant synchronous generators, it is possible to expect the total virtual inertia to exceed the synchronous inertia. For instance, when $H_v / H = 2.5$, the Bode plots are shown in Fig. 3-15.

Fig. 3-15. Bode diagram of the loop gain $G(s)H(s)$ with $H_v / H = 2.5$.

As seen in Fig. 3-15, the maximum crossover frequency exceeds the bandwidth of $G_{f_pu}(s)$ and $G_{clv_pu}(s)$, while becoming higher than the previous case. Moreover, both MAF-PLL-based and centralized virtual inertia control schemes show negative phase margins, indicating the system is unstable. For demonstration, Fig. 3-16 illustrates the stability margin of the MAF-PLL-based virtual inertia control with various H_v / H .

Fig. 3-16. The stability margin of the MAF-PLL-based virtual inertia control with various H_v / H .

Similarly, for the centralized virtual inertia control, the critical delay time $\tau_{\max}$ with various H_v / H is shown in Fig. 3-17.

Fig. 3-17. Critical communication delay $\tau_{\max}$ for the centralized virtual inertia control.

3.5 Modified Virtual Inertia Control

3.5.1 Modified MAF-PLL-Based Virtual Inertia Control

Based on the above analysis, we identify that when the total virtual inertia is close to or exceeds the synchronous inertia, the phase lag introduced by the frequency measurements and the DC-link voltage loops are responsible for the instabilities. In this section, the original virtual inertia control is modified for stability improvement.

For the MAF-PLL, several researchers have modified its structure for dynamic improvements [16, 46]. However, those changes make the design and implementation

Fig. 3-18. Block diagram of the modified MAF-PLL-based virtual inertia control.

more complex. For the purpose of the minimized filter structure changes, the block diagram of a modified MAF-PLL-based virtual inertia control is shown in Fig. 3-18, where K_m represents the proportional gain of the additional forward term. In the conventional virtual inertia control, the MAF-PLL frequency ω_{rm_pu} is directly transported to the virtual inertia controller. However, as seen in Fig. 3-18, the modified MAF-PLL frequency ω_{k_pu} is adopted for the inertia emulation in the replacement of ω_{rm_pu} . It is noted that this control scheme will not influence the frequency tracking because θ_{r_pu} equals θ_{rm_pu} in the steady state. As such, the Bode plots of $G(s)H(s)$ with various K_m is shown in Fig. 3-19.

Fig. 3-19. Bode diagram of the loop gain $G(s)H(s)$ with various K_m .

As shown in Fig. 3-19, the loop gain magnitude in the mid-frequency band is attenuated due to the additional gain K_m , and thus the stability enhancement can be observed. Specifically, without the modified virtual inertia control, i.e., $K_m = 0$, the system is unstable due to a negative phase margin. When $K_m = 0.28K_{pll_p_pu}$, both phase and gain margin are improved. The case of $K_m = 0.64K_{pll_p_pu}$ leads to a system with a 2.4 dB gain margin and a phase margin of 15 degree.

Although the proposed modified MAF-PLL-based virtual inertia control is effective for system stability enhancement, its effect is limited. In this case, the phase and gain

margins with various $K_m / K_{pll_p_pu}$ are depicted in Fig 3-20. As seen, the maximum stability margin enhancement is achieved when $K_m / K_{pll_p_pu} = 0.64$.

Fig. 3-20. The stability margin of the modified MAF-PLL-based virtual inertia control with various K_m .

3.5.2 Modified Centralized Virtual Inertia Control

Similarly, the stability enhancement for the centralized virtual inertia control can also be realized through a feedforward gain K_n in the voltage loop, shown in Fig. 3-21.

Fig. 3-21. Block diagram of the modified centralized virtual inertia control.

Notice that the proposed modified control scheme will also not change the voltage regulation dynamic due to $\Delta v_{dc_ref} = 0$ in the steady state. When $H_v / H = 2$ and $\tau = 35$ ms, the phase and gain margins with various K_n / K_{vp} are depicted in Fig. 3-22. In this case, K_n / K_{vp} is designed as 0.5 to achieve a phase margin of 51 degree and gain margin of 3.7 dB.

Fig. 3-22. The stability margin of the modified centralized virtual inertia control with various K_n .

The Bode plot of the loop gain with the modified virtual inertia control is shown in Fig. 3-23, where stability margin improvements can be clearly observed.

Fig. 3-23. Bode diagram of the loop gain $G(s)H(s)$ with various K_n .

3.6 Experimental Verifications

The stability of GCCs with virtual inertia in the islanded mode will be experimentally investigated in this section. Furthermore, the effectiveness of the proposed modified virtual inertia control for system stability enhancement will also be verified.

As for the grid frequency regulation test, the conventional synchronous generator is emulated by a virtual synchronous generator (VSG), which exhibits the same terminal

characteristics as a conventional synchronous generator. The VSG aims to control the grid voltage and provide the system inertia as well as the droop characteristic. The control and parameter design of VSG have been discussed in [37]. Fig. 3-24 shows the system structure of the testing islanded system.

Fig. 3-24. Schematic diagram of the testing system.

A dSPACE (Micolabbox) control platform was used to control the whole system, and an oscilloscope (TELEDYNE LECROY: HDO8038) was adopted to capture the waveforms. The system parameters of the VSG are shown in Table II, while the GCC parameters are shown in Table III. Moreover, the figure of the experimental test-bed is shown in Fig. 3-25.

Fig. 3-25. A photo of the experimental test-bed.

3.6.1 MAF-PLL-Based Virtual Inertia Control

When $H_v / H = 2.5$, Fig. 3-26 demonstrates the steady-state waveforms of the three-phase power converters with MAF-PLL-based virtual inertia control, where the grid voltage v_{gabc} , grid currents i_{cabc} , DC-link voltage change Δv_{dc} , and system frequency change Δf_r are presented. As seen, the current waveforms i_{cabc} are totally distorted. Additionally, the oscillations in the frequency change Δf_r and DC-link voltage change Δv_{dc} indicate that the virtual inertia control, which links f_r to v_{dc} directly, should be blamed for this instability. Because the saturation units in the controller limit the variation ranges of DC-link voltage v_{dc} and frequency f_r to prevent over voltages, v_{dc} and f_r only oscillate within certain ranges, rather than being totally unstable.

Fig. 3-26. Steady-state experimental results of the power converter with conventional MAF-PLL-based virtual inertia control (v_{abc} : the grid voltage, i_{cabc} : the converter currents, Δv_{dc} : the DC-link voltage change, and Δf_r : the frequency change).

When $K_m / K_{pll_p_pu}$ is set as 0.64, Fig. 3-27 shows the steady-state waveforms with modified MAF-PLL-based virtual inertia control implementation. As seen, the oscillations disappear, indicating the system is stable with modified virtual inertia control. The effectiveness of the proposed modified MAF-PLL-based virtual inertia control can be further verified in Fig. 3-28. As seen, when the saturation units are disabled, the instability appears after the modified virtual inertia control is replaced by the conventional virtual inertia control, i.e., $K_m = 0$. To protect semiconductor devices, the

power converter stops working when the converter currents go beyond 15 A, and therefore v_{abc} and i_{cabc} reach zero eventually.

Fig. 3-27. Steady-state experimental results of the power converter with modified MAF-PLL-based virtual inertia control.

Fig. 3-28. Experimental results of the power converter with various K_m .

3.6.2 Centralized Virtual Inertia Control

When $H_v / H = 1$, Fig. 3-29 illustrates the influence of communication delay on the stability of centralized virtual inertia control. As observed, the oscillations appear after the delay time increase from 0 ms to 100 ms.

Fig. 3-30 displays the experimental results of the system with various integral gain K_{vi} when $\tau = 100$ ms. As seen, the system is stable if K_{vi} is set as 1. However, the DC-link voltage and frequency begin to oscillate when $K_{vi} = 2$, which agrees well with the previous analysis (shown in Fig. 3-14).

As the virtual inertia level increases to $H_v / H = 2$, the critical delay time is 28 ms, according to Fig. 3-17. To verify the effectiveness of the theoretical analysis, Fig. 3-31 illustrates the experimental results when delay time increases from 0 ms to 20 ms. As can be seen, the system is stable due to the relatively low time-delay.

Fig. 3-29. Experimental results of the power converter with centralized virtual inertia control and various τ . ($H_v / H = 1$)

Fig. 3-30. Experimental results of the power converter with centralized virtual inertia control and various K_{vi} . ($H_v / H = 1$)

Fig. 3-31. Experimental results of the power converter with centralized virtual inertia control and various τ . ($H_v / H = 2$)

However, when the delay time exceeds the critical delay time, instability issues would appear. As shown in Fig. 3-32, when delay time equals 35 ms, the experimental waveforms have been seriously distorted. The oscillations in the DC-link voltage and frequency indicate that the system is unstable in this case.

Fig. 3-32. Experimental results of the power converter with centralized virtual inertia control and various τ . ($H_v / H = 2$)

Fortunately, once the modified centralized virtual inertia control is enabled, the above instability issue can be successfully addressed. As shown in Fig. 3-33, the oscillations disappear gradually when K_n / K_{vp} is set as 0.5, indicating that the system becomes stable.

Fig. 3-33. Experimental results of the power converter with centralized virtual inertia control and various K_n . ($H_v / H = 2$)

Via the proposed virtual inertia control, the system inertia can be effectively improved, even in the presence of the time-delay effect. Fig. 3-34 shows the grid frequency and DC-link voltage response of the grid-connected converters with and without virtual inertia control under a 2% step-up load change. It is worth emphasizing that it is

necessary to keep the frequency droop and its changing rate below the requirements of grid codes under such a frequency event. In Fig. 3-34, the DC-link voltage in power converters with virtual inertia control varies in proportional to the grid frequency. As such, a reduced maximum frequency deviation and a smaller RoCoF can be observed as expected.

Fig. 3-34. Experimental results with and without virtual inertia control under a 2% step-up load change.

3.7 Summary

This chapter has introduced the fundamental of virtual inertia control and identified its stability threat in an islanded power system. As the dynamics of voltage loop and frequency detection, e.g., phase lock loops and centralized control, are considered, it is revealed that the system stability margin decreases as the virtual inertia level increases. To address this problem, the modified virtual inertia control is proposed subsequently. However, this stability enhancement through the modified method is at the price of the voltage loop / PLL bandwidth reduction. Finally, experimental verifications are provided, which are consistent with the theoretical analysis.

Chapter 4 Impedance Modeling and Stability of Grid-Connected Converters with Virtual Inertia Control in the Grid-Connected Mode

This chapter focuses on the converter-level stability of a GCC with virtual inertia control. Based on harmonic linearization, the sequence impedance model of a GCC with and without virtual inertia control in the grid-connected mode is derived, while considering the interactions between AC and DC side networks and the mirror-frequency coupling effects. Besides, referring to the derived impedance expressions, the system performance under unideal grid conditions are also investigated and shown in this chapter.

4.1 Introduction

4.1.1 Impedance-Based Modeling Methods

As the fundamental building blocks of future environment-friendly power systems, GCCs are expected to grow in scope and importance in coming decades due to the rising large-scale adoption and deployment of RESs. However, the interactions between GCCs and electrical grids at their terminals may lead to instability or resonance issues [47-50]. To list a few examples, a 25th order harmonic resonance event was recorded in China southern power grid, and the grid voltage feedforward control for the VSC-HVDC system under the weak grid condition is blamed for this event [51]. Reference [52] presented a subsynchronous resonance event in a system with two wind farms in Texas, US, which is due to the tripping of a transmission line. Additionally, a 451 Hz resonance event occurred in the North Sea offshore wind farms. The author pointed out that the resonance is attributed to the improper control system parameters and the AC network configuration [49]. Moreover, vehicle-grid low-frequency oscillation phenomenon is also attracting growing attention [53-55].

To avoid these accidents, an accurate modeling tool is required for pinpointing the sources of resonances and instabilities. In 1976, Middlebrook and Cuk proposed averaged state-space model to deal with the nonlinearity brought by switching behaviors [56]. This approach is effective and widely accepted while modeling the low-frequency dynamics [57, 58]. However, to derive the conventional state-space model, it is supposed to have prior knowledge of all detailed parameters and configurations of the entire system, which precludes its practical applications.

Alternatively, besides state-space approaches, the frequency-domain method serves as another way to model the power electronic circuits. It targets to obtain the linearized system behaviors and to describe system dynamics in a transfer function form [59]. To get the system transfer function, a small-signal perturbation is usually injected on the equilibrium points, which is the well-known small-signal model. Among the frequency-domain small-signal models, the impedance-based model has been found wide acceptance during these years due to its effectiveness and feasibility [60, 61]. As for its mechanism, the impedance-based analysis injects a sinusoidal perturbation into the system input variable on the steady-state operating point and collects its response in the output at the perturbation frequency.

However, due to the absence of fixed system operating points, the conventional small-signal impedance methods are difficult to be utilized in AC power electronic systems such as GCCs. To solve this problem, one common approach is that applying a proper coordinate transformation, converting a time-periodic system into a time-invariant system, e.g., dq reference frame system. The original sinusoidal signals are mapped into DC signals in the d -axis and q -axis, which enables the linearization and small-signal impedance derivations [1]. However, several limitations still exist. For instance, for an unbalanced three-phase system, a negative-sequence AC component is mapped into a second harmonic component in the dq -domain [62]. Additionally, as the impedance

models are built in an artificial reference frame, the developed impedance expressions are quite difficult to measure [63, 64].

To overcome aforementioned limitations and disadvantages, the harmonic linearization method targets to characterize the three-phase GCCs directly in the phase domain without being tied to any artificial reference frames [18, 65]. With this approach, a three-phase system is described with a positive-sequence and a negative-sequence component. As such, the grid-connected system is decomposed into a positive-sequence and a negative-sequence subsystem. The sequence impedance model of a grid-connected voltage source converter (VSC) is shown in [66]. Additionally, the sequence impedance has also been developed for MMCs [67], type-III and type-IV turbines [68-70] and HVDC converters [71], serving as a promising tool to analyze and mitigate the resonance problems.

4.1.2 Impedance-Based Stability Criterion

For state-space models, the system stability can be evaluated by characteristic equations and eigenvalues. In contrast, the impedance-based stability criterion is a frequency-domain method based on the input/output impedance of two cascaded subsystems, e.g., converters and grids. The impedance-based stability criterion is firstly proposed by Middlebrook to handle DC-DC converters [72]. Due to its effectiveness, this criterion has also been successfully expended to AC systems. For illustration, a three-phase GCC can be described by its positive-sequence and negative-sequence subsystems with the developed sequence impedance model, and thus grid-converter interactions are analyzed in each sequence subsystem separately. As such, The GCC is simplified into its

Fig. 4-1. Impedance-based equivalent circuit.

corresponding Thevenin or Norton equivalent circuits, and the grid-converter impedance ratio is supposed to satisfy the Nyquist criterion with sufficient margins [47]. Fig. 4-1 summarizes the impedance-based equivalent circuit for voltage and current source systems. As seen, the detailed power converter circuits are eclipsed, whose dynamics are only reflected at the point of common coupling (PCC). Therefore, if each subsystem is individually stable, the stability of the interconnected system can be analyzed through Nyquist criterion with the impedance ratio $Z_s(s) / Z_l(s)$ (for voltage source systems) or $Z_s(s) / Z_l(s)$ (for current source systems). Whichever impedance ratio is adopted, the essential principle is to avoid $1 / (Z_s(s) + Z_l(s))$ having any right-half plane (RHP) poles [73].

With the help of the impedance criterion, one can predict the potential resonance issues when the grid-converter impedance ratio satisfies the Nyquist criterion without a sufficient stability margin. For example, reference [74] studied the VSC operation characteristics under unbalanced weak impedance. The effects of control delay are investigated in [75] based on the impedance criterion. The voltage stability of offshore wind farms in analyzed with sequence impedance modeling is shown in [71].

4.1.3 Mirror-Frequency Effects

As for the conventional sequence impedance models, only the response at the perturbation frequency is considered, while neglecting the responses at other frequencies [66]. However, considering the nonlinear behaviors of power electronic circuits and the corresponding controls, the current response under a voltage perturbation contains other

Fig. 4-2. Schematic diagram of a three-phase GCC with DC-link voltage control

frequencies besides perturbation frequency [76-78]. To further illustrate this effect, a case study is conducted as follows. Fig. 4-2 shows the schematic diagram of a three-phase GCC with DC-link voltage control, where the phase voltage is represented as v_{abc} ; the phase current is i_{abc} ; L corresponds to the equivalent filter inductance; the DC-link voltage is denoted as v_{dc} ; C and R refer to the DC-link capacitor and the resistance load, respectively; I_o designates the constant current load. To collect and derive the response impedance, a small-signal voltage perturbation at one certain frequency is injected into the phase voltage firstly. To show as an example, a $f_p + f_1$ Hz positive sequence perturbation is injected into the grid phase voltage and Fig. 4-3 presents the FFT analysis results for the current response with $f_p = 150$ Hz.

Fig. 4-3. FFT spectrum of the phase current under voltage perturbations.

As seen, the phase current responds as a positive sequence component at $f_p + f_1 = 200$ Hz and a negative sequence component at $f_p - f_1 = 100$ Hz. Generally speaking, in a three-phase system, when the injected signal frequency is $f_p + f_1$ (positive sequence), the corresponding mirror-frequency is located at $f_p - f_1$ (negative sequence). Similarly, when the injected signal frequency is $f_p - f_1$ (negative sequence), the corresponding mirror-frequency is $f_p + f_1$ (positive sequence). These mirror-frequency coupling effects are usually caused by DC-link voltage control or fast PLL dynamics and extensive attention should be attached during the modeling. Reference [79] has studied this phenomenon brought by PLL dynamics and asymmetrical dq -frame control. Moreover, A unified

method called multi-harmonic linearization is proposed in [80] to refine the original model.

4.2 Impedance Modeling for GCCs with Virtual Inertia Control

4.2.1 Effects of DC-Link Voltage Control

Before we start to investigate the virtual inertia control implementations, it is of importance to mathematically analyze a GCC with DC-link voltage control firstly. The three-phase GCC is depicted in Fig. 4-2. In the time domain, the phase A voltage v_a and current i_a , considering the mirror-frequency, are given as

$$v_a = V_1 \cos(2\pi f_1 t + \phi_{v1}) + V_p \cos[2\pi(f_p + f_1)t + \phi_{vp}] + V_n \cos[2\pi(f_p - f_1)t + \phi_{vn}], \quad (4-1)$$

$$i_a = I_1 \cos(2\pi f_1 t + \phi_{i1}) + I_p \cos[2\pi(f_p + f_1)t + \phi_{ip}] + I_n \cos[2\pi(f_p - f_1)t + \phi_{in}], \quad (4-2)$$

where V_1 with ϕ_{v1} and I_1 with ϕ_{i1} represent the magnitudes and phases of the fundamental voltage and current at frequency f_1 , respectively. V_p with ϕ_{vp} and I_p with ϕ_{ip} refer to the magnitudes and phases of the positive-sequence perturbations at frequency $f_1 + f_p$, respectively. V_n with ϕ_{vn} and I_n with ϕ_{in} correspond to the magnitudes and phases of the negative-sequence perturbations at frequency $f_1 + f_p$, respectively. Moreover, the DC-link voltage is written as [76]

$$v_{dc} = V_{dc_ref} + V_{dp} \cos(2\pi f_p t + \phi_{dp}), \quad (4-3)$$

where V_{dp} and ϕ_{dp} are unknown variables that will be derived later. Based on Bilateral Fourier Transform, the aforementioned variables are described in frequency domain as

$$V_a[f] = \begin{cases} \dot{V}_1, & f = \pm f_1 \\ \dot{V}_p, & f = \pm(f_p + f_1), \\ \dot{V}_n, & f = \pm(f_p - f_1) \end{cases} \quad (4-4)$$

$$I_a[f] = \begin{cases} \dot{I}_1, & f = \pm f_1 \\ \dot{I}_p, & f = \pm(f_p + f_1), \\ \dot{I}_n, & f = \pm(f_p - f_1) \end{cases} \quad (4-5)$$

$$V_{dc}[f] = \begin{cases} V_{dc_sat}, & \text{dc} \\ \dot{V}_{dp}, & f = \pm f_p \end{cases}, \quad (4-6)$$

where $\dot{V}_p = (V_p / 2)e^{\pm j\phi_p}$, $\dot{V}_n = (V_n / 2)e^{\pm j\phi_n}$, $\dot{I}_p = (I_p / 2)e^{\pm j\phi_p}$, $\dot{I}_n = (I_n / 2)e^{\pm j\phi_n}$, $\dot{V}_{dp} = (V_{dp} / 2)e^{\pm j\phi_{dp}}$. Moreover, the interactions between the AC and DC side networks can be described with the following averaged model.

$$L \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = v_{dc} \begin{bmatrix} d_a \\ d_b \\ d_c \end{bmatrix} - \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}, \quad (4-7)$$

$$i_{dc} = d_a i_a + d_b i_b + d_c i_c = -(I_o + \frac{v_{dc}}{R} + C \frac{dv_{dc}}{dt}), \quad (4-8)$$

where d_{abc} represents the averaged duty cycles of the upper switches $S_1 - S_6$. In this case, the voltage/current controllers are implemented in the dq -domain, which are based on the Park's transformation defined as follows:

$$T_{abc-dq}(\theta_{pll}) = \frac{2}{3} \begin{bmatrix} \cos \theta_{pll} & \cos(\theta_{pll} - 2\pi/3) & \cos(\theta_{pll} + 2\pi/3) \\ -\sin \theta_{pll} & -\sin(\theta_{pll} - 2\pi/3) & -\sin(\theta_{pll} + 2\pi/3) \end{bmatrix}. \quad (4-9)$$

The PLL dynamics are ignored in this case so θ is equal to $2\pi f_1 t + \phi_{v1}$ in (4-9). As such, the system plant (4-7) and (4-8) can be changed into rotating dq -frame as

$$L \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = v_{dc} \begin{bmatrix} d_d \\ d_q \end{bmatrix} - \begin{bmatrix} v_d \\ v_q \end{bmatrix} + \begin{bmatrix} 0 & \omega_1 L \\ -\omega_1 L & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}, \quad (4-10)$$

$$i_{dc} = \frac{3}{2}(d_d i_d + d_q i_q) = -(I_o + \frac{v_{dc}}{R} + C \frac{dv_{dc}}{dt}), \quad (4-11)$$

where v_d and v_q refer to the d -axis and q -axis component of the phase voltages, respectively, which are expressed in frequency domain as [66]

$$V_d[f] = \begin{cases} V_1 \cos \phi_{v1}, & \text{dc} \\ \dot{V}_p + \dot{V}_n, & f = \pm f_p \end{cases}, \quad V_q[f] = \begin{cases} V_1 \sin \phi_{v1}, & \text{dc} \\ \mp j \dot{V}_p \pm j \dot{V}_n, & f = \pm f_p \end{cases}. \quad (4-12)$$

Similarly, i_d and i_q represent the d -axis and q -axis component of the phase currents, respectively

$$I_d[f] = \begin{cases} I_1 \cos \phi_{i1}, & \text{dc} \\ \dot{I}_p + \dot{I}_n, & f = \pm f_p \end{cases}, \quad I_q[f] = \begin{cases} I_1 \sin \phi_{i1}, & \text{dc} \\ \mp j \dot{I}_p \pm j \dot{I}_n, & f = \pm f_p \end{cases}. \quad (4-13)$$

The d -axis averaged duty ratio d_d and q -axis averaged duty ratio d_q can be obtained by the controller block diagram, shown as in Fig. 4-4,

where $H_v(s)$ and $H_c(s)$ are voltage/current PI controllers, respectively. Since we mainly focus on the low- and medium-frequency dynamics, the control delay is ignored in this case. Referring to Fig. 4-4, the frequency domain expressions of d_d and d_q are

Fig. 4-4. Block diagram of the GCC with DC-link voltage control.

$$D_d[f] = \frac{1}{V_{dc_ref}} \begin{cases} V_{d_sat}, & \text{dc} \\ \dot{V}_{dp} H_v(s) H_c(s) - (\dot{I}_p + \dot{I}_n) H_c(s), & f = \pm f_p \end{cases}, \quad (4-14)$$

$$D_q[f] = \frac{1}{V_{dc_ref}} \begin{cases} V_{q_sat}, & \text{dc} \\ -(\mp j\dot{I}_p \pm j\dot{I}_n)H_c(s), & f = \pm f_p \end{cases} \quad (4-15)$$

It should be noted that because both $H_v(s)$ and $H_c(s)$ are proportional-integral controllers, the DC component of $D_d[f]$ and $D_q[f]$ should be written as V_{d_sat} and V_{d_sat} instead of $V_{dc_sar}H_v(0)H_c(0) - I_1\sin\phi_{v1}H_c(0)$ and $-I_1\sin\phi_{v1}H_c(0)$, respectively, where V_{d_sat} and V_{d_sat} represent the steady-state converter output voltages in the rotating dq -frame. Let the derivation terms in (4-10) and (4-11) equal to zero, we can obtain the equilibrium points

$$\begin{cases} \phi_{v1} = \phi_{i1} = 0 \\ V_{d_sat} = V_1 \\ V_{q_sat} = \omega_1 L I_1 \\ I_1 = -\frac{2}{3V_1} (V_{dc_ref} I_o + \frac{V_{dc_ref}^2}{R}) \end{cases}, \quad (4-16)$$

where $\omega_1 = 2\pi f_1$. Furthermore, based on harmonic linearization [65], (4-10) and (4-11) can be expressed in the frequency-domain, at frequency f , as

$$j2\pi Lf \begin{bmatrix} I_d[f] \\ I_q[f] \end{bmatrix} = V_{dc}[f] \otimes \begin{bmatrix} D_d[f] \\ D_q[f] \end{bmatrix} - \begin{bmatrix} V_d[f] \\ V_q[f] \end{bmatrix} + \begin{bmatrix} 0 & \omega_1 L \\ -\omega_1 L & 0 \end{bmatrix} \begin{bmatrix} I_d[f] \\ I_q[f] \end{bmatrix}, \quad (4-17)$$

$$\frac{3}{2} (D_d[f] \otimes I_d[f] + D_q[f] \otimes I_q[f]) = -(I_o[f] + 1/R + j2\pi Cf)V_{dc}[f], \quad (4-18)$$

where “ $\otimes$ ” represents the convolution symbol. It should be mentioned that I_o is the constant current load which means if $f \neq 0$, $I_o[f] = 0$. Therefore, at the positive frequency $+f_p$, (4-17) and (4-18) are changed as

$$j2\pi Lf_p \begin{bmatrix} I_d[f_p] \\ I_q[f_p] \end{bmatrix} = V_{dc}[f_p] \begin{bmatrix} D_d[0] \\ D_q[0] \end{bmatrix} + V_{dc}[0] \begin{bmatrix} D_d[f_p] \\ D_q[f_p] \end{bmatrix} - \begin{bmatrix} V_d[f_p] \\ V_q[f_p] \end{bmatrix} + \begin{bmatrix} 0 & \omega_1 L \\ -\omega_1 L & 0 \end{bmatrix} \begin{bmatrix} I_d[f_p] \\ I_q[f_p] \end{bmatrix}, \quad (4-19)$$

$$\frac{3}{2} (D_d[0]I_d[f_p] + D_d[f_p]I_d[0] + D_q[0]I_q[f_p] + D_q[f_p]I_q[0]) = -(1/R + j2\pi Cf_p)V_{dc}[f_p]. \quad (4-20)$$

Substituting (4-4) - (4-6) and (4-12) - (4-16) into (4-20), it yields

$$\begin{aligned} & \frac{3}{2V_{\text{dc_ref}}} \{V_1(\dot{I}_p^+ + \dot{I}_n^+) + I_1[\dot{V}_{dp}^+ H_v(s) H_c(s) - H_c(s)(\dot{I}_p^+ + \dot{I}_n^+)] + \omega_1 L I_1(-j\dot{I}_p^+ + j\dot{I}_n^+)\} \\ & = -(1/R + sC)\dot{V}_{dp}^+, \end{aligned} \quad (4-21)$$

where the superscripts “+” and “-” are the variable at the positive or negative frequency.

For example, $\dot{I}_p^+ = (I_p / 2)e^{+j\phi_p}$, $f = +f_p$, $\dot{I}_n^+ = (I_n / 2)e^{+j\phi_n}$, and $f = +f_p$. Further on, (4-21) can be simplified as

$$\dot{V}_{dp}^+ = P(s)\dot{I}_p^+ + Q(s)\dot{I}_n^+, \quad (4-22)$$

where

$$P(s) = 3 \frac{j2\pi f_1 L I_1 - V_1 + I_1 H_c(s)}{2V_{\text{dc_ref}}(sC + 1/R) + 3I_1 H_v(s) H_c(s)}, \quad (4-23)$$

$$Q(s) = 3 \frac{-j2\pi f_1 L I_1 - V_1 + I_1 H_c(s)}{2V_{\text{dc_ref}}(sC + 1/R) + 3I_1 H_v(s) H_c(s)}. \quad (4-24)$$

With the derived expression of $V_{\text{dc}}[+f_p]$, we can begin to derive the system sequence impedance model. Substitution of (4-22), (4-4) - (4-6) and (4-12) - (4-16) into (4-19) and let $\dot{V}_n^+ = 0$ firstly, one can derive that

$$Ls(-j\dot{I}_p^+ + j\dot{I}_n^+) = \dot{V}_{dp}^+ \frac{\omega_1 L I_1}{V_{\text{dc_ref}}} - (-j\dot{I}_p^+ + j\dot{I}_n^+) H_c(s) - \omega_1 L(\dot{I}_p^+ + \dot{I}_n^+) + j\dot{V}_p^+, \quad (4-25)$$

$$Ls(\dot{I}_p^+ + \dot{I}_n^+) = \dot{V}_{dp}^+ \frac{V_1}{V_{\text{dc_ref}}} - (\dot{I}_p^+ + \dot{I}_n^+) H_c(s) + \dot{V}_{dp}^+ H_v(s) H_c(s) + \omega_1 L(-j\dot{I}_p^+ + j\dot{I}_n^+) - \dot{V}_p^+. \quad (4-26)$$

Assuming $Z_1 = \frac{\omega_1 L I_1}{V_{\text{dc_ref}}}$, $Z_2(s) = \frac{V_1}{V_{\text{dc_ref}}} + H_v(s) H_c(s)$, (4-26) is rearranged as

$$\dot{I}_n^+ = M_1(s)\dot{I}_p^+ + N_1(s)\dot{V}_p^+, \quad (4-27)$$

where $M_1(s) = \frac{jLs + P(s)Z_1 + jH_c(s) - \omega_1 L}{jLs - Q(s)Z_1 + jH_c(s) + \omega_1 L}$, $N_1(s) = \frac{j}{jLs - Q(s)Z_1 + jH_c(s) + \omega_1 L}$. Hence, the sequence impedance $\dot{V}_p^+ / \dot{I}_p^+$ can be obtained by replacing $\dot{I}_n^+$ by (4-27) in (4-25), derived as

$$\begin{aligned} Z_{pp-dq}(s) &= \frac{\dot{V}_p^+}{-\dot{I}_p^+} \\ &= \frac{Ls(1 + M_1(s)) - [P(s)Z_2(s) + Q(s)Z_2(s)M_1(s) - j\omega_1 L + j\omega_1 LM_1(s)]}{1 + N_1(s)[Ls + H_c(s)] - j\omega_1 LN_1(s) - Q(s)Z_2(s)N_1(s)}, \end{aligned} \quad (4-28)$$

Similarly, other three sequence impedance expressions can also be derived as

$$\begin{aligned} Z_{np-dq}(s) &= \frac{\dot{V}_n^+}{-\dot{I}_p^+} \\ &= \frac{Ls(1 + M_1(s)) - [P(s)Z_2(s) + Q(s)Z_2(s)M_1(s) + j\omega_1 L - j\omega_1 LM_1(s)]}{1 - N_1(s)[Ls + H_c(s)] + j\omega_1 LN_1(s) + Q(s)Z_2(s)N_1(s)}, \end{aligned} \quad (4-29)$$

$$\begin{aligned} Z_{pn-dq}(s) &= \frac{\dot{V}_p^+}{-\dot{I}_n^+} \\ &= \frac{Ls(1 + M_2(s)) - [Q(s)Z_2(s) + P(s)Z_2(s)M_2(s) + j\omega_1 L - j\omega_1 LM_2(s)]}{1 - N_2(s)[Ls + H_c(s)] - j\omega_1 LN_2(s) + P(s)Z_2(s)N_2(s)}, \end{aligned} \quad (4-30)$$

$$\begin{aligned} Z_{nn-dq}(s) &= \frac{\dot{V}_n^+}{-\dot{I}_n^+} \\ &= \frac{Ls(1 + M_2(s)) - [Q(s)Z_2(s) + P(s)Z_2(s)M_2(s) + j\omega_1 L - j\omega_1 LM_2(s)]}{1 + N_2(s)[Ls + H_c(s)] + j\omega_1 LN_2(s) - P(s)Z_2(s)N_2(s)}, \end{aligned} \quad (4-31)$$

where $M_2(s) = 1 / M_1(s)$, and $N_2(s) = \frac{j}{jLs + P(s)Z_1 + jH_c - \omega_1 L}$. It is notable that (4-

28) - (4-31) are derived from the rotating dq -frame based system plant, which should be changed to the stationary frame as

$$Z_{pp}(s) = \frac{V_p(s)}{-I_p(s)} = Z_{pp-dq}(s - j\omega_1), \quad (4-32)$$

$$Z_{pn}(s) = \frac{V_p(s)}{-I_n(s - 2j\omega_1)} = Z_{pn-dq}(s - j\omega_1), \quad (4-33)$$

$$Z_{nn}(s) = \frac{V_n(s)}{-I_n(s)} = Z_{nn-dq}(s + j\omega_1), \quad (4-34)$$

$$Z_{np}(s) = \frac{V_n(s)}{-I_p(s + 2j\omega_1)} = Z_{np-dq}(s + j\omega_1). \quad (4-35)$$

(4-32) - (4-35) can be integrated as

$$-\begin{bmatrix} I_p(s) \\ I_n(s) \\ I_p(s + 2j\omega_1) \\ I_n(s - 2j\omega_1) \end{bmatrix} = \begin{bmatrix} 1/Z_{pp}(s) & 0 \\ 0 & 1/Z_{nn}(s) \\ 0 & 1/Z_{np}(s) \\ 1/Z_{pn}(s) & 0 \end{bmatrix} \begin{bmatrix} V_p(s) \\ V_n(s) \end{bmatrix}. \quad (4-36)$$

Note that the impedance expressions (4-32) - (4-35) are simply the ratios of the perturbation voltage to the response current at the same frequency or its mirror-frequency. Accordingly, the impedance verification can be easily realized by the frequency scanning without the need for two independent voltage perturbations like [76]. For verification, a point-by-point frequency scanning is conducted by simulations, and the modeled and measured impedance i.e., $Z_{pp}(s)$, $Z_{pn}(s)$, $Z_{nn}(s)$ and $Z_{np}(s)$, are depicted in Fig. 4-5. To investigate the impacts of the operation modes, the blue, red and yellow lines denote the modeled impedance for standby mode ($I_1 = 0$), rectifier mode ($I_1 = -10$) and inverter mode ($I_1 = 10$), respectively. Meanwhile, the dotted lines are their corresponding measured impedance. The following system parameters are used in this chapter: $V_{dc_ref} = 400$ V, $L = 4$ mH, R is open-circuit, $V_1 = 155$ V, $f_s/f_{sw} = 10k/10k$, $K_{vp} = 2$ A/V, $K_{vi} = 300$ A/(V·s), $K_{cp} = 30$ V/A, $K_{ci} = 300$ V/(A·s).

Fig. 4-5. Impedance response with DC-link voltage control. Solid lines: developed impedance expressions; Dots represent frequency response measurements.

As can be seen, the results show that the measured response well matches the modeled impedances. Specifically, In the case of the rectifier mode, the magnitude of its coupling impedance $Z_{pn}(s)$ and $Z_{np}(s)$ are smaller than the other two cases, indicating that this mode would generate more mirror-frequency components under voltage perturbations. Meanwhile, as for the inverter operation mode, notice that the magnitudes of its positive and negative impedance $Z_{pp}(s)$ and $Z_{nn}(s)$ are the smallest among the three cases, which means more current would respond in the perturbation frequency in this mode. At last, it

is also found that the converter is relatively not sensitive to the voltage perturbations when there is no power exchange between the grid.

4.2.2 Effects of PLLs and Virtual Inertia Control

In this section, the influence of PLL dynamics, as well as the virtual inertia control, on the system impedance model will be discussed. At first, Fig. 4-6 shows the structure of a conventional synchronous reference frame PLL (SR-PLL),

Fig. 4-6. Block diagram of a SR-PLL.

where the PLL loop filter, i.e. a PI controller $G_{pll_pi}(s)$, is

$$G_{pll_pi}(s) = K_{pll_p} + \frac{K_{pll_i}}{s}. \quad (4-37)$$

In this case, K_{pll_p} and K_{pll_i} are tuned as 3 (rad/s)/V, 300 (rad/s)/(V·s), respectively. Under small perturbations, the transfer function between the perturbed PLL phase angle $\Delta\theta_{pll}$ and q -axis voltage Δv_{q_pll} can be represented as

$$H_{pll}(s) = \frac{\Delta\theta_{pll}(s)}{\Delta v_{q_pll}(s)} = \frac{2\pi}{s} (K_{pll_p} + \frac{K_{pll_i}}{s}). \quad (4-38)$$

Further on, according to [66], the linear response of $\Delta\theta_{pll}$ in the frequency domain is given by

$$\Delta\theta_{pll}[f] = \begin{cases} 0 & \text{dc} \\ \mp jT_{pll}(s)\dot{V}_p \pm jT_{pll}(s)\dot{V}_n & f = \pm f_p \end{cases}, \quad (4-39)$$

where $T_{\text{pll}}(s)$ is defined as $H_{\text{pll}}(s) / (1 + V_1 H_{\text{pll}}(s))$. In the case of perturbations and system dynamics, the PLL phase θ_{pll} is not equal to the grid phase angle $2\pi f_1 t + \phi_{v1}$ in this case. To evaluate its effects, the dq/abc transformation under a small-signal disturbance $\Delta\theta_{\text{pll}}$ is formulated as

$$\begin{aligned} T_{abc-dq}(\theta_{\text{pll}} + \Delta\theta_{\text{pll}}) &= \begin{bmatrix} \cos \Delta\theta_{\text{pll}} & \sin \Delta\theta_{\text{pll}} \\ -\sin \Delta\theta_{\text{pll}} & \cos \Delta\theta_{\text{pll}} \end{bmatrix} T_{abc-dq}(\theta_{\text{pll}}) \\ &\approx \begin{bmatrix} 1 & \Delta\theta_{\text{pll}} \\ -\Delta\theta_{\text{pll}} & 1 \end{bmatrix} T_{abc-dq}(\theta_{\text{pll}}). \end{aligned} \quad (4-40)$$

Because the PLL phase $\theta_{\text{pll}} + \Delta\theta_{\text{pll}}$ is embedded in the abc/dq and dq/abc transformation matrices, all the transformations are inevitably be affected by the PLL phase disturbance $\Delta\theta_{\text{pll}}$. For instance, the phase currents are detected and then transformed from the stationary frame into the dq -frame by the abc/dq transformation. As such, the measured dq -frame under the PLL dynamics can be formulated by [81]

$$\begin{bmatrix} \Delta i_{d_pll} \\ \Delta i_{q_pll} \end{bmatrix} \approx \begin{bmatrix} I_1 \sin \phi_{i1} \Delta\theta_{\text{pll}} + \Delta i_d \\ -I_1 \cos \phi_{i1} \Delta\theta_{\text{pll}} + \Delta i_q \end{bmatrix}. \quad (4-41)$$

Similarly, the calculated d -axis and q -axis duty ratios are transformed back into the stationary frame by the dq/abc transformation, so PLL effects during this process are given by [81]

$$\begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix} \approx \begin{bmatrix} -V_{q_sat} \Delta\theta_{\text{pll}} + \Delta V_{d_pll} \\ V_{d_sat} \Delta\theta_{\text{pll}} + \Delta V_{q_pll} \end{bmatrix}. \quad (4-42)$$

Moreover, to emulate synchronous inertia, the PLL frequency change Δf_{pll} is directly linked to DC-link voltage reference change $\Delta v_{\text{dc_ref}}$ through a virtual inertia gain K_{fi} [14]. Therefore, the relationship between $\Delta\theta_{\text{pll}}$ and $\Delta v_{\text{dc_ref}}$ is given by

$$\Delta v_{\text{dc_ref}} = \frac{K_{fi} s}{2\pi} \Delta\theta_{\text{pll}}. \quad (4-43)$$

As seen, the virtual inertia control inherently introduces a differential operator between $\Delta\theta_{\text{pll}}$ and $\Delta v_{\text{dc_ref}}$. According to the above analysis and the steady-state condition (4-16), the block diagram of the GCC with PLL effects and virtual inertia control is shown in Fig. 4-7.

Fig. 4-7. Block diagram of the GCC with PLL dynamics and virtual inertia control.

According to (4-14), (4-15) and Fig. 4-7, the expressions of d_d and d_q are rewritten as

$$D'_d[f] = \frac{1}{V_{\text{dc_ref}}} \begin{cases} V_{d_sat}, & \text{dc} \\ D_d[\pm f_p] - T_{\text{pll}}(s) \left(\frac{K_{fv}s}{2\pi} H_v(s) H_c(s) + \omega_1 L I_1 \right) (\mp j \dot{V}_p \pm j \dot{V}_n), & f = \pm f_p \end{cases}, \quad (4-44)$$

$$D'_q[f] = \frac{1}{V_{\text{dc_ref}}} \begin{cases} V_{q_sat}, & \text{dc} \\ D_q[\pm f_p] + T_{\text{pll}}(s) (I_1 H_c(s) + V_1) (\mp j \dot{V}_p \pm j \dot{V}_n), & f = \pm f_p \end{cases}. \quad (4-45)$$

Substituting (4-44) and (4-45) into (4-20) yields

$$\dot{V}_{dp}^{t+} = P(s) \dot{I}_p^+ + Q(s) \dot{I}_n^+ - A(s) \dot{V}_p^+ + A(s) \dot{V}_n^+, \quad (4-46)$$

where

$$A(s) = 3j \frac{I_1 T_{\text{pll}}(s) \left[\frac{K_{fv}s}{2\pi} H_v(s) H_c(s) + \omega_1 L I_1 \right]}{2V_{\text{dc_ref}} (sC + 1/R) + 3I_1 H_v(s) H_c(s)}. \quad (4-47)$$

Similarly, the sequence impedance with PLL effects and virtual inertia control is

$$Z_{vpp_dq}(s) = \frac{\dot{V}_p^+}{-\dot{I}_p^+} \quad (4-48)$$

$$= \frac{Ls[M_1(s)+1] - [P(s)Z_2(s) + Q(s)Z_2(s)M_1(s) - j\omega_1 L + j\omega_1 LM_1(s)]}{1 + A_1(s) - jT_{pll}[\omega_1 LI_1 + \frac{K_{fv}s}{2\pi} H_v(s)H_c(s)] + N'_1(s)[Ls + H_c(s) - j\omega_1 L - Q(s)Z_2(s)]}$$

$$Z_{vnp_dq}(s) = \frac{\dot{V}_n^+}{-\dot{I}_p^+} \quad (4-49)$$

$$= \frac{Ls[M_1(s)+1] - [P(s)Z_2(s) + Q(s)Z_2(s)M_1(s) - j\omega_1 L + j\omega_1 LM_1(s)]}{1 - A_1(s) + jT_{pll}[\omega_1 LI_1 + \frac{K_{fv}s}{2\pi} H_v(s)H_c(s)] - N'_1(s)[Ls + H_c(s) - j\omega_1 L - Q(s)Z_2(s)]}$$

$$Z_{vpn_dq}(s) = \frac{\dot{V}_p^+}{-\dot{I}_n^+} \quad (4-50)$$

$$= \frac{Ls[M_1(s)+1] - [Q(s)Z_2(s) + P(s)Z_2(s)M_2(s) + j\omega_1 L - j\omega_1 LM_2(s)]}{1 + A_1(s) - jT_{pll}[\omega_1 LI_1 + \frac{K_{fv}s}{2\pi} H_v(s)H_c(s)] - N'_2(s)[H_c(s) + Ls + j\omega_1 L - P(s)Z_2(s)]}$$

$$Z_{vnn_dq}(s) = \frac{\dot{V}_n^+}{-\dot{I}_n^+} \quad (4-51)$$

$$= \frac{Ls[M_1(s)+1] - [Q(s)Z_2(s) + P(s)Z_2(s)M_2(s) + j\omega_1 L - j\omega_1 LM_2(s)]}{1 - A_1(s) + jT_{pll}[\omega_1 LI_1 + \frac{K_{fv}s}{2\pi} H_v(s)H_c(s)] + N'_2(s)[H_c(s) + Ls + j\omega_1 L - P(s)Z_2(s)]}$$

where $N'_1(s) = \frac{j - A_1(s)Z_1 - jT_{pll}(I_1H_c + V_1)}{jLs - Q(s)Z_1 + jH_c + \omega_1 L}$, $N'_2(s) = \frac{j - A_1(s)Z_1 - jT_{pll}(I_1H_c + V_1)}{jLs + P(s)Z_1 + jH_c - \omega_1 L}$.

Accordingly, the sequence impedance can be formulated as

$$-\begin{bmatrix} I_p(s) \\ I_n(s) \\ I_p(s+2j\omega_1) \\ I_n(s-2j\omega_1) \end{bmatrix} = \begin{bmatrix} 1/Z_{vpp}(s) & 0 \\ 0 & 1/Z_{vnn}(s) \\ 0 & 1/Z_{vnp}(s) \\ 1/Z_{vpn}(s) & 0 \end{bmatrix} \begin{bmatrix} V_p(s) \\ V_n(s) \end{bmatrix}. \quad (4-52)$$

To explore the impacts of the virtual inertia control, the sequence impedances are analyzed with three various virtual inertia gains, i.e., $K_{fv} = 0, 1, 10$ V/Hz. Other system parameters remain unchanged and $I_1 = I_o = 0$ A.

Fig. 4-8. Impedance response with virtual inertia control. Solid lines: developed impedance expressions; Dots represent frequency response measurements.

In Fig. 4-8, the blue, red, yellow lines correspond with the sequence impedance with different virtual inertia gain 0, 1, 10 V/Hz, respectively. As can be seen, the analytical model matches the point-by-point simulation results, and the virtual inertia control dramatically reduces the magnitude of the system impedance. In the case of $K_{fv} = 10$, the

magnitudes of the impedance are almost below the 0-dB line, which means the GCC is extremely sensitive to the grid voltage imbalance. Moreover, it should be mentioned that the virtual inertia gain is normally set larger than 100 V/Hz for inertia emulation [14].

Due to directly linking the DC-link voltage with PLL frequency derivation, the PLL bandwidth plays a critical role in the virtual inertia control. To investigate its impacts, Fig. 4-9 shows three cases with $K_{fv} = 10$ V/Hz and various K_{pll_p} .

Fig. 4-9. Impedance response with various PLL bandwidths. Solid lines: developed impedance expressions; Dots represent frequency response measurements.

In Fig. 4-9, the blue, red and yellow lines correspond with the impedance with various K_{pll_p} 0.3, 3 and 30 (rad/s)/V. As seen, a narrower PLL bandwidth helps to mitigate the impedance magnitude reduction in the high frequency band. However, the magnitudes of $Z_{vp}(s)$ and $Z_{vnn}(s)$ further decrease in the low frequency band with a slow PLL.

4.3 Effects of Nonideal Grid Conditions

4.3.1 Effects of Distorted Grids

As the sequence impedance is directly modeled in the phase domain, it is convenient to investigate the impacts of distorted grid voltages on the GCC with virtual inertia control. According to [82], the voltage distortion in a distributed power system is below 3%, typically speaking. Therefore, assuming the grid voltage is

$$\begin{cases} v_a = 155 \cos(50 \times 2\pi t) + 0.1 \cos(300 \times 2\pi t) \\ v_b = 155 \cos(50 \times 2\pi t - \frac{2}{3}\pi) + 0.1 \cos(300 \times 2\pi t + \frac{2}{3}\pi) \\ v_c = 155 \cos(50 \times 2\pi t + \frac{2}{3}\pi) + 0.1 \cos(300 \times 2\pi t - \frac{2}{3}\pi) \end{cases} \quad (4-53)$$

As seen, the grid voltage contains a negative sequence component at 300 Hz. Fig. 4-10 and Fig. 4-11 show the simulation waveforms of phase currents as well as the DC-link voltage under the unbalanced grid voltage. In this case, $K_{fv} = 10$ V/Hz and $I_1 = I_0 = 0$ A, which are consistent with the previous analysis.

Fig. 4-10. Simulation waveforms of the phase currents under a distorted grid.

Fig. 4-11. Simulation waveforms of the DC-link voltage under a distorted grid.

Fig. 4-12. FFT results for the phase currents.

Fig. 4-13. FFT results for the DC-link voltage.

According to Fig. 4-8, without the virtual inertia control, the magnitude of $Z_{vm}(s)$ is 31.65

dB at 300 Hz. However, it will decrease to around -25 dB when $K_{fv} = 10$ V/Hz. Consequently, it is reasonable to infer the phase currents will be totally distorted even under a 0.1V unbalanced voltage and a 10 V/Hz virtual inertia gain. As validated in Fig. 4-10, the simulation results well match the analytical results, as the phase current waveforms are seriously distorted when the virtual inertia control is enabled. This current distortion will trigger the protection system and violate the grid code requirements.

Moreover, Fig. 4-12 and Fig. 4-13 show that the FFT results for the phase currents and DC-link voltage ripple, where the mirror-frequency effects can be validated. Notice that we ignore the DC-link ripple at $2f_p$ Hz in our previous discussions, as its impacts are much weaker than that at f_p Hz.

4.3.2 Effects of Weak Grids

The weak grid, characterized by a variable grid impedance, often causes instability problems. Under a weak grid condition, the grid impedance may deteriorate the control of grid-connected power converters in terms of passive component resonance, grid synchronization instability, and excessive power transfer. This is because the resonance frequency of passive filters should be kept within certain frequency ranges with specific active damping control. In addition, grid-connected power converters normally measure the PCC voltages for grid synchronization. However, a large grid impedance may distort this voltage, thereby causing grid synchronization instability unless the synchronization unit is designed deliberately for a specific grid impedance. Another possibility for instability lies in the violation of power transfer limitations, which have a close relationship with the grid impedance.

Although the stability of GCCs with virtual inertia control on weak grids has been discussed in [19], the author used conventional methods to analyze. In the following contents, we will adopt the impedance-based criterion to further confirm and evaluate this problem.

As mentioned before, based on harmonic linearization, the converter-grid system is decomposed into a positive-sequence and a negative-sequence subsystem. Hence, with the grid-converter impedance ratio in each subsystem, the Nyquist stability criterion can be utilized to evaluate the interconnected system stability.

For illustration, if $\mathbf{I} = -[I_p(s) \ I_s(s) \ I_p(s + 2j\omega_1) \ I_p(s - 2j\omega_1)]^T$, $\mathbf{V} = [V_p(s) \ V_s(s)]^T$, the GCC admittance $\mathbf{Y}_i$ is given as

$$\mathbf{I} = \mathbf{Y}_i \mathbf{V}, \quad (4-54)$$

where

$$\mathbf{Y}_i = \begin{bmatrix} 1/Z_{vpp}(s) & 0 \\ 0 & 1/Z_{vnn}(s) \\ 0 & 1/Z_{vnp}(s) \\ 1/Z_{vpn}(s) & 0 \end{bmatrix}. \quad (4-55)$$

Similarly, a balanced and linear grid impedance matrix can be formulated as

$$\mathbf{V} = \mathbf{Z}_g \mathbf{I}, \quad (4-56)$$

where

$$\mathbf{Z}_g = \begin{bmatrix} Z_g(s) & 0 & 0 & 0 \\ 0 & Z_g(s) & 0 & 0 \end{bmatrix}. \quad (4-57)$$

In (4-57), Z_g represents the grid impedance and it equals sL_g for a series inductor branch in the s -domain. Therefore, the grid-converter impedance ratio is derived as

$$\mathbf{Y}_i \mathbf{Z}_g = \begin{bmatrix} Z_g(s)/Z_{vpp}(s) & 0 & 0 & 0 \\ 0 & Z_g(s)/Z_{vnn}(s) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (4-58)$$

As seen, (4-58) is a diagonal matrix and thus its eigenvalues can be adopted for the stability analysis. Here we define

$$G_{\text{sta}_p}(s) = \frac{1}{Z_{vpp}(s) + Z_g(s)}, \quad G_{\text{sta}_n}(s) = \frac{1}{Z_{vnn}(s) + Z_g(s)}. \quad (4-59)$$

$$H_{\text{sta}_p}(s) = \frac{Z_g(s)}{Z_{vpp}(s)}, \quad H_{\text{sta}_n}(s) = \frac{Z_g(s)}{Z_{vnn}(s)}. \quad (4-60)$$

Before embarking on the stability analysis for the integrated system, we have to make sure the individual subsystem is stable. Let $Z_g(s) = 0$ and thereby the pole-zeros maps of (4-59) with various K_{fv} are plotted in Fig. 4-14 and Fig. 4-15.

Fig. 4-14. Pole-zeros maps of $G_{\text{sta}_p}(s)$ with various K_{fv} ($L_g = 0$ mH).

Fig. 4-15. Pole-zeros maps of $G_{\text{sta}_n}(s)$ with various K_{fv} ($L_g = 0$ mH).

Although a zero gradually approaches to the origin, it shows that the system is always stable as there is no pole drifts to the RHP as K_{fv} increases.

Another case study is conducted as follows, where the virtual inertia control is disabled, i.e., $K_{fv} = 0$ V/Hz. Fig. 4-16 and Fig. 4-17 show the Nyquist plots of the impedance ratio defined by (4-60) with various grid impedance.

Fig. 4-16. Nyquist plots of grid-converter impedance ratio $H_{sta,p}(s)$ with various L_g ($K_{fv} = 0$ V/Hz).

Fig. 4-17. Nyquist plots of grid-converter impedance ratio $H_{sta,n}(s)$ with various L_g ($K_{fv} = 0$ V/Hz).

As can be seen, notice that the Nyquist plots of the impedance ratio (4-60) encircle the critical point $(-1,0j)$ zero time, indicating that the system is stable in the presence of grid impedance without the virtual inertia control.

Therefore, based on the above analysis, the stability of GCCs with virtual inertia control under the weak grid condition is studied as follows. With various K_{fv} and $L_g = 1$ mH, Fig. 4-18, and Fig. 4-19 show the pole-zeros map and the Nyquist plot of $G_{sta_p}(s)$ and $H_{sta_p}(s)$, respectively.

Fig. 4-18. Pole-zeros maps of $G_{sta_p}(s)$ with various K_{fv} ($L_g = 1$ mH).

Fig. 4-19. Nyquist plots of grid-converter impedance ratio $H_{sta_p}(s)$ with various K_{fv} ($L_g = 1$ mH).

Similarly, the zero-poles map and Nyquist plot of $G_{\text{sta}_n}(s)$ and $H_{\text{sta}_n}(s)$ are shown in Fig. 4-20 and Fig. 4-21, respectively. Notice that whether for the positive-sequence subsystem or the negative-sequence subsystem, the critical virtual inertia gain is the same, which can be explained by

$$Z_{vpp}(s) = Z_{vnn}^*(-s) \quad (4-61)$$

A positive-sequence component at frequency f_p is equivalent to a negative-sequence component at $-f_p$ with a negative phase [75].

Fig. 4-20. Pole-zeros maps of $G_{\text{sta}_n}(s)$ with various K_{fv} ($L_g = 1$ mH).

Fig. 4-21. Nyquist plots of grid-converter impedance ratio $H_{\text{sta}_n}(s)$ with various K_{fv} ($L_g = 1$ mH).

As K_{fv} increases, the system becomes unstable due to the encirclement of the critical point $(-1,0j)$ by the eigenvalue curves. Specifically, when $K_{fv} = 0.5$, the Nyquist plot does not circle the critical point $(-1,0j)$, indicating the system is stable. When K_{fv} increases to 1, the locus circles $(-1,0j)$ once. For further illustration, Fig 4-20 shows the corresponding movements of the closed-loop poles and zeros. As can be seen, the blue ones are the poles and zeros when $K_{fv} = 0.5$, and none of them locates at the RHP. However, when K_{fv} increases to 1, one pole drifts to the RHP, which is consistence with the Nyquist plot. This instability is validated by the following simulation waveforms, where the phase currents and the PLL frequency are shown in Fig. 4-22 and Fig. 4-23, respectively.

Fig. 4-22. Simulation waveforms of the phase currents with various K_{fv} .

Fig. 4-23. Simulation waveforms of the PLL frequency with various K_{fv} .

It can be seen that the system is stable when K_{fV} equals zero. However, the oscillation appears after setting the virtual inertia gain $K_{fV} = 5$ V/Hz at t_1 . The PLL frequency also begins to oscillate due to the grid impedance. When K_{fV} further increase to 50 V/Hz, the increasing phase currents and PLL frequency indicate that the system becomes completely unstable. These simulation results agree well with the pole-zero maps and the Nyquist plots shown in Fig. 4-14 – Fig. 4-21.

4.4 Summary

This chapter has briefly reviewed the modeling methods for power electronics. Among the various small-signal models, the impedance-based method has been found wide acceptance due to its effectiveness and practicality. To investigate the effects of the virtual inertia control in the grid-connected mode, the sequence impedance modeling method is adopted in this chapter.

Firstly, without the virtual inertia control, it is found that the DC-link voltage control would cause mirror-frequency effects. Through harmonic linearization, the derived impedance expressions indicate that this effect is relatively significant in the rectifier mode operation. Furthermore, virtual inertia control brings in issues to the GCCs in the grid-connected mode. When the virtual inertia control is enabled, the PLL frequency is directly linked to the DC-link voltage reference. Therefore, this additional interaction dramatically reduces the impedance magnitudes and thereby distorting the phase currents under grid voltage imbalance. Moreover, through impedance-based stability criterion, it is confirmed that the virtual inertia control will bring potential instability issues under the weak grid condition.

Chapter 5 Conclusions and Future Research

This chapter concludes this thesis and provides recommendations for future research in terms of virtual inertia control and system frequency stability improvement. The conclusions of this thesis are listed below:

- (1) In an islanded power system with virtual inertia implementations, the DC-link voltage loop and frequency measurements can bring system-level instability concerns.
- (2) Through harmonic linearization, it is revealed that the GCC with DC-link voltage control will cause mirror-frequency effects.
- (3) In the grid-connected mode, the derived sequence impedance expressions indicate that the GCC would become extremely sensitive to the grid imbalance owing to the virtual inertia control.
- (4) Based on the impedance-based stability criterion, the virtual inertia control will cause converter-level instability issues in the grid-connected mode.

After emerging challenges are identified, this chapter further points out the future research directions as follows:

- (1) Stability analysis under unbalanced grid impedance condition.
- (2) Advanced virtual inertia controller design.

5.1 Conclusions

This thesis explores the modeling as well as the stability of grid-connected power converters with virtual inertia control. Through extensive theoretical analyzes, the conclusion can be readily obtained as follows.

For an islanded power system, the virtual inertia cannot perfectly supplant the synchronous inertia due to the delay effects brought by grid frequency measurements and DC-link voltage loops. Originally, the existing virtual inertia control has simplified these dynamics as unit gains, which is valid when the total virtual inertia is sufficiently smaller than synchronous inertia. However, as for more-electronics power systems with a high virtual inertia level, things are changed. Through the mathematical derivations and Bode diagrams, it is revealed that when the total virtual inertia is close to or exceeds the system inertia, the phase lag introduced by the delay effects can destabilize the system. To address this problem, modified virtual inertia controls are proposed for system stability enhancement. However, there is a tradeoff between stability improvement and the bandwidths of the frequency tracking / DC-link voltage regulation.

Unlike the islanded mode, the grid frequency is assumed unchanged in the grid-connected operation mode. However, the virtual inertia can still introduce converter-level instability concerns. Based on the harmonic linearization, the sequence impedance model successfully predicts the mirror-frequency effects due to the DC-link voltage control. Next, when the virtual inertia control, as well as the PLL effects, are involved, the impedance magnitude reduces significantly, thus indicating that the GCC becomes seriously sensitive to the grid imbalance. Additionally, through the impedance-based stability criterion, it is confirmed that the virtual inertia control might destabilize the system under weak grids.

5.2 Future Works

5.2.1 Stability Analysis Under Unbalanced Grid Impedance Conditions

As investigated before, the mirror-frequency impedance expressions, i.e. $Z_{vpn}(s)$ and $Z_{vnp}(s)$, are of importance with respect to the system behaviors. However, in (4-58) we notice that only $Z_{vpp}(s)$ and $Z_{vnn}(s)$ influence the eigenvalues of the grid-converter impedance ratio matrix. However, it is worth to mention that $Z_{vpn}(s)$ and $Z_{vnp}(s)$ will also

impact system stability. For example, for an unbalanced grid impedance, its impedance matrix is rewritten as

$$\mathbf{Z}'_g = \begin{bmatrix} Z_g(s) & Z_m(s) & 0 & 0 \\ Z_m(s) & Z_g(s) & 0 & 0 \end{bmatrix}, \quad (5-1)$$

where $Z_m(s)$ represents the mutual coupling impedance between phases [74]. In this sense, the grid-converter impedance ratio matrix is changed as

$$\mathbf{Y}_i \mathbf{Z}'_g = \begin{bmatrix} Z_g(s)/Z_{vpp}(s) & Z_m(s)/Z_{vpp}(s) & 0 & 0 \\ Z_m(s)/Z_{vnn}(s) & Z_g(s)/Z_{vnn}(s) & 0 & 0 \\ Z_m(s)/Z_{vnp}(s) & Z_g(s)/Z_{vnp}(s) & 0 & 0 \\ Z_g(s)/Z_{vnn}(s) & Z_m(s)/Z_{vnn}(s) & 0 & 0 \end{bmatrix}. \quad (5-2)$$

As seen, we should calculate the eigenvalues of this new matrix to evaluate the effects of the mirror-frequency impedance $Z_{vpn}(s)$ and $Z_{vnp}(s)$. Therefore, investigation of the impacts of the mirror-frequency impedance expressions on the system stability serves as one of our future works.

5.2.2 Advanced Virtual Inertia Controller Design

Although being effective for frequency nadir and RoCoF reduction, a large inertia constant increases the frequency recovery time [14]. To further reap the advantages of more-electronics power systems, it is needed to design other advanced virtual inertia controllers. To give an example, one might conceive of the adaptive virtual inertia control, which targets to emulate a large inertia constant during the frequency drop, while a small one after the frequency nadir. The similar works have been done, such as [83], [84]. However, it turns out that the adaptive virtual inertia implantations suffer from the complexity and the need for RoCoF detection. Additionally, as mentioned in [13], the nonlinear nature of the adaptive control might destabilize the system.

Alternatively, based on the infinite-norm of the closed-loop transfer function, there is another way to reduce the frequency nadir during a frequency event. Instead of proportionally linking the grid frequency change to the DC-link voltage reference, an additional first order low-pass-filter is added into the virtual inertia loop. Therefore, the transfer function of the equivalent virtual inertia controller becomes

$$G_{fv}(s) = \frac{K_{fv}}{\tau_0 s + 1} \quad (5-3)$$

Based on Table I and Table II, the frequency regulation closed-loop transfer function with various τ_0 is shown in Fig. 5-1.

Fig. 5-1. Bode diagram of the frequency regulation closed-loop transfer function with various τ_0 .

As seen, the case of $\tau_0 = 2.08$ features the minimum infinite norm, indicating a smaller overshoot during a step change response [85]. For verification, Fig. 5-2 shows the simulation results of the frequency regulation curves for the above three cases.

As can be seen, the simulation results match the Bode diagrams well. Notice that the frequency nadir is smallest when the virtual inertia is tuned based on the minimum infinite norm, even better than the conventional virtual inertia control. In addition, this

infinite norm-based virtual inertia controller is a linear type, which facilitates its practical applications. However, as observed in Fig. 5-2, the frequency nadir reduction is at the

Fig. 5-2. Simulation waveforms of the frequency regulation with various τ_0 .

cost of RoCoF increment due to the low pass filter characteristic of (5-3), as companied with the conventional virtual inertia.

The flexibility and fast-responding features of the more-electronics system provide various and numerous approaches to control the grid frequency and improve system stability. Therefore, it is worth investigating more effective and practical ways concerning the advanced virtual inertia control.

Author's Publication

H. Yang, J. Fang, and Y. Tang, "On the Stability of Virtual Inertia Control Implemented by Grid-Connected Power Converters with Delay Effects", in *ECCE*, Baltimore, MD, Sep. 2019, pp. 2881-2888.

H. Yang, J. Fang, and Y. Tang, "Exploration of Time-Delay Effect on the Stability of Grid -Connected Power Converters with Virtual Inertia", in *ICPE-2019 ECCE Asia*, Bexco, Busan, Korea, May. 2019, pp. 2573-2578.

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