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Real-time Robust Multi-lane Detection and Tracking in Challenging Urban Scenarios

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Abstract—In this paper, we consider multi-lane detection in challenging urban scenarios such as emerging, ending, spitting and merging of lane markings, heavily curved lanes, zig-zag lanes, on/off ramp and disturbance of other road writings. We present a fast robust multi-lane detection and tracking framework to address these challenges. In this method, lane feature elements are first extracted and then grouped into clusters, and clusters are associated through energy minimization. Probabilistic decision making is adopted to track individual lane considering lane cluster measurements and prior lane state. A multi-lane tracking strategy is also presented to manage lane tracks from their appearance to disappearance, which can reduce false detection and improve robustness of the algorithm. Real driving data are used to verify the effectiveness of our algorithm in all mentioned challenging scenarios.

I. INTRODUCTION

Localizing lanes in real-time on highway and in urban areas has been active research over the last few decades. It is an important component for various intelligent vehicle applications, such as lane level localization [1]–[3] and navigation [4], [5], Lane Departure Warning Systems (LDWS) as well as Advanced Driver Assistance Systems (ADAS) [6], [7].

Lane detection in urban areas with uniform width, close to parallel lane or flat road assumption have been studied extensively [8]–[13]. All adjacent lanes can be detected in [10] using RANSAC spline fitting by assuming all the lanes are parallel and the ground is flat. [11] fused edge feature and high intensity feature to fit lane into straight line, and adopted Kalman filter for information fusion and tracking. [12] also needed inverse perspective mapping and adopted straight line model, which is not flexible to represent complex lane pattern. Multiple visual features (gradient-based, intensity-based and texture-based features) are combined in [13] to improve robustness. The lane markers in [13] are also assumed parallel on the flat ground plane.

However, multi-lane detection becomes sophisticated when considering the real driving situations (e.g. emerging, ending, spitting and merging of lane markings, heavily curved lanes, zig-zag lanes, on/off ramps and disturbance

of other road writings). To describe a wider range of lane structures, [14] established a B-Snake based lane model and used a CHEVP algorithm to initialize the B-Snake lane model. But it required the assistance of vanishing point detection, which may not exist in urban roads. [15] also used the property of vanishing point to reduce false detections. There are some works which don't have limitation of the applicable road situations [16]–[19]. [16] convolved original image with horizontal matched filter and used distance transform to guide the spline search for lane detection. A multi-object Bayes filter approach was proposed in [17] to directly integrate possible lane states constraints for multi-lane detection. [18] combined spline model with extended Kalman filter without assumption about parallelism for multi-lane detection and tracking. A illumination robust lane detection algorithm in [19] was presented using gradient-enhancing conversion method, adaptive Canny edge detection, Hough transform, and curve model fitting method. But they all didn't fully demonstrate the algorithm performance on challenging scenarios or on disturbance situations mentioned above. To address these issues, [20] proposed to learn spatial context of lane markings through boosting and track the individual lane using particle filter framework without knowledge of vehicle motion. The results were attractive on challenging daylight and night-time road situations or even under inclement weather conditions. But it required collecting large data in advance to train a classifier to distinguish lane markings with nonlane markings. [21] adopted Conditional Random Field (CRF) to model the multi-lane detection as multiple association task, which could deal with many complex and challenging urban situations but sometimes failed caused by disturbing road arrows and widely spaced dash lanes. A multi-agent model instead of geometric fitting based on a confidence map was presented in [22] for lane detection and tracking. However, it is not applicable to irregular geometric lane patterns (e.g. zig-zag lanes) since it bounds the agent's angle formed between influence and last orientation.

Lane markings can be grouped into two categories: solid and dash lane. Inspired by the work in [21], we find it convenient to represent lane feature clusters as nodes and connectivity of these clusters as edges. Thus, lane detection is a energy minimization problem to associate lane clusters in a CRF graph.

Tracking lane can be modeled as the problem of tracking polynomial shaped extended objects, which was first discussed in [23] with the application to road map estimation using radar measurements. However, the targets in [23], such as guard rails, were assumed stationary in the world

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frame thus resulting in very simple linear motion model of the polynomials. Since our approach performs tracking in image frame without knowledge of vehicle's ego motion, Kalman filter based tracking framework is not applicable to our problem. Particle filter requires large number of particles to evaluate the posterior distribution over states. Since it will be computationally intensive for all the particles to perform lane appearance matching we introduce in this paper and it is inefficient for particle filter to handle high dimensional search space resulting from multiple lanes, our approach uses the "hypothesize and verify" strategy and probabilistic reasoning to choose the most likely lane from all the lane candidates.

With all these considerations, a multi-lane detection and tracking framework is presented in this paper to address the challenges mentioned above.

The main contribution of this work can be summarized as follows:

- 1) Lane feature clusters association through energy minimization are used without any assumption of lane and road model.
- 2) To facilitate lane tracking, we introduce lane appearance matching and propose probabilistic decision making combining prior lane state with lane feature cluster candidate measurements.
- 3) A multi-lane detection and tracking framework is presented which exploits the temporal coherence of lane appearance to improve the smooth detection and robustness to outliers.

The remainder of this paper is organized as follows. Lane features are clustered and associated to obtain the lane curves in Section II. In Section III, posterior lane state estimation is conducted by probabilistic inference. Section IV deals with the multi-lane tracks. We show experiment results in real driving situations in Section V, and Section VI make a conclusion of this paper.

II. LANE STATE SPACE REPRESENTATION

In this paper, we describe all models in the image plane coordinate $[x \ y]^T$. Lane model can be represented by an n^{th} order polynomial as

$$y = a_0 + a_1x + a_2x^2 + \dots + a_nx^n \quad (1)$$

where the parameters $\mathbf{x}_a = [a_0 \ a_1 \ a_2 \ \dots \ a_n]^T$ defines the polynomial coefficients; the curve x is in the interval $[x_p, x_q]$, hence the state vector of lane curve is defined as $\mathbf{x}_t = [\mathbf{x}_a \ x_p \ x_q]^T$. t is the timestamp index. For simplicity, n is set to 2 in this paper.

Lane detection and tracking can be deemed as state estimation problem. In the following part, we show how to obtain the state vector of lane curve.

A. Lane Markings Representation

In this paper, ridge property is defined as the lane features we want to extract. Left filter f_l (2) and right filter f_r (3) as shown in Fig. 1b are used to locate the left and right edge position of lane feature elements in each row x_h of image

Algorithm 1 Generating left and right edge position

```

1: procedure INPUT(Image $I(x, y)$ )
2:   for each row  $x_h$  in ROI of input image  $I(x, y)$  do
3:      $w_{f_h} \leftarrow a + b * \frac{x_h - h_{ROI}}{h_{image} - h_{ROI}}$ 
4:      $e_w \leftarrow$  vector of edges
5:     for  $w \leftarrow w_{f_h} + 1$  to  $w_{image} - w_{f_h} - 1$  do
6:        $l_{intensity} \leftarrow 0$ 
7:        $r_{intensity} \leftarrow 0$ 
8:       for  $i \leftarrow -w_{f_h}$  to 0 do
9:          $l_{intensity} \leftarrow l_{intensity} + I(x_h, w + i)$ 
10:      end for
11:      for  $i \leftarrow 1$  to  $w_{f_h}$  do
12:         $r_{intensity} \leftarrow r_{intensity} + I(x_h, w + i)$ 
13:      end for
14:      if  $(r_{intensity} - l_{intensity})/w_{f_h} >$ 
         $Tresh(l_{intensity}/w_{f_h})$  then
15:         $e_w \leftarrow 1$  (left edge)
16:      end if
17:      if  $(l_{intensity} - r_{intensity})/w_{f_h} >$ 
         $Tresh(r_{intensity}/w_{f_h})$  then
18:         $e_w \leftarrow -1$  (right edge)
19:      end if
20:    end for
21:  end for
22: end procedure

```

Region of Interest (ROI) (Fig. 1a), respectively. Algorithm 1 shows the method for obtaining the left and right edge position.

$$f_l = \begin{cases} -1 & \text{if } x_h \in [-w_{f_h}, 0] \\ 1 & \text{if } x_h \in [1, w_{f_h} + 1] \end{cases} \quad (2)$$

$$f_r = \begin{cases} 1 & \text{if } x_h \in [-w_{f_h}, 0] \\ -1 & \text{if } x_h \in [1, w_{f_h} + 1] \end{cases} \quad (3)$$

(a) Original image

(b) Left and right filters

Fig. 1. Left and right filters to generate left and right edge position in image ROI.

Lane feature elements $\mathbf{E}_i = \{(x_i, y_i)\}$ (Fig. 2a) can be obtained with its start point x_{str_i} and end point x_{end_i} in each row, which should satisfy $x_{end_i} - x_{str_i} \in (w_{min}, w_{max})$ in order to constrain the extracted lane feature width to lay between minimum estimate lane width w_{min} and maximum estimate lane width w_{max} .

Fig. 2. Illustration of the procedure for lane detection and tracking. Same color indicates the same cluster.

Connected components in $\mathbf{E}_i = \{(x_i, y_i)\}$ are grouped as shown in Fig. 2b. Due to the smooth property of lane markings, variance of $x_{end_i} - x_{str_i}$ of all elements in connected components should not exceed a threshold T , which is set empirically considering the size of elements in connected components. The valid connected components can be seen in Fig. 2c. For those elements size under s_{min} (12 in the experiments), we also remove them if there is no strong support in the same direction (image coordinates) by other larger connected components. Finally, the lane feature cluster candidates can be obtained (Fig. 2d), helping to reduce the computation time in the subsequent procedure.

B. CRF Graph Association

From Fig. 2d and Fig. 2e, we can infer most of the lane feature cluster candidates can be obviously grouped due to the local direction consistency. Hence, we associate the cluster candidates c_{c_i} with c_{c_j} when the association probability is the highest among all other alternatives. The association probability $P(c_{c_i}, c_{c_j})$ is defined as (4).

Fig. 3. The red ellipses are used to illustrate the CRF association process, and the same color indicates the same cluster. a) Left: lane feature cluster candidates; b) middle: lane feature clusters; c) right: CRF association result.

$$P(c_{c_i}, c_{c_j}) = \exp\left(-\frac{(\theta_{c_i} - \theta_{c_i c_j})^2}{\sigma^2}\right) + \exp\left(-\frac{(\theta_{c_j} - \theta_{c_i c_j})^2}{\sigma^2}\right) \quad (4)$$

where θ_{c_i} and θ_{c_j} are the center direction of cluster candidates c_{c_i} and c_{c_j} , respectively. $\theta_{c_i c_j}$ is the direction of vector between center position of each cluster candidate c_{c_i}, c_{c_j} . σ

is set to 0.25, and the probability threshold is set to 1.96 in the experiments.

The lane feature cluster candidates in the same color after grouping indicates the same cluster, which can be seen in Fig. 2e and Fig. 3b. Given a set of lane feature clusters, a CRF graph can be used to model the relationship between clusters. Nodes can be used to indicate possible association of two clusters, and the edge donates the possible correlation among the nodes. The energy of a CRF graph composes of two parts: unary potentials and pairwise potentials, the sum of which should be minimized to determine the final association results. The energy function $E(C)$ is defined as

$$\begin{aligned} E(C) &= \sum_{C_i} U(l_i|C_i) + \sum_{C_i, C_j \in cl(G)} \phi_C(l_C|C_C) \\ &= -\left(\sum_{C_i} \ln(P(l_i|C_i)) + \sum_{C_i, C_j \in cl(G)} \ln(P(l_C|C_C))\right) \end{aligned} \quad (5)$$

where $U(l_i|C_i)$ is the unary potential; $\phi_C(l_C|C_C)$ is the pairwise potential in a CRF model.

Here, a Sigmoid function (6) is used to design the unary potential $P(l_i|C_i)$, and a quadratic regression model f_{dist} is adopted to fit the distance function between two clusters. The distance composes of two parts: the sum of difference between end direction of cluster i and link direction of cluster i, j as well as head direction of cluster j and link direction of cluster i, j ; the lateral distance between cluster i and cluster j .

$$P(l_i|C_i) = \frac{1}{1 + \exp(-f_{dist})} \quad (6)$$

To model the pairwise potential $P(l_C|C_C)$, we use a second order polynomial to fit all the lane feature elements position of the possible clusters in the same clique of graph $cl(G)$. The average error f_{err} is then calculated between the fitted value and the real elements position.

$$P(l_C|C_C) = \exp(-\frac{f_{err}^2}{\sigma^2}) \quad (7)$$

where σ is set to 50 during the experiments.

To solve the energy minimization problem, Hungarian method is used to produce the final association results between clusters. We only retain those cliques above lane length threshold. Example of CRF association result can be seen in Fig. 3c.

Finally, we can obtain the state vector of lane curves $\mathbf{x}_t = [x_a \ x_p \ x_q]^T$ by fitting into a second order polynomial on each clique.

III. PROBABILISTIC INFERENCE

There are two main reasons to conduct probabilistic inference with prior lane state estimation:

- The miss detection due to few supporting lane feature cluster candidates, which are usually caused by faded lane markings, largely curved lane, widely space dash lane and occlusion.
- The false detection due to road disturbance.

Lane appearance matching is conducted to evaluate the similarity between the prior lane shape and all possible lane hypotheses generated from lane feature cluster candidates in the ROI of prior lane curve. We can estimate the probability of event E_i given the prior state $\mathbf{x}_{t-1}$ and lane hypotheses L_i as

$$P(E_i|\mathbf{x}_{t-1}, L_i) = \frac{s(\mathbf{x}_{t-1}, L_i)}{\sum_{i=1}^n s(\mathbf{x}_{t-1}, L_i)} \quad (8)$$

where $s(\mathbf{x}_{t-1}, L_i)$ is the lane candidate score indicating how well the lane candidate fits to the state transition model, which can be evaluated through lane appearance matching. Note that

$$\sum_{i=1}^n P(E_i|\mathbf{x}_{t-1}, L_i) = 1 \quad (9)$$

Lane candidate score is defined as

$$s(\mathbf{x}_{t-1}, L_i) = \exp(-\lambda d_{err}) \quad (10)$$

where d_{err} is defined as distance average error between lane hypotheses L_i and the prior lane curve; λ is set to 0.1.

The lane hypotheses L_i with highest probability will be selected as the posterior lane curve estimation. Thus, $\mathbf{x}_t$ can be obtained.

The effect of the probabilistic inference step is demonstrated in Fig. 4 when there are few supporting lane feature cluster candidates. In this figure, the lane detection failed while lane tracking can still work with prior lane state.

IV. MULTI-LANE TRACKING

Multi-lane tracking is more difficult than single-lane tracking in two aspects: the unknown number of lanes over time indicating the varying size of state space; the size of state space grows proportionally with lane targets. We assign a unique identity to manage each track initiation and termination for handling the varying number of lanes. Each cycle of

Fig. 4. Illustration of probabilistic inference effect.

lane estimation starts with hypotheses generated from lane detection results until the consecutive observed measurement counts from lane hypotheses reach preset score. Then we initialize the track. The current tracks are updated with their corresponding associated measurements or valid lane detection results if any. New hypotheses will be generated if the detected lane can not be associated to any existing tracks. The track will be terminated or deleted when it has not been associated to any measurements or detections for several successive frames. This strategy helps to smooth detection and improve the algorithm robustness to outliers.

V. EXPERIMENTS:

LANE DETECTION AND TRACKING FOR GROUND VEHICLES

We have tested our algorithm on several video clips with resolution of 640×480 (including gray or color) in challenging urban streets. The platform is on a normal PC, Intel(R) Core i7-8550U CPU @ 1.80GHz $\times$ 8 and 8 GB RAM. The processing speed is quite fast and can reach around 55 frames per second (fps).

Lane detection and tracking results for different challenging road situations are demonstrated in Fig. 5. We can see the algorithm proposed can handle the real driving situations (e.g. emerging, ending, spitting and merging of lane markings, heavily curved lanes, zig-zag lanes, on/off ramp and disturbance of other road writings). To our best of knowledge, this is the first work that mainly focus on dealing with all these challenges. However, there are still some failed cases shown in Fig. 6a where the algorithm can not address, which are very difficult unless we collected and trained on specific data to classify the confusing disturbance with the same pattern of lane markings. Another scenario in Fig. 6b is a inherent problem in tracking which can not be dealt with.

VI. CONCLUSION

We introduced a fast robust multi-lane detection and tracking algorithm for various challenging urban scenarios. The algorithm run without any assumptions about the lane model and road model. Experiments show that the proposed algorithm can address all the challenges we focus on. The result is promising. However, training and classification are needed if we want to distinguish the real lane markings

(a) Widely spaced dash lane at a junction

(b) Lane stopping at a junction

(c) Disturbing road arrows

(d) Faded lane

(e) Disturbing crosswalk

(f) Lane splitting

(g) Lane merging

(h) Lane switching

(i) Zig-zag lane

(j) Lane at a roundabout

(k) Disturbing road writings

(l) Lane occluded by vehicle

(m) Disturbing road writings

(n) Disturbing road writings

(o) Disturbing hump

(p) Double zig-zag yellow lane

(q) Largely curved lane at a turn

(r) Lane on ramp

Fig. 5. Examples of multi-lane detection and tracking results for different challenging road situations (left: original image; middle: lane detection result; right: lane tracking result).

(a) Confusing curb tracked as lane

(b) Tracking not work at the first frame which lane detected

Fig. 6. Examples of failed cases (left: original image; middle: lane detection result; right: lane tracking result).

with confusing ones of similar lane pattern. This can be investigated in the future work.

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