Distance-based cooperative relative localization for leader-following control of MAVs

Nguyen, Thien-Minh; Qiu, Zhirong; Nguyen, Thien Hoang; Cao, Muqing; Xie, Lihua

2019

Nguyen, T.-M., Qiu, Z., Nguyen, T. H., Cao, M., & Xie, L. (2019). Distance-based cooperative relative localization for leader-following control of MAVs. *IEEE Robotics and Automation Letters*, 4(4), 3641-3648. doi:10.1109/LRA.2019.2926671

<https://hdl.handle.net/10356/141922>

<https://doi.org/10.1109/LRA.2019.2926671>

© 2019 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works. The published version is available at:
<https://doi.org/10.1109/LRA.2019.2926671>.

Downloaded on 26 Apr 2023 17:32:45 SGT

Distance-based Cooperative Relative Localization for Leader-following Control of MAVs

Thien-Minh Nguyen, Zhirong Qiu, Thien Hoang Nguyen, Muqing Cao, and Lihua Xie, *Fellow, IEEE*

Abstract—In multi-robot systems, the capability of each robot to relatively localize its neighbors is a crucial requirement, which needs to be resolved as a prerequisite for almost any distributed scheme of operation. Notably, this problem proves to be quite challenging in GPS-denied environments. In this paper, we investigate a problem of simultaneous relative localization and leader-following control of aerial robots by using only ranging and odometry sensors, waiving the need of external positioning systems. To tackle this challenge, we propose a cooperative estimation-control scheme where specialized agents called *orbiters* are tasked with maintaining *persistently exciting* trajectories to facilitate *exponential convergence* of both *relative localization* and *tracking errors* for itself and others. Numerical simulations and experiments on quadcopters in a GPS-denied environment are carried out to validate the theoretical findings.

Index Terms—Multi-Robot Systems, Localization, Adaptive Control of Robotic Systems

I. INTRODUCTION

IN the recent decade, Micro Aerial Vehicles (MAVs) have been gaining great popularity in many civilian, industrial and military applications. To fulfill the cooperative operation of multi-MAV systems, it is necessary and crucial that each robot can obtain the relative position to their neighbors. This is often realized with external localization systems [1]–[4], or by onboard vision-based techniques [5]–[8]. However, the reliance on external systems restricts the operation range, while vision-based techniques hinder the system’s versatility and scalability due to their limited field of view (FOV).

To address these issues, relative localization has been a very active research topic over the years, especially those employing inter-agent relative distance [9]–[16] as this is the most practically realizable relative sensing method between agents. In this work, we propose a technique leveraging distance and displacement measurements which are respectively obtained from ultra-wideband (UWB) ranging and low-cost optical flow sensors (or any other visual odometry system if it provides better accuracy). Our method only relies on onboard sensors, thus is not affected by the coverage of external localization

systems or restricted to specific environments. Also, since UWB is almost omni-directional (see [17] for the analysis on the radiation pattern of UWB), it avoids the issue of limited FOV in vision-based techniques.

Some related works on distance based relative localization can be found in the literature. Notable works with real-flight experiments can be listed as [9], [10], [13], [14], [18]. In [9], [10], multiple UWB nodes were installed on the MAV with known offsets from the body’s center, which are combined with distance measurements to infer the location of the target. Due to constraints on the MAV’s size, this scheme may be ineffective beyond a few meters while requiring additional sensors. In [13], displacement measurements from the MAV’s own motion together with the distance measurements were used to estimate the relative position to a target. In [14], a heading-independent leader following scheme was proposed by studying the observability matrix. However, in these works the convergence condition for the relative localization was left unaddressed.

Some other researchers have studied the theoretical guarantee on the *exponential convergence* (EC) of the relative localization based on the *persistently exciting* (PE) assumption. Most notably, with the PE assumption on the relative velocity, both [19] and [20] obtained EC for the relative position estimate. Specifically, [19] employed distance, derivative of distance, and relative velocity measurements in the relative localization scheme for agents with single integrator dynamics, while [20] employed distance, relative velocity, and relative acceleration measurements to propose a different localization scheme for agents with double integrator dynamics. Note that [19] only focused on the estimation problem, while in [20] the authors also proposed a control strategy to track the target by embedding a PE term in the control law. However, the introduced PE term was not explicitly designed but rather acted as a source of disturbance to the system, thus resulting in non-zero tracking error.

Based on the insights from previous studies, in this paper, we propose a cooperative estimation-control scheme to solve the leader-following control problem for MAVs. The follower agents are separated into two groups as *orbiters* and *followers*: the orbiters are to track the leader with a predefined periodic trajectory that satisfies a PE condition, and the followers are to track the leader at a relative setpoint. We first study the basic case when there exist one orbiter and one follower. For the orbiter which can communicate with and range to the leader, we design the corresponding estimation-control laws by using distance and displacement measurements, and show that both the localization error and the tracking error would

This research is supported by National Research Foundation (NRF) Singapore, ST Engineering-NTU Corporate Lab under its NRF Corporate Lab@ University Scheme.

The authors are with School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, 50 Nanyang Avenue (e-mail: thienminh.npn@ieee.org, qiuz0005@e.ntu.edu.sg, e180071@e.ntu.edu.sg, caom0006@e.ntu.edu.sg, elhxie@ntu.edu.sg). (*Corresponding author: Zhirong Qiu.*)

This letter has supplemental downloadable multimedia material available at <http://ieeexplore.ieee.org>, provided by the authors. The Supplementary Materials contain a video summarizing the main results in the paper, and MATLAB code for the simulation program. This material is 19.3 MB in size.

Digital Object Identifier (DOI): 10.1109/LRA.2019.2926671

converge to zero exponentially fast. For the follower which can communicate with and range to the other two agents, we design similar estimation-control laws and achieve similar EC results. The extension to a general hierarchy is also discussed.

Several contributions are achieved when compared with previous works. Most importantly, the proactive introduction of the orbiter enables the PE condition by maintaining a predefined trajectory, which aids the cooperative localization for all the following agents and results in EC of both localization and tracking errors. Moreover, while previous works required quite a few types of continuous-time measurements, we propose a discrete-time relative localization algorithm using only discrete-time distance and displacement, which can be directly obtained from practical sensors. Finally, real-life experiments are conducted to validate the proposed method.

The remainder of the paper is organized as follows: we first focus on a basic structure of three agents, by formulating the problem in Section II, presenting the estimation and control laws of the cooperative relative localization scheme in Section III, and conducting the convergence analysis in Section IV. Then we discuss the extension of this basic structure to a general hierarchical structure in Section V. Simulation and experiment results are respectively provided in Sections VI and VII to validate the theoretical findings and demonstrate the practicality of the proposed algorithm. We conclude our work in Section VIII.

Notations: in this paper we respectively use $\mathbb{R}$, $\mathbb{N}$, and $\mathbb{N}^+$ to denote the sets of natural numbers, nonnegative integers, and positive integers. $\mathbb{R}^m$ denotes the m -dimension real vector space with $m = 2$ or 3 . For $k \in \mathbb{N}$, denote $k^+ = k + 1$. For a vector $v \in \mathbb{R}^m$, $\|v\|$ stands for the Euclidean norm and v' denotes its transpose. For a matrix W , $\varsigma_m(W)$ and $\varsigma_M(W)$ respectively denote the smallest and the largest singular values of W . Finally, we use the abbreviations EC, e.c., PE, p.e. for the terms *exponential convergence*, *exponentially convergent*, *persistent excitation* and *persistently exciting*, respectively.

II. PROBLEM FORMULATION

As introduced earlier, we will first consider a basic structure with 3 agents labelled as 0, 1, and 2. Agent 0 is called the *leader* and serves as a reference point, to which all other agents will estimate their relative positions via a cooperative localization scheme. Moreover, agent 1 is referred to as the *orbiter* and agent 2 as the *follower*. The ranging and communication relationships between these agents are illustrated in Fig. 1. Note that we call agent j as a neighbor of agent i if either agent can range to the other.

Below, we will give the detailed definitions of some basic concepts before stating our objectives.

A. Basic definitions

1) *Dynamic model:* Denote $p_i(k) \in \mathbb{R}^m$ as the position of agent i in some fixed frame of reference at time kT , where T is the sampling period. We also denote $\bar{u}_i(k)$ as the velocity of agent i , and assume that it has the following discrete-time single integrator dynamics with maximum velocity $\bar{U}_i$:

$$p_i(k^+) = p_i(k) + T\bar{u}_i(k), \quad \|\bar{u}_i(k)\| \leq \bar{U}_i < \infty, \quad \forall k. \quad (1)$$

Fig. 1: An illustration of the collaborative ranging and communication scheme executed by three agents: at time kT agent 1 measures the distance $d_{10}(k)$ to agent 0 and receives agent 0's displacement $\phi_0(k-1)$ via the UWB communication; on the other hand, agent 2 would obtain such measurements from both agent 0 and agent 1.

2) *Sensing models:* For an agent i , it can obtain the relative distance to a neighbor agent j as follows:

$$d_{ij}(k) = \|p_i(k) - p_j(k)\|. \quad (2)$$

In addition, we assume that each agent can obtain the displacement measurement of itself and its neighbors (via communication) to calculate the relative displacement:

$$\phi_{ij}(k) \triangleq \phi_i(k) - \phi_j(k), \quad \phi_i(k) \triangleq p_i(k^+) - p_i(k). \quad (3)$$

In implementation, ranging and communication can be realized by using existing commercial UWB hardware [10], [12], while displacement measurement can be obtained from optical flow, visual-magneto-inertial odometry or simultaneous localization and mapping (SLAM) techniques, which essentially calculate the robot's motion between two consecutive camera frames.

Remark II.1. It should be noted that the estimation methods in previous works require at least three types of continuous-time measurements: distance, derivative of distance, and relative velocity as in [19]; or distance, relative velocity, and relative acceleration as in [20]. In continuous-time formulation, the requirement on acceleration and/or velocity from neighbors implies that an agent needs to know the instantaneous control input of its neighbor, which is not very practical. In comparison, our method only requires two types of discrete-time measurements that can be directly obtained from generic sensors. Also, it can be seen from (3) that in the absence of noise and disturbance, the displacement measurement at the time step k is equal to the control input at the previous step $k-1$, which is essentially different from communicating instantaneous control input in [19] and [20].

Remark II.2. To convey the main ideas more efficiently, we model the MAV as a discrete-time single integrator with bounded velocity. In the literature, MAVs are usually modeled as double integrator dynamics. However, it should be noted that we have intentionally imposed a bounded input constraint $\bar{u}_i(k) \leq \bar{U}_i$ in the dynamics (1). Thus, under an acceleration-controlled double integrator dynamics, one can use control

techniques such as optimal control or model predictive control to find an input that drives the agent from $p_i(kT)$ to $p_i(kT) + T\bar{u}_i(k)$. As $T\bar{u}_i(k)$ is bounded, a feasible solution would exist given sufficiently large bound for acceleration input.

B. Relative Localization and Tracking Objectives

Define the relative position between two agents i and j at time kT as $q_{ij}(k) \triangleq p_i(k) - p_j(k)$. Moreover, denote $\hat{q}_{ij}(k)$ as the estimate of $q_{ij}(k)$, and the relative localization error as $\tilde{q}_{ij}(k) \triangleq \hat{q}_{ij}(k) - q_{ij}(k)$. The relative localization and tracking objectives are stated as follows: given predefined relative position setpoints q_{10}^* and $q_{20}^* \in \mathbb{R}^m$, as well as a predefined trajectory $\{\sigma(k)\}_{k=0}^\infty \subset \mathbb{R}^m$, design a set of update laws for $\hat{q}_{i0}(k)$ and $\bar{u}_i(k)$ so that the following can be achieved:

$$\lim_{k \rightarrow \infty} \|\tilde{q}_{i0}(k)\| = \lim_{k \rightarrow \infty} \|\hat{q}_{i0}(k) - q_{i0}(k)\| = 0, \quad \forall i; \quad (4)$$

$$\lim_{k \rightarrow \infty} \|q_{10}(k) - q_{10}^* - \sigma(k)\| = 0; \quad (5)$$

$$\lim_{k \rightarrow \infty} \|q_{20}(k) - q_{20}^*\| = 0. \quad (6)$$

Henceforth, we refer to $\bar{q}_{10}(k) \triangleq q_{10}(k) - q_{10}^* - \sigma(k)$ and $\bar{q}_{20}(k) \triangleq q_{20}(k) - q_{20}^*$ as the *tracking error*.

In the sequel, for the aforementioned agents, as well as those in an extended system that will be introduced in Section V, we will show that all localization errors are e.c., i.e. there exist constants $\tilde{\eta}_{i0} > 0$ and $\tilde{\lambda}_{i0} \in (0, 1)$, whose values depend on the initial localization and tracking errors $\tilde{q}_{j0}(0)$ and $\bar{q}_{j0}(0)$ of all agents j , such that $\|\tilde{q}_{i0}(k)\| \leq \tilde{\eta}_{i0}\tilde{\lambda}_{i0}^k$ for all $k \in \mathbb{N}$. Based on the EC of the localization error, tracking errors can also be shown to be e.c.

Remark II.3. The use of the setpoint q_{i0}^* in (5) and (6) resembles a geometric formation control scheme; however, this resemblance only holds for the followers, but not the orbiters which are to track the leader with a predefined trajectory instead of at a setpoint.

C. Main Assumptions

We adopt the following assumptions in this paper:

Assumption II.1. There exist $N \in \mathbb{N}^+$ and $\rho \in \mathbb{R}^+$ such that $\sigma(k+N) = \sigma(k)$ and $\max_{W \in \mathcal{W}} g(W) = \rho > 0$, where

$$g(W) = \varsigma_M(W) - \sqrt{\varsigma_M^2(W) - \varsigma_m^2(W)}; \quad (7)$$

$$\mathcal{W} = \{W \triangleq [\Delta\sigma(k_1) \dots \Delta\sigma(k_m)] \in \mathbb{R}^{m \times m} : \quad (8)$$

$$0 \leq k_1 < \dots < k_m \leq N-1, \det(W) \neq 0\};$$

$$\Delta\sigma(k) = \sigma(k^+) - \sigma(k), \quad \forall k \in \mathbb{N}. \quad (9)$$

Remark II.4. Assumption II.1 is motivated by a practical interpretation of the PE condition. Specifically, to achieve PE in the displacement of the orbiters, we employ a periodic trajectory $\{\sigma(k)\}_{k=0}^\infty$ so that the set of predetermined displacements $\{\Delta\sigma(k)\}_{k=0}^\infty$ will consist of recurring bases of $\mathbb{R}^m$ for every N steps. By finding the basis W with the largest level of excitation ρ , we can choose proper parameters in the

tracking control law to preserve the PE condition, which will be shown in Theorem IV.1.

Assumption II.2. At time k , agent i can obtain the relative position setpoint q_{j0}^* and the position estimate $\hat{q}_{j0}(k-1)$ of the neighbor agent j , as well as the team velocity command $v(k) \triangleq \bar{u}_0(k)$ from the leader.

In the above, the team velocity $v(k)$ is to coordinate the movement of the MAV team. In practice this can be implemented by using the broadcast mode of UWB, or by using a predefined trajectory running in the software program of all agents [21]. Note that it also implies that at time k , all agents can calculate the displacement of the leader by multiplying the previous team velocity with the time step, i.e. $T\bar{u}_i(k-1)$.

III. INTEGRATED LOCALIZATION-NAVIGATION

In this section we shall design the estimator for the relative localization and the controller for the navigation task. Specifically, the estimator and the controller are constructed by adaptive control techniques, which depends on the following parametric model for agents i and j :

$$\phi'_{ij}(k)q_{ij}(k) = \zeta_{ij}(k), \quad (10)$$

where $\zeta_{ij}(k)$ is a scalar observation combining the distance and displacement measurements as follows:

$$\zeta_{ij}(k) \triangleq \frac{1}{2}[d_{ij}^2(k^+) - d_{ij}^2(k) - \|\phi_{ij}(k)\|^2]. \quad (11)$$

Let us also define the following normalizing functions $\mu_\nu(\cdot)$ and $\pi_U(\cdot)$ that will be used in subsequent estimation and control laws:

$$\mu_\nu(x) \triangleq \sqrt{\max\{\nu, \|x\|^2\}}/\nu, \quad \nu \in (0, 2); \quad (12)$$

$$\pi_U(y) \triangleq s_U(y)y, \quad s_U(y) \triangleq U/\max\{U, \|y\|\}, \quad U > 0. \quad (13)$$

Moreover, let us introduce in advance the main parameters that will be used in the estimation and control laws, along with their conditions as follows:

$$\begin{cases} \nu \in (0, 2), \gamma > 0; \end{cases} \quad (14a)$$

$$\begin{cases} U > 0, \beta \in (0, 1/T); \end{cases} \quad (14b)$$

$$\begin{cases} \rho^* \triangleq \rho - \sqrt{m}TU > 0. \end{cases} \quad (14c)$$

Remark III.1. Note that conditions (14a) and (14b) are required to respectively ensure the stability of the estimator and the controller. In addition, ρ is the level of excitation for the trajectory $\sigma(k)$ defined in Assumption II.1, and condition (14c) is to preserve its p.e. feature in the presence of the tracking term $\pi_U(u_i(k))$ in the subsequent control law (16a). Thanks to this PE feature, the stable estimators and controllers can achieve EC.

A. Orbiter

Based on the model (10), we can calculate the innovation $\epsilon_{10}(k)$ and use the gradient algorithm [22] to update the relative position estimate of the orbiter as follows:

$$\begin{cases} \epsilon_{10}(k) \triangleq \zeta_{10}(k) - \phi'_{10}(k)\hat{q}_{10}(k); \end{cases} \quad (15a)$$

$$\begin{cases} \hat{q}_{10}(k^+) = \hat{q}_{10}(k) + \phi_{10}(k) + \frac{\gamma\phi_{10}(k)\epsilon_{10}(k)}{\mu_\nu^2(\gamma\phi_{10}(k))}, \end{cases} \quad (15b)$$

Based on the estimate, the control input is designed as

$$\begin{cases} \bar{u}_1(k) = \pi_U(u_1(k)) + T^{-1}\Delta\sigma(k) + v(k); \\ u_1(k) = -\beta(\hat{q}_{10}(k) - q_{10}^* - \sigma(k)). \end{cases} \quad (16a) \quad (16b)$$

Remark III.2. Note that the control law (16) is executed before the estimation law (15). The control input $\bar{u}_1(k)$ is generated at time step k from the relative position estimate $\hat{q}_{10}(k)$, which is updated from the measurements $d_{10}(k)$, $d_{10}(k-1)$ and $\phi_{10}(k-1)$ that are available at time k . On the other hand, the estimate $\hat{q}_{10}(k+1)$ is based on the measurements at time step $k+1$, which are available after the control $\bar{u}_1(k)$ has been executed.

B. Follower

Notice the following parametric model for the follower:

$$\begin{aligned} \zeta_{21}(k) &= \phi'_{21}(k)q_{21}(k), \\ \zeta_{20}(k) &= \phi'_{20}(k)q_{20}(k) = [\phi_{21}(k) + \phi_{10}(k)]'q_{20}(k) \\ &= \phi'_{21}(k)[q_{21}(k) + q_{10}(k)] + \phi'_{10}(k)q_{20}(k) \\ &= \phi'_{21}(k)q_{21}(k) + \phi'_{21}(k)q_{10}(k) + \phi'_{10}(k)q_{20}(k), \\ \zeta_{20}(k) - \zeta_{21}(k) &= \phi'_{10}(k)q_{20}(k) + \phi'_{21}(k)q_{10}(k). \end{aligned}$$

Thus we can calculate the innovation $\epsilon_{20}(k)$ using available measurements and the orbiter's estimate to update the relative position estimate of the follower as follows:

$$\begin{cases} \epsilon_{20}(k) \triangleq \zeta_{20}(k) - \zeta_{21}(k) \dots \\ \quad - \phi'_{10}(k)\hat{q}_{20}(k) - \phi'_{21}(k)\hat{q}_{10}(k); \\ \hat{q}_{20}(k^+) = \hat{q}_{20}(k) + \phi_{20}(k) + \frac{\gamma\phi_{10}(k)\epsilon_{20}(k)}{\mu_\nu^2(\gamma\phi_{10}(k))}. \end{cases} \quad (17a) \quad (17b)$$

The control input of the follower is designed as follows:

$$\begin{cases} \bar{u}_2(k) = \pi_U(u_2(k)) + v(k); \\ u_2(k) = -\beta(\hat{q}_{20}(k) - q_{20}^*). \end{cases} \quad (18a) \quad (18b)$$

Remark III.3. For the orbiter, the control law $\bar{u}_1(k)$ consists of three terms: the first one is the tracking term $\pi_U(u_i(k))$, the second one is the p.e. velocity $T^{-1}\Delta\sigma(k)$, and the third one is the team velocity $v(k)$. The control law $\bar{u}_2(k)$ of the follower forgoes the p.e. velocity term. Note that we use the projection $\pi_U(\cdot)$ in the tracking term for two purposes. First, it is to ensure that the control input does not exceed the maximum speed $\bar{U}_i$, since the bound for $u_i(k)$ cannot be predetermined, which is different from the team velocity and the p.e. velocity. Clearly, if $\|v(k)\| \leq U_0$ and $T^{-1}\|\Delta\sigma(k)\| \leq U_\sigma$, the projection $\pi_U(\cdot)$ is invoked to guarantee that $\|\bar{u}_i(k)\| < U_0 + U_\sigma + U \leq \bar{U}_i$ with $U \leq \bar{U}_i - U_0 - U_\sigma$. Second, the use of $\pi_U(\cdot)$ also ensures that $u_i(k)$ does not cancel out the p.e. components of $\Delta\sigma(k)$ (see Remark II.4), as will be explained in more details in Section IV and the proof of Theorem IV.1.

Remark III.4. It can be inferred from the proposed control laws the rationale of naming agents 1 and 2 as orbiter and follower. Actually, by employing the control law (16), agent 1 is tasked with maintaining a p.e. trajectory $\sigma(k)$ as shown in Theorem IV.2. With the access to the p.e. displacement measurements from the orbiter, agent 2 can achieve EC for its

position estimates as shown in Theorem IV.1, which leads to the asymptotic tracking of the leader at the setpoint as shown in Theorem IV.2.

IV. CONVERGENCE ANALYSIS

In this section we shall study the convergence of localization error and tracking error respectively for the orbiter and the follower. To begin, notice that $q_{10}(k+1) = q_{10}(k) + \phi_{10}(k)$, $\tilde{q}_{10}(k) = \hat{q}_{10}(k) - q_{10}(k)$ and $\epsilon_{10}(k) = -\phi'_{10}(k)\tilde{q}_{10}(k)$. Substituting these to (15b) we find that the relative localization error has the following dynamics:

$$\tilde{q}_{10}(k+1) = \left[I - \frac{\gamma\phi_{10}(k)\phi'_{10}(k)}{\mu_\nu^2(\gamma\phi_{10}(k))} \right] \tilde{q}_{10}(k). \quad (19)$$

Moreover, after recalling (16b) and that $q_{10}(k+1) = q_{10}(k) + T\bar{u}_1(k)$, the tracking error $\bar{q}_{10}(k) = q_{10}(k) - q_{10}^* - \sigma(k)$ can be described by the following dynamics:

$$\bar{q}_{10}(k+1) = \bar{q}_{10}(k) + T\pi_U(u_1(k)). \quad (20)$$

For the follower, the dynamics of the localization error $\tilde{q}_{20}(k) = \hat{q}_{20}(k) - q_{20}(k)$ and the tracking error $\bar{q}_{20}(k) = q_{20}(k) - q_{20}^*$ can be similarly found respectively as follows:

$$\tilde{q}_{20}(k+1) = A(k)\tilde{q}_{20}(k) + B(k), \quad (21)$$

$$\bar{q}_{20}(k+1) = \bar{q}_{20}(k) + T\pi_U(u_2(k)), \quad (22)$$

where $A(k) = I - \mu_\nu^{-2}(\gamma\phi_{10}(k))\gamma\phi_{10}(k)\phi'_{10}(k)$ and $B(k) = -\mu_\nu^{-2}(\gamma\phi_{10}(k))\gamma\phi_{10}(k)\phi'_{20}(k)\tilde{q}_{10}(k)$.

A. Localization Error Convergence

The EC of the relative localization error is established in the following theorem, whose proof can be found in Appendix A:

Theorem IV.1. Under Assumptions II.1, II.2 and condition (14), $\phi_{10}(k)/\mu_{10}(k)$, $\mu_{10}(k) \triangleq \mu(\gamma\phi_{10}(k))$ is p.e., i.e.:

$$S(n) \triangleq \sum_{k=n}^{n+N-1} \frac{\phi_{10}(k)\phi'_{10}(k)}{\mu_{10}^2(k)} \geq \rho^* I, \quad \forall n \in \mathbb{N}^+. \quad (23)$$

As a result, the estimation errors $\tilde{q}_{10}(k)$ and $\tilde{q}_{20}(k)$ are e.c.

B. Tracking Error Convergence

Based on Theorem IV.1, we can show the EC of the tracking error the following theorem, whose proof can be found in Appendix B.

Theorem IV.2. Under Assumptions II.1, II.2 and condition (14), the control laws (16) and (18) ensure that

$$\lim_{k \rightarrow \infty} \|\bar{q}_{i0}(k)\| = 0, \quad i = 1, 2. \quad (24)$$

Moreover, the convergence rate is exponential.

V. EXTENSION

In this section, we discuss the extension of the basic structure by adding new agents with similar leader-following objectives. For each newly added agent, we would design the corresponding estimation and control laws depending on whether it is an orbiter or a follower. An illustration for such extension can be seen in Fig. 2 in the simulation.

First, consider a new agent j which is an orbiter (e.g. node 4 in Fig. 2). It can connect to either the leader or a follower i , and the following estimation and control laws can be used to track agent i , using the relative setpoint $q_{ji}^* \triangleq q_{j0}^* - q_{i0}^*$:

$$\begin{cases} \epsilon_{ji}(k) \triangleq \zeta_{ji}(k) - \phi'_{ji}(k) \hat{q}_{ji}(k); & (25a) \end{cases}$$

$$\begin{cases} \hat{q}_{ji}(k^+) = \hat{q}_{ji}(k) + \phi_{ji}(k) + \frac{\gamma \phi_{ji}(k) \epsilon_{ji}(k)}{\mu_\nu^2(\gamma \phi_{ji}(k))}; & (25b) \end{cases}$$

$$\begin{cases} \hat{q}_{j0}(k^+) = \hat{q}_{ji}(k^+) + \hat{q}_{i0}(k) + \phi_{i0}(k); & (25c) \end{cases}$$

$$\begin{cases} \bar{u}_j(k) = \pi_U(u_j(k)) + T^{-1} \Delta \sigma(k) + v(k); & (25d) \end{cases}$$

$$\begin{cases} u_j(k) = -\beta(\hat{q}_{ji}(k) - q_{ji}^* - \sigma(k)). & (25e) \end{cases}$$

Second, consider a new agent x which is a follower (e.g. agent 5 in Fig. 2). It will need to connect to an orbiter y and another follower/leader agent z . Similar to Section III-B, we can obtain the following parametric model and scalar observation for the new follower:

$$\begin{aligned} \zeta_{xz}(k) - \zeta_{xy}(k) &= \phi'_{yz}(k) q_{xz}(k) + \phi'_{xy}(k) q_{y0}(k) \dots \\ &\quad - \phi'_{xy}(k) q_{z0}(k). \end{aligned}$$

The following estimation and control laws are given to the new follower to track agent z , using the relative setpoint $q_{xz}^* \triangleq q_{x0}^* - q_{z0}^*$:

$$\begin{cases} \epsilon_{xz}(k) = \zeta_{xz}(k) - \zeta_{xy}(k) - \phi'_{yz}(k) \hat{q}_{xz}(k) \dots & (26a) \\ \quad - \phi'_{xy}(k) \hat{q}_{y0}(k) + \phi'_{xy}(k) \hat{q}_{z0}(k); & (26b) \end{cases}$$

$$\begin{cases} \hat{q}_{xz}(k^+) = \hat{q}_{xz}(k) + \phi_{xz}(k) + \frac{\gamma \phi_{xz}(k) \epsilon_{xz}(k)}{\mu_\nu^2(\gamma \phi_{xz}(k))}; & (26c) \end{cases}$$

$$\begin{cases} \hat{q}_{x0}(k^+) = \hat{q}_{xz}(k^+) + \hat{q}_{z0}(k) + \phi_{z0}(k); & (26d) \end{cases}$$

$$\begin{cases} \bar{u}_x(k) = \pi_U(u_x(k)) + v(k); & (26e) \end{cases}$$

$$\begin{cases} u_x(k) = -\beta(\hat{q}_{xz}(k) - q_{xz}^*). & (26f) \end{cases}$$

Remark V.1. Note that extra estimation laws (25c) and (26d) have been introduced to estimate the agent's relative position to the leader when it is not connected to the leader. The reason why these laws are necessary can be observed from the perspective of agent 5 in Fig. 2, which needs to localize itself relative to agent 3 with the aid of agent 4. However, agent 4 does not have any information related to agent 3, hence we need to set a common reference point for all agents, which is the leader.

Hence, we can keep adding new agent to network using either set of estimation and control laws, depending on the role assigned to it. Similar to the analysis of the basic structure in Section IV, we can define the localization and tracking errors of the newly added agents as $\tilde{q}_{ji}(k) \triangleq \hat{q}_{ji}(k) - q_{ji}(k)$, $\bar{q}_{ji}(k) \triangleq \bar{q}_{ji}(k) - q_{ij}^* - \sigma(k)$, $\tilde{q}_{xz}(k) \triangleq \hat{q}_{xz}(k) - q_{xz}(k)$, $\bar{q}_{xz}(k) \triangleq \bar{q}_{xz}(k) - q_{xz}^*$. The following theorem establishes the EC of the localization and estimation error for each added agent under

this extension scheme, and the proof for it can be found in Appendix B.

Theorem V.1. Under Assumptions II.1, II.2 and condition (14), the estimation and control laws (25) and (26) ensure that all localization and tracking errors $\tilde{q}_{ji}(k)$, $\tilde{q}_{j0}(k)$, $\bar{q}_{ji}(k)$, $\tilde{q}_{xz}(k)$, $\tilde{q}_{x0}(k)$, $\bar{q}_{xz}(k)$ are e.c.

Remark V.2. In practice one would like to minimize the number of orbiters, as they need to consume more energy in tracking a p.e. trajectory and possibly communicating with multiple followers. However, it should be noted that the orbiters and the followers are not necessarily fixed and they can swap the roles by negotiation. A more detailed study of these issues will be deferred to future work.

VI. SIMULATION

In this section we will verify the theoretical findings by numerical simulation for a network of six agents as shown in Fig. 2. Here we focus on a simple 2D scenario. However, the algorithm can also be applied in the 3D case by choosing a p.e. trajectory $\sigma(k)$ as in [12].

Fig. 2: A network of 6 agents with one leader (red), two orbiters (green), three followers (blue) and their trajectories. The direction of the arrows is interpreted similarly as in Fig. 1. The square markers indicate the initial positions, and the round ones are for the final positions in simulation.

The values of the parameters and the relative setpoints can be seen in Fig. 2. After computing and comparing all matrices $W \in \mathcal{W}$, we can find that $\rho = \max_{W \in \mathcal{W}}(g(W)) \approx 0.083916$ and $\sqrt{2}TU \approx 0.033941$, thus condition (14c) is satisfied. Moreover, as $T\beta \approx 0.25 < 1$, by Theorems IV.1 and IV.2, both localization and tracking errors converge to zero exponentially fast, which is indeed reflected in Fig. 3.

The trajectories of all agents are shown in Fig. 2. As we can see, all the followers are initially far away from the leader, but as time progresses, they are able to track the leader and finally approach the desired relative setpoints q_{i0}^* . The orbiters can also track the leader while executing the p.e. motion, as shown in green.

Fig. 3: Localization error (top) and tracking error (bottom) of the agents converge to 0 exponentially fast. The subscript ij takes value from 10, 20, 32, 42, 53. We use the green color to distinguish the localization and tracking errors of the two orbiters from those of the followers.

VII. EXPERIMENT

In this section we implement our algorithm in real world condition to further validate the theoretical results. Video recording of our flight tests can be viewed at <https://youtu.be/df25A4jRDO8>, or downloaded in the attached multimedia material package.

Three MAVs are employed to form a basic structure in this case. A set of high accuracy commercial UWB ranging sensors¹ are employed to provide both distance measurements and the communication network. To obtain the displacement measurements, data from inexpensive commercial optical flow sensors [23] are fused with IMU and magnetometer data by using the PX4 flight stack. The displacement information is also sent to the neighbor agents along with the UWB messages. Note that before communication, the optical flow measurements are to be calibrated to the corresponding local inertial frame (for convenience we initialize the MAVs with the same orientation for all the inertial frames, otherwise we can use the absolute orientation in reference to the earth's magnetic field). The algorithm is implemented on ROS (Robot Operating System). All the parameters in the estimation and control laws, as well as the PE trajectory, are the same as in the simulation. The relative position setpoints are chosen as $q_{10}^* = [1, -2.5]'$ and $q_{20}^* = [0.25, 2]'$, which are also used in the deployment and the initialization of the relative position estimates. This accurate initialization is to avoid the unpredictable behavior by the MAVs, e.g. flying outside of the testing area or colliding with each other.

The orbiter and the follower are controlled to take off manually and then switch to the autonomous mode when the velocity control command $\bar{u}_i(k)$ is executed to maintain the relative position with the leader. For safety, the leader is manually controlled, and its velocity is broadcast to all agents. An overhead VICON² motion capture system is used to record the trajectories of the agents for ground truth. These data are only recorded for analysis and not used in the flight test.

A total of 5 flight tests have been conducted. To quantify the performance of the relative localization scheme, we compare

the onboard estimates calculated in each flight test with ground truth from VICON, then calculate the root mean square (RMS) and standard deviation (SD) of the estimation error in all flight tests (see T1 columns in Table I).

At the first look, we found that the localization error is typically around 1 m in either direction. After examining the data, we suppose that this error could be attributed to the measurement errors. To verify this reasoning, we collected the data from the VICON motion capture system (millimeter accuracy) and carried out an “offboard” rerun of the estimation algorithm with *VICON-derived displacement* and the original UWB distance measurements. The RMS and SD of the estimation error are reported under the T2 columns in Table I. Indeed, it can be seen that almost all RMS and SD values on the offboard estimation error show significant improvement (except a slight increase in the RMS and SD values of $\tilde{q}_{20}(k)$ in dataset 3, which is due to the temporary corruption of VICON data when the UAV flew over the boundary of the testing area). Moreover, as shown in Fig. 4, with a better displacement measurements, the offboard relative position estimates are quite close to the true value.

Furthermore, we conducted a similar offboard rerun with the *VICON-derived distance* and the original displacement measurements and reported the RMS and SD values of the estimation errors under T3 columns in Table I. It can be seen that no significant improvement is shown in this offboard estimation process. This can be attributed to the fact that the UWB ranging sensor used in the experiment is already quite accurate, and hence not much improvement can be gained from better ranging accuracy.

Therefore, we infer that the quality of the displacement measurements plays a major role in deciding the accuracy of the relative localization scheme. For improvement in the future, better visual odometry methods should be used instead of the current optical flow sensor, which is prone to error and the lack of features on the ground.

We also conducted some flight tests in the outdoor condition, which can be viewed at the video recording in the multimedia package accompanying this paper. Since no ground truth was available, and the results have been conclusively analysed in the indoor test, we forgo the analysis on these tests. Nevertheless, in all flight tests, we find that the orbiter and follower are able to track the leader with the orbiter performing the predefined trajectory around the setpoint, demonstrating the efficacy of the relative localization and leader-following control scheme in GPS-less environments.

VIII. CONCLUSION

In this paper we have proposed a cooperative scheme to achieve simultaneous relative localization and tracking control for a leader-following task. Our method only requires distance and displacement measurements from onboard sensors and basic communication capability between the MAVs. Rigorous analysis has been conducted to find out the parametric conditions for the convergence of the localization and tracking errors. The theoretical findings were verified by simulation results, and successful implementation has demonstrated the efficacy of the proposed scheme.

¹<http://www.timedomain.com/products/pulson-440/>

²<https://www.vicon.com>

TABLE I: RMS and SD values of the relative localization error from onboard and offboard estimation processes.

#	rms($\hat{q}_{10,x}$) [m]			rms($\hat{q}_{10,y}$) [m]			sd($\hat{q}_{10,x}$) [m]			sd($\hat{q}_{10,y}$) [m]			rms($\hat{q}_{20,x}$) [m]			rms($\hat{q}_{20,y}$) [m]			sd($\hat{q}_{20,x}$) [m]			sd($\hat{q}_{20,y}$) [m]		
	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3	T1	T2	T3
1	.259	.123	.212	.816	.535	.592	.255	.123	.201	.386	.175	.280	.183	.106	.189	.800	.302	.689	.178	.045	.181	.428	.194	.351
2	.325	.128	.347	.671	.353	.622	.325	.097	.347	.393	.121	.397	.488	.255	1.047	.388	.198	.527	.475	.148	.990	.275	.152	.399
3	.259	.123	.553	.816	.535	.656	.255	.123	.457	.386	.175	.365	.183	.106	.312	.800	.302	.338	.178	.045	.275	.428	.194	.202
4	.803	.115	.590	.940	.523	.944	.676	.112	.564	.380	.208	.492	.422	.400	.636	1.102	.173	1.233	.403	.212	.600	.712	.120	.967
5	.803	.154	.770	.227	.280	.250	.333	.094	.323	.223	.131	.226	.238	.221	.239	.513	.111	.386	.221	.130	.207	.324	.111	.310

(a) Relative position estimates and the ground truth of the orbiter.

(b) Relative position estimates and the ground truth of the follower.

Fig. 4: Time evolution of the onboard and offboard relative position estimates of the orbiter and the follower compared with ground truth.

APPENDIX

A. Proof of Theorem IV.1

From (3), (16) and recalling $v(k) = \bar{u}_0(k)$, we get that

$$\phi_{10}(k) = \Delta\sigma(k) + T\bar{v}_1(k), \quad \bar{v}_1(k) = \pi_U(u_1(k)). \quad (27)$$

Clearly, (27) implies that $\|\phi_{10}(k)\| \leq TU + \delta_\sigma$, where $\delta_\sigma = \max_{k \in \mathbb{N}^+} \{\|\Delta\sigma(k)\|\} = \max_{0 \leq k \leq N-1} \{\|\Delta\sigma(k)\|\}$ holds as a result of the periodicity in Assumption II.1. Hence:

$$1/\mu_{10}(k) \geq \mu^* \triangleq 1/\sqrt{\max\{\nu, \gamma(TU + \delta_\sigma)^2\}}/\nu > 0.$$

Consequently, $S_{10}(n) \geq \sum_{k=n}^{n+N-1} (\mu^*)^2 \phi_{10}(k) \phi'_{10}(k) \triangleq (\mu^*)^2 \Phi(n : n+N-1)$; moreover, by the periodicity we only need to check $\Phi(0 : N-1) \triangleq \Phi$, and the PE condition (23) holds if there exists $\rho_1 > 0$ such that $\Phi \geq \rho_1 I > 0$. Equivalently, we need to show that there exists $\rho_1 > 0$ such that for any unit vector $x \in \mathbb{R}^m$, it holds that $x' \Phi x \geq \rho_1$.

Indeed, for any $W = [\Delta\sigma(k_1) \Delta\sigma(k_2) \dots \Delta\sigma(k_m)] \in \mathcal{W}$, we can find that

$$\begin{aligned} x' \Phi x &\geq x' \sum_{j=1}^m \left[\Delta\sigma(k_j) \Delta\sigma'(k_j) + T \Delta\sigma(k_j) \bar{v}'_1(k_j) \right. \\ &\quad \left. + \bar{v}_1(k_j) \Delta\sigma'(k_j) + T^2 \bar{v}_1(k_j) \bar{v}'_1(k_j) \right] x \\ &= \|W'x\|^2 + 2Tx'VWx + T^2 \|V'x\|^2 \\ &\geq \varsigma_m^2 - 2T\varsigma_M \|V'x\| + T^2 \|V'x\|^2, \end{aligned} \quad (28)$$

where $V = [\bar{v}_1(k_1), \bar{v}_1(k_2) \dots \bar{v}_1(k_m)]$, $\varsigma_m = \varsigma_m(W)$, and $\varsigma_M = \varsigma_M(W)$. Therefore, it suffices to let $\varsigma_m^2 - 2T\varsigma_M \|V'x\| + T^2 \|V'x\|^2 > \rho_1 > 0$ for all unit vectors $x \in \mathbb{R}^m$, which can be satisfied by $T \|V'x\| \leq \varsigma_M - \sqrt{\varsigma_M^2 - (1 - \rho_2)\varsigma_m^2}$ with $\rho_1 = \rho_2\varsigma_m^2$ and $\rho_2 \in (0, 1)$. Moreover, note that $\|V'x\| \leq \|V\| \leq \sqrt{m}U$ and the arbitrariness of $\rho_2 \in (0, 1)$, we only need that

$$T\sqrt{m}U < \varsigma_M - \sqrt{\varsigma_M^2 - (1 - \rho_2)\varsigma_m^2}.$$

Finally, as W is arbitrary, we can select W^* to maximize the right hand side of the above inequality as stated in (14c).

2) With the PE condition (23), we can apply Lemma 4.1.1 of [22] to obtain the EC of $\hat{q}_{10}(k)$ under the dynamics (19). Furthermore, considering the dynamics (21) of $\hat{q}_{20}(k)$, as the term $B(k)$ is e.c. due to the EC of $\hat{q}_{10}(k)$, we conclude that $\hat{q}_{20}(k)$ is also e.c.

B. Proof of Theorem IV.2

By (20) and (22) we can obtain

$$\bar{q}_{i0}(k^+) = [1 - T\beta s_i(k)] \bar{q}_{i0}(k) - T\beta s_i(k) \tilde{q}_{i0}(k), \quad (29)$$

where $s_i(k)$ and $u_i(k)$ are given by

$$u_i(k) = -\beta(\bar{q}_{i0}(k) + \tilde{q}_{i0}(k)), \quad s_i(k) = s_U(u_i(k)). \quad (30)$$

We will show the existence of $\bar{K}(\|\bar{q}_{i0}(0)\|, \|\tilde{q}_{i0}(0)\|)$ such that the dynamics (29) is simplified to the following:

$$\bar{q}_{i0}(k^+) = (1 - T\beta) \bar{q}_{i0}(k) - T\beta \tilde{q}_{i0}(k), \quad k > \bar{K}. \quad (31)$$

Let us denote $D_i(k) = \|\bar{q}_{i0}(k)\|^2$ and consider its rate of change $\Delta D_i(k) = D_i(k+1) - D_i(k)$. Substituting (29) to $\Delta D_i(k)$, one has (henceforth we may omit the time argument k and the subscripts where it is inferable from the context):

$$\begin{aligned} \Delta D &= [\bar{q} - T\beta s(\bar{q} + \tilde{q})]' [\bar{q} - T\beta s(\bar{q} + \tilde{q})] - \bar{q}' \bar{q} \\ &\leq T\beta s[-a \|\bar{q}\|^2 + \varepsilon(k)], \end{aligned} \quad (32)$$

where $a \triangleq 2 - T\beta$ and $\varepsilon(k) \triangleq T\beta s \|\bar{q}\|^2 + 2(1 + T\beta s) \|\bar{q}\| \|\tilde{q}\|$. By noticing that $\|\bar{q}\|$ increases at most linearly fast while

$\|\tilde{q}\|$ is e.c., it can be seen that ε is also e.c. Thus, there exist some positive constants $\eta > 0$ and $\lambda \in (0, 1)$ such that $\max\{\|\tilde{q}(k)\|, \varepsilon(k)\} \leq \eta\lambda^k$. Hence we get that $\max\{\|\tilde{q}(k)\|, \varepsilon(k)\} \leq \delta \triangleq \min\{\frac{U}{4\beta}, \frac{aU^2}{(4\beta)^2}\}$ for all $k \geq K^* \triangleq \lceil \ln(\delta/\eta) / \ln(\lambda) \rceil$.

We now seek to show that there exists $\bar{K} \geq K^*$ such that $\|\tilde{q}(k)\| \leq \frac{U}{2\beta}$. Indeed, for all $k > K^*$ if $\|\tilde{q}(k)\| > \frac{U}{2\beta}$, from (32) we have $\Delta D < T\beta s \left(-a\frac{U^2}{(2\beta)^2} + \varepsilon(k)\right) \leq \frac{-3T\beta s a U^2}{(4\beta)^2} < 0$ and $\|\tilde{q}(k+1)\| < \|\tilde{q}(k)\|$, which follows that $\|\tilde{q}(k)\| < \bar{Q} \triangleq \max\left\{\frac{U}{2\beta}, \|\tilde{q}(0)\| + K^*TU\right\}$ for $k \geq K^*$. Thus $s > s^* \triangleq U/\max\{U, \beta(\bar{Q} + \delta)\}$ which leads to $\Delta D < \Delta D^* \triangleq \frac{-3T\beta s^* a U^2}{(4\beta)^2} < 0$. Hence, there exists $\bar{K} < K^* + \left\lceil \left[\bar{Q}^2 - \frac{U^2}{(2\beta)^2}\right] / |\Delta D^*| \right\rceil$ such that $\|\tilde{q}(\bar{K})\| \leq \frac{U}{2\beta}$. As a result, we have $\beta \|\tilde{q}(\bar{K}) + \tilde{q}(\bar{K})\| \leq \beta \left(\frac{U}{2\beta} + \frac{U}{4\beta}\right) = \frac{3U}{4} < U$, which leads to $s(\bar{K}) = 1$ by (30). Substituting this and $\|\tilde{q}(\bar{K})\| \leq \frac{U}{2\beta}$ and $\|\tilde{q}(\bar{K})\| \leq \frac{U}{2\beta}$ in (29), we obtain

$$\begin{aligned} \|\tilde{q}(\bar{K}+1)\| &\leq (1-T\beta)U/(2\beta) + T\beta U/(4\beta) \\ &= U/(2\beta) - TU/4 \leq U/(2\beta). \end{aligned}$$

By induction we conclude that $\|\tilde{q}(k)\| \leq \frac{U}{2\beta}$ and $s(k) = 1$ for all $k \geq \bar{K}$. Then the dynamics (29) will become (31). Since $1-T\beta < 1$ we can see that $\|\tilde{q}(k)\| \leq \bar{Q}\lambda^{k-\bar{K}} = \bar{Q}\lambda^{-\bar{K}}\lambda^k \triangleq \bar{\eta}\lambda^k$ for all $k \geq 0$.

We only consider the case when a new orbiter is added, and the other case of a follower can be shown similarly. Note that the dynamics of localization error $\tilde{q}_{ji}(k)$ for this orbiter will be similar to (19).

Moreover, $\phi_{ji}(k) = T\pi_U(u_j(k)) + \Delta\sigma(k) - T\pi_U(u_i(k))$, where $\pi_U(u_i(k)) = \pi_U(-\beta\tilde{q}_i(k) - \beta\tilde{q}_i(k))$ converges to 0 exponentially fast since both localization and tracking errors of agent i are e.c.

Thus, there exists K_{ji} such that $\sqrt{m}T(U + \pi_U(u_i(k))) \leq \rho$ for all $k \geq K_{ij}$ and $\phi_{ji}(k)$ is p.e., and $\tilde{q}_{ji}(k)$ becomes e.c.

Note that from (25c) we have $\tilde{q}_{j0}(k^+) = \tilde{q}_{ji}(k^+) + \tilde{q}_{i0}(k)$. Since $\tilde{q}_{ji}(k)$ and $\tilde{q}_{i0}(k)$ are e.c., $\tilde{q}_{j0}(k)$ is also e.c., which induces the e.c. localization error of other agents added later.

On the other hand, from (25d) and (25e) we obtain the following tracking error dynamics:

$$\tilde{q}_{ji}(k^+) = [1 - T\beta s_j(k)] \tilde{q}_{ji}(k) - T\beta s_j(k) \tilde{q}_{ji}(k) + T\tilde{v}_i(k),$$

where $\tilde{v}_i(k) = -\pi_U(u_i(k))$ and $s_j(k) = s_U(u_j(k))$. It can be seen that $\tilde{v}_i(k)$ is e.c., and so is $\tilde{q}_{ji}(k)$, thus the dynamics of $\tilde{q}_{ji}(k^+)$ will evolve similarly to (29). Therefore we can repeat the same procedure in the proof of Theorem IV.2 to establish the EC of $\tilde{q}_{ji}(k)$.

REFERENCES

- [1] A. Kushleyev, D. Mellinger, C. Powers, and V. Kumar, "Towards a swarm of agile micro quadrotors," *Autonomous Robots*, vol. 35, no. 4, pp. 287–300, 2013.
- [2] Y. Mulgaonkar, G. Cross, and V. Kumar, "Design of small, safe and robust quadrotor swarms," in *2015 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2015, pp. 2208–2215.
- [3] K. Guo, Z. Qiu, C. Miao, A. H. Zaini, C.-L. Chen, W. Meng, and L. Xie, "Ultra-wideband-based localization for quadcopter navigation," *Unmanned Systems*, vol. 4, no. 01, pp. 23–34, 2016.
- [4] T. M. Nguyen, A. H. Zaini, K. Guo, and L. Xie, "An ultra-wideband-based multi-uav localization system in gps-denied environments," in *2016 International Micro Air Vehicles Conference*, 2016.
- [5] T. Stirling, J. Roberts, J.-C. Zufferey, and D. Floreano, "Indoor navigation with a swarm of flying robots," in *2012 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2012, pp. 4641–4647.
- [6] J. F. Roberts, T. Stirling, J.-C. Zufferey, and D. Floreano, "3-d relative positioning sensor for indoor flying robots," *Autonomous Robots*, vol. 33, no. 1-2, pp. 5–20, 2012.
- [7] T. Ngeli, C. Conte, A. Domahidi, M. Morari, and O. Hilliges, "Environment-independent formation flight for micro aerial vehicles," in *Intelligent Robots and Systems (IROS 2014), 2014 IEEE/RSJ International Conference on*. IEEE, 2014, pp. 1141–1146.
- [8] S. A. Roelofsen, D. Gillet, and A. Martinoli, "Reciprocal collision avoidance for quadrotors using on-board visual detection," in *International Conference on Intelligent Robots and Systems*, no. EPFL-CONF-212951, 2015.
- [9] B. Hepp, T. Ngeli, and O. Hilliges, "Omni-directional person tracking on a flying robot using occlusion-robust ultra-wideband signals," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2016, pp. 189–194.
- [10] T.-M. Nguyen, A. H. Zaini, C. Wang, K. Guo, and L. Xie, "Robust target-relative localization with ultra-wideband ranging and communication," in *2018 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2018, pp. 2312–2319.
- [11] T.-M. Nguyen, Z. Qiu, M. Cao, T. H. Nguyen, and L. Xie, "An integrated localization-navigation scheme for distance-based docking of uavs," in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2018, pp. 5245–5250.
- [12] —, "Single landmark distance-based navigation," *IEEE Transactions on Control Systems Technology*, to appear, 2019.
- [13] M. Coppola, K. N. McGuire, K. Y. Scheper, and G. C. de Croon, "On-board communication-based relative localization for collision avoidance in micro air vehicle teams," *Autonomous Robots*, pp. 1–19, 2018.
- [14] S. van der Helm, M. Coppola, K. N. McGuire, and G. C. de Croon, "On-board range-based relative localization for micro air vehicles in indoor leader-follower flight," *Autonomous Robots*, pp. 1–27, 2019.
- [15] J. Aspnes, T. Eren, D. K. Goldenberg, A. S. Morse, W. Whiteley, Y. R. Yang, B. D. Anderson, and P. N. Belhumeur, "A theory of network localization," *IEEE Transactions on Mobile Computing*, vol. 5, no. 12, pp. 1663–1678, 2006.
- [16] F. Arrichiello, G. Antonelli, A. Aguiar, and A. Pascoal, "An observability metric for underwater vehicle localization using range measurements," *Sensors*, vol. 13, no. 12, pp. 16 191–16 215, 2013.
- [17] J. Chen, D. Raye, W. Khawaja, P. Sinha, and I. Guvenc, "Impact of 3d uwb antenna radiation pattern on air-to-ground drone connectivity," in *2018 IEEE 88th Vehicular Technology Conference (VTC-Fall)*, Aug 2018, pp. 1–5.
- [18] T.-M. Nguyen, T. H. Nguyen, M. Cao, Z. Qiu, and L. Xie, "Integrated uwb-vision approach for autonomous docking of uavs in gps-denied environments," in *2019 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2019.
- [19] G. Chai, C. Lin, Z. Lin, and M. Fu, "Single landmark based collaborative multi-agent localization with time-varying range measurements and information sharing," *Systems & Control Letters*, vol. 87, pp. 56–63, 2016.
- [20] I. Sarras, J. Marzat, S. Bertrand, and H. Piet-Lahanier, "Collaborative multiple micro air vehicles localization and target tracking in gps-denied environment from range-velocity measurements," *International Journal of Micro Air Vehicles*, vol. 10, no. 2, pp. 225–239, 2018.
- [21] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: Algorithms and theory," *IEEE Transactions on automatic control*, vol. 51, no. 3, pp. 401–420, 2006.
- [22] P. Ioannou and B. Fidan, *Adaptive control tutorial*. SIAM, 2006.
- [23] D. Honninger, L. Meier, P. Tanskanen, and M. Pollefeys, "An open source and open hardware embedded metric optical flow cmos camera for indoor and outdoor applications," in *2013 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2013, pp. 1736–1741.