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Design and Optimization of A 3-DOF Spatial Motion Compliant Parallel Mechanism with Fully-Decoupled Motion Characteristics

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Abstract

This paper presents a novel three degrees-of-freedom (DOF) compliant parallel mechanism (CPM) with a fully-decoupled spatial motion ($\theta_X - \theta_Y - Z$) and optimized mechanical properties. To design the CPM using the beam-based structural optimization method, several novel criteria for synthesizing 3-legged CPMs with fully-decoupled motions are derived. The obtained results suggest that the synthesized CPM delivers a diagonal compliance matrix, a large workspace of $10^\circ \times 10^\circ \times 7 \text{ mm}$, fast dynamic response of $\sim 100 \text{ Hz}$ and good stiffness performance whereby the translational and rotational stiffness ratios are ~ 3600 and ~ 570 respectively. A prototype of the synthesized CPM is fabricated using one of the three-dimensional (3D) printing technologies, electron beam melting (EBM). Experimental results have shown that the 3D printed CPM can produce the full workspace with deterministic mechanical properties whereby the highest deviations between the theoretical and experimental results are 11.2% and 1% for stiffness and dynamic behaviors respectively. Importantly, the decoupled-motion characteristic is also verified via an energy approach, i.e., the energies of the undesired parasitic motions are minor ($< 1\%$) as compared to the energy of the desired motion. In addition, several comparisons are conducted to clarify the advantages of the syn-

thesized CPM to the existing designs. All these investigations suggest that the proposed CPM can be used in precision positioning systems due to the good stiffness characteristics, large workspace, fast dynamic response and decoupled output motions.

1 Introduction

Compliant mechanism, which delivers frictionless and highly repeatable motion through the elastic deformation of flexures, has been an essential component within the precision motion systems. Together with other benefits inherited by the parallel kinematic configuration such as closed-loop structure architecture, high non-actuating stiffness, and less sensitive to external disturbances etc., the compliant parallel mechanism (CPM) is widely preferred over its serial counterpart [1]. For the past two decades, CPM has become a potential solution for various applications such as nanopositioning actuators [2, 3], precision manipulators [4–15] and MEMS devices [16–18]. In particular, numerous manipulators with different degrees-of-freedom (DOF), e.g., 1-DOF [4], 2-DOF linear-motion [5–7], 3-DOF planar-motion [8–10], 3-DOF spatial-motion [11–13] and 6-DOF [14, 15], have been developed for a variety of industrial applications such as micro/nano fabrication/assembly systems, manipulation of cells, micro surgery and optical alignment devices,

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etc. To meet the new requirements of manufacturing processes for next generation products, devices that are developed based on CPMs need to deliver larger working range, higher non-actuating stiffness, better dynamic response as well as more precise output motions [1,19]. Wang and Zhang have obtained optimal parameters that determine the performances of 3-DOF planar-motion ($X - Y - \theta_Z$) CPMs in terms of stiffness/dynamic properties, accuracy and decoupling of output motions [20,21]. For CPMs with out-of-plane motions ($\theta_X - \theta_Y - Z$), only the mechanical characteristics have been optimized [22–24] while other behaviors, especially the motion decoupling of output motions, have not been taken into account.

Past literatures have shown that several 3-DOF CPMs, particularly those with spatial motion ($\theta_X - \theta_Y - Z$), had demonstrated large workspace as compared to most multi-DOF CPMs that achieved limited workspace of few hundred micrometers in the translation axes [6–10] and less than a degree in the rotation axes [8–11]. Examples include the 3-DOF spatial-motion CPM synthesized by traditional task-oriented method [25], which achieved a workspace of $5^\circ \times 5^\circ \times 5 \text{ mm}$, and the 3-DOF spatial-motion CPM synthesized by the structural optimization method [22], which achieved a workspace of $8^\circ \times 8^\circ \times 5.5 \text{ mm}$. In addition, the CPM presented in ref. [22] also achieved a high natural frequency of 84.4 Hz . More recently, 3D printing/additive manufacturing technology, i.e., electron beam melting (EBM), has shown the capability in fabricating industrial components with complex structures [26,27], and this technique was even applied to fabricate the same CPM presented in ref. [22] monolithically without any assembly process. An EBM-printed CPM is able to eliminate assembly error and the dynamics performance, as a result, can be enhanced [23]. Based on the merits offered by the structural optimization method and EBM technique, a new design of 3-DOF out-of-plane-motion CPM with large workspace (up to 7 mm for the translation and 10° for the rotations) and fast dynamic behavior via the first resonance frequency of 100 Hz has been introduced in ref. [24].

Other than large working range, high stiffness ratio, and good dynamics performance, a CPM with fully-decoupled motion is another key factor to achieve precise output motions. Here, the decoupled-motion characteristic refers to the output motion decoupling of the CPM [6,12], i.e., one actuating force in a specific direction only produces one output displacement in the same direction without affecting displacements in other directions. Therefore, a CPM with fully-decoupled motion will not generate any undesired parasitic motion along/about the non-actuating axes when any of the desired actuating axis moves. In theory, the fully-decoupled motion characteristic can be obtained if the 6×6 stiffness/compliance matrix of entire CPM only contains the diagonal components and all non-diagonal components are zeros; this matrix form is also commonly known as a diagonal matrix. However, state-of-the-art multi-DOF CPMs demonstrate that only the 2-DOF ($X - Y$) designs can deliver decoupled motions due to the kinematics merit offered by the four symmetrical legs [5–7] while the motion char-

acteristic of other existing CPMs based on 3-legged configuration was neglected [8,9,11,14,15]. This is mainly because most of these CPMs only focus on achieving the desired DOF while other performance factors are seldom taken into account. Even those present in recent works could not deliver fully-decoupled motions [22,23,25].

This paper presented the design criteria for synthesizing a novel 3-DOF spatial-motion CPM with optimal mechanical properties and fully-decoupled motions. Note that the design of CPM was presented in ref. [24] by the authors and used directly here as the optimized CPM. The focus in this work is on the derivation of the design criteria that needed to be observed during the design stage. A 3D printed prototype of the synthesized CPM was built by EBM and some experiments were conducted to evaluate its actual performance. Importantly, the correctness of the proposed design criteria was also experimentally demonstrated.

The remaining of this paper is organized as follows: Section 2 presents the derivation of the design criteria for designing a 3-legged CPM with fully-decoupled motions. Section 3 describes the synthesis of the CPM based on the design criteria together with the desired stiffness and dynamics characteristics. Section 4 presents the experimental evaluation of the 3D printed prototype and Section 5 provides detailed discussions of the experimental results.

2 Design criteria for 3-legged CPM to achieve fully-decoupled motions

In this work, a novel 3-DOF spatial-motion ($\theta_X - \theta_Y - Z$) CPM was synthesized using the beam-based structural optimization method [22] and a 3-legged parallel kinematic configuration was used as the base configuration. The CPM was targeted to deliver fully-decoupled motions. Hence, the design criteria for designing such CPMs were derived mathematically and subsequently used to synthesize the CPM. This section presents the derivation of these design criteria.

Figure 1a illustrates the general structure of a 3-legged CPM that is synthesized using the beam-based method. Three legs of the CPM are symmetrical and 120° apart about the Z -axis, which is located at the center of the end effector. Here, each leg is constructed by a pair of mirrored curved-and-twisted (C-T) beams with one fixed end and one free end that is rigidly connected to the end effector. As the beam-based method is developed for synthesizing multi-DOF CPMs, the C-T beam is proposed as a basic design module since it is able to generate up to 6 DOF at its free end. Referring to Fig. 1b, the C-T beam is a thin flexure created based on a cubic Bezier center curve and two twist angles, α_A and α_B , at both ends. Elastic deformation of the C-T beams in each leg generates the desired motions for the end effector of the CPM.

As the output motions of the CPMs are generated by the elastic deformation of those C-T beams, the analysis of their structure is the main focus. In theory, fully-decoupled motion characteristic can be obtained only through one condition, i.e., the 6×6 stiffness/compliance matrix of the entire CPM must be in a diagonal matrix form where it only con-

Fig. 1. General model of 3-legged CPM synthesized by beam-based method (a) overall structure of the CPM, (b) geometry of a C-T beam and (c) design variables of the dynamic optimization process.

tains the diagonal components and all non-diagonal components are zeros. With K^l representing the stiffness of each leg along the Y -axis as shown in Fig. 2a, the stiffness of entire CPM, K^m , is expressed as

$$K^m = \sum_{i=1}^3 J_i^l R_i^l K^l (R_i^l)^{-1} (J_i^l)^T \quad (1)$$

In Eq. (1), R_i^l and J_i^l represent the rotation and translation matrices of the i^{th} leg respectively, and are written as

$$R_i^l = \begin{bmatrix} R_z(\theta_i) & 0 \\ 0 & R_z(\theta_i) \end{bmatrix}, \quad (2)$$

$$\text{where } R_z(\theta_i) = \begin{bmatrix} \cos \theta_i & -\sin \theta_i & 0 \\ \sin \theta_i & \cos \theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$J_i^l = \begin{bmatrix} I & 0 \\ T_i^l & I \end{bmatrix}, \text{ where } T_i^l = \begin{bmatrix} 0 & D_{z_i} & -D_{y_i} \\ -D_{z_i} & 0 & D_{x_i} \\ D_{y_i} & -D_{x_i} & 0 \end{bmatrix} \quad (3)$$

Here, θ_i represent the orientation of the i^{th} leg ($i = 1, 2, 3$), which is 0° , 120° and 240° respectively due to the symmetrical structure of the CPM. D_{x_i} , D_{y_i} and D_{z_i} components from vector $\mathbf{D}$ in Fig. 2a represent the distances along the X , Y and Z axes respectively from the local frame $X'Y'Z'$; attached at free end of the leg, to the global frame XYZ ; attached at the

center of the end effector. Note that the axes' directions of XYZ and $X'Y'Z'$ are similar.

In the most general case, K^l can be written as

$$K^l = \begin{bmatrix} k_{11}^l & k_{21}^l & k_{31}^l & k_{41}^l & k_{51}^l & k_{61}^l \\ k_{22}^l & k_{32}^l & k_{42}^l & k_{52}^l & k_{62}^l & \\ k_{33}^l & k_{43}^l & k_{53}^l & k_{63}^l & & \\ k_{44}^l & k_{54}^l & k_{64}^l & & & \\ k_{55}^l & k_{65}^l & & & & \\ k_{66}^l & & & & & \end{bmatrix} \quad \text{SYM} \quad (4)$$

By substituting Eqs. (2), (3) and (4) into Eq. (1), the stiffness property of any 3-legged CPM is expressed by the following stiffness matrix

$$K^m = \begin{bmatrix} k_{11}^m & 0 & 0 & k_{41}^m & k_{51}^m & 0 \\ 0 & k_{22}^m & 0 & k_{42}^m & k_{52}^m & 0 \\ 0 & 0 & k_{33}^m & 0 & 0 & k_{63}^m \\ k_{44}^m & k_{42}^m & 0 & k_{44}^m & 0 & 0 \\ k_{51}^m & k_{52}^m & 0 & 0 & k_{55}^m & 0 \\ 0 & 0 & k_{63}^m & 0 & 0 & k_{66}^m \end{bmatrix} \quad \text{SYM} \quad (5)$$

The first row of the matrix shown in Eq. (5) shows that under an external force along the X axis, the CPM can have two parasitic rotations about the X and Y axes beside the desired translation along the X axis. Similar interpretation can be drawn for the translation along the Y axis corresponding to the second row of the matrix. It is seen also that for the rotations about the X and Y axes (the fourth and fifth rows) under the effect of external moments, the CPM can have two parasitic translations along the X and Y axes. In addition, the third row of the matrix shows that a parasitic rotation about the Z axis could be produced when an external force is applied to the CPM along the same axis and vice versa as reflected in the sixth row. It is observed that the five non-diagonal components within Eq. (5) will generate undesired parasitic motions and the expressions of these components are given as

$$k_{41}^m = k_{52}^m = -\frac{3}{2} (dk_{31}^l - k_{41}^l - k_{52}^l) \quad (6)$$

$$k_{51}^m = -k_{42}^m = \frac{3}{2} (k_{51}^l + dk_{32}^l - k_{42}^l)$$

$$k_{63}^m = 3 (dk_{31}^l + k_{63}^l)$$

Here, d represents the size of the end effector as illustrated in Fig. 2a. In order to achieve fully-decoupled motions, all non-diagonal components of K^m must be zeros and the components of K^l are considered as variables. As a result, Eq. (6) has multiple solutions since there are seven variables with only three equations. Therefore a special case, corresponding to making the seven variables to be zeros, is used as the

potential solution as follows

$$k_{31}^l = k_{32}^l = k_{41}^l = k_{42}^l = k_{51}^l = k_{52}^l = k_{63}^l = 0 \quad (7)$$

Note that the other non-diagonal components in K^l are canceled by the symmetrical configuration of the CPM.

As each leg consists of two parallel reflecting C-T beams and each C-T beam is meshed by a number of serial beam elements as shown in Fig. 1b, K^l is given as

$$K^l = (C^{CT})^{-1} + M(C^{CT})^{-1}M^T, \quad (8)$$

where $C^{CT} = \sum_{j=1}^n J_j^e R_j^e C^e (R_j^e)^{-1} (J_j^e)^T$

Here, n indicates the number of beam elements in a C-T beam, C^{CT} and C^e represent the compliance matrix of the C-T beam and the beam element respectively. M is a 6×6 matrix that represents the reflection of two C-T beams in each leg about the $Y'Z'$ plane as illustrated in Fig. 2a. J_j^e and R_j^e are 6×6 matrices that represent the position and orientation of the j^{th} beam element in the C-T beam respectively. In particular, M , R_j^e and J_j^e are written as

$$M = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

$$R_j^e = \begin{bmatrix} R_{z_j}^e & R_{y_j}^e & R_{x_j}^e & 0 \\ 0 & R_{z_j}^e & R_{y_j}^e & R_{x_j}^e \end{bmatrix},$$

where $R_{x_j}^e = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_j & -\sin\alpha_j \\ 0 & \sin\alpha_j & \cos\alpha_j \end{bmatrix}$

$$R_{y_j}^e = \begin{bmatrix} \cos\beta_j & 0 & \sin\beta_j \\ 0 & 1 & 0 \\ -\sin\beta_j & 0 & \cos\beta_j \end{bmatrix}$$

$$R_{z_j}^e = \begin{bmatrix} \cos\gamma_j & -\sin\gamma_j & 0 \\ \sin\gamma_j & \cos\gamma_j & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$J_j^e = \begin{bmatrix} I & D_j^e \\ 0 & I \end{bmatrix}, \text{ where } D_j^e = \begin{bmatrix} 0 & r_{z_j} & -r_{y_j} \\ -r_{z_j} & 0 & r_{x_j} \\ r_{y_j} & -r_{x_j} & 0 \end{bmatrix} \quad (11)$$

In Eqs. (10) and (11), α_j , β_j and γ_j represent the orientations of the j^{th} beam element about the X'' , Y'' and Z'' axes respectively. With the $X''Y''$ plane of the local frame of each beam element lying on the parallel plane with the XY plane of the global frame as shown in Fig. 2a, α_j and β_j represents the rotations of the beam out of the XY plane while γ_j represents the rotation of the beam in the XY plane. In relation to the Euler angles, α_j , β_j and γ_j can be considered as the roll, pitch and yaw angles respectively. The geometries of the original and the oriented beam element are illustrated in Fig. 2b. r_{x_j} , r_{y_j} and r_{z_j} components from vector $\mathbf{R}$ in Fig. 2a represent the relative position of the frame $X''Y''Z''$ of the j^{th} element to the frame $X'Y'Z'$ of the leg. Note that the directions of the frames $X'Y'Z'$ and $X''Y''Z''$ are similar. The stiffness matrix derived at the local frame $X''Y''Z''$ attached at the free end of a standard beam element, K^e , is given as

$$K^e = (C^e)^{-1} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} \\ 0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 \\ 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} \end{bmatrix} \quad (12)$$

where E and G are the Young's modulus and shear modulus of the material respectively. The beam elements employed to create the C-T beams have rectangular cross-sectional area as shown in Fig. 2b. The geometrical parameters of the beam element are given as: L , b , and t represent the length, width and thickness respectively, $A = bt$ is the area of the rectangular cross section, $J = \frac{bt^3}{3}$ is the torsion constant when $b \gg t$, $I_y = \frac{bt^3}{12}$ and $I_z = \frac{b^3t}{12}$ represents the moments of inertia about the Y'' and Z'' axes respectively.

Fig. 2. (a) Modeling of a leg and (b) beam element in C-T beam before and after orientation.

After substituting Eqs. (9), (10), (11) and (12) into Eq. (8), the expression of each component within K^l can be obtained and Eq. (8) is then analyzed using statistical solution with a number of samples. Note that there are seven equations in Eq. (7) with α_j , β_j , γ_j , r_{x_j} , r_{y_j} and r_{z_j} being variables. The computed results that satisfy the condition for achieving fully-decoupled motions are written as follows

$$\text{or } r_{zj} = 0, \alpha_j = 0^\circ \text{ and } \beta_j = 0^\circ, 180^\circ; \forall r_{xj}, r_{yj} \text{ and } \gamma_j$$

$$r_{zj} = 0, \alpha_j = 90^\circ \text{ and } \beta_j = 0^\circ, 180^\circ; \quad (13)$$

Referring to Fig. 2, the results in Eq. (13) imply that in order to achieve the diagonal matrix K^m , all beam elements must be distributed in the XY plane ($r_{zj} = 0$ and $\beta_j = 0^\circ$ or 180°) without twist angles ($\alpha_A = \alpha_B$). Furthermore, the orientation of C-T beams must be horizontal (parallel to the XY plane that represented by $\alpha_j = 0^\circ$) or vertical (perpendicular to the XY plane that represented by $\alpha_j = 90^\circ$).

3 Synthesis of the 3-DOF ($\theta_x - \theta_y - Z$) CPM

The beam-based structural optimization method [22] was employed to design this 3-DOF spatial-motion ($\theta_x - \theta_y - Z$) CPM, which targeted a workspace of at least $8^\circ \times 8^\circ \times 5.5 \text{ mm}$ and a natural frequency of 100 Hz . Most importantly, the CPM must produce fully-decoupled motions to avoid undesired parasitic motions generated from the non-actuation axes. Ti6Al4V was chosen as the building material, its mechanical properties are given as follows: elastic modulus of 111 GPa , Poisson ratio of 0.34 , and density of 4.5 g/cm^3 . The 3-legged CPM as shown in Fig. 1a was used as the base configuration and the design domain of each leg was chosen as $50 \times 50 \times 50 \text{ mm}^3$. The C-T beams were meshed by fifteen beam elements and finite element method (FEM) was employed to analyze the stiffness and dynamic properties of the entire structure. Note that the coefficient factor of 1.27 found in ref. [23] was implemented in the synthesis process to factor in the uncertainty in the thickness of the EBM-printed prototype, and the proposed criteria were applied in order to achieve the desired fully-decoupled motion characteristic.

Fig. 3. Synthesis process of the beam-based structural optimization method.

The CPM was synthesized via two optimization steps as shown in Fig. 3. The stiffness optimization is first carried out to define the geometry of the C-T beams so that the CPM is able to obtain the desired DOF and optimal stiffness characteristic. This is followed by the dynamic optimization to determine the size and mass distribution within the CPM in order to achieve the targeted dynamic behavior.

Supposing the end effector to be under effect of an arbitrary external load vector $P = \{F_x, F_y, F_z, M_x, M_y, M_z\}$, the resulting displacement vector $U = \{\Delta_x, \Delta_y, \Delta_z, \theta_x, \theta_y, \theta_z\}$ is generated as shown in Fig. 4. The energy of a specific motion in the i^{th} direction is represented by its work done, W_i , defined by P_i and U_i written as follows

$$W_i = \int_0^{P_i} U_i dP_i = \int_0^{P_i} \frac{P_i}{K_{ii}} dP_i = \frac{1}{2} \frac{P_i^2}{K_{ii}}, \quad (14)$$

where $P_i = K_{ii} \cdot U_i$

Fig. 4. Load-displacement relation at the end effector of the CPM.

Here, K is the 6×6 stiffness matrix of the entire CPM, K_{ii} represents the diagonal component of K that corresponds with P_i and U_i . The CPM needs to be as flexible as possible in the actuating directions and as stiff as possible in any other non-actuating directions. These requirements can be represented by minimizing the ratios between the energies of the undesired motions ($W_{\Delta_x}, W_{\Delta_y}, W_{\theta_z}$) and those of the desired motions ($W_{\theta_x}, W_{\theta_y}, W_{\Delta_z}$). Referring to ref. [22] and Eq. (14), the objective function of the stiffness optimization process is defined as

$$\text{minimize } f = \frac{W_{\Delta_x}^3 \cdot W_{\Delta_y}^3 \cdot W_{\theta_z}^3}{W_{\theta_x}^3 \cdot W_{\theta_y}^3 \cdot W_{\Delta_z}^3} = \kappa \cdot \frac{K_{\theta_x}^3 \cdot K_{\theta_y}^3 \cdot K_{\Delta_z}^3}{K_{\Delta_x}^3 \cdot K_{\Delta_y}^3 \cdot K_{\theta_z}^3}, \quad (15)$$

where $\kappa = \frac{F_x^6 \cdot F_y^6 \cdot M_z^6}{M_x^6 \cdot M_y^6 \cdot F_z^6}$

Note that Eq. (15) is dimensionless and $\kappa = 1$ if P is a unit vector. By applying the criteria for achieving fully-decoupled motions as presented in Section 2, the C-T beams within the CPM must be located in the XY plane with two possible orientations, i.e., parallel or perpendicular to the XY plane. Hence, the design variables of the stiffness optimization process consist of the coordinates in the X and Y directions of the four control points of the C-T beam's center curve and the C-T beam must have zero twist angle ($\alpha_A = \alpha_B$) with its orientation can be 0° or 90° (see Fig. 1b). Genetic Algorithm (GA) was employed to solve Eq. (15) and the CPM constructed from the obtained result is illustrated in

Fig. 5a. It is observed that the C-T beams located in the XY plane and two reflecting C-T beams in each leg cut each other at the free ends thus creating a V-shape flexure that satisfies the proposed criteria.

Fig. 5. The CPM obtained from (a) the stiffness optimization, (b) the dynamics optimization and (c) EBM printing technique.

Subsequently, the dynamic behavior of the CPM was improved via the dynamic optimization process. The design variables of the dynamic optimization process are shown in Fig. 1c. In particular, b_1 and h_1 represent the cross-sectional area of the C-T beam while r and t determine the size of the end effector. In addition, an additional mass having the size of $b_2 \times h_2 \times l$ is added in each C-T beam at a specific position (p) measured from the start point of the beam in order to optimize the mass distribution within the structure. The objective function of this step consists of two equations as shown in Eq. (16). The first equation is to optimize the first resonant frequency of the CPM, F_1 , to the targeted value of 100 Hz while the second equation guarantees the large working range by maximizing the total energy in the desired DOF.

$$\begin{cases} \text{minimize } (|F_1 - 100|) \\ \text{maximize } (W_{\theta_x} + W_{\theta_y} + W_{\Delta_z}) \end{cases} \quad (16)$$

GA was also used to solve the multiple-objective problem in Eq. (16). The final design of the CPM is shown in Fig. 5b. The synthesis results suggested that the CPM has a first resonant mode of 99 Hz , which is nearly equal to the targeted value. Lastly, the compliant matrix, C , of the synthesized CPM is

$$C = \text{diag}[5.53\text{e-}8 \quad 5.53\text{e-}8 \quad 1.99\text{e-}4 \quad 1.32\text{e-}1 \quad 1.32\text{e-}1 \quad 2.33\text{e-}4] \quad (17)$$

The diagonal compliance matrix shown in Eq. (17) suggests that the design criteria derived in Section 2 are feasible and the synthesized CPM is able to produce fully-decoupled motions. Furthermore, Eq. (17) also represents the good stiffness characteristic of the CPM via high stiffness ratios. In particular, the non-actuating stiffness is larger than the actuating stiffness of about 3599 times and 567 times for the translations and rotations respectively. Such large ratios suggest that the CPM is insensitive to external disturbances in non-actuating directions. With the Ti6Al4V material having a yield strength of 850 MPa , the analysis results show that the synthesized CPM can achieve the large workspace up to $10^\circ \times 10^\circ \times 7 \text{ mm}$, which well satisfies the requirement.

4 Experimental investigation and results

A prototype of the synthesized CPM was fabricated by the EBM technology with the coefficient factor of 1.27 [23] being implemented to compensate the difference between the designed and actual thicknesses caused by the surface roughness. Several experiments were conducted to evaluate the stiffness and dynamics characteristics of the CPM as well as the decoupled motion characteristic.

4.1 Stiffness evaluation

Figure 6 shows the experimental setup for measuring the compliance about the Y -axis. Two contact points distributed along the Y -axis were placed below the end effector to constrain the motion along the Z -axis and the pure rotation about the Y -axis, hence, can be created. A micrometer was used to apply the input displacement to the CPM while a force/torque (F/T) sensor (ATI Mini-40) was used to measure the actuating force. The products of the input displacement, actuating force and the distance between the loading point to the Y -axis defined the rotation angle and the bending moment respectively.

Fig. 6. Experimental setup for evaluating the compliance about the Y -axis.

The setup for measuring the compliance about the X -axis is based on similar concept. The experimental com-

pliance about the X and Y axes, which compiled from five measurements, are shown in Fig. 7b, and 7c respectively. To evaluate the translational compliance along the Z -axis, the two constraints along the Z -axis were removed and the loading point was shifted to the center of the end effector. The results of the compliance along the Z -axis are plotted in Fig. 7a.

The predicted compliance in Fig. 7 is plotted based on the values in Eq. (17). Within a small deformation range of $< 2^\circ$ for the rotation and $< 1 \text{ mm}$ for the translation, the predicted data well matches the measurements. As FEM was used to determine the predicted compliance, they are only accurate for small deformations. At larger deformation range, the compliance becomes nonlinear and goes far away from the predictions due to the over-constrained structure of the CPM. In order to achieve more accurate predictions, the predicted data is compensated by the pseudo-rigid-body (PRB) model [22] where the elongation of the C-T beams is considered. Details of the proposed PRB model are provided in Appendix A. After considering the elongation of the flexures, the compensated values match the experimental results with the deviations of 9.5%, 8% and 11.2% for the translation along the Z -axis, the rotations about the X and Y axes respectively.

4.2 Dynamics evaluation

A standard modal analysis equipment, which includes an impact hammer (PCB PIEZOTRONICS 086C03), an accelerometer (PCB PIEZOTRONICS 353B15) and an acquisition module (DATA TRANSLATION DT9837), was used to evaluate the dynamics behavior of the 3D printed CPM. To measure the first resonant mode along the Z -axis, an input excitation along the same direction was applied by the hammer while the resulting acceleration was recorded by the accelerometer that was attached at the bottom of the end effector as shown in Fig. 8a. The output signals were then processed by the signal acquisition device and the obtained result after analysis is plotted in Fig. 8b. The prototype demonstrated a good dynamic behavior as the experimental resonance frequency (98 Hz) agrees well with the predicted value (99 Hz) with a small deviation of 1%.

4.3 Evaluation of decoupled motions

With the stiffness/compliance matrix of the CPM being symmetrical, only five non-diagonal components possibly exist as shown in Eq. (5). In this work, due to the difficulties in measuring the parasitic rotations about the X and Y axes, only the parasitic rotation about the Z -axis was evaluated. In particular, the component K_{63}^m will be proved to be nearly zero. Moreover, the parasitic motions along the X and Y axes were also experimentally measured to demonstrate that they are very small and have no effect on the desired motion along the Z -axis, even they are zeros in theory. Here, the decoupled motions are proven based on energy approach where work done is used to represent the energy of a motion. Eq. (14) can be rewritten as

Fig. 8. (a) Experimental setup to measure the resonance frequency along the Z -axis and (b) experimental result.

$$W_i = \int_0^{U_i} P_i dU_i = \int_0^{U_i} K_{ii} U_i dU_i = \frac{1}{2} K_{ii} U_i^2 \quad (18)$$

Assuming the non-diagonal components within the stiffness matrix of the CPM exist, Eq. (18) can be modified to represent the total energy in the i^{th} direction as

$$W_i = \sum_{j=1}^6 \frac{1}{2} K_{ij} U_j^2 \quad (19)$$

Referring to Eq. (19), W_i is the combination of the energy of the desired motions (determined by K_{ii} and U_i) and the energies of the undesired parasitic motions (determined by K_{ij} and U_j , where $j \neq i$). In this work, a motion is considered as decoupled if the energies of the parasitic motions have minor contribution over the total energy. Based on Eq. (5), when applying a force along the Z -axis, a parasitic rotation about the Z -axis could be generated. Fig. 9 shows the experimental setup to measure the parasitic motions when applying the input displacement along the Z -axis. A solid block that was rigidly mounted to the end-effector was used to transmit the motion along the Z -axis from the micrometer to the CPM. The parasitic motions along the X and Y axes, and about the Z -axis were detected by three individual capac-

Fig. 7. Measured compliance (a) along the Z-axis, (b) about the X-axis and (c) about the Y-axis.

Fig. 9. Experimental setup for measuring the parasitic motions along the Z-axis.

itive sensors respectively. The sensors measured the distance between their sensing areas to the surfaces of the solid block. The experiment was conducted five times and Fig. 10 plots the ratios between the energy of each parasitic motion and the total energy used along the Z-axis.

Fig. 10. The ratios between the energies of parasitic motions over the total energy along the Z-axis.

From Fig. 10, the highest energy of the parasitic mo-

tions only contributed to less than 1% over the total energy along the Z-axis. These results show that almost all the input energy (along the Z-axis) were used to create the desired motions while the energies that generated the parasitic motions were minimal. The parasitic motions along the X and Y axes exist could be due to the Abbe error caused by the fabrication tolerances and the assembly imperfection, i.e., the surfaces of the solid block are not ideally perpendicular to the measuring directions of the capacitive sensors. In general, the reading from the sensor, ϵ , affected by the Abbe error can be estimated based on the slope angle of the surface of the block, ψ , and the input displacement along the Z-axis, δ_z , given as

$$\tan(\psi) = \frac{\epsilon}{\delta_z} \quad (20)$$

By assuming that the slope angle of the surface of the target block was $\sim 0.2^\circ$, the readings from the sensor that correspond with the input displacements calculated by Eq. (20) are listed in Table 1. These results show that a slight angular misalignment of just 0.2° is enough to cause the capacitive sensors to return the Abbe errors as the parasitic motions instead. Hence, it is possible that the Abbe errors have contributed to the readings even though the CPM may not have produce any parasitic motion. Hence, these experimental results and considering the Abbe errors demonstrate that the synthesized CPM is capable of producing decoupled motions. Although the other motions were not evaluated, the results are expected to be similar.

5 Discussion

Large-workspace CPMs often possess poor dynamics behavior due to the low actuating stiffness. Therefore, designing of CPMs with large working range and fast dynamic response is a common challenge. Among the existing large-stroke CPMs, the synthesized CPM in this work has significantly larger workspace ($10^\circ \times 10^\circ \times 7 \text{ mm}$) and better dynamics behavior (via the first resonance frequency of 98 Hz). The merit of using EBM in fabricating compliant structures to eliminate assembly errors has been also demonstrated via

Table 1. Readings of capacitive sensors caused by Abbe error.

Displacement along the Z axis	Sensor's reading
200 μm	0.698 μm
400 μm	1.396 μm
600 μm	2.094 μm
800 μm	2.793 μm
1000 μm	3.491 μm

the small deviation of only 1% between the predicted and experimental dynamic behaviors of the 3D printed prototype. Furthermore, the lowest stiffness ratio (~ 567) from the synthesized CPM is high so that the CPM is insensitive to external disturbances in the non-actuating directions and very flexible in the actuating directions.

As motion-decoupling characteristic plays an important role in precision positioning systems, some criteria for synthesizing 3-legged CPMs with any DOF and fully-decoupled motions have been derived and applied to synthesize a 3-DOF ($\theta_X - \theta_Y - Z$) CPM. The results show that the derived design criteria are feasible because this new CPM has a diagonal compliance matrix while the existing CPMs with $\theta_X - \theta_Y - Z$ motion have five non-diagonal components as shown in Eq. (5) [22, 23, 25]. It is noticed that the designs in refs. [22, 23] have an offset gap between two reflecting C-T beams ($r_{zj} \neq 0$) which are unsatisfied the criteria for achieving decoupled motions. Importantly, the proposed design criteria are also applicable in determining the motion characteristic of other 3-legged CPMs synthesized by different methods. If a beam element in the beam-based method is considered as a flexure joint in the traditional task-oriented method, the coupled motions of the design in ref. [25] can be easily explained because the prismatic and revolute flexure joints in each leg are not located in the XY plane.

6 Conclusion

This paper presents a novel 3-DOF ($\theta_X - \theta_Y - Z$) CPM with fully decoupled motion capability synthesized by the beam-based optimization method. Such capability is made possible due to some novel design criteria, which are firstly derived in this work. These design criteria, which can be used to analyze the motion characteristics of any 3-legged CPM, have been proved and verified. The synthesized CPM is able to achieve a diagonal compliance matrix, a large working range of $10^\circ \times 10^\circ \times 7 \text{ mm}$ with a high translational and rotational ratios of ~ 3600 and ~ 570 respectively. The obtained dynamic response of 98 Hz also satisfies the targeted value of 100 Hz.

A prototype of the synthesized CPM was fabricated by the EBM technology with a coefficient factor of 1.27 being implemented into the analyses to compensate the fabrication tolerance of the thickness of the 3D printed flexures. Experimental results show that the 3D printed CPM provides

the predicted mechanical properties since the highest deviations between the measurements and predictions are 11.2% and 1% for the stiffness and dynamics properties respectively. The full workspace of the CPM was also achieved. The decoupled-motions capability of the synthesized CPM has been demonstrated since the energy used to generate the parasitic motions is very minimal, i.e., $< 1\%$ over the total energy along the Z-axis. The obtained results suggest that the derived design criteria are feasible. A discussion is also conducted to clarify the improvement of the synthesized CPM compared to the previous designs in terms of workspace, stiffness ratios, dynamics response and decoupled-motion capability. Consequently, the proposed CPM has good performance and can be used in precise positioning tasks.

The future work will focus on generalizing the design criteria for designing decoupled-motion CPMs analytically. Furthermore, the capability of the beam-based method in determining nonlinear stiffness of CPMs during the synthesis process will be improved.

Appendix A. The equivalent PRB model for over-constrained CPMs

The nonlinear experimental compliance as shown in Fig. 7 is caused by the large deformation of the legs along the Z axis. If each leg is represented by a fixed-clamped beam as illustrated in Fig. 11, its parameters can be expressed as

Fig. 11. Equivalent PRB model of the leg.

- The equivalent length, l_e , is the distance from the fixed end to the loading point.
- The equivalent axial force per unit strain, $(EA)_e$, and bending flexural rigidity, $(EI)_e$, are derived respectively based on the first and the third diagonal components within the stiffness matrix of the leg.

Here, the PRB model of the linear spring under large elastic deformation is used to analyze the nonlinear stiffness characteristics of the over-constrained CPM. The S-shape of

the deformed equivalent beam can be represented by two torsion springs at both ends connected by a linear spring as illustrated in Fig. 11. In each leg, the relationship between the actuating force (F_z) and displacement along the Z axis (δ_z) can be expressed as [2]

$$F_z = \left[K_A \Delta l_e \sin(\varphi) + 4K_T \frac{\varphi}{\zeta l_e \cos(\varphi)} \right],$$

where

$$\Delta l_e = \frac{\zeta l_e}{\cos(\varphi)} - \zeta l_e$$

$$K_T = \frac{\zeta}{l_e} K_\theta (EI)_e \quad (21)$$

$$K_A = \frac{(EA)_e}{\zeta l_e + \Delta l_e}$$

$$\varphi = \tan^{-1} \left(\frac{\delta_z}{\zeta l_e} \right)$$

where λ represents the length change in between two torsional springs, i.e., $\zeta l_e + \Delta l_e$ and φ indicates the deflection angle of the deformed beam. K_T represents the stiffness of the torsion spring and K_A represents the stiffness of the linear spring. Based on past literatures, the spring constant, K_θ , is selected as 2 [28] and ζ is derived as 2/3 [29].

Depending on the specific deformation, i.e., translation along the Z axis or rotation about the X or Y axes, the displacement δ_z of each leg is specified and then the corresponding actuating force F_z of each leg can be determined. Such force is used to compensate the elongation of the flexure under large elastic deformation and the predicted compliance with compensation of the CPM is plotted in Fig. 7.

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