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Feedforward multichannel virtual-sensing active control of noise through an aperture: Analysis on causality and sensor-actuator constraints

Dongyuan Shi,¹ Woon-Seng Gan,^{1,a)} Bhan Lam,^{1,b)} Rina Hasegawa,² and Yoshinobu Kajikawa^{2,c)}

¹School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, 639798, Singapore

²Department of Electrical, Electronic and Information Engineering, Kansai University, Osaka, 564-8680, Japan

ABSTRACT:

The multichannel implementation of the auxiliary-filter-based virtual-sensing (AF-VS) technique for active noise control applications is revisited and realized in the paper. Frequency-domain analysis based on random primary noise proves that the multichannel virtual-sensing active noise control (MVANC) technique can achieve optimal control at the desired virtual locations even if the signals at the physical and virtual microphones are not causally related. Further analysis on a number of sensor-actuator configurations shows that the MVANC technique achieves optimal control at the desired locations as long as the number of secondary sources does not exceed that of the physical error microphones. Furthermore, the simulations with measured transfer functions and real-time experiments conducted on a four-channel system validate the frequency domain analyses. © 2020 Acoustical Society of America.

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I. INTRODUCTION

Active noise control (ANC), a technique widely implemented in consumer headphones and automobiles,^{1–10} is increasingly adapted for larger spaces, such as in room interiors, to tackle complex urban noises.^{11–17} The feedforward ANC system precisely actuates the “anti-noise,” which destructively attenuates the disturbance, based on *a priori* and *a posteriori* information from sensors.¹⁸ In most ANC applications, however, the “quiet zone” is localized around the monitoring “error sensor” that provides *a posteriori* information to the adaptive algorithm. The quiet zone is defined as the area on one axial plane where more than 10 dB reduction is attained, and its diameter is about one-tenth of the acoustic wavelength of the highest frequency.¹⁹ Hence, the conventional ANC approach where the quiet zone is dependent on the placement of the physical error sensor seems impractical in scenarios where physical placement is inconvenient, such as at the eardrums,^{20–22} or in domestic living spaces.^{23,24}

At present, different types of virtual-sensing active noise control (VS-ANC) techniques have been proposed to alleviate the restriction on error sensor placement.^{25–28} These approaches effectively generate the quiet zone at desired “virtual” locations, which are located at some distance away from the physical error sensors.²⁹ The VS-ANC techniques are generally categorized into two classes. Class I techniques do not require offline training and directly

estimate the sound characteristics at the virtual location with the physical error microphone based on acoustical models^{30,31} or an extrapolating polynomial.^{32,33} However, the noise reduction performance of these approaches is highly sensitive to the accuracy of the models,^{30,34} and they are most suitable for low-frequency tonal sound fields.³⁵

Class II VS-ANC techniques require a preliminary training stage to obtain a filter of the system model from the physical to the virtual error sensor location^{36,37} or the information of the optimum noise control filter.^{38,39} Subsequently, the pre-trained filter assists the ANC system in achieving satisfactory control of the disturbance at the virtual error sensor location.

Currently, several practical algorithms have been developed to implement class II VS-ANC techniques.^{36,40} The remote microphone technique estimates an “observation filter” from the physical sensor to the desired location by initially placing sensors *in situ* at both locations.^{19,41–45} Another class II VS-ANC algorithm similarly estimates plant states to the virtual sensor location by the Kalman filtering method.⁴⁶ After the sensor at the virtual sensor location is removed, the observation filter or plant states are utilized to predict the error signal at the virtual sensor location with the physical error sensor to achieve noise reduction at the desired location.³⁶ However, the remote microphone technique requires a causal relation^{46,47} between the physical and virtual error sensors. This implies that the virtual primary disturbances have to be causally estimated from the physical primary disturbances. Hence, the physical arrangement of both the virtual and physical error sensors is critical to the performance of the remote microphone technique. For instance, if the location of the virtual error sensor is closer to the secondary actuator than the physical error sensor position, the causal constraint will be violated.

^{a)}ORCID: 0000-0002-7143-1823

^{b)}Electronic mail: bhanlam@ntu.edu.sg, ORCID:0000-0001-5193-6560.

^{c)}ORCID: 0000-0002-1735-2518

The modified remote sensing technique slackens the causal restriction on the microphones by introducing several delays into the estimates of the secondary path and observation filter.^{27,48} However, the delay selection process is vital to the estimation accuracy of the observation filter.⁴⁹ Moreover, the number of delays somewhat strains the step-size bound of the FxLMS algorithm.⁵⁰

The third class II VS-ANC technique is the so-called auxiliary-filter-based virtual-sensing (AF-VS) method.^{51–56} In contrast to the remote microphone technique, the AF-VS replaces the observation filter with the auxiliary filter. This auxiliary filter indirectly records the relationship between the physical sensor and the desired virtual position. Since the transfer functions from the physical sensor to the virtual sensor are omitted, the AF-VS method avoids the causality constraint. Following the above discussion, this paper analyzes the multichannel implementation of the AF-VS ANC algorithm, which is multichannel virtual-sensing active noise control (MVANC). In the paper, first, the tuning and control stages of the MVANC technique in the time domain are detailed. Second, the frequency domain analysis based on two variants of random noises shows that the controllers of the tuning and control stages converge to the same optimal solution, which indicates that the MVANC technique could achieve optimal noise reduction at the virtual positions. Furthermore, this analysis also implicates that the virtual primary path can be independent of the physical primary path, which overcomes the causal constraint between error microphones in other class II algorithms. Third, the limitations of the MVANC technique are explored in both the under-determined (the number of secondary sources exceeds that of the error sensors) and over-determined (the number of error sensors exceeds that of the secondary sources) formulations. Finally, both simulations based on measured transfer functions from an actual system and real-time experiments of a four-channel MVANC system are conducted to verify the theoretical analyses.

II. THE MVANC TECHNIQUE

The multichannel active noise control (MCANC) technique has been recently used to control the noise propagating through a large dimension aperture in a steady-state sound field.⁵⁷ However, most of the time, it is challenging to achieve noise cancellation at desired locations where it is impractical to place the physical error microphones, e.g., in domestic living spaces. In contrast, the MVANC technique increases the flexibility of microphone placement. In this paper, we formulate MVANC based on the context of active control through an open aperture with J reference microphones, K loudspeakers, and M physical microphones, as shown in Fig. 1. The MVANC technique aims to reduce the disturbance, which propagates through the aperture from L noise sources at N desired locations. For brevity and without loss of generality, the number of reference microphones is assumed to be greater than the number of primary sources ($L < J$).

There are two stages in the MVANC technique: the tuning stage and the control stage. In the tuning stage, as shown in Fig. 2, the output signal vector is given by

$$\mathbf{y}(n) = \mathbf{w}^T(n)\mathbf{x}(n) \in \mathbb{R}^{K \times 1}, \quad (1)$$

where $\mathbf{y}(n) = [y_1(n), y_2(n), \dots, y_k(n), \dots, y_K(n)]^T$, and $y_k(n)$ denotes the k th output signal at time sample n ; $(\cdot)^T$ and $\mathbb{R}$ denote the matrix transpose operator and the real number, respectively. The stacked reference vector $\mathbf{x}(n)$ is expressed as

$$\mathbf{x}(n) = [\mathbf{x}_1^T(n), \mathbf{x}_2^T(n), \dots, \mathbf{x}_J^T(n)]^T \in \mathbb{R}^{JN_x \times 1}, \quad (2)$$

where $\mathbf{x}_j(n) \in \mathbb{R}^{N_x \times 1}$ is the j th reference vector with N_x samples and given by

$$\mathbf{x}_j(n) = [x_j(n), x_j(n-1), \dots, x_j(n-N_x+1)]^T. \quad (3)$$

The control filter matrix $\mathbf{w}(n) \in \mathbb{R}^{JN_x \times K}$ in Eq. (1) is given by

$$\mathbf{w}(n) = \begin{bmatrix} \mathbf{w}_{11}(n) & \mathbf{w}_{21}(n) & \cdots & \mathbf{w}_{K1}(n) \\ \mathbf{w}_{12}(n) & \mathbf{w}_{22}(n) & \cdots & \mathbf{w}_{K2}(n) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{w}_{1J}(n) & \mathbf{w}_{2J}(n) & \cdots & \mathbf{w}_{KJ}(n) \end{bmatrix}, \quad (4)$$

where $\mathbf{w}_{kj}(n) \in \mathbb{R}^{N_x \times 1}$ represents the coefficient vector of the control filter from the j th input reference signal to the k th output control signal,

$$\mathbf{w}_{kj}(n) = [w_{kj,0}(n), w_{kj,1}(n), \dots, w_{kj,N_x-1}(n)]^T. \quad (5)$$

Based on the conventional multichannel FxLMS (MCFxLMS) algorithm, the new control filter from the j th input to the k th output is defined as

$$\mathbf{w}_{kj}(n+1) = \mathbf{w}_{kj}(n) - \mu_1 \sum_{i=1}^N \mathbf{x}'_{v,jki}(n) e_{v,i}(n), \quad (6)$$

where $e_{v,i}(n)$ represents the error signal at the i th virtual microphone, and μ_1 denotes the step size of FxLMS algorithm. The filtered reference $\mathbf{x}'_{v,jki}(n)$ is a convolution between the j th reference signal $x_j(n)$ and the virtual secondary path estimate $\hat{g}_{v,ik}(n)$, which is from the k th secondary source to the i th virtual microphone. From Eq. (6), it can be seen that the final control filter in the tuning stage converges to the same optimal control filter obtained with the conventional MCFxLMS.⁵⁸

Once the control filter converges, the auxiliary filters are trained, as illustrated in Fig. 3. Utilizing the least mean square (LMS) algorithm, the m th auxiliary filter vector is obtained by

$$\mathbf{h}_m(n+1) = \mathbf{h}_m(n) + \mu_2 e_{h,m}(n) \bar{\mathbf{x}}(n) \in \mathbb{R}^{JN_x \times 1}, \quad (7)$$

where μ_2 denotes the step size of the LMS algorithm, and

FIG. 1. (Color online) A simplified schematic of the MVANC technique applied on an open aperture. J reference microphones are placed in the front facade of the aperture, and M physical microphones are placed inside the room. K secondary sources installed in the aperture create anti-noise waves to cancel the disturbance, which is generated by L primary noise sources, at the N virtual locations.

$$\mathbf{h}_m(n) = [\mathbf{h}_{m1}^T(n), \mathbf{h}_{m2}^T(n), \dots, \mathbf{h}_{mj}^T(n), \dots, \mathbf{h}_{mJ}^T(n)]^T. \quad (8)$$

The expression $\mathbf{h}_{mj}(n) \in \mathbb{R}^{N_h \times 1}$ denotes the auxiliary filter from the j th reference to the m th physical microphone with a length of N_h taps. Therefore,

$$\mathbf{h}_{mj}(n) = [h_{mj,0}(n), h_{mj,1}(n), \dots, h_{mj,N_h-1}(n)]^T. \quad (9)$$

The corresponding stacked reference vector $\bar{\mathbf{x}}(n)$ is given by

FIG. 2. Training the control filter in the tuning stage of the MVANC technique (Ref. 58).

$$\bar{\mathbf{x}}(n) = [\bar{\mathbf{x}}_1^T(n), \bar{\mathbf{x}}_2^T(n), \dots, \bar{\mathbf{x}}_J^T(n)]^T \in \mathbb{R}^{JN_h \times 1}, \quad (10)$$

where $\bar{\mathbf{x}}_j(n) \in \mathbb{R}^{N_h \times 1}$ is the j th reference vector with N_h samples and given by

$$\bar{\mathbf{x}}_j(n) = [x_j(n), x_j(n-1), \dots, x_j(n-N_h+1)]^T. \quad (11)$$

The expression $e_{h,m}(n)$ is stated as

FIG. 3. Training the auxiliary filter in the tuning stage of the MVANC technique.

FIG. 4. Training the control filter in the control stage of the MVANC technique.

$$e_{h,m}(n) = e_{p,m}(n) - \mathbf{h}_m^T(n) \bar{\mathbf{x}}(n), \quad (12)$$

where $e_{p,m}(n)$ is the error signal at the m th physical microphone.

In the control stage, the microphones at the virtual locations are removed, while the physical error microphones are untouched. As depicted in Fig. 4, the new control filter is given by

$$\mathbf{w}_{kj}(n+1) = \mathbf{w}_{kj}(n) - \mu_3 \sum_{m=1}^M \mathbf{x}'_{p,jkm}(n) e_{h,m}(n), \quad (13)$$

where $\mathbf{x}'_{p,jkm}(n)$ is the filtered reference signal resulting from the convolution between the physical secondary path estimate $\hat{g}_{mk}(n)$, which is from the k th secondary source to the m th physical microphone, and the j th reference signal $x_j(n)$. The symbol μ_3 denotes the step size of the FxLMS algorithm, and the error signal $e_{h,m}(n)$ in Eq. (13) is given by

$$e_{h,m}(n) = e_{p,m}(n) - \mathbf{h}_{o,m}^T \bar{\mathbf{x}}(n), \quad (14)$$

where $\mathbf{h}_{o,m}$ represents the m th optimal auxiliary filter obtained from Eq. (7).

Through the procedure above, a $J \times K \times N$ MVANC system can achieve noise cancellation at N virtual positions. In summary, Eqs. (1), (6), and (7) describe the tuning stage of the MVANC algorithm, while Eqs. (13) and (14) implement the noise control at the virtual positions. Based on these equations, we can determine the number of additions and multiplications of the MVANC algorithm in one period, as listed in Table I, where L_s denotes the length of the secondary path model. Compared to the conventional MCANC, the MVANC algorithm requires more multiplication and addition, which are used in the training and execution of the auxiliary filter in the tuning and control stages, respectively.

TABLE I. The number of additions and multiplications used in the MCANC and MVANC algorithms for one period.

Algorithm	Multiplications	Additions
MCANC	$J[(N+2)N_x + NL_s]$	$J[(N+2)N_x + NL_s + 2N - 1]$
MVANC (tuning)	$J[(N+2)N_x + NL_s + 3MN_h]$	$J[(N+2)N_x + NL_s + 2MN_h + 2N - 1]$
MVANC (control)	$J[(N+2)N_x + NL_s + MN_h]$	$J[(N+2)N_x + NL_s + MN_h + 2N - 1]$

III. FREQUENCY DOMAIN ANALYSIS OF THE MVANC TECHNIQUE

This analysis initially assumes that there are fewer secondary sources than both the physical and virtual microphones ($K \leq N, K \leq M$). The influence of alternate configurations on the optimal control in both the tuning and control stages is analyzed in Sec. IV. It is worth noting that all the derivations on the frequency domain are under the assumption that all components in the MCANC system are ideally linear and additional disturbances are ignored. For clarity and simplicity, the frequency index of the frequency domain symbols is henceforth hidden.

A. Tuning stage

In the tuning stage, the primary noise is defined as a random noise $\mathbf{V}_1 \in \mathbb{C}^{L \times 1}$, where $\mathbb{C}^{L \times 1}$ denotes a $L \times 1$ vector of complex values and comes from L individual primary sources. The reference signal $\mathbf{X}_1 \in \mathbb{C}^{J \times 1}$ is modified by the control filter $\mathbf{W} \in \mathbb{C}^{K \times J}$ and the virtual secondary path $\mathbf{G}_v \in \mathbb{C}^{N \times K}$ to cancel the disturbance $\mathbf{D}_{v1} \in \mathbb{C}^{N \times 1}$ at the virtual positions, as shown in Fig. 5. The virtual error signal vector is thus given by

$$\mathbf{E}_{v1} = \mathbf{D}_{v1} + \mathbf{G}_v \mathbf{W} \mathbf{X}_1 \in \mathbb{C}^{N \times 1}. \quad (15)$$

The virtual secondary path matrix is given by

$$\mathbf{G}_v = \begin{bmatrix} G_{v,11} & G_{v,12} & \cdots & G_{v,1K} \\ G_{v,21} & G_{v,22} & \cdots & G_{v,2K} \\ \vdots & \vdots & \ddots & \vdots \\ G_{v,N1} & G_{v,N2} & \cdots & G_{v,NK} \end{bmatrix}_{N \times K}, \quad (16)$$

where $G_{v,NK}$ represents the transfer function of the secondary path from the k th secondary source to the n th virtual microphone. $\mathbf{G}_v$ is assumed to be a full-rank matrix, and hence, $\mathbf{G}_v^H \mathbf{G}_v \in \mathbb{C}^{K \times K}$ is invertible.

The power spectral density of the virtual error signal can be expressed as

$$J_{v1} = \mathbb{E}(\mathbf{E}_{v1}^H \mathbf{E}_{v1}) = \text{tr}[\mathbb{E}(\mathbf{E}_{v1} \mathbf{E}_{v1}^H)], \quad (17)$$

where $\mathbb{E}(\cdot)$ and $\text{tr}[\cdot]$ denote the expectation and trace operators, respectively; $(\cdot)^H$ stands for the conjugate transpose of the matrix. Expanding Eq. (17) yields

$$J_v = \text{tr}[\mathbb{E}(\mathbf{D}_{v1} \mathbf{D}_{v1}^H + \mathbf{D}_{v1} \mathbf{X}_1^H \mathbf{W}^H \mathbf{G}_v^H + \mathbf{G}_v \mathbf{W} \mathbf{X}_1 \mathbf{D}_{v1}^H + \mathbf{G}_v \mathbf{W} \mathbf{X}_1 \mathbf{X}_1^H \mathbf{W}^H \mathbf{G}_v^H)]. \quad (18)$$

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1k} \\ G_{21} & G_{22} & \cdots & G_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ G_{M1} & G_{M2} & \cdots & G_{MK} \end{bmatrix}_{M \times K}, \quad (31)$$

where G_{mk} represents the transfer function of the secondary path from the k th secondary source to the m th physical microphone. $\mathbf{G}$ is assumed to be a full-rank matrix, and hence, $\mathbf{G}^H \mathbf{G} \in \mathbb{C}^{K \times K}$ is invertible. The power spectral density of Eq. (30) is expressed as

$$J_{h1} = \mathbb{E}(\mathbf{E}_{h1}^H \mathbf{E}_{h1}) = \text{tr}[\mathbb{E}(\mathbf{E}_{h1} \mathbf{E}_{h1}^H)]. \quad (32)$$

Expanding Eq. (32) yields

$$J_{h1} = \text{tr} \left[\mathbf{E}(\mathbf{D}_{p1} \mathbf{D}_{p1}^H + \mathbf{D}_{p1} \mathbf{X}_1^H \mathbf{W}_{o,t}^H \mathbf{G}^H - \mathbf{D}_{p1} \mathbf{X}_1^H \mathbf{H}^H + \mathbf{G} \mathbf{W}_{o,t} \mathbf{X}_1 \mathbf{D}_{p1}^H + \mathbf{G} \mathbf{W}_{o,t} \mathbf{X}_1 \mathbf{X}_1^H \mathbf{W}_{o,t}^H \mathbf{G}^H - \mathbf{G} \mathbf{W}_{o,t} \mathbf{X}_1 \mathbf{X}_1^H \mathbf{H}^H - \mathbf{H} \mathbf{X}_1 \mathbf{D}_p^H - \mathbf{H} \mathbf{X}_1 \mathbf{X}_1^H \mathbf{W}_{o,t}^H \mathbf{G}^H + \mathbf{H} \mathbf{X}_1 \mathbf{X}_1^H \mathbf{H}^H) \right]. \quad (33)$$

By invoking the properties of the trace operator, Eq. (33) can be rewritten as

$$\begin{aligned}
J_{h1} = & \text{tr}(\mathbf{S}_{d,p1} + \mathbf{S}_{xd,p1} \mathbf{W}_{o,t}^H \mathbf{G}^H - \mathbf{H}^H \mathbf{S}_{xd,p1} + \mathbf{G} \mathbf{W}_{o,t} \mathbf{S}_{dx,p1} \\
& + \mathbf{G} \mathbf{W}_{o,t} \mathbf{S}_{x1} \mathbf{W}_{o,t}^H \mathbf{G}^H - \mathbf{H}^H \mathbf{G} \mathbf{W}_{o,t} \mathbf{S}_{x1} \\
& - \mathbf{H} \mathbf{S}_{dx,p1} - \mathbf{H} \mathbf{S}_{x1} \mathbf{W}_{o,t}^H \mathbf{G}^H + \mathbf{H}^H \mathbf{H} \mathbf{S}_{x1}), \quad (34)
\end{aligned}$$

where the cross spectral density matrix $\mathbf{S}_{x_d,p}$ is given by

$$\mathbf{S}_{xd,p1} = \mathbb{E}(\mathbf{D}_{p1} \mathbf{X}_1^H) \in \mathbb{C}^{M \times J}. \quad (35)$$

To obtain the optimal auxiliary filter $\mathbf{H}_o \in \mathbb{C}^{M \times J}$, the gradient of Eq. (34) is set to zero to give

$$\frac{\partial J_{h1}}{\partial \mathbf{H}^*} = -2\mathbf{S}_{x_d p1} - 2\mathbf{G}\mathbf{W}_{o,t}\mathbf{S}_{x1} + 2\mathbf{H}\mathbf{S}_{x1} = 0. \quad (36)$$

As the disturbance $\mathbf{D}_{p1}$ at the location of the physical microphone is the result of random noise $\mathbf{V}_1$ propagating through the physical primary paths $\mathbf{P} \in \mathbb{C}^{M \times J}$, as illustrated in Fig. 5, the cross spectral density matrix $\mathbf{S}_{xd,p1}$ can be rewritten as

$$\mathbf{S}_{xdl,p1} = \mathbf{P}\mathbf{S}_{v1}\mathbf{R}^H. \quad (37)$$

The matrix of primary path responses, $\mathbf{P}$, is given by

$$\mathbf{P} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1L} \\ P_{21} & P_{22} & \cdots & P_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ P_{M1} & P_{M2} & \cdots & P_{ML} \end{bmatrix}_{M \times L}, \quad (38)$$

where P_{ml} denotes the transfer function of the primary path from the l th primary source to the m th physical microphone.

By substituting Eqs. (29) and (37) into Eq. (36), the optimal auxiliary filter is derived as

$$\begin{aligned}
\mathbf{H}_o &= \mathbf{P}\mathbf{S}_{v1}\mathbf{R}^H(\mathbf{R}\mathbf{S}_{v1}\mathbf{R}^H)^{-1} + \mathbf{G}\mathbf{W}_{o,t} \\
&= \mathbf{P}(\mathbf{R}^H\mathbf{R})^{-1}\mathbf{R}^H\mathbf{R}\mathbf{S}_{v1}\mathbf{R}^H(\mathbf{R}\mathbf{S}_{v1}\mathbf{R}^H)^{-1} \\
&\quad - \mathbf{G}(\mathbf{G}_v^H\mathbf{G}_v)^{-1}\mathbf{G}_v^H\mathbf{P}_v(\mathbf{R}^H\mathbf{R})^{-1}\mathbf{R}^H \\
&= [\mathbf{P} - \mathbf{G}(\mathbf{G}_v^H\mathbf{G}_v)^{-1}\mathbf{G}_v^H\mathbf{P}_v](\mathbf{R}^H\mathbf{R})^{-1}\mathbf{R}^H. \tag{39}
\end{aligned}$$

B. Control stage

In the control stage, a different random noise $\mathbf{V}_2 \in \mathbb{C}^{L \times 1}$ is assigned as the primary noise, which also propagates from L individual noise sources. Since the optimal auxiliary filters assist in the updating of the control filters, as shown in Fig. 6, the error signal vector $\mathbf{E}_{h2} \in \mathbb{C}^{M \times 1}$ is expressed as

$$\mathbf{E}_{h2} = \mathbf{E}_{p2} - \mathbf{Y}_{h2} = \mathbf{D}_{p2} + \mathbf{G}\mathbf{W}\mathbf{X}_2 - \mathbf{H}_\theta\mathbf{X}_2, \quad (40)$$

where $\mathbf{E}_{p2} \in \mathbb{C}^{M \times 1}$ and $\mathbf{D}_{p2} \in \mathbb{C}^{M \times 1}$ denote the error signal and disturbance at the physical microphones, respectively. The power spectral density of Eq. (40) is defined as

$$\begin{aligned}
J_{h2} = & \text{tr}(\mathbf{S}_{d,p2} + \mathbf{W}^H \mathbf{G}^H \mathbf{S}_{x_d,p2} - \mathbf{S}_{x_d,p2} \mathbf{H}_o^H + \mathbf{GWS}_{dx,p2} \\
& + \mathbf{W}^H \mathbf{G}^H \mathbf{GWS}_{x2} - \mathbf{GWS}_{x2} \mathbf{H}_o^H - \mathbf{H}_o \mathbf{S}_{dx,p2} \\
& - \mathbf{W}^H \mathbf{G}^H \mathbf{H}_o \mathbf{S}_{x2} + \mathbf{HS}_{x2} \mathbf{H}_o^H). \quad (41)
\end{aligned}$$

To obtain the optimal control filter, the gradient of Eq. (41) is set to zero to give

$$\frac{\partial J_{h2}}{\partial \mathbf{W}^*} = 2\mathbf{G}^H \mathbf{S}_{xd,p2} + 2\mathbf{G}^H \mathbf{G} \mathbf{W} \mathbf{S}_{x2} - 2\mathbf{G}^H \mathbf{S}_{x2} \mathbf{H}_o = 0, \quad (42)$$

where the power spectral density matrix $\mathbf{S}_{x2}$ and the cross spectral density matrix $\mathbf{S}_{xd,p2}$ are given by

$$\mathbf{S}_{x2} = \mathbf{R}\mathbf{S}_{y2}\mathbf{R}^H \in \mathbb{C}^{J \times J} \quad (43)$$

and

FIG. 6. Block diagram of the control stage in the MVANC technique.

$$\mathbf{S}_{xd,p2} = \mathbf{P}\mathbf{S}_{v2}\mathbf{R}^H \in \mathbb{C}^{M \times J}, \quad (44)$$

where $\mathbf{S}_{v2} = \mathbb{E}(\mathbf{V}_2\mathbf{V}_2^H) \in \mathbb{C}^{L \times L}$ denotes the cross spectral density matrix of primary noise $\mathbf{V}_2$. By solving Eq. (42), the optimal solution $\mathbf{W}_{o,c} \in \mathbb{C}^{K \times J}$ for the control filter in the control stage is given by

$$\mathbf{W}_{o,c} = (\mathbf{G}^H\mathbf{G})^{-1}(\mathbf{G}^H\mathbf{H}_o\mathbf{S}_{x2} - \mathbf{G}^H\mathbf{S}_{xd,p2})\mathbf{S}_{x2}^{-1}, \quad (45)$$

where $\mathbf{S}_{x2}$ is assumed to be invertible.¹⁹ By substituting Eqs. (39), (43), and (44) into Eq. (45) yields

$$\mathbf{W}_{o,c} = \mathbf{W}_{o,t} = -(\mathbf{G}_v^H\mathbf{G}_v)^{-1}\mathbf{G}_v^H\mathbf{P}_v(\mathbf{R}^H\mathbf{R})^{-1}\mathbf{R}^H. \quad (46)$$

Therefore, in both the tuning and control stages, the control filter converges to the same optimal solution in Eqs. (29) and (46), respectively.

It is worth noting that the auxiliary filter is essential to obtain the optimal control filter in the control stage of the MVANC technique. The final auxiliary filter, as defined in Eq. (39), is composed of the primary path and virtual primary path transfer functions, which are independently defined by the positions of the physical microphones and virtual microphones, respectively. Furthermore, the optimal control filter of MVANC, as defined in Eq. (46), does not contain any transfer function related to the physical microphone location. Therefore, compared to the conventional virtual-sensing (VS) active control methods,^{36,47} the MVANC technique does not require the causal transfer function from the physical microphone to the virtual microphone and, thus, allows more flexibility in the selection of the locations of virtual and physical microphones.

IV. EFFECT OF ALTERNATIVE SENSOR-ACTUATOR CONFIGURATIONS

The optimal control filter in Sec. III has been derived on the pretext of an over- or fully determined system setup, where there are either more or an equal number of sensors and actuators. Although the effect of under-determined formulations on the optimal solution has been defined for multichannel ANC systems, it is not directly applicable to the MVANC technique due to the presence of the tuning stage. In the interest of minimizing the computational complexity, it is, therefore, worthwhile to investigate the influence of a partially under-determined implementation within the MVANC technique. Hence, four alternative configurations considering various combinations of virtual and physical sensors, as listed in Table II, are analyzed. In this section, the primary noise in both the tuning stage and control stage is denoted by random noise vector $\mathbf{V}$.

To achieve optimal attenuation of the disturbance at the virtual locations, $\mathbf{D}_v$ is given by⁵⁹

$$\mathbf{D}_v = -\mathbf{G}_v\mathbf{Y}_o \in \mathbb{C}^{N \times 1}, \quad (47)$$

where $\mathbf{Y}_o \in \mathbb{C}^{K \times 1}$ stands for the optimal control signal.

TABLE II. System characteristics of alternative sensor-actuator configurations in the tuning and control stages.

Number	Tuning stage	Control stage
Case I	$K \leq N$ Over- or fully determined	$K \leq M$ Over- or fully determined
Case II	$K \leq N$ Over- or fully determined	$K > M$ Under-determined
Case III	$K > N$ Under-determined	$K > M$ Under-determined
Case IV	$K > N$ Under-determined	$K \leq M$ Over- or fully determined

In the tuning stage, the error signal from the physical error microphones is given by

$$\mathbf{E}_p = \mathbf{G}\mathbf{Y}_o + \mathbf{D}_p \in \mathbb{C}^{M \times 1}, \quad (48)$$

where $\mathbf{D}_p \in \mathbb{C}^{M \times 1}$ is the disturbance at the physical microphones. In the control stage, with the aid of the auxiliary filter, the physical error signal remains unchanged and is given by

$$\mathbf{E}_p = \mathbf{G}\mathbf{Y}_{o,c} + \mathbf{D}_p \in \mathbb{C}^{M \times 1}, \quad (49)$$

where $\mathbf{Y}_{o,c} \in \mathbb{C}^{K \times 1}$ denotes the steady-state control signal in the control stage.

A. Case I: Over- or fully determined tuning ($K \leq N$) and control stages ($K \leq M$)

The first case shows the optimal output control signal of the over- or fully determined formulation in the tuning and control stages. The transfer function matrix of virtual secondary paths $\mathbf{G}_v \in \mathbb{C}^{N \times K}$ is assumed to be a full-rank matrix, and hence, $\mathbf{G}_v^H\mathbf{G}_v$ is invertible. In the tuning stage, the optimal control signal in Eq. (47) is derived through the least-squares (LS) approach as

$$\mathbf{Y}_o = -(\mathbf{G}_v^H\mathbf{G}_v)^{-1}\mathbf{G}_v^H\mathbf{D}_v. \quad (50)$$

The steady-state control signal of the control stage is the LS solution of Eq. (49) and is obtained as

$$\mathbf{Y}_{o,c} = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H(\mathbf{E}_p - \mathbf{D}_p), \quad (51)$$

where we also assumed the transfer function of the physical secondary path $\mathbf{G} \in \mathbb{C}^{M \times K}$ to be a full-rank matrix, and hence, $\mathbf{G}^H\mathbf{G}$ is invertible. By substituting Eq. (48) into Eq. (51), the steady-state control signal becomes

$$\mathbf{Y}_{o,c} = \mathbf{Y}_o. \quad (52)$$

Therefore, when there are fewer secondary sources than the physical and virtual microphones, the MVANC technique is able to achieve optimal control at the virtual locations.

B. Case II: Over-determined tuning stage ($K \leq N$) and an under-determined control stage ($K > M$)

In the second case, the tuning stage is over- or fully determined, where there are more control sources than sensors at the virtual locations. Hence, the optimal control signal in the tuning stage is given by Eq. (50), as defined above in case I. Since $\mathbf{G}$ is assumed to be a full-rank matrix, $\mathbf{G}\mathbf{G}^H$ is invertible. In the control stage, the steady-state control signal of the control stage is derived from the LS solution of Eq. (49) as

$$\mathbf{Y}_{o,c} = \mathbf{G}^H(\mathbf{G}\mathbf{G}^H)^{-1}(\mathbf{E}_p - \mathbf{D}_p) + [\mathbf{I}_K - \mathbf{G}^H(\mathbf{G}\mathbf{G}^H)^{-1}\mathbf{G}]\mathbf{z}, \quad \forall \mathbf{z} \in \mathbb{C}^{K \times 1}. \quad (53)$$

Substituting Eq. (48) into Eq. (53) yields

$$\mathbf{Y}_{o,c} = \mathbf{G}^H(\mathbf{G}\mathbf{G}^H)^{-1}\mathbf{G}\mathbf{Y}_o + [\mathbf{I}_K - \mathbf{G}^H(\mathbf{G}\mathbf{G}^H)^{-1}\mathbf{G}]\mathbf{z}, \quad \forall \mathbf{z} \in \mathbb{C}^{K \times 1}. \quad (54)$$

Hence,

$$\mathbf{Y}_{o,c} \neq \mathbf{Y}_o. \quad (55)$$

Therefore, when the number of secondary sources is greater than the number of physical microphones but less than the number of virtual microphones, the control system is unable to achieve optimal control at the virtual locations.

C. Case III: Under-determined tuning ($K > M$) and control stage ($K > M$)

In the third case, both the tuning and control stages are under-determined. The steady-state control signal in the control stage is given by Eq. (54) as derived in case III, which does not equate to the under-determined control signal in the tuning stage as shown in Eq. (55).

Hence, it can be concluded that once the number of secondary sources exceeds that of physical microphones, the MVANC technique fails to achieve optimal control at the desired virtual positions.

D. Case IV: Under-determined tuning stage ($K > M$) and an over-determined control stage ($K \leq M$)

The final case illustrates an under-determined tuning stage, where there are less sensors at the virtual locations than control sources. The optimal control signal in the tuning stage is derived from the LS solution of Eq. (47) as⁶⁰

$$\mathbf{Y}_o = -\mathbf{G}_v^H(\mathbf{G}_v\mathbf{G}_v^H)^{-1}\mathbf{D}_v + [\mathbf{I}_K + \mathbf{G}_v^H(\mathbf{G}_v\mathbf{G}_v^H)^{-1}\mathbf{G}_v]\mathbf{z}, \quad \forall \mathbf{z} \in \mathbb{C}^{K \times 1}, \quad (56)$$

where $\mathbf{z}$ is an arbitrary vector, and $\mathbf{I}_K$ denotes a $K \times K$ identity matrix. Since $\mathbf{G}_v$ is a full-rank matrix, $\mathbf{G}_v\mathbf{G}_v^H$ is

invertible. In the control stage, the steady-state control signal is derived from the LS solution of Eq. (49) given by

$$\mathbf{Y}_{o,c} = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H(\mathbf{E}_p - \mathbf{D}_p). \quad (57)$$

By substituting Eqs. (48) and (56) into Eq. (57), the steady-state control signal also reduces to

$$\mathbf{Y}_{o,c} = \mathbf{Y}_o. \quad (58)$$

Therefore, when the number of secondary sources are greater than the number of virtual microphones but less than the number of physical microphones, the control system is still able to achieve optimal control at the virtual locations.

V. SIMULATION WITH MEASURED TRANSFER FUNCTIONS

To verify the causal independence, as well as the effect of the alternative configurations, a series of numerical simulations are conducted with measured transfer functions from a real system. All the impulse responses of the secondary paths and primary paths used are measured from a four-channel ANC system, as shown in Fig. 7. It worth noting that the length of the control filter and the secondary path model should be longer than T_1f_s and T_2f_s , respectively; T_1 and T_2 denote the acoustic delays of the primary path and the secondary path, respectively, and f_s is the sampling rate. In the MVANC system, a relatively long length is adopted to increase the frequency resolution to deal with broadband noise. Hence, the length of the control filters and secondary path models are set to 512 and 256 taps, respectively. The system operates at a sampling rate of 16 kHz. The primary noise sources in the tuning and control stages are Gaussian white noise and uniformly distributed random noise, respectively. These noises are filtered by a bandpass filter with a frequency range of 500–1500 Hz.

In simulation A, a fully determined system is investigated, where there are four secondary sources ($K=4$), four virtual microphones ($N=4$), and four physical microphones ($M=4$), as shown in Fig. 7(a). The coefficients of the control filters in the tuning and control stages of simulation A in the steady-state are displayed in Fig. 8. The control filters in both stages, as well as their noise reduction levels, are visibly similar, as illustrated in Figs. 8 and 9(a), respectively. This result verifies the abovementioned conclusion that the MVANC technique can achieve the optimal noise control at virtual positions in a causal and fully determined setup.

To investigate the causal relation between the physical microphones and virtual microphones, the positions of the physical microphones and virtual microphones are swapped in simulation B, as illustrated Fig. 7(b). In this situation, the virtual microphones are closer to the secondary sources, and the disturbance and anti-noise waves propagate through the virtual microphone to physical microphone positions. Hence, a causal transfer function that describes the path

FIG. 8. Coefficients of the four steady-state control filters in simulation A.

signal at virtual microphone 1. The virtual microphone had apparent noise reduction when ANC is on. Furthermore, MVANC achieved almost the same error power spectra at the first virtual microphone as the conventional MCANC system.

Figure 15(a) displayed the noise reduction levels at all of the physical and virtual microphones for the MCANC, MVANC, and MCANC without VS. As expected, MVANC achieved almost the same noise reduction level at the error

microphones as the conventional MCANC. The result implied that MVANC converged to a similar control filter as the conventional MCANC, which used real microphones at the virtual locations. MVANC achieved a higher level of noise reduction performance at the virtual positions than MCANC without VS, which just adopted the physical microphones as error microphones. Furthermore, compared to MCANC without VS, MVANC achieved higher noise reduction level at the virtual microphones than at the

FIG. 9. Noise reduction in dB at the virtual microphone positions of the six simulation conditions.

physical microphones, which proved the argument that MVANC could project the quiet zone from the physical microphone to the desired location.

B. Four secondary sources, 4 virtual, and 4 physical microphones

In experiment B, the locations of the physical microphones and virtual microphones of experiment A were swapped, as illustrated in Fig. 13(b). The virtual microphones

were closer to the secondary sources than the physical microphones were. Hence, there were no causal filters that can represent the transfer functions from the physical microphones to the virtual microphones. However, as shown in Fig. 14(b), MVANC obtained a similar error power spectrum to MCANC at the first virtual microphone. Moreover, the virtual microphones in MVANC achieved similar noise reduction levels to MCANC, as shown in Fig. 15(b). Compared to MCANC without VS, MVANC still gained better noise reduction levels at the virtual microphones than the physical microphones.

FIG. 10. Coefficients of the four steady-state control filters in simulation B.

Hence, MVANC achieved satisfactory noise reduction performance even though the physical microphones did not have a causal relation to the virtual microphones.

C. Three secondary sources, four physical, and four virtual microphones

In experiment C, the number of secondary sources was reduced to three, and the number of virtual microphones and physical microphones remained at four, as

illustrated in Fig. 13(c). Hence, this experiment illustrated case I. The MVANC still obtained a similar error power spectrum to the conventional MCANC at the first virtual microphone, as shown in Fig. 14(c). The error microphones of the MCANC and MVANC reflected similar reduction levels, as illustrated in Fig. 15(c). These results indicated that MVANC still functioned well despite having fewer secondary sources than physical and virtual microphones.

FIG. 11. Steady-state control filters in the tuning (blue line) and control (yellow line) stages in simulation E.

FIG. 12. (Color online) A noise chamber with four secondary sources, four physical microphones, and four virtual microphones.

D. Three secondary sources, three physical, and four virtual microphones

In experiment D, both the numbers of secondary sources and physical microphones were set to three, and the number of virtual microphones was maintained to four, as illustrated in Fig. 13(d). Although the physical microphones and secondary sources were less than the virtual microphones, MVANC also achieved almost the same noise reduction performance as MCANC, as shown in Figs. 14(d) and 15(d). Therefore, the results of experiments C and D confirmed the conclusion of case I.

E. Three secondary sources, two physical, and four virtual microphones

In experiment E, MVANC was composed of three secondary sources and two physical and virtual microphones, as shown in Fig. 13(e). Since the number of physical microphones was less than the number of secondary sources, MVANC became under-determined in the control stage. In

FIG. 13. Top view of the noise chamber installed by a $4 \times 4 \times 4$ MVANC with different configurations: (a) $K=4$ (secondary sources), $M=4$ (physical microphones), $N=4$ (virtual microphones); (b) $K=4$, $N=4$, $M=4$; (c) $K=3$, $M=4$, $N=4$; (d) $K=3$, $M=3$, $N=4$; (e) $K=3$, $M=2$, $N=4$; (f) $K=3$, $M=4$, $N=2$.

FIG. 14. Power spectra of the error signals at the first virtual microphones of experiments A–F.

FIG. 15. Noise reduction levels at microphones in experiments A–F.

this situation, the power spectrum of MVANC deviated from that of MCANC at the first virtual microphone, as illustrated in Fig. 14(e). Moreover, noise reduction levels of MVANC at the virtual microphones were well below those of MCANC, as shown in Fig. 15(d). Therefore, this confirmed the conclusions in cases II and III, that if there were more secondary sources than physical microphones, MVANC could not achieve satisfactory noise control performance at the virtual locations.

F. Three secondary sources, four physical, and two virtual microphones

In experiment F, the number of secondary sources and physical and virtual microphones was set to three, four, and two, respectively, as shown in Fig. 13(f). In this case, MVANC exhibited a similar error power spectrum to MCANC at the first virtual error microphone, as illustrated in Fig. 14(f). The noise reduction levels of virtual and physical microphones in the MVANC were almost the same as

MCANC, as shown in Fig. 15(f). Hence, the experiment result supported the conclusion of case IV.

VII. CONCLUSIONS

The multichannel multichannel virtual-sensing active noise control (MVANC) technique revisited in this paper is a practical approach to project the quiet zone from the physical microphones to the virtual positions where it is impractical to place the real error microphones. The MVANC algorithm utilizes the auxiliary filters in a preliminary stage to record the information between the physical microphones and the locations of the virtual microphones. Subsequently, the auxiliary filter assists in noise control at the virtual locations. The frequency-domain analysis based on random primary noise shows that the MVANC technique converges to the same optimal control filter as in the FxLMS-based MCANC, which utilizes the real error microphones at the virtual locations. Moreover, as MVANC does not require the causal transfer function from the physical microphone to the virtual microphone, it can overcome the causality constraint between the locations of the physical and virtual microphones. Further analysis with an over- or fully determined system revealed that the MVANC technique is unaffected by the causality restriction. Analysis of underdetermined formulations in either or both the tuning and control stages exposes the limitations of the MVANC technique. The MVANC technique fails to control the disturbance at the desired virtual positions when there are fewer physical microphones than secondary sources. A series of simulations conducted under a number of conditions verifies the conclusions obtained in the frequency domain analysis. Furthermore, a four-channel MVANC was implemented on an open aperture, and its real-time experimental results validated the theoretical analysis and numerical simulations.

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