Improvements of FDK-type helical cone-beam CT imaging

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Improvements of FDK-type Helical Cone-beam CT Imaging

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in fulfillment of the requirements for the degree of
Doctor of Philosophy

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Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Date

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Liang Hong Zhu

To my parents

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Summary

Computed Tomography (CT) is one of the most popular medical diagnostic modalities. Besides the amazing development of CT hardware, people have also witnessed the evolution of CT imaging algorithms. At present, the CT image reconstruction algorithms remains as a hot topic in CT research area.

This thesis presents the author's research work on the 3D cone-beam CT imaging algorithms which are popular in both biomedical research and engineering applications. The FDK-type cone-beam algorithms are commended for their achievements, nevertheless they still suffer from two main limitations of the data collection: the cone-shaped projection volume and the helical scanning pitch. Once the cone-beam angle becomes relatively large or the helical scanning is relatively fast, it is difficult to avoid the artifacts on the reconstructed images. To improve the image reconstruction, research work of cone-beam imaging algorithms has been done and presented in the thesis.

To reduce the approximate imaging artifacts in large cone-angle and high speed helical scanning, this thesis presents a new algorithm named as CARP which is motivated by the PI-line based exact 3D CT reconstruction algorithms. Being different from the existing algorithms that reconstruct all the points in one plane, the CARP algorithm reconstructs each point on a unique PI-plane which is customized based on the PI-line. This algorithm can reduce the shading artifacts to a considerable extent, especially in the rim area of the image which is easier to suffer from artifacts

than the central area.

Incorporating exact reconstruction features into the approximate algorithm to improve imaging performance is another objective of author's research work. The thesis presents a new approximate cone-beam reconstruction algorithm named as ExFDK. This new 3D CT imaging algorithm is based on the well known Tuy's condition which acts as an exact reconstruction prerequisite. Instead of using the conventional full-scan or 180° half-scan data collection trajectory, a new extended half-scan trajectory is presented and utilized that satisfies the Tuy's condition. The minimum length of this extended half-scan trajectory is worked out to optimize the tradeoff between the computation workload and the imaging performance. Therefore the ExFDK algorithm can improve the image quality considerably with very limited additional workload.

The author's efforts in improving the image reconstruction of gantry-tilted CT special scanning are also presented in this thesis. A new reconstruction algorithm named as TExFDK is presented. Motivated by the ExFDK, the TExFDK algorithm intends to improve the data collection trajectory which is considered one of the main causes of image degradation of gantry-tilted helical reconstruction. This new FDK-type reconstruction algorithm is able to enhance the imaging performance of gantry-tilted system in terms of higher accuracy and fewer artifacts.

In addition, this thesis also presents the research work on the local tomography applied on the gantry-tilted system. Incorporating the local tomography is intended to meet the high safety requirement of gantry-tilted scanning. In order to improve the local imaging accuracy in the helical gantry-tilted reconstruction, the thesis presents a new data correction method which corrects the projection data by geometrical compensation.

The algorithms presented in the thesis are all based on the existing CT machine hardware. Compared to the existing algorithms, these presented algorithms are

able to improve the cone-beam imaging performance in terms of higher imaging accuracy and less severe artifacts. In addition, they reconstruct images with good computation efficiency compared with the existing cone-beam imaging algorithms. Most of the additional computations required by new algorithms can be conducted before data collection, therefore they would not affect the real-time imaging performance.

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Abbreviations

CT	Computed Tomography
1D	1 dimensional
2D	2 dimensional
3D	3 dimensional
SSR	Single Slice Rebinning
ASSR	Advanced Single Slice Rebinning
FDK	Feldkamp, Davis and Kress
FBP	Filtered Backprojection
CARP	Cone-beam approximate reconstruction on PI-plane
ExFDK	FDK-type reconstruction with extend half-scan trajectory
TExFDK	FDK-type gantry-tilted reconstruction with extend half-scan trajectory
ROI	Region of Interest
FT	Fourier Transform
IFT	Inverse Fourier Transform
SSP	Slice Sensitivity Profile
FWHM	Full Width at Half Maximum
MSE	Mean Square Error
SNR	Signal-to-Noise Ratio

Abbreviations

x

NcFDK	FDK reconstruction on nutating curved surface
FsFDK	Full scan FDK reconstruction
HsFDK	Half scan FDK reconstruction
HU	Hounsfield Unit
WT	Wavelet Transform
IWT	Inverse Wavelet Transform
TFsFDK	FDK-type gantry-tilted reconstruction with full-scan trajectory
THsFDK	FDK-type gantry-tilted reconstruction with half-scan trajectory

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Chapter 1

Introduction

Computed Tomography (CT), originally known as Computed Axial Tomography (CAT), is a medical imaging method for reconstructing the cross-section images of an object using a series of X-ray projections taken from different angles around the object [1,3–5]. The word “tomography” is derived from the Greek *tomos* (slice) and *graphein* (to write). The invention of CT is considered one of the most exciting innovations of radiology. Since Hounsfield patented the first CT scanner in 1972, a lot of researches have been done to accelerate the scanning, to improve the image quality and to enhance the scanning safety. Today, CT is an important modality of the radiological diagnosis. CT imaging can provide better visualization of specific structured soft-tissue regions, which could otherwise not be visualized satisfactorily.

1.1 Computed Tomography

A typical modern CT system is shown in Fig.1.1. It mainly consists of a projection gantry, a translation table and a high-end computer for image reconstruction and display. An X-ray projector and detector move along the circular gantry to fully scan the object on the table, as illustrated in Fig.1.2. In the past decades, CT

Figure 1.1: The modern medical CT system (a picture from Siemens website).

system experienced several significant evolutions on both hardware and software:

- **First generation:** The CT scanners in this stage used a pencil-thin X-ray beam directed to one or two detector cells. Images were achieved with *step-and-shoot* mode: the table and object have to keep still during the X-ray projecting, and a patient has to hold breath until the next translation. This seriously limits the scanning efficiency and degrades the image quality.
- **Second generation:** The detector size was increased, and the pencil-thin X-ray beam was replaced by a fan shaped beam. The step-and-shoot mode was still used but the scanning speed was increased significantly [8, 9].
- **Third generation:** The step-and-shoot scanning mode was replaced by a continuous translation mode, i.e. the X-ray source moves along a helical trajectory with respect to the object. The first helical CT system is the

Figure 1.2: CT structure: gantry, source, detector.

SIEMENS SOMATOM Plus[®] system invented in 1989 [2]. In the meantime, the fan shaped X-ray beam and single-row detector were substituted by the cone shaped beam and multi-row detector. The SIEMENS SOMATOM Volume Zoom[®] multi-slice system invented in 1998 is able to reconstruct 4 slices per rotation, and the SOMATOM Sensation 16[®] system invented in 2002 can reconstruct up to 16 slices per rotation [2]. Most of the present medical CT scanners are of the 3rd generation [1]. The *helical + volumetric* mode significantly improves the scanning efficiency and image quality. Consider the thorax examination as an example (referred to Fig.1.3): an examination that required 10 separate breath-holds if using the step-and-shoot scanning can be easily completed in only one breath-hold [8]. Therefore the artifacts yielded by the breathing motion are greatly reduced.

- **Fourth generation:** A 360° detector ring is adopted in 4th generation CT system which is fixed in the gantry other than rotating together with the X-ray source. This can avoid the detector mis-calibration (causing ring artifacts) in the 3rd generation and former CT scanners, and benefit the high speed source

Figure 1.3: The thorax cross-section images reconstructed by CT (a picture from Siemens website).

Figure 1.4: Helical CT scanning: the relative motion of X-ray source and object leads to a helical scanning trajectory of source.

rotation which is a critical requirement of cardiac tomography [10, 12–14]. Nevertheless, the 4th generation scanners are required further improvement in which the inefficient use of detectors made them considerably more expensive than 3rd generation scanners. In addition, they are more sensitive to noise because the non-fixed relationship to the x-ray source made it impossible to reject scattered radiation.

Figure 1.5: 3D CT geometry: gantry plane and off-gantry plane.

1.2 Imaging Algorithms

The evolution of CT scanner is not only in the hardware but also in image reconstruction algorithms. In terms of the raw data dimension, the reconstruction algorithms can be classified into 2D reconstruction algorithms and 3D reconstruction algorithms.

The 2D reconstruction algorithms are suited to the 1st and 2nd generation scanners which are equipped with single-row detectors and can only collect 2D data set [3]. Typically, the fan-beam reconstruction of the 2nd generation CT can be divided into two categories, the equally-spaced reconstruction and the equiangular reconstruction. Their geometry and mathematics will be presented in Section 2.2 of next chapter.

As the cone-shaped projection appeared in the 3rd generation CT scanner, the 3D reconstruction algorithms are developed and applied which differ from the 2D algorithms in the utility of the *off gantry projections*. As shown in Fig.1.5, the off gantry projections are not directed to the central row of the detector. They pass through the off gantry planes and can collect the information outside the objective plane which can significantly improve the image quality in helical and circular scanning [16–28].

The 3D reconstruction algorithms can be classified into two categories: approximate

reconstruction algorithms and exact reconstruction algorithms. Some approximate algorithms only require single-row data sets which are extracted from the original multi-row data set. The *SSR* (Single Slice Rebinning) [16] developed by Noo *et al.* and *ASSR* (Advanced Single Slice Rebinning) [17] developed by Kachelriess *et al.* are the typical rebinning algorithms. The main drawback of these algorithms is that they produce serious artifacts in the large pitch helical reconstructions [15–30]. Some approximate algorithms use multi-row 3D data sets to reconstruct images. The successful *FDK* algorithm is a typical 3D approximate algorithm, which was developed by L. A. Feldkamp, L. C. Davis and W. J. Kress in 1984 [39]. FDK algorithm is extended from the 2D fan-beam reconstruction scheme. It utilizes the off gantry projections and manage to produce much better image quality than the rebinning algorithms, especially in the case of large pitch helical scanning [39, 63, 66–70]. The mathematics of FDK reconstruction will be presented in the next chapter.

The 3D exact reconstruction algorithms are classified into two categories in terms of the mathematical basics: the algorithms based on the 3D inverse Radon Transform [40–45, 49] and the algorithms based on the 1D filtered backprojection (FBP) [51–57]. The first exact algorithm using 3D reverse Radon Transform was proposed by Grangeat in 1991 [40]. The Grangeat-type exact algorithms provide a good insight into the helical cone-beam reconstruction problem, but they easily yield discretionary errors during the numerical implementation. Although some of them had been adopted in practice, they are more of theoretical than of practical significance. Exact algorithms of the 2nd category are based on the algorithm developed by A. Katsevich in 2002 [51]. This algorithm is an important breakthrough of helical cone-beam reconstruction since it adopts 1D shift-invariant filtering that is much more computationally efficient than the Grangeat-type reconstruction [56, 57]. In 2004 Pan and Zou developed an alternative exact 1D FBP algorithm [56–59], and Sidky proposed an extended work in 2005 [60]. These algorithms reconstruct the data volume by calculating the values of the PI-lines, which extended Katsevich’s idea.

Chapter 4 and Chapter 5 present author's work on incorporating the exact reconstruction into the approximate algorithm to improve the reconstruction accuracy.

1.3 Motivations and Objectives of the Thesis

1.3.1 Motivations

The 3D CT reconstruction algorithm is one of the most attractive topics in medical imaging research. So far, the existing approximate algorithms still suffer from the imaging artifacts in large cone-beam angle and large helix pitch scanning, while the exact algorithms are being concerned about the overwhelming computation workload and imaging noise. Both the approximate and exact reconstruction algorithms are necessary to be further improved in many aspects.

Most of the modern CT systems adopt the approximate reconstruction algorithm, because the scanning time and working efficiency is much more concerned in real applications than in academic research. Approximate reconstruction algorithms have become an active topic of CT imaging research since last century. The FDK-type algorithms are popular in both the theoretical research and the practical applications. Due to the extremely rapid development of CT hardware, the existing FDK-type algorithms have to be further improved to satisfy the relatively large volume and high speed scanning. In the past decade, a lot of efforts have been done in improving the FDK-type algorithms. In 2003 Sourbelle and Kalender proposed an FDK-type reconstruction scheme incorporating the helix tangential filtering [66]. In 2006 Tang and Hsieh proposed a new 3D weighting scheme to improve the FDK reconstruction [67], and Hu *et al.* suggested to reconstruct nutating curved surfaces instead of the normal transversal planar slice [68]. The pursuit of better image quality, in terms of less severe artifacts and noise, will continue inspiring the CT algorithm

research.

On the other hand, research on the exact 3D reconstruction algorithms have been carried out since the last decade. The exact algorithms can produce much higher reconstruction accuracy than the approximate algorithms. The main differences between these two type of algorithms are how to conduct the data filtering and how to handle the backprojection. The features of exact algorithms, such as the PI-line filtering and data collection requirement, are helpful in improving the approximate reconstruction accuracy without large amount of computation required by the exact algorithms. In the past decade, some probing research work has been done to improve the approximate algorithm by incorporating some features of exact reconstruction [30, 33, 45, 68]. The research work on this topic is presented in this thesis.

The rapid development of data processing capability inspires people to invent and use large volumetric scanners. The SIEMENS SOMATOM Sensation-16® scanner built in 2002 is only equipped with a 16-row detector, while a new SIEMENS scanner with 96-row detector will soon be applied in real medical field [2]. The continuous pursuit of larger volumetric scanning has raised more and more concern on the X-ray radiation which is seriously harmful to patient body. Local tomography is one of the most effective techniques for reducing the radiation dose. In both laboratory and real medical field, local tomography has been proved reducing the X-ray dose over 40% in some cases of cerebrum and thorax scanning [2, 3]. Local tomography will remain as an appealing research topic in near future. The author's work on this topic is presented in the thesis.

In many modern medical scanning, the tilted CT scanning is required with respect to the patient and couch. For instance, in the cerebrum scanning the projections directed to patient's eyeballs or teeth should be avoided. Most of the modern medical scanners are equipped with tiltable gantry. Due to the tilting of scanning geometry,

the scanning data has to be corrected and the conventional reconstruction has to be modified. A few efforts have been done to correct the scanning data [103–106]. The author’s work on this topic focuses on improving the accuracy of gantry-tilted helical FDK-type reconstruction.

1.3.2 Objectives

Motivated by the above topics of CT research, the author’s work is to achieve these following objectives:

- Improve the FDK-type approximate reconstruction performance geometrically in terms of reducing the artifacts in the large cone-angle and high speed helical scanning;
- Incorporate the exact reconstruction into the FDK-type reconstruction to improve the approximate reconstruction accuracy;
- Improve the gantry-tilted CT imaging quality in terms of reducing artifacts and enhancing reconstruction accuracy;
- Improve the local tomography in FDK-type approximate reconstruction.

1.4 Contributions of the Thesis

The research work presented in this thesis is mainly on the improvement of FDK-type approximate CT image reconstruction at the following aspects:

- **Improve the FDK reconstruction geometry therefore reduce the approximate imaging artifacts at the large cone-angle and high speed helical scanning.** The main limitation of the existing FDK-type imaging

algorithms is that they would unavoidably produce shading artifacts in the large cone-angle and large helical pitch scanning. The research work to further improve the reconstruction performance is presented in Chapter 3. A new FDK-type algorithm named as CARP algorithm is proposed. This algorithm reconstructs each points on its unique PI-plane which is decided by each point's unique PI-line. Compared to the existing algorithms, the CARP algorithm produces less shading artifacts therefore it can improve the large cone-angle and high helical pitch scanning.

- **Improve the approximate reconstruction by incorporating the exact reconstruction.** Chapter 4 presents the ExFDK algorithm which is for more accurate approximate reconstruction. This cone-beam helical CT algorithm is based on an extended half-scan helix which satisfies the Tuy's exact reconstruction condition. Compared to the existing approximate and exact reconstruction algorithms, ExFDK can considerably enhance the helical cone-beam reconstruction accuracy with very limited additional computation.
- **Produce better image quality of gantry-tilted scanning.** Chapter 5 presents the research work on gantry-tilted helical cone-beam imaging. The new TExFDK algorithm is presented in this chapter. Motivated by the ExFDK, this algorithm also incorporates the exact reconstruction features to produce a better gantry-tilted helical reconstruction performance.
- **Enhance the local tomography performance.** Chapter 5 also presents the research work on improvement of the gantry-tilted local tomographic imaging. The work aims at these three objectives: to improve gantry-tilted local scanning trajectory, to improve the local projection accuracy, and to improve the local data correction. To implement these, the scanning mechanism is investigated in order to optimize the local scanning and minimize the projection dose, and an efficient data correction scheme is proposed based on the

relatively small size of Region of Interest (ROI).

1.5 Outline of the Thesis

The outline of this thesis is as follows:

In Chapter 1, the background of the research work is described, followed by the research motivation and objectives. The main contributions of this thesis are listed thereafter.

In Chapter 2, the mathematics of CT image reconstruction is reviewed. The approximate and exact reconstruction algorithms, especially the FDK algorithm are described in details.

In Chapter 3, the CARP algorithm is presented. The mathematical details of reconstruction are presented in this chapter, followed by the phantom imaging simulations with CARP algorithm and the existing approximate and exact reconstruction algorithms.

In Chapter 4, the ExFDK algorithm is presented. The derivation of the proposed scanning trajectory is described, followed by the new reconstruction procedures. Various phantom imaging simulations and noise analysis are presented to compare the ExFDK algorithm with the existing approximate and exact algorithms.

Chapter 5 first presents the special geometry and reconstruction algorithm of gantry-tilted helical cone-beam scanner. Then it is separated into two main sections. The first section presents the author's work on the local tomographic imaging applied to the gantry-tilted scanning. The second section proposes the TExFDK algorithm. The derivation of the proposed scanning trajectory is described. The phantom imaging simulations and noise analysis are presented and compared the TExFDK with the existing gantry-tilted imaging algorithms.

In the last chapter, the research work is concluded and discussed, including their advantages and limitations. In the end, the future research work in this area is recommended.

Chapter 2

Tomographic Image Reconstruction

This chapter presents the mathematics of computed tomographic reconstruction. Although the basic formula of CT image reconstruction was developed by Radon in 1900s, people had interests on the tomographic imaging algorithms and started the research work in this area only after the first CT machine was born in 1972.

2.1 Filtered Backprojection

This section focuses on the mathematics of tomographic image reconstruction. The details of how to achieve an cross-section image from X-ray projections are presented.

2.1.1 Line Integral

The X-ray projections would attenuate while passing through the object. Since the attenuation depends on each tissue's properties, the X-ray projection can collect

the information inside the object. Denote $f(x, y)$ as object, a single beam's attenuation while passing through $f(x, y)$ can be presented, which is the well known Beer-Lambert law [3], as:

$$\int_L f(x, y) dl = -\ln \frac{I_{\text{out}}}{I_{\text{in}}}, \quad (2.1)$$

where I_{in} and I_{out} respectively denote the ray beam's intensity before and after penetrating the object, and $\int_L dl$ denotes a line integral. Referring to Fig.2.1 and denoting θ as the source rotation angle, the line integral through $L : x \cos \theta + y \sin \theta = t$ can be presented as an integral with the well known Dirac Delta function, as follows [5, 6]:

$$D(t, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - t) dx dy, \quad (2.2)$$

where for any continuous function $f(t)$ the Delta function $\delta(t)$ satisfies $\int_{-\infty}^{\infty} f(t) \delta(t - a) dt = f(a)$ [7]. One projection consists of a set of parallel or fan-shaped or cone-shaped line integrals. Projections from different rotation angles can achieve a cross-section image. The parallel-beam projection is the simplest one which is formed by a set of parallel ray beams at a constant angle. A cross-section image can be directly achieved via “Fourier Transform (FT) + Inverse Fourier Transform (IFT)” of a parallel-beam projection data set, according to the well-known *Fourier Slice Theorem* [3–6].

Figure 2.1: Line integral of object $f(x, y)$.

2.1.2 Fourier Slice Theorem and Parallel-beam Reconstruction

Recall the 1D and 2D Fourier Transform:

$$F(u) = \text{FT}_1[f(x)] = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx, \quad (2.3)$$

$$F(u, v) = \text{FT}_2[f(x, y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(ux+vy)} dx dy. \quad (2.4)$$

Fourier Slice Theorem can be stated that *the 1D Fourier Transform of a parallel-beam projection of object $f(x, y)$ at θ , $\text{FT}_1[D(t, \theta)]$, gives the values of $\text{FT}_2[f(x, y)]$ at a line [3].* This theorem indicates that the values of all points of $\text{FT}_2[f(x, y)]$ can be achieved from the projections at $\theta_1, \theta_2, \dots, \theta_k$. Therefore $f(x, y)$ can be achieved through the IFT.

The 1D Fourier Transform of parallel-beam projection data at θ is as follows

$$F(\omega, \theta) = \int_{-\infty}^{\infty} D(t, \theta) e^{-j2\pi\omega t} dt. \quad (2.5)$$

Referred to Eq.2.4, the object $f(x, y)$ can be obtained by 2D IFT of $F(u, v)$, as

$$f(x, y) = \text{IFT}_2[F(u, v)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv. \quad (2.6)$$

If using the polar coordinates, one can have $u = \omega \cos \theta$, $v = \omega \sin \theta$, $du dv = \omega d\omega d\theta$.

As a result the Eq.2.6 can be written as

$$\begin{aligned} f(x, y) &= \int_0^{2\pi} \left[\int_0^{\infty} F(\omega, \theta) e^{j2\pi\omega t} \omega d\omega \right] d\theta \\ &\quad (t = x \cos \theta + y \sin \theta) \\ &= \int_0^{\pi} \left[\int_{-\infty}^{\infty} F(\omega, \theta) |\omega| e^{j2\pi\omega t} d\omega \right] d\theta. \end{aligned} \quad (2.7)$$

Therefore, the object $f(x, y)$ can be reconstructed following: (1), parallel-beam projection (Eq.2.1); (2), 1D Fourier Transform (Eq.2.5); (3), 2D IFT (Eq.2.7). The operation in Eq.2.7 is known as *Filtered Backprojection* (or FBP) which means the filtered projection data (filtered by $|\omega|$) is projected inversely and angle-wisely to construct the object [3].

The filtered backprojection can also be implemented by the means of convolution in time domain, as follows:

$$f(x, y) = \int_0^{\pi} Q(x \cos \theta + y \sin \theta, \theta) d\theta, \quad (2.8)$$

where

$$Q(t, \theta) = D(t, \theta) * h(t), \quad (2.9)$$

and $h(\cdot)$ is the ramp filter, which is the IFT of $|\omega|$ in Eq.2.7.

Figure 2.2: Fan shaped beam CT scanning. a, equiangular; b, equally-spaced.

2.2 Fan-beam Reconstruction

The parallel-beam projection is more of research works than practical application [3]. In the 2nd development stage of CT scanner, the fan-beam scanning and reconstruction was introduced. For reconstruction with FBP method, the fan-beam projection data set can be easily rebinned into a parallel-beam projection data set.

In terms of the detector designs, the fan-beam CT scanners can be classified into the *Equiangular* scanners and the *Equally-spaced* scanners, as shown in Fig.2.2. For the equiangular detector, the X-ray source is exactly the circle center of the detector arc. Therefore the projection data is in the form of $D_{FA}(\beta, \theta)$, where β denotes the fan-angle of a ray beam. $D(\beta, \theta)$ can be rebinned into a parallel-beam data set $D(t, \theta)$ as follows:

$$\begin{aligned} D_{FA}(\beta, \theta) &= D(R \sin \beta, \theta + \beta), \\ D(t, \theta) &= D_{FA}\left(\arcsin \frac{t}{R}, \theta - \arcsin \frac{t}{R}\right), \end{aligned} \quad (2.10)$$

where R is the X-ray source rotation radius. Based on the parallel-beam recon-

struction (Eq.2.8 and Eq.2.9), the equiangular fan-beam reconstruction formula is as follows:

$$f(x, y) = \int_0^{2\pi} \frac{R}{t^2 + (R - s)^2} Q_{FA} \left(\arctan \frac{t}{R - s}, \theta \right) d\theta, \quad (2.11)$$

where $t = x \cos \theta + y \sin \theta$, $s = -x \sin \theta + y \cos \theta$, and Q_{FA} can be presented as a convolution [6],

$$Q_{FA}(\beta, \theta) = \left(\cos \beta D_{FA}(\beta, \theta) \right) * \left(\frac{\beta^2}{2 \sin^2 \beta} h(\beta) \right). \quad (2.12)$$

The equally-spaced fan-beam scanner uses a planar detector. The projection data is recorded in the form of $D_{FS}(k, \theta)$, which can be rebinned into a parallel-beam data set $D(t, \theta)$ as follows:

$$\begin{aligned} D_{FS}(k, \theta) &= D \left(\frac{Rk}{\sqrt{k^2 + d_{sd}^2}}, \theta + \arctan \frac{k}{d_{sd}} \right), \\ D(t, \theta) &= D_{FS} \left(\frac{td_{sd}}{\sqrt{R^2 - t^2}}, \theta - \arcsin \frac{t}{R} \right). \end{aligned} \quad (2.13)$$

where d_{sd} is the distance between X-ray source and the planar detector. Similarly as Eq.2.11, the equally-spaced fan-beam reconstruction is as follows:

$$f(x, y) = \int_0^{2\pi} \frac{1}{U^2} Q_{FS} \left(\frac{t d_{sd}}{R - s}, \theta \right) d\theta, \quad (2.14)$$

where $t = x \cos \theta + y \sin \theta$, $s = -x \sin \theta + y \cos \theta$, $U = \frac{R-s}{R}$, and

$$Q_{FS}(k, \theta) = \left[\frac{d_{sd}}{\sqrt{k^2 + d_{sd}^2}} D_{FS}(k, \theta) \right] * \frac{1}{2} h(k). \quad (2.15)$$

The ratio $d_{sd}/\sqrt{k^2 + d_{sd}^2}$ denotes the cosine of the fan-angle corresponding to the objective point (x, y) . Note that the minimum angular interval of a complete fan-

Figure 2.3: Cone-beam projection. A cone-shaped beam can be looked as a convergent stack of fan-shaped beams.

beam data set is “ $\pi + 2\sigma$ ” (σ is the maximum fan-angle, the minimal σ to cover the object equals to $\arcsin \frac{r}{R}$), which ensures the data set to be rebinned into a $[0, \pi]$ parallel-beam data set for a reconstruction [21].

2.3 FDK Cone-beam Reconstruction

The 3rd generation CT scanner is equipped with multi-row detector which enables the cone-shaped volumetric projection and, more importantly, enables the off-gantry plane reconstruction. However, the conventional 2D fan-beam reconstruction schemes would unavoidably show artifacts in the reconstructed off-gantry image. A lot of efforts have been done to improve the 2D reconstruction performance in cone-beam scanning [16–25]. However, those algorithms still yield serious artifacts in the 3D cone-beam volumetric reconstruction.

To improve the off-gantry reconstruction performance, the 3D approximate and exact reconstruction algorithms have been developed. The FDK algorithm developed in 1984 [39] is a typical and most popular approximate algorithm. FDK algorithm is extended from 2D fan-beam reconstruction scheme. Referred to Fig.2.3, the cone-

shaped beam can be considered a stack of fan shape beams and each fan-beam projection is recorded by the multi-row detector. The cone-beam projection data set is denoted by $D(\beta, n, \theta)$ where (β, n) index the cells of the multi-row equiangular detector. Based on Eq.2.11 and Eq.2.12, the FDK reconstruction of cone-beam projection data is as follows:

$$f(x, y, z) = \int_0^{2\pi} \frac{R}{t^2 + (R - s)^2} Q\left(\arctan \frac{t}{R - s}, \frac{z d_{sd}}{R - s}, \theta\right) d\theta, \quad (2.16)$$

where $t = x \cos \theta + y \sin \theta$, $s = -x \sin \theta + y \cos \theta$, and Q is the weighted and filtered data set of projection data $D(\beta, n, \theta)$, as

$$Q(\beta, n, \theta) = \left[\frac{d_{sd} \cos \beta}{\sqrt{d_{sd}^2 + n^2}} \cdot D(\beta, n, \theta) \right] * \left[\frac{\beta^2}{2 \sin^2 \beta} h(\beta) \right]. \quad (2.17)$$

For the equally-spaced cone-beam projection data, the reconstruction can be extended from equally-spaced fan-beam reconstruction (Eq.2.14 and Eq.2.15), as follows:

$$f(x, y, z) = \int_0^{2\pi} \frac{1}{U^2} Q\left(\frac{t d_{sd}}{R - s}, \frac{z d_{sd}}{R - s}, \theta\right) d\theta, \quad (2.18)$$

where

$$t = x \cos \theta + y \sin \theta, \quad s = -x \sin \theta + y \cos \theta, \quad U = \frac{R + x \sin \theta - y \cos \theta}{R}, \quad (2.19)$$

and

$$Q(m, n, \theta) = \left[\frac{d_{sd}}{\sqrt{d_{sd}^2 + m^2 + n^2}} \cdot D(m, n, \theta) \right] * \frac{1}{2} h(m). \quad (2.20)$$

The ratio $(d_{sd} \cos \beta / \sqrt{d_{sd}^2 + n^2})$ in Eq.2.17 and $(d_{sd} / \sqrt{d_{sd}^2 + m^2 + n^2})$ in Eq.2.20 are the approximation ratios and denote the cosine of the angle between the central ray beam and the beam passing through the objective point (x, y, z) . To reconstruct an off-gantry point, FDK algorithm adopts these ratios to approximate the

Figure 2.4: Helical cone-beam reconstruction with 2D rebinning algorithm (SSR and ASSR) and FDK algorithm. The results of left column are of helical scanning with pitch=0.3 and a 6-row detector, while the results of right column are of helical scanning with pitch=0.5 and a 8-row detector.

off-gantry projections to 2D fan-beam projections which can be reconstructed with 2D rebinning algorithms. Many simulations have demonstrated the advantages of FDK algorithm in handling the helical cone-beam reconstruction over those 2D rebinning algorithms. Fig.2.4 presents the helical cone-beam reconstructed images, which is displayed in grey window $[0.95, 1.05]$, of a simulated Turbell Clock Phantom with FDK and 2D rebinning algorithms. It can be seen that the FDK reconstruction results show much less shade and stripe artifacts than those of 2D rebinning algorithms in helical volumetric reconstruction.

It is noted that the FDK reconstruction will be exact if and only if the scanning is circular and the objective slice is on the circular plane [39] (in this case it becomes a 2D fan-beam reconstruction). The FDK-type reconstruction algorithms presented in the thesis are all for the helical scanning. The circular FDK reconstruction is not

Figure 2.5: Helical cone-beam scanning.

covered in the thesis.

2.4 Helical Cone-beam Scanning

The 3rd generation CT scanners enables the X-ray source moving at a helical trajectory with respect to the object. The scanning helix can be presented as

$$\begin{cases} x = R \cos(\theta + \theta_0); \\ y = R \sin(\theta + \theta_0); \\ z = h(\theta - \theta_0) \end{cases} \quad (2.21)$$

where θ_0 is the source rotation angle when $z = 0$, and h presents the ratio of translation distance over the rotation angle. Denote W as the axial width of detector.

Figure 2.6: The sampling region of a 180° half-scanning, illustrated in terms of the fan-angle β and source rotation angle θ . The bold-lined trapezoid denotes the minimum scanning interval for each beam within $[-\sigma, \sigma]$.

The helix pitch p can be calculated as

$$p = \frac{2\pi h}{W}. \quad (2.22)$$

The helical cone-beam scanner is illustrated in Fig.2.5. In last decade the modern medical CT scanner experienced a quick development. The modern helical volumetric scanner can adopt high pitch value and equip large detector with row number up to 96. The basic FDK helical reconstruction algorithm is hard to satisfy such quick and large volume scanning. A lot of efforts have been done to pursue a higher performance of approximate cone-beam reconstruction [63, 66–70].

2.4.1 Cone-parallel Rebinning

The minimum angular interval of a data set for approximate fan-beam reconstruction is $\pi + 2\sigma$, where σ is the maximum fan angle. This angular interval ensures that the scanning range of every objective point is not less than 180° , which is well known as *180° Half-scan* [15, 61, 62, 78–80]. Fig.2.6 presents the relationship between the fan angle β and the rotation angle θ in half-scanning. The bold-lined trapezoid denotes

Figure 2.7: Conjugate ray beams in 2D fan-beam scanning. A beam fan-angle β is set to be positive while the beam is at the anti-clockwise side of the central beam, and be negative at the other side.

the minimum data set needed for a short-scan reconstruction. The data in the two dark triangles are *conjugate* and redundant to each other. As show in Fig.2.7, a pair of conjugate ray beams coincide with each other in opposite directions. For a ray beam with index (β_1, θ_1) , its conjugate beam has index (β_2, θ_2) where

$$\beta_2 = -\beta_1, \quad \theta_2 = \begin{cases} \theta_1 + \pi - 2\sigma, & \beta_1 > 0 \\ \theta_1 - \pi - 2\sigma, & \beta_1 < 0 \end{cases} \quad (2.23)$$

A ray beam's fan-angle β is assumed positive while the beam is at the anti-clockwise side of the central beam (the beam through the isocenter), and negative at the other side.

This redundancy of the conjugate beams would yield image distortion if the conventional reconstruction scheme was adopted. To solve this problem, Dennis Parker proposed a weighting scheme before the FBP [78]. Compared to other weighting schemes, *Parker's Weighting* has better performance by minimizing the discontinu-

ities of the weighting factors at the borders. Now Parker's Weighting is popularly used, and the weighting factors are as follows (in the case of equiangular cone-beam projection):

$$w(\beta, \theta) = \begin{cases} \sin^2 \left(\frac{\pi}{4} \cdot \frac{\theta - \theta_0}{\sigma - \beta} \right), & \theta_0 \leq \theta \leq \theta_0 + 2\sigma - 2\beta \\ 1, & \theta_0 + 2\sigma - 2\beta \leq \theta \leq \theta_0 + \pi - 2\beta \\ \sin^2 \left(\frac{\pi}{4} \cdot \frac{\pi + 2\sigma - \theta + \theta_0}{\beta} \right), & \theta_0 + \pi - 2\beta \leq \theta \leq \theta_0 + \pi + 2\sigma \end{cases} \quad (2.24)$$

where θ_0 is the initial angle of the half-scan. For two conjugate beams (β_1, θ_1) and (β_2, θ_2) , their weighting factors are complementary to each other,

$$w(\beta_1, \theta_1) + w(\beta_2, \theta_2) = 1. \quad (2.25)$$

Based on Parker's Weighting, in 2000 Silver proposed the *Silver's Weighting* for the short scanning whose data collecting trajectory is longer than $\pi + 2\sigma$ [79, 80]. It is presented as follows:

$$D_{pw}(m, n, \theta) = w(m, \theta) D_w(m, n, \theta), \quad (2.26)$$

where

$$w(m, \theta) = \begin{cases} \sin^2 \left(\frac{\pi}{4} \cdot \frac{\theta - \theta_1}{\sigma - \Gamma} \right), & \theta_1 \leq \theta \leq \frac{\theta_2 + \theta_1}{2} - \frac{\pi - 2\sigma}{2} - 2\Gamma \\ 1, & \frac{\theta_2 + \theta_1}{2} - \frac{\pi - 2\sigma}{2} - 2\Gamma \leq \theta \leq \frac{\theta_2 + \theta_1}{2} + \frac{\pi - 2\sigma}{2} - 2\Gamma \\ \sin^2 \left(\frac{\pi}{4} \cdot \frac{\pi + 2\sigma - \theta + \theta_1}{\Gamma} \right), & \frac{\theta_2 + \theta_1}{2} + \frac{\pi - 2\sigma}{2} - 2\Gamma \leq \theta \leq \theta_2 \end{cases} \quad (2.27)$$

where $\Gamma = \frac{1}{4}(\theta_2 - \theta_1 - \pi) + \frac{1}{2} \arctan \frac{m}{d_{sd}}$.

The weighting procedure of data set $D(\beta, n, \theta)$ is as follows:

$$D_w^f(\beta, n, \theta) = [w(\beta, \theta) D(\beta, n, \theta)] * h(\beta). \quad (2.28)$$

Figure 2.8: Rebin the cone-beam data set to cone-parallel data set: (a), original cone-beam data set; (b), new data set collected by virtual detector.

It is noted in Eq.2.24 and Eq.2.27 that the initial angle θ_0 is one of the variables of the Parker's and Silver's weighting factors, which means the weighting factors are related with the positions of data sets. Since this weighting is also related with β and should be implemented before the filtering, filtering of projection data has to be repeated. Chapter 3 presents a new cone-beam approximate reconstruction algorithm that reconstructs different objective points with different half-scan data sets. This algorithm avoids the pointwisely β -dependent weighting by adopting the *cone-parallel rebinning* before the FBP [25, 64, 65, 67, 70]. With this technique the native cone-beam projection data set is rebinned row-by-row into a "3D parallel" data set, as illustrated in Fig.2.8. This approximate cone-beam imaging algorithm is developed from FDK-type cone-beam algorithms. However, since the cone-beam data set has been rebinned to a parallel-beam data set before the FBP, this algorithm is not properly referred to FDK-type algorithm which always conducts the backprojection with the cone-beam data set.

The cone-parallel rebinning of an equiangular cone-beam data set $D(\beta, n, \theta)$ is as

follows:

$$\begin{aligned} D(\beta, n, \theta) &= D_p(R \sin \beta, n - h\beta, \theta - \beta), \\ D_p(t, n, \theta) &= D\left(\arcsin \frac{t}{R}, n + h \arcsin \frac{t}{R}, \theta + \beta\right). \end{aligned} \quad (2.29)$$

After the whole data set being rebinned into the cone-parallel projection data, each point can be reconstructed from a cone-parallel data set of 180° angular interval. No redundant (conjugate) data exists in this 180° cone-parallel data set, except the data at the beginning and ending angle. Thus the θ -dependent weighting such as the Parker's weighting and Silver's weighting are not necessary for the half-scan reconstruction. It simply need to weight the data at these two angles with 0.5, and weight the others with 1.0.

The cone-parallel rebinning is considered a data pre-processing before the filtered backprojection (FBP). It would benefit the image reconstruction by avoiding the pointwise conjugate data weighting which may cost large computation source for this reconstruction. Furthermore, after the cone-parallel rebinning the row-wise filtering of the cone-parallel data set can be considered the helix tangential filtering onto the original cone-beam data [67], which can improve the reconstruction accuracy significantly. Refer to Eq.2.29, the rebinning reforms the data with very slight and simple computation, which would not yield additional risk of computational error or imaging noise onto the cone-beam FBP reconstruction.

2.4.2 3D Conjugate Weighting

For the helical cone-beam reconstruction with FDK-type algorithms, the conjugate ray beam can be weighted similarly as in 2D fan-beam reconstruction. In FDK reconstruction of a point (x, y, z) , every ray beam passing through that point would be measured. Because of the helical source trajectory, the conjugate ray beams will

Figure 2.9: Two conjugate ray beams in helical scanning. The conjugate beams may not coincide with each other in helical scanning.

no longer coincide with each other, as shown in Fig.2.9. The angle between the ray beam and the objective transversal slice is named as *off-plane angle*, denoted by α . This angle directly determines how far the beam's projection on detector leaves the gantry plane, i.e. determines the 2nd index of that beam's projection data. Therefore two conjugate ray beams indexed as (β_1, n_1, θ_1) and (β_2, n_2, θ_2) may also be indexed as $(\beta_1, \alpha_1, \theta_1)$ and $(\beta_2, \alpha_2, \theta_2)$.

Recall Eq.2.17 and Eq.2.20 in Section 2.3, FDK algorithm approximates the off-gantry projection as 2D fan-beam projection inside the gantry plane. It is clear that the off-plane angle of a ray beam directly determines the approximation ratio, i.e. a small off-plane angle leads to a good approximation while a large off-plane angle leads to a rough approximation. Tang proposed an α -dependent weighting scheme for the conjugate ray beam pairs, which gives a favorable weight to the beam with small off-plane angle and an unfavorable weight to the beam with large off-plane angle [67]. The weighting operation is the same as Eq.2.28 except that the 2D weighting factor $w(\beta, \theta)$ is replaced by a 3D weighting factor $w_3(\beta, n, \theta)$.

The 3D weighting factor $w_3(\beta, n, \theta)$ of the conjugate projection data (indexed as

Figure 2.10: Data filtering at the helix-tangential direction.

(β_1, n_1, θ_1) and (β_2, n_2, θ_2) can be expressed in the form

$$\begin{aligned} w_3(\beta_1, n_1, \theta_1) &= w_3(\beta_1, \alpha_1, \theta_1) = \frac{w(\beta_1, \theta_1) g(\alpha_2)}{w(\beta_1, \theta_1) g(\alpha_2) + w(\beta_2, \theta_2) g(\alpha_1)}, \\ w_3(\beta_2, n_2, \theta_2) &= w_3(\beta_2, \alpha_2, \theta_2) = \frac{w(\beta_2, \theta_2) g(\alpha_1)}{w(\beta_1, \theta_1) g(\alpha_2) + w(\beta_2, \theta_2) g(\alpha_1)}. \end{aligned} \quad (2.30)$$

where $w(\beta, \theta)$ is the conventional 2D weighting factor, e.g. the Parker's Weighting factor. $g(\alpha)$ is a monotonic increasing function of off-plane angle α , and $g(\alpha) \in (0, 1)$

$$0 < g(\alpha_1) < g(\alpha_2) < 1, \text{ while } \alpha_1 < \alpha_2. \quad (2.31)$$

The weighting factors of conjugate projection data are also complementary to each other,

$$w_3(\beta_1, \alpha_1, \theta_1) + w_3(\beta_2, \alpha_2, \theta_2) = 1. \quad (2.32)$$

2.4.3 Helix Tangential Filtering

The helix-tangential filtering technique was originally suggested by Yan and Leahy [63] in 1992, and developed by Sourbelle and Kalender [66] in 2003. The idea is

referred to Fig.2.10. This technique can effectively enhance the axial resolution of helical cone-beam reconstruction. It has been adopted in a few works on cone-beam approximate reconstruction [64–68], and it is also adopted in the new algorithms presented in Chapter 3 and 4.

Referred to Section 2.4.1, it is noted that the cone-parallel rebinning could naturally produce the helix-tangential filtering. In the rebinned cone-parallel data set, the data of the parallel beams with the same source rotation angle and same cone-angle will be filtered together, referred to Fig.2.8. These data's positions at axial direction vary along with the rotating source. Look at Eq.2.29, the first and second index of $D_p(\cdot)$ change following

$$\left. \frac{\delta n}{\delta t} \right|_{[D_p]} = \left. \frac{\delta(n_0 + h \arcsin \frac{t}{R})}{\delta(\arcsin \frac{t}{R})} \right|_{[D]} = h. \quad (2.33)$$

This means the parallel data in one projection and with the same cone-angle change positions along the source moving direction, i.e. the helix-tangential direction. Therefore the helix-tangential filtering is a favorable byproduct of cone-parallel rebinning.

In following contents of the thesis, several approximate helical cone-beam reconstruction algorithms will be presented. All these algorithms are pursuing: a, better imaging performance with less severe artifacts in higher pitch and larger cone-angle scanning; b, smooth imaging with less noise; c, imaging with better computation efficiency.

Chapter 3

Approximate Algorithm for Cone-beam Reconstruction on PI-plane

In this chapter an improved approximate cone-beam reconstruction algorithm is presented, namely the *CARP* algorithm. To suppress the cone-beam artifacts in the relatively large cone-angle scanning, this algorithm conducts a pointwise reconstruction on *PI-planes*. Section 3.2 defines this reconstruction planes, and Section 3.3 presents the implementation of this algorithm. In Section 3.4 this algorithm is evaluated with computer simulated comparisons. Section 3.5 and 3.6 ends up this chapter with discussions and conclusions.

3.1 Introduction

As depicted in Section 2.4, the helical cone-beam CT scanner is developed for rapid scanning with high axial image resolution for medical diagnosis. Most of the mod-

ern CT scanners in medical field adopt the approximate algorithms for cone-beam reconstruction. Compared to the exact algorithms, the approximate algorithms can provide feasible compromise between the image quality and computational efficiency.

There have been many approximate reconstruction algorithms developed in the last decade, including the 2D rebinning algorithms and the Feldkamp-type (FDK) 3D reconstruction algorithms. To enhance the reconstruction performance some non-transversal or non-planar reconstruction methods have been suggested [15–18]. However, these existing algorithms would unavoidably produce artifacts in the reconstruction of large cone-angle scanning. During the helical scanning, the distance between the ray source and the reconstruction slice would affect the performance of approximate reconstruction [15–18]. The FDK algorithm is good at 3D cone-beam reconstruction. However, the transversal slice reconstruction with FDK algorithm would unavoidably yield visible artifacts while the helix pitch or the projection cone-angle is considerably large.

Recall the definition of *off-plane angle* in Section 2.4.2: the angle between the ray beam and the objective transversal slice. The off-plane angles of points at slice rim area is larger than that of the points at central area, which can be seen in Fig.2.5. The off-plane angle directly affects the accuracy of approximate reconstruction, referred to Section 2.2 and 2.3. The large off-plane angle would result serious artifacts. This directly limits the employment of the large cone-angle scanning and therefore limits the scanning speed of the 3rd generation scanners.

To solve this problem, M. Kachelriess *et al.* proposed the well known ASSR algorithm which implements the 2D rebinning reconstruction on a tilted plane which is close to 180° half-scan helix [17]. ASSR algorithm can be applied in a faster helical scanning and can yield less severe artifacts if comparing with conventional transversal reconstruction. However, since ASSR is based on the simple 2D data rebinning, it would still produce visible artifacts while being applied in the scanning with large

helix pitch and large cone-angle.

In Section 3.2 an improved approximate cone-beam imaging algorithm is presented. This algorithm reconstructs transversal objective slice directly by calculating each point at its unique customized plane, which is named as PI-plane. Each PI-plane passes through the corresponding point's *PI-line* and the mid point of the corresponding *PI-helix*.

1. *PI-line* is defined as a straight line segment passing this point and connecting other two points on the helical trajectory. PI-line has an interesting property: *any point inside helix belongs to one and only one PI-line* [30, 83].
2. *PI-helix* is the helix segment connecting the two terminals of PI-line.

For each point's reconstruction the data are collected at a half-scan helix centered by the mid point of the corresponding PI-helix. Therefore the ray source is close to the PI-plane and, therefore, the off-plane angle of corresponding point can be effectively reduced. This would especially benefit the rim area of transversal slice which always suffer from lots of artifacts under other existing reconstruction scheme, and enhance the universal image quality.

3.2 Geometry of PI-plane Cone-beam Reconstruction

In this section the new pointwise reconstruction algorithm is presented. Three aspects of this algorithm are explored, as follows:

1. Considering the transversal slice reconstruction as objective, each point is reconstructed based on a unique PI-plane;

2. The reconstruction plane passes through the corresponding point's PI-line and is close to the source moving trajectory;
3. This algorithm produces very limited additional workload compared to the FDK-type reconstruction.

The approximate reconstruction with the cone-parallel data set is as follows:

$$f(x, y, z) = \int_0^\pi D_{pw}^f \left(x \cos \theta + y \sin \theta, \frac{Rz}{R + x \sin \theta - y \cos \theta}, \eta \right) d\eta. \quad (3.1)$$

where η is the virtual source rotation angle of the rebinned cone-parallel data set.

To perform the reconstruction pointwisely on corresponding PI-planes, the cone-parallel rebinning is adopted [64, 67] and the raw cone-beam projection data set is to be rebinned into a cone-parallel data set before the filtered backprojection (FBP). The cone-parallel rebinning can benefit the image reconstruction from the following two aspects:

1. Referred to Sec.2.4.1 in Chapter 2, adopting the cone-parallel rebinning before reconstruction can avoid the pointwise conjugate data weighting which may cost large computation source for this reconstruction.
2. After the cone-parallel rebinning, the row-wise filtering of the cone-parallel data set can be considered a helix tangential filtering onto the original cone-beam data [67], therefore such simple row-wise filtering can improve the reconstruction accuracy significantly.

This approximate cone-beam imaging algorithm is developed from FDK-type cone-beam algorithms. However, it is noted that since the cone-beam data set has been rebinned to a parallel-beam data set before the FBP, the presented algorithm is not referred to FDK-type algorithm which always conducts the backprojection with the cone-beam data set.

Refer to Eq.3.1. After cone-parallel rebinning, the minimum angle interval of cone-parallel data set for reconstruction is π , instead of the 180° cone-beam half-scan interval $\pi + 2\sigma$. Data $D_{pw}^f(\cdot)$ is the filtered and weighted data set of rebinned cone-parallel data $D_p(\cdot)$ (Eq.2.29),

$$D_{pw}^f(t, n, \eta) = \frac{d_{sd}}{\sqrt{d_{sd}^2 + s^2}} \left[\omega'(\eta) D_p(t, s, \eta) \right] * h(t), \quad (3.2)$$

$$t = x \cos \theta + y \sin \theta, \quad s = -x \sin \theta + y \cos \theta. \quad (3.3)$$

Ratio $d_{sd}/\sqrt{d_{sd}^2 + s^2}$ presents the cosine of the off-plane angle of each parallel beam. $w(\eta)$ denotes the conjugate weighting factor for cone-parallel projection data set. Due to the cone-parallel rebinning, the conjugate redundant data only exist at source rotation angle 0 and π . Therefore $w(\eta)$ can be 0.5 at these two ends of the trajectory and can be 1 at the other position [15].

The geometry of helical multi-slice CT scanning can be referred in Section 2.4. Referred to Fig.2.5, any point's off-plane angle, denoted by α , is defined as the angle between the reconstruction plane and the ray beam passing through this point. Because of the mathematical nature of the approximate cone-beam FBP algorithms, the approximate imaging performance would be deteriorated by large off-plane projection angle. Therefore the imaging performance at the rim area would be worse than that of the central area since the rim points have larger off-plane angles than the central points.

To reduce the off-plane distance of the objective point without affecting the object translation speed, a new pointwise reconstruction on uniquely customized PI-planes is proposed in this chapter. This PI-plane is essentially related with the properties of PI-line. Referred to Fig.3.1, a point is randomly chosen for demonstration, namely point λ with coordinates $(x_\lambda, y_\lambda, z_\lambda)$. Its unique PI-line connects two points $s_1(\theta_1)$ and $s_2(\theta_2)$ on the source helical trajectory while $0 < \theta_2 - \theta_1 < 2\pi$. The reconstruction

Figure 3.1: The proposed PI-plane Ω is determined by the PI-line s_1s_2 and the midpoint s_3 of PI-helix.

plane, denoted by Ω , is determined by the PI-line $\overrightarrow{s_1s_2}$ connecting $s_1(\theta_1)$ and $s_2(\theta_2)$ and the mid point s_3 of the PI-helix $\widehat{s_1s_2}$ whose source rotation angle is $\theta_3 = \frac{\theta_1 + \theta_2}{2}$. Note that the plane Ω is of the family of κ -plane [85]. It is named as PI-plane in this thesis.

Consider a point of the helix with coordinates $(R \cos \theta, R \sin \theta, h\theta)$ (referred to Eq.2.21, and consider the initial angle $\theta_0 = 0$ without loss of generality). The PI-plane Ω has the following formula:

$$\begin{vmatrix} x - R \cos \theta_1 & y - R \sin \theta_1 & z - h\theta_1 \\ R(\cos \theta_2 - \cos \theta_1) & R(\sin \theta_2 - \sin \theta_1) & h(\theta_2 - \theta_1) \\ R(\cos \theta_3 - \cos \theta_1) & R(\sin \theta_3 - \sin \theta_1) & h(\theta_3 - \theta_1) \end{vmatrix} = 0 \quad (3.4)$$

The normal vector $\vec{\gamma} = (a, b, c)^T$ of PI-plane Ω can be calculated as follows:

$$\begin{aligned} a &= \frac{1}{\mathbb{N}} \begin{vmatrix} 1 & R \sin \theta_1 & h\theta_1 \\ 1 & R \sin \theta_2 & h\theta_2 \\ 1 & R \sin \theta_3 & h\theta_3 \end{vmatrix}, \\ b &= \frac{1}{\mathbb{N}} \begin{vmatrix} R \cos \theta_1 & 1 & h\theta_1 \\ R \cos \theta_2 & 1 & h\theta_2 \\ R \cos \theta_3 & 1 & h\theta_3 \end{vmatrix}, \\ c &= \frac{1}{\mathbb{N}} \begin{vmatrix} R \cos \theta_1 & R \sin \theta_1 & 1 \\ R \cos \theta_2 & R \sin \theta_2 & 1 \\ R \cos \theta_3 & R \sin \theta_3 & 1 \end{vmatrix}, \end{aligned} \quad (3.5)$$

where

$$\mathbb{N} = \begin{vmatrix} R \cos \theta_1 & R \sin \theta_1 & h\theta_1 \\ R \cos \theta_2 & R \sin \theta_2 & h\theta_2 \\ R \cos \theta_3 & R \sin \theta_3 & h\theta_3 \end{vmatrix}. \quad (3.6)$$

Since each point inside the scanning helix has a unique PI-line, the PI-plane can be considered a function of point λ , written as $\Omega(\lambda)$.

The off-plane angle of point λ at source rotation angle θ with respect to PI-plane $\Omega(\lambda)$ can be calculated as follows:

$$\alpha = \arcsin \left(\frac{d_s(\theta)}{\sqrt{(x_s - x_\lambda)^2 + (y_s - y_\lambda)^2 + (z_s - z_\lambda)^2}} \right). \quad (3.7)$$

The denominator $\sqrt{(x_s - x_\lambda)^2 + (y_s - y_\lambda)^2 + (z_s - z_\lambda)^2}$ is the distance between point λ and the ray source. The source off-plane distance to the PI-plane $\Omega(\lambda)$ can be presented as a product of the normal vector $\vec{\gamma}$ and source-to-point vector $\overrightarrow{ss_3}$, as

Figure 3.2: The comparison of source off-plane distance to different reconstruction planes. The horizon denotes the off-plane distance equals to zero. The curves denote the off-plane distance of the four PI-planes, while the straight lines denote the off-plane distance of the objective transversal plane.

follows

$$\begin{aligned}
 d_s(\theta) &= \frac{|\vec{\gamma} \cdot \vec{ss}_3|}{|\vec{\gamma}|} \\
 &= \frac{|ax_s + by_s + cz_s - aR \cos \theta_3 - bR \sin \theta_3 - ch\theta_3|}{\sqrt{a^2 + b^2 + c^2}}. \quad (3.8)
 \end{aligned}$$

$d_s(\theta)$ is a positive value if the ray source is on the same side of the PI-plane as its normal vector $\vec{\gamma}$ and negative if it is on the opposite side. The off-plane distance of the source to PI-plane is much less than that between the source and the objective transversal plane. Therefore the off-plane angle α can be effectively reduced.

To show this, a scanning helix with parameters $[R = 50\text{cm}, h = 5/\pi \text{ cm}]$ is simulated. Four objective points at the objective transversal slice are selected and their PI-planes are found out. While the source moves along each of the four PI-helices, the source off-plane distance to the corresponding PI-plane is recorded, as shown in Fig.3.2. For comparison, the off-plane distance to the transversal objective plane is also calculated respectively. One can see in the figure that in the range of each PI-helix the off-plane distance of the PI-plane is obviously less than that of the transversal plane at $\theta = \theta_3$.

3.3 Reconstruction on PI-plane

The following are the procedure of reconstructing an objective point $\lambda(x_\lambda, y_\lambda, z_\lambda)$ on the PI-plane $\Omega(\lambda)$. This new algorithm is named as Cone-beam Approximate Reconstruction on PI-plane, abbreviated to CARP Algorithm.

1. Collect the cone-beam projection data set D , and rebin the whole data set into cone-parallel data set (referred to Section 2.4.1 and Fig.2.8), as follows:

$$D_p(t, s, \eta) = D\left(\frac{tR}{\sqrt{R^2 - t^2}}, s, \eta + \arcsin \frac{t}{R}\right), \quad (3.9)$$

Different from Eq.2.29, the rebinning in Eq.3.9 is for equally-spaced cone-beam projection data which is adopted in this reconstruction.

2. Filter the cone-parallel data D_p row-by-row. Note that after the cone-parallel rebinning, the row-wise filtering of the cone-parallel data set can be considered a helix tangential filtering of the original cone-beam data D [67]. Denote Q as the filtered data set.
3. Calculate the PI-planes:

(3-a) Determine the PI-line $\overrightarrow{s_1 s_2}$ to which λ belongs.

(3-b) Determine the normal vector $\vec{\gamma} = (a, b, c)^T$ of PI-plane $\Omega(\lambda)$, following Eq.3.5.

(3-c) Build a new Cartesian system $x' - y' - z'$ determined by $\Omega(\lambda)$. The axis x' and y' are positioned at the $\Omega(\lambda)$ plane and axis z' has the same direction as $\vec{\gamma}$. The transformation matrix can be divided into two parts, as follows:

$$\begin{bmatrix} x' & y' & z' \end{bmatrix}^T = T_2 T_1 \begin{bmatrix} x & y & z \end{bmatrix}^T, \quad (3.10)$$

where

$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta_1 & \sin \beta_1 \\ 0 & -\sin \beta_1 & \cos \beta_1 \end{bmatrix}, \quad T_2 = \begin{bmatrix} \cos \beta_2 & 0 & \sin \beta_2 \\ 0 & 1 & 0 \\ -\sin \beta_2 & 0 & \cos \beta_2 \end{bmatrix}. \quad (3.11)$$

β_1 and β_2 is the rotation angle of the coordinate system, and can be achieved from vector $\vec{\gamma} = (a, b, c)^T$, as

$$\beta_1 = -\arctan \frac{c}{b}, \quad \beta_2 = \arctan \frac{a}{\sqrt{b^2 + c^2}}. \quad (3.12)$$

Therefore the ray source $S(x_s, y_s, z_s)$ has new coordinates as

$$\begin{bmatrix} x'_s & y'_s & z'_s \end{bmatrix}^T = T_2 T_1 \begin{bmatrix} x_s & y_s & z_s \end{bmatrix}^T, \quad (3.13)$$

and point λ has new coordinates as

$$\begin{bmatrix} x'_\lambda & y'_\lambda & z'_\lambda \end{bmatrix}^T = T_2 T_1 \begin{bmatrix} x_\lambda & y_\lambda & z_\lambda \end{bmatrix}^T. \quad (3.14)$$

4. A half-scan data set ranging at $[\theta_3 - \frac{\pi+2\sigma}{2}, \theta_3 + \frac{\pi+2\sigma}{2}]$ ($\theta_3 = \frac{\theta_1+\theta_2}{2}$) is selected from filtered data set Q .

5. Extract the data needed for point λ reconstruction,

$$Q_\lambda(\eta) = Q\left(t_\lambda, \frac{R(z_\lambda - h\eta - h \arcsin \frac{t_\lambda}{R})}{R - s_\lambda}, \eta\right), \quad (3.15)$$

$$t_\lambda = x_\lambda \cos \eta + y_\lambda \sin \eta, \quad s_\lambda = -x_\lambda \sin \eta + y_\lambda \cos \eta,$$

where (t_λ, s_λ) are coordinates of the point λ in the rotating coordinates system. Next the data Q_λ is multiplied with new cone-parallel weighting factors which are calculated in $x' - y' - z'$ system and with respect to PI-plane $\Omega(\lambda)$. The cone-parallel weighting is as follows:

$$\bar{Q}_\lambda(\eta) = \frac{R' - s'_\lambda}{\sqrt{(R' - s'_\lambda)^2 + d_s^2(\eta + \varrho)}} Q_\lambda(\eta), \quad (3.16)$$

where R' and s'_λ are respectively the source-detector distance and the point coordinates calculated in $x' - y' - z'$ system. ϱ is the difference between the source angles of ray beam before and after the cone-parallel rebinning,

$$R' = \sqrt{x_s'^2 + y_s'^2}, \quad \varrho = \arcsin \frac{t'_\lambda}{R'}. \quad (3.17)$$

$d_s(\cdot)$ is the source off-plane distance (referred to Eq.3.8),

$$t'_\lambda = x'_\lambda \cos \eta + y'_\lambda \sin \eta, \quad s'_\lambda = -x'_\lambda \sin \eta + y'_\lambda \cos \eta. \quad (3.18)$$

6. Refer to Eq.3.1, implement backprojection to achieve the value of point λ , as follows:

$$f(\lambda) = \int_{\eta_{\min}}^{\eta_{\max}} \omega'(\eta) \bar{Q}_\lambda(\eta) d\eta, \quad (3.19)$$

$$\eta_{\min} = \theta_3 - \frac{\pi + 2\sigma}{2} - \arcsin \frac{t_\lambda}{R}, \quad \eta_{\max} = \eta_{\min} + \pi. \quad (3.20)$$

Although the CARP algorithm introduces additional computation to determine the

PI-plane and new $x' - y' - z'$ coordinates system for each objective point, these additional computations can be pre-calculated and stored in a look-up table in memory before scanning. One only needs search the look-up table before the reconstruction on the PI-plane can be implemented. Since only the PI-planes' parameters are to be looked up, this online search does not yield too much signal flow. Next, this algorithm will be compared with several existing approximate algorithms.

3.4 Evaluation with Computer Simulations

In this section, the CARP algorithm is tested with simulated phantom at simulated helical CT system, compared with ASSR 2D-rebinning reconstruction [17] and the FDK transversal reconstruction [39]. The simulated helical scanning system is defined as Table 3.1 shown. Note that the helical pitch p is defined in Eq.2.22.

3.4.1 Phantom Reconstruction

A $256 \times 256 \times 256$ mm Turbell Clock Phantom is adopted for simulation. The well known Turbell Clock phantom [16, 17, 98] is very sensitive to the artifacts by large cone-angle or large helix pitch. For this simulation, different sized detectors (16 rows, 32 rows, 64 rows, 96 rows) simulate the different cone-angle scanning, in which the

Table 3.1: Parameters of simulated CT scanner

Source-axis distance R	600mm
Source-detector distance d_{sd}	1000mm
Detector cell	1×1 mm
Projections per rotation	600 (with 600 cells per row)
Helical pitch p	0.8

cone-angle can be measured as

$$2 \arctan \left(\frac{[\text{Total number of row}] \times 1}{d_{sd} \times 2} \right). \quad (3.21)$$

Fig.3.3 and Fig.3.4 show the reconstruction results, which are displayed in the window $[0.95, 1.05]$. One can see that in the large cone-angle reconstruction, the CARP algorithm can effectively suppress the shading artifacts and maintains the high imaging performance compared to other approximate algorithms.

A $256 \times 256 \times 256$ mm Defrise Disk Phantom is adopted in the evaluation of CARP algorithm. The well known Defrise Disk Phantom, in which there are five white ellipsoids in dark peripheral, is sensitive to the artifacts occurred in FDK-type reconstructions (“FDK Killer”) [16–18, 67]. The 32 rows detector is used in this evaluation. The sagittal images of Defrise phantom are reconstructed with CARP, ASSR and FDK algorithm, and displayed in Fig.3.5. One can clearly see that CARP results have less shade in the ellipses than the results of the other algorithms.

Another evaluation is implemented with a modified Defrise Phantom, in which those five ellipsoids are replaced with five cylindrical disks. This modified Defrise Phantom contains higher frequency information than the original one does, and it is more rigorous in evaluating the cone-beam FBP approximate reconstruction [17, 67]. In Fig.3.6 the transversal images of the phantom reconstructed with different algorithms are display. One can see that the results with the CARP algorithm have less shading artifacts than that of the FDK transversal reconstruction.

3.4.2 Analysis of Slice Sensitivity Profile

The analysis of *Slice Sensitivity Profile* (SSP) is conducted to evaluate the axial resolution (or z -resolution) of CARP algorithm. A dark solid phantom is adopted

Figure 3.3: Computer simulation imaging results (with 16-row and 32-row detector), with CARP, ASSR and FDK algorithm.

Figure 3.4: Computer simulation imaging results (with 64-row and 96-row detector), with CARP, ASSR and FDK algorithm.

Figure 3.5: Reconstructed sagittal images of Defrise Disk Phantom with different approximate algorithms in the simulations of different cone-angle scanning [32-row, 64-row, 96-row].

within which a 1mm thick bright disk is transversally positioned. Several transversal slices near the central bright disk are reconstructed under pitch=0.5 and 0.8. Four rim area points (with coordinates $[\pm 120, 0]$ and $[0, \pm 120]$) and the center point ($[0, 0]$) of the bright disk are chosen, and the reconstructed grey values of these five points on different slices are displayed as the SSPs. The SSPs of the CARP, the ASSR and the half-scan FDK transversal reconstructions are shown in Fig.3.7 and Fig.3.8. The solid curves present the axial profiles of the center point, while those dotted and dashed curves present the profiles of the four points at the rim area. To compare these SSPs, the Full Width at Half Maximum (FWHM) of these curves are averaged and listed in Table 3.2.

Figure 3.6: Reconstructed transversal images of modified Defrise Disk Phantom with different approximate algorithms.

From Fig.3.7 and 3.8 and corresponding FWHM comparison in Table 3.2 it can be seen that CARP can enhance the imaging z -resolution of the slice's rim area compared to the ASSR and the FDK transversal reconstruction. It is consistent with the simulated results of the previous phantom experiments. The ASSR and FDK transversal reconstruction produce better imaging quality at the central area than at the rim area of the slice, therefore the center point's SSP curves (solid) are narrower than the other curves. Differently, the CARP SSP curves of the rim area points look as narrow as the curve of center point.

Figure 3.7: Slice Sensitivity Profiles (SSP) of the reconstructed images (pitch=0.5). (A) SSP measured in central point; (B) SSP measured in rim point.

3.4.3 Noise Analysis

In order to evaluate the reconstruction noise of CARP algorithm, this section presents the Mean Square Error (MSE) and Signal-Noise Ratio (SNR) of CARP and other algorithms. Refer to Fig.3.9, a $256 \times 256 \times 256$ mm Shepp-Logan head phantom is adopted. The MSE and SNR values are calculated within the five little square sampling areas (a,b,c,d,e) whose grey values are 1.02.

$$\text{MSE} = \frac{1}{N} \sum_N e_i^2, \text{SNR} = \frac{\text{signal mean}}{\sqrt{\text{MSE}}}. \quad (3.22)$$

Figure 3.8: Slice Sensitivity Profiles (SSP) of the reconstructed images (pitch=0.8). (A) SSP measured in central point; (B) SSP measured in rim point.

As shown in Table 3.3, the CARP reconstruction has less MSE and larger SNR values compared to the ASSR and FDK transversal reconstruction, although their reconstructed values are at the same level.

In real CT scanner, the X-ray projection would produce noise due to the X-ray's nature, the object tissue and the CT scanner. To further evaluate the algorithm, a simulated reconstruction with noise is conducted [17, 19, 44]. After scanning, a noise is added to the projection data of each angle. The noise satisfies the Gaussian distribution [99, 100]. The mean and variance of this Gaussian noise is

$$\mu_i = 0, \quad \sigma_i = \frac{\exp(D_i)}{I_0}, \quad (3.23)$$

Table 3.2: The FWHM of the SSP curves in Fig.3.7A and 3.8A.

	CARP	ASSR	FDK
pitch=0.5	0.991mm	1.003mm	1.533mm
pitch=0.8	1.226mm	1.278mm	1.704mm

Figure 3.9: Five 10×10 mm little squares are sampled from standard Shepp-Logan Phantom (grey value ranges $[1.0, 2.0]$) for the noise analysis. Grey value of sampling areas is 1.02.

where D_i is the single projection datum and I_0 is the estimated total amount of the incident photon. In practice the incident photon number is a very large number, and in this simulation it is set $I_0 = 10^5$. Thus this noise is signal dependent.

From these numerical comparisons shown in Table 3.4 it can be seen that the CARP helical cone-beam reconstruction still has less MSE and larger SNR values than the ASSR and FDK transversal reconstruction even if the field noise is taken into account.

Table 3.3: Mean reconstructed values of the sampling areas in noise-free simulations

	a	b	c	d	e
Mean Reconstructed Values					
CARP	1.021	1.021	1.019	1.020	1.020
ASSR	1.022	1.021	1.019	1.018	1.020
FDK	1.020	1.019	1.019	1.021	1.020
MSE Values ($\times 10^{-4}$)					
CARP	7.733	7.638	7.812	7.466	7.094
ASSR	11.54	12.38	12.06	11.97	11.25
FDK	10.65	11.07	10.89	11.23	10.14
SNR Values					
CARP	47.56	48.01	47.86	48.33	48.95
ASSR	32.79	33.05	33.17	32.56	33.63
FDK	38.35	37.96	37.87	38.14	38.89

3.5 Discussions

The main advantage of CARP algorithm is that it can suppress the cone-beam artifacts occurring in large cone-angle scanning, and therefore allows the large cone-angle scanning to be adopted in helical CT system. Motivated by the PI-line reconstruction used in the exact algorithms [56, 57], the CARP algorithm conducts the reconstruction at customized PI-planes to reduce the off-plane angle which can result the cone-beam artifacts.

Differently from those existing customized plane reconstruction algorithms, CARP constructs the more practical transversal plane and ensures that every objective point is calculated at a uniquely customized plane. Therefore the off-plane angle of each objective point can be reduced. Especially for the Region of Interest (ROI) at the rim area of slice which has relatively larger off-plane angle, CARP can effectively

Table 3.4: Mean reconstructed values of the sampling areas in noise-free simulations

	a	b	c	d	e
Mean Reconstructed Values					
CARP	1.020	1.021	1.019	1.020	1.020
ASSR	1.022	1.020	1.018	1.018	1.019
FDK	1.021	1.019	1.020	1.021	1.020
MSE Values ($\times 10^{-4}$)					
CARP	8.961	8.645	9.032	8.448	7.923
ASSR	17.31	13.09	18.04	12.88	9.423
FDK	14.71	15.78	14.66	14.13	9.023
SNR Values					
CARP	46.71	48.41	47.33	47.18	46.09
ASSR	27.17	32.56	25.08	34.64	41.05
FDK	30.76	29.35	30.89	31.49	42.79

reduce the shading artifacts and enhance the reconstruction performance, which is shown in the simulation imaging results and numerical statistics in Section 3.4. As a result, this new algorithm can benefit the larger cone-angle scanning in practice which can speed up the helical scanning and enhance the diagnosis efficiency.

Compared to the FDK transversal reconstruction, CARP requires additional computation for constructing the PI-planes and tilted coordinate system. Fortunately these additional calculations do not involve the projection data, so they can be pre-calculated and stored as a looking-up table in memory before scanning. Based on the pre-calculation of the new coordinate system, the workload involved in the source's new coordinates calculations (Eq.3.13) and the new cone-weighting factors calculations (Eq.3.16, 3.17) can also be implemented off line.

The cone-parallel rebinning adopted in CARP reconstruction can naturally produce the helix tangential filtering which enhances the axial resolution of volumetric re-

Table 3.5: Workload of an $N \times N$ pixel slice reconstruction with CARP and the FDK transversal imaging algorithm.

CARP	FDK
data collection ($\varepsilon_1 M$ projections)	data collection ($\varepsilon_2 M$ projections)
cone-parallel rebinning	not applicable
ramp filtering ($\varepsilon_1 M \times H$ filterings)	ramp filtering ($\varepsilon_2 M \times H$ filterings)
PI-plane calculation coordinates transformation (pre-calculated and stored)	not applicable
backprojection	backprojection

construction. Moreover, due to the cone-parallel rebinning, the whole projection data need only to be filtered one time before reconstructions. Therefore, the online workload of new algorithm is further reduced.

Table 3.5 presents the overall computation workload for reconstructing an $N \times N$ pixel slice with CARP and with the FDK transversal imaging algorithm. There are M projections in one rotation, and data are collected with an H -row detector. For CARP reconstruction, it is required $\varepsilon_1 M$ ($0 < \varepsilon_1 \leq 1$) projections. For FDK transversal reconstruction, the 180° half-scan data are needed, in which there are $\varepsilon_2 M$ ($0 < \varepsilon_2 \leq 1$, $\varepsilon_2 = \frac{\pi+2\sigma}{2\pi}$) projections. Although $\varepsilon_1 M > \varepsilon_2 M$, from the table one can see that CARP has no significant additional online computing workload compared with the FDK transversal reconstruction.

It is worth to mention that the CARP can be used directly to reconstruct a volumetric ROI instead of achieving it slice-wisely like the ASSR method does. During the volumetric reconstruction implemented by CARP, it is noted that if points on different reconstruction planes are found being in a same PI-line, duplicated PI-

line searching will be avoided and different points can share one PI-segment and therefore one PI-plane.

In practical medical scanning the motion artifacts is sometimes unavoidable, especially the breath caused motion artifacts. Since at most of the time breath-caused motion follows the objective transversal direction, the motion would affect the conventional transversal reconstruction more seriously than the CARP reconstruction in which the breath-caused motion is weakened on the tilted PI-planes.

3.6 Conclusions

This chapter presents the approximate cone-beam FBP reconstruction algorithm *CARP* for helical cone-beam CT system. For constructing a transversal slice with less cone-beam artifacts, this algorithm proposes to reconstruct each point on a unique customized *PI-plane*. The PI-plane passes through the corresponding point's PI-line and be close to the corresponding PI-helix. The half-scan data adopted for reconstructing each point are collected along the helix extended from corresponding PI-helix. Compared to the existing algorithms (ASSR [17] and FDK [39]), the CARP algorithm suppresses the cone-beam artifacts more efficiently and provide better imaging results at the large cone-angle scanning.

Chapter 4

Approximate Reconstruction Using Extended Scanning Helix

This chapter presents a new FDK-type approximate algorithm for helical cone-beam CT image reconstruction, namely ExFDK algorithm. To reconstruct the transversal image slices at a more accurate level, this algorithm proposes an extended helical scanning trajectory which satisfies the Tuy's exact reconstruction condition. In Section 4.2 the scanning geometry of this algorithm is illustrated. Section 4.3 presents the new reconstruction algorithm mathematically. In Section 4.4 this algorithm is evaluated in simulated phantom image reconstructions, which are followed by the discussions in Section 4.5 and the conclusions in Section 4.6.

4.1 Introduction

The approximate cone-beam CT reconstruction with modified scanning trajectory is focused in this chapter. In practice the approximate algorithms are employed in most of the medical cone-beam scanners [3]. Compared to the exact reconstruction

algorithms, approximate algorithms can provide more flexible tradeoff between the image quality and computational efficiency.

Due to their mathematics nature, these approximate algorithms would unavoidably exhibit considerable artifacts in the case of large cone-angle or large scanning pitch. In the last decade, uncountable efforts have been made for reducing the artifacts and improving the approximate reconstruction performance [16–19, 24–26, 39, 63–67]. In Chapter 3 of this thesis, a new FDK-type approximate algorithm CARP is presented which can enhance the cone-beam reconstruction to some extent. The CARP algorithm reconstructs the Region Of Interest (ROI) pointwisely on a tilted PI-plane, with which the cone-beam artifacts at the slice’s rim area can be suppressed effectively.

It is well known that the “ $\pi + 2\sigma$ ” is the minimum scanning angular interval for approximate reconstruction [3], where σ is the maximum fan-angle. This limitation is due to the mathematical nature of approximate reconstruction. In 2002 Noo and Kudo *et al.* [46, 47] proposed their work on how to reconstruct an ROI image by a scanning less than π . In 2005 they slightly extended this work by swapping the order of derivative and the backprojection [48]. These works which are for exact reconstruction are referred as the “super-short-scan” algorithms. The super-short-scan is based on the condition which can be presented that **an ROI can be exactly reconstructed if and only if all the planes intersecting the ROI also intersect the source scanning trajectory**. This reconstruction condition was originally proposed by Tuy in 1983 [50], therefore it is well known as *Tuy’s exact reconstruction condition* [50, 83, 84, 91–94].

In 2006 Hu *et al.* proposed an FDK-type approximate algorithm which considered the reconstruction of nutating curved surfaces using short scan data [68]. It was shown that all points of the nutating curved surface satisfy the Tuy’s condition. Therefore all the points of the nutating curved surface have potential to be exactly

reconstructed. In practical diagnosis only the classical planar slices are usable for application. After the nutating curved surfaces are reconstructed by FDK scheme, the planar transversal slices can be obtained by interpolation or extrapolation of the curved surfaces.

The proposed algorithm in this chapter is motivated by Hu *et al.*'s consideration for approximate reconstruction of the image slice which satisfies the Tuy's exact reconstruction condition. Instead of constructing the nutating curved surface as proposed in Hu *et al.*'s work, the direct FDK-type reconstruction of transversal plane is taken into account, which is more feasible and practical. A shortest helical scanning trajectory is derived to satisfy the Tuy's condition. Computer simulations show that the proposed algorithm can achieve improvement in computational efficiency compared to the existing algorithm.

4.2 Geometry and Reconstruction Condition

4.2.1 Helical Scanning Geometry

The geometry and parameters of the helical cone-beam CT scanning can be referred in Fig.2.5. Referred to Eq.2.21, the coordinates of X-ray source in “ $x-y-z$ ” system can be represented as $(x, y, z)^T = (R \cos \theta, R \sin \theta, h\theta)^T$. It is assumed without loss of generality that the initial source angle $\theta_0 = 0$.

Consider a helix segment between two source rotation angles θ_1 and θ_2 ($\theta_1 < \theta_2$). As shown in Fig.4.1, the PI-lines emitting from the helix terminal point at θ_2 form a surface, named as S_2 . Similarly, PI-lines emitting from the terminal point at θ_1 form another surface, named as S_1 . In view of Fig.4.1, it can be seen that only the object being totally covered between surfaces S_1 and S_2 can satisfy Tuy's condition and, therefore, has potential to be exactly reconstructed.

Figure 4.1: Helical CT scanning geometry. Surface S_1 and S_2 are formed by the PI-lines emitting from the two terminals of helix. The ROI sandwiched between S_1 and S_2 satisfies the Tuy's condition.

It is noted that the 180° half-scan scheme requires a " $\pi + 2\sigma$ " scanning, where σ is the maximum fan-angle. While it uses reduced amount of data in order to save the radiation dose and computational workload, some part of the reconstruction slice may not satisfy the Tuy's condition for exact reconstruction. This can be geometrically illustrated as in Fig.4.2(a), where some parts of the reconstruction slice can not be covered between the surfaces S_1 and S_2 . As a result, these parts may not be satisfactorily reconstructed by exact or approximate algorithms.

To solve this problem, in Hu *et al.*'s works [68] it was proposed to reconstruction a nutating curved surface between S_1 and S_2 , other than constructing a transversal planar slice. The nutating curved surface is obtained by averaging the intersection part of $S_{1,2}$ and the objective cylindrical support. Surely there is no part of the nutating curved surface outstretching the S_1 and S_2 . It is demonstrated in Fig.4.2(b). If denoting C_1 as the intersection surface between S_1 and the cylindrical object, C_2 as the surface of S_2 , therefore the reconstructed nutating curved surface has the

Figure 4.2: Geometry of conventional 180° helical half-scan. (a), some parts of the transversal planar slice can not be covered between the surfaces S_1 and S_2 . The PI-lines pass through the transversal slice at those solid and hollow points. (b), the nutating curved surface is obtained by averaging the intersection surfaces C_1 and C_2 and, therefore it can be totally covered by S_1 and S_2 .

following formula:

$$\begin{cases} x^2 + y^2 \leq r^2 \\ z(x, y) = \frac{C_1(x, y) + C_2(x, y)}{2} \end{cases} \quad (4.1)$$

To obtain a transversal planar slice, two methods are suggested in Hu *et al.*'s paper: 1, by conducting the interpolation and extrapolation of several nutating curved surfaces; 2, by reconstructing the slice pointwisely (each point is reconstructed on the corresponding nutating curved surface). Both methods are computationally expensive.

Figure 4.3: Geometry of the proposed extended helical scanning. The PI-lines from point A and B pass through the virtual slice and leave the grey objective slice untouched. Due to the helix extension, the objective slice can be totally covered between S_1 and S_2 .

4.2.2 Tuy's Condition for Transversal Reconstruction Plane

Tuy's condition for exact reconstruction of the transversal plane can be satisfied by extending the length of the half-scan helix. Given a transversal planar slice, there exists a minimum helix length for such a condition to be satisfied. With respect to the helix segment within $\theta \in [\theta_1, \theta_2]$ and the object slice positioned at $z = \frac{h(\theta_2 + \theta_1)}{2}$ as shown in Fig.4.3, the minimum helix angular interval $\theta_{\min} = \theta_2 - \theta_1$ is worked out for all points of the object slice to satisfy Tuy's condition, as follows:

$$\theta_{\min} = \max \left\{ \theta + \theta \sqrt{\frac{\frac{2r^2}{R^2} - \cos \theta - 1}{1 - \cos \theta}} \right\}, \theta \in \left(2 \arccos \frac{r}{R}, 2\pi - 2 \arccos \frac{r}{R} \right). \quad (4.2)$$

It can be easily verified that $(\theta + \theta \sqrt{\frac{\frac{2r^2}{R^2} - \cos \theta - 1}{1 - \cos \theta}})$ in Eq.4.2 is a convex function within the range $\theta \in [2 \arccos \frac{r}{R}, 2\pi - 2 \arccos \frac{r}{R}]$ and has a unique maximum. Thus the computation of $\theta_{\min}$ is simple and straightforward.

Derivation: To derive the formula Eq.4.2, it needs to take a look at the helical scanning geometry, as shown in Fig.4.4. Consider that a PI-line starts from point A at source rotation angle θ_1 , passes through the object support and meets the helix at point C with angle θ ($\theta_1 < \theta < \theta_2$).

Referred to Fig.4.4(a), $(x, y, z)^T$ denotes one of the two intersection points (the one be far from the point A) between PI-line AC and the surface of the objective support. Let the initial angle θ_1 equals to 0 without loss of generality, then the coordinate of point C can be presented as $(R \cos \theta, R \sin \theta, h\theta)^T$.

The scanning helix and the PI-lines in Fig.4.4(a) is projected onto a transversal plane where the helix starting point A locates, as shown in Fig.4.4(b). The projections of the helix $\widehat{AB}$ and the PI-line AC on that plane are respectively $\widehat{AB'}$ and AC' .

According to the triangular proportional relationships lying within Fig.4.4, one can have:

$$\frac{|AJ|}{|AK|} = \frac{|F'J|}{|C'K|} = \frac{|AF'|}{|AC'|} = \frac{|FF'|}{|CC'|}, \quad (4.3)$$

$$\frac{R - x}{R - R \cos \theta} = \frac{y}{R \sin \theta} = \frac{z}{h\theta}. \quad (4.4)$$

Since the point $(x, y, z)^T$ locates at the cylindrical support, $x^2 + y^2 = r^2$ is satisfied. According to Eq.4.3 and 4.4, $x = R - \frac{Rz(1 - \cos \theta)}{h\theta}$ and $y = \frac{Rz \sin \theta}{h\theta}$. Thus

$$\left(R - \frac{Rz(1 - \cos \theta)}{h\theta}\right)^2 + \left(\frac{Rz \sin \theta}{h\theta}\right)^2 = r^2, \quad (4.5)$$

$$\frac{2R^2}{h^2\theta^2}(1 - \cos \theta)z^2 - \frac{2R^2(1 - \cos \theta)}{h\theta}z + (R^2 - r^2) = 0. \quad (4.6)$$

This is a quadratic equation with respect to z and its feasible solution as a function in θ is

$$z(\theta) = \frac{h\theta}{2} \left(1 + \sqrt{\frac{2\frac{r^2}{R^2} - \cos \theta - 1}{1 - \cos \theta}}\right). \quad (4.7)$$

Figure 4.4: The PI-line's intersection with the object cylindrical support. (a), helix segment $\widehat{AB}$ and PI-line AC' ; (b), projection of helix and PI-line onto a transversal plane where point A locates.

While $\theta \in [2 \arccos \frac{r}{R}, 2\pi - 2 \arccos \frac{r}{R}]$ which implies that the PI-line AC' passing through the object, the function $z(\theta)$ in Eq.4.7 is a convex function and has a unique maximum, named as z_m .

Because z_m is the maximal z -position of the PI-line's intersection curve with the object cylindrical support, it can be concluded that *if the transversal object slice with radius r is positioned at z_m (i.e. $h\theta_2/2 = z_m$), the PI-lines emitting from point A will no longer pass through the slice, and there is one and only one PI-line contacting the slice's circular rim.* According to the geometrical symmetry, the PI-

lines emitting from point B do not pass through the slice either and there is only one PI-line contacting the circular slice's rim. Once the slice's z -position is lower than z_m there will be PI-lines of surface S_1 passing through the slice; Once the slice's z -position is higher than z_m there will be PI-lines of surface S_2 passing through the slice. Therefore the helix interval $\theta_2 - \theta_1 = 2z_m/h$ is the minimal interval for the objective slice with radius r to satisfy this condition. Refer to Eq.4.7 one can have

$$\begin{aligned}\theta_2 - \theta_1 = \frac{2z_m}{h} &= \max \left\{ \frac{2}{h} \cdot \frac{h\theta}{2} \left(1 + \sqrt{\frac{2\frac{r^2}{R^2} - \cos \theta - 1}{1 - \cos \theta}} \right) \right\} \\ &= \max \left\{ \theta + \theta \sqrt{\frac{2\frac{r^2}{R^2} - \cos \theta - 1}{1 - \cos \theta}} \right\}.\end{aligned}\quad (4.8)$$

This immediately yields Eq.4.2. □

Note that this derivation is conducted in the assumption of $\theta_1 = 0$. Due to the helix consistency, the value of θ_1 would not affect the minimal angular interval. So the result of Eq.4.2 can be adopted in transversal reconstruction at any axial position.

4.3 Reconstruction Using Extended Scanning Helix

The following are the reconstruction steps for a transversal planar slice using data from the extended helical scanning.

1. Collect the data set along the helix segment. Denote $D(m, n, \theta)$ as the data set, where (m, n) is the index of equally-spaced detector cell. θ ranges within $[\theta_1, \theta_2]$ which is the angular interval of extended helical scanning. d_{sd} is the source-detector distance, and ratio τ is defined as $\tau = R/d_{sd}$. Before the FBP procedure, a standard

cone-beam weighting is applied to $D(m, n, \theta)$ as follows:

$$D_w(m, n, \theta) = \frac{R}{\sqrt{R^2 + \tau^2 m^2 + \tau^2 n^2}} D(m, n, \theta). \quad (4.9)$$

2. Apply the Silver's Weighting scheme to $D_w(m, n, \theta)$. The Silver's Weighting is designed for the scanning longer than " $\pi + 2\sigma$ " 180° short scan [79, 80], while the Parker's Weighting is appropriate for the exact " $\pi + 2\sigma$ " short scan.

$$D_{pw}(m, n, \theta) = w(m, \theta) D_w(m, n, \theta), \quad (4.10)$$

where

$$w(m, \theta) = \begin{cases} \sin^2 \left(\frac{\pi}{4} \cdot \frac{\theta - \theta_1}{\sigma - \Gamma} \right), & \theta_1 \leq \theta \leq \frac{\theta_2 + \theta_1}{2} - \frac{\pi - 2\sigma}{2} - 2\Gamma \\ 1, & \frac{\theta_2 + \theta_1}{2} - \frac{\pi - 2\sigma}{2} - 2\Gamma \leq \theta \leq \frac{\theta_2 + \theta_1}{2} + \frac{\pi - 2\sigma}{2} - 2\Gamma \\ \sin^2 \left(\frac{\pi}{4} \cdot \frac{\pi + 2\sigma - \theta + \theta_1}{\Gamma} \right), & \frac{\theta_2 + \theta_1}{2} + \frac{\pi - 2\sigma}{2} - 2\Gamma \leq \theta \leq \theta_2 \end{cases} \quad (4.11)$$

And $\Gamma = \frac{1}{4}(\theta_2 - \theta_1 - \pi) + \frac{1}{2} \arctan \frac{m}{d_{sd}}$.

3. Filter the data set with the helix tangential filtering technique, referred to Section 2.4.3 in Chapter.2, as follows:

$$D_{pw}^f(m, n, \theta) = D_{pw}(m_t, n_t, \theta) * \frac{1}{2} h(\tau m_t). \quad (4.12)$$

$h(\cdot)$ denotes the ramp filter, and $D_{pw}(m_t, n_t, \theta)$ is the data set extracted along the helix tangential direction from $D_{pw}(m, n, \theta)$, as follows:

$$\begin{bmatrix} m_t \\ n_t \\ \theta \end{bmatrix} = \begin{bmatrix} \cos \vartheta & \sin \vartheta & 0 \\ -\sin \vartheta & \cos \vartheta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \\ \theta \end{bmatrix} \quad (4.13)$$

ϑ is the angle between the helix tangential unit vector and the $x - y$ plane, $\vartheta =$

$$\arctan(\delta z/R\delta\theta) = \arctan(h/R).$$

4. Conduct the filtered back-projection (FBP) to obtain the object $f(\cdot)$, as follows:

$$f(\vec{r}) = \int_{\theta_1}^{\theta_2} \frac{R^2}{(R-s)^2} \cdot D_{pw}^f\left(\frac{Rt}{\tau(R-s)}, \frac{R(z_0-h\theta)}{\tau(R-s)}, \theta\right) d\theta, \quad (4.14)$$

where $\vec{r} = (t, s, z_0)^T$ is the rotated coordinates of objective point, as follows:

$$\begin{bmatrix} t \\ s \\ z_0 \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z_0 \end{bmatrix}. \quad (4.15)$$

This algorithm is to enhance the approximate reconstruction performance using extended helical scanning data set which satisfies the Tuy's condition. It is named as FDK-type Reconstruction with Extended Half-scan Trajectory, abbreviated to *ExFDK*. In the following section, the *ExFDK* algorithm is tested by different phantoms, and compared with Hu *et al.*'s nutating curved reconstruction algorithm [68] and conventional half-scan FDK reconstruction.

4.4 Computer Simulation

In this section, the *ExFDK* algorithm is tested in Head phantom [90–93], Clock phantom [98] and Defrise phantom [16, 17]. The performance of *ExFDK* is compared with that of Hu *et al.*'s nutating curved half-scan FDK reconstruction [68] (abbreviated as *NcFDK*), that of the conventional full scan FDK reconstruction (abbreviated as *FsFDK*) and that of conventional half scan FDK reconstruction (abbreviated as *HsFDK*). The helix tangential filtering is adopted in these reconstruction. Parameters of the simulated CT scanning system are listed in Table 4.1. Since the length of extended helical scanning is related to the size of scanning ra-

dus and object support, two scanners with different gantry size are simulated and adopted as shown in Table 4.1.

Table 4.1: Parameters of simulated CT scanner

Source – axis distance (R)	600mm
Source – detector distance	1000mm
Detector size	1×1 mm
Number of rows	64
Number of detectors per row	600
Projections per rotation	800
Helical pitch (h)	0.5 and 0.8

4.4.1 Phantom Reconstruction

First these algorithms are tested with a $256 \times 256 \times 256$ mm low contrast Head Phantom. The central transversal slice is reconstructed in the simulated scanner. The length of 180° half scan helix is $\pi + 2\sigma = \pi + 2 \arcsin\left(\frac{128 \times 1.414}{600}\right) = 1.1951\pi$. The extended half scan helix of ExFDK is calculated following Eq.4.2 and the result is 1.3872π . With NcFDK algorithm, the transversal image slice is reconstructed pointwisely. Each point is reconstructed with data collected from the corresponding PI-helix segment. The position of each PI-helix segment is pre-calculated and saved in a looking-up table.

The reconstructed transversal images using the above algorithms are shown in Fig.4.5 and Fig.4.6 with the display window being $[0.95, 1.05]$. It is shown that the result of the ExFDK can provide good image quality in terms of sharp edge and less stripe artifacts, which is about the same quality as that of NcFDK and is better than that of HsFDK.

A $256 \times 256 \times 256$ mm Turbell Clock phantom is adopted for evaluation. The results of the ExFDK, NcFDK and conventional FsFDK and HsFDK reconstruction are

Figure 4.5: Helical reconstruction of Head phantom. Images (a, b, c, d) are respectively the result of ExFDK, NcFDK, FsFDK and HsFDK under pitch=0.5.

Figure 4.6: Helical reconstruction of Head phantom. Images (a, b, c, d) are respectively the result of ExFDK, NcFDK, FsFDK and HsFDK under pitch=0.8.

shown in Fig.4.7 and Fig.4.8 with the display window being $[0.95, 1.05]$. Since the Clock phantom is more sensitive to the cone-beam artifacts, these results can more clearly show the imaging advantage of ExFDK over the conventional FDK algorithm.

The well known Defrise Phantom is adopted for the evaluation of new algorithm [17, 67]. In Fig.4.9 the transversal images of the phantom reconstructed with ExFDK, NcFDK, conventional FsFDK and HsFDK algorithms are display. Similarly as the previous simulations, one can see that the results with the ExFDK algorithm from different reconstructions have less shading artifacts than that of the conventional FDK reconstruction, and have as good image quality as that of the NcFDK.

4.4.2 Analysis of Slice Sensitivity Profile

The SSP analysis is conducted to evaluate the reconstruction z -resolution of ExFDK algorithm. Same as Section 3.4.2, a dark solid phantom is adopted within which a 1mm thick bright disk is transversally positioned. Several transversal slices near the central bright disk are reconstructed under pitch=0.5 and 0.8. The reconstructed values of four rim area points (with coordinates $[\pm 120, 0]$ and $[0, \pm 120]$) and one center point $([0, 0])$ of the bright disk are adopted to produce the SSP curves.

The SSPs of the ExFDK, NcFDK, FsFDK and HsFDK reconstruction are displayed in Fig.4.10 and Fig.4.11. To compare these SSPs, the full width at half maximum (FWHM) of these curves are averaged and listed in Table 4.2. One can see from these SSPs that ExFDK has good z -resolution compared with the NcFDK algorithms, and achieves an enhancement compared to the conventional FDK reconstructions.

Figure 4.7: Helical reconstruction of Clock phantom. Images (a, b, c, d) are respectively the result of ExFDK, NcFDK, FsFDK and HsFDK under pitch=0.5.

Figure 4.8: Helical reconstruction of Clock phantom. Images (a, b, c, d) are respectively the result of ExFDK, NcFDK, FsFDK and HsFDK under pitch=0.8.

Figure 4.9: Helical reconstruction of Defrise Phantom. Images from left to right are respectively of ExFDK, NcFDK, FsFDK and HsFDK. Images in left column are under pitch=0.5 while the right column images are under pitch=0.8.

Figure 4.10: Slice Sensitivity Profiles (SSP) of the reconstructed images (pitch=0.5). (A) SSP measured in central point; (B) SSP measured in rim point.

4.4.3 Noise Analysis

A cylindrical water phantom is simulated to evaluate the ExFDK algorithm. This phantom is often adopted in noise analysis [99–102]. Every material has its X-ray attenuation coefficient μ and the corresponding CT Number [3], and the coefficient of water is adopted to calculate the others' CT Number following $CT_Number = (\mu - \mu_{\text{water}}) / \mu_{\text{water}} \times 1000$ Hounsfield Unit (HU). Therefore the water's CT number equals to 0 HU. The air has attenuation coefficient 0, so its CT number is -1000 HU.

The $256 \times 256 \times 256$ mm simulated water phantom consists of cylindrical water with radius 124mm and the peripheral air. In this simulation the grey value of water

Figure 4.11: Slice Sensitivity Profiles (SSP) of the reconstructed images (pitch=0.8). (A) SSP measured in central point; (B) SSP measured in rim point.

phantom is set as 1.0 and the value of the peripheral air as 0. The central transversal slice image of the phantom is reconstructed, with the ExFDK, NcFDK and conventional FsFDK and HsFDK algorithm. To evaluate the reconstruction, five 10×10 mm little squares (indexed by a, b, c, d, e, and locate at $[110, 0]$, $[-110, 0]$, $[0, 110]$, $[0, -110]$, $[0, 0]$) on the transversal slice are chosen and the Mean Square Value (MSE) and Signal-Noise-Ratio (SNR) are calculated within the squares, similarly as the noise analysis in Chapter 3. The results are shown in Table 4.3. As shown in the table, these four FDK-type algorithms have the mean reconstructed values at the same level, but the MSE and SNR values are able to show the advantage of ExFDK over the full scan FsFDK reconstruction and half scan HsFDK reconstruction.

Table 4.2: The FWHM of the SSP curves in Fig.4.10A and 4.11A.

	ExFDK	NcFDK	FsFDK	HsFDK
pitch=0.5	1.112mm	1.090mm	1.407mm	1.343mm
pitch=0.8	1.143mm	1.137mm	1.532mm	1.471mm

To evaluate the ExFDK algorithm in noise-added simulation, a simulated Gaussian noise is added into the water phantom and the above reconstructions are repeated. The Gaussian distributed noise is added to the projection data of each angle, referred to Section 3.4.3. The mean and variance value of the noise are

$$\mu_i = 0, \sigma_i = \frac{\exp(D_i)}{I_0}, \quad (4.16)$$

where D_i is the single projection datum and I_0 is the estimated total amount of incident photon. I_0 is set to 10^5 in this simulation. The results are shown in Table 4.4. One can see that ExFDK algorithm can improve the approximate reconstruction accuracy in terms of less MSE and higher SNR values compared to FsFDK and HsFDK.

4.5 Discussions

This section provides some discussions on the ExFDK algorithm to elaborate its advantages in these following aspects:

4.5.1 Scanning Helix Length

The objective of CT image reconstruction is to produce transversal planar image slices of an object. Considering the NcFDK algorithm, to obtain a transversal planar slice from the nutating curved slices it needs a large group of data sets from

Table 4.3: Mean reconstructed values of the sampling areas in noise-free simulations

	a	b	c	d	e
Mean Reconstructed Values					
ExFDK	1.005	1.004	1.004	1.005	1.002
NcFDK	1.004	1.002	1.004	1.003	1.002
FsFDK	1.005	1.005	1.004	1.006	1.003
HsFDK	1.005	1.004	1.003	1.003	1.003
MSE Values ($\times 10^{-4}$)					
ExFDK	10.37	9.898	9.935	10.24	9.431
NcFDK	9.242	9.316	9.197	9.285	8.693
FsFDK	12.58	12.75	13.05	12.94	12.49
HsFDK	14.87	15.58	14.96	15.39	14.38
SNR Values					
ExFDK	38.94	38.83	39.65	38.99	39.86
NcFDK	45.38	44.79	45.19	45.18	45.27
FsFDK	33.37	33.75	32.95	33.14	33.74
HsFDK	30.46	29.85	30.13	29.88	30.62

different PI-helices corresponding to different objective points. Differently, ExFDK algorithm directly reconstructs the planar transversal slice using only one data set from an extended helix segment.

To provide a quantitative evaluation of the data range for the transversal planar slice reconstruction, the involved helix length of ExFDK, NcFDK and 180° half-scan are compared and listed in Table 4.5 with respect to different sized objects. Demonstratively, only four PI-helices corresponding to four given points are selected and shown as a , b , c and d in Fig.4.12. Length of unions of the four helices for different r/R is listed in Table 4.5 in comparison with that of 180° half-scanning and that of ExFDK. It is noted that each union is only a subset of the whole helix union and is shorter than the whole union. Thus it can be seen in Table 4.5 that

Table 4.4: Mean reconstructed values of the sampling areas in noise-added simulations

	a	b	c	d	e
Mean Reconstructed Values					
ExFDK	1.004	1.004	1.005	1.003	1.002
NcFDK	1.003	1.002	1.005	1.003	1.002
FsFDK	1.004	1.003	1.005	1.006	1.003
HsFDK	1.005	1.005	1.003	1.004	1.004
MSE Values ($\times 10^{-4}$)					
ExFDK	12.14	11.75	11.80	12.09	11.64
NcFDK	10.64	10.33	10.79	10.63	10.14
FsFDK	14.67	14.33	14.86	14.54	14.35
HsFDK	16.77	16.84	16.75	16.92	16.58
SNR Values					
ExFDK	35.38	34.96	34.98	35.21	35.43
NcFDK	42.44	42.18	41.97	42.39	42.60
FsFDK	30.78	30.85	30.62	30.68	31.04
HsFDK	27.34	27.14	27.33	27.05	27.48

the helix of ExFDK for reconstruction of a transversal planar slice is always shorter than the union of all involved PI-helices for NcFDK.

ExFDK algorithm also have advantages over the exact reconstruction algorithms. Consider the typical exact algorithm proposed by Zou and Pan which requires the 1D shift-invariant filtering on each PI-line [56,57]. For constructing a transversal planer slice, the exact algorithm needs a large volume of projection data since each point is calculated through 1D filtered backprojection along the corresponding PI-line. The data within the *Tam-Danielsson Window* [83], which is the part of detector segmented by the projections of two neighboring helix, are mostly needed. Thus the involved data amount is significantly greater than that required by the ExFDK algorithm.

Figure 4.12: Reconstruction of the points $[a, b, c, d]$ with NcFDK will involve data sets from the four corresponding PI-helices. The thick helix is the union of the four separate PI-helices.

Table 4.5: Length of involved helices in different algorithms.

r/R	180° half-scan	NcFDK	ExFDK
0.1	1.0638π	1.1412π	1.1216π
0.2	1.1282π	1.2978π	1.2502π
0.3	1.1940π	1.6086π	1.3873π
0.4	1.2620π	1.8696π	1.5352π
0.5	1.3333π	1.8966π	1.6972π

4.5.2 Memory Cost and Computational Workload

With NcFDK, reconstruction of a transversal slice involves using a large amount of PI-helices. Registrations and indices of these PI-helices require large amount of memory usage. In the case of 256×256 pixels reconstruction, it will involve 256×256 PI-helices whose starting and terminating positions are saved in memories to form a look up table. In contrast, the proposed ExFDK only requires a single helix segment for constructing a transversal planar slice, and it is straightforward to obtain the exact length and position of this helix segment. Therefore, ExFDK requires much less memory resource than that required by NcFDK.

All short-scan reconstructions require conjugate beam weighting to reduce the im-

age distortion [37, 38, 61, 62, 78, 79]. It is known that some θ -dependent weighting schemes are more competent than the conventional θ -independent weighting methods in helical reconstruction. Since reconstructions with different data sets require separate θ -dependent weighting, reconstructing a transversal slice by NcFDK will involve considerable workload for these pointwisely conjugate weighting.

Referring to Eq.4.10 and 4.12, it can be seen that the conjugate weighting should be applied before the ramp filtering, as:

$$D_{pw}^f(m, n, \theta) = [w(m, \theta)D_w(m, n, \theta)] * \frac{1}{2}h(\tau m).$$

Therefore the ramp filtering should be repeated in each PI-helix data set. Given a 256×256 pixels reconstruction, it needs 256×256 times ramp filtering. If the ramp filtering is implemented in terms of convolution, one can estimate the total amount of involved multiplications as follows. Consider a detector with 16 rows and 500 cells per row and there are 1000 projections per rotation. The number of multiplications required by the ramp filtering for a 256×256 pixels reconstruction is

$$\sum_{i=1}^{256 \times 256} \left[500 \times 500 \times \frac{\theta_{\pi}^i}{2\pi} \times 1000 \right] \approx 1 \times 10^{13}$$

where $(\frac{\theta_{\pi}^i}{2\pi} \times 1000)$ denotes the projection amount in each PI-helix.

On the contrary, ExFDK requires only one helix segment and only one weighting and filtering operation per transversal planar slice reconstruction. In the same case as above, the number of multiplications for the ramp filtering is estimated to be

$$500 \times 500 \times \frac{\theta_{\min}}{2\pi} \times 1000 \times 16 < 4 \times 10^9$$

where $(\frac{\theta_{\min}}{2\pi} \times 1000)$ is the projection amount in the extended helical scanning. Hence the computational workload of ExFDK is much less than that of NcFDK.

In medical practice (e.g. the cerebral CT imaging), it often needs to construct a group of transversal slices that are continuous at axial direction for 3D image rendering and volumetric reconstruction. Since every point at each slice has its unique PI-helix segment, to reconstruct a series of z -continuous transversal planar slices using NcFDK is computationally expensive in determining all the required helices. On the contrary, the length of the extended scanning helix is independent of the z -position as shown in Eq.4.2, so it is not necessary to repeatedly compute the helix segment length slice by slice in using ExFDK.

Compared to the typical exact algorithm [56, 57], the ExFDK algorithm can save a large amount of computation. Since the exact algorithm suggests implementing the FBP along the PI-line for each objective point's reconstruction, it requires much more computations than ExFDK. Consider the case of 256×256 pixels reconstruction with a $16 \times 500 \times N$ data set, which is collected by a 16-row detector with 500 cells per row at the N projections. The exact algorithm requires filtering along PI-line at every projection angle, there it needs $256 \times 256 \times N$ filtering (by the means of convolution if implementing in time domain). For the ExFDK, since it only needs row-wise filtering at each projection, the filtering amount is $16 \times N$. The difference is obvious.

4.5.3 Tuy's Condition for Approximate Reconstruction

The Tuy's condition presents the prerequisite for the exact reconstruction. Referred to the Fig.4.2 and 4.3, the reconstruction slice satisfying this condition in the short scan is sandwiched between the surface S_1 and S_2 . As shown in Fig.4.13(a), if the slice is totally sandwiched by S_1 and S_2 the whole slice is projected at relatively small off-plane angles. If the slice is too large to be totally covered (as shown in Fig.4.13(b)), the outstretched part would be projected at relatively large off-plane angles. If the slice is located higher or lower than the halfway of the scanning helix

Figure 4.13: Different slices in half-scan helix. The thick line denotes the sagittal projection of helix, while the dotted curves denotes the projections of surface S_1 and S_2 . (a), the slice is exactly sandwiched between S_1 and S_2 ; (b), the slice has exceeded size; (c), the slice is not located in the halfway of scanning helix.

(as shown in Fig.4.13(c)), some part of the slice may also be projected at large off-plane angles.

Consider the FDK-type approximate reconstruction in Section 2.3. The off-plane angle of ROI would seriously affect the performance of approximate cone-beam reconstruction. The ROI with smaller off-plane angle can have better reconstructed image quality in terms of less severe artifacts. This can geographically explain why the scanning satisfying Tuy's condition could have better approximate reconstruction, which is the initiative of ExFDK algorithm.

4.6 Conclusions

This chapter presents the ExFDK algorithm. Instead of using the conventional full-scan or 180° half-scan data collection trajectory, a new extended helical scanning trajectory is applied which satisfies the well known Tuy's exact reconstruction condition. To optimize the tradeoff between computation workload and imaging performance, the minimum length of this helical scanning trajectory is calculated and applied. Compared to the conventional half-scan FDK reconstruction, the ExFDK algorithm can improve the image quality and reduce artifacts with cost of a slight increase of helix length and computation. Compared to the previously proposed

NcFDK algorithm [68] which also adopts the data satisfying Tuy's condition, the ExFDK adopts a more direct reconstruction concept and can achieve a much higher reconstruction efficiency.

Chapter 5

Gantry-tilted Cone-beam CT Reconstruction

This chapter presents the author's research work on the gantry-tilted cone-beam scanning. Efforts have been done to improve the reconstruction accuracy of the helical gantry-tilted imaging which possesses different geometry with the normal scanning. Section 5.2 proposes a new imaging algorithm which is motivated by the ExFDK algorithm of Chapter 4 and is pursuing a more accurate gantry-tilted reconstruction. Section 5.3 presents a new method to improve the gantry-tilted local tomography by correcting the local scanning data.

5.1 Introduction

In many medical scanning it is necessary to tilt the CT scanning gantry with respect to the patient and couch. The reasons include reducing the metal artifacts caused by tooth fillings, avoiding unnecessary projection to eyeballs during a cerebrum scanning, or simply for some special image plane which may not lie in the normal

Figure 5.1: A gantry-tilted CT scanner.

transversal plane [103] (e.g. in the cervical spine tomography the gantry is tilted to follow the curvature of the spine to ensure the optimal spatial resolution.) The SIEMENS SOMATOM DRH[®] Scanner born in 1983 can conduct maximum $\pm 25^\circ$ tilted scanning [2]. Nowadays most of the medical scanners are equipped with tiltable gantry. Fig.5.1 shows a SIEMENS[®] scanner working in gantry-tilted mode.

Imaging algorithms for the gantry-tilted scanners is an attractive research topic. In 2000 Hsieh proposed an error correction scheme for the 2D rebinning approximate reconstruction of the gantry-tilted helical cone-beam CT [103]. In 2003 Hein *et al.* proposed an FDK-type reconstruction algorithm for gantry-tilted cone-beam scanning [104, 105], which is based on the gantry-normal FDK reconstruction and without significant additional computation. In 2004 Noo *et al.* proposed a general reconstruction scheme for the gantry-tilted cone-beam CT [106]. It suggested to transform the gantry-tilted projection data into a virtual gantry-normal projection data which can be adopted for any 3D reconstruction algorithm.

All these research works focus on how to correct the data deviation occurring in

Figure 5.2: Gantry-tilted scanning. (a), 2D fan-beam scanning; (b), cone-beam scanning.

the gantry-tilted helical cone-beam scanning. The data deviation is produced by the change of relative position of Region of Interest (ROI) to the gantry axis during helical scanning. This deviation would result the motion artifacts and degrade the image. As shown in Fig.5.2(a), the tilting gantry introduces no significant change to the 2D fan-beam circular scanning. The projection of ROI inside the gantry plane remains at same position at the detector for every rotation angle, which has no difference compared to the gantry-normal scanning. For the helical cone-beam scanning shown in Fig.5.2(b), however, the relative position of ROI to the gantry axis varies with the object translation due to the gantry tilting. Denote ν as the *gantry tilting angle* (the angle between the gantry axis and the object axis), and Δ as the change of relative transversal position of ROI to the gantry axis, the position

Figure 5.3: Transversal images of Head Phantom. a, reconstructed in gantry-normal helical cone-beam scanner; b, reconstructed in 10° tilted helical cone-beam scanner. White arrows clearly point out the shading artifacts caused by the gantry tilting.

change during one source rotation is

$$\Delta = 2\pi h \tan \nu. \quad (5.1)$$

This relative movement would monotonically increase with the helix pitch and with the angle ν , which can also be obviously seen in Fig.5.2(b).

All the conventional approximate reconstruction algorithms are based on the assumption that the ROI remains the same relative position to the gantry axis during the scanning [3, 15, 39]. Therefore they are not appropriate for the gantry-tilted helical reconstruction. Fig.5.3 show the reconstructed transversal images of Head Phantom [16, 17]. Figure.5.3(a) is from a simulated gantry-normal helical cone-beam scanner, and Figure.5.3(b) is from a simulated 10° tilted helical cone-beam scanner. Both are reconstructed with the same conventional FDK algorithm. The motion artifacts can be clearly seen in figure b. To avoid this artifacts, some correction schemes should be done in the conventional reconstruction.

Figure 5.4: The geometry of gantry-tilted helical cone-beam scanner. “ $x - y - z$ ” denotes the universal coordinates system while z axis is parallel with the object translation direction; “ $x' - y' - z'$ ” denotes the gantry coordinates system while z' axis is parallel with the source rotation axis.

Fig.5.4 presents the system geometry of the gantry-tilted cone-beam scanner. The universal coordinate “ $x - y - z$ ” and the tilted gantry coordinate “ $x' - y' - z'$ ” are adopted. Referred to the figure, z -axis is parallel with the object translation direction, x -axis is normal to the ground and y is parallel with the ground. In the tilted gantry coordinates $x' - y' - z'$, the z' -axis is parallel with the gantry rotation axis, while y' -axis is parallel with the ground.

5.2 Extended Half-scan Reconstruction for Gantry-tilted CT

Motivated by the ExFDK algorithm presented in Chapter 4, the extended half scanning is applied in the gantry-tilted helical cone-beam scanning to pursue a more accurate approximate cone-beam reconstruction. Similarly as in the gantry-normal cone-beam scanner, the Tuy's reconstruction condition [50] can also benefit the gantry-tilted helical scanning in the terms of a more accurate cone-beam reconstruction.

5.2.1 Extended Half-scan Helix

Due to the gantry tilting, the geometry of scanning is much more complex than that of gantry-normal helical scanning. Referred to Fig.5.5, the z -axis of the $x - y - z$ coordinates system is the object translation axis, and the z' -axis of the $x' - y' - z'$ coordinates system is the gantry rotation axis. Denote ν as the gantry tilting angle. The coordinates of the ray source, in $x - y - z$ and $x' - y' - z'$ system, are presented as follows:

$$\begin{cases} x = R \cos \theta \cos \nu \\ y = R \sin \theta \\ z = h\theta + R \cos \theta \sin \nu, \end{cases} \quad (5.2)$$

$$\begin{cases} x' = R \cos \theta \cos 2\nu - h\theta \sin \nu \\ y' = R \sin \theta \\ z' = R \cos \theta \sin 2\nu + h\theta \cos \nu \end{cases} \quad (5.3)$$

In Eq.5.3 the rotation starting angle is assumed zero, and h denotes the translation-to-rotation ratio. $x' - y' - z'$ coordinates of a point can be calculated from $x - y - z$

Figure 5.5: Geometry of the proposed extended half-scan applied in gantry-tilted helical scanning, shown in $x' - y' - z'$ coordinates system. The PI-lines from point A and B pass through the virtual slice and leave the grey objective slice untouched. Due to the helix extension, the objective slice can be totally covered between S_1 and S_2 .

coordinates as follows

$$\begin{cases} x' = x \cos \nu - z \sin \nu \\ y' = y \\ z' = x \sin \nu + z \cos \nu. \end{cases} \quad (5.4)$$

Compared to the gantry-normal system, the source movement in gantry-tilted helical system is different. Source movement at the translation axis would vary with the source rotation angle θ , as follows

$$\frac{\delta z}{\delta \theta} = \frac{\delta}{\delta \theta} [h(\theta - \theta_0) + R \sin(\theta + \theta_0) \sin \nu] = h + R \sin \nu \cos \theta. \quad (5.5)$$

While the $\cos \theta > 0$, $\frac{\delta z}{\delta \theta} > h$, this part of helix is named as p^+ helix. While the $\cos \theta \leq 0$, $\frac{\delta z}{\delta \theta} > h$, this part of helix is named as p^- helix.

Recall the well known Tuy's exact reconstruction condition: an ROI can be ex-

actly reconstructed if and only if any plane intersecting the ROI also intersects the scanning trajectory. The Tuy's condition has been referred as the theory basic by some exact reconstruction algorithms, such as the works by Noo's group [46–48]. In Chapter 4 the ExFDK algorithm is presented for approximate reconstruction with higher accuracy. ExFDK algorithm adopts an extended half-scan scheme to enable the object slice which satisfies the Tuy's condition, referred to Section 4.2 in the previous chapter. The transversal reconstruction plane is totally sandwiched between two surfaces formed by PI-lines, therefore any plane intersecting the ROI slice will unavoidably cut the scanning helix. According to Tuy's condition, such transversal slice has potential to be exactly reconstructed. Benefited by this well designed scanning helix, the ExFDK is able to improve the reconstruction with very limited additional computation and data storage, compared to the existing FDK-type methods.

Motivated by the ExFDK algorithm, in this section an extended half scanning is proposed and applied in the gantry-tilted helical scanning, and the length of the scanning helix is calculated. Due to the gantry-tilted system has different geometry, the formula and derivation of the trajectory's length is different with that of the ExFDK which is shown in Section 4.2.2.

Derivation: Refer to Fig.5.5. The point A has coordinates $[x'_a, y'_a, z'_a]^T$ and the B has coordinates $[x'_b, y'_b, z'_b]^T$. Assume a random point C on the helix with source rotation angle θ_c and coordinates $[x'_c, y'_c, z'_c]^T$. Thus the PI-line BC is on the straight line whose formula is:

$$\frac{x' - x'_b}{y' - y'_b} = \frac{x'_c - x'_b}{y'_c - y'_b}, \quad \frac{x' - x'_b}{z' - z'_b} = \frac{x'_c - x'_b}{z'_c - z'_b}. \quad (5.6)$$

Note that the reconstruction plane is set on the plane $z' = 0$. The intersection point

between PI-line BC and plane $z' = 0$ can be determined, which has coordinates as

$$\begin{cases} x' = \frac{z'_b}{z'_b - z'_c}(x'_c - x'_b) + x'_b \\ y' = \frac{z'_b}{z'_b - z'_c}(y'_c - y'_b) + y'_b \\ z' = 0 \end{cases} \quad (5.7)$$

Denote d_b as the distance between this intersection point and the coordinate center, $d_b = x'^2 + y'^2$.

$$\begin{aligned} d_b &= x'^2 + y'^2 \\ &= [\mathbb{A}R(\sin \theta_c - \sin \theta_2) + R \sin \theta_2]^2 \\ &\quad + [\mathbb{A}(R(\cos \theta_c - \cos \theta_2) \cos 2\nu - h(\theta_c - \theta_2) \sin \nu) \\ &\quad + R \cos \theta_2 \cos 2\nu - h\theta_2 \sin \nu]^2 \end{aligned} \quad (5.8)$$

$$\left(\mathbb{A} = \frac{R \cos \theta_2 \sin 2\nu + h\theta_2 \cos \nu}{R(\cos \theta_2 - \cos \theta_c) \sin 2\nu + h(\theta_c - \theta_2) \cos \nu} \right) \quad (5.9)$$

Assuming θ_2 is fixed and θ_c is varying, it can be verified that d_b has a unique minimum value while θ_c varies in a reasonable range. Denote $d_{b,m}$ as this minimum d_b value.

$$d_{b,m} = \min_{\theta_c} d_b, \quad \theta_c \in [\theta_2 - \frac{3}{2}\pi, \theta_2 - \pi] \quad (5.10)$$

Therefore $d_{b,m}$ can be considered a function of single variable θ_2 , i.e. $d_{b,m} = f(\theta_2)$. Let $d_{b,m}$ equals to the object radius r (the transversal slice is still considered a circular disk with radius r since the ν is always a small angle), then θ_2 can be determined from the following equation

$$f(\theta_2) = r. \quad (5.11)$$

With the same procedure the θ_1 can also be determined based on the PI-line emitting from point A . Therefore the length of the extended half-scan helix can be worked

Table 5.1: Length of extended short-scan helix in the cases of different tilted angles and different objective size.

r/R	5°	10°	15°
0.1	1.1297π	1.1143π	1.1357π
0.2	1.2468π	1.2291π	1.2207π
0.3	1.3792π	1.3537π	1.3679π
0.4	1.5218π	1.4839π	1.4753π
0.5	1.6827π	1.6970π	1.6634π
0.6	1.7341π	1.7033π	1.6839π

out.

Meanwhile, with the following Eq.5.12 the relative position of the reconstruction plane with respect to the proposed scanning helix can be determined.

$$z' = R \cos \theta \sin 2\nu + h\theta \cos \nu = 0. \quad (5.12)$$

□

In Table 5.1 there listed the length of several extended helices in tilted helical scanning in the cases of three different tilted angles and six different size objects.

5.2.2 Reconstruction and Simulation

In Section 5.2.1 an extended half-scan scheme is proposed for gantry-tilted helical cone-beam scanning. This reconstruction method is named as FDK-type Gantry-Tilted Reconstruction with Extended Half-scan Trajectory, abbreviated to TExFDK Algorithm. The TExFDK reconstruction is presented as follows:

1. Collect the data set along the tilted helix. Note that once the tilting angle and objective size are decided, the length and the relative position of the proposed short-

scan helix can be calculated following Section 5.2.1. Therefore the data set can be extracted from the extended half-scan helix.

2. Transform the data set to the tilted $x' - y' - z'$ coordinates system. The transformation is as follows:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \nu & 0 & \sin \nu \\ 0 & 1 & 0 \\ \sin \nu & 0 & \cos \nu \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (5.13)$$

3. Apply a conjugate weighting on the short-scan data set. Similar to the ExFDK algorithm presented in Chapter.4, the Silver Weighting is applied on the data set due to the scanning trajectory is more than the 180° half-scan length $(\pi + 2\sigma)$. The weighting formula is referred to Eq.4.10.

4. Filter the data set along the helix tangential direction. Referred to Section 2.4.3 in Chapter.2 and Section 4.3 in Chapter.4, the angle between the gantry plant and the helix tangential direction is named as ϑ_t ,

$$\begin{aligned} \vartheta_t = \arctan \frac{\delta z'}{R \delta \theta} &= \arctan \left(\frac{\delta}{R \delta \theta} (R \cos \theta \sin 2\nu + h \theta \cos \nu) \right) \\ &= \arctan \left(-\sin \theta \sin 2\nu + \frac{h}{R} \cos \nu \right). \end{aligned} \quad (5.14)$$

The data is “tilted” to match the helix tangential direction, and the transformation factor is

$$\begin{bmatrix} \cos \vartheta_t & \sin \vartheta_t & 0 \\ -\sin \vartheta_t & \cos \vartheta_t & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (5.15)$$

5. Apply the FDK-type filtered back-projection (FBP) to obtain the tilted transversal image. The reconstruction formula is referred to Eq.4.14 in Chapter.4. Note that the FDK reconstruction includes the gantry-tilted data correction.

The proposed TExFDK algorithm is applied in simulated gantry-tilted helical cone-beam system, and is compared with the FDK-type reconstruction with full circle scan data (referred as TFsFDK) and 180° ($\pi + 2\sigma$) half-scan data (referred as THsFDK). The parameters of the simulate gantry-tilted scanner are listed in Table 5.2.

Table 5.2: Parameters of simulated gantry-tilted scanner

Source-to-center distance R	600mm
Center-to-detector distance	400mm
Detector size	1×1 mm
Number of rows	64
Number of detectors per row	600
Projections per rotation	800
Helical pitch h	0.5 and 0.8
Gantry tilting angle ν	5° and 15°

Table 5.3: Full-scan helix parameters

Helix length	2π
Start / End position	$0 / 2\pi$
Slice position	π

Table 5.4: Half-scan helix parameters

Helix length	1.1951π
Start / End position	$0 / 1.1951\pi$
Slice position	0.598π

Table 5.5: Extended half-scan helix parameters

Angle ν	Length	Start	End	Slice position
5°	1.1951π	0.2133π	1.4084π	0.8431π
10°	1.3679π	0.1853π	1.5532π	0.8784π

Figure 5.6: Reconstruction results of gantry-tilted helical cone-beam scanning (pitch=0.5). a, with TExFDK reconstruction, $\nu = 5^\circ$. b, with THsFDK reconstruction, $\nu = 5^\circ$. c, with TExFDK reconstruction, $\nu = 15^\circ$. d, with THsFDK reconstruction, $\nu = 15^\circ$.

Figure 5.7: Reconstruction results of gantry-tilted helical cone-beam scanning (pitch=0.8). a, with TExFDK reconstruction, $\nu = 5^\circ$. b, with TFsFDK reconstruction, $\nu = 5^\circ$. c, with TExFDK reconstruction, $\nu = 15^\circ$. d, with TFsFDK reconstruction, $\nu = 15^\circ$.

5.2.2.1 Phantom Reconstruction

A $256 \times 256 \times 256$ mm low contrast Head Phantom is adopted for this simulation. The central transversal slice is reconstructed in the simulated scanner. Refer to Table 5.2.2. The length of 180° half-scan helix is $\pi + 2\sigma = \pi + 2 \arcsin \frac{128 \times 1.414}{600} = 1.1951\pi$. Referred to Section 5.2.1, length of the proposed scanning helix is calculated and the results are 1.3792π at $\nu = 5^\circ$ and 1.3679π at $\nu = 15^\circ$.

The reconstruction results are shown in Fig.5.6 and Fig.5.7. The advantages of TExFDK algorithm can be seen more clearer in the focused area. In the cases of different tilting angles or different helix pitch value, the FDK-type reconstruction with proposed extended half-scan helix yields less shading artifacts, than those of the full-scan and 180° half-scan FDK reconstruction.

5.2.2.2 Analysis of Slice Sensitivity Profile

The following is the SSP analysis for evaluating the reconstruction z -resolution of TExFDK algorithm. Same as the simulations in previous chapters, a dark solid phantom is adopted within which a 1mm thick bright disk is transversally positioned. Several transversal slices near the central bright disk are reconstructed. Several transversal slices near the central bright disk are reconstructed under tilted angle $\nu = 15^\circ$ and pitch=0.5 and 0.8. The reconstructed values of four rim area points (with coordinates $[\pm 120, 0]$ and $[0, \pm 120]$) and one center point $([0, 0])$ of the bright disk are adopted to produce the SSP curves.

Table 5.6: Averaged FWHM of the SSP curves in Fig.5.8A and 5.9A.

	TExFDK	THsFDK	TFsFDK
pitch=0.5	0.992mm	1.314mm	1.644mm
pitch=0.8	1.032mm	1.453mm	1.746mm

Figure 5.8: Slice Sensitivity Profiles (SSP) of the reconstructed images (pitch=0.5). (A) SSP measured in central point; (B) SSP measured in rim point.

The SSP curves of the TExFDK, TFsFDK and THsFDK reconstruction are displayed in Fig.5.8 and 5.9. The FWHM of these curves are averaged and listed in Table 5.6.

5.2.2.3 Noise Analysis

To conduct a noise analysis, a Gaussian distributed noise is added to the projection data of low contrast Head Phantom, referred to Section 3.4.3 and Section 4.4.3. The mean value and variance value of the noise is calculated as follows

$$\mu_i = 0, \sigma_i = \frac{\exp(D_i)}{I_0} (I_0 = 10^5), \quad (5.16)$$

Figure 5.9: Slice Sensitivity Profiles (SSP) of the reconstructed images (pitch=0.8). (A) SSP measured in central point; (B) SSP measured in rim point.

where D_i is the single projection datum and I_0 is the estimated total amount of incident photon.

The noise analysis results of TExFDK reconstruction and half-scan FDK reconstruction are shown in Fig.5.10. Compared to the half-scan FDK reconstruction, it can be seen the TExFDK achieves much better imaging performance in reconstruction with noise in terms of much less shading artifacts. Referred to Table 5.2.2, the TExFDK reconstruction only requires very limited projection data compared to the half-scan FDK reconstruction.

Figure 5.10: Simulation (with noise) results of gantry-tilted helical cone-beam scanning (pitch=0.8). a, with TExFDK reconstruction, $\nu = 5^\circ$. b, with half-scan FDK reconstruction, $\nu = 5^\circ$. c, with TExFDK reconstruction, $\nu = 15^\circ$. d, with full-scan FDK reconstruction, $\nu = 15^\circ$.

Table 5.7: Length of scanning helices by different algorithms.

r/R	TExFDK	THsFDK	TFsFDK
0.1	1.1143π	1.0638π	2π
0.3	1.3537π	1.1940π	2π
0.5	1.6970π	1.3333π	2π

5.2.3 Discussions

The TExFDK algorithm is motivated by the ExFDK algorithm of Chapter 4, which intends to reconstruct a transversal image slice by collecting the cone-beam projection data along an extended short-scan helical trajectory. The above reconstruction simulation results proved that the TExFDK has advantages over the traditional FDK gantry-tilted reconstruction in the aspect of imaging accuracy and shading artifacts.

TExFDK algorithm can improve the gantry-tilted helical imaging with moderate computation compared to the other FDK-type reconstruction. To evaluate the computational load quantitatively, the comparison of scanning helices with different algorithms is done and shown in Table 5.7. The comparison is done in the condition of $\nu = 10^\circ$ and the objective transversal slice with different relative size with the gantry size ($r/R = 0.1, 0.3, 0.5$). Length of involved helices of TExFDK algorithm are referred to Table 5.1. As Table 5.7 shown, the additional helical scanning of TExFDK algorithm over the 180° half scanning is very limited.

5.3 Data Correction for Gantry-tilted Local CT

Gantry-tilted CT scanning is specially for the diagnosis which requires higher safety. Compared to the gantry-normal scanning, it is more necessary to limit the radiation dose in the gantry-tilted scanning than in the gantry-normal scanning.

Figure 5.11: Local projection and reconstruction.

Local Computed Tomography (or *local CT*) is a technique to achieve the ROI's image while limiting the projection within the ROI. As illustrated in Fig.5.11, with this technique the ROI is reconstructed from only the local projection of ROI [3,108–111].

With the development of the 3rd generation CT scanner, the volumetric scanner has been more and more concerned. SIEMENS SOMATOM Sensation-16[®] scanner was born in 2002 possesses a 16-row detector [2]. In near future a scanner with 96-row detector will be applied in medical field [3]. The pursuit of larger volumetric scanning yields more and more concerns on the X-ray projection dose, since the overdose X-ray projection is seriously harmful to human bodies. In the past decade, local tomography has been proven capable to limit the projection dose and enhance the safety in the diagnosis. In some cases of the cerebral CT imaging, for instance, the local CT technique can save the X-ray dose up to 50% [3].

Due to the tilted gantry, the helical scanning geometry becomes more complex. The image quality may be degraded since the conventional imaging algorithms are not compatible with the gantry-tilted CT image reconstruction. Efforts have been

made to improve the gantry-tilted helical imaging and the research on this problem has been active. In 2000, J. Hsieh proposed an error correction scheme for the 2D rebinning approximate reconstruction of the gantry-tilted helical cone-beam CT [103]. In 2003, I. Hein *et al.* proposed an FDK-type reconstruction algorithm for gantry-tilted cone-beam scanning [104, 105], which is based on the gantry-normal FDK reconstruction and without significant additional computation. In 2004, F. Noo *et al.* proposed a general reconstruction scheme for the gantry-tilted cone-beam CT [106]. It suggests to transform the gantry-tilted projection data into a virtually normal projection data which can be adopted by some 3D reconstruction algorithms.

Due to its geometry, the gantry-tilted helical cone-beam CT has an inherent problem that the projection's relative shifting would degrade the ROI's image. In this section the local tomography is applied in the gantry-tilted cone-beam projection for a safer diagnosis. In order to solve the problem and improve the imaging performance, this section presents a new data correction method which intends to correct the projection data by geometrical compensation.

In the followings, Section 5.3.1 presents the principles and implementation of the local tomography. In Section 5.3.2, the geometry of local gantry-tilted helical imaging and the caused projection deviation are investigated, and a new data correction method is presented in the following Section 5.3.3. The new method is evaluated in comparison with those existing solutions in Section 5.3.4.

5.3.1 Local Tomography

A local ROI image can be achieved from 2D or 3D local projection data. However, the conventional FBP method would yield error in local image reconstruction because the ramp filter $h(t) = \text{IFT}[|\omega|]$ (referred to Eq.2.9) does not support the local

operation.

Referred to Section 2.1, the ramp filter $h(t)$ is the IFT of $|\omega|$ in frequency domain, as follows

$$h(t) = \int_{-\infty}^{\infty} |\omega| e^{j2\pi\omega t} d\omega = \int_{-\infty}^{\infty} [\omega \cdot \text{sign}(\omega)] e^{j2\pi\omega t} d\omega. \quad (5.17)$$

The IFT of “ ω ” is a derivative operator, while the IFT of the Sign Function “ $\text{sign}(\omega)$ ” is a *Hilbert Transform* [3, 108, 110, 111]. Therefore the filtering with $h(t)$ can be presented as a Hilbert Transform of the derivative of a function, as follows

$$f(t) * h(t) = \text{Hilbert} \left[\frac{\delta}{\delta t} f(t) \right]. \quad (5.18)$$

It is easily proved that the derivative is a “local” operation, i.e. the derivative of a signal $f(t)$ at $t = t_0$ only involves the data within a small range around t_0 ,

$$\frac{\delta}{\delta t} f(t) \Big|_{t=t_0} = \frac{1}{2} \left[\frac{f(t_0 + 1) - f(t_0)}{t_0 + 1 - t_0} + \frac{f(t_0) - f(t_0 - 1)}{t_0 - t_0 + 1} \right]. \quad (5.19)$$

However, the Hilbert Transform will introduce a discontinuity at the origin of the frequency domain,

$$\text{sign}(\omega) = \begin{cases} 1, & \omega \geq 0 \\ -1, & \omega < 0 \end{cases} \quad (5.20)$$

$$\begin{aligned} \frac{\delta}{\delta \omega} \text{sign}(\omega) \Big|_{t=0^+} &= \frac{\text{sign}(1) - \text{sign}(0)}{1 - 0} = 0, \\ \frac{\delta}{\delta \omega} \text{sign}(\omega) \Big|_{t=0^-} &= \frac{\text{sign}(0) - \text{sign}(-1)}{0 - (-1)} = 2 \neq \frac{\delta}{\delta \omega} \text{sign}(\omega) \Big|_{t=0^+}. \end{aligned} \quad (5.21)$$

This discontinuity yields a result that the Hilbert Transform of a neatly supported function (a function sharply falls to zero once the variable being out of a range) would expand its support [108–112]. It means that the filtered value of $f(t)$ at $t = t_0$ will involve large amount of the data, not just the data near t_0 . This also

implies that most of the ray beams passing through the slice have to be measured to reconstruct one point.

To circumvent the nonlocal nature of the conventional FBP reconstruction, there have been a few methods invented for the local tomography. These algorithms can be classified into two main categories: the wavelet-based local reconstruction algorithms [108–111] and the Λ -tomography algorithms [112, 113, 116, 117].

Λ -tomography: This technique does not aim to reconstruct the image $f(x, y)$, but to reconstruct a filtered image $\Lambda f(x, y)$, as follows:

$$\Lambda f(x, y) = \text{IFT}_2 \left[|\omega| \cdot F(\omega, \theta) \right], \quad (5.22)$$

where IFT_2 is the 2D Inverse Fourier Transform, $F(\omega, \theta)$ is the 2D Fourier Transform of $f(x, y)$ being presented in polar coordinates. The function $\Lambda f(x, y)$ is able to be reconstructed from the locally projection data. But Λ -tomography can only present the tissue edges instead of the actual grey value of each tissue [111–113]. Therefore it is often applied when tissue's edge raises more interest than the actual intersection image.

Wavelet-based local CT: Another category of the local tomography algorithms are of the wavelet-based algorithms. The *wavelet* is a short waveform segment, denoted by $\psi(t)$, which possesses the following mathematical properties [114, 115]:

$$\int_{-\infty}^{\infty} \psi(t) dt = 0, \quad \int_{-\infty}^{\infty} t^l \psi(t) dt = 0 \quad (l = 0, 1, 2, \dots) \quad (5.23)$$

It is known that the wavelet function possesses many zero moments. The Wavelet Transform (WT) of a function $f(x)$ is similar with the Fourier Transform, defined as follows

$$\text{WF}(j, k) = \text{WT}[f(x)] = \int_{-\infty}^{\infty} f(t) \psi_{j,k}(t) dt, \quad (5.24)$$

where $\psi_{j,k}(t)$ is a scaled and shifted wavelet,

$$\psi_{j,k}(t) = j^{-0.5} \psi(jt - k). \quad (5.25)$$

The Inverse Wavelet Transform (IWT) is defined as follows,

$$f(t) = \text{IWT}[\text{WF}(j, k)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \text{WF}(j, k) \psi_{j,k}(t) dj dk. \quad (5.26)$$

Any wavelet $\psi(t)$ has an orthogonal wavelet pair $\phi(t)$ which is known as the scale wavelet. The $\psi(t)$ and $\phi(t)$ are also known as Bi-orthogonal pair [109, 111]. There are two important characteristics of the Bi-orthogonal wavelets pairs, as follows:

1. Similarly with the Fourier Transform which decomposes any function into sine and cosine components, Wavelet Transform can decompose any function into the Bi-orthogonal wavelets components [109, 111, 114, 115].
2. Similarly with the low-pass filter and high-pass filter, the scale wavelet $\phi(t)$ can extract out the low frequency part of a function while the wavelet $\psi(t)$ can extract out the high frequency part.

It has been proved that a function with many zero moments, such as the wavelet function, can preserve its support after the Hilbert Transform [108–111]. Since the wavelet transform of a function $f(t)$ can inherit the property of the wavelet function and also possesses many zero moments, the wavelet theory has long been adopted by the researchers to solve the problem of local tomography. The first numerical algorithm using the wavelet for the local tomography was suggested and implemented by DeStefano and Olson [108] in 1994. This algorithm reconstructs the local values of a function $f(t)$ directly from the 1D WT of the projection data. In 1994 A. H. Delaney and Y. Bresler adopted the 2D WT into the tomographic reconstruction to pursue a local recovery [109]. Both algorithms require some amount of global

projection data to reconstruct the low frequency part of image, therefore they are not exactly local.

In 1997 F. R. Farrokhi *et al.* proposed a multiresolution local tomography algorithm based on 2D separable WT [111]. It adopts the scale wavelet $\phi(t)$ and constructs 2D separable wavelets, as follows:

$$\begin{aligned}\Phi(t, s) &= \phi(t)\phi(s) , \quad \Psi_1(t, s) = \phi(t)\psi(s) , \\ \Psi_2(t, s) &= \psi(t)\phi(s) , \quad \Psi_3(t, s) = \psi(t)\psi(s).\end{aligned}\tag{5.27}$$

Since the wavelets $\phi(t)$ and $\psi(t)$ are respectively related with a function's low and high frequency part, the above four 2D separable wavelets can divide a 2D image $f(t, s)$ into four separate images which represent the low and high frequency components of the original image at different directions. Referred to the WT and IWT formulas in Eq.5.24, 5.26 and 5.27, the 2D separable WT of a function $f(t, s)$ can be presented as follows

$$WF_{2^j}(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t, s) \Phi(t - 2^{-j}u, s - 2^{-j}v) dt ds , \tag{5.28}$$

$$WF_{2^j}^i(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t, s) \Psi_i(t - 2^{-j}u, s - 2^{-j}v) dt ds , \quad i = 1, 2, 3 \tag{5.29}$$

where the scaling factor $a = 2^j$ ($j=1,2,3\dots$). The function $f(t, s)$ is decomposed into four parts, each part represents a combination of low and high frequency components of the function. The j value implies the depth of the wavelet decomposition.

F. R. Farrokhi *et al.* proposed to apply the 2D separable WT into the local projection data $D(t, \theta)$ and decompose the projection data into different frequency components. This wavelet-based local reconstruction can be presented as the following four steps:

1. Collect the data via local projection.

2. Decompose the data into four parts, which present the high and low frequency parts respectively, using 2D WT as shown in Eq.5.28 and Eq.5.29.
3. Filter the four data sets separately and then use the backprojection to obtain the four subimages.
4. Compose the four subimages using 2D WT and then obtain the whole image, as follows:

$$F(u, v) = WF(u, v) * \Phi(u, v) + \sum_i [WF^i(u, v) * \Psi^i(u, v)] \quad (i = 1, 2, 3) . \quad (5.30)$$

Benefited by the wavelets, the FBP reconstruction can be completed while maintaining the local ROI values. To further improve the gantry-tilted local scanning safety, in next section the scanning is explored in two aspects, the X-ray focusing and the data correction.

5.3.2 Isocenter Deviation and Data Correction

The gantry-tilted helical cone-beam CT has an inherent problem that the relative transversal shifting of the region of interest (ROI) blurs the reconstructed image. Refer to the gantry-tilted scanning geometry shown in Fig.5.4. The coordinates system “ $x' - y' - z'$ ” is defined in gantry rotation plane where axis z' is the gantry rotation axis. Recall the fan-beam reconstruction in Section 2.2 and define the coordinates system “ $t - s - z'$ ” in which the axis t and s rotate with the X-ray source, as shown in Fig.2.1.

Referring to the geometry of gantry-tilted helical scanning shown in Fig.5.4, the z -axis deviates from the gantry rotation axis z' in the gantry-tilted scanning system. This is referred as the *isocenter* deviation which affects the sampling consistency of the original data set along the z direction [103–106]. The isocenter deviation of the

Figure 5.12: The projection shifting happened in gantry-tilted helical cone-beam scanning, while an objective point moves against the “ $x - y$ ” axis from C_0 to C_1 .

gantry-tilted helical scanning can cause the ROI relative shifting during the helical scanning. Refer to Fig.5.12. Consider C_0 and C_1 as the position of one ROI point at different moment T_0 and T_1 . The $x'y'$ -plane projection of the ROI's center at the objective slice z'_0 and z'_1 . The relative shifting amount C_0C_1 can be measured as $|C_0C_1| = (z'_0 - z'_1) \tan \nu$, where $(z'_0 - z'_1)$ denotes the axial moving distance between t_0 and t_1 , and ν denotes the gantry tilting angle. Consider the point C_0 and C_1 have coordinates (x'_0, y'_0) and (x'_1, y'_1) in the $x' - y'$ system, and have coordinates (t_0, s_0) and (t_1, s_1) in the $t - s$ system. Thus

$$t_0 = x'_0 \cos \theta + y'_0 \sin \theta, \quad s_0 = -x'_0 \sin \theta + y'_0 \cos \theta, \quad (5.31)$$

$$t_1 = x'_1 \cos \theta + y'_1 \sin \theta, \quad s_1 = -x'_1 \sin \theta + y'_1 \cos \theta. \quad (5.32)$$

Refer to Fig.5.12. Denote the point D_0 and D_1 as the projections of the C_0 and C_1 onto the detectors. Following the triangular relationship, the projection deviation can be measured as:

$$\begin{aligned} D_0 D_1 &= D_{sd} \left[\frac{t_0}{D_{so} - s_0} - \frac{t_1}{D_{so} - s_1} \right] \\ &= D_{sd} \left[\frac{x'_0 \cos \theta + y'_0 \sin \theta}{D_{so} - (-x'_0 \sin \theta + y'_0 \cos \theta)} \right. \\ &\quad \left. - \frac{x'_1 \cos \theta + y'_1 \sin \theta}{D_{so} - (-x'_1 \sin \theta + y'_1 \cos \theta)} \right], \end{aligned} \quad (5.33)$$

where D_{sd} denotes the distance from source to detector, and D_{so} denotes the source to the rotation center. Due to $y'_0 = y'_1$, the difference between $x'_0 \sin \theta$ and $x'_1 \sin \theta$ is relatively smaller than D_{so} . Therefore it can be considered that

$$D_{so} - (-x'_0 \sin \theta + y'_0 \cos \theta) \approx D_{so} - (-x'_1 \sin \theta + y'_1 \cos \theta). \quad (5.34)$$

One can calculate the deviation of the local area's center, denoted by Δd_0 , as

$$\begin{aligned} \Delta d_0 = D_0 D_1 &\approx \frac{D_{sd}(x'_0 - x'_1) \cos \theta}{D_{so} + x'_0 \sin \theta - y'_0 \cos \theta} \\ &= \frac{D_{sd}(z'_0 - z'_1) \tan \nu \cos \theta}{D_{so} + x'_0 \sin \theta - y'_0 \cos \theta}. \end{aligned} \quad (5.35)$$

From Eq.5.35 one can see that although the ROI's relatively shifting amount only depends on the gantry tilting angle ν and helical pitch value, the actual shifting amount of the projections on detector would also vary with the source rotation angle θ and the relative position within the ROI. Note that for an arbitrary point (x', y') at the projection path SD_0 , its deviation value Δd can be obtained similarly.

To compensate the projection data deviation, the linear interpolation was adopted

in J. Hsieh's publications [103–105], as follows

$$D_n(t, \theta, z'_0) = \frac{z'_2 - z'_0}{z'_2 - z'_1} D(t, \theta, z'_1) + \frac{z'_0 - z'_1}{z'_2 - z'_1} D(t, \theta, z'_2), \quad (5.36)$$

where $D_n(t, \theta, z'_0)$ is the new data set to be formed, and $D(t, \theta, z'_{1,2})$ is the original data sets sampled from the detector. z'_0 denotes the objective slice, and z'_1 and z'_2 denotes the two closest slices at upper and lower side of z'_0 . This method forms the objective data set by directly combining the two projection sets in two axial positions. Refer to Fig.5.12, while the objective point changes the relative position from C_0 to C_1 , the path of the objective projection would also divert from SD_0 to SD_1 . Therefore the direct interpolation would not completely avoid the data degradation and image blurring.

5.3.3 Local Gantry-tilted Data Correction

Although the points on the neighboring slices have parallel and equal shifts, the deviation of their projections on the detector are different since the fan magnification varies with the different coordinates. Referring to Fig.5.12, to one projection path SD_0 on z'_0 , there is only one virtually ideal corresponding path $S'D'$ on z'_1 , which is parallel to SD_0 with a distance $|C_0C_1| \cos \theta$. It is easily proved that only this ideal path can have projection data which is consistent with that from the path SD_0 . The distance between anyone path SD_1 and the virtual ideal path $S'D'$ is evaluated by the average distance of n pairs of points located on the two paths respectively and at the same y' level:

$$\text{Dist} = \frac{1}{n} \left[\sum_{i=1}^n (\Delta x'_i)^2 \right]^{1/2}, \quad n \geq 2, \quad (5.37)$$

where $\Delta x'_i$ is the distance of two corresponding points at the real and ideal paths. The projection path SD_1 is to be adjusted in order to minimize the Dist . Consid-

ering that the expected SD_1 joins the ideal path $S'D'$ at (x'_c, y'_c) , one can measure the deviation value $\Delta\bar{d}$ on the detector as:

$$\begin{aligned}\Delta\bar{d} &\approx \frac{D_{sd}(z'_0 - z'_1) \tan \nu \cos \theta}{D_{so} + (x'_c + C_0 C_1) \sin \theta - y_c \cos \theta} \\ &= \frac{D_{sd}(z'_0 - z'_1) \tan \nu \cos \theta}{D_{so} + [x'_c + (z'_0 - z'_1) \tan \nu] \sin \theta - y'_c \cos \theta}.\end{aligned}\quad (5.38)$$

The correction amount is shown in terms of the detector cell:

$$\Delta d_n = \frac{N}{L} \Delta\bar{d}, \quad (5.39)$$

where n is slice number, L is the length of detector row, and N is the total cell amount of one detector row.

Referred to Eq.5.38, the above results about the projection deviation depends on the source rotation angle θ and the ROI's position. Furthermore, the ROI's shape should also be taken into account in the deviation measurement. The above calculations do not involve the values of the projection data set, therefore they can be done in advance of the diagnostic scanning. Once the local scanning and reconstruction scheme are decided, these deviation values can be calculated and stored for compensation of the projection data set.

This section presents a new linear interpolation data correction method as follows, which is applicable to “360LI” interpolation,

$$\begin{aligned}D_n(t, \theta, z'_0) &= \frac{z'_2 - z'_0}{z'_2 - z'_1} D(t + \Delta d_1, \theta, z'_1) \\ &\quad + \frac{z'_0 - z'_1}{z'_2 - z'_1} D(t + \Delta d_2, \theta, z'_2),\end{aligned}\quad (5.40)$$

and “180LI” interpolation,

$$D_n(t, \theta, z_0) = \frac{z'_1 - z'_0}{z'_2 - z'_1} D(t + \Delta d_1, \theta, z'_1) + \frac{z'_0 - z'_1}{z'_2 - z'_1} D(t + \Delta d_2, \theta_\phi, z'_2). \quad (5.41)$$

Δd_1 and Δd_2 are respectively the deviation of slice “ $z'_0 - z'_1$ ” and slice “ $z'_0 - z'_2$ ”, which are calculated with Eq.5.38 and Eq.5.39. The θ_ϕ is the projection angle of the conjugated beam.

In addition to the linear interpolation, a “ z -filtering” based method is introduced as follows:

$$D_n(t, \theta, z'_0) = \frac{\sum_{m=1}^M D_m(t, \theta, z'_m) f_z(z'_m - z'_0)}{\sum_{m=1}^M f_z(z'_m - z'_0)}, \quad (5.42)$$

where

$$z'_m = z'_1 + \frac{z'_2 - z'_1}{M} m, \quad M \in \mathbb{Z}, \quad M \geq 2, \quad m = 1, 2, \dots, M, \quad (5.43)$$

$$D_m(t, \theta, z'_m) = \frac{z'_2 - z'_m}{z'_2 - z'_1} D(t_1 + \lambda_1, \theta, z'_1) + \frac{z'_m - z'_1}{z'_2 - z'_1} D(t_2 + \lambda_2, \theta, z'_2), \quad (5.44)$$

$$\lambda_1 = (\Delta d_1 + \Delta d_2) \frac{m}{M}, \quad \lambda_2 = (\Delta d_1 + \Delta d_2) \left(\frac{m}{M} - 1 \right). \quad (5.45)$$

$f_z(z)$ denotes a normal filter [15].

Compared with the nonlinear data correction methods [103] which adopt the nonlinear filtering or frequency domain interpolations, the proposed method adopts the simple linear interpolation in time domain (as shown in Eq.5.40 and Eq.5.41). Its execution time is shorter than the non-linear interpolation methods. In the next section, the proposed method is evaluated by simulated image reconstruction, and compared with the existing methods.

5.3.4 Simulations and Discussions

In this section, the computer simulation is implemented to evaluate the proposed data correction method and to compare with the existing correction methods [103]. The parameters of the simulate gantry-tilted scanner are listed in Table.5.8.

Table 5.8: Parameters of simulated gantry-tilted scanner

Source – axis distance (R)	600mm
Source – detector distance	1000mm
Detector size	1×1 mm
Number of rows	64
Detector cells per row	600
Projections per rotation	800
Helical pitch h	0.8

First, a low-contrast Shepp-Logan phantom is adopted for the simulation. Therefore it is highly sensitive to imaging noise. The local ROI is scanned while the gantry tilts to 10° , 20° and 30° respectively. Fig.5.13 shows the simulation results of the gantry-tilted local image reconstruction. The display window is $[0.95, 1.05]$. From the left column to the right, it can be seen that when the tilted angle is relatively small (e.g., 10°), the reconstructed images by two the methods display little difference. While the gantry tilted angle increases, the difference becomes more visible. The images reconstructed with the existing method have magnificent artifacts, while the images constructed by the proposed data correction and interpolation scheme for local reconstruction have better quality. It is shown that the ellipses in the phantom images remain reasonably clear and sharp at relatively big gantry tilted angles.

Next, a low-contrast Head Phantom is adopted. The central transversal slices at tilted angles $\theta = 5^\circ$ and $\theta = 10^\circ$ are reconstructed with the new and existing linear method, respectively. The comparison result is shown in Fig.5.14 while the display window is $[0.95, 1.05]$. As the gantry tilted angle increases, the artifacts in the

Figure 5.13: Central ROI reconstruction of Shepp-Logan phantom, with different tilted angles. Figures of column (A) are from the existing algorithm, while figures of column (B) are from the proposed algorithm.

reconstructed image of the existing method become more visible, while the artifacts are much less by observation in the results by the proposed method.

An execution time comparison is carried out between the existing data correction methods and the proposed method. The Head Phantom is adopted in this comparison. The reconstruction parameters follow the Table.5.8. Simulations are conducted in different projection density (800×600 and 1200×800 , referred to “projection per rotation” $\times$ “detector cells per row”) and different gantry tilted angle ($\theta = 10^\circ$ and $\theta = 20^\circ$). Denote *LODCOR* (Local CT Data Correction) as the new proposed method, *GLOCOR* (Global Linear Data Correction Method) as the existing linear data correction method, *NODCOR* (Non-Linear Data Correction Method) as the existing nonlinear data correction method. The 4-point linear interpolation is used in *LODCOR* and *GLOCOR*, while the 2-point frequency domain interpolation is used in *NODCOR*. The resource workstation is equipped with Pentium 1.7GHz CPU and 1GB RAM.

Table 5.9: Execution time of simulated reconstructions (unit: sec)

	Density 800×600		Density 1200×800	
	$\theta = 10^\circ$	$\theta = 20^\circ$	$\theta = 10^\circ$	$\theta = 20^\circ$
<i>LODCOR</i>	690 / 492	696 / 510	1656 / 1230	1617 / 1178
<i>GLOCOR</i>	N.A / 523	N.A / 557	N.A / 1379	N.A / 1388
<i>NODCOR</i>	N.A / 687	N.A / 692	N.A / 1753	N.A / 1765

The execution time comparison is shown in Table.5.9. The results are presented in the format “*off-line calculation time* (the time for generating the data compensation matrices) / *online calculation time* (the time for reconstructing the image)”. The results in Table.5.9 show that the proposed Local data correction method can provide faster online data correction and image reconstruction than that of the other two algorithms, since the correction data matrices can be computed and stored before the online computation. It is noted, however, when the scanned ROI is changed,

Figure 5.14: Local gantry-tilted tomographic image of Head phantom. a, result from new method (5° tilted). b, result from existing method (5° tilted). c, result from new method (10° tilted). d, result without existing method (10° tilted).

the correction data matrices should be recomputed and, in such a case, the total executive time is the sum of the off line computation time and online computation time.

Investigate the root mean squares (RMS) error of the reconstructed image as $\sqrt{\overline{\varepsilon_{ij}^2}}$ with ε_{ij} being error of the pixel at i th row and j th column of the image and $\overline{\varepsilon_{ij}^2}$ being the mean value of all ε_{ij}^2 . A comparison of the RMS errors of the three algorithms, *LODCOR*, *GLOCOR* and *NODCOR* for reconstruction of the ROI of the Head Phantom is given in Table 5.10. Compared with the existing algorithms, the proposed *LODCOR* corrects the gantry-tilted scanning data with more accurate compensation matrices. It is further proved by the RMS investigation that the proposed algorithm yields less reconstruction error than the *GLOCOR* and *NODCOR*.

Table 5.10: The RMS of reconstruction errors of ROI

	<i>LODCOR</i>	<i>GLOCOR</i>	<i>NODCOR</i>
10°	0.023	0.061	0.042
15°	0.028	0.071	0.048
20°	0.038	0.088	0.061

One can see from the above simulation results that the new approach is able to provide a better gantry-tilted local imaging performance in comparison with the existing methods. In addition to its computational efficiency, the proposed method can be adaptive with respect to the ROI's position and shape which leads to a more accurate data correction. Furthermore, the data compensation matrices can be implemented and stored off-line, right after the gantry tilting angle and the ROI's shape and position is decided. As a result, the real-time computing workload can be reduced and the operational efficiency can be improved.

5.4 Conclusions

This chapter focuses on the gantry-tilted helical cone-beam scanning. First the gantry-tilted helical scanning trajectory is investigated and an extended half-scanning trajectory is proposed which satisfies the Tuy's exact reconstruction condition. The minimum length of this extended half-scan trajectory is worked out and applied in the new gantry-tilted reconstruction. Compared to the conventional half-scan gantry-tilted FDK reconstruction in phantom imaging simulation and noise analysis, the proposed extended half-scan helix is able to improve the gantry-tilted imaging performance in terms of reducing artifacts and reducing noise, and with a very limited additional workload.

Next the local tomography for gantry-tilted helical scanning is observed. To further improve the accuracy of local scanning and reconstruction, the conventional half-scan and local projection focusing are investigated, and a simplified gantry-tilted data correction method is proposed for local ROI imaging. Simulation results show that the proposed method can assist the

Chapter 6

Conclusions and Recommendations of Future Work

6.1 Conclusions

The research work presented in this thesis is on the development of more efficient and more accurate imaging algorithms for modern helical cone-beam CT scanner. New algorithms are presented for helical cone-beam image reconstruction with both the normal and tilted gantry scanner. Compared with the existing popular CT algorithms, it is shown that these new algorithms are able to improve the image quality and reconstruction efficiency.

- CARP algorithm: To further improve the FDK reconstruction accuracy, this algorithm reconstructs each points on the corresponding PI-plane. The proposed PI-plane is decided by each point's unique PI-line and PI-helix. The pointwise reconstruction based on the PI-plane is able to reduce the shading artifacts of the helical cone-beam imaging, compared to the existing ASSR and FDK reconstruction.

- ExFDK algorithm: This reconstruction is based on a proposed extended half-scan helix with which the helical scanning can satisfy the Tuy's exact reconstruction condition. Compared to the FDK reconstructions with full-scan and 180° short-scan, this algorithm can enhance the image quality considerably with very limited additional workload.
- TExFDK algorithm: This algorithm adopts a proposed scanning trajectory with which the gantry-tilted helical scanning can satisfy the Tuy's exact reconstruction condition. Compared to the existing gantry-tilted FDK reconstructions with full-scan and half-scan trajectory, the TExFDK algorithm can enhance the imaging quality in terms of less shading artifacts and noise.

The advantages of these algorithms are as follows:

- All these new algorithms are based on the 3D FDK reconstruction and can enhance the helical cone-beam imaging performance in relatively large cone-angle scanning. The development of the CARP algorithm is motivated by the PI-line reconstruction which is used in those exact algorithms. It is proved that the reconstruction based on the customized PI-plane can reduce the cone-beam artifacts and enhance the image quality to a considerable extent. The ExFDK algorithm adopts the Tuy's exact reconstruction condition and improves the helical cone-beam transversal reconstruction using a designed helical short scanning. Chapter 5 presents the TExFDK algorithm which applies the idea of extended half-scan helix in gantry-tilted helical scanning and achieves an improved gantry-tilted reconstruction in terms of less severe artifacts and noise.
- The second advantage is that these three algorithms are computationally efficient. Compared to the existing FDK reconstruction algorithms, these new algorithms can provide considerably better image quality with limited additional computation workload. Compared to the exact reconstruction, these al-

gorithms require much less computation resource while producing good image quality. For example, the main additional computation of CARP algorithm beyond the conventional FDK reconstruction is for constructing the PI-planes. However, this computation will not affect the reconstruction speed since it can be done off line before the scanning.

In addition, research work is also carried out on the local tomography applied in gantry-tilted scanning. Local tomography is very useful in improving the X-ray scanning safety, especially in the gantry-tilted scanning which requires higher scanning safety. The new data correction scheme, the LODCOR, presented in Chapter 5 is able to improve the accuracy of the gantry-tilted local imaging which can be referred to the reconstruction comparisons with the existing algorithms.

The presented algorithms are evaluated by computed phantom simulations which have been popularly used by researchers. The computed phantoms adopted in this thesis, such as Shepp-Logan Phantom, Head Phantom, Turbell Clock Phantom, Defrise Phantom, are all well known and often adopted in this research area. Being aware of the lack of the real medical application testing, the work is also evaluated by the computed noise analysis which can simulate the projection environment of the real application.

6.2 Recommendations of Future Work

To continue on this research work on the cone-beam CT reconstruction algorithms, the following future work is recommended in the end of this thesis.

1. Features of exact reconstruction algorithms should be adopted to further improve the approximate algorithm, without increasing the computation workload too much. The CARP reconstruction presented in Chapter 3 is benefited

by the PI-line which was only used in exact reconstruction algorithms developed by Zou and Pan [57]. In Chapter 4 the ExFDK algorithm adopts the extended half-scan helix which is usually used in exact reconstruction algorithms [53] since it satisfies the Tuy's exact reconstruction condition. Note that these algorithms are still based on FDK reconstruction and do not require a large amount of additional computation.

It is noted that the FBP-type exact algorithms by Zou and Pan can achieve relatively high reconstruction efficiency therefore they have evoked interests in this research area [56, 57]. These algorithms reconstruct the ROI values of the PI-lines with filtered backprojection technique. Their computation load is comparable or less than the Katsevich's algorithm [53]. The PI-line concept also play a critical role in the CARP algorithm of Chapter 3 and the ExFDK algorithm of Chapter 4. Research will be continued in the topic that how the PI-line based FBP-type algorithms benefit the helical cone-beam reconstruction with acceptable workload.

2. More efforts are recommended to facilitate the applications of the local tomography in real machine. Local CT is a very useful technique for enhancing the CT diagnostic safety. To overcome its mathematical limitation (the Hilbert Transform does not support the localized backprojection), other tools (such as Wavelet or λ -tomography) have to be incorporated which would yield a lot of additional workload and limit the local CT application. In Chapter 5 the Wavelet is used in gantry-tilted local image reconstruction. Due to the mathematical implicity and large workload of Wavelet, future research work may be done to improve the λ -tomography application in local CT which is much easier but currently can only recover the image edges.
3. Recently in worldwide CT research area many researches are focused on the exact reconstruction algorithms. Due to the quick development of the com-

putation hardware, the exact reconstruction is possible to be applied in real machine. Besides the tremendously heavy workload, the main pending work on the exact reconstruction before its real application is to reduce the noise level. A low-pass filter applied on the images may interfere the useful information. Therefore research work must be carried on on how to reduce the imaging noise from the root of the exact reconstruction.

4. More work should be carried out in the noise added model for computer simulation. Since it may not be easy to test each new algorithm on actual CT machine, a good computerized model becomes very important to the CT imaging research. For noise analysis, it is important for the model to simulate the noise yielding in the actual scanning.

Author's Publications

- [1] H. Liang, C. Zhang, M. Yan, "Data correction for gantry-tilted local CT ," Computerized Medical Imaging and Graphics, Vol. 32, pp. 251-257 (2008)
- [2] H. Liang, C. Zhang, M. Yan, "An approximate algorithm for helical multislice CT using extended scanning helix," Computerized Medical Imaging and Graphics, under review.
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- [4] H. Liang, C. Zhang, M. Yan, "A reconstruction algorithm for helical CT imaging on PI-planes," Proceeding of 28th IEEE EMBS Annual International Conference (2006)
- [5] H. Liang, C. Zhang, M. Yan, "An extended half-scan Feldkamp-type CT reconstruction," Proceeding of SPIE International Symposium on Medical Imaging (2006)
- [6] H. Liang, C. Zhang, M. Yan, "Data correction for gantry-tilted local CT ," Proceeding of 27th IEEE EMBS Annual International Conference (2005)
- [7] M. Yan, C. Zhang, H. Liang, "A new scheme and reconstruction algorithm for dual source circular CT," Proceeding of 28th IEEE EMBS Annual International

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