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Volumetric Calibration for Single-Camera Light-Field PIV

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Abstract

This work presents a volumetric calibration method for single-camera light-field Particle Image Velocimetry (Light-field PIV or LF-PIV). The proposed technique makes use of the unique point-like feature of particle light-field images to accurately determine affected pixels for a spatial voxel over a relative large measurement volume. A calibration model is derived based on Gaussian optics, which relates a spatial point light source with its confusion circle produced on microlens array (MLA), and optical distortions are accounted for by introducing five calibration parameters. By taking lens defects and misalignment between MLA and image sensor into account, the calibration method can calculate weighting coefficient for particle image reconstruction more accurately than the theoretical ray tracing method, especially for regions further away from focal plane where light ray deflections are significant due to optical distortions. The volumetric calibration method was validated by simulation tests using synthetic light-field images, and has been successfully applied to a classic vortex-ring LF-PIV measurement, where the measurable range in depth direction was successfully extended and the quality of reconstructed volumetric velocity field was greatly improved.

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Nomenclature

S_o Object distance

S_i	Image distance
f_m	Focal length of the main lens
$f_{\#}$	F-number of the main lens
p_m	Effective aperture of the main lens
M	Magnification of the plenoptic camera, $M = -\frac{S_i}{S_o}$
f_l	Focal length of lenslet
p_l	Lenslet pitch
S_y	Lenslet centre (relative to optical axis)
y_l	Offset of a light ray from the lenslet centre
y_p	Intersection of a light ray made with the pixel plane
y_c	Intersection of a light ray made with the main lens, light ray passes lenslet centre
y_m	Intersection of a light ray made with the main lens
$C_{l(i)}$	Coordinate of the i -th lenslet centre in pixel plane
d_l	Lenslet pitch (in pixel plane), $d_l = p_l \frac{S_i + f_l}{S_i} = C_{l(i)} - C_{l(i-1)} $
$O(X, Y, Z)$	3D coordinate of a point light source
$E(X, Y, Z)$	Intensity of a voxel at (X, Y, Z)
$I(x_i, y_i)$	Pixel intensity of a light-field image at image coordinate (x_i, y_i)
$w = w_1 \times w_2$	Weighting coefficient for MART reconstruction
w_1	Lenslet weighting coefficient
w_2	Pixel weighting coefficient
C'_{df}	Coordinate of the confusion circle centre (in MLA plane)
D'_{df}	Diameter of the confusion circle (in MLA plane)
C_{df}	Coordinate of the confusion circle centre (in pixel plane), $C_{df} = C'_{df} \frac{S_i + f_l}{S_i}$

D_{df}	Diameter of the confusion circle (in pixel plane), $D_{df} = D'_{df} \frac{s_i + f_l}{s_i}$
$\mathcal{M}_{4 \times 3}$	Mapping matrix that relates a spatial point $O(X, Y, Z)$ with its corresponding C_{df}
p_c	Centre of point-like feature beneath each lenslet
ppm	Particle per microlens

1 Introduction

As an alternative to multi-camera Tomographic Particle Image Velocimetry (Tomo-PIV), the light-field imaging based single-camera LF-PIV has received much attention recently (Ding et al. 2015, Fahringer et al. 2015, Shi et al. 2016). By capturing three-dimensional information of seeding particles with a single plenoptic camera, LF-PIV can achieve volumetric velocity measurements with only one optical window. Such uniqueness is very attractive in terms of simplifying experimental setups for three-dimensional flow measurements. Moreover, it is particularly useful for space-constraint applications where multi-camera configuration is infeasible (Chen and Sick 2017). With continuous efforts devoted to more efficient and higher-accuracy light-field reconstruction algorithms (Fahringer et al. 2016, Shi et al. 2017), it has been demonstrated that single-camera LF-PIV can achieve measurement accuracy levels similar to multi-camera Tomo-PIV when the plenoptic camera resolutions are relatively high (Shi et al. 2018).

Despite rapid developments in LF-PIV, one major challenge remains and that is how to accurately retrieve three-dimensional information of seeding particles from a single light-field particle image. A key step in resolving such a challenge is to establish the mapping relation between a spatial voxel and its affected pixels correctly. Under ideal conditions (i.e. thin-lens assumption and zero optical distortions), such a mapping relation could be accurately derived from Gaussian ray-tracing method. However, any imperfection in the main lens as well as acrylic or glass walls used in the experimental test section would result in light ray deflections. **Effect caused by such refractive index changes is a classic multimedia photogrammetry problem (Mass 1995), and has been successfully dealt with for Stereoscopic PIV (Soloff et al. 1997), Tomo-PIV (Wieneke 2008, 2018) and synthetic aperture PIV (SAPIV, Belden et al. 2010, Belden 2013). In addition to such common problem, LF-PIV faces an unique challenge, that is** plenoptic camera relies on precise installation of a MLA and an image sensor for accurate light-field image recording. As such, any misalignment between the MLA and image sensor, inevitable due to

minute mechanical assembly uncertainties in plenoptic camera, will result in discernible deviation of light rays from theoretical estimations.

In view of accurate three-dimensional geometry measurement from single light-field image, several calibration methods have been developed for single-camera light-field imaging, where polynomial mapping functions based on thin-lens model and pinhole model are typically sought after. Thomason et al. (2014) developed a geometric model which is solved by the Nelder-Mead Simplex Method to estimate the position and orientation of microlens array as well as the distance between main lens and microlens array. Heinze et al. (2016) proposed a calibration method for Raytrix plenoptic cameras, which aims to relate disparity map with metric space so that accurate three-dimensional geometry measurement can be performed by a single plenoptic camera. Similarly, Bok et al. (2017) developed a geometric calibration method for Lytro plenoptic cameras, which solves camera intrinsic parameters for three-dimensional metric measurements. More recently, Hall et al. (2018) built 3rd order polynomial mapping functions to relate the real world coordinates with light-field coordinates, in order to more accurately estimate three-dimensional geometric information from an in-house plenoptic camera. These light-field calibration methods are effective and particularly useful for three-dimensional geometry measurement. However, such techniques cannot be directly applied to light-field particle image reconstruction for volumetric LF-PIV measurements, since multiplicative algebraic reconstruction technique (MART) based reconstruction requires information on spatial voxels and their affected microlens and pixels instead of the metric relation between spatial coordinate and light-field coordinate.

The present work seeks to tackle this problem by developing a volumetric calibration method for plenoptic cameras such that the relation between a spatial voxel and its affected microlens and pixels can be accurately determined, regardless of distortions caused by defects in the main lens, acrylic and glass walls, as well as misalignments between the MLA and image sensor. To achieve that, the proposed technique makes use of the distinctions of light-field particle images, which is particles at different spatial position produce unique circle of confusion patterns on microlens and point-like features on pixels. Based on such information, a model that can predict the centre and diameter of confusion circle is developed, such that weighting coefficient could be accurately calculated for a high-quality MART light-field particle image reconstruction. This paper is structured as follows: in next section, principle of the volumetric calibration method utilising confusion circle and point-like features will be described. Validation analysis through the use of synthetic light-field images will be presented in Section 3, after which Section 4 will demonstrate

the performance of the proposed calibration technique by applying it to a classical vortex-ring LF-PIV experiment. Finally, Section 5 will summarise and conclude the main findings of the current study.

2 Confusion Circle and Point-like Feature Based Calibration Technique

A plenoptic camera captures the light-field of objects (e.g. PIV seeding particles) through a form of “optical coding”. That is, light rays from different incident angles would be refracted by lenslet and recorded by different pixels. Figure 1 shows schematically how light rays from a point source (e.g. a seeding particle) are “coded” by the MLA (Fig. 1 (b) and (c)) and eventually produce point-like features on the sensor plane (Fig. 1 (d)), which is distinctive for each position in the measurement volume. To retrieve the three-dimensional information of such a point light source, or to reconstruct a three-dimensional seeding particle image by MART approach, a key procedure will be to establish the relation between the seeding particle $O(X, Y, Z)$ and its corresponding point-like features (Shi et al. 2017). Inspect the following MART equation (Eq. 1), to correctly calculate the intensity of j -th voxel $E(X_j, Y_j, Z_j)$ from the pixel intensity of light-field particle image $I(x_i, y_i)$, it is important to accurately determine how much light intensity from the j -th voxel are captured by the i -th pixel (i.e. $w_{i,j}$ in Eq. 1). Fundamentally, the weighting coefficient $w_{i,j}$ can be precisely calculated if the centre C_{df} and diameter D_{df} of confusion circle, as well as the centre p_c of each point-like feature can be correctly identified (Fig. 1 (b), (c) and (d)). In the following, it will be demonstrated how these parameters can be accurately determined for each voxel even if optical distortions are present.

$$E(X_j, Y_j, Z_j)^{k+1} = E(X_j, Y_j, Z_j)^k \left(\frac{I(x_i, y_i)}{\sum_{j \in N_i} w_{i,j} E(X_j, Y_j, Z_j)^k} \right)^{\mu w_{i,j}} \quad (1)$$

Fig. 1 (a) Ray propagation inside a plenoptic camera (only **one**-dimensional schematic is shown here); (b) **circle of confusion** formed on the MLA plane (shown for a hexagonal lenslet, which offer higher spatial resolution over a square one (Shi et al. 2016)); (c) **confusion circle** formed on the **sensor** plane; (d) synthetic light-field image showing distinctive point-like features (generated with ray tracing method detailed in Shi et al. (2016))

2.1 Determination of **Confusion Circle** Diameter and Centre

Without any lens distortion and offset between the MLA and image sensor, the diameter (D'_{df}) of the **confusion circle** formed on the MLA plane and the diameter (D_{df}) of **confusion circle** formed on the **sensor** plane by a point light source can be derived based on Gaussian optics using thin-lens model (Eq. 2 and 3). **Please refer to Fig. 1 and nomenclature for specific meaning of the symbols.**

$$D'_{df} = p_m \left(\frac{S_i}{S_o} - \frac{S_i}{z+S_o} \right) = p_m \left(-M - \frac{1}{\frac{z}{S_i} - \frac{1}{M}} \right) \quad (2)$$

$$D_{df} = \frac{S_i + f_l}{S_i} \cdot p_m \left(-M - \frac{1}{\frac{z}{S_i} - \frac{1}{M}} \right) \quad (3)$$

For a perfect thin lens, we have:

$$S_i = f_m(1 - M) \quad (4)$$

$$f_{\#} = f_m/p_m \quad (5)$$

It should be noted that Eq. 3 only works for distortion free thin-lens model. In PIV experiments however, a fixed focal length or zoom lens comprising of a complex combination of a number of individual lenses is typically used. Hence, a correction factor α is introduced to compensate ray prediction errors caused by optical aberrations and thin-lens model. In addition, deflection of ray propagation resulting from offset and rotation between the MLA and image sensor can be accounted for by incorporating four parameters $\beta, \omega, \varphi, \gamma$ into Eq. 3. Consider the fact that the focal length of lenslet f_l is negligible as compared to the image distance S_i , the diameter of **confusion circle** D_{df} can be modelled as following:

$$D_{df} = \alpha \cdot p_m \left(-M - \frac{1}{\frac{\beta z + \omega x + \varphi y + \gamma}{S_i} - \frac{1}{M}} \right) \quad (6)$$

The centre of **confusion circle** C_{df} , is where the main light ray hits the pixel plane (e.g. the red light ray in Fig. 1 (a)). Generally, the position where a random light ray (e.g. the green light ray in Fig. 1(a)) hits the pixel plane y_p , can be calculated with Gaussian optics:

$$y_p = S_y + \frac{y_l + S_y - y_m}{S_i} f_l = C_{l(i)} - \frac{f_l}{S_i} y_m + \frac{y_l}{S_i} f_l \quad (7)$$

Using triangle similarity, the **centre of confusion circle** C_{df} , and the point-like feature centre, p_c , can be calculated from following equations:

$$p_c = C_{l(i)} - \frac{f_l}{S_i} y_c \quad (8)$$

$$C_{df} = \frac{y_c}{p_m} D_{df} + C_{l(i)} \quad (9)$$

$$\frac{p_m}{S_i} = \frac{d_l}{f_l} \quad (10)$$

Collating Eqs. 8 to 10, the relation between the diameter and centre of **confusion circle**, centre of each point-like feature, lenslet centre and lenslet pitch can be formulated as:

$$(C_{l(i)} - p_c)D_{df} + d_l(C_{l(i)} - C_{df}) = 0 \quad (11)$$

For a point light source (e.g. $O(X, Y, Z)$ in Fig. 1 (a)), there are more than one lenslet that are fully covered by the scattered pattern, each of which will produce one point-like feature in the pixel plane. That is to say, a system of equations can be formulated for Eq. 11 using information from the point-like features. For instance, there are five lenslets fully covered by the **confusion circle** in Fig. 1 (c), each of the fully covered lenslet produces one point-like feature in sensor plane (Fig. 1 (d)). The point-like feature centre p_c can be calculated using Gaussian fitting with sub-pixel accuracy. The lenslet centre coordinate $C_{l(i)}$ can be predetermined by stepping down the main lens aperture to its minimum and capture a light-field image for a white plate. The image captured would be an array of white dots in a black ground and the lenslet centre coordinate $C_{l(i)}$ is then calculated by Gaussian fitting the centre of these white dots (Ng 2006). With p_c , $C_{l(i)}$ and d_l known for these five lenslets, five over-determined equations can be used to solve D_{df} and C_{df} for the spatial point $O(X, Y, Z)$.

With the above formulations derived for a spatial point $O(X, Y, Z)$ and its corresponding **confusion circle** diameter D_{df} and centre C_{df} respectively, a calibration board with a white dot array can be used to precisely translate along z-axis in the measurement volume. Such procedure will produce three sets of calibration data: $O(X, Y, Z)$ which are known precisely from the calibration board, and their corresponding D_{df} and C_{df} which can be solved from over-determined equations. The data set $O(X, Y, Z)$ and D_{df} can then be used for Eq. 6 to fit the calibration coefficients $\alpha, \beta, \omega, \varphi$ and γ through the sequence quadratic program (SQP) method. Whereas the data set $O(X, Y, Z)$ and C_{df} can be used to determine the mapping matrix $\mathcal{M}_{4 \times 3}$ according to:

$$\begin{pmatrix} C_{df}(x) \\ C_{df}(y) \\ 1 \end{pmatrix} = \mathcal{M}_{4 \times 3} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (12)$$

Hence, the coefficients $\alpha, \beta, \omega, \varphi$ and γ and mapping matrix $\mathcal{M}_{4 \times 3}$ can be obtained for calibration dots $O(X, Y, Z)$ in the measurement volume. In return, such calibration information (Eqs. 6 and 12) can be used to determine **confusion circle** diameter D_{df} and centre coordinate C_{df} for any points

in the measurement volume, which will be applied in the following section to calculate weighting coefficient w .

2.2 Calculation of MART Weighting Coefficient

According to MART based LF-PIV reconstruction methods (Fahringer et al. 2015, Shi et al. 2017), the weighting coefficient w consists of two parts - lenslet weighting coefficient w_1 and pixel weighting coefficient w_2 ($w = w_1 \times w_2$). The lenslet weighting coefficient w_1 is defined as the overlap area between the affected lenslet and confusion circle (i.e. overlap area between red dotted circle and lenslets in Fig. 1 (c)). With the predetermined lenslet centre $C_{l(i)}$, as well as C_{df} and D_{df} from calibration step, the microlens affected by a voxel can be correctly identified (i.e. the red lenslets in Fig. 1(c)). For each lenslet, its weighting coefficient is calculated using the Monte Carlo method, in which 100 random points are generated inside the confusion circle (e.g. the red dotted circle in Fig. 1 (c)), and w_1 is calculated as the percentage of points that fall into the specific lenslet. Note that more random points were tested for calculating w_1 , it was found that 100 points reaches the best balance between accuracy and computational cost.

As for the pixel weighting coefficient w_2 , it is defined as the overlapping area between the affected pixels and point-like feature. Referring to Figs. 1 (c) and (d), the task at hand is to determine the affected pixels for each point-like feature. For an affected lenslet (e.g. the one highlighted in blue in Fig. 1 (c)), its affected pixels (e.g. the ones highlighted in red in Fig. 1 (d)) locate in the range covered by light rays passing the upper and lower boundary of the lenslet ($-p_l/2 \leq Y_l \leq p_l/2$, see Fig. 1 (a), demonstrate in one-dimension). The position where a light ray hits image plane can be calculated according to Eq. 7. Typically, $S_i \gg f_l$ and $S_i \gg p_l$, thus $\frac{y_l}{S_i} f_l$ can be neglected in Eq. 7. For instance, microlens focal length and lenslet pitch of a typical plenoptic camera is $f_l = 0.3mm$, $p_l = 0.08mm$. Further assume that a $f_m = 200mm$ main lens is used for the PIV experiment. Under such a situation, $\frac{y_l}{S_i} f_l < \frac{0.08}{2 \times 200} \times 0.3mm = 0.00006mm$, which is less than 0.01pixel (note that the pixel size of our in-house plenoptic camera is 0.0055mm). In addition, following equation can be derived using triangle similarity.

$$\frac{y_m}{p_m} = \frac{C_{df} - \left(C_{l(i)} + y_l \frac{S_i + f_l}{S_i} \right)}{D_{df}} \quad (13)$$

Collating Eqs. 7, 10 and 13, the location where a light ray hits on the sensor plane can be calculated according to Eq. 14.

$$y_p = C_{l(i)} + \frac{d_l \left(y_l \frac{S_i + f_l}{S_i} + C_{li} - C_{df} \right)}{D_{df}} \quad (14)$$

Therefore, the region of affected pixels can be determined by using light rays passing the edge of lenslet. Take the one-dimensional schematic in Fig. 1 (a) as an example, by substituting $y_l = y_{p_dn} = -\frac{d_l}{2}$ and $y_l = y_{p_up} = \frac{d_l}{2}$, the affected pixels would locate in the region confined by Eqs. 15 and 16. To calculate the weighting coefficient for each affected pixel, similar Monte Carlo method is used, that is 100 random points are generated inside the determined region, w_2 is then calculated as the percentage of points that fall into the specific pixel.

$$y_{p_dn} = \begin{cases} C_{l(i)} + \frac{d_l \left(-\frac{d_l}{2} + C_{l(i)} - C_{df} \right)}{D_{df}}, & z < 0 \\ C_{l(i)} + \frac{d_l \left(\frac{d_l}{2} + C_{l(i)} - C_{df} \right)}{D_{df}}, & z > 0 \end{cases} \quad (15)$$

$$y_{p_up} = \begin{cases} C_{l(i)} + \frac{d_l \left(\frac{d_l}{2} + C_{l(i)} - C_{df} \right)}{D_{df}}, & z < 0 \\ C_{l(i)} + \frac{d_l \left(-\frac{d_l}{2} + C_{l(i)} - C_{df} \right)}{D_{df}}, & z > 0 \end{cases} \quad (16)$$

To summarise, flowchart of the volumetric calibration method is listed in Fig. 2, where the MART reconstruction step is also included for the sake of completeness.

Fig. 2 Flowchart of the volumetric calibration method (steps in black boxes), the reconstruction step is also shown in dotted box for completeness.

3 Simulation Tests

To validate the proposed volumetric calibration method, a set of synthetic light-field images were generated using distortion free, thin-lens model. The derived calibration mathematical model can be verified if affected pixels predicted by the calibration model match well with synthetic images. With reference to the schematic shown in Fig. 1, synthetic light-field particle images were generated by propagating a large number of light rays (e.g. 5,000,000 light rays) from a tracer particle, through the main lens and MLA, until they reach the pixel plane. Detailed procedures and related mathematic formulas can be found in Georgiev et al. (2006), Ng (2006) and Shi et al. (2016).

Subsequently, synthetic light-field images of a dot array were generated by traversing an artificial calibration board along the z-axis in the range of $-10\text{mm} \leq z \leq 10\text{mm}$ with a step-interval of 0.5mm. The artificial calibration board measures $36.3 \times 24.2\text{mm}$ and consist of 13×9 white dots (each one was simulated by a point light source) with 2.5mm grid spacing. A total of 41 synthetic calibration light-field images were generated using optical parameters listed in Table 1, which means there are in total $13 \times 9 \times 41 = 4797$ spatial points were used for determining coefficients in Eq. 6 and mapping matrix in Eq. 12. Figure 3 (a) shows an example of a light-field image simulated for one calibration dot, where these images were subsequently processed through the procedures shown in Fig. 2. In the first step, point-like features for each dot were processed using Gaussian fitting to find the centre coordinates p_c . Substituting p_c and lenslet centre coordinate C_l to Eq. 11, confusion circle diameter D_{df} and centre coordinate C_{df} could be calculated for each calibration dot. In the second step, the calculated D_{df} and C_{df} together with the known spatial coordinates $O(X, Y, Z)$ of the dot array were fed to Eq. 6 and 12, so as to retrieve the calibration coefficients and mapping matrix, which are listed below. With the calibration coefficients and mapping matrix available, the confusion circle diameter D_{df} and centre coordinate C_{df} could now be obtained using Eq. 6 and 12 for each calibration dot, and its affected pixels could be determined according to Eqs. 15 and 16. Subsequently, the weighting coefficient could then be calculated using method detailed in Section 2.2.

$$\alpha = 1.009, \beta = 1.0021, \omega = -0.0007, \varphi = 0.0016, \gamma = 0.0086$$

$$\mathcal{M}_{4 \times 3} = \begin{bmatrix} -1.5467e + 002 & +1.1081e - 004 & +7.6924e + 000 & +3.2995e + 003 \\ -1.6632e - 003 & +1.5467e + 002 & +5.0254e + 000 & +2.1995e + 003 \\ -9.0152e - 007 & +1.4232e - 007 & +2.3104e - 003 & +1.0000e + 000 \end{bmatrix}$$

Table 1 Key parameters for synthetic light-field image generation and LF-PIV image reconstruction

Parameter	Value	Parameter	Value
Pixel resolution	6600×4400	Simulation volume	36.3×24.2×20mm
Pixel size	5.5μm	Reconstruction volume	3300×2200×364voxel
MLA resolution	450×350	Particle density	0.5ppm
Lenslet size	77μm	Pixel voxel ratio	2:1 (x, y);10:1 (z)
Lenslet focal length	308μm	Voxel size	11×11×55μm
Main lens focal length	200mm	1 st interrogation window	256×256×64 voxel
Main lens f number	4	2 nd interrogation window	128×128×32 voxel
Magnification	-1 (for both synthetic calibration board image and simulated flow image)		

To validate the calibration process, the weighting coefficient and affected pixel clusters determined from above calculation are inspected. Let's take a calibration dot located at $O(X, Y, Z) = (0, 0, -2.0)mm$ as an example, whose light-field image is shown in Fig. 3 (a). For a clear comparison, its corresponding weighting coefficient calculated by the calibration method is converted to a grey scale image and shown in Fig. 3 (b). It can be seen that the calculated weighting coefficient matches well with the original simulated light-field image. To further quantify the calibration performance, pixel clusters affected by every calibration dot are determined by the calibration method, and their centre coordinates p_c are calculated using weighted centroid method. If the calibration process is correct, the determined pixel clusters would overlap with point-like features produced by the calibration dots. Hence, the centre coordinates of these point-like features are calculated directly from the synthetic calibration light-field image, and serve as ground truth to compare with centre coordinates p_c estimated by the calibration method. Such calculation is performed for every calibration dot recorded at the 41 position, and the cumulative distribution function (CDF) of the absolute errors is shown in Fig. 3 (c). As can be seen from the figure, more than 95% of the predicted p_c has errors of less than 0.5 pixels, indicating affected pixels are correctly identified by the calibration method.

Fig. 3 (a) Synthetic light-field image of a calibration dot at $(O(X,Y,Z) = (0,0,-2.0)mm)$; (b) corresponding weighting coefficient calculated with the proposed calibration method (plotted as grey scale image for better comparison); (c) error distribution of the calculated point-like feature centre p_c

To evaluate the performance of the proposed calibration method further, additional simulation test was performed on a synthetic Gaussian vortex-ring. Equation 17 defines the displacement field, where the vortex-ring size is regulated by the length scale $l = 2mm$, and R is the distance from a spatial point to vortex-ring centre. To generate two consecutive LF-PIV images, the measurement volume centre (which is also the vortex-ring centre) was artificially located at $(X,Y,Z) = (0,0,0)mm$. A number of seeding particles ($ppm = 0.5$) were randomly distributed within the measurement volume, and the first frame light-field particle image was generated using ray tracing method. Thereafter, these particles were offset with displacements calculated from Eq. 17, and the second frame light-field particle image was then generated using the new spatial location of the particles.

$$d = \left\| \begin{pmatrix} u \\ v \\ w \end{pmatrix} \right\| = \frac{8R}{l} e^{-\frac{R}{l}} \quad (17)$$

The synthetic light-field particle images were reconstructed with the DRT-MART method, using weighting coefficient calculated with the calibration method as well as the ray tracing method (Shi et al. 2017). The iteration number and relaxation factor were set at 200 and 2.5 for both synthetic and real light-field image reconstructions. These parameters have effects on the MART light-field reconstruction different from Tomo-PIV, and readers are referred to Shi et al. (2017) for more details. Since there is neither optical distortion nor misalignment between MLA and image sensor in current simulation test, data from the ray tracing method represent the ideal

results. The reconstructed particle images were processed by three-dimensional multi-grid cross-correlation method, with initial and final interrogation window sizes of $256 \times 256 \times 64$ *voxel* and $128 \times 128 \times 32$ *voxel* respectively (Table 1). The interrogation window overlap ratio was 75%. Outlier vectors were detected through a median filter and replaced by a $3 \times 3 \times 3$ linear interpolation. The reconstructions and cross-correlation calculations were enhanced with GPU parallel processing (NVIDIA GeForce 1080Ti, 11GB graphics memory, installed on a Dell Precision T7910 workstation). It took about 185mins to reconstruct a pair of synthetic light-field images and each reconstructed volume takes up 9GB disk space. The cross-correlation took around 34mins for processing one instantaneous velocity volume.

The simulation results are plotted in Fig. 4, which shows the reconstructed Gaussian vortex-ring and the probability density function (PDF) of absolute velocity error distribution. It can be observed that measurement accuracy of the proposed calibration method matches well with the ideal ray tracing method. Due to elongation effects in the z-direction (Shi et al. 2016, Shi et al. 2017), the measurement accuracy of the two methods for velocity component in depth direction is lower than the other two velocity components.

Fig. 4 (a) Gaussian vortex-ring reconstructed with the calibration method (b) measurement error of the calibration method, as compared to ray tracing method

4 Experimental Tests

After satisfactory validations using synthetic data, the proposed volumetric calibration method was applied to a vortex-ring LF-PIV experiment. A circular vortex-ring was produced via the “cylindrical slug” technique using experimental setup shown in Fig. 5, where a Delrin-based

cylindrical piston was pushed through a hollow stainless-steel cylinder by an Origa ball-screw linear-actuator. The linear-actuator was driven by an Applied Motion Products high-torque stepper-motor, which was controlled by a workstation with a resolution of 51,200 microsteps per revolution. The stepper-motor was programmed using trapezoidal velocity profiles to achieve a Reynolds numbers of $Re = 2000$. The piston diameter was $d = 20\text{mm}$ and a stroke-length of $L = 30\text{mm}$ was used during the experiment to ensure a stroke-ratio of $L/d=1.5$ in order to avoid trailing-jet formations. Readers are referred to New et al. (2016) for more details on vortex-ring setup system.

Fig. 5 Schematic of the vortex-ring experimental setup

For the LF-PIV experiments, the water tank was homogenously seeded with polyamide seeding particles (0.4ppm, Dantec Dynamics, $50\mu\text{m}$, $\rho = 1.03\text{g/cm}^3$). A test volume measuring $37.9 \times 25.3 \times 30\text{ mm}$ was uniformly illuminated by a double pulse Nd:YAG laser (Beamtech, 200 mJ/pulse, 532 nm). Particle images were captured by an in-house 29M pixel plenoptic camera with a Micro-NIKKOR 200mm lens (Shi et al. 2016, Shi et al. 2018). The laser and in-house plenoptic camera were synchronised by a pulse generator (Microvec, SM-Micropulse725) with a time-interval of 2 ms, such that the maximum displacement between two frames was limited to 60 voxels.

The calibration procedure was performed **right** after the LF-PIV images were taken, by using the setup shown in Fig. 6 (a). To accomplish that, a $55 \times 40\text{ mm}$ calibration board was placed inside the water tank and traversed along the optical axis of the plenoptic camera through the use of a high-precision motorised translation stage (Thorlabs LNR50S/M, resolution $0.1\mu\text{m}$). The

aluminium calibration board has 21×15 white dots (0.2 mm diameter) with a grid spacing of 2.5 mm , which were laser-painted on its matt black surface (Fig. 6 (b)). The calibration board was traversed to-and-fro of the focal plane in the range of $-10 \sim 10 \text{ mm}$ with a 0.5 mm step-interval, such that 41 light-field dot images were acquired for calibration ($21 \times 15 \times 41 = 12915$ points were used for solving Eq. 6 and 12). Note that calibration images were taken by the in-house plenoptic camera with optical settings (e.g. camera position, focal distance, main lens aperture) exactly same as those for the LF-PIV experiment, in order to faithfully record any optical distortions during the experiments. For each of the calibration image, procedures described in Section 2 were performed to first calculate D_{df} and C_{df} for every calibration dot, and then determine the calibration coefficient and mapping matrix by fitting the known spatial coordinates $O(X, Y, Z)$ of calibration dots with the associated D_{df} and C_{df} to Eq. 6 and 12. The calibration coefficient and mapping matrix for the current experimental setup are:

$$\alpha = 0.7322, \beta = 0.9882, \omega = 0.0145, \varphi = 0.0231, \gamma = -0.2$$

$$\mathcal{M}_{4 \times 3} = \begin{bmatrix} +1.7765e+002 & +1.2227e-001 & +4.9384e+000 & +3.3106e+003 \\ -3.7039e-001 & +1.7740e+002 & +1.6402e+000 & +2.2286e+003 \\ +6.5267e-006 & -2.3101e-005 & +8.2910e-004 & +1.0000e+000 \end{bmatrix}$$

Fig. 6 (a) Schematic of the calibration setup, and (b) the calibration board

It is worthwhile to examine its accuracy before applying the calibration result to light-field particle image reconstruction. To do that, pixel clusters affected by calibration dots are estimated separately by the ray tracing and calibration methods, and compared with the point-like feature in the real calibration image. The ray tracing method does not consider any optical distortions, to locate affected pixel clusters for a calibration dot, it randomly propagates 5,000,000 light rays from each point on the calibration board and trace them to the sensor plane (in a similar way as generating a synthetic light-field image). Pixels collect these light rays will produce ideal point-

like feature images, and the centre p_c of these affected pixels are calculated using the weighted centroid method. If there was no distortion during the calibration process, pixel clusters located by the ray tracing method would be identical to the point-like features in the captured calibration light-field image, and their centre coordinates p_c would match with the ones calculated directly from the real calibration image. For the volumetric calibration method, pixel clusters affected by calibration dots are determined by Eq. 15 and 16 through using the calibration data. The centre p_c of the pixel clusters is calculated by weighting centroid. If the calibration performs well, the centre coordinate p_c should coincide with the ones calculated directly from the real calibration image.

Figure 7 shows the point-like feature image captured for one of the calibration dots, together with the centre coordinates p_c calculated by the ray tracing (yellow cross) and volumetric calibration (green circle) methods. As Fig. 7 (b) shows, the volumetric calibration method can better predict the affected pixels than the ray tracing method, which in turn will increase the accuracy for weighting coefficient calculation. For a quantitative assessment, centre coordinate of every real point-like feature are calculated for 41 calibration images, and compared with the ones calculated by the ray tracing and calibration methods. The p_c error distribution for all calibration images are plotted in Fig. 8 (a), which clearly indicates that over 96% of the affected pixel clusters can be accurately identified (absolute p_c error less than 1 pixel) when the optical distortions are accounted for. To illustrate the calibration accuracy in the entire measurement volume, the errors of calculated point-like feature centres p_c are plotted along optical axis in Fig. 8 (b). It shows that the volumetric calibration method can generally better locate the affected pixel clusters than the ray tracing algorithm. However, there is an evident increase of calibration error near the focal plane. This is due to the fact that, for a point light source in this region, light rays tend to focus on fewer lenslets, meaning fewer lenslets are fully covered (refer to Fig. 1 (b)). As a result, there are fewer numbers of Eq. 11 could be used to calculate D_{df} and C_{df} . The worst scenario happens at the origin of focal plane ($O(0,0,0)$), where light rays are focused on only one lenslet and there is only one equation can be established for Eq. 11, as such the D_{df} and C_{df} could not be solved for this case.

Fig. 7 (a) An example of light-field image for **one** calibration dot, showing together the real centre of affected pixels, together with results from ray tracing and volumetric calibration method, (b) magnified view for a better appreciation of the differences **between** two methods

Fig. 8 (a) CDF error distribution of the detected point-like feature centre, (b) error distribution of the detected point-like feature centre along depth direction

With the available calibration data, the weighting coefficient was calculated using the method detailed in Section 2.2. The light-field particle images were reconstructed by the DRT-MART method with a pixel voxel ratio of 2:2:10 in x-, y- and z-direction, which results in a reconstruction domain of $3200 \times 2100 \times 550$ voxels ($0.011 \times 0.011 \times 0.055 \text{mm}^3/\text{voxel}$). Reconstructing a pair of light-field image with such resolution costed 200mins GPU time

(NVIDIA GeForce 1080Ti). Each reconstructed volume takes up about 13.77GB disk space. For comparison purposes, reconstruction with weighting coefficient calculated using ray tracing method was also performed. The reconstructed particle volume by the two methods are shown in Fig. 9 (plot by Tecplot with the same isosurface value), which demonstrates that the reconstruction volume is greatly extended in the depth direction by using the volumetric calibration method. However, to which extent a seeding particle can be reconstructed is mainly limited by two factors, intensity of its affected pixels and whether these pixels can be accurately located. When a seeding particle is too far away from the focal plane such that its light intensity captured by each affected pixel is lower than the background noise, this particle is basically impossible to reconstruct. The proposed volumetric calibration method does not aim to solve such challenge however.

On the other hand, for particles away from the focal plane but not too far away that the intensity of its affected pixels is much higher than the background noise, the successful reconstruction of such particle highly depends on the accurate weighting-coefficient calculation for each affected pixels. Inspecting the MART equation (Eq. 1), when many pixels are involved in the reconstruction, which is the case for particles away from the focal plane, a slight miscalculation of the weighting coefficient would cause the MART iteration to diverge. The proposed volumetric calibration method aims to tackle this problem. For particles further away from the focal plane, light rays will be spread out to more lenslets, especially lenslets further away from the optical axis. Therefore, the prediction error introduced by optical distortion will become more significant and as a result, the affected pixel clusters cannot be correctly identified for these particles if no calibration information was available. On the other hand, more lenslets being affected in regions away from focal plane means that there would be more numbers of Eq. 11 available (overdetermined), which is helpful to more accurately solve D_{af} and C_{af} and thus resulting a better calibration performance in these regions. Unfortunately, it is noted that current volumetric calibration method does not help to mitigate the particle elongation effect in the depth direction, which will limit the measurement resolution in z-direction as what have been quantitatively analysed in earlier studies (Shi et al., 2017 and 2018).

Fig. 9 (a) and (b) Three-dimensional particle images reconstructed by the **volumetric** calibration method; (c) and (d) three-dimensional particle images reconstructed by the ray tracing method

To further investigate how the velocity volume reconstruction can benefit from the volumetric calibration, the reconstructed particle volumes were processed using multi-grid three-dimensional cross-correlation using parameters similar to those outlined in Section 3. **Each instantaneous velocity volume took around 51mins of GPU time.** Figure 10 shows the instantaneous velocity fields reconstructed with and without calibration information. By using the proposed volumetric calibration, the measurable range in depth direction has been successfully extended to over 20mm, which facilitates full reconstruction of the vortex-ring. In contrast, the effective measurement range in depth direction was limited to 10mm if no calibration was used. As very few particles were reconstructed beyond $-5 \leq z \leq 5\text{mm}$ (see Fig. 9 (c)), cross-correlation failed to calculate

valid velocity vectors in this region (see Fig. 10 (d)). To further quantify the efficiency of the volumetric calibration method, divergence of the reconstructed velocity data were calculated and plotted in Fig. 11. For high-quality measurements of an incompressible flow, divergence of the velocity data should close to zero as governed by the Navier-Stokes equations. As can be seen from Figs. 11 (a) and (c), the velocity data calculated with volumetric calibration converge near the 45° line, whereas velocity data calculated without volumetric calibration scattered over a very significant region (see Fig. 11 (b) and (d)).

Fig. 10 (a) and (b) Instantaneous velocity field reconstructed with volumetric calibration (c); and (d) instantaneous velocity field reconstructed without volumetric calibration

Fig. 11 (a) and (c) Divergence of the instantaneous velocity field reconstructed with volumetric calibration; (b) and (d) divergence of the instantaneous velocity field reconstructed without volumetric calibration

5 Conclusion

A volumetric calibration method is proposed for single-camera light-field PIV, based on the **circle of confusion** and point-like features formed on the **MLA and** sensor plane of a plenoptic camera. Optical distortions caused by lens defects, thick glass wall and misalignment between MLA and image sensor are compensated using **five** correction coefficients, which is beneficial in identifying the affected pixel clusters more accurately for a spatial point. Through simulation and experimental tests, it has been demonstrated that the volumetric calibration method can more accurately reconstruct particles further away from the focal plane, thereby greatly increase the

measurable range in the depth direction and improve data quality for the reconstructed velocity field.

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