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Distributed Adaptive Control for Distance-based Formation and Flocking control of Multi-Agent Systems

Ying Zou^{1*}, Changyun Wen¹, Mingyang Guan¹

¹ School of Electrical and Electronic Engineering, Nanyang Technological University, 50 Nanyang Ave, Singapore 639798

* E-mail: zouy0011@e.ntu.edu.sg

Abstract: In this paper, a distributed adaptive control strategy for distance-based formation and flocking control of multi-agent system with parametric uncertainty is proposed. A 3-agent Leader-First-Follower (LFF) system is considered first. The agents are supposed to have nonidentical dynamics, whereas with similar structure. By introducing two new variables that involve distance error and velocity error and utilizing them in the controller design, global convergence of all agents to the desired formation and to the velocity of the leader is established. Finally, the 3-agent case is inductively extended to N -agent case. The stability and the effectiveness of the proposed control strategy are analyzed in theory and demonstrated through simulation results, respectively.

1 Introduction

In recent years, much attention has been paid to formation control of multi-agent systems. According to the variables be controlled, the formation control problems can be categorized as position-based, displacement-based, and distance-based formation control problems [1]. Due to the advantage that less global information is required in controller design, distance-based formation control has recently attracted much research interest [2–7]. Although the nonlinearity of distance-based control laws complicates the stability analysis of the multi-agent system, much effort has been made in this research area.

While most of existing control strategies under directed graphs only focus on the formation shape control, however, in practical applications such as moving target enclosure and maneuvering of autonomous vehicles [8, 9], the distance-based control algorithms enables groups of agents to achieve the desired formation and also share a common velocity which can be more useful. Such control algorithms, handling formation shape control and velocity consensus simultaneously, have been proposed in [10–13] under the assumption that there is no leader in the formation. However, in some situations, it may be desirable that the agents share a common velocity of a leader [14]. Therefore, it is important to investigate how a group of agents can achieve the desired formation shape and converge to the velocity of a moving leader simultaneously. This issue has been studied for a group of 3 agents modeled by single-integrator in [15–20]. In these algorithms, the reference leading velocity is assumed to be an unknown constant vector to the followers. Thus adaptive control schemes are proposed to estimate the unknown reference velocity in the controller design. In [16, 17, 19, 20], the global convergence of three agents with triangular structures to the desired formation and to a common velocity is established. On the other hand, by assuming that the constant or time-varying leading velocity is known, distance-based formation and flocking control is achieved for a group of agents modeled by double-integrator in [21–23], though only local stability is established based on the Lyapunov stability theorem and LaSalle's invariance principle [24]. Nevertheless, one important issue that remains open is to realize global stability for agents modeled by second-order dynamics and the leading velocity is time-varying.

Although simple single-integrator or double-integrator models studied in [15–23] are beneficial for investigating fundamental properties, more realistic agent models are required to enhance the practicality of distance-based formation control [1]. Therefore, distance-based formation control for a team of agents modeled by

electric charges is studied in [25]. However, the considered models in [25] are assumed to be accurate and exactly known. Note that, intrinsic model uncertainty is another issue that has been less studied in distance-based formation control, even though it is a hot-spot in other control areas [26–28].

Motivated by the above discussions, an adaptive control strategy for distance-based formation and flocking control of multi-agent systems is proposed in this paper. Since it is really difficult to consider general directed graphs, similar to most existing works, we focus on minimally persistent graph [20]. In this work, we first study a system with LFF type triangular structure, which means the system only consists of three agents, i.e., the leader with time-varying velocity, the first follower and the follower. The three agents are modeled by nonidentical second-order dynamics with parametric uncertainty. To solve our formulated problem, adaptive technique is employed to estimate and compensate for the unknown parameters. Furthermore, by introducing two new variables that involve distance error and velocity error and utilizing them in the controller design, it is established that all agents in the formation globally converge to the desired formation and to the velocity of the leader. Then, the LFF type system is inductively extended to N -agent system, in which the proposed control strategy can be applied.

Consequently, the contributions and novelty of this paper are summarized as follows:

- Global convergence of all agents in the system to the desired formation and to the time-varying velocity of the leader is achieved simultaneously.
- The agents are modeled as nonidentical second-order dynamics with parametric uncertainty. In controller design, adaptive update laws are designed to estimate and compensate for the unknown parameters.

The rest of the paper is organized as follows. In Section 2, the dynamics of the multi-agent system with some notation are given and the problem of distance-based formation and flocking control is formulated. The proposed distributed adaptive control strategy to solve the problem is presented and analyzed in Section 3. Simulation results are provided in Section 4. Finally, conclusion and future works are given in Section 5.

2 Problem Formulation

Consider a group of N agents moving in the 2D Euclidean space, which can be modeled as follows,

$$\begin{aligned}\dot{p}_i &= v_i \\ \dot{v}_i &= u_i + \varphi_i(v_i)^T \theta_i \quad \forall i \in \mathcal{V}\end{aligned}\quad (1)$$

where $p_i \in \mathbb{R}^2$, $v_i \in \mathbb{R}^2$ and $u_i \in \mathbb{R}^2$ are the position, velocity and input of agent i . $\theta_i \in \mathbb{R}^{l_i}$ is a vector of unknown constants and $\varphi_i(v_i)$ is a matrix with appropriate dimensions of known smooth nonlinear functions. The system model described in (1) allows the agents to have nonidentical dynamics with parametric uncertainty. Clearly this class of systems is more general than those described as single- or double-integrator in most of existing work including [19, 20, 22] and those exactly known second-order dynamics in [25].

Suppose that the interactions and information transmission among the N agents can be modeled by the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \dots, N\}$ denotes the set of agents and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of edges between two distinct agents. An edge in $\mathcal{E}$ from j to i indicates that agent j can measure the relative position of agent i and obtain the velocity information transmitted from agent i . In this case, agent i is called a neighbor of agent j . The set of neighbors of agent j is denoted as $\mathcal{N}_j = \{i \in \mathcal{V} | (i, j) \in \mathcal{E}\}$.

In distance-based formation control, the objective is to realize the desired formation which is formulated by the desired distances between agents. For a given graph $\mathcal{G}$, the desired distances are given as $d = (\dots, d_{ij}, \dots)$, $\forall (i, j) \in \mathcal{E}$ where $d_{ij} > 0$ is the desired distance between agent i and agent j . In this paper, $z_{ij} \in \mathbb{R}^2$ and e_{ij} are respectively defined as the relative position and the distance error as follows,

$$z_{ij} = p_i - p_j \quad \forall (i, j) \in \mathcal{E} \quad (2)$$

$$e_{ij} = \|z_{ij}\|^2 - d_{ij}^2 \quad \forall (i, j) \in \mathcal{E} \quad (3)$$

From (3), the distance between agent i and agent j converging to d_{ij} is equivalent to the distance error e_{ij} converging to zero, i.e., $\|z_{ij}\| = d_{ij} \Leftrightarrow e_{ij} = 0$. Therefore, to achieve the desired formation shape, it is required to make all of e_{ij} converge to zero. Furthermore, assign Agent 1 as the leader, to achieve the flocking behavior, the other agents are required to approach the velocity of the leader. Thus, the control objective of this paper can be expressed as designing controller for each agent such that

$$\tilde{v}_{1i} \rightarrow 0 \quad \forall i \in \mathcal{V}, i \neq 1 \quad (4)$$

$$\|z_{ij}\| \rightarrow d_{ij} \quad \forall (i, j) \in \mathcal{E} \quad (5)$$

where $\tilde{v}_{1i} = v_1 - v_i$ is the velocity error between agent 1 and agent i .

Firstly, we focus on the multi-agent system with LFF type triangular structure, as shown in Fig. 1 where Agent 1 is assigned as the leader, Agent 2 is called the first follower and Agent 3 is the follower. The leader is supposed to perform the following dynamics by human operator or self-navigation,

$$\begin{aligned}\dot{x}_1(t) &= v_1(t) \\ \dot{v}_1(t) &= a_1(t).\end{aligned}\quad (6)$$

In this case, the velocity of the leader is time-varying.

Remark 1. Note that global convergence for distance-based formation control of an LFF system is achieved in [20], but it assumes each agent to be a single integrator and the velocity of the leader is a constant vector. Furthermore, it is pointed out that extending such simple dynamics to high-order dynamic systems is very complicated. As addressed in [22, 23], for distance-based formation control of agents modeled as double-integrators, there is no complete solution that can ensure a global convergence for a group of agents in the constant or time-varying leading velocity case. Thus, it is actually an

Fig. 1: The system setup of LFF in Cartesian coordinates (Taken from [20]).

important yet difficult problem to achieve global stability for agents with unknown second-order dynamics in the case that the leading velocity is time-varying.

To solve the mentioned distance-based formation and flocking control problem of a LFF system, we need the following assumptions:

Assumption 1. The information, $v_1(0)$ and $a_1(t)$ expressed in (6), of the leader can be obtained by all the agents in the formation. Moreover, $v_1(t)$ and $a_1(t)$ are assumed to be bounded.

Assumption 1 is equivalent to that $v_1(t)$ is known, as it can be derived from $v_1(0)$ and $a_1(t)$. Actually, for the case of double-integrator agents in [21] and [22], the velocity of the leader is also assumed to be known, but it is a constant which implies a_1 is 0.

Assumption 2. The desired formation for the system given in Fig. 1 is realizable. The necessary and sufficient condition for the existence of a realization is given as follows,

$$\begin{aligned}d_{12} &< d_{13} + d_{23} \\ d_{23} &< d_{12} + d_{13} \\ d_{13} &< d_{12} + d_{23}\end{aligned}$$

Assumption 3. [20, 29] The desired formation is minimally persistent obtained as the result of a Henneberg sequence.

3 Controller Design

In this section, the control laws for the first follower and follower in the LFF system will be designed separately, and then the extension of LFF system to N -agent system will be derived.

3.1 Controller Design for the First Follower

Introduce a new variable $\bar{v}_{12} = \tilde{v}_{12} + k_{p2}h_2$ where

$$h_2 = e_{12}z_{12}, \quad (7)$$

k_{p2} is a positive design parameter.

Remark 2. The introduction of variable $\bar{v}_{12}$ is a key technique to achieve global stability in the proposed control strategy. Note that, $\bar{v}_{12}$ involves distance error e_{12} and velocity error $\tilde{v}_{12}$. If $\bar{v}_{12} \rightarrow 0$ and $h_2 \rightarrow 0$, then $\tilde{v}_{12} \rightarrow 0$. Utilizing $\bar{v}_{12}$, the proposed controller makes h_2 and the velocity error $\tilde{v}_{12}$ converge to zero, which will be established subsequently. Afterwards, based on the stability analysis, it can be proved that the first follower will converge to the desired formation by choosing appropriate control parameters.

Firstly we derive an adaptive law to estimate the unknown parameter in the dynamic model of the first follower. Define $\hat{\theta}_2$ as the estimate of θ_2 .

Choose a Lyapunov function candidate as

$$V_2 = \frac{1}{4}e_{12}^2 + \frac{1}{2}\tilde{v}_{12}^T\tilde{v}_{12} + \frac{1}{2}\tilde{\theta}_2^T\Gamma_2\tilde{\theta}_2 \quad (8)$$

where $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2$ and Γ_2 is a user defined positive definite matrix with appropriate dimension. Then, the time derivative of V_2 can be computed as

$$\begin{aligned} \dot{V}_2 &= \frac{1}{2}e_{12}\dot{e}_{12} + \tilde{v}_{12}^T\dot{\tilde{v}}_{12} + \tilde{\theta}_2^T\Gamma_2\dot{\tilde{\theta}}_2 \\ &= e_{12}z_{12}^T(\tilde{v}_{12} - k_{p2}h_2) \\ &\quad + \tilde{v}_{12}^T(a_1 - u_2 - \varphi_2^T\tilde{\theta}_2 + k_{p2}\dot{h}_2) - \tilde{\theta}_2^T\Gamma_2\dot{\tilde{\theta}}_2. \end{aligned} \quad (9)$$

Based on (9), the control law and the parameter update law are designed as

$$u_2 = k_{v2}\tilde{v}_{12} + a_1 - \varphi_2^T\hat{\theta}_2 + k_{p2}\dot{h}_2 + h_2 \quad (10)$$

$$\dot{\hat{\theta}}_2 = -\Gamma_2^{-1}\varphi_2\tilde{v}_{12} \quad (11)$$

where k_{v2} is a positive design parameter and from (7)

$$\dot{h}_2 = 2z_{12}z_{12}^T\tilde{v}_{12} + e_{12}\tilde{v}_{12}. \quad (12)$$

Substituting (10) and (11) into (9) yields

$$\dot{V}_2 = -k_{p2}\|h_2\|^2 - k_{v2}\|\tilde{v}_{12}\|^2 \leq 0 \quad (13)$$

Lemma 1. *With the application of control law (10) and parameter update law (11) to the first follower, all the signals in the resulted closed-loop system are bounded. In addition, h_2 and the velocity error $\tilde{v}_{12}$ asymptotically converge to zero, i.e., $h_2 \rightarrow 0$ and $\tilde{v}_{12} \rightarrow 0$ as $t \rightarrow \infty$.*

Proof: From (8) and (13), it can be obtained that $V_2 \geq 0$ and $\dot{V}_2 \leq 0$, therefore, e_{12} , z_{12} , $\tilde{v}_{12}$, $\tilde{\theta}_2$ and $\hat{\theta}_2$ are bounded. Then, based on (9) and (13), it can be established that $\dot{V}_2$ is bounded. By applying Barbalat's lemma [30], we can obtain $\dot{V}_2 \rightarrow 0$, i.e.,

$$\lim_{t \rightarrow \infty} (\|h_2\| + \|\tilde{v}_{12}\|) = 0 \quad (14)$$

Hence, the asymptotic convergence of the velocity error (i.e., $\tilde{v}_{12} \rightarrow 0$ as $t \rightarrow \infty$) can be derived from (7). $\square$

Remark 3. *It should be noted that with the existing distance-based gradient control, the results in Lemma 1 cannot be obtained. Technically, if applying such an approach here, the following Lyapunov function would be considered*

$$W = \frac{1}{4}e_{12}^2 + \frac{1}{2}\tilde{v}_{12}^T\tilde{v}_{12} + \frac{1}{2}\tilde{\theta}_2^T\Gamma_2\tilde{\theta}_2$$

which could yield the control law and parameter update law as $u_2 = k_{v2}\tilde{v}_{12} + e_{12}z_{12} + a_1 - \varphi_2^T\hat{\theta}_2$ and $\dot{\hat{\theta}}_2 = -\Gamma_2^{-1}\varphi_2\tilde{v}_{12}$, respectively. Then the resulting time derivative of W would be

$$\dot{W} = -k_{v2}\tilde{v}_{12}^T\tilde{v}_{12} \leq 0,$$

from which only the global convergence of $\tilde{v}_{12}$ to zero could be obtained. In this paper, based on the introduction of the new variable $\tilde{v}_{12}$, the proposed control law (10) with parameter update law (11) ensures the asymptotic convergence of both h_2 and $\tilde{v}_{12}$, which is crucial in establishing the global convergence of the follower to the desired formation and to the velocity of the leader, as seen in the following analysis.

Lemma 2. *The states of the first follower converge to one of the following two equilibrium points,*

$$1) \quad e_{12} = 0, \quad \tilde{v}_{12} = 0; \quad (15)$$

$$2) \quad z_{12} = 0, \quad \tilde{v}_{12} = 0. \quad (16)$$

Proof: Based on Lemma 1, $e_{12}z_{12}$ and $\tilde{v}_{12}$ converge to zero. From the definition of e_{12} in (3), it can be found that e_{12} and z_{12} cannot be zero at the same time. Therefore, the states of the first follower can only converge to one of the two equilibrium points: $e_{12} = 0$ and $\tilde{v}_{12} = 0$ or $z_{12} = 0$ and $\tilde{v}_{12} = 0$. $\square$

From Lemma 2, there are two global equilibrium points in the solution space and the one in (15) is the desired equilibrium point. If the equilibrium point (16) is unstable, then from Lemma 2 the other one (15) is globally stable which will ensure the first follower converges to the desired formation. By analyzing the stability of the system and choosing the design parameters appropriately, this can be achieved, as summarized in following theorem.

Theorem 1. *The convergence of the first follower to the desired formation, i.e., $e_{12} \rightarrow 0$ and $\tilde{v}_{12} \rightarrow 0$ as $t \rightarrow \infty$, is ensured by the control law (10) and parameter update law (11) with the design parameters k_{p2} and k_{v2} chosen according to the following guideline,*

$$k_{p2}d_{12}^2 - k_{v2} > 0.$$

Proof of Theorem 1: Define $\phi = [\phi_1^T \ \phi_2^T]^T = [z_{12}^T \ \tilde{v}_{12}^T]^T$ and consider the following dynamics:

$$\dot{\phi}_1 = \tilde{v}_{12} = f_1$$

$$\dot{\phi}_2 = -\varphi_2^T\tilde{\theta}_2 - k_{v2}\tilde{v}_{12} - k_{p2}\dot{h}_2 - h_2 = f_2.$$

Let $f = [f_1^T \ f_2^T]^T$. To analyze the stability of the equilibrium point (16), the above dynamics is linearized as follows,

$$\begin{aligned} \left. \frac{\partial f}{\partial \phi} \right|_{\phi=0} &= \begin{bmatrix} \frac{\partial f_1}{\partial \phi_1} & \frac{\partial f_1}{\partial \phi_2} \\ \frac{\partial f_2}{\partial \phi_1} & \frac{\partial f_2}{\partial \phi_2} \end{bmatrix} \bigg|_{\phi=0} \\ &= \begin{bmatrix} O_{2 \times 2} & I_2 \\ \gamma_{21}I_2 & \gamma_{22}I_2 \end{bmatrix} = A. \end{aligned}$$

where $\gamma_{21} = (k_{p2}k_{v2} + 1)d_{12}^2$ and $\gamma_{22} = k_{p2}d_{12}^2 - k_{v2}$. Calculating $\det(\lambda I_4 - A) = 0$, the characteristic equation of matrix A is given as

$$\lambda^4 - 2\gamma_{22}\lambda^3 + (\gamma_{22}^2 - 2\gamma_{21})\lambda^2 + 2\gamma_{21}\gamma_{22}\lambda + \gamma_{21}^2 = 0 \quad (17)$$

Utilizing the Routh-Hurwitz stability criterion, it can be obtained that, there is at least one solution which has positive real part if $\gamma_{22} > 0$. Therefore, if k_{p2} and k_{v2} are chosen such that $k_{p2}d_{12}^2 - k_{v2} > 0$, the equilibrium point (16) is unstable. In this case, since the system is asymptotically stable from Lemma 1, the other equilibrium point (15) is globally stable. It means that the first follower will converge to the desired formation stated in (4) and (5) if k_{p2} and k_{v2} satisfy $k_{p2}d_{12}^2 - k_{v2} > 0$. $\square$

Remark 4. *The above proof process is a common way to analyze the stability of such systems which has been used in [16] and [20]. However, in contrast to the approach there, we need to consider how to choose appropriate control parameters based on the stability analysis to make sure that only the equilibrium point (15) is stable in this paper.*

3.2 Controller Design for the Follower

Using a similar controller design process as done for the first follower, a new variable $\bar{v}_{13} = \tilde{v}_{13} + k_{p3}h_3$ is firstly introduced for the follower by defining

$$h_3 = e_{13}z_{13} + e_{23}z_{23} \quad (18)$$

where k_{p3} is a positive design parameter. It can also be noted that, if $\bar{v}_{13} \rightarrow 0$ and $h_3 \rightarrow 0$, then $\tilde{v}_{13} \rightarrow 0$.

The control law and parameter update laws for the follower are designed as

$$u_3 = k_{v3}\bar{v}_{13} + a_1 - \varphi_3^T \hat{\theta}_3 + k_{p3}\dot{h}_3 + h_3 \quad (19)$$

$$\dot{\hat{\theta}}_3 = -\Gamma_3^{-1} \varphi_3 \bar{v}_{13} \quad (20)$$

where k_{v3} is another positive design parameter, Γ_3 is also a user defined positive definite matrix with appropriate dimension, $\hat{\theta}_3$ is the estimate of θ_3 , and based on (18)

$$\begin{aligned} \dot{h}_3 &= 2z_{13}z_{13}^T \tilde{v}_{13} + e_{13}\tilde{v}_{13} \\ &+ 2z_{23}z_{23}^T (v_2 - v_3) + e_{23}(v_2 - v_3), \end{aligned} \quad (21)$$

in which v_2 is the velocity of the first follower.

To analyze the stability of the proposed controller for the follower, consider the following Lyapunov function,

$$V_3 = \frac{1}{4}e_{13}^2 + \frac{1}{4}e_{23}^2 + \frac{1}{2}\bar{v}_{13}^T \bar{v}_{13} + \frac{1}{2}\tilde{\theta}_3^T \Gamma_3 \tilde{\theta}_3 \quad (22)$$

where $\tilde{\theta}_3 = \theta_3 - \hat{\theta}_3$.

Then, the time derivative of V_3 is given as

$$\begin{aligned} \dot{V}_3 &= \frac{1}{2}e_{13}\dot{e}_{13} + \frac{1}{2}e_{23}\dot{e}_{23} + \bar{v}_{13}^T \dot{\bar{v}}_{13} + \tilde{\theta}_3^T \Gamma_3 \dot{\tilde{\theta}}_3 \\ &= e_{13}z_{13}^T \tilde{v}_{13} + e_{23}z_{23}^T (\tilde{v}_{13} - \tilde{v}_{12}) - \tilde{\theta}_3^T \Gamma_3 \dot{\tilde{\theta}}_3 \\ &+ \bar{v}_{13}^T (a_1 - u_3 - \varphi_3^T \theta_3 + k_{p3}\dot{h}_3) \\ &= -k_{p3} (e_{13}z_{13}^T + e_{23}z_{23}^T) h_3 - e_{23}z_{23}^T \tilde{v}_{12} \\ &- \bar{v}_{13}^T \varphi_3^T \tilde{\theta}_3 - k_{v3}\bar{v}_{13}^T \bar{v}_{13} - \tilde{\theta}_3^T \Gamma_3 \dot{\tilde{\theta}}_3 \\ &= -k_{p3}\|h_3\|^2 - k_{v3}\|\bar{v}_{13}\|^2 - e_{23}z_{23}^T \tilde{v}_{12} \\ &\leq -k_{p3}\|h_3\|^2 - k_{v3}\|\bar{v}_{13}\|^2 + \|e_{23}z_{23}\|\|\tilde{v}_{12}\|. \end{aligned} \quad (23)$$

Lemma 3. [20] Applying the proposed control (19) and parameter update law (20) to the follower, all the signals in the resulted closed-loop system are bounded.

Lemma 4. $\|h_3\|$ and $\|\bar{v}_{13}\|$ shown in (23) converge to zero, i.e., $\|h_3\| \rightarrow 0$ and $\|\bar{v}_{13}\| \rightarrow 0$, as $t \rightarrow \infty$.

Proof: From Lemma 3, V_3 is bounded, thus, there exists $\eta > 0$ such that $\|e_{23}z_{23}\| < \eta$. Furthermore from Lemma 1, $\tilde{v}_{12} \rightarrow 0$ as $t \rightarrow \infty$. Therefore, for any $\epsilon > 0$, there exists $T > 0$ such that $\|\tilde{v}_{12}\| < \epsilon$ when $t > T$. Then, it can be obtained that for $t > T$

$$-\mu(t) - \zeta < \dot{V}_3 < -\mu(t) + \zeta$$

where $\mu(t) = k_{p3}\|h_3\|^2 + k_{v3}\|\bar{v}_{13}\|^2$ and $\zeta = \epsilon\eta$. If $\lim_{t \rightarrow \infty} \mu(t) = 0$, then the proof is completed. Otherwise, $\dot{V}_3 \leq 0$ for all sufficiently large t . Therefore, V_3 is a decreasing function and

$$\lim_{t \rightarrow \infty} V_3 = M \geq 0$$

Hence, applying Barbalat's lemma, it can be obtained that $\dot{V}_3 \rightarrow 0$, then $\lim_{t \rightarrow \infty} \|h_3\| = 0$ and $\lim_{t \rightarrow \infty} \|\bar{v}_{13}\| = 0$. $\square$

As it is proved that $\|h_3\|$ and $\|\bar{v}_{13}\|$ converge to zero in Lemma 4, and moreover, it is obvious that e_{13} (e_{23}) and z_{13} (z_{23}) cannot be zero at the same time, the states of the follower will converge to one the following three equilibrium points:

$$1) \ e_{13} \neq 0, \ z_{13} = -\frac{e_{23}}{e_{13}}z_{23}, \ \tilde{v}_{13} = 0; \quad (24)$$

$$2) \ e_{23} \neq 0, \ z_{23} = -\frac{e_{13}}{e_{23}}z_{13}, \ \tilde{v}_{13} = 0; \quad (25)$$

$$3) \ e_{13} = 0, \ e_{23} = 0, \ \tilde{v}_{13} = 0. \quad (26)$$

Note that, the equilibrium point (26) represents the desired formation of the follower. Therefore, the convergence of the follower to the desired formation is equivalent to that the equilibrium points (24) and (25) are unstable. Applying a similar proof process as done for the first follower, we will show that the equilibrium points (24) and (25) are unstable. Before proceeding with stability analysis, the following two lemmas are presented.

Lemma 5. [20] When the states of the follower converge to the equilibrium point (24) or (25), it can be obtained that $z_{13,1}z_{23,2} - z_{13,2}z_{23,1} = 0$, where $z_{13} = [z_{13,1} \ z_{13,2}]^T$ and $z_{23} = [z_{23,1} \ z_{23,2}]^T$.

Lemma 6. [31] When the states of the follower converge to the equilibrium point (24) or (25) and further $e_{12} = 0$, it can be obtained that $e_{13} + e_{23} < 0$.

Then we can establish the results presented in following theorem.

Theorem 2. Under the control law (19) with parameter update law (20), the follower converges to the desired formation, i.e., $e_{13} \rightarrow 0$, $e_{23} \rightarrow 0$ and $\tilde{v}_{13} \rightarrow 0$ as $t \rightarrow \infty$.

Proof of Theorem 2: Define $\psi = [\psi_1^T \ \psi_2^T]^T = [z_{13}^T \ z_{23}^T]^T$, hence, the dynamics of ψ can be given as follows:

$$\ddot{\psi}_1 = -\varphi_3^T \tilde{\theta}_3 - k_{v3}\bar{v}_{13} - k_{p3}\dot{h}_3 - h_3 = g_1$$

$$\ddot{\psi}_2 = u_2 + \varphi_2^T \theta_2 - a_1 - \varphi_3^T \tilde{\theta}_3 - k_{v3}\bar{v}_{13} - k_{p3}\dot{h}_3 - h_3 = g_2.$$

Let $g = [g_1^T \ g_2^T]^T$, the following Jacobian matrix can be obtained

$$\frac{\partial g}{\partial \psi} = \begin{bmatrix} \frac{\partial g_1}{\partial \psi_1} & \frac{\partial g_1}{\partial \psi_2} \\ \frac{\partial g_2}{\partial \psi_1} & \frac{\partial g_2}{\partial \psi_2} \end{bmatrix} = B.$$

From Lemma 1 and Lemma 4, $\tilde{v}_{12} \rightarrow 0$ and $\tilde{v}_{13} \rightarrow 0$, hence, $v_2 - v_3 (= \tilde{v}_{13} - \tilde{v}_{12}) \rightarrow 0$. To analyze the stability of the equilibrium points (24) and (25), substituting $\tilde{v}_{13} = 0$ and $v_2 - v_3 = 0$ into the above Jacobian matrix, we have,

$$B = -k \begin{bmatrix} \gamma_{31} & \gamma_{32} \\ \gamma_{31} & \gamma_{32} \end{bmatrix}$$

where $k = k_{p3}k_{v3} + 1$, $\gamma_{31} = e_{13}I_2 + 2z_{13}z_{13}^T$ and $\gamma_{32} = e_{23}I_2 + 2z_{23}z_{23}^T$. Therefore, the characteristic equation of matrix B can be calculated as

$$\lambda^4 + ks_1\lambda^3 + k^2(s_2 + s_3)\lambda^2 = 0 \quad (27)$$

where

$$s_1 = 2(e_{13} + e_{23} + \|z_{13}\|^2 + \|z_{23}\|^2),$$

$$s_2 = (e_{13} + e_{23})(e_{13} + e_{23} + 2\|z_{13}\|^2 + 2\|z_{23}\|^2),$$

$$s_3 = 4(z_{13,1}z_{23,2} - z_{13,2}z_{23,1})^2.$$

According to Lemma 5, $z_{13,1}z_{23,2} - z_{13,2}z_{23,1} = 0$, i.e., $s_3 = 0$. Therefore, the above characteristic equation can be rewritten as follows,

$$\lambda^4 + k(\alpha + \beta)\lambda^3 + k^2\alpha\beta\lambda^2 = 0 \quad (28)$$

where $\alpha = e_{13} + e_{23}$ and $\beta = e_{13} + e_{23} + 2\|z_{13}\|^2 + 2\|z_{23}\|^2$.

In Theorem 1, the convergence of e_{12} is proved. From Lemma 6, if the states of the system reach the equilibrium point (24) or (25) and further $e_{12} = 0$, then $\alpha < 0$. The system is unstable if at least one of the solutions of the above characteristic equation has positive real part. According to the Routh-Hurwitz stability criterion, the stability of the system with characteristic equation (28) is related to the sign of $k(\alpha + \beta)$ and $k^2\alpha\beta$. Since $\alpha < 0$ and $k > 0$, we consider the following three cases to analyze the stability of the system:

- 1) $\beta > 0$
Thus $k^2\alpha\beta < 0$, there is at least one solution that has positive real part.
- 2) $\beta < 0$
Thus $k(\alpha + \beta) < 0$, there is at least one solution that has positive real part.
- 3) $\beta = 0$
Thus $k(\alpha + \beta) < 0$ and $k^2\alpha\beta = 0$, there is one solution that has positive real part.

In conclusion, when the states of the system converge to the equilibrium point (24) or (25), at least one solution of (28) has positive real part, hence, the equilibrium points (24) and (25) are unstable. Therefore, the states of the follower will converge to the equilibrium point (26). $\square$

Combining Theorem 1 and Theorem 2, it can be concluded that, the proposed control strategy makes the LFF system converge to the desired formation and to the velocity of the leader as described in (4) and (5).

3.3 Extension to N -agent Case

In the previous parts, global convergence of an LFF system to the desired formation and to the velocity of the leader is proved. Now the convergence of an N -agent system to the desired formation can be inferred inductively. Without loss of generality, similar to the inductive procedure done in [20], taking the LFF system as a base case, a cycle-free persistent graph $\mathcal{G}$ is obtained as the result of a Henneberg sequence containing only vertex additions. Technically, agent 4 is obtained by adding an agent into graph of LFF using Henneberg sequence, and the resulted graph is still cycle-free persistent graph, subsequent agents are added one by one similarly.

An example of $\mathcal{G}$ for an extended 5-agent system is shown in Fig. 2. The control law (19) with parameter update law (20) designed for the follower can be applied to the added agent i as,

$$\begin{aligned} u_i &= k_{vi}\bar{v}_{1i} + u_1 + \varphi_1^T \hat{\theta}_{1i} - \varphi_i^T \hat{\theta}_i + k_{pi}\dot{h}_i + h_i \\ \dot{\hat{\theta}}_i &= -\Gamma_i^{-1} \varphi_i \bar{v}_{1i} \end{aligned} \quad (29)$$

where $\bar{v}_{1i} = \bar{v}_{1i} + k_{pi}h_i$, with $k_{pi} > 0$ and

$$h_i = \sum_{j \in \mathcal{N}_i} e_{ij} z_{ij}.$$

Assume the obtained n -agent graph locally converges to desired formation, then the $n + 1$ -agent graph is obtained by adding an agent to the n -agent graph using Henneberg sequence. Applying the control law and parameter update law in (29) to the added agent, then from the above lemmas and theorems, it can be proved that the $n + 1$ agents locally converge to the desired formation. Based on the results established for the LFF system (i.e. the base case), by mathematical induction, the cycle-free persistent graph locally converges to the desired formation by applying the designed control law.

Even though it has been established that the LFF system globally converges to the desired formation, with the same analysis as in [20], the N -agent system locally converges to the desired formation because there are two positions that satisfies $e_{ij} = 0$ for each agent. For example, consider the system in Fig. 3, agent 4 has another position $\bar{p}_4$ satisfies $e_{14} = e_{24} = 0$. If agent 4 converges to $\bar{p}_4$, in

this case, it may have $\|\bar{p}_{34}\| > d_{35} + d_{45}$, which makes it impossible for agent 5 to achieve $e_{35} = e_{45} = 0$, i.e., the system cannot converge to the desired formation.

Fig. 2: The system setup of 5-agent system in Cartesian coordinates.

Fig. 3: An example of two positions of agent 4 that satisfies $e_{14} = e_{24} = 0$.

4 Simulation

To verify the established theoretical results, simulation study has been carried out. In the simulation, a group of five agents, whose interaction structure is shown in Fig. 2, is considered. Besides, the desired distances between agents that represent the desired formation are set as $d_{12} = d_{13} = d_{23} = d_{14} = d_{24} = 4$ and $d_{35} = d_{45} = 5$. The function matrices and constant vectors in the dynamics of each agent (1) are given as follows,

$$\begin{aligned} \varphi_2 &= \begin{bmatrix} 0 & \sin(v_{2,2}) \\ \cos(v_{2,1}) & 0 \end{bmatrix}, \theta_2 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}; \\ \varphi_3 &= \begin{bmatrix} v_{3,1} \sin(t) & 0 \\ 0 & v_{3,2} \cos(t) \end{bmatrix}, \theta_3 = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}; \\ \varphi_4 &= \begin{bmatrix} v_{4,2} & \sin(v_{4,2}) \\ \cos(v_{4,1}) & v_{4,1} \end{bmatrix}, \theta_4 = \begin{bmatrix} 0.6 \\ 0.9 \end{bmatrix}; \\ \varphi_5 &= \begin{bmatrix} \sin^2(v_{5,1}) & 0 \\ 0 & \cos^2(v_{5,2}) \end{bmatrix}, \theta_5 = \begin{bmatrix} 0.8 \\ 0.45 \end{bmatrix}. \end{aligned}$$

where $v_i = [v_{i,1} v_{i,2}]^T$, $i = 2, 3, 4, 5$.

The initial positions of the five agents are set as,

$$\begin{aligned} p_1(0) &= [4 \quad 0.2]^T, p_2(0) = [0.23 \quad -2.1]^T, \\ p_3(0) &= [-0.19 \quad 2.2]^T, p_4(0) = [4.3 \quad -4.2]^T, \\ p_5(0) &= [-1.2 \quad -3.2]^T. \end{aligned}$$

The initial velocity and acceleration of the leader are respectively set as $v_1(0) = [2, 1]^T$ and $a_1(t) = [0.5 \cos(0.5t), 0.5 \cos(0.5t)]^T$. The initial values of the estimates are set as 50% of their true values. Furthermore, the design parameters of the proposed control scheme are set as $k_{pi} = 1$, $k_{vi} = 4$ and $\Gamma_i^{-1} = 0.5I_2$.

Note that no theoretical result is established for this example system by using existing methods and thus so far no method is

applicable it. However, in the simulation, a comparison with the control scheme proposed in [22] is conducted, even though its control scheme is designed for the constant leading velocity case and based on double-integrator model. Also since there is no adaptive law to address the dynamic uncertainty, thus the initial value $\theta_i(0)$ is used in the control law. Let L be the Laplacian matrix of the graph in Fig. 2 with the following form,

$$L = \begin{bmatrix} 0_{1 \times 1} & 0_{1 \times 4} \\ L_{21} & L_{22} \end{bmatrix} \quad (30)$$

There exists a diagonal positive definite matrix P such that $Q := PL_{22} + L_{22}^T P > 0$ [32]. Then the control law in [22] can be given as follows,

$$u = -(\bar{L} \otimes I_2)v + (P^{-1} \otimes I_2)h - \Phi^T \hat{\Theta}(0) + 1_{n-1} \otimes a_1 \quad (31)$$

where $u = [u_2^T, \dots, u_5^T]^T$, $h = [h_2^T, \dots, h_5^T]^T$, $\bar{L} = [L_{21} \ L_{22}]$, $\Phi = \text{diag}([\varphi_2, \dots, \varphi_5])$ and $\hat{\Theta} = [\hat{\theta}_2^T, \dots, \hat{\theta}_5^T]^T$. In the simulation, $P = \text{diag}([2, 2, 2, 2])$.

Fig. 4: The trajectories of the five agents generated by the proposed control scheme.

Fig. 5: The trajectories of the five agents generated by [22].

Overall, the trajectories of the five agents generated by the two control schemes are respectively shown in the Fig. 4 and Fig. 5, from which, it can be seen that the five agents achieve and maintain the formation during their motions with both control schemes. Now the

control performance is assessed by the formation shape control performance and velocity consensus performance. The formation shape control performance of two control schemes in terms of distance errors e_{ij} , $\forall (i, j) \in \mathcal{E}$ are illustrated in Fig. 6 and Fig. 7, respectively. As observed, the distances between pairs of agents converge to zero under the proposed control scheme, however, such control objective is not achieved with the control scheme in [22]. Additionally, from the velocities of agents as shown in Fig. 8 and Fig. 9, it can be seen that all agents approach the velocity of the leader and then maintain the same velocity with the proposed control scheme, whereas, there exists deviations between the velocities of leader and other agents with the control scheme in [22]. These simulations illustrate the effectiveness of the proposed scheme and also verify the theoretical results established.

Fig. 6: The distance errors between agents of the proposed control scheme.

Fig. 7: The distance errors between agents of the control scheme in [22].

5 Conclusion and Future Works

A distributed adaptive strategy is proposed for distance-based formation and flocking control of LFF systems in this paper. Different from existing research works in this area, the global stability is established in the case that the leading velocity is time-varying, and all agents are supposed to have nonidentical second-order dynamics with parametric uncertainty. By employing adaptive technique, the unknown parameters are estimated and compensated. The introduction of two

Fig. 8: The velocities of the five agents of the proposed control scheme. (a) and (b) respectively show the two components, $v_{i,1}$ and $v_{i,2}$, of v_i , $i = 1, 2, 3, 4, 5$.

Fig. 9: The velocities of the five agents of the control scheme in [22].

new variables that involve distance error and velocity error makes sure that the system globally converges to the desired formation and all agents share a common velocity. The stability and effectiveness of the proposed control strategy is analyzed in theory and verified through simulation results, respectively. Additionally, the result is extended from LFF system to N -agent case.

Note that in present paper, the information of the leader needs to be known by each agent. Therefore, how to relax this condition

is worthy for further investigation. Furthermore, collision avoidance is also an important issue in the formation control. However, it is not simple and requires much effort to achieve collision avoidance and global stability simultaneously because of nonlinear dynamics. The proposed control scheme is applicable to unmanned aerial vehicles, but cannot be directly applied to under-actuated systems, such as nonholonomic mobile robot and marine robotic vehicles, thus an extension to control such systems would also be an interesting future research. Finally, adaptive control is employed to address the dynamic uncertainty, however, it does not have any requirement on the transient performance in this paper. Thus, how to improve the transient performance of adaptive control is also worthy for further research, for example by following the approach in [33].

6 References

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