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A Graph-based Model to Measure Structural Redundancy for Supply Chain Resilience

Wen Jun Tan^{a*}, Allan N. Zhang^b and Wentong Cai^a

^aSchool of Computer Science and Engineering, Nanyang Technological University, Singapore;

^bSingapore Institute of Manufacturing Technology, Singapore

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ABSTRACT

Globalization and lean initiatives increase the vulnerabilities of the supply chains (SC), where disruptions in any plant in a supply chain network (SCN) can propagate throughout the whole SCN. Redundancy is part of the SC re-engineering to improve supply chain resilience (SCRES). This paper presents a conceptual model of an SCN using graph theory, considering the relationships between plants and materials. Based on the model, the structural redundancy of the SCN is measured, which is used to assess SCRES. This assessment approach focuses on the resilience of the SCN against disruptions. Case studies are discussed to illustrate the applicability of this model and show that increasing structural redundancy of the SCN improves SCRES against disruptions.

KEYWORDS

Supply chain resilience; graph theory; supply chain network; supply chain redundancy; supply chain risk management

1. Introduction

The recent explosion at the Port of Tianjin resulted in insurance cost of USD\$1.5 billion, with uninsured costs several times higher (Huang and Zhang 2015). While the daily operations resumed after a month, the customers and port operations continued to be affected by the incident. While such events are rare, supply chain (SC) disruptions are common and often have significant impacts (Ward 2016). In recent decades, globalisation and lean production unintentionally have caused the increase in SC risks. To lower manufacturing costs, firms often source materials and components from around the world, and also keep lower inventories and buffer stocks, improve agility in response to market shifts, and simplify quality costs. At the same time, many firms do not have enough visibility of their SCs. They do not always have the information regarding the transportation routes of their suppliers, or the sources of their suppliers. These issues increase the likelihood of disruptions in SCs, and allow the effects of disruptions to ripple throughout the whole supply chain network (SCN) (Dolgui, Ivanov, and Sokolov 2017; Scheibe and Blackhurst 2017; Venkatesan and Goh 2018). Therefore, it is increasingly important to be able to assess supply chain resilience (SCRES)

*Corresponding author. Email: wtan047@e.ntu.edu.sg

and investigate the impact and relative importance of different strategies to enhance SCRES (Kamalahmadi and Parast 2016).

SCRES is defined as ‘the adaptive capability of a supply chain to reduce the probability of facing sudden disturbances, resist the spread of disturbances by maintaining control over structures and functions, and recover and respond by immediate and effective reactive plans to transcend the disturbance and restore the supply chain to a robust state of operations’ (Kamalahmadi and Parast 2016). SC re-engineering has been identified as one of the SCRES’s principles, which includes two practices of flexibility and redundancy. They are considered as the two important approaches for building SCRES in the literature (Sheffi 2005; Tomlin 2006; Ponomarov and Holcomb 2009; Ponis and Koronis 2012; Melnyk 2014; Kamalahmadi and Parast 2016). In this paper, we focus on the redundancy aspect of SCRES.

When the supplies of vital materials are concentrated on a single firm, the SCs are inherently susceptible to disruption risks (Nakatani et al. 2017). For example in the Tohoku earthquake, a specific supplier was severely damaged by the disaster. This caused disruptions across multiple part manufacturers, resulting in major automotive manufacturers unable to continue production (Matsuo 2015). When Toyota was forced to rebuild its SC after the earthquake, managers were surprised to discover just how many parts relied on the same few suppliers far upstream (Ward 2016). This incident demonstrated that firms must understand material flow in the SC, including not only immediate but also indirect links via the SCs, as well as supply disruption risk of each material. Therefore, they can anticipate whether and how disruptions of specific upstream materials will affect their own production processes.

Most literature on SC disruption analysis focused on a single plant or between stages in an SC, even though disruptions can have long and lasting effects throughout the SC (Schmitt and Singh 2012). Some existing models considered redundancy only between two stages in an SCN (Wang and Ip 2009; Salehi Sadghiani, Torabi, and Sahebjamnia 2015). However, disruptions in the production of vital upstream materials can propagate and affect the production processes downstream (Nakatani et al. 2017). The current models from the literature (as summarised in Section 2) are unable to analyse the dependencies across multi-stages in an SCN, where vulnerabilities in the upstream stages of the SCN can have a significant impact downstream.

To enable analytical tractability and simplify the problem, existing research assumed that plants within the SCN or the same stage are equivalent, or the type of material flows between stages are the same (Graves and Tomlin 2003; Xu, Wang, and Zhao 2014; Kim, Chen, and Linderman 2015; Mari et al. 2015; Li et al. 2017). This implies the existing models in the literature do not differentiate the production processes of each type of materials in the SCN. This is unrealistic as there are significant cost and time to set up the same production capabilities in all plants within the same stage. Most enterprises are small and medium-sized in the SCN, they are highly specialised and the labour division is sophisticated (Wang and Xiao 2016). In the fierce market competition, they would not voluntarily change their competitive business activities.

Given the above draw-backs, this paper proposes a graph-based model of a multi-stage SCN to assess SCRES, in terms of the *structural redundancy* at the network level. The model considers both materials and plants in the SCN and captures two essential relationships between the materials and plants (that is, *material-to-product relationships* and *inter-plant relationships*). The two relationships form the basic building blocks for constructing a multi-stage SCN. Based on the topological structure of the SCN, all the SCs in the SCN are identified. Correspondingly an approach to analyse SCRES is proposed to identify the critical plants and measure the number of SCs in

the SCN. The model and the approach for SCRES analysis are illustrated using the real world SCN from the existing samples of SCN (Willems 2007).

The main theoretical contributions of this paper are the proposed conceptual model of an SCN that can represent structural redundancy across multiple stages. Based on this model, critical plants in the SCN can be identified and redundancy of SCRES can be assessed. The practical implications of the model and the approach are demonstrated by using proactive and reactive strategies to build a more resilient SCN. For proactive strategy, critical plants should be the key priority to build resilience in the SCN. For reactive strategy, the impact of disruptions on the critical plants determines the level of response toward the disruptions.

In the remainder of this paper, Section 2 provides a review of SCRES and related works on SCN graph models. The graph-theoretical model of SCN is described in Section 3. This provides an approach to evaluate SCRES in the SCN. Section 4 illustrates the applicability of the model on the real-world SCNs. Section 5 concludes this paper.

2. Literature Review

2.1. Concepts Related to Supply Chain Resilience

Supply Chain Network

SCNs are interconnected structures that emerge from a largely downstream exchange of goods between firms (i.e., manufacturers, distributors, retailers, etc.) that are involved in creating a set of final products (Arora and Ventresca 2018). These network structures can provide high-level representations of the relationships in the SCN, where detailed information regarding SC operations, e.g. capacities and lead time, are not available. By analysing topological characteristics of SCNs, it can provide summarised statistics for describing particular features. This is particularly useful since analysis of topological characteristics of the interconnection structure in an SCN enables managers to analyse various aspects of the SCN. In addition, many real-world complex networks, e.g. SCN, can contain a significant amount of structural redundancy, in which multiple vertices play identical topological roles (MacArthur and Sánchez-García 2009).

An SCN is typically modelled as a graph $G = (V, E)$, V is the set of nodes and E is the set of edges. The nodes represent the firms, and edges represent the set of connections that link these firms together. In a production process, the relationship from the input material to the output product is known as the *material-to-product* relationship. Assume that each firm represents a plant in the production process, the *inter-plant relationship* represents the relation between the plants. SC disruption can be modelled by removing a node or link from the SCN. A network disruption occurs when there is no path between the source node (supplier) and the sink node (customer) due to disruption in the nodes or links. When the network is disconnected, it disrupts the flow in the network, which causes a loss in the function of the SCN.

Supply Chain Resilience

Ponomarov and Holcomb (2009) identified three phases of SCRES as readiness, response and recovery, where responding and recovering to the same or better state is of the key importance. Melnyk (2014) defined SCRES as ‘the ability of a supply chain to both resist disruptions and recover operational capability after disruptions occur’. Similarly, Tukamuhabwa et al. (2015) assessed SCRES on four aspects –

preparation for a disruptive event; response to an event; recovery from the event; and growth/competitive advantage after the event. Kamalahmadi and Parast (2016) also identified four phases for SCRES – anticipation, resistance, recovery and responses. Hence, two critical capacities in SCRES can be identified: (i) resistance capacity – the ability of a system to minimise the impact of a disruption by evading the hazard entirely or reacting early; and (ii) recovery capacity – the ability of a system to return to a steady state of operational capacity after a disruption.

Correspondingly, there are two broad categories of risk mitigation strategies to improve SCRES: proactive and reactive strategies. Proactive strategies plan and design the SCN for anticipating unexpected disruptions (Ponis and Koronis 2012). On the other hand, managers may be reluctant to implement proactive strategies since it becomes difficult to justify investments that mitigate potential disruptive events which may not ultimately occur (Tukamuhabwa et al. 2015). Reactive strategies focus on the ability to identify the risk sources and impacts, and to adapt to the impact of disruption and change accordingly to recover quickly and efficiently. During disruptions, contingency plans must be implemented quickly to expedite stabilisation and recovery in order to ensure continuity of supply and avoid long-term impact (Ivanov 2018).

Supply Chain Redundancy

One way to achieve SCRES is creating redundancies across an SC (Kamalahmadi and Parast 2016). For instance, there are several approaches to build redundancy in an SC:

- (1) Holding safety and emergency stock of materials and finished goods in some facilities to be used if disruption occurs (Chopra and Sodhi 2004; Chopra and Meindl 2015).
- (2) Having redundant plants to allow the SCN to maintain the production process even if one of the plants is down during the disruption (Tomlin 2006).
- (3) Utilizing back-up facilities of a critical facility during disruptions, in order to maintain the production process (Tomlin 2006; Chopra and Meindl 2015; Dada, Petruzzi, and Schwarz 2007).
- (4) Protect suppliers against disruptions by increasing redundancy in their operations (Sawik 2013).

2.2. Graph Models and Measures for SCRES Analysis

Table 1 summarizes the existing graph models of SCN based on topics of ‘SCRES’, ‘Redundancy’ and ‘Graph’ from the *Web of Science*. The type of the graphs is described, along with the representations of nodes and edges. The number of stages that the model can represent is also shown. The last column shows the measure used in the model.

A number of studies have been proposed to assess SCRES using qualitative indicators, e.g., for redundancy and flexibility (Wagner and Neshat 2010; Soni, Jain, and Kumar 2014; Vargas and González 2016; Chowdhury and Quaddus 2017; Jafarnejad Chaghooshi et al. 2018; Ruiz-Benítez, López, and Real 2018). The indicators are usually collected through surveys of the experts. Dependencies between qualitative indicators are then modelled using a graph, and an aggregated index is used to assess the overall SCRES. However, these studies do not model the actual SCN, which is the structural representation of the physical material flows and production processes in

Table 1. Review of existing graph models of SCN

Reference	Graph	Nodes	Edges	Stages	Measure
Graves and Tomlin (2003)	Bipartite graph	Plants & materials	Material-plant links	Multi	Process flexibility
Wang and Ip (2009)	Undirected bipartite graph	Demand & supply	Delivery lines	2	Weighted sum of node resilience
Adenso-Díaz et al. (2012) Adenso-Díaz, Mar-Ortiz, and Lozano (2017)	Directed graph	Suppliers, plants, wholesalers & customers	Product flows	Multi	Network reliability & robustness
Mizgier, Jüttner, and Wagner (2013)	Directed graph	Suppliers, focal firms & customers	Purchasing relationships	Multi	Total average loss
Ma et al. (2014)	Undirected graph	Manufacturers, suppliers, retailers & customers	Trading relationships	Multi	Node centrality
Xu, Wang, and Zhao (2014)	Directed graph	Firms	Demand-supply relations	Multi	Customer satisfaction
Kim, Chen, and Linderman (2015)	Directed graph	Physical locations	Transportation links	Multi	Ratio of redundant nodes to total nodes
Mari et al. (2015)	Directed graph	Suppliers, manufacturers & retailers	Material flows	3	Supply availability, size of LFN, clustering coefficient, supply path length in LFN
Reyes Levalle and Nof (2015)	Directed graph	Firms	Material flows	Multi	Total cost of flow and total quality of service
Salehi Sadghiani, Torabi, and Sahebjamnia (2015)	Bipartite graph	Demand & supply	Material flows	2	Supplier setup costs & transportation costs
Han and Shin (2016)	Undirected graph	Risk factors	SC relations	Multi	Average probability of disruption per risk
Wang and Xiao (2016)	Undirected graph	Suppliers, manufacturers, distributors & retailers	Production relations	4	SFZ-based index of structure, fixation, efficiency & flexibility
Li et al. (2017)	Directed graph	Suppliers, manufacturers, distribution centres & retailers	Materials or product deliveries	4	Recovery time
Nakatani et al. (2017)	Directed graph	Materials	Material-to-product links	Multi	Market concentration

the SCN.

Much of the current literature on SC redundancy focused on optimizing redundancy in an SCN with only a few stages using analytical models (Bertok et al. 2012; Azadeh et al. 2013; Paul, Sarker, and Essam 2017; Kamalahmadi and Parast 2017; Ni, Howell, and Sharkey 2017). Optimizing redundancy for multi-stage SCN quickly becomes intractable due to the complexities of the problem.

There are also existing models on SCRES that consider redundancy only between two stages in an SCN. Wang and Ip (2009) quantified the resilience of logistic networks considering redundancy between the demand and supply. Salehi Sadghiani, Torabi, and Sahebjamnia (2015) proposed a deterministic multiple set-covering model to design resilient SCNs under operational and disruption risks. Redundant supplier is assigned to each demand node in the SCN. These approaches are unable to analyse the dependencies across multi-stages in the SCN, where vulnerabilities in the upstream stages of the SCN can have a significant impact downstream.

To enable analytical tractability and simplify the problem, existing research works on multi-stage SCNs assumed that all plants within the SCN or the same stage play the equivalent role, or the type of material flows between stages are the same. Graves and Tomlin (2003) considered a multi-product, multi-stage SC with materials being processed at each stage. Each plant has a capacity that limits the quantity of materials that can be processed. Redundancies can be measured by the excessive capacity between two stages. Xu, Wang, and Zhao (2014) developed an assessment approach of SCRES based on demand satisfaction. The demand at each stage can only be satisfied by the upstream stage. Kim, Chen, and Linderman (2015) considered SCRES as a network-level attribute to withstand disruptions of nodes in the SCN. SCRES is defined as the ratio of the redundant nodes to the total number of nodes. Mari et al. (2015) proposed various resilience metrics for SCs that measure the largest functional SCN (LFN), which is the remaining SCN after disruption, assuming that the demand can be satisfied through the remaining supply nodes. Li et al. (2017) proposed a new measure of resilience which measured the average system performance, considering recovery time with a strict upper bound. Nodes with the same function are grouped as one stage, e.g., suppliers, manufacturers, distributors, retailers, etc.

In the context of SCRES, there are also related works that use graph theoretical approaches for disruption analysis and vulnerability assessment of SCNs. Adenso-Diaz et al. (2012) considered the impact of the network factors, e.g., source criticality, on the reliability of supply networks. The authors also analysed the robustness of the supply networks under targeted attacks or random disruptions (Adenso-Díaz, Mar-Ortiz, and Lozano 2017). Mizgier, Jüttner, and Wagner (2013) measured SCRES using vulnerability in terms of average loss at each firm due to disruptions. Ma et al. (2014) proposed that resilience can be achieved either through redundancy to the key suppliers or keeping a high level of collaborative working to the key suppliers. The key node can be identified by the critical path analysis. Reyes Levalle and Nof (2015) analysed the impact of SCN design on SCRES, which is measured using the total cost of flow and total quality of service in the SCN. Han and Shin (2016) assessed the structural robustness of an SC considering disruption propagation, which is measured by the average probability of disruption per risk at each node. Wang and Xiao (2016) considered SC recovery under cascading failures. When a node fails, it increases the loads of its upstream and downstream nodes. Based on the load at each node, SCRES is measured using spatial fidelity zones (SFZ) in terms of the structure, fixation, efficiency, and flexibility of the SCN. Nakatani et al. (2017) used a graph model representing the material-to-product relationships to assess the SC's vulnerability to disruption

risks. SC vulnerability indicators are proposed based on the market concentration of domestic and imported commodities.

In summary, some current studies are limited to a few stages in the SCN and unable to analyse large-scale multi-stage SCN. Other research works did not differentiate the production process of each type of material and assumed that all plants, either in the whole SCN or within the same stage, are equivalent. This limited structure complexity hides the in-depth dependencies among the plants and materials. Hence, these models are unable to capture both material-to-product and inter-plant relationships in a multi-stage SCN. Therefore, a graph-based model of an SCN that considers the dependencies between materials and plants needs to be developed, which allows us to apply the existing graph properties to reason about the structural redundancy and connectivity in the SCN.

3. Proposed Model

First, the SC model is discussed before introducing the SCN model. We consider a single-product, multi-stage SC, consisting of plants and materials. By extending on Grave's model (Graves and Tomlin 2003), the dependencies between materials and plants are considered. This allows any general multi-stage production system to be modelled, in which each stage performs a distinct operation and requires its own plants. To represent structural redundancy in the SCN, the model is extended to consider redundant dependencies between the materials and plants. However, there are also additional limitations of the model. The capacity of the plant is not considered in this model. To simplify the analysis, we assume that each plant only produces a single product.

3.1. A Supply Chain Model

A rooted directed tree G is denoted as $G = (M, P, E, m_o)$, where M and P are disjoint sets of nodes, and E is the set of directed edges between nodes in M and P respectively. The root of the tree is m_o , where $m_o \in M$. Leaves of the tree are nodes in P . G is also a bipartite graph, where the nodes can be divided into two disjoint and independent sets M and P .

M is the set of materials and P is the set of plants in an SC. m is a material in the SC, such that $m \in M$. p is a plant in the SC, such that $p \in P$. The *end product* of the whole SC is represented by root m_o , which is assumed as the focal product subjected to analysis. There are two basic components in an SC, which represent the dependencies between the material and plant: *output dependency* and *input dependency*.

Figure 1. Dependency between plant and material

Output dependency is the dependency relationship from a plant p to a material m , such that m is produced or supplied by p . A single **stage** in the SC is represented by an output dependency. Hence, an output dependency represents a single production

Figure 2. Input dependencies of plant p_1 on materials m_1 and m_2

(with input and output materials at p) or supply process (only output material at p). For a production process, the material, such as sub-assembly or intermediate product, is produced from the plant. For a supply process, the material is sourced from the plant. Figure 1a shows an example of the material m_1 that is produced from plant p_1 . The set of output dependencies in the SC are represented by the set of edges E_O .

Input dependency is the dependency relationship from a material m to a plant p , where the material, e.g. raw material, sub-assembly or intermediate products, is required by the plant for the production process. Input dependencies link stages of the SC together. For example in Figure 1b, plant p_1 is dependent on material m_2 . The set of input dependencies in the SC is represented by the set of edges E_I . All input dependencies of a plant are necessary to the plant, as they represent the materials directly required by the plant to process the product. Two input dependencies for plant p_1 are shown in Figure 2: from m_1 to p_1 , and from m_2 to p_1 . Since all the input materials are necessary, this can be represented using a logical *AND* relationship ($\wedge$).

Based on the two basic components, two key relationships in the SC can be modelled simultaneously. First, the input dependency and output dependency represents the *material-to-product relationships* in the production process. Whereas the combination of the output dependency and input dependency forms the *inter-plant relationships* between the plants.

Suppliers provide the raw materials required in the SC. They are not involved in any production processes within the SC. Hence, all the leaves in G represent the suppliers, which are from the set of plants P .

- (1) E_O is a set of distinct directed edges, where each directed edge from p to m is represented by an ordered pair of nodes (p, m) or $p \rightarrow m$. This relation can be expressed as:

$$E_O \subseteq (P \times M) = \{(p, m) | m \in M, p \in P\} \quad (1)$$

where E_O is the subset or equal set of all possible edges from P to M .

- (2) E_I is a set of distinct directed edges, where each directed edge from m to p is represented by an ordered pair of nodes (m, p) or $m \rightarrow p$. This relation can be expressed as:

$$E_I \subseteq (M \times P) = \{(m, p) | m \in M, p \in P\} \quad (2)$$

where E_I is the subset or equal set of all possible edges from M to P .

- (3) Since there are only edges between m and p , there are no edges between nodes from the same set, that is $E = E_O \cup E_I$ and $E_O \cap E_I = \emptyset$.

We denote the set of in-nodes of p as $M(p)$, where $\forall m \in M(p), (m, p) \in E_I$, and the set of in-nodes of m is denoted as $P(m)$, where $\forall p \in P(m), (p, m) \in E_O$,

Figure 3. A two-stage supply chain

Assuming that each plant only produces a single product,

Proposition 1. *For all material $m \in M$, there will be only one plant producing m in an SC, i.e. $\forall m \in M, |P(m)| = 1$.*

Definition 1 (Supply Chain). A supply chain is represented by a rooted directed tree $G = (M, P, E, m_o)$, where M is the set of materials and P is the set of plants. The root of the tree, m_o , represents the *end product*. The leaves of the tree from P , represent the *suppliers*. E represents the dependency relationships between materials and plants.

An SC represents the production system to process the material and produce the end product, where all plants play a role in the production process. The minimum number of nodes in G consists of a root m and a single plant p , connected as the output dependency (p, m) . This represents a single stage in the SC.

Proposition 2. *There is at least one output dependency in a supply chain.*

Since the supply chain is a tree and tree is a connected graph, the following proposition is obvious:

Proposition 3 (Supply Chain Connectivity). *An SC is connected.*

Since the SC is connected, all plants and materials must exist in the set of output dependencies, where each stage contains a plant and the corresponding material that is processed. Stages in the SC are linked together through the input dependencies, where the paths represent the material flow from all plants to the end product.

Example 1. Figure 3 shows an industrial organic chemicals manufacturing SC (H_1) from an existing set of samples (Willems 2007). It is represented as a two-stage SC G for an end product m_1 . Plant p_3 , p_4 , and p_5 produce material m_2 , m_3 and m_4 correspondingly. Material m_2 , m_3 and m_4 are materials required for plant p_1 to produce the end product m_1 . Hence, $\{(m_2, p_1), (m_3, p_1), (m_4, p_1)\}$ are the input dependencies for p_1 , and $\{(p_1, m_1)\}$ is the output dependency.

Figure 4. Redundant output dependencies of material m_1 on plants p_1 or p_2

3.2. Supply Chain Network Model

In order to assess the SC redundancy, we introduce an SCN in which redundant plants can exist. There is more than one plant producing a material in the SCN, i.e. SCN has **redundant output dependencies**. In the SC, it requires that a material is only produced by one plant (Proposition (1)). The SCN model relaxes this constraint, such that there can be multiple plants producing the same material. Hence, an SCN can be defined as a set of SCs (Harland 1996). This redundancy represents sourcing policies such as dual or multiple sourcing, as shown in Figure 4, where there are two plants p_1 and p_2 producing the same material m_1 . As the output dependencies are redundant, this can be represented using a logical *OR* relationship ($\vee$).

Let $\mathbb{G}$ be the set of trees G with the same root m_o , a rooted directed acyclic graph $H = (M_H, P_H, E_H, m_o)$ is the union of trees $G_x \in \mathbb{G}$, such that:

$$\begin{aligned} \forall G_x(M_x, P_x, E_x, m_o) &\in \mathbb{G}, \\ M_H &= \bigcup_x M_x \\ P_H &= \bigcup_x P_x \\ E_H &= \bigcup_x E_x \\ m_o &\in M_H \end{aligned}$$

Definition 2 (Supply Chain Network). Given $\mathbb{G}$ is a set of trees that have the same root m_0 , a supply chain network is a rooted directed acyclic graph H , which is a union of trees in $\mathbb{G}$.

Example 2. Figure 5a shows a two-stage SCN H for a common end product m_1 based on an existing SCN, H_1 , from the dataset (Willems 2007). It comprises two SCs as shown in Figure 5. Since there are two plants producing m_1 , there are redundant output dependencies, (p_1, m_1) and (p_2, m_1) , for material m_1 .

Given the definition of the SCN, the following two propositions are obvious:

Proposition 4. *Given an existing SCN, there is at least one SC exists.*

Proposition 5 (Supply Chain Network Connectivity). *A supply chain network is connected if there is at least one connected supply chain.*

Since an SCN consists of multiple SCs, as long as one of the SCs is still connected, the end product m_o is still available.

Figure 5. Two-stage supply chain network H contains two SCs G_1 and G_2

3.3. Disruptions in a Supply Chain Network

We consider disruption in an SC as the disruption of plants. If a plant p is disrupted, all dependencies of p will be disrupted. To model the disruption of plant p , all edges connected to p will be removed from the SC. When one or more dependencies is removed from the SC, the SC is disconnected, since removing an edge from a tree will disconnect the tree. Hence, we can define SC disruption in terms of graph connectivity.

Definition 3 (Supply Chain Disruption). A supply chain is disrupted if a supply chain is disconnected.

After considering disruptions in the SC, we extend the scope to consider disruptions in the SCN. From Proposition 5, if there is no connected SC, the SCN is disconnected.

Definition 4 (Supply Chain Network Disruption). A supply chain network is disrupted if a supply chain network is disconnected.

Example 3. For example in Figure 5, there are two SCs G_1 and G_2 . When p_1 is disrupted, the SC G_1 is disrupted. When p_2 is disrupted, the SC G_2 is disrupted. If both plants p_1 and p_2 are disrupted, both SCs G_1 and G_2 are disrupted. Hence, the SCN H is disrupted.

3.4. Number of Supply Chains in Generic Supply Chain Network

One way of mitigating the impact of disruption is by increasing the number of SCs in an SCN, i.e. increasing redundancy. A higher number of SCs reduces the probability that the SCN will be disconnected when plants are disrupted. To determine the number

of SCs in an SCN, we use a *generic SCN*, where the SCN structure of a tree can be constructed based on parameters defined.

Definition 5 (Generic Supply Chain Network). A generic supply chain network is a supply chain network where there are α plants producing each material, each plant requires β input materials, and there are L stages.

A generic SCN H is k -ary tree if both $\alpha = k$ and $\beta = k$. The minimum possible generic SCN that is connected is when $\alpha = 1$, $\beta = 1$ and $L = 1$, which is shown in Figure 1a. Hence, for a connected generic SCN, it is constrained by $\alpha \geq 1$, $\beta \geq 1$ and $L \geq 1$. Figure 6 shows an example of a generic SCN with $L = 2$, $\alpha = 2$ and $\beta = 2$.

Figure 6. A generic supply chain network with the parameters $L = 2$, $\alpha = 2$ and $\beta = 2$

Since α represents the number of plants for each material, the number of plants at stage l is α times of the number of materials at stage l . Assume for each stage $l = 0, \dots, L - 1$,

$$\text{Number of materials at stage } l = \alpha^l \times \beta^l$$

$$\text{Number of plants at stage } l = \alpha \times (\alpha^l \times \beta^l)$$

By applying geometric progression, where $\alpha > 1$ and $\beta > 1$,

$$\begin{aligned} \text{Number of materials, } |M| &= \sum_{l=0}^{L-1} \alpha^l \times \beta^l \\ &= \frac{\alpha^L \beta^L - 1}{\alpha\beta - 1} \end{aligned}$$

$$\begin{aligned} \text{Number of plants, } |P| &= \sum_{l=0}^{L-1} \alpha \times (\alpha^l \times \beta^l) \\ &= \alpha \times \sum_{l=0}^{L-1} (\alpha^l \times \beta^l) \\ &= \alpha \times |M| \end{aligned}$$

The number of plants that each material depends on increases the number of SCs linearly by a factor of α . In Figure 6, plant p_1 depends on material m_2 and m_3 . Material

m_2 depends on plant p_3 and p_4 , while m_3 depends on p_5 and p_6 . We can form four chains with respect to p_1 :

- (1) $\{p_3 \rightarrow m_2 \rightarrow p_1, p_5 \rightarrow m_3 \rightarrow p_1\}$
- (2) $\{p_4 \rightarrow m_2 \rightarrow p_1, p_5 \rightarrow m_3 \rightarrow p_1\}$
- (3) $\{p_3 \rightarrow m_2 \rightarrow p_1, p_6 \rightarrow m_3 \rightarrow p_1\}$
- (4) $\{p_4 \rightarrow m_2 \rightarrow p_1, p_6 \rightarrow m_3 \rightarrow p_1\}$

As material m_1 depends on either plants p_1 or p_2 , the total number of SCs is $4 \times 2 = 8$.

Assume that the number of SCs for a generic SCN with L stages is n_{L-1} ,

$$\begin{aligned} n_0 &= \alpha \\ n_l &= \alpha \times (n_{l-1})^\beta \end{aligned}$$

By applying $\log_\alpha$ on both sides of the equation:

$$\begin{aligned} \log_\alpha n_0 &= 1 \\ \log_\alpha n_1 &= 1 + \beta \log_\alpha n_0 \\ &= 1 + \beta \\ \log_\alpha n_2 &= 1 + \beta \log_\alpha n_1 \\ &= 1 + \beta(1 + \beta) \\ &= 1 + \beta + \beta^2 \\ \log_\alpha n_{L-1} &= 1 + \beta + \beta^2 + \dots + \beta^{L-1} \end{aligned}$$

By induction,

$$\log_\alpha n_{L-1} = \frac{1 + \beta^{L-1}}{1 - \beta}$$

$$\therefore n_{L-1} = \alpha^{\frac{1 + \beta^{L-1}}{1 - \beta}} \quad (5)$$

3.5. Identifying Supply Chains from a Supply Chain Network

However, most of the real-world SCNs cannot be represented using the generic SCN, as it often does not have the same number of input and output dependencies for all materials and plants. A more general method is required to determine the number of SCs in an SCN. Given that at least one SC existing in an SCN, an algorithm to find all the SCs in an existing SCN is presented in Algorithm 1.

Algorithm 1 describes a reverse depth-first traversal algorithm to determine the set of SCs $\mathbb{G}$ from an SCN H . The algorithm recursively traverses H from a material m , to find all the SCs with respect to material m .

The function **FindSupplyChains**(m) returns $\mathbb{G}_m$, which is the set of trees with the root m . Each plant p producing m is traversed (Line 3) to find the set of trees $\mathbb{G}_p$ with respect to p . For each plant p , m_i denotes the input material required by the plant p (Line 5). For each input material m_i , the set of trees $\mathbb{G}_i$ with respect to m_i is found by calling **FindSupplyChains**(m_i) (Line 6). Edges from input material m_i to plant p , and from plant p to output product m , are added to each tree in $\mathbb{G}_i$ (Line 7).

Algorithm 1 Find Supply Chains

```
1: function FINDSUPPLYCHAINS( $m$ )
2:    $\mathbb{G}_m \leftarrow \emptyset$ 
3:   for  $p \in P(m)$  do
4:      $\mathbb{G}_p \leftarrow \emptyset$ 
5:     for  $m_i \in M(p)$  do
6:        $\mathbb{G}_i \leftarrow \text{FindSupplyChains}(m_i)$ 
7:        $\mathbb{G}_{i,p} \leftarrow \{(M \cup \{m, m_i\}, P \cup \{p\}, E \cup \{(m_i, p), (p, m)\}), \forall G(M, P, E, m_i) \in \mathbb{G}_i\}$ 
8:        $\mathbb{G}_p \leftarrow \{G_1 \cup G_2, \forall (G_1, G_2) \in (\mathbb{G}_p \times \mathbb{G}_{i,p})\}$ 
9:      $\mathbb{G}_m \leftarrow \mathbb{G}_m \cup \mathbb{G}_p$ 
10:  return  $\mathbb{G}_m$ 
```

The product of $\mathbb{G}_p$ and $\mathbb{G}_{i,p}$ returns a set of paired trees, where each pair (G_1, G_2) are unioned to form the set of trees $\mathbb{G}_p$ with respect to p (Line 8). This generates the combinations of trees where there are redundant output dependencies for the material m (as described in the previous subsection). Each tree in $\mathbb{G}_p$ is then added to $\mathbb{G}_m$. Hence, all the SCs from an SCN H can be found by calling `FindSupplyChains( $m_o$ )` to traverse the SCN from the end product m_o .

Example 4. In Figure 5a, the tree H is traversed starting from the root m_1 .

- (1) There are two plants producing m_1 , such that $P(m_1) = \{p_1, p_2\}$.
- (2) For each plant in $P(m_1)$, it depends on three materials $\{m_2, m_3, m_4\}$, such that $M(p_1) = \{m_2, m_3, m_4\}$ and $M(p_2) = \{m_2, m_3, m_4\}$.
- (3) For each of the materials $\{m_2, m_3, m_4\}$, the trees with respect to each material can be found:
 - (a) $\mathbb{G}_2 = \{G(\{m_2\}, \{p_3\}, \{p_3 \rightarrow m_2\}, m_2)\}$
 - (b) $\mathbb{G}_3 = \{G(\{m_3\}, \{p_4\}, \{p_4 \rightarrow m_3\}, m_3)\}$
 - (c) $\mathbb{G}_4 = \{G(\{m_4\}, \{p_5\}, \{p_5 \rightarrow m_4\}, m_4)\}$
- (4) For the plant p_1 , these trees are added with the edges $\{(m_2, p_1), (m_3, p_1), (m_4, p_1), (p_1, m_1)\}$ to obtain the intermediate set of trees $\mathbb{G}_{2,1}$, $\mathbb{G}_{3,1}$ and $\mathbb{G}_{4,1}$.
 - (a) $\mathbb{G}_{2,1} = \{G(\{m_1, m_2\}, \{p_3, p_1\}, \{p_3 \rightarrow m_2 \rightarrow p_1 \rightarrow m_1\})\}$
 - (b) $\mathbb{G}_{3,1} = \{G(\{m_1, m_3\}, \{p_4, p_1\}, \{p_4 \rightarrow m_3 \rightarrow p_1 \rightarrow m_1\})\}$
 - (c) $\mathbb{G}_{4,1} = \{G(\{m_1, m_4\}, \{p_5, p_1\}, \{p_5 \rightarrow m_4 \rightarrow p_1 \rightarrow m_1\})\}$
- (5) The union of trees in the sets $\mathbb{G}_{2,1}$, $\mathbb{G}_{3,1}$ and $\mathbb{G}_{4,1}$ form the tree G_1 in Figure 5b.
- (6) Similarly for the plant p_2 , these trees are added with the edges $\{(m_2, p_2), (m_3, p_2), (m_4, p_2), (p_2, m_1)\}$ to obtain the intermediate set of trees $\mathbb{G}_{2,2}$, $\mathbb{G}_{3,2}$ and $\mathbb{G}_{4,2}$.
 - (a) $\mathbb{G}_{2,2} = \{G(\{m_1, m_2\}, \{p_3, p_2\}, \{p_3 \rightarrow m_2 \rightarrow p_2 \rightarrow m_1\})\}$
 - (b) $\mathbb{G}_{3,2} = \{G(\{m_1, m_3\}, \{p_4, p_2\}, \{p_4 \rightarrow m_3 \rightarrow p_2 \rightarrow m_1\})\}$
 - (c) $\mathbb{G}_{4,2} = \{G(\{m_1, m_4\}, \{p_5, p_2\}, \{p_5 \rightarrow m_4 \rightarrow p_2 \rightarrow m_1\})\}$
- (7) Then the union of trees in the sets $\mathbb{G}_{2,2}$, $\mathbb{G}_{3,2}$ and $\mathbb{G}_{4,2}$ form the tree G_2 in Figure 5c.

Hence, two SCs G_1 and G_2 can be obtained from the SCN H .

3.6. Assessment Metrics

Based on our SCN model, we proposed a two-level approach to assess the SCRES using structural redundancy. The measures are validated through two case studies on real-world SCN in Section 4.

3.6.1. Non-redundant Plants

The first level identifies plants within the SCN without redundancy. When a plant without redundancy is disrupted, it will have a significant impact on the SCN.

Definition 6 (Non-redundant Plant). A non-redundant plant is a plant in an SCN for a material m , such that there is only one plant producing the material m .

The set of non-redundant plants can be determined by:

$$P_{nr} = \bigcup_{m \in M_H} \begin{cases} P(m) & \text{if } |P(m)| = 1 \\ \emptyset & \text{otherwise} \end{cases} \quad (6)$$

If there is no *non-redundant plants*, all plants have at least one redundant plant. Hence, the SCN can tolerate at least one plant disruption without disrupting any SCs. If the firm aims to achieve redundancy on all plants, the set of non-redundant plants will be the key areas for building additional redundancy. On the other hand, it is possible for an SCN to have non-redundant plants for materials that are required by redundant plants downstream.

Example 5. Given two SCs G_1 and G_2 in an SCN, G_1 's set of edges $E_1 = \{p_2 \rightarrow m_1 \rightarrow p_1 \rightarrow m_0\}$, and G_2 's set of edges $E_2 = \{p_4 \rightarrow m_2 \rightarrow p_3 \rightarrow m_0\}$. Material m_0 has redundant plants p_1 and p_3 . Whereas for m_1 only depends on p_2 , and m_2 depends on p_4 . Both p_2 and p_4 are non-redundant plants, but m_0 has redundancy with respect to p_2 and p_4 .

Definition 7 (Critical Plant). A critical plant is a plant that is shared by all the SCs in an SCN.

The set of critical plants P_{crit} is the intersection of the vertex set P of all trees $G \in \mathbb{G}$. If any $p \in P_{crit}$ is disrupted and all edges connected to p are removed from SCN H , all trees $G \in \mathbb{G}$ will be disconnected.

$$P_{crit} = \bigcap_{G(M,P,E) \in \mathbb{G}} P \quad (7)$$

The set of critical plants is a subset or equal set of the *non-redundant plants*, such that $P_{crit} \subseteq P_{nr}$.

If there is no *critical plant*, there will be no single point of failure that may disrupt the entire SCN. Hence, the critical plants are the key areas to focus on to improve the SCRES.

3.6.2. Redundancy

However, the number of critical plants does not measure the amount of redundancies in the SCN, where there may be more than one plant for each material. A large number of critical plants in an SCN indicates large overlapping amongst the SCs (and hence fewer redundancies in the SCN). So, to capture the structural redundancy (i.e., to capture the redundancy at the network level instead of nodes), the second level measures the number of SCs in an SCN. This indicates the SCN's ability to resist disruptions and maintain operations.

Definition 8 (Supply chain network redundancy). Supply Chain Network Redundancy is the number of SCs in an SCN H .

$$n_{scn} = |\mathbb{G}| \quad (8)$$

A large number of SCs can indicate that there are more alternative production processes for the end product. This redundancy allows the SCN to maintain the production process even when some plants are disrupted, or provide backup plants that can be utilised during disruptions.

The advantage of this measurement approach is the ability to represent structural redundancy in a multi-stage SCN. In addition, upstream disruptions in the SCN may not be felt as quickly as downstream disruptions. But the impact of upstream disruptions can be amplified and outlasting the disruptions themselves (Schmitt and Singh 2012). Hence, the redundancies of plants at the lower stages have a larger impact on the SCRES compared to redundancies of plants at higher stages.

One limitation of the measure is that two SCN designs, one with critical plants and one having redundancies for all plants, may have the same number of SCs. Therefore, this requires a **two-level assessment**: (i) the number of critical plants, and (ii) the number of SCs. It is more important to improve the redundancies for the critical plants, before improving the redundancies of the remaining non-redundant plants in the SCN.

Example 6. In Figure 5a, there is only one plant producing each material m_2 , m_3 and m_4 , i.e. p_3 producing m_2 , p_4 producing m_3 and p_5 producing m_4 . For the first level analysis, we consider the non-redundant plants in the SCN. The set of non-redundant plants is $P_{nr} = \{p_3, p_4, p_5\}$. The set of critical plants P_{crit} is the same set as P_{nr} .

For the second level of analysis, we measure the number of SCs in the SCN. There are two plants p_1 and p_2 producing the same material m_1 . Two SCs (Figure 5b and Figure 5c) can be found from the SCN, i.e. $n_{scn} = 2$.

4. Case Studies

To illustrate the applicability of the conceptual model, we conduct analysis using two multi-stage SCNs, H_3 and H_{21} , from an existing set of real-life cases (Willems 2007). H_3 is a real-world computer peripheral equipment manufacturing SCN, consisting of nine plants. This SCN is chosen as it is small enough for visual analysis yet contains redundancies within the SCN. H_{21} is another real-world SCN which manufactures perfumes, cosmetics, and other toilet preparations, consisting of 98 plants. This case is used to illustrate the scalability of the model, and to demonstrate the limitation of using visual analysis due to the complexities of the network structure.

4.1. Case I – Reactive Risk Management

First, we consider a reactive strategy by performing the contingency actions after an SC disruption has occurred. When a plant disruption has occurred, we first determine whether the disrupted plant is a critical plant. If the disrupted plant is not a critical plant, the SCN can still maintain some functionalities due to the redundancies within the SCN. If a critical plant is disrupted, contingency action such as activating the backup is required.

This case study is conducted on H_3 , which consists of nine plants, eight materials and a single end product m_8 . There are three stages in the SCN, with ten input dependencies and nine output dependencies. There are two SCs in H_3 , each containing six plants and six materials.

The first four scenarios are designed based on H_3 to demonstrate the applicability of the model and the reactive strategy to improve the resilience of the SCN against disruptions. The SCNs in the four scenarios are evaluated using the metrics, and the results are tabulated in Table 2. To determine the potential risk within the SCN, the set of non-redundant plants P_{nr} and the set of critical plants P_{crit} are identified.

Table 2. Assessment for H_3

Scenario	$ P $	P_{nr}	P_{crit}	n_{scn}
1	6	$\{p_1, p_2, p_3, p_5, p_7, p_8\}$	$\{p_1, p_2, p_3, p_5, p_7, p_8\}$	1
2	9	$\{p_1, p_2, p_3, p_4, p_5, p_6, p_7\}$	$\{p_2, p_3, p_7\}$	2
3	7	$\{p_1, p_2, p_3, p_4, p_6, p_7, p_9\}$	$\{p_2, p_3, p_7\}$	1
4	7	$\{p_1, p_2, p_3, p_4, p_6, p_7^b, p_9\}$	$\{p_2, p_3, p_7\}$	1
5	12	$\{p_1, p_4, p_5, p_6\}$	$\emptyset$	16
6	16	$\emptyset$	$\emptyset$	64

Scenario 1 is an SC of H_3 , as shown in Figure 7a. It represents an SCN without any redundancy, i.e. there is only 1 SC in this scenario. Hence, all the plants have no redundancies. The lack of redundancies causes the SCN unable to tolerate any disruptions.

Scenario 2 is the original SCN in H_3 as shown in Figure 7b. It contains redundancies in the SCN, such that there are two SCs in H_3 . As such, the SCN has a smaller number of critical plants in the SCN compared to *Scenario 1*.

Considering a scenario where there is a plant disruption in the SCN, whether the disrupted plant is a critical plant needs to be determined. *Scenario 3* is the remaining SCN of H_3 after a plant p_5 is disrupted. In Figure 7c, it shows the shaded plant p_5 is disrupted and the dependencies $\{(m_1, p_5), (m_2, p_5), (m_3, p_5), (p_5, m_6)\}$ are removed from the SCN. As a result, nodes $\{p_1, p_8, m_1, m_6\}$ are affected by the disruption (as drawn in dotted lines). We can see that one of the SCs is disconnected when p_5 is disrupted. Since p_5 is not a critical plant, the SCN is still connected after disruption due to the redundancies within the SCN. As the SCN is still connected, the firm can still afford to passively wait for the disruption to end, and for the plant p_5 to recover.

Scenario 4 represents another scenario where a critical plant p_7 is disrupted in the *Scenario 3*. Since p_7 is a critical plant, it disrupts all the SCs in the SCN, resulting in disruption of the whole SCN. Hence, *contingency policies* need to be taken to recover the SCN from disruption. One possible policy is to use contingent re-routing, which seeks alternative plants after disruption, where the demand will be rerouted to the backup plant (Tomlin 2006). To recover the production capabilities of the SCN, the firm needs to re-route the demand to the backup plant p_7^b to maintain the flow of the material m_5 .

4.2. Case II – Proactive Risk Mitigation

For the proactive approach, we prioritise risk mitigation in the SCN by improving redundancy of non-redundant plants in the SCN. One possible strategy to redesign the SCN is improving the SCRES against disruption by having multiple sourcing options for its products (Ni, Howell, and Sharkey 2017). Multiple plants diversify risks

(a) *Scenario 1* – a single SC in H_3

(b) *Scenario 2* – original SCN H_3

(c) *Scenario 3* – p_5 disrupted

(d) *Scenario 4* – p_5 and p_7 disrupted

(e) *Scenario 5* – redundant critical plants

(f) *Scenario 6* – redundant plants for all non-redundant plants

Figure 7. Six scenarios based on H_3 from samples (Willems 2007), where the disrupted plant is shaded and the dotted nodes represent plants that are affected by the disruption

by decreasing the likelihood that a disruption to a single plant significantly impacts the SCN. If we assume that each plant has the same fixed setup cost, the number of plants in the SCN is proxy for setup costs for the plants. More complex operational costs of dual sourcing, such as variable ordering costs, are part of the future works.

For the proactive mitigation approaches, there are two additional scenarios. To improve the SCRES of *Scenario 2*, dual sourcing is implemented for all critical plants, where one additional plant is added for the critical plants $P_{crit} = \{p_2, p_3, p_7\}$ identified in Table 2. *Scenario 5* is a redesigned SCN after adding the redundant plants for all the critical plants in *Scenario 2*, where it has 12 plants and 8 materials, as shown in Figure 7e. There are in total 10 input dependencies and 12 output dependencies. The additional plants do not affect the total number of materials in the SCN. Another approach to prioritizing risks is by improving the redundancy for all non-redundant plants. *Scenario 6* implements dual sourcing for all the non-redundant plants of *Scenario 5*, i.e. $P_{nr} = \{p_1, p_4, p_5, p_6\}$, as shown in Figure 7f. It has 16 plants and eight materials, with 16 input dependencies and 16 output dependencies. Hence, it has redundancy in the all plants, where there are at least two plants producing each material. This means that all the materials can tolerate at least one plant that is producing it to be disrupted.

Figure 8. Percentage of plants disrupted against number of remaining SCs in H_3

To analyse the resilience of SCN against random disruptions, we simulate disruptions of the plants using Monte-Carlo simulation and evaluate the probability that the SCN remains connected. We randomly generate a set of plants to be disrupted, varying from 0% to 100%. After each percentage of the plants that are disrupted, the number of SCs in the remaining SCN is calculated. 10,000 sets of disrupted plants are generated, and the average number of SCs across the different percentage of plants disrupted are determined. The results are shown in Figure 8, which plots the percentage of plants disrupted against the number of remaining SCs in logarithmic scale. The number of remaining SCs reduces as more plants are disrupted. At around 65% of plants disrupted, the SCNs in all scenarios is disrupted.

Scenario 5 has a better resilience during disruption compared to *Scenario 2*, where

it requires 55% of the plants to be disrupted in order to disrupt the whole SCN, compared to 45% in *Scenario 2*. Hence, the SCN is more resilient against disruptions if redundant plants are added for the three critical plants (33% additional redundant plants). *Scenario 6* demonstrates the best resilience compared to *Scenario 2* and *Scenario 5*, as shown in Figure 8, where it can remain connected up to 65% of the plants disrupted. However, this requires around 77% more plants to achieve the level of the SCRES, which is a significant investment to consider.

4.3. Case III - Evaluation of a Large Supply Chain Network

This case applies the same proactive mitigation approaches as the previous case II, but on a larger SCN. H_{21} consists of 98 plants, 89 materials and a single end product m_1 . There are six stages in the production process, with 187 input dependencies and 98 output dependencies. Figure 9 shows the non-redundant plants and critical plants in H_{21} .

Three scenarios are designed based on H_{21} : *Scenario 7* is the original SCN H_{21} that is implemented by the industry, such that there are 10 SCs. To determine the potential risks within H_{21} , the set of 88 non-redundancy plants and four critical plants are identified. To improve the resilience of *Scenario 7*, four additional plants are added for the critical plants identified in *Scenario 7*. In *Scenario 8*, the new SCN has 102 plants and 89 materials. There are in total 187 input dependencies and 102 output dependencies. The additional plants do not affect the total number of materials in *Scenario 8*. Another possible strategy to improve the SCRES is implementing dual sourcing for all the non-redundant plants by adding 88 new plants in *Scenario 9*. It has 186 plants and 89 materials, with 364 input dependencies and 186 output dependencies.

Table 3. Assessment for H_{21}

Scenario	$ P $	$ P_{nr} $	P_{crit}	n_{scn}
7	98	88	$\{p_{72}, p_{73}, p_{74}, p_{121}\}$	10
8	102	84	$\emptyset$	160
9	186	0	$\emptyset$	3 54,334,820,352

Based on these scenarios, the SCNs are evaluated and tabulated in Table 3. Similarly, a Monte-Carlo simulation is conducted to simulate random disruptions in H_{21} . The results are shown in Figure 10, which plots the percentage of plants disrupted against the number of remaining SCs in logarithmic scale.

From Figure 10, when 30% of the plants in *Scenario 7* are disrupted, the whole SCN will be disrupted. Hence, these non-redundancy plants should be the priority to redesign the SCN to improve redundancy. For *Scenario 8*, it requires around 40% of the SCN to be disrupted, in order to disrupt the whole SCN. Therefore, *Scenario 8* is more resilient against disruptions if dual sourcing is used for the four critical plants. This is just by building additional 4% of redundant plants in *Scenario 7* to achieve this level of resilience.

Whereas for *Scenario 9*, the whole SCN is disrupted only after 60% of the plants are disrupted. *Scenario 9* demonstrates the best resilience against disruption as shown in Figure 10. However, to achieve redundancy for all the plants, this requires additional 90% of the redundant plants in the SCN, compared to *Scenario 8*. This can be considered a huge investment to implement redundancy for all the plants. A more practical

Figure 9. SCN H_{21} from samples (Willems 2007), where non-redundant plants are identified: critical plants in red, and remaining non-redundant plants in blue

Figure 10. Percentage of plants disrupted against the number of remaining SCs in H_{21}

and cost-effective strategy is to just implement redundancy for the critical plants.

5. Discussion and Conclusion

5.1. Theoretical Contributions

The theoretical contribution of this study to the SCRES literature is the graph-based model of an SCN capable of representing the structural redundancy. Previous literature mainly considered redundancy between two stages, and are unable to analyse redundancy or the corresponding vulnerability in the upstream stages of the SCN. In our model, structural redundancy in multi-stage SCNs is represented by the dependencies between the materials and plants.

In this paper, we proposed an approach to assess the SCRES. The number of SCs in the SCN measures the structural redundancy in the SCN. We also highlighted the importance of assessing vulnerability in the SCN by identifying critical plants at the upstream stages of the SCN. Disruptions in the production of vital upstream material can propagate and affect the production processes downstream.

The model also enables a quick analysis of the remaining connectivity in the SCN after disruptions. This is used to assess the resistance phase in the SCRES during the disruptions. The resistance of the SCN against the disruptions is measured using the number of remaining SCs after the disruption.

5.2. Practical Implications

The presented model provides a top-down approach for decision-makers to build a more resilient SCN, where detailed visibility of all SC operations may not be available. The practical applicability of proactive and reactive strategies to mitigate the disruptions has already been illustrated in the case studies and are further discussed in this subsection.

For proactive strategy, the decision-makers can build a more resilient SCN by re-

inforcing the critical plants through building flexibility and redundancy in the SCN. Redundancy can be created through the SCN, such as building additional redundant plants, as described in Section 2.1. The SCN can be designed with multiple plants for each of the materials. These plants diversify risks by decreasing the likelihood that a disruption to a single plant will disrupt the entire SCN. However, this needs to be balanced by the additional costs involved by having additional plants. The additional costs include the construction costs of the plant and reduction of the economics of scale by having lower production quantity at each plant. To achieve redundancies for all the plants, at least two plants are needed to produce each material. A flexible SCN with production flexibility at the critical plants allows the SCN to recover quickly after a disruption. The production flexibility enables the critical plants to ramp up the production capacity in order to clear the back-orders.

For reactive strategy, the decision-makers need to monitor the SCN to determine an appropriate response to the disruption. A two-level response to the disruption depends on whether a critical plant is affected by the disruption. If the non-critical plant is disrupted, the firm can delay the response or passively accept the disruption and wait for recovery. However, when a critical plant is disrupted, an immediate response, such as activation of contingency actions, is required.

5.3. *Conclusion and Future Works*

This paper presents a conceptual model of an SCN based on graph theory. The model considers both the materials and plants in an SC and captures two essential dependency relationships between the materials and plants (that is, input dependency and output dependency). The two dependency relationships form the basic building blocks for constructing a multi-stage SCN. The paper also proposes an approach to assess SCRES in terms of structural redundancy using critical plants and number of SCs in an SCN. The applicability of the model is demonstrated by case studies using samples from real-world SCNs.

Two SCNs (i.e. H_3 and H_{21}) from existing samples are used to illustrate the applicability of the approach for assessing the SCRES. H_3 is used to demonstrate the model and approach for both proactive and reactive mitigation of SC disruptions. The similar proactive approach is also applied to a larger SCN (i.e., H_{21}). To evaluate the proactive mitigation, random disruptions are simulated in the SCN, and it shows that the SCRES can be improved by adding redundant plants. However, adding redundancy to all the non-redundant plants requires significant investments.

Our proposed model assumes that a single output dependency is able to satisfy the demand and it does not consider the quantities of materials for simplicity. A future enhancement to this model will consider the inventory system and material flows, such that the dynamic response of the SCN during disruptions can be analysed. This enables the evaluation of more sophisticated strategies, such as considering both supplier selection and order allocations (Venkatesan and Goh 2018), to improve resilience. Our future work will also consider multiple products in the model. The production for each end product is modelled as a separate SCN. To balance the trade-off between resilience and cost, more complex cost measures will also be considered such as operational costs including production, inventory and transportation costs.

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