Optimal leak factor selection for the output-constrained leaky filtered-input least mean square algorithm

Shi, Dongyuan; Lam, Bhan; Gan, Woon-Seng; Wen, Shulin

2019

Shi, D., Lam, B., Gan, W.-S., & Wen, S. (2019). Optimal leak factor selection for the output-constrained leaky filtered-input least mean square algorithm. *IEEE Signal Processing Letters*, 26(5), 670-674. doi:10.1109/lsp.2019.2903908

<https://hdl.handle.net/10356/143203>

<https://doi.org/10.1109/lsp.2019.2903908>

© 2019 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works. The published version is available at:
<https://doi.org/10.1109/lsp.2019.2903908>

Downloaded on 26 Apr 2023 22:08:27 SGT

Optimal Leak Factor Selection for the Output-constrained Leaky Filtered-Input Least Mean Square Algorithm

Dongyuan Shi, *Student Member, IEEE*, Bhan Lam, *Student Member, IEEE*, Woon-Seng Gan, *Senior Member, IEEE*, Shulin Wen

Abstract—The leaky filtered-input least mean square (LFXLMS) algorithm is widely used in active noise control applications to minimize the degradation of attenuation performance due to output saturation distortion. However, the leak factor, which is critical in determining the steady-state error and robustness of the algorithm, is usually selected through trial and error. This Letter proposes a leak factor selection approach, which ensures the LFXLMS algorithm converges to its optimal solution under the average-output-power constraint and can be readily derived in practice. Both broadband and narrowband cases are considered in the derivation without the independence assumption, and the simulations are conducted based on real primary and secondary paths to verify its effectiveness.

Index Terms—Active Noise Control, leaky FxLMS, Saturation.

I. INTRODUCTION

ACTIVE noise control (ANC), generating an anti-noise to suppress the primary disturbance at a specific location, is usually regarded as an efficient solution to deal with low-frequency noise [1]–[4]. However, its performance is also hampered by some hardware imperfections, such as output saturation [5]–[7]. In particular, when the hardware has insufficient output power to attenuate the disturbance, saturation distortion from the overdriven amplifier and actuator will deteriorate both the attenuation performance and residual sound quality [6]. It may result in the divergence of the adaptive algorithm in severe cases [6], [8]. Hence, numerous adaptive algorithms have been developed to solve this problem [9]–[13]. For instance, the minimum output variance least mean square (MOV-LMS) adaptive estimator handles the saturation nonlinearities by minimizing the adaptive filter output power [14]. Other algorithms, such as the rescaling algorithm [15], [16] and the weight-constrained FxLMS (CFxLMS) [17], [18], avoid the output saturation effect by adaptively scaling the weights of the control filter and output power. Owing to its stability and low complexity, however, the leaky filtered-input least mean square (LFXLMS) algorithm is the most widely implemented in ANC applications [19]–[23].

The LFXLMS algorithm introduces a leak factor in its cost function, which forces the algorithm to seek the optimal control filter with the minimal cost [8] while confining the weight of the filter. Hence, the leak factor is vital to the performance of the algorithm [20]. A significantly large leak factor will tightly restrain the weight of the filter, but often results in poor noise reduction due to a significant bias in the

Fig. 1. Block diagram of ANC with the saturation effect [24].

control filter. On the contrary, a small leak factor is usually insufficient in confining the power of the output signal within a safe range, resulting in saturation nonlinearities [15].

Due to ambiguity in the leak factor selection criterion, many trials and errors are required for different applications. Hence, a constant leak factor has been proposed based on a specific nonlinear saturation model [24], [25]. However, this deterministic leak factor requires the system saturation model and an impulse response estimate of the primary path – the acoustic path from the reference sensor to the error sensor where the noise is to be attenuated. An alternative [14] based on the assumption that the reference signal is independent of the control filter is ineffective for the narrowband noise. There are also modified approaches, with a variable and appropriately-sized leak factor, do not require an offline prediction on the leak factor [24], [26]–[28]. However, these modified approaches are computationally complex, hindering their practical implementation. This Letter presents an optimal leak factor selection algorithm, which guarantees that the LFXLMS algorithm reaches the optimal control filter within a specified output power constraint. The proposed leak factor selection algorithm does not require any system saturation model nor the primary path information and can be readily calculated for real applications.

II. THE OUTPUT-CONSTRAINED LFXLMS ALGORITHM

An ANC system with the output saturation function $f(\cdot)$ is considered in this Letter, as illustrated in Fig. 1. The reference signal $x(n)$ is directly obtained from the primary source; the primary disturbance $d(n)$ is the convolution result of the reference signal and the primary path p , and the vector s represents the secondary path model. For alleviating saturation

distortion, the LFXLMS algorithm is applied to update the control filter $\mathbf{w}(n)$ [15],

$$\mathbf{w}(n+1) = (1 - \mu\gamma)\mathbf{w}(n) + \mu e(n)\mathbf{x}'(n) \quad (1)$$

where μ denotes the step size, which is a positive real value, and γ represents the leak factor, which is a positive real value or a real positive definite matrix [29]. The error signal in Fig. 1 is expressed as

$$e(n) = d(n) - f \left[\sum_{l=0}^{L-1} s_l \mathbf{w}^T(n-l)\mathbf{x}(n-l) \right] \quad (2)$$

where s_l is the l th coefficient of the secondary model, which is represented by an FIR filter with L taps. The filtered reference signal [30] is stated as

$$\mathbf{x}'(n) = \sum_{l=0}^{L-1} s_l \mathbf{x}(n-l). \quad (3)$$

Since the leak factor γ in (1) determines the “tightness” of the constraint on the control filter weights, its value is critical for the LFXLMS algorithm to control the maximum output power [15]. However, there are few guidelines on the selection of γ with a specified output constraint [24]. Hence, a trial-and-error routine is usually employed [15].

III. DERIVATION OF THE OPTIMAL LEAK FACTOR

To deal with this issue, we define an active noise control cost function with a specified output constraint given by

$$\begin{aligned} \min_{\mathbf{w}} J(\mathbf{w}) &= \mathbb{E} \left[\left| d(n) - \sum_{l=0}^{L-1} s_l \mathbf{w}^T(n-l)\mathbf{x}(n-l) \right|^2 \right] \\ \text{s.t. } g(\mathbf{w}) &= \mathbb{E} \left[|\mathbf{w}^T(n)\mathbf{x}(n)|^2 \right] \leq \rho^2 \end{aligned} \quad (4)$$

where ρ^2 denotes the maximum average output power or the rated power of the actuator and can be measured from experiments or directly refer to the datasheet of the output amplifier. $\mathbb{E}[\cdot]$ represents the expectation operation. The solution of (4) achieves the minimum mean square error (MMSE) under the condition that the output power of the control filter does not exceed the constraint ρ^2 . Since the constraint forces the ANC to operate in the linear region, the nonlinearity function $f(\cdot)$ can be ignored.

To modify (4) to an equality constraint, a slack variable θ^2 is introduced as

$$\theta^2 = \rho^2 - g(\mathbf{w}). \quad (5)$$

The Lagrangian function [31], based on (4) and the slack variable in (5), is defined as

$$\mathbb{L}(\mathbf{w}, \lambda, \theta) = J(\mathbf{w}) + \lambda [g(\mathbf{w}) + \theta^2 - \rho^2] \quad (6)$$

where λ ($\lambda \in \mathbb{R}$) is the Lagrangian factor. According to the Lagrangian necessary condition [31], (6) is differentiated with respect to $\mathbf{w}(n)$, λ and θ , respectively, and set to 0,

$$\frac{\partial \mathbb{L}(\mathbf{w}, \lambda, \theta)}{\partial \mathbf{w}} = \frac{\partial J(\mathbf{w})}{\partial \mathbf{w}} + \lambda \frac{\partial g(\mathbf{w})}{\partial \mathbf{w}} = 0 \quad (7a)$$

$$\frac{\partial \mathbb{L}(\mathbf{w}, \lambda, \theta)}{\partial \lambda} = g(\mathbf{w}) + \theta^2 - \rho^2 = 0 \quad (7b)$$

$$\frac{\partial \mathbb{L}(\mathbf{w}, \lambda, \theta)}{\partial \theta} = 2\lambda\theta = 0. \quad (7c)$$

The optimal control filter [32] of (4) is derived from (7a), (7b), and (7c) to

$$\mathbf{w}_o = (\lambda_o \mathbf{R}_x + \mathbf{R}_{x'})^{-1} \mathbf{P}_{dx'} \quad (8)$$

where $\mathbf{R}_x$ and $\mathbf{R}_{x'}$ represent the autocorrelation matrices of the reference signal and the filtered reference signal, respectively. $\mathbf{P}_{dx'}$ denotes the cross-correlation vector of the filtered reference signal and the disturbance. The optimal Lagrangian factor λ_o is obtained from (7a), (7b), and (7c) as

$$\begin{aligned} \lambda_o &= \frac{\mathbf{w}_o^T \mathbf{P}_{dx'} - \mathbf{w}_o^T \mathbf{R}_{x'} \mathbf{w}_o}{\rho^2} \\ &= \frac{\mathbb{E}\{\mathbf{w}_o^T \mathbf{x}'(n)d(n)\} - \mathbb{E}\{\mathbf{w}_o^T \mathbf{x}'(n) [\mathbf{x}'(n)]^T \mathbf{w}_o\}}{\rho^2} \\ &= \frac{\mathbb{E}\{y_o'(n)d(n)\} - \mathbb{E}\{y_o'(n)^2\}}{\rho^2}. \end{aligned} \quad (9)$$

Furthermore, as proven in [33], (4) has a unique solution when $\lambda_o \geq 0$. The resultant system operation can be described in two cases.

Case 1 : As indicated by (9), when $y_o'(n)$ equals $d(n)$, λ_o is 0. It indicates that the system possesses enough power to mitigate the disturbance, and the system operates linearly.

Case 2 : The final anti-noise $y_o'(n)$ is smaller than $d(n)$, which refers to the case when the system is incapable of canceling the disturbance entirely.

To investigate the relationship between $y_o'(n)$ and $d(n)$, we analyze the system in frequency domain [34]. As such, the error signal can be expressed as

$$E(\omega) = X(\omega)P(\omega) - X(\omega)W(\omega)S(\omega) \quad (10)$$

where $P(\omega)$, $W(\omega)$ and $S(\omega)$ denote the transfer functions of the primary path, control filter, and secondary path, respectively. $X(\omega)$ is the reference signal. Hence, the optimal control filter is similarly obtained by solving

$$\begin{aligned} \min_W J(W) &= \mathbb{E}[E(\omega)E^*(\omega)] \\ \text{s.t. } g(W) &= \mathbb{E}[W(\omega)X(\omega)X^*(\omega)W^*(\omega)] \leq \phi^2 \end{aligned} \quad (11)$$

where ϕ^2 represents the output constraint. To solve (11), we define a Lagrangian-like function given by

$$\mathbb{L}(W, \lambda) = J(W) + \lambda [g(W) - \phi^2] \quad (12)$$

where the Lagrangian factor λ is a real scalar. According to the Karush-Kuhn-Tucker condition [35],

$$\begin{cases} \nabla_W \mathbb{L}(W_o, \lambda_{\text{opt}}) = 0 \\ \nabla_\lambda \mathbb{L}(W_o, \lambda_{\text{opt}}) \leq 0 \\ \lambda_{\text{opt}} [g(W_o) - \phi^2] = 0 \\ \lambda_{\text{opt}} \geq 0 \end{cases} \quad (13)$$

where ∇ represents the gradient, we obtain

$$(1 + \lambda_{\text{opt}})\phi^2 = P(\omega)S_x(\omega)W_o^*(\omega)S^*(\omega) \quad (14)$$

and

$$\phi^2 = S(\omega)W_o(\omega)S_x(\omega)W_o^*(\omega)S^*(\omega) \quad (15)$$

where $W_o(\omega)$ and λ_{opt} represent the optimal control filter and the leak factor, respectively. Combining (14) and (15) yields

$$(1 + \lambda_{\text{opt}})S(\omega)W_o(\omega) = P(\omega). \quad (16)$$

So, the disturbance can be represented as

$$\begin{aligned} d(n) &= x(n) * \mathcal{F}^{-1}[P(\omega)] \\ &= x(n) * \mathcal{F}^{-1}[(1 + \lambda_{opt})W_o(\omega)S(\omega)] \\ &= (1 + \lambda_{opt}) \times y'_o(n) \end{aligned} \quad (17)$$

where $\mathcal{F}^{-1}[\cdot]$ and $*$ denote the inverse Fourier transform and convolution operation, respectively. Since the secondary path is the non-minimum phase in practice, it is only to obtain an approximation of the perfect control filter $W_o(\omega)$ in (17) [36], [37]. Therefore, the optimal anti-noise $y'_o(n)$ is approximately linearly correlated to the disturbance $d(n)$, and their correlation coefficient is given by

$$\frac{\mathbb{E}\{[y'_o(n) - \mathbb{E}[y'_o(n)]] [d(n) - \mathbb{E}[d(n)]]\}}{\sqrt{\sigma_{y'_o}^2} \sqrt{\sigma_d^2}} \approx 1 \quad (18)$$

where $\sigma_{y'_o}^2$ is the variance of the final anti-noise. As the expectation of the $y'_o(n)$ and $d(n)$ equates to 0 (ANC can only deal with the bipolar signal), we obtain $\mathbb{E}[y'_o(n)d(n)] = \sqrt{\sigma_{y'_o}^2} \sqrt{\sigma_d^2}$ from (18), and $\mathbb{E}[y'_o(n)^2] = \sigma_{y'_o}^2$. Hence, (9) can be rewritten as

$$\lambda_o = \frac{\sigma_{y'_o}^2}{\rho^2} \left(\sqrt{\frac{\sigma_d^2}{\sigma_{y'_o}^2}} - 1 \right). \quad (19)$$

The variance of the final anti-noise is given by

$$\sigma_{y'_o}^2 = \mathbb{E} \left\{ \left[\sum_{l=0}^{L-1} s_l y_o(n-l) \right]^2 \right\} = \sum_{l=0}^{L-1} s_l^2 \sigma_{y_o}^2 \quad (20)$$

where the output signal $y(n)$ is assumed to be zero-mean white noise, and $\sigma_{y_o}^2$ presents the variance of the final output signal $y_o(n)$. Substituting (20) into (19) yields

$$\lambda_o = \frac{\sigma_{y_o}^2 \sum_{l=0}^{L-1} s_l^2}{\rho^2} \left(\sqrt{\frac{\sigma_d^2}{\sum_{l=0}^{L-1} s_l^2 \sigma_{y_o}^2}} - 1 \right). \quad (21)$$

It should be noted that when the actuator is saturated, the final output signal has the maximum average power ($\sigma_{y_o}^2 = \rho^2$). By considering both the cases above, we define the degree of nonlinearity η^2 of the system as

$$\eta^2 = \max \left(\frac{\sigma_d^2}{\sum_{l=0}^{L-1} s_l^2 \rho^2}, 1 \right) \quad (22)$$

where $\max(\cdot)$ is the maximum value of arguments. Hence, the optimal Lagrangian factor (21) can be expressed as

$$\lambda_o = \sum_{l=0}^{L-1} s_l^2 (\eta - 1). \quad (23)$$

To simplify this analysis, we assume that the reference signal $x(n)$ is zero-mean white noise. So, the optimal control filter (8) with the output constraint can be rewritten as

$$\mathbf{w}_o = (\lambda_o \sigma_x^2 \mathbf{I} + \mathbf{R}_{x'})^{-1} \mathbf{P}_{dx'} \quad (24)$$

where σ_x^2 and $\mathbf{I}$ denote the variance of the reference signal and an identity matrix. Based on the derivation in [20], [29], the optimal solution of the LFXLMS algorithm (1) is given by

$$\mathbf{w}_o = (\gamma \mathbf{I} + \mathbf{R}_{x'})^{-1} \mathbf{P}_{dx'} \quad (25)$$

Fig. 2. The convergence paths of the leaky FxLMS algorithm with the optimal leak factor, for different primary paths with a sinusoid as the reference signal.

when γ is a positive scalar, or

$$\mathbf{w}_o = (\gamma + \mathbf{R}_{x'})^{-1} \mathbf{P}_{dx'} \quad (26)$$

when γ is a positive definite matrix [29]. To ensure that the LFXLMS algorithm has the same optimal solution as (4), (25) should equate to (24). Hence, the sufficient condition is given by

$$\gamma = \lambda_o \sigma_x^2 \quad (27)$$

Substituting (23) into (27) yields the optimal leak factor as

$$\gamma_o = \sigma_x^2 \sum_{l=0}^{L-1} s_l^2 (\eta - 1). \quad (28)$$

In consideration of practical cases where the reference signal and the output signal are not white noise, their auto-correlation matrices will not be diagonal. To ensure that the LFXLMS algorithm has the same solution as (4), (26) should equate to (8). Hence, the sufficient condition is given by

$$\gamma = \lambda_o \mathbf{R}_x. \quad (29)$$

Substituting (19) into (29) yields the optimal leak factor as

$$\gamma_o = G_s (\eta - 1) \mathbf{R}_x. \quad (30)$$

where $G_s = \sigma_{y'_o}^2 / \rho^2$ is the power gain of the secondary path. If the spectrum of the control signal is flat or narrowband, such as a tonal noise, G_s can be written as

$$G_s = \frac{1}{2\pi} \int_{\omega_1}^{\omega_2} |S(e^{j\omega})|^2 d\omega \quad (31)$$

where the signal frequency ranges between $[\omega_1, \omega_2]$. The degree of system nonlinearity also changes to

$$\eta^2 = \max \left(\frac{\sigma_d^2}{G_s \rho^2}, 1 \right). \quad (32)$$

Furthermore, the parameters in either (28) or (30) can be readily measured in a real-world system. Therefore, the optimal leak factor proposed in the Letter is applicable in practice.

Fig. 3. The convergence paths of the leaky FxLMS algorithm with the optimal leak factor, for different primary paths with white noise as the reference signal.

IV. EXPERIMENTAL RESULTS

In the first simulation, the reference signal is a sinusoid $x(n) = 2.15 \sin(0.16\pi n)$, and the power constraint of the system is $\rho^2 = 0.75^2/2$. The secondary path is set to $\mathbf{s} = [1, 0]^T$, and the step size of algorithms is $\mu = 0.005$. Figure 2 depicts the convergence paths of the LFXLMS algorithm under four different primary paths. In Fig. 2 (a), the primary path is $\mathbf{p}_1 = [0.1, 0.2]^T$, and the optimal solution falls within the constraint, which is the case where ANC cancels the disturbance without violating the output constraint. Hence, the optimal leak factor γ_o is 0, and the LFXLMS algorithm reaches the optimal solution. When the primary path changes to $\mathbf{p}_2 = [0.1, 0.2834]^T$, the optimal solution of the control filter coincides with the output constraint, as illustrated in Fig. 2 (b). In this situation, the leak factor γ_o remains as 0, and the LFXLMS converges to the optimal control filter. When the primary path changes to $\mathbf{p}_3 = [0.35, 0.3]^T$ and $\mathbf{p}_4 = [0.15, 0.35]^T$, the optimal solution of the control filter without the output constraint are located outside the constraint, as shown in Fig. 2 (c) and (d). In these cases, the leak factor derived from (30) still guarantees LFXLMS algorithm to the optimal solution without violating the output constraint.

Figure 3 exhibits the LFXLMS algorithm convergence paths of the second simulation with different primary paths using White Gaussian noise $N(0, 2.5)$ as the reference signal. The secondary path and the step size are set to $\mathbf{s} = [0, 0.8]^T$ and $\mu = 0.001$, respectively, and the average-power constraint is set to $\rho^2 = 0.75^2$. The LFXLMS algorithm converges to the optimal solutions with the leak factor $\gamma_o = 0$ under the primary paths $\mathbf{p}_1 = [0, 0.8, 0.16]^T$ and $\mathbf{p}_2 = [0, 0.8, 0.2267]^T$, as shown in Fig. 3 (a) and (b), respectively. Their optimal solutions are located within the constraint and at the edge of the constraint, respectively. In contrast, when the primary path changes to $\mathbf{p}_3 = [0, 0.28, 0.24]^T$ and $\mathbf{p}_4 = [0, 0.13, 0.28]^T$, the optimal solutions without output constraint are outside of the feasible area. Meanwhile, the LFXLMS algorithms with

Fig. 4. The spectrum of the error signal and the control signal of different algorithms.

the optimal leak factor $\gamma_o = 2.1$ and $\gamma_o = 1.1$ successfully approach their corresponding restrained optimal solutions, which are located at the edge of the constraint, as illustrated in Fig. 3 (c) and (d). It is worth noting that all the optimal leak factors of this simulation are derived with (28).

In the final simulation, we used the real measured primary and secondary paths of an air duct. To simulate the saturation effect of the ANC system, we also introduced a clipping function cascaded with the control filter that truncates parts of the output signal (above 0.75 or below -0.75) [33]. The spectrum and time domain plots of the disturbance signal (a sinusoidal reference $x(n) = 0.32 \sin(0.1\pi n)$ modified by the primary path with the measurement noise $v(n) \sim N(0, 0.001)$) are shown in Fig. 4 (a) and (b), respectively. If the FxLMS algorithm is adopted, the control signal will violate the constraint and result in severe distortion, as illustrated in Fig. 4 (c) and (d). In contrast, the LFXLMS algorithm with the optimal leak factor derived with (30) can successfully restrain the output signal within the constraint and mitigate the disturbance, as shown in Fig. 4 (e) and (f).

V. CONCLUSION

This Letter revisited the optimal leak factor selection issue in the LFXLMS algorithm when it is adopted to avoid the output saturation distortion and to obtain the satisfying noise reduction performance in the active noise control (ANC) application. An optimal leak factor is proposed for the LFXLMS algorithm, which assists ANC system to achieve the best noise reduction while restraining the average output power within a specified range to prevent saturation distortion. Furthermore, compared to the previous related works, this optimal leak factor does not require any system saturation model and impulse response of the primary path and can be readily calculated in practice. The simulation results based on the real primary and secondary paths verify its performance.

REFERENCES

- [1] Y. Kajikawa, W. Gan, and S. M. Kuo, "Recent applications and challenges on active noise control," *APSIPA Transactions on Signal and Information Processing*, vol. 1, 2012.
- [2] S. J. Elliott and P. A. Nelson, "Active noise control," *IEEE Signal Processing Magazine*, vol. 10, no. 4, pp. 12–35, Oct 1993.
- [3] C. Shi, H. Li, D. Shi, B. Lam, and W. Gan, "Understanding multiple-input multiple-output active noise control from a perspective of sampling and reconstruction," in *2017 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA ASC)*, Dec 2017, pp. 124–129.
- [4] C. H. Hansen, *Understanding active noise cancellation*. CRC Press, 1999.
- [5] M. H. Costa, J. C. M. Bermudez, and N. J. Bershad, "Stochastic analysis of the LMS algorithm with a saturation nonlinearity following the adaptive filter output," *IEEE transactions on signal processing*, vol. 49, no. 7, pp. 1370–1387, 2001.
- [6] D. Shi, C. Shi, and W. Gan, "Effect of the audio amplifier's distortion on feedforward active noise control," in *2017 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA ASC)*, Dec 2017, pp. 469–473.
- [7] M. H. Costa, J. C. M. Bermudez, and N. J. Bershad, "Stochastic analysis of the filtered-X LMS algorithm in systems with nonlinear secondary paths," *IEEE Transactions on Signal Processing*, vol. 50, no. 6, pp. 1327–1342, June 2002.
- [8] O. J. Tobias and R. Seara, "Performance comparison of the FXLMS, nonlinear FXLMS and leaky FXLMS algorithms in nonlinear active control applications," in *2002 11th European Signal Processing Conference*, Sep. 2002, pp. 1–4.
- [9] W. J. Kozack and T. Ogunfunmi, "An active noise control algorithm with gain and power constraints on the adaptive filter," *EURASIP Journal on Advances in Signal Processing*, vol. 2013, no. 1, p. 17, 2013.
- [10] B. Rafaely and S. J. Elliott, "A computationally efficient frequency-domain lms algorithm with constraints on the adaptive filter," *IEEE Transactions on Signal Processing*, vol. 48, no. 6, pp. 1649–1655, June 2000.
- [11] W. J. Kozack and T. Ogunfunmi, "An adaptive IIR filter with constraints on the output power level," in *2010 Conference Record of the Forty Fourth Asilomar Conference on Signals, Systems and Computers*, Nov 2010, pp. 984–987.
- [12] Z. Zhang, F. Hu, and J. Wang, "On saturation suppression in adaptive vibration control," *Journal of Sound and Vibration*, vol. 329, no. 9, pp. 1209–1214, 2010.
- [13] W. J. Kozack and T. Ogunfunmi, "A cascaded IIR–FIR adaptive ANC system with output power constraints," *Signal Processing*, vol. 94, pp. 456–464, 2014.
- [14] J. C. M. Bermudez and M. H. Costa, "Optimum leakage factor for the MOV-LMS algorithm in nonlinear modeling and control systems," in *2002 IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 2, May 2002, pp. II–1393–II–1396.
- [15] X. Qiu and C. H. Hansen, "A study of time-domain FXLMS algorithms with control output constraint," *The Journal of the Acoustical Society of America*, vol. 109, no. 6, pp. 2815–2823, 2001.
- [16] —, "Applying effort constraints on adaptive feedforward control using the active set method," *Journal of Sound and Vibration*, vol. 260, no. 4, pp. 757–762, 2003.
- [17] H. Lan, M. Zhang, and W. Ser, "A weight-constrained FxLMS algorithm for feedforward active noise control systems," *IEEE Signal Processing Letters*, vol. 9, no. 1, pp. 1–4, 2002.
- [18] S. J. Elliott and K. H. Back, "Effort constraints in adaptive feedforward control," *IEEE Signal Processing Letters*, vol. 3, no. 1, pp. 7–9, Jan 1996.
- [19] S. M. Kuo and D. R. Morgan, "Active noise control: a tutorial review," *Proceedings of the IEEE*, vol. 87, no. 6, pp. 943–973, June 1999.
- [20] O. J. Tobias and R. Seara, "Leaky-FXLMS algorithm: stochastic analysis for gaussian data and secondary path modeling error," *IEEE Transactions on Speech and Audio Processing*, vol. 13, no. 6, pp. 1217–1230, Nov 2005.
- [21] H. Zhao, X. Zeng, Z. He, T. Li, and W. Jin, "Nonlinear adaptive filter-based simplified bilinear model for multichannel active control of nonlinear noise processes," *Applied Acoustics*, vol. 74, no. 12, pp. 1414 – 1421, 2013.
- [22] K. Chen, R. Paurobally, J. Pan, and X. Qiu, "Adaptive feedback noise control with leaky FeLMS algorithm," in *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, vol. 249, no. 3. Institute of Noise Control Engineering, 2014, pp. 4196–4205.
- [23] B. Subudhi, P. Ray, and S. Ghosh, "Variable leaky least mean-square algorithm-based power system frequency estimation," *IET Science, Measurement & Technology*, vol. 6, no. 4, pp. 288–297, 2012.
- [24] O. J. Tobias and R. Seara, "On the LMS algorithm with constant and variable leakage factor in a nonlinear environment," *IEEE Transactions on Signal Processing*, vol. 54, no. 9, pp. 3448–3458, Sep. 2006.
- [25] R. Srazhidinov and R. Kamil, "Performance comparison of LFXLMS, MOVFXLMS and THF-NLFXLMS algorithms for hammerstein NANC," in *2016 International Conference on Instrumentation, Control and Automation (ICA)*, Aug 2016, pp. 12–15.
- [26] B. Huang, L. Wen, Y. Xiao, J. Sun, and Y. Shen, "A complete parallel narrowband active noise control system based on residual error separation using variable leaky LMS," in *2015 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA)*, Dec 2015, pp. 981–985.
- [27] O. J. Tobias and R. Seara, "Variable leakage FXLMS algorithms for nonlinear active control applications," in *2004 12th European Signal Processing Conference*, Sep. 2004, pp. 933–936.
- [28] M. Kamenetsky and B. Widrow, "A variable leaky LMS adaptive algorithm," in *Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers*, 2004., vol. 1, Nov 2004, pp. 125–128 Vol.1.
- [29] L. Wu, X. Qiu, and Y. Guo, "A generalized leaky FxLMS algorithm for tuning the waterbed effect of feedback active noise control systems," *Mechanical Systems and Signal Processing*, vol. 106, pp. 13–23, 2018.
- [30] C. Chang, S. M. Kuo, and C. Huang, "Secondary path modeling for narrowband active noise control systems," *Applied Acoustics*, vol. 131, pp. 154–164, 2018.
- [31] D. P. Bertsekas, *Nonlinear programming*. Athena scientific Belmont, 1999.
- [32] F. Taringoo, J. Poshtan, and M. H. Kahaei, "Analysis of effort constraint algorithm in active noise control systems," *EURASIP Journal on Advances in Signal Processing*, vol. 2006, no. 1, p. 054649, 2006.
- [33] D. Shi, W. Gan, B. Lam, and C. Shi, "Two-gradient direction FXLMS: An adaptive active noise control algorithm with output constraint," *Mechanical Systems and Signal Processing*, vol. 116, pp. 651–667, 2019.
- [34] S. Elliott, *Signal processing for active control*, ser. Signal Processing and its Applications. London: Academic Press, 2001.
- [35] M. A. Hanson, "On sufficiency of the Kuhn-Tucker conditions," *Journal of Mathematical Analysis and Applications*, vol. 80, no. 2, pp. 545–550, 1981.
- [36] S. D. Snyder and C. H. Hansen, "The effect of transfer function estimation errors on the filtered-x LMS algorithm," *IEEE Transactions on Signal Processing*, vol. 42, no. 4, pp. 950–953, April 1994.
- [37] A. A. Milani, I. M. S. Panahi, and R. Briggs, "Performance analysis of sub-band nLMS, APA and RLS in fMRI ANC with a non-minimum phase secondary path," in *2007 IEEE International Conference on Acoustics, Speech and Signal Processing - ICASSP '07*, vol. 2, April 2007, pp. II–353–II–356.