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A comparative study on the use of Calibrated and Rainbow Schlieren techniques in axisymmetric supersonic jets

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Abstract

Detailed experimental comparisons had been conducted between calibrated and rainbow schlieren on perfectly- and under-expanded axisymmetric jets through a modified Z-type schlieren system. The techniques were implemented by using a weak lens in the field-of-view to provide calibration information for the extraction of quantitative density gradients from the experimental schlieren images. Sixth-order polynomial curve fits were obtained for both calibrated and rainbow schlieren respectively. The effects of light inhomogeneity caused by the mirrors and system diaphragm aperture had been evaluated for the colour images and results indicate that averaging the background hue is an acceptable approach for minimizing light variations with less than 2% experimental error. Density gradients as calculated via Abel transform have also been evaluated to validate the two different set-ups. Additionally, experimental results have been compared to validated numerical results and they show that calibrated schlieren is able to predict density gradients within 2% of the numerical results. This is significantly more superior to rainbow schlieren, where errors in the estimated density gradients are closer to 20%. It is shown here that rainbow schlieren results are more adversely impacted by the system diaphragm aperture, especially for vertical light cut-off configuration. This is partly due to the loss of sensitivity of the schlieren system, as well as potential light diffusion caused by the filter.

Keywords: Rainbow Schlieren, Calibrated Schlieren, Optical Flow

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1 Introduction

While schlieren imaging has been used in the study of compressible flow and heat transfer problems since the mid-20th century, its scientific applications remain largely qualitative and quantitative applications are far less common. On the other hand, quantitative schlieren imaging allows one to determine the gas densities from the refractive index gradient fields captured in schlieren images based on standard Toepler-type or z-type schlieren setups. Such a non-intrusive measurement technique is highly attractive for many compressible flow and heat transfer problems, particularly if instantaneous global measurements with minimal or zero flow disruptions are desired. There are currently two different types of quantitative schlieren imaging techniques – firstly, calibrated schlieren which makes use of conventional monochrome schlieren images and secondly, rainbow schlieren which makes use of a colour filter as light cut-off in lieu of the knife edge (Settles, 2001).

Determining density and temperature field information by measuring the refraction of a light beam was already possible as early as the 1930s, though these early efforts were severely hampered by computational limitations. Nevertheless, Howe (1984) and Greenberg (1995) made the first successful attempts on quantitative schlieren techniques. In particular, Greenberg (1995) discussed at length the challenges in obtaining quantitative data from schlieren images, primarily because of inhomogeneous light absorption in the test section, image diffraction due to a filter comprised of colour strips with sharp transitions, and measurement non-linearities. To avoid these issues and to improve the detection sensitivity, the use of a graded colour filter was suggested and that has since become the typical approach. Greenberg (1995) also introduced the concept of defining a colour distribution as a function of its hue, since the latter is insensitive towards variations in light intensities.

More recently, Agrawal et al. (1998, 1999, 2002), Al-Ammar et al. (1998, 2000), Albers and Agrawal (1999) and Kasyap and Agrawal (2003) made extensive use of quantitative colour schlieren technique for studies involving flames and micro-jets. These studies possess commonalities in that a one-

dimensional vertical colour band, rainbow-type filter was used, and a system calibration by traversing the filter was adopted. These studies demonstrated the feasibility of determining scalar flow properties such as density, temperature and pressure from a rainbow schlieren system. It should be highlighted that Elsinga et al. (2004) conducted a comparison between calibrated quantitative colour schlieren and background-oriented schlieren, in which tri-colour segmented and graded filters were evaluated. The graded filter used in the study differed from the conventional one-dimensional filter in that there were no distinguishable colour strips. The advantage of such a two-dimensional graded filter lies in its ability to evaluate gradients in two orthogonal directions, which provides clear benefits in more complex scenarios such as shock structures and interactions.

Satti et al. (2007) designed a small-scale schlieren system for the study of axisymmetric micro-jets and applied quantitative colour schlieren technique similar to that adopted by Al-Ammar et al. (2000), where the results demonstrated good correlations with large-scale jet behaviour and theory. Khole and Agrawal (2009) extended the technique to supersonic micro-jets and results again demonstrated the potential of colour schlieren technique to obtain quantitative data from an experimental method that most considered to be qualitative. Hargather and Settles (2017) took an in-depth look at the two calibration methodologies first described by Greenberg (1995), as well as conducted a comparison between calibrated and rainbow schlieren techniques. They observed that calibration by lens is much simpler than and as precise as the filter-traverse approach. Settles and Hargather (2017) completed an extensive overview of schlieren systems, their applications, as well as their unique advantages and disadvantages, the outcome of which supports the notion that there exist significant opportunities for quantitative schlieren techniques to be further developed and more widely adopted. It should not be surprising then, that further studies were undertaken by Stevenson and Skews (2014) in the area of shock reflections, where they made use of lens-based calibration approach and a disk-type filter instead of the conventional one-dimensional band-type rainbow filters. Results showed that quantitative data can be determined from high-resolution, high-quality schlieren images, though their

accuracy levels remain limited for such high-speed flow scenarios. Most recently, Takano et al. (2016) introduced a three-dimensional rainbow schlieren technique which sought to address some of the accuracy issues, among other studies.

Along with developments in rainbow schlieren, further developments in non-intrusive optical flow measurements have been carried out, in particular in the use of colored background-oriented schlieren (CBOS) and color interferometry. Leopold (2007) presented work focusing on the use of CBOS for both free-flight tests in a wind tunnel and a first-of-a-kind in-flight test. For both tests, Leopold (2007) used a coloured speckled background based on the red-green-blue combination of colours, showing promising results and ease of implementation in particular in a real-life case. Mier and Hargather (2016) also evaluated the performance of CBOS, introducing the concept of a graded background rather than a speckled background, showing high-quality results. In the field of color interferometry, Desse (2006) and Desse and Olchewsky (2017) conducted extensive evaluations on the application of color interferometry, in particular for hot and cold supersonic jets, showing a high level of accuracy in the results, with the downside of the complexity of properly implementing these techniques.

It should be clear from the preceding overview that significant efforts had been expended to develop rainbow schlieren techniques, with little emphasis on the use of calibrated schlieren techniques. However, the case to understand and develop the latter technique is significant, due to the fact that many existing schlieren-imaging systems make use of high-speed monochromatic cameras for qualitative flow visualizations. Hence, there is a compelling case to be made that extending these monochromatic cameras towards quantitative endeavours based on calibrated schlieren technique will be a cost-effective flow measurement solution. Furthermore, calibrated schlieren offers the possibility of better accuracy levels if monochromatic cameras of sufficient bit-depth were to be used. However, there have been few direct comparisons between calibrated and rainbow schlieren

techniques to investigate their relative accuracy levels, based on a similar experimental setup, as well as a well-understood and robust test scenario. As such, the objective of the present investigation is to address this knowledge gap by conducting an experimental comparison between calibrated and rainbow schlieren techniques based on classical supersonic round jet flows. The advantage of utilizing supersonic jet flows for the present comparison lies not only in the significant compressibility effects to be encountered, but that these jets also produce well-documented shock structures and behaviour that ease better comparisons.

2 Experimental setup and procedures

2.1 Experimental apparatus

The experiments were conducted using the supersonic jet and schlieren imaging facilities described in Wu and New, 2017, and hence only a brief description will be provided here. Free supersonic jets were produced via exhausting compressed air through a jet apparatus with flow conditioning devices to improve flow quality at a design Mach number of $Ma_d=1.45$, where the round jet nozzle has a diameter of $d=12.7\text{mm}$. The nozzle was designed to be of the converging-diverging type, and a detailed description is provided in Wu and New (2017). The total pressure and temperature of the jet flows were 8.3bar and 300K respectively. Lastly, two nozzle pressure ratios (NPR) of 3.4 and 5.0 were used, where they correspond to perfectly-expanded and under-expanded jets respectively. Readers are referred to Wu and New (2017) for more details.

2.2 Schlieren imaging system

As shown in Fig. 1, the schlieren imaging system used in the present study consist of a modified Z-type schlieren system. The particularity of this type of system is its inherent compactness in comparison to a traditional z-type schlieren system, making it very attractive for exploiting the advantages of high quality, comparatively large mirrors with long focal lengths in limited space environments. At the same time, the major disadvantage is in the flow not being located in a test region of parallel light rays. As

Figure 1 Calibrated (top) and rainbow (bottom) schlieren experimental setups with details on lens positioning (right)

such, a preliminary validation of the set up was completed using a standard z-type schlieren, and the comparison of the flows captured by the two systems has shown that, in this particular case, the divergence of the light can be considered negligible.

An LED light-source produced a 200W broadband light beam that passed through a variable source aperture diaphragm, before it was reflected and collimated by two 300mm diameter, f/10 parabolic mirrors located 1.2m apart from each other, and passed through the test section. Both calibrated and rainbow schlieren experiments were conducted using the same setup, except that a standard knife-edge was placed at the focal point of the collimating mirror for the former, instead of a graded rainbow filter for the latter. Schlieren images were captured using a 6000px by 4000px resolution digital single-reflex lens camera with a 200mm lens, 1/8000s exposure time and ISO1000 sensitivity. System calibration for both calibrated and rainbow schlieren techniques was carried out by locating a weak 10m focal length plano-convex lens in front of the jet nozzle along its central axis and is schematically shown in Fig. 1.

Figure 2 Rainbow filter (left) and designed hue distribution (right)

2.3 Rainbow filter design

The rainbow filter was based on a VIBGYOR (Violet-Indigo-Blue-Green-Yellow-Orange-Red) colour distribution (Panigrahi and Muralidhar, 2012) and was designed using an image editing software to have a continuous, linearly varying colour spectrum. This was done by grading the regions between two distinct colour bands to avoid any abrupt colour variations which would affect the sensitivity of the schlieren system (Hargather and Settles, 2012). The digital colour spectrum file was subsequently printed as a 35mm slide by Gammatech (Hargather and Settles, 2012) and several filter sizes were designed with widths ranging from 3mm to 18mm. Tests demonstrated that the 6mm wide filter provided the optimum filter width for the current study, as it made full use of the calibration lens diameter. The 6mm filter and the designed hue distribution are shown in Fig. 2.

2.4 System calibration and data reduction

Obtaining quantitative data from schlieren images requires the schlieren imaging system to be carefully calibrated. As such, a 50.8mm diameter, 10m focal-length weak plano-convex calibration lens was placed within the test section where the jet nozzle was located, such that the intensity gradients or the full spectrum of colour gradients would be visible upon the lens. Calibration images were obtained for both the vertical and horizontal light cut-offs, as shown in Fig. 3. To evaluate the

Figure 3 Calibration lenses for calibrated schlieren - horizontal (CHKE) and vertical (CVKE) knife edge orientation (left); rainbow schlieren - horizontal (RHKE1) and vertical (RVKE1) for the first diaphragm opening (centre), and horizontal (RHKE2) and vertical (RVKE2) for the second diaphragm opening (right)

light loss due to the presence of the filter, the impact of the diaphragm aperture was investigated and the outcome can be appreciated in Fig. 3. The same 6mm filter was tested at two different diaphragm apertures (RVKE1, RVKE2) for vertical filter placement configuration, as well as two different diaphragm apertures (RHKE1 and RHKE2) for the horizontal filter placement configuration. It can be observed that RVKE2 and RHKE2 configurations led to higher sensitivity and sharpness of the colour gradients. On the other hand, the effects of diaphragm aperture upon calibrated schlieren technique was not studied, as the light loss due to the presence of the knife edge is negligible and the system would be at its most sensitive configuration. Each calibration image was analysed separately, with the pixel intensity or hue value interpolated along the central vertical or horizontal diameter line of the calibration lens. The positions, r_0 , of the background intensity and hue values, which correspond to zero light refraction at ambient conditions, were identified as a function of the lens radius r . The

Figure 4 Calibration curves for the present (a) calibrated and (b) rainbow schlieren techniques for the vertical knife edge (VKE) light cut-off

refraction angle, ε , was corrected for background intensity/hue and subsequently calculated as a function of the location along the lens radius and focal length, before been plotted against intensity or hue (Hargather and Settles, 2012). The current calibration procedures provided refraction angle ranges of -1.9×10^{-3} radian to 1.95×10^{-3} radian, and -2.4×10^{-3} radian to 2.5×10^{-3} radian for the present calibrated and rainbow schlieren techniques respectively. The resulting calibration curves are shown in Fig. 4(a), where it incidentally highlights one of the limitations associated with calibrated schlieren. Specifically, calibrated schlieren has a narrower measurement range in terms of refraction angles due to light saturation caused by light cut-off.

This is in fact discernible along the edges of the calibration lens in Fig. 3. This limitation may introduce potentially significant errors in the prediction of density gradients and demonstrate the importance of fine-tuning the light cut-off. Based on the datasets, a 6th-order polynomial curve fitting was applied to the calibrated schlieren calibration data, with corresponding construction error of approximately 4%. The hue curve in Fig. 4(b) indicates that the gradients of the colour bands in the filter are non-optimal, requiring the optimization of the calibration polynomial curve fitting. With the theoretical hue distribution shown in Fig. 2 indicating the optimum distribution, a 2nd order curve fitting was applied and compared with higher order, 4th and 6th, curve fittings that would take into account the

Figure 5 Effects of the variation in the calibration curve fit polynomial order. (a) Cross-sectional data at Y/D=0.25, (b) 2nd (2OP), (c) 4th (4OP), and (d) 6th (6OP) order polynomial

deviations from linearity, as shown in Fig. 5. **Polynomial curve fittings based on orders higher than the 6th-order showed gradual overall deteriorations in the prediction capabilities of the models.** Extrapolated data in Fig. 5(a) shows that the 2nd order curve fitting generally predicts higher maximum peaks at the compression regions with respect to the 4th and 6th order polynomials, with a minimal higher prediction in the expansion regions of the flow. With an increase in the order of the polynomial the estimation of the maximum compression peaks is gradually reduced, with the estimation of the density gradients in the expansion regions of the flow becoming coherent. The results shown in Fig. 5 indicate that the 6th order polynomial more accurately takes into consideration the deviation from linearity in the hue distribution with a maximum reconstruction error of approximately 6%, the closest value of the intercept of the polynomial to the zero refraction angle, the best value of R^2 .

Figure 6 Background profiles at NPR=5.0 jet flow condition. (a) Schlieren image, (b) background hue, (c) density gradient variation due to background hue value and (d) maximum averaged difference of the three density gradients

Another source of uncertainty in the current analysis is due to the non-homogeneity in the light distribution across the mirror, a characteristic particularly noticeable in the rainbow schlieren colour images obtained with a vertical light cut-off, where fringes are visible in the background colour variation, as shown in the example in Fig. 6(a) for the RVKE2 set up. To quantify the light distribution variations, data points were extracted across the background of the experimental images of the nozzle at NPR=5.0 jet flow condition and plotted as a function of pixel position in Fig. 6(b). Three background hue values (i.e. maximum, minimum and average) were analysed. Three calibration curves were obtained and three corresponding sets of density gradients were determined, as shown in Figs. 6(c). Closer inspections of the density gradients in Fig. 6(d) reveal that the maximum variation between the

averaged and non-averaged background hue data is $\frac{\partial \rho}{\partial y} \sim 10$, or an error of less than 2% with respect to the density gradient range allowable by the lens. Background analysis of the jet at NPR=3.4 led to similar conclusions, as the setup remained unchanged. As such, these results indicate that averaging of the background hue values is an acceptable approach to minimizing background light inhomogeneity.

2.5 Numerical simulation

The present RANS simulations have been performed using the compressible solver in OpenFOAM with fully structured mesh of approximately 24million cells in total and shear stress transport $k-\omega$ turbulence model. Mesh refinement was applied along the jet core and shear layers, such that the estimated y^+ at the nozzle exit based on the exit velocity was approximately 3. A detailed assessment of the present RANS simulations has been reported in a recent study by Zang et al. (2018), where both mesh independence check and validation with available experimental measurements were carried out.

3 Results and Discussion

3.1 Abel Transform Algorithm Validation

Prior to conducting a full performance comparison between calibrated and rainbow schlieren, the input parameters for the Abel Transform algorithm were tested and optimised. In particular, attention was given to the definition of the upper frequency value, as it is known that a low value of Upper Frequency (UPF) reduces noise but may also potentially filter out important flow features (Pretzler, 1991, Killer, 2013). Conversely, while a high UPF value will map the flow features more accurately, it may introduce a significant amount of noise and hence, variations in the results. A full comparison is shown in Fig. 7 in terms of data and contour plot. Extrapolated data at $Y/D=0.25$ in Fig. 7(a) shows a rapid improvement in capturing the peaks associated with the presence of the shock waves as the

Figure 7 Effects of the variation in upper frequency value for data calculation in Abel's Transform algorithm. (a) Cross-sectional data at $Y/D=0.25$, (b) relative differences, (c) pictorial representation at UPF 6 and (d) UPF 20

value of UPF is increased. This is a consequence of the increase in the number of the Fourier-based radial interpolation, particularly up to a UPF value of 12. Beyond a UPF value of 12, little or no improvement is detected with the penalty of a longer data-processing time. This trend is confirmed in Fig. 7(b) in terms of relative differences between the results as the UPF value is varied. Note that the magnitudes of the relative differences past UPF 12 to UPF 14 values practically collapse on top of one another. These results indicate that a UPF value of 12 is suitable to be used for the analysis of both calibrated and rainbow schlieren data. A pictorial representation of the data is presented in Figs. 7(c) and 7(d) through the density gradient contours at UPF=6 and 20 (the two evaluated extremes), showing a more accurate data representation at higher UPF values.

Figure 8 Vertical knife edge cut-off jet geometric validations at NPR=5.0 for (a) calibrated and (b) rainbow schlieren

Figure 9 Horizontal knife edge cut-off jet geometric validations at NPR=5.0 for (a) calibrated and (b) rainbow schlieren

Once the UPF value for the analysis was identified with respect to density gradient relative variation, further validation was completed to ensure that the algorithm would correctly capture the structure of the perfectly and under-expanded jets by comparing the contour plots with raw schlieren images. Examples taken at NPR=5.0 are shown in Figs. 8 and 9 for vertical and horizontal light cut-off respectively. These figures demonstrate that the Abel transform algorithm used and UPF=12 value is capable of representing the geometric properties of supersonic jets satisfactory. They also show consistently good qualitative repeatability in the data between calibrated and rainbow schlieren.

Figure 10 (a) Perfectly- and (b) under-expanded jets with a horizontal light cut-off

3.2 Horizontal knife edge

Similar to qualitative schlieren, the choice of the light cut-off (or knife edge orientation) in quantitative schlieren is dependent upon the flow features of interest. In a horizontal light cut-off, the flow features highlighted are primarily along the longitudinal axis. In the case of a supersonic jet, they primarily highlight the jet shear layers for both the perfectly and under-expanded configuration shown in Figs. 10(a) and 10(b), as well as some oblique structures in the under-expanded jet shown in Fig. 10(b). The major drawback of the horizontal light cut-off lies in its inability to distinguish between oblique shock waves and expansion fans, as can be observed in Fig. 10(b) as well. As mentioned earlier, the geometric properties of the perfectly and under-expanded jets are captured correctly by both experimental techniques. Where calibrated and rainbow schlieren differ is in the prediction of the density gradient magnitudes across the jet. As shown graphically in Figs. 11 and 12, calibrated schlieren tends to predict higher values of density gradient as compared to rainbow schlieren, especially across the shear layer regardless of the diaphragm aperture.

To further quantify the variations between the two techniques, cross-sectional data was extracted at $X/D=1$ and 2 locations, as shown in Figs. 13 and 14. Results of the perfectly-expanded jet in Fig. 13 confirm the higher $\partial\rho/\partial y$ magnitudes estimated by the calibrated schlieren as compared to rainbow schlieren method at $X/D=1$ location. However, the shear layer thickness and average density gradient

Figure 11 (a) Calibrated and (b) rainbow schlieren density gradients at NPR=3.4 jet flow conditions

Figure 12 (a) Calibrated and (b) rainbow schlieren density gradients at NPR=5.0 jet flow conditions

Figure 13 Density gradients data comparison at NPR=3.4 jet flow conditions

Figure 14 Density gradients data comparison at NPR=5.0 jet flow conditions

magnitudes across the jet are comparable. Along cross-sections taken farther downstream, and in correspondence to the transition and dissipation of the shear layer, the predictions of the density gradients by the two set-ups are more consistent. Figure 13 also shows that RHKE1 and RHKE2 perform similarly in the prediction of the density gradient at the shear layer, where a maximum difference of $\partial\rho/\partial y \sim 15$ exists at $X/D=2$ location at the peak corresponding to the shear layer. Hence, this indicates an acceptable repeatability between the two set-ups.

Results for the under-expanded jet, as shown in Fig. 14, follow similar trends observed for the perfectly-expanded jet. For instance, calibrated schlieren predicting higher density gradients across the shear layer by approximately 90kg/m^4 in comparison to those predicted by rainbow schlieren, with the latter producing density gradient estimates consistently independent of the diaphragm opening. Interestingly, variation in the nozzle pressure ratio does not appear to affect the absolute difference in density gradient predictions between calibrated and rainbow schlieren. Taking into account the errors introduced by the calibration curves and the background intensity/hue variations, as well as the fact that the shear layer in the calibrated schlieren may be contaminated by saturated light, results nonetheless indicate that differences in peak values are within the experimental uncertainties. No

Figure 15 (a) Perfectly-expanded and (b) under-expanded jets with a vertical light cut-off

direct comparison has been conducted against numerical results as the RANS $k-\omega SST$ model used for the simulations is known for its difficulties in predicting shear layer flow accurately.

3.3 Vertical knife edge

In addition to the results obtained with the horizontal light cut-off, experiments were conducted with the vertical light cut-off to highlight flow variations across shock waves which the horizontal light cut-off was unable to differentiate. In the case of a perfectly-expanded jet with no shock waves, the vertical light cut-off appears to be ineffective and produces an image (see Fig. 15(a)) where no distinct flow is visible. Conversely, a vertical light cut-off is very effective in visualizing the density gradients variations across shock waves and expansions for an under-expanded jet (see Fig. 15(b)). As little quantitative information can be extracted for the perfectly-expanded jet using a vertical light cut-off, subsequent quantitative analysis focuses solely on the under-expanded jet. As shown in Fig. 16, calibrated schlieren predicts stronger density gradient variations both in the compression and expansion regions than rainbow schlieren, which has already been observed in the results for the horizontal light cut-off. Figure 16(b) also shows a significant variation in the density gradient magnitudes as a function of the diaphragm aperture, with RVKE2 showing stronger variations as compared to RVKE1. These results indicate that the performance of rainbow schlieren is significantly affected by the aperture of the diaphragm at the light source when vertical light cut-off is used, at

Figure 16 (a) Calibrated and (b) rainbow schlieren density gradients at NPR=5.0 jet flow conditions

Figure 17 Density gradient result comparisons at NPR=5.0 jet flow conditions

least for the current schlieren set-up. To quantify the variations between techniques and set-ups, data was extracted along $Y/D=0.25$ location and compared with numerical results in Fig. 17. Firstly, Fig. 17(b) shows considerable differences between RVKE1 and RVKE2 configurations, where there exist significant differences between the two set-ups at peaks 1 and 2. In addition, Fig. 17 also demonstrates that RVKE1 configuration under-predicts the density gradients significantly, as compared to CVKE configuration and numerical results. Therefore, the former configuration should not be considered for the analysis. These issues can also be noticed in Fig. 16(b) earlier, where the second shock cell is only partially mapped. Results in Fig. 17 indicate that both CVKE and RVKE2 configurations predict compression/expansion peak locations that agree well with numerical results,

while calibrated schlieren predicts the maximum values at the compression peaks 1 and 2 more accurately. For peak 1 located at $X/D=1.47$ where it corresponds to the reflected shock of the first cell, there is only a 2% difference between calibrated schlieren and numerical results. In contrast, an approximately 40% difference remains present between RVKE2 rainbow schlieren and the numerical results. For peak 2 located at $X/D=3.20$, calibrated schlieren and numerical results again agree well with each other with a difference in the peak magnitude in the order of 10%, with RVKE2 configuration results showing differences greater than 20%. At the same time, calibrated schlieren results appears to be less sharp around peaks 1 and 2, which lead to large discrepancies from the numerical results before and after the density gradient peaks. With respect to the expansion regions, both calibrated and rainbow schlieren can be observed to predict the more gradual density variations well. This can be seen for expansion regions A and B in Fig. 17(b), where rainbow schlieren performing marginally better in that the difference between RVKE2 configuration and numerical results are more consistent (i.e. within the order of $\pm 10 \text{ kg/m}^4$). On the other hand, calibrated schlieren results show more significant deviations from the numerical results.

4 Conclusions

A study has been conducted on the use of calibrated and rainbow schlieren techniques to determine the density gradients of perfectly- and under-expanded axisymmetric jets. An initial evaluation of the effects due to the diaphragm aperture upon the colour images was conducted, with the objective of minimizing light inhomogeneity in the form of fringes. Results indicate that averaging the hue values across the background is an acceptable approach and introduces an experimental error of less than 2%. Experimental results also show that the present calibrated schlieren implementation can predict density gradient variations across shock waves as within a 2% range of the validated numerical results. Typical accuracy remains within 10% of the numerical results, with larger discrepancies at the peaks corresponding to the maximum density variations. In contrast, while rainbow schlieren can predict the trends in the density variations, it generally underperforms in capturing the sharp density

gradients across shock waves. Instead, it is better in predicting areas of gradual density variations than calibrated schlieren.

Current work also reveals challenges inherent to rainbow schlieren. While designing a filter with a theoretically ideal hue distribution is now easily achievable using image editing software, the experimental hue distribution may not be ideal due to light distortion. Furthermore, presence of the filter decreases the sharpness and sensitivity of the system, potentially increasing light distortions within the schlieren system. Such technical issues do not afflict the calibrated schlieren system. It should also be noted that the filter used in the study – a photographic slide rather than a glass or gelatine filter – introduces additional errors in the light diffraction and may potentially act as a weak light diffuser. Lastly, detailed comparisons show that there are no significant advantages in using rainbow schlieren with a banded filter over calibrated schlieren, and that calibrated schlieren may be a more robust, sensitive, and easier to set up technique that provides more accurate data.

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