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Receding Horizon Control of 3 DOF Helicopter Using Online Estimation of Aerodynamic Parameters

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Abstract

This study presents a numerical implementation of fast nonlinear model predictive control (NMPC) and nonlinear moving horizon estimation (NMHE) for the trajectory tracking problem of 3 degree of freedom (DOF) helicopter. The motivation behind using the NMPC instead of its linear counterpart is that the helicopter is operated over nonlinear regions. Moreover, this system has cross-couplings that make the control of the system even more complicated. What is more, according to our simulation scenario, the system has a time-varying dynamical model because of having time-varying parameters which are estimated online using NMHE and extended Kalman filter (EKF) throughout the control. Although NMHE is computationally more demanding, its capability of incorporating the constraints encourages us to utilize NMHE rather than EKF. Two reference trajectories, namely, sinusoidal and square-like, are tracked, and owing to the better learning capability of NMHE over EKF, the NMPC-NMHE closed-loop control framework is able to track both reference signals with more accuracy than NMPC-EKF control framework, even under parameter uncertainties. Thanks to ACADO toolkit, the combined average execution time is 4 milliseconds which demonstrates the potential of the proposed framework for real-time aerospace applications using relatively cheaper processors.

Keywords

Unmanned Aerial Vehicles, Quanser 3 DOF Helicopter, Nonlinear Model Predictive Control, Nonlinear Moving Horizon Estimation, Extended Kalman Filter.

1. Introduction

Being unmanned and controlled remotely from thousands of miles away, and simultaneously, placing no risk on human lives even in hostile situations, unmanned aerial vehicles (UAVs) have become popular in both military and civilian applications. They have been used in performing a wide variety of tasks including surveillance (1), search and rescue at disaster sites (2) or damage evaluation after an earthquake (3), etc. In general, UAVs boil down to two categories: fixed wing and rotary

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wing. For the civilian applications in particular, rotary wing UAVs are preferred on account of their vertical take-off and landing, and hover capabilities. Such capabilities make rotary-wing UAVs more versatile aerial vehicles when compared to their fixed wing counterparts, for short distance transportation (4).

Helicopter UAV is amongst extensively utilized rotary-wing UAVs and has always received noticeable attention within the control community. Its control is considerably challenging because of its highly nonlinear dynamics with significant cross-couplings as well as the presence of inherent instabilities (5; 6). Conventional approaches such as proportional-integral-derivative (PID) controller, for the control of a helicopter, involve the linearization of the nonlinear dynamics about a set of aforethought equilibrium points, within the application domain. In addition, such a decentralized approach (PID for each axis) behaves in a selfish manner which may eventually result in a substandard performance. Other linearized approaches, such as linear-quadratic-regulator (LQR), is able to handle multi-input-multi-output (MIMO) systems in a structured way (7). However, the unmodelled uncertainties, which may arise due to changing environmental conditions, may substantially degrade the performance of these linearized controllers and induce oscillations leading even to instability (8; 9). Moreover, by conservative tuning of these linearized controllers, robustness could be achieved but the price to pay is the performance (10).

Model predictive control (MPC) finds its roots in the control of industrial process control systems including chemical and petroleum industries (11; 12; 13). The reason being is their slow dynamics which gives the controller ample time for computing optimized solutions underlying MPC scheme. Nevertheless, the technological advancements in modern computation power have opened the doors of MPC applications, even for aerospace systems with very small time constants, i.e. of the order of 10 ms. In addition, researchers from all over the world are utilising MPC including, linear MPC (14; 15) and explicit MPC (16; 17; 18), for the control of helicopter systems. Recently, there have also been advances in MPC applications to more complex aerospace systems such as quadcopters (19; 20; 21; 22) and tilt-rotor tricopters UAVs (23; 24). Apart from linear MPC, nonlinear model predictive control (NMPC) is also gaining popularity on account of its ability to obtain an optimal solution, in contrasts to a suboptimal solution that is obtained in the case of linear MPC. NMPC, although previously considered computationally very demanding has been applied to numerous UAVs including, quadcopters with a heavy slung load in (25; 26), a small unmanned helicopter for collision-free guidance control in (27) and a more complicated planar vertical take-off and landing (PVTOL) aircraft in (28). Apart from trajectory tracking, NMPC has also been incorporated in path planning applications of UAVs (29). In all the aforementioned applications, the most substantial advantages of MPC are: (i) to be able to handle MIMO systems, and (ii) to be able to employ system constraints, in a systematic way.

The core of MPC scheme is basically to find a solution online to an optimal control problem (OCP) and implement it in a receding horizon manner. Without loss of generality, it requires the knowledge of full state vector at each sampling instant. However, practically it is often infeasible to measure all the states, and hence, the state estimators such as MHE and EKF are employed to estimate the unmeasured states and unknown parameters (if any). The EKF is based on the linearization of the nonlinear system at the current estimate and is suitable only for unconstrained problems, involving disturbances of Gaussian nature (30). On the contrary, MHE strategy exploits the past measurements available over a window and solve an optimization problem in order to estimate the system's states and unknown parameters (31; 32). In addition, MHE is a powerful nonlinear estimator that is not only suitable for non-Gaussian disturbance, but also is competent in handling constraints, explicitly. The recent applications of MHE on aerospace systems are (33; 34; 35) and of NMHE on other autonomous systems are (36; 37).

The contributions of the proposed study are:

- Trajectory tracking for a 3 DOF helicopter, incorporating two closed-loop control frameworks, namely, NMPC-EKF and NMPC-NMHE.

- Investigation of the learning abilities of EKF and NMHE in estimating uncertainties in the dynamical model. To facilitate this, we explicitly varied one of the system's aerodynamic parameters with time and estimated it online with both EKF and NMHE.
- Performance evaluation of NMPC-EKF and NMPC-NMHE frameworks for high-speed aggressive flight manoeuvres, where the system explicitly operates in a nonlinear region.

This paper is organized as follows: Section 2 starts with the description and dynamical model of the 3 DOF helicopter. In Section 3, the definition and problem formulation of NMPC are presented. In Section 4, the description and formulation of NMHE are provided. The NMPC-NMHE solution methods are presented in Section 5. The closed-loop problem formulation and the numerical results are summarized in Section 6. Lastly, in Section 7, some conclusions are drawn from this study.

2. 3 DOF Helicopter Model

The numerical investigation setup, which is the model of a helicopter plant manufactured by Quanser Consulting Inc., is used to test the proposed control algorithm. The system has 3 DOF: the elevation ϵ , pitch θ and travel λ which are shown in Fig. 1. This system, by virtue of its highly nonlinear dynamics and intercouplings, has been widely used as a benchmark system in order to test nonlinear control algorithms (14; 18; 38; 39).

The helicopter setup consists of a base, a long arm attached to a pendulum and two DC motors vertically installed at each end of a rectangular frame with their thrust vector normal to it. The rectangular frame is suspended from an instrumented joint mounted at one end of the long arm and is free to pitch about its centre. On other end of the arm, a counterweight is fixed in order to make the effective weight lighter. Three encoders are used to measure the elevation, pitch and travel angles of the helicopter setup, whereas two voltage amplifiers provide front-motor voltage V_f and back-motor voltage V_b , given as the control input to the system. A positive voltage applied to the both motors causes an elevation of the body, while a positive voltage applied to either front or back motor causes a negative or positive pitch, respectively. Furthermore, if the body pitches, the thrust vectors also result in a travel of the body (i.e., yaw of the arm). Various symbols used for mathematical modelling and their numerical values are given in Table 1. The movement range of the elevation and pitch angle is restricted within $[-1, +1]$ rad on account of the hardware constraints (39).

Using Euler-Lagrange formulation, the complete nonlinear and coupled dynamics of the helicopter system can be described by elevation, pitch and travel differential equations, similar to (39), as

Fig. 1. Free-body diagram of 3 DOF helicopter.

Table 1. System parameters and their numerical values (40)

Symbol	Description	Value
K_f	Propeller force-thrust constant	0.1188 N/V
m_h	Mass of helicopter	1.15 kg
m_w	Mass of counterweight	1.87 kg
m_f	Mass of front-propeller assembly	0.5750 kg
m_b	Mass of back-propeller assembly	0.5750 kg
L_a	Distance between travel axis and helicopter body	0.6604 m
L_h	Distance between pitch axis and each motor	0.1778 m
L_w	Distance between travel axis and counterweight	0.4699 m
L_d	Length of pendulum for elevation axis	0.05 m
L_e	Length of pendulum for pitch axis	0.02 m
J_ϵ	Moment of Inertia about elevation axis	0.86 kg m ²
J_θ	Moment of Inertia about pitch axis	0.044 kg m ²
J_λ	Moment of Inertia about travel axis	0.82 kg m ²
g	Gravitational acceleration	9.81 m/s ²

$$J_\epsilon \ddot{\epsilon} = [-(m_f + m_b)gL_a + m_wgL_w]\cos(\epsilon) - (m_f + m_b)gL_a \tan(\delta_a) + m_wgL_w \tan(\delta_w) + K_f L_a \cos(\theta)(V_f + V_b), \quad (1)$$

$$J_\theta \ddot{\theta} = K_f L_h (V_f - V_b), \quad (2)$$

$$J_\lambda \ddot{\lambda} = K_f L_a \sin(\theta)(V_f + V_b), \quad (3)$$

where,

$$\delta_a = \tan^{-1}\left(\frac{L_d + L_e}{L_a}\right); \quad \delta_w = \tan^{-1}\left(\frac{L_d}{L_w}\right).$$

In order to be more realistic for the numerical study, effects of unmodelled uncertainties in the form of process noise (w_k), and measurement errors in the form of measurement noise (ν_k), are included in the helicopter model. Finally, the considered nonlinear model in a discretized form can be represented as

$$x_{k+1} = f(x_k, u_k, p) + w_k, \quad (4)$$

$$y_k = x_k + \nu_k, \quad (5)$$

where the state vector is $x = [\epsilon, \theta, \lambda, \dot{\epsilon}, \dot{\theta}, \dot{\lambda}]^T$, the input vector is $u = [V_f, V_b]^T$ and the output vector is $y = [\epsilon, \theta, \lambda, \dot{\epsilon}]^T$; p denotes a constant parameter (can be time-varying); and the process and measurement noises are defined by the covariance matrices Q and R , respectively

$$w \approx (0, Q); \quad \nu \approx (0, R); \quad \mathbb{E}[w\nu^T] = 0. \quad (6)$$

3. Nonlinear Model Predictive Control

NMPC is an advanced optimization based control strategy that solely relies on the accuracy of the mathematical model for its optimum performance. In NMPC strategy, the optimization problem is formulated in the form of a least square (LSQ) function, in order to penalize the deviation of predicted system's trajectory (including states and control inputs) from the specified reference. To keep the computational burden realizable for a real-time application (especially for fast dynamical systems like UAVs), the optimization problem is solved over a finite window, commonly known as *prediction horizon* (N_c).

In NMPC, the dynamic optimization problem is recursively solved for the control inputs (u_k), over the prediction horizon ($t_j \leq t \leq t_{j+N_c}$) at each sampling instant (t_j). The optimization problem formulation is of the form (13):

$$\min_{x_k, u_k} \frac{1}{2} \left\{ \sum_{k=j}^{j+N_c-1} \left\| \tilde{h}(x_k, u_k) - \tilde{y}_k \right\|_W^2 + \left\| \tilde{h}_{N_c}(x_{N_c}) - \tilde{y}_{N_c} \right\|_{W_{N_c}}^2 \right\} \quad (7a)$$

$$\text{s.t. } x_j = \hat{x}_j, \quad (7b)$$

$$x_{k+1} = f(x_k, u_k, p), \quad k = j, \dots, j + N_c - 1 \quad (7c)$$

$$x_{k,\min} \leq x_k \leq x_{k,\max}, \quad k = j, \dots, j + N_c \quad (7d)$$

$$u_{k,\min} \leq u_k \leq u_{k,\max}, \quad k = j, \dots, j + N_c - 1, \quad (7e)$$

where $x_k \in \mathbb{R}^{n_x}$ is the differential state, $u_k \in \mathbb{R}^{n_u}$ is the control input and $\hat{x}_j \in \mathbb{R}^{n_x}$ is the current state estimate; time-varying references are denoted by $\tilde{y}_k \in \mathbb{R}^{(n_x+n_u)}$ and $\tilde{y}_{N_c} \in \mathbb{R}^{n_x}$; the discrete time dynamical model is represented by $f(\cdot)$; $\tilde{h}(\cdot)$ and $\tilde{h}_{N_c}(\cdot)$ are the measurement models with full-state (output of EKF/NMHE) and control inputs feedback, defined as, $\tilde{h}(\cdot) = [\epsilon, \theta, \lambda, \dot{\epsilon}, \dot{\theta}, \dot{\lambda}, V_f, V_b]$ and $\tilde{h}_{N_c}(\cdot) = [\epsilon, \theta, \lambda, \dot{\epsilon}, \dot{\theta}, \dot{\lambda}]$; the weight matrices are given by $W \in \mathbb{R}^{(n_x+n_u) \times (n_x+n_u)}$ and $W_{N_c} \in \mathbb{R}^{n_x \times n_x}$. Furthermore, $x_{k,\min} \leq x_{k,\max} \in \mathbb{R}^{n_x}$ and $u_{k,\min} \leq u_{k,\max} \in \mathbb{R}^{n_u}$ specify the lower and upper bounds on the states and control inputs, respectively.

Once the solution to the optimal control problem (7) at t_j is available, the first computed control input (u_j) is applied to the system for a short time period, that typically coincides with the sampling time (41). Thereafter, a new optimization problem is solved for the prediction window $[t_{j+1}, t_{j+N_c+1}]$, which itself is moving forward with time. Due to this shifting property of the prediction window, the NMPC is also known as *receding horizon* control technique.

The last expression in (7a) represents the final cost incurred due to control variables at the end of the prediction horizon (t_{j+N_c}) and is generally referred as the *terminal penalty* term. This term is often included in the problem formulation for stability reasons (41). One may note that due to space constraints, the stability and optimality proofs of NMPC is not explicitly included in this paper, however, the interested readers are referred to (42; 43; 44).

The selected 3 DOF helicopter setup has few restrictions imposed on its working by the DC motors. They include the maximum applied voltages V_f and V_b (both motors), which if exceeded may result in the burning of their armature coil. Therefore, the enforced constraints to the control inputs (in volts) are:

$$-15 \leq V_f \leq 15, \quad (8)$$

$$-15 \leq V_b \leq 15. \quad (9)$$

In addition, following weight matrices are selected:

$$W = \text{diag}[2000, 15, 1000, 10^{-2}, 10^{-2}, 10^{-2}, 10^{-2}, 10^{-2}], \quad (10)$$

$$W_{N_c} = \text{diag}[8000, 1, 1100, 10^{-1}, 10^{-1}, 10^{-1}], \quad (11)$$

by trial-and-error, such that, a less aggressive controller is obtained and the above constraints are obeyed. Moreover, it is observed that if the weight matrices are reduced, the controller is not able to track the reference well, while if the weights are increased, the controller is becoming very aggressive and is resulting in unacceptable overshoots, particularly in the case of square-like reference tracking. Furthermore, the prediction horizon length N_c of 70 is chosen, to account for the real-time applicability of the controller, particularly for UAVs.

Remark: In 3 DOF Quanser helicopter model, either travel or pitch response can be individually tracked. In our study, we prefer tracking the former, however, some non-zero weights are still given to pitch in (10) and (11), because in this setup, travel and pitch are coupled and hence, the travel cannot be independently controlled.

4. Nonlinear Moving Horizon Estimation

In general, NMHE is considered as a *dual problem* of NMPC (13; 45). Therefore, in a similar manner, the NMHE scheme is also formulated using a least square function to penalize the deviation of estimated outputs ($h(\cdot)$) from measurements (y). The performance of NMHE also relies on the availability of an accurate state model, while a mismatch in the form of process noise between the state model and the real plant may deteriorate the optimal control solution that eventually may lead to an unstable closed-loop. To address this issue, a suitable component (*arrival cost*) is included in the final optimization problem formulation of NMHE, as done in (45).

The NMHE formulation includes a horizon (also known as *estimation horizon*) containing M measurements $y_L, \dots, y_j$, taken at time $t_L < \dots < t_j$, where the length of the horizon is given by $T_E = t_j - t_L$, and $j - M + 1 \stackrel{\text{def}}{=} L$ is taken for notational convenience. Finally, the dynamic optimization problem to estimate the constrained states ($\hat{x}$) as well as the unknown parameter ($\hat{p}$) at time t_j using the process model $f(\cdot)$, measurement model $h(\cdot)$ and available measurements $y_L, \dots, y_j$, within the horizon, is of the form (45):

$$\min_{\hat{x}_k, \hat{p}} \left\{ \left\| \begin{bmatrix} \hat{x}_L - \bar{x}_L \\ \hat{p} - \bar{p}_L \end{bmatrix} \right\|_{P_L}^2 + \sum_{k=L}^j \|y_k - h(\hat{x}_k, u_k)\|_V^2 + \sum_{k=L}^{j-1} \|w_k\|_W^2 \right\} \quad (12a)$$

$$\text{s.t. } \hat{x}_{k+1} = f(\hat{x}_k, u_k, p) + w_k, \quad k = L, \dots, j-1 \quad (12b)$$

$$\hat{x}_{k,\min} \leq \hat{x}_k \leq \hat{x}_{k,\max}, \quad k = L, \dots, j \quad (12c)$$

$$\hat{p}_{\min} \leq \hat{p} \leq \hat{p}_{\max}, \quad (12d)$$

where $h(\cdot)$ includes the measured plant states and control inputs, and is defined as, $h(\cdot) = [\epsilon, \theta, \lambda, \dot{\epsilon}, V_f, V_b]$; w_k represents the added process noise; $\hat{x}_{k,\min} \leq \hat{x}_{k,\max}$ and $\hat{p}_{\min} \leq \hat{p}_{\max}$ specify the lower and upper bounds on the estimated states and parameter, respectively; $\bar{x}_L$ and $\bar{p}_L$ denote the estimated state and parameter values (arrival cost data, defined later) at the start of estimation horizon, i.e. at t_L (45). The weight matrices P_L , V and W , are interpreted as the inverse of the covariance matrices and are evaluated as:

$$P_L = Q_0^{-\frac{1}{2}} = \begin{bmatrix} Q_0^x & 0 \\ 0 & Q_0^p \end{bmatrix}^{-\frac{1}{2}}; \quad V = R^{-\frac{1}{2}}; \quad W = Q^{-\frac{1}{2}} = \begin{bmatrix} Q^x & 0 \\ 0 & Q^p \end{bmatrix}^{-\frac{1}{2}}, \quad (13)$$

where Q_0 is the initial covariance matrix (incorporating state and parameter, both); R is the measurement noise covariance matrix; and Q is the process noise covariance matrix. With the above choice of weight matrices, it is assured that the NMHE scheme results in a maximum-likelihood estimate for the very likely trajectories (13).

Even though the new measurements continuously appear with time, the size of the estimation window (M) remains bounded, i.e., the number of measurements utilized in estimation by solving the optimization problem (12), is kept constant (41). As new measurement becomes available, the measurement, which is present at the beginning of the estimation window (y_L), is dropped. In this way, the estimation window moves forward in time, which gives this technique the name *moving horizon* estimation.

The first term in (12a) is analogous to the terminal penalty term of NMPC and is generally referred as the arrival cost. This term is incorporated into the objective function in order to accommodate the effect of past measurements (before the beginning of estimation horizon), in the current state and parameter estimate. According to (45), the NMHE's performance as a full-information estimator and its stability are closely related to a good choice of the arrival cost.

Another parameter that affects the performance of NMHE is the choice of estimation window length M , which in general is problem-specific. It basically represents a trade-off between computational liability and estimation accuracy that simultaneously grow with M . In the case of small but fast dynamical systems like UAVs, we cannot indefinitely increase M as a limited computational power is available on-board.

Moreover, it is not necessarily true that the estimation accuracy always increases with M , as the plant model mismatches degrade the significance of model prediction which adversely effect the estimation performance (41). That is, selecting too high value of M for a system in which the unknown parameter (to be estimated) is radically changing, plant model anomalies may arise that eventually may result in deteriorated overall estimation quality.

For the selected helicopter model, the weight matrices P_L , V and W , are chosen to be:

$$P_L = \text{diag}[0.0032, 0.0032, 0.0032, 0.0032, 0.0032, 0.0032, 0.01], \quad (14)$$

$$V = \text{diag}[100, 141.4214, 70.7107, 100, 10, 10], \quad (15)$$

$$W = \text{diag}[22.3607, 14.1421, 15.8114, 15.8114, 31.6228, 11.1803, 15.8114]. \quad (16)$$

The above values of the weight matrices are decided based on the definition in (13) and exploiting the availability of the true values of the noise covariance matrices (specified later in (20) and (21)). Furthermore, the estimation window length M is selected to be equal to 70, in order to achieve decent computational burden and hence, facilitate real-time applicability of this framework.

Remark: The formulation and theory of EKF is not explicitly mentioned here as it is considered to be well-known in the literature. For its theory and working algorithm, the interested reader may refer to (46). Moreover, it is initialized with the following values of initial state covariance matrix P_0 , process noise covariance matrix Q and measurement noise covariance matrix R :

$$P_0 = \text{diag}[10^{-5}, 10^{-5}, 10^{-5}, 10^{-5}, 10^{-5}, 10^{-5}, 10^{-4}], \quad (17)$$

$$Q = \text{diag}[0.002, 0.005, 0.004, 0.004, 0.001, 0.008, 0.004], \quad (18)$$

$$R = \text{diag}[10^{-4}, 5 \times 10^{-5}, 2 \times 10^{-4}, 10^{-4}, 10^{-2}, 10^{-2}]. \quad (19)$$

It may be worth noting that the above values of Q and R matrices are selected based on trial-and-error, such that, a less noisy estimation is obtained from EKF.

5. NMPC-NMHE Solution Methods

Owing to the similarity between the optimization problems of NMPC and NMHE, same solution methods can be applied to both at each measurement instant. Amongst various available solution methods, we incorporate the direct multiple shooting method to discretize the continuous time OCP into a sequential quadratic program (SQP), which is further solved utilizing the generalized Gauss-Newton (GGN) method. Moreover, this GGN method is a tailored version of classical Newton method, modified for least-square problems and with an added advantage of not requiring explicit calculations of second order derivatives. However, being iterative in nature, it requires moderate number of iterations which cannot be known beforehand, that apparently limits its real-time feasibility (41). To overcome this, (47) proposed a *real-time iteration* (RTI) scheme, in which, the maximum number of Gauss-Newton iterations are constrained to 1, while each new optimization problem is intelligently initialized based upon the outputs of the previous one (41). As a result, the overall convergence property of this method is significantly improved in comparison to the classical method.

Finally, the optimization problems in (7) and (12) are solved using direct multiple shooting method and RTI approach incorporated in ACADO (Automatic Control and Dynamic Optimization) toolkit, which is an open-source software environment written in C++ for automatic control and dynamic optimization applications (48). It provides an exhaustive code library that links to the qpOASES (49) solver that applies an active set strategy in order to solve sequential, quadratic sub-problems (25). In addition, it has a code generation package which creates tailored (well structured to avoid redundant computations), self-contained C-codes based on a user specified symbolic NMPC or NMHE problem formulation.

Moreover, it is capable of running on low computational power embedded systems and can also be interfaced with other software platforms including MATLAB/SIMULINK (50). For the detailed information regarding its algorithm and built-in function, the interested user is referred to (48).

6. Numerical Results

In the numerical study, we implement NMPC-EKF and NMPC-NMHE control frameworks as displayed in Fig. 2, for closed-loop control, and online state and parameter estimation of a Quanser 3 DOF helicopter system model which is in the form of (4) and (5). All the simulations are implemented using MATLAB/SIMULINK installed on a hardware consisting of 3.4-GHz Intel Core i7 processor and 8 GB RAM.

Consider the true initial state of the helicopter $x(0) = [0, 0, 0, 0, 0, 0, 0, 0.1188]^T$ and let the simulation time step $\Delta t = 0.02$ s. The covariance matrices for added process and measurement noise are given by:

$$Q = \text{diag}[0.002, 0.005, 0.004, 0.004, 0.001, 0.008, 0.0008], \quad (20)$$

$$R = \text{diag}[10^{-4}, 5 \times 10^{-5}, 2 \times 10^{-4}, 10^{-4}, 10^{-2}, 10^{-2}]. \quad (21)$$

The above values of covariance matrices (Q and R) are obtained by making realistic estimation of state and measurement noise in the first place and thereafter, modifying these estimated matrices in order to obtain satisfactory estimation (and control) performance. The state estimators EKF and NMHE are initialized as $\hat{x}_0 = [0, 0, 0, 0, 0, 0, 0, 0.1]^T$. It may be noted that for simultaneous state and parameter estimation, the constant parameter K_f , is taken to be the last state in the state vector, i.e., $\hat{x} = [\epsilon, \theta, \lambda, \dot{\epsilon}, \dot{\theta}, \dot{\lambda}, \dot{K}_f]^T$. In addition, for the discretization of nonlinear helicopter model using multiple shooting method, a fixed integration process consisting of $2N_c$ steps in case of NMPC (and $2M$ steps for NMHE), is performed based on Runge-Kutta 4th order method (implemented in ACADO).

In the subsequent part of this numerical study, we employ sinusoidal and square-like reference trajectories in order to individually examine: (i) the ability of NMPC in handling highly nonlinear and coupled dynamics of the helicopter system model, and (ii) the capability of EKF and NMHE to provide full-state information (including unknown parameter, K_f) to NMPC, based on noisy measurements. The unknown parameter is considered to be time-varying such that, its true value changes between $[0.08, 0.16]$ in a sinusoidal and square-like manner for the respective cases. Additionally, a constrained boundary is set for K_f between $[0.07, 0.17]$, which represents $\pm 40\%$ variation from its mean value. It is important to emphasize here that both the trajectories are selected so that the operation regime explicitly lies outside the linear region.

Fig. 2. The closed-loop framework.

Fig. 3. The attitude tracking response of the system for sinusoidal reference: elevation angle (ϵ), pitch angle (θ), travel angle (λ), respectively.

Furthermore, mean-square error (MSE) is used as a measure of the tracking accuracy, and is defined by

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N e_i^2, \quad (22)$$

where e_i is the tracking error for the i^{th} sample, and N represents the total number of samples.

Remark: Although we are not explicitly providing any reference for the pitch, its results are still presented for completeness.

6.1. Sinusoidal reference trajectory

A sinusoidal reference trajectory of frequency 0.05 Hz is given in both, elevation and travel. Additionally, the parameter K_f is chosen to be varying sinusoidally with frequency 0.05 Hz. Based on the specified constraints in (8) and (9), and weight matrices in (10) and (11), the tracking performance of NMPC in both the control frameworks is shown in Fig. 3 and its control inputs along with constraints are given in Fig. 4. From Figs. 3 and 4, it can be seen that elevation angle and travel angle by NMPC-EKF and NMPC-NMHE, both are well tracked. This is due to the selected weight matrices which result in smooth and robust closed-loop control. Moreover, NMPC-NMHE control framework results in a less oscillating pitch response (not explicitly controlled but affects travel performance) over NMPC-EKF. This observation can be explained based on the more noisy estimation results of EKF, which eventually gets the controller oscillating. In addition, the MSEs in tracking of elevation and travel angles by NMPC-EKF and NMPC-NMHE for the sinusoidal reference are listed in Table 2(a).

The results of EKF and NMHE in estimating the attitude angles from noisy measurements are shown in Fig. 5, whereas the results of parameter estimation are provided in Fig. 6. As evident from Figs. 5 and 6, NMHE shows a good performance over EKF in both state and parameter estimation. Particularly in the case of pitch angle estimation, it is visible that EKF thoroughly follows the noisy measurements without any rejection effect. On the other hand, NMHE is illustrated to show more noise filtering and hence, results in a smoother estimation.

Furthermore, the unknown time-varying parameter is well assessed by NMHE, while complying with the specified constraints, but EKF results in an inferior estimation, always violating the constraints. This apparently results in a degraded

Fig. 4. The control inputs to the system for sinusoidal reference.

Fig. 5. The attitude angle estimation for the sinusoidal reference: elevation angle (ϵ), pitch angle (θ), travel angle (λ), respectively.

control performance by NMPC-EKF control framework, as also justified in Table 2(a) (specifically for travel). It may be worth noting here that the EKF's performance is very much dependent on the selected covariance matrices. That is, its parameter estimation accuracy may improve by further reducing the covariance matrices provided in (18) and (19). However, the lower covariance matrices result in more noisy estimates from EKF, which eventually introduces more oscillation in NMPC-EKF control framework. Hence, in the trade-off between better estimation or control performance, the latter is given more preference. Furthermore, MSEs for parameter estimation by EKF and NMHE for the sinusoidal reference trajectory are listed in Table 2(b).

Remark: In the interest of space, the estimation results of other three states, i.e., rotational rates are not explicitly presented in this paper. Moreover, nothing additional is deduced from those results.

Fig. 6. The parameter estimation response for the sinusoidal reference.

Fig. 7. The attitude tracking response of the system for square reference: elevation angle (ϵ), pitch angle (θ), travel angle (λ), respectively.

6.2. Square-like reference trajectory

Next, we present the NMPC-EKF and NMPC-NMHE tracking results for a square-like reference of frequency 0.03 Hz, provided along both, elevation and travel. Also, the parameter K_f is considered to be varying in a square-like manner with frequency 0.03 Hz. Incorporating the specified constraints and weight matrices, tracking performance of NMPC in both the frameworks for a square-like trajectory is shown in Fig. 7 and its control inputs are displayed in Fig. 8, along with the constraint boundary mentioned in (8) and (9). From Figs. 7 and 8, it can be seen that the response of NMPC-NMHE is more smooth as compared to the response of NMPC-EKF. This observation can also be explained based on the noisy estimation process of EKF, which on account of providing noisy feedback to the controller, makes it behave more oscillating. It may be worth noting that this noisy feedback may be catastrophic to controller's performance, as it induces oscillations and depending upon how we have tuned the controller, it may try to compensate the noise by some aggressive control and may eventually get unstable. Furthermore, the tracking MSEs of NMPC-EKF and NMPC-NMHE in following the square-like elevation and travel references are provided in Table 2(a).

Another point to note in Table 2(a) is that generally for ground systems, it is more challenging to follow a sinusoidal reference in comparison to a square reference, on account of the friction coming from ground. However, in aerial systems, this friction is absent which causes frequent overshoots and hence, makes it more strenuous for them to follow the step (or square) signals. Because of this reason, the MSEs in Table 2(a) are more for the square-like reference as compared to the sinusoidal reference trajectory.

The attitude angles estimation results of EKF and NMHE are shown in Fig. 9, while the parameter estimation response is displayed in Fig. 10. Figures 9 and 10, justify that: (i) the estimation accuracy of NMHE for a square-like reference is

Fig. 8. The control inputs to the system for square reference.

Fig. 9. The attitude angle estimation for the square reference: elevation angle (ϵ), pitch angle (θ), travel angle (λ), respectively.

better than EKF in a way that, besides the measurements are well followed by state estimates of NMHE, it also showed noise filtering effect, which at the end resulted in a smoother estimation, and (ii) the NMHE is more capable of learning a parameter whose true value changes rapidly (square-like) without violating any constraint, while, the EKF resulted in a sub-graded learning with frequent constraint violations. This observation is as anticipated because there is no explicit way of handling constraints in EKF, which in turn is one of the biggest strength of NMHE. Moreover, Table 2(b) lists the MSE value for parameter estimation by EKF and NMHE for the square-like reference.

Lastly, the results of execution time incurred by NMPC, EKF and NMHE in both the control frameworks are obtained. Overall average execution times for the considered references are listed in Table 3(a). In addition, to be sure about the real-time applicability, the maximum execution times for each are also obtained and are given in Table 3(b). It is worth emphasising here that in the optimization formulation of NMHE, the control constraints of (8) and (9) are not incorporated. This is done in order to bring down the execution of NMHE, as they are observed not to have any affect on its estimation performance. Moreover, the combined average and maximum execution times of NMPC-NMHE are approximately 4 ms

Fig. 10. The parameter estimation response for the square reference.

Table 2. Mean-square error (MSE)

(a) Trajectory tracking MSE				
	Sinusoidal reference		Square reference	
	NMPC-EKF	NMPC-NMHE	NMPC-EKF	NMPC-NMHE
ϵ ($^\circ$)	1.6435	3.8590	3.5640	5.8009
λ ($^\circ$)	26.3857	8.9891	27.9675	21.6469

(b) Parameter estimation MSE				
	Sinusoidal reference		Square reference	
	NMPC-EKF	NMPC-NMHE	NMPC-EKF	NMPC-NMHE
K_f	5.6969×10^{-4}	5.1906×10^{-5}	7.1252×10^{-4}	2.0034×10^{-4}

Table 3. Execution time for each run for the considered trajectories

(a) Average time (ms)			
NMPC-EKF		NMPC-NMHE	
NMPC	EKF	NMPC	NMHE
1.8	1.8	1.8	2.2

(b) Maximum time (ms)			
NMPC-EKF		NMPC-NMHE	
NMPC	EKF	NMPC	NMHE
2.8	3.0	7.3	3.8

and 11 ms, respectively, which are considerably less as compared to the utilized sampling time, 0.02 s (or 20 ms). Thus, this observation supports the real-time applicability of the presented NMPC-NMHE framework.

Remark: Since NMHE is a dual problem to NMPC, combining it with NMPC is advantageous as it is fairly straightforward to design NMHE using ACADO, after one has already designed NMPC. However, combining NMHE with some other controllers like adaptive controller, would require considerably extra effort as compared to the benefits achieved in terms of performance. In that scenario, utilization of EKF is expected to be more beneficial over NMHE.

7. Conclusion

The NMPC-EKF and NMPC-NMHE control frameworks for the trajectory tracking application of 3 DOF helicopter model have been elaborated in this study. In addition to already complex dynamical model of the helicopter, a time-varying parameter is considered, in order to investigate in depth, the learning capabilities of EKF and NMHE for the examined reference trajectories. The results have illustrated that both the explored frameworks, when well tuned, are capable of controlling such a nonlinear system with a decent amount of tracking accuracy. However, the superior learning quality and explicit constraint handling ability of NMHE makes NMPC-NMHE framework more versatile for the control of helicopter UAVs.

Owing to RTI scheme of ACADO toolkit, a combined average execution time of 4 ms is achieved that apparently demonstrates the real-time-feasibility of the optimization based NMPC-NMHE framework for aerospace applications. In future work, the aim is to further explore the real-time applicability of the considered framework by real-time implementation on complex aerospace systems including tilt-rotor tricopter UAVs.

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