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A TRANSACTIVE ENERGY MANAGEMENT SYSTEM FOR FUTURE COMMUNITY NEEDS

AMRIT PAUDEL

School of Electrical and Electronic Engineering
2020

A Transactive Energy Management System for Future Community Needs

Amrit Paudel

School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2020

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Amrit Paudel

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Assoc. Prof. H. B. Gooi

Authorship Attribution Statement

The thesis contains materials from three published journal papers, in the following peer-reviewed journals, where I was the first and the corresponding author.

Chapter 3 is published as: A. Paudel, K. Chaudhari, C. Long, and H. B. Gooi, "Peer-to-Peer Energy Trading in a Prosumer-Based Community Microgrid: A Game-Theoretic Model," in *IEEE Transactions on Industrial Electronics*, vol. 66, no. 8, pp. 6087-6097, Aug 2019.

The contributions of the co-authors are as follows:

- I developed the concept, produced the results and prepared the manuscript.
- Dr. Kalpesh Chaudhari and Dr. Chao Long assisted in editing the manuscript. They helped me during revision of the manuscript.
- Assoc. Prof. Hoay Beng Gooi closely supervised all the research work, gave suggestions for the organisation of the manuscript and reviewed the manuscript for the final submission.

Chapter 4 is published as: A. Paudel, M. Khorasany, and H. B. Gooi, "Decentralized Local Energy Trading in Microgrids with Voltage Management," in *IEEE Transactions on Industrial Informatics*.

The contributions of the co-authors are as follows:

- I developed the concept, produced the results and prepared the manuscript.
- Mr. Mohsen Khorasany assisted in editing the manuscript. He helped me during revision of the manuscript.
- Assoc. Prof. Hoay Beng Gooi closely supervised all the work, gave sugges-

tions for the organisation of the manuscript and reviewed the manuscript for the final submission.

Chapter 5 is published as: A. Paudel, L. P. M. I. Sampath, Y. Jiawei, and H. B. Gooi, "Peer-to-Peer Energy Trading in Smart Grid Considering Power Losses and Network Fees," in *IEEE Transactions on Smart Grid*.

The contributions of the co-authors are as follows:

- I have developed the basic concept, implemented the idea in the simulation platform, and prepared the manuscript.
- Mr. L. P. M. I. Sampath and Mr. Yang Jiawei helped me to prepare the manuscript.
- Assoc. Prof. Hoay Beng Gooi closely supervised all the work, gave suggestions for the organisation of the manuscript and reviewed the manuscript for the final submission.

2020-08-31

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Amrit Paudel

To My Family.

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Abstract

Transactive energy (TE) is emerging as one of the most innovative approaches for the transformation of existing electricity grids towards the future smart grid as the penetration level of distributed generation (DG) is increasing in power systems. TE based local energy trading (LET) is one of the novel concepts in the area of distribution networks. Peer-to-peer (P2P) energy trading is a kind of LET, and it is one of the promising approaches for implementing decentralized electricity market paradigms. A proper business model is required to manage LET. The pricing mechanism is crucial because the agreed energy price determines the benefits of LET. Since the physical network or grid is used for energy transfer, power losses are inevitable, and grid-related costs always occur during energy transactions. These grid-related aspects should be considered while designing a business model for LET. A proper market clearing mechanism is necessary to facilitate energy transactions among different parties. Designing a proper market clearing mechanism with a specific objective considering the privacy of agents and different aspects of physical networks is a challenging task.

This thesis presents the study of different types of TE based energy trading and management schemes for future smart grids. The focus of the thesis is to develop market clearing algorithms for TE based LET and P2P energy trading. The contributions in this thesis are broadly divided into three parts. The first part proposes a novel game-theoretic approach for P2P energy trading among the prosumers in a community microgrid. The objective of trading is to maximize individual welfare.

The proposed approach models the price competition among the sellers as a non-cooperative game. The evolutionary game theory is used to model the dynamics of the buyers for selecting sellers. The proposed approach is applied to a small community microgrid with photovoltaic (PV) and energy storage systems. Simulation results show that P2P energy trading provides significant financial and technical benefits to the community, and it is emerging as an alternative to cost-intensive energy storage systems.

The second part proposes a decentralized algorithm for LET in microgrids with an integrated pricing mechanism considering welfare maximization and network voltage management through local information exchange among neighbors. The proposed algorithm ensures that the energy transactions do not violate voltage constraints in a physical network, and agents' privacy is preserved. A two-stage approach is proposed to achieve fast convergence and increase the practicability of the algorithm. The efficacy of the proposed approach is demonstrated using illustrative case studies.

The final part presents a decentralized market-clearing mechanism for the P2P energy trading considering the privacy of the agents, power losses as well as the utilization fees for using the third party owned network. Grid-related costs in the P2P energy trading are considered by calculating network utilization fees using an electrical distance approach. The effectiveness of the proposed decentralized approach for market clearing in the P2P energy trading is demonstrated using illustrative scenarios.

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Nomenclature

Acronyms

ADMM	alternating direction method of multiplier
BS	bill sharing
DER	distributed energy resources
DG	distributed generation
DOPF	distributed optimal power flow
DR	demand response
DRM	demand response management
EDC	equivalent daily cost
FIT	feed-in tariff
GDR	generation-to-demand ratio
GWAC	Grid Wise Architecture Council
ISO	independent system operator
LEM	local energy market
LEMS	local energy management system
LET	local energy trading
LMO	local market operator
MMR	mid-market rate
NE	Nash equilibrium
OPF	optimal power flow
P-EMS	prosumer energy management system

P2G	peer-to-grid
P2P	peer-to-peer
PTD	power transfer distance
PTDF	Power Transfer Distribution Factor
RES	renewable energy sources
SDR	supply-to-demand ratio
SE	Stackelberg equilibrium
TE	transactive energy
TEMS	transactive energy management system

Sets and Indices

$\mathcal{C}$	Set of consumers/buyers.
$\mathcal{E}$	Set of lines
$\mathcal{N}$	Set of all agents/prosumers.
$\mathcal{N}_l$	Set of neighbor nodes of agent l .
$\mathcal{P}$	Set of producers/sellers.
$\mathcal{T}$	Set of operational time slots.
$\mathcal{V}$	Set of edges in communication topology.
i	Index of sellers/producers.
j	Index of buyers/consumers.
l	Index of agents.
n	Index of prosumers.
o	Index of lines/edges.
t	Index of time slots.

Symbols

$\Delta \hat{p}_l$	Local estimation of average of Δp by agent l .
Δp	Total power mismatch.
$\hat{\lambda}_l$	Local estimation of λ by agent l .
ξ_l	Solution for agent l from decentralized algorithm for voltage management.

Nomenclature

$(\cdot)^t$	Corresponding parameter value at time slot t .
$[x]^+$	$\max(0, x)$.
β, θ	Utility function parameters.
χ_i	Supply-to-demand ratio of seller i .
γ_i	Probability of buyer j choosing a seller i .
$\hat{v}_l$	Voltage at node l before voltage management.
λ	Energy price in LEM/dual variable.
λ_i	Per unit price of energy from producer/seller i .
$\mathbf{B}$	Admittance matrix of order $N \times N$.
$\mathbf{E}$	Doubly stochastic matrix for communication graph $\mathcal{G}_c$.
$\mathbf{H}$	Network matrix of order $L \times N$.
$\mathbf{I}$	Identity matrix of appropriate dimensions.
$\mathbf{p}_c$	Vector of power demands.
$\mathbf{P}_L$	Matrix of active power flows.
$\mathbf{p}_p$	Vector of power generations.
$\mathbf{p}$	Vector of active power injection at nodes.
$\mathbf{Q}_L$	Matrix of reactive power flows.
$\mathbf{q}$	Vector of reactive power injection at nodes.
$\mathbf{v}$	Vector of nodal voltages.
$\mathcal{G}_c$	Undirected graph for the communication topology.
$\mathcal{G}$	Undirected graph for the microgrid physical network.
$\mu_{ch,n}/\mu_{dch,n}$	Battery charging/discharging power of prosumer n .
Ω^l	Feasible region of p_l .
$\bar{\nu}_l, \underline{\nu}_l$	Available capacities of agent l to support other agents.
$\bar{d}_n/\underline{d}_n$	Maximum/minimum limits on demand (consumption) of prosumer n .
$\bar{E}_n/\underline{E}_n$	Limits on the energy level of the battery of prosumer n .
$\bar{P}_{ch,n}/\underline{P}_{dch,n}$	Maximum charging/discharging power of prosumer n .
$\bar{p}_{ji}/\underline{p}_{ji}$	Maximum/minimum power consumer/buyer j can buy from pro-

	ducer/seller i .
$\bar{v}_l, \underline{v}_l$	Lower and upper limit of voltage at node l .
ψ_l	Amount of power request of agent l to other agents for voltage management.
$\tilde{p}_l$	Required amount of power need to inject at node l to clear voltage violation.
$\underline{p}_i/\bar{p}_i$	Minimum/maximum generation of producer i .
$\underline{p}_j/\bar{p}_j$	Minimum/maximum demand of consumer j .
$\underline{p}_l, \bar{p}_l$	Lower and upper limit of p_l .
φ	Network usage charge per unit electrical distance.
ϱ_i	Coefficient of losses for producer i .
$ \cdot $	Cardinality of a set.
a, b, c	Cost function parameters.
$C(\cdot)$	Cost function of producer.
C_c	Capital cost of the battery system.
C_m	Annual maintenance cost of the battery.
D_n	Nominal demand of prosumer n .
d_n	Actual total demand (consumption) of prosumer n .
d_{ji}	PTD between consumer j and producer i .
E_n	Battery energy level of prosumer n .
G_n	PV generation of prosumer n .
K	Lipschitz constant.
L	Total number of lines in the network.
N	Total number of agents/prosumers.
n_c	Total number of consumers/buyers ($n_c \triangleq \mathcal{C} $).
n_p	Total number of producers/sellers ($n_p \triangleq \mathcal{P} $).
N_{days}	Number of days in a year.
p_i	Total generation of producer i .
p_j	Total demand of consumer j .

Nomenclature

p_l	Power demand/generation of agent l .
$P_{ch,n}/P_{dch,n}$	Battery charging/discharging power of prosumer n .
$P_{ex,i}$	Amount of power prosumer $\mathfrak{B} \in \mathcal{P}$ (seller) can export.
p_{ji}	Power demand of consumer/buyer j from producer/seller i .
r_{lm}/x_{lm}	Resistance/reactance of line connecting nodes l and m in pu.
S_i	Total demand comes to seller i .
T	Total number of time slots.
TF_j	Total network utilization fees for consumer j .
$U(\cdot)$	Utility function consumer/prosumer.
v_l	Voltage at node l .
$v_{l,ref}$	Reference voltage at node l .
W	Welfare function.
w_{ji}	Welfare of consumer/buyer j from the trading producer/seller i .
x	Discount rate.
X_n, Ψ_n	Binary variables related to charge and discharge states.
y_n	Demand from non-flexible loads of prosumer n .
z_n	Demand from flexible loads of prosumer n .

Chapter 1

Introduction

1.1 Background

Recent advancements in the energy sector reveal that renewable-based distributed generation (DG) seems to be sustainable and copes relatively well with rapidly increasing electricity demand. The use of renewable energy addresses the increasing environmental and climate change concerns and supply the increasing electricity demand in a more flexible and modular way. In order to grab this potential, the existing electricity infrastructures need to be re-engineered and transformed into an integrated and intelligent system called the smart grid.

Environmental benefits, financial incentives, and reduction in electricity bills motivate house owners to install renewable based distributed generators and energy storage units at residential buildings [1]. Increasing deployment of distributed generators and energy storage systems with intelligent infrastructures enables the residential consumers to harness energy and inject into the distribution systems. This advancement changes the residential consumers into prosumers. An entity which has the capability to produce, consume, and possibly also has demand response (DR) capacities is known as prosumer [2]. A group of prosumers can be integrated as a prosumer energy community. The small-scale power system in a

house is known as a prosumer nanogrid [3]. The terms prosumer nanogrid and prosumer are used interchangeably. Several nanogrids serving in close proximity can be combined to form a community microgrid. Since the concept of prosumers in the electricity market is new, there may be many unknowns and risks that need to be identified and managed. Policymakers and planners should develop strategies to integrate prosumers effectively and efficiently into the competitive electricity market [4].

Currently, all the excess energy is supplied back to the upstream grid at feed-in tariff (FIT) rates. However, the continuous reduction in FIT rates shows that it is no more beneficial for DG owners to supply the energy back to the grid at FIT rates. The concept of energy sharing among neighbors has been already introduced in the modern prosumer dominated community microgrid. The surplus energy after fulfilling the own demand of prosumers can be either stored or can be shared with neighbors to fulfill their needs. It can be sold to the upstream grid if the neighbor also does not need the energy.

Similarly, in the case of energy shortage in any nanogrid, either stored energy can be utilized or energy sharing can be sought from the neighbors and bought from the upstream grid. Economic price signals mainly drive all these transactions, which is also known as the transactive energy (TE) system. The Grid Wise Architecture Council (GWAC) was formed by the US Department of Energy to promote and enable interoperability among the many entities that interact with the electric power system. GWAC defines TE as [5]:

"The techniques for managing the generation, consumption, or flow of electric power within an electric power system through the use of economic or market-based constructs while considering grid reliability constraints. TE allows the balance of supply and demands dynamically across the entire electrical network using the economic value as a key operational parameter."

As a part of the evolving electricity market, DR, which engages customer price

1.1. Background

bidding and deals with flexible loads in buildings and homes, is being considered. At the same time, the price of energy storage is dropping steadily and its use for grid energy storage is more attractive than before. It is envisaged that the future community not only wishes to have electricity at the affordable/low price but also needs to gain the ability to withstand supply interruption by upstream networks. It should be able to operate as a standalone entity that relies on its generators, distributed energy resources (DER), and energy storage systems. These are some of the challenges and opportunities for investors, governments, and researchers. The winners are those who know how to take advantage of the opportunity by preparing early to overcome any challenges along their way.

The proliferation in the deployment of DER and flexible loads along with the digitization of the grids (smart grids) are shaping the future of the distribution networks [6]. The distribution network no longer exists as a passive unidirectional circuit supplying consumer loads when DER are integrated into it. It becomes an active system having bidirectional power flow capability [7]. The integration of DER into distribution networks has several advantages such as loss reduction, reduced dependency on the grid, etc. However, the increasing penetration of DER raises various operational and market challenges on the existing distribution networks such as voltage limit violations, line congestions, lack of visibility of DER, intermittent energy imbalances, etc. [8]. These technical issues usually vary geographically or locally. Coordinated and controlled use of DER could provide substantial benefits for the market as a whole, and commercial value to consumers, DER owners, and distribution networks across the electricity sector. However, due to various technical, economic, communication, regulatory barriers, it is challenging, and sometimes impractical to design a centralized market system that serves several locations to solve such local problems from DER integration. A centralized market limits opportunities for DER to offer their benefits to broader electricity markets and networks, or to be rewarded for providing these benefits.

Due to these reasons, existing incentives and tariffs designed to engage DER in the energy trading are no longer attractive to motivate them to participate in the market. The concept of local energy trading (LET) seems to be a potential option to cope with issues arising due to DER integration into the distribution networks. Thus, the design of the alternative energy markets for LET in the distribution level which supports active DER participation has become important.

1.2 Motivation

The local energy market (LEM) is an emerging energy market platform in distribution networks for the implementation of the LET concept. LET helps to cope with the challenges posed by the increasing penetration of renewable energy sources (RES), the decreasing FIT rates, and the increasing retail prices. LET has the potential to elevate the benefits of all the participants in the LEM using a TE concept. It also assists in the local balance of the demand and intermittent generation from RES, thus improving the health of the distribution network.

Proactive consumers and producers¹ are the prime stakeholders in the LEM. They are always encouraged to trade energy in the local market. Both the financial as well as non-financial factors, motivate the producers/consumers to enter the LEM. The financial factor is mainly the monetary benefit whereas the social responsibility to alleviate the emissions can be taken as an example of non-financial factors.

LET cannot be applied without an integrated platform, which enables the information exchange among peers, and also assists the system operators to monitor and control the distribution network. In addition, different trading rules defined by the platform also have significant influences on the decisions made by peers when trading with other peers. Therefore, smart energy management services are required to enable the actors or agents to make decisions on whether or not they want to trade, with whom and when and at what price, while negotiating

¹Those entities participating in the LEM are called agents or actors or peers in this thesis.

1.3. *Specific Contributions*

with other actors in the energy market. Development of a well-designed energy management system to manage these complex energy transactions in the LEM, considering the technical aspects of the network is the fundamental motivation behind this study.

1.3 Specific Contributions

The prime objective of the research work done for this thesis is to develop a trans-active energy management system (TEMS) for future smart grids. This research work mainly focuses on the market clearing and energy pricing mechanism design for TE based LET. The contributions of the research work presented in this thesis are summarized as follows:

1. A novel game-theoretic model is proposed for peer-to-peer (P2P) energy trading using direct interactions between buyers and sellers in a community microgrid considering the DR capability and privacy of prosumers. The buyers can adjust the energy consumption behavior based on the price and quantity of the energy offered by the sellers. The pricing competition among the sellers is modeled as a non-cooperative game. The dynamics of buyers in the process of selecting sellers are modeled as an evolutionary game. Moreover, an M-leader and N-follower Stackelberg game approach is used to model the interaction between buyers and sellers. Two iterative algorithms are proposed for the implementation of the games such that an equilibrium state exists in each of the games.
2. A fully decentralized algorithm, which neither requires any third-party nor reveals any private information of the agents, to determine the energy trading price for LET is proposed. Besides, a novel method to internalize the external effects of the decision of agents on node voltages within the energy price so that the agents can trade in the LEM without affecting the

network performance is also proposed. Since the convergence speed is one of the important aspects of any algorithm design, a two-stage approach is proposed to achieve faster convergence. The two-stage approach improves the convergence speed and increases the practicability of the algorithm for real-time implementation.

3. A P2P energy trading is formulated as a social welfare maximization problem with consideration of network/grid usage, i.e., power losses and network utilization fees, in P2P energy transactions. An electrical distance approach is proposed to calculate the network fees for P2P energy transactions, where network fees are proportional to the electrical distance between producers and consumers. Besides, a novel decentralized approach is proposed to solve the P2P market clearing problem without sharing the agents' preferences and respects the privacy of agents.

1.4 Outline of the Thesis

This thesis is organized into six different chapters. The background and motivation behind this study, the scientific contributions of the research work and the outlines of the thesis are presented in Chapter 1.

Chapter 2 presents a comprehensive review of various topics related to the TE based LET concept. The basics of TE and TE based LET with its advantages are also reviewed. Key aspects of LET such as different market designs, different market clearing approaches, key elements and requirements for the implementation of LET as well as challenges and barriers for LET, are identified and discussed.

Chapter 3 proposes a game-theoretic model for P2P energy trading in a prosumer based community microgrid. The goal of each prosumer participating in the energy trading along with a framework for P2P energy trading is defined. The implementation of a game theory approach for energy trading on a simple and

1.4. Outline of the Thesis

realistic platform is also presented.

Chapter 4 presents a decentralized algorithm for LET with an integrated pricing mechanism considering the welfare maximization and the network voltage management through local information exchange among neighbors. The LET problem is formulated as an aggregated welfare maximization problem. The proposed method is applied to a microgrid having the IEEE 33-node network structure to verify the efficacy of the proposed method.

Chapter 5 presents a fully decentralized market clearing mechanism for P2P energy trading. A P2P energy trading is formulated as an aggregated welfare maximization problem with the consideration of grid usage aspects in P2P energy transactions. The proposed method is applied to the IEEE 9-bus system and IEEE 39-bus system. The convergence and scalability are demonstrated via numerical results.

Chapter 6 concludes the thesis with some recommendations for future research.

Chapter 2

Transactive Energy Based Local Energy Trading

2.1 Transactive Energy

TE is emerging as one of the most innovative approaches for the transformation of the existing electricity grid towards the future smart grid as the penetration level of DGs is increasing in power systems. In recent years, TE catches the attention of both industry and academia throughout the world, although it is a relatively new concept [9]. Several sources have defined the TE system. The 5th edition of the Smart Grid Dictionary, provides the following definition [10]:

"A software-defined grid managed via market-based incentives to ensure grid reliability and resiliency. This is done with software applications that use economic signals and operational information to coordinate and manage devices' production and/or consumption of electricity in the grid. TE describes the convergence of technologies, policies, and financial drivers in an active prosumer market where prosumers are buildings, EVs, microgrids, virtual power plants, or other assets."

The definition of TE provided by GWAC [5] is given in Chapter 1. Unlike in

the past, where the control is possible only on the supply side, the GWAC's new definition implied the concept of control mechanism on the demand side. In addition, GWAC uses the following key terms to explain the concept of TE.

- **Dynamic balance:** TE deals with the real-time balance between the demand and supply in the context of operations of the power system, unlike the context of its planning in the past.
- **Economic value:** TE uses the value-based information captured during the transaction between the participants to render the operational decisions.
- **Entire electrical network:** The TE approach is feasible across the entire electricity system, from the transmission system with traditional generators to the distribution system with varieties of connected customers as well as varieties of newly introduced assets such as customer-owned DGs, energy storage devices and flexible loads.

Based on the definition of key terms, TE can be considered as an innovative technique to establish a short-term balance between the real-time supply and the demand on the electricity grid. The short-term prices drive the production and consumption decisions of producers and consumers, respectively. TE is recognized as a basis for the formation of the decentralized electricity market for energy transactions among various producers and consumers. TE is based on buying and selling known as transactions of energy among parties that consume, produce, store, and transport electrical energy [11]. In TE, medium to small-sized electricity consumers or producers automatically negotiate their actions with one another. They also negotiate with devices in the physical network. The transaction is performed using a dispatch system of energy suppliers through efficient and scalable electronic market algorithms [12]. TE helps to make the electricity system more reliable, resilient, and secure through the real-time and decentralized decision-making [13].

2.2 Concept of TE based LET

In the conventional electricity system, the electricity generated by large-scale centralized power plants is transmitted through the high-voltage transmission networks, voltage transformed at the substations, and finally distributed through the distribution networks to the varieties of end-users or consumers. The power flow is unidirectional from the generating stations to consumer locations [14]. The generation companies sell electricity to retailers in the wholesale market in bulk and retailers sell to the end consumers in the retail market in a relatively small amount. The increasing adaptations of DER in distribution networks contribute to change the scenario of conventional electricity systems in both technological and commercial aspects.

The integration of renewable-based DG results in bi-directional power flows in the network as well as leads to severe uncertainties in the electricity market and grid operation. These operational and market issues raised by the proliferation of DER constitute significant challenges for the planning, operation, and protection of modern power systems. Practically, DER are integrated to the edge of the existing networks and could affect the system voltage and frequency [6]. Assuming that these issues created by the integration of DER confined locally in a certain area and vary geographically, a design of a new energy trading mechanism called LET based on the TE concept is becoming an effective approach to cope with such local and diverse challenges.

In recent years, LET among producers and consumers is becoming a topic of interest in both academia and industry. LET plays an effective role in both, energy management as well as better delivery of the DG output to the network as DGs are connected to the edge of the network [15]. LET is the practical aspect of the TE concept. The market platform which implements the LET concept is called LEM. LET is often called the micro-market concept in electricity systems.

2.2.1 Key Actors in LET

There are various participants or agents or actors involved in LET. Some of the key actors involved in LET are described below:

- **Producers or Sellers:** Each producer has a generation capability. Every producer as well as agents having storage capability can act as sellers in LET. Some examples of producers or sellers involved in LET are DGs [16–22], EVs [18], energy storage systems [21–24], prosumers with excess energy, etc.
- **Consumers or Buyers:** Buyers or consumers in LET are those agents who buy energy from producers or sellers to meet their energy demand. Conventional inflexible loads [25], prosumers in energy deficit [26–29], and flexible loads with load adjustment capability [30] can act as consumers or buyers in LET.
- **Mediators:** Any other agents involved in LET other than producers and consumers fall under this category. Generally, these agents are known as mediators in LET, and they are mainly responsible for the distribution of electrical energy in the network. Some agents are capable of bearing the responsibility of mediator along with acting as a seller/buyer in LET. Some examples of mediators in LET are aggregator [18,31,32], auctioneer [30,33,34], local market operator (LMO) [25,35–37]. LMO is responsible for maintaining the supply-demand balance in the network. The roles of mediator in LET are also summarized in Chapter 4.

2.2.2 Benefits of LET

LET offers several advantages to asset owners, utilities as well as community and consumers. The nature of LET benefit ranges from direct economic incentives to the asset owners and utilities to the social and economic benefits to the consumers and society. Some crucial benefits are enumerated here.

2.2. Concept of TE based LET

1. LET can reduce the system operational cost by optimizing the use of DER [38, 39]. Also, the transmission losses are reduced when the energy is used at or closed to the place of production. It increases the efficiency of the system [40].
2. LET creates a platform for various distributed energy producers and consumers to trade energy and to manage the uncertainty in the supply and demand locally. It helps to reduce the dependency on the main grid and increase the reliability [38] and, improve the network performance.
3. LET provides the direct economic benefit to the asset owners and market participants through the cost minimization or profit maximization. The economic benefit to the participants can be achieved from the optimal DER deployment, the minimization of energy cost as well as the minimization of transmission losses. Hence, TE based LET is a major impulsive force behind the power grid modernization and the proliferation in DER deployment [41, 42].
4. LET allows producers and consumers to share and trade energy within the local energy community. Such gestures promote solidarity in society and improve the sense of community. It is regarded as a social benefit of LET.
5. LET helps to cope with sensitive bottlenecks in distribution systems during peak period by providing strong incentives to DER owners and consumers to participate in local energy transactions. It results in the deferral of grid upgrades.
6. In the conventional wholesale market, producers and consumers cannot participate directly and they trade through a retailer by paying an extra fee. However, in LET, they can trade directly among one another and they do not have to pay an extra fee to the retailer.

2.3 Framework for LET

A proper framework is required to achieve the desired outcome from energy trading. As reported in the existing literature, there are various aspects in the LET framework such as trading objective, energy pricing, market clearing approach, and energy trading structure or market structure. These four aspects are the essential elements each LET framework should have in order to be a viable one. These key aspects of viable LET framework are discussed here.

2.3.1 Market Structure

An innovative market arrangement is essential to perform LET in modern power systems with the rapidly increasing share of DER. Based on the degree of decentralization, market structure for LET found in the existing literature can be classified as centralized market, decentralized market, and distributed market.

Centralized Market

The centralized market is designed with a centralized coordinator or mediator or LMO who communicates with producers and consumers participating in LET and manages all energy and financial transactions in the market. The centralized coordinator is also responsible for the interaction with other neighboring local markets and existing wholesale markets. The LMO decides the energy price based on the objective of the trading [43]. One of the major advantages of centralized markets is that the market outcome has less uncertainties since the behavior of agents are under the supervision and control of the centralized coordinator. Another advantage of the centralized market is that it is easier to achieve the trading objective as the coordinator makes the trading decisions for the participating agents and the global optimization can be performed.

A major drawback of the centralized market is that the computational and communication burden increase exponentially as the number of agents increased in

2.3. Framework for LET

the market. Since the coordinator collects all the information of agents and controls their decision, it may compromise the privacy and autonomy of the agents. Besides, there is a risk of a halt in market operation since centralized markets are susceptible to single-point failures.

In [44], a centralized agent called P2P trader decides the trading prices for local energy transactions to maximize the economic benefits of households with PV and batteries. The utility providers and prosumers do not interact directly for energy sharing; they interact through a mediator called energy sharing provider in [45]. A contribution-based energy trading mechanism has been proposed in [46], where a distributor plays a mediator role and performs the energy allocation among providers and consumers in a microgrid. The distributor collects the excess energy from providers and fairly distributes it to consumers. In [36], the distribution system operator coordinates to schedule the supply and demand in order to establish a market balance and maximize social welfare in the local distribution area. Tushar *et al.* in [33] have proposed a double-auction method, where a third party auctioneer manages the auction process for the sharing of energy storage. The LET based on the centralized market structure has been reported in [15, 47, 48] also.

Decentralized Market

Unlike in the centralized market, there is no mediator or centralized coordinator in the decentralized market. Agents directly negotiate with each other to make trading decisions. The decentralized market structure respects the privacy and autonomy of agents as there is no centralized coordinator to collect the information and influence the trading decisions of agents. The decentralized market is more scalable as compared to the centralized market. In the decentralized market, agents can independently enter and leave the market easily.

On the other hand, since there is no mediator to perform global optimization as

in the case of the centralized market, the efficiency of the decentralized market may not be as high as that of the centralized market. Also, the market outcome may not be feasible for the network operators and it is challenging to address the network constraints in decentralized markets.

A fully decentralized market has been proposed for LET in [26, 49–52]. Khorasany *et al.* in [49] has proposed a fully decentralized P2P market to maximize the welfare of each agent. A relaxed consensus cum innovation approach is proposed for a multi-bilateral economic dispatch in a fully decentralized P2P energy market in [50]. A blockchain-based fully decentralized mechanism for LET under demurrage is proposed by Devine *et al.* in [51]. In [52], a multi-agent based coalition mechanism has been proposed for energy trading negotiations in a fully decentralized manner. In the Brooklyn microgrid [53], small agents trade energy in the local microgrid market without any central entity. Compared to the centralized market, diverse methods for energy trading and allocation can be proposed with the decentralized market. However, in the decentralized market, it is challenging to achieve a global optimal point.

Distributed Market

The design of distributed markets lies in between centralized markets and decentralized markets. The distributed market combines the features of both the centralized market and decentralized market to give a mid-way solution. The distributed market involves a coordinator in market operations. However, the coordinator does not control and influence the trading decisions of agents directly, unlike in the centralized market, where the coordinator directly controls the trading decisions and actions. The coordinator only requires minimal information, such as pricing signals, to establish coordination among agents in the distributed market. Hence, the privacy and autonomy of agents are respected to a certain extent. Since the distributed market involves a coordinator, agents in the market can have better coordination as compared to the fully decentralized market.

2.3. Framework for LET

Several studies [54–58] proposed a distributed market for LET in energy networks. Wang *et al.* proposed an agent-based distributed mechanism for bi-directional energy trading between the smart building and utility grid in [54]. In [55], a distributed intelligent energy management system is proposed for local power trading among microgrids.

A distributed approach proposed in [56, 57] allows responsive demands, the DER, and the storage unit to buy, sell, and store energy at appropriate times. A distributed market for P2P economic dispatch is created using a consensus alternating direction method of multipliers in [58]. In general, pricing signals are used to coordinate the decisions and actions of agents in the distributed market. Therefore, designing a proper pricing mechanism is a major factor behind the implementation of the distributed market structure for LET.

2.3.2 Trading Objectives

The objective of the trading defines the ultimate target that the participating agents aim to achieve from the LET. The target could be economic benefits such as the generation cost minimization, the social welfare maximization, technical benefits such as loss minimization, reliability enhancement, or environmental interests. As reported in the existing literature, minimization of the operational cost and maximization of social welfare are two popular trading objectives used for market clearing in LET. Each of them is discussed briefly here.

Operation Cost Minimization

Some existing studies [20, 21, 27, 32, 59–61] consider the operational cost minimization as the trading objective for market clearing in LEM. The operational cost mainly includes two major components: the energy cost and the system operation cost for all agents in the market. The cost function defined for different agents may have different characteristics. In practical implementation, the cost functions

of all agents are added together regardless of their characteristics to obtain the total cost of operation, and thus the obtained total cost is minimized subject to various constraints. The balance between the total demand and total supply is essential for the stable operation of the grid. Thus, the power balance constraint is the compulsory constraint to be considered while minimizing the operational cost. If $\mathcal{N} = \{1, 2, \dots, N\}$ is the set of agents in the market with index l and $C_l(p_l)$ is the cost function of agent l , then the total operational cost is given by

$$\text{Total Cost} = \sum_{l=1}^N C_l(p_l) \quad (2.1)$$

If $p_l > 0$, agent l is a seller or producer and if $p_l < 0$, agent l is a buyer or consumer. The optimization problem can be defined as

$$\underset{p_1, \dots, p_N}{\text{minimize}} \sum_{l=1}^N C_l(p_l) \quad (2.2)$$

$$\text{s. t. } \sum_{l=1}^N p_l = 0 \quad (2.3)$$

Social Welfare Maximization

The social welfare is defined as the sum of the producer's surplus and the consumer's surplus [62]. The works in [16–19, 22, 25, 29–31, 34, 35, 37, 46, 63, 64] have considered maximization of the social welfare as the objective of market clearing. The profit of the individual agent is concurrently maximized when the social welfare maximization is considered as the objective of the LET. If $\mathcal{C} = \{1, 2, \dots, n_c\}$ and $\mathcal{P} = \{1, 2, \dots, n_p\}$ are the sets of consumers and producers in the market, respectively, then social welfare is defined as follows

$$\text{Social Welfare} = \sum_{j=1}^{n_c} U_j(p_j) - \sum_{i=1}^{n_p} C_i(p_i) \quad (2.4)$$

where $U_j(p_j)$ is the utility function of consumer $j \in \mathcal{C}$ and $C_i(p_i)$ is the production cost function of producer $i \in \mathcal{P}$. Since the power balance is essential for the stable operation of the power grid, the optimization problem can be defined as follows:

$$\underset{\mathbf{p}_c, \mathbf{p}_p}{\text{maximize}} \sum_{j=1}^{n_c} U_j(p_j) - \sum_{i=1}^{n_p} C_i(p_i) \quad (2.5)$$

$$\text{s. t. } \sum_{j=1}^{n_c} p_j = \sum_{i=1}^{n_p} p_i \quad (2.6)$$

2.3. Framework for LET

Utility Function

The utility function is a method to quantify the level of satisfaction or happiness that the consumer receives when it consumes a certain amount of electrical energy. The utility function of consumers in the community will have two main arguments. The first one is the power consumption and the second one is the parameter to distinguish a consumer from the others.

If p is the power consumption level and β is the parameter to distinguish a consumer from the others, the utility function can be represented by $U(p, \beta)$. The utility functions should possess the following properties [65].

Property 1: The utility functions should be non-decreasing. It is assumed that consumers are always interested on consuming more until they reached their maximum consumption level. Mathematically,

$$\frac{\partial U(p, \beta)}{\partial p} \geq 0 \quad (2.7)$$

The marginal utility as the additional satisfaction a consumer receives from consuming an additional unit of electricity is defined as

$$V(p, \beta) \triangleq \frac{\partial U(p, \beta)}{\partial p} \quad (2.8)$$

Property 2: The marginal utility should be non-increasing function. This applies that the utility functions are concave and the level of satisfaction gets saturated over time. Mathematically,

$$\frac{\partial V(p, \beta)}{\partial p} \leq 0 \quad (2.9)$$

Property 3: For a fixed consumption level p , a larger β brings a larger utility.

$$\frac{\partial U(p, \beta)}{\partial \beta} > 0 \quad (2.10)$$

Property 4: Zero power consumption brings zero utility (i.e., no consumption-no satisfaction).

$$U(0, \beta) = 0, \quad \forall \beta > 0 \quad (2.11)$$

Any function which satisfies the above four properties can be used as an utility function. There are two types of utility functions frequently used for modeling the electricity consumers. They are logarithmic utility function [66–70] and quadratic utility function [65, 71, 72]. A quadratic utility function [65] to quantify the usage benefit of consumer $j \in \mathcal{C}$ from consuming demand of p_j is defined as follows:

$$U_j(p_j) = \begin{cases} \beta_j p_j - \frac{1}{2} \theta_j p_j^2 & : 0 \leq p_j \leq \frac{\beta_j}{\theta_j} \\ \frac{\beta_j^2}{2\theta_j} & : p_j \geq \frac{\beta_j}{\theta_j} \end{cases} \quad (2.12)$$

where $\beta_j > 0$ is a preference parameter characterizing the consumer's behaviours, which vary from consumer to consumer and $\theta_j > 0$ is a predetermined constant.

Cost Function

The cost function $C_i(p_i)$ of producer $i \in \mathcal{P}$ is usually approximated by a quadratic convex function of generated power p_i [73] as

$$C_i(p_i) = \frac{1}{2} a_i p_i^2 + b_i p_i + c_i \quad (2.13)$$

where a_i , b_i , and c_i are the positive cost coefficients of producer i . These coefficients depend on the type of generation of each producer.

2.3.3 Energy Pricing and Market Clearing Approaches

The energy pricing and market clearing mechanism for LET depend on the market structure, specific rules of the market as well as on the trading objective. As mentioned earlier, compared to the centralized market, diverse methods for market clearing can be proposed with the decentralized market. But, in the decentralized market, it is challenging to achieve a global optimal point. Several market clearing methods for LET adopted in the literature are discussed in this section.

Decomposition Method

The decomposition methods of market clearing for LET are based on the distributed market structure. The decomposition methods split a complex optimization problem into a set of several sub-problems corresponding to various agents

2.3. Framework for LET

in the market. The obtained sub-problems are easier to solve as compared to the main problem. The decomposition methods distribute the computational tasks among various agents in the market instead of solving them by a single centralized coordinator. The computational burden at the coordinator is redistributed and this method is applicable to larger systems. However, this method is always prone to single-point failure at the coordinator. There are two major decomposition methods: primal decomposition and dual decomposition. In the primal decomposition method, a coordinator directly assigns a resource to each agent or subproblem and agents optimize the use of the assigned resource. Therefore, the primal decomposition method is also called a direct resource allocation method. On the other hand, in the dual decomposition method, a coordinator decides the price and agents optimize its resource based on the price set by a coordinator, and it is also called an indirect resource allocation method or a resource allocation via pricing. The authors in [18, 22, 23, 37, 61, 74, 75] used decomposition methods for market clearing in LET with various objectives and constraints.

Game-theoretic Approach

A mathematical framework to study the conflict and cooperation between intelligent decision-makers is called Game Theory [76]. It presents a mathematical model to explain the situation, where an individual decision of an agent affects the decision of other agents. Cooperative games and non-cooperative games are the two popular game-theoretic techniques used in the field of economics, social sciences, biology, engineering, and more recently, smart grid [77–79]. In non-cooperative games, players make decisions independently, whereas, in cooperative games, players collaborate to achieve a common goal. Players are encouraged with some form of incentives to participate in the game. The benefit of the collaboration among players in a cooperative game is shared among themselves. There have been some studies [27, 47, 64, 68, 77, 80–84] using different game-theoretic models in various aspects of LET and P2P energy trading.

A Stackelberg game is a popular tool to model a scenario where the actions of a certain set of players are followed by another set of players [85]. The first set of players are called leaders and the latter are called followers in the game. The evolutionary game extends the formulation of the non-cooperative game by introducing the concept of population, which refers to a set of players [86]. These games are popular in the area of energy trading due to their efficiency. The concept of Nash equilibrium (NE) is used in a non-cooperative game to find the solution of the game. The more details on the Stackelberg game, Evolutionary game, and NE will be discussed in Chapter 3.

Auction Theory

An auction is a well-specified market mechanism for trading negotiations between sellers and buyers. An intermediary agent called auctioneer facilitates the auction process. It is not necessary that the auctioneer should be a real entity. It can be a set of predefined rules executed automatically [87]. The ascending-bid auction, the descending-bid auction, the first-price sealed-bid auction, and the second-price sealed-bid auction are the basic types of auctions [88]. The final price and the quantity to trade are the outcomes of the auction.

Auction mechanism is very popular for market clearing in the wholesale and retail electricity market. Recently, the auction method catches the popularity in LET to maximize economic efficiency by discovering the lowest-cost matching between supply and demand. A double auction mechanism is used for the energy trading among storage units and smart grid elements in [64]. In [35], an iterative double auction mechanism to maximize the welfare in LEM for electricity and hydrogen is proposed. A P2P energy trading mechanism proposed in [18] use an iterative double auction to maximize the social welfare of electric vehicles. In addition, the authors in [30, 33, 34, 89, 90] have also used auction-based approach for market clearing in small-scale energy trading in LEM.

2.3. Framework for LET

Consensus Approach

The consensus-based approach for market clearing is the backbone of the decentralized market structure for LET. In this method, agents in the market exchange information only with the immediate neighbors in the communication network. The LEM can be modeled by a graph with agents at its vertices, where an edge between vertices indicates the information exchangeability between agents in the graph. The structure of the communication graph determines how agents coordinate with each other to make decisions. The consensus-based approach is a special kind of decomposition method, where the decomposition of the main problem depends on the structure of the communication graph. Since this method is associated with graph theory, it is also called the graph theory approach of market clearing. There have been several studies using a consensus approach for energy management in smart grids.

The authors in [17] use the directed graph to model the interaction among demands and generations and employ a consensus based approach for distributed energy management. In [63], the communication topology is modeled with the undirected graph to perform the consensus-based distributed economic dispatch. An incremental welfare-based consensus algorithm is proposed for generation/demand response in [16], where interaction among agents is modeled using the undirected graph. Meng *et al.* [19] propose a consensus-based distributed energy management algorithm to maximize the social welfare, which ensures the privacy of agents and enables the plug-and-play features. Besides, the works in [21, 50, 59, 91] also use the graph theory and consensus-based approach to propose a distributed energy management algorithm in various scenarios.

Distributed Optimal Power Flow

The optimal power flow (OPF) problem determines a network operating point that minimizes a certain operational objective subject to various technical, oper-

ational, and network constraints together with power flow equations [92]. It is crucial to include network constraints in LET for DER coordination. A standard approach to consider network constraints in LET formulation is to solve the OPF approach with a large number of market participants [93]. Traditionally, the independent system operator (ISO) collects all the necessary parameters and performs a central computation to solve the OPF problem [94]. The centralized approaches of the OPF solution are most prevalent in current power systems and schedule a relatively small number of generators. However, with increasing penetration of DER in future networks, we need to optimize the operation of a large number of intelligent and controllable devices. In this case, the centralized approaches for the OPF solution are not scalable because of its communication and computational overhead. As a solution, distributed algorithms for solving OPF problems have been the subject of significant research interest and is called distributed optimal power flow (DOPF).

In DOPF, each agent communicates only with its neighbors and will have to share limited information to solve the OPF problem without the central controller [94]. DOPF respects the privacy of agents and computationally superior to conventional centralized OPF. Therefore, DOPF is emerging as a promising approach for DER coordination in the future power system.

The alternating direction method of multiplier (ADMM) is one of the most used techniques to solve the DOPF because of its simplicity [95]. Using this technique, the original OPF problem can be decomposed into several subproblems that can be solved in parallel. A fully decentralized AC-OPF algorithm based on the ADMM is proposed in [96] but the non-convexity of the OPF problem is completely ignored and the convergence of the algorithm is not guaranteed. In [97], the ADMM is applied to solve the convexified DOPF problem with convergence guarantee. There have been several studies on ADMM based DOPF in power systems [98–100].

2.4. Challenges and Barriers for LET

Additional Methods

In addition to the four methods discussed above, several other methods have been used for market clearing in LET. The agent-based method, which is a special kind of the distributed method, has been used for modeling the large-scale electricity systems with participants of diverse characteristics [101]. An agent-based model is suitable for the computer simulation of energy management algorithms. Agent-based methods have been used for various energy management applications [29, 56, 102–105]. Mortsyn *et al.* [26] design a scalable market for P2P energy trading by implementing a bilateral contract network. A contract game in the short-term market for direct energy trading in the smart grid is proposed in [106]. A consensus cum innovation approach is used for the market clearing in LET in [107].

2.4 Challenges and Barriers for LET

The concept of LET is still not fully matured and has not been widely adopted. There are several challenges and barriers which prevent LET to be widely acceptable and implementable. Those challenges and barriers could be either technical or non-technical. Some challenges and barriers identified from the literature review are discussed here in brief.

Information Security and Privacy

Since LET involves the collection of sensitive and/or confidential data of agents, security, and privacy of agent's information are crucial in any LEM. The LEM should have two essential features: security and transparency, but it is still unclear how such markets can be established. The application of blockchain technology could be one of the solutions since it has several merits. The blockchain technology not only helps in maintaining privacy and transparency but also allows a large number of distributed transactions as well as continuous tracking of negotiations among agents [108]. However, blockchain technology has issues of scalability,

complexity, and implementation with current components. These issues have not been studied in detail yet. Besides, data protection from cyber attacks in the smart grid is another important aspect of LEM implementation. Hence, more research is required to enhance the privacy and data protection in LEM.

Scalability and Robustness of Market Clearing Algorithms

In the design of LEM, scalability to accommodate the increasing number of market participants is a prominent issue. In general, markets designed with decentralized market clearing methods are more scalable as compared to the markets which need a centralized coordinator. The barriers to the implementation of markets with a centralized coordinator with a large number of agents are communication systems and computational burdens. The large-scale implementation of any market design requires sufficient communication infrastructures and computation facilities. Although decentralized market designs are more scalable, proper supporting facilities should be developed accordingly as the size of the market increases. These two factors always become a matter of concern as the size of power systems expands. In addition, robust and lightweight market clearing algorithms represent LET scenarios more efficiently. Therefore, more intensive research is required to enhance the robustness and efficiency of market clearing algorithms for LET.

Network Constraints and Power Quality Issues

Once the trading negotiations reached an agreement, the agreed amount of energy should be exchanged between the trading parties. Agents in the LEM need power networks for the energy exchange. Such power networks could be the existing power network or a newly constructed network in conjunction with the existing network.

There are several identifiable constraints, such as line capacity, voltage limits, etc., associated with the electrical power network. Voltage limit violation is the power quality issues and it affects the health of the consumer's appliances, whereas line

2.4. Challenges and Barriers for LET

congestions affect the network health itself. LET would not be feasible when such factors related to the physical network are excluded. Hence, the electrical network should be considered in each transaction in LEM. There have been some efforts in the literature on the design of LEM considering physical networks. However, more detail research is needed in this aspect.

Interaction Between LEM and Existing Wholesale/Retail Markets

With recent developments and contributions in designing LEM, it can be foreseen that the LEM and the existing wholesale/retail markets co-exist together in the near future. Therefore, the relationship between LEM and the existing markets should be properly explored. The aspect of ancillary services is not considered in LEM design so far. Besides, renewable-based generations participating in LET bring uncertainties and may cause an intermittent imbalance between supply and demand. The existing markets cannot be isolated completely from the LEM although even if there is enough generation in LEM. More intensive research is required on the interaction between newly designed LEM and the existing markets.

Social Factors and Policy Issues

LET involves a large number of small-scale producers and consumers in a community or society. LET would not be possible until there is the active participation of such small scale entities. The participation of small-scale producers and consumers in LET depends on both monetary (financial) returns and non-monetary or intangible returns. The integration of energy and society from non-monetary aspects such as willingness to install DER, psychological motivation to participate in LET, etc. are not intensively studied yet. In order to make LET widely acceptable for real implementation, a proper research on the integration of energy and society from the intangible aspect is essential. Hence, the anthropological approaches should be considered while designing the LEM.

In addition to social aspects of the adaptation of LEM, various legal and policy barriers obstruct the development of LEM. The existing regulatory schemes for the electricity sector are based on the conventional electricity system. A lack of an established framework for LEM regulation imposes a legal barrier for the market participation of DER through LET. Since DER are connected to the existing distribution network, there exists an issue of payment of connection charges to the distribution network owner for the network reinforcement. The policy reform is necessary for the large-scale implementation of LET.

2.5 Summary

A detailed review of the TE based LET concept in the future electricity systems with high penetration of DGs and flexible demands is presented in this chapter. Key aspects of LET are identified and discussed. The key aspects include, but not limited to, the study of market design for LET, different market clearing approaches, key actors in LET, and requirements for the implementation of LET as well as challenges and barriers for LET. From the challenges and barriers discussed above, it is not difficult to conclude that there are still a lot of subjects for research by both academia and industry.

Chapter 3

Game-Theoretic Model for P2P Energy Trading

3.1 Introduction

The P2P network is a widely used model for resource sharing in the field of computer science where resources are located in and provided by computers (i.e., peers) at the edge of the network [109]. Since a community microgrid consists of several prosumers in close proximity having their own generation and demand, it can be modelled as a P2P network. A P2P energy trading model seems to be suitable for energy trading in the prosumer based community microgrid. P2P energy trading is flexible between peers, where excess energy from various small-scale DER is traded locally [110]. The P2P model encourages LET and DR to the available resources in a community [111].

Since P2P energy trading in a community microgrid is a new concept, a proper modelling framework is required to define the business models, to determine energy prices, as well as to implement DR programs in a community. The business model and energy pricing play a vital role because they determine the suitability of P2P trading in a community microgrid in terms of financial benefits. The business

model also forms a basis for the implementation of DR programs.

3.2 Related Works

In [41], the authors have proposed a P2P energy trading model for smart households. The optimal microgrid energy and price for P2P trading are determined in order to minimize the total energy cost. A customer-to-customer business based on the architecture model for P2P energy trading is introduced in [27]. The bidding system called *Elecbay* is proposed for P2P trading and simulation is done using game theory. An efficient and privacy-preserving P2P energy exchange scheme in a smart grid environment is presented in [112], with the novel optimization approach to increase efficient energy transfer without information leakage. In [113], a new algorithm has been proposed for automating electricity trading by prosumers in the P2P electricity market. A framework for performance evaluation using several indices is proposed in [114] to assess the economic performance of P2P energy sharing models. A P2P index is developed to assess the feasibility of P2P energy trading in low voltage distribution system in [111]. In [115], the authors have introduced different P2P market paradigms using bill sharing (BS), mid-market rate (MMR), and auction based pricing strategies but DR is not considered.

In line with the above works on P2P energy trading, many researchers have carried out researches on demand response management (DRM), energy sharing management, and real-time pricing using various game theoretical approaches in [67–69, 116]. In [68], the authors have proposed a competitive market based distributed mechanism for energy trading among microgrids using multileader-multifollower Stackelberg game. A Stackelberg game theory is used to study the DRM from various aspects in [66, 71, 72]. A hybrid approach using stochastic programming and Stackelberg game is presented in [67] to coordinate energy sharing and energy consumption behavior of prosumers by using internal prices.

3.3. Major Contributions

In these works [45, 67, 111, 115, 116], a separate entity acted as an energy trading coordinator and was responsible for the execution of energy trading. There was no direct communication between buyers and sellers. However, since prosumers in a community are proactive, it is possible to perform P2P energy trading in a community with less involvement of the energy trading coordinator (i.e., using the direct communication or interaction between buyers and sellers). The direct involvement of each prosumer during the trading process is the main feature of P2P energy trading. It poses the challenge of modeling the decision-making process of each participant for the greater benefit of the entire community microgrid while considering human factors such as rationality, motivation, and environmental friendliness [117]. In situations where there are many prosumers with conflicting interests, it would be quite challenging either to capture such conflicting interests in the decision-making process of each participant or to motivate them to cooperate for achieving the goals of P2P trading. Therefore, there should be a proper tool for modeling the decision-making process such as deciding energy price, incorporating DR in P2P energy trading in a community microgrid. The modeling tool should be able to deliver the energy management solution considering a diverse set of constraints in a trading process. In this context, considering the interactive and conflicting nature of energy trading in a community microgrid, game theory is a very effective tool for modeling the decision-making process of prosumers participating in the P2P energy trading.

3.3 Major Contributions

The main contributions of this chapter are summarized as follows:

1. A novel game-theoretic model is proposed for P2P energy trading using direct interactions between buyers and sellers in a community microgrid considering the DR capability and privacy of prosumers.

2. The pricing competition among the sellers is modeled as a non-cooperative game. Dynamics of buyers in the process of selecting sellers are modeled as an evolutionary game, and interaction between sellers and buyers is modeled as a Stackelberg game.
3. A novel iterative algorithm is proposed to reach the stable state in the evolutionary game among buyers for the seller selection.
4. A novel distributed algorithm is proposed to obtain the equilibrium in both non-cooperative game among sellers and Stackelberg game between buyers and sellers.

3.4 System Model

3.4.1 Structure of Community Microgrid

Fig. 3.1 shows a structure of a smart community microgrid comprising several prosumers. Each prosumer comprises loads and PV systems. Prosumers may or may not have BESS since deploying energy storage systems in the residential level is still costly. The PV system of a prosumer is connected to the load and AC system through DC/AC converter which is also known as PV inverter. If any prosumer has both a PV system and a battery, they can be connected via a DC coupled or an AC coupled topology. In the DC coupled topology, the battery is connected to the PV system at the DC side of the PV inverter through the DC/DC bi-directional converter as shown in Prosumer 1 in Fig. 3.1. In the AC coupled topology, the battery is connected to the PV system at the AC side of the PV inverter through DC/AC bi-directional converter as shown Prosumer 2 in Fig. 3.1.

All the prosumers in a community are connected to each other through the bi-directional power and communication links, and a whole community microgrid is connected to the upstream utility grid via a one grid connection point. Smart

3.4. System Model

Figure 3.1: Structure of a community microgrid for P2P trading

meters are installed at each prosumer. Also, each prosumer has a local workstation with an energy management system called prosumer energy management system (P-EMS). P2P energy trading algorithm is integrated with the P-EMS software. The smart meter measures the prosumer's generation, consumption, and energy transaction with other prosumers or with the grid and sends information to the local workstation for processing.

It is assumed that there is an agent called P2P market operator (P2PMO) which assists in energy trading via a P2P market in a community. P2PMO is a part of the distribution system operator and is assumed to serve free of charge to assist the P2P trading because P2P trading helps to reduce the congestion in the distribution system. Since the prosumers are serving nearby and the amount of energy traded in the P2P market is small, we assume that distribution losses and distribution cost are negligible. Smart meters are capable of communicating with other entities and work stations are powerful enough to carry out the computational tasks. During the trading process, all the communication tasks are done through the smart meters, and computations are performed in local workstations. With the existing infrastructures explained, the main aim of the study is to develop an algorithm

for the P2P energy trading.

Let $\mathcal{N} = \{1, 2, \dots, N\}$ denote the set of the prosumers in the community with $n \in \mathcal{N}$ and $N \triangleq |\mathcal{N}|$ gives the total number of prosumers in the community. It is assumed that the total operation time is divided into different slots of equal interval Δt . In this study we have considered $\Delta t = 1$ hour. Let $\mathcal{T} = \{1, 2, \dots, T\}$ denote the set of all operation time slots with $t \in \mathcal{T}$ and $T \triangleq |\mathcal{T}|$ gives the total number of operation time slots.

The PV generation profile of prosumer n during a day can be defined as:

$$\mathbf{G}_n = \{G_n^1, G_n^2, \dots, G_n^T\}, \quad n \in \mathcal{N} \quad (3.1)$$

The nominal demand profile of prosumer n during the same period is defined as:

$$\mathbf{D}_n = \{D_n^1, D_n^2, D_n^3, \dots, D_n^T\}, \quad n \in \mathcal{N} \quad (3.2)$$

The nominal demand represents the demand from regular appliances before the implementation of any DR program and the installation of BESS.

3.4.2 Battery Energy Storage Systems

Let $E_n^{t-\Delta t}$ and E_n^t denote the battery energy level of prosumer n at the beginning and end of the time slot t ; $P_{ch,n}^t$ denote the battery charging power of prosumer n at time slot t ; and $P_{dch,n}^t$ denote the battery discharging power of prosumer n at time slot t . Let $\eta_{ch,n}^t$ and $\eta_{dch,n}^t$ denote battery charging and discharging efficiency of prosumer n . Assuming the self-discharge of the battery is negligible and the charge/discharge power remains constant during a time slot, the dynamics of the battery level can be modeled as [43]

$$E_n^t = E_n^{t-\Delta t} + \left(X_n^t P_{ch,n}^t \eta_{ch,n}^t - \frac{\Psi_n^t P_{dch,n}^t}{\eta_{dch,n}^t} \right) \times \Delta t \quad (3.3)$$

where X_n^t and Ψ_n^t are binary variables of prosumer n related to charge and discharge states in time slot t .

To avoid simultaneous charging and discharging of the battery

$$X_n^t + \Psi_n^t \leq 1 \quad (3.4)$$

3.4. System Model

In practical applications, the energy stored in a battery is restricted within a certain range, i.e.,

$$\underline{E}_n \leq E_n^t \leq \overline{E}_n \quad (3.5)$$

where $\underline{E}_n$ is the maximum capacity of the battery and $\overline{E}_n$ is the minimum amount of energy that prevents the battery from deep discharge. The limits of charging and discharging power are determined by the size of the inverter as follows

$$\begin{aligned} 0 \leq P_{ch,n}^t &\leq \overline{P}_{ch,n} \\ 0 \leq P_{dch,n}^t &\leq \overline{P}_{dch,n} \end{aligned} \quad (3.6)$$

where $\overline{P}_{ch,n}$ and $\overline{P}_{dch,n}$ are the maximum charging and discharging power.

In the conventional peer-to-grid (P2G) trading, the excess PV energy of the prosumer is used to charge the battery until the battery is fully charged and the remaining is fed back to the grid. The stored energy is used to meet the demand when the PV generation is lower than the demand. The prosumer buys energy from the grid when there is insufficient energy from the PV and battery system.

In P2P energy trading, if any prosumer has excess PV energy, the first priority is to supply to the neighbors who have unmet consumption in the community and if there is any remaining power, that is used to charge the battery. When the demand is higher than the PV generation, some of the unmet demand is satisfied by buying the PV power from the neighbors through the P2P market first, and the remaining, if there is any, is met by discharging the own battery and buying power from the grid. Furthermore, the stored energy in a particular prosumer's battery can be used to supply its own demand only when that prosumer participates in the P2P market. This is because if any prosumer can sell stored energy in the P2P market, the prosumer having high storage capacity and high excess generation during certain hours can influence the market in other hours, which discourage other prosumers to participate in the P2P market.

The use of battery systems should be justified by cost saving due to its use. as explained below. If C_c is the capital cost of the battery system (i.e., the cost of

the battery and converter); C_m is the annual maintenance cost of the battery; and N_{days} is the number of days in a year, then the equivalent daily cost (EDC) of the battery is given by [118]

$$EDC = C_c \times \frac{x(1+x)^M}{(1+x)^M - 1} \times \frac{1}{N_{days}} + \frac{C_m}{N_{days}} \quad (3.7)$$

where x is the discount rate and M is the lifetime of the battery in years. In this work, the expected value of the lifetime of the battery provided by the manufacturer is considered to calculate the EDC of the battery. However, in the dynamic approach, the battery lifetime is estimated online using the accumulated usage cycles, and the predefined depth-of-discharge [119]. It is quite obvious that the battery with the longer expected lifetime has less EDC. The detailed study on the trade-off between the EDC and the expected lifetime of the battery is beyond the scope of this thesis. The use of the battery is beneficial only if the saving in a day is higher than the EDC.

3.4.3 Classification of Prosumers as a Seller and a Buyer

The generation-to-demand ratio (GDR) of a prosumer $n \in \mathcal{N}$ in a given time period $t \in \mathcal{T}$ is defined as

$$GDR_n^t = \frac{G_n^t}{D_n^t} \quad (3.8)$$

Let $\mathcal{P} = \{n \in \mathcal{N} | GDR_n^t \geq 1\}$ represent the set of sellers at time slot t with index $i \in \mathcal{P}$ and $n_p = |\mathcal{P}|$ gives the total number of sellers at time t .

Let $\mathcal{C} = \{n \in \mathcal{N} | GDR_n^t < 1\}$ represent the set of buyers at time slot t with index $j \in \mathcal{C}$ and $n_c = |\mathcal{C}|$ gives the total number of buyers at time t .

The amount of power the prosumer $i \in \mathcal{P}$ can sell (export) at time t is

$$P_{ex,i}^t = (GDR_i^t - 1)D_i^t \quad (3.9)$$

3.4.4 Non-flexible and Flexible Demand of Prosumer

Suppose that d_n^t , y_n^t and z_n^t represent the total demand, non-flexible demand and flexible demand of the prosumer $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ respectively. The total

3.4. System Model

demand of prosumer n at any time t can be defined as:

$$d_n^t = y_n^t + z_n^t \quad (3.10)$$

Let $\mathbf{Y}_{o,n}$ and $\mathbf{Z}_{o,n}$ denote the nominal non-flexible and flexible demand profile of the prosumer $n \in \mathcal{N}$. According to (3.10), the nominal demand of the prosumer n can be expressed as

$$\mathbf{D}_n = \mathbf{Y}_{o,n} + \mathbf{Z}_{o,n} \quad (3.11)$$

The critical (non-flexible) loads are fixed and have no contribution to DR. The total demand of the prosumer can be modified by adjusting the flexible demand.

The adjusted demand of the prosumer $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ can be defined as

$$d_n^t = y_{o,n}^t + z_n^t \quad (3.12)$$

where, $y_{o,n}^t$ is the nominal non-flexible demand of prosumer n at time t .

Assume that all the flexible loads are continuous in nature. Their reduction affects the comfort level and reduces the energy cost of the prosumers. If the amount of flexible load in nominal case at any time t is $\varrho_f D_n^t$, then

$$0 \leq z_n^t \leq \varrho_f D_n^t, \quad 0 \leq \varrho_f \leq 1 \quad (3.13)$$

where ϱ_f is a fraction of the flexible demand, the minimum value of z_n^t is $\underline{z}_n^t = 0$, and maximum value of z_n^t is $\bar{z}_n^t = \varrho_f D_n^t$.

The minimum demand occurs when all the flexible loads are off, i.e., $z_n^t = \underline{z}_n^t$. The amount of non-flexible demand $y_{o,n}^t$, which is also the minimum demand at the same time after adjustment and is given by

$$\underline{d}_n^t = y_{o,n}^t = (1 - \varrho_f) D_n^t \quad (3.14)$$

Similarly, the maximum demand occurs when all the flexible loads are on, i.e., $z_n^t = \bar{z}_n^t$ and given by

$$\bar{d}_n^t = y_{o,n}^t + \bar{z}_n^t = (1 - \varrho_f) D_n^t + \varrho_f D_n^t = D_n^t \quad (3.15)$$

The maximum demand after the adjustment is capped at the nominal demand. They will not achieve more satisfaction by using more than they want at that particular time. Those prosumers willing to participate in the DR program has a

provision to adjust their demand between $\underline{d}_n^t$ and $\bar{d}_n^t$, i.e.,

$$\underline{d}_n^t \leq d_n^t \leq \bar{d}_n^t \quad (3.16)$$

The terms d_n^t , $\underline{d}_n^t$ and $\bar{d}_n^t$ are the actual power consumption, its lower limit and upper limit for prosumer n at time t , respectively. In this study, only the buyers will participate in the DR program because the sellers already have enough generation to satisfy their own demand with full satisfaction.

3.4.5 Utility Function

A quadratic utility function is considered to quantify the satisfaction of prosumer $n \in \mathcal{N}$ at time $t \in \mathcal{T}$. Based on (2.12), prosumer's utility function can be defined as

$$U_n^t(d_n^t) = \begin{cases} \beta_n d_n^t - \frac{1}{2} \theta_n (d_n^t)^2 & : 0 \leq d_n^t \leq \frac{\beta_n}{\theta_n} \\ \frac{\beta_n^2}{2\theta_n} & : d_n^t \geq \frac{\beta_n}{\theta_n} \end{cases} \quad (3.17)$$

where $\beta_n > 0$ is a preference parameter characterizing the prosumers's behaviours, which vary from prosumer to prosumer and $\theta_n > 0$ is a predetermined constant. The utility should have all the properties given in Section 2.3.2.

3.4.6 Welfare Function for Buyers and Sellers

If any buyer $j \in \mathcal{C}$ chooses seller $i \in \mathcal{P}$, the welfare of the buyer j is defined as

$$w_{ji}^t = U_j^t(d_j^t) - \lambda_i^t p_{ji}^t \quad (3.18)$$

For any buyer $j \in \mathcal{C}$,

$$\begin{aligned} \underline{d}_j^t &\leq d_j^t \leq \bar{d}_j^t \\ d_j^t &= p_{ji}^t + G_j^t \end{aligned} \quad (3.19)$$

Based on (3.19), the condition (3.20) is always true.

$$\begin{aligned} \underline{p}_{ji}^t &= \underline{d}_j^t - G_j^t \\ \bar{p}_{ji}^t &= \bar{d}_j^t - G_j^t \\ \underline{p}_{ji}^t &\leq p_{ji}^t \leq \bar{p}_{ji}^t \end{aligned} \quad (3.20)$$

where λ_i^t , p_{ji}^t , $\underline{p}_{ji}^t$ and $\bar{p}_{ji}^t$ are the price decided by seller i , actual power the buyer

3.5. Methodology

j buys from seller i and its lower and upper power limits buyer j can buy from seller i at time t , respectively. Now, the welfare function of any buyer j when it chooses only seller $i \in \mathcal{P}$ can be expressed as

$$w_{ji}^t(p_{ji}^t) = U_j^t(p_{ji}^t + G_j^t) - \lambda_i^t p_{ji}^t \quad (3.21)$$

The welfare function of the seller $i \in \mathcal{P}$ can be defined as

$$W_i^t = U_i^t(d_i^t) + \lambda_i^t \min(P_{ex,i}^t, S_i^t) \quad (3.22)$$

S_i^t is the total demand of electricity comes to seller i at time t and is defined in (3.24). For any seller the nominal demand is always satisfied so that $d_i^t = D_i^t$.

3.5 Methodology

3.5.1 P2P Energy Trading Between Buyers and Sellers

The aim of P2P trading is to maximize the welfare of both buyers and sellers, and reduce the dependency on the upstream grid. P2P energy trading is carried out in the following steps:

1. Prosumers register into a P2P market as a seller or a buyer based on their GDR.
2. After grouping of prosumers as a seller or a buyer, P2PMO assigns a unique and encrypted identity to each buyer and seller, which maintains the anonymity of sellers and buyers ensuring privacy.
3. The P2PMO sends uniquely assigned buyers identities to all sellers and sellers identities to all buyers. This establishes direct communication between sellers and buyers.
4. Each anonymous seller and each anonymous buyer participate in a P2P energy trading game to obtain the stable state.
5. Finally, P2PMO receives information about the final price and amount of

energy traded anonymously by different prosumers to settle the financial transactions.

There are multiple sellers and multiple buyers in the community. Two levels of competition exist in this multi-seller and multi-buyer P2P energy trading market as shown in Fig. 3.2.

Figure 3.2: Two-levels of competition in P2P market

The competition at the higher level is among the sellers to sell their excess energy to the group of buyers at the lower level. The competition among the sellers is modeled as a non-cooperative game. The competition at the lower level among multiple buyers is to select the sellers to buy energy offered by them. An evolutionary game approach is used for competition among the buyers.

In addition to the separate competition at two levels of the P2P market, there is an interaction between the two levels. The main aim of this interaction between two levels is to set the appropriate strategies for both sellers and buyers to maintain the balance between supply and demand in the P2P market. The interaction between the two levels is modeled as a Stackelberg game.

3.5.2 Evolutionary Game Among Buyers

In general, the players of the game are arranged in multiple groups. In this work, without loss of generality, all the buyers are arranged in a single group and there is only one population scenario in the game. The buyers should possess the following behaviors to participate in the evolutionary game.

1. A buyer chooses the seller from whom to buy power such that the welfare is maximized.
2. A buyer learns and gradually changes the decision on the seller selection.
3. A buyer does not have any intention to influence the decision of other buyers in the future and behaves independently.

The evolutionary game for P2P energy trading in the community can be precisely defined as follows:

- Players: buyers $j \in \mathcal{C}$
- Population: set of buyers
- Strategy: selection of seller
- Utility: welfare function of the buyers

All buyers are arranged in a single population group, so all the buyers will exhibit the same behavior, i.e., the strategy of each buyer in the population is identical. Buyers select a seller to purchase power when they receive the price announced by sellers. Each buyer then gradually adjusts its selection strategy and acts independently in the selection process [70].

Let γ_i^t be the probability of the buyer $j \in \mathcal{C}$ choosing a seller $i \in \mathcal{P}$ at time t , where $0 \leq \gamma_i^t \leq 1$ and $\sum_{i=1}^{n_p} \gamma_i^t = 1$. Since the strategies of all the population (buyers) are identical, the population state can be denoted as $\gamma^t = [\gamma_1^t, \gamma_2^t, \dots, \gamma_{n_p}^t]$.

When buyer j selects seller i , the optimal amount of power buyer j buying from

seller i can be achieved by maximizing the welfare given by (3.21) subject to (3.20).

Mathematically,

$$p_{ji}^{t*} = \arg \max_{p_{ji}^t} w_{ji}^t \quad (3.23)$$

The total demand of electricity comes to the seller i at time t is given by

$$S_i^t = \gamma_i^t \sum_{j=1}^{n_c} p_{ji}^{t*} \quad (3.24)$$

The supply-to-demand ratio (SDR) for seller i at time t can be defined as

$$\chi_i^t = \frac{P_{ex,i}^t}{S_i^t} = \frac{P_{ex,i}^t}{\gamma_i^t \sum_{j=1}^{n_c} p_{ji}^{t*}} = \frac{\alpha_i^t}{\gamma_i^t} \quad (3.25)$$

where, $\alpha_i^t = \frac{P_{ex,i}^t}{\sum_{j=1}^{n_c} p_{ji}^{t*}}$ and α_i^t , $P_{ex,i}^t$, and λ_i^t are constant during the evolution. The actual amount of power that buyer j buys from the seller i at time t is

$$p_{ji,actual}^t = \begin{cases} \gamma_i^t p_{ji}^{t*} & \chi_i^t \geq 1 \\ \chi_i^t \gamma_i^t p_{ji}^{t*} & \chi_i^t < 1 \end{cases} \quad (3.26)$$

Assume that the net utility σ_i^t associated with the seller i at time t can be defined as the accumulated welfare of all buyers obtained from seller i . There are two possibilities:

Case 1: If $P_{ex,i}^t \geq S_i^t$, then the available power is more than the demand from the buyers and all the demand can be satisfied.

Case 2: If $P_{ex,i}^t < S_i^t$, then all the demand cannot be satisfied and buyer j can receive only the $\chi_i^t p_{ji}^{t*}$ amount of power from seller i .

The replicator dynamics is designed to depict the selection dynamics of the buyers as follows:

$$\frac{\partial \gamma_i^t}{\partial t} = \gamma_i^t (\sigma_i^t - \bar{\sigma}^t) \quad (3.27)$$

where, $\bar{\sigma}^t$ denotes the average utility, and it can be calculated as

$$\bar{\sigma}^t = \sum_{i=1}^{n_p} \gamma_i^t \sigma_i^t \quad (3.28)$$

A stable condition at which population will not change its selection is referred to as an evolutionary stable strategy. Mathematically,

$$\frac{\partial \gamma_i^t}{\partial t} = \dot{\gamma}_i^t = 0 \quad (3.29)$$

The condition for stable state in evolutionary game can also be written as

$$\sigma_1^t = \sigma_2^t = \dots = \sigma_{n_p}^t = \bar{\sigma}^t \quad (3.30)$$

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If we consider the dynamics of $\sum_{i=1}^{n_p} \gamma_i^t$, then

$$\begin{aligned} \frac{\partial \sum_{i=1}^{n_p} \gamma_i^t}{\partial t} &= \sum_{i=1}^{n_p} \gamma_i^t (\sigma_i^t - \bar{\sigma}^t) \\ &= \sum_{i=1}^{n_p} \sigma_i^t \gamma_i^t - \bar{\sigma}^t \sum_{i=1}^{n_p} \gamma_i^t \\ &= \bar{\sigma}^t - \bar{\sigma}^t \sum_{i=1}^{n_p} \gamma_i^t \end{aligned} \quad (3.31)$$

Using (3.29) and (3.31), it can be proved that $\sum_{i=1}^{n_p} \gamma_i^t = 1$ is always valid during the game. The equilibrium in the evolutionary game is denoted by $\gamma^{t*} = [\gamma_1^{t*}, \gamma_2^{t*}, \dots, \gamma_{n_p}^{t*}]$. The convergence of the game to the evolutionary equilibrium given in (3.29) using the replicator dynamics given in (3.27) can be proved via Lyapunov theory [120].

The replicator dynamics can be approximated by using the discrete replicator in iterative way though it is time continuous. The discrete replicator for approximation of replicator dynamics is defined as follows:

$$\gamma_i^t(k+1) = \gamma_i^t(k) + \rho_1 \gamma_i^t(k) (\sigma_i^t(k) - \bar{\sigma}^t(k)) \quad (3.32)$$

The termination criterion is defined as

$$|\sigma_i^t(k) - \bar{\sigma}^t(k)| < \epsilon \quad (3.33)$$

where k is the iteration number, ρ_1 is the adjustment parameter, and ϵ is a small positive constant. An algorithm to reach the evolutionary equilibrium is given in Algorithm 1. The results from evolutionary game among buyers are illustrated in Section 3.6.1.

3.5.3 Non-cooperative Game Among Sellers

Each seller aims at maximizing its own welfare by selling the power to buyers who need it. They are non-cooperative and behave in rational manner. A non-cooperative game is used to model the competition among the sellers as follows:

- Players: selling role prosumers or sellers $i \in \mathcal{P}$
- Strategy: the price λ_i^t , $\forall i \in \mathcal{P}$ and $\forall t \in \mathcal{T}$

Algorithm 1 Algorithm for evolutionary game among buyers**Input:** Price vector $\lambda^t = [\lambda_1^t, \lambda_2^t, \dots, \lambda_{n_p}^t]$ from the sellers**Output:** Equilibrium state $\gamma^{t*} = [\gamma_1^{t*}, \gamma_2^{t*}, \dots, \gamma_{n_p}^{t*}]$

- 1: **Initialize:** Initial population state $\gamma^t(1) = [\gamma_1^t(1), \gamma_2^t(1), \dots, \gamma_{n_p}^t(1)]$, such that $\sum_{j=1}^{n_p} \gamma_j^t(1) = 1$;
- 2: $k = 0$;
- 3: **repeat** ▷ Executed by all buyers in parallel for all sellers.
- 4: $k = k + 1$;
- 5: Choose seller $i \in \mathcal{P}$ and compute the optimal amount of power p_{ji}^{t*} that buyer j can buy from seller i according to (3.23);
- 6: Seller i computes $\chi_i^t(k)$ according to (3.25);
- 7: Compute the net utility of seller i either in *Case 1* or *Case 2*;
- 8: Compute the average value of net utility $\bar{\sigma}^t(k)$ according to (3.28);
- 9: Update the replicator dynamics according to (3.32);
- 10: **until** $(|\sigma_i^t(k) - \bar{\sigma}^t(k)| > \epsilon)$

- Utility: the welfare of the sellers

The welfare function of the seller $i \in \mathcal{P}$ given in (3.22) can be re-written as

$$W_i^t = \begin{cases} U_i^t(d_i^t) + \lambda_i^t S_i^t & P_{ex,i}^t > S_i^t \\ U_i^t(d_i^t) + \lambda_i^t P_{ex,i}^t & P_{ex,i}^t \leq S_i^t \end{cases} \quad (3.34)$$

The solution of the game is called NE which includes the energy price and amount of energy sold. NE exists in the game if [121]

- The player set is finite.
- The strategy sets are closed, convex, and bounded.
- The utility functions are continuous and quasi-concave in the strategy space.

There are n_p number of sellers, so that the player set is always finite. The game first starts by announcing the price, λ_i^t by the seller. There are limits on λ_i^t such that $\lambda_i^t \in [\underline{\lambda}_i^t, \bar{\lambda}_i^t]$. The lower bound $\underline{\lambda}_i^t$ is normally determined by the generation cost and the operating expenses. The upper bound $\bar{\lambda}_i^t$ is fixed by the government policies. The sellers must consider the price limits during the game. The FIT (ρ_{sell}) is chosen as the lower limit and the grid buying price (ρ_{buy}) as the upper limit such that $\lambda_i^t \in [\rho_{sell}, \rho_{buy}]$. Hence, the strategy sets are nonempty, closed, bounded, and convex.

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The utility function given in (3.34) is a continuous function of λ_i^t . Since the welfare W_i^t of seller i increases with the amount of power for a given λ_i^t , the seller shall assume that all the available power to export ($P_{ex,i}^t$) can be sold to the buyers. The game will be stopped when all the $P_{ex,i}^t$ is sold or all S_i^t is satisfied.

If $P_{ex,i}^t \leq S_i^t$, then

$$\frac{\partial^2(W_i^t)}{\partial(\lambda_i^t)^2} = 0 \quad (3.35)$$

If $P_{ex,i}^t > S_i^t$, then

$$\frac{\partial^2(W_i^t)}{\partial(\lambda_i^t)^2} = -2\lambda_i^t \sum_{j=1}^{n_c} \frac{1}{\theta_j} < 0 \quad (3.36)$$

Combining (3.35) and (3.36), we have $\frac{\partial^2(W_i^t)}{\partial(\lambda_i^t)^2} \leq 0$, so that W_i^t is always quasi-concave in λ_i^t . Therefore, we can conclude that NE exists in the non-cooperative game among the sellers.

3.5.4 Stackelberg Game Between Sellers And Buyers

The interaction behaviors between the sellers and buyers can be modeled as an M-leader and N-follower Stackelberg game. The sellers are the multiple leaders and the buyers are the multiple followers. The Stackelberg game establishes a relationship between the evolutionary game and the non-cooperative game. The output of the non-cooperative game, i.e., price vector, is used as an input to the evolutionary game to update the seller selection strategy. The output of the evolutionary game, i.e., evolutionary stable strategy or seller selection probability, is used as an input to the non-cooperative game to update the price vector. So, all the three games are related to one another. All the buyers receive the announced price vector from the sellers and participate in the evolutionary game. Once the evolutionary stable strategy is obtained in the evolutionary game, sellers update their price to obtain an NE, i.e., price.

The sellers are involved in a non-cooperative game and each seller does not know the information of other sellers. In this scenario, it is not possible to obtain the

NE via analytical method and an iterative approach must be used. An iterative distributed algorithm is designed to obtain the NE among the sellers such that the Stackelberg equilibrium (SE) is reached in the Stackelberg game between sellers and buyers. The price updating strategy of the seller i is designed as follows:

$$\lambda_i^t(r+1) = \lambda_i^t(r) + \rho_2(S_i^t(r) - P_{ex,i}^t) \quad (3.37)$$

The termination criterion is defined as

$$|\lambda_i^t(r+1) - \lambda_i^t(r)| < \epsilon \quad (3.38)$$

where r is the iteration counter; ρ_2 is the speed adjustment parameter; and ϵ is a small positive number.

The energy trading game starts with an announcement of the price vector from the sellers. The buyers then receive the announced price vector and play the evolutionary game. When the buyers obtain evolutionary equilibrium for a given price vector, they send the information of evolutionary stable strategy back to the sellers. Once the sellers receive the updated strategy from the buyers, they participate in the non-cooperative game and update their price, and the process is repeated until the final equilibrium state is reached. The detail process is given in Algorithm 2. The results from the proposed algorithm are presented in Section 3.6.2

The proposed P2P energy trading algorithm uses an iterative pricing mechanism. The demand from the buyers would change in response to the prices provided by the sellers, and this change will, in turn, affects the sellers' prices. If there are large numbers of prosumers, a change in the energy demand may be too high so that the prices may oscillate heavily. This fact may lead to an outcome where the seller selection and pricing iteration never end, i.e., the algorithm does not converge. The convergence of the algorithm is essential for its implementation. If the convergence cannot be guaranteed, control measures should be taken to enhance the convergence of the algorithm.

The step length control method [122] is used to deal with the effect of random

3.6. Simulation Results

exploration for better convergence of the algorithm. The step length control at each iteration of price update can be achieved by

$$\max(\lambda_i^t(r) - \Delta, \underline{\lambda}_i^t) \leq \lambda_i^t(r+1) \leq \min(\lambda_i^t(r) + \Delta, \bar{\lambda}_i^t) \quad (3.39)$$

$$\Delta = \zeta |\lambda_i^t(r)|$$

where ζ is a non-negative ramping rate.

This technique limits the ramping rate of the price in the iterative process. The buyers reject any price that exceeds the prescribed ramping rate so that all the sellers have to obey the ramping rate limit. Since the buyers' strategies are the response to the sellers' price, a step length control method for price update in Algorithm 2 ultimately improves the convergence of Algorithm 1.

Algorithm 2 Algorithm for Stackelberg Equilibrium

Input: Initial strategy of the sellers

Output: Stackelberg Equilibrium state

```

1: for  $t \in \mathcal{T}$  do
2:   Randomly initialize  $\lambda^t(1) = [\lambda_1^t(1), \lambda_2^t(1), \dots, \lambda_{n_p}^t(1)]$ ;
3:    $r = 0$ ;
4:   repeat
5:      $r = r + 1$ ;
6:     Execute Algorithm 1;
7:     Compute the power demand  $S_i^t(r)$  of all the sellers according to (3.24);
8:     Update the price according to (3.37) and (3.39);
9:   until  $(|\lambda_i^t(r+1) - \lambda_i^t(r)| > \epsilon)$ 
10: end for
```

3.6 Simulation Results

This section presents the results of simulation studies to assess the performance of the proposed game-theoretic model for P2P energy trading in a prosumer based community microgrid. A community microgrid consists of five prosumers is considered. The community microgrid is connected to the utility grid, thus the prosumers can trade with each other as well as the the retailers. Each prosumer has a solar PV system. Each prosumer has both the flexible and non-flexible demand. The generation and load profiles of each prosumers in the community are taken

from [45]. For one day study $\mathcal{T} = \{1, 2, 3, \dots, 24\}$.

The value of β is selected randomly from [5,10], and θ is taken as 0.5. Based on the actual electricity price of Singapore, the buying price of electricity from the grid (ρ_{buy}) is taken as 20 cents/kWh. The selling price to the grid (ρ_{sell}) is assumed 2 cents/kWh. Since the solar PV is used as a local generation, it is assumed that the P2P market is active only during the day time from 9:00 to 18:00. Prosumer 2 and Prosumer 5 have installed the battery of 20 kWh capacity. Lithium-ion batteries are considered in this work. The minimum battery capacity $\underline{E}_n$ is 4 kWh, the charging/discharging efficiency is taken as 90%, and the charging and discharging capacity assumed to be 3 kW. The capital cost (C_c) of 20 kWh Tesla Powerwall is 7,800 SGD. The discount rate x is taken 5% in this paper and the lifetime is set to 15 years and maintenance cost (C_m) for a given size of the battery is 150 SGD. Based on the data and the proposed algorithm, all the simulations are conducted in the MATLAB 2016a environment with an Intel(R) Xeon (R) E5-1630 v4@3.7 GHz processor, 16 GB RAM.

Based on the value of GDR, the prosumers can behave as sellers or buyers during the day time. The GDR of the prosumers at each time slot is shown in Fig. 3.3. The time slot 11 is chosen to demonstrate the performance of the proposed method. At time slot 11, Prosumer 1 and Prosumer 5 are treated as sellers and the remaining three prosumers are buyers.

3.6.1 Convergence of Evolutionary Game Among Buyers

Each buyer participates in evolutionary game to find the probability of buying power from a particular seller, which is an evolutionary stable strategy. Fig. 3.4 shows the convergence characteristics of the evolutionary game and it depicts that buyers converge to the stable strategy by executing Algorithm 1. The convergence characteristics of the average net utility is shown in Fig. 3.5. The average net utility is settled at a certain value after it reaches to a maximum value. The

3.6. Simulation Results

Figure 3.3: GDR of all prosumers

convergence process of the mismatch between the individual net utility and average net utility is shown in Fig. 3.6. Fig. 3.5 and Fig. 3.6 ensure that buyers achieve better welfare at the evolutionary stable strategy.

Figure 3.4: Convergence of evolutionary game at $t = 11$

3.6.2 Convergence of Non-cooperative Game and Stackelberg Game

Algorithm 2 is proposed for two purposes: i) to perform the price competition among the sellers using non-cooperative game, and ii) to carry out the negotiation between the buyers and the sellers using Stackelberg game. Since the sellers are

Figure 3.5: Convergence of average net utility at $t = 11$ Figure 3.6: Convergence of mismatch between individual net utility and average net utility at $t = 11$

leaders of the game, the solution of the Stackelberg game is an optimal response for the price announced by the sellers. Fig. 3.7 shows the convergence characteristics of sellers price to the NE. As the seller price approaches NE, the power demand from the buyers to the seller also converges to a certain value and ensures the existence of the SE in the trading process. The convergence characteristics of power demand of sellers are shown in Fig. 3.8. The welfare of the sellers gradually increases and settles at a certain maximum value as shown in Fig. 3.9. The convergence characteristics of SDR is given in Fig. 3.10. It ensures that power demand and supply match at the end of the game.

3.6. Simulation Results

Figure 3.7: Convergence of price of sellers at $t = 11$

Figure 3.8: Convergence of power demand of sellers at $t = 11$

Figure 3.9: Convergence of welfare of sellers at $t = 11$

Figure 3.10: Convergence of SDR of sellers at $t = 11$

The amount of power (in kW) buyer j can buy from seller i at the equilibrium state is given as

$$\begin{array}{c} \text{Buyers (2, 3, 4)} \\ \text{Sellers (1, 5)} \end{array} \begin{bmatrix} 3.31 & 84.38 & 43.56 \\ 3.31 & 84.45 & 43.56 \end{bmatrix}$$

The actual amount of power (in kW) the buyer buys from the seller is given by (3.26) is

$$\begin{array}{c} \text{Buyers (2, 3, 4)} \\ \text{Sellers (1, 5)} \end{array} \begin{bmatrix} 0.8141 & 20.7529 & 10.7130 \\ 2.4959 & 63.6771 & 32.8470 \end{bmatrix}$$

The total power procured by Prosumer 2, Prosumer 3, and Prosumer 4 from the P2P market is 3.31 kW, 84.43 kW, and 43.56 kW respectively, which satisfy the power requirement of the buying role prosumers.

3.6.3 Comparison of Results

In this section, the results of proposed method are analyzed and compared with existing methods from various aspects to evaluate its performance. At first, three

3.6. Simulation Results

cases are considered: P2G trading without BESS, P2P without both DR and BESS, and P2P with DR but without BESS using the proposed game theoretic model. The total power import from the grid and the total power export to the grid in above three cases are shown in Fig. 3.11 and Fig. 3.12 respectively. The effect of P2P trading and DR application can be observed from $t = 9$ to $t = 18$ in Fig. 3.11 and Fig. 3.12. This is the assumed operation time of the P2P market. In Fig. 3.11, the power import from grid reduces with the application of P2P method and reduces further with the application of the P2P method with DR using the proposed method. Similarly, power export to the grid is reduced with the application of the P2P with DR using the proposed method compared to P2G but can be seen nearly equal when compared with P2P without DR. Hence, Fig. 3.11 and Fig. 3.12 highlight that the proposed methodology reduces the dependency on the grid by efficiently allocating all the available energy within the community microgrid.

Figure 3.11: Total power import from the grid without BESS

The overall cost of each prosumer using various approaches of P2P trading is given in Table 3.1. The following six scenarios/methods are considered for the numerical studies:

- *Scenario 1*: P2G trading (Base case).
- *Scenario 2*: P2P with DR (proposed method).

Figure 3.12: Total power export to the grid without BESS

- *Scenario 3*: P2P without DR (proposed method).
- *Scenario 4*: P2P without DR using BS method [115].
- *Scenario 5*: P2P without DR using MMR method [115].
- *Scenario 6*: P2P without DR using SDR method [45].

The total cost of the community microgrid without BESS is shown in Fig. 3.13. It can be observed that the total cost from P2G trading is the highest among all methods, and P2P trading with DR using the proposed method is the lowest. Column 3 of Fig. 3.13 indicates that the total cost from the proposed method is less than the cost obtained by the existing methods, even if P2P trading is implemented without DR. Furthermore, in Fig. 3.13, the cost of the microgrid with DR using the proposed method can be seen significantly less compared to the cost of the microgrid using the P2G method. The cost of the microgrid with DR using the proposed is only 88.13% of the cost in the P2G trading. Therefore, the proposed game-theoretic model is effective in handling the P2P trading in a prosumer based community microgrid.

The EDC of the battery is 2.45 SGD as given by (3.7). The deployment of BESS is feasible only if the cost saving due to the use of BESS is higher than the EDC. Compared to the base case (i.e., P2G trading without BESS), Prosumer 2 and

3.6. Simulation Results

Table 3.1: Overall daily cost of prosumers in various scenarios/methods

Scenario	Overall cost without BESS (SGD)					Overall cost with BESS (SGD)				
	P_1	P_2	P_3	P_4	P_5	P_1	P_2^*	P_3	P_4	P_5^*
1	371.25	460.75	814.96	285.24	444.99	371.25	458.22	814.96	285.24	442.42
2	314.63	390.71	699.64	258.95	431.17	314.63	389.85	699.64	258.95	428.20
3	348.17	433.36	747.29	254.78	421.85	348.17	432.63	747.29	254.78	420.57
4	350.43	435.12	764.65	268.29	452.53	-	-	-	-	-
5	343.64	444.92	776.00	272.89	406.57	-	-	-	-	-
6	337.35	455.50	740.33	268.05	442.79	-	-	-	-	-

*Indicate the prosumer with energy storage.

Figure 3.13: Total cost of community microgrid without BESS

Prosumer 5 can save 0.08 SGD and 0.12 SGD respectively in P2G trading with BESS. When all the prosumers in the community participate in P2P trading using the proposed method without both DR and BESS, Prosumer 2 and Prosumer 5 can save 27.39 SGD and 23.14 SGD respectively. If they participate in P2P without DR but with BESS, Prosumer 2 can save 25.67 SGD and Prosumer 5 can save 21.97 SGD. It can be seen that the saving for the prosumers who installed BESS is higher when they participate in P2P trading without BESS. This is because when they participate in the P2P market, the priority is to sell the excess energy in the P2P market. They cannot fully utilize the battery capacity. It is not necessary that all the prosumers have flexible loads, so the case without DR is chosen for analysis. This study shows that participating in the P2P energy trading in a community could be an alternative to installing the cost-intensive energy storage systems. But, the final conclusion cannot be drawn from the single case study because the P2P market is not matured yet. Several intensive studies and modifications regarding energy trading structures, current policies and legal obligations are needed before it comes in wide applications.

3.7 Summary

In this chapter, a game-theoretic model for real-time P2P energy trading in a prosumer based community microgrid is presented. The prosumer in a community involved in a P2P trading is either a seller or a buyer. The interaction between the sellers and buyers is modeled as a Stackelberg game, where sellers are leaders and buyers are followers. The seller selection competition among buyers is modeled as an evolutionary game and an iterative algorithm is proposed to reach the stable state in a game. Also, the price competition among sellers is modeled as a non-cooperative game. A distributed iterative algorithm is used to reach the equilibrium states in a non-cooperative game and Stackelberg game. The proposed method is applied to a small community microgrid with PV and energy storage systems. Simulation results show that each game converges to the stable state using the proposed algorithms. Simulation results also show that the proposed model is effective to handle the P2P energy trading in a community microgrid. The cost of the microgrid is significantly reduced with P2P energy trading in both cases: (i) with BESS and (ii) without BESS, as compared to the cost in conventional P2G trading. The stochastic nature of prosumers, distribution losses, and distribution costs are not considered in this chapter.

Chapter 4

Decentralized LET in Microgrids with Voltage Management

4.1 Introduction

Microgrids provide one avenue for sharing and monetizing values from the increasing volume of DER through the LEM. A microgrid can be defined as the part of a distribution network that accommodates a variety of small-scale DER and different types of energy users [123]. For simplicity, producers refer to DER, and consumers refer to energy users, and they are jointly called agents. The agents in the microgrid can take the necessary actions to manage their consumption/generation. The producers can provide energy to the consumers locally, i.e., they can trade energy locally within the microgrid [124]. LET in microgrids is a viable option to handle local problems due to the integration of DER into the distribution networks.

LET is one of the new concepts of growing importance in the area of distribution networks [125]. LET is beneficial to both producers and consumers compared to the trading with the utility grid. For LEMs to become an attractive option for energy management in microgrids, a suitable business model with proper pricing mechanism or market clearing mechanism is necessary to facilitate the energy

transactions among different parties [126, 127]. The pricing mechanism is crucial because an energy price determines the benefits of LET and plays a significant role in shaping agents' behavior. The pricing mechanism should be computationally efficient and set with a defined objective of the trading. Agents in the market behave independently with their interest and have a set of private information which they do not want to reveal to the public. Therefore, designing a proper pricing scheme while maintaining privacy is a challenging task.

On the other side, a physical network is used for energy transfer, and hence excluding the physical network from LET is not practical. In power networks, any local change in decisions of the users will result in global effects and cause external costs associated with it [128]. In distribution networks, node voltages are more sensitive to changes in active power due to the high value of resistance to reactance ratio of the lines [129]. Since the pricing mechanism plays a vital role in changing the decision of agents, any change in the energy price will change node voltages in the network and may also cause other external problems. Such voltage deviations can be managed using different voltage regulating devices, but may not be cost-effective for small-scale DER and consumers. The pricing mechanism should be designed in such a way that agents' internal decisions will not create any external problems in the network and avoid the cost associated with it during the LET. It makes the LET concept more implementable. The pricing mechanism should be able to handle the external effects of the internal decisions of the agents. In a nutshell, designing a proper pricing mechanism with a specific objective considering the privacy of the agents and respecting the physical network constraints is a major challenge in implementing the LET in microgrids. The focus of this chapter is on the design of a proper pricing mechanism for LET in microgrids with consideration of the privacy of the agents along with voltage management in the underlying power distribution network.

4.2 Related Works

Li *et al.* [130] presented a contract theoretic approach for energy trading between small-scale renewable energy suppliers and the aggregator. An aggregator acts as a coordinator of the energy trading process. An asymptotic Shapley value-based fair revenue division scheme is derived. A game-theoretic model for P2P energy trading is proposed in [126] to maximize the social welfare of the buyers and sellers in a prosumer based community microgrid. The centralized entity called P2PMO is required to initiate the P2P energy trading. In [127], a hybrid energy market comprising an external utility company and a local trading market managed by a local trading center is investigated. The consumers and sellers adjust their behaviors to maximize their benefits after the local trading center sets the energy price. An incentive-compatible energy trading framework based on Stackelberg game is proposed in [131]. The electricity retailer acts as a mediator between energy users and shared energy storage.

A reinforcement learning-based indirect customer-to-customer energy trading in the distribution level is proposed in [132], where an energy broker facilitates the market operation and participants update strategies with learning capability. Laihyuk *et al.* [133] proposed an unsupervised learning algorithm for clustering of prosumers in LEMs and auction mechanism for truthful electricity trading with local power exchange center as a broker.

A fog computing based novel architecture for real-time vehicle-to-vehicle energy trading on the Internet of vehicles is proposed in [134]. Energy trading is controlled by a local market manager called the fog computing energy center. The outcome of the trading depends on the interest of fog computing energy center, whether it is profit-driven or non-profit-driven, whereas vehicles are always profit-driven. A forward bilateral contract network for real-time P2P energy trading is proposed in [26], where suppliers act as intermediaries between sellers and buyers. A grid influenced P2P energy trading is proposed in [135] to reduce the peak demand on

centralized power system. The centralized power system decides the energy price which incentivizes the prosumers to participate in P2P energy trading to meet their demand and not to import any energy from the grid. Zhang *et al.* [136] proposed a contract game for direct energy trading between small-scale electricity suppliers and consumers. The energy management system for microgrids considering the physical network constraints such as node voltages, line-flow limits, etc., to ensure the effective operation of the microgrid is studied in [137–139].

The core objective in LET is to use price signals to incentivize agents in the market to provide efficient generation and demand decisions by managing their flexibilities while taking into account the technical limitations of the network. Hence, the pricing mechanism should incorporate both economic incentives of agents linked to their welfare/cost/benefit as well as technical constraints linked to the physical limitation of the grid. The business model for LET should consider both technical and economic aspects while designing the pricing mechanism. Most of the reviewed works [26, 126, 127, 130–136] focused on the economic aspect for increasing welfare or decreasing costs. But, they ignore the underlying electrical network for power distribution in the LET model, and thus the outcome may not be feasible in practice. The network constraints should be considered in designing the trading mechanism since the physical network is used for actual energy transfer. However, limited research effort has been focused on the electrical constraints of the network, which is also a crucial requirement for any successful implementation of the LET concept. The works in [137–139] consider network constraints in the efficient operation but ignore the energy pricing mechanism.

In addition, the works in [26, 126, 127, 130–136], consider a centralized coordinating agent to facilitate the LET. The energy pricing and consideration of network constraints with a centralized coordinating entity are more straightforward since all the required information can be collected centrally. However, the presence of a centralized entity could endanger the privacy of the agents, since they have a

4.2. Related Works

set of private information and they may not be willing to reveal. Moreover, the centralized system has issues of scalability and computational burden with a large number of agents. So, there is a need to design a decentralized solution for efficient LET in microgrids. Since agents in the market behave independently to fulfill their interest, the individual optimal solution may not be socially optimal. Therefore, the pricing mechanism in LET needs to be carefully designed such that individual optimal solutions align with the socially optimal solution, while the privacy of the agent is preserved, and network constraints are respected.

To this end, there is a lack of a unified framework for decentralized energy pricing in LET considering both technical and economic aspects together with the privacy of agents. The work in this chapter aims to present a decentralized algorithm for supporting LET in microgrids with an integrated pricing mechanism considering welfare maximization and network voltage management through the local information exchange among neighbors. The proposed method ensures that the energy transactions do not violate voltage constraints in a physical network and agents' privacy is preserved.

4.2.1 Major Contributions

The major contributions are summarized as follows.

1. A LET in microgrids is formulated as a social welfare maximization problem, and a fully decentralized algorithm, which neither requires any third-party nor reveals any private information of the agents while determining the energy trading price is designed.
2. A fully decentralized approach is proposed to adjust the contribution of the agents to maintain node voltages within the acceptable ranges during LET. A novel method to internalize the external effects of decision of agents on node voltages within the energy price so that the agents can trade in the LEM without affecting the network performance is also proposed.

3. An important aspect of designing any iterative algorithm is the convergence speed. In this regard, a two-stage approach is proposed to achieve faster convergence while considering voltage management in the energy pricing mechanism. The two-stage approach improves the convergence speed and increases the practicability of the algorithm for real-time implementation.

4.3 Problem Formulation

4.3.1 System Description

Consider a microgrid with several DER and responsive consumers who participate in LET as producers and consumers. All DER (consumers) loads have electronic regulators to control the power generation (consumption) level. Smart meters are installed at each consumer and DER locations. Also, each agent has a local workstation with an energy management system called a local energy management system (LEMS). The LET algorithm is integrated with the LEMS software. The microgrid is divided into two layers: the physical layer and the virtual layer. The physical layer includes electrical connection of all players, whereas the virtual layer is the communication network that can be used for information exchange among agents. Smart meters are capable of communicating with other entities, and workstations are powerful enough to carry out the computational tasks. All the communication tasks are done through the smart meters, and computations are performed in local workstations. The structure of microgrid for LET is shown in Fig. 4.1. Consider a microgrid with n_p producers and n_c consumers. The producer set is denoted by $\mathcal{P}$ and the consumer set is denoted by $\mathcal{C}$, and thus $\mathcal{N} = \mathcal{P} \cup \mathcal{C}$ and $N = n_p + n_c$. It means any agent l in the market is either a consumer, i.e., $l \in \mathcal{C}$ or a producer, i.e., $l \in \mathcal{P}$. Assume that there is one agent at each node of the microgrid network, and indices of the agents are identical to the indices of the nodes. There are N nodes as there are N agents in the microgrid.

4.3. Problem Formulation

Figure 4.1: LET platform for microgrid

4.3.2 Consumer Agent Model

A quadratic utility function given by (2.12) is considered to quantify the usage benefit of consumer $l \in \mathcal{C}$ from consuming demand of p_l . The utility $U_l(p_l)$ is the private information of consumer $l \in \mathcal{C}$ and not known to any other agents in the energy trading platform. The welfare function of the consumer $l \in \mathcal{C}$ is

$$W_l(p_l) = U_l(p_l) - \lambda p_l \quad (4.1)$$

Each consumer $l \in \mathcal{C}$ chooses demand p_l to maximize its own welfare given by (4.1) subject to its demand constraints. The minimum demand represents the demand from *must run* appliances and also called baseline demand. Assume that flexible loads of the consumers are continuous. Consumers with flexible loads adjust their demand as a function of LET platform price. Consumers with inflexible loads can be modeled by setting minimum and maximum demands equal to each other and the corresponding p_l becomes constant instead of the optimization variable [49].

4.3.3 Producer Agent Model

The cost function $C_l(p_l)$ of the DER owned by producer $l \in \mathcal{P}$ is usually approximated by a quadratic convex function of generated power p_l [73] as given by (2.13). The cost function $C_l(p_l)$ is the private information of producer $l \in \mathcal{P}$ and not known to any other agents in the energy trading platform. The welfare function of producer $l \in \mathcal{P}$ is modeled as

$$W_l(p_l) = \lambda p_l - C_l(p_l) \quad (4.2)$$

Each producer $l \in \mathcal{P}$ determines its generation p_l based on the energy trading platform price λ to maximize its welfare given by (4.2) subject to its generation constraints. If any producer has renewable-based DER such as solar PV, wind turbine, etc. with zero marginal cost, it can be modeled as *must take* generation by setting maximum and minimum generation limits equal to each other. The corresponding p_l variables become constant instead of optimization variable.

4.3.4 Microgrid Network Model

The underlying physical network of a microgrid can be modeled as a radial system represented by the connected undirected graph/tree $\mathcal{G}(\{0, \mathcal{N}\}, \mathcal{E})$. A line in $o \in \mathcal{E}$ is denoted by the pair (l, m) of nodes it connects. The incidence matrix $\mathbf{A}^0$ of size $N \times (N + 1)$ is defined for graph $\mathcal{G}$. The entries are defined as $A_{ol}^0 = 1$ if line o leaves node l ; $A_{ol}^0 = -1$ if line o enters node l ; and otherwise $A_{ol}^0 = 0$. Let $\mathbf{a}_0$ denote the first column of $\mathbf{A}^0$ and $\mathbf{A}$ be the rest of $\mathbf{A}^0$, i.e. $\mathbf{A}^0 = [\mathbf{a}_0 \ \mathbf{A}]$. The LinDistFlow model [140] can be summarized as

$$-\mathbf{A}^T \mathbf{P}_L = -\mathbf{p} \quad (4.3)$$

$$-\mathbf{A}^T \mathbf{Q}_L = -\mathbf{q} \quad (4.4)$$

$$\mathbf{a}_0 + \mathbf{A}\mathbf{v} = \mathbf{D}_r \mathbf{P}_L + \mathbf{D}_x \mathbf{Q}_L \quad (4.5)$$

where $\mathbf{D}_r$ and $\mathbf{D}_x$ are $N \times N$ diagonal matrices whose diagonal entries are constituted by r_{lm} and x_{lm} , respectively. Solving for $\mathbf{P}_L$ and $\mathbf{Q}_L$ and plugging them

4.3. Problem Formulation

into (4.5) yields

$$\mathbf{v} = \mathbf{R}\mathbf{p} + \mathbf{X}\mathbf{q} + v_o\mathbf{1} \quad (4.6)$$

where $\mathbf{R} := \mathbf{A}^{-1}\mathbf{D}_r\mathbf{A}^{-T}$ and $\mathbf{X} := \mathbf{A}^{-1}\mathbf{D}_x\mathbf{A}^{-T}$. Define $\tilde{\mathbf{v}} = v_o\mathbf{1} + \mathbf{X}\mathbf{q}$ as a vector of reference voltages that include the substation voltage and the voltage contribution from the reactive power injections. This allows us to write the nodal voltage $\mathbf{v} \in \mathbb{R}^N$ as a linear function of the nodal power injection $\mathbf{p} \in \mathbb{R}^N$:

$$\mathbf{v} = \tilde{\mathbf{v}} + \mathbf{R}\mathbf{p} \quad (4.7)$$

A matrix $\mathbf{R} \in \mathbb{R}^{N \times N}$ depends on the topology of the network and the line resistances [141].

4.3.5 Network Communication Model.

An undirected connected graph $\mathcal{G}_c = \{\mathcal{N}, \mathcal{V}\}$ is used to represent the communication topology of the network. A edge between nodes l and m is denoted by a pair $(l, m) \in \mathcal{V}$, which means agents l and m can exchange information with each other. It is assumed that self-loops are not allowed, i.e., $(l, l) \notin \mathcal{V}$ and the graph is balanced. The neighbor set of node (agent) l is defined as $\mathcal{N}_l = \{m \in \mathcal{N} | (l, m) \in \mathcal{V}\}$. The nodes can communicate bidirectionally with their physically connected neighbors. Hence, the topology of the communication network is the same as that of the power network. However, it is allowed to have a different communication network topology than the topology of the power network.

A doubly stochastic matrix $\mathbf{E} = (e_{lm})_{N \times N}$ associated with $\mathcal{G}_c$ is defined as follows

$$e_{lm} = \begin{cases} \frac{1}{\max(|\mathcal{N}_l|, |\mathcal{N}_m|) + 1} & : m \in \mathcal{N}_l \\ 1 - \sum_{m \in \mathcal{N}_l} \frac{1}{\max(|\mathcal{N}_l|, |\mathcal{N}_m|) + 1} & : l = m \\ 0 & : \text{otherwise} \end{cases} \quad (4.8)$$

4.3.6 Optimization Problem

The energy price depends on the desired objective of LET. In this work, the goal is to maximize the social welfare of the microgrid given by (4.9),

$$W(\mathbf{p}_p, \mathbf{p}_c) \triangleq \sum_{l \in \mathcal{P}} W_l(p_l) + \sum_{l \in \mathcal{C}} W_l(p_l) \quad (4.9)$$

Mathematically,

$$\begin{aligned} & \underset{\mathbf{p}_p, \mathbf{p}_c}{\text{maximize}} W(\mathbf{p}_p, \mathbf{p}_c) \\ & \text{s. t. } \underline{p}_l \leq p_l \leq \bar{p}_l \end{aligned} \quad (4.10)$$

where $\mathbf{p}_p = [p_1, p_2, \dots, p_{n_p}]^T$, & $\mathbf{p}_c = [p_1, p_2, \dots, p_{n_c}]^T$.

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4.4.1 Energy Pricing Mechanism

The optimization problem (4.10) can be reformulated as a convex problem as

$$\begin{aligned} & \underset{\mathbf{p}_p, \mathbf{p}_c}{\text{minimize}} \left[\sum_{l \in \mathcal{P}} C_l(p_l) - \sum_{l \in \mathcal{C}} U_l(p_l) \right] \\ & \text{s. t. } \mathbf{A}_p \mathbf{p}_p + \mathbf{A}_c \mathbf{p}_c = 0 \end{aligned} \quad (4.11)$$

$$\underline{p}_l \leq p_l \leq \bar{p}_l, \forall l \in \mathcal{N}$$

where $\mathbf{A}_p = [-1, -1, \dots, -1]_{1 \times n_p}$, $\mathbf{A}_c = [1, 1, \dots, 1]_{1 \times n_c}$. The Lagrangian function corresponding to (4.11) is

$$\mathcal{L} = \sum_{l \in \mathcal{P}} C_l(p_l) - \sum_{l \in \mathcal{C}} U_l(p_l) + \lambda(\mathbf{A}_p \mathbf{p}_p + \mathbf{A}_c \mathbf{p}_c) \quad (4.12)$$

The dual variable λ corresponding to the global equality constraint in (4.11) represents the price of energy in the LEM. The inequality constraints are not included in (4.12) as they are local constraints and can be treated as the boundaries of the feasible region of the local problems.

Solving (4.11) in a centralized fashion violates the privacy of the agents because the centralized coordinator needs complete information of all agents. It should be solved with minimal information exchange between the agents. Since the objective function in (4.11) is separable among agents, problem (4.11) can be solved in a

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distributed manner using the principle of dual decomposition¹ [95]. Based on the principle of dual decomposition, a distributed iterative approach for solving (4.11) can be defined as

$$\lambda^k = [\lambda^{k-1} + \rho \Delta p^{k-1}]^+, \Delta p^{k-1} = \mathbf{A}_p \mathbf{P}_p^{k-1} + \mathbf{A}_c \mathbf{P}_c^{k-1} \quad (4.13)$$

$$p_l^k = \begin{cases} \arg \min_{p_l \in \Omega^l} [C_l(p_l) - \lambda^k p_l] & : l \in \mathcal{P} \\ \arg \min_{p_l \in \Omega^l} [\lambda^k p_l - U_l(p_l)] & : l \in \mathcal{C} \end{cases} \quad (4.14)$$

where k is the iteration counter and $(\cdot)^k$ is the corresponding parameter value at k^{th} iteration; and ρ is the step size.

Each agent $l \in \mathcal{N}$ can solve (4.14) in parallel locally. But, the local optimizations in (4.14) require knowledge of the global (i.e., dual) variable λ . Updating the global variable λ using (4.13) requires the knowledge of total power mismatch Δp . The coupling between local problems and global variables in an iterative approach still requires a central coordinating agent to gather the local information about power consumption and generation levels, update the dual variable, and broadcast it to all agents. Although it can be achieved without any private cost/utility function parameters of agents, the presence of a centralized agent is undesirable as it requires all agents to communicate with the central coordinator. Inspired by [142], a fully decentralized algorithm for LET is proposed to overcome this issue. The proposed algorithm requires information exchange among neighboring agents only and constitutes a decentralized solution. Each agent can estimate the value of the required global variables by collaborating with its neighbors.

Two parameters λ and Δp are variables of concern in this problem. The update equation for agent l to estimate Δp^k locally can be written as

¹The basic principle of dual decomposition method and related examples are given in Appendix A.

$$\Delta \hat{p}_l^k = \begin{cases} \Delta \hat{p}_l^{k-1} + \sum_{m \in \mathcal{N} \setminus \{l\}} e_{lm}(\Delta \hat{p}_m^{k-1} - \Delta \hat{p}_l^{k-1}) + p_l^{k-1} - p_l^k & : l \in \mathcal{P} \\ \Delta \hat{p}_l^{k-1} + \sum_{m \in \mathcal{N} \setminus \{l\}} e_{lm}(\Delta \hat{p}_m^{k-1} - \Delta \hat{p}_l^{k-1}) + p_l^k - p_l^{k-1} & : l \in \mathcal{C} \end{cases} \quad (4.15)$$

Summing $\Delta \hat{p}_l^k$ over all producers and consumers

$$\begin{aligned} \sum_{l \in \mathcal{N}} \Delta \hat{p}_l^k &= \sum_{l \in \mathcal{N}} \Delta \hat{p}_l^{k-1} + \underbrace{\sum_l \sum_{m \in \mathcal{N} \setminus \{l\}} e_{lm}(\Delta \hat{p}_m^{k-1} - \Delta \hat{p}_l^{k-1})}_{\text{second term}} \\ &\quad + \underbrace{\left(\sum_{l \in \mathcal{C}} p_l^k - \sum_{l \in \mathcal{P}} p_l^k \right)}_{\Delta p^k} - \underbrace{\left(\sum_{l \in \mathcal{C}} p_l^{k-1} - \sum_{l \in \mathcal{P}} p_l^{k-1} \right)}_{\Delta p^{k-1}} \end{aligned} \quad (4.16)$$

Since $e_{lm} = e_{ml}$ for all l and m , the second term in the right hand side equals to zero. Then, (4.16) can be written as

$$\sum_{l \in \mathcal{N}} \Delta \hat{p}_l^k = \sum_{l \in \mathcal{N}} \Delta \hat{p}_l^{k-1} + \Delta p^k - \Delta p^{k-1} \quad (4.17)$$

It is reasonable to assume that the algorithm can be initiated from an initially balance stage, i.e., for all l , $\Delta p^0 = \Delta \hat{p}_l^0 = 0$. Based on (4.17)

$$\begin{aligned} \sum_{l \in \mathcal{N}} \Delta \hat{p}_l^1 &= \sum_{l \in \mathcal{N}} \Delta \hat{p}_l^0 + \Delta p^1 - \Delta p^0 = \Delta p^1 \\ \sum_{l \in \mathcal{N}} \Delta \hat{p}_l^2 &= \sum_{l \in \mathcal{N}} \Delta \hat{p}_l^1 + \Delta p^2 - \Delta p^1 = \Delta p^2 \\ \sum_{l \in \mathcal{N}} \Delta \hat{p}_l^3 &= \sum_{l \in \mathcal{N}} \Delta \hat{p}_l^2 + \Delta p^3 - \Delta p^2 = \Delta p^3 \end{aligned} \quad (4.18)$$

Now we can write

$$\sum_{l \in \mathcal{N}} \Delta \hat{p}_l^k = \Delta p^k \quad (4.19)$$

According to (4.19), the sum of local estimations of global power mismatch $\Delta \hat{p}_l^k$ by all agents equals the total power imbalance. As the algorithm converges, estimations by each agent converge towards each other, i.e.,

$$\lim_{k \rightarrow \infty} \Delta \hat{p}_1^k = \Delta \hat{p}_2^k = \dots = \Delta \hat{p}_N^k \quad (4.20)$$

From (4.19) and (4.20), for all $l \in \mathcal{N}$

$$\lim_{k \rightarrow \infty} \Delta \hat{p}_1^k = \frac{\Delta \hat{p}^k}{N} \quad (4.21)$$

It means local estimation $\Delta \hat{p}_l^k$ by all agent l is corresponding to the average total power imbalance Δp^k . Now, each agent can update the local estimation of the dual variable using the estimated value of power imbalance. But, to ensure that

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the local estimations of the dual variable by all units converge to the same value as the estimations of power imbalance converge, they need to be coordinated among neighbors. Using the same concept used in (4.15), the update equation for the local estimation of the dual variable can be written as

$$\forall l \in \mathcal{N} : \hat{\lambda}_l^k = \left[\hat{\lambda}_l^{k-1} + \sum_{m \in \mathcal{N} \setminus \{l\}} e_{lm} (\hat{\lambda}_m^{k-1} - \hat{\lambda}_l^{k-1}) + \rho \Delta \hat{p}_l^{k-1} \right]^+ \quad (4.22)$$

Each agent updates its generation/consumption in a greedy manner once it has the estimation of dual variable as

$$p_l^k = \begin{cases} \arg \min_{p_l \in \Omega^l} [C_l(p_l) - \hat{\lambda}_l^k p_l] & : l \in \mathcal{P} \\ \arg \min_{p_l \in \Omega^l} [\hat{\lambda}_l^k p_l - U_l(p_l)] & : l \in \mathcal{C} \end{cases} \quad (4.23)$$

Hence, the updating rules for agents' coordination can be written in a concise form as follows:

$$\forall l \in \mathcal{N} : \hat{\lambda}_l^k = \left[\sum_{m \in \mathcal{N}} e_{lm} \hat{\lambda}_m^{k-1} + \rho \Delta \hat{p}_l^{k-1} \right]^+ \quad (4.24)$$

$$p_l^k = \begin{cases} \arg \min_{p_l \in \Omega^l} [C_l(p_l) - \hat{\lambda}_l^k p_l] & : l \in \mathcal{P} \\ \arg \min_{p_l \in \Omega^l} [\hat{\lambda}_l^k p_l - U_l(p_l)] & : l \in \mathcal{C} \end{cases} \quad (4.25)$$

$$\Delta \hat{p}_l^k = \begin{cases} \sum_{m \in \mathcal{N}} e_{lm} \Delta \hat{p}_m^{k-1} + p_l^{k-1} - p_l^k & : l \in \mathcal{P} \\ \sum_{m \in \mathcal{N}} e_{lm} \Delta \hat{p}_m^{k-1} + p_l^k - p_l^{k-1} & : l \in \mathcal{C} \end{cases} \quad (4.26)$$

The proposed approach has the following merits:

1. It removes the need of the central coordinator to gather information from all the agents and broadcast the updated dual variables.
2. It distributes the communications and thus removes the need for global communications requirements.
3. Each agent uses its private information and power mismatch estimation from neighbors to update its state, which ensures the privacy of the agents in the market.

By comparing welfare definitions in Section 4.3 with (4.25), $\hat{\lambda}_l$ can be interpreted

as the local prices of energy offered to agents. They are used for energy transactions. These are the prices that the producers are paid for providing energy, and consumers pay for consuming energy. Since $\hat{\lambda}_l$ are the local estimations of the global dual variable λ , all $\hat{\lambda}_l$ throughout the network should converge to a unique value at the optimal point, which gives the energy price in the LEM. Once the local energy prices equate with one another, the power mismatch becomes zero, and the optimal solution is achieved. Based on the local energy price $\hat{\lambda}_l$, each agent updates its generation/consumption level greedily according to (4.25). If a particular agent solves the optimization problem independently in a selfish way, the components that depend on the private information of other agents in the iteration step are not available. The feasibility region is also reduced for a particular agent only. Each agent uses its private information and power mismatch estimation from neighbors to update its state, which ensures the privacy of the agents in the market. For sufficiently small positive value of ρ , the algorithm is stable and all the variables converge, i.e., $\hat{\lambda}_l^k \rightarrow \lambda^*$, $p_l^k \rightarrow p_l^*$, $\Delta \hat{p}_l^k \rightarrow 0$, as $k \rightarrow \infty$, $\forall l \in \mathcal{N}$. The convergence characteristics of the proposed algorithm are discussed in Section 4.4.2. The process for energy price update is summarized in Algorithm 3. The implementation procedure of the energy price update algorithm and the corresponding results are given in Section 4.4.6 and Section 4.5, respectively.

Algorithm 3 Decentralized Algorithm for Price Update

Require: Step-size and tolerance levels for termination

Determine: Energy price for LET λ^*

Each agent $l \in \mathcal{N}$

- 1: Initializes all $\hat{\lambda}_l$, p_l , and $\Delta \hat{p}_l$.
 - 2: Sets the tolerance levels ϵ_p and ϵ_λ for the power mismatch and price, respectively.
 - 3: Sets step size ρ ($0 < \rho < 1$)
 - 4: **repeat**
 - 5: Broadcasts $\hat{\lambda}_l^{k-1}$ and $\Delta \hat{p}_l^{k-1}$ to neighbors and receives neighbors' information;
 - 6: Updates $\hat{\lambda}_l^k$ according to (4.24);
 - 7: Updates p_l^k according to (4.25);
 - 8: Updates $\Delta \hat{p}_l^k$ according to (4.26);
 - 9: **until** $\Delta \hat{p}_l$ converges as $|\Delta \hat{p}_l^k| < \epsilon_p, \forall l \in \mathcal{N}$ and λ converges as $|\hat{\lambda}_l^k - \hat{\lambda}_l^{k-1}| < \epsilon_\lambda, \forall l \in \mathcal{N}$;
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4.4.2 Convergence of decentralized algorithm for LET

The updating rules of consensus based decentralized algorithm (4.24) and (4.26) can be rewritten in the following form:

$$\hat{\lambda}^k = \mathbf{E}\hat{\lambda}^{k-1} + \rho\Delta\hat{\mathbf{p}}^{k-1} \quad (4.27)$$

$$\Delta\hat{\mathbf{p}}^k = \mathbf{E}\Delta\hat{\mathbf{p}}^{k-1} + \mathbf{u}^{k-1} \quad (4.28)$$

where $\hat{\lambda}$ and $\Delta\hat{\mathbf{p}}$ are the column vectors of $\hat{\lambda}_l$ and $\Delta\hat{p}_l$, respectively. Writing (4.27) and (4.28) in vector form

$$\begin{bmatrix} \hat{\lambda}^k \\ \Delta\hat{\mathbf{p}}^k \end{bmatrix} = \begin{bmatrix} \mathbf{E} & \rho\mathbf{I} \\ 0 & \mathbf{E} \end{bmatrix} \begin{bmatrix} \hat{\lambda}^{k-1} \\ \Delta\hat{\mathbf{p}}^{k-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{I} \end{bmatrix} \mathbf{u}^{k-1} \quad (4.29)$$

The l^{th} element of the vector $\mathbf{u}^{k-1}$ is defined as

$$\mathbf{u}^{k-1}(l) = \begin{cases} p_l^{k-1} - p_l^k & : l \in \mathcal{P} \\ p_l^k - p_l^{k-1} & : l \in \mathcal{C} \end{cases} \quad (4.30)$$

$$\mathbf{u}^{k-1}(l) = \begin{cases} \arg \min_{p_l \in \Omega^l} [C_l(p_l) - \hat{\lambda}_l^{k-1} p_l] - \arg \min_{p_l \in \Omega^l} [C_l(p_l) - \hat{\lambda}_l^k p_l] & : l \in \mathcal{P} \\ \arg \min_{p_l \in \Omega^l} [\hat{\lambda}_l^k p_l - U_l(p_l)] - \arg \min_{p_l \in \Omega^l} [\hat{\lambda}_l^{k-1} p_l - U_l(p_l)] & : l \in \mathcal{C} \end{cases} \quad (4.31)$$

Denote $p_l^* = \arg \min_{p_l \in \Omega^l} [\hat{\lambda}_l^{k-1} p_l - U_l(p_l)]$, $\forall l \in \mathcal{C}$ and define $\Delta\hat{\lambda}_l = \hat{\lambda}_l^k - \hat{\lambda}_l^{k-1}$, then

$$p_l^* + \mathbf{u}^{k-1}(l) = \arg \min_{p_l \in \Omega^l} [(\hat{\lambda}_l^{k-1} + \Delta\hat{\lambda}_l) p_l - U_l(p_l)] \quad (4.32)$$

The derivative at the minimum point would be zero if the local limits are not considered, i.e.,

$$\hat{\lambda}_l^{k-1} + \Delta\hat{\lambda}_l - \frac{\partial U_l}{\partial p_l} \Big|_{p_l^* + \mathbf{u}^{k-1}(l)} = 0 \quad (4.33)$$

Also,

$$\hat{\lambda}_l^{k-1} - \frac{\partial U_l}{\partial p_l} \Big|_{p_l^*} = 0 \quad (4.34)$$

The dynamics of the algorithm around the fixed point can be analyzed using the first-order Taylor expansion for $\frac{\partial U_l}{\partial p_l}$ as follows:

$$\hat{\lambda}_l^{k-1} + \Delta\hat{\lambda}_l - \left(\frac{\partial U_l}{\partial p_l} \Big|_{p_l^*} + \frac{\partial^2 U_l}{\partial p_l^2} \Big|_{p_l^*} \times \mathbf{u}^{k-1}(l) \right) = 0 \quad (4.35)$$

With the help of (4.34), (4.35) yields

$$\Delta\hat{\lambda}_l = \frac{\partial^2 U_l}{\partial p_l^2} \Big|_{p_l^*} \times \mathbf{u}^{k-1}(l) \quad (4.36)$$

Since $\frac{\partial^2 U_l}{\partial p_l^2}|_{p_l^*} = -2\theta_l$,

$$\mathbf{u}^{k-1}(l) = \left(\frac{-1}{\theta_l}\right) \Delta \hat{\lambda}_l \quad (4.37)$$

Since there are local limits, the absolute value of $\mathbf{u}^{k-1}(l)$ would be smaller than the one presented in (4.37). Therefore,

$$\forall l \in \mathcal{C} : \mathbf{u}^{k-1}(l) = -\tilde{b}_l \Delta \hat{\lambda}_l, \quad 0 \leq \tilde{b}_l \leq \frac{1}{\theta_l} \quad (4.38)$$

With similar analysis for producers,

$$\forall l \in \mathcal{P} : \mathbf{u}^{k-1}(l) = -\tilde{b}_l \Delta \hat{\lambda}_l, \quad 0 \leq \tilde{b}_l \leq \frac{1}{a_l} \quad (4.39)$$

Since $\Delta \hat{\lambda}_l = \hat{\lambda}_l^k - \hat{\lambda}_l^{k-1}$, the expression for $\Delta \hat{\lambda}_l$ can be written as

$$\Delta \hat{\lambda}_l = \mathbf{e}_l \hat{\boldsymbol{\lambda}}^{k-1} + \rho \Delta \hat{\mathbf{p}}_l^{k-1} - \hat{\lambda}_l^{k-1} \quad (4.40)$$

where $\mathbf{e}_l$ is the l^{th} -row of the matrix $\mathbf{E}$. Combining (4.38), (4.39), and (4.40) gives

$$\mathbf{u}^{k-1} = \tilde{\mathbf{B}}((\mathbf{I} - \mathbf{E})\hat{\boldsymbol{\lambda}}^{k-1} - \rho \Delta \hat{\mathbf{p}}^{k-1}) \quad (4.41)$$

where $\tilde{\mathbf{B}} = \text{diag}([\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_N])$, and

$$\begin{cases} 0 \leq \tilde{b}_l \leq \frac{1}{a_l}, & \forall l \in \mathcal{P} \\ 0 \leq \tilde{b}_l \leq \frac{1}{\theta_l}, & \forall l \in \mathcal{C} \end{cases} \quad (4.42)$$

If both p_l^k and p_l^{k-1} are saturated, then the value of $\tilde{b}_l$ is zero. If both of them are within the limits, then $\tilde{b}_l$ is set to maximum as given by (4.42). And, if either of them is saturated, then $\tilde{b}_l$ lies in the range defined by (4.42). Now (4.29) can be written as

$$\begin{bmatrix} \hat{\boldsymbol{\lambda}}^k \\ \Delta \hat{\mathbf{p}}^k \end{bmatrix} = \begin{bmatrix} \mathbf{E} & \rho \mathbf{I} \\ (\tilde{\mathbf{B}}\mathbf{I} - \mathbf{E}) & (\mathbf{E} - \rho \tilde{\mathbf{B}}) \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{\lambda}}^{k-1} \\ \Delta \hat{\mathbf{p}}^{k-1} \end{bmatrix} \quad (4.43)$$

Define $\tilde{\mathbf{H}}$ as

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{E} & \rho \mathbf{I} \\ (\tilde{\mathbf{B}}\mathbf{I} - \mathbf{E}) & (\mathbf{E} - \rho \tilde{\mathbf{B}}) \end{bmatrix} \quad (4.44)$$

The eigenvalue approach [143] is used to analyze the convergence properties. If ρ is chosen small enough to be neglected, $\tilde{\mathbf{H}}$ has the repeated eigenvalue set of eigenvalues of $\mathbf{E}$, i.e.,

$$|\sigma \mathbf{I} - \tilde{\mathbf{H}}| = |\sigma \mathbf{I} - \mathbf{E}|^2 \quad (4.45)$$

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Since $\mathbf{E}$ is a doubly stochastic matrix and irreducible, 1 is a simple eigenvalue of $\mathbf{E}$ and the others have moduli smaller than 1. So, $\sigma_1 = \sigma_2 = 1$ is the double eigenvalue of $\tilde{\mathbf{H}}$, and the remaining eigenvalues have moduli smaller than 1. The eigenvalue 1 leads the evolution of consensus variables to a common value. It can be verified that $[\mathbf{1} \ \mathbf{0}]^T$ is the eigenvector of $\tilde{\mathbf{H}}$ associated with eigenvalue $\sigma_1 = 1$, i.e.,

$$\begin{bmatrix} \mathbf{E} & \rho \mathbf{I} \\ (\tilde{\mathbf{B}}\mathbf{I} - \mathbf{E}) & (\mathbf{E} - \rho \tilde{\mathbf{B}}) \end{bmatrix} \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix} \quad (4.46)$$

Since the eigenvalue 1 leads the algorithm to convergence, regardless of the value of ρ , $[\hat{\lambda}^k \ \Delta \hat{\mathbf{p}}^k]^T$ converges to span $[\mathbf{1} \ \mathbf{0}]^T$, i.e.,

$$\lim_{k \rightarrow \infty} \begin{bmatrix} \hat{\lambda}^k \\ \Delta \hat{\mathbf{p}}^k \end{bmatrix} = \begin{bmatrix} \lambda^* \\ \mathbf{0} \end{bmatrix} \quad (4.47)$$

4.4.3 Voltage Management in LET

Let $\mathbf{v}^k$ and $\mathbf{p}^k$ denote the vector of node voltage magnitudes and agents' decision on active power injections at k^{th} iteration of the decentralized algorithm, respectively. If $\Delta \tilde{\mathbf{p}}^k = \mathbf{p}^k - \mathbf{p}^{k-1}$ is the vector describing small variations in active power injections at nodes between iterations $k-1$ and k , then based on (4.7), the vector of the corresponding variations in node voltage magnitudes $\Delta \mathbf{v}^k = \mathbf{v}^k - \mathbf{v}^{k-1}$ can be expressed as

$$\Delta \mathbf{v}^k = \mathbf{R} \Delta \tilde{\mathbf{p}}^k \quad (4.48)$$

The relation that describes how the bus voltage magnitudes evolve with the change in active power injections in successive iterations is given by

$$\mathbf{v}^k = \mathbf{v}^{k-1} + \mathbf{R} \Delta \tilde{\mathbf{p}}^k \quad (4.49)$$

The active power deviations $\Delta \tilde{\mathbf{p}}$ required to bring the voltage deviations of $\Delta \mathbf{v}$ is given as

$$\Delta \tilde{\mathbf{p}} = \mathbf{R}^{-1} \Delta \mathbf{v} \quad (4.50)$$

The aim is to maintain each node voltage magnitudes within the acceptable ranges as

$$\underline{v}_l \leq v_l \leq \bar{v}_l \quad (4.51)$$

The agent at node l sets the reference voltage at the k^{th} iteration as

$$v_{l,ref}^k = \begin{cases} \underline{v}_l & : \hat{v}_l^k < \underline{v}_l \\ \hat{v}_l^k & : \underline{v}_l \leq \hat{v}_l^k \leq \bar{v}_l \\ \bar{v}_l & : \hat{v}_l^k > \bar{v}_l \end{cases} \quad (4.52)$$

The voltage violation at node l from its reference value at the k^{th} iteration is

$$\vartheta_l^k = v_{l,ref}^k - \hat{v}_l^k \quad (4.53)$$

The agent l estimates the additional amount of the active power needed to clear the violation given by (4.53) as

$$\delta_l^k = \frac{\alpha_c \vartheta_l^k}{R_{ll}} \quad (4.54)$$

where $\alpha_c > 0$ is some constant. The value of α_c should be chosen carefully to ensure the stability of the algorithm. The detailed analysis of the selection of the value of α_c is given in Section 4.4.4. The total estimated active power to be injected by the agent at node l to clear the voltage violation is

$$\tilde{p}_l^k = p_l^k + \delta_l^k \quad (4.55)$$

If $\underline{p}_l \leq \tilde{p}_l^k \leq \bar{p}_l, \forall l \in \mathcal{N}$, every agent can provide the estimated amount of active power to clear the voltage violation at its node by itself. But, if $\tilde{p}_l^k > \bar{p}_l$ or $\tilde{p}_l^k < \underline{p}_l$, agent l cannot provide the estimated amount of active power to clear the voltage violation at its node by itself. If there is at least one agent which cannot clear the violation by itself, other agents having additional capacity will decide an extra amount of power they can provide to support other agents through a decentralized algorithm. Each agent shares a set of variables $\psi_l, \bar{\nu}_l, \underline{\nu}_l$ with its neighbors with the initial values set as follows:

$$\psi_l^0 = \begin{cases} \tilde{p}_l^k - \bar{p}_l & : \tilde{p}_l^k > \bar{p}_l \\ \tilde{p}_l^k - \underline{p}_l & : \tilde{p}_l^k < \underline{p}_l \\ 0 & : \text{otherwise} \end{cases} \quad (4.56)$$

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$$\bar{\nu}_l^0 = \begin{cases} \bar{p}_l - \tilde{p}_l^k & : \tilde{p}_l^k < \bar{p}_l \\ 0 & : \text{otherwise} \end{cases} \quad (4.57)$$

$$\underline{\nu}_l^0 = \begin{cases} \underline{p}_l - \tilde{p}_l^k & : \tilde{p}_l^k > \underline{p}_l \\ 0 & : \text{otherwise} \end{cases} \quad (4.58)$$

Each agent updates the corresponding variables based on its own and the exchanged information with its neighbors. The updating rules for variables are as follows:

$$\begin{aligned} \psi_l^r &= \sum_{m \in \mathcal{N}} e_{lm} \psi_m^{r-1} \\ \bar{\nu}_l^r &= \sum_{m \in \mathcal{N}} e_{lm} \bar{\nu}_m^{r-1} \\ \underline{\nu}_l^r &= \sum_{m \in \mathcal{N}} e_{lm} \underline{\nu}_m^{r-1} \end{aligned} \quad (4.59)$$

Then, at each iteration r , agent l with $\underline{\nu}_l^r \neq 0$ or $\bar{\nu}_l^r \neq 0$ calculates its contribution ξ_l^r as

$$\xi_l^r = \begin{cases} \frac{\psi_l^r}{\underline{\nu}_l^r} \underline{\nu}_l^0 & : \psi_l^r < 0 \\ \frac{\psi_l^r}{\bar{\nu}_l^r} \bar{\nu}_l^0 & : \psi_l^r \geq 0 \end{cases} \quad (4.60)$$

where ξ_l^r will asymptotically converge to

$$\xi_l = \begin{cases} \frac{\sum_{l=1}^N \psi_l^0}{\sum_{l=1}^N \underline{\nu}_l^0} \underline{\nu}_l^0 & : \lim_{r \rightarrow \infty} \psi_l^r < 0 \\ \frac{\sum_{l=1}^N \psi_l^0}{\sum_{l=1}^N \bar{\nu}_l^0} \bar{\nu}_l^0 & : \lim_{r \rightarrow \infty} \psi_l^r \geq 0 \end{cases} \quad (4.61)$$

The details behind this result are given in Section 4.4.5. Once the algorithm converged, agent l updates its power generation/consumption decision to

$$p_l^k = \begin{cases} \bar{p}_l & : \tilde{p}_l^k + \xi_l > \bar{p}_l \\ \underline{p}_l & : \tilde{p}_l^k + \xi_l < \underline{p}_l \\ \tilde{p}_l^k + \xi_l & : \text{otherwise} \end{cases} \quad (4.62)$$

The algorithm for voltage management in LET is given in Algorithm 4.

Algorithm 4 Algorithm for Voltage Management**Require:** Node voltages: $v_1^k, \dots, v_l^k, \dots, v_M^k$ **Determine:** Contribution for voltage management ξ Each agent $l \in \mathcal{N}$

- 1: Sets the tolerance level ϵ ;
- 2: Sets reference node voltage $v_{l,ref}^k$ according to (4.52);
- 3: Computes the voltage violation at node v_l^k using (4.53);
- 4: Estimates the additional power needed to clear the violation δ_l^k by (4.54) and total estimated power $\tilde{p}_l^k$ by (4.55);
- 5: Initializes the parameters of the distributed algorithm ψ_l , ν_l , and $\bar{\nu}_l$ according to (4.56), (4.57), and (4.58), respectively;
- 6: **repeat**
- 7: Broadcasts the parameters ψ_l^r , ν_l^r , and $\bar{\nu}_l^r$ to neighbors;
- 8: Updates parameters according to (4.59) after receiving neighbors' information;
- 9: Calculates contribution ξ_l^r according to (4.60);
- 10: **until** contribution parameter ξ_l^r converges to (4.61) as $|\xi_l^r - \xi_l^{r-1}| < \epsilon$;

4.4.4 Choice of α_c for Stability

From (4.54), we can write

$$\boldsymbol{\delta}^k = \alpha_c \mathbf{S}(\mathbf{v}_{ref}^k - \hat{\mathbf{v}}^k) \quad (4.63)$$

where $\mathbf{S}$ is a diagonal matrix with $s_{ll} = 1/R_{ll}$. Based on (4.49), the relation between $\mathbf{v}^k$ and $\hat{\mathbf{v}}^k$ can be written as

$$\mathbf{v}^k = \hat{\mathbf{v}}^k + \mathbf{R}\boldsymbol{\delta}^k \quad (4.64)$$

Substituting (4.63) into (4.64) gives

$$\mathbf{v}^k = (\mathbf{I} - \alpha_c \mathbf{R}\mathbf{S})\hat{\mathbf{v}}^k + \alpha_c \mathbf{R}\mathbf{S}\mathbf{v}_{ref}^k \quad (4.65)$$

The system represented by (4.65) is a linear time-invariant system with bounded inputs $\mathbf{v}_{ref}^k$. In order to ensure the internal stability of this system $\hat{\mathbf{v}}^k$ should remain bounded [144]. For the internal stability of the system (4.65), we should ensure that $|\sigma_i| < 1$, $\forall i$, where σ_i denotes the i^{th} eigenvalue of $\mathbf{I} - \alpha_c \mathbf{R}\mathbf{S}$. If $\chi_i = \text{Re}\{\chi_i\} + j\text{Im}\{\chi_i\}$ denotes the i^{th} eigenvalue of the matrix $\mathbf{R}\mathbf{S}$, then $\sigma_i = 1 - \alpha_c \chi_i$, $\forall i$. Therefore, $\sigma_i = (1 - \alpha_c \text{Re}\{\chi_i\}) + j\alpha_c \text{Im}\{\chi_i\}$, and $|\sigma_i|^2 = 1 - 2\alpha_c \text{Re}\{\chi_i\} + \alpha_c^2 |\chi_i|^2$. Then, $|\sigma_i|^2 < 1$, if $\alpha_c < 2\text{Re}\{\chi_i\}/|\chi_i|^2$. Thus, $|\sigma_i| < 1$, $\forall i$ can be ensured by choosing α_c such that $0 < \alpha_c < \min_i \{2\text{Re}\{\chi_i\}/|\chi_i|^2\}$.

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4.4.5 Convergence of voltage management algorithm

The update rules in (4.59) can be generalized at any iteration r in a compact form as

$$\begin{aligned}\boldsymbol{\psi}^r &= \mathbf{E} \boldsymbol{\psi}^{r-1}; \\ \bar{\boldsymbol{\nu}}^r &= \mathbf{E} \bar{\boldsymbol{\nu}}^{r-1}; \\ \underline{\boldsymbol{\nu}}^r &= \mathbf{E} \underline{\boldsymbol{\nu}}^{r-1}\end{aligned}\tag{4.66}$$

where $\boldsymbol{\psi}$, $\bar{\boldsymbol{\nu}}$, and $\underline{\boldsymbol{\nu}}$ are the column vectors of ψ_l , $\bar{\nu}_l$, and ν_l , respectively. After r iterations, the relation between the vectors $\boldsymbol{\psi}^r$, $\bar{\boldsymbol{\nu}}^r$, $\underline{\boldsymbol{\nu}}^r$ can be expressed in terms of the initial vectors $\boldsymbol{\psi}^0$, $\bar{\boldsymbol{\nu}}^0$, $\underline{\boldsymbol{\nu}}^0$, respectively, as

$$\begin{aligned}\boldsymbol{\psi}^r &= \mathbf{E}^r \boldsymbol{\psi}^0 \\ \bar{\boldsymbol{\nu}}^r &= \mathbf{E}^r \bar{\boldsymbol{\nu}}^0 \\ \underline{\boldsymbol{\nu}}^r &= \mathbf{E}^r \underline{\boldsymbol{\nu}}^0\end{aligned}\tag{4.67}$$

Since $\mathbf{E}$ is doubly stochastic, non-negative, irreducible, and aperiodic matrix [145], $\lim_{r \rightarrow \infty} \mathbf{E}^r$ converges to a rank-one deterministic matrix. This ensures that variables in (4.67) converge to the unique solutions $\lim_{r \rightarrow \infty} \psi_l^r = \zeta_l \sum_{l=1}^N \psi_l^0$, $\lim_{r \rightarrow \infty} \bar{\nu}_l^r = \zeta_l \sum_{l=1}^N \bar{\nu}_l^0$, and $\lim_{r \rightarrow \infty} \nu_l^r = \zeta_l \sum_{l=1}^N \nu_l^0$, where ζ_l is the l -th element of the unit eigenvector corresponding to eigenvalue 1 of the matrix $\mathbf{E}$ and satisfies $\sum_{l=1}^N \zeta_l = 1$ and $\zeta_l > 0$ [145]. If $\sum_{l=1}^N \psi_l^0 \geq 0$, then $\lim_{r \rightarrow \infty} \psi_l^r \geq 0$, and if $\sum_{l=1}^N \psi_l^0 < 0$, then $\lim_{r \rightarrow \infty} \psi_l^r < 0$. Therefore, for $\lim_{r \rightarrow \infty} \psi_l^r \geq 0$, as $r \rightarrow \infty$

$$\lim_{r \rightarrow \infty} \frac{\psi_l^r}{\bar{\nu}_l^r} = \frac{\zeta_l \sum_{l=1}^N \psi_l^0}{\zeta_l \sum_{l=1}^N \bar{\nu}_l^0} = \frac{\sum_{l=1}^N \psi_l^0}{\sum_{l=1}^N \bar{\nu}_l^0}\tag{4.68}$$

$$\xi_l = \lim_{r \rightarrow \infty} \xi_l^r = \frac{\sum_{l=1}^N \psi_l^0}{\sum_{l=1}^N \bar{\nu}_l^0}, \forall l\tag{4.69}$$

Similarly, for $\lim_{r \rightarrow \infty} \psi_l^r < 0$, as $r \rightarrow \infty$

$$\xi_l = \lim_{r \rightarrow \infty} \xi_l^r = \frac{\sum_{l=1}^N \psi_l^0}{\sum_{l=1}^N \nu_l^0}, \forall l\tag{4.70}$$

4.4.6 Implementation Procedure

In real applications, it is not practical to change the energy price λ continuously in the electricity market. It is more practical and feasible to update the energy price λ in a fixed period. So, the proposed energy trading algorithm needs to be run at the beginning of each period and the outcome is kept constant throughout that period. The proposed LET algorithm uses an iterative approach which computes the trading price through the interaction among agents. The responses from agents would change in response to the energy trading price, and this change will, in turn, affects the energy price. The algorithm starts with the initialization of the required variables by each agent l and broadcast of initial values to other agents. The selection of initial values has a significant influence on the convergence of the algorithm. Since the cost/utility function parameters and generation/demand limits do not change abruptly from one period to another period, it is reasonable to consider stable solution of the previous time period as initial values for the next period. Upon receiving the latest update from neighboring agents, each agent l updates local price $\hat{\lambda}_l^k$ according to (4.24) and calculates optimal energy generation/demand p_l^k using (4.25) based on the recently updated local price. Then, each agent l updates the local estimate of average global power mismatch $\Delta \hat{p}_l^k$ according to (4.26).

If the network voltage management is considered in the energy pricing, the sequence of execution is slightly different from the one explained before. An intermediate step will take place between calculating p_l^k using (4.25) and updating $\Delta \hat{p}_l^k$ using (4.26) by each agent. Each agent l executes voltage management algorithm given in Algorithm 4 before calculating $\Delta \hat{p}_l^k$ after it calculates p_l^k . The value of p_l^k used for updating $\Delta \hat{p}_l^k$ in this case is not the same as given by (4.25). Each agent l calculates its contribution for voltage management ξ_l from Algorithm 4 and updates p_l^k according to (4.62). Then, $\Delta \hat{p}_l^k$ is calculated using the latest updated value of p_l^k . Assume that each agent has the complete knowledge of the

4.5. Results and Discussions

Figure 4.2: Flowchart for decentralized LET

network topology, and the real-time voltage measurement is available. The concept of virtual measurement using the power flow engine is used for simulation purpose [146]. After each iteration, the stopping criteria are examined. If $|\Delta \hat{p}_l^k|$ and $|\hat{\lambda}_l^k - \hat{\lambda}_l^{k-1}|$ are lower than ϵ_p and ϵ_λ , respectively, and when there is no voltage violation, the iterative algorithm is stopped. Otherwise, the process is repeated until the stopping criteria are fulfilled. The overall flow of the implementation procedure is summarized in Fig. 4.2

4.5 Results and Discussions

This section presents numerical case studies to demonstrate the virtue of the proposed LET mechanism. A microgrid with 16 producers and 16 consumers as shown in Fig. 4.3 is considered. The microgrid has the same network structure as that of the IEEE 33-bus distribution system. Each agent can communicate with its physically connected neighbors. The simulation results are based on the assumption of perfect and fast bi-directional communication and real-time measurements are available. There are five solar PV systems in the microgrid at nodes 1, 2, 19, 30, and 32, which are non-dispatchable generation. Only one time period is considered in this case study². The minimum and maximum generation

²Since the multi-period problem can be easily decoupled into multiple problems at a separate period using relaxation techniques, the problem is solved for a single period to demonstrate the

Figure 4.3: 33-bus microgrid network

for producers are in the range of $[0, 3]$ kW and $[3, 8]$ kW, respectively. Similarly, the minimum and maximum demand for consumers are assumed to be in the range of $[0, 3]$ kW and $[3, 8]$ kW, respectively. The utility function parameters of consumers β_l and θ_l are considered to be $\beta \in [10, 20]$ cents/kWh and $\theta \in (0, 1]$ cent/kWh², respectively whereas the cost function parameters for producers a_l and b_l are selected randomly from the interval of $(0, 1]$ cents/kWh² and $[0, 10]$ cents/kWh, respectively, and c_l is set to 0. The required tolerances for termination are set to $\epsilon_\lambda = 0.001$ and $\epsilon_p = 0.001$. It is assumed that the voltage magnitudes must lie within $\pm 5\%$ of 1 p.u. at all nodes. Based on the data and the proposed algorithm, all the simulations are conducted in the MATLAB 2016a environment with an Intel(R) Xeon(R) CPU E5-1630 v4@3.70 GHz, 16 GB RAM. The results in different cases are discussed in the following sections. The initial values are set as $\hat{\lambda}_l^0 = C'_l(\underline{p}_l)$, $\forall l \in \mathcal{P}$; $\hat{\lambda}_l^0 = U'_l(\bar{p}_l)$, $\forall l \in \mathcal{C}$ and $\Delta \hat{p}_l^0 = 0$.

4.5.1 Without Voltage Management

Fig. 4.4 shows the evolution of the total demand and supply during LET using the proposed decentralized algorithm without considering the network voltage management. In Fig. 4.4, there exists a mismatch between the total demand and supply initially. The evolution of local energy prices $\hat{\lambda}_l$ is shown in Fig. 4.5 and the evolution of local estimation of mismatch power $\Delta \hat{p}_l$ is shown in Fig. 4.6. As performance of the proposed method in a clearer manner.

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the algorithm proceeds, each local price converges towards a common value λ^* and all the local estimation of power mismatch $\Delta \hat{p}_l, \forall l \in \mathcal{N}$ get close to zero as shown in Fig. 4.6. The total power imbalance Δp approaches zero, as shown in Fig. 4.8 as the local estimation of power mismatches get close to zero. The total supply meets the total demand, and thus, the power balance condition given in (4.11) is satisfied gradually, as shown in Fig. 4.4. The proposed decentralized algorithm satisfies the termination conditions $|\hat{\lambda}_l^k - \hat{\lambda}_l^{k-1}| < \epsilon_\lambda$ and $|\Delta \hat{p}_l| < \epsilon_p$ when the number of iterations $k = 187$ and the results converge. The voltage magnitudes at some nodes are less than 0.95 p.u. and violate the safe operating limits, as shown in Fig. 4.9 when the voltage management strategy is not considered in the energy pricing mechanism. In real-time applications, it may not be practical to run the algorithm for a large number of iterations to reach a very precise solution. So, the algorithm should be stopped after a certain number of iterations. The compromise between time and accuracy is necessary. Fig. 4.7 shows the variation of the error in local power mismatches, local prices, and global power mismatch with the number of iterations. The required number of iterations depends on the accuracy required.

Figure 4.4: Evolution of total demand and supply without voltage management

Figure 4.5: Evolution of local prices without voltage management

Figure 4.6: Evolution of estimation of power mismatch without voltage management

Figure 4.7: Accuracy vs. iterations without voltage management

4.5.2 With Voltage Management

Fig. 4.10 shows the evolution of the total supply and demand in the LEM while incorporating the voltage management strategy within the energy pricing mechanism.

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Figure 4.8: Evolution of power imbalance Δp without voltage management

Figure 4.9: Node voltages with and without voltage management

Figure 4.10: Evolution of total demand and supply with voltage management

nism. The effect of considering the network voltages in the energy pricing mechanism can be seen in Fig. 4.10. The evolution of the supply and demand curves in Fig. 4.10 shows different nature compared to the evolution curves without voltage management in Fig. 4.4. The decision of agents depends only on the energy

Figure 4.11: Evolution of power imbalance Δp with voltage management

Figure 4.12: Evolution of local prices with voltage management

price in the previous case without voltage management. But, with the voltage management, agents' decision depends on both the energy price and node voltage magnitudes. Unlike the case without voltage management shown in Fig. 4.8, with the same initial conditions, the imbalance between the supply and demand with voltage management shown in Fig. 4.11 has different characteristics. It can be observed in Fig. 4.11 that spikes occur at several places as the algorithm proceeds. These spikes arise from the fact that voltage limits violations force the agents to change their decision abruptly from one iteration to another to bring back the voltages in an acceptable range. It results in a change in the magnitude of power imbalance. Such changes in agents' decisions on demand/generation due to voltage violations in the current iteration will be reflected in energy price in the next iteration. The termination conditions are satisfied when $k = 277$, and the total

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supply and demand are balanced, as shown in Fig. 4.10. The evolution of local energy prices $\hat{\lambda}_l$ and the evolution of local estimation of mismatch power $\Delta\hat{p}_l$ with voltage management are shown in Fig. 4.12 and Fig. 4.13, respectively. It can be seen in Fig. 4.9 that voltage magnitudes fall within the specified operating limits of $\pm 5\%$ of 1 p.u. at all nodes when voltage management is considered in the energy pricing mechanism.

Figure 4.13: Evolution estimation of power mismatch with voltage management

Figure 4.14: Evolution of total demand and supply in two-stage approach

4.5.3 Two-stage Approach

It is clear that the iterative algorithm takes a longer time to converge with voltage management ($k = 277$) as compared to the case without voltage management ($k = 187$). It is due to the inclusion of the network voltage management strategy

in the energy pricing mechanism. It is required to run the voltage management algorithm, i.e., Algorithm 4, in each iteration of the proposed decentralized algorithm as shown in Fig. 4.2, and the convergence time is increased.

As explained in Section 4.5.1, it may not be practical to run the algorithm for a large number of iterations to reach a very precise solution in real-time applications. The increase in convergence time reduces the practicability of the algorithm for real-time implementation. A two-stage approach for energy pricing is proposed considering these factors to increase the viability of the algorithm for real-time implementation. At the first stage, the energy pricing algorithm is executed without considering the voltage management until the equilibrium condition is reached. The intermediate block containing Algorithm 4 of Fig. 4.2 is skipped at this stage. All the node voltages are examined for possible violations at the beginning of the second stage. If there are no voltage violations in the network, then the solution from the first stage is the final solution. Otherwise, the algorithm proceeds to the second stage considering the voltage management strategy in the energy pricing mechanism and is executed until the final equilibrium condition is achieved. The solution obtained from the first stage is taken as an initial input to the second stage which helps in fast convergence of the second stage.

The evolution of the supply and demand curves in a two-stage approach is shown in Fig. 4.14. The final power balance condition obtained from the two-stage approach shown in Fig. 4.14 is the same as that obtained from the single-stage approach with voltage management, as shown in Fig. 4.10. The termination conditions are satisfied when $k = 277$ in two-stage approach. As explained in Section 4.4.6, the proposed energy trading algorithm needs to be run at the beginning of each time period and the outcome is kept constant throughout that period. It means that the communication and computational facilities remain idle for the rest of the time during that period once the algorithm is converged. It makes possible to run the first stage of the algorithm for the next period during the idle duration of the

4.6. Summary

previous period. Practically, only the second stage has to be run in real time which helps to achieve faster convergence and increase the practicability of the algorithm. In this case study, the second stage of the algorithm converges in $277 - 187 = 90$ iterations. This is the required number of iterations that need to be run in real time in a two-stage approach whereas in a single-stage approach with voltage management, all 277 iterations need to be run in real time. Moreover, running the first-stage algorithm during the idle duration helps in effective utilization of available communication and computation facilities. So, the two-stage approach is more suitable for real implementation.

4.6 Summary

In this chapter, an energy pricing mechanism for decentralized LET in microgrids with DGs and responsive demands considering the network voltage management is presented. The LET problem in a microgrid is formulated as an aggregated welfare maximization problem. A fully decentralized approach based on local communication with neighbors is proposed for energy pricing and voltage management. The proposed method is applied to a microgrid with the IEEE 33-node network structure. The convergence of the proposed method is verified via numerical results. The simulation results demonstrated that the proposed decentralized approach through the coordination among the agents establishes the supply-demand balance, decides the energy trading price, and maintains all the node voltages within the safe operating limits. The two-stage approach improves the convergence speed of the algorithm and increases the practicability of the algorithm for real-time implementation.

Chapter 5

P2P Energy Trading Considering Power Losses and Network Fees

5.1 Introduction

The increasing presence of more proactive actors or agents in the current power system has triggered a design and adaptation of a more decentralized paradigm to power systems and electricity market operation [49, 147]. P2P energy trading is one of the promising approaches for implementing decentralized electricity market paradigms. Since the P2P energy trading in smart grids is a new concept, a proper market clearing mechanism is required for the P2P energy trading between different producers and consumers [126]. A market clearing mechanism deals with electricity pricing and energy allocation. The market clearing method should be computationally efficient and set with a defined trading objective. The objective of the trading should be designed in such a way that it incentivizes the participation of agents in the P2P market. Besides, agents in the market behave independently with their interest and have a set of private information which they do not want to reveal. Therefore, designing a proper market clearing mechanism for the P2P energy trading while maintaining privacy is a challenging task.

On the other hand, a physical network or grid, which is used for energy transfer, imposes various grid-related aspects on energy trading. Such networks or grids usually owned by third parties other than agents participating in the market. Grid-related costs always occur during each energy transaction. They mainly account for investment costs to build a network as well as the associated operation and maintenance costs [58]. In real applications, the owner of the grid collects the network utilization fees, paid by consumers corresponding to its network usage during energy transactions, to cover these costs. In traditional electricity markets, such network fees constitute a significant portion of the energy bills of consumers [148]. However, P2P energy trading enables consumers to reduce network utilization fees by strategically choosing producers. To achieve this objective, a factor related to network usage should be considered while designing a market clearing mechanism for the P2P trading. Besides, the transmission losses are inevitable in power grids. It is essential to consider transmission losses in the P2P trading model to make it more practical. In summary, power losses in the network and grid-related costs should be properly integrated into the P2P trading model. Hence, developing a privacy-preserving and fair market clearing mechanism for the P2P energy trading considering various aspects of the grid is the motive behind this work.

5.1.1 Related Works

In the literature, there has been an increasing interest in the area of market design for P2P energy trading. Khorasany *et al.* [49] presented a decentralized bilateral energy trading system for P2P electricity markets. A P2P energy trading under network constraints is proposed in [125] and the impact of P2P transactions on the network is assessed by sensitivity analysis to ensure that the exchange of energy does not violate the network constraints. A game theory based pricing model for the P2P energy trading is proposed in [126] to maximize the social welfare of buyers and sellers in a prosumer based community microgrid. P2P negotiations are initiated with the help of a coordinating agent called P2P market operator.

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In [149], a coordinated market model for P2P energy trading and ancillary services in distribution grids is proposed. The distribution network operator manages ancillary services in the network during P2P energy transactions. A P2P energy trading in virtual microgrids with heterogeneous prosumers is proposed in [148], where interactions among the prosumers are modeled as a non-cooperative game.

An indirect customer-to-customer energy trading in the distribution level is proposed by Chen *et al.* in [132]. Agents update their trading strategies using the reinforcement learning principle and an energy broker acts as a coordinator for managing the market operations. Suppliers act as intermediaries in a forward bilateral contract network designed for the real-time P2P energy trading between sellers and buyers in [26]. Tushar *et al.* [135] presented a P2P energy trading mechanism where a centralized power system decides the energy price to incentivize the prosumers to participate in P2P energy trading to reduce the peak demand on the centralized power system. A novel P2P model for a joint trading of energy and uncertainties in the LEM is proposed in [150]. Liu *et al.* [151] proposed a double auction-based P2P energy trading platform for residential demand response to face disturbances. A consensus-based energy management scheme for smart grids is proposed in [16, 17], where the coordination among the agents through only local information exchange among neighbors establishes the supply-demand balance.

Most of the existing studies on P2P energy trading often neglect the power losses in the network, assuming that the energy is transmitted over a short distance in the distribution system. However, without considering power losses, power flows of P2P trading decisions cannot satisfy the power balance condition for the stable operation of the power system, and the practicability of the P2P trading becomes questionable. Since the power losses occurring in the network directly impact the market outcome, a transparent loss allocation framework is required to ensure the economic fairness among agents in the P2P market. In [152], network losses in a microgrid are allocated to each node and compensated by discharging battery

storage units at the corresponding node. Kim *et al.* [153], has proposed a P2P market considering power losses, where costs are allocated to each trade based upon its grid usage. A graph-based loss allocation framework is proposed in [154] for transactive energy markets in distribution systems. In [155], the bilateral exchange coefficient is used to calculate the losses cost associated with each P2P transaction between nodes.

In these works [26, 49, 125, 126, 132, 135, 148, 150, 151], different approaches for P2P energy trading are studied considering the economic aspect for increasing welfare or decreasing the cost of agents. However, the grid-related costs of using the third party owned network for power distribution as well as power losses in P2P energy transactions are not considered. In addition, [26, 126, 132, 135, 149] consider a separate entity to coordinate the energy trading. The coordinator has different roles, such as initiate the energy trading [126], manage the network services [156], and perform the market clearing [26, 132, 135], based on the market model. The vital role of the coordinator in market clearing endangers agents' privacy as well as the system scalability of the market as all agents need to communicate with the coordinator responsible for market clearing and share sufficient information with it. The role of the coordinator in market clearing is eliminated using a consensus-based approach to enhance the privacy of the agents in [16, 17, 157]. However, there is no direct negotiation among agents. The direct negotiation among agents is the rationale behind P2P trading. Also, the grid-related aspects are not considered.

On the other hand, [152–155] propose different approaches for losses and associated cost allocations but have not studied its impact on the market outcome. The proposed P2P markets in [153–155] rely on the distribution network operator for the compensation of losses, which increases the dependency of the local P2P market with the upstream market. The incurred losses caused by each transaction in the local market should be compensated to achieve a feasible market solution. Therefore, the incurred losses should be included in the market model and compen-

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sated within the local market itself to increase the autonomy of the P2P market. Besides, all agents behave greedily and always try to reach the optimal solution for themselves. The market clearing mechanism should be designed in such a way that the optimal local solution should coincide with an optimal global solution, and it should be fair to all agents.

To this end, there is a lack of a proper framework for the P2P energy trading, which considers the network usage during P2P energy transactions along with the privacy of agents. The focus of this chapter is on the design of a proper market clearing mechanism for P2P energy trading, considering the privacy of the agents, power losses as well as the utilization fees for using the third party owned network during energy transactions. This paper proposes a fully decentralized approach for market clearing in the P2P energy market. The proposed approach uses iterative negotiations and local decision making to eliminate the need of the coordinator for market clearing. In addition, power losses and network utilization fees during P2P energy trading are taken into account.

5.1.2 Major Contributions

The major contributions of this chapter are as follows:

1. A P2P energy trading is formulated as a social welfare maximization problem with consideration of network/grid usage, i.e., power losses and network utilization fees, in P2P energy transactions.
2. An electrical distance approach is proposed to calculate the network fees for P2P energy transactions. The network fees are proportional to the electrical distance between producers and consumers. In this approach, longer distance transactions become more expensive because of higher network fees, so consumers are encouraged to trade with producers at shorter electrical distance.

3. A novel decentralized approach, which neither requires any third-party nor reveals any private information of the agents, is proposed for market clearing in the P2P energy trading. The proposed decentralized algorithm solves the P2P market clearing problem without sharing the agents' preferences and respects the privacy of agents.

5.2 Problem Formulation

5.2.1 System Description

Consider a smart grid consisting of a set $\mathcal{P} \triangleq \{1, \dots, n_p\}$ of producers with index $i \in \mathcal{P}$ and a set $\mathcal{C} \triangleq \{1, \dots, n_c\}$ of consumers with index $j \in \mathcal{C}$. The set of all agents is $\mathcal{N} = \mathcal{P} \cup \mathcal{C}$, and $\mathcal{P} \cap \mathcal{C} = \emptyset$. The total number of agents is $N \triangleq n_p + n_c$. The smart grid has the electrical network and the communication network. The electrical network is used for energy transfer and the communication network is used for information exchange among agents. Smart meters are installed at the premise of each agent. All the communication tasks are done through the smart meters using communication infrastructure. The envisioned electricity market for the P2P energy trading in a smart grid consists of multiple producers and multiple consumers, as shown in Fig 5.1. The producers and consumers have flexible production and consumption, respectively. The market is cleared through P2P interactions among producers and consumers. This work focuses on a deterministic clearing mechanism in the P2P energy market for a single market period of one hour. The single period problem can be extended to a multiple period problem with temporally coupled constraints. The terms power and energy are used interchangeably since the market period considered is one hour.

Since the proposed electricity market has n_p producers and n_c consumers, every possible bilateral trades can be condensed in the demand matrix $\mathbf{P} \in \mathbb{R}^{n_c \times n_p}$ as in (5.1). Each element p_{ji} of the demand matrix $\mathbf{P}$ represents the power demand

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Figure 5.1: The schematic model of P2P energy market

of consumer j from producer i . All entries of the demand matrix are considered as decision variables.

$$\mathbf{P} = \begin{bmatrix} p_{11} & \cdots & p_{1n_p} \\ \vdots & \ddots & \vdots \\ p_{n_c1} & \cdots & p_{n_cn_p} \end{bmatrix} \quad (5.1)$$

The j^{th} row of $\mathbf{P}$, denoted by vector $\mathbf{P}_j \in \mathbb{R}^{1 \times n_p}$ gives the demand schedule of consumer j . The total demand of consumer j is given by

$$p_j = \sum_{i \in \mathcal{P}} p_{ji} \quad (5.2)$$

Similarly, the i^{th} column of $\mathbf{P}$, denoted by $\mathbf{P}_i \in \mathbb{R}^{n_c \times 1}$ gives the supply schedule of producer i . For each producer i

$$p_i - \phi_i(p_i) = \sum_{j \in \mathcal{C}} p_{ji} \quad (5.3)$$

In (5.3), $\phi_i(p_i)$ is the power losses induced by producer i . The power losses are separable and can be approximated as a nonlinear function of p_i as follows [158]:

$$\phi_i(p_i) = \varrho_i p_i^2 \quad (5.4)$$

The value of loss-coefficient ϱ_i depends on the parameters and configuration of the the network model [158].

5.2.2 Consumer and Producer Model

The utility function represents the personal satisfaction or convenience for electricity usage, and it can be expressed as a function of energy demand. The utility function of consumer j is denoted by $U_j(p_j)$, and defined in (2.12). The utility function parameters β_j and θ_j are the private information of consumer j .

The cost function $C_i(p_i)$ of the generator owned by producer i is a quadratic convex function of power p_i [73] as defined in (2.13). The cost function parameters a_i , b_i , and c_i are the private information of producer i . The coefficient of losses in (5.4) satisfies $0 \leq \varrho_i \leq a_i$.

5.2.3 Network Utilization Fees for P2P Transactions

A new network structure called an electrical structure of the network based on the electrical distance is used to calculate the network utilization fees. The electrical structure of the network has the same number of the node to node connections as that in the topological structure [159]. The network owner provides the charging rate for network utilization in advance before the P2P negotiation starts. The network owner considers the capital cost recovery, cost of maintenance and modernization of power lines, taxes and policies, etc. to decide the rate for the network utilization. The detailed study of how the network owner decides the rate for using the network is not included in the scope of this thesis.

Now, if consumer j buys the p_{ji} amount of power from producer i over the electrical distance of d_{ji} , the network fee for consumer j is estimated by

$$T(p_{ji}) = \varphi d_{ji} p_{ji} \quad (5.5)$$

The total network fee to be paid by consumer j is

$$TF_j = \sum_{i \in \mathcal{P}} \varphi d_{ji} p_{ji} \quad (5.6)$$

There are various approaches such as the Thevenin's impedance distance, mutual impedance distance, power transfer distance (PTD), Jacobian distance to estimate

5.2. Problem Formulation

the electrical distance between nodes in the power system [160]. In this work, the PTD is used to estimate the electrical distances between producers and consumers, and PTD is used interchangeably with the electrical distance. PTD between two nodes indicates how much of the network's assets are used in facilitating a P2P transaction between two nodes. PTD is calculated using the Power Transfer Distribution Factor (PTDF). The PTDF is the fraction of transacted power from one node to another node that flows over a given line $o \in \mathcal{E}$. The detail method to calculate PTDF is given in [161] and summarized here. The PTDF matrix is calculated as

$$\mathbf{PTDF} = \mathbf{H}_r \mathbf{B}_r^{-1} \quad (5.7)$$

where $\mathbf{H}_r$ is the sub-matrix of matrix $\mathbf{H}$ obtained by deleting the column corresponding to the slack node, and $\mathbf{B}_r$ is the sub-matrix of matrix B obtained by removing the row and column corresponding to the slack node. The matrices $\mathbf{H}$ and $\mathbf{B}$ are defined as

$$\mathbf{B}_{mn} = -\frac{1}{x_{mn}} \text{ for } m \neq n, \mathbf{B}_{mm} = \sum_{m \neq n} \frac{1}{x_{mn}} \quad (5.8)$$

$$\mathbf{H}_{om} = -\mathbf{H}_{on} = \frac{1}{x_{mn}}, \mathbf{H}_{or} = \sum_{m \neq n} \frac{1}{x_{mn}} \text{ for } r \neq n, m \quad (5.9)$$

where indices m , n , and r represent nodes. The PTD between node m and node n denoted as d_{mn} is calculated by

$$d_{mn} = \sum_{o \in \mathcal{E}} |\mathbf{PTDF}_{o,mn}| \quad (5.10)$$

The PTD between consumer j and producer i is

$$d_{ji} = \sum_{o \in \mathcal{E}} |\mathbf{PTDF}_{o,ji}| \quad (5.11)$$

5.2.4 Welfare of Consumer and Producer

In the P2P market, consumers negotiate with each producer at the same time. It means each consumer can buy energy from different producers with various marginal costs, and for each trade, there is a different network utilization fee. Therefore, a consumer has different valuation or welfare for each trade. If consumer

j trades energy with producer i in P2P manner, the welfare of consumer j is given by the utility of the demand p_{ji} minus the sum of the paid money for this energy and the network utilization fee to be paid for this trade. Mathematically,

$$w_{ji} = U_j(p_{ji}) - \lambda_i p_{ji} - T(p_{ji}) \quad (5.12)$$

where λ_i is the per unit price of energy from producer i . The total welfare of consumer is given by the sum of the welfare from all possible P2P trades, i.e., $\sum_{i \in \mathcal{P}} w_{ji}$, and can be expressed as

$$W_j = \sum_{i \in \mathcal{P}} U_j(p_{ji}) - \sum_{i \in \mathcal{P}} \lambda_i p_{ji} - TF_j \quad (5.13)$$

The welfare of producer i is modeled as

$$W_i = \lambda_i(p_i - \phi_i(p_i)) - C_i(p_i) \quad (5.14)$$

The first term in (5.14) indicates the revenue collected by selling energy to the consumers and the second term represents the corresponding generation cost.

5.2.5 Optimization Problem

The P2P energy trading in smart grids is formulated as a social welfare maximization problem as follows:

$$\underset{\mathbf{P}, \mathbf{p}_\mathbf{p}}{\text{maximize}} \quad \sum_{j \in \mathcal{C}} \sum_{i \in \mathcal{P}} \hat{U}_j(p_{ji}) - \sum_{i \in \mathcal{P}} C_i(p_i) \quad (5.15a)$$

$$\text{s. t.} \quad \sum_{j \in \mathcal{C}} p_{ji} = p_i - \phi_i(p_i), \forall i \in \mathcal{P} \quad (5.15b)$$

$$\underline{p}_i \leq p_i \leq \bar{p}_i, \forall i \in \mathcal{P} \quad (5.15c)$$

$$\underline{p}_j \leq \sum_{i \in \mathcal{P}} p_{ji} \leq \bar{p}_j, \forall j \in \mathcal{C} \quad (5.15d)$$

where $\hat{U}_j(p_{ji}) = U_j(p_{ji}) - T(p_{ji})$; and $\mathbf{p}_\mathbf{p} = [p_i]_{i \in \mathcal{P}}$. A balance between the supply and demand in the system is essential for a stable operation of the power grid. The constraint (5.15b) represents the power balance constraint for producer i , i.e., the total power demanded by consumers from producer i should match the total generation less the loss contribution of producer i . For the feasibility of (5.15), Assumption 1 given below must be satisfied.

Assumption 1. *The total demand from consumers to producer $i \in \mathcal{P}$ must be*

5.2. Problem Formulation

higher than its minimum generation capacity, i.e.,

$$\sum_{j \in \mathcal{C}} p_{ji} \geq \underline{p}_i - \phi_i(\underline{p}_i) \quad (5.16)$$

A dual variable corresponding (5.15b), i.e., λ_i represents the energy price of producer i . A power balance in the system is established when individual producer satisfies (5.15b). Hence, (5.15b) is a global constraint, and (5.15c) and (5.15d) are the local capacity constraints of each producer and consumer, respectively. The nonlinear equality constraint (5.15b) makes the optimization problem (5.15) non-convex, which is difficult to solve directly. However, the same optimal solution can be recovered by transforming the non-convex problem (5.15) into a strictly convex problem (5.17) by relaxing the non-convex equality constraint.

$$\underset{\mathbf{P}, \mathbf{P}_\mathbf{P}}{\text{maximize}} \quad \sum_{j \in \mathcal{C}} \sum_{i \in \mathcal{P}} \hat{U}_j(p_{ji}) - \sum_{i \in \mathcal{P}} C_i(p_i) \quad (5.17a)$$

$$\text{s. t.} \quad \sum_{j \in \mathcal{C}} p_{ji} \leq p_i - \phi_i(p_i), \forall i \in \mathcal{P} \quad (5.17b)$$

(5.15c) and (5.15d)

Theorem 1. *The transformed problem (5.17) has the same optimal solution as the original problem (5.15).*

Proof. The Lagrangian of transformed problem (5.17) is

$$\begin{aligned} \mathcal{L} = & \sum_{j \in \mathcal{C}} \sum_{i \in \mathcal{P}} \hat{U}_j(p_{ji}) - \sum_{i \in \mathcal{P}} C_i(p_i) + \lambda_i \left(p_i - \varrho_i p_i^2 - \sum_{j \in \mathcal{C}} p_{ji} \right) \\ & + \underline{\vartheta}_i (p_i - \underline{p}_i) + \bar{\vartheta} (\bar{p}_i - p_i) + \sum_{j \in \mathcal{C}} \underline{\mu}_j \left(\sum_{i \in \mathcal{P}} p_{ji} - \underline{p}_j \right) \\ & + \sum_{j \in \mathcal{C}} \bar{\mu}_j \left(\bar{p}_j - \sum_{i \in \mathcal{P}} p_{ji} \right) \end{aligned} \quad (5.18)$$

The KKT conditions of (5.17) are given below in (5.19).

$$\frac{\partial \mathcal{L}}{\partial p_i} = -\frac{\partial C_i(p_i)}{\partial p_i} + \lambda_i (1 - 2\varrho_i p_i) - \bar{\vartheta}_i + \underline{\vartheta}_i, \forall i \in \mathcal{P} \quad (5.19a)$$

$$\frac{\partial \mathcal{L}}{\partial p_{ji}} = \frac{\partial \hat{U}_j(p_{ji})}{\partial p_{ji}} - \lambda_i - \bar{\mu}_i + \underline{\mu}_i, \forall j \in \mathcal{C} \quad (5.19b)$$

$$\lambda_i \left(\sum_{j \in \mathcal{C}} p_{ji} - p_i + \varrho_i p_i^2 \right) = 0, \lambda_i \geq 0; \forall i \in \mathcal{P} \quad (5.19c)$$

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$$\bar{\vartheta}_i(\bar{p}_i - p_i) = 0, \bar{\vartheta}_i \geq 0; \forall i \in \mathcal{P} \quad (5.19d)$$

$$\underline{\vartheta}_i(p_i - \underline{p}_i) = 0, \underline{\vartheta}_i \geq 0; \forall i \in \mathcal{P} \quad (5.19e)$$

$$\bar{\mu}_j(\bar{p}_j - p_j) = 0, \bar{\mu}_j \geq 0; \forall j \in \mathcal{C} \quad (5.19f)$$

$$\underline{\mu}_j(p_j - \underline{p}_j) = 0, \underline{\mu}_j \geq 0; \forall j \in \mathcal{C} \quad (5.19g)$$

As per the KKT conditions (5.19c), $\lambda_i \geq 0$ for all $i \in \mathcal{P}$. If $\lambda_i > 0$, (5.17b) is strictly satisfied by the optimal solution due to the KKT condition (5.19c).

Since $\bar{p}_i > \underline{p}_i$ for all $i \in \mathcal{P}$, either $\bar{\vartheta}_i$ or $\underline{\vartheta}_i$ equals to zero due to the KKT conditions (5.19d) and (5.19e). Assume $\lambda_i = 0$ for some $i \in \mathcal{P}$. Then, $\underline{\vartheta}_i$ should be positive to satisfy the KKT condition (5.19a), as $\frac{\partial C_i(p_i)}{\partial p_i} > 0$ (strictly positive) for all $i \in \mathcal{P}$. This means λ_i can be equal to 0 only when $p_i = \underline{p}_i$. Then, the KKT condition (5.19c) implies either (5.20a) or (5.20b).

$$\sum_{j \in \mathcal{C}} p_{ji} < \underline{p}_i - \phi_i(\underline{p}_i) \quad (5.20a)$$

$$\sum_{j \in \mathcal{C}} p_{ji} = \underline{p}_i - \phi_i(\underline{p}_i) \quad (5.20b)$$

As per Assumption 1, (5.20a) is a contradiction and hence, (5.16) is the only possible condition. Therefore, the optimal solution which satisfies the KKT conditions strictly satisfies (5.17b). Hence, the two problems (5.15) and (5.17) are equivalent. $\square$

5.3 Design of Market Clearing Algorithm

Problem (5.17) can be solved in a centralized fashion having a coordinator with all the information of agents in the market. But, the presence of a coordinator may breach the privacy of agents and affect the fairness in the market clearing process, which is undesirable. Hence, a decentralized approach is proposed to solve (5.15), where each agent needs to solve its sub-problem locally with a limited amount of information from other agents. Due to the presence of spatially coupled constraints (5.15d) and (5.17b), problem (5.17) cannot be solved directly. Firstly, problem (5.15) is decomposed into a set of single producer multi-consumer sub-problems

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based on the principle of dual decomposition [95] and the sub-problems are solved distributively. Since the electricity price is a crucial variable used by all agents in the market, it is used to realize the coordination among producers and consumers. The detailed methodology is explained in the following sections.

5.3.1 Decoupling into Sub-problems

Let us define a Lagrangian of transformed primal problem (5.17) by relaxing the spatially coupled constraint (5.15d) and (5.17b) as follows:

$$\begin{aligned} \mathcal{L}(\mathbf{P}, \mathbf{p}_p, \mathbf{\Lambda}, \underline{\boldsymbol{\mu}}, \bar{\boldsymbol{\mu}}) &= \sum_{j \in \mathcal{C}} \sum_{i \in \mathcal{P}} \hat{U}_j(p_{ji}) - \sum_{i \in \mathcal{P}} C_i(p_i) \\ &+ \sum_{j \in \mathcal{C}} \underline{\mu}_j \left(\sum_{i \in \mathcal{P}} p_{ji} - \underline{p}_j \right) + \sum_{j \in \mathcal{C}} \bar{\mu}_j \left(\bar{p}_j - \sum_{i \in \mathcal{P}} p_{ji} \right) \\ &+ \sum_{i \in \mathcal{P}} \lambda_i \left(p_i - \phi_i(p_i) - \sum_{j \in \mathcal{C}} p_{ji} \right) \end{aligned} \quad (5.21)$$

where $\lambda_i \geq 0$ is the Lagrangian multiplier or dual variable for producer i corresponding to constraint (5.17b); $\underline{\mu}_j, \bar{\mu}_j \geq 0$ are Lagrangian multipliers for consumer j corresponding to (5.15d); and $\mathbf{\Lambda} \triangleq [\lambda_i]_{i \in \mathcal{P}}$, $\bar{\boldsymbol{\mu}} \triangleq [\bar{\mu}_j]_{j \in \mathcal{C}}$ and $\underline{\boldsymbol{\mu}} \triangleq [\underline{\mu}_j]_{j \in \mathcal{C}}$ are vectors of Lagrangian multipliers. From an economic point of view, the dual variable λ_i represents the energy price of producer i to maintain the balance between supply and demand. The constraint in (5.15c) is not included in the Lagrangian equation as they are local constraints and can be treated as the boundaries of the feasible region of the local problems.

The supremum of the Lagrangian over the variables $\mathbf{P}$ and $\mathbf{p}_p$ gives a dual function as

$$\begin{aligned} \mathcal{D}(\mathbf{\Lambda}, \underline{\boldsymbol{\mu}}, \bar{\boldsymbol{\mu}}) &= \sup_{\mathbf{P}, \mathbf{p}_p} \mathcal{L}(\mathbf{P}, \mathbf{p}_p, \mathbf{\Lambda}, \underline{\boldsymbol{\mu}}, \bar{\boldsymbol{\mu}}) \\ &= \sum_{i \in \mathcal{P}} \sum_{j \in \mathcal{C}} \mathcal{G}_{ji}(\lambda_i, \underline{\mu}_j, \bar{\mu}_j) + \sum_{i \in \mathcal{P}} \mathcal{H}_i(\lambda_i) \\ &+ \sum_{j \in \mathcal{C}} (\bar{\mu}_j \bar{p}_j - \underline{\mu}_j \underline{p}_j) \end{aligned} \quad (5.22)$$

where $\mathcal{H}_i(\lambda_i)$ is the sub-problem to be solved by producer i and $\mathcal{G}_{ji}(\lambda_i, \underline{\mu}_j, \bar{\mu}_j)$ is the subproblem to be solved by consumer j to trade energy with producer i . The

sub-problems are defined as follows

$$\mathcal{H}_i(\lambda_i) \triangleq \arg \max_{\underline{p}_i \leq p_i \leq \bar{p}_i} [\lambda_i(p_i - \phi_i(p_i)) - C_i(p_i)] \quad (5.23)$$

$$\mathcal{G}_{ji}(\lambda_i, \underline{\mu}_j, \bar{\mu}_j) \triangleq \arg \max_{0 \leq p_{ji} \leq \bar{p}_j} [\tilde{U}_j(p_{ji}) - \lambda_i p_{ji}] \quad (5.24)$$

where $\tilde{U}_j(p_{ji}) \triangleq \hat{U}_j(p_{ji}) + (\underline{\mu}_j - \bar{\mu}_j)p_{ji}$. The social welfare maximization concurrently maximizes the individual welfare of the consumers and producers. Now, the dual problem is defined as

$$\begin{aligned} & \underset{\Lambda, \underline{\mu}, \bar{\mu}}{\text{minimize}} \mathcal{D}(\Lambda, \underline{\mu}, \bar{\mu}) \\ & \text{s. t. } \lambda_i, \underline{\mu}_j, \bar{\mu}_j \geq 0, \quad \forall i \in \mathcal{P}, \quad \forall j \in \mathcal{C} \end{aligned} \quad (5.25)$$

Theorem 2. *The transformed problem (5.17) holds the strong duality.*

Proof. Let $\Psi_i \geq \varrho_i p_i^2 > 0$ for all $i \in \mathcal{P}$ be a decision variable of (5.17) such that

$$\sum_{j \in \mathcal{C}} p_{ji} = p_i - \Psi_i; \quad \forall i \in \mathcal{P} \quad (5.26a)$$

$$\left(\frac{\Psi_i + 1}{2} \right)^2 \geq \left(\frac{\Psi_i - 1}{2} \right)^2 + \left(\sqrt{\varrho_i} p_i \right)^2; \quad \forall i \in \mathcal{P} \quad (5.26b)$$

where (5.26a) is a linear constraint and (5.26b) is a convex second-order cone constraint. Accordingly, (5.26) can replace the power balance constraint (5.17b) in (5.17). Therefore, (5.17) is a convex (second-order cone) program which holds a strong duality. $\square$

The dual problem (5.25) can be solved iteratively by deploying the sub-gradient projection method. The Lagrangian multipliers are updated in the opposite direction to the sub-gradient¹ of the dual function as

$$\underline{\mu}_j^{k+1} = \left[\underline{\mu}_j^k - \kappa \nabla_{\underline{\mu}_j} \mathcal{D} \right]^+ \quad (5.27a)$$

$$\bar{\mu}_j^{k+1} = \left[\bar{\mu}_j^k - \kappa \nabla_{\bar{\mu}_j} \mathcal{D} \right]^+ \quad (5.27b)$$

$$\lambda_i^{k+1} = \left[\lambda_i^k - \kappa \nabla_{\lambda_i} \mathcal{D} \right]^+ \quad (5.27c)$$

where k is the iteration counter; and κ is the step-size for Lagrangian multiplier

¹Suppose $f : \mathbf{R}^n \rightarrow \mathbf{R}$ is convex. To minimize f , the sub-gradient method uses the iteration $x^{k+1} = x^k - \alpha_k g^k$, where x^k is the k^{th} iterate, g^k is any sub-gradient of f at x^k , and $\alpha_k > 0$ is the k^{th} step-size. Thus, at each iteration of the sub-gradient method, we take a step in the direction of a negative sub-gradient.

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update. Each consumer updates $\underline{\mu}_j$ and $\bar{\mu}_j$ in each iteration. These values are not shared with any other entities in the market. But, each producer updates λ_i in each iteration and shares with consumers in the market.

5.3.2 Sub-problem Solution

At each iteration k , given the value of dual variable, i.e., price λ_i^k , each consumer obtains energy demand from producer i , $\hat{p}_{ji}^k$ by locally (independently) solving the sub-problem (5.24),

$$\hat{p}_{ji}^k = \left[u_j^{-1}(\lambda_i^k) \right]_0^{\bar{p}_j}, \quad \forall j \in \mathcal{C} \quad (5.28)$$

where $u_j(p_{ji}) = \frac{\partial \tilde{U}_j(p_{ji})}{\partial p_{ji}}$.

Similarly, each producer i independently solves (5.23) for a given λ_i^k to determine the energy production. Define $v_i(p_i) = \frac{\partial C_i(p_i)}{\partial p_i} \left(1 - \frac{\partial \phi_i(p_i)}{\partial p_i} \right)^{-1}$, then

$$\hat{p}_i^k = \left[v_i^{-1}(\lambda_i^k) \right]_{\underline{p}_i}^{\bar{p}_i}, \quad \forall i \in \mathcal{P} \quad (5.29)$$

The update rule for the dual variable λ_i becomes

$$\lambda_i^{k+1} = \left[\lambda_i^k - \kappa \left((\hat{p}_i^k - \phi_i(\hat{p}_i^k)) - \sum_{j \in \mathcal{C}} \hat{p}_{ji}^k \right) \right]^+ \quad (5.30)$$

The update rules for the Lagrangian multipliers $\underline{\mu}_j$ and $\bar{\mu}_j$ are:

$$\underline{\mu}_j^{k+1} = \left[\underline{\mu}_j^k - \kappa \left(\sum_{i \in \mathcal{P}} \hat{p}_{ji}^k - \underline{p}_j \right) \right]^+ \quad (5.31a)$$

$$\bar{\mu}_j^{k+1} = \left[\bar{\mu}_j^k - \kappa \left(\bar{p}_j - \sum_{i \in \mathcal{P}} \hat{p}_{ji}^k \right) \right]^+ \quad (5.31b)$$

In order to ensure the convergence, the value of κ in (5.30) and (5.31) should be sufficiently small such that $0 < \kappa < 2/K$, where K is Lipschitz constant [162] for the dual function:

$$\|\nabla \mathcal{D}(\mathbf{\Omega}_1) - \nabla \mathcal{D}(\mathbf{\Omega}_2)\|_F \leq K \|\mathbf{\Omega}_1 - \mathbf{\Omega}_2\|_F \quad (5.32)$$

where $\mathbf{\Omega} = (\mathbf{\Lambda}, \mathbf{\underline{\mu}}, \mathbf{\bar{\mu}})$ is the single Lagrangian multiplier, and $\|\cdot\|_F$ is the matrix Frobenius norm. The stopping criterion are $|\lambda_i^{k+1} - \lambda_i^k| < \epsilon_\lambda$, $|\underline{\mu}_j^{k+1} - \underline{\mu}_j^k| < \epsilon_\mu$, and $|\bar{\mu}_j^{k+1} - \bar{\mu}_j^k| < \epsilon_\mu$. The process of the price update by producer i and the demand update by consumer j is summarized in Algorithm 5 and Algorithm 6, respectively.

The proposed algorithms are executed through two-way communications between

producers and consumers. Energy price and energy demand are the two pieces information need to be exchanged between each producer and consumer during P2P trading negotiation. Fig. 5.2 illustrates the interaction between producers and consumers during P2P trading negotiation.

Figure 5.2: Illustration of information exchange between producers and consumers during P2P trading negotiation

Algorithm 5 Price Update by Producer i

Require: Termination criteria ϵ_λ

Determine: Energy price for P2P energy trading λ_i

- 1: Assign the initial value of energy price λ_i^0 ;
 - 2: **repeat**
 - 3: Broadcast energy price λ_i^k to all consumers;
 - 4: Receive all consumers' demand $\hat{p}_{ji}^k$ for the given price;
 - 5: Update production $\hat{p}_i^k$ by (5.29);
 - 6: Update the energy price by (5.30);
 - 7: **until** Energy price λ_i converges as $|\lambda_i^{k+1} - \lambda_i^k| < \epsilon_\lambda$.
-

5.4 Results and Discussions

This section presents numerical case studies to show the feasibility and effectiveness of the proposed method for the P2P energy trading. For numerical case studies, an IEEE 9-bus system with three producers and six consumers, as shown

Algorithm 6 Demand Update by Consumer j

Require: Energy price $\Lambda^k = [\lambda_1^k, \lambda_2^k, \dots, \lambda_{n_p}^k]$

Determine: Power consumption p_j^k

- 1: **repeat**
 - 2: Receive energy price λ_i^k from producer i ;
 - 3: Update demand $\hat{p}_{ji}^k$ from producer i by (5.28);
 - 4: Broadcast $\hat{p}_{ji}^k$ to corresponding producer i ;
 - 5: Update Lagrangian multipliers $\underline{\mu}_j^{k+1}$ and $\bar{\mu}_j^{k+1}$ by (5.31);
 - 6: **until** Lagrangian multipliers $\underline{\mu}_j$ and $\bar{\mu}_j$ converge as $|\underline{\mu}_j^{k+1} - \underline{\mu}_j^k| < \epsilon_\mu$, and $|\bar{\mu}_j^{k+1} - \bar{\mu}_j^k| < \epsilon_\mu$.
-

in Fig 5.3 is considered. The parameters for consumers and producers in the IEEE 9-bus system are taken from [91] and given in Table 5.1. Based on the data and the proposed algorithm, all the simulations are conducted in the MATLAB 2016a environment with an Intel(R) Xeon(R) CPU E5-1630 v4@3.70 GHz, 16 GB RAM. The step-size for the Lagrangian multiplier update κ is chosen as 0.005. The required tolerances for termination are set to $\epsilon_\lambda = 0.001$ and $\epsilon_\mu = 0.001$. The initial values are set as $\lambda_i^0 = \frac{\partial C_i}{\partial p_i}|_{p_i=p_i^0}, \forall i \in \mathcal{P}$ and the charging rate for the network usage is assumed to be $\varphi = 0.2$ \$/MWh per electrical distance unit. The following four cases are considered for the numerical studies:

- *Case 1:* P2P trading without losses and network fees.
- *Case 2:* P2P trading with losses.
- *Case 3:* P2P trading with network fees.
- *Case 4:* P2P trading with both losses and network fees.

It is worth mentioning that the results from the centralized approach are used as a benchmark to validate the results from the proposed decentralized approach. The centralized approach is implemented using the interior-point method in *Gurobi 8.1.1*.

Fig. 5.4 shows the development of the objective value for different cases using proposed decentralized approach. It can be seen that the optimal objective value obtained from the proposed decentralized algorithm in all cases match the global

5.4. Results and Discussions

Figure 5.4: Evolution of objective value in different cases

Figure 5.5: Evolution of total demand and supply in different cases

cases. Once the power losses are taken into account, the total supply should meet the sum of the demand and losses. The mismatch between the total supply and

Figure 5.6: Producers prices update during P2P negotiation in different cases

Figure 5.7: Supply adjustment of producers in response to prices in different cases

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Figure 5.8: Evolution of primal residuals $\|\mathbf{P}^k - \mathbf{P}^*\|_2$ in different cases.

the total demand is the total losses in the system. Fig. 5.6 shows the evolution of the producers prices in the above mentioned four cases. The evolution of the MW supply of producers in different cases are shown in Fig. 5.7. It shows that producers update their production decision in response to the evolution in their prices. Fig. 5.8 shows the evolution of primal residuals given by $\|\mathbf{P}^k - \mathbf{P}^*\|_2$ in different cases, where $\mathbf{P}^k$ is the demand matrix $\mathbf{P}$ at the k^{th} iteration. $\mathbf{P}^*$ is the global optimal solution obtained by using the centralized approach, and $\|\cdot\|_2$ is the Euclidean norm. The primal residual or Euclidean norm indicates how close the optimal solution obtained from the proposed decentralized algorithm with the optimal global solution obtained from the centralized approach. The solution is considered to be more accurate if the value of the Euclidean norm is small. The value of the Euclidean norm is less than 0.01 for all cases, which is negligible. Hence, the solution from the proposed method converges to the global optimum solution despite the individuals behave in a greedy manner.

Table 5.2: PTD For IEEE 9-Bus System in Fig. 5.3

Producer	Consumer					
	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉
P ₁	1.00	2.50	2.54	3.72	4.00	3.77
P ₂	3.72	2.95	4.00	1.00	2.42	3.51
P ₃	3.77	4.00	3.00	3.51	2.59	1.00

Table 5.3: Price of Producers For P2P Energy Trading

Producer	Energy price (\$/MWh)			
	Case 1	Case 2	Case 3	Case 4
P ₁	5.7586	6.3935	5.4205	6.0017
P ₂	6.2853	6.9535	5.9940	6.5830
P ₃	6.0765	6.5523	5.7671	6.2071

Table 5.4: Demand of Individual Consumer From Different Producers

Cases	Producers	Consumers					
		C ₄	C ₅	C ₆	C ₇	C ₈	C ₉
Case 1	P ₁	34.602	32.445	34.022	40.752	26.551	50.919
	P ₂	27.284	24.465	26.498	31.176	19.529	39.215
	P ₃	30.187	27.628	29.480	34.972	22.313	43.855
Case 2	P ₁	27.785	22.826	33.423	29.209	19.861	36.818
	P ₂	18.008	14.342	25.424	19.028	12.395	24.368
	P ₃	23.579	20.419	31.154	26.321	17.744	33.281
Case 3	P ₁	36.521	29.994	36.208	33.263	20.393	41.679
	P ₂	20.993	19.952	23.845	32.836	16.952	30.099
	P ₃	24.013	20.195	29.947	27.843	19.526	46.286
Case 4	P ₁	28.728	22.607	35.573	22.796	17.510	28.764
	P ₂	13.091	12.446	23.098	22.127	13.964	17.010
	P ₃	18.181	14.947	31.329	19.843	18.525	36.509

Table 5.2 shows the PTD for different trading pairs of P2P energy transactions in the IEEE 9-bus system shown in Fig. 5.3. These PTDs are used to calculate the network utilization fee for every possible P2P transaction. The energy prices of different producers for the P2P energy trading in different cases are shown in Table 5.3. The electricity prices in *Case 2* are higher compared with those of *Case 1* due to power losses as shown in Fig. 5.6. The producers set higher prices because they have to bear the cost of losses from the revenue of the actual energy sold. Table 5.4 shows the trading amount between different pairs in the above mentioned four different cases. It is obvious that the amount of energy transacted between the trading pair is less in *Case 2* as compared with those in *Case 1* because of the higher prices in *Case 2*. But, when the network utilization fee is considered in the

5.4. Results and Discussions

P2P trading, the amount of energy transacted between the trading pairs depends not only on the energy price but also on the network fee. The trading of consumer C_9 with different producers in *Case 3* is chosen to explain the effect of the network utilization fee on P2P trading decisions. Consumer C_9 buys the largest amount of power from producer P_3 despite the price offered by producer P_1 is the cheapest. Such alteration in trading decisions is because of the electrical distance between the trading pairs. It is clear from Table 5.2 that producer P_3 is electrically near to C_9 as compared to P_1 , i.e., PTD between P_3 and C_9 is 1.00 whereas PTD between P_1 and C_9 is 3.77. Hence, consumer C_9 has to pay more network fees if it buys more power from P_1 . So it prefers to buy from P_3 . Similar observations can be made in other consumers too. The combined effects of power losses and network fees on P2P trading decisions can be observed in *Case 4*. It is clear that if network utilization fees are considered in P2P energy trading, the decision of consumers depends on the prices offered by producers as well as the electrical distance from producers.

Figure 5.9: Convergence characteristics of the proposed method in IEEE 39-bus system without considering losses and network fees (*Case 1*).

Figure 5.10: Evolution of primal residuals $\|\mathbf{P}^k - \mathbf{P}^*\|_2$ in different cases for IEEE 39-bus system.

The proposed algorithm is also applied on the IEEE 39-bus system with 18 consumers and 10 producers to demonstrate the scalability. The parameters of producers and consumers are taken from [16, 17]. Fig. 5.9 shows the evolution of the different parameters, i.e., the total supply and total demand, objective value, producers' prices, and supply, using the proposed method in the IEEE 39-bus system without considering losses and network fees (*Case 1*). The total supply equates the total demand, and the power balance condition is being satisfied when the number of iterations $k = 589$ for the same level of accuracy as in the case of the IEEE 9-bus system. Fig. 5.9(c) and 5.9(d) show an evolution of producers' prices update and supply decisions update, respectively. These figures consist of 10 curves in each as there are 10 producers in the system. As the prices converge, the objective value, i.e., social welfare, converges to its optimal value, as shown in Fig. 5.9(b). Fig. 5.10 shows the evolution of the primal residuals. It demonstrates the convergence of the proposed decentralized algorithm in the IEEE 39-bus sys-

5.5. Summary

tem. The optimal objective value from the proposed approach is the same as the one obtained from the centralized method. The small value of the primal residual and the matching of the optimal objective value are evidence of the convergence of the proposed algorithm. It is obvious that the extent of the computations being performed by each agent is determined by the number of iterations required for convergence. However, in the proposed algorithm, the computational burden is fairly shared among all agents in the system. Hence, the proposed decentralized algorithm for market clearing in the P2P energy trading is scalable in terms of computational burden and convergence.

5.5 Summary

In this chapter, a fully decentralized market clearing mechanism for the P2P energy trading is proposed. A P2P energy trading is formulated as an aggregated welfare maximization problem with consideration of the grid usage aspect in P2P energy transactions. The original non-convex problem is transformed into a convex problem by relaxing the non-convex equality constraint under mild assumptions. The electricity prices and generation/demand are adjusted to maximize the social welfare and to achieve the balance between the supply and demand in the market. The proposed method is applied to the IEEE 9-bus system and IEEE 39-bus system. The convergence and scalability of the proposed method are verified via numerical results. It is found that if the network fees are considered in the P2P energy trading, the decision of consumers depends on the prices offered by producers as well as the electrical distance from producers.

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

LET is the basis of the formation of the decentralized electricity market for energy transactions among various producers and consumers in future smart grids. TE is based on the buying and selling of energy known as transactions among parties that consume, produce, store, and transport electrical energy. TE helps to make the electricity system more reliable, resilient, and secure through real-time and decentralized decision-making. Proactive consumers and producers are the prime stakeholders in the LEM. Both the financial as well as non-financial factors, motivate the prosumers and consumers to enter the local market. A platform or market with properly defined trading rules, which enables agents to make trading decisions and exchange information with other agents, is necessary to implement the LET concept. On the other side, a physical network is used for energy transfer, and hence excluding the physical network from LET is not practical.

This thesis studied various aspects of energy transactions in the LEM in future power systems considering the technical aspects of the network. The research work

mainly focused on developing the privacy-preserving decentralized market clearing algorithms for LET. Besides, grid-related aspects such as power losses, network usage charges, and grid voltage constraints were also considered. The research work conducted in this thesis has shown several prospects for future studies. A chapter-wise summary of the significance, objectives and outcomes of this research are summarized in the following paragraphs.

Chapter 1 introduced the thesis with the background and motivation behind the research together with contributions made as well as the organization of the thesis. A detailed review of key aspects of TE and LET such as market design for LET, different market clearing approaches, key elements, and requirements for the implementation of LET as well as challenges and barriers for LET, were discussed in Chapter 2.

Chapter 3 described a game-theoretic model for real-time P2P energy trading in a prosumer based community microgrid. Prosumers in a community involved in P2P trading is either a seller or a buyer. There were two separate competitions during the trading process: (i) price competition among the sellers and (ii) seller selection competition among the buyers. An M-leader and N-follower Stackelberg game approach was used to model the interaction between buyers and sellers. The proposed method was applied to a small community microgrid with PV and energy storage systems. Simulation results showed that each game converges to a stable state using the proposed algorithms. They also showed that the cost of the microgrid is significantly reduced with P2P energy trading in both cases: (i) with BESS and (ii) without BESS, as compared to the cost in conventional P2G trading. The stochastic nature of prosumers and the physical network aspects were not considered in the proposed game-theoretic model for P2P energy trading.

Chapter 4 presented a decentralized formulation of decentralized LET in microgrids. LET in microgrids with DGs and responsive demands was formulated as an aggregated welfare maximization problem. The network voltage management was

6.2. Recommendations

also considered in LET. A fully decentralized approach based on local communication with neighbors was proposed for energy pricing and voltage management. The application of the proposed decentralized algorithm for LET on the IEEE 33-node microgrid network showed that the supply-demand balance in the microgrid was established through the coordination among the agents. At the same time, the energy price for LET was decided and all the node voltages were maintained within the safe operating limits. The two-stage approach improved the convergence speed of the algorithm. The effects of power losses and network charges in LET were not considered in this chapter.

Chapter 5 considered a grid usage aspect in P2P transactions and formulated a P2P energy trading as an aggregated welfare maximization problem. A fully decentralized market clearing mechanism for the P2P energy trading was proposed. The non-convexity that exists in the social welfare maximization problem was removed by relaxing the non-convex equality constraint under mild assumptions. The proposed method was applied to the IEEE 9-bus system and IEEE 39-bus system. Numerical results were used to verify the convergence and scalability of the proposed method. The network fees for each P2P transaction were calculated using an electrical distance approach. The trading decision of consumers was affected by the prices offered by producers as well as the electrical distance from producers.

6.2 Recommendations

The works presented so far in this thesis have been focused on the development of various models for LET and P2P energy trading. The LET concept is not matured yet, and several aspects need to be considered to make the LET concept widely acceptable. Some essential aspects to be considered to make LET widely acceptable for real applications are information privacy and security, power losses, grid constraints and dynamics, uncertainties and measurement errors. Based on the

challenges and barriers for LET identified in Chapter 2, as well as the contributions made in this thesis, following recommendations are made for future research directions.

1. As discussed before, information security and data privacy are crucial in any LEM. The decentralized algorithms proposed in this thesis increase the privacy as agents do not have to reveal their private information to other agents. The application of blockchain technology for LET would further enhance the information security and data privacy in LEM. It could also provide a strong barrier against potential cyber-attacks and help to maintain transparency. There are several barriers to implement blockchain technology with current components of LEM. Hence, the detailed study of blockchain application in LEM could be one of the possible future research directions.
2. The proposed decentralized algorithms in this thesis do not consider uncertainties in the generation and load forecast, as well as the measurement errors. Uncertainties in the renewable-based generations participating in LET could affect the system performance. The robust and fast converging market clearing algorithms represent LET scenarios more efficiently. The research work can be extended by considering uncertainties to enhance the robustness and efficiency of the market clearing algorithms to solve the stochastic optimization problems.
3. There are several issues related to electrical power networks. Some of them are power losses in the network, line congestions, voltage limits, grid stability, and power flow dynamics. Since agents in the LEM need power networks for energy exchange, LET would not be feasible when such factors related to physical networks are excluded. This thesis has put some efforts to include power losses and voltage constraints in LET algorithms. It is assumed that there is no congestion in the illustrated system during simulation studies. However, in real systems, line capacity constraints are considered to avoid

6.2. Recommendations

the overloading of the power lines. Apart from this, stability and power flow dynamics are also not studied. Therefore, the research work in this thesis can be extended to include network constraints using an optimal power flow.

4. In the near future, the co-existence of the conventional wholesale/retail markets and the newly designed LEM can be envisaged. Therefore, agents should have the freedom to choose between different markets. Besides, ancillary services are not considered in recent designs of LEM. Therefore, the existing markets cannot be isolated completely from LEM, although there is enough generation in LEM. The interaction of LEM with the wholesale market is not studied in this thesis. Future research should focus on defining protocols for interactions between the newly designed LEM and the existing markets.
5. In simulation studies, it is assumed that the communication system is ideal. In order to implement the proposed algorithms in actual systems, real information and communication infrastructures should be designed to provide a secure path for information exchange. The implementation of LET algorithms in practical cases with a suitably designed communication systems to analyze their real-time behavior could be the possible extension of this research in the future.

List of Publications

The Author's contribution in this Ph.D thesis are summarised in the following publications:

Journal Publications:

1. **A. Paudel**, K. Chaudhari, C. Long, and H. B. Gooi, "Peer-to-Peer Energy Trading in a Prosumer Based Community Microgrid: A Game-Theoretic Model," *IEEE Transactions on Industrial Electronics*, vol. 66, no. 8, August 2019.
2. **A. Paudel**, M. Khorasany, and H. B. Gooi, "Decentralized Local Energy Trading in Microgrids with Voltage Management," *IEEE Transactions on Industrial Informatics*, early access, March 2020, doi: 10.1109/TII.2020.2980160.
3. **A. Paudel**, L. P. M. I. Sampath, Y. Jiawei, and H. B. Gooi, "Peer-to-Peer Energy Trading in Smart Grid Considering Power Losses and Network Fees," *IEEE Transactions on Smart Grid*, early access, May 2020, doi: 10.1109/TSG.2020.2997956.
4. Y. Jiawei, **A. Paudel**, and H. B. Gooi, "Compensation for Power Loss by A Proof-of-Stake Consortium Blockchain Microgrid," *IEEE Transactions on Industrial Informatics*, early access, July 2020, doi:10.1109/TII.2020.3007657.
5. M. Khorasany, **A. Paudel**, R. Razzaghi, and P. Siano, "Non-Mediated Negotiation for Peer-to-Peer Energy Trading in Smart Grids," *IEEE Transactions on Smart Grid*, (Under revision).

Conference Publications:

1. **A. Paudel**, Y. Jiawei, and H. B. Gooi, "Peer-to-Peer Energy Trading Considering Network Utilization Fees," *IEEE Power and Energy Society General Meeting 2020*, Montreal, Canada, Aug. 2-68, 2019.
2. **A. Paudel** and H. B. Gooi, "Pricing in Peer-to-Peer Energy Trading Using Distributed Optimization Approach," *IEEE Power and Energy Society General Meeting 2019*, Atlanta, GA, USA, Aug. 4-8, 2019.
3. Y. Jiawei, **A. Paudel** and H. B. Gooi, "Blockchain Framework for Peer-to-Peer Energy Trading with Credit Rating" *IEEE Power and Energy Society General Meeting 2019*, Atlanta, GA, USA, Aug. 4-8, 2019.
4. **A. Paudel** and H. B. Gooi, "A Hierarchical Peer-to-Peer Energy Trading in Community Microgrid Distribution Systems," *IEEE Power and Energy Society General Meeting (PESGM)*, Portland, OR USA, Aug. 5-9, 2018.
5. S. M. Tan, B. Wang, U. Manandhar, K. Chaudhari, **A. Paudel**, and H. B. Gooi, "Model Predictive Control for Hybrid Energy Storage System using Single Inductor Dual-Input Single-Output Converter," *ISGT Asia*, Singapore.

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Appendix A

A.1 Principle of Dual Decomposition Method

The dual decomposition method solves the convex optimization problem with N ($N \geq 2$) blocks of variables [95]:

$$\underset{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N}{\text{minimize}} \sum_{i=1}^N f_i(\mathbf{x}_i) \quad \text{s. t.} \quad \sum_{i=1}^N A_i \mathbf{x}_i = c, \quad (\text{A.1})$$

where $\mathbf{x}_i \in \mathbb{R}^{n_i}$, $A_i \in \mathbb{R}^{p \times n_i}$, $c \in \mathbb{R}^p$, and $f_i : \mathbb{R}^{n_i} \rightarrow (-\infty, +\infty]$ are closed proper convex functions, $i = 1, 2, \dots, N$. Its corresponding Lagrangian is:

$$\mathcal{L}(\mathbf{x}_1, \dots, \mathbf{x}_N, \lambda) = \sum_{i=1}^N f_i(\mathbf{x}_i) + \lambda^T \left(\sum_{i=1}^N A_i \mathbf{x}_i - c \right) \quad (\text{A.2})$$

where $\lambda \in \mathbb{R}^m$ represents the Lagrangian multiplier corresponding to constraint in (A.1). The dual decomposition method consists of the following iterations:

$$\begin{aligned} (\mathbf{x}_1^k, \dots, \mathbf{x}_N^k) &:= \arg \min_{\mathbf{x}_i} \mathcal{L}(\mathbf{x}_1, \dots, \mathbf{x}_N, \lambda^{k-1}) \\ \lambda^k &:= \left[\lambda^{k-1} + \rho^k \left(\sum_{i=1}^N A_i \mathbf{x}_i - c \right) \right]^+ \end{aligned} \quad (\text{A.3})$$

where $\rho^k > 0$ is a step size, and $[\cdot]^+$ denotes the projection onto the non-negative orthant. The dual variable λ will converge to the dual optimal λ^* as $k \rightarrow \infty$. Since the Lagrangian function (A.2) is separable in $\mathbf{x}_i$, the $\mathbf{x}$ -minimization step splits into N separate x_i -subproblems:

$$\mathbf{x}_i^k := \arg \min_{\mathbf{x}_i} f_i(\mathbf{x}_i) + \langle \lambda^{k-1}, A_i \mathbf{x}_i \rangle \quad (\text{A.4})$$

The $\mathbf{x}$ -minimization step (A.4) can be carried out independently, in parallel, for each $i = 1, \dots, N$. It makes dual decomposition method effective for distributed optimization.

A.2 Example of Dual Decomposition Method

Consider the following optimization problem¹:

$$\begin{aligned}
 & \underset{x_1, x_2, x_3}{\text{minimize}} \quad \underbrace{(2.5x_1^2 - 20x_1)}_{f_1(x_1)} + \underbrace{(x_2^2 - 15x_2)}_{f_2(x_2)} + \underbrace{(x_3^2 + 3x_3)}_{f_3(x_3)} \\
 & \text{s. t. } x_1 + x_2 - x_3 = 0 \\
 & x_1, x_2, x_3 \geq 0
 \end{aligned} \tag{A.5}$$

The Lagrangian function is

$$\begin{aligned}
 \mathcal{L}(x_1, x_2, x_3, \lambda) = & \underbrace{(2.5x_1^2 - 20x_1)}_{f_1(x_1)} + \underbrace{(x_2^2 - 15x_2)}_{f_2(x_2)} \\
 & + \underbrace{(x_3^2 + 3x_3)}_{f_3(x_3)} + \lambda \underbrace{(x_1 + x_2 - x_3)}_{\Delta x}
 \end{aligned} \tag{A.6}$$

where λ is a Lagrangian multiplier for the equality constraint. The inequality constraints are not included in (A.6) as they can be treated as the local constraints of the subproblems. The given optimization problem can be solved using the principle of dual decomposition.

A.2.1 With Coordinator

Based on the explanation in Section A.1, iterative equations for solving the given optimization problem can be written as

$$\begin{aligned}
 x_1^k &= \arg \min_{x_1 \geq 0} \mathcal{L}(x_1, x_2, x_3, \lambda^{k-1}) = \arg \min_{x_1 \geq 0} [f_1(x_1) + \lambda^{k-1}x_1] \\
 x_2^k &= \arg \min_{x_2 \geq 0} \mathcal{L}(x_1, x_2, x_3, \lambda^{k-1}) = \arg \min_{x_2 \geq 0} [f_2(x_2) + \lambda^{k-1}x_2] \\
 x_3^k &= \arg \min_{x_3 \geq 0} \mathcal{L}(x_1, x_2, x_3, \lambda^{k-1}) = \arg \min_{x_3 \geq 0} [f_3(x_3) - \lambda^{k-1}x_3] \\
 \lambda^k &= [\lambda^{k-1} + \rho \Delta x^k]^+
 \end{aligned} \tag{A.7}$$

If we assume that the variable x_1 is associated with Agent 1, x_2 is associated with Agent 2, and x_3 is associated with Agent 3, then x -minimization in (A.7) can be carried out independently, in parallel, for each agent. The iterative approach starts with the initialization of the value of λ by the coordinator and ends when the termination criterion $|\Delta x| < \epsilon$ is satisfied. In each iteration, the coordinator collects

¹All symbols and variables used in this problem are not applicable to other parts of this thesis.

the decision of each agent, updates the Lagrangian multiplier λ , and broadcasts updated λ to each agent. The process is repeated until the solution converged. The result of the iterative process with $\lambda^0 = 1$ and $\epsilon = 0.00001$ is summarized in Table A.2.1. The termination criterion is satisfied when the number of iterations $k = 111$ and the solution converged.

Table A.1: Dual decomposition method with coordinator

Iteration (k)	x_1^k	x_2^k	x_3^k	Δx^k	λ^k
1	3.800	7.000	0.000	10.800	2.080
2	3.584	6.460	0.000	10.044	3.084
3	3.383	5.958	0.042	9.299	4.014
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
111	1.833	2.083	3.916	9.39×10^{-6}	10.833

A.2.2 Without Coordinator

The presence of a centralized coordinator is undesirable as it requires all agents to communicate with the central coordinator. This example shows how the role of the central coordinator can be eliminated using local communication among the agents.

Assume that the given problem has the communication topology as shown in Fig. A.1. Based on Section 4.3.5, the communication matrix for the given problem is

Figure A.1: Communication topology for three agents problem

obtained as

$$c = \begin{bmatrix} 2/3 & 1/3 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/3 & 2/3 \end{bmatrix}$$

The updating rules for agents' coordination for this particular problem can be

written as

$$\begin{aligned}\hat{\lambda}_1^k &= \left[\sum_{m=1}^3 c_{1m} \hat{\lambda}_m^{k-1} + \rho \Delta \hat{x}_1^{k-1} \right]^+ \\ \hat{\lambda}_2^k &= \left[\sum_{m=1}^3 c_{2m} \hat{\lambda}_m^{k-1} + \rho \Delta \hat{x}_2^{k-1} \right]^+ \\ \hat{\lambda}_3^k &= \left[\sum_{m=1}^3 c_{3m} \hat{\lambda}_m^{k-1} + \rho \Delta \hat{x}_3^{k-1} \right]^+\end{aligned}\tag{A.8}$$

$$\begin{aligned}x_1^k &= \arg \min_{x_1 \geq 0} [f_1(x_1) + \hat{\lambda}_1^k x_1] \\ x_2^k &= \arg \min_{x_2 \geq 0} [f_2(x_2) + \hat{\lambda}_2^k x_2] \\ x_3^k &= \arg \min_{x_3 \geq 0} [f_1(x_3) - \hat{\lambda}_3^k x_3]\end{aligned}\tag{A.9}$$

$$\begin{aligned}\Delta \hat{x}_1^k &= \sum_{m=1}^3 c_{1m} \Delta \hat{x}_m^{k-1} + x_1^k - x_1^{k-1} \\ \Delta \hat{x}_2^k &= \sum_{m=1}^3 c_{2m} \Delta \hat{x}_m^{k-1} + x_2^k - x_2^{k-1} \\ \Delta \hat{x}_3^k &= \sum_{m=1}^3 c_{3m} \Delta \hat{x}_m^{k-1} + x_3^k - x_3^{k-1}\end{aligned}\tag{A.10}$$

where $(\hat{\lambda}_m^k)$ and $(\Delta \hat{x}_m^k)$ represent the local estimation of (λ_m) and (Δx_m) by agent m at the k^{th} iteration, respectively. The iterative approach starts with the initialization of $(\hat{\lambda}_m)$ and $(\Delta \hat{x}_m)$ by each agent and broadcasts initial values to other agents. The process ends when the termination criterion $|\Delta \hat{x}_m| < \epsilon$ is satisfied. The result of the iterative process with $\hat{\lambda}_m^0 = 1$, $\Delta \hat{x}_m^0 = 0$ and $\epsilon = 0.00001$ is summarized in Table A.2. The termination criteria is satisfied when the number of iterations $k = 318$ and the solution converged.

Table A.2: Dual decomposition method without coordinator

k	$\hat{\lambda}_1^k$	$\hat{\lambda}_2^k$	$\hat{\lambda}_3^k$	x_1^k	x_2^k	x_3^k	$\Delta \hat{x}_1^k$	$\Delta \hat{x}_2^k$	$\Delta \hat{x}_3^k$
1	1.000	1.000	1.000	3.800	7.000	0.000	3.800	7.000	0.000
2	1.380	1.700	1.000	3.724	6.650	0.000	4.791	3.250	2.333
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
9	3.713	3.635	3.571	3.257	5.612	0.285	3.011	2.854	2.788
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
318	10.833	10.833	10.833	1.833	2.083	3.916	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$