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Design and experimental validation of a Robust output feedback control for the coupled dynamics of a micro air vehicle

K. Harikumar *, Sidhant Dhall and M. Seetharama Bhat

Abstract: This paper addresses the design and experimental validation of a linear robust static output feedback controller for a 150 mm span fixed wing micro air vehicle (MAV). Severe coupling between longitudinal and lateral dynamics of the MAV lead to the design of a multivariable controller for the combined dynamics. The control design problem is posed in the framework of static output feedback (SOF) due to the inexpensive computational requirements for implementation. The multiobjective control design problem including stability requirements, closed loop damping ratio requirements and H_∞ norm minimization is solved using the hybrid technique of linear matrix inequalities (LMI) and genetic algorithm (GA). The design is carried out in the discrete time domain, facilitating in direct implementation of the multivariable controller in the onboard autopilot hardware. The robustness of the resulting closed loop system under parametric uncertainties is evaluated using structured singular value analysis. The effectiveness of the proposed controller is demonstrated through outdoor flight trial of the micro air vehicle with a customized lightweight autopilot hardware.

Keywords: Coupled dynamics, Flight testing, Genetic algorithm, H_∞ control, Linear matrix inequalities, Micro air vehicle, Robustness, Static output feedback.

1. INTRODUCTION

Fixed wing MAVs come under the category of low Reynolds number flyers with a maximum wingspan of 0.15 m and a typical cruise speed less than 15 m/s [1]. Black Widow MAV is the first one among the 0.15 m class of fixed wing MAVs to achieve stabilized flight [2]. Black Widow had a PID control based closed loop stabilized flight for a few minutes. Open loop and remotely piloted flight trials are reported for other fixed-wing MAVs of wingspan 0.15 m [3], 0.12 m [4] and 0.15 m [5]. A lot of literature have been devoted to the study of the aerodynamic efficiency of fixed-wing MAVs operating at low Reynolds number [6]. Unlike bigger UAVs, the issues of stability and closed loop control of fixed wing MAV flight is seldom addressed in the literature. The control design for slightly bigger fixed wing air vehicles (wingspan ranging from 0.214 m to 3.1 m) is addressed in the literature, [7] to [14]. A proportional feedback control law is designed and flight tested for the control of pitching motion of 0.214 m wingspan fixed wing air vehicle in [7]. A μ synthesis based longitudinal controller for pitch angle control and a lateral controller for roll angle control is tested on a 0.87 m wingspan air vehicle through hard-

ware in loop simulations (HILS) [8]. A sliding mode controller is designed and validated in HILS for the longitudinal dynamics of a 0.3 m wingspan aircraft in [9]. A fractional order PID controller is designed and experimental results are presented for the control of roll angle of a 1.5 m wingspan aircraft [10]. A randomized based probabilistic robust control through state feedback is designed and simulated for the longitudinal control of a 1 m wingspan aircraft [11]. A robust simultaneously stabilizing H_2 controller is designed and validated through HILS for a 0.6 m wingspan aircraft in [12]. An adaptive control law is designed and simulated using full state information for a 3.1 m wingspan UAV in [13]. A simultaneously stabilizing PID controller is designed and simulated for a 0.3 m wingspan aircraft in [14].

The control techniques presented above for slightly bigger aircraft are difficult to apply directly for the closed loop control of MAVs. The linear control laws presented above are designed for decoupled longitudinal and lateral dynamics. But there is severe coupling between longitudinal and lateral dynamics for MAVs. The coupling between the angle of attack (α) and the rolling moment is analyzed in [15]. It is shown that when MAVs perform steady turn and climb, the linear state space model for decoupled dy-

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namics fail to capture unstable spiral mode [16]. The low rolling moment of inertia, asymmetric propeller wake and change in motor-propeller counter torque with forward speed leads to the instability of spiral mode. Analysis of the aerodynamic coupling for a 0.075 m wingspan MAV is presented in [17]. The nonlinear control laws presented above for bigger aircraft assume full state information and also not robust enough to handle high uncertainties present in the dynamic model of MAV. Full state information is not available for MAVs due to lack of lightweight sensors for measuring aerodynamic angles like α and β . Unlike bigger UAVs, the flight of MAVs is highly affected by wind gusts due to their light weight, the small moment of inertia and magnitude of flight velocity comparable to wind velocities [18]. MAVs with low power and low bandwidth actuators have poor disturbance attenuation capability [19]. Considering the above factors, a multivariable output feedback controller is to be designed for the coupled dynamics of MAV that stabilizes the unstable spiral mode, provide robustness against high parametric uncertainties and also possess excellent disturbance rejection capabilities.

H_∞ control method is appropriate for handling systems with uncertainties when combined with structured singular value (μ) analysis. The static output feedback (SOF) controller is an ideal choice for MAVs where only limited onboard computational resources are available. The most popular approach for solving SOF control problems is through LMI techniques [20]. Incorporation of multiobjective constraints to the control design problem often leads to bilinear matrix inequalities (BMI). In this paper, the multi-objective H_∞ control problem is solved iteratively using a combination of GA and LMI techniques. The significant contributions of this paper are the following. The algorithm for the control design for the unstable coupled dynamics of MAV is presented in detail in this paper. The control design method is generic and can be used for any multi-input multi-output (MIMO) systems with unstable poles and non-minimum phase zeros. A systematic weight selection method is utilized to provide a good roll off in the complementary sensitivity function around the frequency of short period mode and Dutch roll mode. The resulting controller is shown to be robust using structural singular value (μ) technique. A plot of sensitivity and complementary sensitivity indicate tracking and disturbance rejection ability of the controller. Flight test results recorded in the customized autopilot hardware are presented validating the tracking and disturbance rejection properties of the controller.

The organization of the paper is as follows. A brief description of linear state space model of the aircraft and coupling between longitudinal and lateral dynamics for MAV is discussed in section 2. In section 3, the control system design specifications are given. The control design problem for the MAV is formulated as a multivariable H_∞

control problem in section 4. The weight selection for H_∞ control design is presented in section 5. The algorithm for obtaining the static output feedback gains is presented in section 6. The robust stability analysis of the closed loop system using structured singular value is presented in section 7. Numerical simulation results are presented in section 8 along with comparison with other flight control techniques. The results of flight trial of MAV in semi-autonomous mode are given in section 9, followed by conclusions in section 10.

2. LINEAR STATE SPACE EQUATIONS OF AIRCRAFT

The linearized equations of motion are derived from the nonlinear equations by assuming small perturbations from equilibrium flight (or trim conditions). The linear state space model equations of aircraft are categorized into linear longitudinal dynamic equations, linear lateral dynamic model equations and linear coupled longitudinal-lateral model equations. The details of variables and symbols used in this section and subsequently is given in Appendix B. The linear longitudinal dynamic model equations are having the state variables $\tilde{u}$, $\tilde{w}$, $\tilde{q}$, $\tilde{\theta}$, $\tilde{h}$ and control inputs elevator deflection $\tilde{\delta}_e$, throttle $\tilde{\delta}_{th}$. The linear lateral dynamic model equations are having state variables $\tilde{v}$, $\tilde{p}$, $\tilde{r}$, $\tilde{\phi}$ and the control input rudder deflection $\tilde{\delta}_r$. In general, the linear coupled longitudinal-lateral model state space equations are given below.

$$\dot{X} = AX + BU_c \quad (1)$$

$$Y = CX \quad (2)$$

where the states are

$$X = [\tilde{u}, \tilde{w}, \tilde{q}, \tilde{\theta}, \tilde{h}, \tilde{v}, \tilde{p}, \tilde{r}, \tilde{\phi}]^T \quad (3)$$

and inputs

$$U_c = [\tilde{\delta}_e, \tilde{\delta}_r, \tilde{\delta}_{th}]^T \quad (4)$$

The outputs are

$$Y = [\tilde{q}, \tilde{\theta}, \tilde{h}, \tilde{p}, \tilde{r}, \tilde{\phi}]^T \quad (5)$$

The matrix A in (1) can be written as

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \quad (6)$$

A_{11} and A_{22} represents the system matrices of conventional decoupled longitudinal and lateral dynamics respectively. The matrix A_{12} consists of the coupling terms affecting the longitudinal dynamics due to variation in the states of lateral dynamics. Similarly, the matrix A_{21} consists of the coupling terms affecting the lateral dynamics due to variation in the states of longitudinal dynamics. The degree of coupling between longitudinal and lateral

dynamics can be determined by the shift in the eigenvalues and eigenvectors of linear coupled longitudinal-lateral state space equations when compared to the decoupled linear longitudinal and lateral state space equations. The next section describes the degree of coupling between linear longitudinal and lateral dynamics for the case of a 150 mm wingspan MAV.

2.1. Linear coupled state space model of MAV

The photograph of 150 mm wingspan MAV is given in Fig. 1 [21]. The MAV is of rectangular planform with elevator and rudder control surfaces. The specifications of the MAV are given in Table 6 in Appendix A [21]. The details of wind tunnel testing and nonlinear modeling of the MAV can be found in [21].

Fig. 1. Photograph of 150 mm wing span MAV

The eigenvalues of coupled and decoupled linear state space model for straight and constant altitude flight condition where flight velocity $V_a = 8$ m/s, turn rate $\dot{\psi} = 0$ rad/s and climb rate $\dot{h} = 0$ m/s is given in Table 1. The eigenvalues of decoupled linear state space model are the eigenvalues of A_{11} and A_{22} matrices. The eigenvalues of coupled linear state space model are the eigenvalues of the A matrix. From Table 1, the eigenvalues of coupled linear state space model and decoupled linear state space model are same. Hence the coupling between the longitudinal dynamics and lateral dynamics is not significant for straight and constant altitude flight condition. The eigenvalues of coupled and decoupled linear state space model for steady turning flight with constant altitude and steady turning flight with a constant climb rate are given in Tables 2 and 3 respectively. From Tables 2 and 3, the coupled linear state space model shows that spiral mode is unstable whereas the decoupled linear state space model shows that the spiral mode is stable. Hence the coupling between the longitudinal dynamics and lateral dynamics is very significant for steady turning flight condition. The control system design presented in sections 4 to 6 utilizes combined longitudinal and lateral dynamics. The selection of linear nominal state space model for control system design is performed using v -gap metric analysis. Further details on the coupling between longitudinal and lateral

dynamics and the selection of nominal model are given in [21]. The linear state space model at a flight velocity of $V_a = 8$ m/s, turn rate $\dot{\psi} = 0.8$ rad/s and climb rate $\dot{h} = 1$ m/s is taken as nominal model for control system design with the measured outputs as given in (5).

Table 1. Eigenvalues for straight and constant altitude

Mode	Coupled and Decoupled
Short period (ω_{sp}, ζ_{sp})	35.9 , 0.254
Phugoid (ω_{ph}, ζ_{ph})	1.39 , 0.157
Dutch roll (ω_{dr}, ζ_{dr})	42.3 , 0.303
Roll subsidence	-2.08
Spiral	-0.87
Pole for $\dot{h}$	0

Table 2. Eigenvalues for steady turn and constant altitude

Mode	Coupled	Decoupled
Short period (ω_{sp}, ζ_{sp})	35.6, 0.27	35.6 , 0.27
Phugoid (ω_{ph}, ζ_{ph})	2.65, 0.28	1.20, 0.24
Dutch roll (ω_{dr}, ζ_{dr})	43.2, 0.35	43.2, 0.35
Roll subsidence	-3.47	-2.37
Spiral	1.39 (unstable)	-0.67
Pole for $\dot{h}$	0	0

Table 3. Eigenvalues for steady turn and constant climb

Mode	Coupled	Decoupled
Short period (ω_{sp}, ζ_{sp})	35.6, 0.29	35.6, 0.29
Phugoid (ω_{ph}, ζ_{ph})	1.15, 0.22	2.65, 0.24
Dutch roll (ω_{dr}, ζ_{dr})	43.8, 0.34	43.9, 0.34
Roll subsidence	-2.44	-3.51
Spiral	1.51 (unstable)	-0.57
Pole for $\dot{h}$	0	0

The matrices A_{11} , A_{12} , A_{21} and A_{22} corresponding to flight velocity of $V_a = 8$ m/s, turn rate $\dot{\psi} = 0.8$ rad/s and climb rate $\dot{h} = 1$ m/s are given in (7), (8), (9) and (10) respectively. The input matrix, B is given in (11).

$$A_{11} = \begin{pmatrix} -0.293 & -0.327 & -1.91 & -9.34 & 0 \\ 0.747 & -5.92 & 7.46 & -2.51 & 0 \\ 18.9 & -154.0 & -14.5 & 0.0 & 0 \\ 0.0 & 0.0 & 0.837 & 0.0 & 0 \\ 0.31 & -0.79 & 0.0 & 0.44 & 0 \end{pmatrix} \quad (7)$$

$$A_{12} = \begin{pmatrix} 0.677 & 0.0 & -0.416 & 0.0 \\ 0.386 & 0.416 & 0.0 & -5.11 \\ 2.67 & -0.617 & 0.244 & 0.0 \\ 0.0 & 0.0 & -0.547 & -0.76 \\ -0.52 & 0.0 & 0.0 & 1.36 \end{pmatrix} \quad (8)$$

$$A_{21} = \begin{pmatrix} -0.747 & -0.272 & 0.0 & -1.64 & 0 \\ 2.57 & 0.658 & 0.183 & 0.0 & 0 \\ 13.9 & 3.54 & -0.24 & 0.0 & 0 \\ 0.0 & 0.0 & 0.175 & 0.84 & 0 \end{pmatrix} \quad (9)$$

$$A_{22} = \begin{pmatrix} -3.34 & 1.93 & -7.55 & 7.82 \\ -40.5 & -2.22 & 2.48 & 0.0 \\ 234.0 & -2.84 & -27.0 & 0.0 \\ 0.0 & 1.0 & 0.268 & 0.0 \end{pmatrix} \quad (10)$$

$$B = \begin{pmatrix} 4.08 & 0.0 & 0.251 \\ -6.33 & 0.0 & 0.0 \\ -717.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & -8.41 & 0.0 \\ 0.0 & 59.7 & 0.0 \\ 0.0 & 793.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{pmatrix} \quad (11)$$

3. CONTROL SYSTEM DESIGN SPECIFICATIONS

The control system design specifications for the piloted aircraft are given by MIL-F-8785C standard [22]. The damping ratios specified for bigger aircraft are unacceptable for micro air vehicles, since they have higher natural frequencies (30-40 times) when compared to the piloted aircraft. Moreover, the unstable spiral mode with a time to double less than 0.5 s (Table 3) will lead to mission failure if not stabilized by feedback control. The specifications for closed loop poles, error bounds while tracking a command and actuator constraints are discussed in this section.

3.1. Damping ratio required for aircraft modes

Since the phugoid mode is the slowest, a minimum damping ratio of 0.30 is taken as the required value. For achieving higher phugoid damping [23], velocity feedback is required and the velocity measurements are not reliable in these classes of vehicles due to low precision differential pressure sensors. The minimum required closed loop damping ratios for Dutch roll mode and the short period mode is taken as 0.40 as they have higher natural frequencies when compared to the phugoid mode. The closed loop spiral mode should be stabilized by the feedback control.

3.2. Tracking specifications

The tracking specifications for roll angle hold and altitude hold is adapted from [24]. The root means square (RMS) attitude deviations should not exceed 5 degrees in

pitch and 10 degrees in roll, in the presence of wind disturbances. The amplitude of total wind and the crosswind is taken as 5 m/s and 4 m/s respectively.

3.3. Actuator dynamics and saturation

Thrust is generated using a 4x2.5 inch GWS propeller and brush-less direct current (BLDC) motor controlled by an electronic speed controller. The motors used for elevator and rudder are HK-5320 Ultra-Micro Digital servo weighing 1.7 grams. The experimentally identified transfer function of the servo motor is given in (12) with the input and output normalized to unit magnitude. The state vector corresponding to actuator dynamics is given in (13).

$$G_a(s) = \frac{2367}{s^2 + 72.22s + 2367} \quad (12)$$

$$X_a = [\tilde{\delta}_{ep}, \tilde{\delta}_{ep1}, \tilde{\delta}_{er}, \tilde{\delta}_{er1}]^T \quad (13)$$

The available elevator deflection is in the range of 0.26 rad to -0.61 rad (+15° to -35°), rudder deflection is in the range of ±0.44 rad (±25°) and for the motor-propeller, the maximum available RPS is 200. The motor-propeller actuator is modeled as a unity transfer function.

4. MULTIVARIABLE H_∞ CONTROL

In this section, a mixed sensitivity H_∞ control problem is formulated where the sensitivity function (S) is shaped along with complementary sensitivity function (T) and control input sensitivity function (KS) [25]. The block diagram of the H_∞ control problem is given in Fig. 2. In the Fig. 2, $Y_r = [\tilde{h}, \tilde{\phi}_r]^T$ are the tracking outputs and $Y_f = [\tilde{q}, \tilde{\theta}, \tilde{p}, \tilde{r}]^T$ are the other outputs feedback to the controller. The objective of the control design problem is to perform a stabilized flight maintaining a near constant desired altitude ($\tilde{h}_r$) and roll angle ($\tilde{\phi}_r$) in the presence of wind disturbances. Weighting functions are used for control input ($W_3(s)$), measured output ($W_2(s)$) and tracking error ($W_1(s)$).

Fig. 2. Block diagram of H_∞ control problem

The generic form of the weighting functions $W_1(s)$, $W_2(s)$ and $W_3(s)$ are given in (14), (15) and (16) respectively. The detailed weight selection procedure is given in section 5.

$$W_1(s) = \text{diag} (W_{\tilde{h}_e}(s), W_{\tilde{\phi}_e}(s)) \quad (14)$$

$$W_2(s) = \text{diag} (W_{\tilde{q}}(s), W_{\tilde{\theta}}(s), W_{\tilde{h}}(s), W_{\tilde{p}}(s), W_{\tilde{r}}(s), W_{\tilde{\phi}}(s)) \quad (15)$$

$$W_3(s) = \text{diag} (W_{\tilde{\delta}_e}(s), W_{\tilde{\delta}_r}(s), W_{\tilde{\delta}_h}(s)) \quad (16)$$

The performance outputs Z_1 , Z_2 and Z_3 , indicated in the Fig. 2 are given in (17), (18) and (19) respectively.

$$Z_1 = [Z_{\tilde{h}_e}, Z_{\tilde{\phi}_e}]^T \quad (17)$$

$$Z_2 = [Z_{\tilde{q}}, Z_{\tilde{\theta}}, Z_{\tilde{h}}, Z_{\tilde{p}}, Z_{\tilde{r}}, Z_{\tilde{\phi}}]^T \quad (18)$$

$$Z_3 = [Z_{\tilde{\delta}_e}, Z_{\tilde{\delta}_r}, Z_{\tilde{\delta}_h}]^T \quad (19)$$

Let X_g be the state vector, Y be the vector of measurements, U_c be the vector of control inputs, W_d be the vector of disturbance inputs, G_p be the plant transfer function matrix and G_d be the transfer function matrix from disturbance input to the plant output. The state space equation of generalized plant is given below.

$$\dot{X}_g = A_g X_g + B_u U_c + B_w W_d \quad (20)$$

$$Z_1 = C_1 X_g + D_{11} U_c + D_{12} W_d \quad (21)$$

$$Z_2 = C_2 X_g + D_{21} U_c + D_{22} W_d \quad (22)$$

$$Z_3 = C_3 X_g + D_{31} U_c + D_{32} W_d \quad (23)$$

$$Y = C X_g \quad (24)$$

Where, $X_g \in R^{n_g}$, $Y \in R^{p_g}$, $U_c \in R^{m_g}$ and $W_d \in R^{d_g}$. Here n_g is the order of the generalized plant, p_g is the number of measurements, m_g is the number of control inputs and d_g is the number of disturbance inputs. For MAV, $n_g=36$, $p_g=6$, $m_g=3$ and $d_g=6$. The state vector $X_g = [X, X_a, X_w]^T$, where X is given in (3), X_a are the actuator states given in (13), and X_w are the states corresponding to the weighting transfer functions given in (14) to (16). The disturbance input W_d given in (20) represents the wind disturbances acting on the MAV and the reference inputs $\tilde{h}_r$, $\tilde{\phi}_r$. The matrix B_w contains the columns of A_g matrix corresponding to the states $\tilde{u}$, $\tilde{v}$, $\tilde{w}$ and $\tilde{p}$ for accommodating wind disturbances [26] and columns corresponding to the reference inputs $\tilde{h}_r$, $\tilde{\phi}_r$. The high order of the generalized

plant ($n_g=36$), makes the implementation of full order dynamic compensator impractical. The control design objective posed here is to find a static output feedback gain $U_c = FY$ that minimizes the norm given below.

$$\|T_{zw_d}\|_{\infty} = \left\| \begin{pmatrix} W_1 S \\ W_2 T \\ W_3 K S \end{pmatrix} \right\|_{\infty} \quad (25)$$

The problem posed here is of multiobjective in nature where the norm given in (25) is minimized along with closed loop stability and damping requirements. The multiobjective H_{∞} control problem is solved here using a combination of linear matrix inequalities (LMI) and genetic algorithm (GA). GA is a popular evolutionary search method used to find a solution that optimizes a performance index [27]. The performance index is selected based on the closed loop stability and damping ratio requirements, and H_{∞} norm minimization.

5. WEIGHT SELECTION

For good tracking performance and disturbance rejection at low frequencies, the sensitivity function should be small. For error outputs $\tilde{h}_e$ and $\tilde{\phi}_e$, low pass filter is used as weighting function as given in (26) and (27) so as to improve the tracking performance at lower frequencies.

$$W_{\tilde{h}_e}(s) = \frac{10}{(s+0.1)} \quad (26)$$

$$W_{\tilde{\phi}_e}(s) = \frac{10}{(s+0.1)} \quad (27)$$

The weight $W_2(s)$ is taken as a high pass filter to reduce the effect of sensor noise and unmodeled dynamics at higher frequencies. Let $G_u(s)$ be a transfer function with minimum damping ratio and maximum natural frequency and let $G_n(s)$ be the transfer function matrix of the nominal plant. Then $W_2(s)$ is given below, [28].

$$W_2(s) = (G_u(s) - G_n(s))(G_n(s))^{-1} \quad (28)$$

The symbols used for stability derivatives in this section are the standard ones found in aircraft literature, and can be found in [29]. The transfer function for simplified second order system with a zero for $\frac{\tilde{q}(s)}{\tilde{\delta}_e(s)}$ is given by $G_q(s)$ as shown below.

$$G_q(s) = \frac{k_y(s+a_y)}{s^2 + 2\zeta_y \omega_y s + \omega_y^2} \quad (29)$$

Where, $k_y = -1665.5$, $a_y = 1.98$, $\zeta_y = \zeta_{sp} = 0.286$, $\omega_y = \omega_{sp} = 35.6 \text{ rad/s}$. For the terms dependent on static derivatives, 20% uncertainty is taken and for the terms dependent on dynamic derivatives, 40% uncertainty is taken. Using (28), the weighting function $W_{\tilde{q}}(s)$ obtained is given below.

$$W_{\tilde{q}}(s) = \frac{0.19(s+1.25)(s^2+41.9s+5501)}{(s+1.98)(s^2+17.1s+2484)} \quad (30)$$

Since $S+T=I$, the real zero at -1.25 is moved closer to the origin to reduce the low frequency gain and the complex conjugate zeros are augmented to form real zeros to increase the gain of $W_{\bar{q}}(s)$ at higher frequencies. Final weight $W_{\bar{q}}(s)$ is given below.

$$W_{\bar{q}}(s) = \frac{1.9(s+0.25)(s+13.6)(s+405)}{(s+1.98)(s^2+17.1s+2484)} \quad (31)$$

The weighting function $W_{\bar{q}}(s)$ has high amplitude around the open loop short period mode frequency of 35.6 rad/s, with a peak amplitude at 40% perturbation. Pitch rate perturbations due to unmodeled dynamics and high frequency noises are minimized around and above a frequency of 35.6 rad/s by the proper selection of $W_{\bar{q}}(s)$. Following similar procedure, the weighting transfer functions $W_{\bar{\theta}}(s)$, $W_{\bar{p}}(s)$, $W_{\bar{\phi}}(s)$ and $W_{\bar{r}}(s)$ are obtained. For a maximum control input range of U_{max} and the saturation occurring at a reference input of R_{max} , the weighting function for the control input is a scalar [28] and is given below.

$$W_u = \frac{R_{max}}{U_{max}} \quad (32)$$

A maximum reference input amplitude change of 0.35 rad (20°) for pitch angle and roll angle is considered while deciding the weights $W_{\bar{\delta}_e}=0.75$ and $W_{\bar{\delta}_r}=0.75$. A large variation in the flight velocity is not expected during flight. So a lower value of $W_{\bar{\delta}_h}=0.25$ is selected when compared to $W_{\bar{\delta}_e}$ or $W_{\bar{\delta}_r}$.

6. MULTIVARIABLE SOF- H_∞ CONTROLLER SYNTHESIS

Controller designed in discrete time domain handles sampling inaccuracies and quantization effects to a large extent. The implementation of the controller in autopilot hardware is performed in the discrete time domain as given in this section. The discretization of the continuous time plant is done using MATLAB *c2d* command with Zero Order Hold (ZOH) at the actuator input. A sampling time of 0.02 s is used for discretization, matching with servo motor actuation rate of 50 Hz. The discrete time representation for the generalized plant given in (20) to (24) is written as

$$X(k+1) = A_d X(k) + B_{ud} U_c(k) + B_{wd} W_d(k) \quad (33)$$

$$Z_1(k) = C_{1d} X(k) + D_{11d} U_c(k) + D_{12d} W_d(k) \quad (34)$$

$$Z_2(k) = C_{2d} X(k) + D_{21d} U_c(k) + D_{22d} W_d(k) \quad (35)$$

$$Z_3(k) = C_{3d} X(k) + D_{31d} U_c(k) + D_{32d} W_d(k) \quad (36)$$

$$Y(k) = C_d X(k) \quad (37)$$

In (33)-(37), matrices A_d , B_{ud} , B_{wd} , C_{1d} , C_{2d} , C_{3d} , D_{11d} , D_{12d} , C_{2d} , D_{21d} , D_{22d} , D_{31d} and D_{32d} are the discretized version of the matrices used in the continuous version

given in (20)-(24). The Lyapunov condition for the stability of the discrete time plant given in (33) with a controller $U_c(k) = FY(k)$ can be written as given below.

$$(A_d + B_{ud} F C_d)^T P (A_d + B_{ud} F C_d) - P < 0 \quad (38)$$

where $P > 0$.

Lemma1 [30]: The Lyapunov condition for stability for a system in discrete time is given in Eq. 39.

$$A_d^T P A_d - P < 0 \quad (39)$$

The inequality stated above can be written as the matrix inequality given below.

$$\begin{pmatrix} P & A_d^T M^T \\ M A_d & -P + M + M^T \end{pmatrix} > 0 \quad (40)$$

where M is any square matrix

A proof for lemma 1 is given in [30]. Using lemma 1, the (38) can be written as given below.

$$\begin{pmatrix} P & Q^T \\ Q & -P + M + M^T \end{pmatrix} > 0 \quad (41)$$

where $Q = M(A_d + B_{ud} F C_d)$

$$\begin{pmatrix} -P & -Q^T \\ -Q & P - M - M^T \end{pmatrix} < 0 \quad (42)$$

(42) follows from (41) with a change of sign. The Bilinear Matrix Inequality (BMI) given in (38) is transformed into a Linear Matrix Inequality (LMI) for a fixed value of M as in (42). The initial value of M is the decision variable for GA. While formulating the objective function, maximum penalty is given for violating the closed loop stability requirements. The expression for the objective function is given below.

$$J_{obj} = P_{t1}(10000) + P_{t2}(5000) + P_{t3}(\|T_{zw}\|_\infty) \quad (43)$$

In (43), the variables P_{t1} , P_{t2} and P_{t3} takes the value 0 or 1 based on closed loop stability and damping ratio requirements. A high value is given as penalty for a feedback gain that does not stabilize the closed loop plant. The second largest penalty is given for violating the closed loop damping ratio requirements. The values of P_{t1} , P_{t2} and P_{t3} are obtained as follows. Let λ_{cli} denote the i^{th} eigenvalue of the closed loop system matrix $A_d + B_{ud} F C_d$. Let ζ_{cli} denote the damping ratio of the i^{th} complex conjugate eigenvalue of the closed loop system matrix $A_d + B_{ud} F C_d$ and ζ_{clri} be the required closed loop damping ratio as given in section 3.

CASE I : IF $|\lambda_{cli}| > 1$, for any $i = 1, 2, \dots, n_g$, THEN $P_{t1} = 1$, $P_{t2} = 0$ and $P_{t3} = 0$.

CASE II : IF $|\lambda_{cli}| < 1$, $\forall i = 1, 2, \dots, n_g$ AND $\zeta_{cli} < \zeta_{clri}$ for any $i = 1, 2, \dots, n_g$, THEN $P_{t1} = 0$, $P_{t2} = 1$ and $P_{t3} = 0$.

CASE III : IF $|\lambda_{cli}| < 1$, $\forall i = 1, 2, \dots, n_g$ AND $\zeta_{cli} > \zeta_{clri}$ $\forall i = 1, 2, \dots, n_g$, THEN $P_{t1} = 0$, $P_{t2} = 0$ and $P_{t3} = 1$.

The algorithm for obtaining the SOF gains is given below.

1. Choose M as the decision variable for GA.
2. For an initial M given by the genetic algorithm, solve the LMI given in (42) to obtain P and F .
3. Obtain the closed loop matrix $A_d + B_{ud}FC_d$ and evaluate the objective function as given in (43).
4. Run the steps 1,2 and 3 until a satisfactory solution is obtained.

6.1. Computed feedback gains

Controller is designed for stabilized flight of the MAV in loiter mode where the MAV follows a circular path while maintaining the specified altitude. The order of the generalized plant is 36. The matrix M given in (42) is taken as a diagonal matrix. The controller gain matrix obtained after running the algorithm is given below.

$$F = \begin{bmatrix} F_1 & F_2 \end{bmatrix} \quad (44)$$

Where

$$F_1 = \begin{bmatrix} -0.007 & -0.57 & -1.01 \\ -0.001 & 0.336 & 0.008 \\ -0.766 & 47.8 & -0.034 \end{bmatrix} \quad (45)$$

and

$$F_2 = \begin{bmatrix} 0.009 & 0.005 & -0.13 \\ -0.038 & 0.006 & -0.004 \\ -9.67 & -5.25 & 15.9 \end{bmatrix} \quad (46)$$

The poles of the closed loop system in continuous domain is given in Table 4. The closed loop system in discrete time domain is converted to continuous time domain using MATLAB *d2c* command.

Table 4. Poles of closed loop system

ω_{sp}, ζ_{sp}	32.00, 0.57
ω_{ph}, ζ_{ph}	2.26, 0.42
ω_{dr}, ζ_{dr}	34.72, 0.41
Roll subsidence mode pole	-9.52
spiral mode pole	-2.27
Pole corresponding to $\tilde{h}$	-1.91
Elevator servo	40.4, 0.55
Rudder servo	55.7, 0.64

From the Table 4, we can see that the spiral mode is stable and the damping ratios of all the dynamic modes are above 0.4. The controller satisfies the closed loop stability and damping ratio requirements. The value of $\|T_{zwd}\|_\infty = 0.7563$. The singular value plot of the S and T for the closed loop system is given in Fig. 3. From Fig. 3, it is evident that there is high roll off for T after a frequency of 5 rad/s. The closed loop system has a very low magnitude of S below a frequency of 2 rad/s and a very low

magnitude of T above 20 rad/s. Hence the closed loop system has very good tracking performance below 2 rad/s and good noise attenuation above 20 rad/s. There is only a narrow band of frequency range between 2 rad/s to 7 rad/s where the magnitude of S and T are comparable.

Fig. 3. S and T plot for closed loop system

7. ROBUST STABILITY ANALYSIS USING STRUCTURED SINGULAR VALUE

For a real or complex matrix M_p with all the perturbations represented by a matrix Δ possessing a block diagonal structure, the structured singular value (μ) is defined as given below.

$$\mu(M_p) = \frac{1}{\min(k_u / \det(I - k_u M_p \Delta) = 0, \forall \bar{\sigma}(\Delta) \leq 1)} \quad (47)$$

Fig. 4. $M_p \Delta$ structure for robust stability analysis

A larger value of $\mu(M_p)$ means that a smaller perturbation can make the system unstable. A smaller value of $\mu(M_p)$ means the system can handle larger uncertainty. If the nominal system M_p and the perturbations Δ are stable, then the $M_p \Delta$ system shown in Fig. 4 is stable if and only if $\mu(M_p) < 1, \forall \bar{\sigma}(\Delta) \leq 1$.

Reference [31], suggests uncertainty levels of 5% for longitudinal static derivatives, 30% for longitudinal dynamic

derivatives, 10% for lateral static derivatives, 20% for lateral dynamic derivatives and 5% for all control derivatives. But the MAV listed in the reference [31] has a span of 840 mm. The uncertainty increases with reduction in wingspan due to the flight in lower Reynolds number regime. So for a 150 mm span MAV, uncertainty levels of 20% is considered for all static derivatives and 40% for all dynamic derivatives. The μ analysis performed for robust stability using MATLAB for the $M_p\Delta$ structure is shown in Fig. 4. The result is plotted in Fig. 5. From the Fig. 5 we can see that the peak value of $\mu(M_p) = 0.7494 < 1$. Hence the closed loop system is robustly stable and can handle 26% uncertainty in static derivatives and 52% uncertainty in dynamic derivatives.

Fig. 5. Structured singular value plot

8. NUMERICAL SIMULATION RESULTS

A numerical simulation is conducted using the high fidelity 6-DOF non-linear model of the MAV given in [21]. The altitude reference is increased in steps of size 1 m from initial value of 0 m to a final value of 5 m. When the altitude reaches near the value of 5 m, a roll angle command of 5° is issued. The aerodynamic angles, velocity components, and MAV altitude are shown in Fig. 6. The sudden jump in the response of the states is due to the step change in altitude reference. The climb rate of the MAV is around 0.125 m/s and the maximum deviation in the angle of attack is 0.3° from the trim value. The maximum deviation of β is -0.2° from the trim value. The Euler angles and body angular rates are shown in Fig. 7. Initially, till 40 s, ϕ oscillates at a frequency of 0.8 rad/s as in figure 7. This is due to the change in airspeed while climbing and the corresponding change in the counter torque. The MAV is able to track the required altitude of 5 m and roll angle of 5° . The elevator, rudder and thrust input is given in Fig. 8. The rudder input oscillates initially to compensate for the counter torque generated while climbing. The rudder input becomes constant after MAV starts loitering. The thrust input is higher during climb and reduces thereafter. The position of MAV in the inertial frame is shown in Fig.

9. Figures 6 to 9 shows that the MAV can maneuver precisely within a short range flight envelope.

Fig. 6. MAV states - altitude, α , β , u , v and w

Fig. 7. MAV states - ϕ , θ , ψ , p , q and r

Fig. 8. MAV control input - elevator, rudder and thrust

Fig. 9. MAV position for simulation

A comparison of various control laws available in the literature for small UAVs with the present H_∞ -SOF control for MAV is given in Table 5. The attributes considered for comparison are closed loop stability robustness and performance. The dynamics considered fall into longitudinal (Long), lateral (Lat) and coupled. The variables used for feedback control is also given. Robustness is quantified using phase margin (PM), maximum parametric uncertainty (Δ) or μ value. Performance is evaluated in terms of percentage overshoot for tracking or closed loop damping ratio. From Table 5, it is evident that the proposed H_∞ -SOF control law gives higher robustness and good performance when compared to most of the other control laws.

Table 5. Comparison of control laws for small UAVs.

Control law	Dynamics, (Feedback)	Robustness	Performance
P- control [7]	Long, $(\tilde{w}, \tilde{h})$	PM = 106°	$\zeta_{sp}=0.61$
μ -Synthesis [8]	Lat, $(\tilde{p}, \tilde{r}, \tilde{\phi})$	$\mu=0.8$	30% overshoot
Sliding mode [9]	Long, $(\tilde{\gamma}, \tilde{V})$	-	50% overshoot
Fractional-PI [10]	Lat, $(\tilde{p}, \tilde{\phi})$	PM = 80°	40% overshoot
State feed-back [11]	Long, $(\tilde{V}, \tilde{\alpha}, \tilde{q}, \tilde{\theta})$	$\Delta=10\%$	$\zeta_{sp}=0.7$, $\zeta_{ph}=0.2$
H_2 control [12]	Lat, $(\tilde{p}, \tilde{r}, \tilde{a}_y)$	$\Delta=20\%$	$\zeta_{dr}=0.5$
Adaptive control [13]	Coupled, full state	$\Delta=20\%$	30% overshoot
PID [14]	Lat, $(\tilde{p}, \tilde{r}, \tilde{\phi})$	PM= 57.5°	50% overshoot
H_∞ -SOF	Coupled, $(\tilde{q}, \tilde{\theta}, \tilde{h}, \tilde{p}, \tilde{r}, \tilde{\phi})$	$\Delta=26\%$, $\mu=0.74$	$\zeta_{sp}=0.57$, $\zeta_{ph}=0.42$, $\zeta_{dr}=0.41$

9. FLIGHT TESTING OF 150 MM MAV

The flight tests are conducted with a customized autopilot hardware. A survey of commercial autopilot hardware available for UAVs are given in [32]. But here due to the weight constraints, a customized lightweight autopilot hardware of 8 grams is used. The autopilot consists of a 32 bit ARM Cortex M3 microcontroller running at 120 MHz, 3 axis accelerometer (± 4 g), 3 axis rate gyro (± 500 deg/s), 3 axis magnetometer (± 4 gauss), altimeter (260-1260 mbar), flash storage (16 MB), GPS with integrated antenna (2.5 m CEP) and a bi-directional RF communication module with antenna (range 1 km line of sight). Photographs of the top side and bottom side of the autopilot hardware are given in Fig. 10 and Fig. 11 respectively.

Fig. 10. Top side of autopilot hardware

Fig. 11. Bottom side of autopilot hardware

9.1. Tracking Performance

In order to verify tracking performance, a step reference input is given to altitude and roll angle. The flight tests are conducted under mild wind gust. The plot of altitude obtained from flight testing is given in Fig. 12. The reference altitude to be achieved is 985 m and is 10 m from the initial altitude of 975 m from Mean Sea Level (MSL). The MAV climbs to a maximum altitude of 987 m. There is a maximum overshoot of 2 m from the set point. The plot of $\tilde{\phi}$ is given in Fig. 13, for tracking $\tilde{\phi}_r = 10^\circ$ reference. The error in tracking $\tilde{\phi}$ is higher due to cross wind.

9.2. Disturbance Rejection

The wind disturbances affect the body angular rates significantly as they are dominant in high frequency modes. The frequency spectrum of $\tilde{p}$, $\tilde{q}$ and $\tilde{r}$ are given in Figs. 14 to 16. The peak amplitude of $\tilde{p}$ and $\tilde{q}$ is occurring close to 8 rad/s and that of $\tilde{r}$ is close to 2 rad/s. The high frequency Dutch roll and short period dynamics are not excited. The

controller gives high roll off in the $\tilde{p}$ and $\tilde{q}$ channel after a frequency of 10 rad/s. The roll off in $\tilde{r}$ channel is after a frequency of 7 rad/s.

9.3. Control Effort

The elevator, rudder and thrust inputs are given in Figs. 17 to 19. Initially, the MAV utilizes more thrust to climb. The maximum throttle RPM required is 7675 and is 64% of the maximum amplitude of 12000 RPM. The maximum amplitude of elevator deflection is -15% degrees and that of rudder deflection is 10%. The elevator and rudder deflections are well within the maximum amplitude of -35% degrees and 25%.

Fig. 12. Altitude for the flight test

Fig. 13. Plot of angle $\tilde{\phi}$ for the flight test

Fig. 14. Frequency spectrum of roll rate ($\tilde{p}$)

Fig. 15. Frequency spectrum of pitch rate ($\tilde{q}$)

Fig. 16. Frequency spectrum of yaw rate ($\tilde{r}$)

Fig. 17. Plot of elevator input for the flight test

Fig. 18. Plot of rudder input for the flight test

Fig. 19. Plot of thrust input for the flight test results

10. CONCLUSIONS

The current work mainly focuses on the control system design using the coupled longitudinal and lateral linear state space model of the micro air vehicle. The coupling between longitudinal and lateral dynamics lead to the requirement of multivariable techniques to be applied for control design. The control design specifications emphasize on the requirement on stabilizing the unstable spiral mode, improve the damping ratio and tracking performance while operating within the actuator constraints. The design of the static output feedback controller is carried out using a hybrid approach of linear matrix inequalities and genetic algorithm. The static output feedback controller stabilizes the spiral mode and improves the damping ratios of short period mode, phugoid mode, and Dutch roll mode. The robustness analysis of the closed loop system against parameter uncertainty is carried out using μ analysis. The control system is able to handle 1.3 times higher uncertainty level than used for μ analysis. The plot of sensitivity and complementary sensitivity functions indicate the disturbance attenuation capabilities of the closed loop system. The robustness and tracking properties of the controller are verified through flight test with customized autopilot hardware. The flight results show that the proposed controller satisfies the control system design objectives. Wind disturbances are not measured during the flight tests, due to lack of accurate lightweight sensors. Estimating the magnitude and direction of the wind disturbances will enable the micro air vehicle to perform better during fully autonomous missions. The extension of this work would be to develop a waypoint navigation algorithm that takes care of the coupled dynamics of the MAV to perform fully autonomous missions. During fully autonomous missions, preventing the MAV from abrupt stall in the presence of wind disturbances and handling constraints on turn rate with limited actuator bandwidth is a challenging task.

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12. APPENDIX A

Table 6. Specifications of 150 mm MAV

Planform	Rectangular
Airfoil	E387
Weight (grams)	53
Velocity(m/s)	6-13
Chord(m)	0.11
Span (m)	0.15
Control surfaces	Elevator and rudder
$J_{xx}(Kgm^2)$	1182.68×10^{-7}
$J_{yy}(Kgm^2)$	773.56×10^{-7}
$J_{zz}(Kgm^2)$	431.24×10^{-7}
$J_{xz}(Kgm^2)$	42.49×10^{-7}

13. APPENDIX B

This appendix provides the details of symbols, variables and abbreviations used in this manuscript.

$\tilde{u}$ —Linearized velocity along the body X axis (m/s)

$\tilde{v}$ —Linearized velocity along the body Y axis (m/s)

$\tilde{w}$ —Linearized velocity along the body Z axis (m/s)

$\tilde{p}$ —Linearized roll rate (rad/s)

$\tilde{q}$ —Linearized pitch rate (rad/s)

$\tilde{r}$ —Linearized yaw rate (rad/s)

$\tilde{\phi}$ —Linearized roll angle (rad)

$\tilde{\theta}$ —Linearized pitch angle (rad)

$\tilde{h}$ —Linearized altitude (m)

$\tilde{\delta}_e$ —Linearized elevator deflection (rad)

$\tilde{\delta}_r$ —Linearized rudder deflection (rad)

$\tilde{\delta}_{th}$ —Linearized thrust input (RPS)

α —Angle of attack (rad)

β —Sideslip angle(rad)

μ —Structured singular value

Δ —Parametric uncertainty

ω_{sp} —Short period mode frequency (rad/s)

ζ_{sp} —Short period mode damping ratio

ω_{ph} —Phugoid mode frequency (rad/s)

ζ_{ph} —Phugoid mode damping ratio

ω_{dr} —Dutch roll mode frequency (rad/s)

ζ_{dr} —Dutch roll mode damping ratio

Long—Longitudinal dynamics

Lat—Lateral dynamics

GA—Genetic Algorithm

HILS—Hardware In Loop Simulations

LMI—Linear Matrix Inequality

MAV—Micro Air Vehicle

PID—Proportional Integral Derivative

RPS—Rotations Per Second

SOF—Static Output Feedback

UAV—Unmanned Air Vehicle

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