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Robust Model Predictive Control for Discrete T-S Fuzzy Systems with Nonlinear Local Models

Long Teng¹, Youyi Wang², Wenjian Cai², and Hua Li³

Abstract—This paper presents a robust model predictive control method for discrete nonlinear systems. Instead of conventional T-S fuzzy system where linear local models are used, T-S fuzzy system with nonlinear local models is adopted that the number of fuzzy rules is decreased and the computational burden is reduced. Meanwhile, persistent external disturbances are also considered in the T-S fuzzy systems that input-to-state stability is realized. Based on the concept of robust positively invariant set, the terminal constraint set for T-S fuzzy systems with nonlinear local models is built. The advantages of the developed method is demonstrated in simulation by comparison with an existing fuzzy model predictive control method with linear local models.

I. INTRODUCTION

Model predictive control (MPC) attracts lots of research interests and has been widely used in industrial process control for decades. Usually, an optimal control input is got via optimization of a cost function and satisfaction of some constraints. In recent years, great progresses have been made on many aspects of MPC. Control scheme of nonlinear model predictive control (NMPC) is discussed in [1] and [2]. In [1], [3]-[4], terminal constraint set is adopted in MPC. In [5], by designing explicit control laws corresponding to a set of asymptotically stable invariant ellipsoids, an off-line algorithm of robust model predictive control (RMPC) is realized to reduce the computational burden. MPC handling external disturbances can be seen in [6] and several more recent works [7]-[9]. The industrial processes are mostly nonlinear systems, however, NMPC may not be easily implemented due to the large computational cost and difficulties in acquisition of exact nonlinear model [10]-[11].

The T-S fuzzy model is firstly proposed in [12] and has been widely utilized in the control of nonlinear systems, see [13] and references therein. To utilize the advantages of both MPC and T-S fuzzy systems, in recent years, some works can be seen on combinations of the two methods [8], [11], [14]-[17]. However, the aforementioned fuzzy control methods and fuzzy model predictive control (FMPC) methods only adopt T-S fuzzy systems with linear local models. In [18] and [19], T-S fuzzy system with nonlinear local models

is proposed. Compared with the conventional T-S fuzzy systems with linear local models, the number of fuzzy rules can be decreased and the computational burden can be alleviated by adopting the nonlinear T-S fuzzy systems. Thus it is more appropriate for practical applications especially when the systems are with complex nonlinearities.

Based on the existing results, robust model predictive control for discrete-time T-S fuzzy systems with nonlinear local models is proposed in this paper. Meanwhile, external disturbances and input constraints are considered. Compared with existing FMPC where MPC of T-S fuzzy systems with linear local models is considered, the advantages of proposed method are that: (1) As industrial processes usually contain severe nonlinearities and MPC is a kind of optimization control methods that usually a large number of constraints or optimization matrix inequalities are introduced, it is easy to see that the computational burden and time consumption which are essential for engineering applications become even worse when combinations of MPC and T-S fuzzy systems are adopted. However, the computational burden and time consumption can be greatly reduced when proposed FMPC with nonlinear T-S fuzzy systems is adopted. (2) Compared with the robust fuzzy model predictive control (RFMPC) method proposed in [8] where linear local models are adopted, a better control performance can be achieved via the proposed method.

The rest of the paper is structured as follows. In Section II, some preliminary knowledge about nonlinear T-S fuzzy system and RMPC are introduced. In Section III, the computation of RPI set and terminal constraint set for nonlinear T-S fuzzy system are given, the control algorithm is introduced in sequel. And the feasibility and input-to-state stability (ISS) are also analyzed. In Section IV, a numerical example is illustrated to verify the effectiveness of the proposed method. Finally, some conclusions are given in Section V.

II. PRELIMINARIES

A. T-S Fuzzy System with Nonlinear Local Models

In this paper, we consider the following discrete-time nonlinear system with disturbances:

$$x(k+1) = f(x(k)) + g(x(k))u(k) + h(x(k))d(k) \quad (1)$$

where $x(k+1)$, $x(k) \in R^n$ are the states, $u(k) \in R^m$ and $d(k) \in R^s$ are the control input and disturbance, respectively. $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ are nonlinear functions. $f(\cdot)$ often contains a lot of nonlinear terms. Then the aforementioned system is described by the following T-S fuzzy system with nonlinear local models:

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Plant Rule l :

IF: z_1 is F_1^l and $\dots z_v$ is F_v^l

THEN:

$$x(k+1) = A_l x(k) + B_l u(k) + G_l \phi(k) + E_l d(k) \quad (2)$$

where, $z := [z_1, \dots, z_v]$ are premise variables. $F_1^l, \dots, F_v^l$ are fuzzy sets. $l \in \mathbb{Z}_{[1, L]}$, and L is the number of fuzzy rules. $\phi(k)$ is a nonlinear term which is separated from the following equation

$$f(x(k)) = \sum_{l=1}^L \mu_l(z) [A_l x(k) + G_l \phi(k)] \quad (3)$$

Make the following definitions:

$$A_\mu := \sum_{l=1}^L \mu_l(z) A_l$$

$$G_\mu := \sum_{l=1}^L \mu_l(z) G_l$$

$$B_\mu := \sum_{l=1}^L \mu_l(z) B_l$$

$$E_\mu := \sum_{l=1}^L \mu_l(z) E_l$$

where $\phi(k) \in \text{co}\{0, Wx(k)\}$, $\mu_l(z)$ is normalized membership function. Then the system can be rewritten as follows,

$$x^+ = A_\mu x + B_\mu u + G_\mu \phi + E_\mu d \quad (4)$$

where x^+ represents the system state in the next time instant.

Remark 1. The reason of separating $f(x(k))$ into two parts is that we try to apply MPC to this kind of T-S fuzzy systems with nonlinear local models. In this fuzzy control scheme, $\phi(k)$ is retained as a nonlinear term rather than being used for fuzzification.

Control method for above T-S fuzzy system is given below [19]:

Control Rule l :

IF: z_1 is F_1^l and $\dots z_v$ is F_v^l

THEN: $u_l(k) = K_{al}x(k) + K_{bl}\phi(k)$

By using the fuzzy inference method with a singleton fuzzier, product inference and center average defuzzifiers, the final control output can be obtained as follows:

$$u(k) = K_{a\mu}x(k) + K_{b\mu}\phi(k) \quad (5)$$

where

$$K_{a\mu} = \sum_{l=1}^L K_{al}, K_{b\mu} = \sum_{l=1}^L K_{bl}$$

B. Robust Fuzzy Model Predictive Control

The prediction model for the above T-S fuzzy system is

$$x(k+i+1|k) = A_\mu x(k+i|k) + B_\mu u(k+i|k) + G_\mu \phi(k+i|k) + E_\mu d(k+i|k) \quad (6)$$

MPC optimize a cost function under some constraints. Usually, a finite horizon cost function which can approximate

an upper bound of the infinite horizon cost function is adopted, which is shown in (7).

$$J(k) = \sum_{i=0}^{N-1} \ell(k+i|k) + V_m(x(k+N|k)) \quad (7)$$

where $\ell(k+i|k)$ and $V_m(x(k+N|k))$ are called the stage cost and the terminal cost, respectively. The stage cost is chosen as

$$\ell(k+i|k) = x^T(k+i|k)Qx(k+i|k) + u^T(k+i|k)Ru(k+i|k) - \tau d^T(k+i|k)d(k+i|k) \quad (8)$$

where Q, R are positive matrices, τ is a positive scalar. Since persistent disturbance is involved, the input-to-state stability (ISS) [20] rather than asymptotical stability is considered. A terminal constraint set is adopted that the system states enter the terminal constraint set at the end of prediction. The online optimization problem is shown as follows,

$$\begin{aligned} \min_{u(k+i|k)} \quad & \max_{d(k+i|k)} J(k), \\ u(k+i|k) \in U, \\ d(k+i|k) \in D, \\ x(k+N|k) \in \Omega_m, \end{aligned}$$

where $i \in \mathbb{Z}_{[1, N-1]}$, Ω_m is the terminal constraint set. The disturbance and control input satisfy that

$$d \in D := \{d | d^T d \leq \gamma^2\}$$

$$u \in U := \{u | |u_t| \leq u_{t, \max}\}$$

C. Useful Lemmas

Lemma 1 ([22]). For $\phi(k) = \begin{bmatrix} \phi_1(k) & \phi_2(k) & \dots & \phi_s(k) \end{bmatrix}^T$, $W = \begin{bmatrix} W_1 & W_2 & \dots & W_s \end{bmatrix}$, $\Lambda^{-1} = \text{diag}[\rho_1 \ \rho_2 \ \dots \ \rho_s]$, $\rho_i > 0$, $1 \leq i \leq s$, given $\phi_i(k) \in \text{co}\{0, W_i x(k)\}$, then

$$\phi^T \Lambda^{-1} W x - \phi^T \Lambda^{-1} \phi \geq 0 \quad (9)$$

III. MAIN RESULTS

In this section, the constraint of RPI set for RFMPC with nonlinear local models is firstly discussed, based on which the computation of terminal constraint set is given in sequel.

A. RPI Set

Denote Ω as an RPI set and $h(x)$ as the corresponding control law. The robust invariant set property can be described as, $\forall x \in \Omega$, $x^+ \in \Omega$ for all allowable uncertainties and disturbances. Define Ω as follows:

$$\Omega := \{x | x^T P_\mu x \leq \xi\} \quad (10)$$

The controller for T-S fuzzy systems with nonlinear local models is selected as

$$h(x) := K_{a\mu}x + K_{b\mu}\phi \quad (11)$$

Lemma 2 [8]. The set Ω is an RPI set if and only if there exists a positive scalar $\lambda \in \mathbb{R}_{(0,1)}$ such that

$$\frac{1}{\xi} x^{+T} P_\mu^+ x^+ - \frac{1-\lambda}{\xi} x^T P_\mu x - \frac{\lambda}{\gamma^2} d^T d \leq 0 \quad (12)$$

for all $x^+ \in A_\mu x + B_\mu u + G_\mu \phi + E_\mu d$, $u \in U$, and $d \in D$.

Proof. A simple proof is also given here. Move the last two items in (12) to the right side. As $x^T P_\mu x \leq \xi$ and $d^T d \leq \gamma^2$, then $\frac{1}{\xi} x^{+T} P_\mu^+ x^+ \leq (1 - \lambda) + \lambda = 1$, thus $x^{+T} P_\mu^+ x^+ \leq \xi$.

The RPI set property and input constraints will be given as bilinear and linear matrix inequalities respectively in the following theorem.

Theorem 1. Consider the fuzzy system (2), if there exist positive definite matrices X_i (or X_j , X_l), matrices L_j , Y_{aj} , Y_{bj} , Z , positive scalar $\lambda \leq 1$, $\Lambda^{-1} = \text{diag}[\rho_1 \ \rho_2 \ \cdots \ \rho_s]^{-1}$ with $\rho_1, \rho_2, \dots, \rho_s > 0$, such that the following matrix inequalities are feasible

$$\begin{bmatrix} \Phi & * & * & * \\ WL_j & -2\Lambda & * & * \\ 0 & 0 & -\lambda/\gamma^2 & * \\ A_i L_j + B_i Y_{aj} & G_i \Lambda + B_i Y_{bj} & E_i & -X_l \end{bmatrix} \leq 0 \quad (13)$$

$$\begin{bmatrix} Z & * & * \\ Y_{aj}^T & \Delta & * \\ Y_{bj}^T & -WL_j & 2\Lambda \end{bmatrix} \geq 0, Z_{tt} \leq u_{t,\max}^2, t \in \mathbb{Z}_{[1,m]} \quad (14)$$

where $\Phi = (-1 + \lambda)(L_j + L_j^T - X_i)$, $\Delta = L_j + L_j^T - X_j$. Z_{tt} is the t -th diagonal element of matrix Z , and $i, j, l \in \mathbb{Z}_{[1,L]}$, then the set $\Omega = \{x | x^T P_\mu x \leq \xi\}$ is an RPI set for the fuzzy system (2) corresponding to the feedback control law $h(x) = K_{a\mu} x + K_{b\mu} \phi$, where $P_\mu = \sum_{i=1}^L \mu_i(x) P_i$, $K_{a\mu} = \sum_{l=1}^L K_{al}$, $K_{b\mu} = \sum_{l=1}^L K_{bl}$, with $P_i = \xi X_i^{-1}$ and $K_{ai} = Y_{ai} L_i^{-1}$, $K_{bi} = Y_{bi} \Lambda^{-1}$.

Proof. Resorting to the dilation lemma [21],

$$-L^T X^{-1} L \leq X - L^T - L \quad (15)$$

then multiplying $\text{diag}\{L_j^{-T}, \Lambda^{-1}, I, I\}$ and its transpose from both sides of (13) respectively, substituting $Y_{ai} L_i^{-1}$ with K_{ai} , and $Y_{bi} \Lambda^{-1}$ with K_{bi} , the following inequality is got

$$\begin{bmatrix} (-1 + \lambda) X_i^{-1} & * & * & * \\ \Lambda^{-1} W & -2\Lambda^{-1} & * & * \\ 0 & 0 & -\lambda/\gamma^2 & * \\ A_i + B_i K_{aj} & G_i + B_i K_{bj} & E_i & -X_l \end{bmatrix} \leq 0 \quad (16)$$

Substituting X_i^{-1} with P_i/ξ , (16) can be written as

$$\begin{bmatrix} (-1 + \lambda) P_\mu/\xi & * & * & * \\ \Lambda^{-1} W & -2\Lambda^{-1} & * & * \\ 0 & 0 & -\lambda/\gamma^2 & * \\ A_\mu + B_\mu K_{a\mu} & G_\mu + B_\mu K_{b\mu} & E_\mu & -X_l \end{bmatrix} \leq 0 \quad (17)$$

Applying schur complement to (17), then

$$\begin{bmatrix} (-1 + \lambda) P_\mu/\xi & * & * \\ \Lambda^{-1} W & -2\Lambda^{-1} & * \\ 0 & 0 & -\lambda/\gamma^2 \end{bmatrix} + \Psi^T X_l^{-1} \Psi \leq 0 \quad (18)$$

where $\Psi = [A_\mu + B_\mu K_{a\mu} \ G_\mu + B_\mu K_{b\mu} \ E_\mu]$. Multiplying $[x^T \ \phi^T \ d^T]$ and its transpose from both sides

of (18) respectively, the following inequality can be got,

$$x^{+T} X_l^{-1} x^+ - \frac{1}{\xi} x^T P_\mu x - \lambda(\frac{1}{\gamma^2} d^T d - \frac{1}{\xi} x^T P_\mu x) + 2(\phi^T \Lambda^{-1} W x - \phi^T \Lambda^{-1} \phi) \leq 0 \quad (19)$$

where $x^+ = (A_\mu + B_\mu K_{a\mu})x + (G_\mu + B_\mu K_{b\mu})\phi + E_\mu d$. Substituting X_l^{-1} with P_l/ξ into the above inequality, and since $2(\phi^T \Lambda^{-1} W x - \phi^T \Lambda^{-1} \phi) \geq 0$, then

$$\frac{1}{\xi} (x^+)^T P_l x^+ - \frac{1}{\xi} x^T P_\mu x - \lambda(\frac{1}{\gamma^2} d^T d - \frac{1}{\xi} x^T P_\mu x) \leq 0 \quad (20)$$

Thus the robust positively invariant set property of the set Ω can be established if (13) holds. Moreover, the input constraint requirements can be realized by (14), which are proved below.

According to (15), the following inequality can be got if (14) holds,

$$\begin{bmatrix} Z & * & * \\ Y_{aj}^T & L_j^T X_j^{-1} L_j & * \\ Y_{bj}^T & -WL_j & 2\Lambda \end{bmatrix} \geq 0 \quad (21)$$

Multiplying $\text{diag}\{I, L_j^{-1}, \Lambda^{-1}\}$ and its transpose to (21), substituting $Y_{ai} L_i^{-1}$ with K_{ai} , and $Y_{bi} \Lambda^{-1}$ with K_{bi} , the following inequality is got

$$\begin{bmatrix} Z & * & * \\ K_{aj}^T & X_j^{-1} & * \\ K_{bj}^T & -\Lambda^{-1} W & 2\Lambda^{-1} \end{bmatrix} \geq 0 \quad (22)$$

(22) can be written as

$$\begin{bmatrix} Z & * & * \\ K_{a\mu}^T & X_\mu^{-1} & * \\ K_{b\mu}^T & -\Lambda^{-1} W & 2\Lambda^{-1} \end{bmatrix} \geq 0 \quad (23)$$

Applying schur complement to (23), then

$$\begin{bmatrix} X_\mu^{-1} & * \\ -\Lambda^{-1} W & 2\Lambda^{-1} \end{bmatrix} - \frac{1}{Z} \begin{bmatrix} K_{a\mu}^T \\ K_{b\mu}^T \end{bmatrix} \begin{bmatrix} K_{a\mu} & K_{b\mu} \end{bmatrix} \geq 0 \quad (24)$$

Multiplying $[x^T \ \phi^T]$ and its transpose to (24), substituting X_μ^{-1} with P_μ/ξ , then

$$\frac{1}{Z} (K_{a\mu} x + K_{b\mu} \phi)^T (K_{a\mu} x + K_{b\mu} \phi) - \frac{1}{\xi} x^T P_\mu x + 2(\phi^T \Lambda^{-1} W x - \phi^T \Lambda^{-1} \phi) \leq 0 \quad (25)$$

Thus

$$\frac{1}{Z} (K_{a\mu} x + K_{b\mu} \phi)^T (K_{a\mu} x + K_{b\mu} \phi) - \frac{1}{\xi} x^T P_\mu x \leq 0 \quad (26)$$

Since $u = K_{a\mu} x + K_{b\mu} \phi$, (27) can be got.

$$\frac{1}{Z} u^T u \leq \frac{1}{\xi} x^T P_\mu x \quad (27)$$

Since $\frac{1}{\xi} x^T P_\mu x \leq 1$ which is ensured by the feature of RPI set, thus $u^T u \leq Z$. Proof is completed.

Lemma 3. The terminal constraint set Ω_t should satisfied two requirements. Firstly, it is a RPI set. Secondly, there exists a positive definite function (terminal cost function) $V(x)$ that $\forall x \in \Omega_t$

$$\alpha(\|x\|) \leq V(x) \leq \beta(\|x\|) \quad (28)$$

$$V(x^+) - V(x) < -(x^T Q x + h^T(x) R h(x) - \tau d^T d) \quad (29)$$

where α and β are K_∞ functions, the positive definite function $V(x)$ is defined as

$$V(x) := \sum_{l=1}^L \mu_l(x) x^T P_l x \quad (30)$$

Theorem 2. Consider the (4), if the following matrix inequality is feasible

$$\begin{bmatrix} \Xi & * & * & * & * & * \\ WL_j & -2\Lambda & * & * & * & * \\ 0 & 0 & -\tau\xi & * & * & * \\ \Omega & \Theta & \xi E_i & -X_l & * & * \\ QL_j & 0 & 0 & 0 & -\xi Q & * \\ RY_{aj} & RY_{bj} & 0 & 0 & 0 & -\xi R \end{bmatrix} < 0 \quad (31)$$

where $\Xi = X_i - L_j - L_j^T$, $\Omega = A_i L_j + B_i Y_{aj}$, $\Theta = G_i \Lambda + B_i Y_{bj}$. X_i (or X_j , X_l), L_j , Y_{aj} , Y_{bj} are defined in Theorem 1, then Ω_t is a terminal constraint set regarding the terminal cost function $V(x)$.

Since (28) can be easily got by resorting to the eigenvalues. (31) which guarantees (29) can be got by following a similar proof for (13). The proof is thus omitted.

C. Control Algorithm

Based on the results of terminal constraint sets presented in the last section, online control algorithm is discussed in this section to complete the FMPC controller design.

From the aforementioned computation of terminal constraint sets, the terminal cost function $V(x(k))$, see (30), should satisfied the following condition,

$$V(x(k)) = \sum_{l=1}^L \mu_l(x(k)) x^T(k) P_l x(k) \leq \xi \quad (32)$$

thus $x(k) \in \Omega_t$. It can be seen that (32) also guarantees that ξ is an upper bound of $V(x(k))$. Thus the following problem is considered to minimize $V(x(k))$,

$$\min \xi, \text{ subjected to } V(x(k)) \leq \xi \quad (33)$$

A sufficient condition for $V(x(k)) \leq \xi$ is

$$\begin{bmatrix} 1 & x^T(k) \\ x(k) & X_l \end{bmatrix} \geq 0 \quad (34)$$

Consider the terminal constraint sets, τ is also can be optimized. Taking the same idea as stated in [19], a new

variable ε is defined as: $\varepsilon = \tau\xi$, thus ε is minimized instead.

Then inequality (31) is rewritten as

$$\begin{bmatrix} \Xi & * & * & * & * & * \\ WL_j & -2\Lambda & * & * & * & * \\ 0 & 0 & -\varepsilon & * & * & * \\ \Omega & \Theta & \xi E_i & -X_l & * & * \\ QL_j & 0 & 0 & 0 & -\xi Q & * \\ RY_{aj} & RY_{bj} & 0 & 0 & 0 & -\xi R \end{bmatrix} < 0 \quad (35)$$

In summary, the MPC optimization problem is executed by solving (13), (14), (34), and (35), where ε is minimized.

Theorem 3: For system (4), if the constrained optimization problem has a solution at time 0, it will always be solvable. Furthermore, recursive feasibility can be achieved.

Proof: Assume that the optimization problem is satisfied at time k , then $x(k) \in \Omega(k)$, where $\Omega(k)$ represents the set Ω obtained at time k . Since $\Omega(k)$ is an RPI set, $x(k+1) \in \Omega(k)$ is got. Then it can be concluded that it is solvable at time $k+1$. Moreover, the solution obtained at time k is feasible at time $k+1$. Applying the same process, it is observed that the optimization problem is feasible at all time.

Theorem 4: If the constrained optimization problem is feasible at the initial time 0, system (4) is ISS with respect to disturbance d .

Proof: Consider the $V(k, x)$, that is $V(k, x) = x^T(k) P_\mu^*(k) x(k)$, where $P_\mu^*(k) = \sum_{l=1}^L \mu_l(z) P_l^*(k)$, $P_l^*(k)$ is an optimal value of $P_l(k)$ at time k , then it can be easily got

$$\rho_{\min}^* \|x(k)\|^2 \leq V(k, x) \leq \rho_{\max}^* \|x(k)\|^2 \quad (36)$$

where

$$\rho_{\min}^* := \min \{ \rho_{\min}(P_l^*(k)) | l \in \mathbb{Z}_{[1, L]}, k \in \mathbb{N} \}$$

$$\rho_{\max}^* := \max \{ \rho_{\max}(P_l^*(k)) | l \in \mathbb{Z}_{[1, L]}, k \in \mathbb{N} \}$$

where $\rho_{\max}(\cdot)$ and $\rho_{\min}(\cdot)$ represent the maximal and minimal eigenvalues respectively.

Moreover, the constraints (29) imply

$$V_k(x(k+1)) - V(k, x) < -(x(k)^T Q x(k) + u^T(k) R u(k) - \tau d^T d) \quad (37)$$

where $V_k(x(k+1)) = x(k+1)^T P_\mu^*(k) x(k+1)$. Thus one has

$$V_k(x(k+1)) - V(k, x) < -x(k)^T Q x(k) + \tau d^T d \quad (38)$$

Owing to the optimality of the problem at time $k+1$, it holds

$$V_{k+1}(x(k+1)) \leq V_k(x(k+1)) \quad (39)$$

Then it holds that

$$V_{k+1}(x(k+1)) - V(k, x) < -x(k)^T Q x(k) + \tau d^T d \quad (40)$$

(36) and (40) together yield that $V(k, x)$ is an ISS Lyapunov function. Hence the close-loop system is ISS with respect to disturbances. The proof is completed.

IV. ILLUSTRATIVE EXAMPLE

In this section, an example is given to show the effectiveness of the proposed methods. The simulations are implemented with MATLAB and all matrix inequalities are solved by YALMIP toolbox [22]. The simulations are executed on HP desktop (3.40 GHz Intel Core i7, 8GB RAM).

The model is given by

$$\begin{cases} x_1(k+1) = 1.15x_1(k) + 0.35\sin(x_1(k))x_1(k) + 2x_2(k) \\ x_2(k+1) = 1.15x_2(k) + 0.5\sin(x_2(k))\cos(x_2(k)) \\ \quad + u(k) + d(k) \end{cases}$$

The input constraint is $|u(k)| \leq 5$. And the disturbance is a white noise satisfying $|d(k)| \leq 0.02$.

It can be seen that there are two nonlinear terms $z_1(k) = \sin(x_1(k))$, $z_2(k) = \frac{\sin(x_2(k))\cos(x_2(k))}{x_2(k)}$. The model is rewritten as follows

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} f_1(x_1(k)) & 2 \\ 0 & 1.15 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} \phi(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d(k)$$

where $f_1(x_1(k)) = 1.15 + 0.35\sin(x_1(k))$, $\phi(k) = \sin(x_2(k))\cos(x_2(k))$, $x_1(k), x_2(k) \in \{-\frac{\pi}{2}, \frac{\pi}{2}\}$. From Fig. 1, it can be find that $\phi(k)$ satisfying $\phi(k) \in \text{co}\{0, x_2(k)\}$.

Fig. 1. The bound of ϕ .

Then the following T-S fuzzy model is got:

Rule 1:

IF $z_1(k)$ is 1,

THEN $x(k+1) = A_1x(k) + B_1u(k) + G_1\phi(k) + E_1d(k)$

Rule 2:

IF $z_1(k)$ is -1,

THEN $x(k+1) = A_2x(k) + B_2u(k) + G_2\phi(k) + E_2d(k)$

where $A_1 = \begin{bmatrix} 1.5 & 2 \\ 0 & 1.15 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0.8 & 2 \\ 0 & 1.15 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $G_1 = G_2 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$, $E_1 = E_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

The membership functions are defined as follows,

$$\begin{cases} F_1 = \frac{\sin(x_1(k)) + 1}{2} \\ F_2 = 1 - F_1 \end{cases}$$

Let $Q = \text{diag}\{1, 1\}$, $R = 1$.

Meanwhile, the "0-step" (online) linear FMPC (LFMPC) method proposed in [8] is also executed for comparison purpose (see Section IV. A in [8]). Simulation results of the two methods are shown in Figs. 2-5. The differences of the number of matrix inequalities and consumption of cputime are shown in Table 1.

Fig. 2. Curves of x_1 .

Fig. 3. Curves of x_2 .

From Figs. 2-5, it can be seen that the control performance of nonlinear FMPC (NFMPC) is better than that of LFMPC. Table I shows that the number of matrix inequalities adopting proposed method is much less than that of LFMPC, as a result, the time consumption is significantly reduced by the proposed method.

Fig. 4. Curves of u .

Fig. 5. Upper bound of infinite horizon cost function.

TABLE I
COMPUTING TIME OF DIFFERENT METHODS

Methods	LFMPC	NFMPC
The number of MIs	150	30
The number of decision variables	16	13
Total cpu time	706.1791s	89.4869s

V. CONCLUSIONS

Robust model predictive control method for nonlinear systems which are modeled as T-S fuzzy systems with nonlinear local models is investigated. External disturbances are also considered in the above systems that the input-to-state stability is guaranteed. Simulation result shows that the proposed FMPC method has advantages in control perfor-

mance and computational cost compared with the existing FMPC method where only linear local models are adopted.

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