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Effect of oblique retro-reflection from a vibrating mirror on laser Doppler shift

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A laser Doppler vibrometer (LDV) fails to measure a large out-of-plane vibration of a rotating mirror when the mirror obliquely reflects the laser beam away, causing a signal loss from detection. To solve this problem, an external retro-reflective tape was used to recover the oblique reflection. However, the reading of LDV obtained from the recovered signal is not right because the retro-reflection adds extra Doppler frequency shifts to the oblique reflection. Here, we first derive the relationship of Doppler shift to the oblique angle of retro-reflection. For the first time with the help of retro-reflection, a standard LDV can measure the largely vibrating mirror as good as a high-speed camera, albeit without the needs for heavy computation. © 2019 Optical Society of America

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1. INTRODUCTION

A laser Doppler vibrometer (LDV) is widely used for non-contact vibration measurement [1, 2] in the industries of automotive[3], micro-electro-mechanical system[4–9], manufacturing[10], medical diagnosis[11] and even vehicle navigation[12]. An LDV can measure a translational velocity of a vibrating object. This velocity measurement is based on the Doppler effect of light [1]. During measurement, an LDV sends a light beam on a vibrating target, and simultaneously receives the reflected light. Superimposed of the original and the reflected light wave signals yield an interference pattern and the determination of the Doppler frequency shift [13, 14]. Since LDV sensor head has a laser source and a photodetector sharing the same objective lens, the best signal detection is achieved by focusing light beam on a moving target and collecting most of the reflected light. Signal detection of LDV is sensitive to the surface finish of the target[15]. A shiny surface such as a mirror, or a polished metal, with normal reflection yields a clear signal. Whereas a diffuse surface yields omnidirectional reflections and thus an average level of signal detection.

Figure 1a shows that a standard LDV sensor head can measure the pure translation of a vibrating mirror M from the normal reflection. A static tilt θ of the vibrating mirror causes an oblique reflection, diverging the signal away from the detection (Fig. 1b). Specifically, this happens when the tilt angle θ is greater than the angle δ , which is defined as the maximum tilt angle detectable

by the collecting optics:

$$\delta = \frac{1}{2} \tan^{-1} \left(\frac{r}{f_c} \right) \quad (1)$$

where r is the radius of the objective lens and f_c is the focal length as shown in Fig. 1a. This tilt can be manually adjusted to restore to the normal reflection. However, this manual alignment method does not help in the presence of dynamic rotation of a mirror that tilts continuously between oblique and normal reflections. For example, LDV fails to properly measure the out-of-plane vibration of a rotating mirror, or a deflecting cantilevered mirror (Fig. 1c and Fig. 1d). When the dynamic tilt causes the oblique reflection beyond the LDV's view angle, the LDV signal detection is lost [7, 16].

The maximum detectable tilt angle of a vibrating mirror-surface target is limited by the numerical aperture (N.A.) of the objective lens (of the LDV sensor head) [17]. A microscope objective lens with a large N.A. lens (with a small focal length) can widen the angle of detection for an LDV [5, 6, 9]. For example, an LDV with $20\times$ microscope objective lens can measure 12 degrees dynamic rotation of micromirror [9]. However, this approach only works for small micro-mirrors. Also, the small focal length limits the working access by probes [4]. Unfortunately, a higher N.A. microscope lens ($>20\times$) compromises the accuracy of the measured vibration amplitude; for example, a $60\times$ (N.A.=0.8) lens causes a maximum error of 5% [18].

Diffuse coatings can be applied on a vibrating mirror to im-

Fig. 1. Effect of mirror motions on reflected signals to LDV sensor: (a) a normal reflection on a translating mirror. (b) an oblique reflection on a translating mirror. (c) an oblique reflection on a rotating mirror. (d) an oblique reflection on a deflecting mirror (coupled translation and rotation).

prove the signal detection by an LDV. For example, a removable paint was applied on a shiny car body [3] to help modal testing by an LDV, or powder coatings were sprayed on a fan blade [19] to help strain measurement by an LDV. However, diffuse coating is not always feasible and practical. It needs to be removed [10] from the finished product of mirrors after the sampling check. Furthermore, diffuse coating adds extra mass and may alter dynamics of micro-mirrors. Similarly, a retro-reflective tape [20] was applied on a rotating shaft for measuring the radial vibrations, but it introduces cross-axis noises [21]. While measuring the vibration of a rotating shaft, a tangential motion of the micro-rough retro-reflective surface can be mistaken as the radial vibration.

An external retro-reflective tape can solve the above mentioned problems by recovering the oblique reflection. However, the velocity reading of LDV from the recovered signal is wrong because retro-reflection adds extra Doppler frequency shift to the oblique reflection. Here, we first derive the dependence of Doppler shift on the oblique angle of retro-reflection. In addition, we showed that a standard LDV with the help of external retro-reflection can measure the large vibration of a cantilevered mirror which undergoes dynamic tilt or rotation. The subsequent sections will describe in detail the method and the results of this retro-reflection assisted LDV.

2. PRINCIPLE

A standard LDV can measure out-of-plane motion of a moving object. Figure 2a shows that an object M is moving obliquely at velocity $\vec{v}$ by an angle α off the incident laser beam at point p . The LDV sensor at point o measures a total Doppler frequency shift f_D proportional to the projected velocity along the direction of the beam with [1]:

$$f_D = 2 \frac{v \cos \alpha}{\lambda} \quad (2)$$

where v is the magnitude of the object velocity $\vec{v}$, and λ is the optical wave length of the laser beam. This is twice the Doppler shift as observed by the moving object.

In a special case that the object moves along the laser beam ($\alpha = 0^\circ$), the magnitude of the measured velocity v_m by the standard LDV is defined as:

$$v_m = \frac{\lambda}{2} f_D \quad (3)$$

However, if the moving object is shiny (like a mirror) and tilted with $\theta > \delta$ (see Fig. 2b), the oblique reflection (at an angle θ off the surface normal $\hat{n}$) will miss from being detected by the LDV sensor head.

Figure 2c shows a simple setup devised by using an external retro-reflective tape to recover the diverged laser beam (i.e. oblique reflection). The incident laser beam from a standard LDV is obliquely reflected by the moving mirror M . The outgoing light hits the retro-reflective tape at point q . By retro-reflection from spherical glass beads partially embedded in a reflective substrate on the tape (Fig. 2d), the diverged laser beam will return to the sensor at point o . In this way, the LDV sensor can always detect the oblique reflection off the moving mirror even in the presence of a large tilt angle.

However, the retro-reflection from q to p causes extra Doppler frequency shifts Δf_D :

$$\Delta f_D = 2 \frac{v \cos \beta}{\lambda} \quad (4)$$

where β is the angle between the object velocity $\vec{v}$ and the retro-reflected beams (i.e. $\beta = 2\theta - \alpha$). This is twice the Doppler shift as observed by the retro-reflective tape q (see Fig. 2e and Appendix A for detailed derivation of Eq. 4).

As a result, the LDV observes a sum of Doppler frequency shifts f_D' from both oblique and retro-reflections:

$$\begin{aligned} f_D' &= f_D + \Delta f_D \\ &= 2 \frac{v}{\lambda} (\cos \alpha + \cos \beta) \end{aligned} \quad (5)$$

This Doppler frequency shift is more than the one measured from normal reflection (see Eq. 2). The magnitude of the measured velocity v_m' from the retro-reflection-assisted LDV is defined as:

Fig. 2. Effect of oblique and retro-reflections on the Doppler frequency shifts (see **Visualization 1**). Standard method: (a) a normal reflection as observed by a standard LDV. (b) an oblique reflection undetected by the LDV sensor. Proposed method: (c) an oblique reflection retro-reflected back to the LDV sensor. (d) Left: a drawing of a retro-reflection on a spherical glass bead partially embedded in a reflective substrate and Right: a retro-reflective tape under a laser illumination at 10X magnification (right). (e) the light path and the Doppler frequency shifts, f_D and Δf_D in oblique and retro-reflections (see Appendix A for details).

$$v_m' = \frac{\lambda}{2} f_D' \quad (6)$$

Hence, the measured velocity by LDV needs to be corrected by substituting Eq. 5 into Eq. 6 to yield the mirror (object) velocity using the following relationship:

$$v = \frac{v_m'}{(\cos \alpha + \cos \beta)} \quad (7)$$

In the case that the object velocity is normal to the surface ($\theta = \alpha = \beta$), Eq. 7 reduces to:

$$v = \frac{v_m'}{2 \cos \theta} \quad (8)$$

3. EXPERIMENTS WITH VIBRATION CALIBRATOR

The objective lens of a standard LDV has a limited view angle to detect oblique reflection from a vibrating mirror. Here, we measure the limit of view angle of the LDV. Subsequently, we investigate the method of external retro-reflection to recover the oblique reflection. Then, we correct the measured mirror velocity from the oblique reflected signals at various tilt angles.

Figure 3a shows the experiment setup using an out-of-plane LDV (LV-S01, Sunny Instruments Singapore) method to measure the velocity v_m of a vibrating mirror mounted on a vibration calibrator. The LDV has a vari-focal objective lens of radius $r = 2.4$ cm. The laser beam was focused on the target of the vibrating mirror at a focal length $f_c = 50$ cm. In this setup, the numerical aperture is calculated as $\text{N.A.} = \sin \delta = 0.048$ [17], where $\delta = \frac{1}{2} \tan^{-1}(\frac{r}{f_c})$. The controller of LDV indicates the level of detected light signal.

Figure 3b shows the experiment setup of a retro-reflection-assisted LDV. In this setup, the laser beam is incident obliquely ($\theta > 0^\circ$) to the vibrating mirror. To recover the oblique reflection,

Fig. 3. An experimental setup of LDV to measure the vibration of a mirror (top view). (a) at normal reflection ($\theta = 0^\circ$) (b) at oblique reflection ($\theta > 0^\circ$) with retro-reflection method. (c) The instruments used in the experiment.

we introduced a retro-reflective adhesive tape (3M Scotchlite™) along the light path. The retro-reflective tape was mounted externally on a support column (Fig. 3c). The retro-reflective tape with a wide entrance angle [22] can always return the incident light back to the source, without the need for precise angular adjustment. However, the laser beam needs to be focused on the retro-reflective tape instead of the target by adjusting the focus of the vari-focal lens of the LDV in order to maximize the signal detection. The signal level indicator can help to confirm when the returned signal is optimum. Here, the measured velocity using the retro-reflection method is denoted as v_m'

A. View angle of LDV measurement

To measure the maximum view angle of an LDV sensor (see Eq. 1), we turn a vibrating mirror at angular increment until the signal detection (from oblique reflection) was lost. A rotary measuring stage was turned at angular increments below: $\theta = 0, \pm 1, \pm 1.5, \pm 2, 3, 4, 5, 10, 22.5, 30$ and 45 degrees. A vibration calibrator (VC110, Metra Mess-und Frequenztechnik) was used to drive the mirror. The vibration calibrator (with a piezo stack actuator) generates a sinusoidal acceleration of 1 m/s^2 constant amplitude. When the mirror is excited by the calibrator at 220 Hz , its velocity amplitude is 0.72 mm/s .

Figure 4 shows the view angle within which the LDV can accurately measure the mirror velocities of a vibrating mirror on the rotary stage. The experiment shows that this LDV can accurately measure the mirror velocity when the mirror is positioned nearly normal to the laser beam not more than 1.5° tilt. This maximum detectable tilt angle δ is close to the predicted value of 1.4° tilt from Eq. 1.

Fig. 4. Dependence of the measured velocity on the tilt angle θ using LDV with a standard method and retro-reflection method.

In contrast, we showed that an external retro-reflective tape can recover the oblique reflection from a vibrating mirror over a wide range of tilt angles. Interestingly, the tilt angle can be as wide as 45° and the retro-reflective tape can help recover the signal for LDV measurement. The corrected velocity is identical to the driving velocity of the calibrator.

B. Correction of the measured velocity

Here, we correct the LDV measured velocities by substituting the tilt angle into the Eq. 8. The correction test is conducted

Fig. 5. Tilt angular dependence & correction of the LDV measured velocities from oblique and retro-reflections. (a) Tilt angle θ dependence of controlled mirror vibrations. (b) Frequency dependence of controlled mirror vibrations. (c) The true mirror velocities after correction.

Fig. 6. (a) a front-view of the cantilevered ruler (the measuring ruler is inserted for visualization purpose). The setup (top-view) for the measurement of large deflection of a cantilevered mirror: (b) using a standard out-of-plane LDV (see **Visualization 2**) and (c) using a retro-reflection-assisted LDV ($\theta = 20^\circ$) (see **Visualization 3**). (d) the relative position of the retro-reflective tape with respect to the cantilevered mirror.

over a range of driving frequency. The vibration calibrator is tuned to drive a mirror from 150Hz to 250Hz. Given a constant acceleration amplitude, the amplitude of calibrator velocity is inversely proportional to the driving frequency. Figure 5a shows that the velocities measured by LDV from oblique and retro-reflected signals are different from the true velocities of vibrating mirror. The measured velocity is only correct when the mirror is tilted to cause normal reflection ($\theta = 0^\circ$). Figure 5b shows that the deviation between the measured velocity to the true one is frequency independent (i.e. at constant proportionality). Upon angular correction, Eq. 8 yields the correct velocity as shown in Fig. 5c.

4. MEASUREMENT OF A CANTILEVERED VIBRATION

A cantilevered mirror translates and rotates when excited by a shaker at the cantilever base. The translation of the cantilevered mirror is large but its rotation is small. However, the vibrating mirror with a small rotation can deflect a laser beam like a scanning mirror [6]. An out-of-plane LDV cannot properly measure the large translation of a cantilevered mirror in the presence of a small rotation. To illustrate this problem and feasible solution, we measure the vibration of a cantilevered mirror using three different methods. These methods are: 1) a standard out-of-plane LDV (LV-S01), 2) a retro-reflection-assisted LDV, and 3) imaging method using a high-speed camera.

A. Setup and methodology

The sample under test is a cantilevered mirror (see Fig. 6a). The cantilever is a ruler of 135 mm long, and 7.9 grams in weight. On

the tip of the cantilever is a silicon mirror (of $8 \times 20 \text{ mm}^2$ in size & < 0.3 grams in weight) attached adhesively. The cantilever base is mounted on an electrodynamic shaker (TK2004E01, The Modal Shop, USA). In the test, vibration levels of shaker were increased until a dynamic tilt of the cantilevered mirror exceeds the detection angle of a standard LDV. And, the driving frequency input signal was set at 14 Hz. The base motion x_{base} caused by the shaker excitation is described as:

$$x_{base}(t, u) = X(u) \sin(2\pi ft) \quad (9)$$

where t is the time, and f is the frequency of the periodic input signal, X is the vibration amplitude (in mm), and u is the applied voltage to the shaker. Meanwhile, the translation of the cantilever tip (mirror) y is described as:

$$y(t, u) = A(u) \sin(2\pi ft) \quad (10)$$

where A is the amplitude.

In the test, we used a high-speed camera (FPS1000, Imagetec, UK) to capture the video of the cantilevered vibration. The camera has an image resolution of 1280×720 at 550 frames per second. To assist in our measurement, a needle acting as a pointer was attached on the cantilever tip. While the cantilever vibrates, the tip (pointer) scans over a measuring ruler beneath it. Tracker software [23] was used to process the video frame-by-frame. This yields the displacement waveforms, y_h , over the time, and thus the velocity waveforms, v_h as the time derivative $v_h = dy_h/dt$. The displacement amplitude A_h is taken as half of the peak-to-peak value of the periodic waveforms. The inset in Fig. 6b shows an example of the captured image of the pointer.

Neutral orientation of the vibrating mirror needs to be adjusted to suit to the measuring methods. To achieve normal reflection detectable by a standard LDV, the mirror surface is positioned to be normal to the incident light from LDV (as seen in Fig. 6b). To show the recovery of oblique reflection by the retro-reflective tape, the mirror surface is oriented at an angle, say $\theta=20^\circ$ (as shown in Fig. 6c). A retro-reflective tape of 50 mm long was placed at a distance of 35 mm away from the target mirror (Fig. 6d) to cover the scanning range of the mirror's reflection. To maximize the signal detection by LDV, the laser beam was focused on the retro-reflective tape, instead of the vibrating target. The focus can be readily done at a point on the tape, say near pivotal mount of angular adjustable column. Next, the retro-reflective tape was turned about the mounting column and bent to nearly equally focus the laser beam on the extremes of scanning line on the retro-reflection tape.

The LDV acquires raw data in the form of velocity waveforms (time-domain). Here, a Fast Fourier Transform (FFT) function was used to transform the time varying-signal data into frequency spectrum data. The peak value in the spectrum curve is the velocity amplitude V . Here, V_m denotes the value obtained by standard LDV method, and V_r denotes the value obtained by retro-reflection method using Eq. 7 with $\theta=20^\circ$. Time integrals of the LDV's measured velocity yields the displacement waveforms, y_m and y_r from standard and retro-reflection method respectively. Ideally, the amplitudes A of the harmonic displacement can be calculated in terms of velocity amplitudes following: $A = V / (2\pi f)$.

B. Results of measurements

The high-speed camera can capture the cantilever tip's vibration from low (1.5 mm amplitude) to high (6.1 mm amplitude) while the voltage driving the shaker was raised from 0.1 V to 0.34 V. Images in Fig. 7a show the extreme positions of the vibrating cantilevered mirror (as indicated by the pointers above the measuring ruler) when a shaker excites the base of the cantilever at 14 Hz under 0.34 V input. The displacement waveforms obtained from the processed images appear harmonic with a displacement amplitude A_h is found to be at 6.1 mm (see Fig. 7a). The velocity waveforms as the derivative of the displacement's (v_h) is also harmonic. However, the velocity obtained by out-of-plane LDV is full of noises due to signal truncation beyond the LDV's view angle. This leads to anharmonic displacement waveforms with inaccurate amplitude A_m . On the other hand, our retro-reflection method avoid the problem by fully recovering the oblique reflected signals. This yields the accurate measurement of large vibration of the cantilevered mirror.

Figure 8a shows the base excitation X and the vibration amplitudes of the cantilevered mirror increase when the driving voltage to the shaker was ramped up. Figure 8a and 8b show the limit of a standard LDV to measure the large vibration amplitudes. Below the limit (vibration amplitude of 4.7 mm, and tilt angle of 1.8 degrees), the standard LDV works well. Beyond the limit, the measurement becomes inaccurate, being about 21% lower than the "true" value as measured by the high-speed camera method and the retro-reflection method.

5. CONCLUSION

We showed that an external retro-reflection method can recover oblique reflection from a large vibration of a mirror. The reading by a standard LDV however deviates from the true velocity because the oblique retro-reflection introduces extra Doppler

Fig. 7. Measurement results (see **Visualization 2** and **3**): (a) Snapshot of high-speed camera showing pointer's position at Left: minimum amplitude, Right: maximum amplitude, giving $A_h=6.1\text{mm}$ (b) Time plots of the mirror vibration measurement as measured by a high-speed camera, a standard LDV (see **Visualization 4**) and an LDV with retro-reflection (see **Visualization 5**)

Fig. 8. (a) The measured vibration amplitudes of the cantilevered tip, and the corresponding tilt angles ($\Delta\theta$) as a function of the voltage input to the shaker. (b) The equality plot for the vibration amplitude measurement from high-speed camera, and two methods of LDV. The line of equality is with respect to the measurement from high-speed camera, A_h

frequency shifts to the detected signal. Here we successfully derived the contribution of oblique retro-reflection to the extra Doppler frequency shifts. In this way, we correct the measured velocity of a vibrating mirror from the recovered signal, even in the presence of large deflection. For the first time with the help of retro-reflection, a standard LDV can measure the largely vibrating mirror as good as a high-speed camera, albeit without the needs for heavy computation.

APPENDIX A

Here, we present the detailed derivation of the Doppler frequency shift in Eq. 4. Figure 2e shows the light path and the Doppler frequency shifts, f_D and Δf_D in oblique and retro-reflections. There are three local observers namely: point o on the LDV, point p on the moving target M , and point q on the retro-reflective tape. Let us first consider that target M is moving closer toward o and q with a velocity $\vec{v}$ by an angle α off the incident laser beam at p (Fig. 2c).

The first incremental Doppler shift in oblique reflection happens to the emitted light traveling along the path $\overline{op}$ as observed by p [1]:

$$f_{D_{op}} = \frac{v \cos \alpha}{\lambda} \quad (\text{A1})$$

where v is the absolute velocity of $\vec{v}$. The sign in the above expression is positive because the observer p is moving closer toward the stationary source of light at o . The second incremen-

tal Doppler shift in oblique reflection happens to the reflected light along the path $\overline{po}$ as observed by o :

$$f_{D_{po}} = \frac{v \cos \alpha}{\lambda} \quad (\text{A2})$$

The sign in the above expression is also positive because the source of light at p is moving closer toward the stationary observer at o . We can observe that the sum of $f_{D_{op}}$ and $f_{D_{po}}$ is equivalent to the Doppler shift f_D as measured in normal reflection in Eq. 2, and write this as:

$$f_D = f_{D_{op}} + f_{D_{po}} = 2 \frac{v \cos \alpha}{\lambda} \quad (\text{A3})$$

The first incremental Doppler shift in retro-reflection happens to the reflected light traveling along the path $\overline{pq}$ as observed by q :

$$f_{D_{pq}} = \frac{v \cos \beta}{\lambda} \quad (\text{A4})$$

where β is the angle between the reflected light along the path $\overline{pq}$ and the velocity vector $\vec{v}$. The sign in the above expression is positive because the source of light at p is moving closer toward the stationary observer at q . The second incremental Doppler shift in retro-reflection happens to the retro-reflected light along the path $\overline{qp}$ as observed by p :

$$f_{D_{qp}} = \frac{v \cos \beta}{\lambda} \quad (\text{A5})$$

The sign in the above expression is also positive because the observer at p is approaching the stationary observer at q . And the sum of the $f_{D_{pq}}$ and $f_{D_{qp}}$ is expressed as Δf_D to denote the total Doppler shifts due to retro-reflection, and is written as:

$$\Delta f_D = f_{D_{pq}} + f_{D_{qp}} = 2 \frac{v \cos \beta}{\lambda} \quad (\text{A6})$$

Our derivation remains the same in the case of the target M moving away from o and q . In such a case, the sign in the expression Eq. A1-A6 becomes negative.

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