Acoustic signal processing for well integrity monitoring systems

Ang, Yi Yang

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**NANYANG
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**ACOUSTIC SIGNAL PROCESSING FOR WELL
INTEGRITY MONITORING SYSTEMS**

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SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING

2019

ACOUSTIC SIGNAL PROCESSING FOR WELL INTEGRITY MONITORING SYSTEMS

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Supervisor Declaration Statement

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Professor Gan Woon Seng

Authorship Attribution Statement

This thesis contains material from 4 paper(s) published in the following peer-reviewed journal(s)/ from papers accepted at conference/ from technical disclosures in which I am listed as an author and/or corresponding author.

It also contains materials from 1 paper prepared for journal publications

Published

Chapter 3 is published as Y. Y. Ang, N. Nguyen, W. S. Gan, “Multiband Grid-Free Compressive Beamforming,” *Mech. Syst. Signal Process.*, vol. 135, p. 106425, 2020.

The contributions of the co-authors are as follows:

- I conceived the idea and conducted all the numerical experiments and analyzed all the experimental data
- I prepared the manuscripts. Dr. Nguyen and Prof Gan revised the manuscript.
- Dr. Nguyen and Prof Gan provided advice in the mathematical formulation and guidance in the interpretation of the results

Chapter 5 is published as Y. Y. Ang, N. Nguyen, J. P. Lie, and W. S. Gan, “Localization of harmonic source using a single moving sensor of known trajectory,” in *2017 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA ASC)*, 2017, pp. 324–328.

The contributions of the co-authors are as follows:

- I prepared the manuscripts. The manuscript was revised by Dr. Nguyen, Dr. Lie, and Prof Gan.
- I conducted all the numerical experiments and analyzed all the experimental data
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- Dr. Taware and Dr. Lu provided the direction to the well integrity problems.
- I conceived the idea and prepared the technical disclosures documents.
- Dr. Nguyen and Dr. Lie provided advice in the mathematical formulation and guidance in the interpretation of the results

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The contributions of the co-authors are as follows:

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- I prepared the manuscript drafts. A. R. Venna, Dr. Nguyen, and Dr. Lu

revised the manuscript.

- A. R. Venna performs the collection and testing of the experimental data
- I performed the post-processing of the experimental data and literature survey
- The post-processed data was then used to train the classifier by A. R. Venna.

In preparation for submission

Chapter 4 contains materials prepared for Y. Y. Ang, N. Nguyen, W. S. Gan, “Aliasing-free Nearfield Localization using Linear array,” in preparation for submission

The contributions of the co-authors are as follows:

- I conceived the idea and conducted all the numerical experiments and analyzed all the experimental data
- I prepared the manuscripts. Dr. Nguyen and Prof Gan revised the manuscript.
- Dr. Nguyen and Prof Gan provided advice in the mathematical formulation and guidance in the interpretation of the results

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Singapore, January 2019

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Summary

This thesis presents a systematic study of the processing of acoustic radiation to address the complex practical challenges involved in acoustic signal processing and its application to a passive well integrity monitoring system.

The conventional approach to passive well integrity evaluation is to deploy single or multiple independent sensors to detect leaks within the wellbore. Even though these approaches have been proven to be successful in detecting leaks, they do not determine other crucial parameters such as the directions, the radial locations, and the types of leak in the wellbore. These parameters are important because they can enable the development of enhanced diagnostics of the condition of a wellbore. Accordingly, an array processing approach was recently developed to provide an additional dimension of information to localize the source of a leak.

At the beginning of this thesis, the conventional beamforming used in array signal processing algorithms is studied to investigate the advantages of using sensor array for passive well integrity evaluation. Even though an array can provide localization information, an array's spatial aliasing frequency limits the type of localizable passive source that can be used. The array spatial aliasing frequency is usually below the typical time-domain aliasing limit, which is one-half the sampling rate. This limitation prevents the array from localizing passive sources with frequency contents above the array spatial aliasing frequency.

Accordingly, the far-field source model is investigated. If the wideband source satisfies a specific condition, an aliasing-free direction-of-arrival estimation can be achieved via sparse beamforming. However, this condition is not applicable to the

recently proposed grid-free compressing beamforming technique because it was derived for sources with a single frequency. To alleviate this deficiency, a multiband grid-free technique is developed, and a new cost-function for a multiband source is derived from a theoretical investigation of a multiband grid-free model, where this function can be solved through semidefinite programming (SDP). The method generalizes the original single-band grid-free technique to a multiband representation. This is validated through simulations and experimental test results.

Next, a study is performed on the nearfield source model, which reveals that a signal received from a point source in the nearfield is not bandlimited in the spatial frequency domain. Although many solutions are available in the literature, they all require modification of the array's aperture, and this modification may not be practical in the passive well integrity evaluation problem. Thus, via investigation of the theoretical array's steering vector, a condition is derived that guarantees an aliased-free localization. This condition is validated by extensive numerical simulations and experimental analyses. Effectively, the condition extends the operating frequency range of the array without requiring modification of the aperture.

In most situations, it is desirable to use few sensing devices because the use of numerous sensing devices will drive up the cost and complexity of circuitry. To optimize this trade-off, the third study focuses on performing localization with a single sensing device. Here, it is found that a recording from a single moving sensor of a known trajectory can be used to form a one-sensor array (OSA) that achieves localization performance close to that of the Cramer-Rao bound (CRB) of a stationary multi-element array. However, this performance is restricted to harmonic sources. For non-harmonic sources, an innovative approach called the differential received-signal strength one-sensor array (dROSA) is developed. Notably, the dROSA approach does not require

phase compensation, and its use is not restricted to harmonic sources.

Finally, the classification of sources is also essential to a passive well integrity evaluation system. To that end, a support vector machine (SVM) learning approach is developed to classify the types of leaks into four categories, as follows: (i) liquid-to-liquid (L2L); (ii) liquid-to-gas (L2G); (iii) gas-to-liquid (G2L); and (iv) gas-to-gas (G2G). It is found that this approach correctly categorizes 93% of leaks from 1800 acoustic recordings collected from ten different oilfields.

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List of Abbreviations

CRB	Cramer-Rao bound
CS	compressive sensing
DOA	direction of arrival
dROSA	differential received-signal strength one-sensor array
G2G	gas-to-gas
G2L	gas-to-liquid
JSM	joint sparsity model
L2G	liquid-to-gas
L2L	liquid-to-liquid
LLS	linear least squares
MD	multiple dictionary
MP	moving platform
MUSIC	multiple signal classification
MVDR	minimum variance distortionless response
OMP	orthogonal matching pursuit
OSA	one-sensor array
PCA	principal component analysis
PSD	power spectral density
RMSE	root-mean-square error
SA	static array
SDP	semidefinite programming
SBL	sparse Bayesian learning
SMF	spatial matched filter
SNR	signal-to-noise ratio
SSR	sparse signal representation
SVM	support-vector machine
TSE	Taylor series expansion
ULA	uniform linear array

List of Notations

Chapter 2

$\mathbf{p}(\omega)$	pressure-field measurement of the array
$w_n(\alpha, \omega)$	scan vector of the n th sensor
$b(\alpha, \omega)$	beamformer output
$\beta(\alpha, \omega)$	beamforming power
$\mathbf{R}(\omega)$	array correlation matrix
d	intersensor spacing of the sensor array
c	acoustic-propagation speed in the medium
θ_s	source angle-of-arrival to an array
θ	scan-vector angle
$s(\omega)$	amplitude of the source
$a_n(\alpha, \omega)$	transfer function between the n th sensor and source $s(\omega)$ from parameter α
$\mathbf{a}(\alpha_s, \omega)$	steering vector
$\mathbf{r}_s$	source location
r_n	Euclidean distance between the source located at $\mathbf{r}_s$ to the n th sensor

$\mathbf{s} \in \mathbb{C}^M$	signal vector for sparse beamforming reconstruction
$\mathbf{p} \in \mathbb{C}^N$	measurement vector for sparse beamforming reconstruction
$\mathbf{A} \in \mathbb{C}^{N \times M}$	dictionary matrix
ϵ	norm of error

Chapter 3

$p_{n,\ell}$	ℓ th subband response on the n th sensor
$s_{\ell,k}$	ℓ th subband coefficient of the k th source
$\mathbf{p}_\ell \in \mathbb{C}^N$	ℓ th subband sensor response
$\mathbf{a}_{\ell,k} \in \mathbb{C}^N$	ℓ th subband steering vector of the k th source
$\Theta = \{\theta_1, \dots, \theta_M\}$	set of search grids for beamforming
$\beta(\theta_m)$	SMF beamforming power at θ_m
$\mathbf{A}_{s_\ell} \in \mathbb{C}^{N \times M}$	sparse beamforming dictionary matrix
$\mathbf{s}_\ell \in \mathbb{C}^M$	K sparse source vector at the ℓ th frequency bin with nonzero elements for indices containing the K DOA
$\mathbf{p} \in \mathbb{C}^{LN}$	L frequencies-stacked measurement vector
$\mathbf{A}_s \in \mathbb{C}^{LN \times M}$	L frequencies-stacked sparse beamforming dictionary matrix
$\mathbf{s} \in \mathbb{C}^M$	K sparse source vector common at all the L frequencies
ϵ	norm of error

$\mathbf{s}(t, \mathbf{f}) \in \mathbb{C}^{1 \times L}$	continuous-model signal model with L discretized multiband frequencies
$\mathbf{f} \in \mathbb{R}^{1 \times L}$	discretized multiband frequencies
$t_k = \sin \theta_k$	support on the continuous sine spectrum $\mathbb{T} = [-1, +1]$ with $t_k \subset \mathbb{T}$
$\partial(t)$	Dirac delta function
$\mathbf{p}_n \in \mathbb{C}^{1 \times L}$	L frequencies pressure field measurement from the n th sensor
$s_{\ell k}$	complex amplitude of the s th source at the ℓ th frequency bin
$\mathbf{P} \in \mathbb{C}^{N \times L}$	array measurement from the continuous signal space $\mathbf{s}(t, \mathbf{f})$
$\mathbf{S}_k \in \mathbb{C}^{L \times L}$	diagonal matrix of the complex amplitude of source, i.e., $diag([s_{1k} \quad \cdots \quad s_{Lk}])$
$\mathbf{A}_k \in \mathbb{C}^{N \times L}$	linear operator that maps the k th continuous signal $\mathbf{s}(t, \mathbf{f})$ to observation $\mathbf{P}$
$\mathbf{a}_{\ell k} \in \mathbb{C}^N$	column vector containing the ℓ th column of the linear operator $\mathbf{A}_k$
$\mathbf{S} \in \mathbb{C}^{LK \times L}$	row concatenation of all of the k th $\mathbf{S}_k$, i.e., $\mathbf{S} = [\mathbf{S}_1^T \quad \cdots \quad \mathbf{S}_K^T]^T$
$\mathbf{A} \in \mathbb{C}^{N \times LK}$	column concatenation of all of the k th $\mathbf{A}_k$, i.e., $\mathbf{A} = [\mathbf{A}_1 \quad \cdots \quad \mathbf{A}_K]$
$\mathbf{W} \in \mathbb{C}^{N \times L}$	array measurement noise matrix
$\ \cdot\ _{\mathcal{M}}$	sparsity measure for the atom of the continuous complex amplitude
$\mathbf{C} \in \mathbb{C}^{N \times L}$	Lagrangian multiplier

I identity matrix of an appropriate dimension

Chapter 4

$2\pi\tau_n(\mathbf{r}_s, \lambda_\ell)$ phase of the steering vector

$\mathbf{r}_s = (r_s, \theta_s)$ source location

$\lambda_\ell = c/f_\ell$ source wavelength at the frequency f_ℓ

$\mathbf{a}_n(\mathbf{r}_s, \lambda_\ell)$ steering vector of the n th sensor from at $\mathbf{r}_s$ with wavelength λ_ℓ

$r_{\min}$ minimum Fresnel approximation region

$r_{\max}$ maximum Fresnel approximation region

D array aperture

$\mathbf{p}_\ell$ complex pressure field measurement received at the N sensor-array for a source with a frequency f_ℓ

s_s amplitude of the source located at $\mathbf{r}_s$

$\mathbf{a}_\ell(\mathbf{r}_s)$ ℓ th frequency steering vector of the source located at $\mathbf{r}_s$

$\mathcal{R} = \{\mathbf{r}_1 \cdots \mathbf{r}_M\}$ set of vectors denoting the position of the M scan vector's position

$\beta_\ell(\mathbf{r}_m)$ ℓ th frequency beamforming power at source located at $\mathbf{r}_m$

k, k_1, k_2, k_3 arbitrary integer constant indicating that aliasing has occurred

J number of iterations before OMP is halted

Chapter 5

$\mathbf{r}_s = (x_s, y_s)$ actual source location

$s(t)$ acoustic pressure of the source

$p(t)$	acoustic pressure measured by the moving sensor
$r(t) = \ \mathbf{r}_s + \mathbf{v}t\ _2$	Euclidean distance between the source and the sensor at time t moving with velocity $\mathbf{v}$
$q(t)$	additive white Gaussian noise
T	total recording duration
$p_n(t)$	n th frame of the moving sensor measurement $p(t)$
$\Delta T = T/N$	duration of each n th frame
$\mathbf{p}(t)$	temporally corrected OSA pressure measurement
$\mathbf{a}(\mathbf{r}_m)$	steering vector to steer the array to the position $\mathbf{r}_m = (x_m, y_m)$
$\mathbf{R}_{OSA}$	correlation matrix of the temporally corrected OSA pressure measurement $\mathbf{p}_{OSA}(t)$
$\mathbf{R}_{pp}$	correlation matrix of the OSA pressure measurement $\mathbf{p}(t)$
$\hat{\mathbf{r}}_s$	estimated source location
$\hat{\mathbf{U}}_{OSA_s}$	eigenvectors of the OSA correlation matrix signal subspace
$\hat{\mathbf{\Lambda}}_{OSA_s}$	eigenvalues of the OSA correlation matrix signal subspace
$\hat{\mathbf{U}}_{OSA_q}$	eigenvectors of the OSA correlation matrix noise subspace
$\hat{\mathbf{\Lambda}}_{OSA_q}$	eigenvalues of the OSA correlation matrix noise subspace
$\mathbf{w}(\mathbf{r}_m)$	weights for the OSA localization

$\beta(\mathbf{r}_m, \ell)$ ℓ th frequency beamforming power at the location $\mathbf{r}_m = (x_m, y_m)$

$\mathbf{P}_{ave}$ average power vector of the N OSA measurement

P_s average transmitted acoustic signal power of the source

r_n average distance from the source to the moving sensor at the n th frame position

$P_{n,1}$ differential received-signal power of the OSA

$r_{n,1}$ alternate form of the differential received-signal power where its expectation equals r_n^2 / r_1^2

$\mathbf{C}_q$ noise covariance matrix of the location estimate

$\mathbf{C}_{\hat{\theta}}$ estimate covariance matrix

Chapter 6

$\mathbf{X} \in \mathbb{R}^{L \times M}$ data matrix with L PSD frequency bins and M experimental data

$\mathbf{R}_x \in \mathbb{R}^{L \times L}$ data covariance matrix

$\mathbf{P}_K \in \mathbb{R}^{L \times K}$ projection matrix

$\mathbf{X}_{red} \in \mathbb{R}^{K \times M}$ reduced-data matrix

Chapter 1 Introduction

Recent accidents in the oil and gas industry, such as British Petroleum's Macondo blowout in the Gulf of Mexico in April 2010, the Chevron rig fire in Nigeria in January 2012, and the Elgin platform gas leak in the North Sea in March 2012 have highlighted the dangers inherent to this key economic sector [1].

Given that the operational phase of a wellbore is approximately 30 years, it is crucial to ensure that it functions optimally throughout its lifespan. Hence, the integrity of a wellbore must be evaluated using relevant technologies and by developing innovative technologies where necessary. The concept of well integrity is thus explored by many researchers. In this chapter, the environmental effect of oil leaks, well barriers, existing technologies, and our proposed framework are discussed.

1.1 Environmental Effects of Wellbore Integrity Failures

Oil spills and leaks due to a loss of wellbore integrity are harmful to both people and the environment. In addition, it is expensive to remedy and restore operations to normal, and it is difficult to estimate the damages caused to the environment by blowouts and non-catastrophic leaks in both onshore and offshore situations.

In the Macondo blowout incident, more than 40,000 responders [2] were deployed over 89 days to resolve the situation. Clean-up technologies were used to absorb and slow the oil's spread. This was achieved by combustion of the oil on the water surface and by dilution of the oil by addition of chemical dispersants onto the surface and into the subsea zone. By the time the wellbore was capped and sealed on 15 July 2010, the Flow Rate Technical Group estimated that 4.9 million barrels of oil had been discharged

into the Gulf of Mexico [3]. The physical-chemical reaction between the oil and seawater led to the formation of an oil-water emulsion, as illustrated in Figure 1.1. This emulsion covered and smothered marine vegetation, causing severe environmental damage.

Figure 1.1: Emulsification of crude oil (Extracted from [2]).

Therefore, it is essential that methods for diagnosis of well integrity are developed to allow such accidents to be prevented. Most of the research published by the Society of Petroleum Engineers on well integrity management systems is focused on risk assessment using a risk matrix to grade the integrity of the wellbore [4]–[6]. Stuart et al. [7] acknowledged that recent high-profile blowout incidents have made it inevitable that regulators would press for the development of better systems. He concluded that a systematic well integrity management system and a database to forecast and predict potential problems were needed.

1.2 Mitigation Strategies from a Wellbore Perspective

The NORSOK D-010 standard defines well integrity as the “application of technical, operational, and organizational solutions to reduce the risk of uncontrolled release of formation fluids throughout the life cycle of well” [8]. The NORSOK standard also states that as a minimum condition, an operational wellbore should have two qualified and independent well barrier envelopes to prevent hydrocarbons from leaking toward the surface, as depicted in Figure 1.2. This is also required by most government enforcement agencies [9].

Figure 1.2: Block diagram of a completed wellbore (Extracted from [10]).

Well barriers are separated by annuli. As shown in Figure 1.2, the first separation is annulus “A” (green), which is the void between the production tubing (brown) and the smallest casing string. The production tubing is where the hydrocarbon is extracted. Annulus “B” (red) then ensures the integrity of the casing string.

In 2006, the Petroleum Safety Association conducted a statistical study on seven operating companies and evaluated 12 offshore sites and 406 wellbores [11]. Their results revealed that 18% of the studied samples had well integrity problems and that

7% were shut-in due to these problems. Although the generalizability of these conclusions is limited due to the sample size, the data provided a useful statistical distribution of the types and number of failures, as shown in Figure 1.3.

Figure 1.3: Statistical distribution of barrier failures (Extracted from [11]).

Another study was performed by SINTEF, an independent petroleum research company, of one operator's eight fields of 217 wellbores in 2008 [12]. The SINTEF research found three main categories of leaks: (1) tubing to annulus A leaks, (2) wellhead leaks, and (3) downhole safety valve leaks. Figure 1.4 shows the statistical distribution of these three categories.

Figure 1.4: Statistical distribution of barrier failures (Extracted from [12]).

This study also had a small sample size, but its results were consistent with those of the Petroleum Safety Association study and suggested that there is a high probability

that a wellbore will suffer from leaks between the production tubing and annulus A.

In summary, [11] and [12] emphasized the need for the industry to be able to identify any leaks that occur inside a wellbore in their early stages to enable the operator to isolate the leaks and resolve the problem before the condition deteriorates further.

1.3 Existing Methods for Leak Detection

Ultrasonic energy analysis is widely used in leak detection because ultrasonic energy propagates through the steel structure of a wellbore [13]. For example, Farooqui et al. [14] introduced an acoustic-based tool, Johns et al. [15] illustrated an ultrasonic method of leak detection for casing repair, and Johns et al. [16] studied the attributes of ultrasound and leaks in Alaskan north slope wells. Other approaches were later based on noise measurement [17] and more recent integrated approaches [18], [19] in which noise and temperature measurements are coupled with a turbine or spinner flowmeter information to correlate data, which are then used to infer the location of leaks.

Overall, these proposed approaches rely on the measurement of a single sensor or a combination of multiple independent measurements of various parameters to detect leaks. In efforts to provide more information and to improve robustness, Walters et al. [20] extended Farooqui et al.'s approach [14] by using an array of sensors to capture the acoustic event simultaneously from multiple points. They demonstrated that their method could suppress inaccurate and false echo generated at the leak location as the standing waves from the sound of the leak travel up and down a wellbore. In addition, with the added dimension of sensors, their method allows localization of the leaks in wellbore tubing, Annuli A, Annuli B, or its outer casing by generating a spatial spectrum map of the acoustic energy.

1.4 Motivation

The literature review indicates that array processing is a novel and practical way of performing well integrity evaluation to localize leaks inside a wellbore. Specifically, array processing with additional sensors can suppress inaccurate and false echo, and the multiple sensors provide an additional dimension of information to localize the position of leaks inside a wellbore.

However, gaps remain in the research on the use of sensor arrays in a leak evaluation context. First, a single sensor can capture a propagating acoustic signal up to one-half of the sampling rate. This implies that if the sampling rate of a sensor is set arbitrarily high, the sensor will be able to capture a wide range of frequencies. This is a promising feature for passive leak detection because a leak signal can span a wide range of frequencies, from 15 to 60 kHz [17]. However, a sensor array can be interpreted as a spatial sampling system that is limited by its spatial aliasing frequency. Thus, attempts to localize a signal with frequency content beyond an array's spatial aliasing limit will result in aliasing and ambiguity. This prevents a sensor array from localizing a wide range of signals, which is a disadvantage when performing passive localization of potential wellbore leaks of unknown frequency.

Second, the use of additional sensors increases the complexity of the circuitry and the time required to perform a well integrity evaluation. Finally, because well integrity evaluation is costly, it is desirable to obtain as much information on the leak as possible during the evaluation to assist well operators in optimal monitoring of well integrity. These factors serve as the motivation of this thesis in addressing how to improve leak localization in well integrity evaluation.

1.5 Major Contribution of the Thesis Structure

In this thesis, array processing algorithms are studied, analyzed, and applied to solve the passive localization problems likely to be faced in well integrity leak localization. Several new algorithms are derived from different signal assumptions and evaluated in a practical context. The contributions of this thesis are as follows:

- *Investigation of the spatial aliasing of a uniform linear array (ULA) subject to a wideband far-field source:* The parameters of far-field source spatial sampling are investigated. A theorem is found that was derived for a far-field wideband source under the sparse beamforming framework of Tang et al. [21]. Such sparse beamforming can be viewed as a dictionary-based sparse beamforming problem and is subject to basis mismatch when the signal subspace is not an element of the discretized dictionary. Previously, the grid-free compressive beamforming of Xenaki et al. [22] was proposed to be an effective algorithm to combat this basis mismatch problem. However, the grid-free compressive beamforming of Xenaki et al. [22] was formulated for only a single frequency and the theorem of Tang et al. [21] cannot be used to extend the operating frequency of the algorithm. To this end, the first contribution shows that the algorithm established by Xenaki et al. [22] can be formulated in a multiband grid-free compressive form to yield a semidefinite programming problem. Effectively, this generalization extends the grid-free compressive beamforming [22] algorithm to a wideband form and enables the theorem of Tang et al. [21] to be applied. Hence, this approach extends the operating frequency of the algorithm beyond the spatial aliasing limit of the array.

- *Investigation of the spatial aliasing of a ULA subject to a wideband nearfield source:* Here, the spatial aliasing condition of a nearfield source is reviewed. It is found that a nearfield source is not bandlimited, such that the standard half-wavelength spacing between the sensors is insufficient to achieve an aliasing-free result. Conventionally, this can be easily solved by modification of the array's aperture. However, this is not practical due to space and time constraints of the leak localization problem in well integrity evaluation. To address this challenge, the second contribution extends the theorem of Tang et al. [21] using the second-order Taylor series expansion (TSE) model [23], [24]. This reveals that an aliasing-free localization can be achieved if a source has two frequencies with an absolute difference in wavelengths that is more than twice r_{min} and an array aperture smaller than r_{min} , where r_{min} is the minimum Fresnel approximation distance of the array.
- *Development of an array processing method using a single sensor:* As part of the effort to bridge the research gap between single-sensor and sensor-array processing for passive source localization, a signal processing method for a one-sensor array (OSA) is developed. It is found that the method approaches the Cramer-Rao bound (CRB) of a stationary array when the source is harmonic. To make the algorithm more practical, it is shown that the OSA method can be reformulated into a differential received-signal strength one-sensor array (dROSA) form and applied to a general wideband acoustic source. The dROSA approach is found to approach the theoretical CRB when the wideband source is approximately the same distance as the tubing from the center of the wellbore.

- *Classification of multiphase well leaks using support vector machine (SVM):*

To maximize the amount of diagnostic information about leaks in a well, an SVM-based classification method is studied. It is found that the SVM is a practical means to enable the use of array recordings to classify the leaks into one of these four categories, as follows: (i) liquid-to-liquid (L2L), (ii) liquid-to-gas (L2G), (iii) gas-to-liquid (G2L), (iv) gas-to-gas (G2G).

The major contributions of the thesis are summarized in Figure 1.5.

Figure 1.5: Organization of major contributions in this thesis.

1.6 Thesis Structure and Organization

The organization of the thesis is as follows. In Chapter 2, the array processing method for passive localization is introduced. The conventional beamforming methods and sparse beamforming methods are then reviewed.

In Chapter 3, the spatial aliasing problem of a linear array with an impinging far-field

source is investigated. A multiband grid-free compressive beamforming method is then developed to generalize the recently proposed grid-free compressive beamforming method to manage sources with multiple frequencies. This generalization allows the theorem previously established for aliasing-free wideband beamforming to be used for sparse signal representation (SSR) in a grid-free context.

In Chapter 4, the spatial aliasing problem of a linear array with an impinging nearfield source is investigated. It is found that nearfield spatial sampling is not bandlimited, and thus the use of the standard half-wavelength sensor distance cannot guarantee an aliasing-free reconstruction. To extend the aliasing-free reconstruction of the far-field source to a nearfield source, a new theorem is derived that guarantees that an aliasing-free nearfield localization is achievable if the conditions of the theorem are satisfied.

In Chapter 5, an array processing technique using a single sensor is investigated. An innovative approach based on the OSA is proposed, the solution for which has performance approaching the CRB of a stationary array when the source is harmonic. By utilizing the received signal strength, the OSA can be further generalized into a dROSA that can localize a stationary wideband source in the free-field context.

Additional processing using the recording of the array, such as classification, is explored in Chapter 6. An SVM is shown to be a practical and efficient method to classify the captured recordings into the categories of (i) L2L, (ii) L2G, (iii) G2L, and (iv) G2G.

Finally, Chapter 7 concludes the thesis and includes a discussion of possible future work related to the research topic and contributions.

Chapter 2 Fundamentals of Array

Processing

This chapter provides the basis for the rest of this thesis. In Chapter 1, the evolution of acoustic signal processing for well integrity monitoring system in the past few decades is reviewed. This shows that the acoustic evaluation of well integrity began with leak detection along the depth of a wellbore using a single sensor. Integrated approaches using multiple independent sensors were then developed, which also detected leaks only along the wellbore. Recently, new approaches have moved beyond detection to perform leak localization using an array of hydrophone sensors. Using such a sensor array not only enables detection of leaks along wellbore depth but also the localization of leaks radially along the barrier of wellbore depth.

However, this localization ability comes at a cost. To evaluate the implications of using an array in wellbore leak detection, this chapter concentrates on the established array processing methods and their limitations.

2.1 Introduction

A signal propagating through space and time is often corrupted by noise and interference [25]. A great deal of research has been conducted on this problem and focuses on detecting the presence of the desired signal and reducing the effect of noise when using a single sensor [17], [26], [27]. However, Johnson and Dudgeon [25] acknowledged that it is often advantageous to use an array of sensors for better performance. However, although sensor arrays can be planar, circular, or linear [28], these designs may be

unsuitable for the wellbore environment. In fact, due to the limited space and great length of wellbores, only linear arrays are suitable for wellbore leak evaluation [20]. Consequently, the conventional beamforming used in linear array signal processing, comprising the conventional beamforming used for far-field sources, nearfield sources, and wideband sources, is reviewed.

Beamforming is based on the principle that the pressure field measured by a sensor contains information about the wavefront that has propagated in space and time. This spatiotemporal information of the acoustic signal can be represented by parameters such as the shapes of the wavefront and the strength and locations of the acoustic sources [29]. Conventional beamforming attempts to extract parameters related to the source location using scan vectors [29].

The accuracy and robustness of parameter estimation are adopted as the criteria by which to evaluate the performance of beamforming. In this context, accuracy reflects the ability to estimate and discriminate the position of a source positioned in space with minimal error, whereas robustness represents the consistency of the estimates in the presence of noise [29]. In the following section, we describe some of the conventional beamforming approaches.

2.2 Conventional Beamforming

Conventional beamforming is a form of spatial matched filtering (SMF) that matches the recorded pressure field with a set of scan vectors. Each scan vector corresponds to a position in space and is designed to match the wavefront of the sound source as the wavefront propagates through space.

First, the case of a narrowband sound source is discussed. Next, two cases are

considered in which sound will impinge on the array: (a) at the far-field of the array or (b) at the nearfield of the array. Finally, the analysis is extended to a wideband sound source.

2.2.1 Narrowband Processing

Ideally, a sound source with an angular frequency ω propagating from an arbitrary position in space will arrive at the sensor array, and each sensor will receive a delayed signal of the sound source. This propagation delay depends on the distance between the position of the source and the position of the sensor. By designing the scan vector to match the propagation delay, the location that matches the position of the sound source will compensate exactly [29]. Conventional beamforming estimates the position for a far-field or nearfield source by finding the corresponding scan vector where the signal is maximized.

Figure 2.1: General beamforming system with N sensors, $\mathbf{p}(\omega)$, and N beamforming weights, $\mathbf{w}(\alpha, \omega)$, with α being the parameter to be extracted from the measured pressure field.

Figure 2.1 depicts a general beamforming system. Here, the pressure field measurement from an acoustic source with frequency ω measured by a ULA of N sensors in a uniform medium can be expressed in the vector form as:

$$\mathbf{p}(\omega) = [p_1(\omega) \quad \cdots \quad p_N(\omega)]^T. \quad (2.1)$$

The pressure field contains information about how the wavefront propagates from the source to the sensors. The beamforming weight $w_n(\alpha, \omega)$ is the n th element of the scan vector used to extract a certain parameter α from the pressure field.

Using the defined scan vector and measured pressure field, the beamformer output can now be defined as:

$$b(\alpha, \omega) = \mathbf{w}^H(\alpha, \omega) \mathbf{p}(\omega), \quad (2.2)$$

where $\mathbf{w}(\alpha, \omega) = [w_1(\alpha, \omega) \quad \cdots \quad w_N(\alpha, \omega)]^T$ is a vector collection of the N scan vector weights for the N sensors at parameter α . Note that α can be any variable representing the location of the modeled sound source; for example, for wellbore leak detection, α can represent either (a) the direction-of-arrival, θ_s , of a leak in the far-field of the array, or (b) the location, $\mathbf{r}_s$, of a leak in the nearfield of the array.

The beamforming power, which is the expected value of the squared beamformer output (2.2), is a useful measure to evaluate the sound power distribution at parameter α . Mathematically, the beamforming power can be expressed as:

$$\beta(\alpha, \omega) = \mathbb{E}\{|b(\alpha, \omega)|^2\} = \mathbf{w}^H(\alpha, \omega) \mathbf{R}(\alpha, \omega) \mathbf{w}(\alpha, \omega), \quad (2.3)$$

where $\mathbf{R}(\omega) = \mathbb{E}\{\mathbf{p}(\omega) \mathbf{p}^H(\omega)\}$ is the expected value of the sensor correlation matrix.

Essentially, the scan vector represents the discretized grid space within the search area

of the array. In each scanning space α , the beamforming power is computed using (2.3). The power is then analyzed, and the α that has the highest power is interpreted as the estimated source location.

2.2.1.1 Far-Field Sources

In general, if the source is located in the far-field of the array, the wavefront arriving at the sensor agrees well with the plane wave model, as illustrated in Figure 2.2.

Figure 2.2: Plane wave propagation model for a ULA with N sensors, spaced d apart, with a signal impinging the array at an angle θ_s from the broadside.

In the plane wave model, the pressure field measurement at the n th sensor is given by:

$$p_n(\omega) = s(\omega) e^{-j\omega(n-1)\left(\frac{d \sin \theta_s}{c}\right)} = s(\omega) a_n(\theta_s, \omega), \quad (2.4)$$

where θ_s is the direction-of-arrival (DOA) of the plane wave, $s(\omega)$ is the amplitude of the source, and $a_n(\theta_s, \omega)$ denotes the transfer function between the n th sensor and

source $s(\omega)$.

In this case, the beamformer output in the direction of θ can be found by designating the scan vector $\mathbf{w}(\alpha, \omega)$ to have $\alpha = \theta$ and substituting the result into (2.4), as follows:

$$b(\theta, \omega) = \sum_{n=0}^{N-1} p_n(\omega) w_n^*(\theta, \omega) = \frac{s(\omega)}{N} \sum_{n=0}^{N-1} e^{-j\omega(n-1)\frac{d}{c}(\sin\theta_s - \sin\theta)}. \quad (2.5)$$

2.2.1.2 Nearfield Sources

Alternatively, the source may be located in the nearfield of the array, and thus the wavefront perceived by the sensors will be perceptively curved, as shown in Figure 2.3.

Figure 2.3: Spherical wave propagation model for a ULA with N sensors, spaced d apart, with a signal impinging the array from $\mathbf{r}_s$.

This is a spherical wavefront, and the measured pressure field of the n th sensor from a source located at $\mathbf{r}_s = (x, y)$ can then be modeled as:

$$p_n(\omega) = s(\omega) \frac{e^{j\frac{\omega}{c}r_n}}{r_n} = s(\omega) a_n(\mathbf{r}_s, \omega), \quad (2.6)$$

where $r_n = \sqrt{(x - x_n)^2 + (y - y_n)^2}$ is the Euclidean distance from the source located at $\mathbf{r}_s$ to the n th sensor.

2.2.2 Wideband Processing

However, the processing methods in Section 2.2.1 is only valid for a narrowband signal. Because leaks within a wellbore are usually wideband, wideband processing is investigated in this section. From Parseval's theorem, the total beamforming power in the time domain is equivalent to the total power in the frequency domain. Applying the theorem to a signal of finite length affords the following expression :

$$\mathbb{E}\{\beta(\alpha, t)\}_t = \frac{1}{T} \int_{t_0}^{t_0+T} \beta(\alpha, t) dt = \sum_{\ell=-\infty}^{+\infty} \beta_\ell(\alpha), \quad (2.7)$$

where $\beta_\ell(\alpha)$ is the squared Fourier series coefficient, as given by:

$$\mathbb{E}\{\beta(\alpha, t)\}_t = \frac{1}{T} \int_{t_0}^{t_0+T} \beta(\alpha, t) dt = \sum_{\ell=-\infty}^{+\infty} \beta_\ell(\alpha),$$

where $\omega_\ell = 2\pi \frac{\ell}{T}$ is the ℓ th discretized angular frequency.

If we consider a measured pressure field $p_n(\omega)$ bandlimited within $\omega \in [\omega_l, \omega_h]$, then the right-hand side of (2.7) can be expressed as:

$$\mathbb{E}\{\beta(\alpha, t)\} = \frac{2}{T} \sum_{\ell=\ell_l}^{\ell_h} \beta(\alpha, \omega_\ell), \quad (2.8)$$

where $\omega_l = 2\pi \frac{\ell_l}{T}$, and $\omega_h = 2\pi \frac{\ell_h}{T}$. Note that (2.8) is doubled to capture the negative

frequency component as a one-sided spectrum. Effectively, (2.8) implies that the expected beamforming power equals the finite summation of the beamforming power at discrete frequencies. Now, by substituting (2.8) into the beamforming power of (2.3), we arrive at the following expression to compute the wideband beamforming power:

$$\sum_{n=n_l}^{n_h} \beta(\alpha, \omega_n) = \sum_{n=n_l}^{n_h} \mathbf{w}^H(\alpha, \omega_n) \mathbf{R}(\omega_n) \mathbf{w}(\alpha, \omega_n). \quad (2.9)$$

2.3 Adaptive Beamforming

The scan vectors in the previous section are data-independent weights based on how the sound field propagates in space: far-field or nearfield. The performance of the beamforming power obtained by using the scan vector can be improved by using the data-dependent correlation matrix $\mathbf{R}(\omega)$, which contains the spatial statistical correlation of the measured data [30].

In the following section, the conventional data-dependent approach is reviewed. First, the minimum variance distortionless response (MVDR) algorithm is investigated. The subspace approach that separates the correlation matrix $\mathbf{R}(\omega)$ into the source and noise contribution is then studied. Finally, the next section ends with a review of beamforming using the sparsity constraint.

2.3.1 MVDR algorithm

The MVDR algorithm provides a distortionless response to the desired location while minimizing the output power of the beamformer in (2.3). Mathematically, this can be represented by:

$$\min_{\mathbf{w} \in \mathbb{C}^N} \mathbf{w}^H \mathbf{R}(\omega) \mathbf{w}, \quad \text{subject to } \mathbf{w}^H \mathbf{a}(\alpha_s) = 1, \quad (2.10)$$

where $\mathbf{a}(\alpha_s) = [a_1(\alpha_s, \omega) \ \cdots \ a_N(\alpha_s, \omega)]^T$ is the steering vector that is a function of the source-location parameter. The minimization problem in (2.10) can be solved by first formulating the Lagrangian, given by:

$$L(\mathbf{w}, \lambda) = \mathbf{w}^H \mathbf{R}(\omega) \mathbf{w} + \lambda (\mathbf{w}^H \mathbf{a}(\alpha_s) - 1), \quad (2.11)$$

where λ is the Lagrangian multiplier. Solving (2.11) arrives at the optimal $\mathbf{w}_{opt}$, as follows:

$$\mathbf{w}_{opt} = \frac{\mathbf{R}^{-1}(\omega) \mathbf{a}(\alpha_s)}{\mathbf{a}^H(\alpha_s) \mathbf{R}^{-1}(\omega) \mathbf{a}(\alpha_s)}. \quad (2.12)$$

Practically, because $\mathbf{a}(\alpha_s)$ is unavailable, and because it is the steering vector of the source located at α_s to the sensor array, the scan vector can be designed by modifying (2.12) to give:

$$\mathbf{w}_{opt}(\alpha, \omega) = \frac{\mathbf{R}^{-1}(\omega) \mathbf{a}(\alpha, \omega)}{\mathbf{a}^H(\alpha, \omega) \mathbf{R}^{-1}(\omega) \mathbf{a}(\alpha, \omega)}. \quad (2.13)$$

Substituting (2.13) back into (2.3) gives the following beamforming power for the MVDR beamformer at location α :

$$\beta_{MVDR}(\alpha, \omega) = \frac{1}{\mathbf{a}^H(\alpha, \omega) \mathbf{R}^{-1}(\omega) \mathbf{a}(\alpha, \omega)}. \quad (2.14)$$

Note that (2.14) requires all location parameters α to be evaluated, distinct from (2.12), in which only the source location is evaluated.

2.3.2 Multiple signal classification (MUSIC)

The MUSIC approach is a subspace-based approach invented by Schmidt [31] that provides super-resolution beamforming. Assuming that the number of K impinging sources is known *a priori*, the eigendecomposition of the correlation matrix $\mathbf{R}$ will arrive at:

$$\mathbf{R} = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_q \mathbf{\Lambda}_q \mathbf{U}_q^H, \quad (2.15)$$

where $\mathbf{U}_s \in \mathbb{C}^{N \times K}$ contains the K impinging signal-subspace eigenvectors of $\mathbf{R}$ and $\mathbf{\Lambda}_s \in \mathbb{C}^{K \times K}$ contains the corresponding eigenvalues of $\mathbf{R}$. Likewise, $\mathbf{U}_q \in \mathbb{C}^{N \times (N-K)}$ and $\mathbf{\Lambda}_q \in \mathbb{C}^{(N-K) \times (N-K)}$ contain the noise-subspace eigenvectors and eigenvalues, respectively.

MUSIC utilizes the orthogonal relationship between the signal and noise subspace by focusing on the noise subspace. Because $\mathbf{U}_q$ is orthogonal to the basis vectors of the signal subspace, the beamforming power from the noise subspace, defined as:

$$\beta_q(\alpha, \omega) = \mathbf{w}^H(\alpha, \omega) \mathbf{U}_q \mathbf{U}_q^H \mathbf{w}(\alpha, \omega), \quad (2.16)$$

will be a minimum for the scan vector in the signal subspace, i.e., $\mathbf{w}(\alpha, \omega) \in \mathbf{U}_s$. Let the noise subspace correlation matrix be defined as:

$$\mathbf{R}_{ss}^{-1} = \mathbf{U}_q \mathbf{U}_q^H, \quad (2.17)$$

then, the inverse of the noise subspace beamforming power, which is given as:

$$\beta_{ss}(\alpha, \omega) = \frac{1}{\mathbf{w}^H(\alpha, \omega) \mathbf{R}_{ss}^{-1} \mathbf{w}(\alpha, \omega)}, \quad (2.18)$$

will be a maximum when the scan vector $\mathbf{w}(\alpha, \omega)$ subsumes the signal subspace. Thus, (2.18) is known as the MUSIC spectrum, and it provides a pseudo-power spectrum of the source.

2.4 Sparse Beamforming

Recent advances in the compressive sensing (CS) framework have enabled much progress in the reconstruction of signals from a limited number of measurements [32], [33]. By exploiting the sparsity in a signal, a sparse signal can be reconstructed from a few measurements by solving a convex minimization problem. CS has been proven to outperform the traditional beamforming methods discussed in the previous section [34]. Reconstruction of the signal or estimating its parameters from a set of small observations is an underdetermined problem, and CS ensures that accurate recovery is possible if the following two conditions are met [35], [36]:

- (a) The underlying signal is sparse, and
- (b) There is sufficient incoherence of the process that maps the underlying signal to the observations.

Let $\mathbf{s} \in \mathbb{C}^M$ be the unknown signal vector to be reconstructed. In CS, the elements in $\mathbf{s}$ are assumed to be sparse; that is, only K nonzero elements with $K \ll M$. Accordingly, let $\mathbf{p} \in \mathbb{C}^N$ be a vector of measurement that is linearly related to $\mathbf{s}$ by a dictionary $\mathbf{A} \in \mathbb{C}^{N \times M}$. Mathematically, sparsity can be imposed on $\mathbf{s}$ by minimizing its ℓ_0 -norm, as follows:

$$\min_{\mathbf{s} \in \mathbb{C}^M} \|\mathbf{s}\|_0, \quad \text{subject to } \mathbf{p} = \mathbf{A}\mathbf{s}. \quad (2.19)$$

However, the minimization problem in (2.19) is nonconvex. Nonetheless, it was proven in [37]–[39] that (2.19) is equivalent to:

$$\min_{\mathbf{s} \in \mathbb{C}^M} \|\mathbf{s}\|_1, \quad \text{subject to } \mathbf{p} = \mathbf{A}\mathbf{s}. \quad (2.20)$$

The ℓ_1 -norm relaxation in (2.20) is commonly known as the basis pursuit [40].

Assuming the observation vector is contaminated with additive noise $\mathbf{q} \in \mathbb{C}^N$, (2.20) can be reformulated as:

$$\min_{\mathbf{s} \in \mathbb{C}^M} \|\mathbf{s}\|_1, \quad \text{subject to } \|\mathbf{A}\mathbf{s} - \mathbf{p}\|_2 \leq \epsilon, \quad (2.21)$$

where ϵ is the upper bound for noise, i.e., $\|\mathbf{q}\|_2 \leq \epsilon$. Both (2.20) and (2.21) can be solved using the interior-point solvers in the CVX toolbox [41], [42].

2.5 Observations and Research Gaps

In the presented studies, the algorithms are all derived under the condition that the source frequency does not exceed the spatial aliasing frequency limit of the array. Correspondingly, the array is typically designed with fixed intersensor spacings to satisfy the spatial aliasing condition. However, as discussed in the previous section, leak detection in a wellbore based on a single sensor typically spans a wide range of frequencies (15 to 60 kHz) [17]. The detection capability of such a sensor is limited by the Nyquist sampling theorem of $0.5F_s$, where F_s is the sensor sampling frequency.

In comparison, the recently proposed approach by Walter et al. [20], which uses an acoustic sensor array with fixed intersensor spacing, is limited by the spatial aliasing

frequency of an array that is smaller than $0.5F_s$; in other words, the spatial aliasing frequency prevents an array from localizing a wider range of passive sources.

From the above, it can be seen that to achieve effective leak localization using an array, the intersensor spacing of an array must be designed with reference to the type of source to be localized. However, leak localization, as a form of passive source localization, may encounter acoustic signatures with frequency content that is beyond the spatial aliasing frequency of a fixed array. In such a situation, it is impractical and expensive to redesign another array to localize such sources. Accordingly, in an effort to close the research gaps between the existing methods that use a single sensor and those that use a sensor array, this thesis addresses the following questions:

- In the event that a source is impinging from the far-field of an array, how can DOA estimation be achieved for a source with a frequency beyond the spatial aliasing limit of an array?
- In the event that a source is impinging from the nearfield of an array, how can passive source localization be achieved for a source with a frequency beyond the spatial aliasing limit of an array?
- Given that a single sensor is typically used for wellbore-leak detection, can a single sensor provide localization up to an array's spatial aliasing limit?
- What is a practical way of using a priori knowledge to extract more information about such leaks?

Chapter 3 Spatial Aliasing in a Linear Array with a Far-field Source

In Chapter 2, it can be seen that array processing algorithms are derived based on the assumption that the source frequencies do not exceed the spatial aliasing frequency of an array. As a result, in passive leak localization, the usable frequency range for DOA estimation using an array is limited by an array's intersensor spacing. Therefore, this chapter focuses on the development of an array processing algorithm that does not require modification of intersensor spacing and that enables aliasing-free passive wideband far-field source DOA estimation. Essentially, this chapter serves to extend the usable frequency range using signal processing.

3.1 Introduction

Recently, much attention has been focused on formulating the DOA estimation as an SSR problem, in which the estimation is made by addressing an underdetermined problem using convex optimization subject to a sparsity constraint. Solving the SSR problem can provide a robust and super-resolution solution [43], [44] that outperforms the traditional methods for DOA estimation.

Nonetheless, SSR has its limitations: basis mismatch [45] occurs when the discrete angular spectrum dictionary does not contain the source direction and thus causes inaccurate reconstruction. Typically, basis mismatch is addressed using a finer dictionary and an increased computational effort [7]. However, there is a limit to the amount of fine discretization, because its introduction increases the mutual coherence

between the columns of the dictionary [44]. In contrast, the columns of the dictionary must be nearly orthogonal (possess restricted isometry property) [35] for a robust recovery.

Other methods have since been proposed to overcome basis mismatch, such as the atomic norm approach of Xenaki and Gerstoft [22] and the discretization-free and parametric approach of Yang et al. [46]. Yet few have considered the possibility of (a) extending the estimation for a wideband source, or (b) a source with frequency content beyond the spatial aliasing frequency of an array, i.e., $f_{\text{signal}} > f_{\text{aliased}} = c/2d$, where c is the speed of propagation in the medium, and d is the intersensor spacing. These extensions are necessary for a wideband source because it has a large bandwidth relative to its center frequency [13], and the DOA estimation performance of a wideband source that uses the existing approaches from a single center frequency will degrade significantly.

Previously, Tang et al. [21] considered a wideband signal model and derived a theorem that, when its propositions are met, guarantees the DOA estimation to be free of spatial aliasing. These propositions require the far-field sources to have at least two frequencies (possibly beyond the array's spatial aliasing frequency) with an absolute difference of less than $c/(2d)$. Tang et al. [21] also proposed an SSR approach following the joint sparsity model-2 (JSM-2) [48] to perform DOA estimation using an overcomplete dictionary. Nannuru et al. [49], [50] have since proposed the sparse Bayesian learning (SBL) and beamforming framework that iteratively updates itself to combine the observations from the dictionaries. Antoni et al. [51] recently extended the (SBL) framework to an iterative Bayesian-focusing approach that showed promising performance in nearfield acoustic holography. Many studies have also examined multiple discretized dictionaries. Nonetheless, to overcome the need for a dictionary,

this chapter focuses on formulating the DOA estimation of a wideband source using a multiband continuous angular spectrum. It is demonstrated that grid-free compressive beamforming can provide an accurate DOA reconstruction.

The remainder of the chapter is organized as follows. The working principle of the proposed multiband grid-free compressive beamforming method is given in Section 3.2. Specifically, Section 3.2.1 provides a brief review of the discrete signal model used in the classical beamforming and the SSR approach. Section 3.2.2 discusses the proposed multiband grid-free compressive beamforming. Section 3.2.3 addresses the necessary conditions for a solution to be determined. Section 3.2.4 examines how the proposed method can be extended for an aliasing-free DOA estimation. Section 3.2.5 further extends the proposed approach to a general signal model with noise, and Section 3.3 discusses the acoustic data from measurement and simulation processed with the proposed approach to demonstrate that the method provides high-resolution aliasing-free DOA estimation in a practical situation.

3.2 Multiband Grid-Free Compressive Beamforming

This section shows that the grid-free compressive beamforming approach of Xenaki and Gerstoft [22] can be extended to a multiband formulation. This is useful because multiband formulation allows the algorithm to be extended to sources that satisfy Theorem 1 of Tang et al. [21], which enables aliasing-free estimation of these sources' DOA. Traditionally, the multiband approach is used to locate passive underwater mechanical signals such as a diver's open-circuit breathing [21], [52] or water turbulence generated by passing ship [53]. Simulations of multiband sources are also used to study the aliasing-free DOA of a wideband source using the SSR-based approach

[21]. Overall, the proposed approach generalizes the framework of grid-free compressive beamforming [22] to extend its use beyond a single-tone source.

3.2.1 Discrete Model

Consider a ULA with N sensors indexed by $n \in [0, N-1]$, with an intersensor spacing d , receiving K point sources from the far-field region such that each source can be considered to be a plane wave. For wideband processing, the received signal at each channel is further processed into L frequency subbands by discrete Fourier transformation. If the bandwidth of each subband is less than its central frequency f_ℓ , then the signal received by the n th sensor for the ℓ th bin is expressed as:

$$p_{n,\ell} = \sum_{k=1}^K s_{\ell,k} e^{j \frac{2\pi f_\ell}{c} d n t_k}, \quad (3.1)$$

where c is the acoustic propagation speed in the medium, $t_k = \sin \theta_k$, θ_k is the DOA of the k th source, and $s_{\ell,k}$ is the ℓ th subband coefficient of the k th source. Now, stacking all of the N sensors response gives:

$$\mathbf{p}_\ell = \sum_{k=1}^K s_{\ell,k} \mathbf{a}_{\ell,k}, \quad (3.2)$$

where

$$\mathbf{a}_{\ell,k} = \begin{bmatrix} e^{j \frac{2\pi f_\ell}{c} d 0 t_k} & \cdots & e^{j \frac{2\pi f_\ell}{c} d (N-1) t_k} \end{bmatrix}^T. \quad (3.3)$$

Thus, the objective of this work is to estimate the source DOAs given the array measurement $\mathbf{p}_\ell$ for $\ell = [0 \ \cdots \ L-1]^T$.

3.2.1.1 Conventional Beamforming

As discussed in Chapter 2.2, the DOA estimation problem is conventionally solved using the far-field scan vectors and the SMF algorithm. Accordingly, the scan vectors are designed by discretizing the angle space in front of the array into M search grids, i.e., $\Theta = \{\theta_1, \dots, \theta_M\}$ with $\Theta \in \left\{-\frac{\pi}{2}, +\frac{\pi}{2}\right\}$. Next, the beamforming power of each m th search grid-point in the angle space is computed as:

$$\beta(\theta_m) = \left| \frac{1}{N} \sum_{\ell=0}^{L-1} \mathbf{a}_{\ell,m}^H \mathbf{p}_\ell \right|^2. \quad (3.4)$$

Finally, the location corresponding to the highest energy is the estimated DOA of the source.

3.2.1.2 Sparse Beamforming

Similarly, as discussed in Chapter 2.4, the DOA estimation problem can be formulated using SSR, in which the angle space is also divided into M search grid points, Θ . Typically, a $N \times M$ dictionary matrix is constructed for each frequency f_ℓ , as follows:

$$\mathbf{A}_{s_\ell} = \begin{bmatrix} e^{j\frac{2\pi f_\ell}{c}d_0 t_1} & \dots & e^{j\frac{2\pi f_\ell}{c}d_0 t_M} \\ \vdots & \ddots & \vdots \\ e^{j\frac{2\pi f_\ell}{c}d(N-1)t_1} & \dots & e^{j\frac{2\pi f_\ell}{c}d(N-1)t_M} \end{bmatrix}, \quad (3.5)$$

where $\mathbf{A}_{s_\ell}$ is commonly known as an overcomplete dictionary [54], [55]. Assuming that the DOA of the K sources are subsumed by the search grid in $\mathbf{A}_{s_\ell}$, then the received pressure field can be written as:

$$\mathbf{p}_\ell = \mathbf{A}_{s_\ell} \mathbf{s}_\ell, \quad (3.6)$$

where $\mathbf{s}_\ell \in \mathbb{C}^M$ is a K sparse vector with nonzero elements for each index that contains the DOA of the k th source. Using (2.20), the sparse minimization problem is equivalent to:

$$\min_{\mathbf{s}_\ell \in \mathbb{C}^M} \|\mathbf{s}_\ell\|_1, \quad \text{subject to } \mathbf{p}_\ell = \mathbf{A}_{s_\ell} \mathbf{s}_\ell. \quad (3.7)$$

For a wideband signal with L frequency subbands, (3.7) can be modified by stacking the pressure field measurements from $\mathbf{p}_\ell$ into $\mathbf{p} = [\mathbf{p}_1^\top \cdots \mathbf{p}_L^\top]^\top \in \mathbb{C}^{LN}$ such that $\mathbf{p} = \mathbf{A}_s \mathbf{s}$, where $\mathbf{A}_s = [\mathbf{A}_{s_1}^\top \cdots \mathbf{A}_{s_L}^\top]^\top \in \mathbb{C}^{LN \times M}$ and $\mathbf{s} = \mathbf{s}_1 = \cdots = \mathbf{s}_L \in \mathbb{C}^M$ [21] and:

$$\min_{\mathbf{s} \in \mathbb{C}^M} \|\mathbf{s}\|_1, \quad \text{subject to } \mathbf{p} = \mathbf{A}_s \mathbf{s}. \quad (3.8)$$

For noisy measurement, the constraint in (3.8) becomes $\|\mathbf{p} - \mathbf{A}_s \mathbf{s}\|_2 < \epsilon$, where ϵ is the ℓ_2 -norm of the measurement noise [56]. Then, the minimization problem becomes:

$$\min_{\mathbf{s} \in \mathbb{C}^M} \|\mathbf{s}\|_1, \quad \text{subject to } \|\mathbf{p} - \mathbf{A}_s \mathbf{s}\|_2 < \epsilon. \quad (3.9)$$

Various studies have examined such multiple dictionary problems, and (3.8) is one of the strictest forms found in collaborative spectrum-sensing [57]. Other forms, such as the multichannel-image problem [58], place a less strict constraint on $\mathbf{s}_\ell$ by requiring $\sum_k h_\ell(\mathbf{s}_\ell) = 0$, where $h_\ell(\cdot)$ is some differentiable sparsity-inducing function. In its loosest form, (3.8) can be generalized as a JSM-2; that is, the signals share a common sparse structure but are measured by different dictionaries. JSM-2 problems can be solved using the orthogonal matching pursuit (OMP) approach [48]. Even though recent breakthroughs using SBL showed promising performance [49]–[51], SBL still requires

the discretization of the dictionaries. Hence, in the experimental example, the proposed method is also compared with the OMP solution of (3.8) [21].

3.2.2 Continuous Multiband Model

Here we develop a grid-free solution that models the wideband signal in a continuous space using a continuous multiband model. By extending the continuous approach of a source with a single frequency [22], a K sparse wideband signal $\mathbf{s}(t, \mathbf{f}) \in \mathbb{C}^{1 \times L}$, that has L discretized multibands with central subband frequencies $\mathbf{f} = [f_1 \ \cdots \ f_L] \in \mathbb{R}^{1 \times L}$, can be expressed as:

$$\mathbf{s}(t, \mathbf{f}) = \sum_{k=1}^K [s_{1k} \ \cdots \ s_{Lk}] \partial(t - t_k), \quad (3.10)$$

where $s_{\ell k} \in \mathbb{C}$ is the complex amplitude of the k th source at the ℓ th frequency bin, $t_k = \sin \theta_k$ is the source support on the continuous sine spectrum $\mathbb{T} = [-1, +1]$ with $t_k \subset \mathbb{T}$ being the set of K DOA sources, $\partial(t)$ is the Dirac delta function. Note that in the continuous multiband model, the assumption of $s_{ik} = s_{\ell k}$ for $i \neq \ell$ as in (3.8) is not required.

Then, the signal received at the n th sensor will be the superposition of the plane waves from all of the possible directions on the continuous sine spectrum, which is calculated as follows:

$$\mathbf{p}_n = \int_{-1}^{+1} \mathbf{s}(t, \mathbf{f}) \circ e^{j \frac{2\pi \mathbf{f}}{c} d(n-1)t} dt = \sum_{k=1}^K \begin{bmatrix} e^{-j \frac{2\pi f_1}{c} d(n-1)t_k} \\ \vdots \\ e^{-j \frac{2\pi f_L}{c} d(n-1)t_k} \end{bmatrix}^H \mathbf{S}_k, \quad (3.11)$$

where $\circ$ is the Hadamard product [28], that is the element-by-element multiplication,

and

$$\mathbf{S}_k = \text{diag} \left(\begin{bmatrix} s_{1k} \\ \vdots \\ s_{Lk} \end{bmatrix} \right) = \begin{bmatrix} s_{1k} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{Lk} \end{bmatrix},$$

is a diagonal matrix of dimension L . Note that the received n th sensor signal $\mathbf{p}_n \in \mathbb{C}^{1 \times L}$

is now a row vector that contains the L discretized multiband frequencies of the source.

With all of the N sensors included, the array measurement matrix $\mathbf{P} \in \mathbb{C}^{N \times L}$ can now be expressed as:

$$\begin{aligned} \mathbf{P} &= \mathbf{A} \mathbf{S} \\ &= \sum_{k=1}^K \begin{bmatrix} e^{j \frac{2\pi f_1}{c} d 0 t_k} & \cdots & e^{j \frac{2\pi f_L}{c} d 0 t_k} \\ \vdots & \ddots & \vdots \\ e^{j \frac{2\pi f_1}{c} d (N-1) t_k} & \cdots & e^{j \frac{2\pi f_L}{c} d (N-1) t_k} \end{bmatrix} \begin{bmatrix} s_{1k} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{Lk} \end{bmatrix} \\ &= [\mathbf{A}_1 \quad \cdots \quad \mathbf{A}_K] \begin{bmatrix} \mathbf{S}_1 \\ \vdots \\ \mathbf{S}_K \end{bmatrix}, \end{aligned} \quad (3.12)$$

where $\mathbf{A} \in \mathbb{C}^{N \times LK}$, with:

$$\mathbf{A}_k = \begin{bmatrix} e^{j \frac{2\pi f_1}{c} d 0 t_k} & \cdots & e^{j \frac{2\pi f_L}{c} d 0 t_k} \\ \vdots & \ddots & \vdots \\ e^{j \frac{2\pi f_1}{c} d (N-1) t_k} & \cdots & e^{j \frac{2\pi f_L}{c} d (N-1) t_k} \end{bmatrix}.$$

Each column vector in $\mathbf{A}_k$ can be written as $\mathbf{a}_{\ell k} = [z_{\ell k}^0 \quad \cdots \quad z_{\ell k}^{N-1}]^T \in \mathbb{C}^N$, where

$z_{\ell k} = e^{j \frac{2\pi f_\ell}{c} d t_k}$. Here, $\mathbf{A}$ can be interpreted as a linear operator (inverse Fourier

transform) that maps the continuous signal $\mathbf{s}(t, \mathbf{f})$ to the observation $\mathbf{P} \in \mathbb{C}^{N \times L}$.

In the presence of additive noise, $\mathbf{W} \in \mathbb{C}^{N \times L}$, the final sensor array measurement, can be described as:

$$\mathbf{P} = \mathbf{A}\mathbf{S} + \mathbf{W}. \quad (3.13)$$

3.2.2.1 Primal Problem

Using the following sparsity measure for the atom of the continuous complex amplitude matrix [21], [60], [61]:

$$\|\mathbf{S}\|_{\mathcal{M}} = \sum_{k=1}^K \text{Tr} \left\{ \begin{bmatrix} |s_{1k}| & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & |s_{Lk}| \end{bmatrix} \right\},$$

the DOA estimation for the continuous angular space can be solved by a sparsity-promoting minimization formulation for a L dimension continuous signal $\mathbf{s}(t, \mathbf{f})$ of:

$$\min_{\mathbf{s}} \|\mathbf{S}\|_{\mathcal{M}}, \quad \text{subject to } \mathbf{P} = \mathbf{A}\mathbf{S}. \quad (3.14)$$

Because $\mathbf{s}(t, \mathbf{f})$ in (3.10) is a continuous parameter; the primal problem in (3.14) is L infinite-dimensional and cannot be solved. The following section will demonstrate that the L infinite-dimensional problem can be dealt with by solving its dual problem.

3.2.2.2 Dual Problem

The dual problem can be formulated by constructing the Lagrangian of (3.14), as

$$L(\mathbf{S}, \mathbf{C}) = \|\mathbf{S}\|_{\mathcal{M}} + \text{Re} \left[\text{Tr} \{ \mathbf{C}^H (\mathbf{P} - \mathbf{A}\mathbf{S}) \} \right], \quad (3.15)$$

where

$$\mathbf{C} = \begin{bmatrix} c_{0,1} & \cdots & c_{0,L} \\ \vdots & \ddots & \vdots \\ c_{N-1,1} & \cdots & c_{N-1,L} \end{bmatrix} = [\mathbf{c}_1 \quad \cdots \quad \mathbf{c}_\ell \quad \cdots \quad \mathbf{c}_L] \in \mathbb{C}^{N \times L},$$

is the Lagrange multiplier, and the dual function $g(\mathbf{C})$ is the greatest lower bound of

(3.15) and thus is given by:

$$\begin{aligned} g(\mathbf{C}) &= \inf_{\mathbf{S}} L(\mathbf{S}, \mathbf{C}) \\ &= \text{Tr}\left\{\text{Re}\left[\mathbf{C}^H \mathbf{P}\right]\right\} + \inf_{\mathbf{S}} \left\{\text{Re}\left[\|\mathbf{S}\|_{\mathcal{M}}\right] - \text{Re}\left[\text{Tr}\left(\mathbf{C}^H \mathbf{A} \mathbf{S}\right)\right]\right\}. \end{aligned} \quad (3.16)$$

Note that the operation of $\text{Tr}(\text{Re}(\cdot)) = \text{Re}(\text{Tr}(\cdot))$ is interchangeable, thus $\text{Re}\left\{\text{Tr}\left[\mathbf{C}^H \mathbf{A} \mathbf{S}\right]\right\} = \text{Tr}\left\{\text{Re}\left[\mathbf{C}^H \mathbf{A} \mathbf{S}\right]\right\}$.

For the k th element, the second term of (3.16) can be interpreted as meaning that for every $\mathbf{S}_k$, $\text{Tr}\left\{\text{Re}\left[\mathbf{C}^H \mathbf{A} \mathbf{S}\right]\right\}_k = \text{Re}\left\{\text{Tr}\left[\mathbf{C}^H \mathbf{A} \mathbf{S}\right]\right\}_k = \text{Tr}\left\{\left|\mathbf{A}^H \mathbf{C}\right|_k \odot \left|\mathbf{S}_k\right| \cos \phi_k\right\}$, where ϕ_k is the element-wise angle between $\left|\mathbf{A}^H \mathbf{C}\right|_k$ and $\left|\mathbf{S}\right|_k$. Then, the following equation can be formulated:

$$\begin{aligned} \text{Tr}\left[\left|\mathbf{S}_k\right| - \text{Re}\left[\text{Tr}\left(\mathbf{C}^H \mathbf{A} \mathbf{S}\right)\right]\right]_k &= \text{Tr}\left\{\left[\mathbf{I} - \left|\mathbf{A}^H \mathbf{C}\right| \odot \cos \phi\right] \cdot \left|\mathbf{S}_k\right|\right\} \\ &\geq \text{Tr}\left\{\left[\mathbf{I} - \left|\mathbf{A}^H \mathbf{C}\right|\right] \cdot \left|\mathbf{S}_k\right|\right\} \\ &\geq \sum_{\ell=1}^L \left\{\left[1 - \left|\mathbf{a}_{\ell k}^H \mathbf{c}_{\ell}\right|\right] \cdot \left|s_{\ell k}\right|\right\}, \end{aligned} \quad (3.17)$$

where $\mathbf{I}$ is the identity matrix of size L . Next, by using the following inequality:

$$\sum_i \inf x_i \leq \inf \sum_i x_i, \quad (3.18)$$

and substituting (3.17) into (3.16), the lower bound of (3.16)'s second term is obtained, as represented by:

$$\inf_{\mathbf{S}} \sum_{\ell=1}^L \left\{\left[1 - \left|\mathbf{a}_{\ell k}^H \mathbf{c}_{\ell}\right|\right] \cdot \left|s_{\ell k}\right|\right\} \geq \sum_{\ell=1}^L \inf \left\{\left[1 - \left|\mathbf{a}_{\ell k}^H \mathbf{c}_{\ell}\right|\right] \cdot \left|s_{\ell k}\right|\right\}.$$

Note that the lower bound is nonnegative, and infimum is attained at zero when each ℓ th element is $\left|\mathbf{a}_{\ell k}^H \mathbf{c}_{\ell}\right| \leq 1$ and $-\infty$ otherwise. Hence, the dual function is given by:

$$g(\mathbf{C}) = \begin{cases} \operatorname{Re}\{\operatorname{Tr}[\mathbf{C}^H \mathbf{P}]\} & |\mathbf{a}_{\ell k}^H \mathbf{c}_\ell| \leq 1 \\ -\infty & \text{otherwise.} \end{cases} \quad (3.19)$$

However, this approach requires LK constraints, and because it is not in a standard form; it cannot be efficiently solved by any known solver. In the following, it is demonstrated that by using two propositions, the constraints in (3.19) can be rewritten in a standard form with a concise constraint.

Each k th source has $\ell = 1:L$ frequency components, which are captured in the diagonal elements of the $|\mathbf{A}_k^H \mathbf{C}|$ matrix for the k th source. Furthermore, the constraint can be relaxed by bounding the diagonal entries of $|\mathbf{A}_k^H \mathbf{C}|^2$ to be less than or equal to one. This allows the relaxed problem to be solvable using the semidefinite programming (SDP) formulation. This approach can be summarized by the following two propositions:

Proposition 3.1: Bounding the diagonal elements of $|\mathbf{A}_k^H \mathbf{C}|^2$ to be less than or equal to one is sufficient to bound the diagonal elements of $|\mathbf{A}_k^H \mathbf{C}|$ to be less than or equal to one.

Proof: Consider the following matrix $|\mathbf{A}_k^H \mathbf{C}|^2 = \mathbf{A}_k^H \mathbf{C} \mathbf{C}^H \mathbf{A}_k \in \mathbb{C}^{L \times L}$. Note that $\mathbf{C} \mathbf{C}^H = \mathbf{c}_1 \mathbf{c}_1^H + \dots + \mathbf{c}_L \mathbf{c}_L^H = \mathbf{Q} \in \mathbb{C}^{N \times N}$ is the Gram matrix associated with the diagonal elements of $\mathbf{A}_k^H \mathbf{C} \mathbf{C}^H \mathbf{A}_k$.

Remark 3.1: By inspection, each basis of $\mathbf{Q}$ is a positive semidefinite matrix satisfying $\mathbf{a}_{ik}^H \mathbf{c}_\ell \mathbf{c}_\ell^H \mathbf{a}_{ik} \geq 0$ for all nonzero vectors $\mathbf{a}_{ik}$.

Remark 3.2: Furthermore, without loss of generality, let $z = e^{j \frac{2\pi f}{c} dt}$ and consider the

following vector, $\mathbf{a}(z) = [1 \quad \cdots \quad z^{N-1}]^T \in \mathbb{C}^N$. Then, $\mathbf{H}(z) = \mathbf{a}^H(z) \mathbf{C} \in \mathbb{C}^{1 \times L}$ and $R(z) = \mathbf{H}(z) \mathbf{H}^H(z) = \mathbf{a}^H(z) \mathbf{C} \mathbf{C}^H \mathbf{a}(z) \in \mathbb{C}$. This represents the type of polynomial in each diagonal entry of $\mathbf{A}_k^H \mathbf{C} \mathbf{C}^H \mathbf{A}_k$. i.e.:

$$\begin{aligned} R(z_{1k}) &= [\mathbf{A}_k^H \mathbf{C} \mathbf{C}^H \mathbf{A}_k]_{11} = \sum_{\ell=1}^L \mathbf{a}^H(z_{1k}) \mathbf{c}_\ell \mathbf{c}_\ell^H \mathbf{a}(z_{1k}), \\ &\vdots \\ R(z_{Lk}) &= [\mathbf{A}_k^H \mathbf{C} \mathbf{C}^H \mathbf{A}_k]_{LL} = \sum_{\ell=1}^L \mathbf{a}^H(z_{Lk}) \mathbf{c}_\ell \mathbf{c}_\ell^H \mathbf{a}(z_{Lk}), \end{aligned} \quad (3.20)$$

where $[\mathbf{A}_k^H \mathbf{C} \mathbf{C}^H \mathbf{A}_k]_{\ell\ell}$ is the ℓ th diagonal entry of $\mathbf{A}_k^H \mathbf{C} \mathbf{C}^H \mathbf{A}_k$.

Because $R(z)$ is a generalized representation of each diagonal element of $\mathbf{A}_k^H \mathbf{C} \mathbf{C}^H \mathbf{A}_k$, and $\mathbf{A}_k^H \mathbf{C} \mathbf{C}^H \mathbf{A}_k$ is related to each diagonal element of $\mathbf{A}_k^H \mathbf{C}$ by squaring, $R(z)$ is also related to each diagonal element of $\mathbf{A}_k^H \mathbf{C}$ by a square relation. Bounding $R(z_{ik})$ to be greater or equal to an arbitrary constant γ is equivalent to the following inequality:

$$\mathbf{a}_{ik}^H \mathbf{c}_i \mathbf{c}_i^H \mathbf{a}_{ik} \leq \mathbf{a}_{ik}^H [\mathbf{c}_1 \mathbf{c}_1^H + \cdots + \mathbf{c}_L \mathbf{c}_L^H]_{N \times N} \mathbf{a}_{ik} \leq \gamma, \quad \forall i \in [1, L]. \quad (3.21)$$

Because of the observation of Remark 3.1, then:

$$\mathbf{a}_{ik}^H \mathbf{c}_\ell \mathbf{c}_\ell^H \mathbf{a}_{ik} \geq 0, \quad \forall i, \ell \in [1, L], \quad (3.22)$$

and in the summation operation, the right side is larger than the left side. Further, bounding

$$R(z) \leq \gamma, \quad (3.23)$$

implies that:

$$\begin{aligned}\mathbf{a}_{ik}^H \mathbf{c}_i \mathbf{c}_i^H \mathbf{a}_{ik} &\leq \gamma, \\ \left| \mathbf{a}_{ik}^H \mathbf{c}_i \right|^2 &\leq \gamma, \\ \left| \mathbf{a}_{ik}^H \mathbf{c}_i \right| &\leq \sqrt{\gamma}.\end{aligned}$$

Hence, by setting $\gamma = 1$, bounding $R(z) \leq 1$ is equivalent to bounding each diagonal entry of $\mathbf{A}_k^H \mathbf{C} \mathbf{C}^H \mathbf{A}_k$, which is equivalent to bounding each absolute diagonal entry of $\mathbf{A}_k^H \mathbf{C}$, that is:

$$R(z) \leq 1 \rightarrow \left| \mathbf{a}_{ik}^H \mathbf{c}_i \right| \leq 1, \quad \forall i \in [1, L].$$

Proposition 3.2: The constraint in (3.19) can be represented as a linear matrix inequality constraint.

Proof: Proposition 3.1 established that bounding $R(z)$ is equivalent to bounding $\left| \mathbf{a}_{ik}^H \mathbf{c}_i \right|$. Further, a bounded $\left| R(z) \right|$ can be expressed as:

$$\begin{aligned}\left| R(z) \right| &\leq \left| R_B(z) \right|, \\ \left| \mathbf{H}(z) \right|^2 &\leq \left| \mathbf{B}(z) \right|^2,\end{aligned}$$

where $\left| \mathbf{B}(z) \right|^2$ is another polynomial that bounds $\left| \mathbf{H}(z) \right|^2$. Using Lemma 4.25 of [59] we obtain:

$$\left| R(z) \right| \leq \left| R_B(z) \right|,$$

which implies that:

$$\mathcal{Q} \succeq \mathcal{Q}_H,$$

where $\mathcal{Q} = \mathbf{b} \mathbf{b}^H$, and $\mathcal{Q}_H = \mathbf{C} \mathbf{C}^H$, which can be represented using the Schur complements as:

$$\begin{bmatrix} \mathbf{Q}_{N \times N} & \mathbf{C}_{N \times L} \\ \mathbf{C}_{L \times N}^H & \mathbf{I}_{L \times L} \end{bmatrix} \succeq 0,$$

$$\sum_{n=1}^{N-j} \mathbf{Q}_{n,n+j} = \begin{cases} 1 & j = 0 \\ 0 & j = 1, \dots, N-1. \end{cases}$$

Combining Proposition 3.1 and 3.2 with (3.19) gives the following conclusion that the dual problem can be solved with:

$$\begin{aligned} & \max_{\mathbf{C}, \mathbf{Q}} \operatorname{Re} \left\{ \operatorname{Tr} \left[\mathbf{C}^H \mathbf{P} \right] \right\}, \\ & \text{subject to } \begin{bmatrix} \mathbf{Q}_{N \times N} & \mathbf{C}_{N \times L} \\ \mathbf{C}_{L \times N}^H & \mathbf{I}_{L \times L} \end{bmatrix} \succeq 0, \\ & \sum_{n=1}^{N-j} \mathbf{Q}_{n,n+j} = \begin{cases} 1 & j = 0 \\ 0 & j = 1, \dots, N-1. \end{cases} \end{aligned} \quad (3.24)$$

Note that (3.24) is in the standard SDP form, which can be solved with interior-point algorithms [41], [59]. These interior-point algorithms are iterative, and each iteration has the computational complexity of $O(N^2 L^2)$.

3.2.2.3 Support Detection using the Dual Polynomial

Because the primal problem has linear equality constraints, it is convex, and strong duality holds. Hence, the maximum of the dual problem is equivalent to the minimum of the primal problem. The optimal dual solution nevertheless provides an optimal dual but not the primal solution. Using (3.23) and noting that the infimum in (3.16) is attained if and only if all of the diagonal entries of $\left| \left(\mathbf{A}_k^H \mathbf{C} \right)_{\ell\ell} \right| = 1$, the primal solution can be estimated by locating z when the amplitude of the dual polynomial is one, as follows:

$$\left| \mathbf{a}_{\ell k}^H \mathbf{c}_\ell \right| \leq 1, \quad \forall t \in \mathbb{T} \rightarrow \begin{cases} \left| \mathbf{a}_{\ell k}^H \mathbf{c}_\ell \right| = 1 & \forall \ell, t_k \in \hat{T}, \\ \left| \mathbf{a}_{\ell k}^H \mathbf{c}_\ell \right| < 1 & \forall \ell, t_k \in \mathbb{T} \setminus \hat{T}. \end{cases}$$

This estimation can be performed in two steps: first, by locating the roots of the nonnegative polynomials that lie on the unit circle $|z| = 1$, as follows:

$$P(z) = 1 - R(z) = 1 - \sum_{n=-(N-1)}^{N-1} r_n z^n, \quad (3.25)$$

where $R(z) = \mathbf{H}(z) \mathbf{H}^H(z) = |\mathbf{H}(z)|^2$ with coefficients $r_n = \sum_{\ell=1}^L \sum_{i=0}^{N-1} [\hat{\mathbf{c}}_\ell]_i [\hat{\mathbf{c}}_\ell^*]_{i+n}$, $n \geq 0$

and $r_{-n} = r_n^*$ is the autocorrelation of $\mathbf{C}$. Notice that the degree of the polynomial $P(z)$ is $2(N-1)$, as:

$$P_+(z) = z^{N-1} P(z) = (1 - r_0) z^{N-1} - \sum_{n=-(N-1), n \neq 0}^{N-1} r_n z^{N+n-1}, \quad (3.26)$$

which has only positive powers of the variable z and the same roots as $P(z)$. Second, the source support is recovered by locating the roots of $P_+(z)$ on the unit circle and finding the intersection over the known $1:L$ frequencies, as follows:

$$t_k = \frac{c \arg P_+(z_{1k})}{2\pi f_1 d} \cap \frac{c \arg P_+(z_{2k})}{2\pi f_2 d} \cap \dots \cap \frac{c \arg P_+(z_{Lk})}{2\pi f_L d}, \quad (3.27)$$

where $\cap$ is the intersection operation.

3.2.2.4 Reconstruction of the Primal Solution $\mathbf{S}$

Once the supports are recovered by locating the intersecting roots in (3.27), the source amplitude $\mathbf{S} \in \mathbb{C}^{KL \times L}$ can be determined by solving the following inverse least-squares problem:

$$\mathbf{S} = \mathbf{A}_T^+ \mathbf{P}, \quad (3.28)$$

where $(\cdot)^+$ is the pseudo-inverse of $\mathbf{A}_T = [\mathbf{A}_{T_1} \cdots \mathbf{A}_{T_K}] \in \mathbb{C}^{N \times LK}$ with column

$\mathbf{a}_\ell(t_k) = e^{j \frac{2\pi f_\ell}{c} d [0 \cdots N-1]^T t_k}$ and $\mathbf{P} \in \mathbb{C}^{N \times L}$ is the observation matrix.

Figure 3.1 shows the DOA estimation with the proposed approach. The simulation uses an array of $N = 16$ sensors with an intersensor spacing $d = 0.06$ m and two sources submerged in water with an acoustic propagation speed $c = 1500$ m/s. The two sources are multiband sources with two central frequencies of 7 and 12 kHz, $[42.067^\circ, 15.962^\circ]$ from the broadside with amplitude $[1, 0.5]$ ($\mathbf{S}_{42.067^\circ} = \text{diag}([0.25 \ 0.75])$ and $\mathbf{S}_{15.962^\circ} = \text{diag}([0.35 \ 0.15])$).

Figure 3.1a shows the support T that is estimated by the angle of the roots, z_{ℓ_k} , of $P(z_{\ell_k})$ where $z_{\ell_k} = 1$. Figure 3.1b compares the proposed approach with the conventional SMF approach at their respective frequencies and the wideband approach from (3.4). It can be seen that the proposed approach produces extremely accurate localization compared to the conventional SMF approach.

Figure 3.1: Support detection using the dual polynomial. A ULA is used with $N = 16$ sensors, an intersensor spacing $d = 0.06$ m, a speed of acoustic propagation $c = 1500$ m/s with $K = 2$ sources at 42.067° and 15.962° with amplitude $|s_k| = [1, 0.5]$, and two frequency bins at $\mathbf{f} = [07, 12]$ kHz. (a) The support is estimated by the intersection angle of the roots z_{t_k} of $P(z_{t_k})$, and (b) shows a reconstruction with a multiband grid-free (*) and spatial matched filter applied at each respective frequency.

3.2.3 Maximum Resolvable DOA

As with any algorithm, there is a resolution limit. In this case, the maximum number of roots of $P_+(z)$ determines the maximum number of resolvable DOAs. Because the coefficients of the polynomial $P_+(z)$ are conjugate symmetric around z^{N-1} , the roots

appear in pairs. In essence, one root will be inside the unit circle and another outside, at the same angular direction of z . Hence, there are at most $N - 1$ possible pairs of roots on the unit circle. Also, because the roots z are a combination of K DOAs and L frequencies, the necessary condition for the dual polynomial to satisfy condition $|\mathbf{a}_{\ell k}^H \mathbf{c}_\ell| < 1$ is that the number of sources and frequencies should not exceed [22], [62], as expressed by:

$$K_{\max} L_{\max} = \left\lfloor \frac{N-1}{2} \right\rfloor, \quad (3.29)$$

where $\lfloor \cdot \rfloor$ is the floor operator. Note that (3.29) inherently implies that if N is small, it will not be able to resolve sources that have many frequency bins.

3.2.4 Aliasing Suppression

Typically, linear arrays are designed with fixed intersensor spacing, which determines the spatial sampling frequency and sets an upper limit on the frequency of the source that is discriminable by the array. However, in many situations, a passive wideband source may have a frequency range beyond an array's spatial aliasing limit. In this situation, it would be advantageous to have an approach to extend the array's operating frequency without necessitating the design of a new array [21], [49], [51].

To identify a solution to this problem, it is necessary to understand the other two parameters that affect the spatial aliasing limit of the array: the acoustic propagation speed in the medium and the frequency of the acoustic source. Because the acoustic propagation speed is also fixed, as it results from the physical property of the medium, the remaining property that could be used is the source's frequencies. In this regard, one observation may show that spatial aliasing for the various source frequencies may not

completely overlap; therefore, suppression of aliasing may be achievable by combining the solutions for various frequencies emitted by a wideband source.

Tang et al. [21] proved that if two corresponding frequencies of a source (for example, f_i and f_j) satisfy the following:

$$0 < |f_i - f_j| < \frac{c}{2d}, \quad (3.30)$$

then the intersection of the support exclusively contains the source DOA. The inequality can be regarded as another relaxed constraint that avoids the region in which different frequencies of spatial aliasing overlap. The relaxed constraint guarantees that only the support of the true source angle will intersect, resulting in an aliased-free reconstruction. By inspection, the solution from (3.27) also allows localization of sources beyond the array spatial aliasing frequency if the maximum difference of the frequency for $\mathbf{f}$ in (3.10) satisfies (3.30).

Figure 3.2 shows the DOA estimation when (3.30) is satisfied. This simulation uses an array of $N = 16$ sensors with an intersensor spacing $d = 0.06$ m and two sources submerged in water with an acoustic propagation speed $c = 1500$ m/s. The acoustic propagation speed is determined by the depth and temperature of the water [22]. The two sources are multiband sources with three central frequencies of 25, 30 and 35 kHz, impinging with an angle of $[42.067^\circ, 15.962^\circ]$ from the broadside with amplitude $[1, 0.5]$, $\mathbf{X}_{42.067^\circ} = \text{diag}([0.15, 0.75, 0.1])$ and $\mathbf{X}_{15.962^\circ} = \text{diag}([0.12, 0.13, 0.25])$.

Figure 3.2a shows that the support T , which is estimated by the angle of the roots, $z_{\ell k}$, of $P(z_{\ell k})$ where $z_{\ell k} = 1$. Figure 3.2b compares the proposed approach with the conventional SMF approach at their respective frequencies and the wideband approach from (3.4). It can be seen that the proposed approach produces extremely accurate

localization, whereas the conventional SMF approach results in multiple false peaks due to spatial aliasing.

(a)

(b)

Figure 3.2: Support detection using the dual polynomial. A ULA is used with $N = 16$ sensors, an intersensor spacing $d = 0.06$ m, a speed of acoustic propagation $c = 1500$ m/s with $K = 2$ sources at 42.067° and 15.962° with amplitude $|s_k| = [1, 0.5]$, and three frequency bins at $\mathbf{f} = [25, 30, 35]$ kHz. (a) The support is estimated by the intersection angle of the roots $z_{\ell k}$ of $P(z_{\ell k})$, and (b) reconstruction with a multiband grid-free (*) and spatial matched filter at each respective frequency.

3.2.5 Reconstruction with Noise

In general, a noiseless signal model is not representative of the signal captured in a practical situation. Assuming that the measurements are contaminated with additive noise as in (3.13) such that $\|\mathbf{W}\|_F \leq \epsilon$, the matrix atomic norm minimization problem can be reformulated as:

$$\min_{\mathbf{S}} \|\mathbf{S}\|_{\mathcal{M}}, \quad \text{subject to } \|\mathbf{P} - \mathbf{A}\mathbf{S}\| \leq \epsilon. \quad (3.31)$$

Similar to (3.14), this primal is an infinite-dimensional problem that can be solved using its dual, which is formulated by constructing the following Lagrangian:

$$L_w(\mathbf{S}, \mathbf{C}, \lambda) = \|\mathbf{S}\|_{\mathcal{M}} + \text{Re} \left\{ \text{Tr} \left[\mathbf{C}^H (\mathbf{P} - \mathbf{A}\mathbf{S} - \mathbf{W}) \right] \right\} + \text{Re} \left[\lambda \left(\|\mathbf{W}\|_F^2 - \epsilon^2 \right) \right], \quad (3.32)$$

where $\mathbf{C} \in \mathbb{C}^{N \times L}$, and $\lambda \in \mathbb{C}$ is the dual variable. The dual function $g_w(\mathbf{W}, \mathbf{C}, \lambda)$ is the greatest lower bound of the infimum of (3.32), which can be expressed as:

$$\begin{aligned} g_w(\mathbf{W}, \mathbf{C}, \lambda) &= \inf_{\mathbf{S}} L_w(\mathbf{S}, \mathbf{C}, \lambda) \\ &= \text{Re} \left[\text{Tr} (\mathbf{C}^H \mathbf{Y} - \mathbf{C}^H \mathbf{W}) \right] + \text{Re} \left\{ \lambda \left[\text{Tr} (\mathbf{W} \mathbf{W}^H - \epsilon^2) \right] \right\} \\ &\quad + \inf_{\mathbf{S}} \text{Re} \left[\|\mathbf{S}\|_{\mathcal{M}} - \text{Tr} (\mathbf{C}^H \mathbf{A}\mathbf{S}) \right]. \end{aligned} \quad (3.33)$$

Minimizing the dual over the unknown noise $\mathbf{W} \in \mathbb{C}^{N \times L}$ gives:

$$\frac{\partial L_w(\mathbf{S}, \mathbf{C}, \lambda)}{\partial \mathbf{W}} = -\mathbf{C} + 2\lambda \mathbf{W} = 0, \quad (3.34)$$

and thereby leads to the optimal noise matrix $\mathbf{W}_{opt} = \frac{\mathbf{C}}{2\lambda}$. Next, evaluating the dual at

$\mathbf{W}_{opt}$ and maximizing it over λ gives:

$$\left. \frac{\partial L_w(\mathbf{S}, \mathbf{C}, \lambda)}{\partial \mathbf{W}} \right|_{\mathbf{W}_{opt}} = \frac{1}{4\lambda^2} \sum_{\ell=1}^L \sum_{n=0}^{N-1} c_{\ell n} c_{\ell n}^* - \epsilon^2 = 0, \quad (3.35)$$

where the optimal value for the dual is $\lambda_{opt} = \frac{1}{2\epsilon} \|\mathbf{C}\|_F$. The dual function at the optimal

$\mathbf{W}_{opt}$ and λ_{opt} affords:

$$g(\mathbf{C})|_{\mathbf{W}_{opt}, \lambda_{opt}} = \text{Re} \left[\text{Tr}(\mathbf{C}^H \mathbf{P}) \right] - \epsilon \|\mathbf{C}\|_F + \inf_{\mathbf{S}} \text{Re} \left[\|\mathbf{S}\|_{\mathcal{M}} - \text{Tr}(\mathbf{C}^H \mathbf{A} \mathbf{S}) \right]. \quad (3.36)$$

Finally, maximizing the dual function of (3.36) over the dual variable $\mathbf{C} \in \mathbb{C}^{N \times L}$ generates:

$$\max_{\mathbf{C}, \mathbf{Q}} \text{Re} \left\{ \text{Tr}[\mathbf{C}^H \mathbf{P}] \right\} - \epsilon \|\mathbf{C}\|_F, \quad \text{subject to } \max_{\ell, k} \left| \mathbf{a}_{\ell k}^H \mathbf{c}_{\ell} \right| \leq 1. \quad (3.37)$$

Following (3.24), this is equivalent to the SDP problem of:

$$\begin{aligned} & \max_{\mathbf{C}, \mathbf{Q}} \text{Re} \left\{ \text{Tr}[\mathbf{C}^H \mathbf{P}] \right\} - \epsilon \|\mathbf{C}\|_F, \\ & \text{subject to } \begin{bmatrix} \mathbf{Q}_{N \times N} & \mathbf{C}_{N \times L} \\ \mathbf{C}_{L \times N}^H & \mathbf{I}_{L \times L} \end{bmatrix} \succeq 0, \\ & \sum_{n=1}^{N-j} \mathbf{Q}_{n, n+j} = \begin{cases} 1 & j = 0 \\ 0 & j = 1, \dots, N-1, \end{cases} \end{aligned} \quad (3.38)$$

by which the dual coefficient $\mathbf{C} \in \mathbb{C}^{N \times L}$ can be efficiently obtained. Similar to (3.24), the DOAs of the existing source can be determined by first locating all of the points where the dual polynomials have unit amplitudes, by following the process in Section 3.2.2.3.

Figure 3.3 shows the DOA estimation using the reconstruction incorporating a noise approach. This simulation uses an array of $N = 16$ sensors with an intersensor spacing $d = 0.06$ m and two sources submerged in water with an acoustic propagation speed $c = 1500$ m/s. The two sources are impinging the array at $[42.067^\circ, 15.962^\circ]$ from the broadside with amplitude $[1, 0.5]$ at a signal-to-noise ratio (SNR) of 0 dB. Here, the

SNR is defined as $SNR = 10 \log_{10} \left| \mathbf{p}_{rms}^2 / \sigma^2 \right|$, with $\mathbf{p}_{rms} \in \mathbb{R}^N$ being the root-mean-square of the time-domain recorded array measurement and $\sigma^2 \in \mathbb{R}^N$ being the corresponding variance of the additive white Gaussian noise at the N channel.

Figure 3.3a illustrates the non-aliased case in which the signal is a multiband source with two central frequencies of 7 and 12 kHz. Figure 3.3a shows that both the proposed approach and the conventional SMF can arrive at the DOA estimate without ambiguity. Nonetheless, the proposed approach has a norm error of 0.6322, which is attributable to noise. Figure 3.3b illustrates the aliased case in which the signal is a multiband source with three central frequencies of 25, 30, and 35 kHz. Figure 3.3b shows that the proposed method arrives at an estimate close to the true DOA, with a norm error of 0.0675. The conventional SMF, however, fails to provide a meaningful DOA estimation because the SMF suffers from spatial aliasing that results in multiple ghost sources, which means that conventional SMF cannot be used for DOA estimation of the true source.

(a)

(b)

Figure 3.3: DOA estimation using a ULA with $N = 16$ sensors, an intersensor spacing $d = 0.06$ m and a speed of acoustic propagation $c = 1500$ m/s with $K = 2$ sources at 42.067° and 15.962° with amplitude $|s_k| = [1, 0.5]$: (a) and two frequency bins at $\mathbf{f} = [07, 12]$ kHz, (b) and three frequency bins at $\mathbf{f} = [25, 30, 35]$ kHz. Both are reconstructed with a multiband grid-free (*) and spatial matched filter at each respective frequency, and the SNR is 0 dB.

3.3 Experimental Results

To demonstrate that the derived multiband grid-free approach is also applicable to a general wideband source, here we describe the measurement of water-turbulence from a pressurized water hose, where the data were collected in a water tank using acoustic sensors recorded with at a sampling rate of $F_s = 500$ kHz. The water jet was placed at a depth of 4.572 m, 0.2032 m from the sensor. Figure 3.4 depicts the spectrogram of the recording, which has an extremely large bandwidth covering up to 150 kHz.

Figure 3.4: Spectrogram of the data characterizing water-turbulence generated by water hose in a water tank at a depth of 4.572 m and recorded using an acoustic sensor at 0.2032 m with a sampling frequency $F_s = 500$ kHz.

In this example, the recorded water turbulence is processed using the acoustic propagation model in free-field water to create the effect of different angles impinging an array. In particular, the acoustic propagation models an array with 32 sensors, and the water-turbulence recording is considered to be impinging the array broadside, at an angle of 52° . The signal is then Fourier-transformed with 2^9 samples, and frequencies of interest are selected to form the $\mathbf{P}$ matrix. Because this recording is from an experiment, the noise level and the SNR are unavailable. Here, it is proposed to use

1.0 times the condition number of $\mathbf{P}$ as the estimate of ϵ [60].

Figure 3.5 and Figure 3.6 depict the DOA estimation given by the SMF, the OMP [21], and the proposed method. The DOA estimation at each frequency bin using the grid-free approach [22] is also included for comparison. For both the SMF and OMP, data are post-processed on the discrete DOA grid $[-90^\circ : 0.1 : +90^\circ]$.

For Figure 3.5, frequencies from 3 to 12 kHz are used to illustrate the performance at standard array-operating conditions ($f_{\text{aliased}} = 12.5$ kHz). It is observed that the SMF arrives at an estimate of $\hat{\theta}_{\text{SMF}} = 52.3^\circ$, closely followed by the proposed method's estimate of $\hat{\theta} = 52.511^\circ$. Last, the OMP with a sparsity level of 5 gives an estimate of $\hat{\theta}_{\text{OMP}} = 52.8^\circ$. Evidently, under this comparison, the SMF is superior at non-aliased frequencies. It can also be seen that the grid-free approach [22] reveals many estimated sources congregating around the angle of 52° . This result suggests that the approach from Xenaki and Gerstoft [22] may be applicable to wideband processing by judiciously combining the estimates.

For Figure 3.6, frequencies from 26 to 35 kHz are used to study the performance at the spatial-aliased condition that satisfies (3.30). Observe that the SMF gives the closest estimate of the true DOA at $\hat{\theta}_{\text{SMF}} = 51.3^\circ$ and multiple aliased angles. The proposed method performs better, giving only $\hat{\theta} = 51.266^\circ$, without ambiguity. Next, the OMP, with a sparsity level of 5, gives at $\hat{\theta}_{\text{OMP}} = -4.8^\circ$.

Notably, the estimation precision of $\hat{\theta}_{\text{OMP}}$ can be increased with the use of more frequency bins; poorer performance is due to the small number of dictionaries being used [21]. Evidently, this shows that the proposed method performs better for sources at the aliased frequency range, arriving at a non-ambiguous DOA estimation.

Figure 3.5: DOA estimation using a ULA with $N = 32$ sensors, an intersensor spacing $d = 0.06$ m, a speed of acoustic propagation $c = 1500$ m/s, $K = 1$ sources at 52° , and frequency $\mathbf{f} = [03 : 12]$ kHz. Reconstruction was performed by application of a multiband grid-free (*) and spatial matched filter at each respective frequency, and ϵ is estimated to be the condition number of $\mathbf{P}$.

Figure 3.6: DOA estimation using a ULA with $N = 32$ sensors, an intersensor spacing $d = 0.06$ m, a speed of acoustic propagation $c = 1500$ m/s, $K = 1$ sources at 52° , and frequency $\mathbf{f} = [26 : 35]$ kHz.

Reconstruction was performed with a multiband grid-free (*) and spatial matched filter at each respective frequency, and ϵ is estimated to be the condition number of $\mathbf{P}$.

Finally, it can be seen that the approach from Xenaki and Gerstoft [22] is no longer successful in estimating the angle around 52° . This may be attributable to the fact that the frequency bins are individually evaluated, which means that this approach suffers from the same spatial aliasing problem faced by the SMF. Overall, the result suggests that the single-frequency grid-free approach [22] is inadequate for DOA estimation of wideband sources with frequency content beyond the array's spatial aliasing range.

3.4 Conclusions

This section addresses the problems of multiband grid-free compressive beamforming. It demonstrates that with sufficient sensors, the DOA estimation problem can be formulated as a sparsity-promoting optimization program that can be solved in the dual domain with SDP. The approach consists of four steps: (1) construct the dual as an SDP problem by first retaining the frequency of interest, $\mathbf{f}$, in the frequency domain, $\mathbf{P}$; (2) estimate the noise level with the condition number of $\mathbf{P}$; (3) estimate the DOA by solving the dual and determining the intersection of the support of $\mathbf{f}$; and (4) reconstruct the source amplitude by solving an inverse least-squares problem.

It is further demonstrated that the formulation allows an aliased-free reconstruction if the absolute difference for the frequency $\mathbf{f}$ is less than $c/(2d)$. The numerical simulations and test data confirm that the proposed technique can be applied to estimate the DOA of a wideband signal, even if the source has frequency content beyond the spatial aliasing frequency of the array. Overall, this work advances grid-free compressive beamforming technique beyond a single frequency and improves aliasing-free DOA estimation using grid-free compressive beamforming.

Chapter 4 Spatial Aliasing in a Linear Array with a Nearfield Source

Array processing algorithms are designed based on a wavefront model of the impinging source. In Chapter 3, a grid-free algorithm for the DOA estimation of a far-field source impinging onto the array is derived that uses an existing theorem for aliasing-free wideband DOA estimation. However, sound sources may also impinge from the nearfield of the array. Consequently, the wavefront of a nearfield source is spherical when measured from the array, and application of algorithms based on the far-field model will result in a significant amount of error in this context [63]. Accordingly, this chapter investigates the nature of the spatial aliasing problem for a source in the nearfield of the array, and conditions are derived that allow aliasing-free localization of a wideband source with frequencies beyond the spatial aliasing limit of a standard array, i.e., $f_{\text{aliased}} = c/(2d)$.

4.1 Introduction

Determining the spatial aliasing frequency f_{aliased} of an array is challenging when the source is in the nearfield. Notably, it was concluded by Abhayapala et al. [64] that the received signal from a point source in the nearfield is not bandlimited in the spatial-frequency spectrum. Hence, spatial sampling of a ULA with elements spaced a half-wavelength apart may still introduce undesirable spatial aliasing effects. In recent years, a great deal of research has been conducted on enabling aliasing-free localization via physical changes in array design, involving the development of methods such as limiting

the steering angle [65], using sensors with large membranes, designing the array with $\lambda/4$ spacing [66], and using directional sensors [67] or irregular arrays [68]. However, few have considered developing aliasing-free localization via measurement and signal-processing techniques. One such exploration was performed by Cigada et al. [69], who used a moving microphone array to reduce spatial aliasing during beamforming and demonstrated that beamforming the measurements of a rotating spiral array reduced the spatial aliasing effect.

From Chapter 3, it is clear that many have researched aliasing-free wideband DOA estimation, for which Tang et al. [21] derived a theorem that guarantees the DOA estimation to be aliasing-free if the far-field source has at least two frequencies with an absolute difference of less than $c/(2d)$. Furthermore, Tang et al. [21] also proposed a wideband SSR approach that uses an overcomplete dictionary, which enables the DOA estimation of a source beyond the spatial aliasing frequency-limit of an array. Nannuru et al. [49], [50] recently proposed the sparse Bayesian learning (SBL) and beamforming framework, which iteratively updates itself to combine the observations from all dictionaries, obtaining the DOA estimates of far-field sources. However, these studies are all focused on far-field sources.

In this chapter, a rigorous ambiguity analysis is performed to achieve aliasing-free localization of a nearfield source via signal processing. A theorem is derived from the analysis that enables aliasing-free localization using a ULA and shows that some properties can be exploited via a signal-processing technique to achieve unambiguous source-localization. Effectively, this theorem suggests that localization is possible even when the nearfield source has frequency content beyond the spatial aliasing frequency of a standard array. Finally, this theorem is theoretically studied, numerically investigated, and experimentally tested.

This chapter is organized as follows. Section 4.2 discusses the nearfield data model as seen by an array and its approximation model in the array's nearfield Fresnel region. Section 4.3 begins with a recap of the conventional beamforming technique described in Section 2.2 and its implications for the spatial aliasing problem in localization. Section 4.4 continues with a discussion of the SSR model and derives the conditions for aliasing-free localization and the proposed approach in which the SSR technique is used to achieve aliasing-free localization. Section 4.5 contains numerical results from the simulations. Section 4.6 contains experimental verification, and Section 4.7 concludes the chapter.

Figure 4.1: Geometry of a source in the nearfield of the array.

4.2 Nearfield Signal Model

Consider a ULA, along the y-axis, with N sensors and an intersensor spacing d , measuring a stationary point source located in the nearfield of the array, as shown in Figure 4.1. The steering vector of the source located at $\mathbf{r}_s = (r_s, \theta_s)$ to the n th sensor can then be modeled as [23]:

$$a_n(\mathbf{r}_s, \lambda_\ell) = e^{j2\pi\tau_n(\mathbf{r}_s, \lambda_\ell)} = e^{j\frac{2\pi}{\lambda_\ell}\sqrt{r_s^2 + (nd)^2 - 2r_s \cdot nd \cdot \sin\theta_s} - r_s}, \quad (4.1)$$

where $\lambda_\ell = c/f_\ell$ is the wavelength of the source with frequency f_ℓ and c is the acoustic propagation speed in the medium, and r_s and θ_s are the range and angle parameters of the source, respectively. Note that r_s and θ_s can be easily transformed to or from a Cartesian coordinate representation of the source position. When the source is located in the Fresnel region defined by r_s , in the range $r_{\min} < r_s < r_{\max}$, with D representing the aperture of the array, $r_{\min} = 0.62(D^3/\lambda)^{1/2}$, $r_{\max} = 2D^2/\lambda$, and $\lambda = 2d$, the angle of (4.1) can be approximated with the second-order Taylor series expansion (TSE) [24], as follows:

$$2\pi\tau_n(\mathbf{r}_s, \lambda_\ell) = -\frac{2\pi}{\lambda_\ell} \cdot (nd) \cdot \sin\theta_s + \frac{2\pi}{\lambda_\ell} \frac{(nd)^2 \cos^2\theta_s}{2r_s} + O\left(\frac{d^2}{r_s^2}\right), \quad (4.2)$$

where $O(d^2/r_s^2)$ represents terms of order greater or equal to d^2/r_s^2 . The second-order Taylor series expansion approximation has been used extensively in many studies on the nearfield localization of sources using a ULA [23], [70]–[73]. However, TSE has not been extensively studied in aliasing-free nearfield localization.

Substituting (4.2) into (4.1) yields the following approximation of the nearfield steering vector:

$$e^{j2\pi\hat{\tau}(\mathbf{r}_s, \lambda_\ell)} = e^{j\left(-\frac{2\pi}{\lambda_\ell}(nd)\sin\theta_s + \frac{2\pi(nd)^2\cos^2\theta_s}{\lambda_\ell 2r_s}\right)}. \quad (4.3)$$

Then, by using (4.1), a source with frequency f_ℓ measured using an array with N sensors can be expressed in the frequency domain as:

$$\mathbf{p}_\ell = s_s \mathbf{a}_\ell(\mathbf{r}_s), \quad (4.4)$$

where s_s is the source amplitude and $\mathbf{a}_\ell(\mathbf{r}_s) = [a_1(\mathbf{r}_s, \lambda_\ell) \cdots a_N(\mathbf{r}_s, \lambda_\ell)]^T \in \mathbb{C}^N$ is the array steering vector at frequency f_ℓ . The objective of this chapter is to develop a method to localize a wideband nearfield source with frequencies beyond the spatial aliasing frequency, $f_{\text{aliased}} = c/(2d)$, based on the array measurements from a ULA.

4.3 Conventional Beamforming

Conventionally, nearfield localization is achieved using beamforming based on the array measurements. First, the search space directly in front of the array is discretized into grid points, and the spatial spectrum is computed for each grid point. Last, the spatial spectrum energy at each grid point is analyzed. Accordingly, the estimated source location is inferred from the grid point with the highest energy. If we let $\mathcal{R} = \{\mathbf{r}_1 \cdots \mathbf{r}_M\}$ denote the set that contains the discretized grid points, the SMF [74]–[76] of the array at a frequency f_ℓ can be found as:

$$\beta_\ell(\mathbf{r}_m) = \left| \frac{1}{N} \mathbf{a}_\ell^H(\mathbf{r}_m) \mathbf{p}_\ell \right|^2. \quad (4.5)$$

As discussed in Section 2.2, the SMF can also be formulated for a nearfield source. Likewise, it is subject to resolution limitation [76] and spatial aliasing [74]. Spatial

aliasing is detrimental to source localization because it will generate multiple false peaks or ghost sources [69]. Fundamentally, spatial aliasing implies that it is possible to find at least one other position $\mathbf{r}_{s'} = (r_{s'}, \theta_{s'})$, such that:

$$\tau_n(\mathbf{r}_s, \lambda_\ell) - \tau_n(\mathbf{r}_{s'}, \lambda_\ell) = k, \quad (4.6)$$

where k is an arbitrary integer. As a result, $\mathbf{a}_\ell(\mathbf{r}_s) = \mathbf{a}_\ell(\mathbf{r}_{s'})$ and this gives rise to multiple peaks or potential ghost sources in the spatial spectrum.

4.4 Localization using SSR

As discussed in Section 2.4, due to the advancement in CS, a great deal of attention has been placed in recent years on the use of SSR to improve localization resolution [33], [35], [43], [77]. The main difference between SSR and the classical approach to source localization is that the SSR model requires an overcomplete dictionary [44].

4.4.1 Problem Formulation

In essence, for each frequency f_ℓ , a dictionary $\mathbf{A}_\ell \in \mathbb{C}^{N \times M}$ is constructed from the M grid points in $\mathcal{R} = \{\mathbf{r}_1 \cdots \mathbf{r}_M\}$ as follows:

$$\mathbf{A}_\ell = [\mathbf{a}_\ell(\mathbf{r}_1) \cdots \mathbf{a}_\ell(\mathbf{r}_M)]. \quad (4.7)$$

Assuming that the true source location is subsumed in the dictionary, i.e., $\mathbf{a}_\ell(\mathbf{r}_s) \in \mathbf{A}_\ell$, then the model in (4.4) can be written in an overcomplete form as:

$$\mathbf{p}_\ell = \mathbf{A}_\ell \mathbf{s}_\ell, \quad (4.8)$$

where $\mathbf{s}_\ell \in \mathbb{C}^M$ is the sparse signal vector containing the signal amplitude for the

frequency f_ℓ .

4.4.2 Aliasing Suppression

If we consider that the source is wideband and located in the nearfield of the array, then the received signal at the array can be further processed into L frequency subbands by discrete Fourier transformation. Suppose f_ℓ is the central frequency of each subband, and the signal can be modeled with (4.8) using multiple dictionaries constructed from the L subband frequencies. On this basis, it can be shown that the use of multiple dictionaries enables the proposed method to eliminate spatial aliasing.

To facilitate the development of the multiple dictionaries, let Γ_ℓ denote the support of all of the possible source locations in the ℓ th dictionary corresponding to the central subband frequency f_ℓ , thus:

$$\Gamma_\ell := \mathcal{T}, \quad (4.9)$$

where $\mathcal{T}$ denotes the collection of row indices in $\mathbf{s}_\ell$. From (4.6), it is clear that spatial aliasing is both frequency- and location-dependent. This dependency implies that the ambiguity resulting from various subband frequencies will not completely overlap. In fact, by invoking the following theorem, the ambiguity due to spatial aliasing can be eliminated.

Theorem 1: Given the approximation in (4.3), if there are at least two dictionaries whose corresponding frequencies, say f_i and f_o , satisfy the following relation:

$$|f_i - f_o| < \frac{c}{2r_{\min}} \quad \text{and} \quad |n|d < r_{\min}, \quad (4.10)$$

then the intersection of the support related to the different dictionaries, that is:

$$\bigcap_{\ell=i,o} \Gamma_{\ell} = \mathcal{T}, \quad (4.11)$$

will only contain the source location, i.e., $|k| < 1$.

Proof: By using the second-order Taylor series expansion in (4.3), the counterexample of Tang et al. [21] can be rewritten for a nearfield source. Consider $\mathbf{r}_s$ as the source location and $\mathbf{r}_{s'} \neq \mathbf{r}_s$ as the spatial aliasing contained in both dictionaries corresponding to frequencies f_i and f_o , respectively. This counterexample for a nearfield source implies that $\{\mathbf{r}_s, \mathbf{r}_{s'}\}$ belongs to both Γ_i and Γ_o . Now, under (4.6), the following definitions can be obtained:

$$\begin{aligned} \hat{\tau}_n(\mathbf{r}_s, \lambda_i) - \hat{\tau}_n(\mathbf{r}_{s'}, \lambda_i) &= k_1, \\ \hat{\tau}_n(\mathbf{r}_s, \lambda_o) - \hat{\tau}_n(\mathbf{r}_{s'}, \lambda_o) &= k_2, \\ \hat{\tau}_n(\mathbf{r}_s, \lambda_i) - \hat{\tau}_n(\mathbf{r}_{s'}, \lambda_i) - \hat{\tau}_n(\mathbf{r}_s, \lambda_o) + \hat{\tau}_n(\mathbf{r}_{s'}, \lambda_o) &= (k_1 - k_2) \\ &= k_3, \end{aligned}$$

where k_1, k_2 , and k_3 are integers. Note that because of the port/starboard ambiguity in the ULA and the Taylor approximation, the search spaces' angles are limited to $|\theta| < 90^\circ$ and r is bounded by the Fresnel approximation requirement. Expanding k_3 with $\lambda_i = c/f_i$ and $\lambda_o = c/f_o$ gives:

$$\frac{f_i - f_o}{c} 2E \left((\sin \theta_{s'} - \sin \theta_s) + E \left(\frac{\cos^2 \theta_s}{r_s} - \frac{\cos^2 \theta_{s'}}{r_{s'}} \right) \right) = k_3, \quad (4.12)$$

where $E = nd/2$. It is clear from the conclusion of Tang et al. [21] that spatial aliasing will not occur if $|k_3| < 1$. It is thus found that, given (4.12), $|k_3| < 1$ if and only if Theorem 1 is satisfied.

Using the trigonometric identities of $\cos^2 \theta = 1 - \sin^2 \theta$ and let $X = \sin \theta_s$, $A = r_s$,

$Y = \sin \theta_s$, $B = r_s$, (4.12) can be expressed as:

$$\frac{f_i - f_o}{c} 2E \left((X - Y) + E \left(\frac{1 - Y^2}{B} - \frac{1 - X^2}{A} \right) \right) = k_3. \quad (4.13)$$

Next, using the Cauchy-Schwarz inequality, (4.13) can be expanded to:

$$|k_3| < \left| \frac{f_i - f_o}{c} 2E \right| \cdot \left| (X - Y) + E \left(\frac{1 - Y^2}{B} - \frac{1 - X^2}{A} \right) \right|. \quad (4.14)$$

Now, if we let $h(X, Y, E) = \left[(X - Y) + E \left(\frac{1 - Y^2}{B} - \frac{1 - X^2}{A} \right) \right]$, and group the same

variables together, we have:

$$\begin{aligned} h(X, Y, E) &= \left(\frac{E}{A} X^2 + X - \frac{E}{A} \right) + \left(\frac{E}{B} - Y - \frac{E}{B} Y^2 \right) \\ &= g(X, E) + u(Y, E). \end{aligned} \quad (4.15)$$

Lemma 1: $\left| \frac{E}{A} \right| < \frac{1}{2}$ and $\left| \frac{E}{B} \right| < \frac{1}{2}$ are the sufficient and necessary conditions for

$|g(X, E)| < 1$ and $|u(Y, E)| < 1$, respectively.

Proof: Let $g(X, E) = k_g$, and $\left| \frac{E}{A} \right| = \left| \frac{E}{B} \right| = |C|$. Assuming $|C| < \frac{1}{2}$, it can be shown that

$|k_g| < 1$. First, rewrite $g(X, E)$ as follows:

$$\begin{aligned} g(X, E) &= C(X^2 - 1) + X \\ &= k_g. \end{aligned} \quad (4.16)$$

Then, through the triangle inequality, the following can be arrived at:

$$\begin{aligned}
|k_g| &= |C(X^2 - 1) + X| \\
&\leq |C| \cdot |X^2 - 1| + |X| \\
&< \frac{1}{2} \cdot |X^2 - 1| + |X|,
\end{aligned}$$

in other words,

$$|k_g| < \frac{1}{2} |X^2 - 1| + |X|. \quad (4.17)$$

Now, let $f_1(X) = \frac{1}{2} \cdot |X^2 - 1| + X$, and since $X \in [\pm 1]$, therefore it follows that:

$$|k_g| < \max \mathcal{F}_1 = 1, \text{ when } \left| \frac{E}{A} \right| < \frac{1}{2}, \quad (4.18)$$

where $\mathcal{F}_1 = \{f_1(X) | X \in [\pm 1]\} = [0.5, 1]$ is the range of $f_1(X)$. Note that $\mathcal{F}_1$ is the maximum in this situation; that is, $\max \mathcal{F}_1$, as the objective is to determine the upper bound of $|k_g|$, i.e., $|k_g| < 0.5 < 1$. In other words, the largest possible $|k_g|$ is bounded by one, given $\left| \frac{E}{A} \right| < \frac{1}{2}$.

Similarly, assuming $|k_g| < 1$, it can be shown that $\left| \frac{E}{A} \right| = |C| < \frac{1}{2}$. First, $g(X, E)$ is rewritten as follows:

$$g(X, E) = k_g = C(X^2 - 1) + X < 1. \quad (4.19)$$

Then, via the triangle inequality, the following equation can be obtained:

$$\begin{aligned}
|C(X^2 - 1) + X| &< |C| \cdot |X^2 - 1| + |X|, \\
&< 1,
\end{aligned}$$

which means that, in other words,

$$|C| \cdot |X^2 - 1| + |X| < 1. \quad (4.20)$$

Rearranging (4.20) gives:

$$|C|^2 < \left[\frac{1 - |X|}{|X^2 - 1|} \right]^2. \quad (4.21)$$

Note that as $X \in [\pm 1]$, there is a singularity in the denominator when $X = \pm 1$. Then,

letting $f_2(X) = \left[\frac{1 - |X|}{|X^2 - 1|} \right]^2$ and applying L'Hospital's rule when $X = \pm 1$ gives:

$$\lim_{x \rightarrow \pm 1} f_2(X) = \frac{1}{4}. \quad (4.22)$$

Now, asserting $f_2(X)$ affords:

$$|C|^2 < \min \mathcal{F}_2 = \frac{1}{4}, \quad (4.23)$$

where $\mathcal{F}_2 = \{f_2(X) | X \in [\pm 1]\} = [0.25, 1]$ is the range of $f_2(X)$. Note that $\mathcal{F}_2$ is the minimum (i.e., $\min \mathcal{F}_2$) in this situation, because the objective is to determine $|C|$ to

satisfy the entire range of $f_2(X)$, i.e., when $|C|^2 < \frac{1}{4} < 1$. For instance, if $|C|^2 = 0.5$, it

will not satisfy $\min \mathcal{F}_2$. Therefore,

$$|C| < \frac{1}{2}, \text{ when } |k_g| < 1. \quad (4.24)$$

Hence, $|E/A| < 1/2$ is the sufficient and necessary condition for $g(X, E) < 1$. ■

Remark 1: A similar result can be derived for $u(Y, E)$. Let $u(Y, E) = k_u$, then

$$u(Y, E) = - \left[\frac{E}{B} (Y^2 - 1) + Y \right] \\ = k_u.$$

Considering the absolute values gives:

$$\left| \frac{E}{B} (Y^2 - 1) + Y \right| = |k_u|,$$

which is similar to the definition of $|k_g|$ with X and A replaced with Y and B , respectively.

Remark 2: Note that $h(X, Y, E)$ is bounded by the superposition of $g(X, E)$ and $u(Y, E)$. This observation implies that the condition $|h(X, Y, E)| < 2$ can only be achieved when $|E/A| < 1/2$ and $|E/B| < 1/2$ are satisfied simultaneously. This condition occurs if and only if $|E| < \min(|A|, |B|)/2$, or simply:

$$|n|d < \min(A, B). \quad (4.25)$$

Practically, because A and B are generally unknown, to guarantee that $|E|$ is always within the bound, the value of $\min(A, B)$ can be set as the lower limit of the Fresnel approximation of $r_{\min} = 0.62(D^3/\lambda)^{1/2} = 0.62[d^3(N-1)^3/(2d)]^{1/2}$, or:

$$|n|d < r_{\min}. \quad (4.26)$$

Remark 3: Recall from Remark 2 that $|h(X, Y, E)| < 2$ if and only if $|n|d < r_{\min}$. This implies that k_3 is bounded by one, i.e., $|k_3| < 1$, when $|n|d < r_{\min}$ and $\left| \frac{f_i - f_o}{c} 2E \right| < \frac{1}{2}$.

Using the Cauchy-Schwarz inequality, we obtain:

$$\left| \frac{f_i - f_o}{c} \cdot 2E \right| < |f_i - f_o| \cdot \frac{2|E|}{c} < \frac{1}{2},$$

or simply:

$$|f_i - f_o| < \frac{c}{4|E|} < \frac{c}{2 \min(|A|, |B|)}. \quad (4.27)$$

Similar to Remark 2, as A and B are generally unknown, to guarantee that $|E|$ is within the bound, the value of $\min(A, B)$ can also be set as the lower limit of the Fresnel approximation $r_{\min}$ to give:

$$|f_i - f_o| < \frac{c}{2r_{\min}}. \quad (4.28)$$

Therefore, with (4.28) and (4.26), this concludes the proof of Theorem 1. ■

Remark 4: Theorem 1 generalizes the aliasing-free wideband DOA estimation of far-field sources according to Tang et al. [21]. It is noted that in the event of a far-field source, r_s and $r_{s'}$ will occur at $+\infty$. Accordingly, limiting A and B of (4.13) to $+\infty$ gives:

$$\lim_{A, B \rightarrow \infty} \left| \frac{f_i - f_o}{c} \cdot 2E \right| \cdot \left| (X - Y) + E \left(\frac{1 - Y^2}{B} - \frac{1 - X^2}{A} \right) \right| = \left| \frac{f_i - f_o}{c} \cdot 2E \right| \cdot |X - Y| < \lim_{A, B \rightarrow \infty} |k_3|,$$

or simply,

$$\left| \frac{f_i - f_o}{c} \cdot 2E \right| \cdot |h(X, Y)| < |k_3|, \quad (4.29)$$

where $|h(X, Y)|$ of (4.15) relinquishes its dependency on E to give:

$$\begin{aligned} h(X, Y) &= X - Y \\ &= g(X) + u(Y). \end{aligned} \quad (4.30)$$

It should be noted that $g(X)$ and $u(Y)$ in (4.30) are no longer dependent on E . Then, using the triangle inequality to enlarge (4.30) gives:

$$\begin{aligned} |h(X, Y)| &= |g(X) + u(Y)| \\ &\leq |g(X)| + |u(Y)|. \end{aligned} \quad (4.31)$$

Now, the maximum range of $|h(X, Y)|$ is:

$$\max |h(X, Y)| \leq \max(X) + \max(-Y) = 2. \quad (4.32)$$

Asserting (4.29) over the $\max |h(X, Y)|$ gives:

$$\left| \frac{f_i - f_o}{c} \cdot 2E \right| \cdot \left[\max_{X, Y} |h(X, Y)| \right] < \max_{X, Y} |k_3|,$$

or, in other words:

$$\left| \frac{f_i - f_o}{c} \cdot 2E \right| < \left| \frac{1}{2} \right|. \quad (4.33)$$

Once more, by enlarging the left-hand side of (4.33) with the triangle inequality we obtain:

$$\left| \frac{f_i - f_o}{c} \cdot 2E \right| < \frac{|f_i - f_o|}{|c|} \cdot |2E| < \left| \frac{1}{2} \right|,$$

or simply,

$$|f_i - f_o| < \left| \frac{1}{2} \right| \cdot \frac{|c|}{\left| \frac{nd}{2} \right|} = \frac{|c|}{2|n|d}, \quad (4.34)$$

which is identical to the condition derived in Tang et al. [21]. Hence, Theorem 1 in (4.10) generalizes Tang et al. [21] to cover both far-field and nearfield sources.

4.4.3 Aliasing-Free SSR Recovery

Nannuru et al. [49] noted that even though observations might be from different dictionaries, they will share the same sparsity as long as they are generated by the same wideband source. This observation suggests that aliasing-free localization can be achieved as the solution to a multiple-dictionary (MD) joint-optimization problem. Accordingly, if we let ℓ denote the ℓ th frequency component in Theorem 1, then the array observation in (4.8) can be stacked into $\mathbf{p} = [\mathbf{p}_1^T \cdots \mathbf{p}_L^T]^T \in \mathbb{C}^{NL}$, such that:

$$\mathbf{p} = \mathbf{A}_s \mathbf{s}, \quad (4.35)$$

where $\mathbf{A}_s = [\mathbf{A}_1^T \cdots \mathbf{A}_L^T]^T \in \mathbb{C}^{NL \times M}$ is the corresponding MD response formed by stacking all of the L dictionaries together, and $\mathbf{s} = \mathbf{s}_1 = \cdots = \mathbf{s}_L \in \mathbb{C}^M$ represents the sparse vector that captures the location of the true source. With (4.35), the MD joint-optimization problem can now be formulated as:

$$\min_{\mathbf{s} \in \mathbb{C}^M} \|\mathbf{s}\|_0, \quad \text{subject to } \mathbf{p} = \mathbf{A}_s \mathbf{s}, \quad (4.36)$$

whose solution will be free of any ambiguity under Theorem 1. Note that given (4.35), the minimized objective function (4.36) is equivalent to the number of nonzero rows in $\mathbf{s}$, that is, the number of elements in $\mathcal{T}$ of (4.11).

Extensive work has been done on MD joint-optimization problems, such as their management in the collaborative-sensing problem [57] or as a multichannel-image problem [58] and a generalized form of JSM-2 [48], where different signals with common sparse support and distinct dictionaries are studied. Even though the OMP method is used to solve the joint-sparsity problem [48], it requires a stopping criterion [78]. In practice, the OMP is halted after a fixed number of J iterations, and it can be

designed from the upper-bound of the possible number of sources. Comparatively, the OMP is preferred over other joint-optimization approaches due to its faster implementation [21] and is used in the numerical examples and experimental analysis later in this thesis. Fast implementation is understandably essential to many practical applications.

Essentially, the multichannel-image problem [58] has a per-iteration computational complexity on the order of $M \cdot \log(M)$. The total number of iterations required is the number required for convergence, which can be much greater than $J \times NL$. However, the MD OMP optimization that uses Theorem 1 is dominated by $O(J \times NL \times M)$ [79], [80], which can be optimized up to a further 40 times faster using graphics processing unit [81].

4.5 Numerical Analysis

This section presents several numerical analyses using the proposed method to achieve aliasing-free nearfield localization via signal processing. First, it is validated that the inequality $k_3 < 1$ is true if and only if the conditions of Theorem 1 are satisfied by multiple Monte Carlo trials. Next, the localization performance of the conventional SMF is compared with that of the proposed approach. The result shows that the conventional SMF fails, whereas the proposed signal-processing approach produces an aliasing-free localization when the source satisfies the conditions of Theorem 1.

4.5.1 Monte Carlo Trial, $k_3 < 1$

This section presents numerical simulations to validate the theorem derived in Section 4.4.2. In the first simulation, 10,000 Monte Carlo trials are conducted using the steering vector model approximated from the 2nd order TSE in (4.3) to validate the theorem. Similarly, the same set of Monte Carlo trials are also applied to (4.1) to test the performance of the theorem on the actual steering vector model. The simulations are performed with the assumption that a wideband source emits two subbands with central frequencies that satisfy the conditions of Theorem 1.

Figure 4.2: Monte Carlo of 10,000 trials: (a) using the second-order TSE of (4.3), and (b) on the actual steering vector of (4.1).

The Monte Carlo trials are generated from 10,000 uniformly generated angles from $-\pi/2$ to $+\pi/2$ and A and B from $r_{\min}$ to $r_{\max}$, where $r_{\min}$ and $r_{\max}$ is the lower and upper boundary of the Fresnel approximation, respectively. For brevity, n is set equal to one. Figure 4.2 depicts the set of trials when the source has two central subband frequencies that satisfy the conditions of Theorem 1. Specifically, Figure 4.2a shows the value of k_3 using the TSE approximation model in (4.3), and Figure 4.2b shows the value of k_3 using the actual steering vector model in (4.1). It can be seen that $|k_3| < 1$ (with $|k_{\text{limit}}| = 0.99$) for both situations when the conditions are met, which is expected because of Theorem 1.

Note that the TSE is only valid for the Fresnel region. The theorem does not guarantee that $|k_3| < 1$ when the source is located in the region of the array where $|r| < r_{\min}$. This is due to the fact that the reactive region (i.e. $|r| < r_{\min}$) contains evanescent wavenumber components, and thus a source in the reactive region cannot be described solely by the TSE [29]. Thus, Theorem 1 is not valid for $|r| < r_{\min}$.

4.5.2 Aliasing-Free Localization

This section presents numerical simulations to validate that aliasing-free localization can be achieved in an array-processing environment. Here, a ULA of $N = 4$ sensors is used, with an intersensor spacing $d = 10.16$ cm centered at the origin and along the y -axis. The source is placed at $r_s = 35.921$ cm and $\theta_s = \pi/4$ rad. Note that the source location can also be represented in the Cartesian coordinates of $(x_s, y_s) = (25.4, 25.4)$ cm. Both the array and the source are submerged in water. The acoustic propagation speed, c , in the medium is 1500 m/s. The SNR, which is defined as

$SNR = 20 \log_{10} |\mathbf{p}_{rms} / \boldsymbol{\sigma}|$, with $\mathbf{p}_{rms} \in \mathbb{R}^N$ being the root-mean-square of the recorded array measurement and $\boldsymbol{\sigma} \in \mathbb{R}^N$ being the corresponding standard deviation of the additive white Gaussian noise at the N channels, is set as 3 dB as in [21].

Here, the simulated source is a Gaussian sequence, and it is processed using the acoustic propagation in free-field water to create the effect of the source propagating toward the array [25], [28]. The propagation model produces the effect of having the source located at $r_s = 35.921$ cm and $\theta_s = \pi/4$ rad. The frequency subbands used for processing are on the continuous frequency spectrum from 20 to 23 kHz, a frequency range of leak [19]. Note that the subband frequencies are beyond the spatial aliasing frequency of a standard array, i.e., $f_{aliased} = c/(2d)$.

Figure 4.3 depicts the localization results using conventional SMF. Figure 4.3a and Figure 4.3b depict the localization results at the 20 kHz and 23 kHz subband frequency bins, respectively. Figure 4.3c depicts the localization results of the conventional wideband SMF by summing the L subband responses using (4.5). The conventional SMF approach results in multiple ghost sources in the spatial spectrum because of spatial aliasing.

As $|f_1 - f_2| = 3 \text{ kHz} < c/(2r_{\min})$, it satisfies Theorem 1. Hence, an aliasing-free localization can be found in the intersection of the support. With the halting of the OMP algorithm fixed at $J = 5$, Figure 4.4 depicts the normalized OMP response when Theorem 1 is applied to design the MD joint-optimization problem. Comparing Figure 4.3 with Figure 4.4, it is clear that the aliased locations are removed. Clearly, Figure 4.4 illustrates the ability of the proposed method with Theorem 1 to achieve an aliasing-free localization when the conditions are satisfied.

Figure 4.3: Spatial spectra of a source with an array of intersensor spacing $d = 10.16$ cm, a far-field spatial aliasing frequency $f_{\text{aliased}} = 7.382$ kHz, at (a) 20 kHz, (b) 23 kHz, and (c) a continuous frequency spectrum of 20-23 kHz.

Figure 4.4: Aliasing-free localization using the proposed OMP approach based on Theorem 1.

4.6 Experimental Results

Experimental data were collected and evaluated using the proposed method to demonstrate that the proposed signal-processing approach can achieve aliasing-free localization in a practical context. Specifically, experiments were conducted using an array of $N = 4$ hydrophones, with an intersensor spacing $d = 10.16$ cm. The data were collected in a water tank with an acoustic propagation speed in the medium $c = 1500$ m/s. The source was a pressurized water hose located in a fixed Fresnel region of the array, and the array was hoisted by a winch, as shown in Figure 4.5. The proposed method was validated using multiple sets of data collected from the hydrophone array. Each set was sampled at a rate $F_s = 500$ kHz for $T = 2$ s. In the experiments, the acoustic source was generated by a water hose at various pressures, i.e., $[1.379, 2.758, 4.137, 5.516, 6.895, 8.274]$ MPa, and the source was placed at the Cartesian coordinate system $\mathbf{r}_s = (20.32, 15.24)$ cm.

Figure 4.6 shows the spatial spectrum responses of the conventional wideband SMF in (4.5) using the continuous frequency spectrum from $f = 30 - 33$ kHz when the water hose was at different pressures. Similar to the numerical simulations in the previous section, the spatial spectrum response using the conventional wideband SMF from the recording also suffered from ambiguity due to spatial aliasing. These experimental results illustrate that the spatial aliasing problem is present even if the source measured from a realistic environment has a different acoustic intensity. Practically, in passive localization operation, this implies that the conventional wideband SMF will not be able to identify the location of the source.

Figure 4.5: Experimental setup of the recording system, where a winch was used to hoist the array of hydrophones.

Figure 4.6: Conventional wideband SMF spatial spectra of the recording at the aliased frequency of $f = [30 : 33]$ kHz, with a sampling rate of $F_s = 500$ kHz, $NFFT = 1024$, $N = 4$ sensors, the source at $(20.32, 15.24)$ cm, and pressurized at (a) 1.379 MPa, (b) 2.758 MPa, (c) 4.137 MPa, (d) 5.516 MPa, (e) 6.895 MPa, and (f) 8.274 MPa.

Figure 4.7: Proposed OMP spatial spectra of the recording at the aliased frequency $f = [30 : 33]$ kHz, with $F_s = 500$ kHz, $NFFT = 1024$, $N = 4$ sensors, the source at $(20.32, 15.24)$ cm, and pressurized at (a) 1.379 MPa, (b) 2.758 MPa, (c) 4.137 MPa, (d) 5.516 MPa, (e) 6.895 MPa, (f) 8.274 MPa.

Figure 4.7 illustrates the localization performance using the proposed OMP method. The OMP spatial spectra are constructed from the region in which the Fresnel approximations are valid; that is, $r_{\min} = 23.114$ cm to $r_{\max} = 91.44$ cm, $\theta = [-90^\circ : +90^\circ]$ and using the frequencies between 30 and 33 kHz as supports. Note that to satisfy Theorem 1, the maximum frequency difference between the continuous spectrum must be $3 \text{ kHz} < c/(2r_{\min})$, and the OMP responses from this experiment are also computed

at a normalized magnitude. Although the proposed OMP approach localizes the source with error, which can be attributed to noise and experimental conditions, it does not suffer from the ambiguity problems visible in Figure 4.6.

Nonetheless, as compared to Figure 4.6, the results in Figure 4.7 demonstrate that aliasing-free localization of a wideband nearfield source can be achieved using the proposed method when Theorem 1 is satisfied. Effectively, this allows localization of sources with frequency content beyond the spatial aliasing limit of an array. Practically, this is particularly useful in passive-localization operation, because the proposed approach will extend the array's operating frequency range and thus enable localization of a broader range of sources from a small subset of each source's frequencies [21], [49].

4.7 Conclusions

This chapter addresses the problem of spatial aliasing that arises when a wideband source is in the nearfield region of a ULA. In particular, a theorem that guarantees an aliasing-free localization is derived based on the second-order Taylor series expansion of the array steering vector.

The theorem is valid for any wideband sources within the Fresnel distance of the array. Furthermore, the theorem suggests that when the conditions are satisfied, observations from multiple dictionaries can be used to remove ambiguity due to spatial aliasing. Notably, each dictionary corresponds to a judiciously chosen frequency based on the theorem.

Consequently, the wideband nearfield localization problem is formulated to find the support of the observations in the frequency domain. Furthermore, it is proposed that

the problem can be solved with the OMP approach.

Finally, the proposed approach is verified numerically and experimentally. It achieves aliasing-free localization for an acoustic source in the nearfield of the array in a practical context. Crucially, this work advances the framework of array processing, achieving aliasing-free nearfield localization via a signal-processing technique by applying an understanding of the derived theorem and sparse beamforming. In addition, the proposed approach also generalizes the solution previously derived by Tang et al. [21] to include both the far-field and nearfield source.

Chapter 5 One-Sensor Array

Localization Methods

Localization using an array of acoustic sensors provides an additional dimension of information for well operators to use to diagnose the condition of a wellbore. However, as discussed in Section 1.3, well operators traditionally use a single acoustic sensor to detect abnormal acoustic sound within a wellbore. This single-sensor approach indicates where a leak might be present along the depth of a wellbore, but due to physical limitations, a single sensor will not be able to achieve results as good as a sensor array.

Nonetheless, the next natural question is to ask whether there are single-sensor-based methods to approximate the localization of sound sources. Accordingly, in this chapter, the utility of the OSA in this context is explored. In particular, two localization methods are examined for (a) a harmonic, and (b) a non-harmonic source using a single sensor moving in a piecewise-linear trajectory with a known velocity. Numerical simulations and experimental verifications of the proposed approach illustrate the effectiveness of OSA in a practical context. In short, this chapter bridges the research gaps between well integrity evaluation using a sensor array and evaluation using a single sensor.

5.1 Introduction

A key observation from Chapter 2 is that although each algorithm has different strengths and weaknesses, multiple sensors are required to form an array for processing. Accordingly, the localization performance of the algorithms is directly proportional to the number of sensors.

In recent years, a great deal of research has been conducted on extending array apertures by using moving sensor arrays [82]. Such methods have included the extended towed-array measurement approach (ETAM) [83]–[85], which uses the correlation between the overlapping positions of each sensor to correct the mismatch attributable to its movement to form a larger synthetic-array aperture. The moving circular-array approach has also been studied [69], which requires a static sensor working in tandem with a rotating array. Still, few have considered the possibility of extending the array aperture using only a single sensor.

Hioka and Kishida [86], [87] proposed the circularly moving sensor for the use of modulation effect (CAROUSEL) method, which demonstrated that DOA estimation could be achieved with a single sensor moving around the circumference of a circle. Hioka et al. [88] further improved their approach by designing a practical recording device that had a microphone embedded on a rotating disc and added an instantaneous frequency-estimation method using the center of gravity algorithm [89]. Nevertheless, Hioka et al. [88] did not consider multi-tone sources.

This chapter proposes two methods to exploit the piecewise-linear trajectory of a sensor to form a linear synthetic array for source localization where the trajectory of the sensor is either known or deduced from its known initial position and velocity. In brief, the methods extend the work of Hioka et al. [86]–[88] such that the sensor is not limited to a circular trajectory. First, applying a harmonic-signal model assumption, a synthetic array is formed using the recordings from a single sensor mounted on a moving platform (MP). Next, because acoustic perturbation due to leaks is non-harmonic, an alternative approach is developed based on a non-harmonic-signal model assumption and using a single sensor moving in a linear trajectory.

This chapter is organized as follows. Section 5.2 presents an introduction to the

proposed method that considers only an acoustic point source signal and the approach to reformulate the single channel into a multichannel waveform. Next, in Section 5.3, the OSA algorithm based on a harmonic-signal model is derived. This commences with a discussion of how the temporal-correction factor is derived from the multichannel waveforms to extract the embedded spatial information and how it is applied to approximate the recording as a virtual static array in Section 5.3.1. Section 5.3.2 shows the numerical analyses conducted for a point source and a harmonic source, and Section 5.3.3 presents the experimental analysis to validate the proposed OSA approach for localization of a harmonic source based on a set of experimental data collected with a sensor mounted on an MP. After this, in Section 5.4, the dROSA is discussed, and based on a non-harmonic signal model assumption, the dROSA approach is derived in Section 5.4.1. The numerical simulation for the dROSA is presented in Section 5.4.2, and the experimental results are given in Section 5.4.3. Finally, Section 5.5 concludes the chapter.

Figure 5.1: Moving sensor configuration.

5.2 Problem Formulation

Consider a sensor moving along the x-axis, or any appropriate axis, measuring the signal of a stationary point source, as shown in Figure 5.1. The source at the spatial position $\mathbf{r}_s = (x_s, y_s)$ is denoted as:

$$s(t) = s_s e^{j(\omega t + \psi)}, \quad (5.1)$$

where s_s , ω , and ψ are the amplitude, angular frequency, and phase of the point source, respectively. Note that s_s and ψ are assumed to be time-invariant because of the stationary assumption. Then, with the moving sensor initially located at the origin $(x, y) = (0, 0)$, the signal recorded by the moving sensor $p(t)$, can be denoted as:

$$p(t) = s(t) e^{-j\frac{\omega}{c}r(t)} + q(t), \quad 0 \leq t \leq T, \quad (5.2)$$

where $r(t) = \|\mathbf{r}_s + \mathbf{v}t\|_2$ is the Euclidean distance between the source and the sensor at a time t moving with velocity $\mathbf{v} = (v_x, v_y)$, $q(t)$ is the additive white Gaussian noise used to model the spatial and temporal noise inherent in the moving sensor within the recording window T , and $e^{-j\omega r(t)/c}$ is the phase shift that occurs due to the propagation of the signal toward the moving sensor. The collected samples can now be decomposed into N frames, such that each frame is quasi-stationary [90]. The n th frame can be expressed as:

$$p_n(t) = p(t + (n-1)\Delta T), \quad 0 \leq t \leq \Delta T, \quad (5.3)$$

where $\Delta T = T/N$ is the duration of each n th frame. With (5.3) in-hand, the N frames can now be expressed in a vector form as:

$$\mathbf{p}(t) = [p_1(t) \quad \cdots \quad p_N(t)]^T, \quad 0 \leq t \leq \Delta T. \quad (5.4)$$

5.3 OSA

Two forms of OSA are investigated: (1) the temporal-correction model and (2) the differential received-signal strength model. These forms are derived based on the different signal assumption. In the temporal-correction model, the signal is assumed to be harmonic, and an optimal temporal-correction factor is derived to correct the temporal delay component, which exists in the signal model. Conversely, the differential received-signal strength model assumes a non-harmonic source impinging the array. In this case, the temporal delay is not extractable. Accordingly, an alternative algorithm is derived for source localization based on other parameters that are more readily available, such as the received-signal strength.

5.3.1 Temporal-Correction Model

In this section, the OSA algorithm is derived. First, in Section 5.3.1.1, considering the point source to be stationary, it is shown that a temporal-delay component is inherent in the signal model of (5.4). Next, it is shown that an optimal temporal-correction factor exists and can be used to negate the effects of the temporal delay to arrive at (5.7). Then, in Section 5.3.1.2, the beamforming approach from Chapter 2 is used to localize the source. Accordingly, it is observed that by performing beamforming with the appropriate weights, the temporal-corrected OSA beamformer can be designed using the uncorrected model of (5.4). Finally, in Section 5.3.1.3, the point source model is extended to the harmonic-source model by the superposition of the $L-1$ harmonic components.

5.3.1.1 Design of temporal correction factor

When a source is harmonic, it can be observed that the signal model produces a temporal delay component that needs to be corrected. Omitting $q(t)$ from (5.4) gives:

$$\mathbf{p}(t) = \begin{bmatrix} s(t + 0 \cdot \Delta T) e^{-j\frac{\omega}{c}r(t+0 \cdot \Delta T)} \\ \vdots \\ s(t + (N-1) \cdot \Delta T) e^{-j\frac{\omega}{c}r(t+(N-1) \cdot \Delta T)} \end{bmatrix}. \quad (5.5)$$

Rearranging (5.5) yields:

$$\mathbf{p}(t) = \underbrace{s_s e^{j(\omega t + \psi)}}_{\text{Source Component}} \underbrace{\begin{bmatrix} e^{j\omega 0 \cdot \Delta T} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & e^{j\omega(N-1) \cdot \Delta T} \end{bmatrix}}_{\text{Temporal Delay}} \underbrace{\begin{bmatrix} e^{-j\omega r(t+0 \cdot \Delta T)/c} \\ \vdots \\ e^{-j\omega r(t+(N-1) \cdot \Delta T)/c} \end{bmatrix}}_{\text{Source Propagation}}. \quad (5.6)$$

From (5.6), it is evident that $\mathbf{p}(t)$ consists of three components: (a) the source, (b) the temporal delay, and (c) the source propagation. Here, two observations can be made. First, if the n th $r(t + (n-1)\Delta T)$ in the source with the propagation component is approximately constant, it is similar to the n th sensor of a static-array response. Second, the temporal-delay component is correctable by use of a correction factor. By inspection, the optimal correction factor is found to be $\mathbf{A}_c = \text{diag} \left(\begin{bmatrix} e^{-j\omega 0 \cdot \Delta T} & \dots & e^{-j\omega(N-1) \cdot \Delta T} \end{bmatrix} \right)$.

Approximating (5.6) with the first observation and applying the optimal temporal-correction factor $\mathbf{A}_c$ to correct the temporal delay yields the following OSA with N synthetic sensors:

$$\mathbf{p}_{OSA}(t) = \mathbf{A}_c \mathbf{p}(t) = s_s e^{j(\omega t + \psi)} \begin{bmatrix} e^{-\frac{\omega}{c}r(t+0 \cdot \Delta T)} & \dots & e^{-\frac{\omega}{c}r(t+(N-1) \cdot \Delta T)} \end{bmatrix}^T. \quad (5.7)$$

5.3.1.2 Localization through beamforming

With the OSA measurements from (5.7), the localization problem can now be solved in three steps using the beamforming approach, as discussed in Chapter 2. First, the design of the scan vectors is conducted by discretizing the search space in front of the OSA into grid points. Next, for each grid point, the beamforming power is computed. Finally, the location that has the highest power is interpreted as the source-estimated location.

First, let $\mathcal{R} = \{\mathbf{r}_1 \cdots \mathbf{r}_M\}$ denote the set that contains the search-space pointing to the discretized grid-point within the OSA. Furthermore, let $\hat{\mathbf{v}} = (v_x, v_y)$ be the estimated velocity of the sensor moving along a trajectory. Accordingly, the position of each n th synthetic sensor of the OSA can be approximated by the position of the moving sensor when it is in the middle of each n th frame, that is, at $\mathbf{v} \cdot \Delta T (n - 0.5)$. Now, the steering vector can be constructed as:

$$\mathbf{a}(\mathbf{r}_m) = \begin{bmatrix} e^{-j\frac{\omega}{c}\sqrt{(x_m - v_x \cdot 0.5 \cdot \Delta T)^2 + (y_m - v_y \cdot 0.5 \cdot \Delta T)^2}} \\ \vdots \\ e^{-j\frac{\omega}{c}\sqrt{(x_m - v_x (N-0.5) \cdot \Delta T)^2 + (y_m - v_y (N-0.5) \cdot \Delta T)^2}} \end{bmatrix}, \quad (5.8)$$

where $\mathbf{r}_m = (x_m, y_m)$ is the vector pointing to a discrete point within the OSA immediate search space.

In turn, to compute the beamforming power of each grid point, the $N \times N$ correlation matrix must be constructed. Each element of the correlation matrix represents the correlation of two synthetic sensors and is defined as follows:

$$\mathbf{R}_{OSA} = \mathbb{E} \left\{ \mathbf{p}_{OSA}(t) \mathbf{p}_{OSA}^H(t) \right\} = \mathbf{A}_c \mathbf{R}_{pp} \mathbf{A}_c^H, \quad (5.9)$$

where $\mathbf{R}_{pp} = \mathbb{E} \left\{ \mathbf{p}(t) \mathbf{p}^H(t) \right\}$ is the correlation matrix of the OSA before the temporal

correction. In practical application, because the correlation matrix is unavailable, it is estimated with the sample correlation matrix [91]:

$$\hat{\mathbf{R}}_{OSA} = \mathbf{A}_c \left\{ \frac{1}{\Delta T} \int_0^{\Delta T} \mathbf{p}(t) \mathbf{p}^H(t) dt \right\} \mathbf{A}_c^H = \mathbf{A}_c \hat{\mathbf{R}}_{pp} \mathbf{A}_c^H. \quad (5.10)$$

Finally, the source is localized by finding the argument that maximizes the MUSIC spatial spectrum [31], [92], [93]

$$\hat{\mathbf{r}}_s = \arg \max_{\mathbf{r}_m \in \mathcal{R}} \frac{1}{\mathbf{a}^H(\mathbf{r}_m) \hat{\mathbf{U}}_{OSA_q} \hat{\mathbf{U}}_{OSA_q}^H \mathbf{a}(\mathbf{r}_m)}, \quad (5.11)$$

where $\hat{\mathbf{U}}_{OSA_q}$ in (5.11) is the noise subspace of the sample correlation matrix of the OSA, and it is computed after performing the eigendecomposition on (5.10). In other words, $\hat{\mathbf{R}}_{OSA} = \hat{\mathbf{U}}_{OSA_s} \hat{\mathbf{\Lambda}}_{OSA_s} \hat{\mathbf{U}}_{OSA_s}^H + \hat{\mathbf{U}}_{OSA_q} \hat{\mathbf{\Lambda}}_{OSA_q} \hat{\mathbf{U}}_{OSA_q}^H$ with $\hat{\mathbf{U}}_{OSA_s}$ and $\hat{\mathbf{\Lambda}}_{OSA_s}$ being the signal-subspace eigenvectors and eigenvalues, respectively, and $\hat{\mathbf{U}}_{OSA_q}$ and $\hat{\mathbf{\Lambda}}_{OSA_q}$ being the noise-subspace eigenvectors and eigenvalues, respectively.

However, the noise subspace of $\hat{\mathbf{R}}_{OSA}$ is a translated noise subspace of $\hat{\mathbf{R}}_{pp}$ with the temporal-correction factor $\mathbf{A}_c$, i.e., $\hat{\mathbf{U}}_{OSA_q} = \mathbf{A}_c \hat{\mathbf{U}}_{pp_q}$, where $\hat{\mathbf{U}}_{pp_q}$ is the noise subspace of $\hat{\mathbf{R}}_{pp} = \hat{\mathbf{U}}_{pp_s} \hat{\mathbf{\Lambda}}_{pp_s} \hat{\mathbf{U}}_{pp_s}^H + \hat{\mathbf{U}}_{pp_q} \hat{\mathbf{\Lambda}}_{pp_q} \hat{\mathbf{U}}_{pp_q}^H$. Therefore, (5.11) can also be interpreted as:

$$\hat{\mathbf{r}}_s = \arg \max_{\mathbf{r}_m \in \mathcal{R}} \frac{1}{\mathbf{w}^H(\mathbf{r}_m) \hat{\mathbf{U}}_{pp_q} \hat{\mathbf{U}}_{pp_q}^H \mathbf{w}(\mathbf{r}_m)}, \quad (5.12)$$

where $\mathbf{w}(\mathbf{r}_m) = \mathbf{A}_c^{-1} \mathbf{a}(\mathbf{r}_m) = \mathbf{A}_c^H \mathbf{a}(\mathbf{r}_m)$ being the weights for OSA localization. Effectively, (5.12) implies that the OSA localization algorithm can be performed using (5.4) because the correction term can be coupled as a modified scan-vector weight.

5.3.1.3 Harmonic signal model using beamformer in the frequency domain

Next, (5.12) is extended to the harmonic signal model. This extension can be achieved by the superposition of the pitch and its $I - 1$ harmonic components with an unknown phase-shift at each ℓ th pitch to capture the random superposition [86]. In the harmonic signal model, $s(t)$ is now extended to give:

$$s(t) = \sum_{\ell=0}^{I-1} s_{s_\ell} e^{j(\omega t^\ell + \psi_\ell)}, \quad (5.13)$$

where s_{s_ℓ} and ψ_ℓ are the amplitude and phase of the ℓ th-order harmonic components.

Substituting (5.13) into (5.2) affords the following expression for the moving sensor:

$$p(t) = \sum_{\ell=0}^{I-1} s_{s_\ell} e^{j(\omega t^\ell + \psi_\ell)} e^{-j\frac{\omega}{c}r(t)^\ell} + q(t). \quad (5.14)$$

Following the wideband model in Section 2.2.2, the correlation matrix in the frequency domain can be represented as:

$$\tilde{\mathbf{p}}(\ell) = \mathcal{F}\{\mathbf{p}(t)\} = [\tilde{p}_1(\ell) \quad \cdots \quad \tilde{p}_N(\ell)]^T, \quad (5.15)$$

where $\mathcal{F}\{\cdot\}$ is the Fourier-transform operation along the rows of (5.4). The correlation matrix of each frequency bin in (5.15) can now be defined as:

$$\tilde{\mathbf{R}}_{pp}(\ell) = \mathbb{E}\{\tilde{\mathbf{p}}(\ell)\tilde{\mathbf{p}}^H(\ell)\}. \quad (5.16)$$

Similarly, extending the temporal-correction factor yields:

$$\tilde{\mathbf{A}}_c(\ell) = \text{diag} \left(\begin{bmatrix} e^{-j\omega_\ell 0 \cdot \Delta T} \\ \vdots \\ e^{-j\omega_\ell (N-1) \cdot \Delta T} \end{bmatrix} \right), \quad (5.17)$$

where $\omega_\ell = 2\pi\ell/L$, with L being the total number of frequency bins. Adding (5.17) into

(5.15) gives the following corrected OSA for each frequency bin:

$$\tilde{\mathbf{p}}_{OSA}(\ell) = \tilde{\mathbf{A}}_c(\ell) \tilde{\mathbf{p}}(\ell). \quad (5.18)$$

Accordingly, the $N \times N$ correlation matrix of the corrected frequency domain OSA at the ℓ th bin is constructed using:

$$\begin{aligned} \tilde{\mathbf{R}}_{OSA}(\ell) &= \mathbb{E} \{ \tilde{\mathbf{p}}_{OSA}(\ell) \tilde{\mathbf{p}}_{OSA}^H(\ell) \} \\ &= \tilde{\mathbf{A}}_c(\ell) \tilde{\mathbf{R}}_{pp}(\ell) \tilde{\mathbf{A}}_c^H(\ell). \end{aligned} \quad (5.19)$$

Finally, the localization of the harmonic source can be achieved by substituting (5.19) into (5.12) to yield:

$$\hat{\mathbf{r}}_s = \arg \max_{\mathbf{r}_m \in \mathcal{R}} \sum_{\ell=0}^{L-1} \beta(\mathbf{r}_m, \ell), \quad (5.20)$$

where the beamforming power is given by:

$$\beta(\mathbf{r}_m, \ell) = \frac{1}{\tilde{\mathbf{w}}^H(\mathbf{r}_m, \ell) \tilde{\mathbf{U}}_{pp_q}(\mathbf{r}_m, \ell) \tilde{\mathbf{U}}_{pp_q}^H(\mathbf{r}_m, \ell) \tilde{\mathbf{w}}(\mathbf{r}_m, \ell)}, \quad (5.21)$$

with the n th element of $\tilde{\mathbf{w}}(\mathbf{r}_m, \ell)$ defined as:

$$[\tilde{\mathbf{w}}(\mathbf{r}_m, \ell)]_n = [\tilde{\mathbf{A}}_c(\ell)]_n [\tilde{\mathbf{a}}(\mathbf{r}_m, \ell)]_n.$$

Here, $[\tilde{\mathbf{A}}_c(\ell)]_n$ is the n th diagonal element of (5.17), and the n th steering vector of the ℓ th bin is given by:

$$[\tilde{\mathbf{a}}(\mathbf{r}_m, \ell)]_n = e^{-j \frac{\omega_\ell}{c} \sqrt{(x_m - v_x(n-0.5)\Delta T)^2 + (y_m - v_y(n-0.5)\Delta T)^2}}.$$

As in (5.12), $\tilde{\mathbf{U}}_{pp_q}(\mathbf{r}_m, \ell)$ is the noise subspace at ω_ℓ , extracted through eigendecomposition.

Practically, the frequency ω_ℓ of the source signal in passive-source localization

problems is usually unknown. Nonetheless, this can be easily overcome by performing a power-spectrum analysis of the recorded signal in (5.14) to estimate the fundamental frequency content of the harmonic source.

5.3.2 Numerical Analysis and Experimental Results

This section presents numerical simulations to illustrate the validity of the OSA approach established in Section 5.3.1. In the simulations, a moving sensor initially located at the origin of the Cartesian coordinate system is moved with a velocity of $\mathbf{v} = (v_x, v_y) = [1, 0]$ m/s in the $+x$ direction at a sampling rate of 25.6 kHz. The acoustic-propagation medium is assumed to be air with $c = 343$ m/s, which implies that $1 \gg v/c = 0.003$, resulting in a negligible Doppler effect [25].

First, the OSA localization performance of (5.12) to a point source of $f = 250$ Hz is compared with the CRB of the static array (SA) response [24], [94]. In terms of comparison, both OSA and SA are allocated the same number of synthetic sensors and physical sensors, respectively, and both have the same number of available snapshots for the correlation matrix. The CRB used in the comparison is the theoretical statistical bound of an SA correlation matrix at its given SNR [94]. Next, the performance of the OSA in response to a harmonic source of the same frequency of 250 Hz is evaluated.

Figure 5.2: Standard deviation comparison between the OSA and $\sqrt{CRB}$ of the static array for (a) $\hat{x}_s$ estimation and (b) $\hat{y}_s$ estimation.

Figure 5.2 depicts the standard deviation of 500 localization trials of the OSA using (5.12). Here, the standard deviation is compared with the $\sqrt{CRB}$ for a source in SA at an SNR of -20 to 10 dB. The OSA is constructed with $T = 60$ s and $N = 384$ synthetic sensors, with each sensor having an intersensor spacing $d = 0.15625$ m, and the source located at $\mathbf{r}_s = (30, 15)$ m. Accordingly, perfect knowledge of the signal's frequency is assumed to enable evaluation of only the localization performance.

Figure 5.2 shows that the estimation performance of $\hat{x}_s$ and $\hat{y}_s$ of the OSA follows closely with the SA's CRB over a wide range of SNR. The convergence of $\hat{y}_s$ begins at the SNR of +8 dB. Nonetheless, poorer performance at higher SNR is expected because of the model approximation error. Finally, the estimation performance of OSA degrades if the n th synthetic sensor is not modeled at $\hat{\mathbf{v}} \cdot \Delta T (n - 0.5)$ because of the poorer model approximation of $\hat{\mathbf{R}}_{OSA}$.

Figure 5.3 depicts the evaluation of (5.21) using 500 Monte Carlo trials. Here, a harmonic source with $I = 3$ and fundamental frequency of 250 Hz is used. Each pitch has a variance of one and a random phase of $\psi = [-\pi, +\pi]$. The recording duration of the OSA is constructed from a moving sensor with $\mathbf{v} = (v_x, v_y) = [1, 0]$ m/s, $T = 10$ s, and $N = 20$ synthetic sensors, with each sensor having an intersensor spacing $d = 0.5$ m. The signal is located at $\mathbf{r}_s = (4, 15)$ m, and its SNR varies based on the $SNR = \mathbb{E}\{s(t)\} / \mathbb{E}\{q(t)\}$. The only assumption used in this simulation is that the source is harmonic. Therefore, to acquire additional information such as the frequencies of the signal, the Welch power-spectral density (PSD) is used [95]. Thus, the signal frequencies are first estimated using the PSD before being applied as per (5.20). Because the PSD approach only provides an estimate of the frequencies, numerical optimization is required to determine the true signal frequencies.

Figure 5.3: Standard deviation comparison between the OSA source-location estimation error, $\|\hat{\mathbf{r}}_s - \mathbf{r}_s\|_2$, when the frequencies are known and estimated to the standard deviation of the SA source-location estimation error.

The performance of the OSA is compared with the results when the true frequency information is available, and the estimation performance when an SA is used. When the estimated frequency is accurate, the performance of the OSA is comparable to that of the SA. Also, when the signal frequency of the OSA is estimated and optimized, the localization performance of the OSA approaches the SA at high SNRs. This result illustrates that the localization of a harmonic source is robust when a good estimate of the frequency is available.

5.3.3 Experimental Results

The OSA algorithm is applied to a set of data collected from [96]. In this experiment, the sensor was mounted on top of the MP located at $(x, y) = (+6.605, -0.25)$ m. The MP traverses along a linear path at approximately $\mathbf{v} = (1.411, 0)$ m/s, which creates a negligible Doppler effect [25], i.e., $1 \gg v/c = 0.0041$. The sound was generated by a speaker located at $\mathbf{r}_s = (30, 15)$ m broadcasting a composite square wave at 250, 750,

and 1250 Hz. The recording was performed for $T = 27$ s, at a sampling rate of $F_s = 25.6$ kHz. Using (5.20), $N = 60$ synthetic sensors were created. Similar to the harmonic signal simulation, the analysis assumes no knowledge of the frequency is available. Consequently, the Welch PSD and numerical optimization are used to identify the spectral response before applying (5.20).

Figure 5.4 shows the recorded signal and spatial MUSIC beamforming power spectrum of the OSA model with a beamformer resolution of 0.25 m. Although the source is localized with a source location estimation error, this demonstrates the difficulty of maintaining a constant velocity along the MP trajectory [96]. Nonetheless, the result demonstrates that the OSA localization of a harmonic source is achievable by using a single moving sensor with an approximately known velocity.

Figure 5.4: $T = 27$ s of the recorded signal (top), sampled at $F_s = 25.6$ kHz, and the OSA MUSIC spectrum beamforming power (bottom). The MP has a velocity $\mathbf{v} = (1.411, 0)$ m/s. The OSA has $N = 60$ synthetic sensors.

5.4 dROSA

Herein the proposed dROSA algorithm is derived. Principally, this work serves to extend the OSA approach beyond the localization of harmonic sources. Because leaks in a wellbore are non-harmonic, use of the temporally corrected OSA model will not be sufficient, because non-harmonic sources do not have a periodic component that can be extracted from the signal mode. Hence, another approach is needed to solve the localization problem when using a single moving sensor.

Typically, an array that captures acoustic perturbation deals with two aspects: (a) a phase- or time-delay of the source and (b) the decayed signal strength of the source. As discussed earlier, non-harmonic sources do not have a temporal property that the OSA can use. Therefore, this section focuses on deriving an approach based on the dROSA. Incidentally, the dROSA does not suffer from spatial aliasing but is susceptible to environmental conditions [97], [98].

Figure 5.5 depicts the block diagram of the proposed dROSA approach, which consists of (i) sequence extraction to extract the N frames to form N synthetic sensors, (ii) estimation of power and noise, and (iii) solution of the linear least squares (LLS) model using the power and noise estimate to localize the source. The bandpass filter (BPF) of the signal is an optional component.

Figure 5.5: Block diagram of the proposed dROSA algorithm. It consists of three parts, namely (i) sequence extraction to form the multichannel, (ii) power and noise estimation, and (iii) source-location estimation.

5.4.1 Differential Power Signal Model

Importantly, by expanding (5.4), the source recording from the sensor moving along another axis, that is, the y -axis, can be modeled with:

$$\mathbf{p}(t) = \text{diag} \left(\begin{bmatrix} \frac{1}{r_1(t)} & \cdots & \frac{1}{r_N(t)} \end{bmatrix} \right) \begin{bmatrix} s(t + \tau_1 - r_1(t)/c) \\ \vdots \\ s(t + \tau_N - r_N(t)/c) \end{bmatrix}, \quad 0 \leq t \leq \Delta T, \quad (5.22)$$

where $r_n(t) = \|(t + \tau_n) \mathbf{v} - \mathbf{r}_s\|_2$ is the radial distance between the source at $\mathbf{r}_s$ to the n th frame when the moving sensor is at the position $p_n(t)$, and $\tau_n = (n-1)\Delta T$ is the time instance when the n th frame is extracted, with ΔT being the duration of each extracted frame (as depicted in Figure 5.6).

Figure 5.6: Illustration of how the multi-frames can be extracted from a moving sensor recording $p(t)$, where each n th frame is extracted at a time τ_n for a duration of ΔT .

5.4.1.1 Power and Noise Estimation

Now, the average power of the multichannel frame is given by the following equation:

$$\mathbf{P}_{ave} = \frac{1}{\Delta T} \int_0^{\Delta T} |\mathbf{p}(t)|^2 dt = [P_1 \quad \cdots \quad P_N]^T. \quad (5.23)$$

Expressing the averaging power (5.23), in dB, of the multichannel in log-normal, gives the following equation:

$$P_n = P_s - 10 \log_{10} r_n^2 + q_n, \quad (5.24)$$

where P_s is the average transmitted signal power (in dB), r_n is the average distance from the source to the moving sensor at the n th frame position, and q_n is the average noise (in dB) modeled as a Gaussian normal $q_n \sim \mathcal{N}(0, \sigma_n^2)$.

Note that (5.24) is still too restrictive because it requires knowledge of the average transmitted signal power P_s , which is generally unknown due to the nature of passive-source localization. To avoid using P_s in the signal mode, it is proposed to formulate it in the differential form [99]. Thus, using the first frame as a reference, the differential form is:

$$P_{n,1} = -\ln\left(\frac{r_n}{r_1}\right)^2 + q_{n,1}, \quad n = 2, \dots, N, \quad (5.25)$$

where $P_{n,1} = 0.1 \lceil \ln 10 (P_n - P_1) \rceil$ and $q_{n,1} = 0.1 \lceil \ln 10 (q_n - q_1) \rceil$. Rearranging this differential power equation gives:

$$e^{-P_{n,1}} = \left(\frac{r_n}{r_1}\right)^2 e^{-q_{n,1}}, \quad n = 2, \dots, N, \quad (5.26)$$

where the noise is now multiplicative. Now, evaluating the expectation of (5.26) gives:

$$\mathbb{E}\left\{\left(\frac{r_n}{r_1}\right)^2\right\} = r_{n,1} = e^{-P_{n,1}} e^{-\frac{1}{2}(\lambda_n^2 + \lambda_1^2)}, \quad n = 2, \dots, N, \quad (5.27)$$

where $\lambda_n^2 = 0.01 (\ln 10)^2 \sigma_n^2$. Note that $r_{n,1}$ is an unbiased estimate of r_n^2 / r_1^2 .

5.4.1.2 Estimation of the source

Note that:

$$r_n^2 = (x_n - x)^2 + (y_n - y)^2, \quad n = 1, \dots, N, \quad (5.28)$$

where (x_n, y_n) and $\mathbf{r}_s = (x, y)$ are the average location of the sensor at the n th frame and the true source location, respectively. With (5.28), the differential channel can be expressed in the following LLS form:

$$r_{n,1} r_1^2 = r_n^2 + q_n, \quad n = 2, \dots, N, \quad (5.29),$$

which also has the following matrix form:

$$\mathbf{A}\boldsymbol{\theta} = \mathbf{b} + \mathbf{q}, \quad (5.30)$$

where:

$$\mathbf{A} = \begin{bmatrix} 2x_2 - 2r_{2,1}x_1 & 2y_2 - 2r_{2,1}y_1 & r_{2,1} - 1 \\ \vdots & \vdots & \vdots \\ 2x_N - 2r_{N,1}x_1 & 2y_N - 2r_{N,1}y_1 & r_{N,1} - 1 \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} x \\ y \\ x^2 + y^2 \end{bmatrix},$$

$$\mathbf{b} = \begin{bmatrix} x_2^2 + y_2^2 - r_{2,1}(x_1^2 + y_1^2) \\ \vdots \\ x_N^2 + y_N^2 - r_{N,1}(x_1^2 + y_1^2) \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} q_2 \\ \vdots \\ q_N \end{bmatrix}.$$

Finally, localization can be achieved with the LLS estimate to afford:

$$\hat{\boldsymbol{\theta}} = (\mathbf{A}^\top \mathbf{C}_q^{-1} \mathbf{A})^{-1} \mathbf{A}^\top \mathbf{C}_q^{-1} \mathbf{b}, \quad (5.31)$$

where $\mathbf{C}_q = \mathbb{E} \left\{ (r_{n,i}r_1^2 - r_i^2) \cdot (r_{n,i}r_1^2 - r_i^2)^\top \right\}$ is the noise covariance matrix found to be of the form:

$$\mathbf{C}_q = \boldsymbol{\Sigma} \boldsymbol{\Lambda} \boldsymbol{\Sigma}, \quad (5.32)$$

where

$$\boldsymbol{\Lambda} = \begin{bmatrix} e^{(\lambda_2^2 + \lambda_1^2)} - 1 & e^{\lambda_1^2} - 1 & \cdots & e^{\lambda_1^2} - 1 \\ \vdots & \vdots & \ddots & \vdots \\ e^{\lambda_1^2} - 1 & e^{\lambda_1^2} - 1 & \cdots & e^{(\lambda_N^2 + \lambda_1^2)} - 1 \end{bmatrix},$$

and $\boldsymbol{\Sigma} = \text{diag}(r_2^2, \dots, r_N^2)$. In a practical context, r_n , which is the distance between the source and the n th synthetic sensor, is usually unknown. Nonetheless, because scaling of the noise covariance matrix does not affect the solution, it can be represented with $\boldsymbol{\Sigma} = \text{diag}(r_2^2/r_1^2, \dots, r_N^2/r_1^2)$. Hence, when the noise contribution is small, the noise covariance matrix can be modeled as:

$$\mathbf{C}_q \approx \tilde{\boldsymbol{\Sigma}} \boldsymbol{\Lambda} \tilde{\boldsymbol{\Sigma}}, \quad (5.33)$$

where $\tilde{\boldsymbol{\Sigma}} = \text{diag}(r_{2,1}, \dots, r_{N,1})$.

Likewise, the covariance of the estimate $\mathbf{C}_{\hat{\boldsymbol{\theta}}}$ can be evaluated as:

$$\begin{aligned}
\mathbf{C}_{\hat{\boldsymbol{\theta}}} &= \mathbb{E} \left\{ \left[\hat{\boldsymbol{\theta}} - \boldsymbol{\theta} \right] \cdot \left[\hat{\boldsymbol{\theta}} - \boldsymbol{\theta} \right]^T \right\}, \\
&= \mathbb{E} \left\{ \left(\mathbf{A}^T \mathbf{C}_q^{-1} \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{C}_q^{-1} \mathbf{q} \mathbf{q}^T \mathbf{C}_q^{-1} \mathbf{A} \left(\mathbf{A}^T \mathbf{C}_q^{-1} \mathbf{A} \right)^{-1} \right\}, \\
&\leq \mathbb{E} \left\{ \left(\mathbf{A}^T \mathbf{C}_q^{-1} \mathbf{A} \right)^{-1} \right\} \cdot \mathbb{E} \left\{ \mathbf{A}^T \mathbf{C}_q^{-1} \right\} \cdot \mathbb{E} \left\{ \mathbf{q} \mathbf{q}^T \right\} \cdot \mathbb{E} \left\{ \mathbf{C}_q^{-1} \mathbf{A} \right\} \cdot \mathbb{E} \left\{ \left(\mathbf{A}^T \mathbf{C}_q^{-1} \mathbf{A} \right)^{-1} \right\}, \\
&\approx \mathbb{E} \left\{ \left(\mathbf{A}^T \mathbf{C}_q^{-1} \mathbf{A} \right)^{-1} \right\}.
\end{aligned} \tag{5.34}$$

Next, we let $\mathbf{A}_i = \bar{\mathbf{A}}_i + \Delta_i$ where $\Delta_i = \begin{bmatrix} -2r_{i,1}x_i & -2r_{i,1}y_i & r_{i,1} \end{bmatrix}$ is its noise component. Using Slutsky's theorem [100], (5.34) can then be evaluated as:

$$\mathbb{E} \left\{ \left(\mathbf{A}^T \mathbf{C}_q^{-1} \mathbf{A} \right)^{-1} \right\} = \left(\bar{\mathbf{A}}^T \mathbf{C}_q^{-1} \bar{\mathbf{A}} + \mathbb{E} \left\{ \Delta^T \right\} \mathbf{C}_q^{-1} \mathbb{E} \left\{ \Delta \right\} \right)^{-1}, \tag{5.35}$$

where:

$$\mathbb{E} \left\{ \Delta \right\} = \begin{bmatrix} -2 \frac{d_2^2}{d_1^2} x_1 & -2 \frac{d_2^2}{d_1^2} y_1 & \frac{d_2^2}{d_1^2} \\ \vdots & \ddots & \vdots \\ -2 \frac{d_N^2}{d_1^2} x_1 & -2 \frac{d_N^2}{d_1^2} y_1 & \frac{d_N^2}{d_1^2} \end{bmatrix}. \tag{5.36}$$

However, because the dROSA is modeled along the y -axis, the first column of the matrix $\mathbf{A}$ in (5.30) is null. To mitigate this, a second-step estimation is conducted to estimate $\mathcal{X}$ using the nonzero values of $\hat{\boldsymbol{\theta}}$ in (5.31), i.e.:

$$\hat{x} = g([\boldsymbol{\theta}]_{2:3}) = \sqrt{[\hat{\boldsymbol{\theta}}]_3 - [\hat{\boldsymbol{\theta}}]_2^2}. \tag{5.37}$$

This second-step estimation is possible because there are relationships between the first row of $[\hat{\boldsymbol{\theta}}]_{1:2} = [\hat{\mathbf{x}}, \hat{\mathbf{y}}]^T$ and the third row of $[\hat{\boldsymbol{\theta}}]_3 = [\hat{\mathbf{x}}^2 + \hat{\mathbf{y}}^2]$. Using the transformation of parameters [101], the variance of $\hat{x}$ in (5.37) is modeled as:

$$\begin{aligned}
\text{var}(\hat{x}) &\geq \mathbb{E} \left\{ \frac{\partial g(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \mathbf{C}_{\hat{\boldsymbol{\theta}}} \frac{\partial g^T(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right\} \\
&\approx \mathbb{E} \left\{ \begin{bmatrix} 0 & -\frac{\hat{y}}{\hat{x}} & \frac{1}{2\hat{x}} \end{bmatrix} \right\} \mathbb{E} \left\{ \left(\mathbf{A}^T \mathbf{C}_q^{-1} \mathbf{A} \right)^{-1} \right\} \mathbb{E} \left\{ \begin{bmatrix} 0 \\ -\frac{\hat{y}}{\hat{x}} \\ \frac{1}{2\hat{x}} \end{bmatrix} \right\} \\
&\approx \begin{bmatrix} 0 & -\frac{y}{x} & \frac{1}{2x} \end{bmatrix} \left(\bar{\mathbf{A}}^T \mathbf{C}_q^{-1} \bar{\mathbf{A}} + \mathbb{E} \{ \Delta^T \} \mathbf{C}_q^{-1} \mathbb{E} \{ \Delta^T \} \right)^{-1} \begin{bmatrix} 0 \\ -\frac{y}{x} \\ \frac{1}{2x} \end{bmatrix}.
\end{aligned} \tag{5.38}$$

Finally, the theoretical root mean square (RMSE) for $[\hat{\boldsymbol{\theta}}]_{1:2}$ denoted by $\text{RMSE}([\hat{\boldsymbol{\theta}}]_{1:2})$, is:

$$\text{RMSE}([\hat{\boldsymbol{\theta}}]_{1:2}) = \sqrt{\text{var}(\hat{x}) + [\mathbf{C}_{\hat{\boldsymbol{\theta}}}]_{2,2}}. \tag{5.39}$$

5.4.1.3 Computational Complexity

Because $\tilde{\boldsymbol{\Sigma}}$ is a diagonal matrix, the noise covariance matrix $\mathbf{C}_q$ can be calculated with $O((N-1)^2)$ multiplications. The remaining complexities of the dROSA algorithm using (5.31) are tabulated in Table 5.1. The table shows that the algorithm is dominated by the matrix-inverse operation of $\mathbf{C}_q^{-1}$ with $O((N-1)^3)$.

Notably, the dROSA in (5.31) is computationally more efficient than the harmonic OSA in (5.20) because the harmonic OSA is dominated by $O(LN^3)$, and thus the main computational effort in the harmonic OSA is the L eigendecomposition of the sample correlation matrix at each of the ℓ th frequency bins, $\hat{\mathbf{R}}_{OSA}(\ell)$.

Table 5.1: Computational complexity in the proposed dROSA approach

$\mathbf{C}_q^{-1}$	$(\mathbf{A}^\top \mathbf{C}_q^{-1} \mathbf{A})^{-1}$	$\mathbf{A}^\top \mathbf{C}_q^{-1} \mathbf{b}$	$\hat{\boldsymbol{\theta}}$
$O((N-1)^3)$	$O(3(N-1)^2) + O(3^2(N-1)) + O(3^3)$	$O(3(N-1))$	$O(3^2)$

5.4.2 Numerical Simulation

This section presents four sets of numerical simulations to evaluate the RMSE performance of the proposed dROSA by comparing it with those from the SA and the differential received-signal strength CRB of [99]. The first three sets of simulations use $N = 10$ synthetic sensors, the conventional approach to study a ULA [28]. The synthetic sensors are spaced linearly and uniformly along the y-axis with an intersensor spacing $d = 6$ cm. The fourth set of simulations studies the RMSE performances of the proposed dROSA with respect to the different number of synthetic sensors.

In all of the simulations, the source is assumed to be a white Gaussian sequence $s(t) \sim \mathcal{N}(0, \sigma_s^2)$ located at $\mathbf{r}_s = (x, y)$, in a homogeneous medium, with an acoustic-propagation speed $c = 1500$ m/s. Noise is assumed to be known with identical variances of $\sigma_n^2 = \sigma^2$, and the statistical results are generated from an average of 10,000 independent runs.

5.4.2.1 Test case 1: RMSE vs. SNR

In the first simulation, the dROSA measurements are generated from $T = 24$ s of moving-sensor recording. The moving sensor is initially located at the origin, moving with velocity $\mathbf{v} = [v_x \ v_y]^T = [0 \ 2.54]^T$ cm/s, and $N = 10$ synthetic sensors of dROSA measurements are constructed by uniform decomposition of recording duration T into ten sub-frames. Each sub-frame has a length of $\Delta T = T/N$.

Figure 5.7 shows the RMSE of the location-estimation obtained by using the proposed dROSA, and the SA for a source located at $\mathbf{r}_s = [25.4 \ 25.4]^T$ cm with $\sigma^2 \in [-50, +50]$ dB. The theoretical standard deviation for $\hat{\mathbf{r}}_s$ as derived in (5.39), and the CRB of [99], are also included for comparison.

Figure 5.7 shows that the derived theoretical standard deviation of $\hat{\mathbf{r}}_s$ agrees well with the CRB of [99] when the noise magnitude is small (i.e., $\sigma^2 < 0$ dB) and starts to deviate when it is greater than 0 dB because as the noise magnitude increases, the elements inside $\mathbf{C}_q$ increase exponentially because of $\mathbf{\Lambda}$ in (5.32). However, the CRB of [99] is scaled proportionally to σ^2 . The RMSE of the SA also follows closely with the derived theoretical standard deviation but begins to deviate when the noise magnitude is greater than -15 dB. This RMSE performance is consistent with the results from [99], where the algorithm fails to attain the theoretical standard deviation in the presence of a large noise magnitude when the path loss factor is 2

The RMSE of the proposed dROSA also agrees well with the RMSE using the SA under a similar condition (when $\sigma^2 \in [-30, +50]$ dB) and begins to level off when $\sigma^2 < -30$ dB. The poor dROSA RMSE performance at low noise magnitudes is

expected because of the model approximation error between the SA and dROSA. Nonetheless, the overall dROSA RMSE performance indicates that the proposed approach is comparable to that of the SA over a wide range of σ^2 .

Figure 5.7: RMSE vs. σ^2 at $\mathbf{r}_s = [25.4 \ 25.4]^T$ cm, using $N = 10$ sensors uniformly spaced with an intersensor spacing $d = 6$ cm.

5.4.2.2 Test case 2: RMSE vs. SNR (tubing leak)

Vignes et al. [11] and Randhol et al. [12] found that most well integrity leaks occur in the region of the tubing. Typically, this is located approximately 6 cm (along the x-axis) from the center of the wellbore. In the next simulation, the first test is repeated for a source located at $\mathbf{r}_s = [6.35 \ 25.4]^T$ cm to study the localization's statistical performance when the source is located around the tubing, that is, approximately 6 cm from the center of the wellbore.

The results are plotted in Figure 5.8. Note that the RMSE using the dROSA begin to

level off when $\sigma^2 < -20$ dB; this is different from the previous test. Nonetheless, the estimates attain the derived theoretical standard deviation for $\sigma^2 < 5$ dB. Also, it can be seen that both the RMSE of the dROSA and SA localization performances follow closely with the CRB of [99]. This suggests that the proposed dROSA approach is reasonably efficient for sources located near the tubing region of the wellbore (6 cm from the center of the wellbore) for noise magnitudes greater than -15 dB.

Figure 5.8: RMSE vs. σ^2 at $\mathbf{r}_s = [6.35 \ 25.4]^\top$ cm, using $N = 10$ sensors uniformly spaced with an intersensor spacing $d = 6$ cm.

5.4.2.3 Test case 3: RMSE vs. sensor velocity

Typically, the sensors used in ultrasonic leak-detection can reach a velocity of 15.24 cm/s [27]. The third simulation studies the RMSE performance of the dROSA at varying velocities. Figure 5.9 and Figure 5.10 depict the RMSE performance when the source is located at $\mathbf{r}_s = [25.4 \ 25.4]^\top$ and $\mathbf{r}_s = [6.35 \ 25.4]^\top$, respectively.

Accordingly, for a fair comparison, the recording durations are shortened proportionally. Table 5.2 tabulates the recording variations. In both figures, the noise magnitudes are fixed at (a) $\sigma^2 = -10$ dB and (b) $\sigma^2 = +10$ dB. It can be seen that the RMSE performance is relatively constant within this varying range of moving-sensor velocity. Thus, the proposed approach at the typical moving velocity [27] performs similarly to a static array using the differential received-signal strength approach of [99]. Finally, in Figure 5.10a, the RMSE approaches both the theoretical standard deviation and the CRB of [99], suggesting that the proposed dROSA method is efficient for a source located at $\mathbf{r}_s = [6.35 \ 25.4]^\top$ with a low noise magnitude ($\sigma^2 = -10$ dB).

Table 5.2: Recording duration with various velocities of the moving sensor

v_y (cm/s)	2.54	5.08	7.62	10.16	12.7	15.24
ΔT (s)	24	12	8	6	4.8	4
N sensors	10	10	10	10	10	10

Figure 5.9: RMSE vs. $\mathbf{v}$ at $\mathbf{r}_s = [25.4 \ 25.4]^T$ cm, using $N = 10$ sensors uniformly spaced with an intersensor spacing $d = 6$ cm.

Figure 5.10: RMSE vs. $\mathbf{v}$ at $\mathbf{r}_s = [6.35 \ 25.4]^\top$ cm, using $N = 10$ sensors uniformly spaced with an intersensor spacing $d = 6$ cm.

5.4.2.4 Test case 4: RMSE vs. number of sensors (tubing leak)

Here, the RMSE performance with respect to the number of sensors is examined to evaluate the RMSE reduction performance. Furthermore, to study the robustness, the source is simulated at six locations (i.e., $r_x = 6.35$ cm and $r_y = [0:10.16:50.8]$ cm). Here, r_x is fixed to simulate a leak in the tubing region of the wellbore. For comparison, this involves repeating the simulation in Figure 5.10a with the velocity of the sensor fixed at $\mathbf{v} = [v_x \ v_y]^T = [0 \ 2.54]^T$ cm/s and the noise-variance fixed at $\sigma^2 = -10$ dB.

Figure 5.11 depicts the RMSE performance when the source is closer to the origin, located at $r_x = 6.35$ cm and (a) $r_y = 0.0$ cm, or (b) $r_y = 10.16$ cm. Figure 5.12 depicts the RMSE performance when the source is closer to the center of the synthetic array, with a source located at $r_x = 6.35$ cm and (a) $r_y = 20.32$ cm, or (b) $r_y = 30.48$ cm. Figure 5.13 depicts the RMSE performance when the source is closer to the end of the synthetic array, with a source located at $r_x = 6.35$ cm and (a) $r_y = 40.64$ cm, or (b) $r_y = 50.8$ cm.

A few observations can be made from Figure 5.11–Figure 5.13. First, the derived theoretical standard deviation of $\hat{\mathbf{r}}_s$ agrees well with the CRB of [99] for all six cases. Second, the RMSE decreases as the number of synthetic sensors N increases. However, this decrease in the RMSE comes at the cost of the estimator's efficiency; that is, the RMSE is greater than the theoretical bound. Third, the RMSE of the dROSA is greater than the SA when N is small, due to the error in the model's approximation of the moving sensor's position. Hence, the RMSE of the dROSA approaches the SA as N increases.

(a) $\mathbf{r}_s = [6.35 \ 0]^T$ cm

(b) $\mathbf{r}_s = [6.35 \ 10.16]^T$ cm

Figure 5.11: RMSE vs. N for a source located near the origin at $r_x = 6.35$ cm and
(a) $r_y = 0.0$ cm, or (b) $r_y = 10.16$ cm.

(a) $\mathbf{r}_s = [6.35 \ 20.32]^T$ cm

(b) $\mathbf{r}_s = [6.35 \ 30.48]^T$ cm

Figure 5.12: RMSE vs. N for source located near the center of the synthetic array at $r_x = 6.35$ cm and (a) 20.32 cm, or (b) $r_y = 30.48$ cm.

Figure 5.13: RMSE vs. N for source located near the end of the synthetic array at $r_x = 6.35$ cm and (a) $r_y = 40.64$ cm, or (b) $r_y = 50.8$ cm.

5.4.3 Experimental Results

The dROSA is applied to a set of experimental data collected from a water tank experiment. In the experiment, to simulate various source positions, an array of $N = 6$ sensors was used to perform the recording, to reduce the experimental effort required. Accordingly, the array was hoisted by a winch, as shown in Figure 5.14a, and submerged in a water tank, as shown in Figure 5.14b. The tank was approximately 9.144 m deep, with a length and breadth of 8.2296 m. Subsequently, the array was pulled with a velocity $\mathbf{v} = [0, 3.4]$ cm/s for $T = 22$ s, at a sampling rate of $F_s = 250$ kHz.

Figure 5.15 shows the recorded signal at each of the six sensors, and Figure 5.16 depicts the localization performance of the dROSA. Note that each sensor's recording can be treated as an independent experimental result, with the starting location of each sensor at $T = 0$ fixed as the origin $(0, 0)$; that is, the sensors p_1 to p_6 correspond to a recording of the source placed at $r_x = 25.4$ and $r_y = [0 : 10.16 : 50.8]$ cm, respectively. Although the source is localized with a source localization error, it is attributable to an imperfect recording. Specifically, as can be seen in Figure 5.15, the positive half is not symmetrical with the negative half: in Figure 5.15a, the negative half records up to -15 in amplitude, whereas the positive half barely reaches +5. These asymmetrical waveforms result in a mismatch with the model assumption.

Figure 5.14: Experimental setup of the array, showing the winch used to hoist it into a water tank:
(a) the winch and the sensor array, (b) a plan view of the water tank.

Figure 5.15: Normalized waveform (to standard deviation of 1) of $p_1(t)$ to $p_6(t)$ signal. The sensors were submerged in the water tank with a velocity $\mathbf{v} = [0, 3.4]^T$ cm/s, at a sampling rate of $F_s = 250$ kHz, and a recording duration of $T = 22$ s.

Figure 5.16: Location estimation of the dROSA with $N = 10$ synthetic sensors. The origin $(0,0)$ is fixed at the starting position of each sensor.

As previously studied in Section 5.4.2.4, one way to reduce the error is to increase the number of sensors N [99]. Here, the error of the location estimation is displayed in Table 5.3, where this was calculated as the norm error, $\varepsilon(\hat{\mathbf{r}}_s) = \|\mathbf{r}_s - \hat{\mathbf{r}}_s\|_2$, using a different number of synthetic sensors. Increasing the number of synthetic sensors reduces the estimation error, especially in p_6 , where the error is reduced by

approximately 3.36 cm when $N = 2000$.

Table 5.3: Location estimation error (cm) using various N synthetic sensors at different source positions.

N	10	50	100	200	400	500	750	1000	1500	2000
p_1	3.22	2.61	2.32	2.02	2.08	2.22	1.38	2.09	1.84	2.58
p_2	1.96	1.49	1.55	1.75	1.89	2.07	1.4	2.11	1.18	1.59
p_3	3.11	3.35	3.21	3.14	2.95	2.79	2.7	2.26	2.48	2.15
p_4	5.82	6.14	6.25	6.25	6.44	6.4	6.44	5.55	5.44	4.92
p_5	6.12	6.19	6.39	6.86	6.96	7.02	7.14	6.53	6.57	6.03
p_6	12.54	12.04	12.09	12.31	11.82	11.28	11.62	10.77	10.27	9.18

5.5 Conclusions

This chapter addresses the problems of performing localization when there is insufficient space for sensor placement. Two approaches are devised based on the signal model assumption, which shows that localization can be achieved when the assumptions are valid.

The first approach demonstrates that harmonic source localization is achievable using the OSA approach based on a single sensor moving along a known piecewise-linear trajectory. This approach consists of three steps: (1) data acquisition from a single sensor moving along a trajectory with a known velocity $\mathbf{v}$; (2) data preprocessing to decompose the signal into frames before synchronizing the frames by applying a temporal-correction factor; and (3) conventional beamforming application for source localization. After this numerical simulation is conducted, experimental verification shows that the proposed technique can be applied to localize a harmonic source using a single moving sensor.

The second approach demonstrates that localization is achievable using a non-

harmonic source model using the proposed dROSA approach, which also consists of three steps: (1) power estimation to compute the averaged received power at the localized moving-sensor location; (2) noise estimation to construct the covariance matrix, and (3) application of the LLS model to perform the localization by solving the equations. Both the numerical simulation and experimental verification demonstrate that localization is achievable even when the source is non-harmonic.

Furthermore, the numerical simulation shows that the performance of the proposed dROSA is efficient for sources located near the tubing of the wellbore, which was previously concluded statistically to be one of the most common types of wellbore leak. Also, the numerical simulations show that the estimation performance is approximately constant for moving velocities up to 15.24 cm/s, which corresponds to the conventional sensor-velocity range used in well integrity evaluation to detect wellbore leaks with a single acoustic sensor.

Overall, the approaches presented in this chapter advance the framework of using a single sensor for localization, thus elegantly bridging the research gap between the use of a sensor array and the use of a single sensor.

Chapter 6 Leak Classification with an SVM

Given the long life cycle of a wellbore and the significant repercussions of structural failure, it is important to ensure that well integrity is maintained throughout its operational period [1]. Early detection and localization of any downhole leaks are essential to provide operators with timely information to mitigate and resolve the problem.

In the previous chapters, the problem of bridging the research gap between the noise-logging approach [27], [102] and the acoustic-array approach proposed by Walters et al. [20] was discussed in terms of extending these approaches' operating frequency in arrays (Chapter 3 and Chapter 4) and achieving localization using a single sensor (Chapter 5). Aside from localization, additional knowledge about leak phases can further enhance an operator's ability to resolve any well integrity problems at an early stage.

In this chapter, a phase-classification algorithm based on the source's frequency-magnitude is proposed that can be applied to the acoustic data after leaks have been detected or localized from the single-sensor approach and the acoustic-array approach. The frequency-magnitude based approach avoids the spatial aliasing problem faced in Chapter 3 and Chapter 4, which enables the utilization of the entire source spectrum.

6.1 Introduction

In principle, different leaks exhibit distinct acoustic features that can be used for characterization. For instance, a gas leak will have comparatively higher frequency content than a liquid leak [26]. Therefore, classification can be achieved by learning and extracting features from the distinct types of leaks based on their acoustic signatures.

Figure 6.1 depicts the block diagram of the proposed classification approach, which consists of two stages: training and prediction. During the training stage, the acoustic spectrum of the known leak is first estimated from the power spectrum density (PSD) measured from the hydrophone. Next, principal component analysis (PCA) is applied to extract the dominant features of the estimated PSD. Finally, with the leak labels and their principal acoustic components known, various classifiers are designed via an SVM.

Conventionally, an SVM is derived as a classifier that separates input features into binary classes. To achieve multiple binary classifications, it is proposed to use the “one-vs-one” SVM; here, four classes of leak phase are considered: (a) liquid-to-liquid (L2L), (b) gas-to-gas (G2G), (c) gas-to-liquid (G2L), and (d) liquid-to-gas (L2G). Also, the four scenarios in which these types of leak commonly occur in the downhole are examined: through the tubing, through the casing, outside the tubing, and outside the casing. During the training phase, data from these four leak scenarios was simulated for each of the four classes of leak phase in the laboratory with various orifice sizes and differential pressures, to capture the general characteristics of each leak phase that may occur in the wellbore due to different leak sizes and flow rates.

This chapter is organized as follows. Section 6.2 presents the general concept of PCA, Section 6.3 gives an overview of the SVM, Section 6.4 presents the experimental results and detailed analysis, and Section 6.5 presents the conclusion.

Figure 6.1: Block diagram of the proposed SVM phase-classification approach.

6.2 PCA

PCA is a well-known statistical technique for feature extraction and dimensionality reduction of a dataset. The goal of PCA is to project an L -dimensional dataset into a reduced K -dimensional space, where $K < L$ is sufficient to represent the dataset [103].

Let $\mathbf{X} \in \mathbb{R}^{L \times M}$ be a PSD matrix of the sensor recording, with each row denoting the l th frequency bin of the PSD and each column denoting the m th experimental data. PCA is achieved by first computing the $L \times L$ covariance matrix, $\mathbf{R}_x$. Next, the eigenvectors $\mathbf{U} = [\mathbf{u}_1, \dots, \mathbf{u}_L]$ and their corresponding eigenvalue $\mathbf{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_L)$ are computed and sorted in decreasing order (i.e., $\lambda_1 \geq \dots \geq \lambda_L$). Mathematically, this is represented as:

$$\mathbf{R}_x = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^T. \quad (6.1)$$

Now, the signal $\mathbf{X}$ can be projected onto the K principal subspace of $\mathbf{R}_x$ as:

$$\mathbf{X}_{red} = (\mathbf{U}\mathbf{\Lambda}^{1/2}\mathbf{D}_K)^T \mathbf{X} = \mathbf{P}_K^T \mathbf{X}, \quad (6.2)$$

where $\mathbf{D}_K = [\mathbf{I}_K, \mathbf{0}]^T \in \mathbb{R}^{L \times K}$, with $\mathbf{I}_K$ being a $K \times K$ identity matrix and $\mathbf{0}$ is a $K \times (L - K)$ matrix of zeroes. The projection matrix $\mathbf{P}_K$ projects the data into the K principal subspace. Then, by selecting the K components that have a larger variance, the dimension of the recording can be reduced without significant loss of information from the data.

6.3 SVM Classifier

After dimension reduction via PCA, training of the SVM can be applied to (6.2) to give a classifier. SVM is a linear classifier that separates two groups with an optimal hyperplane in feature space. A high-dimensional hyperplane will always exist if the data are linearly separable. As depicted in Figure 6.2a, which is an example of a two-dimensional classification problem, there are many arbitrary ways to design the hyperplane, such as H_1 and H_2 .

Conversely, as depicted in Figure 6.2b, the SVM selects an optimal hyperplane H_{opt} that maximizes the distance from the margin or the dotted line that spans the nearest data points. These data points on the boundaries are the support vectors. Determining the support vectors is essential for the SVM to find the optimal H_{opt} . For example, consider a set of M training dataset of the following form:

$$(x_1, y_1), (x_2, y_2), \dots, (x_M, y_M), \quad (6.3)$$

where y_m is either +1 or -1, which indicates the class of the data point x_m . Note that after PCA, x_m is a K -dimensional vector ($K \ll L$). Figure 6.2 depicts x_m as a two-

dimensional vector, and $\mathbf{X}_1$, $\mathbf{X}_2$ are the two-dimensional entries of each data point.

Figure 6.2: Two-class classification with (a) an arbitrary separator (left) and (b) an SVM separator, (right).

The problem of finding the optimal SVM hyperplane H_{opt} can now be reduced to the following optimization problem:

$$\begin{aligned} \min_{b, \mathbf{w}} \quad & \frac{1}{2} \mathbf{w}^T \mathbf{w}, \\ \text{subject to} \quad & \mathbf{y}_m (\mathbf{w}^T \mathbf{x}_m + b) \geq 1, \quad \text{for } m = 1, \dots, M, \end{aligned} \quad (6.4)$$

where $\mathbf{w}$ and b are the separating hyperplane, $\mathbf{x}_m$ is the m th data point, and M is the number of data points. The optimization problem is now a quadratic programming (QP) problem and can be solved using conventional solvers, such as CVX [41]. In general, the computational complexity of a QP problem is $O(K^3)$ [104]. The complexity of $O(K^3)$ implies that selecting a small K principal subspace of $\mathbf{R}_x$ will substantially reduce the computational complexity of the training and prediction process.

In practice, most data are nonlinearly separable. To make the SVM more robust for use in such situations, it is necessary to introduce a variable, $\zeta_m \geq 0$, for each data point, to indicate the magnitude of the violation. Thus, ζ_m denotes how far a data point can

extend into the margin before arriving at the new soft margin and is equivalent to:

$$\begin{aligned} \min_{b, \mathbf{w}} \quad & \frac{1}{2} \mathbf{w}^\top \mathbf{w} + C \sum_{m=1}^M \zeta_m, \quad \zeta_m \geq 0, \\ \text{subject to } & \mathbf{y}_m (\mathbf{w}^\top \mathbf{x}_m + b) \geq 1 - \zeta_m, \quad \text{for } m = 1, \dots, M, \end{aligned} \quad (6.5)$$

where C is a penalty factor in the objective function to discourage large violations.

6.4 Experimental Results and Analysis

In this section, experimental data collected from control experiments inside a test facility are used to train the SVM classifiers. The classifiers are then used to classify a set of field data collected from wellbores with known leaks.

Section 6.4.1 contains the experimental setup in the test facility and the required preprocessing to construct the SVM classifier. Section 6.4.2 describes the feature extraction and classification verification using the data from the test facility. The results of applying the proposed method to the field data collected from a set of operational wellbores with known leaks are then discussed.

6.4.1 Setup and preprocessing

In the test facility, a test fixture was designed to simulate multiple flow scenarios in the oilfield. The fixture had two casings with diameters of 20.32 and 12.7 cm and an inner tubing with a diameter of 5.08 cm. A sensor array with $N = 6$ hydrophone sensors, centralized in the tubing, was fixed to measure the simulated leak acoustic. Two main types of leaks were simulated: (i) liquid leaks and (ii) gas leaks:

- (i) liquid leaks were simulated using a water pump with adjustable pressure; and
- (ii) gas leaks were simulated using pressurized cylinders of nitrogen gas with adjustable valve and pressure gauges to modify the outlet pressure.

Figure 6.3 depicts the four different leak scenarios that were considered, which correspond to the “Tubing to annulus A” leaks found in Section 1.2: (a) leak through the casing, (b) leak outside the casing, (c) leak through the tubing, and (d) leak outside the tubing. All four leak scenarios were repeated for the four classes of leak phases: L2L, G2G, G2L, and L2G, for a total of 16 scenarios. Furthermore, four orifice sizes and 10 differential pressures across the orifices were used for each scenario. Different orifice sizes corresponded to various simulated leak sizes. In sum, approximately 620 data points were collected.

Figure 6.3: Diagram of leaks in various simulated situations.

For each scenario, the tubing, inner annulus, and outer annulus were filled with either liquid or air, depending on the scenario and leak phase. For example, in the leak through tubing scenario with a leak phase of G2G, both the tubing and the inner annulus were filled with air. Likewise, for the same scenario, if the leak phase was G2L, the tubing was filled with liquid, and the inner annulus was filled with gas.

The data preprocessing steps comprised a PSD calculation, followed by spectrum averaging and normalizing over the available acquisitions and hydrophones. For each

dataset, data from six hydrophones were recorded for 3 minutes. Data with a 1-s duration were considered as an acquisition, and 180 acquisitions were recorded for each hydrophone. Thus, for each dataset, a total of 180×6 waveforms were captured from the array of six hydrophones. The PSD of each waveform was calculated separately and averaged over the acquisitions and hydrophones, giving a single spectrum PSD for each dataset. Figure 6.4 depicts a PSD from the four classes of leak phase. Each spectrum contains 819 PSD frequency bins corresponding to the frequency bins from 0 to 50 kHz.

Figure 6.4: PSD of the four various leak-phase scenarios: (top left) gas-to-gas; (top right) gas-to-liquid; (bottom left) liquid-to-gas; and (bottom right) liquid-to-liquid.

6.4.2 Feature extraction and classification

The spectra collected from the PSD are divided into four groups based on the leak phase, as shown in Figure 6.4. A significant difference can be seen between each leak phase. For instance, the L2L phase (Figure 6.4, bottom right) has most of its spectral content in the low-frequency region, whereas that of the G2G phase (Figure 6.4, top left) is distributed across the entire frequency region. By inspection, Figure 6.4 suggests that classification can be based on the spectral content of the various leak phases.

Accordingly, the recorded data are divided into segments, with 75% as training data and 25% as validation data. Next, as suggested in Section 6.2, PCA is used to sort the principal components based on their eigenvalues. Figure 6.5a shows the first 100 eigenvalues out of 819, whereas Figure 6.5b depicts the cumulative sum of eigenvalues. It can be seen that the cumulative sum percentage is lower when fewer PCA dimensions are selected, but it nonetheless achieves 75% with just a few PCAs.

Figure 6.5: (a) Eigenvalues analysis of data set, and (b) cumulative sum of eigenvalues from PCA analysis.

Conventionally, the information-theoretic criteria, such as the Akaike information criterion (AIC) or the minimum descriptive length (MDL) [105], can be used to estimate the number of principal components. However, these approaches are susceptible to the subspace swapping effect [106], [107]. Therefore, to decide on the optimal number of principal components, the SVM model is trained using k principal components, where $k \in [1, \dots, 100]$. Accordingly, each model is tested with the validation data, as shown in Figure 6.6. The optimal number of principal components is regarded as the number that results in the best classification performance of the validation data. It is evident in Figure 6.6b that the classification performance begins to decrease as the principal components are increased beyond the optimal principal number. This result contradicts the performance obtained with the training data set, where the classification performance increases as the number of principal increases. The reason for this contraction is due to overfitting; that is, when more principals are used, the classifier becomes overfitted to the training data set. Hence, based on this observation, the optimum principal components are determined to be the first eight eigenvalues, and the remaining eigenvalues are discarded.

Figure 6.6: Classification performance with: (a) training data, (left) and (b) validation data (right) .

Because the principal components preserve the features with the greatest variance,

the spectra can be reconstructed from the selected eight principal components by mapping these back to the original input space using their eigenvectors. Figure 6.7 shows that the spectra reconstructed from the first eight principal components are similar to the original PSD spectrum in Figure 6.4. For instance, G2G still occupies the entire spectrum from 0 to 50 kHz, whereas L2L only occupies the lower frequencies.

Figure 6.7: PSD of the four various leak-phase scenarios after projection of the first eight principal components: (top left) gas-to-gas, (top right) gas-to-liquid, (bottom left) liquid-to-gas, (bottom right) liquid-to-liquid.

Following the PCA dimensional reduction to eight principal components, the reduced dimension 8×620 $\mathbf{X}_{red}$ dataset is used to train the SVM model. The training is performed using the 10-fold cross-validation method, where the dataset is randomly partitioned into ten subsets of equal size. Nine of the ten subsets are used for training the model, and the remaining subset is retained as a validation set to test the model. This process is repeated for 10 iterations, with each of the 10 subsets used only once as the

test data.

The performances of the 10-fold cross-validations are tabulated into a confusion matrix, as shown in Table 6.1. Table 6.1a and Table 6.1b show the 10-fold cross-validations of the SVM-only and the PCA with the SVM classifier, respectively. In summary, the SVM-only and the PCA with SVM have overall accuracies of 44.4% and 89.4%, respectively. This comparison shows that the PCA with SVM performs better as a classifier because by training only with the principal components, the remaining components are considered to be noise and are not involved in the classification process. Training with only SVM introduces overfitting, as seen in Figure 6.6b, where the additional principals result in lower accuracy in the validation data set.

Table 6.1: Ten-fold cross-validation performance of: (a) PCA with SVM, (b) SVM.

		Prediction							Prediction				
		%	L2L	L2G	G2L	G2G			%	L2L	L2G	G2L	G2G
True Value	L2L	100.0	0.0	0.0	0.0	0.0	True Value	L2L	66.7	0.0	0.0	33.3	
	L2G	13.9	76.8	7.9	1.3			L2G	49.7	20.5	0.7	29.1	
	G2L	5.8	1.9	89.6	2.6			G2L	13.6	0.6	63.0	22.7	
	G2G	6.5	1.9	0.6	91.0			G2G	72.3	0.0	0.0	27.7	
(a) Spectrum, PCA, and SVM (89.4%)							(b) Spectrum and SVM (44.4%)						

For comparison, Table 6.2 tabulates two additional 10-fold cross-validation confusion matrices using the PCA with naïve Bayes (NB) classifier and a neural network (NN) classifier with two hidden layers, with both being other methods for acoustic classification [104]. From this comparison, the proposed PCA with SVM has a better 10-fold cross-validation performance (89.4%) than the PCA with the NN (85.6%) and the PCA with the NB (76.6%). Thus, the NB has the lowest performance because its assumptions that the features are independent might not be valid for well-leak signals.

Similarly, the performance of the PCA with SVM is observed to be 4.8% higher than that of PCA with a NN because the SVM's training process, which solves the quadratic optimization problem, overcomes the local minimum problem that is present in the NN classifier. This observation is also consistent with Zhang et al. [108].

Table 6.2: Ten-fold cross-validation performance of: (a) PCA with an NB, (b) PCA with an NN.

	Prediction				
	%	L2L	L2G	G2L	G2G
True Value	L2L	92.7	2.7	0.7	4.0
	L2G	7.9	55.0	31.8	5.3
	G2L	0.0	5.2	87.0	7.8
	G2G	2.6	11.0	14.8	71.6
(a) PCA with NB (76.6%)					

	Prediction				
	%	L2L	L2G	G2L	G2G
True Value	L2L	90.7	2.7	0	6.7
	L2G	5.3	75.5	12.6	6.6
	G2L	0	6.5	82.5	11.0
	G2G	2.6	2.6	5.2	89.7
(b) PCA with NN (84.6%)					

Finally, moving beyond experimental data, Table 6.3 records the performance of the four classifiers from Table 6.1 and Table 6.2 with a set of acoustic data collected from actual wellbores with a known leak phase. The set consisted of 10 field data: nine were L2L leaks, and one was a G2G leak. Each dataset had 180 acquisitions, and each acquisition was evaluated individually with each classifier. Table 6.3 tabulates the number of correct predictions from each of the 180 acquisitions and their respective accuracies.

The overall performances of the classifiers were determined. SVM-only has an accuracy of only 49.8%; NB has an accuracy of 71.8%; NN (with two hidden layers) has an accuracy of 85%; and the proposed PCA with SVM has an overall accuracy of 93.1%. These results are consistent with the performances from the 10-fold cross-validation in Table 6.1 and Table 6.2.

Based on the results from Table 6.3, it is clear that the PCA and the SVM methods of combining spectral estimation are promising approaches for classifying leak-phase flows from the measured acoustic signals in a real-world environment.

Table 6.3: Overall classification performance using actual wellbore leak data.

Phase Type	L2L	L2L	L2L	L2L	L2L	L2L	L2L	L2L	L2L	G2G	Total
Data Size	180	180	180	180	180	180	180	180	180	180	1800
Correct Prediction for SVM-only	44	0	0	150	0	180	180	160	0	180	894
Accuracy for SVM-only (%)	24.4	0	0	83.3	0	100	100	88.9	0	100	49.8
Correct Prediction for PCA with NB	160	180	9	92	180	180	121	119	180	2	1293
Accuracy for PCA with NB (%)	88.9	100.0	5.0	51.1	100.0	100.0	66.1	100.0	40.0	67.2	71.8
Correct Prediction for PCA with NN	180	180	167	180	180	180	101	180	180	2	1530
Accuracy for PCA with NN (%)	100.0	100.0	92.8	100.0	100.0	100.0	56.1	100.0	100.0	1.1	85
Correct Prediction for PCA with SVM	180	180	88	180	180	180	155	177	180	175	1675
Accuracy for PCA with SVM (%)	100.0	100.0	48.9	100.0	100.0	100.0	98.3	100.0	97.2	86.1	93.1

6.5 Conclusions

This chapter addresses the problem of leak classification during a well integrity evaluation. In addition to leak localization, early knowledge of the types of leak that occur inside a wellbore assists operators to remedy a problem before it worsens. Thus, an algorithm is developed to classify the leak-flow phases inside a wellbore, consisting of the PSD estimation of the acoustic measurements, followed by a dimension-reduction using the PCA and classification with an SVM.

Based on the conclusions from Section 1.2, four classes of leak phase in the different scenarios are considered. Experiments were conducted to record the acoustic waveforms for each of the four classes of leak phase in different scenarios using an acoustic array of hydrophone sensors to characterize the acoustics of the leak flow. To design the classifier, the PSD of the acoustic measurements are computed, followed by conducting PCA to select the best features from the spectrum. Finally, SVM is used to design the

classifiers.

In summary, the experimental data are used to train the classifier, and field data are used to test the classifier. The results demonstrate that the developed algorithm achieves an accuracy of 93.1%. In addition, the PCA with SVM performs better than SVM-only and better than PCA with NB or with NN (with two hidden layers) methods, with both the experimental and field data.

Chapter 7 Conclusions

In this chapter, the thesis summary is presented, conclusions are drawn from the research, and future directions for research are suggested. Section 7.1 describes the conclusions of the thesis and its main contributions, and Section 7.2 suggests some potential future work that could be carried out as an extension to the thesis.

7.1 Conclusions

This thesis investigates some of the challenges of well integrity evaluation via acoustic-signal processing of leak signals from the wellbore. The existing methods for well integrity evaluation are leak detection using single acoustic sensors and leak localization using sensor arrays. Even though various industrial well operators have adopted both approaches, each has limitations.

For instance, the single acoustic-sensor approach can only detect leaks along the depth of the wellbore, but it has a wide operating frequency of 15 to 60 kHz. In contrast, the sensor-array approach can localize a leak, both horizontally and vertically, giving the well operator the precise location of the leak inside the wellbore. However, a sensor array is unable to localize sources with frequency content higher than the spatial aliasing of an array, which limits its ability to detect leaks that consist of higher-frequency components (e.g., gas leaks). Also, additional information such as the class of leak can help well operators evaluate the wellbore condition.

The contributions of this thesis extend the operational capability of the existing methods. Accordingly, these contributions can be summarized into three parts: (1) enabling the localization of sources with frequency content beyond the array's spatial

aliasing limit; (2) achieving localization using a single sensor; and (3) providing timely information of leaks via acoustic classification.

First, the spatial aliasing problem in the linear array due to a far-field source is studied. A theorem is found that guarantees aliasing-free DOA estimation of far-field sources using sparse dictionary-based beamforming. However, the sparse beamforming approach is affected by the basis mismatch problem, and the estimation performance will degrade when the source is not subsumed in the dictionary. Recently, grid-free compressive beamforming was found to be an effective approach to avoid basis mismatch because the solution does not require a dictionary; nonetheless, it is formulated for a single frequency. Hence, spatial aliasing will still occur if the source frequency is beyond the array's spatial aliasing limit. This thesis shows that the grid-free compressive beamforming approach can be generalized to a multiband formulation, achieving both (a) an expansion of the application of the theorem, such that an aliasing-free DOA estimation is guaranteed, and (b) an extension of the grid-free approach beyond a single frequency.

Next, the spatial aliasing problem in the linear array due to a nearfield source is examined. It is found that nearfield sources are not spatially bandlimited. The conventional approach to mitigate this is to modify the array's aperture, such as by using a circular or planar array. Nonetheless, these approaches are not suitable in the well integrity evaluation context because the array's aperture is subjected to space constraints in the wellbore. To address this challenge, the thesis extends the theorem derived for the far-field source to a nearfield scenario. By using the second-order Taylor series expansion, it is shown that aliasing-free localization can be achieved if a source has two frequencies with an absolute difference in wavelength that is larger than twice $r_{\min}$ and

an array aperture smaller than $r_{\min}$, where $r_{\min}$ is the minimum Fresnel approximation distance of the array.

Next, the problem of localization using a single sensor is investigated. It is found that when the sensor is moving with a known velocity, it can be approximated as an OSA with multiple synthetic sensors. Further, if the source is harmonic, it is shown that a temporal correction can be derived to correct the drift due to the movement of the sensor. The localization performance under this consideration closely follows the CRB of a stationary array.

The situation of a non-harmonic source is also examined and investigated using the dROSA approach. It is found that the estimation performance of the dROSA follows closely with the CRB of the differential sensory array [99] when the source is located radially at approximately 6 cm, the typical range of the most common leaks (i.e., those in the tubing region). It is also found that the localization performance of the dROSA remains relatively constant for the moving sensor with a velocity of 15.24 cm/s along the sensor axis, the typical operating range of single-leak detection sensors [27]. The performance of the dROSA starts to degrade when the source is located radially further from the sensor axis.

Finally, a classification method for wellbore leak evaluation is considered. It is found that offline training of the experimental recording using an SVM with PCA is a practical approach to design the classifiers. The classifiers can sort the wellbore recordings collected from an oilfield into four categories of leaks: (i) liquid-to-liquid (L2L), (ii) liquid-to-gas (L2G), (iii) gas-to-liquid (G2L), and (iv) gas-to-gas (G2G). Overall, it is found that this approach achieves an accuracy of 93.1% correct predictions on 1800 acoustic recordings collected from 10 different oilfields.

7.2 Future work

Based on the sensor-array processing and single-sensor processing techniques developed in this thesis, several extensions to this work warrant further exploration.

In Chapter 3, it is found that the multiband grid-free compressive beamforming method generalizes the conventional grid-free compressive beamforming to allow the processing of wideband sources. The proposed multiband grid-free approach allows an aliasing-free DOA estimation via the grid-free method. However, increasing the number of frequency bins decreases the number of resolvable sources. This may be resolvable by the use of a nested array or the Khatri-Rao subspace approach [90], because the latter was reported to resolve more sources than the number of available sensors. Incidentally, increasing the number of frequency bins will also enable a better source signal strength reconstruction. Another potential direction is to achieve localization of far-field sources using the triangulation of multiple DOA estimations from a moving sensor [109].

In Chapter 4, a theorem is derived that guarantees nearfield aliasing-free localization. However, the theorem only allows for a limited number of frequencies to be utilized. Future work may extend this to cover the entire source spectrum. One possible approach is to decompose the spectrum into multiple smaller subsets, where the theorem is valid, and combine the contributions efficiently using methods like the multiple-parallel-branch with folding technique [110]. Also, the theorem is applied using the conventional sparse beamforming approach, which is subject to the problem of basis mismatch. Thus, aliasing-free localization using the grid-free method [111] will be an interesting direction to study and bridge the research gap between Chapters 3 and 4.

Next, in Chapter 5, the OSA problem is studied with the assumption that the velocity of the moving sensor is known. However, in actual operation, even though

the velocity is trackable based on the amount of track and wire spooling into the wellbore, it may not be particularly accurate, and the position and velocity may vary due to the swaying of the device inside the wellbore. Therefore, there may be a region of the recording in which the assumption of the dROSA may fail [112]. A future study could illuminate and enhance the dROSA approach for better localization using a single moving sensor. Likewise, although the dROSA approach is not affected by spatial aliasing, they are susceptible to the echo of leaks [20]. Additional studies using acoustic modeling [113] may enhance localization robustness.

Finally, in Chapter 6, the proposed classification method is designed, using the power-spectrum estimate of the signal from the sensor array. It may prove interesting to explore the classification performances obtained by learning [114] or using other well-known acoustic features. Specifically, the Mel-frequency cepstral, non-negative matrix factorization, and feature-engineering techniques [104], [115] may advance leak classification in well integrity evaluation.

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