Estimation of unsaturated shear strength from soil–water characteristic curve

Zhai, Qian; Rahardjo, Harianto; Satyanaga, Alfrendo; Dai, Guoliang

2019

Zhai, Q., Rahardjo, H., Satyanaga, A., & Dai, G. (2019). Estimation of unsaturated shear strength from soil–water characteristic curve. *Acta Geotechnica*, 14(6), 1977-1990.
doi:10.1007/s11440-019-00785-y

<https://hdl.handle.net/10356/144380>

<https://doi.org/10.1007/s11440-019-00785-y>

© 2019 Springer-Verlag Berlin Heidelberg. This is a post-peer-review, pre-copyedit version of an article published in *Acta Geotechnica*. The final authenticated version is available online at: <http://dx.doi.org/10.1007/s11440-019-00785-y>.

Downloaded on 26 Apr 2023 16:10:26 SGT

Estimation of unsaturated shear strength from soil-water characteristic curve

Qian Zhai^{1,2}, Harianto Rahardjo^{3*}, Alfrendo Satyanaga³ and Guoliang Dai¹

¹Key Laboratory for RC and PRC Structures of the Ministry of Education, Southeast University, Nanjing, 210096 China

²Formerly School of Civil and Environmental Engineering, Nanyang Technological University, Block N1, Nanyang Ave., Singapore 639798, Singapore.

³School of Civil and Environmental Engineering, Nanyang Technological University, Block N1, Nanyang Ave., Singapore 639798, Singapore.

* corresponding author. Email: chrahardjo@ntu.edu.sg

Email addresses of authors: zhaiqian@ntu.edu.sg, chrahardjo@ntu.edu.sg, alfrendo@ntu.edu.sg, and daigl@seu.edu.cn

Abstract

Many shallow foundations are constructed within the soil layer above the ground water table (GWT), where the soil remains unsaturated, and the failure of shallow foundation is mostly related to shear failure. The shear strength of the unsaturated soil is one of the main engineering properties required in geotechnical designs. Previous researchers suggested that the shear strength of the unsaturated soil depends on matric suction in the soil. The shape of the soil-water characteristic curve (SWCC) has a significant effect on the characteristics of unsaturated shear strength with respect to matric suction. In this paper, a new model was proposed for the estimation of the unsaturated shear strength from SWCC. In this new model, meniscus was considered to transfer soil suction into both additional net normal stress and

additional cohesion. Based on the categorization from soil science, water in soil can be categorized into three groups: (i), gravity water, (ii), capillary water, and (iii) hygroscopic water. The elemental analysis on the contractile skin indicated that only the capillary water in the soil can transfer stress into soil skeleton. Consequently, the SWCC is modified by considering capillary water only for the estimation of unsaturated shear strength. In the derivation, unsaturated soil is considered as four-phase material. Finally, a new mathematical equation for the estimation of the unsaturated shear strength was proposed and verified with the experimental data from published literatures. In addition, the proposed equation does not consist of any empirical parameter and can be used to predict the shear strength of unsaturated soil.

Key words: unsaturated shear strength, soil-water characteristic curve, meniscus, pore-size distribution function, capillary water

1 Introduction

The overall stability of many structures is dependent upon the strength of the underlying soil. Bearing capacity, lateral earth pressures, and slope stability are examples of common geotechnical applications that depend on the shear strength of the soil. It is important to quantify the shear strength of unsaturated soil and to be able to quantify changes in shear strength that might occur as a result of water infiltration into the soil. (Fredlund et al. 2012)

The soil above the ground water table (GWT) is normally considered as the unsaturated soil and the shear strength of the unsaturated soil is required in different geotechnical designs such as urban slopes in tropical areas, bearing capacity of shallow foundation, open-cut excavation work. Soil-water characteristic curve (SWCC) is commonly considered to be the controlling parameter for the estimation of unsaturated shear strength (Fredlund and Rahardjo, 1993). Therefore, various equations have been proposed by different researchers (Fredlund et

al. 1996, Vanapalli et al. 1996, Khalili and Khabbaz 1998, Toll and Ong 2003, Sheng et al. 2008, Goh et al. 2010, Sheng et al. 2011 and Schnellmann et al. 2014) to estimate the unsaturated shear strength from SWCC. It is observed that previous equations or models only considered the suction to contribute an extra net normal stress regardless of the mechanical property of the contractile skin.

In this paper, based on the elemental analysis on the contractile skin (or air-water interface) in soil, a new mathematical equation was proposed for the estimation of unsaturated shear strength. In the elemental analysis of the meniscus in soil, it is observed that meniscus due to the soil suction contribute to both additional net normal stress into the soil skeleton and additional adhesion between soil particles. Based on the categorization from soil science, water in soil can be categorized into three groups: (i), gravity water, (ii), capillary water, and (iii) hygroscopic water. It is observed that only capillary water in the soil can transfer stress into soil skeleton. The proposed mathematical equation for the estimation of the unsaturated shear strength was also verified with the experimental data from published literatures.

2 Literature Review

The shear strength envelope of soil is a measure of the ability of the soil to withstand the applied shear stresses. The soil will fail when the applied shear stresses equal or exceed the shear strength of the soil (Fredlund et al. 2012). The Mohr-Coulomb failure is a failure criterion that is commonly used to describe the shear strength of a saturated soil by using the effective stress variable (Terzaghi, 1936), as illustrated in Equation (1):

$$\tau_{ff} = c' + (\sigma_f - u_w)_f \tan \phi' \quad \text{----- (1)}$$

where: τ_{ff} = shear stress on the failure plane at failure, c' = effective cohesion, $(\sigma_f - u_w)_f$ = effective normal stress on the failure plane at failure, u_w = pore-water pressure at failure, and

ϕ' = effective angle of internal friction.

The mechanical behavior of an unsaturated soil is affected differently by changes in net normal stress than by changes in matric suction (Jennings and Burland, 1962). Fredlund and Morgenstern, (1977), Fredlund and Rahardjo (1993) and Fredlund et al. (2012) indicated that it is advantageous to recognize an unsaturated soil as a four-phase system when performing a stress analysis on an element. From a behavioral standpoint, an unsaturated soil can be visualized as a mixture with two phases that come to an equilibrium under the applied stress gradients (i.e, soil particles and contractile skin) and two phases that flow under the applied stress gradients (i.e., air and water). From the standpoint of the volume-mass relations for an unsaturated soil, it is possible to consider the soil as a three phase system since the volume of the contractile skin is small and its mass can be considered as part of the mass of water. However, when considering the stress analysis of a multiphase continuum, it is necessary to realize that the air-water interface behaves as an independent phase (Fredlund et al., 2012). In this case, Equation (1) must be extended to describe the shear strength of an unsaturated soil. Bishop (1959) extended Terzaghi's principle of effective stress of saturated soil into unsaturated soil and proposed the shear strength equation for an unsaturated soil as follows:

$$\tau = c' + [(\sigma - u_a) + \chi(u_a - u_w)] \tan \phi' \quad \text{----- (2)}$$

where u_a = pore-air pressure, u_w = pore-water pressure, $u_a - u_w$ = matric suction, χ = the parameter dependent on the degree of saturation.

Fredlund (1987) stated that stress state variables should be chosen such that the variables can be experimentally tested, theoretically justified, and operational in practice. With the concept of continuum mechanics, Fung (1965) and Fung (1977) explained, that variables describing a stress state, should be independent of soil properties. Using two independent stress variables, a linear form of the shear strength for an unsaturated soil was formulated by Fredlund et al. (1978) as follows:

$$\tau_{ff} = c' + (\sigma_f - u_a)_f \tan \phi' + (u_a - u_w)_f \tan \phi^b \quad \text{----- (3)}$$

where, ϕ^b = angle indicating the rate of increase in shear strength with respect to a change in matric suction.

The shear strength equation proposed by Bishop et al. (1960) and that proposed by Fredlund et al. (1978) appear to be similar in form. The χ parameter emerged in the shear strength equation from an extension of the effective stress equation. Attempts have been made to correlate the χ parameter with the degree of saturation of the soil (Bishop et al. 1960). Equating the linear form of the Fredlund et al. (1978) shear strength equation and Bishop et al. (1960) equation yields the following equation:

$$\chi = \frac{\tan \phi^b}{\tan \phi'} \quad \text{----- (4)}$$

Some of the earlier unsaturated soil shear strength data sets (e.g., Bishop et al., 1960) show a close fit to the linear shear strength equation (Fredlund and Rahardjo, 1993). The linear form (i.e., $\tan \phi^b$) is convenient to be used for illustrating the effect of matric suction on shear strength. Vanapalli et al. (1996) extended Fredlund et al. (1978)'s equation to model the non-linearity of the unsaturated shear strength of soil by incorporating the SWCC as follows:

$$\tau_{ff} = c' + \left[(\sigma_f - u_a)_f + \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right) (u_a - u_w)_f \right] \tan \phi' \quad \text{----- (5)}$$

where, θ_s = saturated volumetric water content, θ_r = residual volumetric water content, and θ = volumetric water content in soil with suction of $(u_a - u_w)_f$.

Besides Mohr-Coulomb failure criterion, several critical state shear strength equations have also been proposed for unsaturated soils (Alonso et al. 1990; Wheeler and Sivakumar, 1995; Toll, 1990, Maatouk et al. 1995; Lloret-Cabot et al. 2017 and Lloret-Cabot et al. 2018). The proposed equations attempt to describe the shear strength of an unsaturated soil under critical

state conditions in terms of q-p-r space. In general, the results show an increase in shear strength as matric suction is increased, but the character of the results varies for different equations. On the other hand, Sun et al. (2007a, 2007b, 2010), Sun and Sun (2012) and Gao et al. (2018) incorporated the coupling between hydraulic and stress-strain behavior in the proposed elastoplastic model for the estimation of shear strength and deformation of unsaturated soil.

Vanapalli et al. (1996) divided the entire suction range into three zones including (i) boundary effect zone, (ii) transition zone, and (iii) residual zone, as illustrated in Figure 1. In the boundary effect zone, the water phase is continuous while the air phase is discontinuous. The effect of suction on the shear strength is significant because majority of area in this zone are covered by water. In the transition zone, both water phase and air phase are continuous and the suction effect on the shear strength is dependent on the area of water on the cross section. In the residual zone, the water phase becomes discontinuous and the air phase becomes continuous. As a result, and the suction effect on the shear strength becomes insignificant because of the insignificant change in the area of water on the cross section due to a change in suction.

Figure 1. Illustration of the condition of water phase at different zones (Vanapalli et al. 1996)

3 Theory

The relationship between pore-size distribution function and soil-water characteristic curve is explained in this section. The effect of meniscus in soil on the unsaturated shear strength is explained. Based on elemental analysis on the meniscus in soil, it is recommended to modify the results of SWCC by removing water amount in intra-aggregate pores and hygroscopic water. Consequently, a mathematical equation for unsaturated shear strength is proposed.

3.1 Pore-size distribution function and SWCC

It is known that all pores in the soil have irregular shapes with different sizes. In order to explain the water flow in these pores, engineers simplified these irregular pores into capillary tubes (or circle on 2-D plan), as illustrated Figure 2 with different sizes. The number of tubes with radius of r is dependent on its pore-size density, $f(r)$, as illustrated in Figure 3 (from

Zhai and Rahardjo 2015).

(a) Pores in soil in irregular shape

(b) Simplified pores in soil

Figure 2. Illustration of simplification of pores in soil.

Figure 3. Illustration of conversion between pore-size distribution function and SWCC.

As illustrated in Figure 2(b), after simplification, there are a total of n_1 number of pores with radius of r_1 . The density of pores with an equivalent pore size of r_1 is $f(r_1)$. The total area occupied by pores with radius of r_1 on the cross-sectional area of the element can be calculated as follows:

$$A_1 = n_1 \pi r_1^2 \quad \text{----- (6)}$$

where, A_1 = the total area of pores with radius of r_1 , n_1 = total number of pores with radius of r_1 .

Summing the areas of all the pores gives the porosity of soil, n , as follows:

$$n = \sum_{i=1}^N A_i = \sum_{i=1}^N n_i \pi r_i^2 \quad \text{----- (7)}$$

where, A_i = the total area of the pore with radius of r_i , N = the total number of segments divided for entire pores in soil.

It is known that the ratio between A_1 and total porosity n defines the pore-size density for the pore with radius of r_1 . In this case, the pore-size density can be expressed as follows:

$$f(r_1) = \frac{A_1}{n} = \frac{n_1 \pi r_1^2}{\sum_{i=1}^N n_i \pi r_i^2} = \Delta S_1 \quad \text{----- (8)}$$

where, ΔS_1 = change in degree of saturation by drying pores with radius of r_1 , as illustrated in Figure 3.

3.2 Stresses on the Shear failure plane in soil during different stages

The soil element model during shearing under the application of suctions in three different zones are illustrated in Figure 4. When the suction is within the boundary effect zone, the shear failure band (or possible shear failure surface) does not cut through any contractile skin (air-water interface). The net normal stress is $\sigma - u_a$ and the additional normal stress is $u_a - u_w$. When the suction is within the transition zone, the possible shear failure surface does go through some dry pores whose quantity is dependent on the distribution of dry pores on the projected shear failure plane. The net normal stress is $\sigma - u_a$ and the additional normal stress

can be considered as $\frac{A'_i}{A'_0} (u_a - u_w)$, where, A'_i is the water area on the projected shear failure

plane and A'_0 is the total area of the projected shear failure plane. When the suction is within the residual zone, the possible shear failure surface does go through most of the dry pores whose quantity is still dependent on the distribution of dry pores on the projected shear failure plane. In other words, in the residual zone, suction effect on the shear strength does not disappear immediately. The additional net normal stress still can be calculated from

$$\frac{A'_i}{A'_0}(u_a - u_w).$$

(a) Illustration of sheared soil element when the suction is within boundary effect zone.

(b) Illustration of sheared soil element when the suction is within transition zone.

(c) Illustration of sheared soil element when the suction is within residual zone.

Figure 4. Illustration of shear failure bands within boundary effect, transition and residual

zones

It is noted when the shear plane cuts through dry pores, it will cut through the air-water interface or contractile skin as illustrated in Figure 5. Contractile skin is different from water phase and it has its tensile capacity. In this case, these contractile skin acts as bonding to pull all soil particles. Therefore, this contractile skin (meniscus) can contribute to the additional cohesion of soil.

Figure 5. Illustration of shear failure band with suction in transition and residual zones.

3.3 Estimation of pulling force on soil particles due to matric suction

Once the meniscus is created, the air-water interface curves due to the difference between pore-air pressure, u_a and pore-water pressure, u_w , as illustrated in Figure 6 (a). As the interface is curved, it generates compression force, F_c , on the soil skeleton as illustrated in Figure 6 (b) (where, F_c is the additional compression stress contributed from meniscus on the soil skeleton). In this way, the meniscus due to matric suction contributes to an additional net normal stress. Meanwhile, it generates pulling force, F_p , on solid particles (where, F_p is the additional pulling force contributed from meniscus on the soil skeleton). In this way,

meniscus acts as bonding paste to connect soil particles. As a result, the resultant force F will balance the surface tension force along the edge of meniscus. As explained in Section 3.1, in order to simplify the problem, irregular pores in soil are commonly converted into regular circles (or capillary tube) with different radii. Considering this, Figure 6 can be simplified into capillary tubes as illustrated in Figure 7, where R is radius representing the applied suction and r is radius representing the pore size, p_s is the additional pulling stress and α is the angle defines the relationship between radii R and r .

Figure 6. Illustration of the forces created by meniscus on the soil skeleton.

Figure 7. Illustration of the forces created by meniscus on the simplified capillary tube.

232

233 Based on the geometric relationship as illustrated in Figure 7, the additional normal force due
 234 to matric suction can be calculated by $(u_a - u_w)\pi r^2$ and the pulling force between soil
 235 particles can be calculated by $(u_a - u_w)A'$, where A' is the shaded area in Figure 7 (b). In
 236 fact, the shaded area can be calculated directly from R and r due to their geometrical
 237 relationship as illustrated in Figure 7(b). Based on the geometrical relationship, the following
 238 equation can be obtained to compute the shaded area:

$$239 \quad A' = R^2 \alpha - r \sqrt{R^2 - r^2} \quad \text{----- (9)}$$

240 where, angle α , $\alpha = \arcsin\left(\frac{r}{R}\right)$, R is the equivalent radius corresponding to soil suction, r
 241 is the equivalent radius of pores in soil.

242 Compared with the cross sectional area of the capillary tube, a non-dimensional ratio between
 243 the shaded area and the cross sectional area of the capillary tube η can be obtained as follows:

$$244 \quad \eta = \frac{R^2 \alpha - r \sqrt{R^2 - r^2}}{\pi r^2} \quad \text{----- (10)}$$

245 As it is difficult or impossible to measure the pore radius in soil, the pore radius is commonly
 246 converted into matric suction which can be directly measured in practice. If the contact angle
 247 does not change during the variation of matric suction, based on the capillary law (Fredlund
 248 and Rahardjo, 1993; Fredlund et al. 2012) as shown in Equation (11) and geometrical
 249 relationship as shown in Figure 7(b), the ratio between R and r could be calculated from the
 250 ratio between ψ_i and ψ_m as illustrated in Equation (12):

$$251 \quad r = \frac{2T_s \cos \alpha}{\psi} \quad \text{----- (11)}$$

252 where, r = radius of capillary tube; ψ = matric suction; T_s = surface tension; α = contact
 253 angle.

$$\frac{R}{r} = \frac{\psi_i}{\psi_m} \quad \text{----- (12)}$$

where, ψ_i = equivalent suction corresponding to pore radius of r_i , ψ_m = the applied matric suction.

Substituting Equation (12) into Equation (10), Equation (10) can be expressed into Equation (13) as follows:

$$\eta_i = \frac{1}{\pi} \left[\left(\frac{\psi_i}{\psi_m} \right)^2 \alpha_i - \sqrt{\left(\frac{\psi_i}{\psi_m} \right)^2 - 1} \right] \quad \text{----- (13)}$$

The pulling stress on a particular soil particle is dependent on the effective area of water that provides additional bonding between particles. The effective water area, which contributes the pulling stress between soil particles, can be calculated from pore size density multiple the ratio of η based on the relationship as shown in Figure 7(b). In this case, the pulling stress on soil particles can be calculated using the following equation:

$$p_i = \eta_i (u_a - u_w) f(r_i) = \frac{1}{\pi} \left[\left(\frac{\psi_i}{\psi_m} \right)^2 \alpha_i - \sqrt{\left(\frac{\psi_i}{\psi_m} \right)^2 - 1} \right] (u_a - u_w) f(r_i) \quad \text{----- (14)}$$

The total pulling stress can be obtained by the summation of the pulling stress in each pore. can be calculated as follows:

$$c_s = \sum_{i=m}^N \frac{1}{\pi} \left[\left(\frac{\psi_i}{\psi_m} \right)^2 \alpha_i - \sqrt{\left(\frac{\psi_i}{\psi_m} \right)^2 - 1} \right] (u_a - u_w) f(r_i) \quad \text{----- (15)}$$

3.4 Proposed equation for estimation of unsaturated shear strength from SWCC

As the pores are randomly distributed in soil, the pores will also be randomly distributed on the shear failure plane. In this case, the effective area on the shear failure plane can be calculated using the statistical method with the consideration of pore-size distribution

function.

Under suction of ψ_m , the pores r_1 to r_{m-1} are dry pores and the pores r_m to r_N are wet pores.

The probability of pore r_i to be distributed on the projected shear failure plane is $f(r_i)$. In

this case, the probability of the dry pores to be distributed on the projected shear failure plane

is $\sum_{i=1}^{m-1} f(r_i)$ and the probability of the wet pores to be distributed on the projected shear failure

plane is $\sum_{j=m}^N f(r_j)$. Therefore, the effective area of the compression stress due to matric

suction in the soil element can be calculated from $(1 - \sum_{i=1}^{m-1} f(r_i))$ or $\sum_{j=m}^N f(r_j)$.

The equivalent normal stress on the individual pore due to suction is

$$\sigma_{s_i} = (u_a - u_w) f(r_j) \quad \text{----- (16)}$$

Therefore, the equivalent normal stress converted from matric suction can be calculated from

equations (17) or (18) as follows (S = degree of saturation):

$$\sigma_s = (u_a - u_w) \left(1 - \sum_{i=1}^{m-1} f(r_i) \right) = (u_a - u_w) S \quad \text{----- (17)}$$

$$\sigma_s = (u_a - u_w) \sum_{j=m}^N f(r_j) = (u_a - u_w) S \quad \text{----- (18)}$$

Considering additional normal stress and additional cohesion due to contractile skin (or air-

water interface), the shear strength equation for unsaturated soil can be expressed as follows:

$$\tau = c' + (\sigma - u_a) \tan \phi' + c_s + \sigma_s \tan \phi' \quad \text{----- (19)}$$

Substituting Equations (15) and (18) into Equation (19), the equation for unsaturated shear

strength can be obtained as follows:

$$\tau = c' + (\sigma - u_a) \tan \phi' + S (u_a - u_w) \tan \phi' + \sum_{i=m}^N \frac{1}{\pi} \left[\left(\frac{\psi_i}{\psi_m} \right)^2 \alpha_i - \sqrt{\left(\frac{\psi_i}{\psi_m} \right)^2 - 1} \right] (u_a - u_w) [S(\psi_i + 1) - S(\psi_i)] \quad \text{----- (20)}$$

where, c' = effective cohesion, ϕ' = effective internal friction angle, σ = normal stress, u_a = pore-air pressure, u_w = pore-water pressure, ψ = matric suction, $S(\psi)$ = degree of saturation corresponding to matric suction of ψ and S = degree of saturation corresponding to $u_a - u_w$. Equation (19) is similar as Equation (3) but Equation (19) considers additional adhesion between soil particles due to meniscus.

3.5 Types of water in soil and its role in transferring stress into soil skeleton

Plaster (2009) introduced that water in the soil can be classified into three groups: (i), gravity water, (ii), capillary water, and (iii) hygroscopic water. Gravity water is free water that moves through the soil due to the force of gravity, capillary water moves due to capillary potential and the hygroscopic water forms very thin films around soil particle that is not held in the pores but on the particle surface and is held very tightly by forces of adhesion. Therefore, hygroscopic water that is adhered to the isolated soil particles cannot transfer suction into soil skeleton. Plaster (2009) recommended that when the suction is more than 31 bars, the water that exists in soil is mainly hygroscopic water as illustrated in Figure 8.

Figure 8. Illustration of hygroscopic water in soil (after Plaster 2009).

On the other hand, it is known that soil particles may also contain certain amount of pores,

which are commonly named as intra-aggregate pores. Pepper et al. (2009) explained the intra-aggregate pores in soil particles as illustrated in Figure 9.

Figure 9. Illustration of intra-aggregate pores (from Pepper et al. 2009).

Comparing Figures 1, 2, 4, 8 and 9, both the hygroscopic water and water in intra-aggregate pores cannot transfer stress from suction to soil skeleton because these two forms of water only exist in the isolated soil particles and not in between soil particles. On other hand, it is known that the amount of water measured in SWCC includes both of these two forms of water. Therefore, the measured result of SWCC needs to be modified by removing the hygroscopic water and the water in intra-aggregate pores for the estimation of unsaturated shear strength. In this case, the term S_{cp} is used to define the water content that can be affected by capillary force and the term S' is used to define the water content corresponding to the hygroscopic water and the water in intra-aggregate pores that cannot be affected by the capillary force (where, $S = S_{cp} + S'$). In this study, the degree of saturation corresponding to 3100 kPa (as recommended by Plaster 2009) is considered as S' . To be consistent with the pore-size distribution function, a normalized degree of saturation, S_n , (where, $S_n = \frac{S - S'}{1 - S'}$) is used to re-define the SWCC as illustrated in Figure 10.

Figure 10. Illustration of modified SWCC using normalized degree of saturation.

Therefore, Equation (20) can be revised by using the modified SWCC as illustrated in Figure 10 into Equation (21) as follows:

$$\tau = c' + (\sigma - u_a) \tan \phi' + \frac{S - S'}{1 - S'} (u_a - u_w) \tan \phi' + \sum_{i=m}^N \frac{1}{\pi} \left[\left(\frac{\psi_i}{\psi_m} \right)^2 \alpha_i - \sqrt{\left(\frac{\psi_i}{\psi_m} \right)^2 - 1} \right] (u_a - u_w) [S(\psi_i + 1) - S(\psi_i)] \quad \text{----- (21)}$$

4 Verification of proposed equations using experimental data from literature

Due to the limited experimental data with both SWCC and shear strength, three sets of data including a compacted residual soil from Jurong formation from Rahardjo et al. (2004), compacted kaolin from Thu et al. (2007, 2008) and a reconstituted silty clay from Cunningham et al. (2003) were used for the verification of the proposed equations. As the original SWCC data of the compacted residual soil from Rahardjo et al. (2004) was reported in the form of gravimetric water content, the SWCC for the compacted residual was converted into degree

of saturation by considering certain soil volume changes before it was used for the estimation of the unsaturated shear strength. Thu et al. (2007) reported a full set of SWCC with the confining pressure of 100 kPa (SWCC₁₀₀) and the different air-entry values with the confining pressures of 200 kPa and 300 kPa. The pore size distribution function in soil changed with the increase in the confining pressure. Therefore, the air-entry values (AEV) of these SWCCs are different for the same soil specimen. In this case, SWCCs under the confining pressure of 200 kPa (SWCC₂₀₀) and 300 kPa (SWCC₃₀₀) were converted from SWCC₁₀₀ based on the ratio between these air-entry values as reported by Thu et al. (2007). On the other hand, the silty clay from Cunningham et al. (2003) was isotopically pre-consolidated to 130 kPa prior to the SWCC measurements. There is not much difference between SWCCs with the confining pressures of 50 kPa, 100 kPa, and 200 kPa. In this case, only one SWCC for the silty clay was used for the estimation of its unsaturated shear strength. The results of unsaturated shear strength reported in some literatures were presented in the form of cohesion intercept c and friction angle ϕ' while some were presented in the form of peak deviator stress, q . For consistency purposes, all peak deviator stresses, q , were converted into effective cohesion intercept c' and ϕ' using Mohr circle as illustrated in Figure 11.

Figure 11. Illustration of geometrical relationship between effective cohesion c' and peak

deviator stress.

Based on the geometrical relationship illustrated in Figure 1, the interpreted cohesion can be calculated as follows:

$$c' = \frac{\frac{(\sigma'_1 - \sigma'_3)}{2} - \frac{(\sigma'_1 + \sigma'_3)}{2} \sin \phi'}{\cos \phi'} \quad \text{----- (22)}$$

The SWCCs for the compacted residual soil, SWCC₁₀₀, SWCC₂₀₀, and SWCC₃₀₀ for the compacted kaolin and SWCC for the silty clay are presented in Figure 12.

(a) SWCCs for a compacted residual soil from Rahardjo et al. (2004) and a silty clay from Cunningham et al. (2003)

(b) SWCCs for a compacted kaolin from Thu et al. (2007)

Figure 12. SWCCs for compacted residual soil, silty clay and compacted kaolin.

The fitting parameters of these SWCCs using Fredlund and Xing (1994)'s equations, as shown in Equation (23), were obtained from the regression analysis and they are presented in Table 1. SWCC variables such as the air-entry value (AEV), residual suction (ψ_r), and residual saturation (S_r) were calculated using the equations proposed by Zhai and Rahardjo (2012) and Zhai et al. (2017) and they are listed in Table 2.

$$\theta = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m} = \left[1 - \frac{\ln \left(1 + \frac{\psi}{C_r} \right)}{\ln \left(1 + \frac{10^6}{C_r} \right)} \right] \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m} \quad \text{----- (23)}$$

where: a, n, m = fitting parameters; C_r = an input value related to the residual suction which can be roughly estimated as $C_r = 1500$ kPa for most cases; θ_s = saturated volumetric water content.

Table 1. Fitting parameters of the estimated SWCCs

Soils	a (kPa)	n	m	C _r (kPa)
Compacted residual soil	194.2	2.92	0.68	1500
Silty clay	919.42	2.10	1.11	1500
SWCC ₁₀₀	83.4	3.76	0.74	1500
SWCC ₂₀₀	95	3.8	0.7	1500
SWCC ₃₀₀	101	3.8	0.65	1500

Table 2. SWCC variables for the soils

Soils	AEV(kPa)	ψ_r (kPa)	S _r
Compacted residual soil	125.9	716.2	0.286
Silty clay	465.7	2015	0.229
SWCC ₁₀₀	59.5	256.2	0.204
SWCC ₂₀₀	65.4	327.2	0.22
SWCC ₃₀₀	67.3	407.4	0.242

The experimental data of shear strength with respect to matric suction for these three types of soil are provided in Figure 13 as follows:

(a) Shear strength results for compacted residual soil

(b) Shear strength results for silty clay

(c) Shear strength results for compacted kaolin.

Figure 13. Shear strengths for compacted residual soil, silty clay and compacted kaolin.

5 Results and discussions

The shear strength with respect to matric suction for these three types of soil were estimated from their respective SWCCs using the equation proposed in this study. The predicted results are illustrated in Figure 14.

(a) Predicted results on shear strength for compacted residual soil from Rahardjo et al. (2004)

(b) Estimated results on shear strength for silty clay from Cunningham et al (2003)

(c) Estimated results on shear strength for compacted kaolin from Thu et al. (2007)

Figure 14. Estimated shear strengths for compacted residual soil, silty clay and compacted kaolin.

The coefficient of determination R^2 for the predicted results presented in Figure 14 are shown in Table 3.

Table 3. The coefficient of determination R^2 for the predicted results using the equation proposed in this study.

Soils		R^2		
Compacted residual soil	Silty clay	Confining pressure of 50 kPa	Confining pressure of 100 kPa	Confining pressure of 200 kPa
		96.13%	90.85%	96.80%
Compacted kaolin		Confining pressure of 100 kPa	Confining pressure of 200 kPa	Confining pressure of 300 kPa
		94.79%	94.80%	98.34%

Figure 14 and Table 3 indicate that the estimated results of the shear strength using the proposed equation (Equation 23) agree well with the experimental data for all these three

types of soil (i.e., R^2 is high for all the prediction). Equation (23) was derived based on the statistical theory and concept of the pore-size distribution function and there is no empirical parameter in the equation. In other words, the statistical method, which is commonly used for the estimation of hydraulic conductivity of unsaturated soil, can also be used for the estimation of the shear strength.

If the hygroscopic and intra-aggregate water component is not removed (calculated directly from Equation 20), the estimation results for these three types of soil are illustrated in Figure 15.

(a) Estimated results on shear strength for compacted residual soil

(b) Estimated results on shear strength for silty clay

(c) Estimated results on shear strength for compacted kaolin

Figure 15. Estimated shear strengths for three types of soil considering the entire SWCC.

The estimated shear strength considering the entire SWCC for three types of soil are less accurate than that considering the removal of hydrosopic and intra-aggregate water.

On the other hand, the residual saturation, S_r , is also commonly used for the estimation of the shear strength of unsaturated soil. In comparison, the shear strengths of compacted kaolin are re-estimated using the proposed equation with respect to degree of saturation, S_r (as shown in Table 2) by replacing S' in Equation (23). The re-estimated shear strength results are illustrated in Figure 16.

Figure 16. The re-estimated shear strengths for compacted kaolin by using residual saturation.

Figure 16 indicates that the re-estimated results using the residual saturation underestimate the shear strength of soil. Therefore, it seems that the maximum suction (or minimum saturation) used for the estimation of unsaturated shear strength is different from the conventional residual suction (or residual saturation) which can be defined using the graphical method as introduced by Vanapalli et al. (1998) and Zhai et al. (2012).

6 Conclusions

A new mathematical equation for the estimation of unsaturated shear strength is proposed and verified with experimental data. In the proposed equation, there is no empirical parameters

and the equation was derived based on the statistical theory. In addition, unsaturated soil is considered as “four-phase material” in the derivation. Based on the elemental analysis of the contractile skin (or meniscus) in soil, it is observed that meniscus can transfer stress into soil skeleton as an additional net normal stress and produce an additional bonding connecting soil particles or namely, an additional cohesion. It is also observed that the hygroscopic water and water in intra-aggregate pores should not be considered in the estimation of unsaturated shear strength because these types of water only exist in isolated soil particles and cannot transfer any stress into soil skeleton. As a result, a saturation, S' , which defines the amount of water in soil that cannot transfer stress into soil skeleton or the degree of saturation corresponding to suction of 3100kPa, rather than residual saturation, S_r , is recommended for the estimation of unsaturated shear strength.

7 Acknowledgement

The first author would like to acknowledge the financial supports he received from the National Natural Science Foundation of China (No. 51878160), the National Key Research and development program of China (No. 2017YFC00703408) and the Fundamental Research Funds for the Central Universities (No. 2242018K41046).

Reference:

- Alonso EE, Gens A, Josa A (1990) “*A constitutive model for partially saturated soils.*” *Geotechnique* 40(3):405–430
- Bishop, A. W. (1959) “*The principle of effective stress*” *Teknisk Ukeblad*, Norwegian Geotechnical Institute, Vol. 106, No. 39, pp. 859–863.
- Bishop, A. W., Alpan, I., Blight, G. E., and Donald, I. B. (1960) “*Factors controlling the shear strength of partly saturated cohesive soils*” paper presented at the Research Conference on Shear Strength of Cohesive Soils, ASCE, University of Colorado, Boulder, CO, pp. 503–

532.

Cunningham MR, Ridley AM, Dineen K, Burland JB (2003) "*The mechanical behaviour of a reconstituted unsaturated silty clay.*" *Geotechnique* 53(2):183–194

Fredlund, D. G. (1987). "*The stress state for expansive soils*" Proceedings of the Sixth International Conference on Expansive Soils, New Delhi, pp. 524–534.

Fredlund, D. G., and Morgenstern, N. R. (1977) "*Stress state variables for unsaturated soils*" *Journal of Geotechnical Engineering Division, ASCE*, Vol. 103, No. GT5, pp. 447–466.

Fredlund DG, Morgenstern NR, Widger A (1978) "*Shear strength of unsaturated soils.*" *Can Geotech J* 15:313–321

Fredlund, D.G. and Xing, A. (1994) "*Equations for the soil-water characteristic curve.*" *Canadian Geotechnical Journal*, 31(3): 521-532.

Fredlund, D. G. and Rahardjo H. (1993) "*Soil Mechanics for unsaturated soil*", Wiley, New York.

Fredlund D.G., Rahardjo H. and Fredlund M.D. (2012) "*Unsaturated Soil Mechanics in Engineering Practice.*" Wiley, New York

Fredlund DG, Xing A, Fredlund MD, Barbour SL (1996) "*The relationship of unsaturated soil shear strength to the soil–water characteristic curve.*" *Can Geotech J* 33:440–448

Goh, S.G., H. Rahardjo and E.C. Leong (2010). "*Shear Strength Equations for Unsaturated Soil Under Drying and Wetting*". *ASCE Journal of Geotechnical and Geoenvironmental Engineering*. April, Vol. 136, No.4, pp. 594 –606. DOI: 10.1061/(ASCE)GT.1943-5606.0000261

Fung, Y. C. (1965) "*Foundations of Solid Mechanics*", Prentice-Hall, Englewood Cliffs, NJ.

Fung, Y. C. (1977) "*A First Course in Continuum Mechanics*", 2nd ed., Prentice-Hall, Englewood Cliffs, NJ.

Gao Y, Sun DA, Zhu AC, Xu YF, (2018) "*Hydromechanical behavior of unsaturated soil*"

with different initial densities over a wide suction range” Acta Geotech.

<https://doi.org/10.1007/s11440-018-0662-5>

Jennings J.E.B. and Burland J.B. (1962) “*Limitations to the Use of Effective Stresses in Partly Saturated Soils*” Géotechnique Volume 12 Issue 2, pp. 125-144

Khalili N, Khabbaz MH (1998) “A unique relationship for the determination of the shear strength of unsaturated soils.” Geotechnique 48(5):681–687

Lloret-Cabot M, Wheeler SJ, Sánchez M, (2017) “*A unified mechanical and retention model for saturated and unsaturated soil behavior*” Acta Geotechnica, 12, 1-21

Lloret-Cabot M, Wheeler SJ, Jubert AP, Romero E, Sheng DC (2018) “*From saturated to unsaturated conditions and vice versa*” Acta Geotechnica, 13, 15–37

Maa'touk, A., Leroueil, S., and La Rochelle, P. (1995) “*Yielding and critical state of collapsible unsaturated silty soil.*” Geotechnique, 45(3) 465–477.

Plaster E.J. “*Soil Science and Management*” Clifton Park, NY : Delmar,

Pepper I.L., Gerba C.P., Gentry T. J. and Maier R.M. (2009) “*Environmental Microbiology*” Academic Press; 2nd Edition

Rahardjo, H., B.H. Ong and E.C. Leong (2004). “*Shear Strength of a Compacted Residual Soil from Consolidated Drained and Constant Water Content Triaxial Tests*”. Canadian Geotechnical Journal, June, Vol. 41, No.3, pp. 421-436. DOI: 10.1139/t03-093

Schnellmann, R., H. Rahardjo and H. R. Schneider (2014), “*Controlling Parameter for Unsaturated Soil Property Functions: Validated on The Unsaturated Shear Strength*”, Canadian Geotechnical Journal, August, Vol. 52, pp. 374-381. DOI: 10.1139/cgj-2013-0278

Sheng D, Fredlund DG, Gens A (2008) “*A new modelling approach for unsaturated soils using independent stress variables.*” Can Geotech J 45:511–534

Sheng D, Zhou A., and Fredlund D.G. (2011) “*Shear strength criteria for unsaturated soils*” Geotech Geol Eng (2011) 29: 145-159.

- 526 Sun D.A., Sheng D.C., Sloan S.W. (2007a) *"Elastoplastic modelling of hydraulic and stress-*
527 *strain behaviour of unsaturated compacted soils."* Mech Mater, 39(3):212-21.
- 528 Sun D.A., Sheng D.C., Xu Y.F. (2007b) *"Collapse behaviour of unsaturated compacted soils*
529 *with different initial densities."* Can Geotech J , 44(6):673-86.
- 530 Sun W.J., and Sun D.A. (2012) *"Coupled modelling of hydro-mechanical behavior of*
531 *unsaturated compacted expansive soils."* International Journal for numerical and analytical
532 methods in geomechanics, 36: 1002-1022.
- 533 Sun D.A., and Sun W.J., Li X. (2010) *"Effect of degree of saturation on mechanical behavior*
534 *of unsaturated soils and its elastoplastic simulation."* Computers and Geotechnics, 37: 678-
535 688.
- 536 Terzaghi, K.(1936) *"The shear strength of saturated soils"* Proceedings of the First
537 International Conference on Soil Mechanics and Foundation Engineering, Cambridge, MA,
538 Vol. 1, pp. 54–56.
- 539 Toll DG (1990) *"A framework for unsaturated soil behavior"*. Geotechnique 40(1):31–44
- 540 Toll DG, Ong BH (2003) *"Critical state parameters for an unsaturated residual sandy clay."*
541 Geotechnique 53(1): 93–103
- 542 Thu TM, Rahardjo H, Leong EC (2007) *"Critical state behavior of a compacted silt*
543 *specimen."* Soils Found 47(4):749–755
- 544 Thu TM, Rahardjo H, Leong EC (2008) *"Soil–water characteristic curve and consolidation*
545 *behavior for a compacted silt."* Can Geotech J 44:266–275
- 546 Vanapalli SK, Fredlund DG, Pufahl DE, Clifton AW (1996) *"Model for the prediction of*
547 *shear strength with respect to soil suction."* Can Geotech J 33:379–392
- 548 Vanapalli S.K., W.S. Sillers, and M.D. Fredlund(1998) *"The meaning and relevance of*
549 *residual state to the unsaturated soil."*Can. Geotech. Conf. October 4-7
- 550 Wheeler, S. J., and Sivakumar, V. (1995) *"An elasto-plastic critical state framework for*

551 *unsaturated soils.*” Geotechnique, 45, 35–53.

552 Zhai Q, Rahardjo H. (2012) *"Determination of soil–water characteristic curve variables."*

553 ComputGeotech 2012;42:37–43.

554 Zhai Q, and Rahardjo H. (2015) *“Estimation of permeability function from Soil-Water*

555 *Characteristic Curve”* Engineering Geology 199 (2015) 148-156.

556 Zhai Q., Rahardjo H. and Satyanaga, A. (2017) *“Effects of residual suction and residual*

557 *water content on the estimation of permeability”* Geoderma 303: 165-177