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The role of pore-size distribution function on water flow in unsaturated soil

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Abstract

The hydraulic properties of soil (i.e. soil-water characteristic curve (SWCC) and permeability function) govern the moisture flow in soil. The works from previous research indicated that hydraulic properties of soil are dependent on the pore-size distribution in soil. An improved capillary model was proposed to explain the concept of the pore-size distribution in soil and its relationship to SWCC. A new model named “valve model” was also proposed for the explanation of water flow in unsaturated soil. The pore-size distribution function was incorporated in the “valve model” and used to calculate the relative coefficient of permeability, k_r , (where k_r = unsaturated coefficient of permeability/ saturated coefficient of permeability) for unsaturated soil. In this paper, the role of pore-size distribution function on

the estimation of SWCC and permeability function are explained. Equations were proposed to estimate the pore-size distribution function from the experimental data of relative coefficient of permeability. The results from the proposed equations agree with the experimental data from laboratory measurement and published data from literature.

Key words: soil-water characteristic curve, hydraulic properties, pore-size distribution function, permeability function, valve model

1. Introduction

The flow of fluids through soils is of great significance in the fields of geotechnical engineering, oil technology and agriculture, etc. The hydraulic properties of unsaturated soil (including soil-water characteristic curve and permeability function) are governing factors that control moisture flow in unsaturated soil. Determination of the hydraulic properties of soil (including direct method such as laboratory or field measurement and indirect method such as estimation from index properties of soil) are important to assess the amount of seepage through underground soil.

Multi-fluid flow processes in soil are governed by geometrical pore-space characteristics (Tuli et al. 2005). Therefore, the pore-size distribution in soil plays an important role in water movement in soil such as water drainage from (or storage in) the soil (to be represented as soil-water characteristic curve) and water flow through the soil layer (to be represented as permeability function). Vogel (1997) and Wildenschild et al.(2005) presented that the lack of knowledge on the control of the pore geometry on water flow and transport in soil led to incidental microscopic study of the investigation of water flow and transport properties to be a function of geometrical pore-space properties. In this paper, the microscopic studies on the pores inside the soil were carried out to understand the principle of water flow in unsaturated soil.

The objective of this paper is to explain the role of the pore-size distribution function on the results of SWCC and permeability function. It is observed that the pore-size distribution function acts as a bridge to convert SWCC to a permeability function, and vice versa. A mathematical equation is also proposed to estimate the pore-size distribution function from the experimental permeability data.

2. Literature Review

Soil-water characteristic curve (SWCC) or soil water retention curve defines the relationship between the amount of water in the soil and soil suction. SWCC has been proven to be an interpretive model that uses the elementary capillary model to provide an understanding of the distribution of water in the voids. (Fredlund and Rahardjo 1993, Fredlund et al. 2012). In addition, the work from Childs and Collis-George (1950) and Zhai and Rahardjo (2015) recommended that SWCC to be considered as analogous to the pore-size distribution function. Therefore, SWCC can provide information of pore-size distribution in soil (or pore-size distribution function).

SWCC is expressed using a continuous mathematical equation which is commonly named as SWCC best-fit equation. Various forms of SWCC best fit equations have been proposed by different researchers (Brooks and Corey 1964; Fredlund and Xing 1994; Gardner 1958; Kosugi 1994; Pedroso et al. 2009; van Genuchten 1980). Fredlund and Xing (1994) adopted the concept of pore-size distribution and proposed the best-fit equation by integration of pore-size distribution function. Studies by Leong and Rahardjo (1997) and Zapata (1999) concluded that Fredlund and Xing (1994)'s equation performed best for best fitting the SWCC data for a wide range of soil over the entire range of matric suction. In addition, Zapata (1999) also suggested that van Genuchten's (1980) equation could perform well for best fitting fine-grained soils. Therefore, both Fredlund and Xing (1994)'s equation and van

Genuchten's (1980) equation are selected to describe the pore-size distribution in soil. As SWCC in the form of degree of saturation is used to represent pore-size distribution function, as recommended by Zhai and Rahardjo (2015), both Fredlund and Xing (1994)'s equation and van Genuchten's (1980) equation were expressed in the form of degree of saturation, respectively, as follows:

$$S = 1 - \frac{\ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)} \frac{1}{\left\{ \ln\left[e + \left(\frac{\psi}{a}\right)^n \right] \right\}^m} \quad \text{----- (1)}$$

where,

C_r = input value, can be a roughly estimated value for residual suction, $C_r = 1500$ kPa for most cases as suggested by Fredlund and Xing (1994) and Zhai and Rahardjo (2012a, 2012b)

ψ = matric suction,

e = Euler's number and

a , n and m = fitting parameters.

$$\Theta = \frac{S - S_r}{1 - S_r} = \left[\frac{1}{1 + (ah)^b} \right]^c \quad \text{----- (2)}$$

where,

Θ = normalized degree of saturation,

S = degree of saturation

S_r = residual saturation, Zhai et al. (2017b) recommended to treat S_r in van Genuchten's (1980) equation to be a fitting parameter

h = pressure head and

a , b and c = fitting parameters.

Lambe (1955) indicated that permeability of soil was dependent on soil properties such as soil composition, degree of saturation, void ratio, pore fluid and its structure (including grain

size). Tuli and Hopmans (2004) found that both pore geometry and size distribution were the main factors determining the functional relations between degree of fluid saturation and hydraulic or air conductivity. Childs and Collis-George (1950) adopted the concept of pore-size distribution function and derived the equation for estimation of the coefficient of permeability of soil with respect to different degrees of saturation (or permeability function). Mualem (1976) assumed that the simplified capillary tubes in Childs and Collis-George (1950)'s model to have certain lengths and proposed the pore tortuosity-connectivity parameter to improve Childs and Collis-George (1950)'s method for the estimation of permeability function. However, Mualem (1976) also indicated that there was no procedure available to determine this tortuosity-connectivity parameter and it only can be estimated empirically. On the other hand, Zhai et al. (2018) recommended considering the effect of soil volume change (change in the pore-size distribution function) rather than using the empirical parameter (such as tortuosity-connectivity parameter) to estimate the relative hydraulic conductivity. Therefore, the effect of tortuosity-connectivity on the permeability function is not considered in this paper. Childs and Collis-George (1950)'s model has been improved and extended by Marshall (1958), Kunze et al. (1968), Fredlund et al. (1994) and Zhai and Rahardjo (2015), Zhai et al. (2017a, b). In this case, the unsaturated coefficient of permeability also contains the information about the pore-size distribution in soil. However, there is no valid method or equation that can be used to estimate the pore-size distribution function from the measured unsaturated permeability data. In this paper, a mathematical equation was proposed to estimate pore-size distribution function from measurement data of permeability and the results from this equation agree with experimental data from SWCC measurement.

3. Theory

The limitations of the conventional capillary model were explained in this Section. An improved capillary model for description of the pores in soil was introduced. The pore-size distribution function was incorporated in this improved capillary model. In addition, a new model named “valve model” was proposed to explain water flow in unsaturated soil. With the new model, the relationship between the pore-size distribution function and permeability function can be explained clearly.

3.1 Conventional Capillary model

The conventional capillary model was based on the "bundle of cylindrical capillaries" (BCC) representation of pore space geometry (Millington and Quirk, 1961; Mualem, 1976). The BCC representation postulates that a portion of interconnected cylindrical pores is completely liquid-filled at a given matric potential whereas larger pores are completely empty. The idealization of soil pore space model for the ease of explaining some aspects of unsaturated soil behavior was later found, but this model introduced serious limitations in the general applications to unsaturated soil mechanics (Fredlund and Rahardjo 1993, Fredlund et al. 2012, Zhai et al. 2017c)

In general, the limitations of conventional capillary models can be summarized as follows:

1. There is only a single tube for particular pore size, which cannot represent the pore-size density for different pore sizes.
2. The water height in the tube is based on the capillary height and the total water amount in all these tubes may not be consistent with the water amount in soil with changes in matric suction.
3. The water drainage through a tube is piston-like, either fully filled or fully empty, which cannot represent actual water movement in an irregular pore.

4. The tubes are not connected to each other. As a result, this model cannot be used to explain the water flow in unsaturated soil.

5. As the tubes are not connected to each other, no “ink-bottle” effect can be created which makes this model to be incapable in explaining the wetting process in the soil.

3.2 Improved Capillary model

In order to overcome the limitations of the conventional capillary model as explained in Section 3.1, a new capillary model was proposed. At first an irregular pore is simplified into a series of circular pores with different sizes as illustrated in Figure 1. Secondly, on the cross-sectional area there are numbers of circular pores with different radius, which are converted from different irregular pores, as illustrated in Figure 2 (b). The number of circular pore with radius of r is dependent on its pore-size density, $f(r)$, rather than a single pore. In this case, the soil element as illustrated in Figure 2(a) can be simplified as Figure 2(b) and a 3-D perspective view of simplified soil element is illustrated in Figure 3.

In the simplified element, the length, the width and the height of soil element is considered as

1. The porosity of soil can be defined by total volume of these cylindrical tubes (or capillary tubes) $\sum V_{\text{tube}}$. As the width of soil element is defined as 1, the porosity of soil can also be defined as the area of circular pores on the cross-sectional area, $\sum A_{\text{pore}}$, as follows:

$$\sum A_{\text{pore}} = \sum_{i=1}^N n_i \pi r_i^2 \quad \text{----- (3)}$$

where, $\sum A_{\text{pore}}$ = total area of circular pores on the cross-sectional area, n_i = total numbers of the pores with radius of r_i , r_1 is the maximum radius of pore, r_N is the minimum radius of pore. The pore-size distribution function defines the ratio of the volume of pores with certain radius of r_i to the total volume of pores, and the ratio is commonly defined as pore-size density $f(r_i)$. Therefore, the pore-size density of pores with radius of r_i , $f(r_i)$, can be calculated as follows:

$$f(r_i) = \frac{n_i \pi r_i^2}{\sum_{i=1}^N n_i \pi r_i^2} \quad \text{----- (4)}$$

This improved model can help to define the pore-size distribution function more clearly as compared to the conventional capillary model. In addition, the volumes of pores in the element are consistent with the definition of porosity. As soil elements can be connected to each other, the cylindrical tubes can be assumed to be connected to each other. With random connectivity, the “ink-bottle” effect can be simulated. The detail of calculation of “ink-bottle” effect can be found from Zhai et al. (2017c).

3.3 Relationship between SWCC and pore-size distribution function

If water fully fills the pores from r_j to r_N , then the degree of saturation in soil can be calculated from Equation (5) as follows:

$$S(\psi_j) = \frac{\sum_{i=j}^N n_i \pi r_i^2}{\sum_{i=1}^N n_i \pi r_i^2} \quad \text{----- (5)}$$

where, r_j = the maximum radius of pore which is filled with water.

It is observed that Equation (5) can be obtained by the summation of Equation (3) as follows:

$$S(\psi_j) = \sum_{i=j}^N f(r_i) = \frac{\sum_{i=j}^N n_i \pi r_i^2}{\sum_{i=1}^N n_i \pi r_i^2} \quad \text{----- (6)}$$

Therefore, as illustrated in Equation (6), SWCC in the form of degree of saturation can be obtained from the integration of pore-size distribution function. Equation (6) gives the same conclusion from Fredlund and Xing (1994) that SWCC can be obtained by integrating the pore-size distribution function. It should be noted that Equation (6) was obtained based on the assumption that there is no soil volume change with changes in matric suction.

Therefore, the pore-size distribution function, $f(\psi)$, can be obtained by differentiation of the best fit equations (such as Equation (1) and (2)), which are expressed using the fitting parameters a , n and m , or a , b and c as follows:

$$f(\psi) = \frac{dS}{d \log(\psi)} = \psi \ln(10) \left[\frac{\frac{1}{\ln\left(1 + \frac{10^6}{C_r}\right) \left(1 + \frac{\psi}{C_r}\right) C_r \left\{ \ln\left[e + \left(\frac{\psi}{a}\right)^n\right]\right\}^m} + mn\left(\frac{\psi}{a}\right)^{n-1} \left[\frac{\ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)} \right]}{a \left[e + \left(\frac{\psi}{a}\right)^n \right] \left\{ \ln\left[e + \left(\frac{\psi}{a}\right)^n\right]\right\}^{m+1}} \right] \quad \text{----- (7)}$$

where, a , n and m = fitting parameters in Fredlund and Xing (1994)'s equation.

$$f(h) = \frac{S - S_r}{1 - S_r} = (S_r - 1) h \ln(10) abc \left[\frac{1}{1 + (ah)^b} \right]^{c+1} (ah)^{b-1} \quad \text{----- (8)}$$

where, a , b and c = fitting parameters in van Genuchten's (1980) equation.

3.4 Valve model for unsaturated permeability of soil

The water valve, as illustrated in Figure 4 (a) is commonly used in every house and the valve works based on its internal structural design as illustrated in Figure 4 (b). The working mechanism of a valve can be explained and illustrated in Figure 5.

When the valve is fully opened, the cross-sectional area is completely open for water flow, as illustrated in Figure 5 (a), the effective area allows water to flow is A_0 and the water flow rate reaches its maximum value, q_0 . When the valve is half opened, the effective cross-sectional area is reduced to A_i which results in a smaller flow rate, q_i . When the valve is closed, the effective cross-sectional area becomes zero and the flow rate also reduces to zero. The flow

rate q_i can be calculated from the ratio between A_i and A_0 as illustrated in Equation (9). The reduction in the cross-sectional area can be calculated from the pore-size distribution function using the statistical method.

$$\frac{q_i}{q_0} = \frac{A_i}{A_0} \quad \text{----- (9)}$$

The results of experimental work from Reinson et al. (2005) showed that water only could flow through the water phase and could not flow through the air phase. Assume two soil elements connect each other on section A-A, the water flow through the section is dependent on the moisture condition of two pores as illustrated in Figure 6. If both pores are wet (as illustrated in Figure 6a), then water can flow across the section. On the other hand, if one pore is dry (as illustrated in Figure 6b), then water is blocked by air and cannot flow across the section. The probability of the connection of two pores is dependent on the pore-size density of the pores.

In this case, if the effective area that allows water flow through the section can be calculated, then the relative coefficient of permeability can be calculated. By studying the effective area for each individual pore (from r_m to r_N , where r_m is the maximum radius of wet pore corresponding to matric suction of ψ_m), the total effective area on the cross-sectional area that allows water flow can be calculated as illustrated in Figure 7.

The summation of the effective areas under suction of ψ_m can be divided by the total effective area under the fully saturated condition to determine the relative coefficient of permeability as shown in Equation (10).

$$k_r = \frac{\left\{ (S(\psi_m) - S(\psi_{m+1}))^2 \psi_m^{-2} + \sum_{i=m+1}^N [(S(\psi_m) - S(\psi_i))^2 - (S(\psi_m) - S(\psi_{i-1}))^2] \psi_i^{-2} \right\}}{\sum_{i=1}^N [1 - S(\psi_i)]^2 - [1 - S(\psi_{i-1})]^2] \psi_i^{-2}} \quad \text{----- (10)}$$

where, k_r = relative coefficient of permeability, ψ_m = matric suction in the soil and $S(\psi_m)$ = degree of saturation corresponding to matric suction of ψ_m .

Substituting Fredlund and Xing (1994)'s equation into Equation (10) and relative coefficient of permeability can be obtained using fitting parameters (a , n and m) as follows:

$$k_r = \frac{1}{A_{FX}} \left\{ \left[\frac{C(\psi_m)}{\left\{ \ln \left[e + \left(\frac{\psi_m}{a} \right)^n \right] \right\}^m} - \frac{C(\psi_{m+1})}{\left\{ \ln \left[e + \left(\frac{\psi_{m+1}}{a} \right)^n \right] \right\}^m} \right] \frac{1}{\psi_m^2} + \sum_{j=m+1}^N \left\{ \left[\frac{C(\psi_m)}{\left\{ \ln \left[e + \left(\frac{\psi_m}{a} \right)^n \right] \right\}^m} - \frac{C(\psi_j)}{\left\{ \ln \left[e + \left(\frac{\psi_j}{a} \right)^n \right] \right\}^m} \right] - \left[\frac{C(\psi_m)}{\left\{ \ln \left[e + \left(\frac{\psi_m}{a} \right)^n \right] \right\}^m} - \frac{C(\psi_{j-1})}{\left\{ \ln \left[e + \left(\frac{\psi_{j-1}}{a} \right)^n \right] \right\}^m} \right] \right\}^2 \frac{1}{\psi_j^2} \right\} \quad \text{----- (11)}$$

where,

$$A_{FX} = \sum_{i=1}^N \left\{ \left[1 - \frac{C(\psi_i)}{\left\{ \ln \left[e + \left(\frac{\psi_i}{a} \right)^n \right] \right\}^m} \right]^2 - \left[1 - \frac{C(\psi_{i-1})}{\left\{ \ln \left[e + \left(\frac{\psi_{i-1}}{a} \right)^n \right] \right\}^m} \right]^2 \right\} \frac{1}{\psi_i^2} \quad \text{----- (12)}$$

a , n and m are fitting parameters in Fredlund and Xing (1994)'s equation

On the other hand, if van Genuchten (1980)'s equation is used as the SWCC best fit equation, relative coefficient of permeability can be obtained using fitting parameters (a , b and c) as follows:

$$k_r = \frac{1}{A_{VG}} \left\{ \left(\frac{1}{[1 + (a\psi_m)^b]^c} - \frac{1}{[1 + (a\psi_{m+1})^b]^c} \right)^2 \frac{1}{\psi_m^2} + \sum_{j=m+1}^N \left\{ \left(\frac{1}{[1 + (a\psi_m)^b]^c} - \frac{1}{[1 + (a\psi_j)^b]^c} \right)^2 - \left(\frac{1}{[1 + (a\psi_m)^b]^c} - \frac{1}{[1 + (a\psi_{j-1})^b]^c} \right)^2 \right\} \frac{1}{\psi_j^2} \right\} \quad \text{---- (13)}$$

where,

$$A_{VG} = \left\{ \sum_{i=1}^N \left\{ \left(1 - \frac{1}{[1 + (a\psi_i)^b]^c} \right)^2 - \left(1 - \frac{1}{[1 + (a\psi_{i-1})^b]^c} \right)^2 \right\} \frac{1}{\psi_i^2} \right\} \quad \text{---- (14)}$$

a , b and c are fitting parameters in van Genuchten (1980)'s equation

It is observed from Equations (1), (7) and (11) and Equations (2), (8) and (13) that SWCC, pore-size distribution function, $f(r)$, and relative coefficient of permeability, k_r , can be expressed by the same set of fitting parameters. (e.g. a , n and m in Fredlund and Xing (1994)'s equation or a , b and c in van Genuchten (1980)'s equation). In this case, if any of these properties: SWCC, $f(r)$, or k_r is known, the other two properties can be calculated accordingly.

4. Experimental program for verification

Fredlund and Xing (1994) illustrated that SWCC and $f(r)$ can be calculated from each other. On the other hand, Zhai and Rahardjo (2015), Zhai et al. (2017a, b) demonstrated that k_r can be calculated from SWCC. However, it seems that there is no valid equation that can be used to estimate the pore-size distribution function from the measured permeability data. It is observed that Equations (11) and (13) can be used to calculate SWCC from the experimental

data of permeability. In order to verify the capability of Equations (11) and (13), one specimen of compacted mixture of sand and kaolin K50S50 (50% sand with 50% kaolin) was prepared for both unsaturated permeability test and SWCC measurement. The unsaturated permeability tests were conducted using an unsaturated triaxial permeameter as shown in Figure 8. In this research, the unsaturated triaxial permeameters were assembled following the modifications recommended by Goh (2012), Goh et al. (2015), Rahimi and Rahardjo (2016) and Priono (2016). The schematic diagram of the unsaturated triaxial permeameter used in this study is illustrated in Figure 8.

The unsaturated permeability test was stopped when a steady-state condition was achieved for a period of time. A steady-state condition, i.e. a condition when there is no change in water volume, is usually determined by graphical observation, similar to the determination of equilibrium time in the matric suction equalization stage. After the permeability test is completed at each suction stage, the equilibrium state is reached for the SWCC measurement. Following this procedure, the unsaturated permeability and SWCC can be measured simultaneously.

Laboratory tests were conducted on compacted soils which were produced by mixing L2-grade kaolin and 20-30 Ottawa sand. The compacted soils were used in the research to avoid the heterogeneity of the soils for better analyses. Index properties of the soil samples used in this study are summarized in Table 1.

In addition to the experimental measurements carried out in this study, another five sets of data including volcanic sand, glass beads, fine sand and touch silt loam from Brooks and Corey (1964), superstition sand from Richards (1952) were collected from different literatures. As no index properties of these soil were reported in these literature, only the experimental data of the relative coefficient of permeability and SWCC for these soils are illustrated in Figure 9.

Equations (11) and (13) were used to best fit the measured data of relative coefficients of permeability using curve fitting technique. After obtaining the fitting parameters in Fredlund and Xing (1994)'s equation (i.e., a , n and m) and van Genuchten (1980)'s equation (a , b and c) from the regression procedure, these fitting parameters were then used to calculate SWCC. Consequently, the calculated SWCCs were compared with the measured SWCCs for these six types of soil.

5. Results and discussions

The fitting parameters such as a , n and m in Fredlund and Xing (1994)'s equation and a , b and c in van Genuchten (1980)'s equation obtained from the best fit procedure (by minimizing $\Sigma (\log(k_{ri}) - \log(k'_{ri}))^2$, where k_{ri} is the measured relative hydraulic conductivity and k'_{ri} is the estimated relative hydraulic conductivity from Equation (11) or (13)) are presented in Tables 2 and 3, respectively. The best fitting results for the relative hydraulic conductivity for these soils are illustrated in Figure 10. Subsequently, the determined fitting parameters in Tables 2 and 3 were used to estimate the SWCC for these soils and the estimation results are illustrated in Figure 11. The coefficient of goodness between the estimated results and the experimental results of SWCC for these soils are presented in Table 4.

The results shown in Figure 10 and Table 2 indicated that Equations (11) and (13) performed well (high values of R^2) as the best fit equation for the permeability function. The results illustrated in Figure 11 and Table 3 indicated that the estimation results of SWCC from the measured data of relative coefficient of permeability also agreed with the measured data of SWCC.

6. Conclusions

Conventional capillary model was improved and “valve model” was proposed to explain the water flow in unsaturated soil. The relationships between SWCC, pore-size distribution function and permeability function could be well explained using the proposed models. It is observed that pore-size distribution function acts as a bridge to link SWCC and permeability function. By best fitting the experimental data (including measured data in this study and published data collected from literature) with the proposed equations, the pore-size distribution function can be indirectly obtained from the measured permeability data. Therefore, in the situation that only experimental data of relative coefficient of permeability are available, pore-size distribution function, $f(r)$, still can be indirectly obtained by using proposed equations in this study.

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Table 1: Index properties of soils used in this study.

Index properties	K50S50
Dry Density, ρ_d (Mg/m ³)	1.75
Water Content, w (%)	12.1
Void Ratio, e	0.48
Liquid Limit, LL (%)	46.7
Plastic Limit, PL (%)	27.4
Plasticity Index, PI (%)	19.3
Specific Gravity, G_s	2.59
GSD – Sand (%)	50.0
GSD – Silt (%)	37.5
GSD – Clay (%)	12.5
Unified Soil Classification System (USCS)	SM-ML
Saturated permeability k_s (m/s)	1.12×10^{-8}

Table 2. The fitting parameters obtained by best fitting equations (11) on experimental data of relative coefficient of permeability.

Fredlund and Xing (1994)'s equation					
Soils	a(kPa)	n	m	C_r (kPa)	R^2
Volcanic sand	2.1	6.34	2.02	1500	99.99%
Glass beads	3.09	17.61	1.31	1500	97.73%
Fine sand	6.27	8.57	12.99	1500	98.43%
Touch silt loam	7.76	6.38	0.76	1500	98.80%
Silty sand	3.03	10.19	0.67	1500	99.78%
Mixture of sand and kaolin	43.74	4.95	0.59	1500	99.77%

Table 3. The fitting parameters obtained by best fitting equations (13) on experimental data of relative coefficient of permeability.

Soils	Van Genuchten (1980)'s equation				
	a(kPa-1)	b	c	θ_r	R^2
Volcanic sand	0.487	6.76	0.74	0	99.99%
Glass beads	0.342	31.93	0.2	0	97.06%
Fine sand	0.157	8.61	5.49	0	98.42%
Touch silt loam	0.135	7.74	0.28	0	99.76%
Silty sand	0.37	29.09	0.078	0	99.69%
Mixture of sand and kaolin	0.03	11.31	0.081	0	99.22%

Table 4. The coefficient of goodness, R^2 , for the estimation of SWCC using Equations (11) and (13)

Soils	Using Equation (11)	Using Equation (13)
Volcanic sand	95.65%	94.21%
Glass beads	99.16%	98.91%
Fine sand	86.22%	86.41%
Touch silt loam	95.22%	87.97%
Silty sand	95.70%	83.71%
Mixture of sand and kaolin	99.02%	99.51%

Figure 1. Simplification of an irregular pore into a series of circular pores.

Figure 2. Simplification of pores in soil element.

- (a) irregular pores
- (b) simplified circular pores

Figure 3. Illustration of capillary tubes in soil element.

Figure 4. Illustration of water valve.

- (a) Illustration of water valve.
- (b) Internal structure of water valve

Figure 5. Illustration of working mechanism of a water valve.

- (a) The valve is open
- (b) The valve is half open
- (c) The valve is closed

Figure 6. Illustration of water flow cross the section.

- (a) Two pores are wet
- (b) Big pore is dry

Figure 7. Illustration of effective area for each pore

Figure 8. Schematic diagram of the unsaturated triaxial permeameter setup

Figure 9. Experimental data of relative hydraulic conductivity and SWCC of six types of soils.

- (a) Measured relative hydraulic conductivity for six types of soils
- (b) Measured SWCC for six types of soils

Figure 10. Best fitting results for these soils using Equations (11) and (13).

- (a) Best fitting results using Equation (11)
- (b) Best fitting results of Equation (13)

Figure 11. Estimation of SWCCs for the soils using fitting parameters

- (a) Estimation of SWCCs for the soils using fitting parameters a , n and m
- (b) Estimation of SWCCs for the soils using fitting parameters a , b , and c

Figure 1

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Figure 2(a)

Figure 2(b)

Figure 6(a)

Figure 6(b)

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Figure 7

Effective area for permeability under suction

0
0
0
$\pi m^2 f^2(r_m) r_m^2$
$\pi r^2 f(r_m) f(r_{m+1}) r_{m+1}^2$
$\pi r^2 f(r_m) f(r_{n-1}) r_{n-1}^2$
$\pi m^2 f(r_m) f(r_n) r_n^2$

Figure 8

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Figure 10(a)

Figure 11(b)